Optimizing multi-criteria decision-making: empirical analyses and quantitative insights for reducing the effort of decision-makers in a decision support system

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This dissertation is based on five original studies:

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Peters, Sven; Tönsfeuerborn, Mendy; von Nitzsch, Rüdiger (2024): Integrating Uncertainties in a Multi-Criteria Decision Analysis with the ENTSCHEIDUNGSNAVI. In: *Mathematics*, *12*(11), 1746. https://doi.org/10.3390/math12111746.

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Tönsfeuerborn, Mendy; von Nitzsch, Rüdiger; Siebert, Johannes Ulrich (working paper): Linear transformation of one-dimensional utility functions: an empirical study on the impact on the final ranking of alternatives in personal decisions. Submitted to: Decision Analysis.

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List of abbreviations

AFT	alternative-focused thinking
AI	artificial intelligence
COVID-19	coronavirus disease 2019
DA	decision analysis
DM	decision maker
DSS	decision support system
Freq.	frequency
FU	forecast uncertainty
GroupVA	Group value-anchoring
GroupVN	Group value-nudging
MAUT	multi-attribute utility theory
MAVT	multi-attribute value theory
MCDA	multi-criteria decision analysis
MCDM	multi-criteria decision-making
PU	parameter uncertainty
RWTH	Rheinisch-Westfälische Technische Hochschule
VFT	value-focused thinking
norm	normalized
min	minimum
max	maximum

List of symbols

$\mathbb{A}$	set of alternatives
A_j	alternative j
c	risk aversion parameter
c_i	risk aversion parameter for objective i
$DR(x)$	direct rating function
$EU(A_j)$	expected utility for alternative j
f	index variable denoting individual states
g_q	weight of indicator q
i	index variable denoting individual objectives
I	number of objectives
j	index variable denoting individual alternatives
J	number of alternatives
k	index variable denoting individual states
K_{ij}	number of states of an influence factor for objective i, alternative j
K_δ	number of states of influence factor δ
n	sample size
$\mathbb{O}$	set of objectives
O_i	objective i
$P(s)$	probability of state s
P^{norm}	normalized probability
q	index variable denoting individual indicators
Q	number of indicators
s_{ij}^k	state k of an influence factor for objective i, alternative j

s_δ^f	state *f* of influence factor δ
$U(\,)$	utility function
$U(A_j)$	utility of alternative *j*
$U_i(\,)$	utility function of objective *i*
$U_i(x)$	utility of consequence *x* in objective *i*
v	index variable denoting individual indicators
w_i	weight of objective *i*
w_v	weight of objective *v*
w_i^{norm}	normalized weight of objective *i*
x_i^- / x_i^+	consequence of the lowest/highest utility in objective *i*
x_{ij}	consequence of alternative *j* in objective *i*
x_{ij}^k	consequence of alternative *j* in objective *i* for state *k*
x_{iq}^- / x_{iq}^+	indicator consequence of the lowest/highest utility in objective *i*, indicator *q*
x_{ijq}^k	indicator consequence of alternative *j* in objective *i*, indicator *q* for state *k*
δ	index variable denoting individual influence factors
ε	degree of precision
ε_{c_i}	degree of precision for numerical objectives
ε_{U_i}	degree of precision for verbal objectives
ε_{w_i}	degree of precision for objective weights
$\varepsilon_{P(S_\delta)}$	degree of precision for probabilities
μ	mean
σ	standard deviation

Summary

Making decisions in crucial situations is complicated by various challenges, e.g., the inability to define relevant objectives (Bond et al. 2008) and alternatives (Siebert and Keeney 2015), cognitive biases (Montibeller and von Winterfeldt 2015), or information overload. These factors often lead to impulsive or low-quality decisions, especially under time constraints that limit thorough evaluation of options (Kocher and Sutter 2006). Structured methods like multi-criteria decision analysis (MCDA) have been developed to tackle decision-making challenges by creating quantitative or qualitative models that represent the decision situation. These models help organize information, facilitate trade-offs, and select alternatives, although developing them can be complex and challenging to understand. Decision support systems (DSSs) are designed to assist decision-makers in managing MCDA approaches enhancing decision quality and efficiency (Tripathi 2011). Nevertheless, these tools also often reach their limits when implementing scientifically sound approaches in a user-friendly and simple way.

This dissertation aims to optimize an MCDA-based DSS called ENTSCHEIDUNGSNAVI, designed for decision-makers seeking to enhance their understanding while minimizing the effort required for a scientifically sound approach. The open-source web tool supports reflective decision-making (von Nitzsch and Methling 2021) through a five-step process based on multi-attribute utility theory (Keeney and Raiffa 1976) and value-focused thinking (Keeney 1992). The study provides insights into the theoretical and practical foundations of the ENTSCHEIDUNGSNAVI and examines the extent to which the reflective decision-making process can be simplified for the decision-maker. Therefore, it examines whether value-nudging leads to more value-focused and, thus, better decisions. Moreover, the use of an imprecise information approach is critically analyzed in terms of helpfulness and impact on the decision. In addition, the study investigates how a linear transformation of one-dimensional utility functions affects decision quality by examining its effect on the final ranking of alternatives.

Part A. Introduction of dissertation

1 Multi-criteria decision analysis and decision support systems as solutions for the challenges of complex decisions

Making decisions in crucial situations is a complex process. Many challenges can impede this process, making the decision more difficult or influencing the decision negatively. Researchers found that decision-makers (DMs) are unable to define all relevant objectives (Bond et al. 2008) or alternatives (Siebert and Keeney 2015, Siebert 2016), which prevents them from effectively structuring their decision problem, resulting in a low-quality of the decision. Motivational and cognitive biases are another central challenge in decision-making (Kahneman 2011, Montibeller and von Winterfeldt 2015). These mental errors often lead people to perceive and interpret information selectively, influencing their decisions. Additionally, emotions such as fear or stress can cloud judgment and lead individuals to make impulsive or suboptimal choices (Starcke and Brand 2012, Lerner et al. 2015, Morgado et al. 2015). Even when a decision is well-structured and cognitive biases are avoided, uncertainties arise in nearly all important decisions. These uncertainties may pertain to potential outcomes of alternatives or particular decision parameters. The feeling of ambiguity can be paralyzing and may result in DMs hesitating or failing to decide (Dhar 1997). Added to this is today's information overload. In a world with constant access to data, many DMs find it challenging to filter relevant information and make informed choices (Eppler and Mengis 2008). Furthermore, in today's society, everything should happen quickly and efficiently. Time constraints lead to decisions that must frequently be made quickly, limiting the opportunity to weigh all relevant information carefully (Kocher and Sutter 2006). All these challenges clearly illustrate why many people struggle to make clear and rational decisions.

Structured and transparent methods, such as multi-criteria decision analysis (MCDA), have been established to address these challenges. The term MCDA encompasses a variety of approaches (see Belton and Stewart (2002) and Cinelli et al. (2020)) and the techniques are increasingly recognized and utilized in many application contexts (Haag et al. 2022), e.g., healthcare (Mühlbacher and Kaczynski 2016, Frazão et al. 2018), environment (Hajkowicz and Collins 2007, Huang et al. 2011, Cegan et al. 2017, Adem Esmail and Geneletti 2018), or politics (Kurth et al. 2017). All approaches aim to create quantitative or qualitative models representing the decision situation. These models aid in organizing information, facilitating trade-offs, and selecting alternatives for implementation. However, the development of such a model can be complex and challenging to comprehend.

Decision support systems (DSSs), which are computer-based information systems, have been developed to assist DMs in managing MCDA approaches, improve the quality of decisions, and make the decision-making process more efficient (Tripathi 2011, Razmak and Aouni 2015). They provide a combination of data, analytical models, and user-friendly interfaces to help DMs analyze information and evaluate alternatives. DSS can be applied across various fields, including business (e.g., Hahn and Kuhn (2012), or Barfod et al. (2011)), healthcare (see Sutton et al. (2020) for an overview), agriculture (Zhai et al. 2020), and others, to solve complex problems and make informed decisions. In literature, many different DSSs exist. The International Society on Multiple Criteria Decision Making (MCDM) provides a list[1] of software related to MCDM. Each software uses a specific decision-making approach and offers advantages and disadvantages. Nevertheless, these tools also often reach their limits when implementing scientifically sound approaches in a user-friendly and simple way.

This dissertation aims to optimize an MCDM-DSS for DMs who want to enhance their understanding and reduce the effort of a scientifically sound approach. Therefore, the study focuses

[1] https://www.mcdmsociety.org/content/software-related-mcdm-0

on the DSS ENTSCHEIDUNGSNAVI (von Nitzsch et al. 2020), which employs the multi-attribute utility theory (MAUT) (Keeney & Raiffa, 1976), one of the best-known MCDA approaches. The ENTSCHEIDUGNSNAVI (von Nitzsch et al. 2020, Hannes and Nitzsch 2024, Peters et al. 2024) is an open-source, freely available web tool that supports a reflective decision-making process (von Nitzsch and Methling 2021) and trains decision skills. It is based on the concept of value-focused thinking (VFT) (Keeney 1992) and MAUT (Keeney and Raiffa 1976) and guides the DM through a five-step process. The initial version of the tool was developed in 2017 for educational purposes as part of the 'Decision Theory' course at RWTH Aachen University. In subsequent years, a group of computer science students worked diligently to implement new functionalities, enhance usability, and integrate extensive user feedback. The ENTSCHEIDUNGSNAVI is now utilized by other universities and institutions for teaching and advisory purposes, thanks to its high level of professionalization. Feedback from practical applications, including functionality requests, directly informs the tool's ongoing development and makes it one of the most transparent and user-friendly tools in the field of MCDM-DSS under MAUT. The ENTSCHEIDUNGSNAVI is available in three variants: Starter, Educational, and Professional. The Starter variant is kept simple and aimed at users with no experience in MCDM who want a quick introduction to the decision-making process. The Educational variant guides DMs through reflective decision-making in small steps, providing extensive explanations and background on decision theory, operating instructions, and tips. This version is ideal for DMs looking to deepen their understanding and improve their skills. The Professional variant offers similar features to the Educational version but without guidance or extensive explanations, catering to those who are already familiar with the tools and want to solve decision problems efficiently (Peters et al. 2024). This dissertation is based on the Educational variant.

2 The ENTSCHEIDUNGSNAVI: a decision support system based on the reflective decision-making process

This section introduces the scientific basis of the ENTSCHEIDUNGSNAVI and presents parts of two conceptual papers about the tool. In Section 2.1, Keeney's VFT approach (Keeney 1992, 1996) is described. VFT is used in the tool's decision front-end to structure the decision situation and identify the first pieces of relevant information (objectives and alternatives). In the decision back-end, the concept of MAUT (Keeney and Raiffa 1976) is employed, see Section 2.2, to find the best alternative under uncertainty. Section 2.3 summarizes the paper 'Decision skill training with the ENTSCHEIDUNGSNAVI' (von Nitzsch et al. 2020), in which the reflective decision-making process and tool's functions are described. Section 2.4 gives information on how the tool deals with uncertainties and is based on the paper 'Integrating uncertainties in a multi-criteria decision analysis' (Peters et al. 2024).

2.1 Value-focused thinking by Ralph Keeney

Traditional approaches to decision-making focus on alternatives. People react to problems presented by external factors such as competitors, customers, government actions, or circumstances. They focus on identifying alternatives first before considering the objectives or criteria for evaluation - a process referred to as alternative-focused thinking (AFT). This approach is limited and reactive, hindering control over decision situations because it prioritizes alternatives over articulating values (Keeney 1996).

In 1992, Ralph Keeney revolutionized decision-making with his approach of VFT (Keeney 1992). He emphasizes that values are essential to all actions and should serve as the foundation for decision-making. Values ought to be the basis for the time and effort invested in making decisions. Moreover, VFT harnesses critical reasoning to improve decision-making by prioritizing values. The approach leads to better decisions through insights and specific procedures that emphasize values, allowing for the identification of decision situations as opportunities rather than problems (Keeney 1996).

In contrast to AFT, which aims to solve decision problems, VFT seeks to uncover desirable decision opportunities and generate alternatives. The value-focused paradigm differs from the standard approach in three ways: it emphasizes making values explicit, prioritizes articulating them before other activities, and uses the defined values to identify opportunities and create alternatives. Keeney (1996) argues that identifying and structuring objectives can be challenging, as ends are frequently mistaken for means, objectives may be confused with targets, constraints, or alternatives, and the relationships between different objectives often remain unclear. This process necessitates considerable creativity and collaboration in discussions with DMs and stakeholders involved in the decision. Moreover, decisions typically involve multiple objectives representing a desired outcome within a specific context. Three elements are needed to explicitly state an objective: the decision context, the object, and the direction of preference. For instance, a government agency's objective to 'maximize community well-being' includes creating new measures as the context, social impact as the object, and a preference for more well-being over less. In addition, Keeney differentiates between 'fundamental objectives' and 'means objectives'. Fundamental objectives refer to the ends that DMs value within a specific context, while means objectives are the methods used to achieve those ends. It is important to note that these concepts are not absolute but depend on the context. For example, if the decision context is investing in renewable energy sources to reduce the carbon footprint, minimizing carbon emissions is a fundamental objective. However, driving an electric car becomes a means objective. In strategic decisions - broad decisions an organization faces - fundamental objectives are termed 'strategic objectives'. These strategic objectives provide guidance for all organizational decisions and serve as the foundation for more detailed fundamental objectives tailored to specific situations (Keeney 1996).

Nearly all decision-making experts emphasize the importance of listing objectives. However, they often lack specificity on how to do so or how to utilize these objectives effectively. VFT provides various procedures to aid in this process. First, various techniques are employed to create an initial list of objectives. Second, these objectives are classified as means or ends and structured logically. Third, additional methods aid in utilizing the objectives to develop alternatives. Finally, the objectives are assessed to uncover valuable decision-making opportunities (Keeney 1992, 1996). Figure 1 summarizes the process of VFT.

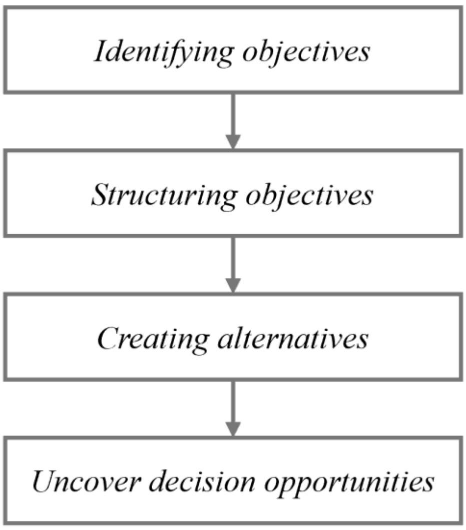

Figure 1. *The process of VFT (Keeney 1992).*

The following explanations of the steps in the VFT process are summaries of Keeney's work (1992, 1996) supplemented with examples.

Identifying objectives

The most straightforward way to identify objectives is through discussions about the decision situation, which requires creativity and critical thinking. Starting by asking DMs what they wish to achieve can generate a list of potential objectives for further exploration. Various techniques can facilitate this identification process, emphasizing that redundancy in listing objectives is beneficial as it helps recognize any missing ones. When prompting individuals to express their objectives, requesting a list without prioritization is important. Questions like ‘If there were no limitations, what would your objectives be?’ can expand the list. Additionally, terms such as trade-offs, consequences, and fairness should trigger deeper inquiries to clarify implicit objectives. For instance, if a DM mentions necessary trade-offs, follow-up questions about what those trade-offs entail can provide insights. Often, clarity on objectives emerges after considering available alternatives. Articulating distinguishing features of these alternatives can lead to specific objectives. Asking respondents to identify desirable and undesirable features of alternatives can further stimulate thought regarding relevant objectives.

Structuring objectives

The initial list of objectives often contains items that are not true objectives, such as alternatives, constraints, and evaluation criteria. Each item can be refined into actual objectives, resulting in a mix of means and fundamental objectives. It is crucial to differentiate between these types and establish their relationships by exploring the reasons behind each objective. Two concepts are utilized: linking objectives through means-ends relationships and specifying fundamental objectives. Tracing specific means objectives should lead to at least one fundamental objective in a decision context. By asking, ‘Why is this objective important?’ DMs can identify whether an objective is fundamental or merely a means objective. For instance, in the context of urban development, ‘increasing green space’ may initially seem like an objective. However, probing further reveals that while more green space enhances community well-being and biodiversity, it may also lead to increased maintenance costs and potential land-use conflicts - highlighting the situation's complexity. Additionally, specification involves breaking down an objective into its logical components for clarity. For example, if ‘maximize community well-being’ is identified as a fundamental objective, asking what specific impacts need attention helps sharpen focus and guide actions effectively. Engaging in multiple questions aimed at specification can yield valuable insights into the nature of each objective.

Creating alternatives

DMs often identify a narrow range of alternatives due to a tendency to shift from vague to well-defined options, seeking immediate progress toward solutions. This quick identification typically leads to focusing on familiar alternatives, which can anchor thinking and limit the exploration of genuinely different options. To uncover new alternatives, DMs should focus on the underlying values guiding the decision.

A systematic approach involves creating alternatives that best align with specified values, probing both qualitative objectives and quantitative priorities. Starting with individual objectives can generate numerous potential alternatives, even if many may not perform well across all objectives. DMs should then consider pairs of objectives and gradually expand to include more until all are addressed, exploring combinations of generated alternatives.

Means objectives also provide valuable insights for generating alternatives since any alternative affecting them will likely influence related fundamental objectives. Strategic objectives serve as a broader foundation for identifying decision opportunities based on values. Reflecting on what can be done to achieve these strategic objectives can yield worthwhile alternatives.

Decision opportunities

DMs should actively control the decision situations they face, as this can significantly influence their ability to achieve objectives. They often view decision situations as problems rather than opportunities, limiting their exploration of potential solutions. VFT encourages DMs to recognize that a decision problem may represent an opportunity and offers two methods for creating decision opportunities: transforming existing problems into opportunities and generating new ones through creative thinking.

Strategic objectives form the foundation for identifying decision opportunities. However, many organizations lack clearly defined and understood strategic objectives. DMs should establish procedures to search for these opportunities regularly, independent of current decisions. For instance, setting aside time monthly to review and refine strategic objectives can facilitate this process.

When stakeholders desire a specific alternative but lack direct control over the decision-making process, they must seize the opportunity to influence the situation. By understanding the values of the actual DM and structuring alternatives that align with those values while maintaining essential features desired by themselves, stakeholders can create modified options that satisfy both parties. This empathetic negotiation involves balancing impacts on both sides to develop win-win alternatives. By leveraging insights about the DMs' values, stakeholders can propose beneficial and fair solutions for everyone involved, ultimately leading to better outcomes in complex decision scenarios.

2.2 The additive model of the multi-attribute utility theory

MAUT is a technique for finding the best alternative in decisions with multiple objectives and limited alternatives under uncertainty (Keeney and Raiffa 1976, von Winterfeldt and Edwards 1993). With this technique, the DM evaluates the defined alternatives with the objectives relevant to the decision to calculate an aggregated utility for each alternative. The alternative with the highest utility is the best and should be chosen by the DM.

To apply MAUT, DMs must define a set of objectives $\mathbb{O} = \{O_1, \dots, O_I\}$ and a set of alternatives $\mathbb{A} = \{A_1, \dots, A_J\}$ first. Then, the DM assesses the consequences x_{ij} of each alternative A_j in every objective O_i with $1 \leq i \leq I$ and $1 \leq j \leq J$. Next, the utility for each alternative $U(A_j)$ is determined. To achieve this, DMs need to establish their utility functions U_i and assign weights w_i to each objective according to their preferences. Keeney and Raiffa (1976) present three distinct methods for calculating utilities and aggregating all objectives within MAUT: multiplicative, multilinear, and additive utility functions. This dissertation concentrates on the additive model, which is the most commonly used approach (Ishizaka and Nemery 2013).

In the additive model, the utility of each alternative is calculated using the additive expected utility (Bernoulli 1954, von Neumann and Morgenstern 1961), as in Formula (1). For the additive expected utility, the objective weights w_i must add up to one, see Formula (1a). To model decisions under uncertainty, Formula (1) considers various states s_{ij}^k that occur with associated probabilities $P(s_{ij}^k)$, leading to specific consequences x_{ij}^k, where $1 \leq k \leq K_{ij}$. If $K_{ij} = 1$, the state s_{ij}^1 has a probability of 100 percent, making x_{ij}^1 a certain consequence. Conversely, if $K_{ij} \geq 2$, the consequence x_{ij} becomes uncertain. This situation arises when influence factors are incorporated into the model. The probabilities for all states for each ij sum to one, see Formula (1b).

$$EU(A_j) = \sum_{i=1}^{I} w_i \left[\sum_{k=1}^{K_{ij}} P(s_{ij}^k) U_i(x_{ij}^k)\right] \tag{1}$$

$$\sum_{i=1}^{I} w_i = 1 \tag{1a}$$

$$\sum_{k=1}^{K_{ij}} P(s_{ij}^k) = 1 \tag{1b}$$

Several types of utility functions exist in the literature (Harel et al. 2018). This dissertation concentrates on exponential utility functions, as these are the most frequently used form (Vilela and Oluyemi 2022), and discrete utilities as an alternative to scales that are not continuously defined. In the ENTSCHEIDUNGSNAVI, DMs can measure their objectives with a numerical or verbal scale. Objectives with a verbal scale (verbal objectives) are evaluated with discrete utilities, as in Formula (2a). Objectives with a numerical scale (numerical objectives) are evaluated with an exponential utility function, as in Formula (2b).

$$U_i(x_{ij}^k) = \begin{cases} 0 & if\ \ x_{ij}^k = x_i^- \\ DR(x_{ij}^k) & if\ x_{ij}^k \in (x_i^-, x_i^+) \\ 1 & if\ \ x_{ij}^k = x_i^+ \end{cases} \tag{2a}$$

$$U_i(x_{ij}^k) = \begin{cases} \frac{1-e^{-c_i \frac{x_{ij}^k - x_i^-}{x_i^+ - x_i^-}}}{1-e^{-c_i}} & if \quad c_i \neq 0 \\ \frac{x_{ij}^k - x_i^-}{x_i^+ - x_i^-} & if \quad c_i = 0 \end{cases} \tag{2b}$$

All consequences x_{ij}^k for objective O_i must be within the interval $[x_i^-, x_i^+]$, defined by the DM. Here, x_i^- indicates the consequence of the lowest utility (zero) and x_i^+ the consequence of the highest utility (one), with utility $U_i(x_{ij}^k)$ increasing as consequences improve. In Formula (2a), the DM determines the exact utility through direct rating, represented by the function $DR(x_{ij}^k)$. In Formula (2b), c_i represents the risk aversion parameter for objective O_i.

If objectives are measured with several indicators in the ENTSCHEIDUNGSNAVI, the consequences x_{ij}^k are calculated as in Formula (3).

$$x_{ij}^k = \sum_{q=1}^{Q} \left[x_i^- + \frac{x_{ijq}^k - x_{iq}^-}{x_{iq}^+ - x_{iq}^-} (x_i^+ - x_i^-) \right] \frac{g_q}{\sum_{v=1}^{Q} g_v} \tag{3}$$

The interval $[x_{iq}^-, x_{iq}^+]$ defines the measurement scale for the q-th indicator of objective O_i. Moreover, g_q represents the weight of the q-th indicator. The utilities for objectives with indicator scales are determined using the exponential utility function in Formula (2b).

2.3 The reflective decision-making process

Von Nitzsch et al. (2020) divide the reflective decision-making process into three phases: 1) structuring of the decision situation, 2) development of the consequences table, and 3) evaluation of the alternatives and the decision. Von Nitzsch and Methling (2021) split the first phase into three steps, resulting in a five-step process, which is the basis of the ENTSCHEIDUNGSNAVI. Each phase confronts the DM with challenges, and studies have shown that DMs need support. In the first phase, the decision statement is often formulated too narrowly (Maule and Villejoubert 2007), the objectives are incomplete (Bond et al. 2008) or not formulated fundamentally, and many alternatives are often not identified (Siebert and Keeney 2015, Siebert 2016). In the second phase, biases can distort the consequences table (Kahneman 2011, Montibeller and von Winterfeldt 2015), and in the third phase, the DM should feel comfortable with the result, i.e., the gut feeling should support the decision. The ENTSCHEIDUNGSNAVI addresses every challenge and guides the DM through the process. Moreover, it helps DMs to improve their decision-making skills. Figure 2 presents an overview of the reflective decision-making process in three phases and the five steps used in the ENTSCHEIDUNGSNAVI.

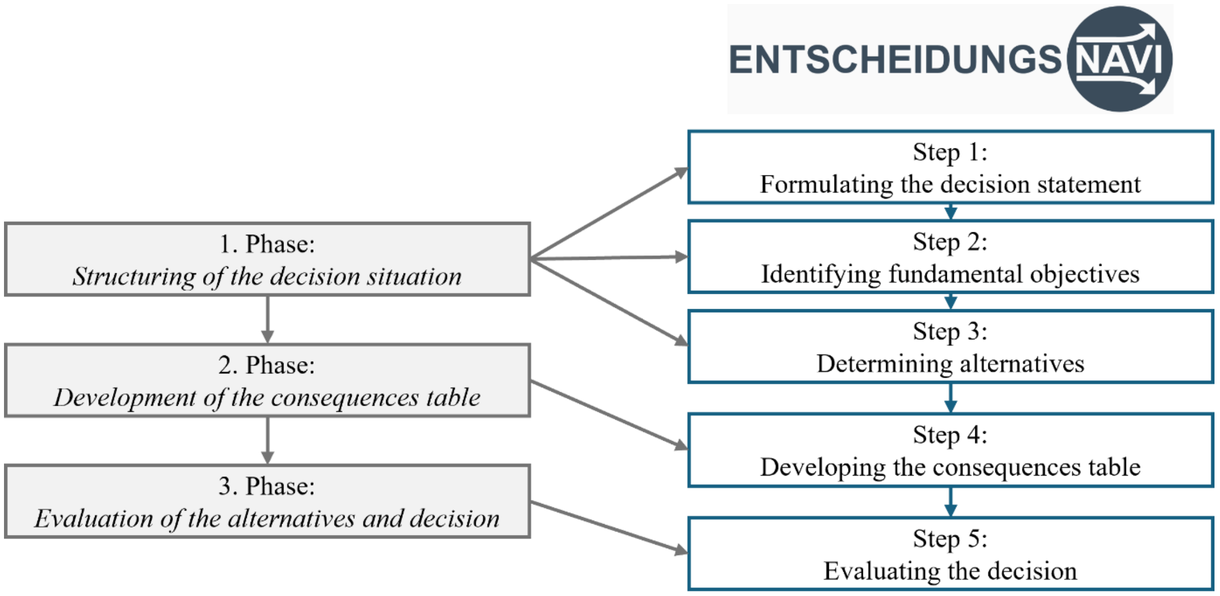

Figure 2. *The reflective decision-making process.*

2.3.1 Structuring of the decision situation

The first phase consists of three steps: formulating the decision statement, identifying the objectives, and determining the alternatives. In this phase, the DM follows a detailed step-by-step guide to acquire relevant skills.

The **formulation of the decision statement** is crucial for clarifying the context of subsequent decision-making steps, especially in group settings (Baer et al. 2013). It involves identifying the DM, understanding who can select alternatives, and what should be achieved with the decision. Additionally, the DM must state assumptions explicitly, including pending decisions

(Keeney 2020). A common issue is that decision statements are often too narrowly defined (Maule and Villejoubert 2007), leading to overlooked alternatives and objectives. DMs need to broaden their perspective when crafting these statements to improve decision quality. This shift from a reactive to a proactive decision can uncover better options. In the ENTSCHEIDUNGSNAVI, DMs initially draft their decision statement but may not consider it broadly enough. Therefore, the tool guides them through a process that encourages reflecting on their core life goals and identifying five key values. Figure 3 shows this substep as it is of central importance for a paper in this dissertation (see Section 5).

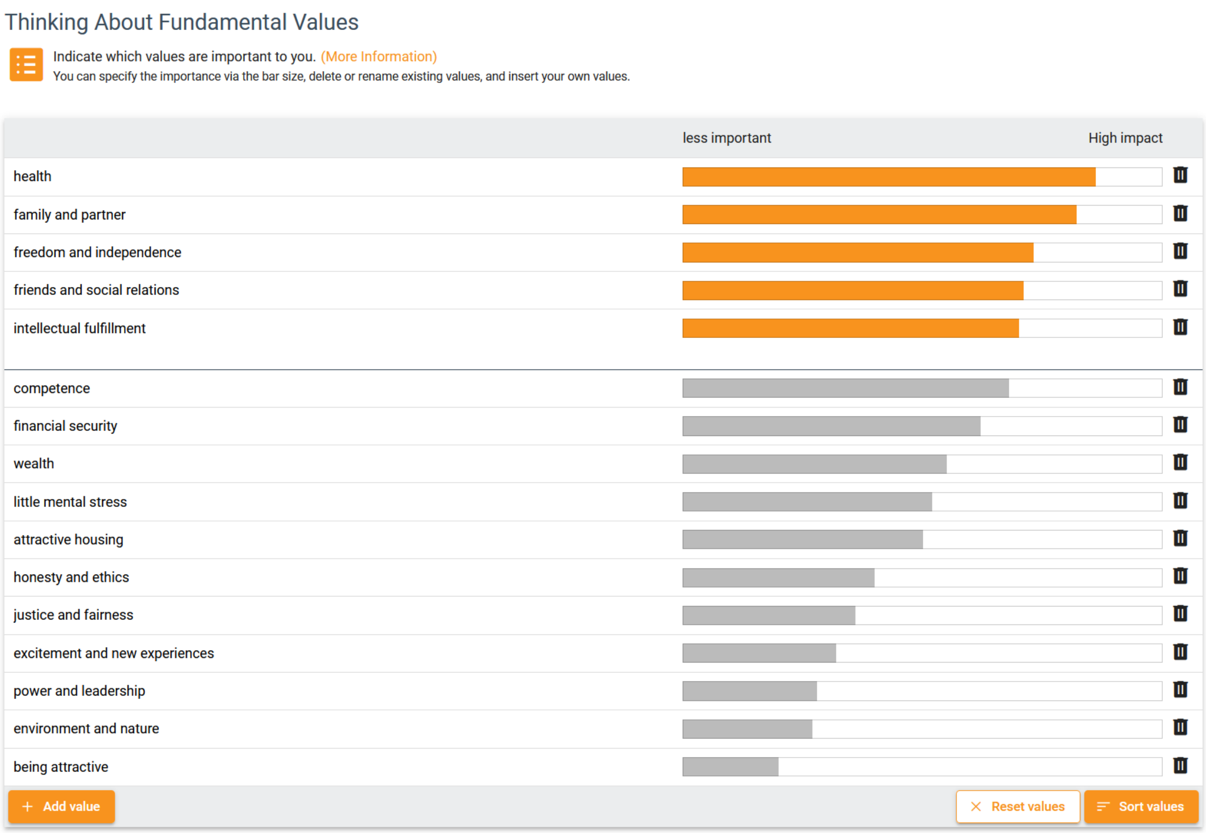

Figure 3. *Prioritizing fundamental values in the ENTSCHEIDUNGSNAVI.*

With prompts to challenge assumptions and think more expansively about the situation, DMs can reformulate their decision statement effectively. By the end of this process, they should have a well-framed and proactive decision statement that serves as a solid foundation for subsequent steps.

Based on VFT, DMs should **identify their objectives** after formulating the decision statement and before determining the alternatives (Keeney 1992, Siebert and Keeney 2020, Keeney 2020). This process ensures that DMs do not limit themselves to obvious options but instead explore new and creative alternatives based on well-defined objectives (Siebert and Keeney 2015). VFT emphasizes distinguishing between fundamental objectives - core aspects of interest with independent value - and means objectives, which merely support these fundamental objectives. Identifying and articulating fundamental objectives can be challenging and is best achieved with the help of an experienced decision analyst. DMs and decision analysts can brainstorm relevant aspects and structure them into a hierarchy, clarifying relationships between means and ends (Keeney 1992). Figure 4 shows an example of an objective hierarchy in the ENTSCHEIDUNGSNAVI.

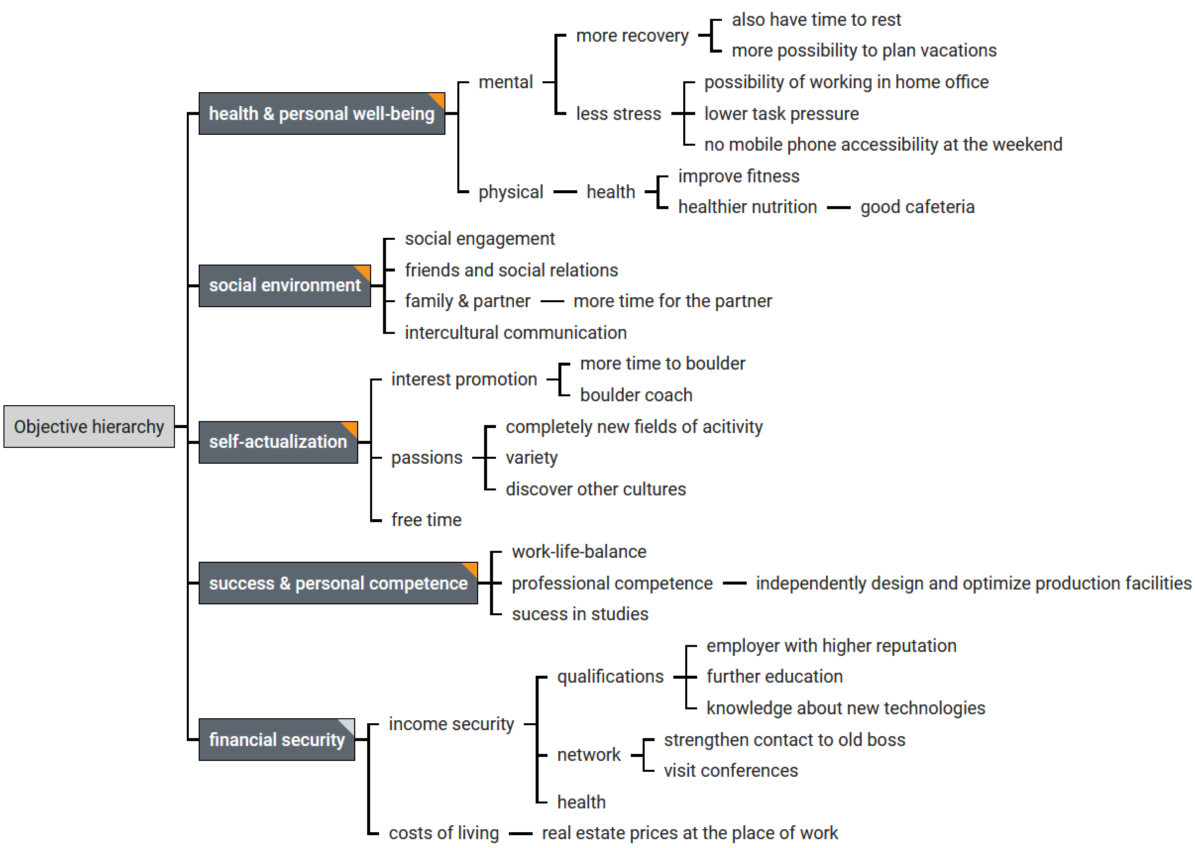

Figure 4. *Example of an objective hierarchy in the ENTSCHEIDUNGSNAVI.*

In the ENTSCHEIDUNGSNAVI, the process begins with a brainstorming session guided by creativity-promoting questions. The DM is then presented with a master list of objectives to ensure comprehensive consideration. The tool provides technical support for structuring these objectives through a user-friendly interface while requiring the DM to identify means-end relationships independently. Throughout this process, the tool indicates additional questions to challenge the significance of each objective and check for preference dependencies among them. Preference dependencies could diminish the validity of the recommendation and should be avoided. Ultimately, by the end of this step, DMs should ideally identify four to six fundamental objectives related to their defined decision statement.

The established objective hierarchy sets the groundwork for creatively **determining new alternatives**. In the ENTSCHEIDUNGSNAVI, the DM begins by listing known alternatives and then follows four sub-steps to explore new options. First, the DM evaluates the weaknesses of existing alternatives and seeks related new options that address these shortcomings. The tool presents the defined objective to help pinpoint where improvements can be made. Next, the DM is encouraged to set aside previous alternatives temporarily and focus on how to achieve each objective effectively. This step emphasizes creativity, allowing the DM to brainstorm new alternatives or combine ideas into one alternative. Moreover, the DM should engage with others familiar with the decision context, soliciting their input on additional alternatives. This approach includes considering perspectives from individuals with differing viewpoints. Finally, the DM identifies two or three key design parameters that differentiate all possible alternatives. These parameters help discover overlooked alternatives and streamline a potentially overwhelming number of alternatives into a manageable set. At the end of this step, DMs rank the determined alternatives based on intuition, setting the stage for later analysis and evaluation. In doing so, they should ensure they have considered all relevant alternatives for the decision.

2.3.2 Development of the consequences table

After structuring the decision situation, the DM must fill an empty consequences table with performance evaluations for each alternative against the defined objectives. This task is challenging due to various biases that can distort subjective estimates, particularly for inexperienced DMs (Tversky and Kahneman 1973, 1983, Arkes and Blumer 1985, Tversky and Koehler 1994, Gilovich et al. 2002, Klein 2007, Winterbottom et al. 2008, Herzog and Hertwig 2009). In this phase, the ENTSCHEIDUNGSNAVI educates the DM about them and provides strategies for prevention.

The DM needs to define attributes and measurement scales for the objectives, which can be complex without prior experience. The ENTSCHEIDUNGSNAVI supports this task by offering guidance on selecting appropriate attributes and scales. The tool adopts the VFT approach, which distinguishes between natural, constructed, and proxy attributes and provides three templates for scale selection: numerical, verbal, and indicator scales. Objectives that can be clearly measured on a natural-numerical scale are typically identifiable by their wording, e.g., objectives referring to a distance should be measured accordingly with a measure of length. In cases where a natural-numerical scale is unavailable, DMs can utilize constructed scales for measurement. One option is to employ a numerical scale with artificial units like points or grades, which is versatile and generally applicable. Another option involves creating a verbal scale with limited possible outcomes. Lastly, DMs can develop an indicator scale based on proxy attributes, including fundamental partial aspects, means objectives, or correlated scales.

To account for uncertainties, the DM can specify an additional influence factor in each table field, defined by a discrete number of states with associated probabilities. The results will also be state-dependent. To assist in determining these probabilities, the ENTSCHEIDUNGSNAVI employs the concept of imprecise information, allowing DMs to associate their specifications with a certain level of imprecision (see Section 2.4.2).

Once the consequences table is fully populated, it automatically highlights fields with color coding - red for worst values and green for best - to give a visual overview of the advantages and disadvantages of each alternative. This visualization aids the DM in identifying dominated alternatives that may be excluded from further consideration.

2.3.3 Evaluation of the alternatives and the decision

The ENTSCHEIDUNGSNAVI is grounded in MAUT (Keeney and Raiffa 1976), facilitating the mapping of multiple objectives within a preference model (see Section 2.2). While MAUT can be complex - especially regarding objective weighting - it helps clarify and analyze preference statements. The ENTSCHEIDUNGSNAVI supports DMs by providing explanations and options to adjust parameters as preferences evolve during decision-making. It also offers various evaluation methods to present calculated results transparently, enabling DMs to critically assess outcomes and explore reasons for any differences between intuition and analytics.

MAUT requires the DM to define utilities by establishing preferences and determining utility functions for each objective (see Section 2.2). The ENTSCHEIDUNGSNAVI assists DMs by explaining utility scales and the differences between linear and non-linear utility functions. Risk-neutral DMs can opt for linear functions, while those with decreasing or increasing marginal utility should choose non-linear ones. To help analyze non-linear utility functions, the ENTSCHEIDUNGSNAVI provides graphical representations and various interpretations, allowing the DM to adjust their statements as needed. Figure 5 shows an example of determining exponential utility functions in the ENTSCHEIDUNGSNAVI.

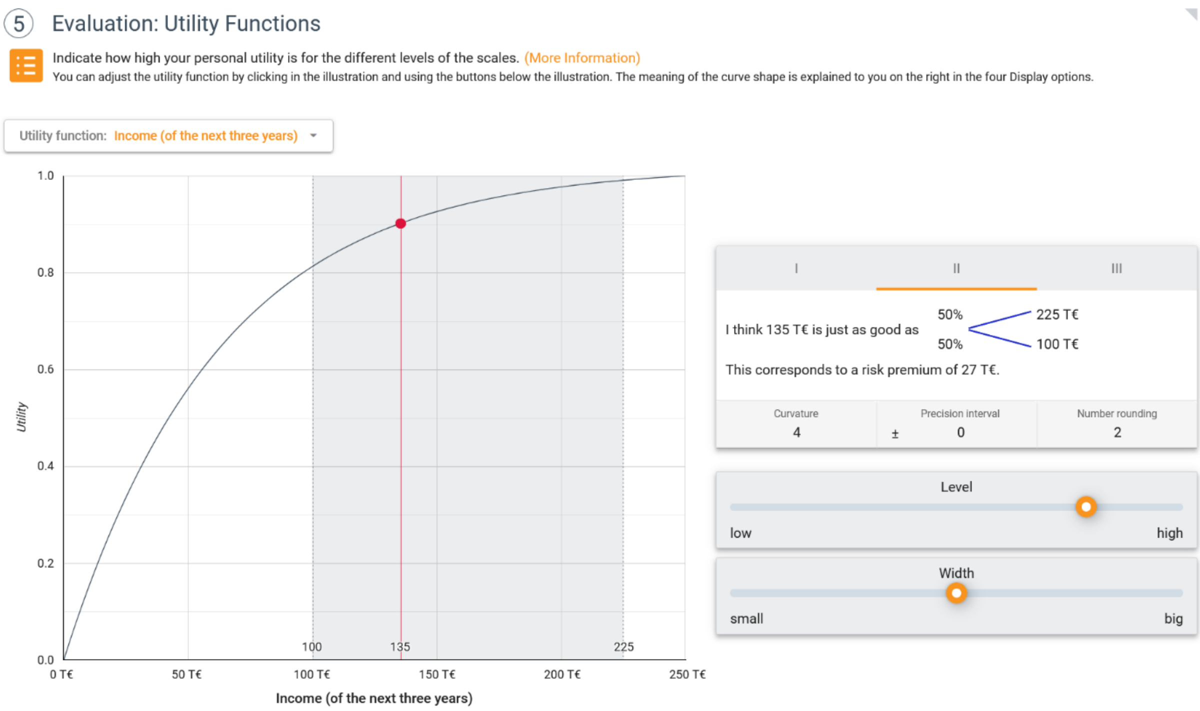

Figure 5. *Determining exponential utility functions in the* E*NTSCHEIDUNGSNAVI.*

After establishing utility functions, the DM must determine objective weights, which serve as scale constants in MAUT. The ENTSCHEIDUNGSNAVI utilizes a trade-off procedure based on Keeney's and Raiffa's (1976) method. To establish weights for multiple objectives, DMs must create I-1 trade-off statements (I: number of objectives) by comparing a reference objective with all others. Using the trade-off statements and the fact that the sum of all objective weights must equal one in MAUT, see Formula (1a) in Section 2.2, the objective weights can be calculated. For the trade-off statements, the DM must specify how improvements in one objective can balance declines in another. The tool provides different formats for checking and modifying preference statements until the DM feels comfortable. Additionally, DMs can enter a certain trade-off, resulting in automatically adjusting ratios of all weights. The ENTSCHEIDUNGSNAVI encourages testing each trade-off with various reference points to ensure suitability.

Once the DM has entered all relevant parameters into the ENTSCHEIDUNGSNAVI - such as the decision statement, objectives, alternatives, consequences table, utility functions, and objective weights - the tool calculates the expected utility for each alternative and presents a ranked list of results. The DM must trust the results, ensuring that analytical outcomes align with gut feelings to implement a decision effectively. After structuring their decision situations, the ENTSCHEIDUNGSNAVI prompts the DM to rank alternatives based on gut feeling, allowing them to identify any discrepancies for further investigation in this step. Discrepancies between intuition and analytics may reveal overlooked objectives or biases in intuition, underscoring the importance of taking gut feelings seriously.

The tool offers various evaluation methods to facilitate critical evaluation and identify discrepancies between intuition and analytics. For example, a pros and cons overview highlights the advantages and disadvantages of each alternative, while a detailed breakdown shows how expected utility is derived for every option. These evaluation methods allow DMs to assess strengths and weaknesses. The DM can also perform sensitivity analyses by adjusting parameters to see how these changes affect rankings. If imprecision is used for any parameter or uncertainties are considered (see Section 2.4), the DM can conduct a robustness check using Monte Carlo simulations to evaluate how stable the rankings are under varying conditions. This test visualizes ranking frequencies and potential ranges of expected utilities for all alternatives, providing an average score based on the expected rank.

2.4 Integrating uncertainties in a multi-criteria decision analysis

In MAUT, there are two types of uncertainties to consider: forecast uncertainty (FU) and parameter uncertainty (PU). FUs involve uncertain forecasts about environmental conditions along with their associated probabilities. PU arises when DMs cannot precisely define specific parameters, such as utilities or objective weights. The ENTSCHEIDUNGSNAVI enables DMs to specify a mean and a degree of precision for parameters to model uncertainties (see Sections 2.4.1 and 2.4.2). Moreover, the tool offers various evaluation methods to analyze and reflect on the final ranking of the alternatives in more detail when uncertainties are given (see Sections 2.4.3 to 2.4.6). These evaluation methods enhance transparency (Peters et al. 2024).

2.4.1 Modeling of forecast uncertainties (FUs)

DMs often struggle to forecast outcomes in a consequences table due to FUs arising from external factors beyond their control. To address this uncertainty, the ENTSCHEIDUNGSNAVI allows DMs to specify an influence factor that impacts the consequence of an alternative for a given objective. While DMs can use multiple influence factors across different cells in the table, each cell can only have one influence factor assigned. The tool provides DMs with two types of influence factors for modeling FUs: user-defined and predefined influence factors.

When DMs can link FUs to specific external factors and events, they can model these using user-defined influence factors. In the ENTSCHEIDUNGSNAVI, the DM can choose between individual and combined user-defined influence factors. Individual influence factors are simpler, requiring DMs to define all possible states and their associated probabilities. In contrast, combined influence factors are more complex, integrating two previously defined individual influence factors with automatically calculated probabilities. These allow DMs to model FUs that depend on multiple external factors.

When DMs cannot link uncertainty regarding the consequences of a cell in the consequences table to specific external factors, they can use a predefined influence factor with a 'worst-median-best' distribution. This approach is practical when deriving likely consequences from extensive data from past or external projects. Predefined influence factors require specifying the p.10, p.50, and p.90 quantiles for the consequences, using 25 % probabilities for p.10 and p.90, and 50 % for p.50 to approximate a normal distribution (Hammond and Bickel 2013). Unlike user-defined influence factors, the probability distributions for predefined influence factors are stochastically independent due to the absence of specific external causes of uncertainty, which can vary across different cells in the consequences table. This independence also applies when assessing consequences through multiple indicators.

2.4.2 Modeling of parameter uncertainties (PUs)

DMs often struggle to specify exact parameters in a decision model, leading to PU. To accommodate this uncertainty, the ENTSCHEIDUNGSNAVI allows DMs to input imprecise information for three types of parameters: utility functions, objective weights, and probability distributions of influence factors. The approach involves identifying a mean μ and a degree of precision ε, which creates an interval for each parameter.

PUs for utility functions

The determination of utilities is based on the scale of each objective, which can be either verbal or numerical, including indicator scales. For verbal scales, utilities are calculated using discrete utilities, while the exponential utility function is employed for numerical scales. The intervals resulting from imprecise information are obtained using Formulas (5a) and (5b) for objectives with verbal scales and Formulas (6a) and (6b) for objectives with numerical scales. The degree of precision for verbal objectives ε_{U_i} can range from 0 to 50 %, and that for numerical objectives ε_{c_i} from 0 to 10.

$$U_i^{min}\left(x_{ij}^k\right) = U_i\left(x_{ij}^k\right) - \varepsilon_{U_i} min\left\{U_i\left(x_{ij}^k\right), 1 - U_i\left(x_{ij}^k\right)\right\} \tag{5a}$$

$$U_i^{max}\left(x_{ij}^k\right) = U_i\left(x_{ij}^k\right) + \varepsilon_{U_i} min\left\{U_i\left(x_{ij}^k\right), 1 - U_i\left(x_{ij}^k\right)\right\} \tag{5b}$$

$$c_i^{min} = c_i - \varepsilon_{c_i} \tag{6a}$$

$$c_i^{max} = c_i + \varepsilon_{c_i} \tag{6b}$$

PUs for objective weights

If the DM opts for imprecise information regarding objective weights, individual weights are determined within specified lower and upper bounds (w_i^{min} and w_i^{max}), calculated using the degree of precision ε_{w_i} for each objective as in Formulas (7a) and (7b). Since imprecise weights can lead to a total that deviates from one, normalization is necessary using Formula (7c) to ensure consistency.

$$w_i^{min} = w_i - \varepsilon_{w_i} \tag{7a}$$

$$w_i^{max} = w_i + \varepsilon_{w_i} \tag{7b}$$

$$w_i^{norm} = \frac{w_i}{\sum_{v=1}^{I} w_v} \tag{7c}$$

PUs for probability distributions

When a DM cannot accurately specify state probabilities for an individual influence factor, they can set a degree of precision $\varepsilon_{P(S_\delta)}$ with $0 \leq \varepsilon_{P(S_\delta)} \leq 50\ \%$. The ENTSCHEIDUNGSNAVI calculates the minimum and maximum probabilities using Formulas (8a) and (8b). However, this may lead to the total sum of probabilities deviating from one. Normalizing these probabilities afterward could result in values that fall outside the defined minimum and maximum ranges if done naively according to Formula (9).

$$P_{min}\left(s_\delta^f\right) = P\left(s_\delta^f\right) - \varepsilon_{P(S_\delta)} min\left\{P\left(s_\delta^f\right), 1 - P\left(s_\delta^f\right)\right\} \tag{8a}$$

$$P_{max}\left(s_\delta^f\right) = P\left(s_\delta^f\right) + \varepsilon_{P(S_\delta)} min\left\{P\left(s_\delta^f\right), 1 - P\left(s_\delta^f\right)\right\} \tag{8b}$$

$$P^{norm}\left(s_\delta^f\right) = \frac{p^{norm}\left(s_\delta^f\right)}{\sum_{v=1}^{K_\delta} p^{norm}\left(s_\delta^v\right)} \tag{9}$$

To prevent this issue, the ENTSCHEIDUNGSNAVI employs an algorithmic approach that arranges the individual probabilities in ascending order while ensuring that their total sum equals one.

For a detailed description, see ‘Paper 2: Integrating uncertainties in a multi-criteria decision analysis with the ENTSCHEIDUNGSNAVI’ in Part B (Peters et al. 2024).

2.4.3 Methods for checking the robustness of the result

Incorporating FUs and PUs can complicate decision models but is often necessary. Due to the additive aggregation method, the expected utility in the MAUT model may not fully capture risks associated with rare events. To address this, the ENTSCHEIDUNGSNAVI conducts Monte Carlo simulations (Kalos and Whitlock 2009) that rank alternatives based on randomly generated scenarios derived from defined PUs and FUs. Figure 6 shows an example of the robustness check in the ENTSCHEIDUNGSNAVI.

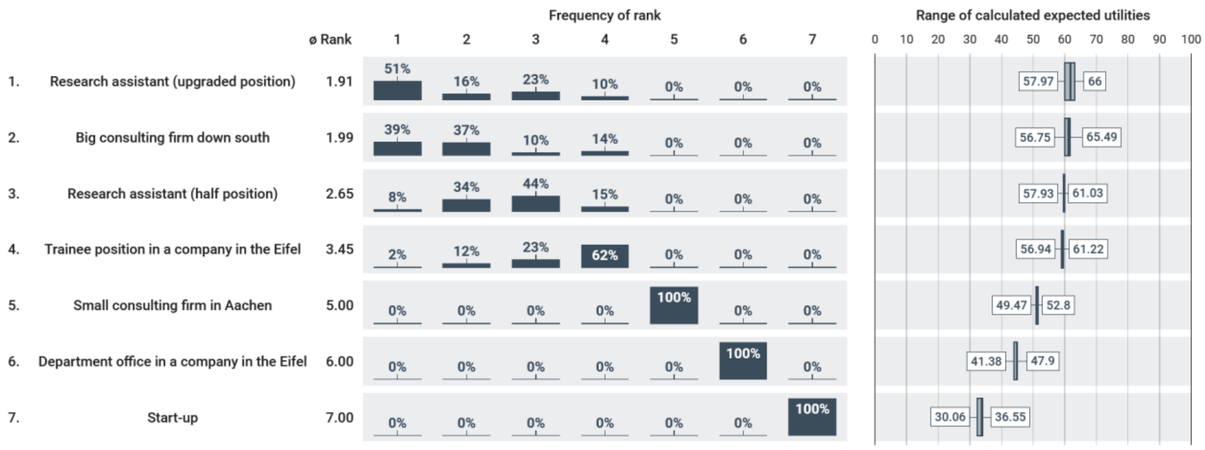

Figure 6. *Robustness check in the ENTSCHEIDUNGSNAVI.*

The simulation allows DMs to assess the rankings' robustness and identifies scenarios where a promising alternative might underperform. Enabling FUs in the simulations allows DMs to see how alternatives respond to external factors beyond their control. For PUs, insights reveal how imprecise parameter specifications impact results, prompting DMs to consider refining their assessments if sensitivity is detected. Simultaneously analyzing both PUs and FUs yields comprehensive insights into uncertainties affecting outcomes. The ENTSCHEIDUNGSNAVI presents the simulation results in a tabular format showing ranking frequencies for each alternative, with scores reflecting weighted averages of these ranks. The tool also identifies conditions for achieving specific ranks and presents expected utility ranges for each alternative. The simulation concludes when changes in ranking frequencies fall below a specified threshold over time, ensuring stable results while minimizing resource use. However, DMs can continue simulations for more accurate outcomes if desired.

2.4.4 Objective weight analysis

Eliciting objective weights is often one of the most challenging aspects of the decision-making process, especially when DMs are uncertain about these weights to the extent that they cannot define them even with precision intervals. When significant PU regarding objective weights exists, an objective weight analysis can help identify which weight intervals might lead to different outcomes. This analysis employs a simulation approach to calculate a range of statistical measures - such as minimum, maximum, p.10, and p.90 quantiles, median, and average - for various objective weights that determine the best alternative. The tool derives these measures from algorithmically selected combinations of objective weights during each calculation step.

2.4.5 Sensitivity analysis

The sensitivity analysis in the ENTSCHEIDUNGSNAVI allows DMs to assess the model's plausibility by examining how different parameters influence the result and the extent of that impact. Unlike many tools that only permit unidimensional analysis, this tool enables simultaneous variation of all parameters, providing a more realistic assessment of multiple uncertainties.

The analysis covers various parameters, including objective weights, utilities, probabilities of FUs, consequences, and indicator weights, with corresponding slider boxes for each. DMs can observe changes in rankings and utilities as they adjust these sliders and choose which alternatives to display. A detailed view of expected utilities breaks down contributions from each objective, with hover functionality to identify utilities of specific objectives. If DMs use imprecise parameters, the specified precision interval is visually marked on the slider with a dark gray area, allowing them to check for effects within defined intervals quickly.

2.4.6 Indicator impacts, tornado diagrams, and risk profiles

The ENTSCHEIDUNGSNAVI offers three methods for visualizing uncertainties and their related risks: indicator impacts, tornado diagrams, and risk profiles.

Indicator impacts

Indicator scales enhance the assessment of consequences by providing DMs with more detailed input. If DMs combine them with influence factors, the ENTSCHEIDUNGSNAVI can visualize how these uncertainties impact the overall consequences of objectives for individual indicators. This approach helps DMs identify key contributing factors and associated risks, guiding them on where to focus efforts for more precise predictions versus where such efforts may be too costly. Each cell in the consequences table that uses an indicator scale features an indicator impact diagram, showing which indicators most significantly affect outcomes based on existing FUs. Depending on whether the DM uses user-defined or predefined influence factors, they receive different information and modification options. For predefined influence factors, the ENTSCHEIDUNGSNAVI calculates impacts initially by assuming either the worst, median, or best state occurs for the relevant indicator while all others are set to the median (simple variant). Alternatively, DMs can opt for a probabilistic variant that considers frequency distributions for other indicators. In cases involving user-defined influence factors, impacts are calculated under the assumption of stochastic independence among indicators, consistently using the probabilistic variant.

Tornado diagrams

Tornado diagrams display changes in expected utility for an alternative based on specific events, known as conditional expected utilities. DMs can analyze these effects in isolation for a single alternative or by comparing two alternatives, highlighting under what circumstances one option is preferable. The diagram ranks influence factors by their impact, with the most significant factor at the top and others following in descending order, resulting in a tornado. This visualization helps DMs identify which uncertainties warrant further analysis or more precise probability estimates to reduce uncertainty in their decision-making process.

Risk profiles

The ENTSCHEIDUNGSNAVI provides various methods for analyzing the risks of selecting an alternative. The tool performs this analysis for each objective individually or aggregated across all objectives. Additionally, when considering all objectives, DMs can assess risk with and without their preferences and objective weights.

Risk profiles indicate the likelihood of exceeding specific results and can be used to check for dominance. They assist DMs in eliminating alternatives to simplify the model and optimize resource use. A DM can generate a risk profile for each cell in the consequences table related to a single objective using the distribution function of its influence factor. This analysis allows comparisons with other alternatives regarding the same objective and enables graphical determination of first-degree stochastic dominance (Hadar and Russell 1969). Additionally, this dominance check is automatically performed across all objectives mathematically after assessing all alternatives without requiring definitions of utility functions or objective weights. The tool highlights alternatives dominated by others in red with an info button. Clicking this button reveals which alternative(s) dominate the one marked, allowing DMs to decide whether to analyze stochastically dominated alternatives further. Even if an alternative is dominated, it may still be optimal in rare scenarios. However, if it is dominated by multiple alternatives or those relying on the same influence factors, DMs can typically exclude it from further analysis. DMs can also assess the risk associated with alternatives based on their preference statements regarding utilities and objective weights. The so-called risk comparison displays the overall utilities of the alternatives, with the x-axis representing possible utilities that depend on specific forecast scenarios. This information is generated through a Monte Carlo simulation, using data similar to that obtained in the robustness check when only FUs are considered.

3 The current state of research on simplifying the reflective decision-making process

The reflective decision-making process, as applied in the ENTSCHEIDUNGSNAVI, is demanding for the DM and requires support, either in the form of a decision analyst or, as implemented in the ENTSCHEIDUNGSNAVI, with many explanatory and helpful texts for the individual steps. But even with this help, many DMs find it challenging to understand and trust the theoretical constructs of the process so that they can decide with a clear conscience. For this reason, researchers have already done much work on how theoretical and scientifically based approaches can be simplified for DMs in practice without compromising the quality of decisions. The following sections present the current state of research on simplifying the reflective decision-making process.

3.1 Structuring of the decision situation

In the first phase, it is crucial to formulate a broad and proactive decision statement (see Section 2.3.1). Hannes and von Nitzsch (2024) analyze how integrating decision science approaches in the ENTSCHEIDUNGSNAVI improves the understanding of the decision problem and, thus, increases the decision quality. The researchers concentrate on three impediments that Baer et al. (2013) identified: a narrow sampling of information, representational gaps, and jumping to solutions. They address these impediments by stating connected decisions (Mingers and Rosenhead 2001, Hammond et al. 2015), dealing with values (Keeney 1996), challenging assumptions (Mitroff and Featheringham 1974, Legrenzi et al. 1993, Montibeller and von

Winterfeldt 2015), and reframing the decision (Maule and Villejoubert 2007, Kahneman 2011, Larrick 2012). The results show that integrating these approaches in the ENTSCHEIDUNGSNAVI helped 87.3 % of the participants to improve their understanding of the decision situation. Moreover, 74.4 % of these participants improved their decision statement through the new implementation.

In the next step, DMs should define their objectives in line with VFT (Keeney 1996). Bond et al. (Bond et al. 2008) analyze whether DMs can articulate what they want. Their study shows that DMs are not able to formulate all relevant objectives. Moreover, they prove that DMs perceived omitted objectives as almost as crucial as those generated by participants alone. Based on their results, the researchers recommend using a 'master list' that provides an abundance of potential decision objectives. They also recommend talking to friends, family, or DMs facing the same decision to define all decision-relevant objectives. Bond et al. (2010) deepen these thoughts and conduct a follow-up study. In this study, they examine a variety of interventions to improve the determination of objectives: the provision of sample objectives, organization of objectives by category, and direct challenges to do better, with or without a warning that essential objectives are missing. The utilization of category names and direct challenges accompanied by a warning both resulted in an increase in the number of objectives generated, without affecting their quality. Other interventions showed less improvement.

In the last step of this phase, DMs should identify their alternatives. Siebert and Keeney (2015) show that DMs recognize fewer than half of their available options, and the average quality of the alternatives they overlook is comparable to those they do identify. This finding is true for decisions where the complete range of potentially desirable alternatives is unclear immediately. Based on VFT, the researchers recommend using an objective master list to create new alternatives. Siebert (2016) shows that the stimulation with objectives increases the quality of alternatives.

3.2 Development of the consequences table

The challenges of the second phase are the avoidance of biases and the handling of uncertainties (see Section 2.3.2). Several researchers point out that informing DMs about potential biases is essential while developing the consequences table e.g., Montibeller and von Winterfeldt (2015). Otherwise, judgments are subject to biases and can reduce the quality of the decision. Biases can influence the direct determination of consequences and the modeling of uncertainty factors, including determining the probabilities of the different states. This fact can unsettle the DM. The approach of imprecise information was developed to support the DM in determining probabilities (Fishburn 1965). With the help of linear programming (Kmietowicz and Pearman 1982, 1984) and the implementation of imprecise information in DSS (Jiménez et al. 2002, Danielson et al. 2007, Mateos et al. 2007), DMs should be able to use uncertainty factors more easily. Furthermore, the approach of imprecise information can be used not only for probabilities but also for determining other parameters, such as utility functions or objective weights.

Peters et al. (2024) present how to integrate FUs and PUs in a DSS to create transparency and understanding for DMs in uncertain decisions (see Section 2.4). They value a simple, user-friendly presentation and the option to analyze and reflect the results for robustness. This approach gives the DM a good feeling even when making uncertain decisions and promotes the implementation of the decision.

3.3 Evaluation of the alternatives and the decision

In the third phase, determining different decision parameters, such as utility functions and objective weights, is essential for using MAUT to find the best alternative.

The approach of imprecise information can help not only in the second phase but also in the third phase in determining the necessary parameters for using the MAUT model. Some researchers deal with the use of imprecise information for utility functions (Weber 1985, Weber

1987, von Nitzsch and Weber 1988, Armbruster and Delage 2015). Others present approaches for unknown or imprecise objective weights (Kirkwood and Sarin 1985, Hazen 1986, Carrizosa et al. 1995). De Almeida et al. (2016) and de Almeida-Filho et al. (de Almeida-Filho et al. 2017) give a broad overview of this topic and categorize the approaches using forms of imprecise information (interval weights, partial/incomplete information on weights, or unknown weights). Many approaches allow imprecise information in more than one parameter in the decision model, e.g., utility functions and objective weights (Park et al. 1996, Eum et al. 2001, Lee et al. 2001, Lee et al. 2002) or probabilities and utility functions (Moskowitz et al. 1993, Danielson et al. 2003, Danielson 2004, Liesiö and Salo 2012). All approaches aim to support the DM in determining parameters, and thus, simplify the decision-making process.

The literature fundamentally questions the use of scientifically theoretical approaches such as MAUT in practice. Several researchers (Keeney and von Winterfeldt 2007, Durbach and Stewart 2012, Katsikopoulos et al. 2018) highlight that employing such a model is not always essential or beneficial. Keeney and von Winterfeldt (2007) contend that a practical value model is sufficient if the analysis conducted with a theoretically superior model does not produce different results compared to the practical one utilized. This thought raises the question of how complex a decision analysis (DA) should be in practice and which preference elicitation methods are most suitable for specific contexts. Keeney and von Winterfeldt (2007) emphasize that selecting an appropriate DA model depends on the situation's characteristics, the DM‘s profile, the time available, and the skills of the decision analyst involved in addressing the decision.

For MAUT to be applied scientifically correctly, the DM must define one-dimensional utility functions and objective weights for each objective (see Section 2.3.3). However, these tasks present the DM with major challenges, as the theory is difficult to understand. This dissertation focuses on the utility functions at this phase and presents the current state of research in the following.

Numerous researchers have focused on evaluating the utility functions to assist DMs in this complex task (e.g., Pratt et al. (1964), Keeney (1972), Keeney and Raiffa (1976), Anderson et al. (1977), Keeney (1977), Farquhar (1984)). Toffano et al. (2022) note that artificial intelligence is increasingly utilized for preference elicitation. Recent studies focus on statistical robustness in utility preference optimization models when preference information is incomplete (Weber 1987, Guo and Xu 2021). However, DMs frequently struggle to articulate their preferences consistently (Cyert and DeGroot 1975, Keeney and Raiffa 1976, Keeney 1982), and the assessment is often subject to errors and biases.

Montibeller and von Winterfeldt (2015) identify various biases affecting this process, including anchoring (Chapman and Johnson 1999), gain-loss bias (Levin et al. 1998), certainty effect (Allais 1953, Kahneman and Tversky 2013), desirability of options bias (von Winterfeldt 1999), and affect-influenced bias (Slovic et al. 2004). Additional research highlights methods to reduce these biases (Hershey et al. 1982, McCord and De Neufville 1986, von Nitzsch and Weber 1988, Wakker and Deneffe 1996), with Bleichrodt et al. (2001) recommending interactive sessions for consistency. However, such sessions can be time-consuming, expensive, and challenging for DMs.

Building on Keeney and von Winterfeldt's (2007) practical considerations, researchers are investigating the importance of accurately determining utility functions and their impact on alternative rankings in MAUT. Stewart (1993) finds that while non-linearities in value functions[2] are significant, they can be simplified using piecewise linear functions. Lahdelma and Salminen (2012) analyze how the shape of utility or value functions affects stochastic multi-criteria acceptability analysis across one real-life problem and 3,600 artificially generated test cases. Their results indicate that rankings are more sensitive to convex than concave functions. While larger decision sets show less variation in identifying the best alternative, sensitivity increases with

[2] Value functions are employed for risk-free decisions in multi-attribute value theory (MAVT) (Keeney & von Winterfeldt, 2007) and serve as the counterpart to utility functions.

more objectives for the whole ranking. However, they caution that their findings are based on simulations and may differ in real-world scenarios.

Finally, in this phase, evaluation methods like those presented by Peters et al. (2024) can strengthen the DM's understanding of the resulting ranking of alternatives (see Sections 2.4.3 to 2.4.6). Numerous approaches and methods in the literature have precisely this objective (Madani and Lund 2011, Spetzler et al. 2016, Baudry et al. 2018, Mukhametzyanov and Pamucar 2018, Shavazipour et al. 2021, Więckowski and Sałabun 2023). However, this dissertation does not focus on these methods, so a detailed explanation is not provided.

4 Research gaps and goals of this study

This work aims to optimize and simplify the reflective decision-making process further so that even DMs without extensive know-how can make sound and reflective decisions. To this end, a research gap is uncovered and examined in the current literature for each phase of the reflective decision-making process. Analyzing these can help simplify the reflective decision-making process and make it more practical. Based on the findings, recommendations for practical application are presented.

In the first phase, structuring the decision situation, the concept of VFT ensures a proactive decision statement, the definition of objectives before the alternatives, and the associated creative search for new alternatives based on the already known objectives. There is already evidence in the literature for this meaningful and helpful approach (see Section 3.1). However, to the best of our knowledge, there is no evidence of how value-oriented DMs make decisions and whether measures can simplify the application of VFT, i.e., whether DMs can be supported in making more value-focused decisions. To help DMs make beneficial decisions, either consciously or unconsciously, concepts like nudging (Keeney 2020) and anchoring (Furnham and Boo 2011) are often employed. These techniques influence decision-making by shaping the environment in which choices are made. ***The first study explores how nudging can facilitate better, value-focused decisions.*** The expectation is that the measures can increase the number of value-based objectives and, thus, improve the quality of decisions.

In the second phase, developing the consequences table, the DM is confronted with uncertainty for the first time. This uncertainty can initially take the form of FU and, in this context and also in the subsequent third phase, PU. In literature, researchers handle PU with the imprecise information approach (see Sections 3.2 and 3.3). To the best of our knowledge, there has been no research conducted to assess the extent to which individuals utilize imprecise information in decision-making situations, how helpful DMs perceive it to be, or whether this approach influences the selection of the best alternative in scenarios where DMs are looking for a single choice. ***The second study analyzes how to deal with an imprecise information approach in a MCDM-Support-System.*** The expectation is that the findings can improve and simplify the determination of parameters, including imprecise information, in a MAUT model.

In the third phase, evaluating the alternatives and the decision, DMs should define their utility function in a way that is consistent with their preferences. Researchers identified major challenges in this step and examined the importance of accurately determining utility functions and their impact on alternative rankings in MAUT. Initial findings have shown that the definition of utility functions for selecting the best alternative is not crucial. However, these statements are based on pure simulations (see Section 3.3). ***The third study analyzes personal decisions and examines whether determining utility functions can be simplified through linear shapes.*** The expectation is that, in some cases, the assessment of utility functions can be simplified with linear shapes without influencing the result and, thus, the quality of the decision.

Figure 7 briefly summarizes how the underlying papers of this dissertation address the different goals.

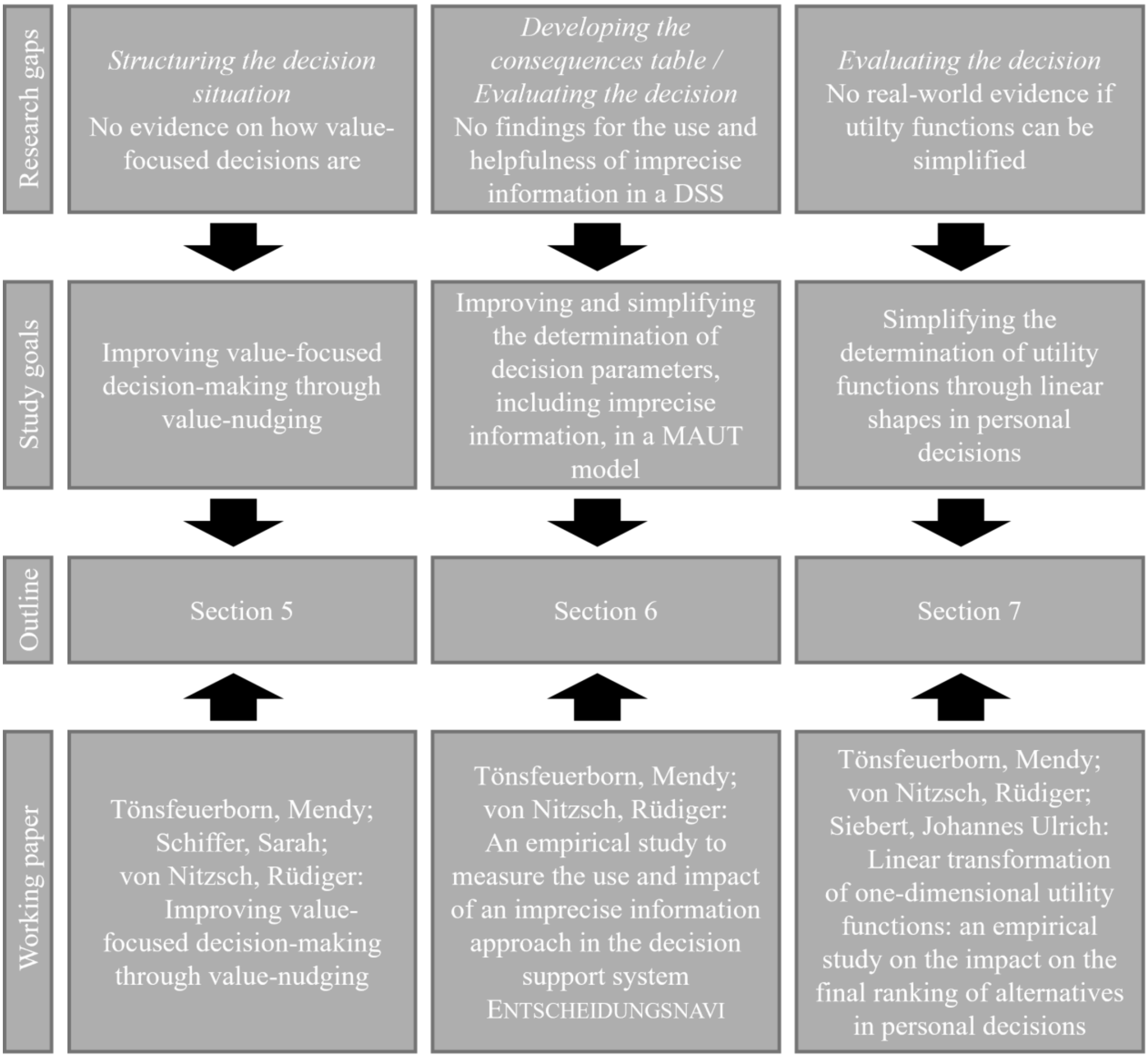

Figure 7. *Outline of this study showing how the papers provided content for the different sections to answer the research questions.*

The data set of the papers was gathered using the DSS ENTSCHEIDUNGSNAVI as part of a voluntary project in the 'Decision Theory' course at RWTH Aachen University. In this project, students were tasked with thoroughly analyzing a significant personal decision while following each of the phases outlined in Section 2.3. Thus, the data set contains all relevant decision parameters for an analysis according to the reflected decision-making process under VFT and MAUT (see Sections 2.1 to 2.3).

5 Improving value-focused decision-making through value-nudging

VFT (Keeney 1992) improves decision-making using personal values. Personal values are an overarching concept shaping people's motives and guiding their decisions and actions. Therefore, important life choices with far-reaching consequences should be considered and made following one's values. In the working paper "Improving value-focused decision-making through value-nudging", we analyze whether DMs can be nudged (Thaler and Sunstein 2021) toward more value-focused and, thus, better decisions.

5.1 Paper approach

This study aims to analyze three research questions: 1) Which values are important to DMs in private decisions?, 2) Does nudging increase the impact of the five most important personal values in private decisions?, and 3) What impact does each of the five most important personal values have on private decisions before and after nudging?.

We divided participants into two groups for comparison: Group value-anchoring (GroupVA) and Group value-nudging (GroupVN). GroupVA, consisting of data from 2019 to 2020, required students to reflect on their values before determining their objectives related to a decision statement (see Section 2.3.1), serving as a control group. In contrast, GroupVN, which includes data from 2021 to 2023, followed the same process but incorporated a nudge through the ENTSCHEIDUNGSNAVI. In this group, the top five ranked values from Step 1 (see Figure 3) were automatically integrated into Step 2 of defining fundamental objectives (see Figure 4), providing participants with a helpful starting point rather than requiring them to begin from scratch as in GroupVA. Additionally, we analyzed various decision topics (career, study, leisure planning, going abroad, and housing situation).

To answer the **first research question**, we analyzed how frequently each value was ranked among the top five by participants in Step 1 across all decision topics for both GroupVA and

GroupVN. For the **second research question**, we compared GroupVA and GroupVN, focusing on the effect of value-based objectives on decision-making through objective weightings. To achieve this, we compared the top five ranked values from Step 1 (see Figure 3) of the ENTSCHEIDUNGSNAVI with the fundamental objectives established in Step 2 (see Figure 4). Three decision experts independently assigned these values to the objectives. Initially, they checked for identical wording, which made assignments straightforward. However, when wording differed, they referred to the objective hierarchy and accompanying explanations in the comment field[3]. If a value was mentioned in an explanation or related to an objective (e.g., as a means objective), it was assigned accordingly. Values could be linked to multiple objectives if referenced more than once, and vice versa. To assess the impact of value-based objectives on decision-making, we analyzed the objective weights established in Step 5 (see Section 2.3.3). We aggregated the weights of objectives based on the five most important values to determine their relative proportion in the decision. An example of the procedure is shown in Figure 8.

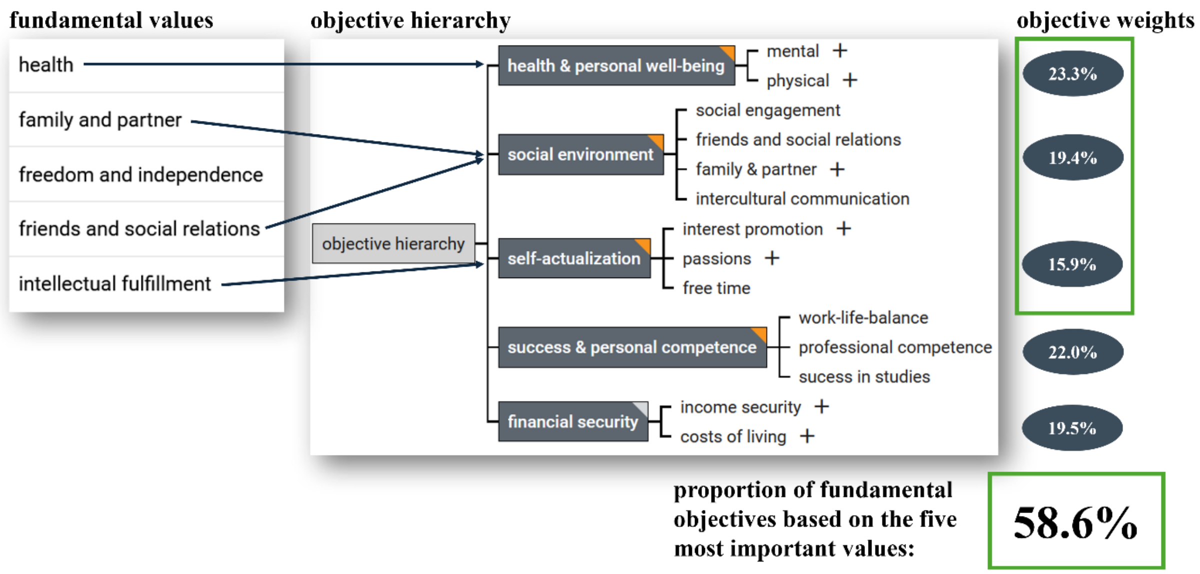

Figure 8. *Example for the assignment of values to fundamental objectives.*

[3] The comments on the objectives made by the participants are shown under the orange triangles in the objective hierarchy, see Figure 8.

We explored the **third research question** to gain deeper insights into the values underlying the objectives and their influence on decisions. Therefore, we calculated the average objective weights for each value in GroupVA and GroupVN. If a value was linked to multiple objectives, its weight was determined by summing the weights of those objectives. Conversely, if several values were associated with a single objective, the weight was distributed equally among those values.

5.2 Results

1) Which values are important to DMs in private decisions?

Overall, we found no major differences between the groups for the values that more than 20 % of the participants selected in the top five. The groups differed in the frequency (Freq.) with which they selected the values only below this threshold. Table 1 summarizes the results.

GroupVA (n = 286)		**GroupVN (n = 325)**	
Value	**Freq.**	**Value**	**Freq.**
family and partner	80.77 %	health	79.08 %
health	80.07 %	family and partner	75.38 %
financial security	65.38 %	financial security	65.54 %
friends and social relations	65.03 %	friends and social relations	65.54 %
freedom and independence	43.36 %	freedom and independence	49.85 %
intellectual fulfillment	36.36 %	intellectual fulfillment	40.92 %
honesty and ethics	30.07 %	honesty and ethics	24.00 %
excitement and new experiences	24.83 %	excitement and new experiences	23.08 %
attractive housing	17.13 %	little mental stress (***)	18.15 %
competence	15.73 %	attractive housing	16.92 %
justice and fairness	13.29 %	competence	16.62 %
little mental stress	8.74 %	justice and fairness	9.85 %
being attractive	5.94 %	environment and nature	4.92 %
power and leadership	5.24 %	power and leadership	4.00 %
environment and nature	4.20 %	being attractive	3.38 %
wealth	3.85 %	wealth	1.54 %

Table 1. *Frequency of values ranked among the top five in Step 1 of the* E*NTSCHEIDUNGSNAVI (*** p<0.001).*

The five most important values ('family and partner', 'health', 'financial security', 'friends and social relations', and 'freedom and independence') were almost stable in the topics. The value of 'intellectual fulfillment' was also relevant in the topics *study* (GroupVA) and *leisure planning* (both groups).

2) *Does nudging increase the impact of the five most important personal values in private decisions?*

Nudging enhanced the impact of the five most important personal values in private decision-making, see Figure 9.

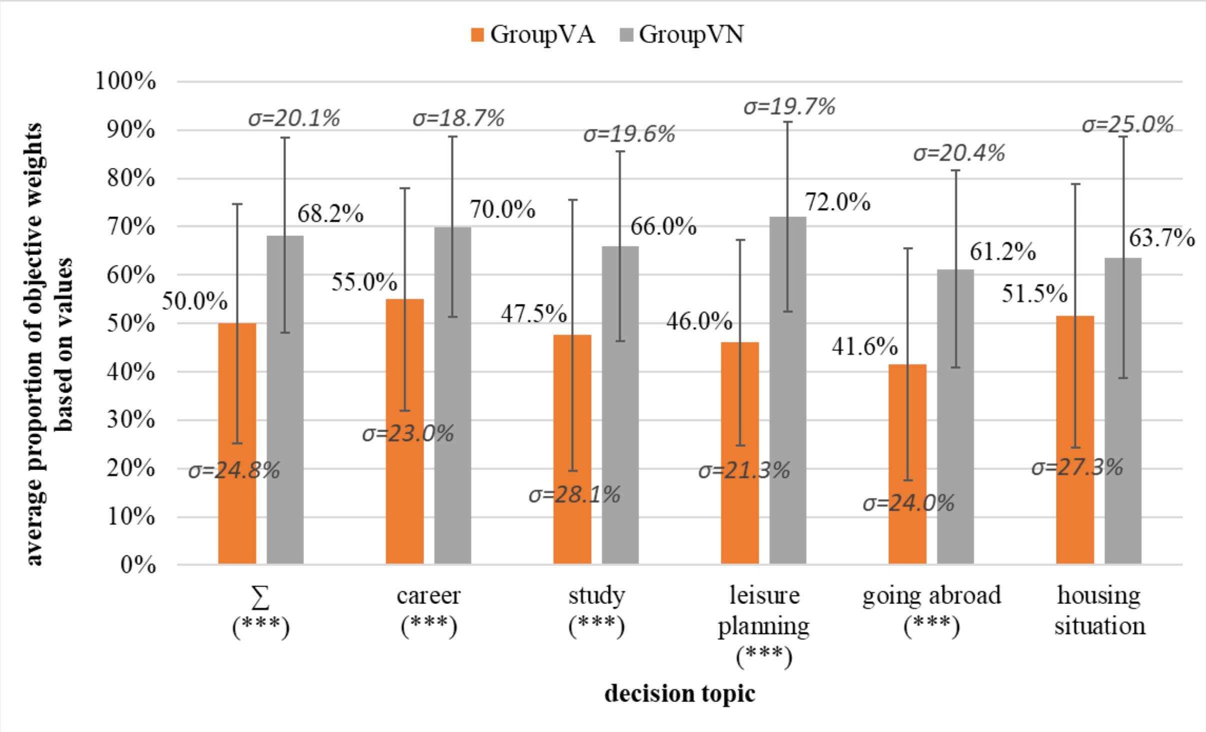

Figure 9. *Average proportion of objective weights based on values in GroupVA and GroupVN (*** p<0.001).*

Overall, the percentage of objective weights derived from values was 18.2 % higher in GroupVN compared to GroupVA. A significant increase in value-based objective weights was observed for the areas of *career*, *study*, *leisure planning*, and *going abroad*. We assume that in GroupVN, the DMs articulated their fundamental objectives more broadly and based on values, which resulted in previously stated non-value-based objectives being referenced more as means objectives. This assumption needs to be proven in further studies.

3) *What impact does each of the five most important personal values have on private decisions before and after nudging?*

The values of 'health', 'family and partner', and 'freedom and independence' significantly influence GroupVN's decisions more than GroupVA's. The most notable difference in the average objective weight between the two groups is observed for the value 'health', which increased from approximately 4 to 12 %. This rise may be attributed to the COVID-19 pandemic, during which participants were exposed to the nudge and, in addition, extensive media coverage regarding health matters. Conversely, we noted a significant decrease in the objective weight associated with the value of 'financial security'. Nudging individuals towards more value-oriented objectives could result in DMs becoming more aware of other values that hold greater importance than money, which they may not have adequately considered before.

Overall, value-nudging is most effective when the relevant values in a situation are not immediately clear. If these values are insignificant to the problem, nudging may lead DMs to consider irrelevant objectives, negatively impacting their identification of fundamental objectives and preferences. Conversely, when important values are present, a well-designed nudge can enhance the clarity of fundamental objectives and improve decision quality.

5.3 Recommendations

DMs should first deal with their personal values to improve value-focused decision-making and identify fundamental objectives aligning with their values. In the ENTSCHEIDUNGSNAVI, value-nudging has enhanced the impact of the five most important personal values in private decision-making and, thus, makes VFT easier for DMs. With value-nudging, they made more value-focused decisions, leading to a higher decision quality. Therefore, we recommend implementing value-nudging in MCDM-support systems that use VFT. This recommendation has already been implemented in the ENTSCHEIDUNGSNAVI and integrated into the process for the long term. However, nudging should always be used cautiously, as unintended or negative effects can arise (Wilkinson 2013, Damgaard and Nielsen 2018, Schmidt and Engelen 2020).

6 Imprecise information for defining quantitative parameters

Imprecise information was developed to support DMs in defining the relevant quantitative parameters for a decision model. The approach avoids specifying exact parameters while still providing a good result. In the working paper "An empirical study to measure the use and impact of an imprecise information approach in the decision support system ENTSCHEIDUNGSNAVI", we analyze how often DMs take this approach in the ENTSCHEIDUNGSNAVI and what impact it has on the decision recommendation.

6.1 Paper approach

This study aims to analyze three research questions: 1) How often is imprecise information used?, 2) How helpful do participants find the use of imprecise information?, and 3) What impact does imprecise information have on the final ranking of alternatives?.

In the analyses, we concentrate on the three categories of PU implemented in the ENTSCHEIDUNGSNAVI (see Section 2.4.2): utility functions, objective weights, and probabilities. Moreover, we divided the participants into two groups and adjusted the default degrees of precision. In Group non-zero, the default setting for the degrees of precision was not zero, leading to imprecise intervals. In contrast, Group zero had all default settings for degrees of precision set to zero. As a result, participants needed to select a degree of precision actively if they wanted.

We analyze the **first research question** by examining the use of imprecise information in total and the three categories. For the **second research question**, we evaluated a questionnaire in which we asked the participants how helpful they found the concept of imprecise information. We used the robustness check (see Section 2.4.3) for the **third research question**. In 100,000 simulations for each decision, we permitted imprecise information across all three categories and examined how frequently the best (second, third) alternative, determined by the mean values of the parameter intervals, ranked first (second, third). To investigate the effect of imprecise information on each category more thoroughly, we conducted additional Monte Carlo simulations where imprecise information was allowed in only one of the three categories at a time.

6.2 Results

1) How often is imprecise information used?

In Group non-zero, 99.90 % of participants utilized imprecise information in at least one category, while in Group zero, this figure was lower at 69.80 %. This trend is evident across all categories. In Group non-zero, the use of imprecise information was fairly uniform across categories (probabilities: 99.72 %, utility functions: 96.97 %, objective weights: 97.91 %). However, in Group zero, there were notable differences; participants used imprecise objective weights less (19.71 %) compared to imprecise probabilities (58.94 %) and utility functions (60.22 %). Overall, when considering all imprecise values regardless of decisions, imprecise probabilities were the most commonly used (86.84 %), followed by utility functions (69.81 %) and objective weights (52.22 %).

2) How helpful do participants find the use of imprecise information?

The questionnaire results indicate that participants found the use of imprecise information on probabilities to be the most helpful ($\mu = 4.24$; $\sigma = 1.3$), followed closely by imprecise utility functions ($\mu = 4.16$; $\sigma = 1.15$). The specification of imprecise preference statements for objective weights was considered the least helpful ($\mu = 3.98$; $\sigma = 1.35$). Participants in Group zero rated the helpfulness significantly lower than those in Group non-zero. Participants who employed imprecise parameters in their decisions rated their helpfulness significantly higher than those who used precise parameters. In each group considered in this analysis, the participants rated imprecise objective weights as the least helpful.

3) *What impact does imprecise information in the three categories have on the final ranking of alternatives?*

Figure 10 shows the impact of imprecise information on the final ranking of alternatives for all decisions and all categories.

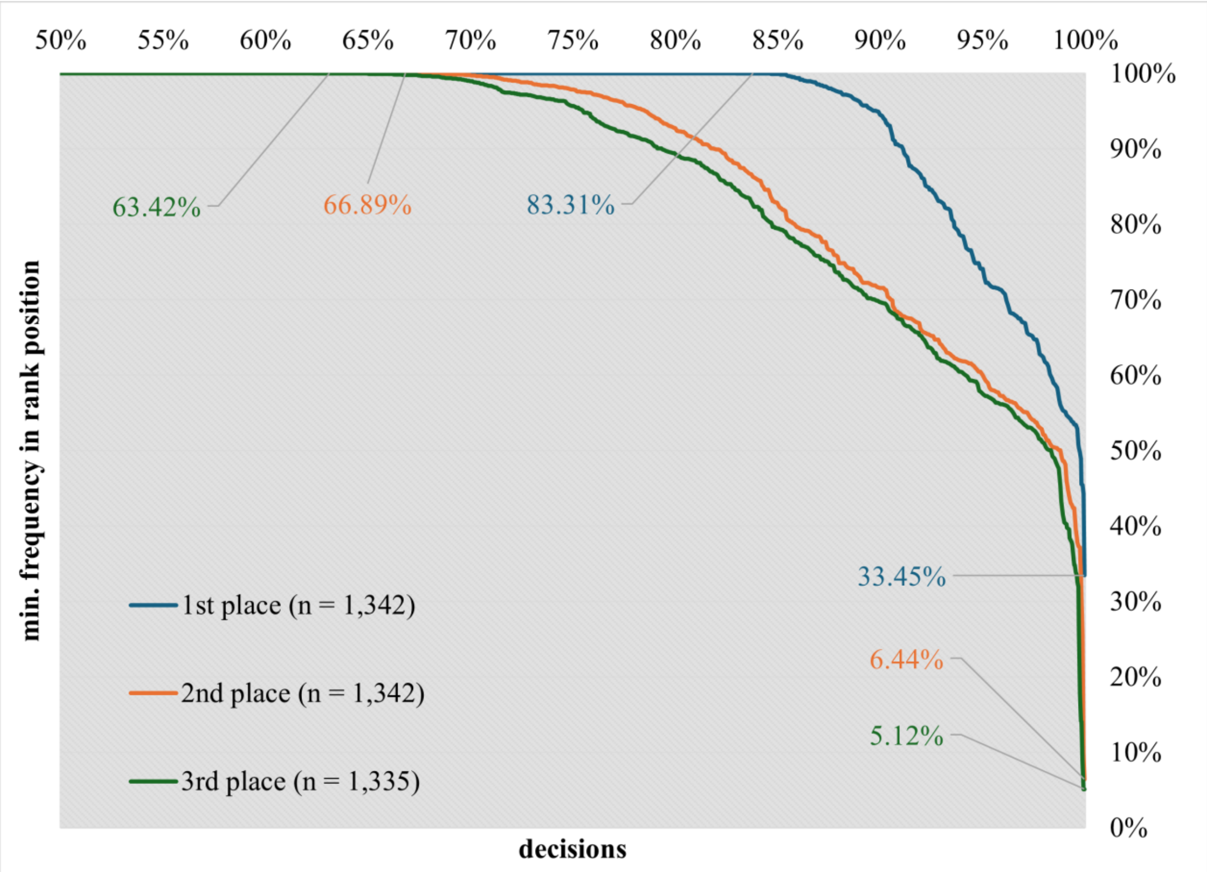

Figure 10. *Impact of imprecise information on the final ranking.*

In 83.31 % of the decisions, the first-place alternative remains stable, meaning that in all 100,000 simulations (100 %), the best alternative calculated from the mean values of the parameter intervals consistently ranks first. In the worst case, imprecise information led to this alternative being identified as the best only 33.45 % of the time. The second- and third-best alternatives held their respective positions in 100 % of simulations for 66.89 % and 63.42 % of decisions.

The comparison of the two groups showed that the first place is more stable (88.83 %) in Group zero than in Group non-zero (81.07 %). Moreover, the analysis of the different categories of PU shows that imprecise utility functions have the greatest impact on the stability of the best alternative, followed by imprecise objective weights and imprecise probabilities. However, there are no major differences between the categories, as the min. frequency for the best alternative varies between 90 % and 94 %.

6.3 Recommendations

Every MCDM-support system should incorporate imprecise information approaches to assist DMs. The study effectively argues that DMs often cannot or prefer not to specify parameters precisely. Therefore, integrating imprecise information approaches with practical evaluation methods is crucial. Evaluation methods, such as the robustness check in the ENTSCHEIDUNGSNAVI, help DMs identify the best alternative even when using imprecise parameters. They also clarify the effects of these imprecise parameters and raise awareness of potentially unstable rankings. If a ranking is stable, DMs can feel confident about their imprecise inputs and may even leverage some degree of imprecision without concern. Conversely, if a ranking is unstable, DMs can work to refine their parameters more accurately. A subsequent robustness check can then reveal whether this refinement leads to a more stable ranking and reduces the risk of not selecting the best alternative. This process provides valuable insights into how their chosen parameters impact decision outcomes.

7 Simplifying the determination of utility functions through linear shapes

Establishing one-dimensional utility functions for each objective in MAUT requires significant time and effort from DMs. They must consider factors such as decreasing or increasing marginal utility and/or their relative risk preferences, leading to a non-linear shape. This evaluation process is susceptible to errors and distortions. In the working paper "Linear transformation of one-dimensional utility functions: an empirical study on the impact on the final ranking of alternatives in personal decisions", we analyze to what extent a linear transformation of one-dimensional utility functions compromises the decision quality. Therefore, we examine the impact of one-dimensional utility functions on the final ranking of alternatives in practice.

7.1 Paper approach

This study aims to analyze three research questions: 1) How often are (non-)linear utility functions used in practice?, 2) How does the use of (non-)linear utility functions impact the final ranking of alternatives in practice?, and 3) To what extent do linear utility functions ensure stability in selecting the best alternatives based on utility differences?.

To answer the **first research question**, we analyzed the use of (non-)linear utility functions in total and the decisions, i.e., how often participants used (non-)linear utility functions in a decision (for an example of utility determination in the ENTSCHEIDUNGSNAVI, see Figure 5). Furthermore, we distinguished between verbal and numerical objectives (see Section 2.2). Additionally, we calculated an average risk aversion parameter c for decisions with numerical objectives only to analyze the level of risk and non-linearity. For the **second research question,** we calculated the expected utilities for every alternative for two scenarios using Formula (1) and compared the resulting rankings. In the first scenario, we considered the utility functions based on DMs' preferences, and in the second scenario, we used simplified linear utility functions. With the help of the two rankings, we could uncover rank reversals of alternatives. For the **third research question**, we analyzed the rank reversals detected in the second research question regarding the utility differences of alternatives under linearity conditions. Therefore, we calculated the utility differences between the alternatives and checked whether or not this utility difference has led to a reversal in rank. In the analyses, by linear objectives, we mean objectives that have been measured using a linear utility function. Conversely, we mean by non-linear objectives those that were measured with non-linear utility functions. The same applies to the type of scale (numerical/verbal objectives).

7.2 Results

1) How often are (non-)linear utility functions used in practice?

In total, the participants evaluated 76.4 % of the objectives with a non-linear utility function. The relative use of non-linearity for numerical and verbal objectives was almost the same. However, the use of numerical objectives (57.7 %) was overall higher than verbal objectives (42.3 %).

Table 2 summarizes the analysis of the decision situations.

Decisions			**Objectives**			**Σ**
			only linear	**only non-linear**	**linear and non-linear**	
Objectives	**only numerical**	absolute	35	258	237	530
		in %	1.4	10.2	9.3	20.9
	only verbal	absolute	10	107	57	174
		in %	0.4	4.2	2.2	6.9
	numerical and verbal	absolute	58	819	955	1,832
		in %	2.3	32.3	37.7	72.2
Σ		absolute	103	1,184	1,249	2,536
		in %	4.1	46.7	49.3	100.0

Table 2. *Use of linear and non-linear objectives in the decisions.*

In the decisions, most participants opted for either exclusively non-linear utility functions (46.7 %) or a combination of non-linear and linear functions (49.3 %). Only 4.1 % of participants relied solely on linear objectives. Thus, 95.9 % of participants incorporated at least one non-linear utility function in their decisions.

Analyzing the risk aversion parameter indicates the following results. Overall, more objectives were evaluated as risk-averse ($c > 0$: 46.7 %) than risk-prone ($c < 0$: 30.7 %). For objectives rated as risk-averse, the average risk aversion parameter was $c = 2.2$, while for those rated as risk-prone, the average risk aversion parameter was $c = -2.5$. On average, 52.8 % of the participants exhibited risk-averse behavior ($c > 0$), while 36.6 % displayed risk-prone behavior ($c < 0$).

2) *How does the use of (non-)linear utility functions impact the final ranking of alternatives in practice?*

Table 3 shows the impact of (non-)linear utility functions on the final ranking of alternatives.

Linearization of non-linear utility functions		**total**	**proportion of non-linear objectives** 67-100 %	34-66 %	1-33 %
N		**2,433**	1,907	359	167
rank reversal (in %)	**1st rank**	**15.5**	16.9	13.4	3.6
	top three ranking[4]	**29.8**	31.3	29.0	13.8
	top three set[5]	**14.0**	15.6	8.7	7.3
	whole ranking	**55.0**	58.6	47.1	31.7

Table 3. *Impact of (non-)linear utility functions on the final ranking.*

In 15.5 % of all decisions, a different optimal alternative would have been chosen if non-linear utility functions had not been used, indicating that non-linearity did not influence the best alternative in 84.5 % of cases. Additionally, in 43.8 % of decisions, one of the top three alternatives changed rank. A detailed analysis revealed that most changes involved only rank exchanges among the top three ranking (29.8 %); the top three set of alternatives is relatively

[4] A rank reversal in the top three ranking means that either the first, second or third rank has changed.
[5] A rank reversal in the top three set means that the group of the first three alternatives has changed, i.e., a lower-ranked alternative has entered this group.

stable (rank reversal in 14 %). Furthermore, in 55.5 % of decisions, at least two alternatives changed rank throughout the whole ranking.

A detailed analysis showed that linearizing discrete utility functions led to rank reversals more often than linearizing exponential utility functions. However, the difference between them was minimal.

3) *To what extent do linear utility functions ensure stability in selecting the best alternatives based on utility differences?*

Figure 11 depicts the frequency of rank reversals for the best alternative as well as for the top three alternatives, based on the absolute utility differences between the top two and the third to fourth-ranked alternatives in a decision.

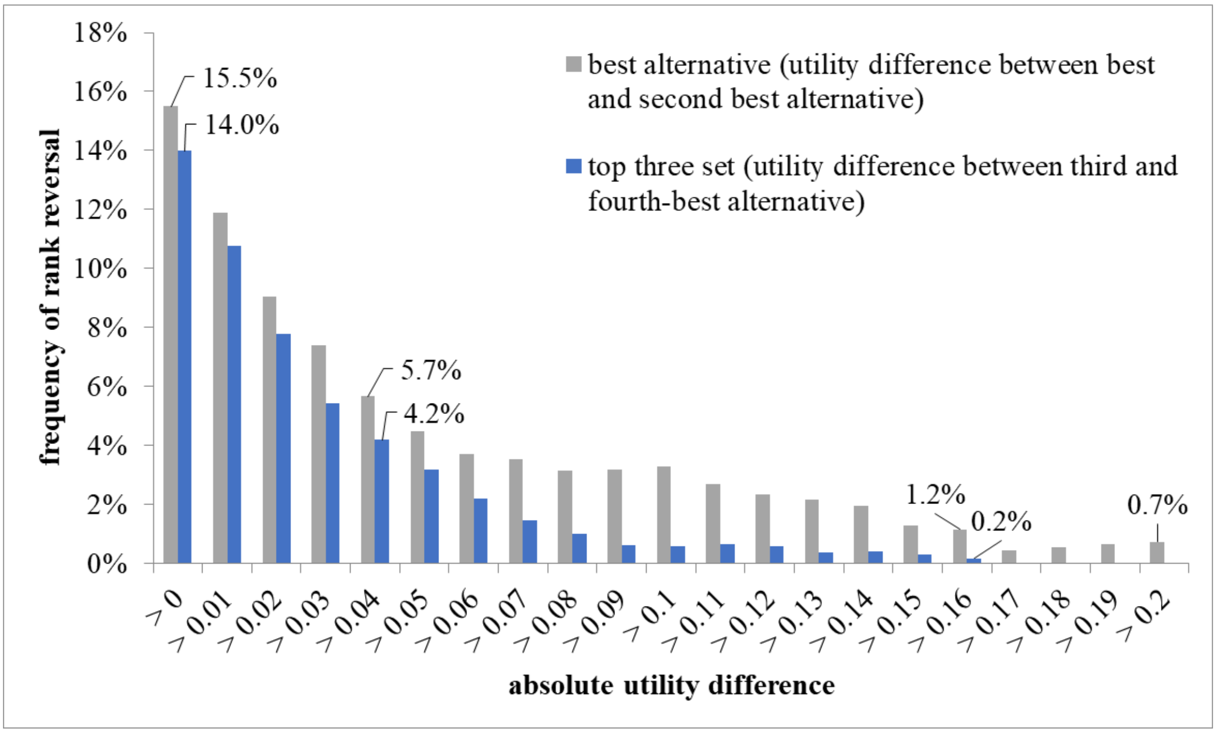

Figure 11. *Frequency of rank reversal depending on the absolute utility difference under linearity.*

As the utility difference between the top two alternatives increased, the occurrence of rank reversals decreased. Initially, there was a rapid decline in the frequency of rank reversals for the best alternative as the absolute utility differences grew. When the utility difference exceeded 0.04, the frequency dropped to just 5.7 %. Beyond this threshold, the decrease in frequency became more gradual. Once the absolute utility difference surpassed 0.27, the best alternative remained consistent, indicating that the type of modeled utility function (linear or non-linear) no longer influenced which alternative was considered best at this level. Similar results could also be shown for relative utility differences.

7.3 Recommendations

When using MAUT for decision-making, we recommend determining one-dimensional utility functions that accurately reflect preferences. This recommendation is especially advisable for important and complex decisions where DMs need a precise ranking of all alternatives. Linearization may distort results and hinder the identification of the best alternative and an accurate ranking. However, if DMs want to save time and effort and are willing to accept the possibility of rank reversals, utilizing simplifications through linearity can be a sensible option.

In this case, DMs can calculate utilities with linear utility functions to create a preliminary ranking. If there is a significant utility gap between the top-ranked alternative and the second-ranked one, the analysis might conclude early. The study's results help estimate the risk of not choosing the optimal alternative. DMs should assess their willingness to accept this risk to save effort but can reduce it by accurately determining the utility function for at least the highest weighted objective. Further efforts may be unnecessary if the utility difference remains large with these changes. If it decreases, all utility functions should be determined precisely.

There may be a significant gap among lower-ranked alternatives if there is no clear gap between the top two. In such cases, DMs can achieve better differentiation in evaluations by determining preference-accurate utility functions (again, starting with the highest-weighted objective). Practically, it also makes sense to use the similarly evaluated alternatives as a basis for further analysis that may not strictly adhere to MAUT modeling. DMs could discuss these alternatives and/or rely on their intuition to choose the best alternative under the similarly performed alternatives.

8 Limitations

The implications of this dissertation should be considered in terms of the overall limitations and those of the individual studies. Each study has limitations that may reduce the validity of the studies or mean that they only apply to specific groups of people or decision-making situations. Overall, our studies and results rely on the MCDM-support system ENTSCHEIDUNGSNAVI, which includes the concepts of VFT and MAUT. The tool concentrates on a specific imprecise information approach and exponential utility functions. In DA, there are numerous other methods and tools available that could lead to different results. Moreover, the data sets of our studies only include students, making the sample of participants very specific. The students possess considerable knowledge about the reflective decision-making process, which may limit their representativeness. In addition, the decision situations are private decisions young students face and, therefore, are not representative of all decisions.

In the first study, we utilized different years and students for the groups. The COVID-19 pandemic and other factors have altered external influences, which may have contributed to the variations in how values were weighted across the groups. Moreover, to maximize the practical benefits for our students, they were allowed to select the decision-making situation they wished to analyze. As a result, this led to varying numbers of participants across different topics. Some topics were only dealt with by a small number of participants. On top of that, we focus on the five most significant values. However, less important values may also influence the formulation of fundamental objectives. Future research is necessary to deepen the robustness of our findings.

In the second and third studies, offering a reward for participating may have employed more imprecision or more non-linear utility functions than they typically would to demonstrate greater effort. However, we instructed them to state their preferences clearly and correctly. While we can assume that participants expressed their preferences accordingly, we cannot confirm this.

Concerning the third study, the context always influences the determination of utility functions (Stewart et al. 2015). Moreover, the dataset may have included dominating alternatives or robust rankings, where non-linear utility functions would logically have no effect. We intentionally chose not to exclude these decisions, as they can occur in real-world scenarios.

These limitations show that our results can and should not be generalized. However, they have helped to improve the ENTSCHEIDUNGSNAVI for future users.

9 Outlook

Further research could confirm the robustness of our results and optimize MDCM in areas other than those we have considered, further reducing the effort for DMs in a DSS. Moreover, future studies should expand beyond student populations to include a more diverse range of participants from various demographics, professions, and age groups, leading to a more diversified data set and different decision-making situations. These studies could provide insights into how different backgrounds influence decision-making processes. Furthermore, this would ensure that all participants are intrinsically motivated to analyze the decision-making situation and that all parameters are given to the best of their knowledge, reflecting their real preferences. In addition, the studies could be carried out with further, similar DSS and compared with the results based on the ENTSCHEIDUNGSNAVI. This comparison would help assess the generalizability of the findings in this dissertation.

Our experience with the ENTSCHEIDUNGSNAVI has shown that DMs have major problems defining objective weights and, thus, with the trade-off method. The tool currently uses indifference curves to help determine trade-offs. However, this method seems counterintuitive for the user, leading to a lack of understanding and unnecessary effort. Further research could evaluate different methods for collecting trade-off data to reduce effort and improve understanding.

Furthermore, using artificial intelligence (AI) could be of central importance in the future in this context. For example, researchers could examine to what extent AI could help to formulate objectives and alternatives in the structuring phase, how consequences in the consequences table can be specified faster and more efficiently with the help of AI, or how AI can help to reflect a final ranking of alternatives.

In any case, the decision-making process should be made as easy as possible for the DM. However, this should not be at the expense of the quality of the decision since the DM should still be able to find and choose the best alternative, even with simplifications in a DA.

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Part B. Studies

Paper 1: Decision skill training with the ENTSCHEIDUNGSNAVI

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Contents

Decision Skill Training with the ENTSCHEIDUNGSNAVI

Decisions with multiple objectives are challenging for many individuals. The decision problem has to be structured appropriately (decision frontend), and the decision makers‘ preferences have to be elicited and aggregated (decision backend). There are dozens of decision support systems helping decision makers deal with their decision problems and thereby promote the quality of one concrete decision. However, most of them require expertise in decision-making. Furthermore, they neglect the improvement of decision-making skills, which leads to better and higher quality decisions in general for DMs with little expertise and experience. In this paper, we introduce the ENTSCHEIDUNGSNAVI, a freely available decision support system for multi-criteria decision-making, which combines the basic functionalities of a decision support system with training to improve the user‘s decision-making skills. Based on the concepts of value-focused thinking, multi-attribute utility theory, and various debiasing techniques, the decision maker can practice his proactive decision-making skills by going through three main phases: structuring the decision situation, developing the consequences table, and evaluating the alternatives. Moreover, we report on the experience gained so far from using the ENTSCHEIDUNGSNAVI and what conclusions can be drawn from it.

Keywords: decision support system; multi-criteria decision making; value-focused-thinking; decision skill training

1 Decision skill training and decision support tools

Individuals often struggle with decisions with multiple objectives. Researchers found out that they are, for example, unable to define all relevant objectives (Bond et al. 2008) or alternatives (Siebert and Keeney 2015, Siebert 2016) and, thus, to structure their decision problem. This leads to a low quality of the decision. Decision support systems (DSS) are developed to support the decision-maker (DM)[1] in solving unstructured problems (Sprague 1980) and promoting the quality of one concrete decision. However, improving decision-making skills, especially for DMs with little expertise and experience, may lead to better and higher quality decisions in the future, which are often neglected. Researcher Ralph Keeney makes clear in his approach of value-focused thinking (VFT) (1992) as well as in his latest book "Give yourself a nudge" (2020) that decision-making is a skill that must be learned. According to our research, the contribution of DSS to this skill improvement is rarely addressed in the literature. Therefore, we combine the basic functionalities of a DSS with training to improve decision-making skills in the design of our new multi-criteria DSS called ENTSCHEIDUNGSNAVI (www.entscheidungsnavi.com). To the best of our knowledge, the tool is the only one that follows this approach explicitly. With the help of this DSS, the DM can run through a self-chosen decision problem based on a detailed step-by-step guide to improve his skills with the concept of 'learning by doing'. To make a high-quality decision, skills must be learned throughout the decision-making process. This process can be roughly divided into three phases: (1) Structuring of the decision situation, (2) Development of the consequences table, and (3) Evaluation of the alternatives and the decision (Raiffa 1970, Keeney 1982). In the conception of the ENTSCHEIDUNGSNAVI, we have formulated our aims for each phase as follows:

[1] The male form chosen in this paper always refers to all genders.

1. Concerning the structuring of a decision-making situation, studies show that there are major deficits in this area. The decision statement is often formulated too narrowly (Maule and Villejoubert 2007), the objectives are incompletely (Bond et al. 2008) or not formulated fundamentally, and likewise, good options are often not identified when the alternatives are specified (Siebert and Keeney 2015, Siebert 2016). Therefore, the user of the tool should learn which methods can be used to specifically address these deficits. The concept of VFT provides a framework for structuring the decision (decision frontend) that has been tested in many case studies (Parnell et al. 2013). Therefore, methods suggested by Keeney‘s VFT should be implemented consistently in a DSS. In his new book, Ralph Keeney has already cited the ENTSCHEIDUNGSNAVI as the only adequate DSS to support this structuring phase with the help of his VFT approach.

2. There are various decision-making biases that can occur when assessing consequences or articulating preferences. Montibeller and von Winterfeldt provide a comprehensive overview of motivational and cognitive biases in decision and risk analysis (Kahneman 2011, Montibeller and von Winterfeldt 2015). Therefore, the user of the tool should improve his decision-making skills by being informed about the essential biases. Additionally, appropriate debiasing methods should be given to him to avoid distortions. With these skills, he is then in a position to assess the consequences with far fewer biases than without this mediation.

3. In the evaluation and decision phase (decision backend), it is essential that the user of the tool should trust the result, which is naturally derived in a DSS in an analytical way, in order to implement the decision. Otherwise, the whole effort was in vain. Therefore, the DM has to feel comfortable with the result, i.e., his gut feeling should support the decision. Thus, the tool should be designed in such a way that the user learns to understand intuitive and analytical decision paths not as opponents but as synergetic paths. This should be made possible by explicitly addressing discrepancies between intuition and analytics and a very transparent

determination of the decision parameters (e.g., objectives, alternatives, consequences prognoses, preference statements), which at the same time support learning effects about the DM's own preferences.

In this article, we present the basic features of the DSS ENTSCHEIDUNGSNAVI. Furthermore, we report on the experience gained so far from using this tool and what conclusions can be drawn from it. Section 2 introduces the basic structuring procedure, Section 3 deals with the development of the consequences table, and Section 4 with the evaluation procedure. Section 5 then presents the experiences and conclusions.

2 Structuring the decision situation with the ENTSCHEIDUNGSNAVI

To improve decision-making skills for structuring decision situations, it is very useful when the DM completely runs through a self-chosen decision problem based on a detailed step-by-step guide. Regardless of whether the decision problem in question is worth the effort or not, all relevant methodologies should be run through once to acquire the skills. Accordingly, the ENTSCHEIDUNGSNAVI contains a guideline in the first phase (structuring the decision situation), which guides the user with a total of 16 sub-steps through the three relevant steps: formulating the decision statement, identifying the objectives, and determining the alternatives. These steps are roughly sketched out below.

2.1 Formulating the decision statement

It is important to formulate the decision statement to make it clear to which context the following decision steps refer. Especially when several people are involved in a group decision, this definition and coordination is a fundamental component of a high-quality decision (Baer et al. 2013). Therefore, the DM must learn to consider all relevant decision aspects, which include determining who the DM is, who can choose between alternatives, and what is to be achieved with the decision. At the same time, the assumptions made must be explicitly formulated, and what is not decided now or what is to be decided at a later date must be stated (Keeney 2020). An at least equally important point in the formulation of the decision statement is the correction of a typical narrow thinking phenomenon, namely that people usually formulate the decision statement too narrowly (Maule and Villejoubert 2007). As a consequence, relevant alternatives or objectives are overlooked and not included in the decision. Therefore, the DM has to learn that the decision statement is formulated for the right context. Good decision quality requires that this bias is broken down and that the decision statement is approached more broadly. This has the effect that even better and more attractive alternatives for action can be found that have not been thought of before. Thus, a reactive approach to a decision statement should be turned into a proactive decision opportunity.

In the ENTSCHEIDUNGSNAVI, the DM can make a first attempt to formulate the decision statement. Our experience has shown that this statement is not to be chosen very broadly at this stage. Therefore, the DM is then consciously led into a very broad thinking process. It is his task to reflect on his basic life goals and to identify the five most important aims. He will be assisted by a list of about 20 aims that are known from relevant research (Maslow and Kruntorad 1981, Reiss 2008) or have often been chosen by other DMs. He is then asked to think about and take notes on four impulse questions. These impulse questions concretely help him to question assumptions made and to think about the decision situation more fundamentally and broadly. Based on this preliminary work, the DM can then reformulate his decision statement. So that he chooses the right frame for his decision situation, the five most important life goals and all notes are presented to him again collectedly.

At the end of this step, the DM should have formulated a proactive, right-framed decision statement in order to ensure an appropriate basis for the following steps.

2.2 Identifying the objectives

In the concept of VFT, it is intended that once the decision statement has been formulated in the first step, the objectives are identified and used as a prompt in the second step to create alternatives (Keeney 1992, 2020, Siebert and Keeney 2020). This is to ensure that the DM does not limit himself hastily to a selection of seemingly obvious alternatives but that the formulated objectives enable him to proactively and creatively find new alternatives (Siebert and Keeney 2015). For this to be possible, the objectives must be well-reflected. In particular, VFT requires the DM to state his so-called fundamental objectives. These are exactly the aspects of core interest and are associated with an independent value for the DM. Means objectives, which only advance fundamental objectives, should be avoided. In this step, the DM learns to distinguish fundamental objectives from means objectives and how to identify his decision-relevant fundamental objectives.

The identification and formulation of fundamental objectives place high demands on a DM. This process is best accomplished with the support of an experienced decision analyst, who, together with the DM, first collects all decision-relevant aspects in a brainstorming session. These are then structured in an objective hierarchy. The decision analyst insists that the DM tells him why he considers each objective important. In this way, all relations between means and end become transparent, and it becomes clear what is important at the core. At the end of the process, the superior objective is at the top of the hierarchy, and the fundamental objectives are directly on the first following hierarchy level. In all other levels, mainly means-end relations are clarified, i.e., means objectives or, sometimes, fundamental aspects that are further differentiated in terms of content are located there.

In the ENTSCHEIDUNGSNAVI, this process is simulated as much as possible equivalently. The DM starts with a brainstorming session, in which he is only supported by creativity-promoting impulse questions, but otherwise can specify all decision-relevant aspects in an unstructured way. He is then presented with a master list of about 70 objectives from which he can add aspects that have been overlooked so far. The subsequent structuring of the objectives is technically supported in the tool by an easy-to-use graphical interface, but in terms of content, the DM is required to recognize the means-end relations himself and to classify them accordingly in the hierarchy (see Figure 1). When creating the hierarchy, possible redundancies between the objectives can be detected and avoided. Furthermore, the ENTSCHEIDUNGSNAVI supports the DM like a decision analyst by presenting more questions that should question the fundamentality of the objectives, e.g., 'What exactly do you understand by this aspect? Could you possibly specify this further?' or 'Why is this aspect important to you? Is there a fundamental objective behind it?'. Moreover, the ENTSCHEIDUNGSNAVI points out that the DM should check whether the relevance of a fundamental objective depends on how well another objective is fulfilled. If this is the case, there is a preference dependency, which reduces the validity of the recommendation. This should be avoided. On top of that, the ENTSCHEIDUNGSNAVI provides a lot of examples and finished decision situations that other DMs with the ENTSCHEIDUNGSNAVI have already analyzed.

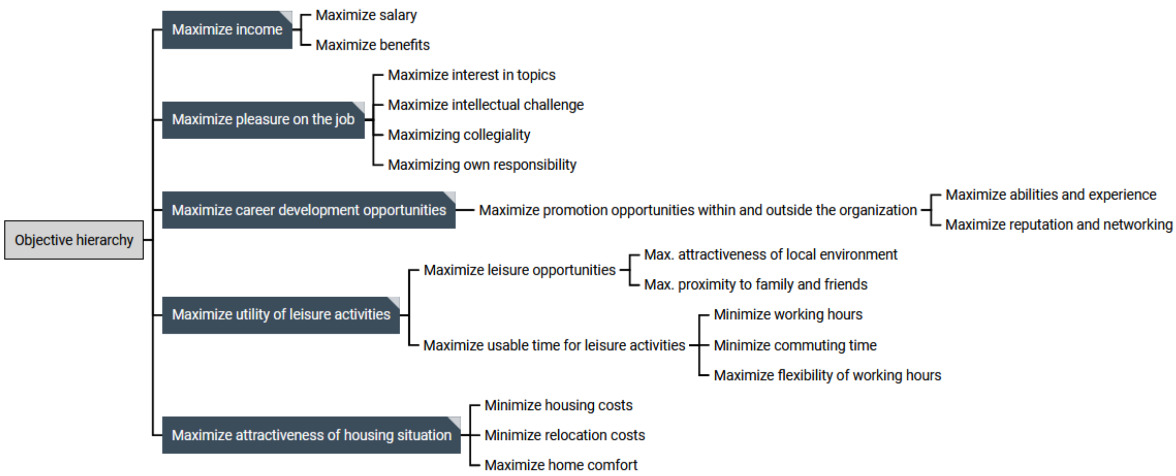

Figure 1. *Objective hierarchy in the ENTSCHEIDUNGSNAVI (The example is taken from the paper by Siebert and von Nitzsch (2020).)*

At the end of this step, the DM should have ideally identified four to six fundamental objectives for the previously defined decision statement.

2.3 Determining the alternatives

With the now-defined objective hierarchy, the foundation stone has been laid to support a creative identification of new action alternatives. The ENTSCHEIDUNGSNAVI focuses on the idea of VFT by using objectives to identify new alternatives. First, the DM can specify the alternatives known to him anyway in the ENTSCHEIDUNGSNAVI. Then, he goes through four sub-steps to find new alternatives. The aim of this step is for the DM to learn to be creative and open-minded to determine all possible alternatives for the decision situation.

In the first step, the DM should identify the greatest weaknesses of the alternatives already defined and try to find new, related alternatives with creative considerations that no longer have these weaknesses or hardly have any. Therefore, all previously defined objectives are presented to him, and the DM can think about which objective the respective alternative would have the greatest weakness in. The weaknesses should be obviously recognizable for the DM and should not require a more detailed analysis.

In the second step, the DM should forget the previous alternatives for a moment and consider how exactly he could fulfill this objective very well for each of the objectives in isolation. The ENTSCHEIDUNGSNAVI indicates that the DM should be very creative and think as openly as possible here. He should try to use these thoughts to identify completely new action alternatives, and it may also be possible to combine several thoughts, creating one new alternative.

In the third step, the DM should ask other people. For this, he should explain the decision situation to suitable people and ask them what additional alternatives they can imagine. These people should either know the DM well or be familiar with the decision situation. Moreover, the ENTSCHEIDUNGSNAVI recommends imagining people who have completely different views in order to find new alternatives. In the fourth step, the DM must try to identify the two or three key design parameters (KDP) in which all possible alternatives differ. On the one hand, he can use this KDP to find new, previously overlooked alternatives in a combinatorial approach. On the other hand, these KDPs also serve as valuable support for reducing the number of alternatives, which may have increased too much, to a manageable number by combining them accordingly.

Finally, the DM's task is to rank the formulated alternatives according to his gut feeling. This order serves as a basis for uncovering possible discrepancies between the head and gut in the later steps after the analytical evaluation.

At the end of this step, the DM should determine all relevant alternatives for the decision situation.

3 Developing the consequences table

After structuring the decision problem, the DM is confronted with a still empty consequences table. The task now is to fill the table with the performance evaluation of each alternative to each objective. The difficulty here is to create a consequences table that is as undistorted as possible.

Researchers point out that there are various biases (Gilovich et al. 2002, Kahneman 2011, Montibeller and von Winterfeldt 2015) that can occur when performance evaluations are given in the consequences table. DMs are often influenced by them, especially if they have little experience and expertise in decision-making. Objective consequences are not affected by this, but subjective estimates, e.g., probability estimates, can be biased, and this leads to a distorted consequences table. For this reason, it is important to inform the DM about the essential biases and teach him how to prevent them. The following biases and hints are explained in the ENTSCHEIDUNGSNAVI:

- *Only observing the future:* The DM is advised not to take into account past, unchangeable results in the consequences but only measure results that he can influence in the future by choosing an action alternative. Otherwise, the result can be falsified if these past results are only taken into account for some alternatives and not for others.
- *Not relying on intuition:* If the DM has a lot of relevant experience, intuition might be very helpful. However, he should be careful stating probabilities simply based on any 'gut feeling', particularly when there is a danger of falling back on pre-conceived thought patterns (see Linda example by Tversky and Kahneman (1983)). The DM is advised to think objectively and to consider the factual situation rather than hastily assuming something probable just because it currently seems feasible.
- *Avoiding overreaction:* Readily available information or events can cause the DM to overreact and misjudge the estimations (availability heuristic (Tversky and Kahneman 1973)). The DM should check whether he is unduly influenced by a certain event or by current media reports while stating his estimations.
- *Not drawing general conclusions from things only heard about:* The so-called 'narrative bias' occurs when the DM is influenced by selected narratives or individual life stories (Winterbottom et al. 2008). The more conclusive the story itself is - or the more vivid a specific individual life story is - the greater the risk that a person will draw general conclusions from it. To avoid this bias, the DM should rather use statistics or data sources as a basis for his probability estimations.
- *Not underestimating marginal events:* 'Narrow thinking' hinders people from being able to conceive large deviations from the norm. This is why DMs, in general, tend to rate the probabilities of the average state/extremely marginal events too high/low. Studies show that with the help of 'time unpacking' (Tversky and Koehler 1994), in which estimation

forecasts are made step by step and not in one go, the range of variation that results is wider and more realistic than without such steps.

- *Not let themselves be manipulated:* If the DM additionally incorporates information from outside parties, then he should bear in mind that in some publications, the data and the results might have been modified in such a way that they imply a stronger effect than is actually the case. For this reason, the DM should be consciously aware of what interested parties are involved and how reliable any source is.
- *Not being too hasty with the estimations:* The 'confirmation bias' (Gilovich et al. 2002) occurs when the DM prefers a specific alternative and states too positive values for this particular alternative. In this case, he often tends to limit his search for information that will support his preferences. To avoid this bias, the DM should make sure that this information does not favor his favorite alternative(s).
- *Watching out for a potential commitment:* The Sunk-Costs Effect (Arkes and Blumer 1985), as well as emotional involvement, can lead to the fact that the DM evaluates the results of a certain alternative too positively. The DM should not be influenced by the fact that efforts have already been wasted but consciously set his estimation of results for the alternatives with potential commitment at a slightly more conservative level.
- *Thinking not only in the success scenario:* If the DM develops an alternative himself, the so-called Inside View can occur. That means that the DM estimates this alternative too positively because he believes in the success of the alternative, and only little thought is given to failure. The prospective hindsight method can help the DM choose the estimations that are more realistic (Klein 2007).
- *Improving estimations through Dialectical Bootstrapping:* The 'Wisdom of the Crowd' method is one option to improve the values. The DM can ask many people and make the average of their opinions in order to get more realistic values. With the help of dialectical

bootstrapping (Herzog and Hertwig 2009), the DM is not dependent on other people and can statistically prove better values.

With these skills, the DM is in a position to make his consequences prognoses with far fewer biases than without this mediation. In order to measure the objectives, the DM has to define the attributes with which he can measure his objectives and, therefore, the scales of measurement. This step is also not entirely trivial for DMs without experience and expertise in decision-making. Therefore, the DM is provided various hints to facilitate the choice of attributes and scales for the individual objectives. The ENTSCHEIDUNGSNAVI follows the approach of VFT, which differentiates between natural, constructed, and proxy attributes (Keeney 1992). Objectives that can be measured unambiguously on a certain natural-numerical scale can usually be recognized by their formulation (e.g., purely monetary objectives should logically be measured in the respective currency, and objectives referring to a distance should be measured accordingly with a measure of length). If there is no natural-numerical scale for an objective, the DM can measure his objectives with the help of constructed scales. One option would be to use a numerical scale with an artificial unit like points or grades. This scale is relatively general but can basically always be used. To make it less vulnerable, it is important that the DM thinks about the degree of fulfillment of every number given on the scale. Another option is to define a verbal scale (e.g., 'bad' to 'good' or with additions like 'very low' to 'very high') with a small amount of possible result states, like 3 to 7. Also, with this scale, it makes sense to describe the verbal statements to make them less vulnerable and general. As a third option, the DM can create an indicator scale based on proxy attributes. These proxy attributes can consist of fundamental partial aspects, means objectives, or correlated scales. The DM can choose how the attributes should be considered in the measurement of the objective: either he can determine weights for every proxy attribute, or he can define his own formula, which allows great design possibilities.

The ENTSCHEIDUNGSNAVI provides a template for the scales mentioned above, allowing the DM to select a suitable scale for each objective.

In addition to the information mentioned above and learning skills, the ENTSCHEIDUNGSNAVI offers further technical functionality to make it easier for the DM to fill out the table and get an overview of the results.

To consider uncertainties, the DM can define a specific, additional influence factor in each table field, which is defined with a discrete number of states. Probabilities for each state are then to be indicated. Moreover, the results are to be specified state dependent. To facilitate the specification of the probabilities, the ENTSCHEIDUNGSNAVI uses the concept of imprecise information. Therefore, the DM can link his specification with a certain degree of precision.

The moment the consequences table is completely filled in for the first time, all fields of the table are automatically colored. The worst (best) possible values, according to the defined bandwidths, get a red (green) background; intermediate values are adapted in color accordingly. This visualization serves as a first, at this point, still rough view of the advantages and disadvantages of the individual alternatives. Dominated alternatives are highlighted and can be excluded by the DM from further analysis.

4 Evaluation

To implement the decision, a fundamental point in this evaluation and decision phase is that the DM trusts the result, i.e., the analytical result and gut feeling should match. Otherwise, the whole effort was in vain. The merging of the head decision and gut feeling is also a good validation for the result of the analytical DSS calculation. The DM has to understand intuitive and analytical decision paths not as competition but as synergetic paths. In case of discrepancies between intuition and analytics, the gut feeling can contain some more information that may have been left out in the analytical model. For example, objectives may have been completely neglected or may not have been formulated fundamentally enough. For this reason, it is important that the DM takes his gut feeling seriously.

The ENTSCHEIDUNGSNAVI already asks the DM for his gut feeling after the structuring phase. There, the DM must rank his defined alternatives according to his gut feeling in a general way and without any analytical calculations. The aim of this ranking is for the DM to identify and investigate any discrepancies between intuition and analytics. These discrepancies could either indicate that the intuition is already influenced by a bias or that important decision parameters have been forgotten in the analytical model. This is discovered in the evaluation phase.

The calculation basis of the ENTSCHEIDUNGSNAVI is the multi-attribute utility theory (MAUT) (Keeney and Raiffa 1976), which enables a DM to map several objectives within a decision situation in a preference model. Furthermore, this preference model offers the possibility of a transparent determination of the decision parameters, which makes the decision-making process and the calculation of the best alternative more comprehensible for the DM. However, the MAUT is not very simple, especially since attribute weighting[2] causes problems for many DMs, but it offers the possibility to understand and analyze the preference statements given by the DM. In the ENTSCHEIDUNGSNAVI, the DM is supported in the development of his preferences by providing explanations and several preference statements for the selected parameters so that the DM can always understand what exactly he is doing. Furthermore, he has the possibility at any point to return to a previous step and adjust or add parameters to the decision situation since preferences often develop after a closer examination of the decision problem.

In the evaluation and decision phase, the ENTSCHEIDUNGSNAVI offers different evaluation variants to provide a detailed and transparent view of the calculated results. Additionally, the DM can take a critical look at the result and identify possible reasons for discrepancies between intuition and analytics.

[2] The term ‘attribute weight’ is used in this paper as a synonym for ‘objective weight’.

4.1 Utility functions

The MAUT requires that the DM indicates utilities. Therefore, he has to define his preferences and determine utility functions for each objective. DMs who are not familiar with this or who find it difficult to formulate their preferences will also receive various hints from the ENTSCHEIDUNGSNAVI in this step. Firstly, the basic understanding of a utility scale is explained, followed by an explanation of linear and non-linear utility functions. If the DM is risk-neutral, he can choose linear utility functions and does not need to concern himself with this step any further. If there is a decreasing or increasing marginal value, he should choose a non-linear utility function.

Figure 2 shows an example of how the utility function of the objective 'Income' can be analyzed in the ENTSCHEIDUNGSNAVI.

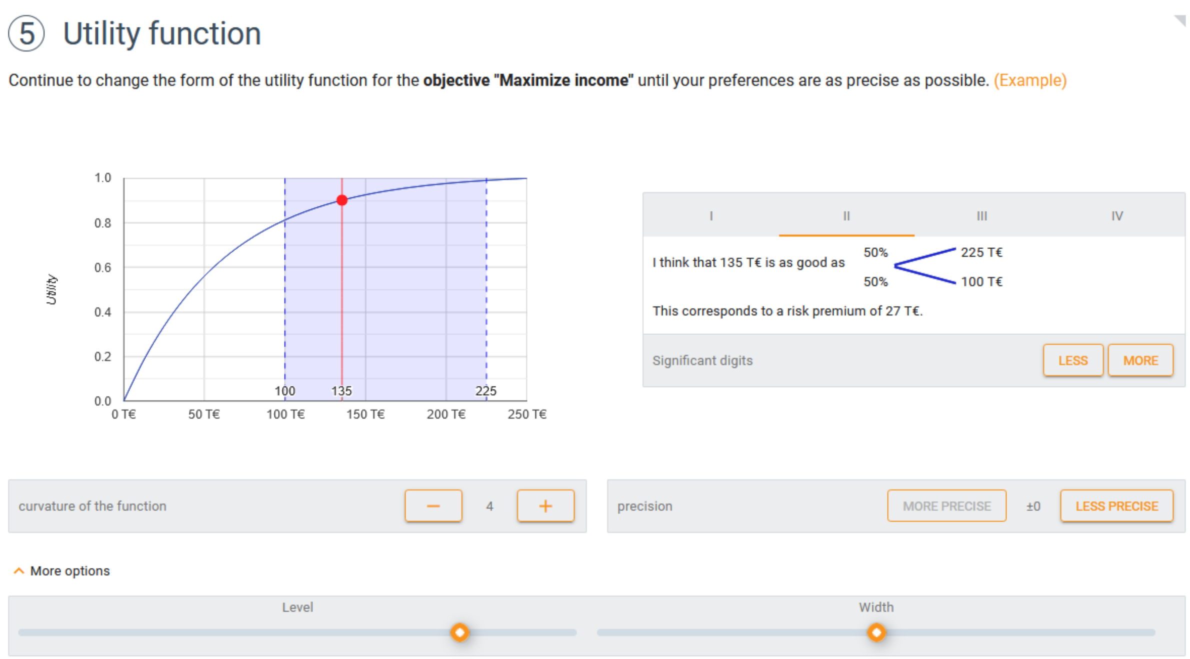

Figure 2. *Determine utility functions in the ENTSCHEIDUNGSNAVI*

The ENTSCHEIDUNGSNAVI supports the DM with graphical representations (see Figure 2 left) and different interpretations (see Figure 2 right) to analyze the non-linear utility functions more closely. Thus, the DM can check his statements and adjust them if necessary. The curvature of the utility function should be chosen so that the DM can identify with the preference statements presented in all explanation variants.

In the first variant, which is suitable for objectives under certainty, risk preferences are not taken into account so that the utility function can be interpreted as a measurable value function. The second variant, as shown in Figure 2, compares a sure option (135 T€) with a lottery with fixed probabilities (50 % chance of 225 T€; 50 % chance of 100 T€) based on the bisection method. The third variant is structured like the second one, with the difference being that the probabilities of the lottery are variable. The fourth variant presents the parameter responsible for the curvature of the utility function.

If the DM is not sure about his preferences and the shown preference statements are too detailed, he can make use of the option 'precision' which offers the DM to use an interval for potential utility functions. The DM should select the interval only wide enough to reflect his uncertain preferences as well as possible.

4.2 Attribute weights

After determining the utility functions, the DM needs to indicate the attribute weights that represent the scale constants of the multi-attribute utility theory. Determining the attribute weights, the ENTSCHEIDUNGSNAVI basically follows the trade-off procedure by Keeney and Raiffa (1976). Figure 3 shows an example of a trade-off in the ENTSCHEIDUNGSNAVI.

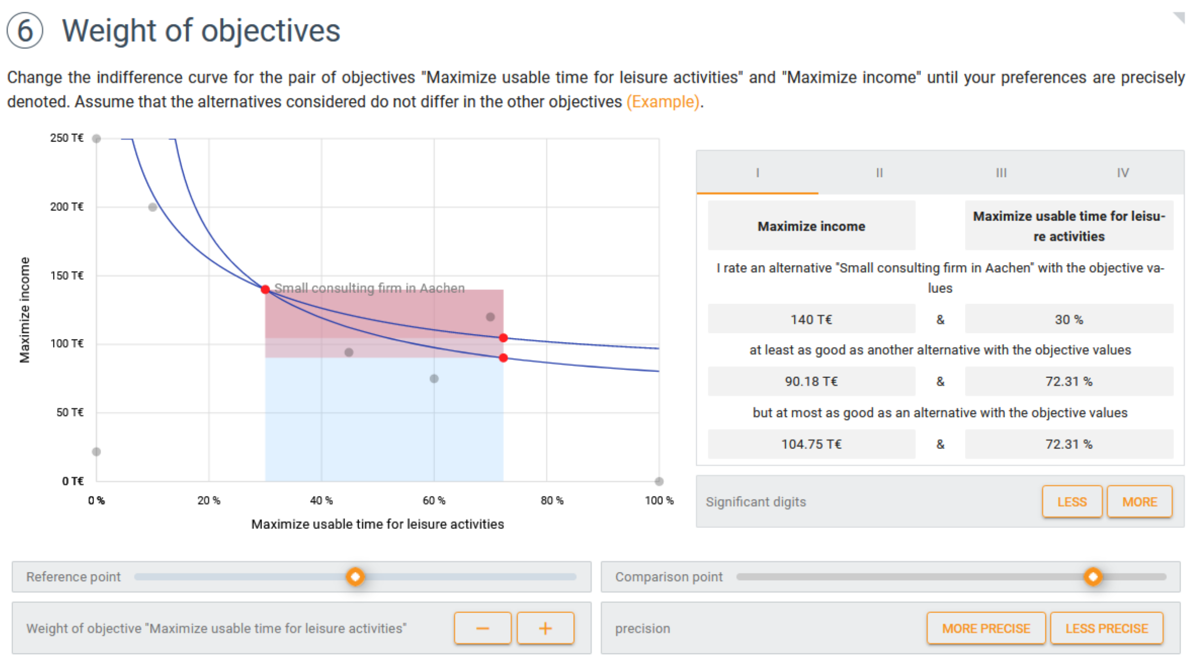

Figure 3. *Trade-off procedure in the ENTSCHEIDUNGSNAVI*

To determine the attribute weights of n objectives, the DM has to make n-1 trade-off statements. Therefore, he chooses a reference objective compared with all other defined objectives in a trade-off. In the ENTSCHEIDUNGSNAVI, one real alternative is compared to a fictitious alternative. These only differ in the results of the two objectives. The DM must specify how deteriorations in one objective can be compensated for by improvements in the other objective. So that the appropriate trade-off statements can be found faster, it is possible to give flat-rate estimations of the attribute weights. The ENTSCHEIDUNGSNAVI then displays the trade-offs as they result from the blanket weights of objectives the DM has specified. With the help of explanation variants I (in table form) and II (in verbal form) on the right-hand side, the DM can check the preference statements and modify the trade-offs by changing the relative weight of the comparison objective. He should do this until he can identify with the statements. Furthermore, the DM also has the possibility in explanation variant III to formulate and enter a new trade-off without regard to already entered weights. In this case, the ratio of the weights is automatically adjusted accordingly. Explanation variant IV presents the calculated weights for both objectives.

The ENTSCHEIDUNGSNAVI recommends checking each trade-off, even with different reference and comparison points, to see whether the resulting statements are suitable for the DM or not. If he has difficulties in determining the exact trade-off, he can set the degree of precision to an interval that allows him to identify with each of the resulting preference statements even with different parameter constellations.

4.3 Evaluation of alternatives

As soon as all relevant parameters (decision statement, objectives, alternatives, consequences table, utility function, attribute weights) have been entered into the tool by the DM, the ENTSCHEIDUNGSNAVI calculates the expected utility for every defined alternative and presents the results in a ranking.

The ENTSCHEIDUNGSNAVI offers different evaluation methods to take a critical look at the result and identify possible reasons for discrepancies between intuition and analytics. The Pros and Cons overview (see Figure 4) points out the advantages and disadvantages of every alternative.

Figure 4. *Pros and cons overview in the ENTSCHEIDUNGSNAVI*

Furthermore, the DM can display a detailed breakdown of the calculation in addition to the overall overview. Moreover, he can analyze the effects of changing parameters on the ranking in a sensitivity analysis and have a robustness test performed if imprecise parameters are used (see Figure 5).

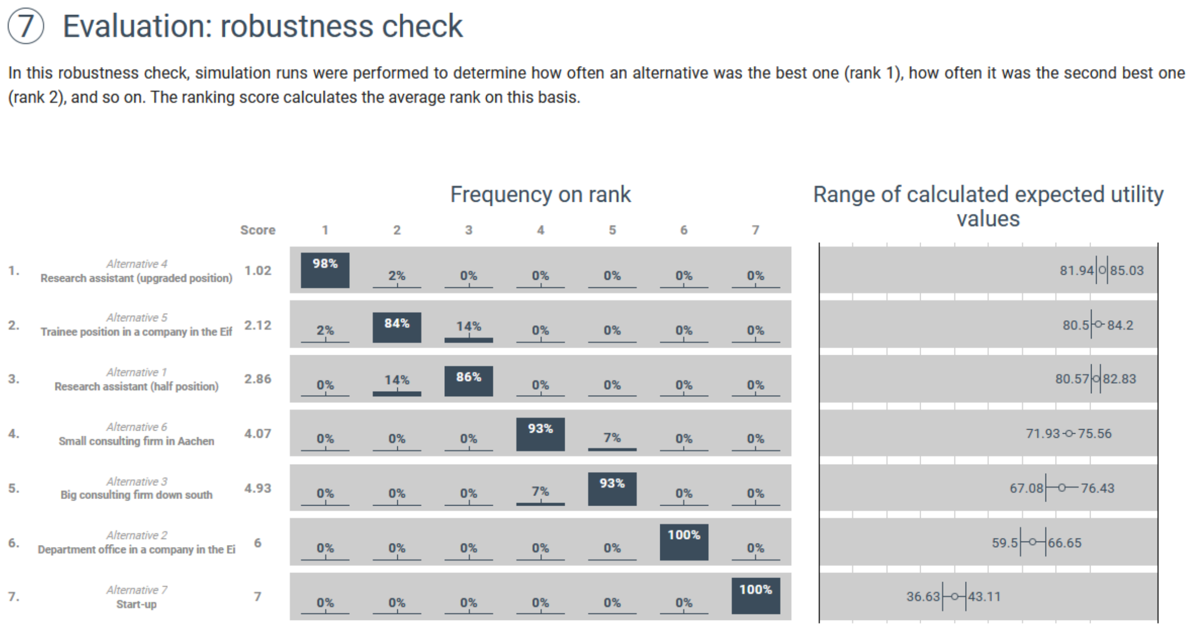

Figure 5. *Robustness check in the ENTSCHEIDUNGSNAVI.*

The Pros and Cons overview offers the DM a clear overview of the calculated results in an easy and understandable way. The DM can decide which alternatives he wants to compare with, and then, the ENTSCHEIDUNGSNAVI displays the utility differences in relation to the average of all selected alternatives for each objective. The bars pointing upwards (down) show the relative advantages (disadvantages) over the other alternatives in the respective objective, considering the attribute weight in the height of the bar. The DM can see directly which weaknesses and strengths the individual alternatives have.

A detailed calculation breakdown shows how the expected utility for every alternative is composed. With the help of this evaluation variant, the DM can see how large the utility is in each objective category for the respective alternatives. In the sensitivity analysis, the DM can change previously set parameters. The ENTSCHEIDUNGSNAVI calculates the new utility of the alternatives and presents the possible new ranking to the decision maker. This sort of playful approach enables the DM to identify sensitive parameters and examine his results even more closely.

If the DM has allowed imprecision for any of his parameters, he has the opportunity to perform an additional robustness test (see Figure 5). This test checks how robust the ranking is with regard to changes in the parameters (utility functions, objective weights, and probabilities) that he has not defined precisely with the help of a Monte Carlo Simulation.

The results of the robustness check visualize the number of ranked positions (frequency on rank) and the ranges of possible expected utilities of all alternatives (range of calculated expected utility values). Furthermore, an average score for every alternative is given, which is calculated as the expected rank.

5 Experiences and conclusions

The development of the tool was started in 2016. Since the first launch, the tool has been in a continuous development process, in which the functionalities are constantly being expanded, and extensive feedback from users is incorporated. At the time of printing this article, version 5.3 is available. It is a freely accessible software that is available in both German and English. The decision problems analyzed by the user can be stored locally on the user's own computer if desired. There is also the possibility of registration and storage on the ENTSCHEIDUNGSNAVI-server. The decision problems there are then used in anonymized form for research purposes.

There are now over 1,300 students at RWTH Aachen University who have analyzed their own real decision problem as part of a voluntary project within the course "Decision Theory". If the students successfully complete the project, they are rewarded with a grade bonus for their careful work. Furthermore, more than 300 students at the Management Center Innsbruck used the

ENTSCHEIDUNGSNAVI to structure an important professional or personal decision situation in groups of 4–5 students. Their grade was based on their detailed report and the ENTSCHEIDUNGSNAVI file.

We have also requested and received detailed feedback from the students. The gut feeling of many students coincided with the analytically calculated decision, which increases the chance that the decision will actually be implemented. Determining the utility functions and attributed weights are the most difficult steps, which shows that students find it difficult to indicate their preferences, and support is necessary, especially in these steps. Overall, the user-friendliness is rated very positively. Both the structure of the tool and the comprehensibility of the instructions were rated by most students with the best school grade 1.

In the meantime, the ENTSCHEIDUNGSNAVI is already being used at several universities in appropriate courses, including in Austria and Switzerland. Examples of use are decision analyses that students carry out in project modules to develop creative solutions for federal or regional political issues or use in courses for personal career development.

In addition, the ENTSCHEIDUNGSNAVI was part of an intervention in a study on the trainability of proactive cognitive skills in decision-making (Siebert et al. 2021). The proactive decision-making scale measures proactive cognitive skills and proactive traits in decision situations (Siebert and Kunz 2016). The more proactive DMs are, the more satisfied they are with their decisions and, ultimately, with their lives (Siebert et al. 2020). In a quasi-experimental field study based on a repeated measures design, Siebert et al. found out that decision training helps to promote individuals' effective decision-making. In two of three decision trainings, the participants also dealt intensely with a decision situation using the ENTSCHEIDUNGSNAVI. For these two groups, the training effect was higher than without using the ENTSCHEIDUNGSNAVI. Therefore, we assume that using the ENTSCHEIDUNGSNAVI intensely positively influences proactive cognitive skills in decision-making.

The ENTSCHEIDUNGSNAVI is also used as a supporting tool in professional consulting projects. One example is the Kopernikus-project ENSURE, which pursues the objective of researching new energy network structures for energy system transformation and comprehensively evaluating them with regard to their social acceptability with the participation of several stakeholder groups (Höfer et al. 2020). Moreover, the tool was also used in a consulting project carried out by the Strategic Decision Group (SDG) for the pharmaceutical division of a global company to evaluate early-stage development projects (Methling et al. 2022).

The experience gained so far can be summarized as follows: Basically, there are no restrictions in the field of application for the ENTSCHEIDUNGSNAVI, and the methodology is independent of context. Only the basic motives used in step 1 (formulating the decision statement) and the master list of fundamental objectives used in step 2 (identifying objectives) make sense in a suitable context. Therefore, we have adapted the two sub-steps for three fields of application, i.e., the user must now choose one of the three categories: (1) Private Life and Career, (2) Politics and Society, and (3) Organizations.

Regardless of the field of application, it is seen as positive that just by dealing with the objectives, one can gain knowledge in the analysis of the decision problem. Especially in group decisions, a coordinated objective system provides high transparency and improves coordination. However, it is also difficult. A typical comment from a user: 'I would not have thought that it is so difficult to formulate objectives.'

As a great challenge, especially for professional or complex applications, we have experienced the appropriate development of scales in addition to the formulation of objectives. This is often associated with a similar effort as the formulation of objectives. In professional applications in which several stakeholders are taken into account, the ENTSCHEIDUNGSNAVI serves only as a supporting tool. The evaluation part is particularly valuable here. The structuring part, as well as the setting up of scales, should better be carried out in workshops with a decision analysis expert.

According to the feedback from previous users, the tool has proven its worth as skill training. For the application area 'Private Life and Career', users need an average of about 8 hours for a complete analysis but consider the time well invested since 84 % would either probably or, in any case, recommend the tool to others. Furthermore, the functionality of steps 2) Developing the consequences table and 3) Evaluation of the alternatives and the decision have proven particularly useful for professional use in group decisions in companies. Regarding the first step, 'Structuring of the decision situation', we found that the methodologies used in the tool are exactly the right ones but that they can be carried out even better in moderated workshops. Therefore, we will continue to develop the ENTSCHEIDUNGSNAVI in the coming years to support the structuring part, especially for group decisions, even better.

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Paper 2: Integrating uncertainties in a multi-criteria decision analysis with the ENTSCHEIDUNGSNAVI

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Contents

Integrating uncertainties in a multi-criteria decision analysis

with the ENTSCHEIDUNGSNAVI

The ENTSCHEIDUNGSNAVI is an open-source decision support system based on multi-attribute utility theory that offers various methods for dealing with uncertainties. To model decisions with uncertainties, decision-makers can use two categories: Forecast and Parameter Uncertainties. Forecast Uncertainty is modeled with (combined) influence factors using discrete, user-defined probability distributions or predefined 'worst-median-best' distributions. Parameter Uncertainty allows imprecision for utilities, objective weights, and probability distributions. To analyze these uncertainties, the ENTSCHEIDUNGSNAVI offers several methods and tools, like a robustness check based on (Monte Carlo) simulations and a sensitivity analysis. The objective weight analysis provides insights into the effects of different objective weight combinations. Indicator impacts, tornado diagrams, and risk profiles visualize the impact of uncertainties in a decision under risk. Risk profiles also enable a check for stochastic and simulation dominance. This article presents the complete range of methods for dealing with uncertainties in the ENTSCHEIDUNGSNAVI using a hypothetical case study.

Keywords: multi-criteria decision making; uncertainty; decision support system; value-focused thinking; multi-attribute utility theory

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1 Introduction

Extensive research over the years has led to a plethora of different Multi-Criteria Decision-Making (MCDM) methods, whose strengths and limitations have been analyzed in numerous studies (e.g., Zavadskas and Turskis (2011), or Taherdoost and Madanchian (2023)) and meta-studies (Sahoo and Goswami 2023). In practice, software tools regularly accompany the application of these methods. The overview of the International Society on MCDM (https://www.mcdmsociety.org/content/software-related-mcdm-0, accessed on 30 March 2024) currently lists 28 tools, the majority of which are freely available. Most tools are based on a particular methodology, but a few offer a choice of methods (e.g., DEFINITE (https://spin-lab.vu.nl/support/tools/definite-bosda/, accessed on 30 March 2024)).

Only a small number of MCDM tools focus on multi-attributive value or utility theory (MAVT/MAUT). MACBETH (https://m-macbeth.com, accessed on 30 March 2024) (Bana e Costa and Vansnick 1999, e Costa and Chagas 2004, De Corte and Vansnick 2012), for example, is a commercial decision support system (DSS) that has already been used in many different application contexts (e.g., Bana e Costa et al. (2002), e Costa et al. (2001), Pereira et al. (2020)). FITradeoff (https://fitradeoff.org/, accessed on 30 March 2024) (de Almeida et al. 2016, de Almeida et al. 2021) and the ENTSCHEIDUNGSNAVI (https://enavi.app/, accessed on 30 March 2024) (von Nitzsch et al. 2020) are available at no cost.

Decision-makers (DMs) seeking a method and software for their MCDM problems face a dilemma between ease of use and the quality of the decision recommendation. In general, the MAUT methodology (Keeney and Raiffa 1976) is considered demanding and difficult due to the trade-offs between objectives that need to be assessed. To reduce the application hurdles and avoid overwhelming the DM, certain methodologies, such as AHP (Saaty 1980) and outranking methods (Belton et al. 2002), are based on clear and simple queries. This also includes fuzzy methods (Mardani et al. 2015, Pereira and Camanho 2024), which deliberately anticipate a certain degree of fuzziness in the model. DMs can use these methods without having knowledge of the underlying algorithms. Although these lower demands on the DM reduce application hurdles and are often methodologically elegant, this can be at the expense of quality. To achieve high decision quality, the DM needs to reflect on the decision problem. First of all, it is important to structure the decision situation, i.e., to clearly formulate the decision statement, analyze the fundamental objectives, and creatively develop alternatives (Keeney 1992, Spetzler et al. 2016). Back in the 1990s, Keeney introduced the concept of value-focused thinking (VFT), which has proven itself in many practical applications (e.g., Hannes et al. (2024), Höfer et al. (2020), Methling et al. (2022)). If DMs are not supported during this phase of structuring, there is a risk that the problem will not be properly defined (Hannes and Nitzsch 2024), the objectives will be incomplete or not fundamental (Bond et al. 2008), and potentially attractive alternatives will be overlooked (Siebert and Keeney 2015).

Secondly, DMs must research and use relevant and reliable information. Furthermore, an evaluation or decision should be based on comprehensible logic and the DM's identified preferences (Spetzler et al. 2016). A transparent and comprehensible process prevents a black-box character, enabling the reflection of the DM's preferences. MAUT has clear advantages over most other methods in this respect. The reason for this can be explained by the comparatively simple and strictly decomposition mathematics combined with a theoretically clean foundation (Keeney and Raiffa 1976). Although the trade-offs between objectives required by MAUT are—as mentioned above—difficult to determine, they can be clearly interpreted in terms of content without further fuzziness. In addition, possible effects on the result are generally easy to understand. In terms of clarity and comprehension, MAUT is an evaluation logic that is highly reflective and transparent.

MAUT enables the consideration of uncertainties in various ways. This concerns not only the integration of risk preferences in utility functions based on axioms (von Neumann and

Morgenstern 1961) but also the possibility of easily integrating various standard methods for dealing with uncertainties into the model. Which methods are suitable here and what an extension of these methods should look like should be based on how beneficial this integration is for the desired reflection of the DM. When faced with uncertainty, DMs can ask themselves, for example, the following questions to reflect on the decision situation: Are the parameters I have entered aggregated in a comprehensible and plausible way to produce a valid result? Are the uncertainties considered in the model exactly the uncertainties that may have a decisive influence on the result? To what extent can the respective uncertainties have a positive or negative impact on the result? Which uncertainties have the greatest influence on the result?

The ENTSCHEIDUNGSNAVI is, as far as we know, the only DSS that comprehensively supports the DM in structuring decision-making situations, applying a transparent mathematical method for calculating the best alternative, and addressing these questions with regard to uncertainties. In the decision front-end, VFT (Keeney 1992) is used to structure the decision situation and identify the first pieces of relevant information (objectives and alternatives). In his recent book, Keeney (2020) also refers to the ENTSCHEIDUNGSNAVI as the only tool comprehensively supporting VFT. In the decision back-end, the concept of MAUT (Keeney and Raiffa 1976) is used to find the best alternative under uncertainty. In addition, the ENTSCHEIDUNGSNAVI offers many different evaluation methods that allow the DM to reflect on and analyze their results.

In this paper, we present how to deal with uncertainties and how to analyze the questions mentioned above in the ENTSCHEIDUNGSNAVI using a simple hypothetical case study. Therefore, we briefly introduce the ENTSCHEIDUNGSNAVI (Section 2) and a case study (Section 3) that is constructed to show all available analysis tools and options for facing uncertainties in the following sections. In Section 4, we explain the implementation of MAUT in the ENTSCHEIDUNGSNAVI, and in Section 5, we show how uncertainties can be modeled in the DSS. In Sections 6–9, we present the following various analysis tools with which uncertainties can be examined more closely by the DM: methods for checking the robustness of the result (Section 6), objective weight analysis (Section 7), sensitivity analysis (Section 8), and indicator impacts, tornado diagrams, and risk profiles (Section 9). In Section 10, we conclude the paper and point out the limitations of dealing with uncertainties in the ENTSCHEIDUNGSNAVI.

2 The ENTSCHEIDUNGSNAVI – a decision support trainer

The ENTSCHEIDUNGSNAVI (von Nitzsch et al. 2020) is an open-source web tool that helps DMs make well-thought-out decisions and improve their decision-making competence. The first version of the tool was developed in 2017 for teaching purposes as part of the 'Decision Theory' course at RWTH Aachen University. In the following years, a group of computer science students worked continuously on implementing functionalities, improving usability, and incorporating extensive user feedback. The ENTSCHEIDUNGSNAVI is now used by other universities and institutions for teaching and advisory purposes due to its high level of professionalization. As a non-profit project, this DSS is available at no cost to all users. To make this possible, the ENTSCHEIDUNGSNAVI is supported by a non-profit organization (Reflektiert Entscheiden e.V. (https://reflektiert-entscheiden.de, accessed on 30 March 2024)) with users from various universities in Germany and Austria and a strategic management consultancy firm that uses the tool for consulting projects. Feedback from practical applications in the form of functionality requests flows directly into further development of the tool. This paper refers to the current version, 8.2.2; further versions with extended functions are already in development.

The ENTSCHEIDUNGSNAVI is based on the approach of reflective decision-making (von Nitzsch and Methling 2021) and consists of five successive steps: the formulation of the decision statement (1), the development of the fundamental objectives (2), the identification of alternatives (3), the establishment of a consequences table (4), and evaluation on the basis of preferences (5). The first three steps support the user in structuring the decision-making situation using the concept of VFT by Ralph Keeney (1992). VFT follows the idea that decisions should be proactively approached, i.e., decision situations should be seen as opportunities to shape something with foresight. Therefore, the first step promotes a broad and open formulation of the decision statement, which paves the way for a correspondingly large scope for action. The ENTSCHEIDUNGSNAVI supports DMs in formulating a proactive decision statement by helping them reflect on their own values and asking several probing questions. In the second step, DMs should develop their fundamental objectives. Therefore, the second step requires DMs to brainstorm important aspects and intensively scrutinize all collected aspects in order to precisely define them. The ENTSCHEIDUNGSNAVI supports DMs in developing the fundamental objectives by providing context-related master lists of objectives and an option to create an objective hierarchy. When identifying possible alternatives in the third step, VFT calls for a high degree of creativity in order to find or design new attractive alternatives. The fundamental objectives that have already been formulated should always be kept in mind so that a high level of attractiveness is achieved when designing new alternatives. The ENTSCHEIDUNGSNAVI supports DMs with the following creativity techniques: finding weaknesses of known alternatives, performing an objective-focused search, and developing a strategy table. In the fourth step, DMs should establish the consequences table, in which biases (Kahneman and Tversky 2013) should be avoided. The ENTSCHEIDUNGSNAVI informs DMs about these biases and thus sensitizes them when evaluating the consequences and possible uncertainties. In the final step, the DMs' preferences are included in the model based on MAUT. This is implemented using utility functions and trade-offs to determine the objective weights. The tool supports DMs with the help of graphic illustrations and explanations of the functions. Finally, the ENTSCHEIDUNGSNAVI offers a wide range of evaluation options to analyze and reflect on the final ranking of the alternatives in more detail. In this paper, we focus on the fourth and fifth steps, as the structuring phase of a decision situation does not affect uncertainties concerning MAUT.

The ENTSCHEIDUNGSNAVI places great emphasis on user-friendliness and an intuitive interface so that everyone can use the web tool. As not all DMs are familiar with the use of MCDM and MAUT, the ENTSCHEIDUNGSNAVI is available in three variants: Starter, Educational, and Professional. The Starter variant is deliberately kept simple and, therefore, has a greatly reduced range of methods and functionalities. It is ideal for DMs who have no prior experience in MCDM and want to quickly obtain a first impression of the process. The focus in this version is on structuring the decision situation. The Educational variant guides DMs through the reflective decision-making process in small steps, supported by extensive explanations and background information. These cover the basics of decision theory, operating instructions, and tips and tricks for the respective step. This version is particularly suitable for DMs who want to deal intensively with all aspects of a reflective decision and improve their decision-making skills. Previous knowledge about MCDM is not necessary in this version either. The Professional variant provides a feature set similar to the Educational variant but without guiding DMs and with fewer explanations. This variant is, therefore, suitable for those who are familiar with the implemented tools and methods and simply want to solve a decision problem efficiently. All variants are based on requests and suggestions from DMs in the field.

3 Case study

To subsequently explain the mathematical models and functionalities of the ENTSCHEIDUNGSNAVI regarding the handling of uncertainties, we would first like to briefly introduce a case study in this section. It is important to note that this case study is a highly simplified model of a logistics decision and was solely created to explain how DMs can deal with and analyze uncertainties using the ENTSCHEIDUNGSNAVI.

258 GmbH, a company that sells on-site-stored wood, would like to expand as part of its growth strategy. For this purpose, it is looking for a new location in North Rhine-Westphalia (NRW),

Germany. The company's decision statement is as follows: 'Which location best suits the new wood warehouse with adjoining wood sales for 258 GmbH?'

The company has already defined the objectives and alternatives of the decision statement and determined how well the alternatives fulfill the objectives in a consequences table (see Figure 1).

Excel Objectives Alternatives	Costs Indicator Scale from 1,100 to 600 T€/year	Ecological Sustainability Verbal Scale from "low" to "high"	Market Volume in the Region Numerical Scale from 30,000 to 140,000 m³	Infrastructure Indicator Scale from 0 to 10 Points
Cologne	1,037.34 T€/year – 1,082 T€/year *Worst-best distribution*	medium	127,600	8.5
Düsseldorf	1,004.39 T€/year – 1,047.66 T€/year *Worst-best distribution*	rather high	136,400	9
Aachen	879.38 T€/year – 926.59 T€/year *Worst-best distribution*	high	58,140 m³ – 71,820 m³ *Scenario: Market Volume in Aache...*	7.91
Oelde	735.93 T€/year – 763.31 T€/year *Worst-best distribution*	rather low	37,500 m³ – 44,000 m³ *Shortage of Skilled Workers in Oel...*	6.91

Figure 1. *Consequences table for the decision statement 'Which location best suits the new wood warehouse with adjoining wood sales for 258 GmbH?'. Good consequences are displayed in green, while average ones are displayed in white, and bad ones are displayed in red.*

258 GmbH defines four fundamental objectives: *Costs, Ecological Sustainability, Market Volume in the Region,* and *Infrastructure*.

The objective *Costs* deals with the expected annual costs for the operation of their wood sales business, including the wood warehouse, which, among other expenses, are costs for rent, the property lease, interest costs, and loan repayment costs for the property, personnel costs, taxes and duties, and maintenance and operating costs. The model does not consider transport costs and sales tax, as they will be passed on to the customer through an increased retail price based on the location of the warehouse. The objective is measured using an indicator scale from 1100 T€/year (worst value) to 600 T€/year (best value), which is made up of five indicators:

Personnel costs (€/month), *Rent/Loan payment* (€/month), *Property tax* (€/quarter), *Electricity costs* (€/year), and *Other operating & maintenance costs* (€/month).

The objective *Ecological Sustainability* evaluates properties at different locations based on their ecological sustainability. It is measured using a verbal scale from low (worst value) to high (best value), with five verbal levels.

The objective *Market Volume in the Region* is measured in terms of the expected volume of construction wood sold in m3. This takes two factors into account: geographical reach and competition. In large cities, we assume that residents travel a maximum distance of 50 km to buy wood. In more rural areas, we assume that residents travel up to 100 km to buy wood, as residents there are used to traveling longer distances to make purchases. To assess competition, we consider the extent of potential competition at the locations. We assume that there is already an abundance of wood merchants, particularly in urban centers. The objective is measured on a numerical scale from 30,000 m3 (worst value) to 140,000 m3 (best value).

The last objective, *Infrastructure*, is made up of two indicators: *Current state of infrastructure* and *Expected future state of infrastructure*. These indicators are used to assess the infrastructure at each location in terms of its quality and possible restrictions (e.g., due to extensive construction sites) now and in the future. Aspects such as highway connections, railways, shipping routes, and the general volume of traffic in the region are evaluated holistically. Airports are not considered in this model, as airplanes are not suitable for transporting the goods offered. The objective is measured using an indicator scale that ranges from 0 (worst value) to 10 points (best value).

Furthermore, 258 GmbH has already limited its choice of alternatives to four locations: *Cologne*, *Düsseldorf*, *Aachen*, and *Oelde*. Each alternative represents a specific property in the city. As the largest city in the Rhineland, *Cologne* offers excellent infrastructure with major highways, the Rhine port for transporting goods, and a lot of craftsmen and construction companies nearby. *Düsseldorf*, the state capital of NRW, has a strong economy, a well-developed road network, and a high density of construction sites. *Aachen* is located in the west of NRW, bordering Belgium and the Netherlands, which offers the strategic advantage of being able to sell to people from three countries. The region is also characterized by a decent amount of construction activity. *Oelde* is a small town in the east of NRW. The town is characterized by its excellent connection to the highway and offers the opportunity to set up a medium-sized wood warehouse.

The performance of the alternatives is assessed with regard to all objectives on the respective objective's scale in the consequences table (see Figure 1). As some consequences are uncertain and depend on external factors, several influence factors are used. Examples of influence factors are the worst–best distribution, which is used to model a kind of general uncertainty regarding consequences, or the 'Shortage of Skilled Workers' in *Oelde*. In the following sections, we will discuss these uncertainties and the different methods with which to analyze them in the ENTSCHEIDUNGSNAVI in more detail.

4 The implementation of MAUT in the ENTSCHEIDUNGSNAVI

The mathematical model of the ENTSCHEIDUNGSNAVI is based on the additive utility function of MAUT, which is used to determine the best alternative in multi-criteria decisions under uncertainty (Fishburn 1965, Keeney 1972, Keeney and Raiffa 1976). In this model, alternatives are compared and ranked according to their utility. The alternative with the highest utility is regarded as the best and should be chosen by the DM. To use the additive model, DMs must first define a set of objectives $\mathbb{O} = \{O_1, \dots, O_I\}$ and a set of alternatives $\mathbb{A} = \{A_1, \dots, A_J\}$ for some natural numbers I, J for the decision situation. Subsequently, they have to evaluate the consequences x_{ij} of all J alternatives in the respective I objectives with $1 \leq i \leq I$ and

$1 \leq j \leq J$ in a consequences table. The utility of each alternative A_j is calculated using Formula (1) for the additive expected utility (Bernoulli 1954, von Neumann and Morgenstern 1961).

$$EU(A_j) = \sum_{i=1}^{I} w_i \left[\sum_{k=1}^{K_{ij}} P(s_{ij}^k) U_i(x_{ij}^k)\right] \quad (1)$$

$$\sum_{i=1}^{I} w_i = 1 \quad (1a)$$

$$\sum_{k=1}^{K_{ij}} P(s_{ij}^k) = 1 \quad (1b)$$

Here, w_i represents the weight of objective O_i. The sum of all objective weights must equal one (1a). Objective weights are determined using the trade-off method (Keeney and Raiffa 1976) in the ENTSCHEIDUNGSNAVI. To model decisions under uncertainty, we have different states s_{ij}^k that occur with corresponding probabilities $P(s_{ij}^k)$ and result in some consequences x_{ij}^k, with $1 \leq k \leq K_{ij}$. So, if $K_{ij} = 1$, the state s_{ij}^1 occurs with a probability of 100 percent, and therefore, x_{ij}^1 is a certain consequence. If $K_{ij} \geq 2$, the consequence x_{ij} is uncertain. This is the case when influence factors are included in the model (see Section 5.1). The probabilities of all states for every ij add up to one (1b). Finally, U_i represents the utility of objective O_i. Utilities are used to map the DM's preferences.

In the ENTSCHEIDUNGSNAVI, the utilities of the consequences are determined differently for objectives with a verbal scale than for objectives with a numerical scale. The utilities for objectives with verbal scales are determined using discrete utilities, as shown in Formula (2a), while for objectives with numerical scales, the utilities are determined using the exponential utility function, as shown in Formula (2b). For a more thorough explanation of how the utilities in the ENTSCHEIDUNGSNAVI are determined, see von Nitzsch et al. (2020).

Discrete utilities for objectives with a verbal scale:

$$U_i\left(x_{ij}^k\right) = \begin{cases} 0 & if \;\; x_{ij}^k = x_i^- \\ DR(x_{ij}^k) & if \; x_{ij}^k \in (x_i^-, x_i^+) \\ 1 & if \;\; x_{ij}^k = x_i^+ \end{cases} \tag{2a}$$

Exponential utility function for objectives with a numerical scale:

$$U_i\left(x_{ij}^k\right) = \begin{cases} \dfrac{1-e^{-c_i \frac{x_{ij}^k - x_i^-}{x_i^+ - x_i^-}}}{1-e^{-c_i}} & if \quad c_i \neq 0 \\ \dfrac{x_{ij}^k - x_i^-}{x_i^+ - x_i^-} & if \quad c_i = 0 \end{cases} \tag{2b}$$

All consequences x_{ij}^k for objective O_i must lie within the interval $[x_i^-, x_i^+]$, which is defined by the DM and represents the measurement scale for the objective. While x_i^- represents the consequence with the lowest utility (zero), and x_i^+ is the consequence with the greatest utility (one), the utility $U_i\left(x_{ij}^k\right)$ increases as the consequences x_{ij}^k improve. The exact utility of the consequence levels is determined by the DM via direct rating; therefore, the utility for the consequence x_{ij}^k is represented by the direct rating function $DR(x_{ij}^k)$. c_i represents the risk aversion parameter for objective O_i. In some instances, the numerical value of x_i^+ can be smaller than that of x_i^-. This happens when inverted scales are used, which is, e.g., the case for cost objectives, where lower costs yield a higher utility.

It is also possible to measure the consequences of every objective using several numerical indicators $1 \leq q \leq Q$ (indicator scale). In this case, the consequences x_{ij}^k are not assessed directly but rather through the consequences for the respective indicators x_{ijq}^k for all Q indicators. Indicator consequences can be aggregated using either an additive-weighted composition or a user-defined formula.

Formula (3) shows the calculation of consequences using an additive-weighted composition:

$$x_{ij}^{k} = \sum_{q=1}^{Q} \left[x_i^- + \frac{x_{ijq}^{k} - x_{iq}^-}{x_{iq}^+ - x_{iq}^-} (x_i^+ - x_i^-) \right] \frac{g_q}{\sum_{v=1}^{Q} g_v} \tag{3}$$

The interval $[x_{iq}^-, x_{iq}^+]$ represents the indicator's measurement scale and defines the possible range of consequences for the q-th indicator of objective O_i. Furthermore, g_q represents the weight of the q-th indicator and can be any positive number. Similarly to the range of the consequence's scale, x_{iq}^- describes the consequence with the lowest utility of the q-th indicator, and x_{iq}^+ describes the consequences with the highest utility. Therefore, x_{iq}^+ can have a lower numerical value than x_{iq}^- for inverted indicator scales. If a user-defined formula is chosen, the DM can decide whether the range of the aggregated scale should be automatically calculated or defined by the DM. The utilities for objectives that are measured using an indicator scale are determined using the exponential utility function in Formula (2b).

In the case study, the objectives *Costs* and *Infrastructure* are measured using an indicator scale. For the latter, the individual indicator consequences are aggregated additively according to their weights, as shown in Formula (3). In the objective *Costs*, the indicator consequences are aggregated according to a user-defined formula to determine the total costs per year. Therefore, five indicators are used: *Personnel costs* (€/month), *Rent/Loan payment* (€/month), *Property tax* (€/quarter), *Electricity costs* (€/year), and *Other operating & maintenance costs* (€/month). Figure 2 shows the input mask for the definition of the indicator scale for the objective *Costs* in the ENTSCHEIDUNGSNAVI. Each indicator is measured on an individual scale. Some of the costs are incurred monthly, while others are incurred quarterly or annually; e.g., *Personnel costs* (Ind1) are measured from 40,000 €/month (worst value) to 0 €/month (best value). Indicator weights are chosen so that we obtain the total yearly costs for all types of costs when we multiply the costs by their indicator weights; i.e., monthly costs have an indicator weight of 12, quarterly costs an indicator weight of 4, and yearly costs an indicator weight of 1.

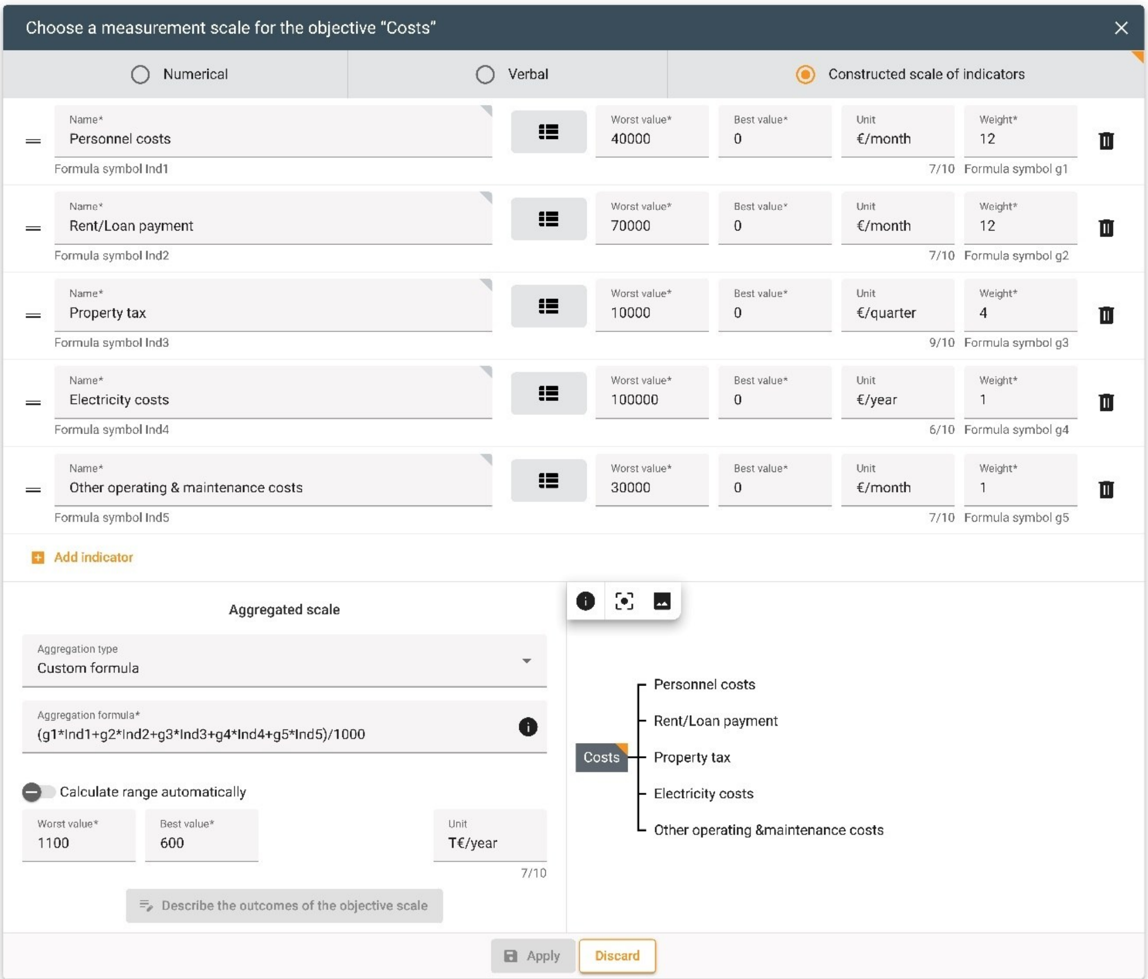

Figure 2. *Indicator scale for the objective Costs in the ENTSCHEIDUNGSNAVI.*

The aggregated measurement scale for costs ranges from 1100 T€/year (worst value) to 600 T€/year (best value), with the indicators being aggregated in the following way:

$$\frac{(g_1 Ind1 + g_2 Ind2 + g_3 Ind3 + g_4 Ind4 + g_5 Ind5)}{1000}$$

5 Modeling uncertainties in the ENTSCHEIDUNGSNAVI

There are two categories of uncertainties to consider in MAUT. On the one hand, there are potentially uncertain forecasts about the environmental circumstances, i.e., forecast uncertainties (FUs) with their corresponding probabilities. On the other hand, there may be fuzziness regarding certain parameters when DMs cannot precisely specify them. This is called Parameter

Uncertainty (PU) and can occur, e.g., for utilities, objective weights, and the probabilities of the environmental circumstances.

PU is commonly dealt with using fuzzy theory, which is particularly useful for translating verbal input into actionable numerical data. While DMs may have problems eliciting a specific probability for an event, they can commonly state that an event is very likely or unlikely. Fuzzy theory is used to assign degrees of membership to sets. In this case, a suitable membership function, e.g., of triangular, trapezoidal, or Gaussian shape, might assign a probability between 70 % and 90 % to the state 'very likely'.

Even though it is possible to include fuzzy theory in the MAUT model (Jimenez et al. 2013) to a certain degree, the philosophical and theoretical foundations of MAUT limit the joint applicability of the two concepts, as MAUT heavily relies on the elicitation of probabilities. It is, however, still possible to account for fuzziness regarding parameters. The ENTSCHEIDUNGSNAVI allows DMs to specify a mean and a degree of precision for parameters. This is comparable to the membership function of fuzzy set theory, as we limit the sets to the interval described by the mean and the degree of precision and assume a uniform distribution. Picking up on the previous example, the DM would have to specify a mean probability of 80 % and a degree of precision to achieve a similar result. While eliciting these values is harder for the DM, it helps them reflect on the decision situation and generates more transparency.

5.1 Modeling of forecast uncertainties (FUs)

Often, DMs cannot easily forecast the consequences in a consequences table. This can be due to consequences being dependent on external factors out of their control, which we call FUs. A common example would be the expected level of competition at a new location, which cannot be precisely specified, having an impact on the number of customers. To account for this kind of uncertainty, the ENTSCHEIDUNGSNAVI allows the DM to specify an influence factor that the consequence of an alternative regarding an objective depends on, i.e., an influence factor can be used to describe the consequence of any cell in the consequences table. While it is possible to use several different influence factors for different cells, it is only possible to use a single influence factor for any cell. The ENTSCHEIDUNGSNAVI offers the DM two kinds of influence factors to model FUs: user-defined influence factors and a predefined influence factor.

5.1.1 User-defined influence factors

When the uncertainty in FUs can be attributed to (a combination of) specific external factors with specific events, the DM can model this by utilizing user-defined influence factors, where the possible events are depicted by different states of the influence factor. In the ENTSCHEIDUNGSNAVI, this is implemented by assigning probabilities to the states $P\left(s_{ij}^{k}\right)$ (Formula (1)) of every influence factor $\widehat{\mathbb{S}}$, from which the DM can choose a set $\mathbb{S}$ to use for their decision. These influence factors can either be individual influence factors $\widehat{\mathbb{S}}_{ind} \subset \widehat{\mathbb{S}}$ or combined influence factors $\widehat{\mathbb{S}}_{com} \subset \widehat{\mathbb{S}}$. Together with $S_{cer} \in \widehat{\mathbb{S}}$, an influence factor with one state and 100 % probability to model certainty, they encompass the entire set of influence factors $\widehat{\mathbb{S}}_{ind} \cup \widehat{\mathbb{S}}_{com} \cup \{S_{cer}\} = \widehat{\mathbb{S}}$. Individual influence factors ($\widehat{\mathbb{S}}_{ind}$) are the simpler form for modeling uncertainties, where the DM specifically defines all the different states and the probability with which they occur. Combined influence factors ($\widehat{\mathbb{S}}_{com}$) are comparatively more complex, as they are a combination of two previously defined influence factors, for which the probabilities are calculated automatically. They can be used by the DM to model FUs when the consequence depends on multiple external factors.

We let $S_{\alpha}, S_{\beta} \in \widehat{\mathbb{S}}$ denote (combined) influence factors without a connection to a consequence; i.e., S_{α}, S_{β} can be any S_{ij} if the DM chooses to assess the consequence of objective O_i for alternative A_j using this influence factor. The DM can combine any S_{α} and S_{β} that have not yet been set in a relationship with one another either directly or indirectly through some previously defined combined influence factor(s). For two influence factors S_α, S_β with K_α and K_β different states, the resulting newly defined combined influence factor $S_\gamma \in \widehat{\mathbb{S}}_{com}$ has $K_\gamma = K_\alpha \times K_\beta$ different states. The state probabilities of the combined influence factor for the case where the d-th state of influence factor S_α and the e-th state of influence factor S_β occur simultaneously are denoted by $P\left(s_\gamma^{de}\right)$. Consequently, they are determined by multiplying the state probabilities $P(s_\alpha^d)$ and $P\left(s_\beta^e\right)$ of the individual influence factors that make up the state of the combined influence factor (see Formula (4)).

$$P\left(s_\gamma^{de}\right) = P(s_\alpha^d) \times P\left(s_\beta^e\right) \tag{4}$$

In the case study, 258 GmbH defines three individual influence factors and one combined influence factor to model the FUs of their decision (see Figure 3).

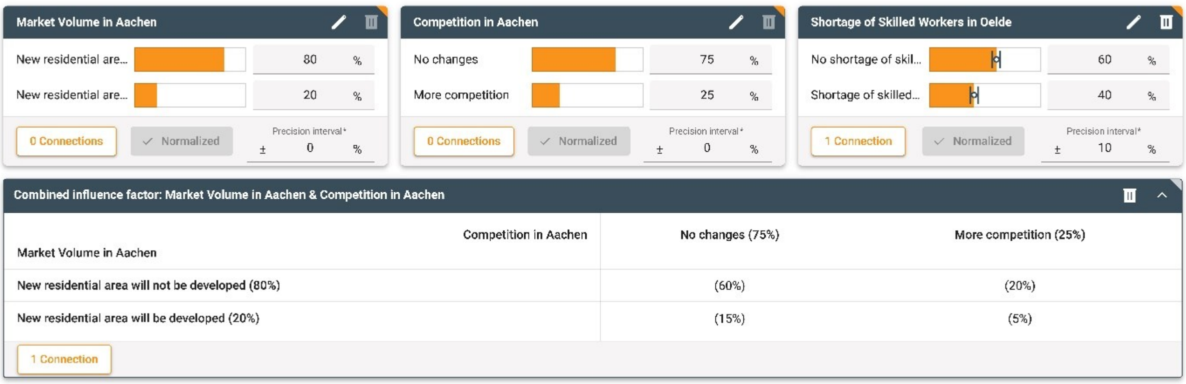

Figure 3. *Influence factors in the* E*NTSCHEIDUNGSNAVI.*

The individual influence factor *Shortage of Skilled Workers in Oelde* is used to assess the consequence for the cell *Oelde*/*Market Volume in the Region*. *Oelde* is located in a very rural area, which is becoming less and less attractive to young people, which leads to problems in recruiting apprentices. Due to the limited attractiveness, only a few people move to *Oelde* from outside the area. In combination, these two factors indicate that a shortage of skilled workers will be expected in the future. The influence factor comprises two states with the following probabilities: No shortage of skilled workers (60 %) and a Shortage of skilled workers (40 %). Therefore, the company defines the consequence for each state, resulting in a range of consequences for the cell *Oelde*/*Market Volume in the Region* in the consequences table (see Figure 1). The combined influence factor merges the two individual influence factors *Market Volume in Aachen* and *Competition in Aachen*.

If necessary, the company can add more states to its influence factors. Moreover, they can choose to base the probabilities on an automatic preset, where they can adjust the probabilities by defining the expected value and dispersion of the distribution. If the given probabilities do not add up to 100 %, as required by Formula (1b), the ENTSCHEIDUNGSNAVI normalizes the probabilities. Additionally, as the company is unsure about the exact probabilities, it has used PUs to define a 'precision interval' for the influence factor Shortage of Skilled Workers in *Oelde*. PUs will be discussed in more detail in Section 5.2.

5.1.2 Predefined influence factor

If the uncertainty regarding the consequences of a cell in the consequences table cannot be attributed to one or a few specific external factors, the DM can model this type of uncertainty by using a predefined influence factor with a 'worst-median-best' distribution. This is often necessary when the uncertainty stems from using a large amount of data from previous or external projects to determine the likely consequences. Predefined influence factors require the p.10, p.50, and p.90 quantiles for the consequences to be specified. Using probabilities of 25 % for p.10 and p.90 and 50 % for p.50 is a good approximation of a normal distribution (Hammond and Bickel 2013).

Contrary to the probability distributions of user-defined influence factors, the probability distributions for the predefined influence factor are stochastically independent. This is necessary due to the lack of specific external factors that cause the uncertainty, which can be different for every cell of the consequences table. If the consequence in a cell is assessed through multiple indicators (see Section 4, Figure 2), this stochastic independence even applies to the consequences of the individual indicators; e.g., while personnel costs could take their best value, the rent might take the median value.

In the case study, 258 GmbH cannot specify the exact costs that would result from choosing the individual alternatives and, therefore, uses the predefined influence factor to assess the consequences for the objective *Costs* for all alternatives. This allows the company to specify the consequences independently for each quantile and indicator, resulting in a range of consequences for each alternative. For example, the consequences range from 1037.34 T€/year to 1082 T€/year for the alternative *Cologne* (see Section 3, Figure 1).

5.2 Modeling of parameter uncertainties (PUs)

It is also common for DMs to have trouble specifying the exact parameters of a decision model, i.e., PU exists. In these cases, it is important to enable DMs to work with this uncertainty and allow imprecise information to be entered. In the ENTSCHEIDUNGSNAVI, PUs can occur for three different types of parameters: the utilities U_i, the objective weights w_i, and the probability distributions $P(S_{ij})$ of the influence factors. The idea of PUs in the ENTSCHEIDUNGSNAVI for all three types of parameters is to identify a mean μ and a degree of precision ε, resulting in an interval for the parameters. If the DM defines $\varepsilon = 0$, they can precisely elicit the parameter, whereas $\varepsilon > 0$ means that they use imprecise information. The handling of imprecise information is different for each type of parameter and will subsequently be explained.

5.2.1 PU regarding utilities

The way in which utilities are determined depends on the scale of the respective objective, which can be either a verbal or a numerical scale. The latter also includes indicator scales. The utility for verbal scales is determined with the help of discrete utilities, as shown in Formula (2a). The utilities for numerical scales are determined by using the exponential utility function shown in Formula (2b). For a deeper explanation of how utilities are determined in the ENTSCHEIDUNGSNAVI, see von Nitzsch et al. (2020).

When PU regarding discrete utilities is present, the utility $U_i(x_{ij}^k)$ of the consequence x_{ij}^k is determined according to a continuous uniform distribution U between the minimum $U_i^{min}(x_{ij}^k)$ and maximum $U_i^{max}(x_{ij}^k)$ utilities of the consequence, i.e., $U_i(x_{ij}^k) \sim U\left(U_i^{min}(x_{ij}^k), U_i^{max}(x_{ij}^k)\right)$. The minimum and maximum utilities are calculated according to Formulas (5a) and (5b) using the degree of precision $\varepsilon_{U_i} > 0$ for a discrete utility scale of the objective O_i, which can range from 0 to 50 %.

$$U_i^{min}(x_{ij}^k) = U_i(x_{ij}^k) - \varepsilon_{U_i} min\{U_i(x_{ij}^k), 1 - U_i(x_{ij}^k)\} \quad (5a)$$

$$U_i^{max}(x_{ij}^k) = U_i(x_{ij}^k) + \varepsilon_{U_i} min\{U_i(x_{ij}^k), 1 - U_i(x_{ij}^k)\} \quad (5b)$$

For numerical scales, the utility $U_i(x_{ij}^k)$ of the consequence x_{ij}^k is determined according to Formula (2b) using $c_i \sim U(c_i^{min}, c_i^{max})$, where ε_{c_i} (between 0 and 10) is the degree of precision for a numerical utility function, and the minimum risk aversion parameter c_i^{min} and maximum risk aversion parameter c_i^{max} for objective O_i are determined according to Formulas (6a) and (6b).

$$c_i^{min} = c_i - \varepsilon_{c_i} \quad (6a)$$

$$c_i^{max} = c_i + \varepsilon_{c_i} \quad (6b)$$

In the case study, 258 GmbH chooses linear utility functions and a linear increase in the discrete utilities for all objectives. However, the company defines precision intervals for the objectives *Costs* and *Ecological Sustainability* since it is unsure about its (risk) preferences for these objectives. Figure 4 shows an overview of the utility determination for the objectives *Costs* and *Ecological Sustainability*. For the objectives *Costs*, 258 GmbH chooses a precision interval of one, leading to a set of possible utility functions. This means that all utility functions within this set reflect the company's preferences. For the objective *Ecological Sustainability*, 258 GmbH chooses a precision interval of 20. This also means that the utility of the individual levels is not precisely defined but lies within an interval. In both cases, the worst and best levels are determined by exact values: zero and one. Figure 4. The determination of utility functions for the objectives *Costs* and *Ecological Sustainability* in the ENTSCHEIDUNGSNAVI.

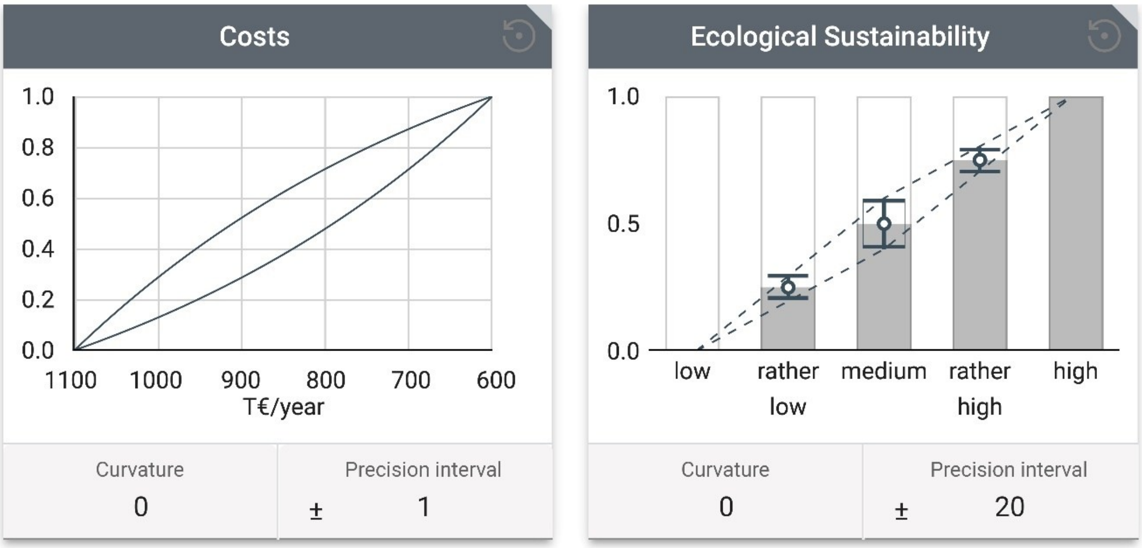

Figure 4. *The determination of utility functions for the objectives Costs and Ecological Sustainability in the ENTSCHEIDUNGSNAVI.*

5.2.2 PU regarding objective weights

The objective weights w_i are determined by eliciting the exchange rates between two objectives through the trade-off method. Therefore, the DM has to choose a reference objective with which all other objectives are compared and then formulate preference statements for all pairs of objectives (trade-offs). The ENTSCHEIDUNGSNAVI supports the DM by visualizing the trade-offs through indifference curves. If the DM chooses to use imprecise information for the objective weights, the individual objective weights w_i are determined within their lower bounds w_i^{min} and upper bounds w_i^{max} according to a continuous uniform distribution $w_i \sim U\left(w_i^{min}, w_i^{max}\right)$. The bounds w_i^{min} and w_i^{max} are determined according to Formulas (7a) and (7b) using the degree of precision ε_{w_i} for every objective O_i. As having imprecise objective weights usually results in the sum of all objective weights deviating from 1, it is necessary to normalize them according to Formula (7c). In this case, the normalized objective weights w_i^{norm} replace w_i in the calculation of the expected utility from Formulas (1) and (1b).

$$w_i^{min} = w_i - \varepsilon_{w_i} \tag{7a}$$

$$w_i^{max} = w_i + \varepsilon_{w_i} \tag{7b}$$

$$w_i^{norm} = \frac{w_i}{\sum_{v=1}^{I} w_v} \tag{7c}$$

Figure 5 shows an overview of the determination of the objective weights in the ENTSCHEIDUNGSNAVI. 258 GmbH chooses *Costs* as the reference objective, with which every other objective is compared in a trade-off. Every trade-off can be analyzed in detail. Figure 6 shows the detailed trade-off view of the objectives *Costs* and *Market Volume in the Region*. In this case, the company chooses a precision interval of $\varepsilon_{w_{Market}} = 5$ %, which leads to an interval from 25 % to 35 % for this objective weight.

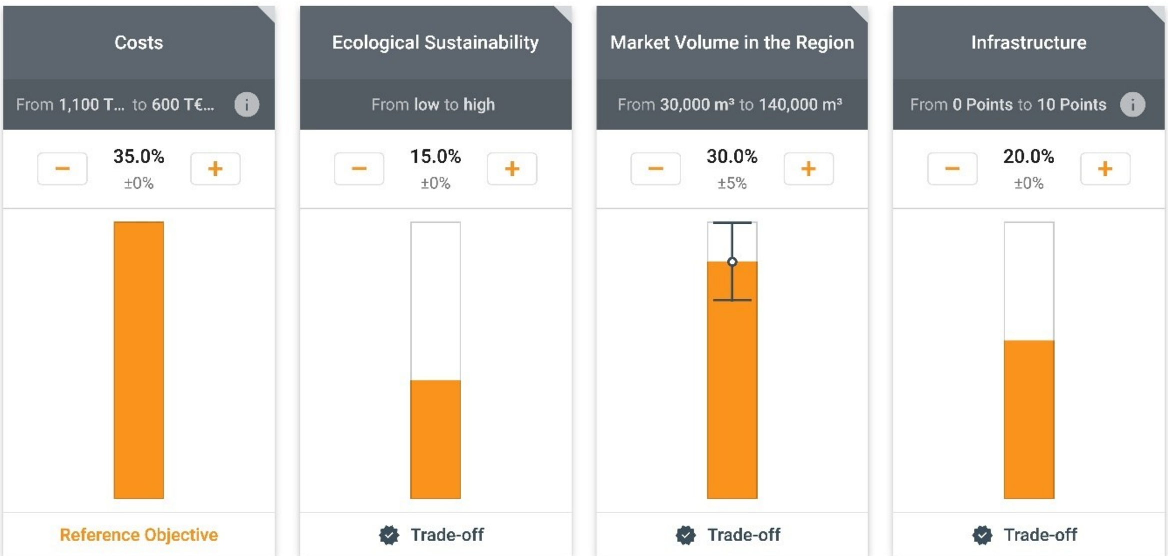

Figure 5. *Overview of determination of objective weights in the ENTSCHEIDUNGSNAVI.*

The detailed view (see Figure 6) shows the indifference curves (left side) resulting from the imprecise trade-off (right side). The company has chosen *Cologne* as a reference point for the trade-off with which two other imaginary alternatives (comparison points) are compared.

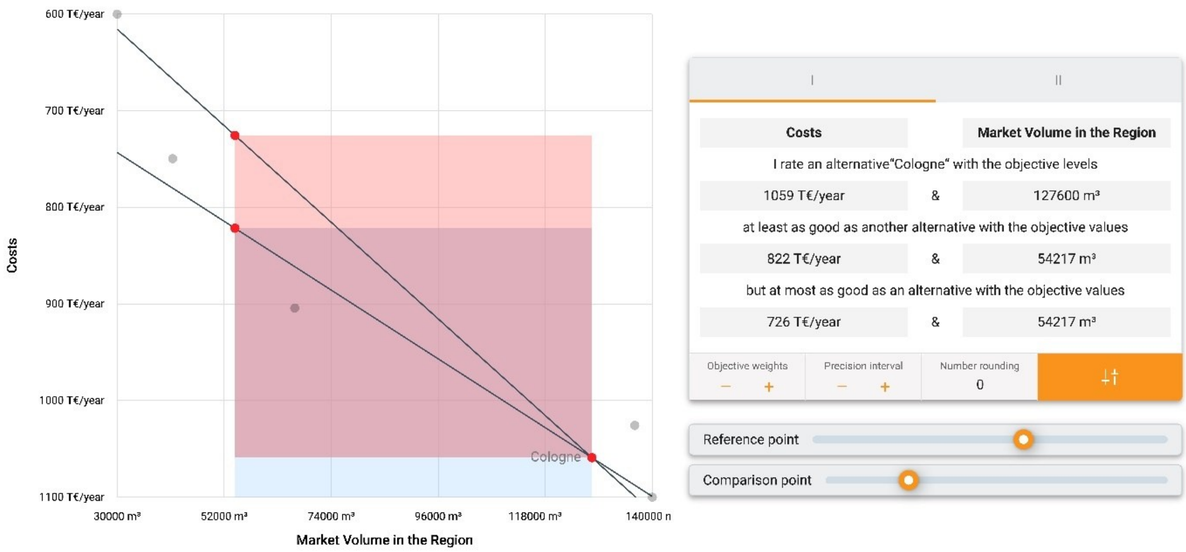

Figure 6. *Detailed view of trade-off determination between Costs and Market Volume in the Region in the* ENTSCHEIDUNGSNAVI*.*

5.2.3 PU regarding probability distributions

In cases where the DM cannot precisely elicit the state probabilities for an individual influence factor $S_\delta \in \widehat{\mathbb{S}}_{ind}$, they can define a degree of precision $0 \leq \varepsilon_{P(S_\delta)} \leq 50\ \%$ for this influence factor. This means that we have to find a probability distribution $P^{norm}(S_\delta)$ where the sum of all probabilities $\sum_{f=1}^{K_\delta} P^{norm}(s_\delta^f)$ equals one, and the probabilities $P^{norm}(s_\delta^f)$ of all $1 \leq f \leq K_\delta$ states are between their minimum probability $P_{min}(s_\delta^f)$ and maximum probability $P_{max}(s_\delta^f)$ according to the given degree of precision. The minimum and maximum probabilities of the individual states are determined according to Formulas (8a) and (8b).

$$P_{min}(s_\delta^f) = P(s_\delta^f) - \varepsilon_{P(S_\delta)} min\{P(s_\delta^f), 1 - P(s_\delta^f)\} \tag{8a}$$

$$P_{max}(s_\delta^f) = P(s_\delta^f) + \varepsilon_{P(S_\delta)} min\{P(s_\delta^f), 1 - P(s_\delta^f)\} \tag{8b}$$

Simply determining the probabilities according to a continuous uniform distribution between the minimum and maximum probabilities will most likely result in the sum of all probabilities deviating from one. The subsequent necessary normalization step could, however, result in probabilities that are below or above the minimum and maximum probabilities if they have been scaled naïvely according to Formula (9).

$$P^{norm}\left(s_\delta^f\right) = \frac{P^{norm}\left(s_\delta^f\right)}{\sum_{v=1}^{K_\delta} P^{norm}\left(s_\delta^v\right)} \tag{9}$$

To avoid this, we use an algorithmic approach where the individual probabilities $P^{norm}\left(s_\delta^f\right)$ are drawn in ascending order, and we ascertain that the sum of all probabilities equals one. For this, we will assume that the probabilities of S_δ are sorted in ascending order: i.e., $S_\delta = \left\{s_\delta^1, s_\delta^2, \ldots, s_\delta^{K_\delta}\right\}$ is partially ordered by the relation $P\left(s_\delta^f\right) \leq P\left(s_\delta^{f+1}\right)$. The probabilities $P^{norm}\left(s_\delta^f\right)$ are uniformly drawn from the interval between a specific minimum probability $P_{min}^{norm}\left(s_\delta^f\right)$ and maximum probability $P_{max}^{norm}\left(s_\delta^f\right)$ with a given degree of precision $\varepsilon_{P(S_\delta)}$. The specific minimum probability is determined according to the recursive Formula (10a) and in a way that ensures that all subsequent probabilities will be between their respective minimum $P_{min}\left(s_\delta^f\right)$ and maximum $P_{max}\left(s_\delta^f\right)$ probabilities. At the same time, they must at least be able to account for the remaining probability to ensure the sum of all probabilities is one.

$$P_{min}^{norm}\left(s_\delta^f\right) =$$

$$\begin{cases} P_{min}\left(s_\delta^f\right) & if\ f = 1 \\ max\left\{P_{min}\left(s_\delta^f\right), 1 - \sum_{g=1}^{f-1} P^{norm}\left(s_\delta^g\right) - \sum_{g=f+1}^{K_\delta} P_{max}\left(s_\delta^g\right)\right\} & if\ 1 < f < K_\delta \\ 1 - \sum_{g=1}^{K_\delta - 1} P^{norm}\left(s_\delta^g\right) & if\ f = K_\delta \end{cases} \tag{10a}$$

The specific maximum probability is determined analogously according to Formula (10b) while ensuring that we can at least allocate the minimum probability $P_{min}(s_\delta^f)$ to all subsequent states.

$$P_{max}^{norm}(s_\delta^f) =$$

$$\begin{cases} P_{max}(s_\delta^f) & if\ f = 1 \\ min\left\{P_{max}(s_\delta^f), 1 - \sum_{g=1}^{f-1} P^{norm}(s_\delta^g) - \sum_{g=f+1}^{K_\delta} P_{min}(s_\delta^g)\right\} & if\ 1 < f < K_\delta \\ 1 - \sum_{g=1}^{K_\delta - 1} P^{norm}(s_\delta^g) & if\ f = K_\delta \end{cases} \tag{10b}$$

Using this, the probabilities are determined by multiplying the possible range of the probability by a random variable $rnd \sim U(0,1)$ and adding this to the specific minimum probability, as in Formula (11).

$$P^{norm}(s_\delta^f) = P_{min}^{norm}(s_\delta^f) + \left(P_{max}^{norm}(s_\delta^f) - P_{min}^{norm}(s_\delta^f)\right) * rnd \tag{11}$$

In the case study, 258 GmbH is unsure about the probabilities of the states of the influence factor *Shortage of Skilled Workers in Oelde*. Therefore, they adjust the degree of precision to use imprecise probabilities with ε = 10 % (see Section 5.1.1, Figure 3). This results in an interval for both states. The probability of the state *No shortage of skilled workers in Oelde* lies between 56 % and 64 %, and the probability of the state *Shortage of skilled workers in Oelde* lies between 36 % and 44 %.

6 Methods for checking the robustness of the result

While including FUs and PUs can greatly increase the complexity of the decision model, it often cannot be avoided. The expected utility of the MAUT model only captures the resulting risk in a limited way, as extreme results for rare events can be concealed by the method of aggregation. The robustness check can, however, reveal these risks and serve as a good starting point to identify alternatives and uncertainties that warrant further consideration. Using a simulation approach, the tool calculates a ranking of the alternatives for randomly generated scenarios that are based on the previously defined PUs and FUs with their given probabilities. It, therefore, helps the DM check how robust the ranking of alternatives is, pointing out scenarios in which an otherwise promising alternative might fall behind.

The analysis is based on a Monte Carlo simulation (Kalos and Whitlock 2009), as even decision models that are otherwise manageable in size can easily result in an infeasible number of calculations. For example, having a decision model with five alternatives and five objectives that are each measured through five indicators, using the predefined influence factor (with three states) for all forecasts results in $35*5*5 \approx 4.36\times1059$ different possibilities due to stochastic independence, making it infeasible to calculate an analytically correct result, even on the fastest computer to date. The simulation approach in the ENTSCHEIDUNGSNAVI provides a very good approximation of this result with only a few million iterations. Furthermore, the DM can select the FUs, i.e., influence factors, and PUs, i.e., uncertainties regarding probabilities, utilities, and objective weights, for which they want to check robustness. This allows for specific analyses to be carried out while also giving the option to reduce computational complexity.

Running the simulation with enabled FUs generates information on how the alternatives react to the given influence factors, i.e., external factors out of the control of the DM. In this kind of simulation, it can be interesting to examine which requirements are necessary for an alternative to be the best and how likely these requirements are to be met. To generate this information, the specific state of every individual influence factor is drawn according to the discrete probability distribution of the influence factor for every simulation iteration. The states of combined influence factors are determined by a combination of states of the individual influence factors.

By enabling PUs for the simulation, DMs gain insights into how their inability to precisely elicit certain parameters affects the result. Based on this information, the DM can consider whether they should spend more time and effort on determining the parameters of the model or not. If the model is sensitive to PU (i.e., the best alternative depends on a randomly drawn parameter within the given interval), the DM should try to give a more precise assessment to aid the overall robustness of the model. To examine the influence of PUs, the uncertain parameters of every simulation iteration are determined according to the methodology explained in Section 5.2.

It is also possible to analyze the robustness of PUs and FUs simultaneously. This generates the most substantial insights, as every kind of uncertainty is considered. Any inferences on the robustness of the outcome should generally be based on a simulation covering PUs and FUs while analyzing only individual PUs and FUs can aid the decision-making process through the generation of specific information. When running the simulation with FUs and PUs, a specific discrete probability distribution is drawn for all selected influence factors with imprecise probabilities for every simulation iteration. These specific probability distributions are then used to draw the specific states for every influence factor, as previously described for the case where only FU occurs.

The outcome of the Monte Carlo simulation is visually presented to the DM (see Figure 7). The ENTSCHEIDUNGSNAVI shows a tabular overview of how often each individual alternative was ranked first, second, third, etc., in the simulation iterations performed. A score is calculated from all the frequencies for every alternative, which reflects the weighted average of ranks achieved on the basis of these frequencies. The lower this score is, the more frequently an alternative is in the top ranks and the more attractive it is. This score is used to check for a new form of dominance, namely, simulation dominance. It can be regarded as similar to stochastic dominance in that it takes probabilities into account. An alternative that achieves a score of exactly one absolutely dominates all other considered alternatives according to the simulation: i.e., it is better in every single iteration of the simulation. If the score deviates from one, no strict simulation dominance exists. The score can, however, still be interpreted as the degree of dominance.

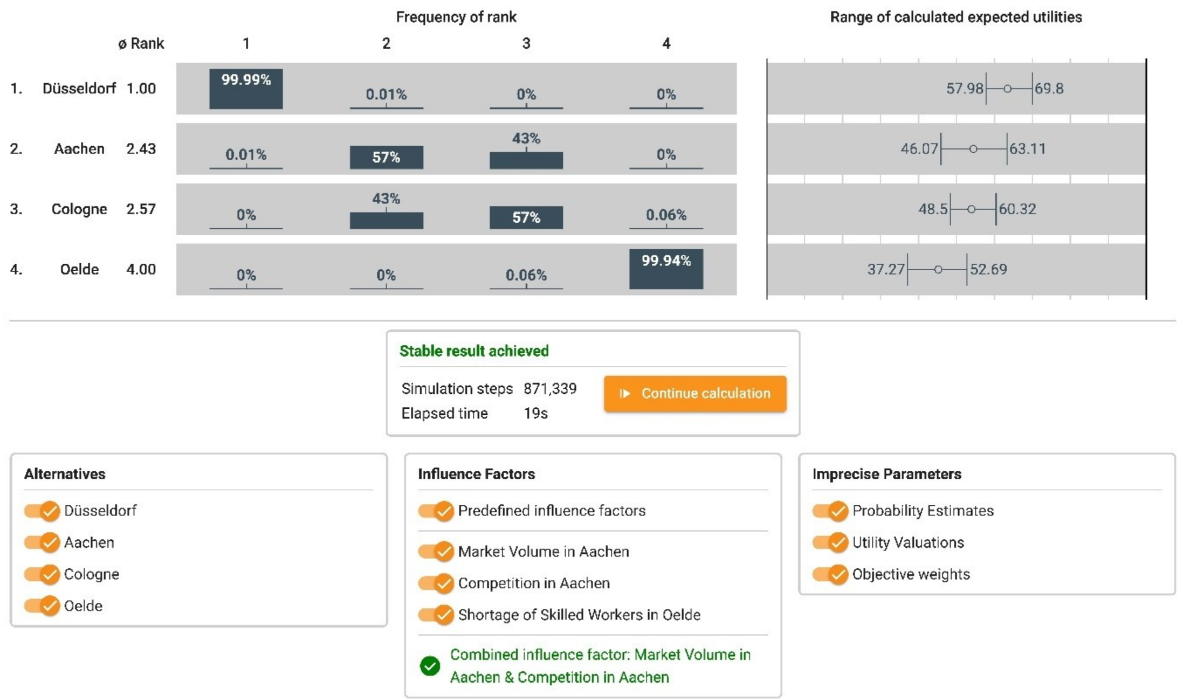

Figure 7. *Robustness check of case study in the ENTSCHEIDUNGSNAVI.*

When only two alternatives are considered, the score for the alternatives can lie between one and two, and their sum adds up to three. As previously mentioned, an alternative with a score of one absolutely dominates the other alternatives, while a score close to one will indicate a strong degree of dominance of one alternative over the other. A score of 1.1 indicates that the alternative is better in 90 % of cases, for example. A complete lack of dominance between the alternatives is ascertained when the scores for both alternatives are 1.5, i.e. when they are both the better alternative in 50 % of the cases.

The ENTSCHEIDUNGSNAVI also collects information on which states or which combination of states are necessary for the rank of an alternative. In the case where a state occurs in all scenarios that lead to a specific rank for an alternative, this information is deemed to be a requirement for the alternative to achieve a specific rank and will be presented to the DM. Additionally, the DM is presented with the range of calculated (conditional) expected utilities for each alternative to better grasp the extent of the impact on the individual alternatives. This can be interesting in cases where a DM wants to avoid risk and opt for an alternative that may be ranked lower in the average rank but shows little deviation in the expected utility across all simulation runs.

For efficiency reasons, the simulation is stopped once the average maximum change in the frequency with which an alternative achieves a rank is below 10−7 per iteration over the course of one second for any alternative. This is considered a stable result that approximates an analytical result well while keeping the necessary resources to a minimum. The DM can, however, always choose to continue the simulation if they want more valid results.

In this case study, the alternative *Düsseldorf* ranks first in almost all simulation iterations (see Figure 7). The expected utility varies between 57.98 and 69.80 and is only surpassed by the utility of *Aachen* in very few constellations. Depending on the values drawn, the second- and third-placed alternatives change, too. *Aachen* ranks second in 57 % and third in 43 % of iterations. *Cologne* ranks second in 43 % and third in 57 % of iterations. *Oelde* ranks last in almost all iterations (99.4 %) of the simulation, while in a few cases, *Cologne* becomes the worst alternative (0.06 %). While the result shows a very strong preference for the warehouse in *Düsseldorf*, there is still a small risk involved where *Aachen* would be the best alternative. Therefore, it is reasonable to further explore the conditions under which *Aachen* should be chosen for the location of the warehouse.

Hovering over the frequency bar for *Aachen* in the first place gives 258 GmbH insights on the necessary requirements (see Figure 8) to reach this rank. The ENTSCHEIDUNGSNAVI provides the DM with information on the necessary influence factor states, objective weights, and utilities. In this case, *Aachen* only becomes the best alternative through a combination of FUs and PUs. This is indicated by the necessity for the influence factors on the left and the deviations of the objective weights and risk aversion parameter c_{costs} from their defined means $w_{Market\ vol.} = 0.3$ and $c_{costs} = 0$, respectively.

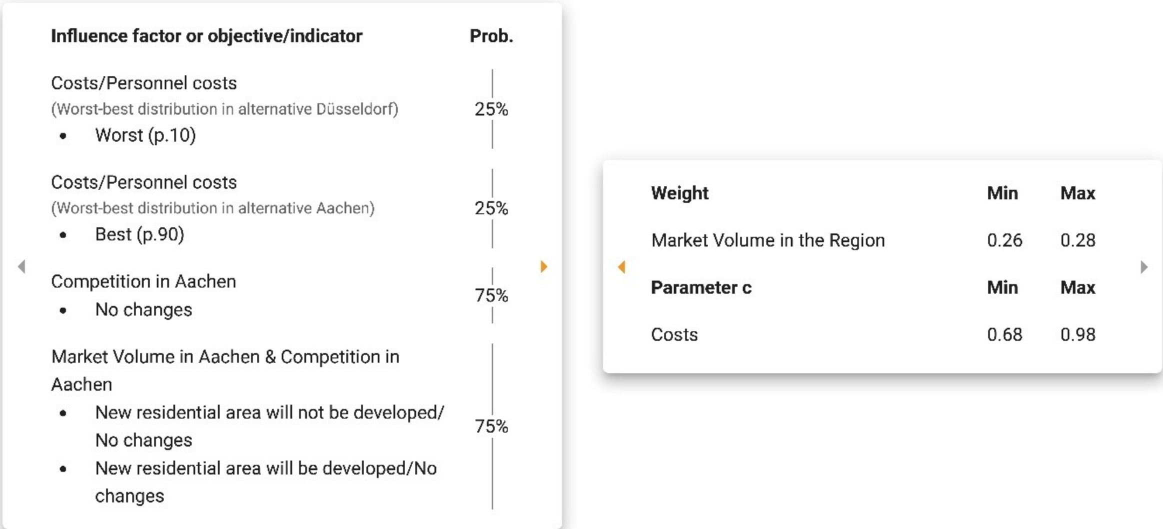

Figure 8. *Necessary requirements for Aachen to become best alternative.*

7 Objective weight analysis

Eliciting one's objective weights is one of the hardest parts of the decision-making process. In many cases, DMs are uncertain about their objective weights, up to the point where they cannot even properly elicit them using precision intervals. In these cases where DMs have a high PU regarding the objective weights, the objective weight analysis can be a helpful starting point to see which objective weight intervals would result in different outcomes, i.e., a different alternative with the highest expected utility.

The objective weight analysis uses a simulation approach and determines a range of statistical measures for the different objective weights that result in an alternative being the best, namely, the minimum, maximum, p.10, and p.90 quantiles, median, and average. These are initially determined by different objective weight combinations algorithmically chosen in each calculation step. The number of increments N_{inc} of the objective weights describes the granularity with which the objective weights are altered during the simulation and depends on the number of objectives $|\mathbb{O}|$. The incrementation values have been chosen to provide a good ratio between the number of necessary calculations and the granularity of the objective weights. The different increment numbers can be seen in Formula (12).

$$N_{inc} = \begin{cases} 1000 & if\ |\mathbb{O}| = 2 \\ 100 & if\ |\mathbb{O}| = 3 \\ 40 & if\ |\mathbb{O}| = 4 \\ 18 & if\ |\mathbb{O}| = 5 \\ 11 & if\ |\mathbb{O}| = 6 \\ 8 & if\ |\mathbb{O}| = 7 \\ 6 & if\ |\mathbb{O}| = 8 \\ 5 & if\ |\mathbb{O}| = 9 \\ 4 & if\ 10 \leq |\mathbb{O}| \leq 11 \\ 3 & if\ 12 \leq |\mathbb{O}| \leq 14 \\ 2 & if\ 15 \leq |\mathbb{O}| \end{cases} \tag{12}$$

The number of iterations for the algorithm depends on the number of objectives and amounts to the number of increments to the power of the number of objectives, i.e., $N_{inc}^{|\mathbb{O}|}$, and ranges between 500 thousand and 4.7 million for decisions with 14 or fewer objectives. For the calculation of the objective weights, we introduce the notation $\lfloor z \rfloor$ for the greatest integer less than or equal to z. Furthermore, we use the binary operation $y\ mod\ z$, as used in computer science (Knuth 1997), which returns the remainder of the division $\frac{y}{z}$ for any real numbers y and z, see Formula (13a).

$$y\ mod\ z = \ y - z\left\lfloor \frac{y}{z} \right\rfloor,\ where\ z \neq\ 0; y\ mod\ 0 = y;\ y, z \in \mathbb{Z} \tag{13a}$$

With this, we calculate the objective weight for objective O_i in iteration λ of the algorithm according to Formula (13b).

$$w_i^{\lambda} = \left(\left\lfloor \frac{\lambda}{N_{inc}^{i-1}} \right\rfloor\ mod\ N_{inc}\right)\frac{1}{N_{inc}-1},\ where\ 1 \leq i \leq |\mathbb{O}|, 1 \leq \lambda \leq N_{inc}^{|\mathbb{O}|} \tag{13b}$$

To generate all possible objective weight combinations with the number of increments, we need a function that iterates through the different incremental values for the different objectives. The term $\left\lfloor \frac{\lambda}{N_{inc}^{i-1}} \right\rfloor$ delays the incrementation of λ by a factor equal to the number of increments from one objective w_i to another objective weight w_{i+1}; i.e., the function will repeat the same value for N_{inc}^{i-1} times before increasing the value by one. Furthermore, the mod function helps us ensure that we always have as many different integer results as we have increments: i.e., for an input value that is a multiple of the increments, the result will be zero, while the results are between one and the number of increments minus one for all other input values. In combination, this allows us to iterate through every possible objective weight combination regarding the number of increments. We can then multiply this value by $\frac{1}{N_{inc}-1}$ to generate equidistant unnormalized objective weights between zero and one.

If, at the end of the algorithm, the change in any of the statistical values recorded is greater than 0.05 % for the calculations during the last ½ second of the algorithm, the ENTSCHEIDUNGSNAVI will continue the simulation with randomly generated objectives weights $w_i^{\lambda} \sim U[0,1]$ until the maximum change in any of the values for all calculations during ½ second is less than 0.05 %. Either way, the normalized objective weights $w_i^{\lambda,norm}$ will be determined for every objective i and every iteration λ according to Formula (14) to obtain a valid set of objective weights for every iteration of the algorithm.

$$w_i^{\lambda,norm} = \frac{w_i^{\lambda}}{\sum_{v=1}^{I} w_v^{\lambda}} \tag{14}$$

Figure 9 shows the objective weight analysis in the ENTSCHEIDUNGSNAVI based on the case study. On the left, the DM can see how often an alternative has ended up in first place across all simulation runs. By default, the alternatives that most frequently reached first place are at the top. In the bar chart in the middle, the DM can see the average objective weights of the entire simulation for which the alternatives were the best (display: average) and the range of objective weights that can occur while an alternative is the best. In the more detailed view (display: distribution), the DM can also see the medians, the maximum and minimum values, and the p.10 and p.90 quantiles of the drawn objective weights with which the corresponding alternative has ended up in the first place. The DM can also change the quantiles in this view to p.25 and p.75 quantiles.

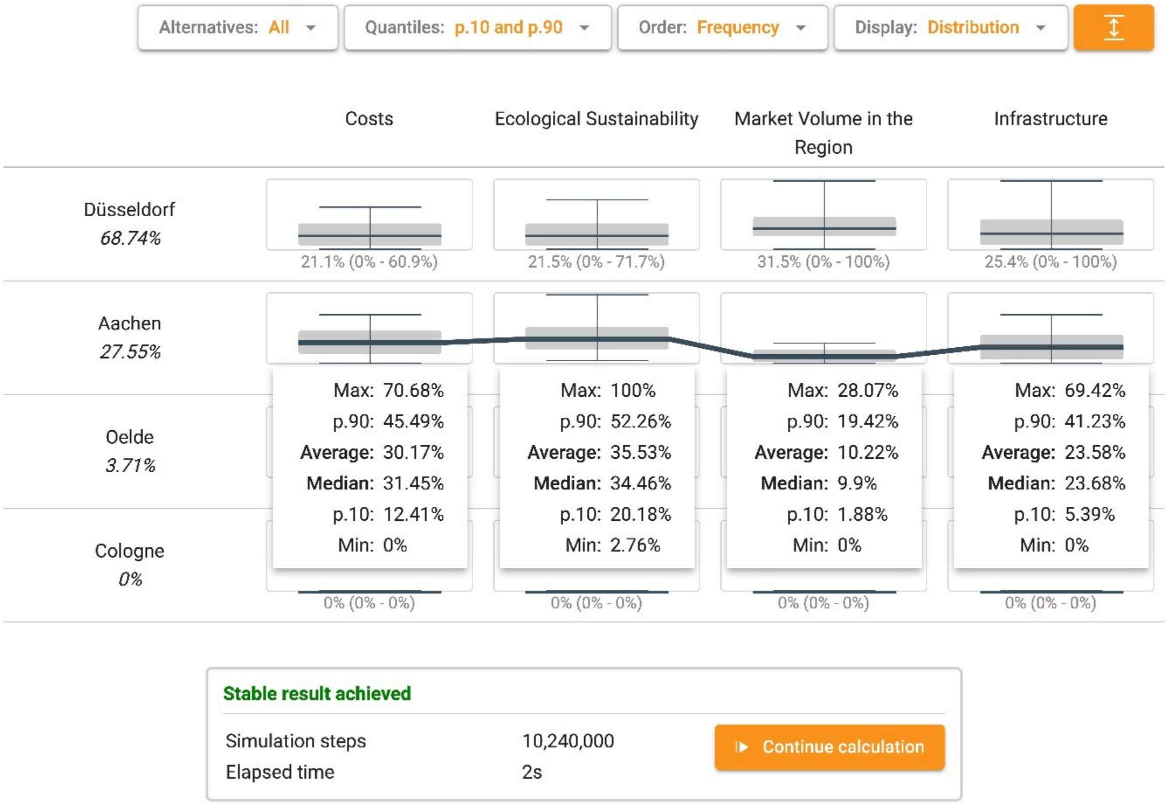

Figure 9. *Objective weight analysis in the ENTSCHEIDUNGSNAVI.*

The objective weight analysis (Figure 9) further strengthens the robustness of the result with *Düsseldorf* as the warehouse's location. Even with objective weights outside the specified uncertainty, it is still the most promising location in 68.74 % of cases. Only objective weights of 61 % and above for *Costs* and above 71.7 % for *Ecological Sustainability* hinder *Düsseldorf* from being in the first place. However, in 27.55 % of cases, *Aachen* becomes the best alternative when varying the objective weights. The narrow interval of possible objective weights for *Market Volume in the Region* can be attributed to *Aachen*'s relative weakness in that regard. The upper limit of 28.07 % is, however, still within the uncertainty in that regard. The upper limit of 28.07 % is, however, still within the uncertainty interval defined by 258 GmbH for this objective. Therefore, further analysis should be conducted.

Above the bar chart, DMs can also select which alternatives should be displayed. In addition, they can change the sorting based on their gut feeling. With the button on the top right, DMs can choose to vertically enlarge the diagram. When the DM hovers over the bars, the exact values of the different parameters are shown (see Figure 9).

8 Sensitivity analysis

The sensitivity analysis helps DMs to check the plausibility of the model, i.e., whether the different parameters have the expected impact on the result and how big the impact is. This is mainly enabled by the multidimensional approach of the ENTSCHEIDUNGSNAVI. While most other tools only allow a unidimensional analysis of the changes caused by varying a parameter, the ENTSCHEIDUNGSNAVI offers the option to vary all parameters simultaneously. This allows the DM to analyze the impact of multiple uncertainties at the same time, which gives more realistic results than changing just one value at a time.

The sensitivity analysis allows the variation in the following parameters of the decision model: objective weights, utilities, probabilities of FUs, consequences, and indicator weights. Corresponding slider boxes are available for each parameter. The effects on the resulting ranking and utilities of alternatives can be observed while changing the slider. It is also possible to select which alternatives are shown.

The detailed display option for expected utilities shows the utility broken down into the contribution of each objective. Hovering over the colored bars shows the DM to which objective the shown utility belongs. All values are multiplied by 100 for better readability.

If imprecise parameters have been chosen, the precision interval specified by the DM is marked graphically on the slider with a dark gray area. The DM can use this to easily check whether there are any effects on the result within the previously defined intervals.

In the case study, 258 GmbH can use the sensitivity analysis to analyze the risks previously identified in the robustness check and objective weight analysis. Figure 10 shows the analysis regarding PU and confirms the findings indicated by the simulation of the robustness check.

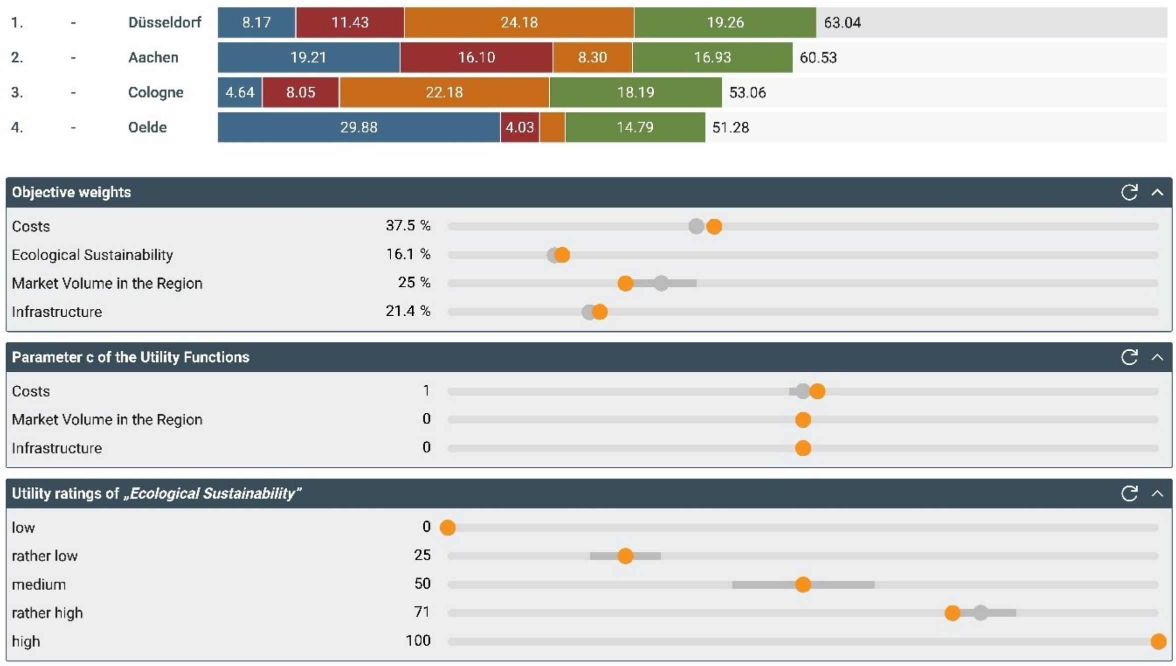

Figure 10. *Sensitivity analysis for PU impact in the* E*NTSCHEIDUNGSNAVI.*

Even when all PUs take the least favorable values for *Düsseldorf* in accordance with the defined uncertainty intervals for the respective parameters, it still remains at the top. 258 GmbH has minimized the objective weight for *Market Volume in the Region* (orange bar), where *Düsseldorf* has a higher utility than *Aachen*. It has also maximized the risk aversion parameter c_{Costs}, benefiting *Aachen's* utility value, as the concave utility function generates a higher utility increase for *Aachen* than it does for *Cologne* as if the utility function were linear. Lastly, it has minimized the utility value for the rather high consequence level of the objective *Ecological Sustainability*, which is the consequence for *Düsseldorf*. As changing the PUs is not sufficient to put *Aachen* in the first place, it can be said that the choice of location must also depend on the FUs identified in the robustness check (Figure 8), i.e., costs associated with the warehouses at the locations, the level of competition, and market volume in *Aachen*. Knowing this, 258 GmbH could try to perform a more precise assessment of the probabilities or the costs to minimize the effect of uncertainty on the model or use other types of analyses to assure themselves that they are making the right choice.

Figure 11 shows the continued analysis, this time regarding the risks from the objective weight analysis. Even though they were only uncertain about the exact objective weight of *Market Volume in the Region*, it is still reasonable to ensure that the result is maximally robust in this regard. Using the average objective weights that put *Aachen* in first place in the objective weight analysis, they conducted an extended investigation into possible thresholds that change the result. The starting point shows a considerable lead in utility for the location in *Aachen* but also a high deviation from the elicited objective weights for *Ecological Sustainability* and *Market Volume in the Region*, pointing to a strength in the first and a weakness in the latter for *Aachen*. Gradually increasing the objective weight for *Market Volume in the Region* shows that upon reaching an objective weight of 19.4 %, *Düsseldorf* becomes the best alternative yet again. The same happens at 9.8 % when the objective weight for *Ecological Sustainability* is decreased. A third analysis shows that even when trying to deviate as little as possible from the original objective weights, there is still a major deviation necessary to put *Aachen* in the first place.

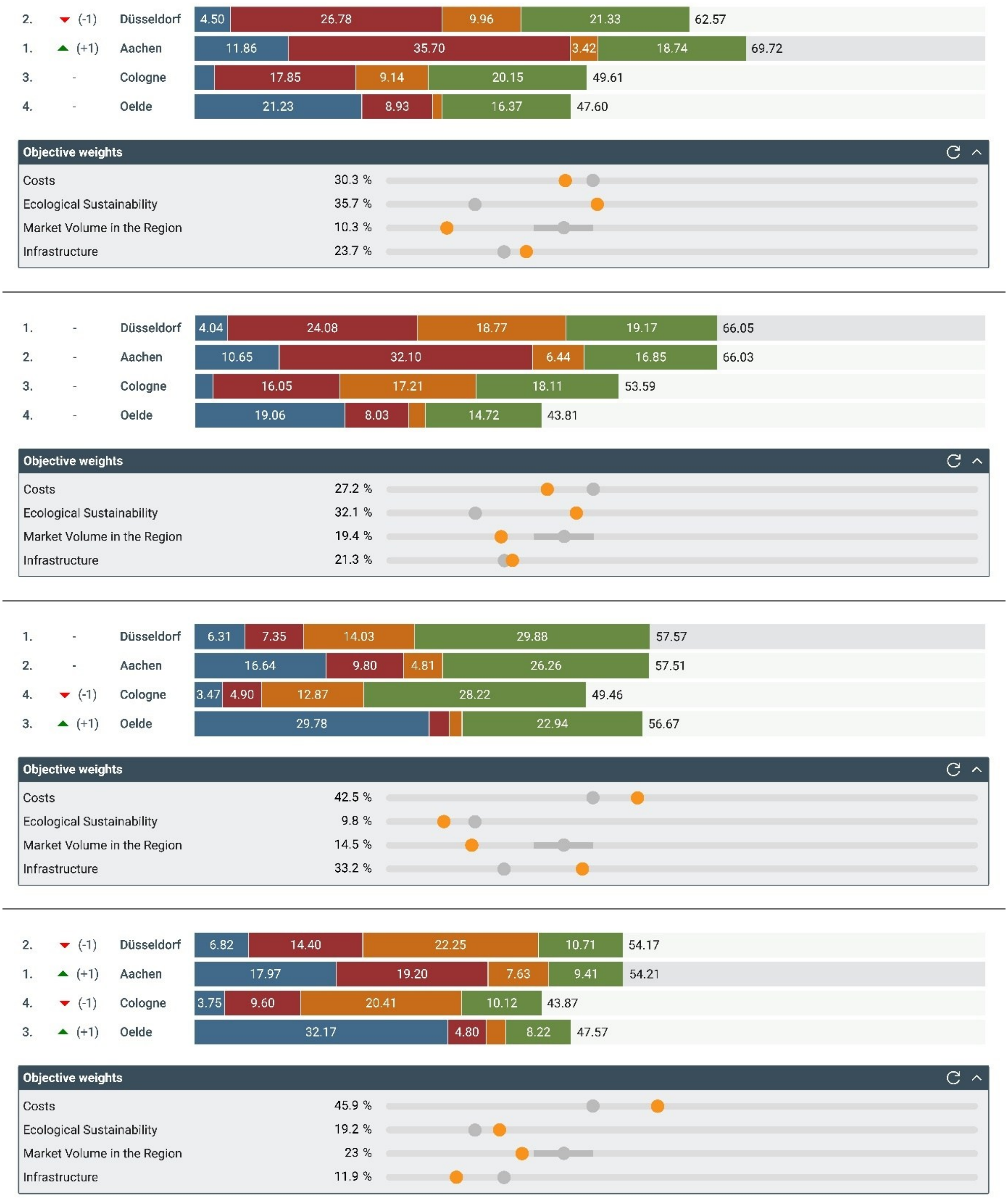

Figure 11. *Sensitivity analysis for the impact of objective weights with detailed utilities for the objectives Costs (blue), Ecological Sustainability (red), Market Volume in the Region (orange), and Infrastructure (green).*

9 Indicator impacts, tornado diagrams, and risk profiles

There are three ways to visualize uncertainties and their associated risks in the ENTSCHEIDUNGSNAVI: indicator impacts, tornado diagrams, and risk profiles.

9.1 Indicator impacts

Indicator scales are helpful in assessing consequences because they give DMs a more granular form of input. When they are combined with influence factors, the ENTSCHEIDUNGSNAVI can visualize the impact of the uncertainty for individual indicators on the overall consequence of an objective. Therefore, using indicator scales also makes it easier for DMs to gain insights into the main contributing factors for the consequences in a matrix cell and their associated risks. This can inform DMs of which uncertainties it would be most beneficial to gather more exact predictions and for which uncertainties the increased effort might be too costly.

The indicator impact diagram is available for every cell of the consequences table where an indicator scale is utilized. The diagram clearly shows which indicators have the greatest, the second greatest, etc., impact on the outcome for such a cell based on the respective existing FU. Depending on the type of influence factor used, i.e., user-defined or predefined, the DM is presented with different information and has different options to modify the diagram.

When the predefined influence factor is used, the impact of the indicators is initially determined under the assumption that either the Worst, Median or Best state occurs for the indicator for which the impact is being determined, and the Median state occurs for all other indicators (selection: simple variant). The DM can, however, choose to have the impact determined under the assumption that either the Worst, Median or Best state occurs for the indicator for which the impact is being determined and the states of all other indicators occur with a frequency according to their defined probability distributions (selection: probabilistic variant). To generate the information regarding the indicator impacts for us-er-defined influence factors, we treat the indicators as if their states were stochastically independent, even though they are not, and always calculate their impacts according to the probabilistic variant.

Figure 12 shows the indicator impacts on the objective *Costs* for the alternative *Cologne* in the ENTSCHEIDUNGSNAVI. 258 GmbH can choose between three different display options for the range of the results scale: total scale (the entire bandwidth of the scale is used as a basis), worst–best scenarios (the limit of the scale results from a combination of the indicated best or worst results of the indicators), and adjusted (the range adjusts to the actual width of the indicator impact diagram). Moreover, the indicators can be sorted by their influence on the result or in the order in which the DM defined them.

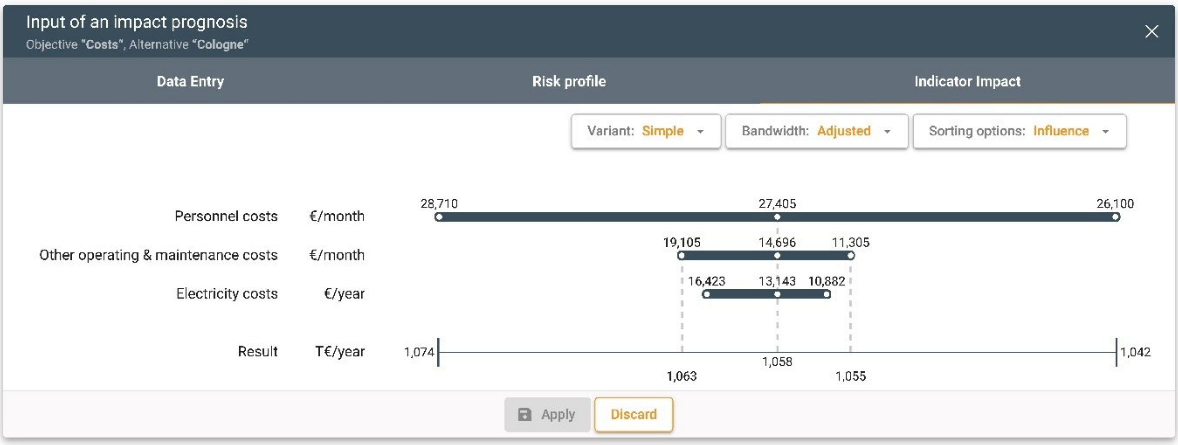

Figure 12. *Indicator impacts for the objective Costs in the alternative Cologne in the ENTSCHEIDUNGSNAVI.*

In this case study, the indicator *Personnel costs* has the biggest impact on the result of the objective *Costs* in the alternative *Cologne*, followed by the indicators *Other operating & maintenance costs* and *Electricity costs*. The indicator *Other operating & maintenance costs* causes the result to vary between 1063 T€/year and 1055 T€/year. Hovering over the bars will show these values to the DM.

9.2 Tornado diagrams

Similarly to indicator impact diagrams, tornado diagrams present the DM with information about the impact of uncertainties on the utility, however, on a larger scale. They show the resulting change in expected utility for an alternative based on certain events occurring, i.e., conditional expected utilities. These effects can be analyzed in isolation when choosing a single alternative or in comparison to another alternative. This comparison is particularly interesting if the decision is to be made between two alternatives. In such a case, the DM can clearly see under which circumstances one alternative is better and for which uncertainties. The influence factor with the greatest impact and, therefore, the widest bar is shown at the top, and the other influence factors follow in descending order of impact. This also sheds light on which uncertainties should preferably be analyzed further, e.g., through more precise probability estimates, if the DM wants to reduce uncertainty in the decision situation.

Figure 13 shows a comparison of two alternatives, *Cologne* and *Aachen*, in the tornado diagram. It helps the DM reflect how the uncertainties from the defined influence factors affect the evaluation of the alternatives.

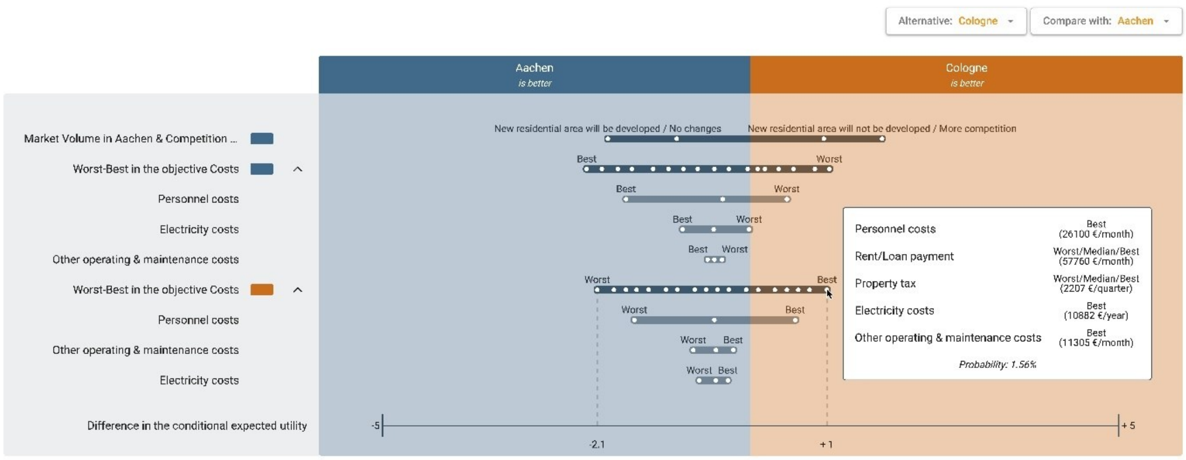

Figure 13. *Tornado diagram for overall result in the ENTSCHEIDUNGSNAVI.*

If an objective is assessed through multiple indicators and the assessment is based on the predefined influence factor, DMs can expand the influence factor on the left and see the isolated effects of uncertainty on the individual indicators.

In the case study, 258 GmbH selects a compared alternative (*Cologne*) on the top right (see Figure 13). It is evident in the comparison of the two alternatives that uncertainties have the greatest impact on the relative advantage. Colored rectangles next to the influence factor names indicate which alternative the uncertainty refers to. Here, the combined influence factor '*Market Volume in Aachen* & *Competition in Aachen*' has the greatest impact on the difference in the conditional expected utility, which varies between −1.9 and +1.8. Furthermore, 258 GmbH has the option to analyze the impact of the predefined influence factor in more depth. For the objective *Costs*, the conditional expected utility ranges between −2.1 and +1.0 because of the worst–best influence factor. By hovering over the individual points, DMs can see which indicator states lead to a certain conditional expected utility and how likely they are. With this illustration, the company can clearly see which states of the influence factor occur for the indicators when an alternative is better.

9.3 Risk analysis

The ENTSCHEIDUNGSNAVI offers multiple ways to analyze the risk associated with choosing an alternative. This analysis can be either conducted individually for every objective or aggregated over all objectives. When considering all objectives, it is also possible to analyze risk with and without the DM's preferences and objective weights.

9.3.1 Individual risk profiles and stochastic dominance

Risk profiles can be considered a tool for experts with profound knowledge in decision theory. They show the likeliness of exceeding results and can, therefore, be used to check for dominance. In some cases, this can help DMs eliminate alternatives from consideration to reduce the complexity of the model and, therefore, efficiently use the resources available.

A risk profile $R(x)$ shows the probability of exceeding a certain result x. This means that $R(x)$ has a complementary relation to the distribution function $F(x)$, i.e., $R(x) = 1 - F(x)$. The DM can generate a risk profile for each cell in the consequences table, i.e., $R_{ij}(x_i)$ with $x_i^- \leq x_i \leq x_i^+$, for an alternative regarding a single objective that is determined using the distribution function for the specific influence factor S_{ij}. Based on this, comparisons can be made with other alternatives regarding the same objective, and first-degree stochastic dominance (for this context, see Hadar and Russell (1969)) can thus be determined graphically for a single objective.

This type of dominance check is also conducted automatically, simultaneously over all objectives on a mathematical basis once the assessments of alternatives in the objectives have been completed. This is referred to as the dominance of alternative ξ over π if ξ stochastically dominates π in all objectives, i.e., if the risk profile for alternative ξ is identical to or above that of alternative π and above it for at least one point (see Formula (15)). Neither utility functions nor objective weights need to be defined for this.

$$R_{i\xi}(x_i) \geq R_{i\pi}(x_i) \mid 1 \leq i \leq |\mathbb{O}|, \forall\, x_i \in \left[x_i^-, x_i^+\right] \quad (15)$$

$$\textit{with at least one strong inequality for one } i_0, x_0$$

Alternatives that are dominated by another alternative are marked in red with an info button. Clicking on this info button shows which alternative(s) dominate(s) the one in question. DMs can use this information to carefully consider whether to analyze alternatives that are stochastically dominated for all objectives further. Even if an alternative is dominated, it can still be the best alternative in rare scenarios. If an alternative is dominated by multiple alternatives or an alternative for which the assessment depends on the same influence factors, DMs can likely exclude it from further analysis.

In the case study, the alternative *Düsseldorf* dominates the alternative *Cologne*, as shown by the red marking in Figure 14. However, *Düsseldorf* only stochastically dominates *Cologne* over all objectives. While *Cologne* also achieves better outcomes than the other two alternatives for most objectives, *Aachen* and *Oelde* achieve better outcomes than *Düsseldorf* for *Ecological Sustainability* and *Costs*, respectively.

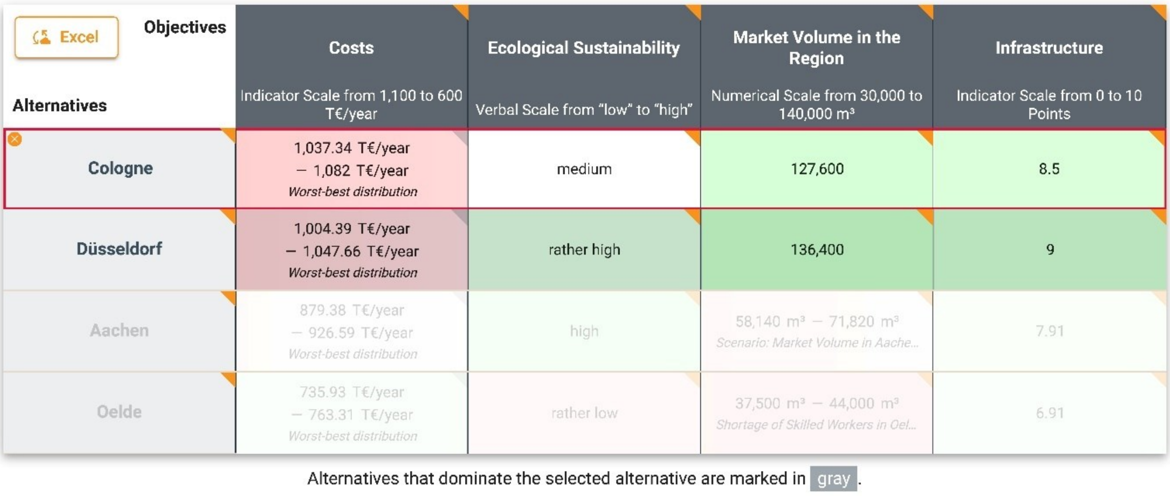

Figure 14. *Checks for stochastic dominance in the* ENTSCHEIDUNGSNAVI*.*

9.3.2 Risk comparison

DMs can also analyze the risk associated with the alternatives on the basis of their preference statements regarding utilities and objective weights. The risk comparison shows the overall utilities of the alternatives, i.e., $R_j^{overall}(\omega)$, with $0 \leq \omega \leq 1$, aggregated across all objectives. The x-axis then shows the possible utilities of the alternatives, each of which only results under the condition that certain overall forecast scenarios occur. In this representation, the utility functions used are, strictly speaking, no longer interpreted as utility functions but as value functions. For the difference between these two functions, see Dyer and Sarin (1979, 1982). The information shown in the diagram is generated via a Monte Carlo simulation. The underlying data are equivalent to those generated in the robustness check when only FUs, i.e., influence factors, are considered.

Figure 15 shows the risk comparison for the case study. It confirms the previous finding that opening the wood warehouse in *Düsseldorf* is a good choice, as it further confirms the previous finding regarding the alternative's robustness. By analyzing the diagram, 258 GmbH can see that *Düsseldorf* can be regarded as the best alternative when looking at the likeliness of attaining an overall utility level, as it has the highest probability of all alternatives exceeding any given utility.

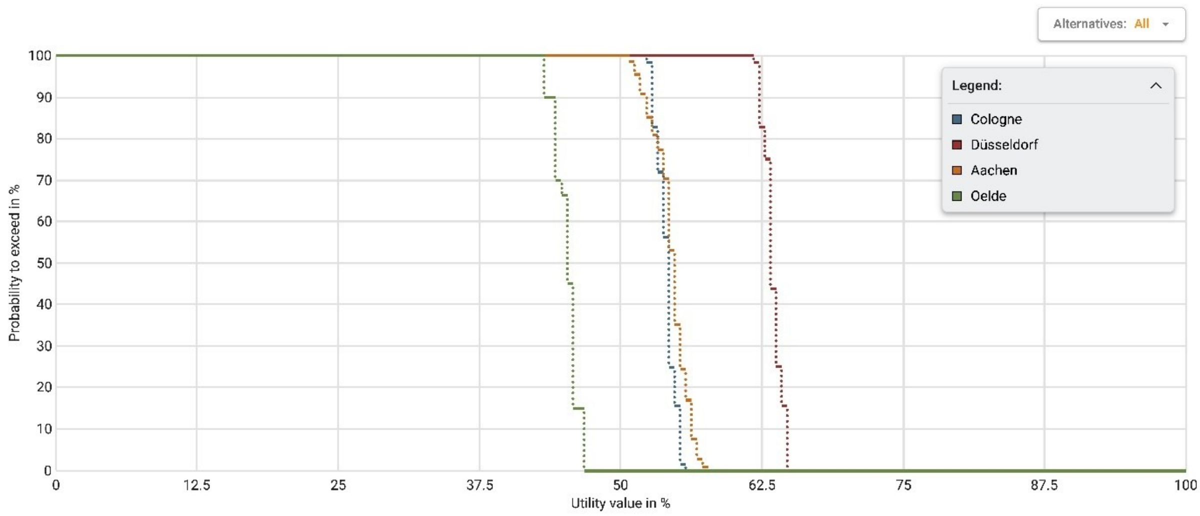

Figure 15. *Risk comparison in the* ENTSCHEIDUNGSNAVI*.*

10 Conclusion

This paper shows how uncertainties are integrated into a multi-criteria decision analysis in the DSS ENTSCHEIDUNGSNAVI and how the methods and analyses can help support the desired reflection of the DM. Therefore, different types of uncertainties are described, and various methods are explained to analyze these uncertainties. The basic mathematical model in the system is the additive utility function of MAUT, which allows two types of uncertainties: forecast uncertainties (FUs) and parameter uncertainties (PUs). FUs can be modeled by influence factors in the consequences table. In the ENTSCHEIDUNGSNAVI, a distinction is made between user-defined and predefined influence factors. User-defined influence factors can either be individual or combined influence factors. The predefined influence factor is based on a 'worst-median-best' distribution. PUs can be used if DMs cannot precisely determine the parameters of a decision model and only imprecise information is available. In the ENTSCHEIDUNGSNAVI, PUs can be modeled for three different parameters: utility functions, objective weights, and the probabilities of the influence factors. With the help of PUs, DMs can define the intervals of these parameters. Furthermore, the paper presents various methods and analyses to check the impact of uncertainties, namely, methods for checking the robustness of the result, objective weight analysis, sensitivity analysis, and indicator impacts, tornado diagrams, risk profiles.

In the robustness check, a Monte Carlo simulation is used to check how robust the result is regarding FUs and PUs. This analysis gives DMs a better insight into how stable the ranking of the alternatives is regarding previously determined uncertainties. The objective weight analysis can be used as a starting point to see which objective weight intervals would result in a different ranking of alternatives. In the sensitivity analysis, DMs can analyze which parameters impact the result of the decision and to what degree. It is possible to vary the following parameters: objective weights, utilities, probabilities of FU, consequences, and indicator weights. Indicator impacts, tornado diagrams, and risk profiles help DMs visualize risks in the decision model. With the help of checks for stochastic and simulation dominance, DMs can identify dominated alternatives without determining utilities or objective weights.

The limitations of the proposed mathematical model under MAUT shown in this paper are the general applicability of the additive model and the use of expected utilities, according to von Neumann and Morgenstern (1961). We assume (and demand) that the objectives of the decision situations are fundamental, measurable, redundancy-free, complete, and preference-independent. Other limitations are of a technical nature with regard to the possible and necessary number of iterations in the Monte Carlo simulation to reach a satisfactory validity of the results. In the case of the objective weight analysis, this is counteracted by gradually decreasing the granularity with which the objective weights are altered with an increasing number of objectives. Furthermore, due to the high development speed of browser engines and their supported technology, the ENTSCHEIDUNGSNAVI officially only supports the last three major browser versions for the most common browsers, i.e., Firefox, Chrome, Safari, and Edge. While it will mostly run on older browsers as well, some functionalities might be limited or missing, as the tool often relies on new browser functions to support the best user experience possible.

In the future, further functionalities will be added to the ENTSCHEIDUNGSNAVI. For example, it should soon be possible to use correlated combined influence factors to display FUs in the consequences table. Furthermore, a team version is currently being developed, and it will be possible to work with several team members on a decision situation and analyze it. The uncertainties in such a team decision result from the different preferences and information bases of the individual team members. It is important to create transparency here so that a considered decision can be made in the interests of all team members.

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Paper 3: Improving value-focused decision-making through value-nudging

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Contents

Improving value-focused decision-making through value-nudging

The Value-Focused hinking approach was developed by Ralph Keeney to improve decision-making using personal values. Personal values are an overarching concept shaping people's motives and guiding their decisions and actions. Therefore, important life choices with far-reaching consequences should be considered and made following one's personal values. In this study, we analyzed whether DMs can be nudged toward more value-focused and, thus, better decisions.

To do this, we analyze data from German students with the help of the ENTSCHEIDUNGSNAVI, a free-to-use decision support system based on value-focused thinking and multi-attribute utility theory. We compared personal decisions of two groups: Group value anchoring (GroupVA), where personal values served as a passive anchor, and Group value nudging (GroupVN), where the five most important personal values were actively incorporated into the objective formulation as a nudge. The analysis focuses on the weighting of objectives based on values. Additionally, we analyzed various decision topics.

Our findings indicate that nudging significantly increased the impact of personal values in private decision-making, with GroupVN showing an 18.2 percentage points higher proportion of value-based objective weights than GroupVA. The value 'health' saw the largest increase (from around 4 to 12 %), probably also due to COVID-19-related discussions about health. By contrast, the importance of 'financial security' decreased, suggesting that nudging can help individuals recognize other important values beyond financial concerns. Value-nudging is particularly effective when the values are not obvious in the decision situation. We assume that value-nudging can improve the quality of the decision analysis.

Keywords: value-focused thinking; personal values; nudging; anchoring; decision support system; objective weights

1 Introduction

Every day consists of a sequence of numerous different decisions and their resulting actions. On average, a person makes about 35,000 decisions per day (Sollisch 2016). Many are commonplace and even happen unconsciously like we decide to stand up every morning or drink when we are thirsty. But some of them are of great importance and may even be life-changing, like what I should do after my studies or where I would like to live in the following years. Our brain controls both types of decisions.

Kahneman (2011) states that our brain works in two systems when we decide: System 1 represents intuition and instinct. The characteristics of this system are fast, subconscious, automatic, and associative. This system handles decisions that we make every day, resulting in routines. System 2 represents the rational thinking. Its characteristics are slow, conscious, effortful, and logical. This system handles all the complex decisions we make in our lives, like career or housing decisions. As System 2 is demanding and overwhelming for many people, researchers have examined how to help people with complex decisions.

According to the principle of alternative-focused thinking, people typically face complex System 2 decisions by identifying obvious alternatives. In most cases, they perceive decisions as specific problems to be addressed rather than recognizing them as general opportunities to be taken advantage of (Ley-Borrás 2015). The problem with alternative-focused decisions is that they focus very narrowly on the possible spectrum of options for action (Keeney 1994). Especially when it comes to far-reaching decisions, such as choosing a course of study and a career, choosing where to live, or purchasing expensive goods, they may not have better alternatives in mind (Keeney 1992).

To address this problem, Keeney (1992) developed the value-focused thinking (VFT) approach, which places the DM's values in the foreground in contrast to alternative-focused thinking: VFT „involves clearly defining and structuring your fundamental values in terms of objectives and using these objectives to guide and integrate decision-making“ (Keeney 1994). Keeney also recommends not only reacting to a problem but actively taking decision opportunities by consciously identifying new and creative alternatives. With Keeney’s approach, DMs can proactively find more alternatives as a means to achieve long-term values (Karelaia and Reb 2015) and, thus, make better and more sophisticated decisions.

To further guide people - either consciously or unconsciously - to a decision that is good for them, the concepts of nudging (Keeney 2020) or anchoring (Furnham and Boo 2011) are frequently used. These lead people to make decisions that are more beneficial to them simply by designing the decision-making environment. With our study, we want to investigate how nudging can be used to make better, more value-oriented decisions. The following subsections provide an overview of the relevant literature and the current study.

1.1 Value-focused decision-making

Many individuals do not think about their objectives or values when they decide. They focus on the given selection of alternatives within the decision-making situation and choose the best alternative from these. Such alternative-focused thinking is reactive and backward because options for action that go beyond the obvious alternatives are usually not considered by the decision-maker (DM) (Keeney 1992). This understanding of human decision-making and action is based on classical motivation theory. The central element here is the current motivation as a result of the interaction of situation-related factors in the form of opportunities and possible incentives and of person-related factors in the form of directly relevant individual needs, values, and objectives. It controls the person's decisions and actions, taking into account the DM’s expectations regarding the consequences (Kehr 2001, Heckhausen and Heckhausen 2018, Rheinberg and Vollmeyer 2018).

In contrast, the VFT approach developed by Keeney (1992) places the DM's values in the foreground. The DM first determines his personal values before deriving his specific objectives and finally considering the alternatives. This is an essential point because, in fact, alternatives are relevant only to address personal values (Keeney 1994). Furthermore, Keeney suggests that individuals should proactively seek out decision opportunities instead of reacting to a problem. Consequently, VFT is a proactive approach in which decisions are not perceived as context-specific problems but are framed as general opportunities that can be seized (Ley-Borrás 2015) and where alternatives are proactively created as means to achieve long-term values (Karelaia and Reb 2015). The VFT decision-making process includes the following steps, whereby Steps 2 and 3 describe an iterative process of articulating values and creating alternatives (Keeney 1992):

1. Identification of a decision opportunity (sometimes based on a decision problem).
2. Specification of one's personal values and formulation of the corresponding objectives.
3. Finding and proactively creating alternatives.
4. Evaluation of the alternatives.
5. Actual decision-making: Selection of one alternative.

Regarding VFT, spending effort defining and structuring a problem and evaluating the relative advantages and disadvantages of different strategic options is essential. However, DMs frequently have no or at least not enough experience using objectives to generate alternatives (Selart and Johansen 2011). Moreover, Thaler and Sunstein (2021) point out that many individuals lack sufficient decision-making skills when faced with poorly structured situations. Most individuals never receive formal training in effective decision-making and often have limited practice in this skill (Bond et al. 2008, Larrick 2012, Hammond et al. 2015, Keeney 2020). Additionally, few people recognize the degree to which their decision-making is biased (Milkman et al. 2009, Morewedge et al. 2015, Scopelliti et al. 2015) and strays from the principles of decision quality (Spetzler et al. 2016). Therefore, it is important to actively train and support more people in making value-focused decisions (Keeney 2004) to help them improve their decision quality.

The general agreement within decision sciences is that enhancing individual decision-making is a valuable objective. An enhancement in terms of VFT is beneficial, as improved decision-making typically raises the likelihood of attaining the desired outcomes (Keren and de Bruin 2003, Hammond et al. 2015, Spetzler et al. 2016) and, therefore, leads to increased satisfaction with the decision or even enhanced life satisfaction (Siebert and Kunz 2016, Siebert et al. 2020). For example, Siebert and Keeney (2015) and Siebert (2016) show that prompting with objectives increases the number and quality of alternatives. Based on Howard's (1988) definition that "a good outcome is a future state of the world that we prize relative to other possibilities" and "a good decision is an action we take that is logically consistent with the alternatives we perceive, the information we have, and the preferences we feel", VFT can help people to make better System 2 decisions.

1.2 Human values

Various definitions of values can be found in the scientific literature. Kluckhohn (1951) defines values as „a conception, explicit or implicit, distinctive of an individual, or characteristic of a group, of the desirable which influences the selection from available modes, means, and ends of action". He distinguishes between two types of personal values: implicit values, which are unobservable, and explicit values, which a person attributes to himself. Additionally, Kluckhohn differentiates between the values of an individual and the values of a group. Rohan (2000) shows a similar differentiation between the so-called 'intrapsychic value systems' that are located within a person (inner values) and certain 'ideological value systems' (external values) regarding value priorities promoted by groups (e.g., religious communities, societies, cultures).

With regard to inner values, he also distinguishes between the ‘personal value system’ that includes the own (implicit or explicit) values of a person and several ‘social value systems’ that refer to others’ expectations. Each of these value systems includes a finite number of universally important values. Differences between two people's values relate to the relative importance they assign to those values. Inner values develop over time as children develop into adolescents and adults. They are shaped by external values through upbringing, training, contact, etc. (Staehle et al. 2014). From adulthood onwards, a person's inner values are largely consolidated and stable (Rokeach 1973, Schuster et al. 2019).

Personal values can be understood as an overarching concept that shapes people's motives and guides their decisions and actions (motivated action as an interaction between person and situation) (Heckhausen and Heckhausen 2018). Schwartz (2012) refers to personal values as a construct that reflects what is important to us in life. Therefore, personal values serve as standards or criteria when evaluating other people, actions, or policies and have a great impact on human attitudes, opinions, and cognitively controlled behavior (represented by System 2). In contrast, personality traits have a greater influence on intuitive, more emotional behavior (represented by System 1) (Roccas et al. 2002). Consequently, values are a dominating force in life and an integral part of decision-making processes and the justification of decisions (Allport 1961, Schwartz 1992, Rokeach 2008).

In the context of the VFT approach, Keeney (1994) defines personal values as follows: “Values, as I use the term, are principles for evaluating the desirability of any possible alternatives or consequences. They define all that you care about in a specific decision situation. It is these values that are fundamentally important in any decision situation, more fundamental than alternatives, and they should be the driving force for our decision making.”

1.3 Anchoring and nudging

The anchoring effect is one of the most robust cognitive heuristics (these are methods that DMs use to quickly reach a decision when having limited knowledge and little time) in behavioral economics. It can be easily observed, especially in estimation tasks or under uncertainty, and has already been studied numerous times (see Furnham and Boo (2011) for an overview of relevant studies). A well-known numerical experiment by Ariely et al. (2003) is the question of willingness to pay for a coffee mug or a bottle of wine, whereas the individuals were previously asked to provide the last two digits of their social security number. Individuals whose social security number ended with a higher number were willing to pay between 60 % and 120 % more than individuals with lower digits. In general, it can be stated that people base their estimates on initial values or reference values, whereby the adjustment for the specific question is insufficient. The result is that estimates based on different anchors are significantly different. The anchor doesn't even have to be thematically related to the actual question; it is generally given too much weight (Tversky and Kahneman 1974, Kahneman et al. 2011). Most anchoring and adjustment studies conducted to date consider situations in which anchors and adjustments are numerical. But this effect can also be observed in perceptual domains, with test subjects orienting themselves on noises, textures, or visual impressions (Jain et al. 2021).

Anchoring is often examined in the context of systematic errors in decision-making behavior, such as biases and fallacies. It has been shown to influence both laboratory and real-world judgments in various domains, such as general knowledge, consumer purchases, forecasting, or legal decisions (e.g., Wansink et al. (1998), Critcher & Gilovich (2008), Englich et al. (2006)). However, anchoring can also be used to unconsciously lead people to make better decisions within the framework of VFT: Simply designing the decision-making environment by actively asking them to reflect on their personal values before formulating their objectives can lead them to make a decision that is more advantageous to them and thus improve the quality of the decision.

While anchoring usually unconsciously influences people in their answers or decisions, nudging represents a more active form of influence by using framing effects to initiate smart decisions and thereby guide people on the right path or, in the best case, to benefit the general public. The concept goes back to Thaler and Sunstein (2021). So-called Nudges are interventions that steer people in certain directions but, at the same time, allow them to go their own (different) way, like recommendations, reminders, or warnings (Thaler and Sunstein 2021). Sunstein (2016) even distinguishes between System 1 and System 2 nudges. System 1 nudges are created in such a way that they address the subconscious, for example, by conveying hope or fear. System 2 nudges are educative nudges and, therefore, “attempts to strengthen the hand of System 2 by improving the role of deliberation and people’s considered judgments” (Sunstein 2016).

The standard example is nudging a healthier diet: By placing fruit and other healthy snacks in an easily accessible and visible location (on the shelf at eye level or at checkout counters), they provide a nudge and can encourage shoppers to eat healthier. Unhealthy snacks, on the other hand, should be placed in less accessible places, for example, at the top or bottom of the cupboard. Hummel and Maedche (2019) give a broad literature review of the effectiveness of nudging by analyzing 100 studies on the topic in different application contexts, e.g., health, environment, or privacy. Moreover, nudges also became very present in the COVID-19 pandemic, e.g., to increase hand hygiene (Weijers and de Koning 2021) or to increase COVID-19 vaccinations (Dai et al. 2021). One aspect is common to all described applications of nudging: people still have a choice and can act autonomously, but there is a friendly nudge in the desired direction of behavior.

1.4 The current study

To the best of our knowledge, no study has yet examined how nudging can help people to make more value-oriented personal decisions. Our study addresses this research gap by analyzing whether DMs can be nudged toward more value-focused and, thus, probably better decisions. In doing so, we compared two groups of participants: One control group in which personal values only serve as an anchor for the formulation of the objectives and one group that was actively nudged towards more value-focused decisions. Such an educative nudge can be understood as a System 2 nudge according to Sunstein's (2016) definition. In particular, we address the following research questions:

- Which values are important to DMs in private decisions?
- Does nudging increase the impact of the five most important personal values in private decisions?
- What impact does each of the five most important personal values have on private decisions before and after nudging?

To collect the data, we use the decision support system ENTSCHEIDUNGSNAVI (von Nitzsch et al. 2020, Hannes and Nitzsch 2024, Peters et al. 2024). The ENTSCHEIDUNGSNAVI, an open-source web tool, supports DMs in making value-focused and reflective decisions by guiding them through a five-step process based on VFT (Keeney 1992) and multi-attribute utility theory (MAUT) (Keeney and Raiffa 1976). The tool aims to train the DMs so that future decisions can be made in a reflective and value-focused manner without the help of a tool.

The paper is structured as follows: Section 2 introduces the five-step process of the ENTSCHEIDUNGSNAVI, and Section 3 explains our study's method. The results are presented in Section 4 and discussed in Section 5. Finally, Section 6 concludes the paper.

2 Problem structuring with the ENTSCHEIDUNGSNAVI

The ENTSCHEIDUNGSNAVI is an open-source (web)tool that supports DMs in a reflective decision-making process. The process is based on VFT (Keeney 1992) and MAUT (Keeney and Raiffa 1976) and consists of five steps: 1. Formulation of the decision statement, 2. Development of the fundamental objectives, 3. Identification of alternatives, 4. Setting up a consequences table, and 5. Evaluation based on preference statements. In the following subsections, we explain the main functionalities relevant to our research. For a detailed conceptual description and explanation of the ENTSCHEIDUNGSNAVI, see von Nitzsch et al. (2020), Peters et al. (2024), and Hannes and von Nitzsch (2024).

2.1 Formulation of the decision statement

The first step, namely the formulation of the decision statement, is based on the work of Hannes and von Nitzsch (2024). It serves to clarify the decision situation. The goal is to create a broad decision statement for new, creative alternatives. This is achieved with the help of four sub-steps: 1. First draft, 2. Fundamental values, 3. Impulse questions, and 4. Review.

In the first draft, DMs can frame the decision statement as it initially comes to mind. In addition, DMs can document all assumptions and preliminary decisions. To further define the decision statement, it is also possible to identify decision statements related to the pending question, which will be considered later.

The second sub-step, fundamental values, removes existing blinders and consciously encourages broader thinking to counteract possibly too narrow-minded thinking. This sub-step is of central importance for the research in this paper. It requires a preoccupation with one's fundamental personal values by presenting several values in a list to be prioritized relative to one another. This is significant because, as described in 1.2, personal values can be understood as an overarching concept that shapes people's motives and, therefore, guides their decisions.

Consequently, it makes sense that this valuation forms the anchor when defining the objectives of the decision-making situation in the second step. The values used in the ENTSCHEIDUNGSNAVI are based on the findings of various research (Rokeach 1973, Beatty et al. 1985, Maio 2010, Schwartz and Cieciuch 2022) and refer to the individual's fundamental 'personal values' (of the personal value system), precisely the self-reported explicit personal values (see Section 1.2) of a single person that determine what they consider important in their life (Friedman et al. 2013, Sagiv and Schwartz 2022). The development team of the ENTSCHEIDUNGSNAVI elaborated on this list in a brainstorming session with several experts. Figure 1 shows a screenshot of this sub-step in the ENTSCHEIDUNGSNAVI, where the values used for this study have already been prioritized in an exemplary way.

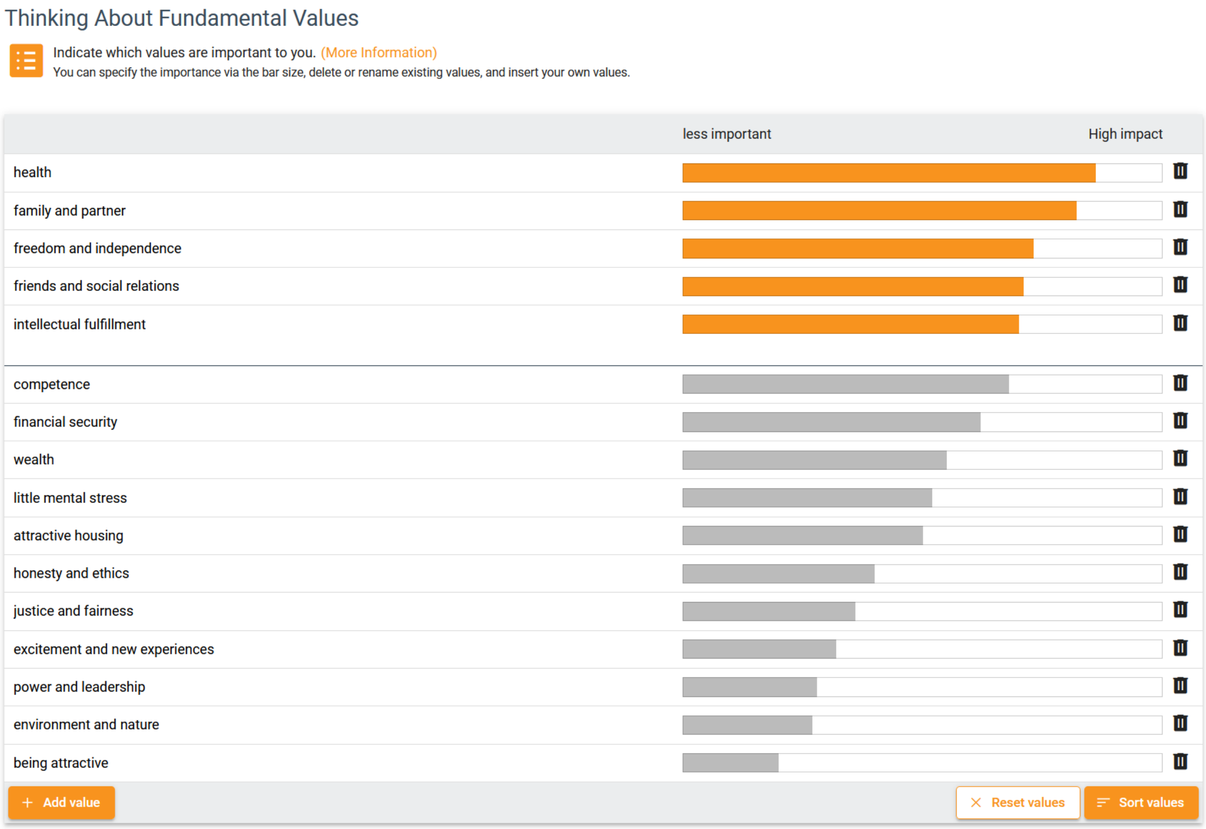

Figure 1. *Prioritizing fundamental values in the ENTSCHEIDUNGSNAVI.*

In the third sub-step, impulse questions, DMs are asked a series of questions designed to help them break down any restricted thought patterns related to the decision situation. Their answer notes can give thought impulses and help them consider a more comprehensive formulation of the decision statement. In the fourth sub-step, review, the results of sub-steps 1 to 3 are summarized. The task of the DMs is to formulate the decision statement more openly and broadly, considering an expanded view based on the previous results. At the same time, assumptions and preliminary decisions can be adjusted accordingly.

2.2 Development of the fundamental objectives

In the second step, the DM develops the fundamental objectives. Fundamental objectives describe the aspects of key importance in the decision-making situation. For a reflective decision-making process, it is necessary to ensure that all relevant aspects are covered, i.e., that the formulated objectives are complete. Additionally, the fundamental objectives should have no or just minimal overlap and be evaluated independently. They should also clearly distinguish between different alternatives.

Developing an objective system that fulfills all these requirements is a challenging task that requires support. The ENTSCHEIDUNGSNAVI assists DMs with various sub-steps. In the first sub-step, DMs start with an initial brainstorming to identify as many relevant aspects as possible. In the second sub-step, DMs should make further considerations, as studies have shown that DMs can name many more important aspects if they are explicitly asked to do so (Bond et al. 2008). In the third sub-step, DMs should build an initial objective hierarchy. The DM examines all aspects already noted to determine whether they are related to each other, e.g., in a means-ends-relationship. With successive incorporation of all aspects into the objective hierarchy, in which these relationships are considered, a first structured representation of the objective system is created. In addition, the fundamental objectives are worked out through these investigations and separated from the means objectives. In the fourth sub-step, examples and suggestions, the ENTSCHEIDUNGSNAVI provides example objective hierarchies and lists with topic-related objectives to add aspects overlooked so far to one's hierarchy. Some objectives are provided with suitable explanatory texts and, in most cases, contain suitable sub-objectives. DMs can transfer suitable objectives from the example objective hierarchy to their objective hierarchy. After this initial structuring, it is usually necessary to revise the objective hierarchy so that the requirements mentioned above are met. The resulting hierarchy ideally comprises five to seven fundamental objectives at the first level. The further hierarchical levels of the revised hierarchy contain the corresponding subordinate objectives and mean objectives, which can still be a valuable aid in subsequent steps, e.g., in the definition of measurement scales. Figure 2 shows an example of an objective hierarchy for job and life planning. DMs can comment on their objectives and explain them in more detail by clicking on the gray triangles (see fundamental objective 'financial security'). The triangles turn orange if a comment has been added (see all other fundamental objectives).

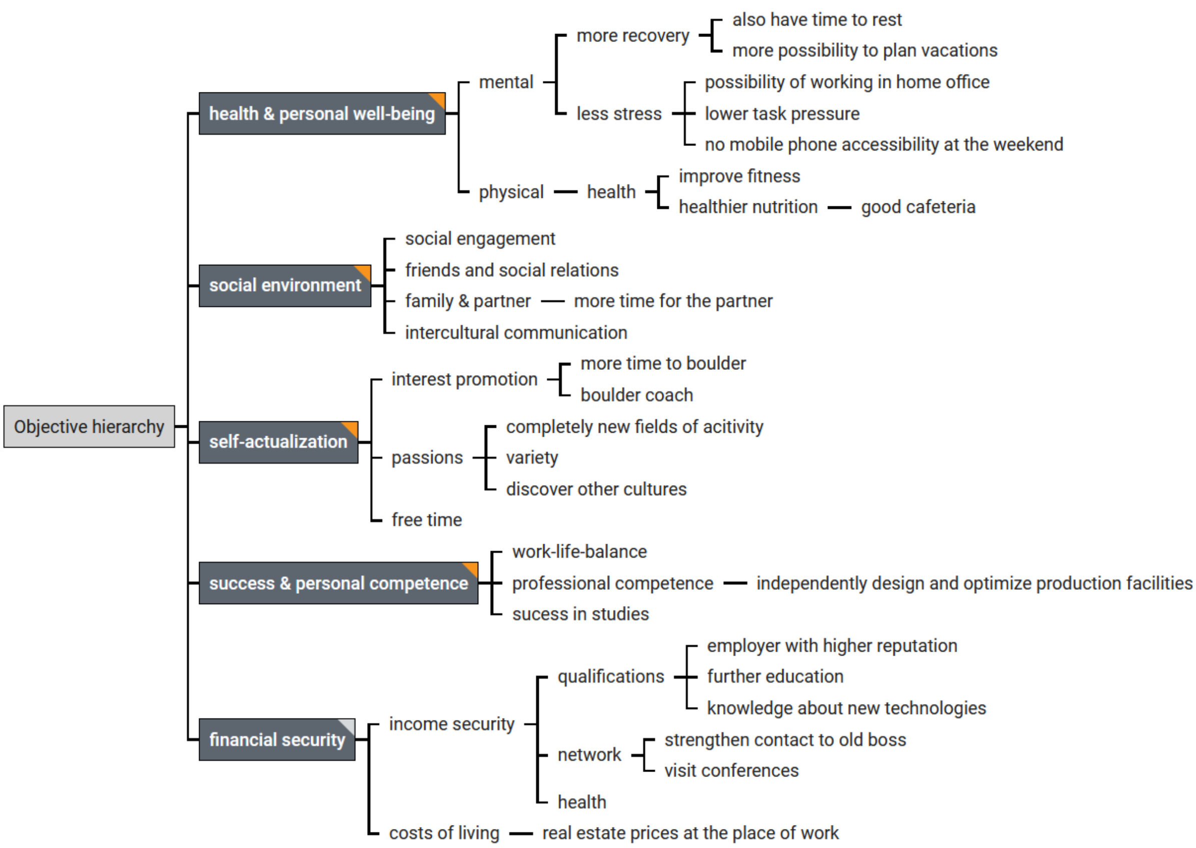

Figure 2. *Objective hierarchy in the* ENTSCHEIDUNGSNAVI.

2.3 Identification of alternatives

Reflective decision-making requires considering the entire scope of action without overlooking any alternatives. Studies have shown that DMs often fail to thoroughly evaluate decision-making situations in this step and neglect to generate attractive alternatives themselves (Siebert and Keeney 2015). For this reason, the sub-steps implemented in the ENTSCHEIDUNGSNAVI concentrate on stimulating creativity while always drawing attention to the fundamental objectives based on the VFT concept so that particularly attractive alternatives are added. As the sub-steps are unimportant for this paper, they are not described in detail here.

2.4 Setting up a consequences table

A consequences table is set up in the ENTSCHEIDUNGSNAVI by filling a matrix with alternatives in the rows and fundamental objectives in the columns. To achieve high quality, DMs must define suitable measurement scales for each objective, relevant uncertainties must be reasonably integrated, and the estimates have to be given without biases based on information sources that are as good as possible. The ENTSCHEIDUNGSNAVI supports DMs by providing different types of measurement scales and influence factors (Peters et al. 2024) as well as information about possible biases (e.g., Montibeller and von Winterfeldt (2015)).

2.5 Evaluation based on preference statements

In the last step, DMs must determine their utility functions and objective weights to enable the calculation of the utilities to evaluate the alternatives. Determining objective weights relies on the trade-off method (Keeney and Raiffa 1976), which is rather challenging for DMs. However, the method is scientifically based and prevents the effect of objective ranges on their weights (von Nitzsch and Weber 1993). In this method, DMs have to make trade-off statements, i.e., determine how much deterioration in one objective they will accept for an improvement in another. If the DM has defined n objectives and must determine n objective weights, n-1 trade-off statements are required from the DM. Using the trade-off statements and the fact that the sum of all objective weights must equal one, the objective weights can be calculated. Thus, the objective weights reflect the relative importance of the individual objectives depending on preferences. The ENTSCHEIDUNGSNAVI supports DMs via diagrams of indifference curves and different verbal explanation variants of the curves. Finally, the ENTSCHEIDUNGSNAVI presents a ranking of alternatives derived from the calculated expected utilities (von Nitzsch et al. 2020, Peters et al. 2024). The DM can analyze the final ranking with a broad range of evaluation tools that enhance understanding of the analysis and assist the DM in explaining potential discrepancies between head and gut feeling.

3 Method

As described above, our study aims to examine how nudging can help people make more value-oriented personal decisions by comparing two groups of participants. The following sections describe our two data sets, the procedure we used to nudge one group of participants actively, and the resulting analyses to evaluate the relevance of nudging regarding more value-focused decision-making.

3.1 Data set

The data set was collected over five winter terms (2019-2023) as part of a voluntary project in the course 'Decision Theory' at a large university in Germany. In the project, students had to meticulously analyze an important personal decision using the decision support system (DSS) ENTSCHEIDUNGSNAVI and work through each of the steps described above in Section 3. Incentives for participating in the study were the professional analysis of an important personal decision and a bonus on their course grade in 'Decision Theory' if they worked on it appropriately. Granting bonuses for submitting voluntary work that enhances students' understanding of lecture material is standard practice at this university and is formally endorsed by the study and examination regulations. To participate in the study, the students had to create an account within the DSS ENTSCHEIDUNGSNAVI. In the registration process, the students were asked to check a box if they allowed us to use their data for scientific purposes. The exact statement is: „I hereby agree that the data entered by me may be used anonymously for scientific purposes and to improve the tool". At the university, utilizing data from voluntary work for research purposes is only allowed when an opt-in procedure is in place, and students are eligible for a bonus if they choose not to share their data, which is the case here.

Overall, 611 full-time students mainly enrolled in business administration and industrial engineering with business administration participated in the study. Among the 611 participants, there were 348 males, 192 females, and 71 participants who did not indicate their gender. Decision experts manually reviewed all decisions, and decisions that were not carefully processed were sorted out. As the participants took part in the lecture 'Decision Theory', they had extensive expertise in the field of value-focused decision-making at the time of the study.

On average, the participants spent 10.72 hours ($\sigma = 7.31$ hours) on their decision situation. They identified 4.77 ($\sigma = 1.21$) objectives and 5.8 ($\sigma = 1.98$) alternatives. The number of objectives in the decisions ranged from 2 to 12 and from 2 to 19 for the alternatives. The students were supported during consultation hours, in the lectures, in workshops, and by a help page in the ENTSCHEIDUNGSNAVI. The comprehensive level of support in the ENTSCHEIDUNGSNAVI, the relatively long processing time, and especially the manual review of the decisions let us assume that the decision situations were carefully elaborated.

3.2 Procedure and analyses

In our study, we want to analyze which values are important to DMs in private decisions, how value-focused decisions are made, and if decisions are made more value-focused if nudging is used. Therefore, we split the participants into two groups and compared them in our study: Group value-anchoring (GroupVA) and Group value-nudging (GroupVN). GroupVA comprises data from the years 2019 to 2020. In this setting, the students went through the process and had to reflect on their values before assessing their objectives regarding their decision statement (see Sections 2.1. and 2.2.). Thus, the values serve as an anchor for the upcoming formulation of the fundamental objectives. This group served as a control group. GroupVN includes data from the years 2021 to 2023. For this group, the process was the same. However, we implemented a nudging measure in the ENTSCHEIDUNGSNAVI to try to improve value-focused decision-making. Therefore, the ranking of values in Step 1 did not only serve as an anchor for value-focused decisions like in GroupVA but also was used as a nudge to improve value-focused decision-making. In GroupVN, the five highest-ranked values in Step 1 were automatically adopted in the second step of setting the fundamental objectives and, thus, actively served as a starting point and initial guide for defining the hierarchy of objectives. Consequently, in contrast to GroupVA, participants of GroupVN did not have to start from scratch in this step.

The study consists of a preliminary analysis and the analyses of the three research questions. The **preliminary analysis** shows the descriptive findings of both groups' data sets. We analyzed the gender distribution, which decision situations were chosen, how often they were chosen, the average, and the minimum and maximum number of fundamental objectives defined by the participants.

To address the **first research question**, 'Which values are important to DMs in private decisions?', we determined how often a value was ranked among the top five most important values by the participants in Step 1. We conducted this analysis for all decision-making situations and the respective decision topics for GroupVA and GroupVN.

To address the **second research question**, 'Does nudging increase the impact of the five most important personal values in private decisions?', we compared GroupVA and GroupVN and analyzed the impact of value-based objectives in decisions based on objective weightings. In contrast to GroupVA, in GroupVN, the five highest-ranked values (Step 1) were automatically suggested as possible objectives (Step 2). We expected that this nudge would increase the proportion of objective weights based on values. We specifically analyzed how much more value-focused decisions were made and whether there were differences in decision-making contexts. To do this, we compared the five highest-ranked values defined in Step 1 of the ENTSCHEIDUNGSNAVI with the fundamental objectives defined in the objective hierarchy in Step 2. Three decision experts independently assigned the values to the fundamental objectives. First, the experts checked the values and objectives for identical wording. Here, the assignment is clear. However, it became more difficult if the wording was not identical. In this case, the experts looked at the objective hierarchy or the additional explanation of the objective in the comment field. If the value was mentioned in the explanation or means objectives, the value was also assigned to the objective. If the value was mentioned in more than one objective, the value was assigned to several objectives. If one objective mentioned several values, these values were assigned to the same objective. The assignment was manually done for every decision situation. After the first assignment, the inter-rater reliability of expert assessments was tested using Fleiss' Kappa (Fleiss 1971), an extension of Cohen's Kappa (Cohen 1960). If the experts' assessments matched, the assignment was clear. If there were discrepancies, these were discussed in more detail and assigned according to the majority principle. The following example in Figure 3 is intended to clarify the procedure. Figure 3 shows the assignment of the five most important values based on Figure 1 to the fundamental objectives of the objective hierarchy in Figure 2. The value 'health' was assigned to the fundamental objective 'health & personal well-being' as the wording is the same. The values 'family and partner' and 'friends and social relations' were assigned to the objective 'social environment' as these are the means of achieving this fundamental objective. The value 'intellectual fulfillment' was assigned to 'self-actualization' because it was mentioned in the objective description.

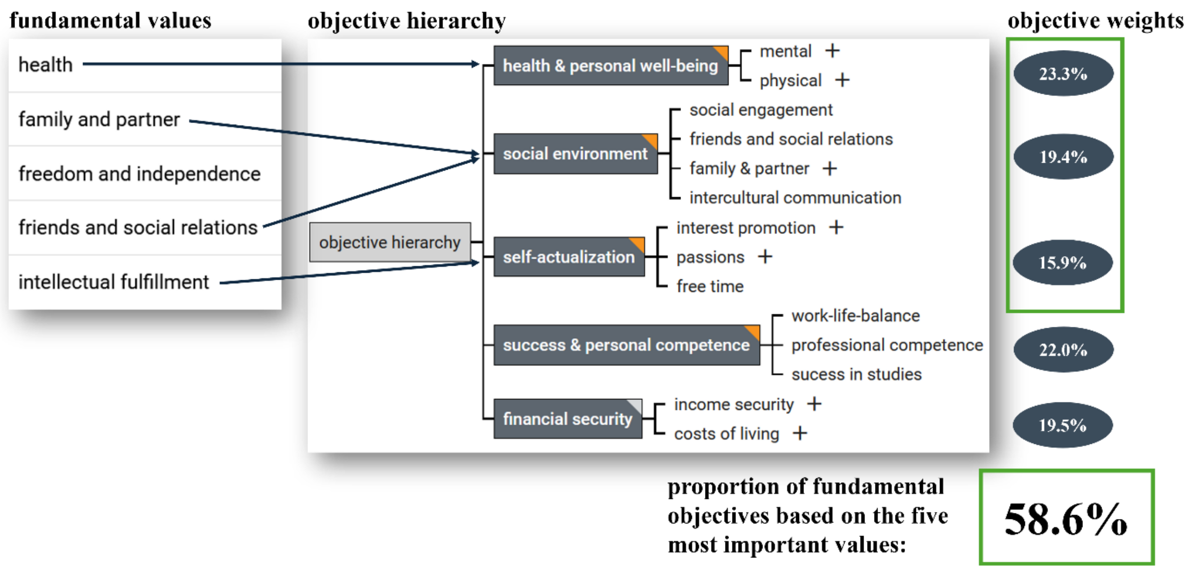

Figure 3. *Example for the assignment of values to fundamental objectives.*

To analyze the impact of value-based objectives on the decision, we examined the objective weights defined in Step 5 (see Section 2.5.). As the ENTSCHEIDUNGSNAVI uses the additive utility function of MAUT to determine the best alternative, all objective weights sum up to 100 % in this model. For the analysis, we added up the weights of value-based objectives and, thus, determined the proportion of fundamental objectives based on the five most important values. In the example above (see Figure 3), we added up the weights of the objectives 'health & personal well-being' (23.3 %), 'social environment' (19.4 %), and 'self-actualization' (15.9 %) because these three objectives are based on values. Thus, the sum (58.6 %) reflects the proportion of objective weights based on the five most important values in the decision. Then, we calculated the average of objective weights based on values in all decisions, in the decision topics, and in the groups.

To get a more detailed insight into the values on which the objectives are based and which values were decisive in the decisions, we addressed the **third research question**, 'Which impact does each of the five most important personal values have on private decisions before and after nudging?'. We calculated the average objective weights and their differences for each value in GroupVA and GroupVN. If one value was assigned to several objectives, the value's objective weight would result in the sum of the objective weights. If several values were assigned to one objective, the objective weight was distributed equally among the corresponding values. In the example in Figure 3, the values 'family and partner' and 'friends and social relations' were both assigned to the objective 'social environment'. Thus, the objective weight for 'social environment' (19.4 %) was distributed evenly to the two values (9.7 % each).

4 Results

4.1 Preliminary analysis

Table 1 summarizes the descriptive findings on the decisions in GroupVA and GroupVN.

		Number of decisions		**Number of objectives in decisions**			
Topic		**Total**	**In %**	**µ**	**σ**	**Min**	**Max**
Σ	GroupVA	286	100.00	4.53	1.33	2	12
	GroupVN	325	100.00	4.98***	1.06	3	10
Career	GroupVA	104	36.36	4.64	1.47	2	12
	GroupVN	118	36.61	4.88	1.04	3	9
Study	GroupVA	67	23.43	4.42	1.09	2	9
	GroupVN	72	22.15	4.94*	1.09	3	10
leisure planning	GroupVA	53	18.53	4.43	1.11	3	7
	GroupVN	66	20.31	4.98**	1.12	3	10
going abroad	GroupVA	44	15.38	4.52	1.23	3	9
	GroupVN	43	13.23	5.12*	0.92	3	7
housing situation	GroupVA	18	6.29	4.67	1.33	3	7
	GroupVN	26	8.00	5.35	0.96	3	7

Table 1. *Descriptive findings on the decisions in GroupVA and GroupVN (*** p<0.001; ** p<0.01; * p<0.05).*

In GroupVA, 286 participants (60.49 % male; 31.47 % female; 8.04 % not specified), and in GroupVN, 325 participants (53.85 % male; 31.38 % female; 14.77 % not specified) took part in the study. The analysis of the decision situations resulted in five topics: career, study, leisure planning, going abroad, and housing situation. All decisions could be assigned to a topic.

The distribution across the decision topics was roughly the same in both groups. Career decisions were most often chosen, followed by study decisions and leisure planning decisions. Only a few participants chose the topics of going abroad and housing situations. Typical decision questions in the field of career were ‘Which career path is best for me after I graduate with my bachelor's degree?’ or ‘What do I want my future to look like the year after my master's degree?’. When planning their studies, participants often think about questions like ‘How do I manage my studies and my private life in such a way that I achieve a high quality of life?’. ‘How do I spend my free time apart from my studies to maximize my financial independence and intellectual fulfillment?’ is one example of the topic of leisure planning. Furthermore, some participants think about if, where, and when they should go abroad during their studies. The housing situation also concerns some participants, who are unsatisfied with the current one and look for an alternative.

On average, the number of objectives in GroupVN was higher than in GroupVA, which applies to all topics. However, the standard deviation was almost always lower in GroupVN. In most cases, the minimum objective number was three, with a few exceptions in GroupVA, where only two objectives were defined. The maximum objective number was 12 in GroupVA and 10 in GroupVN.

4.2 Which values are important to DMs in private decisions?

Table 2 shows how often a value was ranked among the top five most important values in the decisions. The most differences between the ranking of frequencies in the groups could be determined for frequencies below 20 % (see thick line).

GroupVA (n = 286)		**GroupVN (n = 325)**	
Value	**Freq.**	**Value**	**Freq.**
family and partner	80.77 %	health	79.08 %
Health	80.07 %	family and partner	75.38 %
financial security	65.38 %	financial security	65.54 %
friends and social relations	65.03 %	friends and social relations	65.54 %
freedom and independence	43.36 %	freedom and independence	49.85 %
intellectual fulfillment	36.36 %	intellectual fulfillment	40.92 %
honesty and ethics	30.07 %	honesty and ethics	24.00 %
excitement and new experiences	24.83 %	excitement and new experiences	23.08 %
attractive housing	17.13 %	little mental stress (***)	18.15 %
competence	15.73 %	attractive housing	16.92 %
justice and fairness	13.29 %	competence	16.62 %
little mental stress	8.74 %	justice and fairness	9.85 %
being attractive	5.94 %	environment and nature	4.92 %
power and leadership	5.24 %	power and leadership	4.00 %
environment and nature	4.20 %	being attractive	3.38 %
Wealth	3.85 %	wealth	1.54 %

Table 2. *Frequency of values ranked among the top five in Step 1 of the ENTSCHEIDUNGSNAVI (*** $p<0.001$).*

The results in Table 2 show a similar picture for both groups. More than 50 % of the participants ranked the values ‘family and partner’, ‘health’, ‘financial security’, and ‘friends and social relations’ among the top five most important values in their decisions. The groups only differ in the two most frequently mentioned values. In GroupVN, ‘health’ was ranked more often in the top five than ‘family and partner’. In the GroupVA, it was the other way around, although the frequencies of these values are very similar. The values ‘freedom and independence’, ‘intellectual fulfillment’, ‘honesty and ethics’, and ‘excitement and new experiences’ are ranked in the same order in the groups, although the frequencies vary slightly. Looking at the values mentioned as the top five by less than 20 % of the participants, it is noticeable that the order in the groups varies the most here. The chi-square test of independence showed almost no significant difference in the groups for any value. Only the value of 'little mental stress' shows a highly significant difference in frequency in the groups.

Table 3 shows the five most frequent values ranked in the top five for all decision topics. Overall, the five most frequent values ranked among the top five ('health', 'family and partner', 'financial security', 'friends and social relations', and 'freedom and independence') are almost stable regardless of the topic. Compared to the entire data set (see Table 2), the values only change in the topics *study* and *leisure planning*, albeit very slightly. Here, the value of 'intellectual fulfillment' is also important for the participants.

CAREER

GroupVA (n = 104)		**GroupVN (n = 118)**	
Value	**Freq.**	**Value**	**Freq.**
Health	80.77 %	health	83.05 %
family and partner	77.88 %	family and partner	83.05 %
financial security	70.19 %	financial security	73.73 %
friends and social relations	64.42 %	friends and social relations	61.02 %
freedom and independence	39.42 %	freedom and independence	41.53 %

STUDY

GroupVA (n = 67)		**GroupVN (n = 72)**	
Value	**Freq.**	**Value**	**Freq.**
family and partner	85.07 %	health	81.94 %
Health	79.10 %	family and partner	70.83 %*
financial security	71.64 %	financial security	69.44 %
friends and social relations	55.22 %	friends and social relations	66.67 %
intellectual fulfillment	*41.79 %*	freedom and independence	45.83 %

LEISURE PLANNING

GroupVA (n = 53)		**GroupVN (n = 66)**	
Value	**Freq.**	**Value**	**Freq.**
family and partner	83.02 %	health	77.27 %
Health	79.25 %	family and partner	69.70 %
friends and social relations	69.81 %	friends and social relations	65.15 %
financial security	56.60 %	freedom and independence	56.06 %
freedom and independence/ *intellectual fulfillment*	47.17 %	*intellectual fulfillment*	*50.00 %*

GOING ABROAD

GroupVA (n = 44)		GroupVN (n = 43)	
Value	**Freq.**	**Value**	**Freq.**
Health	86.36 %	friends and social relations	74.42 %
family and partner	79.55 %	health	69.77 %
friends and social relations	75.00 %	family and partner	65.12 %
financial security	56.82 %	financial security	65.12 %
freedom and independence	54.55 %	freedom and independence	62.79 %

HOUSING SITUATION

GroupVA (n = 18)		GroupVN (n = 26)	
Value	**Freq.**	**Value**	**Freq.**
family and partner	77.78 %	family and partner	84.62 %
Health	66.67 %	health	73.08 %
friends and social relations	66.67 %	friends and social relations	69.23 %
financial security	61.11 %	financial security	61.54 %
freedom and independence	55.56 %	freedom and independence	61.54 %

Table 3. *The five most frequent values ranked among the top five in different decision topics (* p<0.05).*

In most decision topics, the participants prioritize ‘health’ and social relationships (‘family and partner’ and ‘ friends and social relations’), indicating a strong value placed on personal well-being and social connections. However, there are shifts in emphasis depending on the context. The value of financial security was ranked in the top five in over 70 % of the cases in *career* and *study* topics in both groups. In the other topics, this value was less relevant, and in the topic of *leisure planning* (GroupVN), this value is not even among the five most frequent values. The largest average deviation compared to the complete data set (see Table 2) occurs for the topics *of going abroad* and *housing situations*. In the comparison of the two groups in the different topics, the chi-square test of independence only showed significance in the topic *study* for the value ‘family and partner’. GroupVN ranked this value significantly less among the top five values than GroupVA.

4.3 Does nudging increase the impact of the five most important personal values in private decisions?

In this section, we show the results of our second analysis, in which we analyzed the impact of value-based objectives in decisions based on objective weightings. Therefore, decision experts assigned the values to the fundamental objectives in each decision, as described in section 3.2. The strength of agreement between the three experts' assessments of the first assignment of values to the fundamental objectives can be considered almost perfect (Fleiss' Kappa statistics = 0.94; see Landis and Koch (1977)). Discrepancies were discussed in more detail and assigned according to the majority principle.

Figure 4 shows the average proportion of objective weights based on values in GroupVA and GroupVN in all decisions and different decision topics.

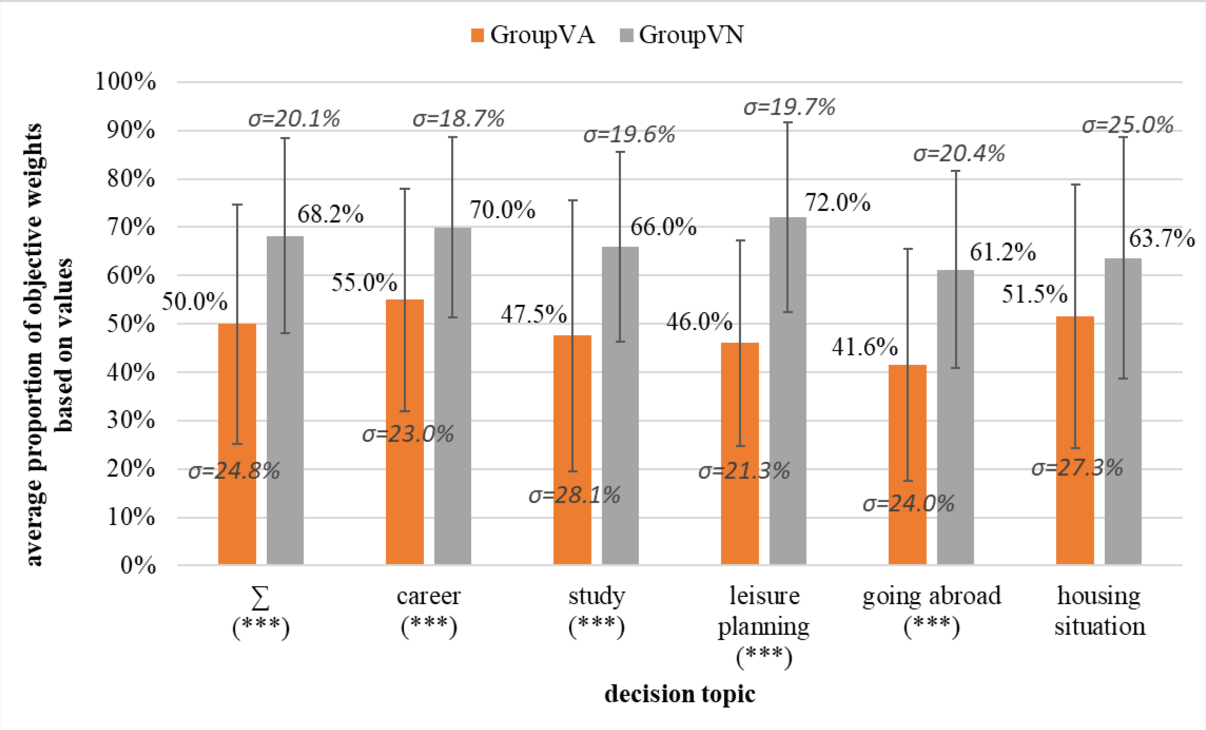

Figure 4. *Average proportion of objective weights based on values in GroupVA and GroupVN (*** p<0.001).*

The results in Figure 4 show that nudging increased the importance of the five most important personal values in private decisions. In GroupVN, the fundamental objectives based on values have a significantly higher impact on the decisions than in GroupVA. This was shown in the whole data set and for *career*, *study*, *leisure planning*, and *going abroad*. In the topic of *housing situation*, an increased impact in GroupVN could be observed, but due to the small data set, no significance could be shown.

Overall, the proportion of objective weights based on values was 18.2 percentage points (p.p.) higher in GroupVN than in GroupVA. The impact of nudging can also be seen in the different topics. The greatest difference in the groups was seen in *leisure planning* (26.0 p.p.), followed by *going abroad* (19.6 p.p.) and *study* (18.5 p.p.). The difference was the smallest in the topics of *career* (15.0 p.p.) and *housing situation* (12.2 p.p.). The error bars based on the standard deviation indicate the variability within the average objective weights of each group, suggesting that individual preferences may vary considerably even within the same topic. The standard deviation was lower in GroupVN than in GroupVA for each topic.

4.4 Which impact does each of the five most important personal values have on private decisions before and after nudging?

In the last section, we showed that nudging significantly increased the importance of the five most important personal values in private decisions and, thus, improved value-focused decision-making. In this section, we analyze which specific values led to this, i.e., which values were given a higher and a lower weighting in the decision based on the defined objectives. Figure 5 shows the average objective weight of every value for GroupVA and GroupVN in all decisions. The values ranked in over 20 % under the top five values (see Table 2) are printed in bold.

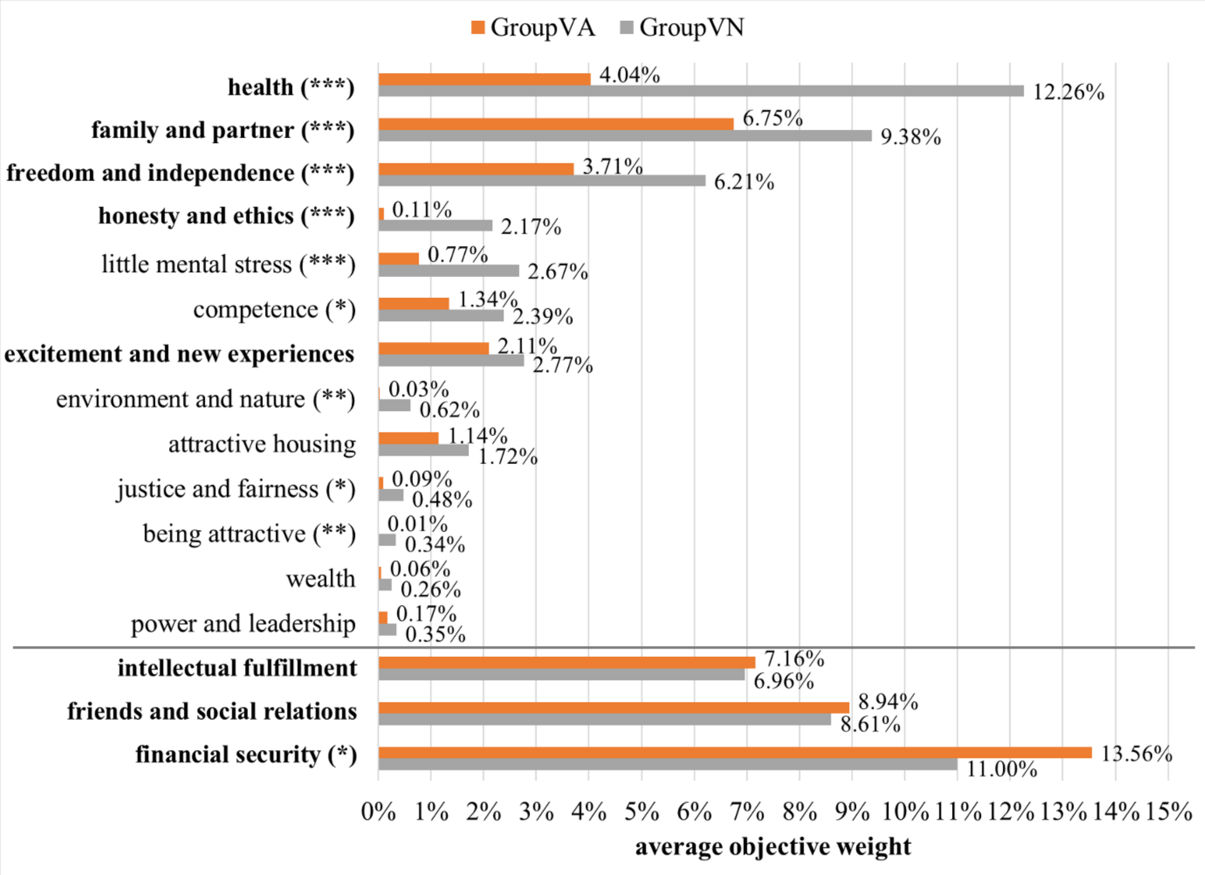

Figure 5. *Average objective weight of every value for GroupVA and GroupVN (*** p<0.001; ** p<0.01; * p<0.05).*

Most of the values had a higher average objective weight in GroupVN than in GroupVA, i.e., nudging increased the impact of these values in decisions. The values ‘health’, ‘family and partner’, ‘freedom and independence’, ‘honesty and ethics’, and ‘little mental stress’ have a significantly higher impact on the decisions in GroupVN than GroupVA ($p<0.001$). The biggest difference in the average objective weight in the groups is shown for the value ‘health’ (8.22 p.p.). The average objective weight tripled after nudging the participants. ‘Family and partner’, as well as ‘freedom and independence’, ‘honesty and ethics’, and ‘little mental stress’ increased their objective weight by about 2 p.p. in GroupVN. The only values for which the average objective weight decreased are ‘intellectual fulfillment’, ‘friends and social relations’, and ‘financial security’. However, significance could only be shown for ‘financial security’ ($p<0.05$).

Figure 6 provides a detailed overview of the changes in the impact of values across the different topics. It shows the change, i.e., the difference between the objective weight of a value in GroupVA and GroupVN. The greener a value is shown, the greater the objective weight has become. The redder a value is shown, the smaller the objective weight of this value has become. The differences for the five most frequent values ranked among the top five in different decision topics (see Table 3) are printed in bold. The results were tested for two-sided significance.

	∑	career	study	leisure planning	going abroad	housing situation
health	8.22%***	8.45%***	9.07%***	8.74%***	5.33%**	7.12%*
family and partner	2.62%***	2.18%	0.07%	4.16%**	1.94%	7.55%**
freedom and independence	2.50%***	1.23%	1.17%	5.20%**	3.52%	1.79%
honesty and ethics	2.06%***	2.46%***	2.45%***	2.24%**	0.69%	0.95%
little mental stress	1.90%***	1.87%*	1.48%	2.50%*	1.52%*	1.46%
competence	1.05%*	0.78%	2.30%*	1.39%	-0.22%	-
excitement and new experiences	0.66%	0.84%	0.65%	2.29%*	-0.48%	-1.02%
environment and nature	0.59%**	0.32%	0.47%	0.81%*	1.27%	0.45%
attractive housing	0.57%	0.89%	0.52%	0.87%	0.71%	-3.26%
justice and fairness	0.38%*	0.04%	0.56%	0.96%*	0.38%	-
being attractive	0.33%**	0.63%*	-	0.50%	-	-
wealth	0.20%	0.35%	-	0.36%	-	-
power and leadership	0.17%	0.19%	0.14%	0.29%	-	0.17%
intellectual fulfillment	-0.20%	-1.05%	0.24%	-1.59%	0.90%	5.03%**
friends and social relations	-0.34%	-1.05%	1.40%	1.76%	1.76%	-4.23%
financial security	-2.56%*	-3.14%	-2.04%	-4.48%	2.35%	-3.84%

Figure 6. *Differences between GroupVA and GroupVN in the objective weights based on values in the topics in p.p. (*** p<0.001; ** p<0.01; * p<0.05).*

Regardless of the topic, no change in the impact of a value with a decreased objective weight (red) was shown to be significant. However, significance could be shown for some values with an increased objective weight (green), e.g., ‘health’ or ‘honesty and ethics’. Compared to the complete data set, the objective weights mostly differ in the topics of *going abroad* and *housing situation*. In *going abroad*, the objectives based on the values ‘competence’ and ‘excitement and new experiences’ have been assigned a lower objective weight in GroupVN than in GroupVA (red). However, the difference is minimal. In *housing situation*, it is the values ‘attractive housing’, ‘friends and social relations’, and ‘financial security’ that have also been given less weighting alongside ‘excitement and new experiences’. It is striking that ‘intellectual fulfillment’ had a significantly higher impact in GroupVN only on this topic. At this point, it should be mentioned that the objective weights must always be seen in relation to each other since the sum of all objective weights in the MAUT model is equal to one (see Section 2.5.). This means that if one objective weight increases, another objective weight must automatically decrease.

5 Discussion

Our study examined which personal values are most important for the students and whether nudging as part of a value-focused decision-making process can help DMs make their decisions more value-oriented.

Regarding private decisions, the **values** ‚family and partner‘, ‚health‘, ‚financial security‘, ‚friends and social relations‘, ‚freedom and independence‘, and ‚intellectual fulfillment‘ are the ones most important to the DMs. Our results show they were also consistently ranked high in the various topics we analyzed (see Tables 2 and 3). This finding supports the definition of values, which states that they can be viewed as relatively stable from young adulthood. Therefore, they do not depend much on the context (Rokeach 1973). The listed values directly address the needs of people, i.e., what motivates people every day. If you look at Maslow's hierarchy of needs, ‘health’ is an overriding value to increase life satisfaction and a prerequisite for developing well in life, which refers to the lowest level of physiological needs. ‘Financial security’ refers to the second level of security needs, ‘family and partner’, and ‘friends and social relationships’ refer to the need for love and belonging (3rd level). ‘Freedom and independence’ addresses the self-esteem needs and ‘intellectual fulfillment’ the cognitive needs (Maslow 1943).

The focus of our study is the question of whether nudging can help people make more value-oriented decisions. Our results suggest that **nudging** as part of a value-focused decision-making process helps DMs make their decisions more value-oriented. The results of our study show that when making decisions in GroupVN, the proportion of weight for objectives based on personal values was significantly higher, at an average of 68.2 %, than in GroupVA at 50.0 % (see Figure 4). This trend can be seen consistently across all topics the students worked on. Concerning the individual topics, except *housing situation*, there is a significant increase between 15 and 26 p.p. in the groups ($p<0.001$). In *housing situation*, the smallest increase (12.2 p.p.) was not significant. At the same time, we observe that the standard deviation has decreased slightly in each topic, and, therefore, the results can be interpreted as somewhat more stable. Our results suggest that the educational nudge (Sunstein 2016) presented here is actually helpful in aligning the decision more closely with one's values. It could be that objectives have been formulated in a more general, value-based way in GroupVN. This could mean that previous fundamental objectives that are not based on values may have become means objectives and been assigned to a more general, value-based fundamental objective. However, this assumption would have to be verified by further investigations.

Looking not at the overall change but at the **nudging effect for each individual value** among the top five (see Table 3), we found that in GroupVN, the objective weights for the values ‘health’, ‘family and partner’, and ‘freedom and independence’ increased significantly ($p<0.001$) (see Figure 5), and many other values also received a (significant) higher proportion of the objective weight. In contrast, there is no increase in the objective weights for the remaining top five values ‘friends and social relations’, ‘intellectual fulfillment’, and ‘financial security’. The average objective weights for ‘friends and social relations’ and ‘intellectual fulfillment’ are slightly lower in GroupVN (no significant change). For ‘financial security’, the average objective weight drops by 2.56 p.p. ($p<0.05$). However, due to the relative dependence of the objective weights on each other (see Section 2.5.), it is logical that some values must lose objective weighting if others are given higher weighting.

At this point, we want to mention that the COVID-19 pandemic could also impact the results of this study. We collected the data sets of the test groups at different times. The data from GroupVA was collected in winter terms 2018-2020 (latest submission of the project on January 20, 2020), i.e., before the COVID-19 pandemic, and that from GroupVN in 2021-2023, i.e., during the pandemic. The topic of health was very present during the pandemic, which may have led to our participants in GroupVN being made aware of the topic of health not only by our nudge but also by the media and current events. Several studies show that the mental health impact can be severe and enduring (Aymerich et al. 2022, Panchal et al. 2023). This fact could also result in a higher priority being given to the value of health in GroupVN, leading to higher weights for objectives based on 'health'.

Regarding the **nudging effect in the different topics,** we can see that only the increase in 'health' is consistently significant in all five topics considered (see Figure 6, columns 2-6). The change in all other objective weights based on values is significant only for particular topics.

The value 'family and partner' shows a significant increase ($p<0.01$) for the topics *housing situation* (+7.55 p.p.) and *leisure planning* (+4.16 p.p.), but no significant change for the other topics. For the *housing situation*, 'family and partner' is also the value that the students mentioned most frequently in the top five (see Table 3). One explanation for the lower objective weights in GroupVA may be that the students focus on standing on their own two feet after moving out. Perhaps that is why this value is initially pushed into the background (narrow framing (Bond et al. 2008)). Through the nudge, they may realize that this value is indeed still relevant to the decision-making situation.

For the value 'freedom and independence', we observed a significant increase in the objective weight based on it by 5.20 p.p. ($p<0.01$) only in *leisure planning*. There is also an increase in the other topics, but it is insignificant. *Leisure planning*, in particular, addresses this value, as the DM has a lot of freedom regarding which activities they would like to pursue in their free time. The nudging of this value, which is obviously addressed and, therefore, the students may not have explicitly considered before, means that people now actively engage with it and perhaps realize that it is essential to them regarding leisure activities.

For the values ‘intellectual fulfillment’ and ‘friends and social relations’, which were also often among the top five, we could hardly see any difference (slight decline, not significant). This result can possibly be explained by the fact that these two values are most present in the students' minds as they study and meet friends daily. However, we see a mixed picture if we look at the change in the individual topics. The value ‘intellectual fulfillment’ shows a significant increase of 5.03 p.p. ($p<0.01$) in the topic *housing situation*, but otherwise, no significant changes. In this case, nudging seems to have a negative effect on the identification of fundamental objectives since it appears to have drawn the DM’s attention to a value that does not seem relevant to the topic. We assume that DMs no longer focus on the specific decision situation but on their general life objectives. This, in turn, could lead to a weighting that does not reflect the correct preferences of the DM in the specific topic, leading to a worse decision analysis and quality. However, follow-up studies would be needed to confirm this assumption and draw better conclusions.

Our results show an opposite effect for the value ‘financial security’, which was also frequently mentioned among the top five values. Here, the proportion of objective weight fell significantly from 13.56 to 11.00 %. However, the change is only slightly significant regarding the overall results ($p<0.05$) but shows no significance in the topics. Nevertheless, the proportion of objective weight attributable to this value decreases in all topics except for *going abroad*. A possible explanation is that nudging made the participants aware of other values that are more important than money and that they had not (sufficiently) considered before. Moreover, because the sum of all objective weights must equal one, this means that if some objective weights rise, others will decrease (see Section 2.5).

In addition to the values mentioned, which were among the top five, we also observed a significant increase in the objective weights for the values ‘honesty and ethics’ in the topics *career*, *study* ($p<0.001$), and *leisure planning* ($p<0.01$). While the value was (almost) not taken into account in GroupVA, nudging here means that the value was taken into account by many students in the decision situation in GroupVN. So, 'narrow framing' may also be an explanation here.

6 Conclusion

In this study, we analyzed whether DMs can be nudged toward more value-focused and, thus, better decisions. To do this, we used the DSS ENTSCHEIDUNGSNAVI (von Nitzsch et al. 2020, Hannes and Nitzsch 2024, Peters et al. 2024), which guides people step by step through the decision-making process based on VFT (Keeney 1992) and MAUT (Keeney and Raiffa 1976), to compare major personal decisions made by two groups of students: GroupVA, in which personal values only represented a passive anchor in that they were reflected upon and evaluated at the beginning of the decision-making process, and GroupVN, in which the five most important personal values were actively adopted in the next step of the objective formulation and, thus, represented a nudge. Furthermore, we analyzed different decision topics (career, study, leisure planning, going abroad, and housing situation). Our results indicate that nudging is indeed a way to make important decisions that are even better aligned with one's values. In the following, we summarize our study's most important findings, analyzing the research questions.

Which values are important to DMs in private decisions?

'Health', 'family and partner', 'financial security', 'friends and social relations', 'freedom and independence', and 'intellectual fulfillment' are the most important values for the participants. There is no difference between the Groups.

Does nudging increase the impact of the five most important personal values in private decisions?

Yes, nudging increased the impact of the five most important personal values in private decisions. Overall, the proportion of objective weights based on values was 18.2 % higher in GroupVN than in GroupVA. A significant increase in the value-based objective weights could be observed for the topics of *career*, *study*, *leisure planning*, and *going abroad*. We assume that, in GroupVN, the DMs formulated the fundamental objectives more generally, value-based, leading to previously formulated, non-value-based objectives being mentioned more as means objectives.

Which impact does each of the five most important personal values have on private decisions before and after nudging?

The values 'health', 'family and partner', and 'freedom and independence' have a significantly higher impact on the decisions in GroupVN than GroupVA. The biggest difference in the average objective weight in the groups is shown for the value 'health' (from about 4 to 12 %). The COVID-19 pandemic could also explain this increase, as participants were exposed to nudges and media coverage of health issues during this time. In contrast, we observed a significant decline in the objective weight for the value 'financial security'. Nudging people towards more value-focused objectives could lead to the fact that DMs are more aware of other values that are more important than money and that they had not (sufficiently) considered before.

The results of our study should be treated with caution. Since we collected our data sets before (GroupVA) and during (GroupVN) the COVID-19 pandemic, we cannot rule out an additional impact of the pandemic. Further studies could be used to test the robustness of our results and their associated independence from the pandemic. Particularly for the second research question, it would be interesting to examine whether the significant increase in the objective weights based on values is attributable only to the nudge in the ENTSCHEIDUNGSNAVI or also to the pandemic. To do this, the value 'health' could be excluded from the current research data, or the research could be repeated with two new data sets collected simultaneously.

In addition, our study addresses the following limitations. First, the data set consists of young German students, so the results are only valid for this group. Second, to achieve the greatest possible practical benefit for our students, they could freely choose the decision-making situation they wanted to analyze with the ENTSCHEIDUNGSNAVI. This, in turn, led to different numbers of participants in the topics. Particularly for the topic of *housing situation*, this was very small, so we could only examine 18 decisions in GroupVA and 26 in GroupVN. Third, we concentrate on the five most important values. It could be that less important values also have an impact on the formulation of the fundamental objectives. Future studies are needed to gain further insights and to check the robustness of the results.

In the meantime, we have further revised the list of values in the ENTSCHEIDUNGSNAVI and adapted it to the 19 values according to Schwartz (Sagiv and Schwartz 2022). In the future, we can investigate whether our results here can be reproduced. In addition, we can conduct a study to check how well our individual items correspond to the results of the Schwartz Value Survey (Schwartz et al. 2012) and, if necessary, derive further improvements. Furthermore, the influence of values on fundamental and means objectives could be examined in more detail in follow-up studies. These studies could support the assumption that value-nudging leads to more generally formulated fundamental objectives based on values, and previous fundamental objectives that are not based on values fade into the background as means.

Overall, it can be said that value-nudging is particularly effective when the values are not obvious in a situation. If these values are not important to DMs (e.g., 'intellectual fulfillment' in *housing situation*), the nudge may cause DMs to consider irrelevant values in their objectives, thereby negatively affecting their identification of the fundamental objectives and, thus, the weighting of objectives no longer reflects their actual preferences. The fact that nudging can have both good and bad effects is already discussed in the literature in many areas (Wilkinson 2013, Damgaard and Nielsen 2018, Schmidt and Engelen 2020). If these values are nevertheless important in the situation (e.g., 'family and partner' in *housing situation*), we can improve the fundamental objectives with a simple, effective nudge in the sense of the VFT. Thus, we can create an added value for the analysis and the quality of the decision.

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Paper 4: An empirical study to measure the use and impact of an imprecise information approach in the decision support system ENTSCHEIDUNGSNAVI

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Contents

An empirical study to measure the use and impact of an imprecise information approach in the decision support system ENTSCHEIDUNGSNAVI

In a rational multi-criteria decision-making process based on multi-attribute utility theory, the decision-maker has to define quantitative parameters to identify the most promising alternative. However, it is often difficult to determine these parameters precisely. Therefore, the literature suggests using imprecise information, which allows the decision-maker to determine imprecise preference statements and parameters. This paper investigates the utilization and impact of an imprecise information approach regarding probabilities, utility functions, and objective weights in the decision support system ENTSCHEIDUNGSNAVI. We analyzed 1,511 personal decision situations. Our results show that most participants chose imprecise parameters in their decisions. Moreover, they found it helpful to use imprecise information in all categories, with imprecise objective weights being the least helpful. In about 83 % of the decisions, the best-ranked alternative is robust against the imprecise intervals the decision-maker chooses. The results show that precise preference statements and parameters are not crucial for identifying the best alternative in the decisions analyzed in the ENTSCHEIDUNGSNAVI. The imprecise information approach facilitates the decision-making process for the decision-maker. However, imprecise trade-off statements for objective weights are not necessarily easier for decision-makers to determine than precise statements, as the methodology is already complex.

Keywords: imprecise information; multi-criteria decision-making; decision support systems; multi-attribute utility theory; Monte Carlo simulation

1 Introduction

A rational multi-criteria decision-making (MCDM) process based on the multi-attribute utility theory (MAUT) presents many difficulties for the decision-maker (DM) both in the decision front end and in the decision back end. In the decision front end, DMs need to structure the decision situation. Therefore, they must formulate the decision statement, identify the objectives, and determine the alternatives. Studies have shown that individuals cannot identify all relevant objectives or alternatives in a decision situation (Bond et al. 2008, Siebert and Keeney 2015). In the decision back end, DMs must develop the consequences table and evaluate the alternatives. Developing the consequences table means each alternative has to be rated on a scale for each objective. In evaluating the alternatives, the DMs indicate their preferences in the model. The difficulty is determining precise values for the quantitative parameters needed to calculate the utility of every alternative in MAUT. This paper focuses on the decision back end and the difficulty of determining quantitative parameters.

The DM has to determine the following quantitative parameters in the decision back end: utility functions, objective weights, and consequences. In the case of uncertainty, several consequences must be defined depending on the state of a possible influence factor. Moreover, DMs need to determine probabilities for all these states. For a high-quality decision, these quantitative parameters should be realistic and not distorted by any biases (see, e.g., Kahneman (2011), Montibeller and von Winterfeldt (2015)). Probabilities are sometimes unknown or cannot be estimated accurately. Many individuals struggle with the interpretation and determination of probabilities and tend to overestimate or underestimate probabilities (see, e.g., Tversky and Kahneman (1973)). While determining utility functions or objective weights, sometimes DMs are unable to determine their preferences precisely and consistently with their values. It may even be that they have no stable preferences at all. This complicates the determination of utility functions and objective weights.

The approach of partial, imprecise, or incomplete information is used in literature to support the DM in determining these quantitative parameters. Because partial, imprecise, and incomplete information are used interchangeably, we only use the term 'imprecise information' in this paper. In MCDM, imprecise information allows the DM not to give exact statements about consequences, probabilities, or preferences, i.e., utility functions and objective weights. It is sufficient, for example, to specify intervals or ranks. With the help of different methods, it is still possible to calculate the best alternative using MAUT.

The approach of imprecise information is already being used in MCDM-Support-Systems to simplify the application for the DM. However, to the best of our knowledge, research has yet to be conducted to determine how much individuals use imprecise information in decision situations, how helpful DMs find it, or whether the approach impacts the best alternative in selection problems where DMs seek one choice. This paper investigates using an imprecise information approach in the decision support system (DSS) ENTSCHEIDUNGSNAVI and analyzes how to deal with this in MCDM-Support-Systems. The MCDM-Support-System ENTSCHEIDUNGSNAVI is a web tool for decision skill training. It supports the DM throughout the whole decision-making process in selection or ranking problems. It offers the DM the possibility to use imprecise information concerning three categories: probabilities, utilities, and objective weights. For a detailed description and explanation of the ENTSCHEIDUNGSNAVI, see von Nitzsch et al. (2020), Peters et al. (2024), and Hannes and von Nitzsch (2024). The idea of imprecise information in the ENTSCHEIDUNGSNAVI for all three categories (probabilities, utilities, and objective weights) is to identify a mean μ and a degree of precision ε, which allows an interval of the parameters.

We examined 1,511 personal selection decisions analyzed by students in the ENTSCHEIDUNGSNAVI regarding three research questions. First, we analyzed how often imprecise information was used. Second, we surveyed how helpful the participants found the use of imprecise information. Third, we examined the impact of imprecise information in the three categories mentioned above on the final ranking of alternatives. We investigated how stable the rank of the best alternative remained using Monte Carlo simulations. We split the participants into two groups to examine the research questions in more detail. In Group non-zero, the default setting in the ENTSCHEIDUNGSNAVI was imprecise parameters for all categories. In Group zero, the default setting was precise parameters. So, the participants had to actively choose an imprecision if they wanted. All research questions were analyzed using the complete data set and the two different groups. By collecting the data with the ENTSCHEIDUNGSNAVI and checking it manually, we can assume that the decision situations are adequately modeled, the consequences are non-distorting, and the DMs have defined their preferences according to their value system.

The paper is structured as follows: after a brief literature review of the application of imprecise information in Section 2, the methodology of the ENTSCHEIDUNGSNAVI, which permits the use of imprecise information concerning probabilities, utility functions, and objective weights, will be explained in Section 3. In Section 4, the results of the empirical study will be presented to answer the research questions. In Section 5, we discuss the results, and in Section 6, we give a conclusion and discuss limitations. Furthermore, we specify conditions under which the imprecise information approach used in the ENTSCHEIDUNGSNAVI facilitates sound decision-making and provide suggestions for further research.

2 Relevant research on imprecise information

Several approaches deal with imprecise information. Some focus on specific categories of imprecise information, such as objective weights, utilities, or probabilities. Others consider a combination of two or all of them. Furthermore, the approaches differ in the implementation of imprecise information.

Fishburn (1965) was one of the first researchers to recognize the problem of determining the exact probabilities of the states of nature. He developed several approaches for four measures of probabilities: null measure, ordinal measure, sets of inequalities, and bounded interval measure. To find the best alternative with imprecise information about the probabilities, the concept of linear programming is used in many approaches (e.g., Kmietowicz and Pearman (1982, 1984), Sarin (1978)). Weber (1985, 1987) contributed to the research on imprecise information in utility functions relying on stochastic dominance or linear programming concepts. Von Nitzsch and Weber (1993) present an interactive procedure on a micro-computer to determine consistent bounds for utility functions. Armbruster and Delage (2015) extend the concept of stochastic dominance by considering more features of the utility functions than those used in the first- and second-order stochastic dominance frameworks like S-Shape or prudence information. Furthermore, they model the decision-making problems as robust utility maximization problems, optimization problems with stochastic dominance constraints, or robust certainty equivalent maximization problems to determine the best alternative. Moreover, many approaches for unknown or imprecise objective weights were developed in the past (see, e.g., Carrizosa et al. (1995), Hazen (1986), Kirkwood and Sarin (1985)). De Almeida et al. (2016) and de Almeida-Filho et al. (2017) give a broad overview of this topic and categorize the approaches using forms of imprecise information (e.g., interval weights (Steuer 1976, Puerto et al. 2000, Park 2004, Mustajoki et al. 2005, Li et al. 2012), partial/incomplete information on weights (Barron 1992, Salo and Hämäläinen 2001, Mármol et al. 2002, Salo and Punkka 2005, Punkka and Salo 2013), or unknown weights (Hazen 1986, Lotfi et al. 1992)). Furthermore, they develop and validate based on the trade-off procedure (Keeney and Raiffa 1976, Keeney 1992) the FITradeoff (Flexible and Interactive Trade-off) approach, which enables a flexible and interactive process handling the problem of imprecise statements of the DM. They even develop a DSS to support the DM in MCDM. However, this approach requires that the DM can rank the objective weights. Additionally, the DM has to define the value functions. Although several options (e.g., linear or non-linear value functions) are offered, these aspects might be difficult for the DM in real-world decision problems. For this reason, many approaches allow the DM to make imprecise statements in more than one area, e.g., utility/value functions and objective weights (Eum et al. 2001, Lee et al. 2001, Lee et al. 2002) or probabilities and utility/value functions (Moskowitz et al. 1993, Danielson et al. 2003, Danielson 2004, Liesiö and Salo 2012). The approach of Sarabando and Dias (2010) allows imprecise information about the weights and values of each alternative in each objective. It uses a Monte Carlo simulation to test the robustness of the ranking of alternatives. However, ordinal information on the weights and cardinal information on the values are required. Nevertheless, these approaches do not help the DM if imprecision exists in all three areas (probabilities, utilities, and objective weights).

Jiménez et al. (2002) developed a DSS for multi-attribute utility evaluation, which allows the use of imprecise statements in all areas. Value ranges can describe the consequences under uncertainty without giving any information about the probability distribution of the state of nature. However, the system uses the mean of the ranges at any point of the evaluation. This means that imprecise information is allowed, but the imprecise ranges are adjusted by the mean value in the evaluation and, therefore, not considered. They offer three ways to assess the utility functions: the certainty equivalent method/probability equivalent method, piecewise linear utility functions, and subjective scales. The DM can give imprecise statements for all of them, which leads to an interval of utility functions or values. Intervals can also be specified for determining objective weights by the trade-off method or pre-emptive ordering. A sensitivity analysis can check the robustness of the final ranking. However, the input parameters for the sensitivity analysis are limited to objective weights and utility functions. The DSS of Mateos et al. (2007) contains no option to check the result for sensitivity or robustness. The DSS introduced by

Danielson et al. (2007) also uses imprecise information in the mentioned areas. The tool works with interval boundaries, comparative value relations, and so-called focal points representing the best value for the variables in an interval. Moreover, the DSS offers different evaluation methods like ordinal ranking, cardinal ranking, or pairwise comparisons. However, due to the complexity, the user might need the help of an expert.

Many researchers have dealt with imprecise information and how to identify the best alternative in selection problems. To the best of our knowledge, the impact and use of imprecise statements in personal decisions in MCDM have not been investigated. For this reason, we examine the imprecise information approach of the DSS ENTSCHEIDUNGSNAVI (von Nitzsch et al. 2020, Hannes and Nitzsch 2024, Peters et al. 2024) in this paper. The ENTSCHEIDUNGSNAVI allows imprecise statements resulting in parameter intervals in all areas and provides different evaluation methods like sensitivity analysis and robustness checks. Moreover, it helps the DM with an intuitive interface to identify the best alternative. In Section 3, the model of the DSS and how imprecise information is incorporated will be explained.

3 The model: imprecise information in the ENTSCHEIDUNGSNAVI

The mathematical model used in the ENTSCHEIDUNGSNAVI for illustrating a MCDM process is the additive model of the MAUT (Fishburn 1967, Keeney 1972, Keeney and Raiffa 1976, Keeney 1992). To support the DM in determining the quantitative parameters for this model, the ENTSCHEIDUNGSNAVI enables the DM to give imprecise statements in three categories: probabilities, utility functions, and objective weights. Moreover, with the help of a Monte Carlo simulation, it is possible to check the robustness of the alternative ranking, including imprecise information. Peters et al. (Peters et al. 2024) give a detailed description of methods for dealing with imprecise information in the ENTSCHEIDUNGSNAVI.

3.1 Additive model

In the additive model, the available alternatives are ranked according to their utilities. The alternative with the highest utility represents the best alternative and should be selected by the DM. To calculate the utilities, DMs must first define a set of objectives $\mathbb{O} = \{O_1, \dots, O_I\}$ and a set of alternatives $\mathbb{A} = \{A_1, \dots, A_J\}$ for some natural numbers I, J for the decision situation. Subsequently, they have to evaluate the consequences x_{ij} of all J alternatives in the respective I objectives with $1 \leq i \leq I$ and $1 \leq j \leq J$ in a consequences table. The utility of each alternative A_j is calculated using Formula (1) for the additive expected utility (Bernoulli 1954, von Neumann and Morgenstern 1961).

$$EU(A_j) = \sum_{i=1}^{I} w_i \left[\sum_{k=1}^{K_{ij}} P(s_{ij}^k) U_i(x_{ij}^k)\right] \tag{1}$$

$$\sum_{i=1}^{I} w_i = 1 \tag{1a}$$

$$\sum_{k=1}^{K_{ij}} P(s_{ij}^k) = 1 \tag{1b}$$

w_i represents the weight of objective O_i. The sum of all objective weights must equal one (1a). To model decisions under uncertainty, different states s_{ij}^k can be determined that occur with a corresponding probability $P(s_{ij}^k)$ and result in some consequence x_{ij}^k with $1 \leq k \leq K_{ij}$. The probabilities of all states for every ij add up to one (1b). Finally, U_i represents the utility function of objective O_i. Utility functions are used to map the DM's preferences.

The ENTSCHEIDUNGSNAVI supports the DM in determining all these parameters by providing different methods. To specify the probabilities $P(s_{ij}^k)$ for all defined influence factors, the DM can directly rate them on a cardinal scale from 0 to 100 %. The utility functions U_i can be defined for two different kinds of objectives: objectives with a verbal and a numerical scale. Utility functions for objectives with a verbal scale can be determined with a direct rating function $DR(x_{ij}^k)$ like in Formula (2a). Utility functions for objectives with a numerical scale can be determined with a predefined exponential utility function with a variable parameter c_i of risk aversion, as in Formula (2b). x_i^- to x_i^+ is the interval of values for the objective O_i.

Discrete utilities for objectives with a verbal scale:

$$U_i(x_{ij}^k) = \begin{cases} 0 & if\ \ x_{ij}^k = x_i^- \\ \mathrm{DR}(x_{ij}^k) & if\ x_{ij}^k \in (x_i^-, x_i^+) \\ 1 & if\ \ x_{ij}^k = x_i^+ \end{cases} \tag{2a}$$

Exponential utility function for objectives with a numerical scale:

$$U_i(x_{ij}^k) = \begin{cases} \frac{1-e^{-c_i \frac{x_{ij}^k - x_i^-}{x_i^+ - x_i^-}}}{1-e^{-c_i}} & if\ \ \ c_i \neq 0 \\ \frac{x_{ij}^k - x_i^-}{x_i^+ - x_i^-} & if\ \ \ c_i = 0 \end{cases} \tag{2b}$$

Furthermore, the objective weights w_i are determined using the trade-off method (Keeney and Raiffa 1976). In contrast to the other methods, this method requires DMs to think in two dimensions since they must weigh what deterioration in one objective they will accept in exchange for an improvement in another (trade-off). This makes the determination of the objective weights more complicated than determining the other parameters where only one dimension is relevant at a time.

3.2 Imprecise information

The idea of imprecise information in the ENTSCHEIDUNGSNAVI for all three categories (probabilities, utilities, and objective weights) is to identify a mean μ and a degree of precision ε, which allows an interval of the parameters (Peters et al. 2024). If DMs define $\varepsilon = 0$, they can precisely elicit the parameter, whereas $\varepsilon > 0$ means that they use imprecise information.

3.2.1 Imprecise probabilities

While determining the probabilities of the states of influence factors S_δ, the degree of precision $\varepsilon_{P(S_\delta)}$ indicates the accuracy with which the probabilities are specified. $\varepsilon_{P(S_\delta)}$ can be between 0 % and 50 % and determined individually for each influence factor. If $\varepsilon_{P(S_\delta)} = 0$ %, the DM decides to determine the probabilities precisely. If $\varepsilon_{P(S_\delta)} \neq 0$ %, the DM makes use of imprecise information and decides to choose intervals for the probabilities instead of exact values. Then, the interval of the probabilities is dynamic based on the value $P(s_\delta^f)$ and calculated as follows:

$$P_{min}(s_\delta^f) = P(s_\delta^f) - \varepsilon_{P(S_\delta)} min\{P(s_\delta^f), 1 - P(s_\delta^f)\} \tag{3a}$$

$$P_{max}(s_\delta^f) = P(s_\delta^f) + \varepsilon_{P(S_\delta)} min\{P(s_\delta^f), 1 - P(s_\delta^f)\} \tag{3b}$$

$1 \leq f \leq K_\delta$ represents the states of the influence factor S_δ. The largest interval results at $P(s_\delta^f) = 50$ %. As described in Formula (1b), the probabilities have to sum up to one. To comply with this condition, the ENTSCHEIDUNGSNAVI normalizes the probabilities. For a detailed description of this step, see Peters et al. (2024). To analyze the research questions, we calculated an average degree of precision $\overline{\varepsilon_{P(S_\delta)}}$ for the defined influence factors.

3.2.2 Imprecise utility functions

Utility functions are determined depending on the type of objective (verbal or numerical scale). The procedure for imprecise utility functions for objectives with verbal scales is analogous to that for probabilities (see Section 3.2.1). ε_{U_i} that can range from 0 to 50 % is the degree of precision for a discrete utility scale of the objective O_i.

The interval of the discrete utility function is determined as follows:

$$U_i^{min}(x_{ij}^k) = U_i(x_{ij}^k) - \varepsilon_{U_i} min\{U_i(x_{ij}^k), 1 - U_i(x_{ij}^k)\} \tag{4a}$$

$$U_i^{max}(x_{ij}^k) = U_i(x_{ij}^k) + \varepsilon_{U_i} min\{U_i(x_{ij}^k), 1 - U_i(x_{ij}^k)\} \tag{4b}$$

For objectives with a numerical scale, the utility functions are determined as an exponential utility function, as in Formula (2b). Imprecise information is given by a possible interval of the parameter c_i, as in Formulas (5a) and (5b). The degree of precision for an exponential utility function ε_{c_i} can range from -10 to 10.

$$c_i^{min} = c_i - \varepsilon_{c_i} \tag{5a}$$

$$c_i^{max} = c_i + \varepsilon_{c_i} \tag{5b}$$

For our analysis, we calculate an average degree of precision for utility functions determined for objectives with verbal scales $\overline{\varepsilon_{U_i}}$ and numerical scales $\overline{\varepsilon_{c_i}}$.

3.2.3 Imprecise objective weights

The objective weights w_i are determined by eliciting the exchange rates between two objectives (trade-off method). Therefore, the DM can choose one reference objective with which all other objectives are compared. Furthermore, the DM has to formulate preference statements for all these pairs of objectives (trade-offs). In contrast to determining a single parameter, the simultaneous consideration of two objectives and their bandwidths is more complex for the DM. Thinking in two dimensions is not trivial for many DMs and can cause difficulties. The ENTSCHEIDUNGSNAVI supports the DM using indifference curves and different explanation variants of the trade-off statements (von Nitzsch et al. 2020). Figure 1 shows the user interface of the ENTSCHEIDUNGSNAVI in this step.

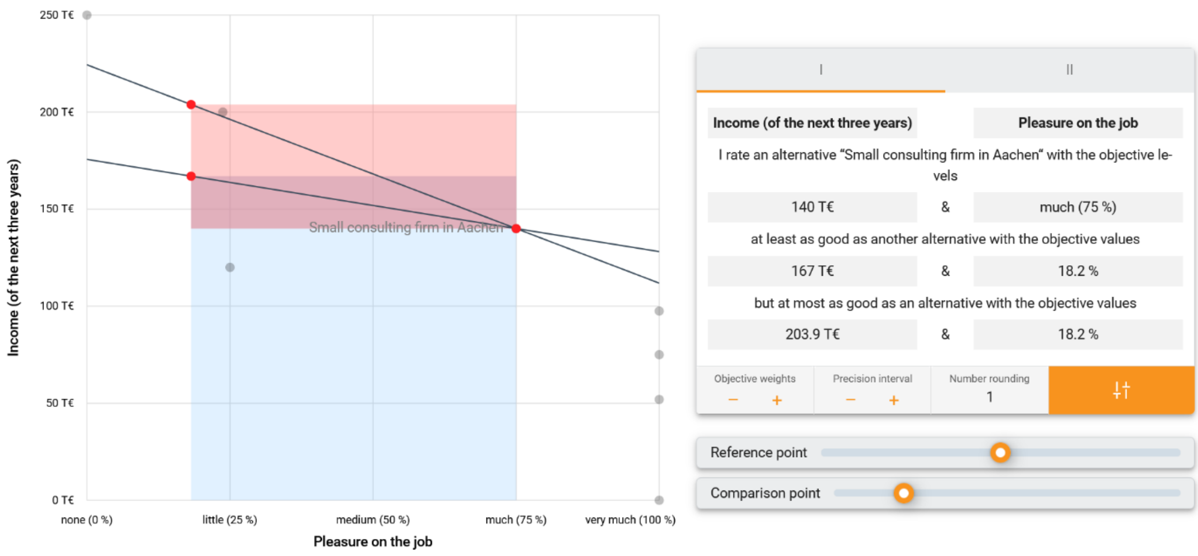

Figure 1. *Determining imprecise trade-off statements in the* ENTSCHEIDUNGSNAVI.

The ENTSCHEIDUNGSNAVI calculates the objective weights based on the trade-offs defined by the DM. Therefore, it determines the appropriate indifference curves for each trade-off. DMs can adjust the trade-off statements or indifference curves until they fit their preferences. If imprecise information is given, a degree of precision $\varepsilon_{w_i} \neq 0$ exists, and the corresponding indifference curve splits into a two-dimensional area (see Figure 1). That might be even more complicated for the DM to understand. After determining all trade-offs and possible imprecisions, the ENTSCHEIDUNGSNAVI calculates the objective weights w_i. If imprecise information is given, it results in an interval, as in Formulas (6a) and (6b).

$$w_i^{min} = w_i - \varepsilon_{w_i} \tag{6a}$$

$$w_i^{max} = w_i + \varepsilon_{w_i} \tag{6b}$$

For our analysis, we calculate an average degree of precision for objective weights $\overline{\varepsilon_{w_i}}$.

3.3 Ranking the alternatives using Monte Carlo simulations

With a robustness check in the ENTSCHEIDUNGSNAVI, the DM can check the stability of their ranking of alternatives. To that end, the ENTSCHEIDUNGSNAVI makes use of a Monte Carlo simulation (Kalos and Whitlock 2009). Evenly distributed random values are drawn from the intervals of parameters, which are imprecisely defined by the degree of precision $\varepsilon_{P(S_\delta)}$, ε_{U_i}, ε_{c_i}, and ε_{w_i} (see, Section 3.2). In this paper, we use 100,000 simulation steps to answer the research questions.

The procedure of drawing random probability values for the influence factors S_δ is iterative. Firstly, the value $P\left(s_\delta^f\right)$ with the greatest probability interval is determined using a random drawing. Secondly, the range of the next smaller probability interval is recalculated so that an empty solution space is excluded, and all other probabilities can lie within their intervals. Thirdly, the value $P\left(s_\delta^f\right)$ of the next smaller probability interval is determined in analogy to step 1. These steps are repeated until every state has been assigned a probability. The utilities are calculated by randomly selecting values from the respective intervals. For discrete utility functions, random values are drawn from the interval bounded by Formulas (4a) and (4b). For exponential utility functions, a random c_i from the interval bounded by Formulas (5a) and (5b) is drawn to calculate the corresponding utility with the help of Formula (2b). The objective weights are randomly drawn from the interval bounded by Formulas (6a) and (6b) and are normalized afterward. After drawing and determining all relevant parameters, the expected utilities are calculated using Formula (1) for every alternative. Finally, the resulting rankings from the Monte Carlo simulations are compared with the ranking of alternatives based on the mean values of the intervals of parameters. Figure 2 shows the user interface of the robustness check in the ENTSCHEIDUNGSNAVI. The robustness check presents three results for every alternative: Ø rank, frequency of rank, and range of calculated expected utilities. The frequency of rank indicates how often an alternative has ended up in the respective ranking positions in the simulations. If the first (second, third, …) ranked alternative results in 100 % of the simulations on the first (second, third, …) rank position, the result is robust to imprecise information. In the example in Figure 2, this applies to the fifth, sixth, and seventh-ranked alternatives. If the first (second, third, …) ranked alternative results in less than 100 % of the simulations on the first (second, third, …) rank position, imprecise information impacts the result, e.g., in Figure 2, the first-ranked alternative is in first place in only 51 % of the simulations. In the remaining simulations, it ended up in second to fourth place, depending on the parameters drawn from imprecise intervals. The Ø rank calculates the average rank position of each alternative after all simulations. The range of calculated expected utilities shows the minimum and maximum utilities the alternatives have achieved in the simulations. For a more detailed description, see Peters et al. (2024).

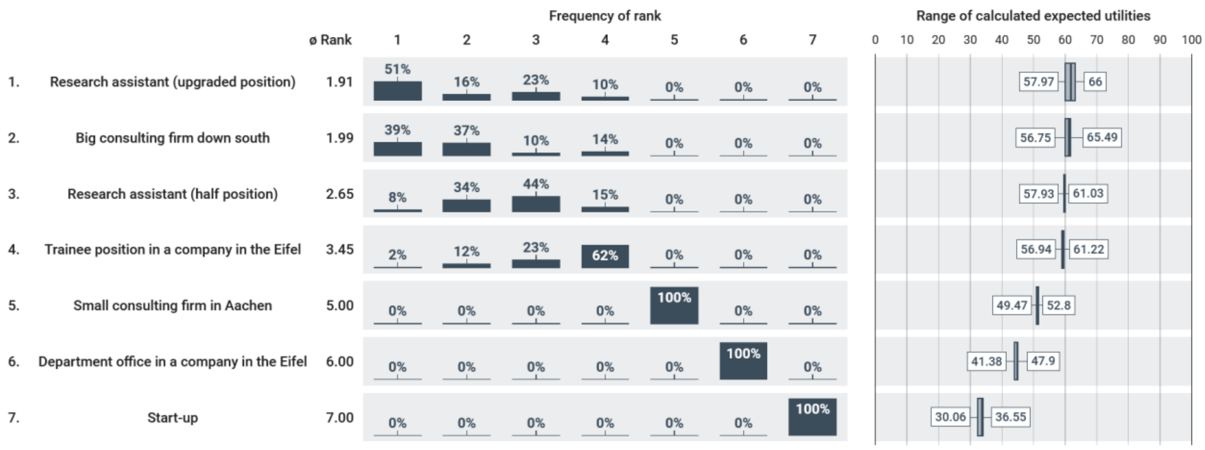

__Figure 2.__ Robustness check in the ENTSCHEIDUNGSNAVI.

4 Empirical findings on the impact and degree of utilization of imprecise information

4.1 Data set and participants

The data set was collected during the winter terms 2018/2019, 2019/2020, and 2020/2021 at a large university in Germany. A voluntary project within the course "Decision Theory" enabled the students to improve their final grade by a third (e.g., B to B+). The task was to analyze a decision situation (selection problem, where the DM seeks one choice) with the ENTSCHEIDUNGSNAVI. The most popular topic for the participants was career planning after studying (28 %), followed by organizing their study program (22 %) and planning leisure activities (18 %). The less frequently chosen topics were, for example, purchase decisions, political decisions, or housing decisions.

The students only received a better grade if all steps in the ENTSCHEIDUNGSNAVI were completed with a reasonable effort. The decision situations had to be fully structured, including formulating the decision statement, identifying all relevant objectives, and determining the alternatives. Furthermore, the consequences table had to be developed as undistorted as possible. Moreover, the alternatives had to be evaluated, which included determining the utility functions and the objective weights. Therefore, the DM had to define their preferences in a way consistent with their values. Content and technical support were given via mail, telephone, or consultation hours. After submission, every decision situation was checked manually by the paper's authors to ensure that the decision situation was completed carefully and appropriately. In addition, participants had to fill in a short questionnaire about the difficulty and usefulness of each step and input options in the ENTSCHEIDUNGSNAVI.

Overall, 1,755 full-time students enrolled in business, engineering, or computer sciences study programs participated. 244 out of 1,755 decision situations could not be considered because the participants did not spend the appropriate effort. Thus, the existing dataset consists of 1,511 carefully analyzed decision situations. The decisions provide information about essential decision parameters (i.e., decision statement, objectives and their weights, alternatives, consequences table, and utility functions), including imprecision regarding probabilities, utility functions, and objective weights.

The participants spent an average of 9.8 hours on their decision situation. They identified, on average, 4.69 ($\sigma = 1.48$) objectives and 5.92 ($\sigma = 2.18$) alternatives; the maximum number for objectives was 24 and for alternatives 23. Moreover, on average, they identified 2.56 ($\sigma = 2.62$) influence factors (maximum: 24; minimum: 0) to model uncertain scenarios.

4.2 Procedure

To examine the research questions in more detail, we split the participants into two groups and varied the default degrees of precision in the ENTSCHEIDUNGSNAVI. For Group non-zero (958 participants), the default setting for the degrees of precision was not zero, which resulted in imprecise intervals for probabilities, utility functions, and objective weights in every newly analyzed decision situation. For Group zero (553 participants), the default setting for all degrees of precision was set to zero. So, the participants had to actively choose a degree of precision if they wanted. To simplify implementation in the ENTSCHEIDUNGSNAVI, we assigned participants in the winter semesters 2018/2019 and 2019/2020 to Group non-zero and participants in the winter semester 2020/2021 to Group zero.

All research questions were analyzed using the complete data set and the two groups. To answer the **first research question**, 'How often is imprecise information used?' the data was analyzed to determine how often the DMs use imprecise information in total and imprecise information in the three categories (probabilities, utility functions, objective weights). For this purpose, the total number of imprecise parameters, the number of decision situations in which imprecise information was used, and the average degrees of precision were examined. The **second research question**, 'How helpful do participants find the use of imprecise information?' can be answered based on the questionnaire. We asked the participants how helpful they found using imprecise information about the three categories (probabilities, utility functions, and objective weights). The participants could answer on a scale from 1 (unnecessary) to 6 (very helpful). To analyze the **third research question**, 'What impact does imprecise information have on the final ranking of alternatives?', we performed a Monte Carlo simulation described in Section 3.3. Therefore, we allowed imprecise information in all three categories and analyzed how often the best (second, third) alternative calculated by the mean values of the intervals of parameters takes first (second, third) place in the 100,000 simulations. To examine the impact of imprecise information on the individual categories more closely, we have again carried out Monte Carlo simulations in which imprecise information is permitted in only one of the three categories. Using this information, we analyze the impact of imprecise information in the three categories on the stability of the best alternative in every decision situation.

4.3 Results

4.3.1 How often is imprecise information used?

The results are summarized in Table 1. In Group non-zero, 99.90 % of the participants used imprecise information in at least one category in their decision situation. In Group zero, the utilization was lower and amounted to 69.80 %. This variation is also reflected in the individual categories. In all categories, participants in Group zero used imprecise information less than participants in Group non-zero. This is illustrated by the number of decisions as well as the total number and the average values. In Group non-zero, the use of imprecise information in the different categories was similar in the decisions (probabilities: 99.72 %, utility func- tions: 96.97 %, objective weights: 97.91 %). However, the use of the various categories differed in Group zero. The participants employed imprecise objective weights less (19.71 %) than imprecise probabilities (58.94 %) or utility functions (60.22 %). A more detailed analysis of the results in Group non-zero shows that most participants chose a degree of precision greater than or equal to the default setting for the categories probabilities and utility functions. However, 95.62 % of participants in Group non-zero used a degree of precision for imprecise objective weights that was lower than the default setting. Considering the number of all imprecise values independent of the decisions in the complete data set, imprecise probabilities were used most (86.84 %), followed by utility functions (69.81 %) and objective weights (52.22 %).

		Group non-zero[2]				Group zero[3]	Complete data set
Imprecise…		**all**	$< \varepsilon$	$= \varepsilon$	$> \varepsilon$		
…information[1] (in %)		99.90	-	-	-	69.80	88.88
…probabilities (in %)	**Participants**	99.72	9.69	40.73	49.30	58.94	87.57
	In total[4]	99.28	8.59	55.84	34.84	52.01	86.84
	$\overline{\varepsilon_{P(S_\delta)}}$	$\pm$12.43 (σ = 5.35)	$\pm$5.84 (σ = 1.86)	$\pm$10 (σ = 0)	$\pm$18.20 (σ = 4.95)	$\pm$6.41 (σ = 8.23)	$\pm$10.84 (σ = 6.78)
…utility functions	**Participants (in %)**	96.97	20.25	58.87	17.85	60.22	83.52
	In total[4] (in %)	88.85	10.47	37.12	41.26	38.42	69.81
	$\overline{\varepsilon_{u_i}}$	$\pm$7.47 (σ = 5.66)	$\pm$3.05 (σ = 1.03)	$\pm$5 (σ = 0)	$\pm$12.08 (σ = 6.23)	$\pm$5.03 (σ = 8.83)	$\pm$6.60 (σ = 7.06)
	$\overline{\varepsilon_{c_i}}$	$\pm$0.60 (σ = 0.59)	$\pm$0.28 (σ = 0.10)	$\pm$0.5 (σ = 0)	$\pm$1.07 (σ = 0.71)	$\pm$0.28 (σ = 0.52)	$\pm$0.48 (σ = 0.58)
…objective weights (in %)	**Participants**	97.91	95.62	0.10	2.19	19.71	69.29
	In total[4]	78.18	75.82	0.11	2.24	9.45	52.22
	$\overline{\varepsilon_{w_i}}$	$\pm$1.21 (σ = 0.02)	$\pm$1.30 (σ = 0.86)	$\pm$5 (σ = 0)	$\pm$9.96 (σ = 0.08)	$\pm$0.12 (σ = 0.01)	$\pm$0.80 (σ = 0.02)

Table 1. *Results - How often is imprecise information used?*

[1] The participants use imprecise information in their decision situation, no matter in which category.

[2] For Group non-zero, the default setting was $\varepsilon_{P(S_\delta)} = \pm 10$ %, $\varepsilon_{u_i} = \pm 5$, $\varepsilon_{c_i} = \pm 0.5$, and $\varepsilon_{w_i} = \pm 5$ %.

[3] For Group zero, the default setting was zero for all degrees of precision.

[4] All defined probabilities/utility functions/objective weights were analyzed.

4.3.2 How helpful do participants find the use of imprecise information?

The results are summarized in Table 2. The results of the questionnaire show that all in all, the participants found the use of imprecise information on probabilities most helpful ($\mu = 4.24$; $\sigma = 1.3$), followed by imprecise utility functions ($\mu = 4.16$; $\sigma = 1.15$). The specification of imprecise preference statements for the objective weights was perceived as least helpful ($\mu = 3.98$; $\sigma = 1.35$). In Group zero, the participants rated the helpfulness significantly lower than in Group non-zero. Moreover, in Group zero, the imprecise probabilities and utility functions were assessed as being about equally helpful (3.93 and 3.96), while in Group non-zero, they were assessed somewhat differently (4.41 and 4.28). However, the helpfulness of imprecise objective weights was rated lowest in both groups. In addition, participants who used imprecise probabilities, utility functions, or objective weights in their decisions rated the helpfulness significantly higher than those who used precise parameters. Both groups again rated imprecise objective weights as least helpful.

Imprecise…	**Group non-zero** **n = 958**	**Group zero** **n = 553**	**Participants using imprecise…** **n = [890[1]; 1262; 1047]**	**Participants using precise…** **n = [126[1]; 249; 464]**	**Complete data set** **n = 1511**
…probabilities	4.41 (σ =1.21)	3.93 (σ =1.39)	4.47 (σ =1.19)	3.94 (σ =1.41)	4.24 (σ =1.30)
…utility functions	4.28 (σ =1.03)	3.96 (σ =1.31)	4.25 (σ =1.08)	3.73 (σ =1.40)	4.16 (σ =1.15)
…objective weights	4.10 (σ =1.35)	3.79 (σ =1.33)	4.11 (σ =1.34)	3.71 (σ =1.34)	3.98 (σ =1.35)

Table 2. *Results - How helpful do participants find the use of imprecise information? The participants could answer on a scale from 1 (unnecessary) to 6 (very helpful). The pairwise comparison is highly significant between Groups non-zero and zero and between participants using imprecise parameters and those using precise ($p<0.001$).*

[1] 495 out of 1511 participants did not use influence factors in their decisions.

4.3.3 What impact does imprecise information in the three categories have on the final ranking of alternatives?

To answer this research question, we focused on the best (three) alternatives ranked by utilities in every decision. The analysis of the Monte Carlo simulation of 1,342 decision situations implementing imprecise information yielded the result shown in Figure 3. In addition to the cumulative course of the min. frequency on a ranking position, the figure also shows two characteristic points of this course for every rank position. The first point describes how many decisions have a stable rank position. The second point describes the minimum frequency of the rank position for all decisions.

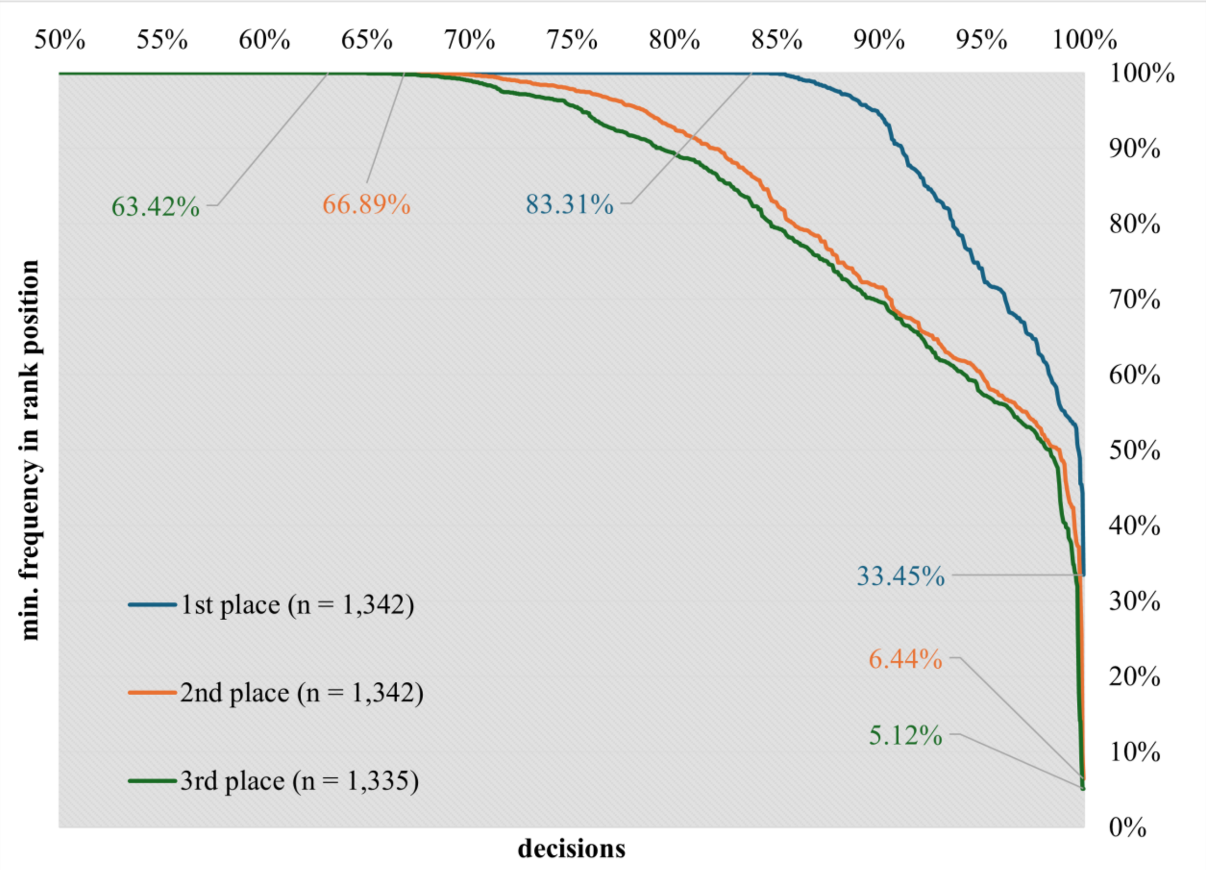

Figure 3. *Impact of imprecise information on the final ranking.*

In 83.31 % of the decisions, the first place is stable. This means that in all 100,000 simulations (100 %), the best alternative calculated by the mean values of the interval of parameters takes first place, and, therefore, the best alternative does not change. In the worst case, imprecise information resulted in this alternative being the best in only 33.45 % of cases. If the first place is not 100 % stable, an additional analysis revealed that the second (third) placed alternative came out in the first place in 17.20 % (3.57 %) of the decisions. The second and the third-placed alternatives are less stable than the first. The second (third) best alternative calculated by the mean values of the intervals of parameters takes second (third) place in 100 % of the simulations for 66.89 % (63.42 %) of the decisions. Among all the decisions, there are some in which the second and third place reacts very sensitively to imprecise information, and a different ranking emerges in almost all simulations compared to the ranking calculated from the mean values. However, most decisions (about 98 to 99 %) have a min. frequency of about 46 % to 51 % in a rank position, depending on the rank position.

Figure 4 shows the results of analyzing the impact of imprecise information on the stability of the best alternative in Group non-zero and zero. In Group zero, the first place is stable for 88.83 % of the decisions. In Group non-zero, the percentage of the decisions in which the first alternative is robust was 81.07 %. Considering up to about 96 % of all decisions, the min. frequency in the best alternative is higher in Group non-zero than in Group zero. In Group non-zero, there is a min. frequency in the best alternative of 33.45 % in all decisions. In Group zero, the value was higher (45.49 %).

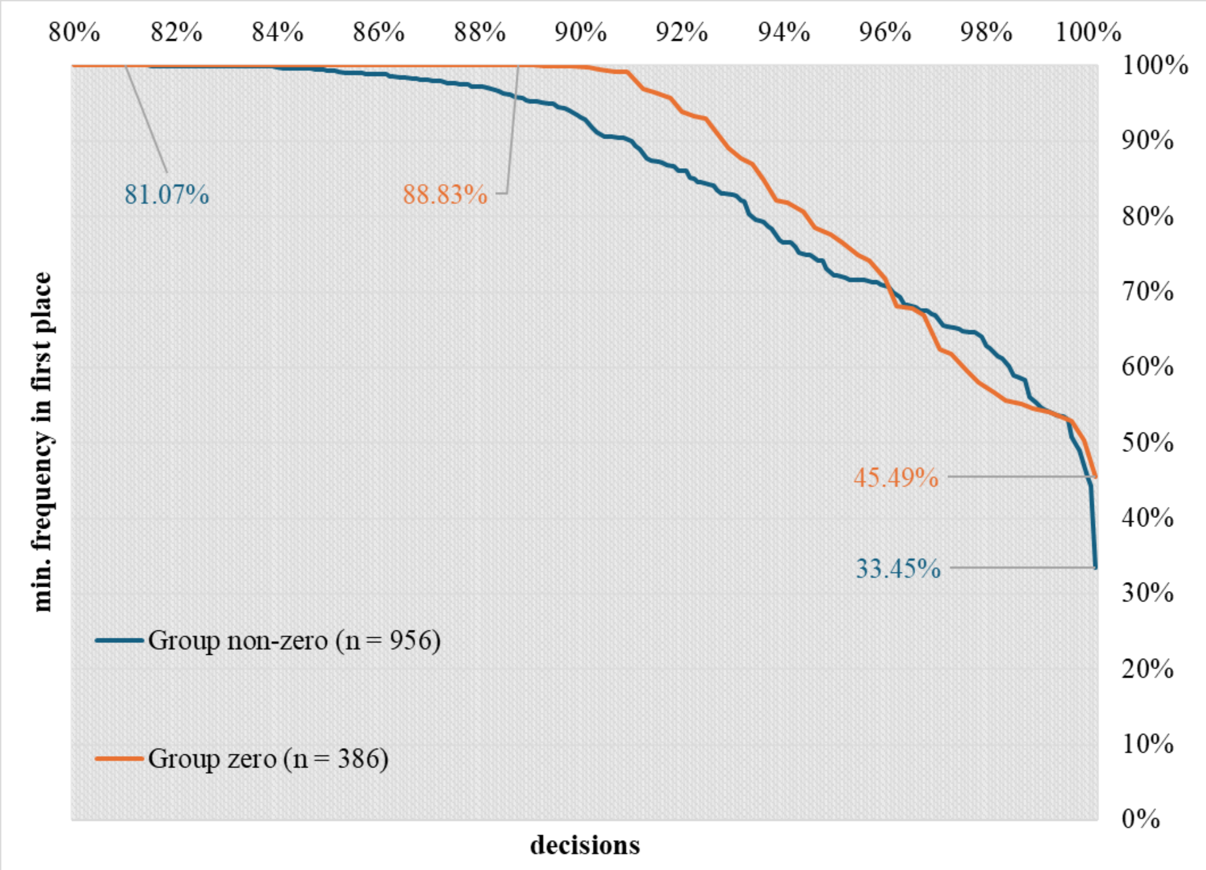

Figure 4. *Impact of imprecise information on the best alternative in Group non-zero and zero.*

Furthermore, we analyzed the three categories of imprecise information more closely and permitted imprecise information in only one of the three categories in the Monte Carlo simulations. The analysis of the Monte Carlo simulation of 888 (1,262; 1,047) decisions implementing imprecise information regarding probabilities (utility functions, objective weights) yielded the results shown in Figures 5a, 5b, and 5c. Imprecise utility functions have the greatest impact on the stability of the best alternative compared to the impact of imprecise probabilities and objective weights, as the lowest number of decisions has a minimum frequency in the first place of 100 %. Imprecise objective weights lead to slightly higher stability, and imprecise probabilities have the lowest impact on 100 % stability of the best alternative. However, there are no major differences between the categories, as the frequencies vary between 90 and 94 %. The lowest minimum frequency in the best alternative is achieved with imprecise probabilities (33.44 %), followed by imprecise utility functions (45.34 %) and imprecise objective weights (45.53 %).

The analyses of the various groups show no major differences in the categories. On top of that, we analyzed that there exists no statistical correlation between the average degrees of precision and the stability of the best alternative.

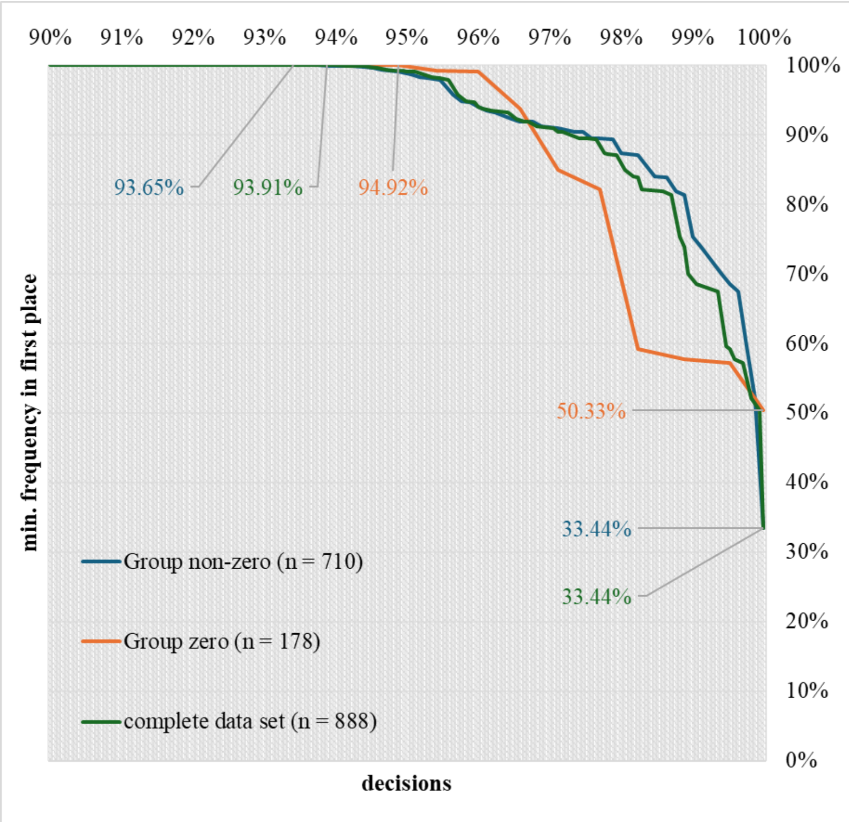

Figure 5a. *Impact of imprecise probabilities on the best alternative.*

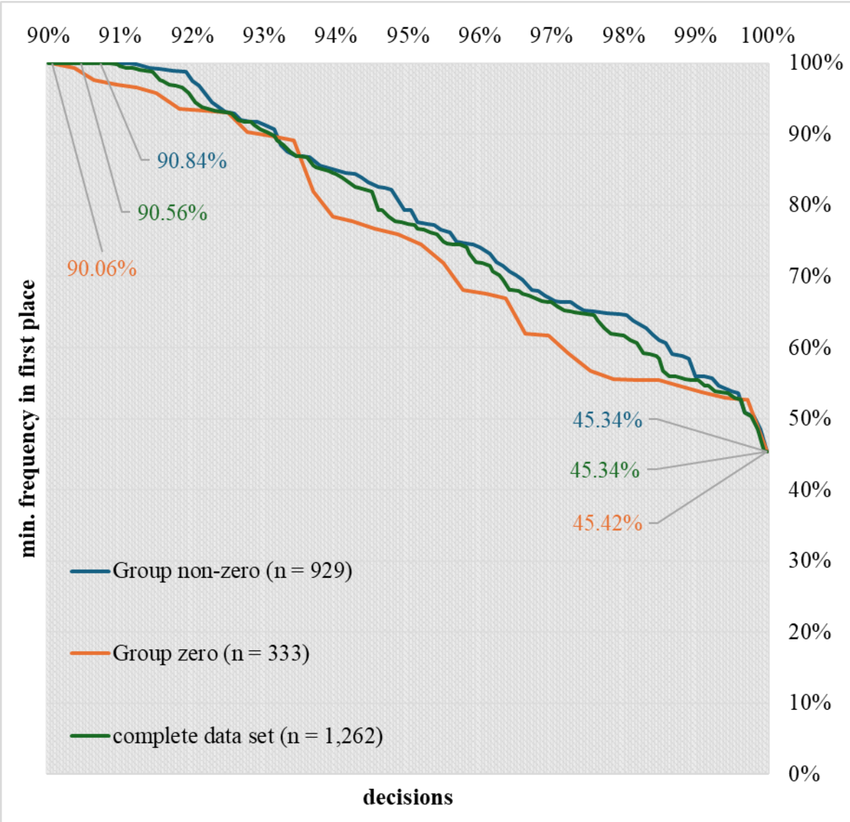

Figure 5b. *Impact of imprecise utility functions on the best alternative.*

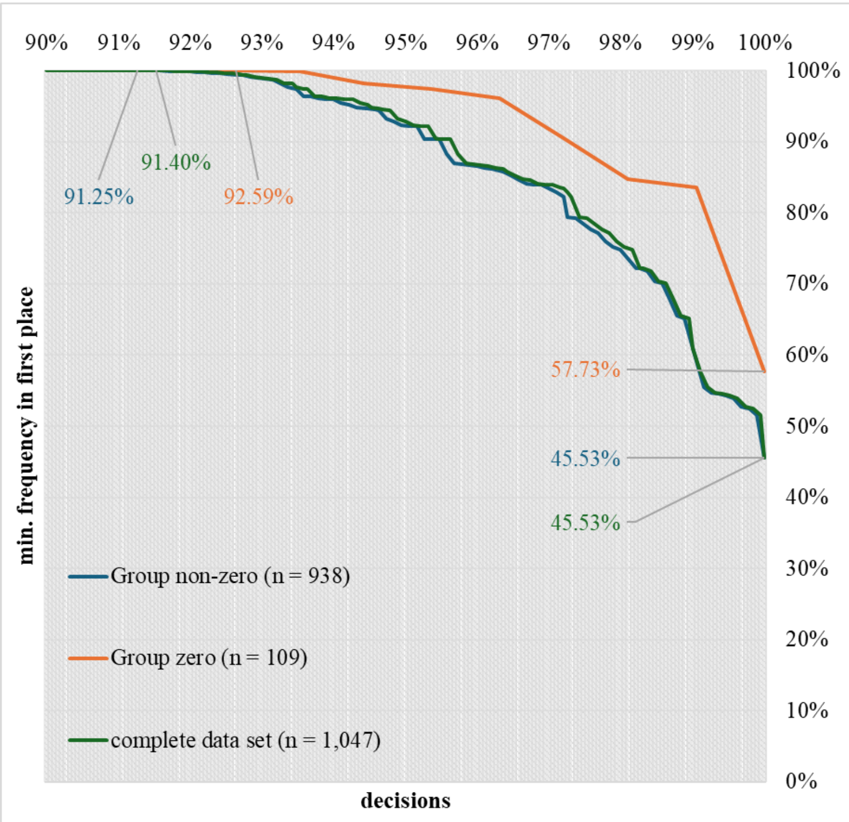

Figure 5c. *Impact of imprecise objective weights on the best alternative.*

5 Discussion

The paper shows that the imprecise information approach used in the ENTSCHEIDUNGSNAVI is a valid and accepted concept for determining parameters in a DSS under MAUT. About 89 % of the participants use imprecise information for at least one parameter in their decision. Even in Group zero, where the participants had to actively choose a degree of precision, almost 70 % made imprecise statements, resulting in intervals of the decision parameters. In Group non-zero, almost all of them used imprecise information. Moreover, the participants generally found using imprecise information more helpful than unnecessary. These results show the need for imprecise information in a DSS to support the DM eliciting the quantitative decision parameters.

Our results also show that the use of the imprecise information approach differed between the categories. The participants found imprecise objective weights less important, as they used them the least (in about 69 % of the decisions), followed by imprecise utility functions (about 84 %) and imprecise probabilities (about 87 %). In Group zero, the use of the categories differed even more. Imprecise objective weights were only used in approx. 20 % of the decisions, with an average epsilon of 0.12 %. The detailed analysis of Group non-zero shows that in contrast to the other categories, most participants reduced the default degree of precision for objective weights.

We also showed that not only the use of imprecise information in the categories differed, but also the perceived support of the approach in the categories. Therefore, imprecise objective weights were not only the least used by the participants but were also perceived as the least helpful. This applies to the participants who have used imprecise parameters and those who have not.

These results could indicate that an imprecise information approach regarding probabilities and utility functions, like in the ENTSCHEIDUNGSNAVI, benefits the participants more than imprecise information regarding objective weights. The lower utilization and evaluation of imprecise objective weights could be explained by the fact that the trade-off method generally requires a higher understanding of the topic than the determination of the other parameters. The DM has to think in two dimensions (trade-off between two objectives), which is not trivial. Moreover, indifference curves for trade-off statements are generally more difficult to interpret than a direct rating of probabilities or utilities. If imprecise information is added, the process becomes much more complicated, and DMs may tend to disregard imprecise information even though they are unsure of their preference statements.

The analysis of the different groups shows the expected effect. Significantly more participants use imprecise information if the default setting in the ENTSCHEIDUNGSNAVI already specified this. In Group non-zero, almost all participants use imprecise information, and there are no big differences in the frequency of use of the different categories. Several researchers have shown that the decision architecture can nudge the DM's decision (see, e.g., Thaler and Sunstein (2021)). In our study, the default setting of imprecision in Group non-zero also led to it being used more frequently. Thus, the determination of the parameters could be influenced by nudging in this group. In addition, the status quo bias (Samuelson and Zeckhauser 1988, Kahneman et al. 1991) could also play a role. Maybe some participants were too lazy or did not think about the default setting and accepted them even though they did not need any imprecision in their decision problem. In Group zero, where the participants had to actively choose a degree of precision, more than half of the participants still used imprecise information regarding probabilities (about 59 %) and utility functions (about 60 %). That means the DMs felt unsure about determining these parameters and, therefore, chose imprecise information. However, the percentages are significantly lower than in Group non-zero.

According to MAUT, the DM should always choose the alternative with the highest utility in selection problems, which makes the ranking of the other alternatives unimportant for the implementation of the decision situation. The results of the third research question show that the parameter intervals resulting from imprecise information in the ENTSCHEIDUNGSNAVI, in general, have no big impact on the stability of the best alternative ranked with MAUT. In only about 17 % of the decisions, there could be a change of the best alternative due to a corresponding constellation of imprecise parameters. If the simulations are carried out only for individual categories (probabilities, utility functions, or objective weights), the percentage of those cases is even lower (about 6 %, 10 %, and 9 %). This could lead to the conclusion that precise preference statements and parameters in MCDM-Support-Systems are not so important since the best alternative does not change with imprecise information anyway in most cases.

However, our results show that, in particular, for objective weights, the best alternative varied very sensitively. The relatively low selected imprecision for objective weights (see, $\overline{\varepsilon_{w_i}}$) led to a similar impact as, e.g., imprecise probabilities, which, on average, were chosen much more generously by the participants. If the participants were to choose larger epsilon values for their objective weights, the impact on the stability of the ranking would be even more significant. This means that DMs are confronted not only with their unsureness regarding the determination of objective weights but also with an unsure ranking and best alternative. This makes decision-making considerably more difficult.

The assumption that the trade-off method prevents the participants from using imprecise parameters due to the two-dimensional thinking and the fact that imprecise objective weights are very sensitive to the result lead us to assume that the ranking would be less stable if a different, easier determination of the objective weights were used. In that case, participants might use imprecise objective weights more often, which leads to a bigger impact on the result. This hypothesis could be investigated in future research work.

6 Conclusion

This paper analyzes the use and impact of an imprecise information approach in the DSS ENTSCHEIDUNGSNAVI. Therefore, 1,511 decision situations analyzed with the DSS were evaluated. Our results show that integrating imprecise information in the ENTSCHEIDUNGSNAVI supports the DM and the decision-making process. Most participants used imprecise information in their decision situations in at least one category and found this helpful. However, we observed different usage behaviors in the categories. Imprecise probabilities and utility functions were used more frequently and with considerably larger intervals than imprecise objective weights. This is probably due to the trade-off method used in the ENTSCHEIDUNGSNAVI, which is already complicated enough and, therefore, not easy to understand for DMs. Moreover, we show that in many cases, the decision remains stable despite imprecise information. In about 83 % of the decisions, the best alternative was robust. This means that no matter which parameters were taken from the defined imprecise intervals in the simulations, the best alternative did not change. However, the second and third-placed alternatives were more sensitive to a change in rank position than the best alternative. As expected, the comparison of Group non-zero and zero showed that the participants in Group zero used imprecise information significantly less than in Group non-zero. This difference can be explained by the status quo bias and nudging. In Group non-zero, only very few participants actively chose precise parameters. In addition, no major differences were identified between the frequency of use in the categories.

Our results are related to the sample of participants, which only consists of students who attended lectures on decision theory, and the DSS ENTSCHEIDUNGSNAVI, which uses specific imprecise information approaches. Therefore, the results cannot and should not be generalized. The students have extensive knowledge about the decision-making procedure and may not be representative. Moreover, they received a reward for careful processing. So, some might use more imprecision than they would usually do to show increased effort, even though they were explicitly told to state their preferences. In particular, the fact that imprecise objective weights are less important for DMs should not be generalized. This may only apply to DSS or approaches that use the trade-off method, like the ENTSCHEIDUNGSNAVI. Other tools that use simpler objective weighting methods may lead to higher use of imprecise objective weights. The approach and methodology are, therefore, very decisive. Future research could investigate this in more detail as the sensitivity of imprecise objective weights on the result presented in this paper also shows the importance of this topic.

Every MCDM support system should consider imprecise information approaches to support DMs. This paper convincingly argues that DMs cannot or do not want to specify the parameters precisely. In any case, linking imprecise information approaches with practical evaluation methods is important. Evaluation methods, like the robustness check in the ENTSCHEIDUNGSNAVI, help DMs find the best alternative even though they choose imprecise parameters. They support DMs in making the impact of imprecise parameters transparent and create awareness of potentially unstable rankings due to imprecise parameters. If the resulting ranking is stable, DMs do not have to worry about their imprecise statements. They can even use imprecision to a certain extent without hesitation to find the best alternative. If the resulting ranking is unstable, DMs can try to determine their parameters more precisely. Then, a renewed robustness check can show whether the ranking has become more stable and whether the DM has minimized the risk of not choosing the best alternative. This gives DMs good insights into the impact of their imprecisely chosen parameters.

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Paper 5: Linear transformation of one-dimensional utility functions: an empirical study on the impact on the final ranking of alternatives in personal decisions

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Contents

Linear transformation of one-dimensional utility functions: an empirical study on the impact on the final ranking of alternatives in personal decisions

Determining one-dimensional utility functions for each objective in multi-attribute utility theory takes time and effort from decision-makers. They must consider including a decreasing or increasing marginal utility and/or their relative risk attitude, resulting in a non-linear shape. This assessment is prone to errors and distortions. We analyze to what extent a linear transformation of one-dimensional utility functions compromises the quality of the decision. Therefore, we examine the impact of one-dimensional utility functions on the final ranking of alternatives in practice, focusing on three aspects: the use of (non-)linear utility functions, their impact on the ranking of alternatives, and the stability of best alternatives concerning utility differences of alternatives assuming linear transformation. We examine 2,536 carefully modeled personal decisions analyzed by students with the decision support tool ENTSCHEIDUNGSNAVI. Our results show that 95.9 % of the participants used at least one non-linear utility function in their decision, and 76.4 % of all objectives were evaluated with non-linear utility functions. Simplifying preference-accurate utility functions with linearization led to a rank reversal of the best alternative in 15.5 % of the decisions. The top three set of alternatives changed in 14 % of the decisions. In 98.9 % of the decisions, the best alternative could be found in the top three alternatives ranked under linearity. Based on our results, we recommend determining the utility functions preference-accurate using (non-)linearity to model the decision as precisely as possible, especially for important decisions. However, no rank reversal for the best alternative was detected in our dataset from an absolute utility difference greater than 0.27 between the best and second-best alternatives under linearity. In these cases, assuming linear utility functions are useful if decision-makers want to save time and effort.

Keywords: decision analysis; utility functions; multi-attribute utility theory; preference elicitation; experiment

1 Introduction

In decision analysis (DA) theory, many different and complex value models exist to support the decision-maker (DM) in finding the best alternative in decisions with multiple objectives. These models use theoretically justifiable assessment procedures based on scientifically sound sources. In practice, however, several researchers (Keeney and von Winterfeldt 2007, Durbach and Stewart 2012, Katsikopoulos et al. 2018) point out that the usage of such a model is not always necessary or useful. Keeney and von Winterfeldt (2007) argue that a practical value model is good enough if the analysis with a theoretically better model does not yield a different result than the practical one used. Baucells et al. (2008) (for a follow-up, see Şimşek (2013), Katsikopoulos et al. (2014)) and Methling et al. (2022) test simple heuristics in multicriteria decision-making and show that these heuristics can help to reduce complexity and still recommend the same alternative as analytical approaches under certain conditions. It raises the question of how complex a DA should be in practice and which preference elicitation methods are best suited in which settings. Keeney and von Winterfeldt (2007) point out that the choice of an appropriate DA model depends on the characteristics of the situation and the DM, the time available, and the skills of the decision analyst facing the decision situation.

The multi-attribute utility theory (MAUT) (Keeney and Raiffa 1976) is a well-established DA model for identifying the best alternative in decisions with multiple objectives (Belton and Stewart 2002). It has been a fundamental component of DA for many years, and its methodology has been extensively studied in various practical contexts (e.g., Dyer and Smith (2021), Liesiö and Vilkkumaa (2021), Caballero et al. (2022)). In MAUT, the alternatives are ranked based on aggregated utilities scaled to a range from zero (worst utility) to one (best utility). The best alternative, with the highest utility, is the one the DM should choose. To apply this concept, the DM first formulates the decision situation, identifies the relevant objectives (Keeney and Gregory 2005, Butler et al. 2006, Bond et al. 2008), and determines the alternatives (Keeney

1992, Siebert and Keeney 2015). Second, the DM evaluates the consequences of the alternatives for each objective. Third, the utility of each alternative is calculated. To do this, DMs must define their one-dimensional utility function and weight for each objective based on their preferences. Keeney and Raiffa (1976) outline three different methods for calculating the utilities and aggregating all objectives in MAUT: the multiplicative, multilinear, and additive utility functions. In this paper, we focus on the latter, which is, in addition to the multiplicative one, the most common decomposition (Keeney and von Winterfeldt 2007).

The basis of the additive multi-attribute utility function is the concept of one-dimensional utility functions, which map the DM's preferences for decisions under risk and uncertainty conditions (Fishburn 1965, Keeney and Raiffa 1976). Furthermore, they normalize the consequences so that the consequences of each objective can be compared with each other. One of the most used utility functions is the exponential one (Vilela and Oluyemi 2022). Here, risk-averse preferences of the DM result in a concave curve, whereas risk-prone preferences result in a convex shape. Linear utility functions express risk-neutral preferences, which are the most straightforward shape to elicit and interpret for DMs (André and Riesgo 2007). To find the best alternative using MAUT, DMs have to determine the shape of the utility function that is consistent with their preferences for every objective.

Determining one-dimensional utility functions is a major challenge and takes time and effort from the DM. Many researchers deal with how to assess these utility functions to support the DM in this challenging task (e.g., Pratt et al. (1964), Keeney (1972), Keeney and Raiffa (1976), Anderson et al. (1977), Keeney (1977), Farquhar (1984)). However, DMs often cannot formulate their preferences consistently (Cyert and DeGroot 1975, Keeney and Raiffa 1976, Keeney 1982).

In many cases, the assessment is prone to errors and distortions. Montibeller and von Winterfeldt (2015) summarize biases influencing the elicitation of utility functions. In addition to the dependence on the design of stimuli and responses (Hershey and Schoemaker 1985, Johnson and Schkade 1989, Schoemaker and Hershey 1992), the assessment of the utility function can be impacted by the anchoring bias (Chapman and Johnson 1999), the gain-loss bias (Levin et al. 1998), the certainty effect (Allais 1953, Kahneman and Tversky 2013), the desirability of options bias (von Winterfeldt 1999), and the affect influenced bias (Slovic et al. 2004). Further researchers (Hershey et al. 1982, McCord and De Neufville 1986, von Nitzsch and Weber 1988, Wakker and Deneffe 1996) investigate more biases from prospect theory and identify methods to reduce their impact. Bleichrodt et al. (2001) recommend interactive sessions to ensure consistency and avoid biases. However, these sessions are time-consuming, expensive, and challenging for DMs.

In the meantime, artificial intelligence is used for preference elicitation (for an overview, see Toffano et al. (2022)), and the latest research deals with statistical robustness in utility preference robust optimization models (Guo and Xu 2021) when preference information is incomplete (Weber 1987). This shows that research on preference elicitation is still relevant and benefits DA.

For these reasons, concerning Keeney and von Winterfeldt's practical considerations, some researchers address how important the exact determination of utility functions is and whether the shape of utility functions influences the ranking of alternatives in MAUT. Stewart (1993) uses a Monte Carlo Study to find out that non-linearities in value functions are essential but can be simplified with piecewise linear functions. Value functions are used for riskless decisions in multi-attribute value theory (MAVT) (Keeney and von Winterfeldt 2007) and, thus, represent the counterpart for utility functions. Lahdelma and Salminen (2012) analyze (1) how the shape of the utility or value function impacts the results of stochastic multi-criteria acceptability analysis, which allows uncertain, imprecise, and partially missing information about criteria measurements and preferences, (2) what level of non-linearity changes the result of a linear model and (3) whether there are decisions in which the shape of the function is irrelevant. They conduct their analysis with one real-life problem and 3,600 artificially generated test problems. The analysis of the real-life problem reveals that the alternative ranking is more sensitive to convex than to concave utility or value functions. The analysis of the test problems shows that the larger the decisions are (increased number of alternatives and objectives), the less the best alternative has changed due to the utility functions. Considering the whole ranking, the authors point out that "the number of alternatives has no significant effect on the sensitivity to non-linearity" (Lahdelma and Salminen 2012), but the sensitivity increases with the number of objectives. However, they criticize the fact that their results are only based on generated test problems and simulations. They point out that the effects could be different in real-life problems.

To address this limitation and contribute to the discussion on using complex, preference-accurate utility functions, this paper analyzes 2,536 personal decisions of students on the impact of one-dimensional utility functions on the final ranking of alternatives in multi-attribute decision-making using MAUT. To do this, we first examine how often (non-)linear utility functions are used in personal decisions. Second, we analyze their impact on the alternatives' ranking, and third, we analyze situations in which the determination of one-dimensional utility functions is irrelevant to give recommendations for practice. Each decision situation in our data set includes all parameters relevant to MAUT (decision statement, objectives, alternatives, consequences table, utility functions, and objective weights) and, thus, represents a real-life application in the context of personal decisions. To the best of our knowledge, no other study is based on such a huge and detailed dataset in this field.

The dataset was collected with the ENTSCHEIDUNGSNAVI, an open-source decision support system (DSS), which supports the DM throughout the decision-making process. Therefore, the framework of value-focused thinking (Keeney 1992) is used in the decision front-end. In the decision back-end, the concept of MAUT (Keeney and Raiffa 1976) is applied. For a conceptual description and explanation of the ENTSCHEIDUNGSNAVI, see von Nitzsch et al. (2020), Peters et al. (2024), and Hannes and von Nitzsch (2024). On average, the DMs spent about eleven hours on their DA. Experts manually reviewed the DAs to ensure that the DMs truly understood their decision problems and that they had consistently elicited and aggregated their preferences with their value system.

The paper is structured as follows. Section 2 reviews the theoretical background of MAUT and the concept of one-dimensional utility functions. It also explains the reasons for (non-) linearity and presents the research questions. Section 3 introduces the model of determining one-dimensional utility functions in the ENTSCHEIDUNGSNAVI. Section 4 demonstrates the empirical findings of the research questions, and Section 5 discusses the results. Section 6 concludes our results, gives recommendations for action, and discusses limitations.

2 Relevant research on MAUT and one-dimensional utility functions

This paper concentrates on the additive utility function model of MAUT since it is the most frequently used model for multi-criteria decisions in practice (Ishizaka and Nemery 2013). Moreover, several researchers argue that an additive value model is simpler than a non-additive model (Stewart 1995, 1996), and if the objectives of a decision situation are fundamental, i.e., they are of key importance and not a means to another objective in the decision context, such a model is sufficient (Keeney and von Winterfeldt 2007). Thus, the additive utility function model forms the basis for our analyses and is implemented in the ENTSCHEIDUNGSNAVI (von Nitzsch et al. 2020). Section 3 explains the implementation of utility functions in the tool in more detail.

2.1 Additive model of MAUT and exponential utility functions

The additive utility function is used to determine the best alternative for the DM in multi-criteria decision-making (Fishburn 1965, Keeney and Raiffa 1976). It calculates the utility for alternatives $\mathbb{A} = \{A_1, \dots, A_J\}$ with $1 \leq j \leq J$ through consequences x_{ij} in several objectives $\mathbb{O} = \{O_1, \dots, O_I\}$ with $1 \leq i \leq I$ for some natural numbers I, J for the decision situation under uncertainty or risk. This utility can be used to rank the alternatives. An appropriate scale $[x_i^-; x_i^+]$ measures every objective and the determined consequences are evaluated using a one-dimensional utility function U_i by mapping each consequence x to a utility $0 \leq U_i(x) \leq 1$. The worst consequence x_i^- in objective O_i is assigned a utility of zero, and the best consequence x_i^+ corresponds to a utility of one. Moreover, specified weights w_i lead to a different weighting of the objectives in the additive utility function. They have to add up to one (1a). To model uncertainties, $\mathrm{P}(s_{ij}^k)$ represents the probability that state s_{ij}^k with $1 \leq k \leq K_{ij}$ occurs and consequence x_{ij}^k results. If $K_{ij} = 1$, the state s_{ij}^1 occurs with a probability of 100 %, and, therefore, x_{ij}^1 is a certain consequence. The probabilities for all states of an uncertainty have to add up to one (1b).

Formula (1) shows the additive expected utility for decisions under uncertainty or risk (Bernoulli 1954, von Neumann and Morgenstern 1961). Formulas (1a) and (1b) show the additional conditions.

$$EU(A_j) = \sum_{i=1}^{I} w_i \left[\sum_{k=1}^{K_{ij}} P(s_{ij}^k) U_i(x_{ij}^k)\right] \tag{1}$$

$$\sum_{i=1}^{I} w_i = 1 \tag{1a}$$

$$\sum_{k=1}^{K_{ij}} P(s_{ij}^k) = 1 \tag{1b}$$

The basis of the additive utility function is the concept of one-dimensional utility functions, which represent the preferences of the DM, the so-called risk attitude. The risk attitude takes into account both the strength of preferences and the relative risk attitude of the DM (Dyer and Sarin 1982). For a detailed explanation, see Section 2.2. Numerous shapes of one-dimensional utility functions in literature can be used to map the DM's preferences. These shapes can be linear or non-linear, monotonous or non-monotonous, exponential or others. In this paper, we focus on linear and non-linear, one-dimensional utility functions in the form of exponential ones since these are the forms most frequently used in practice (Vilela and Oluyemi 2022). Moreover, if DMs choose their objectives fundamentally enough, i.e., mean objectives are not considered, the preferences in these objectives can be described with sufficient precision using exponential utility functions (von Nitzsch 2024). Formula (2) shows the mathematical description of the exponential ($c \neq 0$) and the linear ($c = 0$) utility function $U_i\left(x_{ij}^k\right)$, which assumes a constant risk attitude.

$$U_i\left(x_{ij}^k\right) = \begin{cases} \dfrac{1-e^{-c_i \frac{x_{ij}^k - x_i^-}{x_i^+ - x_i^-}}}{1-e^{-c_i}} & if \quad c_i \neq 0 \\ \dfrac{x_{ij}^k - x_i^-}{x_i^+ - x_i^-} & if \quad c_i = 0 \end{cases} \tag{2}$$

The risk aversion parameter c_i reflects the risk attitude of the DM in objective O_i. A positive c results in a concave utility function, indicating risk aversion, while a negative c results in convexity, indicating risk proneness. The greater the risk aversion parameter, the stronger the DM's risk attitude and the greater the function's curvature. If $c = 0$, the DM is risk-neutral, and the utility function is linear (Pratt 1964).

2.2 Reasons for non-linear utility functions

Dyer and Sarin (1982) argue that risk attitude implicates two different factors, and thus, there are two reasons for non-linearity in utility functions for decisions under uncertainty or risk: strength of preferences (represented by a measurable value function) and the relative risk attitude (risk attitude relative to the strength of preferences). Value functions used for decisions under certainty, e.g., in the multi-attribute value theory (MAVT) model, only provide information about a decreasing, increasing, or constant marginal value and, thus, represent the strength of preferences of the DM. For a detailed description of value functions, see Dyer and

Sarin (1979) or Bell and Raiffa (1988). To determine the risk attitude excluding the strength of preferences, the so-called relative risk attitude, the utility can be compared with the value function. If the measurable value function corresponds to the utility function, Dyer and Sarin (1982) define the DM as relatively risk-neutral. In this case, only the in- or decreasing marginal value leads to a non-linear utility function. If the functions are different, the relative risk attitude is responsible for that difference, and the DM is either relatively risk-averse or relatively risk-prone. See also Krzysztofowicz (1983) for this context.

The exact determination of utility functions takes time and effort. Therefore, some researchers have already dealt with simplifying non-linear utility functions by linear ones. Keeney and von Winterfeldt (2007) state that linear utility functions are appropriate in three situations besides risk neutrality. The first situation is "when the attribute for a fundamental objective measures something that is of value in itself, as opposed to value for its uses" (Keeney and von Winterfeldt 2007). The second situation is when the DM uses expected consequences for an unsure consequence estimate. When using the expected value, it is naturally assumed that linearity is implicit and only a linear utility function should be used. The third situation, in which linearity can be assumed, is when several other decisions are made that contribute towards a common objective. This is because the range of possible outcomes of a single decision is relatively small compared to the range of outcomes for all decisions collectively.

In this paper, we take these statements as an incentive to examine the extent to which linear utility functions can replace non-linear ones in practice so that, on the one hand, the model can be simplified and, on the other hand, the best alternative can still be found.

2.3 Research questions

The following sections and research questions always refer to one-dimensional utility functions. For reasons of readability, the term 'one-dimensional' is no longer used in the following.

Since the literature emphasizes the importance of non-linearities in MAUT, we first want to analyze the significance for DMs in practice. Therefore, we investigate the following first research question.

RQ1 How often are (non-)linear utility functions used in practice?

To find the best alternative using MAUT, DMs must determine utility functions that are consistent with their preferences. We rely on the work of Keeney and von Winterfeldt (2007) and scrutinize if the model can be simplified with linear utility functions to save time and effort. Therefore, we investigate the impact of non-linear utility functions on the final ranking of alternatives and formulate the second research question as follows.

RQ2 How does the use of (non-)linear utility functions impact the final ranking of alternatives in practice?

In addition to the best alternative and the complete ranking, we also examine the first three alternatives explicitly, as we think these are of great importance. Most DMs focus on the best alternative when a decision is made using an approach such as MAUT. In some cases, however, it makes sense not to mindlessly rely on the model and choose the resulting best alternative. For example, gut feeling may not match the analytical recommendation. DMs would find it difficult to choose the recommended alternative against their gut feeling, especially if the alternatives have a small utility difference. Another example is when different preferences or approaches within a group may lead to different best alternatives. Here, it makes sense to enter into the discussion, which possibly no longer strictly follows all model assumptions. With the best three, a reasonable restriction is made to potential best alternatives, which can be discussed. That is why we also focus on the first three alternatives in this paper.

With the help of the third research question, we want to provide actionable practice recommendations. DMs should always choose the best alternative, even with model simplifications like linear utility functions. They should know in which situations non-linearity plays a role and in which it does not. Therefore, we formulate the third research question.

RQ3 To what extent do linear utility functions ensure stability in selecting the best alternatives based on utility differences?

We aim to provide a concrete recommendation for which utility difference is necessary to determine utility functions so that the DM does not choose the wrong alternative. Furthermore, we want to indicate the frequency of a rank reversal of the best alternatives depending on the utility difference under linearity conditions.

3 The model: determination of one-dimensional utility functions in the ENTSCHEIDUNGSNAVI

The ENTSCHEIDUNGSNAVI supports the DM in specifying utility functions that represent their preferences. Therefore, the tool categorizes objectives into two types. Objectives measured with a numerical scale can be determined with an exponential utility function, like in Formula (2). For readability, these objectives are referred to below as numerical objectives. Objectives measured with a verbal scale are specified with discrete utility functions. Here, the DM can assign a utility to each level in a bar chart. For readability, these objectives are referred to below as verbal objectives. Every utility function can be analyzed individually. This means that the DM can decide how to determine the exponential utility function (linear or non-linear) or the discrete utility function (identical or non-identical utility differences) for each objective. By default, linear utility functions and identical utility differences are used in the ENTSCHEIDUNGSNAVI. The tool provides information on utility theory to train the DM. It explains the concept of a utility scale, as well as linear and non-linear utility functions. Furthermore, the DM gets insights into the significance of a non-zero degree of curvature (risk aversion parameter, see Section 2.1.). Additionally, the tool provides graphical representations and various explanations of the defined utility functions to support the DM.

3.1 Numerical objectives

Figure 1 shows how DMs can analyze and determine an exponential utility function for a numerical objective in the ENTSCHEIDUNGSNAVI.

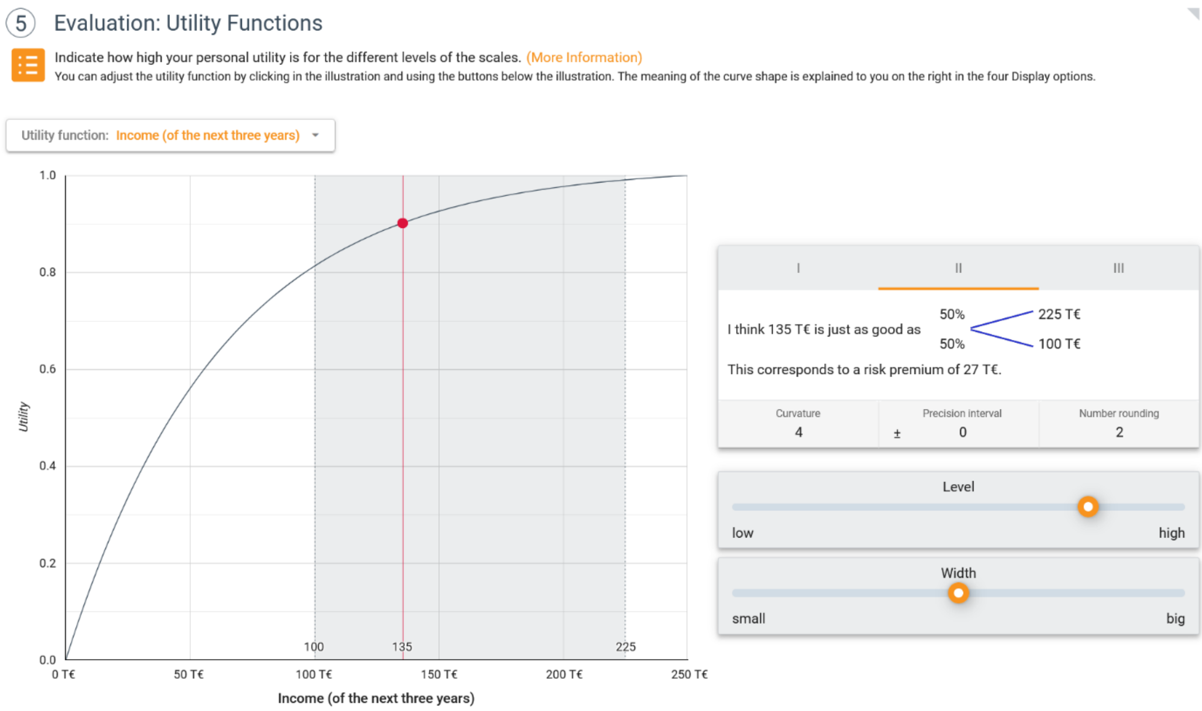

Figure 1. *Determining exponential utility functions in the ENTSCHEIDUNGSNAVI (for the complete DA, see Siebert and von Nitzsch (2020) and von Nitzsch and Siebert (2018).*

A graphical illustration of the utility function is presented on the left-hand side. In this case, the DM selected a concave function with a risk aversion parameter of $c = 4$. The curve's curvature can be altered by using the '-' and '+' buttons or by dragging the curve. A risk aversion parameter between -25 and 25 can be selected. On the right-hand side, three explanation variants are available to provide a more detailed analysis of the utility function. The DM should choose the curvature of the utility function to align with the preference statements outlined in the explanation variants.

In the first variant, risk preferences are not considered, which is suitable for objectives under certainty. In this case, utility functions can be interpreted as measurable value functions. The preference statement of the first variant is, 'If 100 T€ has a low utility and 225 T€ a high utility, then 135 T€ has an average utility for me.' The second variant, as shown in Figure 1, is based on the certainty equivalence method (Keeney and Raiffa 1976, Smidts 1997, Pennings and Smidts 2003) and compares a sure option (135 T€) with a lottery with fixed probabilities (50 % chance of 225 T€; 50 % chance of 100 T€). The third variant is similar to the second option but has variable probabilities. In our example, the preference statement is 'I think that 163 T€ is just as good as the following lottery: 73 % chance of 225 T€; 27 % chance of 100 T€. This corresponds to a risk premium of 27 T€.' The preference statements refer to the parameters 'Level' and 'Width', which define the grey area in Figure 1. The DM can select different settings for them, resulting in thicker or thinner and/or more left or right grey areas and, thus, different preference statements for the same utility function. This enables a thorough examination of statements across different points on the utility function and helps the DM to find the best-fitting utility function. Changing the number of significant digits for all variants is possible by altering the value in 'Number rounding'.

If DMs are unsure about their preferences and the preference statements shown are too detailed, they can use the 'Precision interval' option. The utility function splits and creates a range of potential utility functions by entering an interval. DMs should choose an interval that reflects their uncertain preferences as accurately as possible but not wider than necessary. This paper does not consider imprecise utility functions and uses the mean utility if DMs choose an imprecise utility function.

According to Formula (2), a linear utility function results if the DM chooses a risk aversion parameter of zero.

3.2 Verbal objectives

In Figure 2, an example demonstrates how discrete utility functions of verbal objectives can be analyzed and determined using the ENTSCHEIDUNGSNAVI. The functionalities are identical to those for numerical objectives, but discrete utilities are assigned for each level instead of defining a continuous function. Additionally, the second explanation variant of exponential utility functions is left out and replaced by a verbalization of the c-value in cases where the discrete utilities take the form of an exponential utility function.

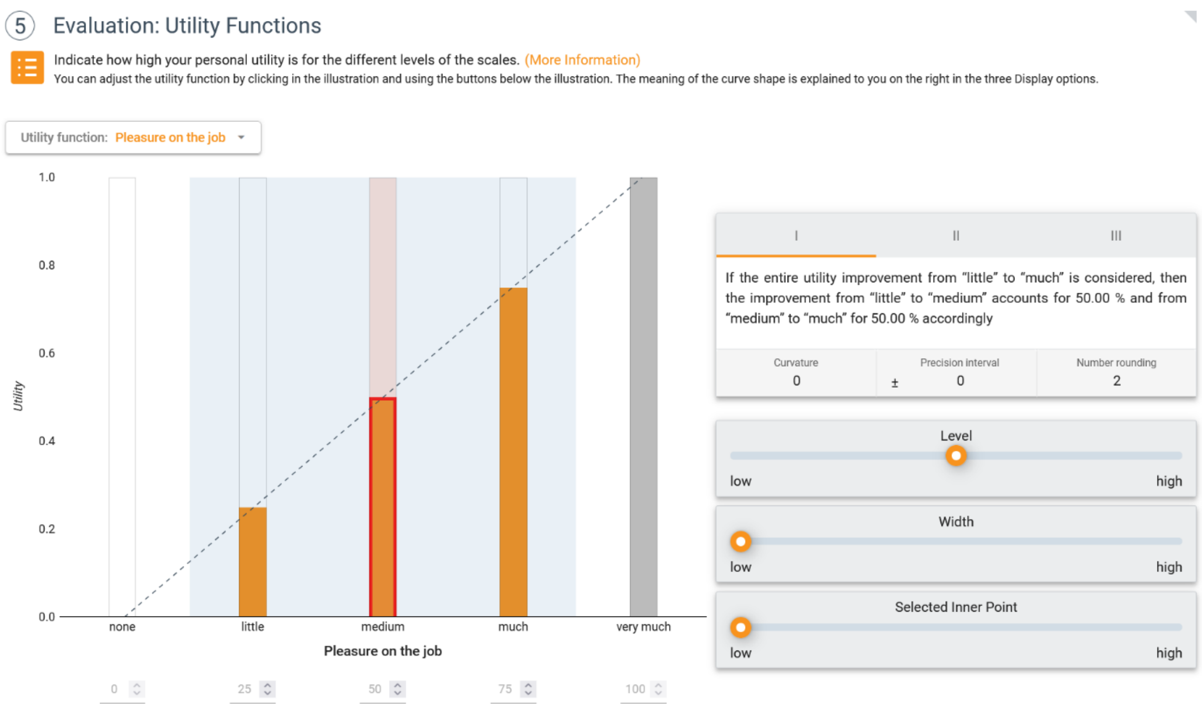

Figure 2. *Determining discrete utility functions in the ENTSCHEIDUNGSNAVI.*

Although it is not a continuous function, for the reader's convenience, we will refer to linear utility functions for verbal objectives if the levels are defined equidistant and the utility differences between those levels are identical, as shown in Figure 2.

In the following sections, we will use the term 'linear objectives' to describe objectives valued with a linear utility function, whether numerical or verbal.

4 Empirical findings on the use and impact of (non-)linear one-dimensional utility functions

This section presents empirical findings on the use and impact of non-linear utility functions to answer our research questions. We use the same dataset for all analyses, i.e., the same decision situations and participants. These are introduced in the following.

4.1 Decision situations and participants

The dataset was collected over five winter terms (2019-2024) as part of a voluntary project in the course "Decision Theory" at a large university in Germany. During the course, students could analyze an important decision they were facing by using the DSS ENTSCHEIDUNGSNAVI meticulously. Most students chose a personal decision-making situation related to their studies or career, such as maximizing international experience or shaping their professional future. Others dealt with their housing situation or how they spent their free time. In these decisions, the students act as DMs in their own decisions. Only some chose a political decision, such as designing a city's traffic concept. Here, the students act as surrogates of DMs. We cannot say whether the personal decisions were implemented or what the consequences were. But we can assume that these issues and topics interest young people in Germany and that they think about them. Every year, we collected new decision situations from new students, resulting in over 3,000 decisions.

Successfully completing the DA resulted in a bonus of one-third of the grade in the Decision Theory lecture. For example, a B grade could be elevated to a B+. To receive a better grade and complete the task, students had to execute all steps in the ENTSCHEIDUNGSNAVI with reasonable effort. Decision experts manually reviewed all decisions, and decisions that were not carefully processed were sorted out.

The study involved a group of 2,536 full-time students enrolled in business, engineering, or computer science programs. The dataset contains information about defined objectives with measuring scales (verbal or numerical), identified alternatives, completed consequences tables, selected utility functions, and specified objective weights. On average, the participants spent 11.23 hours ($\sigma = 7.08\ hours$) on their decision. They identified 4.89 ($\sigma = 1.25$) objectives and defined 6.01 ($\sigma = 2.15$) alternatives. The number of objectives in the decisions ranged from 3 to 15, and the number of alternatives from 3 to 33. The students were supported during consultation hours, in the lectures, in workshops, and by a help page in the ENTSCHEIDUNGSNAVI. The comprehensive level of support in the ENTSCHEIDUNGSNAVI, the relatively long processing time, and especially the manual review of the decisions let us assume that the decision situations were carefully elaborated. Furthermore, we assumed that the students truly understood the importance and topic of fundamental objectives, as it was explained in detail in the lectures and the ENTSCHEIDUNGSNAVI. The students went through several sub-steps, which included brainstorming and the development of an objective hierarchy to identify their fundamental objectives in the decision and to separate them from means objectives.

4.2 RQ1: How often are (non-)linear utility functions used in practice?

4.2.1 Procedure

This descriptive analysis investigates the frequency of used (non-)linear utility functions. We conducted the analysis in two ways. Firstly, we examined the total number of objectives defined in the decisions. We analyzed the use of linear and non-linear objectives and the application of numerical and verbal objectives. Secondly, we counted the number of participants who exclusively used linear, non-linear, or both utility determinations in their decision and those who solely used numerical, verbal, or both objective types. Additionally, we analyzed the level of risk and non-linearity with the risk aversion parameter c for both total objectives and individual decisions. Therefore, we calculated an average risk aversion parameter c for each decision. Since no risk aversion parameter c exists for verbal objectives, only decisions with numerical objectives were considered for the latter analysis.

4.2.2 Results

Table 1 summarizes the results of using linear and non-linear objectives, with either numerical or verbal scales.

Objectives		**linear**	**non-linear**	**Σ**
Numerical	absolute	1,614	5,540	7,154
	in %	13.0	44.7	57.7
Verbal	absolute	1,318	3,933	5,251
	in %	10.6	31.7	42.3
Σ	absolute	2,932	9,473	**12,405**
	in %	23.6	76.4	**100.0**

Table 1. *Total use of linear and non-linear objectives with a numerical or verbal scale.*

The participants defined 12,405 objectives across 2,536 decisions. A majority of these objectives (76.4 %) were defined with non-linear utility functions, and the most frequently used scale was numerical (57.7 %), while the remaining objectives (42.3 %) relied on verbal scales. The most frequently employed objective type (44.7 %) involved a numerical scale and a non-linear utility function. A correlation between non-linearity and measurement scales was not recognizable; the relative use of non-linear utility functions for numerical and verbal objectives was almost the same.

Table 2 displays how the individual participants used (non-)linearity and measuring scales in the decisions. In their decisions, the majority of participants chose either only non-linear utility functions (46.7 %) or a mixture of non-linear and linear ones (49.3 %). Only 4.1 % of participants determined linear objectives exclusively. This means that 95.9 % used at least one non-linear utility function in their decision. Additionally, most participants (72.2 %) used numerical or verbal objectives in their decisions. The most prevalent decisions were those with both measurement scales and non-linear and linear utility functions (37.7 %), followed by those with both numerical and verbal scales and non-linear utility functions (32.3 %).

Decisions			**Objectives**			**Σ**
			only linear	**only non-linear**	**linear and non-linear**	
Objectives	**only numerical**	absolute	35	258	237	530
		in %	1.4	10.2	9.3	20.9
	only verbal	absolute	10	107	57	174
		in %	0.4	4.2	2.2	6.9
	numerical and verbal	absolute	58	819	955	1,832
		in %	2.3	32.3	37.7	72.2
Σ		absolute	103	1,184	1,249	2,536
		in %	4.1	46.7	49.3	100.0

Table 2. *Use of linear and non-linear objectives in the decisions.*

Figure 3 shows the risk aversion parameter c distribution and how strongly non-linearity for objectives or decisions with only numerical objectives was pronounced. It is important to note that this parameter gives information about the risk attitude (relative risk attitude and strength of preferences) and not the relative risk attitude (see Section 2.2.).

7,154 numerical objectives were the basis for analyzing the total distribution of the risk aversion parameter c, represented by blue bars. 530 decisions, including only numerical objectives, were the basis for analyzing the average risk aversion parameter c distribution in the decisions, represented by orange bars. All objectives and decisions with a(n) (average) risk aversion parameter $c = 0$ were left out in this figure. 22.6 % of the numerical objectives were linear (see Table 1), and 10.6 % of decisions had an average risk aversion parameter of zero. Furthermore, all values greater than 5 or less than -5 were summarized as they were not selected individually more than 15 times (0.2 %).

Overall (blue bars), more participants rated their objectives as risk-averse ($c > 0$: 46.7 %) than risk-prone ($c < 0$: 30.7 %). If an objective was rated risk-averse, the average risk aversion parameter was $c = 2.2$. If an objective was rated risk-prone, the average risk aversion parameter was $c = -2.5$

On average (orange bars), 52.8 % of the participants exhibited risk-averse behavior ($c > 0$), while 36.6 % displayed risk-prone behavior ($c < 0$). Most participants (69.4 %) selected an average risk aversion parameter of $-1.1 \leq c \leq 1.9$.

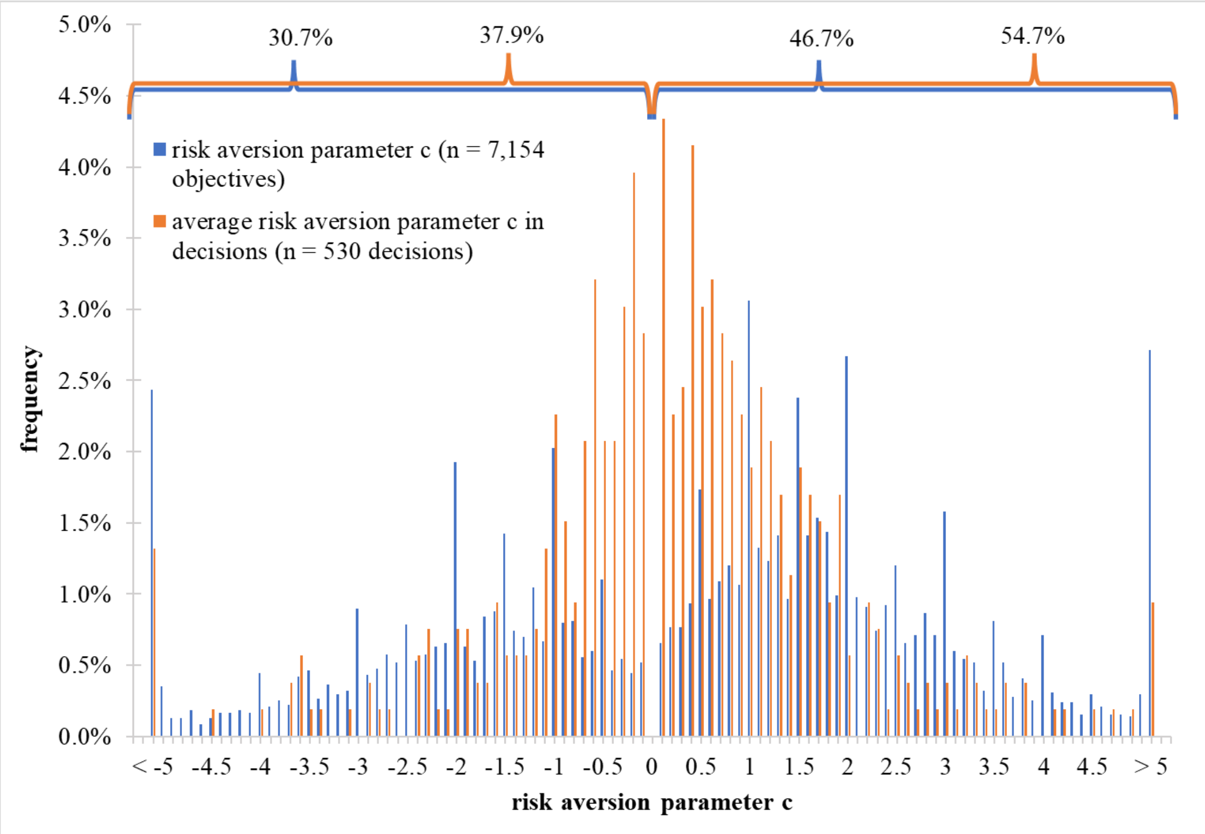

Figure 3. *Distribution of the risk aversion parameter c.*

4.3 RQ2: How does the use of (non-)linear utility functions impact the final ranking of alternatives in practice?

4.3.1 Procedure

For this analysis, we calculated the expected utilities for every alternative for two scenarios using Formula (1): first, under the condition that the (non-)linear utility functions were considered according to the DM's preferences, and second, using the simplified linear utility functions.

For both scenarios, we ranked the alternatives based on the expected utilities. Then, we compared the ranking of alternatives for the participants' stated preferences and the rankings resulting from linearized utility functions and analyzed the impact of non-linearity. To analyze various degrees of impact, we differentiated a rank reversal in the first rank, the first to the third rank, and the whole ranking. For the first to third ranking, we differentiated between a rank reversal within the top three ranking and in the top three set of alternatives. A rank reversal within the top three ranking means that the top three alternatives had only swapped ranks with each other. A rank reversal in the top three set of alternatives means that an originally lower-than-third-ranked alternative had entered the top three. A rank reversal in the whole ranking means that at least one rank reversal occurred and that, at maximum, the entire ranking changed. In addition, we differentiated between the relative proportion of non-linear objectives in a decision. For the second scenario, we linearized the following categories: (1) all utility functions, (2) only exponential utility functions for numerical objectives, and (3) discrete utility functions for verbal objectives. For the first category, we analyzed all decisions. For the second category, we considered decisions with only numerical objectives. Analogously, we only used the decisions with verbal objectives for the third category. Decisions that solely had linear objectives were excluded from the analyses. Logically, in these cases, linearization did not affect the ranking of alternatives because nothing had changed. Therefore, the number of decisions analyzed varied across different linearization categories.

On top of that, we examined how often the second-best, third-best, fourth-best, and so on alternatives under linearity became the best alternative under preference-accurate utility functions. Therefore, we linearized all utility functions and analyzed the decisions concerning their number of defined alternatives. Logically, a decision with only three objectives can not have a fourth-ranked alternative under linearity, which becomes best under preference-accurate utility functions. Thus, the number of decisions (n) varied in the analysis.

4.3.2 Results

The results in Tables 3 to 5 display the proportion of decisions that experienced rank reversal due to the linearization of all utility functions (Table 3), non-linear exponential utility functions (Table 4), and discrete utility functions (Table 5).

Linearization of non-linear utility functions		**total**	**proportion of non-linear objectives**		
			67-100 %	**34-66 %**	**1-33 %**
N		**2,433**	1,907	359	167
rank reversal (in %)	**1st rank**	**15.5**	16.9	13.4	3.6
	top three ranking	**29.8**	31.3	29.0	13.8
	top three set	**14.0**	15.6	8.7	7.3
	whole ranking	**55.0**	58.6	47.1	31.7

Table 3. *Category 1: Impact of (non-)linear utility functions on the final ranking.*

Linearization of non-linear exponential utility functions*		**total**	**proportion of non-linear objectives**		
			67-100 %	**34-66 %**	**1-33 %**
N		**495**	416	60	19
rank reversal (in %)	**1st rank**	**17.2**	17.8	16.7	5.3
	top three ranking	**30.3**	30.5	33.3	15.8
	top three set	**17.4**	16.6	13.0	11.1
	whole ranking	**56.8**	58.4	50.0	42.1

Table 4. *Category 2: Impact of (non-)linear, exponential utility functions on the final ranking. *Decisions with numerical objectives only are analyzed.*

Linearization of non-linear discrete utility functions*		**total**	**proportion of non-linear objectives**		
			67-100 %	**34-66 %**	**1-33 %**
N		**164**	142	16	6
rank reversal (in %)	**1st rank**	**19.5**	21.1	12.5	0.0
	top three ranking	**34.1**	36.6	25.0	0.0
	top three set	**14.4**	14.4	20.0	0.0
	whole ranking	**59.8**	64.1	43.8	0.0

Table 5. *Category 3: Impact of (non-)linear discrete utility functions on the final ranking. *Decisions with verbal objectives only are analyzed.*

In 15.5 % of all decisions (see Table 3), a different optimal alternative would have been selected if non-linear utility functions had not been employed. In other words, non-linearity did not affect the best alternative in 84.5 % of the decisions. Moreover, in 43.2 % of the decisions, one of the top three alternatives changed from rank. The results of the detailed analysis of the best three alternatives clearly show that, in most cases, there was only an exchange of ranks among the top three ranking. This applied to all categories. In the top three set, a rank reversal was rarer than a rank reversal of the best alternative. Even though we limited these percentages to decisions with more than three alternatives. In 55.5 % of decisions, at least two alternatives experienced a rank reversal during the whole ranking. Therefore, a stable ranking of all alternatives could not be achieved through linearization. Analyzing the proportion of non-linear objectives in the decisions resulted in the fewer non-linear objectives in a decision, the lower the impact on the ranking. This applied to all ranking positions.

Comparing the three categories, it was noticeable that linearizing discrete utility functions (Table 5) caused a rank reversal more frequently than linearizing exponential utility functions (Table 4). This held true for the first rank, the top three ranking, and the whole ranking. However, the difference was small.

Table 6 shows the analysis results with the frequency that an alternative at rank X under linearity was the best alternative identified with preference-accurate utility functions.

rank X	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**>12**
n	2433	2433	2344	1937	1306	742	387	219	125	79	52	30
in %	12.3	2.2	0.7	0.4	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 6. *Frequency that an alternative at rank X under linearity was the best alternative.*

Most often, the second-ranked alternative under linearity was the best if preference-accurate utility functions were used (12.3 %). The frequency of a lower-ranked alternative being the best was clearly reduced from the third rank. A fourth-ranked alternative was only the best alternative by 0.7 %. A seventh-ranked or worse alternative was never the best alternative. In 98.9 % of the decisions, the best alternative can be found in the top three set of alternatives under linearity. For this conclusion, only those decisions are considered in which four or more alternatives were defined.

4.4 RQ3: To what extent do linear utility functions ensure stability in selecting the best alternatives based on utility differences?

4.4.1 Procedure

The third analysis should help to give recommendations for practice when defining non-linear utility functions versus using linear modeling. Therefore, we analyzed the rank reversals detected in Section 4.3 regarding the utility differences of alternatives under linearity conditions. First, we assumed linearity in the decisions and linearized all utility functions. Then, we calculated the new expected utilities for each alternative across all decisions, as in Formula (1). Based on these values, we compared the utility of the top two alternatives. We calculated the absolute difference between them and the relative difference compared to the expected utility range used. In addition, we analyzed the top three set of alternatives and calculated the difference between the third and fourth-ranked alternatives. From the second analysis (Section 4.3), we knew which decisions had a rank reversal through linearization. Using this information, we identified differences in utility that led to a change in ranking for the best alternative and the top three set of alternatives. As there were no major differences in the categories, we focused on linearizing all utility functions in all decisions.

4.4.2 Results

Figure 4 illustrates the frequency of a rank reversal for the best alternative and the top three set of alternatives based on the absolute utility differences between the top two and the third to fourth-ranked alternatives in a decision. Figure 5 shows the results depending on the relative utility difference to the expected utility range used.

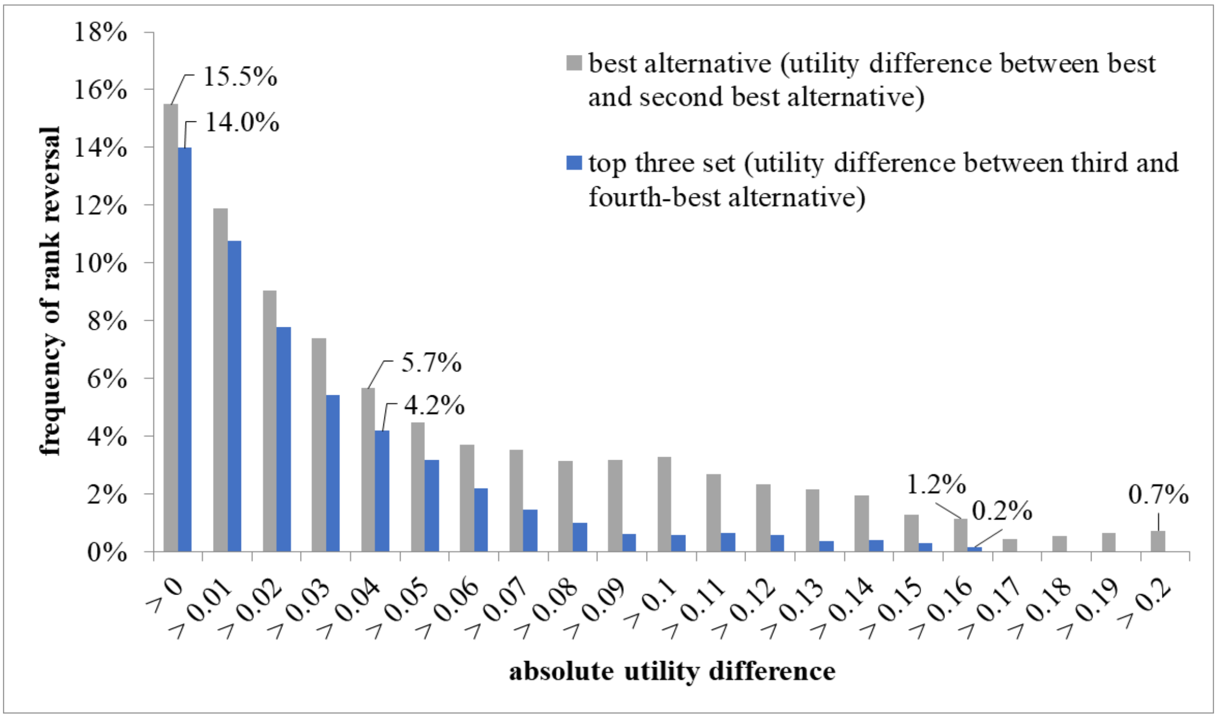

Figure 4. *Frequency of rank reversal depending on the absolute utility difference under linearity.*

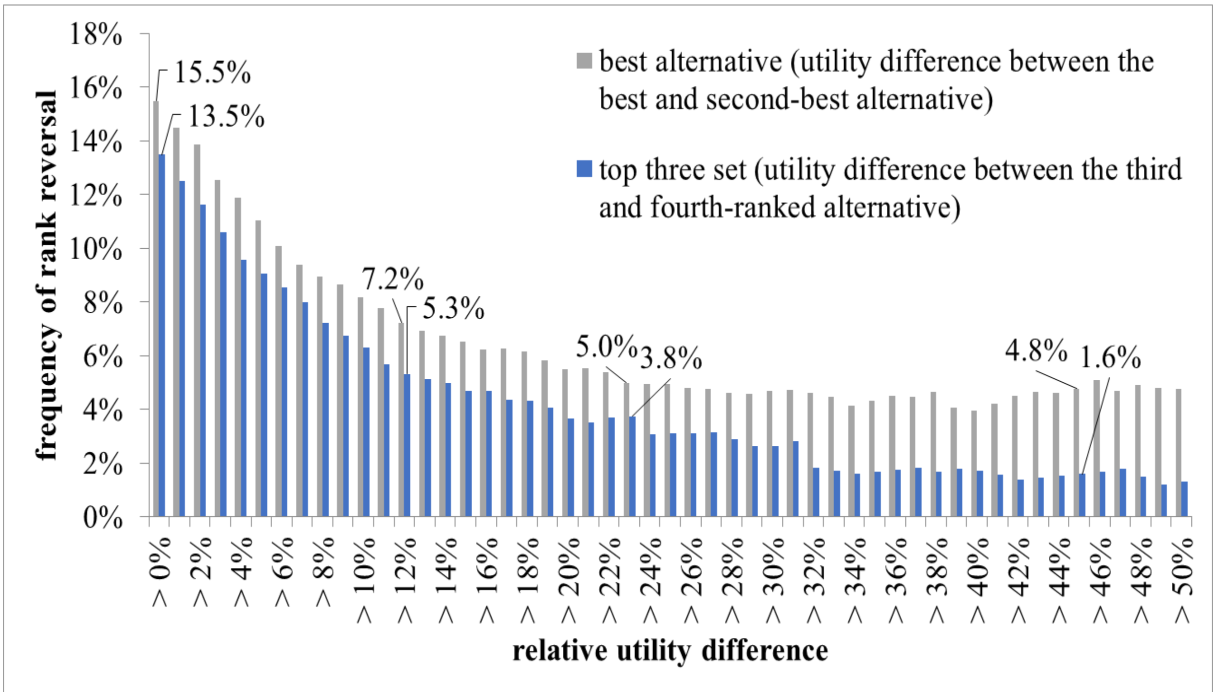

Figure 5. *Frequency of rank reversal depending on the relative utility difference under linearity.*

As the difference in utility between the top two alternatives increased, the frequency of a rank reversal decreased. This could be observed for both absolute and relative values. At first, the frequency of a rank reversal for the best alternative decreased quickly for absolute utility differences. If the utility difference was over 0.04, the frequency was only 5.7 %. From this point on, the frequency decreased slowly. When the absolute difference in utility was over 0.27, the best alternative remained unchanged. At this point, the type of utility function modeled (linear or non-linear) no longer affected the best alternative. For relative utility differences, the frequency of rank reversals decreased more slowly overall. A frequency of 7.2 % was achieved with a relative utility difference of greater than 12 % and a frequency of 5 % with 23 %. A rank reversal still occurred even with a relative utility difference of 50 %. In the data set, no rank reversal could be detected until a relative utility difference of 73 % was reached.

The frequency trend for the top three set of alternatives is similar to that of the best alternative in both figures. However, the frequency is lower in both cases than for the best alternative. With an absolute utility difference over 0.17 between the third and fourth alternatives, the top three set no longer changed. With a relative utility difference of more than 50 %, the frequency of a rank reversal of the top three set was 1.3 %. From a relative utility difference of 56 %, there was no change in our data set.

5 Discussion

5.1 RQ1: How often are (non-)linear utility functions used in practice?

Our first research findings indicate that participants tended to exhibit non-linear utility functions in their decision-making in practice, see section 4.2. Both the absolute number of non-linear objectives (76.4 %) in the data set and the number of participants who used at least one non-linear objective in their decision (95.9 %) were very high. The reasons for choosing non-linear utility functions may be the following, which are listed in section 2.2: relative risk attitude and strength of preferences, and the counterparts of Keeney's and von Winterfeldt's (2007) assumptions for linearity.

In principle, it is possible that the participants wanted to reflect their relative attitude to risk with their non-linear utility functions. However, the results in Figure 3 show that the majority of participants chose convex curved utility functions, which could indicate a risk-prone relative attitude in their decision situations. This would be rather atypical for DMs and leads us to assume that the convexity may not be purely a matter of their relative risk attitude. Rather, it seems more plausible that the convex curved utility functions resulted from an increasing marginal utility and, thus, the strength of preferences. Therefore, we assume that a decreasing or increasing marginal utility could be more decisive here than the relative risk attitude.

Keeney and von Winterfeldt (2007) point out that linear utility functions are appropriate if the fundamental objective measures something that is of value in itself, like loss of lives. Every single human life is worth the same, and non-linearity makes no sense here due to objectivity. Therefore, these objectives should be evaluated objectively and, thus, linearly. Since most participants were dealing with personal (career) decisions, many objectives were used, implying a subjective evaluation. The second assumption for linearity is that expected values implicate linearity, and in these cases, only linear utility functions should be used. Our dataset was collected with the ENTSCHEIDUNGSNAVI, allowing participants to incorporate uncertainty factors into their model so that they were not forced to work with expected values in the consequences table. The third assumption for linearity is when several other decisions are made that contribute towards a common objective. The outcome of a single decision is relatively small compared to the range of outcomes for all decisions collectively, which leads to the irrelevance of the preference-based determination of utility functions in a single decision. This linearity recommendation did not apply either, as the dataset included mainly independent (personal) decisions that did not rely on other decisions with the same objective.

In principle, none of the three situations in which Keeney and von Winterfeldt (2007) recommend linearity occurred in our dataset. The participants dealt with subjective contexts. They could explicitly model uncertainty factors instead of using expected values, and they dealt with independent decisions that could not be compensated by further decisions. In this respect, it is quite plausible that many non-linear utilities could be observed in the analysis.

Another reason for the high frequency of using non-linearity could be attributed to the participants' levels of expertise in the subject matter. Since the dataset was collected as part of a bonus task in a Decision Theory lecture, it is reasonable to assume that the participating students had a solid grasp of the theoretical underpinnings of non-linear utility evaluation, which may have led to their confident use of such functions. Less experienced individuals may tend to rely on linear utility functions or make only slight deviations from them to minimize errors.

5.2 RQ2: How does the use of (non-)linear utility functions impact the final ranking of alternatives in practice?

Analyzing the impact of (non-)linear utility functions (as shown in Tables 3 to 5), the results show that the impact was rather low, considering the best or top three set of alternatives. In our opinion, DMs should focus on exactly these alternatives to decide, making the ranking of other alternatives (worse than third place) less important. Not even every sixth decision has a different best alternative if non-linear utility functions were eliminated. The top three sets changed in 14 % of cases. These results may not seem significant initially and could lead to the statement that preference-accurate utility functions are unimportant. However, if the decisions are crucial and potentially life-changing, we consider a rank reversal of 15.5 % for the best alternative or 14 % for the top three sets as meaningful. Moreover, the results show high sensitivity by analyzing the impact of non-linear utility functions on the exact ranking of alternatives (top three or whole ranking). DMs should, therefore, be careful with linear utility functions set by default if they want to determine the exact ranking.

The comparison between the types of objectives, numerical or verbal, showed that linearizing discrete utility functions caused a rank reversal more frequently than linearizing exponential utility functions. This could be because exponential utility functions guarantee a steady, uniform shape, no matter how pronounced the non-linearity is. With discrete utility functions, clear differentiations can be made explicitly between all levels individually. This can lead to two adjacent levels being evaluated with two very different utilities, which has a greater impact on alternatives evaluated with these adjacent levels. The large utility difference between the levels is relativized by linearization, which, thus, can lead to more frequent rank reversals than with the linearization of exponential utility functions.

The analysis in Table 6 shows that in 98.9 % of the decisions, the best alternative can be found in the top three set of alternatives under linearity. With this insight, the importance of precisely defined utility functions can be reduced, provided that the DM concentrates not on the best-ranked but on the best three-ranked alternatives and chooses the best alternative from them. An alternative that can never be the best can be eliminated from the considered set of alternatives.

5.3 RQ3: To what extent do linear utility functions ensure stability in selecting the best alternatives based on utility differences?

The results in Figure 4 and Figure 5 show which utility differences between the alternatives under linearity were associated with which probabilities of a rank reversal of the best alternative or the top three set under non-linearity. In our dataset, no rank reversal was observed for the best alternative from an absolute utility difference over 0.27 between the best and second-best alternatives under linearity. The top three sets of alternatives did not change for an absolute utility difference over 0.17 between the third and fourth alternatives. However, while the dataset suggests 0 % of rank reversals, there might be excepted cases where they can occur in practice. Nevertheless, the likelihood of this would be very low.

The results are highly dependent on the participants' selected (real) utility functions, as the rank reversals were determined based on these compared to linear ones. The exact utility differences mentioned above are only valid if the dataset contains the DMs' preference-accurate utility functions. This could be criticized since we can only assume that the selected utility functions exactly reflected the preferences of the DMs. Although the participants were trained in decision theory, biases could distort the functions' determination. We cannot prove the exact determination, but we can at least say that the DMs chose plausible utility functions. This is a major advantage over pure simulations. Even if the functions are incorrect, we can show with the results in Figure 4 and Figure 5 how sensitive the results are and at what point the determination of utility functions most likely becomes irrelevant.

6 Conclusion

We analyzed the impact of (non-)linear one-dimensional utility functions on the final ranking of alternatives in practice with the help of three research topics: use of (non-)linear utility functions (RQ1), their impact (RQ2), and the relation between utility differences and rank reversals (RQ3). Therefore, we assumed the additive utility function of MAUT to find the best alternative in decisions with multiple objectives. The dataset consisted of 2,536 real-world decisions, analyzed with an average time expenditure of 11.23 hours in the DSS ENTSCHEIDUNGSNAVI by the participants and manually reviewed by decision experts.

The results show that non-linear utility functions matter for MAUT in practice:

- DMs like to use non-linearity in practice: 95.9 % of the participants used at least one non-linear utility function in their decision. Most objectives (76.4 %) were defined with non-linear utilities.

- The ranking of alternatives can be suboptimal if DMs choose only linear utility functions: Linear instead of (correct) non-linear utility functions led to a rank reversal of the best alternative in 15.5 % of the decisions. The top three set of alternatives changed in 14 % of the decisions.
- Linearising discrete utility functions had a greater impact on the ranking of alternatives than linearising exponential ones.
- In 98.9 % of the decisions, the best alternative could be found in the top three alternatives ranked under linearity.
- The utility difference between the best and second-best alternative must be relatively large under linearity to minimize the likelihood of a rank reversal of the best alternative. In the dataset, no rank reversal for the best alternative was detected from an absolute utility difference greater than 0.27.
- The top three sets of alternatives did not change for an absolute utility difference over 0.17 between the third and fourth-ranked alternatives.

6.1 Limitations

The analysis of the dataset is based on the following limitations. Firstly, the study relies on the ENTSCHEIDUNGSNAVI, which includes the concepts of VFT and MAUT. In DA, many other methods and tools exist and could yield completely different results. The determination of utility functions is always dependent on the setting. Therefore, a generalization is not possible. Secondly, the dataset was collected using a voluntary bonus task at a university, and the participants only consisted of students. Thus, most decisions were private, typical for young DMs, e.g., career decisions. So, the results may not be representative of every decision situation, and for this reason, generalization is also not possible. Furthermore, we could assume but not show that the participants expressed their preferences accordingly. Thirdly, in the dataset, there might have been considered almost dominating alternatives or robust rankings. In these cases, non-linear utility functions logically had no impact. We deliberately did not remove these decisions, as they can occur in practice. Fourthly, the results of this paper referred to the impact and use of non-linear utility functions in the form of exponential or discrete ones. Therefore, one should be careful when drawing conclusions about other utility functions.

6.2 Recommendations

When making decisions using MAUT, we recommend determining one-dimensional utility functions preference-accurate using (non-)linear utility functions. This is particularly recommended if the decisions are important and complex and if DMs require an exact ranking of all alternatives. Linearization can falsify the result and prevent the best alternative and exact ranking of alternatives from being found. However, we support the position of Keeney and von Winterfeldt (2007) and recommend that if DMs want to save time and effort and, in addition, willing to accept the likelihood of a rank reversal, it makes sense to use simplifications through linearity.

The following recommendations can be derived from the results in this paper. Whenever a personal decision is analyzed with MAUT, the utilities of all alternatives can first be calculated using linear utility functions to obtain a first ranking of the alternatives under linearity. If the best-ranked alternative has a clear utility gap with the alternative on the second rank, the analysis can possibly be concluded prematurely. The quantitative results of this study make it easy to estimate the likelihood of not choosing the DM's optimal alternative. Given the importance of the decision, DMs must and can assess for themselves whether they are willing to take the risk of a possibly suboptimal decision to save effort. However, they can reduce the likelihood of such a wrong decision by precisely determining the utility function, at least for the highest weighted objective or, if necessary, the highest weighted objectives. Subsequently, they can consider the utility difference to the second-ranked alternative again. If the utility difference remains high, saving the further effort of determining utility functions is acceptable. If the utility difference is reduced, it is best to determine all utility functions precisely.

If the ranking of alternatives based on linear utility functions does not result in a clear utility gap between the best and second-best alternatives, there may be a greater utility gap at lower-ranked alternatives. Such a constellation would exist if, for example, the utility difference between the second and third-ranked alternatives or, as discussed in this study, the utility difference between the third and fourth-ranked alternatives is high. In this case, achieving greater differentiation in the evaluation is possible by determining preference-accurate utility functions (again, starting with the utility function of the highest-weighted objective). At the same time, however, it also makes sense from a practical point of view to use this top set of similarly well-evaluated alternatives to start an analysis, which may no longer operate strictly in MAUT modeling. Corresponding framework conditions in which this approach makes sense have already been mentioned in this paper. For example, other opinions and interests in group decisions may not be mapped in the MAUT model. DMs could discuss these independently of MAUT based on the top set of alternatives. DMs could also use their gut feeling as a guide, as suboptimal decisions are unlikely anyway due to a similar evaluation of the alternatives. A utility difference of 1 or 2 % should not much matter in practice. Moreover, our results show that in 98.9 %, the DM can find the best alternative under the top three alternatives ranked under linearity.

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