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Rescue vessel allocation in tidal waters of the North and Baltic Sea

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ABSTRACT

In this work, we evaluate the possibility and the effect of optimizing the strategic distribution of naval rescue vessels in the German North and Baltic Sea. These vessels are operated by the German Maritime Search and Rescue Service, who dispatch them in case of distress calls. Generally, the available vessel with the lowest response time is dispatched. However, in the North and Baltic Sea, due to low tides, at predictable times some vessels and stations are not operational. In our work, we build a mathematical model for the allocation of rescue vessels to stations that takes into account these changing availabilities. Then, we show that optimizing expected response time is $\mathcal{NP}$-hard. Next, we provide an Integer Programming formulation and propose two methods of simplifying the model. Finally, we compare the effectiveness of the models in a case study based on real-world data. Results show that the simplified models can be solved to de facto optimality, outperforming the results attained by the full model.

1. Introduction

The water territory of Germany is home to a multitude of maritime traffic. Vessels transporting both people and goods are exposed to a variety of dangers, ranging from human error to extreme weather, all of which may lead to the necessity of assistance by rescue services. For the water territory of Germany, the Deutsche Gesellschaft zur Rettung Schiffbrüchiger (German Maritime Search and Rescue Service) (DGzRS) is primarily responsible for delivering aid [1]. In 2022 alone, over 2000 instances of vessels in need of assistance were recorded [2].

Often, the speed at which an adequate rescue vessel arrives can make a difference between life and death [3]. Thus, ensuring a timely arrival of rescue vessels is an important factor for maritime safety. The following work evaluates the effect of the strategic decision made by the DGzRS of where to place the vessels of their fleet to ensure the fastest possible expected response time for future incidents.

We propose a mathematical optimization model for the assignment of vessels to stations with the aim of reducing expected (weighted) response time. However, computing good solutions for the full model takes too long and requires excessive memory. Therefore, we propose two simplified models that approximate the effect of the tides by aggregating multiple tidal states. We then solve the simplified models and show that the solutions to the simplified cases are also applicable in the full setting.

The rescue process, as far as it is relevant to our work, is the following: A ship somewhere near the German coastline suffers an accident that necessitates help from rescue services. It then calls the control centre of the DGzRS and details both its position and type of incident. The control centre in return checks the list of their available response vessels and decides which of them to send. The selected vessel is then deployed to help the ship with the incident.

Thus, the Rescue Vessel Allocation Problem (RVAP) consists of allocating a set of different vessel types to stations to ensure minimal expected response times to maritime incidents, given a region consisting of zones in which incidents occur as well as stations at fixed positions. Solutions must respect that each station can only house specific types of vessels as well as that at (predictable) times some stations are inoperable due to the tides. Furthermore, some regions are more incident-prone and require more coverage than others.

Additionally, since the DGzRS vessels are manned by local volunteers, each station (harbour) has to house a single rescue craft. This is because not assigning a vessel to a station is equivalent to closing down a local volunteer group, and assigning multiple vessels to a station may lead to under-staffed vessels. Furthermore, tides affect the availability of stations. For example, the harbour of Juist dries up twice a day. During that time, the region their station normally covers needs to be covered by the neighbouring stations. This makes it less desirable to position the fastest available vessel there. However, we cannot permanently place the station from Juist elsewhere since the crew of the vessel consists of volunteers living on the island, who cannot be resettled.

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Table 1
Literature on rescue vessel placement. Entry [x] indicates these modelling elements are considered in the respective work, and [∼] indicates special cases. Means of rescue may be naval or aerial, e.g., boats or helicopters.

Source	Given stations	Means of rescue	Zones	IP	Algorithm	Description
Afshartous et al. (2009) [4]	∼[a]		x	x		Conversion of incident data into probabilities
Ai et al. (2015) [5]	x	∼[b]	x	∼[c]	x	Comparison of heuristics for solution finding
Azofra et al. (2007) [6]	x		x	∼[d]		Basic model for assigning a single vessel
Chen et al. (2021) [7]	x	x	x	x	x	Separation into tactical and operational phase
Jin et al. (2021) [8]		x	x		x	Cover construction and maintenance costs
Jung & Yoo (2019) [9]			x		x	Consider islands & coastline in distance calculation
Feldens & Chen (2020) [10]	x		x	x		Maximization of covered area in Search-and-Rescue (SAR)
Hornberger et al. (2020) [11]	x	∼[e]	x	x		Inclusion of relocation costs
Karatas (2021) [12]	x	x	x	x		Considers response time, working hours and budget
Ma et al. (2024) [13]	∼[f]		∼[g]	x		Robust optimization
Pelot et al. (2015) [14]	x	x	x	x		Multiple modelling approaches
Razi & Karatas (2016) [15]	x	x	x	x		Inclusion of different incident types
Wagner & Radovilsky (2012) [16]	x	x		x		Development of model for practical use
Zhou et al. (2022) [17]	x		x			SAR from a game-theoretical perspective

[a] Stations can be established at predefined positions.
[b] Exactly 2 vessel types.
[c] Model is not linear
[d] Model not explicitly given, but can be inferred.
[e] Exactly 4 vessel types.
[f] Stations can house multiple vessels.
[g] Zones preassigned to stations.

This work's contributions are twofold. First, to the best of our knowledge, there have been no previous attempts to use mathematical optimization for search-and-rescue vessel allocation in either the North or the Baltic Sea. Second, the consideration of tides is a novel attribute that specifically matters in seas with a large tidal range, e.g., the the North Sea. Additionally, we provide a working implementation of our solution algorithms and the corresponding data.

The remainder of the paper is structured as follows: in Section 2 we introduce the RVAP and give a brief overview of related work. Subsequently, in Section 3 we formally introduce the RVAP. Afterward, in Section 4, we prove that RVAP is $\mathcal{NP}$-hard and examine why. In Section 5, we formulate the RVAP as an Integer Program (IP) and consider simplifications that aim to reduce computation times with minimal precision losses. Having done this, in Section 6 we conduct a case study on the effect of these simplifications across several test instances. We discuss the results in Section 7, and summarize our findings and give avenues for further research in Section 8.

2. Related work

Trying to improve the placement of search and rescue vessels is also the basis of Azofra et al. [6], in which the aim is to find suitable criteria to find the optimal placement of a single rescue vessel. To the best of our knowledge, their work is also the first paper to address the problem of rescue vessel placement. The problem of optimal assignment of a single vessel can also be found in Afshartous et al. [4], who place greater emphasis on dividing the area to be covered in zones of different form, as well as on transforming the historical numbers of incidents into probabilistic values. In comparison, Jin et al. [8] do not limit themselves to placement of a single vessel. However, they make the same assumption as us that only a single vessel is assigned to each station. They then solve the problem using a multistage approach based on k-means and nature-inspired heuristics. At the same time, their problem strongly differs from the one in this work, as it focuses on dynamic duty points at sea, i.e., the areas in which vessels operate, not their home stations.

Similar models and solution approaches, including the construction and solving of an IP, are given by Razi and Karatas [15], and Wagner and Radovilsky [16]. While the former analyse historical data given by the Turkish Coast Guard, the latter concentrate on the development of a practical tool for the US Coast Guard. Both differ from our work in that their aim is to minimize the deviation from given values of budget and operating hours instead of minimizing individual response times. The work by Razi and Karatas is further expanded by Karatas [12] who explores several ways of improving the IP in terms of realism. Besides introducing a more diverse arsenal of rescue crafts, they consider how to implement and react to uncertainty in terms of incidents. Another IP-based approach is provided in Chen et al. [7]. Here, the authors use a two-stage approach to solve the IP, since the original formulation does not perform well computationally. For small instances, enumeration may also be possible, as showcased in Jung and Yoo [9].

Recent research by Hornberger et al. [11] focuses on the Pacific Ocean areas under the US Coast Guard's responsibility. While it shares many similarities with our work, it differs in the modelling of incidents. Hornberger et al. assign individual incidents a certain number of hours during which the responding vessel cannot respond to another incident. In comparison, we do not model individual incidents, but incident rates. The approach by Hornberger et al. is closer to a hub location and vehicle routing problem. That is more realistic, if the same vessel has to respond to multiple incidents within a short time frame, and the response time is large. However, in the North and Baltic Seas distances are far smaller than in the Pacific, and accident rates are comparatively low, which leads us to focus on incident rates instead. Their work also includes the possibility of relocating vessels for a certain price. Finally, there is some overlap with Ma et al. [13], who assign a number of homogeneous vessels to sections, but do so addressing uncertainties through robust optimization.

Many works in the field of maritime SAR research have similar motivations but different approaches in terms of models and objectives. Other related work primarily focuses on the possible causes of incidents, and on how to compare and combine their severity [17]. The work by Feldens Ferrari and Chen [10] also deals with maritime SAR but focuses on searching a given area of a single incident. An overview of the different sources and their properties is given in table Table 1.

As the first column shows, almost all sources assume fixed stations, and the *zones* column signals that nearly all of them use zones to

Table 2
Overview variables, parameters and sets used. Here, $\mathcal{P}$ denotes a power set.

Variable	Element of Sets	Meaning
i	$\mathcal{N}$	Index/Set referring to a vessel type
j	$\mathcal{M}$	Index/Set referring to a station
f	$\mathcal{F}$	Index/Set referring to an incident type
z	$\mathcal{Z}$	Index/Set referring to a zone
t	$\mathcal{T}$	Index/Set referring to a time interval
Parameter	Domain	Meaning
a_i	$\mathbb{N}$	Number of vessels of type i available
v_i	$\mathbb{R}^+$	Speed of vessel type i
w_f	$\mathbb{R}^+$	Severity of incident type f
d_{jz}	$\mathbb{R}^+$	Distance between station j and zone z
p_{fzt}	$[0,1]$	Probability of incident of type f occurring in zone z during time t
p_f	$[0,1]$	Probability of incidents of type f happening, if z, t are fixed
p_z	$[0,1]$	Probability of incidents happening in zone z, if f, t are fixed
p_t	$[0,1]$	Probability of incidents happening during time t, if f, z are fixed
Set of tuples	Subset of	Meaning
B	$\mathcal{P}(\mathcal{N} \times \mathcal{F})$	Vessel type i has the equipment required for incident type f
C	$\mathcal{P}(\mathcal{N} \times \mathcal{M})$	Vessel type i can be assigned to station j
$\mathcal{U}(t)$	$\mathcal{P}(\mathcal{N} \times \mathcal{M})$	Vessel type i can be assigned to station j during time t
S	$\mathcal{P}(\mathcal{N} \times \mathcal{M} \times \mathcal{Z})$	Vessel type i can travel from station j to zone z, $(i,j,z) \in S$
$\mathcal{X}$	$\mathcal{P}(\mathcal{N} \times \mathcal{M})$	Set of all possible vessel-station combinations
$\mathcal{Y}$	$\mathcal{P}(\mathcal{N} \times \mathcal{M} \times \mathcal{F} \times \mathcal{Z} \times \mathcal{T})$	Set of all possible vessel-station-incident-zone-time combinations

model the optimization problem. The *means of rescue* column indicates whether a source considers more than a single type of rescue vehicle. These types can be different vessels with different characteristics, as in our work, or other means of transportation, e.g., planes and helicopters. The last two columns reveal that the problems are mostly solved using standard IP-solvers, rather than applying problem-specific algorithms.

For a broader overview of the topic of SAR operations, we refer to the state of the art paper by Raap et al. [18]. Notably, the subject of this work, the allocation of assets (vessels) to stations (harbours), is one of multiple closely related fields, i.e., location modelling of SAR stations, allocation modelling of SAR assets, risk assessment modelling of SAR areas, and search theory and SAR planning modelling [19]. For a general discussion of SAR and its connection to other medical facility location problems, including stochastic variations, we refer to Pelot et al. [14].

3. Model

Consideration of the tides is the most notable difference between our model and the previously listed publications. In the following, we first define all parameters. Second, we define feasibility in terms of a graph matching problem. Third, we motivate and introduce our objective function.

3.1. Parameters

Consider an instance with $n \in \mathbb{N}$ different vessel types, each of which belongs to a certain vessel type. We write $\mathcal{N}$ for the set of all vessel types. Each vessel type has a speed $v_1, \ldots, v_n \in \mathbb{N}$, a total number of available vessels of that type $a_1, \ldots, a_n \in \mathbb{N}$. Similarly, there are $m \in \mathbb{N}$ different stations. We write $\mathcal{M}$ for the set of all station types. The relationship between vessel types and stations is represented by a set of tuples $C \subseteq \mathcal{N} \times \mathcal{M}$, where $(i,j) \in C$ means that a vessel of type i can be positioned at station type j.

Due to the tides, the water level at each of the stations changes regularly, which leads to vessels with a too high draught being unable to leave. To model this, we consider data of the tides in a period $\mathcal{T}$ partitioned into time intervals. For each interval of time $t \in \mathcal{T}$, we collect all usable combinations of vessels and stations $e \subseteq \mathcal{N} \times \mathcal{M}$. We define sets $\mathcal{U}(t) \subseteq \mathcal{P}(\mathcal{N} \times \mathcal{M})$ that contain all usable pairs of vessels and stations e during time t.

The incidents the vessel types respond to are grouped into types given by a set $\mathcal{F}$ with severities $w_1, \ldots, w_{|\mathcal{F}|} \in \mathbb{R}^+$. Since some incident types may require specific features of a vessel type, such as enough weight and power to tow another heavy vessel, the set of tuples $B \subseteq \mathcal{N} \times \mathcal{F}$ represents compatibility between vessel and incident types.

Finally, the territory to be covered is divided into zones $\mathcal{Z}$, which have certain distances to the stations represented by d_{jz} for the distance between station j and zone z. Since not every vessel can reach every zone from every station (e.g., due to tank size or offshore unsuitability), we have a set $S \subseteq \mathcal{N} \times \mathcal{M} \times \mathcal{Z}$ representing the compatible vessel-station-zone-combinations.

An overview of all parameters and variables is given in Table 2.

3.2. Feasible solutions

A feasible solution consists of two parts. The first one represents the allocation of the vessel types to the stations. Let $G = (V, E)$ be a graph with one node for each vessel type and one node for each station. For any pair of vessel and station type, add an edge if and only if the pair is in the set of compatible combinations C. Define the b-values as follows: $b(v) = a_i$ if $v \in V(G)$ represents vessel type i, and $b(v) = 1$ if it represents a station. Then, a feasible solution consists of a (not necessarily optimal) bipartite b-matching M between vessel and station types. A vessel type i is assigned to station j if and only if the edge between their nodes is part of the b-matching M.

The second part of a feasible solution ensures that every incident can be responded to. For that, we search for a mapping $u : \mathcal{F} \times \mathcal{Z} \times \mathcal{T} \to \mathcal{M}$ that assigns every combination of incident type $f \in \mathcal{F}$, zone $z \in \mathcal{Z}$ and tidal state $t \in \mathcal{T}$ to a responding station $j \in \mathcal{M}$ and, combined with M, a responding vessel type i. This mapping has to adhere to the limitations outlined above: The vessel type must be able to help with the incident type meaning, $(i, f) \in B$, it must be able to traverse the distance between station and zone, meaning $(i, j, z) \in S$ and the station must have a high enough water level to be operational for the vessel, so $(i, j) \in \mathcal{U}(t)$ for all $t \in \mathcal{T}$.

3.3. Objective function

For each accident we want a suitable rescue vessel to arrive as soon as possible. Since our model enforces suitability of the vessels as a hard constraint, we can only optimize for response time. Specifically, we minimize an expected weighted response time. To calculate that, let $z \in \mathcal{Z}$ be a zone and j be the station from which a rescue vessel of type i would dispatched to z.[1] Then, we set the response time to $\frac{d_{jz}}{v_i}$. Here, the average avoids effects due to outliers that might otherwise appear, as our model includes the Tiefwasserreede (deep water anchorage), an exclave of Germany's territorial waters significantly beyond the 12 mile limit where d_{jz} is very large.

Since not all incident types are equally urgent their relative importance is denoted by weights $w_f \in \mathbb{R}^{\geq 1}$. Then, we sum over all combinations of accident types f, zones z and time t, and the respective vessel types i and stations j. Thus, we arrive at the following formula:

$$\min \ \mathbb{E}\left(\sum_{\substack{(f,z,t) \in \mathcal{F}\times\mathcal{Z}\times\mathcal{T} \\ (i,j) \in \mathcal{V}(t)}} \frac{d_{jz}}{v_i} w_f \right). \tag{1}$$

We can eliminate the expectation value from (1) by multiplying the objective contribution of each tuple (f, z, t, i, j) with its probability p. Since, in our case study, p is uniquely defined by f, z, t, we write $p = p_{fzt} \in [0, 1]$.

Now, we can reformulate the objective as

$$\min \sum_{\substack{(f,z,t) \in \mathcal{F}\times\mathcal{Z}\times\mathcal{T} \\ (i,j) \in \mathcal{V}(t)}} \frac{d_{jz}}{v_i} w_f p_{fzt}. \tag{2}$$

As noted before, an overview of all parameters and variables is given in Table 2.

4. Complexity

In this section, we examine the complexity of the RVAP. We show that the feasibility problem corresponding to RVAP is $\mathcal{NP}$-hard by reduction from Exact Cover by 3-Sets Problem (X3CP). Based on this, we argue that the RVAP is $\mathcal{NPO}$-complete.

Definition 1. Let A be an instance of the RVAP. We define the Feasibility Rescue Vessel Allocation Problem (f-RVAP) as the problem of finding a feasible solution for A, or proving that no such solution exists.

Note that the f-RVAP is a decision problem, whereas RVAP is an optimization problem. We prove all results for the decision problem and then extend them to the optimization problem.

Lemma 2. *The f-RVAP is in $\mathcal{NP}$.*

Proof. See Appendix A. □

Lemma 3. *The f-RVAP is $\mathcal{NP}$-hard, even for only two vessel types, disregarding differing vessel ranges, vessel-station incompatibilities, incident types and water states.*

Proof. We use the X3CP to prove the $\mathcal{NP}$-hardness of the f-RVAP. Each instance of the X3CP consists of a set X with $q := \frac{|X|}{3} \in \mathbb{N}$ and a set $D \subseteq \{x \subseteq X : |x| = 3\}$. The decision problem is whether there exists a subset $A \subseteq D$ such that $|A| = q$ and $\bigcup A = X$. This problem is a known $\mathcal{NP}$-complete problem, see [20].

Given an instance of the X3CP, we now construct an equivalent RVAP instance. To do so we create one zone per element of X, one station for each set in D, and two vessel types I and II. We create one incident type of severity 1 and probability 1 in every zone. The distance from any station to any zone is set to 1 and the speed of all vessels is set to 1 as well. There are q vessels of type I and $|D| - q$ of type II available. Each vessel is operable in every station at all times. All vessels can be assigned to any of the stations. Vessels of type II cannot reach any zone from any of the stations, vessels of type I positioned at the station corresponding to $d \in D$ can reach all zones corresponding to one of the elements in d. Every vessel type is capable of assisting with the single incident type.

To show such an instance is equivalent to the original we will prove that a feasible solution to the RVAP instance exists if and only if the X3CP instance is a yes-instance.

First, starting with a solution to the RVAP, let A be the set of all elements of D which correspond to stations that have a vessel of type I assigned to them. This is a solution to the X3CP as there are q vessels of type I, so $|A| = q$ and all $3q$ zones are covered, while each station can cover up to 3 zones with a vessel of type I and 0 zones with type II. Due to $|\mathcal{Z}| = 3q$ this means that each station with a vessel of type I assigned covers exactly 3 zones, thereby all of the elements of X occur in exactly one set in A and $\bigcup A = X$.

Second, starting with a solution A to the instance of the X3CP, we assign vessels of type I to all stations corresponding to an element in A and vessels of type II to the remaining stations. Then for each zone with corresponding element $x \in X$ exactly one station with a vessel of type I is capable of responding to incidents of the type in that zone as $x \in \bigcup A$. This gives a feasible (and optimal) solution. As the two instances are equivalent and the reduction to X3CP is polynomial the f-RVAP is $\mathcal{NP}$-hard. □

Theorem 4. *f-RVAP is $\mathcal{NP}$-complete, even for only two vessel types, disregarding differing vessel-speeds, vessel-station incompatibilities, incident types and water states.*

Proof. This follows immediately from and Lemma 3. □

Corollary 5. *RVAP is $\mathcal{NPO}$-complete, even for only two vessel types, disregarding vessel-station incompatibilities, differences in incident types, water states and vessel-speeds.*

Proof. This follows immediately from the $\mathcal{NP}$-completeness of f-RVAP. □

Note that we could equally formulate Lemma 3 and the following results in terms of ranges, not inabilities to reach certain zones. Furthermore, note that the two different vessel types and their (in)ability to reach the incident zones were key to our reduction. This difference can alternatively be replaced by a difference in speed:

Corollary 6. *RVAP is $\mathcal{NPO}$-complete, even for only two vessel types, disregarding vessel-station incompatibilities, differences in incident types, water states and vesselranges.*

Proof. See Appendix A. □

In conclusion, there is no singular aspect of the RVAP responsible for its complexity because any feature used in the reduction to X3CP by itself can be replaced by a combination of the other parameters.

5. Integer program for RVAP

The problem of rescue vessel allocation can be modelled as a (binary) IP. In this context, the variable $x_{ij} \in \{0, 1\}$ represents the decision to assign a vessel of type $i \in \mathcal{N}$ to station $j \in \mathcal{M}$ where i and j represent an allowed combination (meaning $(i, j) \in C$). These variables form the set

$$\mathcal{X} := \{(i, j) \in \mathcal{N} \times \mathcal{M} : (i, j) \in C\}.$$

[1] Note that different types of incidents may require different types of vessels, i.e., the choice of i and j depends on the incident type .

Similarly, we define variables y_{ijfzt} that represent the decision to let a vessel of type i positioned at station j attend to incidents of type $f \in \mathcal{F}$ occurring in zone $z \in \mathcal{Z}$ during time $t \in \mathcal{T}$ via

$$\mathcal{Y} := \{(i,j,f,z,t) \in \mathcal{N} \times \mathcal{M} \times \mathcal{F} \times \mathcal{Z} \times \mathcal{T} : \\ (i,j) \in \mathcal{X}, (i,f) \in B, (i,j) \in \mathcal{U}(t), (i,j,z) \in S\}.$$

Additionally, let

$$\bar{\mathcal{Y}}_{ij} := \{(f,z,t) : (i,j,f,z,t) \in \mathcal{Y}\}.$$

Using the aforementioned sets, an IP for the RVAP is given by

$$\min \sum_{(i,j,f,z,t)\in\mathcal{Y}} \frac{d_{jz}}{v_i} p_{fzt} w_f y_{ijfzt}$$

$$\text{s.t.} \quad \sum_{\substack{i,j:\\(i,j,f,z,t)\in\mathcal{Y}}} y_{ijfzt} \geq 1 \qquad \forall f \in \mathcal{F}, z \in \mathcal{Z}, t \in \mathcal{T} \tag{3a}$$

$$\sum_{(f,z,t)\in\bar{\mathcal{Y}}_{ij}} y_{ijfzt} \leq |\bar{\mathcal{Y}}_{ij}| x_{ij} \qquad \forall (i,j) \in \mathcal{X} \tag{3b}$$

$$\sum_{\substack{j:\\(i,j)\in\mathcal{X}}} x_{ij} \leq a_i \qquad \forall i \in \mathcal{N} \tag{3c}$$

$$\sum_{\substack{i:\\(i,j)\in\mathcal{X}}} x_{ij} \leq 1 \qquad \forall j \in \mathcal{M} \tag{3d}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{X} \tag{3e}$$

$$y_{ijfzt} \in \{0,1\} \qquad \forall (i,j,f,z,t) \in \mathcal{Y}. \tag{3f}$$

Our objective function (which is equal to Eq. (2)) is the sum of the vessel-station-incident-zone-time assignments, each weighted by their respective probabilities p_{fzt} and severities w_f, and the travelling times $\frac{d_{jz}}{v_i}$. Constraint (3a) is used to ensure every zone-time-incident-combination is attended to and Constraint (3b) ensures that vessels are stationed at the station they are sent out from. Constraint (3c) limits the number of vessels assigned to the number of available vessels for every vessel type. Constraint (3d) ensures that every station has at most one vessel assigned to it. We do not enforce having at least one vessel per station, as this is part of any optimal solution.

Initial computational testing showed that the explicit, stochastic version of the IP provided above quickly runs out of memory (and time) if more than five zones are modelled, which is necessary for a realistic model. Thus, we also provide two subject-specific simplifications, which we evaluate in Section 6.

5.1. Corrected water levels for stations and vessels

The Baltic and the North Sea are continuous bodies of water within a limited geographical area. Thus, the tides at different locations are strongly correlated. We validated this for the tide data used in this work, as shown in Appendix B. The data sourcing is covered in more detail in Section 6. Based on the structure of the real-world data, we make the simplifying assumption that vessel-station combinations can be sorted in terms of availability, i.e., if a vessel-station combination that is available 80% of the time is operable, all combinations that are available at least as often, e.g., 85% or 90% of the time, are operable as well.

This means we sort the vessel-station combinations by relative availability and, instead of considering all possible combinations of operable vessels and stations (the set C), we only focus on situations where the most frequently available station-vessel combinations are operable. In practical terms, we calculate

$$\tilde{p}_{(i,j)} := \sum_{t\in\mathcal{T}} \mathbf{1}_{\mathcal{U}(t)}((i,j)) p_t$$

as the relative availability of the station-vessel combination $(i,j) \in \mathcal{M} \times \mathcal{N}$ where $\mathbf{1}$ is the indicator function over $\mathcal{U}(t)$ and p_t is the probability of tidal state t appearing. We define the set of probabilities as

$$P = \{\tilde{p}_{(i,j)} : (i,j) \in \mathcal{N} \times \mathcal{M}\} \cup \{0,1\}$$

and use the intervals

$$\bar{P} = \{[p_1,p_2] \in P^2 : p_1 < p_2 \wedge \nexists p_3 \in P : (p_1 < p_3 < p_2)\}$$

to replace $\mathcal{T}$. This means that for any interval $[p_1,p_2] \in P^2$ we assume that a station-vessel combination is available if its probability of occurring is above p_1. Due to $|\mathcal{T}| \in \mathcal{O}(2^{|\mathcal{N}||\mathcal{M}|})$ and $|\bar{P}| \in \mathcal{O}(|P|) = \mathcal{O}(|\mathcal{N}||\mathcal{M}|)$, this simplification significantly decreases the input size of an instance. The inaccuracy induced by this simplification is based on the difference in water levels across stations at the same point in time, which is relatively small.

To adjust the IP from Section 5 it is necessary to modify $\mathcal{Y}$ to

$$\mathcal{Y} := \{(i,j,f,z,t) \in \mathcal{N} \times \mathcal{M} \times \mathcal{F} \times \mathcal{Z} \times \bar{P} : \\ (i,j) \in \mathcal{X}, (i,f) \in B, \tilde{p}_{(i,j)} \geq \min(t), (i,j,z) \in S\} \tag{4}$$

and to modify all occurrences of y and $\mathcal{T}$ accordingly. Additionally, the objective function must be changed to

$$\min \sum_{(i,j,f,z,t)\in\mathcal{Y}} \frac{d_{jz}}{v_i} p_f p_z (\max(t) - \min(t)) w_f y_{ijfzt},$$

where p_f and p_z represent the (independent) probabilities for accidents of type f or in zone z. Notably, this objective is still a linear function since $\max(t)$ and $\min(t)$ are constants.

5.2. Corrected water levels for stations

The simplification above can be extended by averaging the water level of a station and deleting the dependence on the vessel stationed there. Given the availabilities $\tilde{p}_{(i,j)}$ of the previous section, for a fixed station j we calculate

$$\tilde{p}_j := \frac{\sum_{i\in\mathcal{N}} \tilde{p}_{(i,j)} a_i}{\sum_{i\in\mathcal{N}} a_i}$$

as relative probability by scaling the absolute availability in combination with every vessel type by the vessel type number. These further simplified water levels $\tilde{p}_j$ can be used to replace the inequality $\tilde{p}_{(i,j)} \geq \min(t)$ in Eq. (4) with $\tilde{p}_j \geq \min(t)$. This simplification further reduces the size of P to $\mathcal{O}(|\mathcal{M}|)$ instead of $\mathcal{O}(|\mathcal{N}||\mathcal{M}|)$ by averaging the draught values of all vessel types. The IP can be similarly adjusted to Section 5.1 by changing the corresponding indices from (i,j) to j.

6. Computational study

We tested the IP variations on several instances of the problem. In the following, we give an overview of instance generation, the data used, assumptions made about the data, and the computational setup. We then explain how solutions were validated. The code and data are publicly available through GitHub [21].

6.1. Available data

An instance consists of the vessel types, the stations, the incidents, the tide levels, and the relations thereof. Since we have insufficient access to geolocated and timestamped incident data to estimate a probability distribution for p_{fzt}, for the computational study, we make the simplifying assumption that $p_{fzt} = p_f \cdot p_z \cdot p_t$, i.e., that incident f, zone z and time t are statistically independent. While this approach still captures many critical effects, such as the effect of shipping straits or seasonal changes in recreational boating patterns, it may miss more complex interplays of those factors.

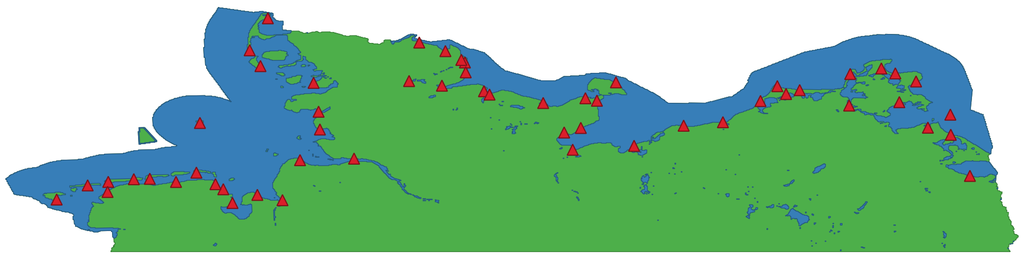

Fig. 1. The red triangles show the positions of the 55 DGzRS-stations in the German coastal waters of the North and Baltic Sea.

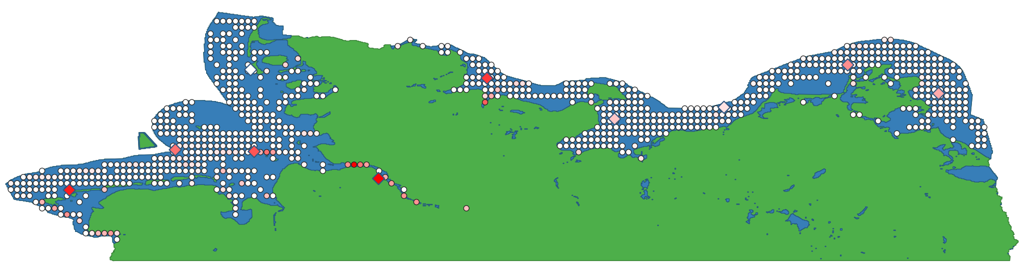

Fig. 2. Incident generation across the German coastal waters of the North and Baltic Sea. Dots represent zones generated that have an average vessel traffic density greater 0, diamonds represent clusters of zones. They are coloured by their average vessel traffic density where white is the lowest and red is the highest average vessel traffic density. Note that these are scaled separately for the clustered and regular points.

6.1.1. Data on vessel types, stations and incident types

The information about the vessel types and stations is based on [22], where the DGzRS lists the vessels and stations currently in use. At the time of writing, there are 11 types of vessels and 55 rescue stations. Every vessel type listed has a specification sheet that details information such as the speed of the vessel in knots, its reach depending on its speed in nautical miles, its draught in metres, and information about its equipment such as material for firefighting or towing vessels of several sizes. As the actual reach is dependent on the speed, which our model does not take into account, we always use the maximum given reach when deciding whether a vessel is capable of reaching a zone from a certain station. The positions of the stations, which we extracted from the data, are shown in Fig. 1.

While the real world data gives indicators for vessel-station compatibility, such as vessel length and current crew, they are insufficient to create clear rules for which combinations are appropriate. Thus, we randomly generated this data by giving each combination a probability of 0.9 of being allowed.

For our strategic model, different incident types matter because not all vessels can address all incidents, i.e., vessels that can address a certain incident type need to be distributed so that they can cover all regions. The German government does not publish standardized data on incidents. Therefore, we constructed incident types based on available data. For that, note that accidents involving non-recreational vessels are comparatively rare. The Global Integrated Shipping Information System (GISIS),[2] reports only 26 incidents in German coastal waters between 2000 – 2024. Instead we used the largest data set on recreational boating accidents publicly available: the US Coast Guard's annual incident reports [23]. We manually mapped the accident types from the 2023 report to the needed equipment types. This resulted in four accident types. Their probabilities, i.e., the fraction of accidents belonging to this class, were estimated based on total frequency. Their severities, i.e., their relative objective weights, were estimated based on the average lethality of each incident type. Table 3 gives an overview of the different incident types.

Table 3
Different incident types, their severities and probabilities.

Incident Type	Special Equipment	Severity (w_f)	Probability (p_f)
General Accident	None	5	0.2
Grounding	Towing, secondary vessel	1	0.1
Fire	Firefighting	8	0.1
Sinking	Pumping	10	0.6

6.1.2. Geographical data

In our study, we focus on the German territorial waters as detailed in [1]. In our implementation we work with the data from [24] using QGIS (see [25]) and its integrated Python-interface PyQGIS. We use their 100 metre grid to filter for those map squares which are completely covered by water and then dissolve this grid by calculating the connected components. The two biggest connected components represent the North and Baltic Sea quite closely because rivers and islands are excluded due to the 100 metre grid. One main limitation, however, is that the *Tiefwasserreede*, a German exclave as detailed in [1], also is excluded.

6.1.3. Generation of zones

To generate zones, we divide the polygon into 1000 equidistant zones in the World Geodetic System (WGS 84) using a PyQGIS script. This gives us a high resolution in regional vessel traffic variations. Note that due to the curvature of the earth, zones vary slightly in size. For the distance calculation between these zones and stations, we use the direct distances between two points on the surface of the earth. This means we disregard possible obstacles like islands and restricted areas,

[2] See https://gisis.imo.org. Module: *Marine Casualties and Incidents*.

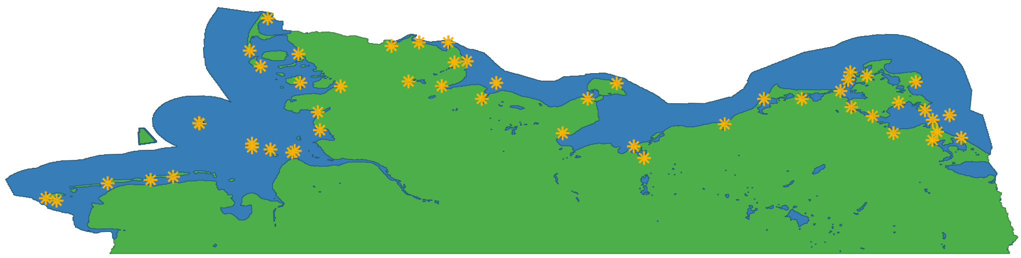

Fig. 3. Locations of the gauges used to determine the water levels for the German coastal waters of the North and Baltic Sea.

possible currents, and differences between tidal levels. To calculate the distances on the surface of the earth, we used the haversine formula (implementation from the Python haversine package[3]) with an earth radius of 6371 kilometres. These distances combined with the range of the vessel types (divided by two to account for the way back) decide whether the vessel can travel between the zones and the stations. To make these instances solvable, similarly to [11], we then apply k-means clustering on the geographical positions. Fig. 2 shows an example of the generated zones (500 metre grid instead of 100 metres for better visualization).

The likelihood of incidents varies between different regions. We assume the frequency of an incident type occurring in a given zone to be proportional to the average vessel traffic density. For that, we used monthly averages of data from the Federal Maritime and Hydrographic Agency of Germany (Bundesamt für Seeschifffahrt und Hydrographie) provided via their *GeoSeaPortal*.[4] This data contains information on the number of vessels entering each ≈ 1 km^2 grid field over a day.

When location points are combined to one point x, we use the mean value of the incident rates of the old points for x. A side effect is smoothing the location values. Previously, a location point close to multiple incident locations would be considered completely harmless since no incidents happened exactly there. After clustering, these incidents indicate that this adjacent point should also be considered dangerous. For the implementation of k-means clustering, a deterministic *k-means* method provided by the *sklearn* library was used [26].

6.1.4. Data on tide levels

For determining adequate tide levels, we consulted government records regarding the measurements of water gauges across both North and Baltic Sea as well as their position [27]. These can be seen in Fig. 3.

Since most stations have no water level gauge at their exact location, we approximate their water level using the three closest gauges, weighted by their distance to the station. This data is recorded every minute and stored for one month. For our tests, we use the data between Monday 20$^{\text{th}}$ November, 2023 and Wednesday 20$^{\text{th}}$ December, 2023. We obtained depth data for the stations from *Bundesamt für Seeschifffahrt und Hydrographie (BSH)*, i.e., the Federal Maritime and Hydrographic Agency of Germany [28]. In cases where no measurement for the exact location is given, a closest point is used. In practical application, a safety margin might be necessary, which we do not consider for now. In total, we arrive at around 8700 different tidal states.

6.2. Setup for the study

The code was written in Python (Version 3.11.2) with Gurobi (Version 10.0.0, [29]) and was executed on the High Performance Cluster of the RWTH Aachen,[5] using Intel Xeon Platinum 8160 Processors (2.1 GHz).

The three different problem formulations are named *best-tidal* (Section 5) for the original model, and *better-tidal* (Section 5.1) and *many-zones* (Section 5.2) for the two modified formulations.

For comparability, all three approaches run on a single core CPU. Since runtime and RAM limitation is needed for the usage of the High Performance Cluster, we cap both. To ensure that Gurobi finds a solution, we set the *TimeLimit* property of Gurobi to 23 hours and the total time per job to 24 hours. All times are measured in wall time.

All instances are initialized with the same 1000 zones. As this number is too high for all approaches, we use the k-means clustering algorithm to shrink them down to a desired number $n_z \in \{1, \dots, 1000\}$. As *better-tidal* is more difficult to solve than *many-zones*, the approach *many-zones* uses $n_z \in \{10, 50, 100\}$ zones for problem solving while *better-tidal* uses $n_z \in \{10, 20, 30\}$ zones. Both approaches have access to 32GB RAM. Because *best-tidal* is very hard to solve, we only use the values $n_z \in \{1, 2, 5\}$ for it with access to 64GB RAM. To compare the different approaches, we calculate all solutions on the objective function of *best-tidal* with 1000 zones, which has the highest resolution and is thus most realistic. For evaluation, we run each configuration with 10 different seeds for the random parameters. Hence, in total, we have 30 instances for every approach, i.e., 90 jobs overall.

6.3. Preliminary computational results

In terms of overall solution time, most algorithm and zone number combinations time out at reaching 23 hours. The only exception is the *many-tidal* model with 10 zones, which takes on average 20.5 min to solve, and some of the 1 and 2 zone *best-tidal* combinations, which display large performance variability, from less than 30 min to more than 23 hours.

In this context, the build times of the models, as can be seen in Fig. 4, already limit the usability of *best-tidal*. Even more so, the memory requirements of *best-tidal* limit its applicability, even if more runtime were provided, as can be seen in Fig. 5.

Out of 10 *best-tidal* runs with 5 zones, 3 ran out of memory, with the first job having reached 64 GB of RAM after 8: 49 hours. This implies that running *best-tidal* without zone clustering is not viable in practice.

Therefore, it is necessary to compare the solution quality of different heuristics and different numbers of zone clusters, as, for example, 100 zone clusters in *many-zones* result in more memory efficiency and a

[3] https://github.com/mapado/haversine

[4] See www.geoseaportal.de/vessel traffic density.

[5] https://web.archive.org/web/20240107173048/https://help.itc.rwth-aachen.de/service/rhr4fjjutttf/article/fbd107191cf14c4b8307f44f545cf68a/

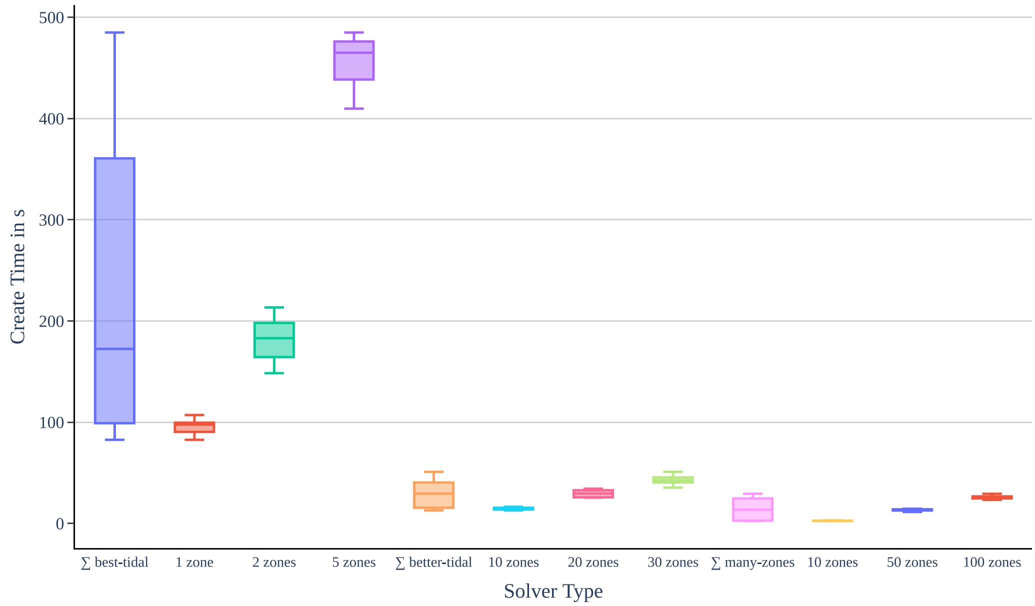

Fig. 4. Time required to build the Gurobi models and cluster the zones.

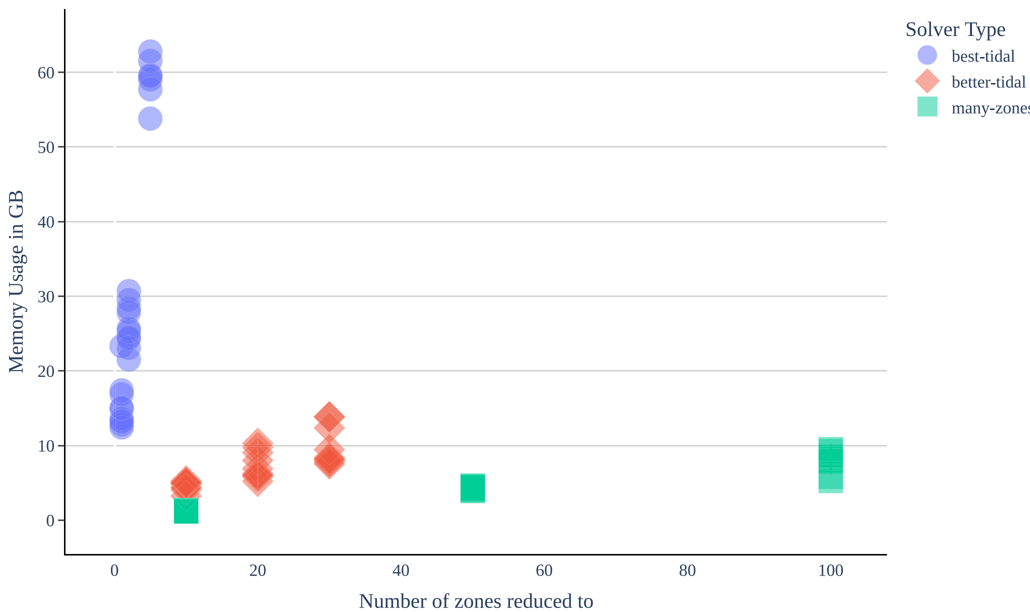

Fig. 5. Memory usage of different levels of tidal information.

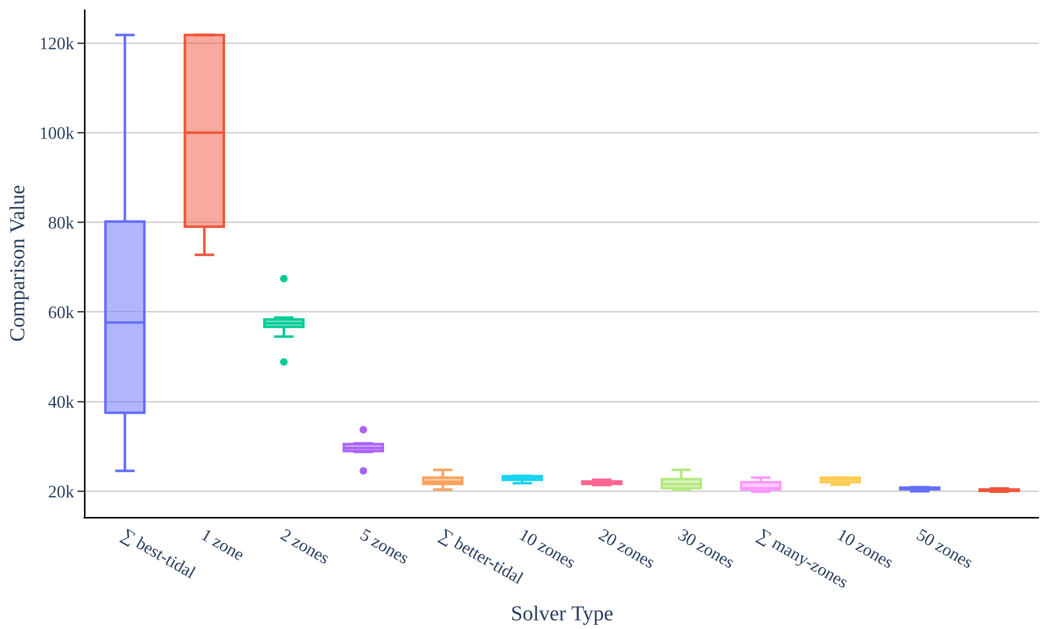

Fig. 6. Comparison value in the complete instances with 1000 zones and the complete tidal information. This represents the quality of an assignment. The objective represents a weighted expected response time, i.e., lower values are to be preferred.

lower run time than 2 clusters in *best-tidal*. To do so, we extract the vessel-station assignment M from every solution. We then validate the second part of the responding station assignment by checking for all combinations of each of the original 1000 zones, each incident type and each tidal state (according to our best available tidal data) which of the stationed compatible vessels can respond the fastest. In this process, we also confirm that all of the produced solutions for the different instances are feasible.

7. Discussion

The quality of the resulting objective values is visualised in Fig. 6. For that, we use a proxy comparison value, which is given by the average response type weighted by incident types.

It is important to note that not every instance has the same optimal solution value, so some variation in comparison value is to be expected.

Clearly, in our instances we gain better results with a higher number of zone clusters in exchange for a reduction in tidal states. This is explained by the correlation between the tidal levels at the different stations, as noted before. Additionally, we see that the difference between *better-tidal* and *many-zones* is rather small. This is most likely due to the fact that the vessels have quite similar draught: the lowest value is 0.5 metres while the highest is 2.7 m. We also observe that too aggressive clustering leads to a strong deterioration in solution quality. The *best-tidal* models with few zones produce solutions that are worse by up to a factor of 6 compared to the other approaches, if evaluated on the full 1000 zones. In comparison, after a certain threshold, the number of zones does not seem to have a big influence on *many-zones* and *better-tidal*.

Furthermore, the final solutions for *best-tidal* display an increasing (average) integrality gap of 13% for 2 zones and 21% for 5 zones. In comparison, for *better-tidal* with 30 zones, the average integrality gap is below 5%, and below 0.5% and 3.5% for *many-zones* with 50 and 100 zones, respectively. Thus, the results for *best-tidal* are bad not just due to the small number of zones but also because the solver can no longer solve the problems to de facto optimality. These results also translate into difference in response times. Table 4 gives the average response time in minutes and its standard deviation for each of the three approaches under different parameter configurations.

Table 4
Response times in minutes sorted by solution approach, number of zones, and incident type. Given values are mean values taken across all zones and instances.

Approach	Zones	General	Grounding	Fire	Sinking
best-tidal	1	124 ± 82	177 ± 95	170 ± 97	124 ± 82
	2	56 ± 31	94 ± 50	90 ± 50	56 ± 31
	5	28 ± 17	58 ± 32	44 ± 28	28 ± 17
better-tidal	10	23 ± 11	51 ± 25	36 ± 17	23 ± 11
	20	23 ± 11	48 ± 25	32 ± 16	23 ± 11
	30	22 ± 11	57 ± 33	41 ± 23	22 ± 11
many-zones	10	24 ± 11	49 ± 24	35 ± 16	24 ± 11
	50	21 ± 10	54 ± 28	36 ± 18	21 ± 10
	100	21 ± 10	53 ± 30	36 ± 20	21 ± 10

We see that the *best-tidal* approach leads to large response times of up two two and a half hours, whereas both *better-tidal* and *many-zones* lead to reasonable response times of an hour at most, with most response vessels arriving within 20 minutes. Notably, the solutions assign higher average response times to grounding and to lesser extent fire. In the case of grounding, this may be a combination of requirements that not every vessel fulfils (towing power, secondary vessel) and the low priority of grounding incident types. For fire, there is a similar need for extra equipment (firefighting), but the incident type has high priority.

8. Conclusion

In this work, we built a mathematical model for the Rescue Vessel Allocation Problem (RVAP). While, in general, RVAP is $\mathcal{NP}$-hard and straightforward IP formulations are large, there are approaches to effectively reduce the problem size. We showed that clustering of incidents as well as simplification of the possible tide states enable us to solve realistically sized instances with appropriate granularity. The degree of simplification and the resolution of the tide states can be varied.

To compare these approaches, we set up a computational study in which we tested all three formulations under restricted resources with respect to memory and runtime. To do so, we used real world data of the DGzRS and other sources. Some of the input, such as incidents and vessel-station compatibility, was generated randomly due to lacking data. Furthermore, this lack of data is also the reason for not comparing our results to the solution currently used, as our results might be infeasible in practice and the real solution in turn infeasible for our instances.

Our computational study establishes the validity of our concept of simplified tide representation and clustering of the incident zones. As shown in Fig. 6, there is no significant difference in quality between *better-tidal* and *many-zones*. Beyond that, *best-tidal* returns worse results. Since, in the context of our computational study, using more zones did not lead to better results, using a k-means clustering algorithm (Section 6.1.3) has proven to be a good approach. Moreover, we obtained feasible solutions, which indicates a practical ability of our model. It might therefore be promising to apply the model to real world data of the DGzRS to improve their current assignment.

8.1. Further research

The model presented in this work can be expanded in multiple ways. First of all, the estimation of incident risk can be further refined. Part of this would be to introduce an uncertainty factor so that solutions are robust to changes in incident data.

Second, our assumption is that every incident is isolated, meaning that a vessel can respond to an infinite number of incidents. While the number of incidents suggests a daily average of less than six incidents across the entire region, making it unlikely that a vessel is required at two location simultaneously, it might be worthwhile to balance the workload across the available vessels.

Third, we assume that the number of vessels is fixed. If this number is reduced, it might be interesting to analyse which vessel should be removed and what the increase in response times would be. Conversely, it is not trivial to quantify which improvement an additional vessel would make and where to place it.

Fourth, we only consider the haversine distance between stations and zones. However, in the real-world, factors like fairway, depth of water, restricted areas and even the location of islands have to be considered when computing a route. Implementing this would serve to make the model more valid for practitioners. Furthermore, as suggested by Siljander et al. [30], wind and wave patterns also influence rescue speed, which lends itself to stochastic optimization.

Fifth, it would be interesting to investigate whether there are yearly patterns in the incident data and how beneficial a seasonal reallocation of the rescue vessels would be.

As a final note, at the moment, tide data is used to determine availability of stations. Of course, zones are also affected by the tides. For the future, it might be worthwhile to also give the zones a tide property and expand route calculation to note temporarily inaccessible zones. Moreover, tides impact the travel time, which could also be integrated into a more complex model.

CRediT authorship contribution statement

Tom Mucke: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Alexander Renneke:** Writing – review & editing, Writing – original draft, Visualization, Validation, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Finn Seesemann:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Felix Engelhardt:** Writing – review & editing, Writing – original draft, Supervision, Project administration, Funding acquisition, Conceptualization.

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Appendix A. Proofs of complexity

In the following, we give proofs of Lemma 2 and Corollary 6.

Lemma. The f-RVAP is in $\mathcal{NP}$.

Proof. A solution consists of a function $M : \mathcal{M} \to \mathcal{N}$ assigning every station in $\mathcal{M}$ a vessel in $\mathcal{N}$ as well as a function $u : \mathcal{F} \times \mathcal{Z} \times \mathcal{T} \to \mathcal{M}$ assigning every combination of time interval in $\mathcal{T}$, zone in $\mathcal{Z}$ and incident type in $\mathcal{F}$ a responding station in $\mathcal{M}$. Given these, in a feasible solution stations only house compatible vessels, meaning $(j, M(j)) \in C$ for all $j \in \mathcal{M}$, which can be checked in $\mathcal{O}(\mathcal{M})$. Furthermore we need to:

- Verify that the vessel of the responding station is operable during the given water state, i.e., the station vessel combination must be in C.
- Ensure that the vessel is equipped to deal with the given incident, meaning $(M(u(f,z,t)), f) \in B$ is true for all $f \in \mathcal{F},\ z \in \mathcal{Z},\ t \in \mathcal{T}$.
- Check that the given zone in $\mathcal{Z}$ is reachable from the specified station by the responding vessel, meaning $(M(u(f,z,t)), u(f,z,t), z) \in S$ must be true for all $f \in \mathcal{F},\ z \in \mathcal{Z},\ t \in \mathcal{T}$.

Since each of these categories requires either $\mathcal{O}(|\mathcal{M}|)$ or $\mathcal{O}(|\mathcal{F}||\mathcal{Z}||\mathcal{T}|)$ checks with a runtime of $\mathcal{O}(1)$ each, in total we have a polynomial runtime of $\mathcal{O}(|\mathcal{M}| + |\mathcal{F}||\mathcal{Z}||\mathcal{T}|)$. □

Corollary 6. RVAP is $\mathcal{NPO}$-complete, even for only two vessel types, disregarding differing vessel-ranges, vessel-station incompatibilities, incident types and water states.

Proof. We assign vessel type I a speed of $v_{\mathrm{I}} = 1$ and vessel type II a speed of $v_{\mathrm{II}} = 0.5$ (and set all distances to $d_{jr} = 1$). The goal equivalent to finding a solution for the X3CP-instance is to find a solution for the constructed RVAP-instance with a total response time of at most $3q$. Since every one of the $3q$ zones contributes either 1 or 2 to the total response time depending on the vessel responding, a total of $3q$ is equivalent to using only the q vessels of type I to respond to the incidents. □

Appendix B. Tide correlation data

See Table B.1.

Data availability

All data is available via https://github.com/veni-vidi-code/RCA.

Table B.1
Correlation between different water level gauges used to measure the tides.

	9610010	9610015	9610020	9610025	9610035	9610040	9610045	9610050	9610066	9610070	9610075	9610080	9630007	9630008	9640015	9650024
9610010	1.00	1.00	1.00	0.24	0.96	0.82	0.99	0.99	0.99	0.97	0.96	0.95	0.94	0.93	0.91	0.43
9610015	1.00	1.00	1.00	0.24	0.96	0.81	1.00	0.99	0.99	0.97	0.97	0.96	0.95	0.94	0.92	0.42
9610020	1.00	1.00	1.00	0.24	0.96	0.81	1.00	1.00	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.41
9610025	0.24	0.24	0.24	1.00	0.18	0.10	0.25	0.25	0.25	0.26	0.25	0.25	0.26	0.26	0.25	-0.18
9610035	0.96	0.96	0.96	0.18	1.00	0.94	0.96	0.96	0.95	0.92	0.93	0.93	0.91	0.90	0.89	0.55
9610040	0.82	0.81	0.81	0.10	0.94	1.00	0.81	0.81	0.81	0.76	0.79	0.80	0.76	0.75	0.74	0.67
9610045	0.99	1.00	1.00	0.25	0.96	0.81	1.00	1.00	1.00	0.98	0.98	0.97	0.96	0.95	0.93	0.42
9610050	0.99	0.99	1.00	0.25	0.96	0.81	1.00	1.00	1.00	0.99	0.98	0.97	0.97	0.96	0.95	0.41
9610066	0.99	0.99	0.99	0.25	0.95	0.81	1.00	1.00	1.00	0.99	0.98	0.97	0.96	0.96	0.94	0.41
9610070	0.97	0.97	0.98	0.26	0.92	0.76	0.98	0.99	0.99	1.00	0.99	0.97	0.98	0.98	0.97	0.36
9610075	0.96	0.97	0.97	0.25	0.93	0.79	0.98	0.98	0.98	0.99	1.00	0.99	0.99	0.99	0.98	0.41
9610080	0.95	0.96	0.96	0.25	0.93	0.80	0.97	0.97	0.97	0.97	0.99	1.00	0.99	0.99	0.97	0.43
9630007	0.94	0.95	0.95	0.26	0.91	0.76	0.96	0.97	0.96	0.98	0.99	0.99	1.00	1.00	0.99	0.38
9630008	0.93	0.94	0.94	0.26	0.90	0.75	0.95	0.96	0.96	0.98	0.99	0.99	1.00	1.00	0.99	0.37
9640015	0.91	0.92	0.93	0.25	0.89	0.74	0.93	0.95	0.94	0.97	0.98	0.97	0.99	0.99	1.00	0.35
9650024	0.43	0.42	0.41	-0.18	0.55	0.67	0.42	0.41	0.41	0.36	0.41	0.43	0.38	0.37	0.35	1.00
9650030	0.52	0.51	0.51	-0.14	0.64	0.72	0.52	0.53	0.52	0.50	0.54	0.55	0.53	0.53	0.53	0.92
9650040	0.90	0.90	0.91	0.18	0.90	0.80	0.91	0.92	0.92	0.93	0.96	0.94	0.95	0.95	0.96	0.47
9650043	0.92	0.93	0.93	0.17	0.93	0.84	0.93	0.94	0.93	0.93	0.95	0.94	0.94	0.93	0.94	0.48
9650070	0.91	0.92	0.92	0.18	0.91	0.82	0.93	0.93	0.93	0.93	0.96	0.95	0.94	0.94	0.95	0.47
9650073	0.89	0.90	0.90	0.19	0.88	0.78	0.91	0.92	0.92	0.92	0.95	0.94	0.95	0.94	0.96	0.43
9650080	0.83	0.83	0.83	0.07	0.89	0.87	0.84	0.85	0.84	0.83	0.87	0.86	0.85	0.85	0.86	0.64
9670046	0.84	0.85	0.85	0.08	0.93	0.92	0.85	0.85	0.84	0.82	0.85	0.84	0.82	0.81	0.82	0.61
9670050	0.88	0.88	0.89	0.15	0.92	0.87	0.89	0.90	0.89	0.89	0.93	0.91	0.91	0.90	0.91	0.55
9670055	0.80	0.81	0.81	0.11	0.87	0.85	0.82	0.83	0.82	0.83	0.86	0.85	0.84	0.84	0.86	0.60
9670063	0.89	0.90	0.90	0.19	0.88	0.79	0.91	0.92	0.91	0.92	0.95	0.94	0.94	0.94	0.95	0.44
9670065	0.83	0.84	0.84	0.24	0.82	0.71	0.86	0.87	0.87	0.88	0.92	0.91	0.93	0.92	0.94	0.39
9670067	0.86	0.87	0.87	0.19	0.85	0.74	0.88	0.89	0.89	0.91	0.93	0.92	0.93	0.93	0.95	0.42
9690077	0.85	0.86	0.86	0.14	0.84	0.75	0.87	0.88	0.88	0.90	0.93	0.91	0.92	0.92	0.94	0.44
9690078	0.80	0.82	0.82	0.24	0.78	0.67	0.84	0.85	0.85	0.87	0.91	0.90	0.91	0.91	0.93	0.36
9690085	0.85	0.86	0.86	0.14	0.87	0.81	0.87	0.88	0.88	0.89	0.91	0.90	0.91	0.91	0.92	0.52
9690093	0.79	0.80	0.81	0.24	0.77	0.65	0.82	0.84	0.84	0.87	0.90	0.89	0.91	0.91	0.93	0.34

	9650030	9650040	9650043	9650070	9650073	9650080	9670046	9670050	9670055	9670063	9670065	9670067	9690077	9690078	9690085	9690093
9610010	0.52	0.90	0.92	0.91	0.89	0.83	0.84	0.88	0.80	0.89	0.83	0.86	0.85	0.80	0.85	0.79
9610015	0.51	0.90	0.93	0.92	0.90	0.83	0.85	0.88	0.81	0.90	0.84	0.87	0.86	0.82	0.86	0.80
9610020	0.51	0.91	0.93	0.92	0.90	0.83	0.85	0.89	0.81	0.90	0.84	0.87	0.86	0.82	0.86	0.81
9610025	-0.14	0.18	0.17	0.18	0.19	0.07	0.08	0.15	0.11	0.19	0.24	0.19	0.14	0.24	0.14	0.24
9610035	0.64	0.90	0.93	0.91	0.88	0.89	0.93	0.92	0.87	0.88	0.82	0.85	0.84	0.78	0.87	0.77
9610040	0.72	0.80	0.84	0.82	0.78	0.87	0.92	0.87	0.85	0.79	0.71	0.74	0.75	0.67	0.81	0.65
9610045	0.52	0.91	0.93	0.93	0.91	0.84	0.85	0.89	0.82	0.91	0.86	0.88	0.87	0.84	0.87	0.82
9610050	0.53	0.92	0.94	0.93	0.92	0.85	0.85	0.90	0.83	0.92	0.87	0.89	0.88	0.85	0.88	0.84
9610066	0.52	0.92	0.93	0.93	0.92	0.84	0.84	0.89	0.82	0.91	0.87	0.89	0.88	0.85	0.88	0.84
9610070	0.50	0.93	0.93	0.93	0.92	0.83	0.82	0.89	0.83	0.92	0.88	0.91	0.90	0.87	0.89	0.87
9610075	0.54	0.96	0.95	0.96	0.95	0.87	0.85	0.93	0.86	0.95	0.92	0.93	0.93	0.91	0.91	0.90
9610080	0.55	0.94	0.94	0.95	0.94	0.86	0.84	0.91	0.85	0.94	0.91	0.92	0.91	0.90	0.90	0.89
9630007	0.53	0.95	0.94	0.94	0.95	0.85	0.82	0.91	0.84	0.94	0.93	0.93	0.92	0.91	0.91	0.91
9630008	0.53	0.95	0.93	0.94	0.94	0.85	0.81	0.90	0.84	0.94	0.92	0.93	0.92	0.91	0.91	0.91
9640015	0.53	0.96	0.94	0.95	0.96	0.86	0.82	0.91	0.86	0.95	0.94	0.95	0.94	0.93	0.92	0.93
9650024	0.92	0.47	0.48	0.47	0.43	0.64	0.61	0.55	0.60	0.44	0.39	0.42	0.44	0.36	0.52	0.34
9650030	1.00	0.64	0.63	0.62	0.61	0.80	0.74	0.71	0.79	0.61	0.57	0.61	0.64	0.56	0.72	0.55
9650040	0.64	1.00	0.98	0.99	0.99	0.94	0.90	0.97	0.94	0.99	0.96	0.99	0.99	0.95	0.98	0.95
9650043	0.63	0.98	1.00	0.99	0.98	0.94	0.93	0.98	0.93	0.98	0.93	0.97	0.97	0.91	0.97	0.90
9650070	0.62	0.99	0.99	1.00	0.99	0.93	0.91	0.97	0.92	0.99	0.95	0.98	0.98	0.94	0.97	0.93
9650073	0.61	0.99	0.98	0.99	1.00	0.92	0.88	0.96	0.92	1.00	0.97	0.99	0.99	0.96	0.97	0.96
9650080	0.80	0.94	0.94	0.93	0.92	1.00	0.96	0.97	0.99	0.93	0.88	0.92	0.93	0.87	0.98	0.86
9670046	0.74	0.90	0.93	0.91	0.88	0.96	1.00	0.96	0.95	0.89	0.81	0.86	0.87	0.79	0.92	0.77
9670050	0.71	0.97	0.98	0.97	0.96	0.97	0.96	1.00	0.97	0.97	0.92	0.96	0.96	0.91	0.97	0.89
9670055	0.79	0.94	0.93	0.92	0.92	0.99	0.95	0.97	1.00	0.92	0.88	0.92	0.93	0.88	0.97	0.87
9670063	0.61	0.99	0.98	0.99	1.00	0.93	0.89	0.97	0.92	1.00	0.97	0.99	0.99	0.96	0.97	0.96
9670065	0.57	0.96	0.93	0.95	0.97	0.88	0.81	0.92	0.88	0.97	1.00	0.97	0.97	0.99	0.94	0.99
9670067	0.61	0.99	0.97	0.98	0.99	0.92	0.86	0.96	0.92	0.99	0.97	1.00	0.99	0.97	0.98	0.97
9690077	0.64	0.99	0.97	0.98	0.99	0.93	0.87	0.96	0.93	0.99	0.97	0.99	1.00	0.97	0.98	0.97
9690078	0.56	0.95	0.91	0.94	0.96	0.87	0.79	0.91	0.88	0.96	0.99	0.97	0.97	1.00	0.94	1.00
9690085	0.72	0.98	0.97	0.97	0.97	0.98	0.92	0.97	0.97	0.97	0.94	0.98	0.98	0.94	1.00	0.94
9690093	0.55	0.95	0.90	0.93	0.96	0.86	0.77	0.89	0.87	0.96	0.99	0.97	0.97	1.00	0.94	1.00

	9340020	9340030	9360010	9390010	9410010	9510010	9510060	9510063	9510066	9510070
9340020	1.00	0.99	0.99	0.94	0.92	0.63	0.34	0.83	0.74	0.84
9340030	0.99	1.00	0.96	0.88	0.85	0.52	0.29	0.74	0.66	0.75
9360010	0.99	0.96	1.00	0.98	0.96	0.73	0.38	0.90	0.80	0.91
9390010	0.94	0.88	0.98	1.00	1.00	0.85	0.42	0.96	0.87	0.97
9410010	0.92	0.85	0.96	1.00	1.00	0.88	0.43	0.98	0.88	0.98
9510010	0.63	0.52	0.73	0.85	0.88	1.00	0.44	0.95	0.86	0.95
9510060	0.34	0.29	0.38	0.42	0.43	0.44	1.00	0.45	0.45	0.44
9510063	0.83	0.74	0.90	0.96	0.98	0.95	0.45	1.00	0.90	1.00
9510066	0.74	0.66	0.80	0.87	0.88	0.86	0.45	0.90	1.00	0.90
9510070	0.84	0.75	0.91	0.97	0.98	0.95	0.44	1.00	0.90	1.00
9510075	0.84	0.75	0.91	0.97	0.98	0.95	0.45	1.00	0.90	1.00
9510095	0.59	0.47	0.69	0.82	0.85	0.99	0.43	0.93	0.84	0.92
9510132	0.43	0.34	0.50	0.59	0.61	0.69	0.29	0.65	0.58	0.65
9530010	0.43	0.31	0.55	0.70	0.73	0.96	0.41	0.84	0.77	0.84
9530020	0.23	0.10	0.36	0.53	0.58	0.89	0.35	0.71	0.65	0.71
9550021	0.36	0.23	0.49	0.64	0.69	0.94	0.39	0.80	0.73	0.80
9570010	0.36	0.23	0.48	0.64	0.68	0.94	0.39	0.80	0.73	0.80
9570040	0.01	-0.11	0.14	0.31	0.36	0.73	0.26	0.49	0.45	0.50
9570050	0.24	0.12	0.37	0.53	0.58	0.87	0.35	0.69	0.64	0.70
9570070	-0.03	-0.14	0.09	0.25	0.30	0.66	0.24	0.42	0.39	0.44

	9510075	9510095	9510132	9530010	9530020	9550021	9570010	9570040	9570050	9570070
9340020	0.84	0.59	0.43	0.43	0.23	0.36	0.36	0.01	0.24	-0.03
9340030	0.75	0.47	0.34	0.31	0.10	0.23	0.23	-0.11	0.12	-0.14
9360010	0.91	0.69	0.50	0.55	0.36	0.49	0.48	0.14	0.37	0.09
9390010	0.97	0.82	0.59	0.70	0.53	0.64	0.64	0.31	0.53	0.25
9410010	0.98	0.85	0.61	0.73	0.58	0.69	0.68	0.36	0.58	0.30
9510010	0.95	0.99	0.69	0.96	0.89	0.94	0.94	0.73	0.87	0.66
9510060	0.45	0.43	0.29	0.41	0.35	0.39	0.39	0.26	0.35	0.24
9510063	1.00	0.93	0.65	0.84	0.71	0.80	0.80	0.49	0.69	0.42
9510066	0.90	0.84	0.58	0.77	0.65	0.73	0.73	0.45	0.64	0.39
9510070	1.00	0.92	0.65	0.84	0.71	0.80	0.80	0.50	0.70	0.44
9510075	1.00	0.93	0.65	0.84	0.71	0.80	0.80	0.50	0.70	0.44
9510095	0.93	1.00	0.68	0.97	0.91	0.96	0.95	0.76	0.90	0.69
9510132	0.65	0.68	1.00	0.64	0.61	0.65	0.64	0.49	0.58	0.43
9530010	0.84	0.97	0.64	1.00	0.96	0.98	0.98	0.85	0.95	0.80
9530020	0.71	0.91	0.61	0.96	1.00	0.99	0.99	0.93	0.98	0.90
9550021	0.80	0.96	0.65	0.98	0.99	1.00	1.00	0.89	0.97	0.84
9570010	0.80	0.95	0.64	0.98	0.99	1.00	1.00	0.90	0.98	0.86
9570040	0.50	0.76	0.49	0.85	0.93	0.89	0.90	1.00	0.95	0.95
9570050	0.70	0.90	0.58	0.95	0.98	0.97	0.98	0.95	1.00	0.93
9570070	0.44	0.69	0.43	0.80	0.90	0.84	0.86	0.95	0.93	1.00

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