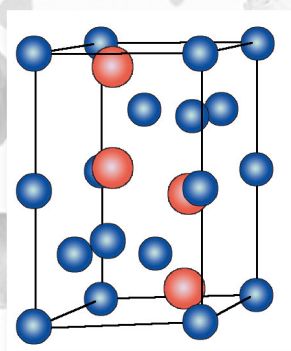


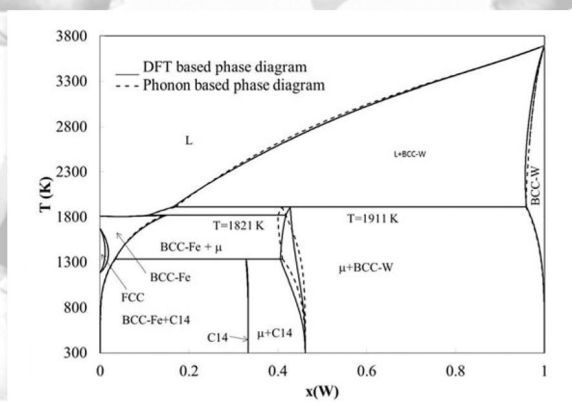
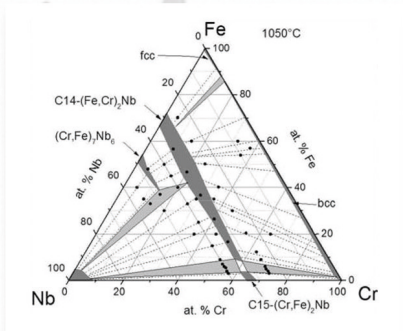
Thermochemical Modeling of Laves Phase Containing Ferritic Steels

Aurélie Jacob

$$\mathcal{H}\psi = \mathcal{H}E$$



$$G = a + bT + cT\ln T + \sum d_n T^n$$



Thermochemical Modeling of Laves Phase Containing Ferritic Steels

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Aachen zur Erlangung des akademischen Grades einer Doktorin der Naturwissenschaften
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Aurélie Jacob

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Abstract

For energy technology, development of new materials is required to be used for boiler and steam turbines with high thermal cyclability, creep strength and excellent corrosion resistance in steam and high temperatures and also at downtime in water. Crofer 22H, a new kind of steel jointly developed at Forschungszentrum Juelich and Thyssen Krupp, has already shown promising results in laboratory tests. The formation of Laves phase particles improves the material's properties such as creep strength while the high Cr-content of 22 % provides excellent corrosion resistance. Unfortunately, the formation of Laves phase competes with the formation of other phases. In order to further improve this type of ferritic steels, knowledge of thermodynamics of the intermetallic systems forming the Laves phases $(\text{Fe,Cr,Si})_2(\text{Nb,W})$ is required for the calculation of phase stability as a function of composition and temperature. The Calphad method is of great help for the calculation of phase stability in technical materials. This kind of calculation is based on thermodynamic databases. Calculations with the commercial steel database have shown to be inaccurate since not all considered alloy systems have been modeled up to now. The present work aims to provide a thermodynamic optimization of the systems containing Laves phases.

Thermodynamic data of Laves phase containing systems were obtained by the Calphad method with an optimization of the sub-systems containing Laves phases. The Calphad method relies on the input data for the considered system, their quality and their quantity are of primary importance for the obtained thermodynamic assessment. In the present work, several methods have been used in order to provide consistent thermodynamic calculations of the phase equilibria.

A combined experimental and modeling approach was chosen to determine the phase equilibria and thermodynamic properties in the systems Fe-W, Cr-Nb, Cr-Fe-Nb and Fe-Nb-Si. DFT calculations were done for all mentioned systems for the determination of the energy of formation of the compounds through the chosen model. In the Fe-W system, in addition to the DFT calculations which are restricted to 0 K, phonon calculations were performed in order to obtain the temperature dependence of the Gibbs energies which were used to improve the Calphad modeling.

As the literature on the Cr-Fe-Nb ternary system is rather scarce, experimental investigations were performed to determine isothermal sections and solidification behavior of selected alloy compositions. The combination of experimental work and DFT simulations was then used as input for the optimization of the phase diagram with the Calphad method.

Kurzfassung

Für Anwendungen in der Energietechnik werden krieCHFeste und korrosionsbeständige Hochtemperaturwerkstoffe auf Eisenbasis benötigt, die zukünftig in Dampfturbinen und Kesseln eingesetzt werden, um möglichst hohe Lastflexibilität, Zykliefähigkeit und Korrosionsfestigkeit sowohl im Betrieb als auch bei Stillstand zu gewährleisten. Während die gegenwärtig eingesetzten Werkstoffe im Wesentlichen auf ferritisch-/martensitischem Gefüge mit 9-12 % Chrom beruhen, bieten Lavesphasenverstärkte ferritische Werkstoffe mit 15-22 % Chrom das Potential nicht nur für Anwendungstemperaturen bis zu 650 °C, sondern weisen auch ausgezeichnete Korrosionsbeständigkeit in Wasserdampf und in wässrigen Lösungen auf.

Das Problem bei der Entwicklung derartiger Werkstoffe besteht darin, dass die entsprechenden Phasendiagramme vielfach nur bedingt bekannt sind, da entsprechende thermochemische Daten über die infrage kommenden Lavesphasen und ihre Zusammensetzung fehlen, um mittels der Calphadmethode die entsprechenden Zustandsdiagramme zu berechnen.

Mittels DFT-Methoden ergänzt durch experimentelle Untersuchungen ist es möglich, die fehlenden Daten und Zusammensetzungen zu bestimmen und die Genauigkeit sowohl der experimentell ermittelten, als auch der mittels Calphad berechneten Zustandsdiagramme erheblich zu verbessern.

Auf diese Weise ist es gelungen, die Phasengleichgewichte in den binären Systemen Fe-W, Cr-Nb sowie den ternären Systemen Cr-Fe-Nb und Fe-Nb-Si zu bestimmen. Für die in diesen Systemen auftretenden Lavesphasen wurden die thermochemischen Eigenschaften mittels DFT bestimmt, für das System Fe-W wurden zusätzlich zu den DFT-Berechnungen Phononen-Berechnungen durchgeführt, um die Temperaturabhängigkeit der Bildungsenergie zu berücksichtigen, was für die Anwendung bei hohen Temperaturen von besonderer Bedeutung ist.

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Introduction

The development of new materials is one of the technology challenges of this century especially in the conversion of energy. Indeed with a decrease of the natural resources and an increase of the demand, there is a necessity of improving the efficiency of the actual technology and to develop new ones. For the provision of energy, fossil fuels remain a secure provision for the next decades to change the energy supply system. Improvement of the efficiency of the actual technology is required to extend the provision of the primary resources. The efficiency of the technology allows also a reduction of the footprint on the environment. An improvement of the efficiency has so two advantages: reduction of the amount of fuel and reduction of CO₂ for the global warming.

For an improvement of the efficiency up to 50 % for steam power plants, the necessary conditions of vapor pressure would be around 700 °C/300 bars. The use of ferritic martensitic steels with respect to creep strength for steam power plants applications is limited to about 600 to 620°C. New steels with a high chromium content of around 20 % with intermetallic Laves phase particles dispersed in a ferritic matrix have shown to be potentially good candidates due to their good creep strength and oxidation behavior. In contrast to the ferritic martensitic steels they have a high corrosion resistance at downtime.

Many technologies are involved in the process of the development of new materials. The development of new materials for an industrial application relies on the prediction of the behavior of the material under working condition. For example, if the material is used under the sea (e.g. offshore wind power) or in high sea level (e.g. gas turbine for plane), the working conditions are different, in the first corrosion issue due to the salt has to be taken into account and in the second high modification of the working temperature. In both cases, it is necessary to know the behavior of the material to predict its working time and its cost. For a fastest development of material with a specific task it is necessary to understand its behavior. The behavior of materials under operating conditions can nowadays easily be predicted using

different simulating tools (calculation of phase diagrams, diffusion and phase field modeling). These tools are based on databases, developed for this purpose.

The time spent on the development of new materials can be shortened by knowing the thermodynamics properties in a first approach. As thermodynamics governs the chemical driving forces, its knowledge is the essential basis for understanding the conditions favoring the formation of certain wanted or unwanted phases and the subsequent control of the mechanisms in microstructure formation. The thermal behavior of the material can be predicted by using thermodynamic software (e.g. Thermo-Calc). The software relies on thermodynamic databases for multicomponent systems. The thermodynamic modeling and the development of these databases is called the Calphad method.

In the present work, the focus was on the improvement of a steel database. Indeed, the IEK-2 at Forschungszentrum Juelich and ThyssenKrupp jointly developed a steel named Crofer 22H ® [1]. This steel has the particularity of precipitation of so-called Laves phases in the ferritic matrix, which are responsible for the steel's favorable properties (creep resistance, strength...). This steel has already shown promising results at the laboratory scale for the use as super heater material in steam power plants. However, the good behavior of the material could not be reproduced at industrial scale. The improvement of the material properties focusses on the precipitation of Laves phase particles in the ferritic matrix that faces competing precipitation of other intermetallic phases (i.e. carbides, σ -phase, and nitrides).

Scope of the work

Iron based alloys can be used for high temperature materials. In presence of carbon, the alloy forms steel. Steels are well known materials since antiquity but it is since the 18th century that industrial production of steels has been done.

There are different kinds of steel, e.g. ferritic, austenitic and ferritic-martensitic. They differ by their composition, microstructure and crystal structures. Ferritic-martensitic steels with 9-12 % Cr are used in steam power plants working below 600°C. Increasing the working temperature would allow an improvement of the efficiency of the process, but these types of materials fail due to an increase of creep and of the oxidation rates. Instead austenitic steels and Ni-alloys can be used but these materials are expensive and prone thermo-mechanical fatigue degradation due to their higher thermal expansion coefficients compared to ferritic and ferritic-martensitic steels.

Ferritic-martensitic steels with higher Cr content than 12 % show to be potentially good candidate for high temperature but exhibit lower creep strength. An interesting approach of steel design is to add some elements forming Laves phases [2]. Indeed the precipitation of Laves phase in high chromium ferritic steels improves tensile, creep and oxidation properties. Such new ferritic steel has been developed in the IEK-2 institute at Forschungszentrum Jülich and commercialized under the name Crofer 22H® [1].

The Laves phases in Crofer 22H are due to the addition of Nb and W as well as Si to compare the Crofer APU [3]. In presence of Fe and Cr, the Laves phases $(\text{Fe,Cr,Si})_2(\text{Nb,W})$ form as fine precipitates dispersed in the ferritic steel matrix improving the materials properties such as creep and yield strength and oxidation behavior. In laboratory, this new kind of steel has shown to be potentially a good candidate for uses in steam power plants. Nevertheless, the promising results obtained were not reproducible at the industrial scale. It is supposed to be due to a competition with the formation of σ phase (σ -FeCr) and nitride (Z phase) [4]. In order to understand why the results were not reproducible, thermodynamic calculations using commercial database (i.e. TCFe6) have been carried out to study the stability of the

different phases involved in the formation of the steel. The calculations do not reproduce the presence of silicon in the Laves phase, pointing out the shortcomings of the commercial steel database (TCFE6) for the modeling of systems containing Laves phase. Nevertheless, many studies [2,4,5] report the great importance of Si in the formation and stability of the Laves phase containing systems.

In the present work, the development of the missing thermodynamic data for the Laves phases containing systems in Crofer 22H® were obtained by using experiments, DFT and phonons calculations where these last two ones can enhance, complete or have to substitute the experimental data. These results are then used as input parameters for the optimization with the calculation of phase diagram (Calphad) method for chosen key binary or ternary systems.

Chapter I – Contents of the study

I. Ferritic steel

A ferritic steel for high temperature applications, Crofer 22H [1], has been developed as an interconnector materials for advanced SOFC systems. It belongs to the second generation of this type of materials developed at Forschungszentrum Juelich.

The first generation Crofer 22 APU exhibits excellent oxidation resistance over long operation time without significant loss in electric conductivity. This can be attributed to the extremely low content of Si and Al, which tend to form internal oxides as well as the subscale oxide layers especially of SiO₂. These oxide layers exhibit very low electric conductivity resulting in high degradation of SOFC components. These high losses in electronic conductivity can be generated by low Al and Si contents in the interconnector steels.

The disadvantage of low Al and Si contents is that Crofer 22 APU has to be produced using technologies like vacuum or inert gas melt which are expensive. The addition of Nb and Si results in formation of Nb-silicides in the steel, which are stable enough to prevent a subscale silica formation below chromia layers and pick up dissolved oxygen. This allows to avoid the use of conventional costly technologies. Addition of W results in increased creep strength of this type of Laves reinforced steels which has been designated Crofer 22H [1].

Crofer 22H [1] is a ferritic steel with high chromium content (>20 mol%). Due to its good creep and corrosion resistance, Crofer 22H is also a potential candidate for application in combustion power plants as steam turbine. In ferritic steel the good strength comes from chromium based carbides [6]. Instead the high temperature strength can be improved by the presence of Laves phases. The precipitation of Laves phase in the matrix of ferritic steel is due the presence of the alloying elements W and Nb. These two elements play a role on the improvement of the creep resistance, oxidation behavior and strength compared with the previous steel of the same type, Crofer 22 APU [7]. These two elements in

combination with the major component of the ferritic steel (i.e. Cr and Fe) form Laves phases of AB₂ type. The composition of Crofer 22H is given in *Table 1*. Furthermore the presence of Si favors the precipitation of Laves phase in the matrix of the ferritic steel.

	Fe	Cr	C	N	S	Mn	Si	Al	W	Nb	Ti	La	P	Cu
At.%	65.68	21.12	0.006	0.007	0.003	0.74	0.29	0.046	9.33	1.57	0.16	0.47	0.02	0.54

Table 1 - Composition of Crofer 22H[®] in atomic %. Source: <http://www.fcdic.com/ja/member/data/Crofer22H.pdf>

It has been shown that the presence of precipitates like Laves phase containing Nb and W improves the yield strength by a factor of 1.4 at 600 °C and 2.7 at 800 °C [7] and improve considerably the creep resistance [2,7]. The good behavior of the Crofer 22H found in the laboratory could not be reproduced at the industrial scale. It has been found that there is a competition between the precipitation of the Laves phase and several unwanted carbides and nitrides [2,4] that deteriorate the material properties.

The composition of the Laves phase forming in the Crofer 22H has been experimentally determined by Kuhn et al. [2] using a model alloy with a composition close to that from the Crofer 22H. The composition of the AB₂ Laves phase is given in the *Table 2*.

	Fe	Cr	Si	Nb	W
Mol %	53.29	12.93	4.128	19.36	11.83

Table 2 – Composition of the Laves phase in the Crofer 22H determined experimentally by Kuhn et al. [2].

Using the Thermo-Calc software [8] it has been shown that there is no perceptible solubility of Si in the Laves phase [2] whereas W-solubility is overestimated [6]. The absence of Si in the Laves phase is explained by the missing thermodynamic data in the database. Nevertheless, silicon plays a significant role on the dissolution, grain growth and stability of the Laves phase.

For further improvement of ferritic steels with high Cr content and Laves phase precipitation, it is necessary to understand the chemical driving force of the formation of the different phases. As the formation of the phases is governed by thermodynamics, good knowledge of phase equilibria is required.

II. Literature survey

1. Laves phases

Laves phases have been discovered by James Friauf in 1927, and then investigation of these intermetallic phases have been done by Laves, Schulze and co-workers. Schulze who was working in Dresden (Democratic Republic of Germany at that time) was very active in the research of Laves phases but the cold war prevented to show the results to the international community and most of the work has been forgotten or lost. A recent review [9] showed some of the results made during this time. More recently the Max Plank Society has run a research initiative called “The Nature of Laves” and focused on different aspects of Laves phases. The work made on the properties of Laves phases has been extensively published, some parts have been reviewed by Stein and co-workers [10,11] and some general trends on the properties of Laves phases have been established.

Laves phases form the largest group of intermetallic phases, more than 1400 for binary and ternary systems [10] are documented. Laves phases show topological close-packed structures and they crystallize within the formula AB_2 . There are three different polytypes: the cubic $MgCu_2$ type (strukturbericht designation C15), the hexagonal $MgZn_2$ type (C14) and the hexagonal $MgNi_2$ type (C36). Their crystal structures are closely related: they can be described by different stacking sequences of the same types of layers. The crystalline structures are represented in *Figure 1*. In each of the three kinds of Laves phase, the coordination number (CN) is 16 for A ($CN_{A-A}=4$, $CN_{A-B}=12$) and 12 for B ($CN_{B-A}=6$, $CN_{B-B}=6$).

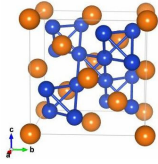
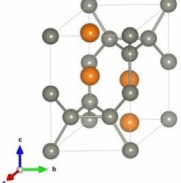
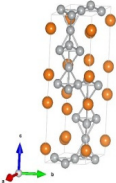
C15 type Cu₂Mg Fd-3m	C14 type MgZn₂ P6₃/mmc	C36 type MgNi₂ P6₃/mmc
		
Cu: 16d (5/8, 5/8, 5/8) Mg: 8a (0, 0, 0)	Mg: 4f (1/3, 2/3, z) Zn: 2a (0, 0, 0) 6h (x, 2x, 1/4)	Mg: 4e (0, 0, z) 4f (1/3, 2/3, z) Ni: 4f (1/3, 2/3, z) 6g (1/2, 0, 0) 6h (x, 2x, 1/4)

Figure 1 – Structure models and crystallography of the three different Laves phase polytypes.

The Laves phases form with specific requirements, e.g. the radii of the two elements forming them are related by $r_A/r_B = (3/2)^{1/2} = 1.225$ (ideal radius ratio), a specific valence electron concentration (see Figure 2) and electroneutrality. These three parameters are the most important properties for the formation of Laves phases. Some relations have been established between the formation of the type of Laves phase, the ideal radius and the electron concentration. In fact Laves phases form with a radius ratio between 1.05 and 1.68 [12] because the true radius differs from the Goldschmidt ideal radius [13]. Depending on the valence electron concentrations (Figure 2), a specific polytype of the Laves phase is preferred, but there are many exceptions. The C36 polytype is often described as metastable (in many systems) and is a transition phase between the C14 and C15 polytypes [14].

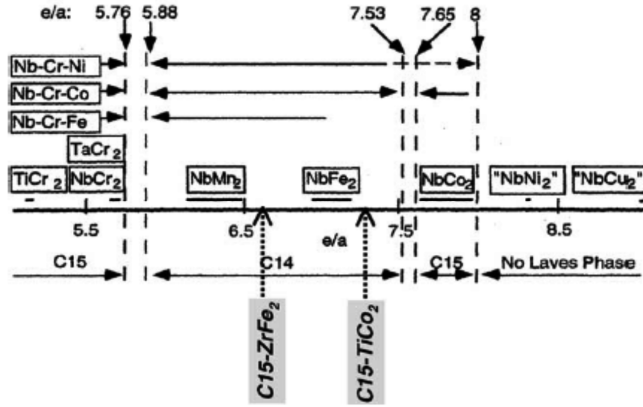


Figure 2 – Scheme of the stability of the Laves phase polytypes as function of the valence electron concentration according to [10].

From binary to ternary systems, different ways are possible for the formation of Laves phases. In the present work, there is evidence of binary C14 or C15 Laves phase $(\text{Fe,Cr,Si})_2(\text{Nb,W})$ depending on the concentration of the elements, and these are the most common Laves phase polytypes. The dissolution of a third element presents different advantages [11]:

- promotion of the formation of Laves phase,
- higher stability of the Laves phase.

There are high solubilities of a third element (Si or Al) in C14 phases (e.g. Fe-Nb-Si [15], Fe-Nb-Al [16]) and a modification of the polytype from C15 in the binary to C14 by addition of Si or Al in different ternary systems (e.g. Cr-Nb-Si [17], Cr-Nb-Al [18]).

The polytypes depend on the elements forming the Laves phase. In both binary and ternary systems, it is possible to find the same polytype of Laves phase, and on the other side it is not possible to extrapolate from ternary to a binary systems [19]. Indeed, a third element may promote the formation of a certain

polytype where in the binary system this polytype would not be found. The addition of a third element may also provoke the formation of Laves phase even if it doesn't exist in the binary system.

The Laves phases have some unique properties such as high melting temperature, low density and high oxidation resistance.

In Crofer 22H, the Laves phases are formed by the elements $(\text{Fe,Cr})_2(\text{Nb,W})$. Silicon as minor element dissolves in the Laves phase (Fe-Nb, Cr-Nb, Fe-W) and substitutes Fe and Cr [3,20]. Different Laves phases can be formed in Crofer 22H containing various element combinations (*Table 2*). Considering the five elements mentioned above, it can be formed the following systems:

- Binary: Fe-Nb, Fe-W, Cr-Nb;

and

- Ternary Fe-Cr-Nb, Fe-Nb-Si, Fe-Si-W and Cr-Nb-Si.

Out of these different mentioned systems a careful literature survey has been done in the following part. In the present work, the interest is the development of a suitable thermodynamic database for the ferritic alloy Crofer 22H containing the different Laves phases as a function of temperature. Our interest has been focused in a first approach on the literature useful for the thermodynamic modeling of the different systems (i.e. experimental data, thermodynamic modeling).

2. Silicon in Laves phase

There are several studies about the role of addition of Si in steel and its role in the formation of Laves phase. Silicon plays a role on the nucleation and growth of the Laves phase [5]. In many ternary systems Si dissolves in the C14 Laves phase, stabilizing it due to the decrease of the electron concentration (e/a) [21,22]. In the case of a C15 Laves phase in a binary system, the presence of Si provokes a structural modification and allows the formation of C14.

The addition of Si promotes the formation of a Laves phase and avoids the formation of carbides, which has beneficial effect on the mechanical properties of steel.

3. Thermodynamic stability of Laves phase containing system

The literature survey includes experimental as well as theoretical knowledge on the different systems previously mentioned.

3.1 Cr-Nb

A lot of studies have been done on the Cr-Nb system due to its importance for Fe and Ni based alloys. The Landolt-Börnstein [23] and the Massalski [24] compilation of phase diagrams show the presence of two different polytypes of Laves phase C15 at low temperature and C14 at high temperature.

Costa Neto et al. [25] have done a thermodynamic assessment of this phase diagram using the available experimental data at that time. They considered the two polytypes of Laves phase (C14 and C15). The most recent calculated phase diagram calculation of this system is given by Pavlů et al. [26] who determined the enthalpies of formation from ab-initio calculations and re-optimized this part of the phase diagram (*Figure 3*).

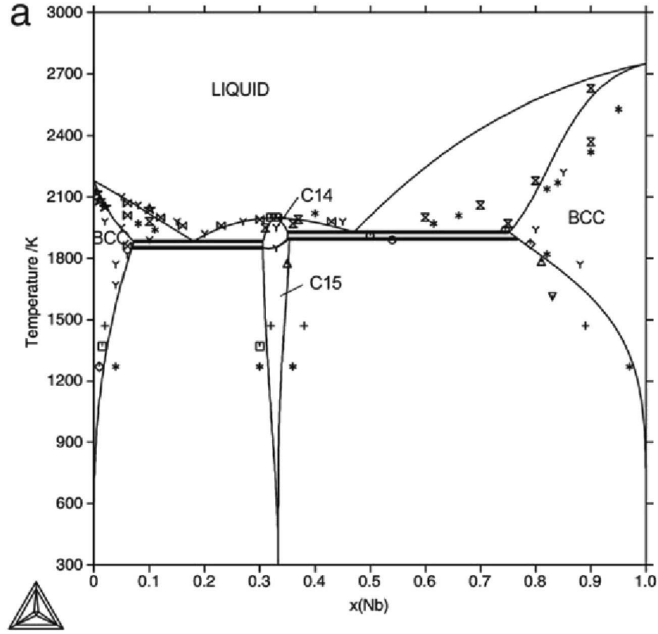


Figure 3 – Calculated phase diagram according to and from Pavlů et al. [26].

Even if many studies report the presence of the C14 Laves phase, the formation of this phase is electronically unfavourable. Indeed, the formation of Laves phase depends on atomic ratio r_A/r_B and valence electron concentration. Looking at the valence electron concentration, the C14 is expected to be unstable, *Figure 2* (or seen in the work of Zhu et al. [27] as well as in the review of Stein et al. [10]). In 2010, Aufrecht et al. [28] showed the metastability of the high temperature phases. Indeed until this time, the presence of C14 phase was extrapolated from higher order systems and some misinterpretation of results were done. In the case of Laves phase, a binary phase cannot be postulated from ternary or quaternary systems. A complete review of this system has been carried out during the time of the present work [29]. A new experimental investigation has been done by Stein et al. [18] following the investigation of Aufrecht [28] in order to check the melting temperatures of this system previously done

by Rudy [30]. Stein et al. [18] confirmed the non-presence of the C14 Laves phase in the Cr-Nb binary system.

The reassessment of this system has been done by Schmetterer et al. [29] taking into account only C15 as stable polytype. For the purpose of this work they used DFT calculations for the enthalpies of formation of the end-members done by myself (details of the calculation can be found in [29] and p. 68).

According to Aufrecht et al. [28], only the low temperature C15 Laves phase and given in *Table 3* is stable.

Phase	Pearson Symbol	Space Group	Prototype	Lattice parameters [Å]	Reference
Cr ₂ Nb	cF24	Fd-3m (227)	Cu ₂ Mg	a=6.96	[23]

Table 3 – Crystallographic information present the Cr-Nb binary system.

3.2 Fe-Nb

The paper of Paul et al. [31] reviewed the system with data which were available before 1983. Using different experimental data from literature, Srikanth et al. [32] optimized the phase diagram and found a rather good agreement with experimental values. Thermodynamic assessment of the iron-niobium system was given using the Calphad method in 2000 by Toffolon and Servan [33]. Due to some disagreement between different assessments [33] and experimental investigations [34] of this phase diagram, Voß et al. [35] completely reinvestigated this system experimentally using EPMA, EDS, SEM and DTA. They determined the Curie temperature at T=1042K for a composition of Nb at 13.5 at. %. The system forms Laves phase (C14) by a congruent reaction at T=1919 K and the μ phase is formed in a peritectic reaction at T=1796 K. The homogeneity range of the Laves phase at 1373 K had been previously studied by Grüner [34].

Enthalpy of formations are given by Barbi and Drobyshev [36] using EMF measurements, whereas Meschel et al. [37] used calorimetric measurements. The two values extrapolated from EMF are in the

same order -20.5 and $-23.7 \text{ kJ.mol}^{-1}.\text{atom}^{-1}$. The calorimetric measurement given by Meschel et al. [37] gives a value of $-5.3 \text{ kJ.mol}^{-1}.\text{atom}^{-1}$.

This phase diagram has been recently intensively reassessed [38–40] by Liu et al. [38], using new knowledge on the experimental data [35] and their calculated ab initio values for the enthalpies of formation. The results of the assessment gave a good agreement with enthalpies of formation from Meschel et al. [37] and did not fit to their ab initio data. Later Khvan et al. [39] have done a new assessment to improve the description of the fcc-austenite phase and to use in higher order system because the previous optimization had failed to be used for ternary systems. The results of this assessment showed agreement at this time with the ab initio data but not with the experimental data of Meschel et al. [37]. Furthermore they have not taken into account the EMF values from Barbi and Drobyshv [36] since they were extrapolated from high temperature measurements. This assessment was then extrapolated to the ternary Fe-Nb-Mn in the same work. The same authors have found out the need of reassessing [40] the same system once more since the description of the Nb-rich part in austenite was not correct. For this work, they decreased the activity of Nb in the austenite at lower temperatures [40]. Their assessment is given in *Figure 4* and compared with experimental data.

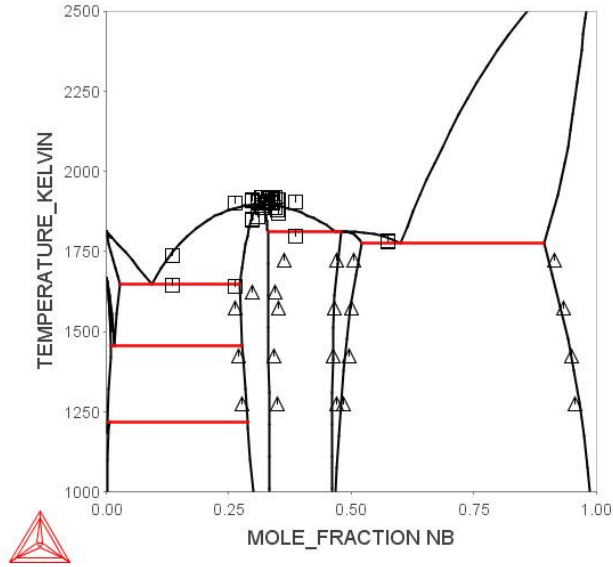


Figure 4 – Fe-Nb phase diagram calculated by Khvan *et al.* [40] using the Calphad method compared with experimental data from EPMA (triangles) and DTA (squares) [35].

In addition to the two pure elements forming BCC structure, there are the C14 Laves (Fe_2Nb) and the μ (Fe_7Nb_6) phases. The C14 Laves phase crystallizes in hexagonal structure as mentioned previously (Laves phases) and the μ phase has a rhombohedral structure. The lattice parameters as well as the Wyckoff position are developed in Table 4.

Phase	Pearson Symbol	Space Group	Prototype	Lattice parameters [Å]	References
Fe_2Nb	hP12	P63/mmc (no. 194)	MgZn_2	$a=4.8414$ $c=7.8933$	[20]
Fe_7Nb_6	hR39	R-3m (no. 166)	W_6Fe_7	$a=4.926$ $c=26.80$	[41]

Table 4 – Crystallographic informations for the intermetallic phases present in Fe-Nb system.

3.3 Fe-W

The Fe-W system is of great interest as refractory alloy. Nevertheless due to the hardness and the high melting point of W, it is rather difficult to handle it experimentally. The literature shows some disagreement, indeed the Massalski's compilation of binary phase diagrams [24] and the Landolt-Börnstein [42,43] show two completely different phase diagrams.

In the original papers, there are also different interpretations of this phase diagram. In the version of Sinha et al. [44], they presented the existence of the Laves phase and of the μ phase. This phase diagram is in agreement with the one from Kirchner et al. [45]. In the paper of Hofman and Henig [46] mentioned in the Landolt-Börnstein [43], they showed the presence of δ -FeW and present the Laves phase as metastable.

Due to the controversy of the existence of certain phases, this system has been recently partially reinvestigated by Antoni et al. [47]. They used two key samples to study the system. In their investigation, they used diffusion couples to study the microstructure of Fe-W system where there were discrepancies between the previous studies. They presented a phase diagram (shown in *Figure 5*) similar to the work of Sinha et al. [44] and Kirchner et al. [45] but did not find the presence of δ -FeW as mentioned by Henig et al. [46].

According to these studies, the Fe-W phase diagram forms a μ phase by a peritectic reaction at 1910 K [44,47] and the Laves phase forms by a peritectoid reaction at $T=1333$ K [44,45,47].

The most recent assessment of this system has been done by Gustafson [48] in 1987 and is shown in *Figure 6*. He based his thermodynamic assessment on the experimental phase diagram of Kostakis [49]. He pointed out that the results presented by Henig and Hofman [46] are dubious. This assessment is a reference for more complex systems and integrated into a commercial database (i.e. TCFe6). Nowadays the intermetallic phases are described by different models than the ones used by Gustafson [48].

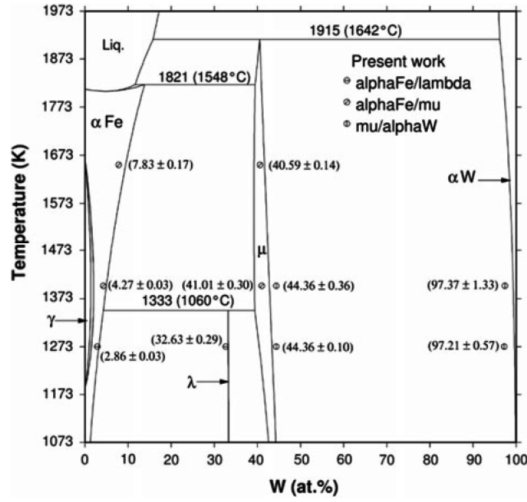


Figure 5 – Experimental phase diagram of Fe-W from [47].

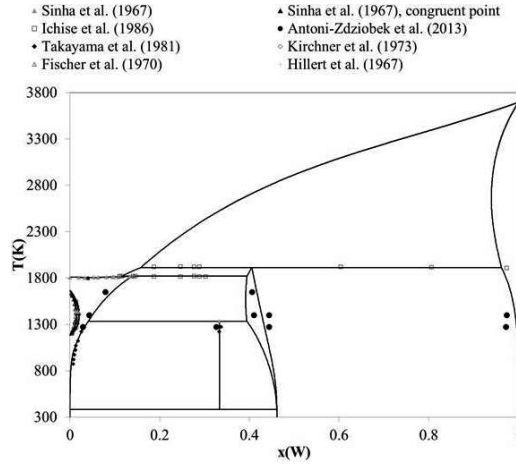


Figure 6 – Calculated phase diagram Fe-W [48] compared with experimental data.

As mentioned in the previous paragraph, there are two intermetallic phases in the Fe-W system, the Laves phase crystallising in C14 polytype and the μ with a rhombohedral settings (Table 5).

Phase	Pearson Symbol	Space Group	Prototype	Lattice parameters [Å]	References
Fe ₂ W	hP12	P63/mmc (no. 194)	MgZn ₂	a=4.731, c=7.70	[50]
Fe ₇ W ₆	hR39	R-3m (no. 166)	W ₆ Fe ₇	a=4.756 c= 25.728	[44]

Table 5 – Crystallographic data of the C14 laves phase in the Fe-W system as given by [44] and [50].

3.1 Cr-Nb-Si

Due to its technological importance the Cr-Nb-Si system has been extensively studied. The first experimental investigations have been carried out by Goldschmidt and Brandt [51]. They postulated in addition to the binary phases the presence of five ternary compounds. Some binary phases show a high extension into the ternary phase diagram. Zhao et al. [52] used diffusion multiple method for the determination of the phase equilibria. They found only three ternary phases. Both versions of phase diagram disagree between themselves on many points as pointed out by Shao [53] who wanted to make a thermodynamic optimization of this system.

In addition to the binary phases, David et al. [17] described this system with four ternary compounds and made a new thermodynamic optimization based on their literature evaluation and some new experimental investigations using diffusion multiples.

Following these studies there were a lot of other investigations on this system to clarify the different controversies from previous studies. Liquidus projection on the Cr-Nb side was proposed by Bewlay et al. [54]. The last assessment on this system was done by David et al. [17] as shown in *Figure 7*. This system will not be further investigated in the present work. Nevertheless the thermodynamic assessment of this system would need to be reconsidered, due to new experimental knowledge (i.e. [54]) and the recent controversy in the literature on the Cr-Nb binary system (see [18,28,29]).

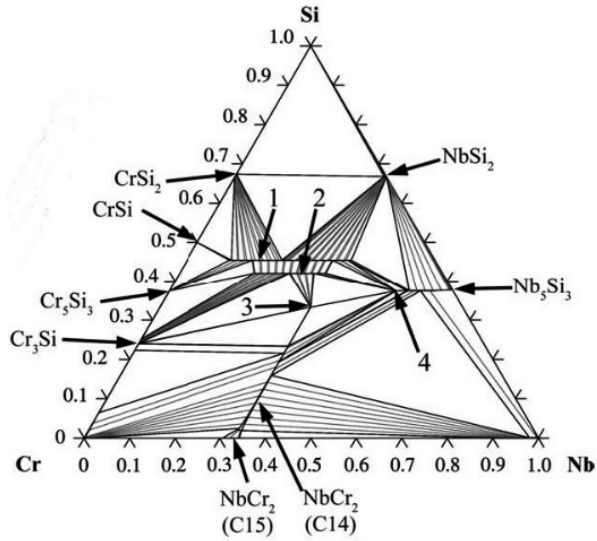


Figure 7 – Calculated isothermal section at 1473 K as given by David et al. [17] with the numbers $1=(Cr,Nb)_6Si_5$, $2=(Cr,Nb)_{11}Si_8$, $3=CrNbSi$, $4=CrNb_4Si_3$.

There are four different ternary compounds according to David et al. [17]. The crystallography of the ternary compounds is given in Table 6.

Phase	Pearson Symbol	Space Group	Prototype	Lattice parameters [Å]	References
CrNbSi	hP9	P-6 ₂ m	ZrNiAl	a=6.598 c=3.359	[55]
CrNb ₄ Si ₃	tI32	I4/mcm	W ₅ Si ₃	a=9.877 c=4.934	[56]
Cr ₄ Nb ₂ Si ₅	oI44	Ibam	Cr ₄ Nb ₂ Si ₅	a=15.81 b=7.497 c=4.879	[57]
Cr ₇ Nb ₄ Si ₈	oP76	Pnma	Cr ₇ Nb ₄ Si ₈	a=13.27 b=15.90 c=4.892	[58]

Table 6 – Crystallography of the ternary compounds of the Cr-Nb-Si system.

3.2 Fe-Nb-Si

Only few articles concern experimental data for this system. Most of the literature deals with crystallographic investigations of the numerous ternary compounds and solid solution phases found in this system. Goldschmidt [59] first proposed a version of the Fe-Nb-Si phase diagram (*Figure 8*). His investigation is incompatible with the actual knowledge of the Fe-Nb (i.e. postulation of σ phase and carbide phases which are not present in Fe-Nb binary phase diagram). Goldschmidt [59] has made the postulation of six ternary compounds and postulated the presence of three more. In some aspects the interpretation of the Fe-Nb-Si given by Goldschmidt [59] does not consider the associated binary phase diagrams and therefore cannot be considered completely accurate. Denham [20] investigated the extension of the Laves phase (see *Figure 9*) into the ternary system and found a smaller extend into it (also in the binary Fe-Nb). They did not find a σ phase. Using new knowledge of Fe-Nb phase diagram, Singh and Gupta (*Figure 10*) studied the extension of the Laves and the μ phases into the ternary phase diagram.

A critical compilation and assessment of this system including other information as well can be found in the Landolt-Börnstein database [60]. An isothermal section at 1423 K is reproduced in *Figure 11*.

As this system has primary importance for steel industry, new experimental investigations have been provided during the time of the present work. Wang et al. [15] studied three isothermal sections of the phase diagram and found six ternary compounds. The proposed isothermal section at 1373 K is given in *Figure 12*. Their investigation does not provide any data for the liquidus projection of the system which are nevertheless relevant informations for the thermodynamic optimization.

No thermodynamic assessment of this system is available in the literature. The calculation of an isothermal section using the commercial database TCFE6 is given in *Figure 13*. This representation of the phase diagram is incompatible with the actual knowledge of the phase diagram. The Laves phase is

shown as mutual solid solution crossing over the phase diagram and there are no ternary compounds shown in this calculation.

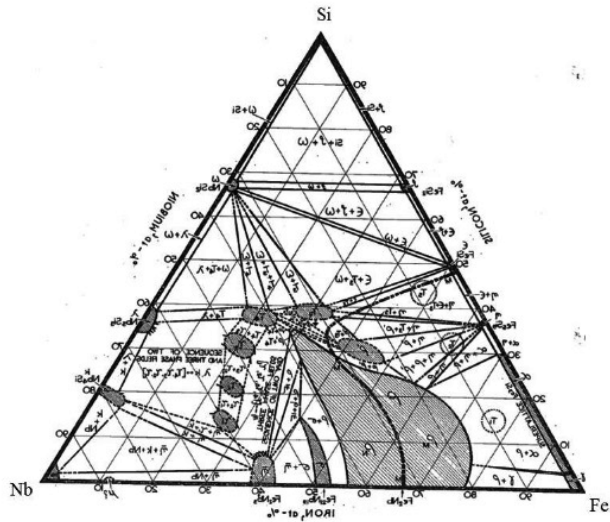


Figure 8 – Experimental phase diagram according to Goldschmidt [59], section at 1273K.

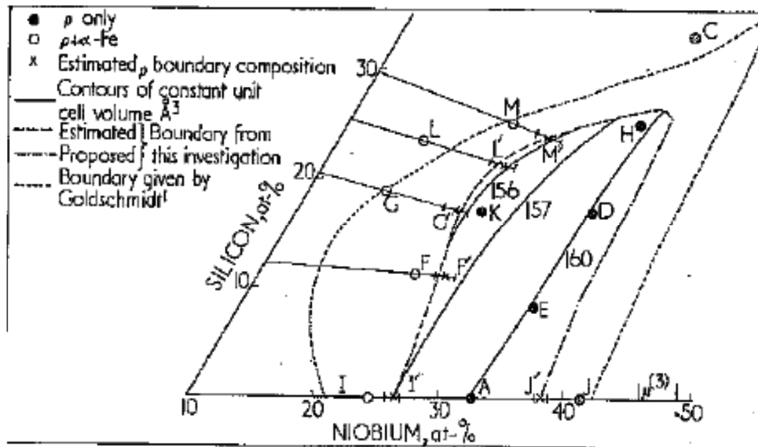


Figure 9 – Experimental phase diagram at 1573 K according to Denham [20].

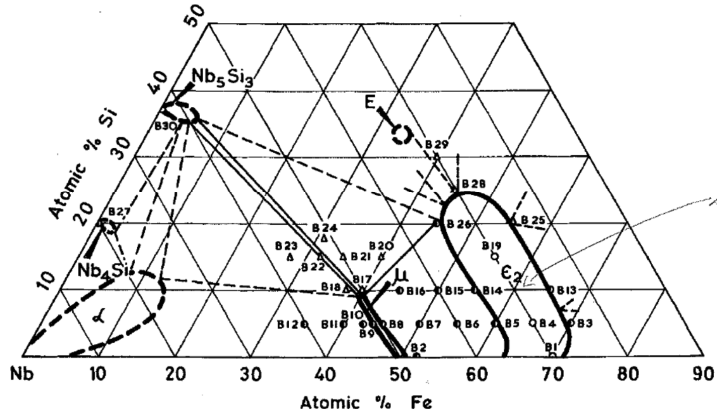


Figure 10 - Experimental phase diagram at 1373 K according to Singh [61].

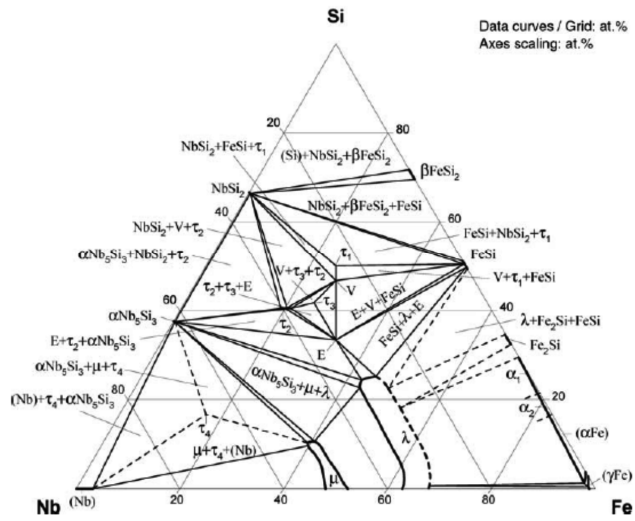


Figure 11 – Isothermal section of Fe-Nb-Si at 1423 K as proposed in Landolt-Börnstein [60].

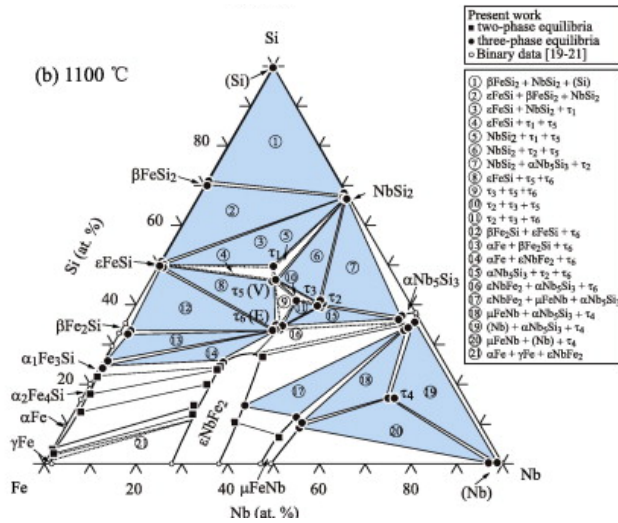


Figure 12 – Experimental phase diagram from Wang et al. [15] at 1373 K.

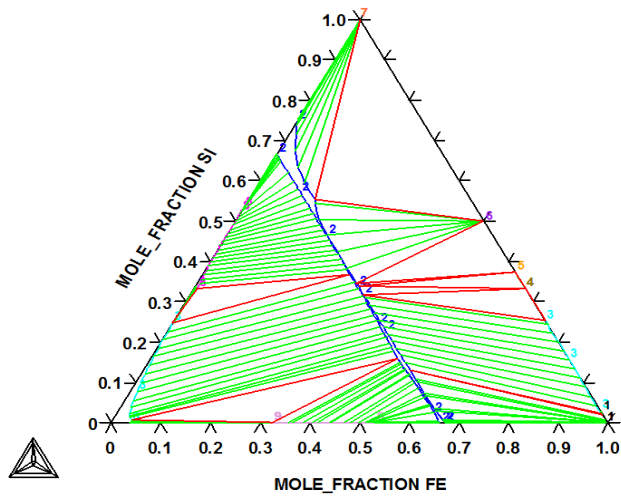


Figure 13 – Calculated phase diagram at 1323 K using the TCF6 database.

Even if not many studies have been performed on the thermodynamic stability of the Fe-Nb-Si, a number of papers report crystallographic data in this ternary system. Denham [20] studied the effect of Si in the Fe₂Nb Laves phase (*Figure 10*). He showed that Si does not affect the symmetry of the crystal structure, only the lattice parameters changed. Si dissolves also in the μ phase.

Steinmetz et al. [55,62] have determined a certain number of ternary phases in this system.

The six different ternary compounds are:

- FeNbSi
- FeNbSi₂
- Fe₃Nb₄Si₅
- FeNb₄Si
- Fe₄Nb₄Si₇
- FeNb₂Si₂

The crystal structures of the different compounds are reported below (*Table 7*) with their respective references.

Phase	Pearson Symbol	Space Group	Prototype	Lattice parameters [Å]	References
FeNbSi ₂	oP48	Pbam (55)	ZrCrSi ₂	a=7.576 b=9.733 c=7.576	[55]
Fe ₄ Nb ₄ Si ₇	tI60	I4/mmm (139)	Zr ₄ Co ₄ Ge ₇	a=12.652 c=4.981	Landolt Börnstein [60] based on [63]
FeNbSi	oP12	Pnma (62)	TiNiSi	a=6.231 b=3.677 c=7.190	[63]
FeNb ₂ Si ₂	tP198	P4 ₂ /mcm (132)	Co ₂ Si	a=23.76 c=4.959	[62]
Fe ₃ Nb ₄ Si ₅	hP12	P63/mmc (194)	MgZn ₂	a=4.868 c=7.758	[64]
FeNb ₄ Si	tP12	P4/mcc (124)	Nb ₄ CoSi	a=6.193 c=5.056	[65]

Table 7 – Crystal structures of the ternary compounds present in the Fe-Nb-Si phase equilibria.

3.3 Cr-Fe-Nb

The first study concerning the Cr-Fe-Nb phase diagram has been proposed by Kaloiev et al. [66] with an isothermal section at 1000°C (*Figure 14*). Later they published a second isothermal section at 700°C [67] (*Figure 15*). This phase diagram is characterized by huge solid solution of the two binary Laves phases formed by Fe_2Nb - Cr_2Nb .

Gujicic et al. [68] studied the influence of Fe in the C15- Cr_2Nb Laves phase. They observed a change of polytype structure at low Fe content (> 7 at. % of Fe). This change of polytype is due to a change in the electron concentration. Nevertheless, in the literature there are no informations for the liquidus projection as well for the reaction scheme diagram. Further experimental investigations are necessary for a good thermodynamic optimization of this system.

There is no ternary compound mentioned in the literature up to now.

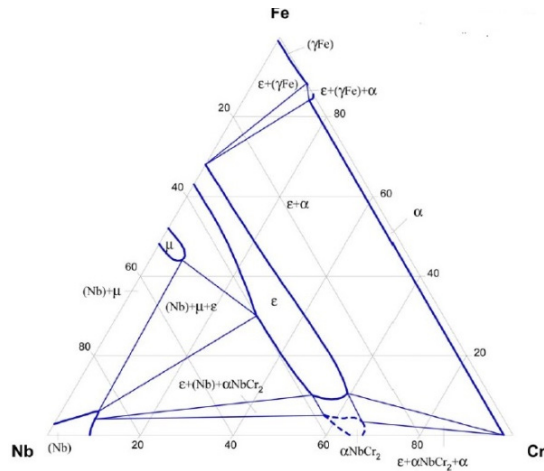


Figure 14 - Isothermal section at 1273K given by Kaloiev [66] according to the Landolt-Börnstein database [69].

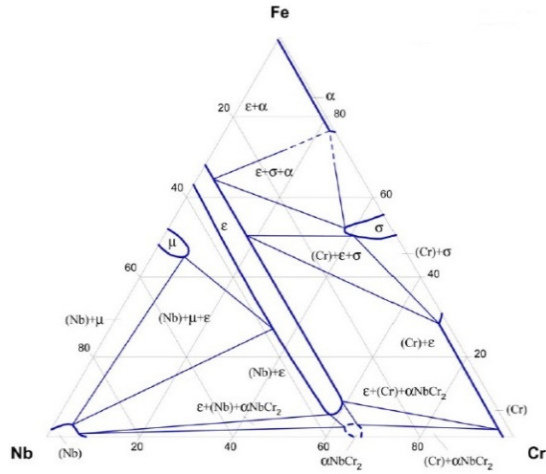


Figure 15 - Isothermal section at 973K given by Kaloiev [67] according to the Landolt-Börnstein database [69].

3.4 Fe-W-Si

The Fe-W-Si phase diagram is of great interest for industrial processes due to the hardness and refractory properties of W. Adding tungsten in ferritic steel improves the material properties like hardness, hot strength and yield strength. The Landolt-Börnstein [70] (*Figure 16*) gives a brief overview of the work that has been done. An older review from Raynor et al. [71] gives a larger overview of the system and shows the complexity of this ternary system. Most of their review is based on the work of Vogel et al. [72]. Using all available information they have made some tentative of an isotherm and a complex reaction scheme based on their actual knowledge. They mentioned the presence of two ternary compounds (FeW_2Si and FeWSi) which form from the reactions between Fe_7W_6 and FeSi .

The calculated phase diagram from the commercial database TCFE6 given in *Figure 17* shows disagreement with the phase diagram proposed in the literature. It has to be mentioned that no assessment of this system exists in the literature and this calculation is an extrapolation from binary systems.

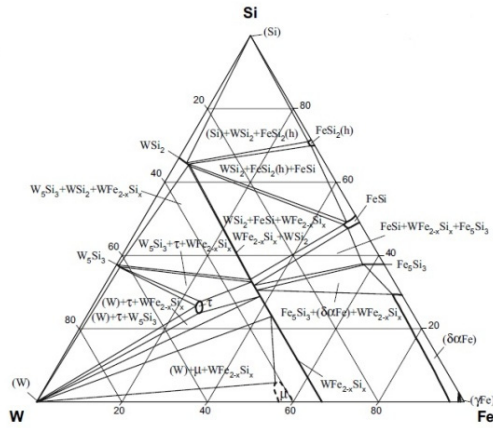


Figure 16 - Experimental isothermal section at $T=1273\text{K}$ of Fe-W-Si according to Landolt-Bornstein [70].

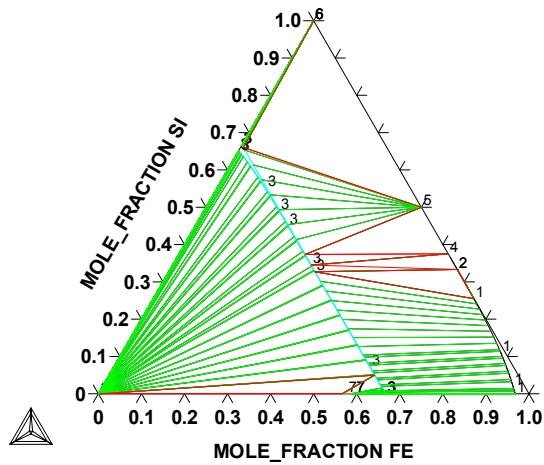


Figure 17 – Calculated phase diagram of Fe-W-Si using the commercial TCF6 database.

The crystallographic structure of FeWSi has been investigated by Gladysvaskii [50]. FeWSi has a C14 Laves phase type structure. From the binary Fe_2W to FeWSi there is a complete solid solution.

Nevertheless according to the Landolt-Börnstein [70], the ternary compound FeWSi is distinct to the Fe₂W dissolving Si. These two compounds have different lattice parameters.

Based on the knowledge from the Landolt-Börnstein [70] and Raynor [71], there are two ternary compounds in the Fe-Si-W system, the ternary compound FeW₂Si crystallizes in orthorhombic structure with 26 atoms per formula unit and FeWSi crystallizes in a C14 Laves phase polytype. The crystal structures based on the literature are reported in *Table 8*.

Phase	Pearson Symbol	Space Group	Prototype	Lattice parameters [Å]	References
FeWSi	hP12	P6 ₃ /mmc (194)	MgZn ₂	a= 4.738 c= 7.66	Landolt Börnstein [70] according to [65]
FeW₂Si	oP26	Pbam (55)	W ₂ FeSi	a=7.817 b=9.283 c=4.755	Landolt Börnstein[70]

Table 8 – Crystallographic data of the compound in the ternary system Fe-W-Si.

3.5 Summary of the literature survey

The assessment of the different systems forming Laves phase in Crofer 22H are given in *Table 9* summarizing the literature survey. In this table, we reported also the associate binary systems of the ternary systems containing Laves phase.

System	Presence of the Laves phases	Last thermodynamic assessment via Calphad method
Fe-Nb	x (C14) Fe ₂ Nb	[40]
Fe-W	x (C14) Fe ₂ W	[48]
Cr-Nb	x (C15, C14*)	[26]
Fe-Cr-Nb	x (C15, C14) (Fe,Cr) ₂ Nb	No assessment
Fe-Nb-Si	x (C14) (Fe,Si) ₂ Nb	No assessment
Fe-Si-W	x (C14) (Fe, Si) ₂ W	No assessment
Cr-Nb-Si	x (C15,C14)	[17]
Fe-Cr	None	[73]
Fe-Si	None	[74]
Nb-Si	None	[75]
Si-W	None	[76]

*Table 9 – Summary of the literature survey of the thermodynamic assessment on the ternary and binary systems containing Laves phase. *The assessment made by Pavlů et al. [26] on the Cr-Nb system take into account the presence of C14 Laves phase, nevertheless it has been experimentally shown by Aufrecht et al. [28] the non-stability of this phase. Reassessment of this system was done during the present study [29].*

The thermodynamic modeling of laves phase containing systems in Crofer 22H which has been reviewed in this literature survey showed that there is a lack of information in many binary and ternary systems. As any thermodynamic modeling of a ternary system can be well achieved with suitable associate binary systems, we first concentrated on the thermodynamic modeling of binary systems which have not been well investigated. Therefore the binary systems were investigated:

- Cr-Nb
- Fe-W

The experimental data of these two systems are abundant in the literature so computer simulations tools were used for the thermodynamic modeling of these two systems. More information about the different methods to calculate these two systems is given later.

The ternary systems Cr-Fe-Nb and Fe-Nb-Si have shown to be very important for the development of steel but were not well investigated in the literature.

For the Cr-Fe-Nb system, it was considered necessary to evaluate the phase diagram experimentally because the amount of experimental data in the literature was restricted. In addition, DFT calculations have been done to calculate the energies of formation of the different compounds according to the models used for the thermodynamic modeling. The combination of experimental data and DFT calculations were then used as input parameters for the thermodynamic optimization of the phase diagram.

The Fe-Nb-Si system was investigated by DFT calculations for the determination of the energy of the compounds (binaries and ternaries). These results have been used as input for the thermodynamic modeling as well as with the experimental data available in the literature. These data are nevertheless probably not sufficient for a suitable thermodynamic modeling due to the complexity of the systems. Furthermore there are no data available of the liquidus projection in the literature. A tentative thermodynamic modeling will be given in the present work.

The present thesis is organized as follows. First the different methods which have been used to carry out the thermodynamic modeling will be explained, and then the results will be given for the different studied systems.

Chapter II – Methods

I. Introduction

In order to develop thermodynamic optimization of phase diagram for Laves phase containing systems in Crofer 22H [1] several methods have been used. Indeed, the optimization of phase diagrams carried out in the framework of the Calphad method relies (see part II) not only on mathematical models but also on several inputs. The primary inputs are the knowledge of the experimental phase diagram and thermodynamic properties of the considered system. It is not always possible to obtain all necessary data from experiments (i.e. enthalpy of formation of the metastable end-members), in this case the recourse to other methods to access these data is necessary. Density Functional Theory (DFT) has been widely used to access the energy of the compounds, especially the metastable ones. DFT results are restricted to 0 K but provide already an accurate approximation to compare the results at 298 K given experimentally. With the extension of the computational power it is nowadays also possible to perform phonon calculations (vibrational properties) to access the thermodynamic functions of compounds over a wide temperature range.

It is a combination of these different methods in different systems that were used in the frame of this investigation. The methods used are:

- Experimental phase diagram determination
- Calphad methods for the optimization of the phase diagram
- DFT calculations
- Phonon calculations
- Recalculation of phase diagram

In this chapter, the different methods used will be explained.

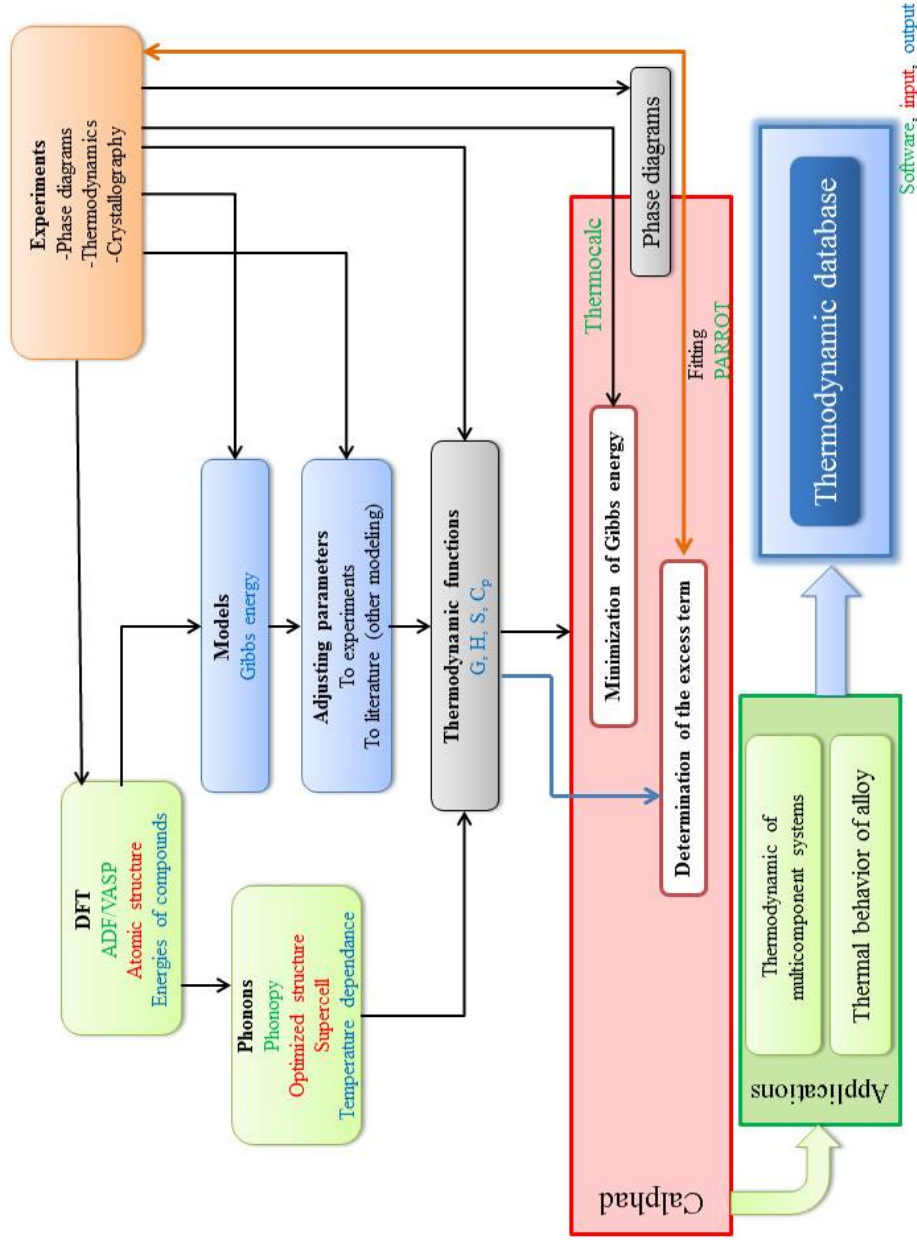


Figure 18 – Flowchart of the different methods used for the optimization of phase diagram.

II. Experimental phase diagram investigations

A phase diagram is a graphical representation of the phase transitions/transformations during heating and cooling. It is of primary importance for the understanding of phase formation. Phase diagrams are very useful for the development of alloys but also in metallurgy. In addition, experimental phase diagrams are the first information necessary for the Calculation of Phase Diagrams (Calphad) method and the optimization of the considered system as it is not possible to predict phases by this method without defining them first.

For the determination of phase diagrams, several methods can be used and several analytical tools are necessary. First, the samples should be prepared from very high purity metals and in high purity atmosphere. In the present work, we used metals with a purity of 99.99 %. The samples were then melted together in a levitation induction furnace under high purity Ar atmosphere. Different techniques are then used to determine specific properties of the phase diagram. Samples which are heat treated at different temperatures (longer when the temperature is low in order to establish thermodynamic equilibrium in samples) and then quenched, allow the construction of isothermal sections. Vertical sections and liquidus projections are constructed from data obtained by a dynamic method (differential thermal analysis – DTA). The samples are then analyzed by analytical methods like metallography or X-ray diffraction (XRD). Metallography allows the analysis of the microstructure (number of phases and invariant reactions) with techniques like scanning electron microscope (SEM). A coupling of the SEM with energy dispersive X-ray spectroscopy (EDX) allows the determination of the chemical composition of the phases. The crystallographic structures of the phases are analyzed by XRD allowing to know the symmetry of the structure and the lattice parameters.

The construction of a phase diagram can also be done by diffusion couple methods. This method can reduce the amount of samples required for phase diagram construction. This method is based on local equilibria at the phase interfaces [77].

III. Calculation of Phase diagrams (Calphad)¹

1. Introduction

Calphad is the common acronym for **CAL**culation of **PHA**se **D**iagrams, or rather, computer coupling of phase diagrams and thermochemistry. This is a method for the calculation of thermodynamic equilibrium of multicomponent systems using a minimization of the Gibbs energy of the system. The Gibbs energy of each possible phase in the system is defined in mathematical form by a Gibbs energy model. The derivation of the Gibbs energy models relies on experimental and ab initio input. Any other thermodynamic property can be derived over temperature, composition and/or pressure from the Gibbs energy.

In the present work, the goal of the thermodynamic modeling is to calculate phase equilibria for different specific systems forming Laves phases (binary or ternary systems) in order to build a suitable thermodynamic database for steel (i.e. Crofer 22H, applicable also for other materials with similar composition and phases) in order to study its stability under working conditions. Additionally such thermodynamic databases can be used to develop new materials (i.e. manufacturing of the material and control of the formation of wanted/unwanted phases) and for other simulation tools (i.e. phase field modeling) for the description of the microstructure of the materials. Thermodynamic modeling has been carried out where no previous modeling has been done (a new modeling of an existing known system is necessary if inconsistent with the present work).

The thermodynamic modeling is called an assessment. A good assessment starts with a careful literature search and a critical review of the considered system (i.e. experimental phase diagram, thermodynamic data, previous thermodynamic modeling). Within different works it can happen that the same kind of data are in disagreement, in this case, a choice (assessment) should be done on which data have to be kept and which should not be considered. The thermodynamic assessment allows the calculation of the phase diagram as well as other thermodynamic properties through an optimization for the determination of the Gibbs energy of all the phases in the system.

¹ Chapter based on the book: Computational thermodynamics, the calphad method [78]

2. Models for the Gibbs energy

The Calphad method [78] is based on statistical thermodynamics. To model a system, it is necessary to choose a mathematical form to describe the thermodynamic function (i.e. Gibbs energy). This mathematical form should rely on physical models in order to provide reasonable extrapolations [79]. The Gibbs energy, rather than other thermodynamic functions, is used to model the phase diagram as it includes pressure and temperature as natural variables.

The Calphad method is based on the minimization of the Gibbs energy allowing the calculation of thermodynamic equilibrium. The general expression of the Gibbs energy is given below:

$$G = \sum_{\alpha} N^{\alpha} G_m^{\alpha}(T, P, x_i) \quad \text{Eq. 1}$$

This expression can be expanded and then is given as:

$$G_m^{\alpha} = {}^{srf}G_m^{\alpha} + {}^{cfg}G_m^{\alpha} + {}^{mgn}G_m^{\alpha} + {}^EG_m^{\alpha} \quad \text{Eq. 2}$$

The three first terms of this equation (Eq. 2) refer to an ideal Gibbs energy of mixing. The first term ${}^{srf}G_m^{\alpha}$ refers to the surface of reference, it is the Gibbs energy of formation of the constituents referring to their stable reference state. ${}^{srf}G^{\alpha}$ is modeled for each phase as given in Eq. 3:

$${}^{srf}G^{\alpha} = \sum_i x_i {}^0G_i^{\alpha}(T) \quad \text{Eq. 3}$$

The second term ${}^{cfg}G_m^{\alpha}$ is linked to the configurational entropy and can be written as:

$${}^{cfg}G_m^{\alpha} = RT \sum_{i=1}^n x_i \ln(x_i) \quad \text{Eq. 4}$$

The term ${}^{mgn}G_m^{\alpha}$ refers to the magnetic contribution of the Gibbs energy.

In an ideal solid solution, only the mixing energy of the two (or more) species present in the solution needs to be taken into account. The Gibbs energy of mixing is therefore equal to the total Gibbs energy and can be written as:

$$G_m = \sum_{i=1}^n x_i {}^0G_i + RT \sum_{i=1}^n x_i \ln(x_i) \quad \text{Eq. 5}$$

For a real solution, the excess terms need to be taken into account to describe the thermodynamics of the system. Atoms are distributed randomly over the solution and have different energies of interaction with each other creating an excess of energy [80]. According to this, the total Gibbs energy of a substitutional regular solution model is:

$$G_m = \sum_{i=1}^n x_i {}^0G_i + RT \sum_{i=1}^n x_i \ln(x_i) + {}^E G_m \quad \text{Eq. 6}$$

And the ${}^E G_m^\alpha$ (Eq. 2, Eq. 6) is the excess Gibbs energy. This parameter is specifically developed in the paragraph dedicated to it (see 4.b Modeling of the Gibbs energy).

Any other thermodynamic quantities can be derived in the same way as the Gibbs energy, since they are temperature derives of Eq. 1:

Entropy
$$S = - \left(\frac{dG}{dT} \right) \quad \text{Eq. 7}$$

Heat capacity
$$C_p = -T \left(\frac{d^2G}{dT^2} \right) \quad \text{Eq. 8}$$

2.1 Unary phase

In order to be consistent with previous work, it is convenient to work on a common basis for the description of the pure elements (also called unary phase in the Calphad community). The unary database has been built for this purpose by Dinsdale [81] and used as a common basis by SGTE (Scientific Group Thermodata Europe) for the community in order to build consistent data. The molar Gibbs energy is expressed as:

$$G_m^\alpha = a + bT + cT \ln T + \sum d_n T^n \quad \text{Eq. 9}$$

This equation allows to have the stable reference (SER) for the pure element. It is used to describe the pure elements in the phase equilibria as well as to be substrate to Gibbs energy of a phase as energy of reference. This equation is also valid for the description of the end-members in the compound energy formalism (see below).

Currently, research projects [82] are carried out in order to improve the unary database. Indeed, the unary phases are the basis of a thermodynamic modeling (i.e. as reference states for the other intermetallic phases present in the studied system) and the present used database is from 1990. Since then, there are more experimental data available in the literature and new computational techniques have been developed (i.e. DFT, molecular dynamics, and phonon). The goal is to improve the fitting and interpolation of the unary database with experimental or computational results but also to provide new mathematical models (i.e. the polynomial temperature dependence, see *Eq. 9*). Most of the unary phases are metastable, but with the development of computational tools, it is nowadays possible to access to them. Using phonon calculations (lattice vibration), new unary data can be obtained for the description of the pure elements in different crystallographic forms. Calculation of metastable/unstable states is encouraged for the description of the pure elements but the question may be asked if there is any physical reliability of such results. Indeed while some of the pure elements in a certain structure can be computed without showing metastability (i.e. pure Cr in σ phase [83]), some show mechanical instabilities (i.e. Re in σ phase [83]). Then the question is raised to know if the thermodynamic resulting from it can be used in Calphad modeling as they are unstable.

2.2 The compound energy formalism= sublattice modeling

The compound energy formalism has been developed in order to model thermodynamic properties of solution phases with homogeneity range and different sublattices [84]. It is a purely mathematical model which allow to give an analytical expression of the Gibbs energy for a phase with two or more sublattices [84]. According to the compound energy formalism, the total Gibbs energy can be written as in *Eq. 2*.

In the Calphad method the thermodynamics of the system is based on the crystal structure for the determination of a suitable model to describe the phase. Even if the thermodynamic stability of a phase has been never measured via the crystalline structure, it is nevertheless directly connected to it, since the interaction between atoms for a certain composition, temperature and pressure depends on a specific arrangement (i.e. crystal structure in solid) of atoms. The so-called sublattice model is directly related to the crystal structure for the compound energy formalism (CEF). Each compound or end-members has its own Gibbs energy according to the CEF. Each equivalent position in a crystal structure (known as Wyckoff position in crystallography) can be described as a sublattice. The different constituents (e.g. A, B) can in principle occupy the different sublattices. By replacing the sublattices with the A, B constituents we obtain $(A,B)_p(A,B)_q$ and the Gibbs energy for all the different configurations. If we consider A_pB_q as the stable phase, some other combinations are metastable. It is possible to get experimental values of thermodynamic data of A_pB_q (e.g. calorimetry, KEMS) for the stable end-members. The hypothetical and metastable end-members are not accessible by experiment. Nevertheless with the development of computational power, it is possible to calculate the energy of formation of the metastable phases by DFT and to obtain accurate values of them.

The sublattice models can become rather complicated for some phases with many different Wyckoff positions and all possible combinations of the different positions. As an example if we take the σ phase (Space group: $P4_2/mnm$, tP30) that contains five different Wyckoff positions and so by definition five sublattice models, $2^5=32$ configurations should be calculated for a binary compound. The same phase in ternary system makes $3^5=243$ configurations, which become rather time consuming to calculate even with DFT. To limit the number of sublattices for higher order systems, there is the possibility to combine different Wyckoff positions depending on their site occupancy, coordination environment and stoichiometry. In order to choose a suitable model for a considered phase, a careful literature survey should be done to look which sublattice model is the most appropriate one. For the σ and μ phases, relevant in the present work, reviews from Joubert and co-workers [85,86] have shown to be valuable sources for choice of a suitable model. In the present work, the studied systems contain Laves, μ and σ phases. The sublattice models used for the present work are described below:

Laves phase:

There are three different polytypes of Laves phases (i.e. named C15, C14 and C36 depending on their crystallographic structure) (see Literature survey). The three different polytypes are closely related but crystallize in different space groups as shown in the introduction (see page 7). The Laves phase can be written AB_2 . In the present work, the addition of a third element occupies preferentially the B site (e.g. Fe-Nb-Si [20]). Commonly a two-sublattice model is used, allowing each atom to go into each sublattice. This model does not change with the type of polytype. It can be written $(A,B,C)(A,B,C)_2$ for a ternary system.

The use of a two-sublattice model is well accepted to describe the three different polytypes, nevertheless it is ongoing discussion to use more complex sublattice models (e.g. three-sublattice model for the C14) as each of the different polytypes has a different set of Wyckoff positions. It has not been proved yet if more complex sublattice models would improve the thermodynamic modeling of the different Laves phases.

μ -phase:

The μ phase crystallizes in space group $R\bar{3}m$. The atoms occupy in five different Wyckoff positions. The description of the crystal structure is given in *Table 10*. This would be rather expensive to calculate the energies of all different end-members using high performance computing. In order to reduce the number of end-members, some sublattices can be combined. Joubert and Dupin [85] therefore suggested a four-sublattice model. The ideal μ phase can be written A_7B_6 . A third element can occupy several positions substituting A or B. According to the site occupancy in different binary and ternary systems, Joubert and Dupin [85] showed that two of the 6c Wyckoff positions are not occupied by A and C (C is defined as the third element). The sublattice model used to describe the homogeneity range is therefore written:



In the present work, we will use this model to describe the μ phase in the different studied systems.

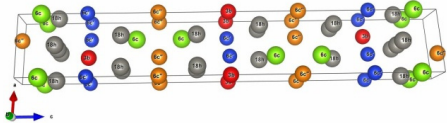
Space group R-3m (no. 166) Pearson symbol hR39 Lattice parameters $a=4.926 \text{ \AA}$, $c=26.80 \text{ \AA}$ Model W_6Fe_7					
Wyckoff position	x	y	z	CN	
3b (red)	0	0	$\frac{1}{2}$	12	
6c (green)	0	0	0.055	14	
6c' (blue)	0	0	0.165	16	
6c'' (orange)	0	0	0.333	15	
18h (grey)	0.5	0.5	0.09	12	

Table 10 – Crystallographic structure information of the μ phase according to [85].

σ phase:

The σ phase crystallizes in the space group $P4_2/mnm$ and the atoms are distributed on five different Wyckoff positions (Table 11). It can in general be written as $A_2^{12}B_4^{15}C_8^{14}D_8^{12}E_8^{14}$ where the lower index indicates the number of atoms per position and the upper index indicates the coordination number [87]. With the point of view of crystallography a three-sublattice model $(A,B)_{10}(B)_4(A,B)_{16}$ is the most convenient [13,86]. The first sublattice is defined by $A_2^{12}D_8^{12}$, the second by B_4^{15} and the third by $C_8^{14}E_8^{14}$. By extension this model can be written $(A,B)_{10}(B,C)_4(A,B,C)_{16}$. This model was recently used for the ternary system Fe-Nb-V [88] and will be used for the description of the σ phase in the system Cr-Fe-Nb (see Chapter Thermodynamic optimization of Cr-Fe-Nb).

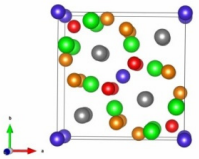
Space group $P4_2/mnm$ (no. 136) Pearson symbol tP30 Lattice parameters $a=8.785$ to $10.060 \text{ \AA}$ [86] $c=4.553$ to $5.23 \text{ \AA}$ [86] Model: FeCr					
Wyckoff position	x	y	z	CN	
2a (blue)	0	0	0	12	
4f (red)	0.399	x	0	15	
8i (orange)	0.464	0.131	0	14	
8i' (green)	0.741	0.066	0	12	
8j (grey)	0.183	0.183	0.251	14	

Table 11 – Crystallographic description of the σ phase.

3. Input data

Any thermodynamic modeling relies on the quality of the input data. The modeling of multicomponent systems requires the knowledge of the different stable phases in the defined system. The Calphad method cannot predict the stability of a phase that has not been experimentally established and defined

for the computation of the phase diagram. While the main part of the semi-empirical Calphad method is to describe the Gibbs energy, it is often not possible to find the Gibbs energy of a system in the literature. Its description should be done using other information that have a complex relation to it (chemical potential, heat capacity, enthalpies...). To describe the Gibbs energy, different input data from experiments can be used:

- **Phase diagram data:** to model the phase equilibria it is important to have a picture of the experimental phase diagram. No thermodynamic modeling of a phase diagram can be done without the knowledge of the experimental phase equilibria. Stable phases and phase boundaries (tie-lines) are the primary informations to be taken into account for modeling. Within the phase diagram information, the invariant equilibria are of primary importance for the optimization of the Gibbs energy model parameters;
- **Thermodynamic data:** Any available thermodynamic information for a phase are of primary importance for modeling of the phase diagram as they directly provide the link to the Gibbs energy of the considered phases;
- **Crystallographic data:** in Calphad modeling, the phases are defined by their crystallography (e.g. BCC, FCC...). Furthermore, this kind of information is necessary to define the sublattice model that would be used to describe the phase in the compound energy formalism (see 2.b *The compound energy formalism*).
- **Ab initio data:** Ab initio data are not experimental data, nevertheless these kind of calculations are based on the electronic structure of the elements and have shown to be a versatile method to get the energies of the compound [89,90] at 0K and is useful information in absence or in complement to experiments. They are of primary importance for the determination of the energies of hypothetical compounds (e.g. end-members of the CEF models). Previously, Gibbs energies of hypothetical compounds were estimated or optimized to fit to experimental data. More details about this method are given in the Chapter Density Functional Theory (p. 46).

- **Phonon calculations:** Ab initio calculations are restricted to 0K and do not allow to give the temperature dependence of the free energy [91]. Phonon (lattice vibrations) are calculated by displacing atoms in a supercell and computing the forces by DFT. (see Chapter Phonon calculations, p. 53).

The combination of the different input parameters will allow the modeling of phase diagrams with more or less accuracy depending on the number of input data and their reliability (measurements errors, purity of the samples...).

For a particular system, many publications can be found in the literature, some of them can disagree. Therefore, a careful choice should be done since the modeling cannot be done by using two contradictory sets of information.

4. Optimization of parameters in the Gibbs energy models

4.1 Principle

In the previous part, the importance of the choice of data and of the models has been discussed. The modeling of phase equilibria requires the knowledge of the Gibbs energy, which is the representation of the interaction between the species. The Gibbs energy is the property that is modeled during an optimization process as it is not often possible to find its values for the considered phase in the literature. All information gathered for the phase diagram of interest (experimental phase diagram data, thermodynamic data, DFT results, etc...) is used as set of input parameters for the modeling. The modeling of the phase diagram is completed, when it has passed a complete optimization process. A successful optimization process allows a correct representation of the phase diagram (i.e. compared to the experimental knowledge of it), nevertheless, even if the representation of the phase diagram seems to be correct it is not always the case for the thermodynamic properties (i.e. enthalpy of mixing, activities). A wrong representation of a low order system can lead to troubles for the description of higher order systems.

4.2 Modeling of the Gibbs energy

The modeling of the Gibbs energy is the key point of Calphad modeling. Its determination is done via an optimization process using for instance the Parrot module in the case of Thermo-Calc [8]. The optimization starts from a set of chosen experimental values and of mathematical models, then the optimization will be done by fitting the calculated Gibbs energy (according to the chosen mathematical model) to the experimental data (according to the critical review). The parrot module uses a least-square method by minimizing the weighted sum of differences between calculated and experimental values. The fitting done provides the excess Gibbs energies of the phases. In Calphad methodology, the excess Gibbs energy is expressed as a summation over the different interactions between species:

$${}^E G_m = \sum_i \sum_{j>i} x_i x_j L_{ij} \quad \text{Eq. 10}$$

Where L_{ij} represents the interaction between i and j.

The determination of these parameters allows a mathematical description of the Gibbs energy to be optimized. A global minimization of the Gibbs energy allows to plot the phase diagram. Even if a representation of the calculated and experimental phase diagram shows a good agreement it is not necessarily a good description of the system. Indeed a modeling of a binary system can result in a good representation of the system, but when this modeling is used for a more complex system, it can show deficiency and the optimization of the binary needs to be reevaluated.

In a binary system, the Redlich-Kister Gibbs energy binary excess model is described as:

$$\Delta G^E = x_i x_j \sum_{i=0}^n L_i (x_i - x_j)^i \quad \text{Eq. 11}$$

During Calphad optimization procedure, the excess energy is often described with a linear temperature dependence $L_i = a_i + b_i T$ (Eq. 12). Using this linear temperature dependence can result in unreasonable boundary conditions and cause an artificial miscibility gap [92,93]. Kaptay [92] proposed to avoid this problem by using an exponential dependence:

$$L_i = h_{0i} \exp \left(-T \times \frac{s_{0i}}{h_{0i}} \right) \quad \text{Eq. 13}$$

Where h_{0i} is the enthalpy part of the interaction parameter and s_{0i} is the entropic interaction energy at $T=0K$.

As for the linear combination (Eq. 12), the exponential dependence of the L parameter can be obtained by a simple fitting through an usual assessment. Nevertheless as previously explained most of the studies forget the physical importance of this parameter and a simple “fitting” can reveal non-physical behavior. It has been shown by Schmid-Fetzer et al. [94] that the Kaptay exponential equation [92] can also cause a miscibility gap artifact at low temperatures as shown in their paper even if this last can well reproduce the boundary conditions at high temperature. Kaptay [95] showed that in order to avoid high temperature miscibility gap by using the linear equation (Eq. 13) the excess entropy is set up to $s \geq -2R$ but can then be in contradiction to experimental data. Due to the low-temperature miscibility gap of the exponential dependence, the community preferred to continue to use the linear dependence. In his latest study, Kaptay [95] explained that the exponential dependence can lead to a low temperature artifact but in much less proportion than the linear combinations lead to a high temperature miscibility gap. In order to reproduce the experimental value, Kaptay [95] proposed to combine the linear and the exponential equations to describe the excess Gibbs energy.

$$L_i = (h_{0i} - Ts_{0i}) \exp \left(-T \times \frac{s_{0i}}{h_{0i}} \right) \quad \text{Eq. 14}$$

So in both cases, the L parameter (e.g. linear and exponential) can lead to artificial miscibility gap at high temperature for the linear dependence and at low temperature for the exponential one. It is probably too early to judge the last proposition of Kaptay [95] (combination of linear and exponential equation) to model the excess Gibbs energy.

In conclusion, none of these models for the Gibbs energy can lead to realistic behavior of the phase diagrams if there are only few available experimental data and they are constrained to realistic behavior. *The optimization of the parameters in the Gibbs energy models should be done with great*

care of the experimental data respecting the phase boundaries as well as the thermodynamic data of the system and the laws of thermodynamics.

4.3 Optimization process in the present work

As explained previously, any optimization can be done with several inputs. The amount of available data will contribute to the determination of the Gibbs energy, with more or less accuracy. Nevertheless, it is difficult to judge the quality of a successful optimization of a phase diagram. The use of an optimized binary system in higher order system can reveal some deficiencies.

As any optimization can only be successfully reached with a sufficient amount of available data, in the present work, several methods to obtain the missing data of the literature have been combined.

At first, experiments have been performed to determine the phase diagrams where the available data were scarce and/or missing (e.g. see Chapter Experimental determination of Cr-Fe-Nb, p. 109).

It can be difficult to obtain experimental data in some systems due to the high melting point and slow diffusivity. In this case, computer simulation tools can be of great help for the determination of missing data. In the present work, DFT calculations were used for the determination of enthalpies of formation.

For the binary Fe-W system, DFT with phonon calculations for the determination of temperature dependence of the Gibbs energy of the end-members have been combined, which was tried for the first time in the optimization of a phase diagram.

IV. Density Functional Theory

1. Introduction

Materials properties are governed by electronic interaction between atoms, at this scale the nature of matter is governed by the law of quantum mechanics. For molecular electronic calculations, density functional theory is one of the most advanced methods and has already shown to be a versatile way of the calculation of materials properties in condensed matter and chemistry.

The main purpose of density functional theory is the solution of the time-independent Schrödinger equation for many electron interactions. For the density functional theory (DFT) calculations, the Vienna Ab initio Software Package (VASP) [96] and Amsterdam Density Functional (ADF) [97,98] have been used.

Based on quantum theory, the starting point of any calculations for density functional theory (DFT) is the many body Hamiltonian function in order to solve the Schrödinger equation.

$$\mathcal{H}\psi = E\psi \quad \text{Eq. 15}$$

Where ψ are many body wave functions for N electrons in the system. The Hamiltonian function $\mathcal{H}$ for many body-electrons is given by the Eq. 16:

$$\begin{aligned} \hat{\mathcal{H}} &= \hat{\mathcal{H}}_e + \hat{\mathcal{H}}_n + \frac{1}{2} \sum_{i=1}^N \sum_{l=1}^{N_n} \frac{Z_l}{|R_l - r_i|} \\ &= \sum_{i=1}^N -\frac{\nabla_i^2}{2} + \frac{1}{2} \sum_{i \neq j}^N \frac{1}{|r_i - r_j|} + \sum_{l=1}^{N_n} -\frac{\nabla_l^2}{2M_l} + \frac{1}{2} \sum_{l \neq j}^{N_n} \frac{Z_l Z_j}{|R_l - R_j|} \\ &\quad + \frac{1}{2} \sum_{i=1}^N \sum_{l=1}^{N_n} \frac{Z_l}{|R_l - r_i|} \end{aligned} \quad \text{Eq. 16}$$

Where $\hat{\mathcal{H}}_e$ is the electronic Hamiltonian and $\hat{\mathcal{H}}_n$ is the nuclear Hamiltonian.

The Hamiltonian function as originally defined is not solvable for more than three electrons. Therefore some approximations have to be taken into account for the solution of this equation (Eq. 16).

2. Born-Oppenheimer approximation

As it is a difficult task to solve the Schrödinger equation itself, some approximation needs to be done to calculate the electronic structure for many electrons. The positions of the nuclei and electrons should be defined. The Born-Oppenheimer approximation considers that due to the great difference of masses between the electrons and nuclei, the movement of the electrons is considered to be much faster as the nuclei's movement. Solution for a fixed position of the atomic nucleus is done. This allows to find the lowest energy configuration of the electron known as the ground state. This approximation reduces drastically the complexity of the calculation in this case the system is considered adiabatic, e.g. the lattice dynamics and the electrons are treated independently.

The two terms containing the ionic coordinates can be rewritten using an external potential (v_{ext}), the Hamiltonian is now written as:

$$\hat{\mathcal{H}} = \sum_{i=1}^N \left(-\frac{\nabla_i^2}{2} + v_{ext} \right) + \frac{1}{2} \sum_{i \neq j}^N \frac{1}{|r_i - r_j|} \quad \text{Eq. 17}$$

Even considering this approximation the Schrödinger equation is rarely solvable analytically with more than two electrons. The Born-Oppenheimer approximation is valid as soon as a small ionic movement does not change the electronic wave function.

3. The Hohenberg-Kohn theorem

Basic theorems of the DFT are directly derived from the postulate of Hohenberg and Kohn [99].

Theorem 1: The external potential $v_{ext}(\vec{r})$ is a unique function of the density of electrons $\rho(\vec{r})$. The Hamiltonian function is then fixed by this external potential.

The energy function can be written as a function of $v_{ext}(\vec{r})$:

$$E[n(r)] = T[n(r)] + v_{ext}[n(r)] + V_{ee}[n(r)] \quad \text{Eq. 18}$$

Theorem 2: The density that minimizes the total energy is the exact ground-state density.

4. The Kohn-Sham approach

For many body electrons, it remains difficult to solve the Schrödinger equation. In 1965, Kohn and Sham [100] made the postulation that one-electron formulation can be applied to a system containing N interacting electrons by introducing a suitable local potential in addition to any external potentials $v_{ext}(r)$ defined previously by Hohenberg and Kohn. It allows to solve the non-interaction-electron Schrödinger equation:

$$[T + v_{ext}(r) + v_c(r) + v_{xc}(r)]\varphi_i = \varepsilon_i \varphi_i \quad \text{Eq. 19}$$

Where T is the kinetic energy. v_{xc} is defined as the differential exchange correlation energy of E_{xc} with respect of the density ρ .

$$v_{xc}(r) = \frac{\delta E_{xc}[\rho]}{\delta \rho(r)} \quad \text{Eq. 20}$$

The exact electronic charge density is defined by the N occupied one-electron molecular orbitals φ_i with corresponding orbital energy ε_i .

$$\rho = \sum_i^N |\varphi_i|^2 \quad \text{Eq. 21}$$

The exchange correlation energy is unknown, and is defined by the functional exchange and correlation energy $E_{xc}[\rho]$.

4.1 The exchange correlation energy

To get accurate values from DFT calculations, the challenge is to determine correctly the exchange correlation energy E_{xc} . The commonly used exchange correlations energies are the Local Density Approximation (LDA) and the Generalized Gradients Approximation (GGA).

In the original paper of Kohn and Sham [100], they have suggested the so-called LDA. LDA is the simplest approximation assuming that the density can be treated locally as an uniform electron gas.

With LDA, it is assumed that the charge density $n(r)$ is uniform over all space at each point:

$$E_{xc}^{LDA}[n(r)] = \int d^3r E_{xc}^{uniform}(n(r)) \quad Eq. 22$$

The LDA approximation doesn't take into account the real charge densities that are varying strongly. This exchange correlation function fails for the description of strongly correlated materials (i.e. materials containing delocalized s- and p- electrons as well as localized partially filled d- and f-shells). As a concrete example, iron (Fe) in bcc lattice is unstable within the LDA [78].

Since the original paper of Kohn and Sham [100], other exchange correlation potentials have been developed as the generalized gradient approximation (GGA), that is an improvement of the local density approximation by incorporating an additional term into the exchange correlation and describes better the systems with inhomogeneous electron density.

$$E_{xc}^{GGA}[n] = \int d^3r E_{xc}(n(r), |\nabla n|) \quad Eq. 23$$

For bulk properties, it is recommended to use the GGA since it gives better agreement with experiments [89].

4.2 Basis sets

To solve the Kohn-Sham equations, there are different approaches. Here explanations for the approaches which have been used in the present work are given. Due to the lack of exact analytical solution, basis sets are used to describe the numerical expansion of the independent-particle wave-function φ_i (Eq. 19). For the calculation the efficiency of the basis set is essential.

The plane wave basis set

The plane wave basis set is appropriate for periodic systems and often used in combination with pseudo-potentials. The periodicity of the system is introduced in the wave function. The number of plane wave depends on the energy cut-off. The plane wave basis is used in VASP [96].

Slater type basis sets

It is a function resembling to the true atomic orbitals. This function is centered at the true nucleus. This basis sets is used in ADF [97,98].

4.3 Self-consistency loop

As it remains difficult to solve numerically these equations, the solution is done via a self-consistency loop as shown below in *Figure 19*. Within each self-consistency loop, a charge density n_i is used to generate a new Kohn-Sham potential (denoted H in the *Figure 19*). Out of this self-consistency, a new charge density is calculated and the loop restarts until the self-consistency is achieved.

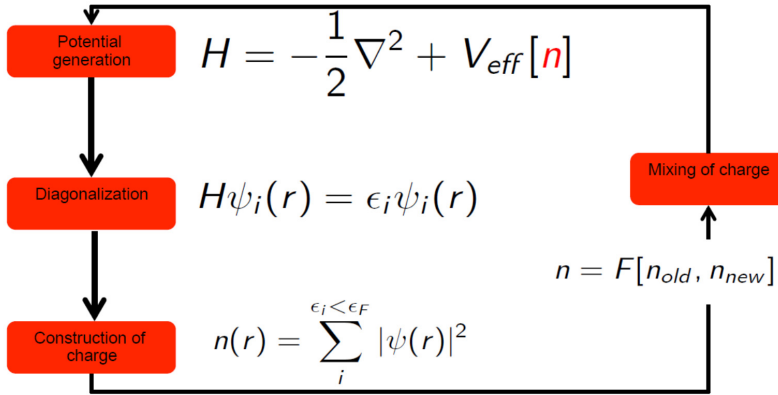


Figure 19 – Scheme of the self-consistency loop as reproduced in ref. [101].

5. Description of the electrons: all-electrons and pseudo-potential methods

5.1 All-electrons method

The wave function is defined for all electrons in the system. The potential is inversely proportional to the distance between electrons and the nucleus ($\sim Z/r$). The core states are treated as completely localized whereas the valence states are rather delocalized.

5.2 Pseudo-potential methods

The use of pseudo-potential method allows to avoid to take into account the core electron explicitly and is less demanding with respect to computational resources. It regards an atom as a perturbation of

the free electron gas [102]. In this approach, there is no information on the charge density and wave function near the nucleus. The all-electron wave function is replaced by a pseudo-wave function. With this, numerical difficulties are avoided.

5.3 The projector augmented wave method (PAW)

The projector augmented wave method (PAW) is an unification between all-electron and pseudo-potential methods. It has been introduced by Blöchl [103]. Depending on the region of the space, the wave functions have different behavior: the wave function is fairly smooth in the bonding region; it oscillates rapidly close to the nucleus. In order to combine these two different aspects of the behavior of the wave function, Blöchl suggested to divide it into the following parts: 1) atom centered sphere, 2) envelope function outside the spheres. This method provides better access to physical properties (i.e. magnetism).

5.4 Periodicity of the system

A solid consists of many atoms ($\sim 10^{24}$ atoms per mole), it would be rather expensive or impossible to compute a real solid. However, a solid consists of an infinite as repeated unit cell (i.e. periodic system). This property is used to reduce the number of atoms necessary to simulate a “real” solid, the number of atoms is now restricted to an unit cell (containing a couple of atoms) or a couple of unit cells (i.e. when studying defects in solids). The three lattice parameters and the angle between the vectors can be used to define the unit cell in the real direct space (Bravais lattice).

Using the Bloch’s theorem, the density is periodic so the wave function is expressed as:

$$\Psi_k^n(r) = u_k^n(r) \exp(ik \cdot r) \quad \text{Eq. 24}$$

Where $u_k^n(r)$ expressed the periodicity of the crystal. The k values are obtained in the reciprocal space of the unit cell. The k vectors than can be chosen are called the first Brillouin zone (BZ).

5.5 Brillouin zone integration

The sampling of k-points within the BZ is a crucial property for the accuracy if the convergence of the calculation. The integration over the Brillouin zone can be reduced by using the symmetry of the structure, the integration is then reduced in the irreducible Brillouin zone. There are several methods to calculate the integration over the Brillouin zone: 1) the tetrahedron method which consists of dividing up the Brillouin zone into tetrahedra, 2) sampling of special k-point as used in the Monkhorst-Pack grids for example. In the Monkhorst-Pack scheme [104], in order to choose a suitable number of k-points, it is proposed to distribute the k-points homogeneously within the first Brillouin zone.

6. DFT calculations in the present work

As previously mentioned, two different codes for the present work (i.e. ADF-BAND [97,98], VASP [96]) have been used. The differences between these two codes are summarized in the *Table 12*.

Code	Basis Set	Potential	BZ integration
ADF	Slater type Orbital	Full electron	Quadratic tetrahedron
VASP	Plane-Wave	Pseudo	Monkhorst-Pack

Table 12 – Summarized of the different approximations used in ADF [97,98] and VASP [96].

In the present work, DFT calculations have been used for the determination of the energy of several compounds in different systems. Referring to the pure elements associated to the considered compound, the enthalpies of formation were calculated. The enthalpies of formation of a compound are given by *Eq. 25*:

$$\Delta_f H (A_x B_y) = E_{A_x B_y} - \frac{x}{x+y} E_A - \frac{y}{x+y} E_B \quad \text{Eq. 25}$$

These enthalpies of formation were then used as input data for the thermodynamic modeling of the considered system according to the compound energy formalism (see Calphad, 2.2).

In addition, DFT calculations were used to calculate the forces in supercell structures for the determination of thermodynamic properties via phonon calculations.

V. Phonon calculations

1. Theory

DFT calculations are very useful at 0 K to determine energy of defined system. Nevertheless, the calculations at 0K do not include vibrational properties generated by temperatures above 0K. In order to be able to calculate properties of solids at real conditions (e.g. temperature), phonon calculations are a useful consideration to take into account. The influence of vibrations creating entropy becomes an important parameter to be taken into account in order to properly simulate materials properties at higher temperature.

To simulate the temperature dependence in a solid, phonon calculations based on short displacement method have been used. The atoms vibrate around their equilibrium positions (the ground state is previously reached by optimizing the geometry of compound using DFT at 0K). This is modeled by using a simple harmonic oscillator. The harmonic approximation provides a good picture to consider temperature dependence without doing too much approximation.

The energy state of a quantum harmonic oscillator E_0 is:

$$E_0 = \frac{1}{2} \hbar \omega \quad \text{Eq. 26}$$

With $\hbar$ is representing the Planck constant and ω natural frequency of the oscillation (i.e. small displacement of atoms in a crystal). This equation provides the zero point energy. For each vibrational mode k , the energy can be computed, thus the energy is:

$$E(k, \nu) = \hbar \omega \left[\frac{1}{2} + n(k, \nu) \right] \quad \text{Eq. 27}$$

Where $n(k, \nu)$ is the number of phonons in the ν -th branch with the wave vector k [105]. The number of phonons $n(k, \nu)$ is related to the temperature:

$$n(k, \nu) = n(\omega, T) = \left[\exp \left(\frac{\hbar \omega(k, \nu)}{k_B T} \right) - 1 \right]^{-1} \quad \text{Eq. 28}$$

According to the two Eq. 27 and Eq. 28, the energy of a crystal within the harmonic approximation is given by, for simplification we can write $(k, \nu) = \omega$:

$$E = \sum \hbar\omega \left[\frac{1}{2} + \left[\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1 \right]^{-1} \right] \quad \text{Eq. 29}$$

This is the representation of the internal energy. Deriving Eq. 29 with respect to the temperature provides the heat capacity at constant volume:

$$C_V = \left(\frac{\partial E}{\partial T} \right)_V = \sum k_B \left(\frac{\hbar\omega}{k_B T} \right)^2 \frac{\exp\left(\frac{\hbar\omega}{k_B T}\right)}{\left[\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1 \right]^2} \quad \text{Eq. 30}$$

At high temperature $k_B T \gg \hbar\omega$, C_V tends to $3Nk_B$ ($\sim 25 \text{ J.mol}^{-1}.\text{K}^{-1}$), this is the Dulong-Petit law. If this rule allows a good approximation of the heat capacity at high temperature, it cannot nevertheless explain the behavior of C_V at low temperature that tends to 0. This is the limit of quantum mechanics to explain the behavior of the heat capacity. In order to explain that heat capacity is tending to 0 at 0K, classical mechanics are required. Furthermore any thermodynamic properties can be computed using the density of state $g(\omega)$. The density of states (DOS) is the number of accessible states at given energy.

The energy of harmonic phonons becomes:

$$E = \int \hbar\omega g(\omega) \left[\frac{1}{2} + \left[\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1 \right]^{-1} \right] d\omega \quad \text{Eq. 31}$$

2. Thermodynamics

2.1 Harmonic approximation

At constant volume, different thermodynamic quantities can be calculated from the phonon density of state:

Free energy

$$F_{Ph}(T) = k_B T \int_0^\infty \ln \left(2 \sinh \left(\frac{\hbar \omega}{2 k_B T} \right) \right) g(\omega) d\omega \quad Eq. 32$$

The vibrational entropy is obtained from the harmonic phonon calculations:

$$S(T) = -T \left(\frac{\partial F}{\partial T} \right)_V = k_B \int \left(-\ln \left[1 - \exp \left(\frac{-\hbar \omega}{k_B T} \right) + \frac{\left(\frac{\hbar \omega}{k_B T} \right)}{\exp \left(\frac{-\hbar \omega}{k_B T} \right) - 1} \right] \right) g(\omega) d\omega \quad Eq. 33$$

Heat capacity at constant volume can be calculated as follows:

$$C_v(T) = \int g(\omega) \frac{(\hbar \omega)^2}{k_B T} \frac{\exp(\hbar \omega / k_B T)}{(\exp(\hbar \omega / k_B T) - 1)^2} d\omega \quad Eq. 34$$

The harmonic approximation works under constant volume. This approximation does not take into account the pressure, whereas experiments are at least carried out under atmospheric pressure. The harmonic approximation does not allow to take into account the thermal expansion, the phonon-phonon interactions as well as the thermal conductivity.

The results obtained can be represented by a simple mathematical model. Phonons provide in domination the heat capacity, and the different models are given for the heat capacity. In the present contribution we give a short overview of the Einstein model, the Debye model and the polynomial function.

2.2 Einstein model

The Einstein model makes the postulation that all modes in the density of state have the same frequency ω_E . The Einstein model allows a good description of the heat capacity over the entire temperature range.

$$\omega_i = \omega_E \quad Eq. 35$$

Therefore the heat capacity is:

$$C_v = \left(\frac{\partial E}{\partial T} \right)_v = \sum k_B \left(\frac{\hbar \omega_E}{k_B T} \right)^2 \frac{\exp \left(\frac{\hbar \omega_E}{k_B T} \right)}{\left[\exp \left(\frac{\hbar \omega_E}{k_B T} \right) - 1 \right]^2} \quad \text{Eq. 36}$$

At high temperatures, this rule follows also the Dulong-Petit rule, nevertheless at low temperatures, this fails and is not equal to zero as it should be.

2.3 Debye model

As the Einstein model failed to represent correctly the behavior of the heat capacity at low temperatures, the Debye model allows a better representation. In the Debye model, each mode has its own frequency. The heat capacity is given by:

$$C_v = 9Nk_B \left(\frac{T}{T_D} \right) \int_0^x \frac{e^x x^4}{(e^x - 1)} dx \quad \text{Eq. 37}$$

Where T_D is the Debye temperature.

2.4 Polynomial function

In thermodynamic databases the Gibbs energy functions are described with a polynomial function. The energy of a compound is described relative to the surface element reference (SER) (i.e. surface of the pure elements in their respective stable phase).

The Gibbs energy of a compound is described as:

$$G_m^\alpha = a + bT + cT \ln T + D_2 T^2 + D_3 T^3 + D_{-1} T^{-1} \quad \text{Eq. 38}$$

This power series can be extended but the results are not improved much. Based on Eq. 38 the heat capacity is given by:

$$C_p = -T \left(\frac{d^2 G}{dT^2} \right) = -C - 2D_2 T - 6D_3 T^2 - 2D_{-1} T^{-2} \quad \text{Eq. 39}$$

This polynomial function fails to describe the thermodynamic function below room temperature.

3. Quasi-harmonic approximation

The harmonic approximation is done at constant volume and cannot reproduce all the physical phenomena of a solid:

- Thermal properties (thermal expansion)
- Phase transitions
- Transport properties

All these properties are referring to the anharmonicity of the crystal. The anharmonicity of the system is the interaction between phonons with other phonons. In a simple approach, the anharmonicity can be modeled using the quasi-harmonic approximation [82] (i.e. phonon calculation are computed over different volumes) as it is small enough that the quasi-harmonic approximation can represent it in many cases (with some exception) [106]. Making the assumption that the anharmonicity is only restricted to the thermal expansion [105], the QHA approximation can be used.

The vibrational Helmholtz energy F in the QHA approximation can then be written:

$$F_0(V, T) = U_0(V) + \frac{1}{2} \hbar \omega_j(k, V) + k_B T \sum \ln \left(1 - \exp \left[-\frac{\hbar \omega_j(k, V)}{k_B T} \right] \right) \quad \text{Eq. 40}$$

Where the first term is the internal contribution, the second the zero point energy and the third the vibrational contribution.

In the quasi-harmonic approximation, the heat capacity at constant pressure (C_P) is obtained from:

$$C_P = C_V + VT\alpha^2 B \quad \text{Eq. 41}$$

Where α and B are the thermal expansion coefficient and the bulk modulus, respectively.

In the same way the entropy is given as:

$$S(V, T) = S_{har} + VT\alpha^2 B \quad \text{Eq. 42}$$

4. Results

Several parameters influence the results of the thermodynamic properties obtained from the phonon calculation:

- Optimization of the structure: any computation of the phonon relies first on the very accurate optimization of the structure. Any structure which is not well relaxed is not in its local ground state (i.e. not in equilibrium) and phonon calculations will lead to the appearance of imaginary frequencies and show that the structure is mechanically unstable. Any parameters influencing a correct geometry optimization (i.e. k-points, energy cut-off) influence the computation of phonons.
- Size of the supercell: the size of the supercell should not be too small to obtain accurate phonon calculations. Nevertheless this one should stay as small as possible for computational reason.

In the present work, phonon calculations have been used to describe the thermodynamic functions of different compounds (stable and hypothetical). The results were then used as input for the thermodynamic modeling as described by the Calphad method.

Chapter III - Pure elements and binary boundary systems

With the development of computational power and simulation tools (i.e. DFT, phonon) it has become clear that these methods will be further used and developed [106] to understand the materials behavior. In the present work, simulation tools as DFT, phonon and Calphad modeling are used to understand the driving forces controlling the formation of the different phases present in Crofer 22H.

I. Pure elements

The thermodynamic modeling of unary phases is important as they are the basis as reference state for other intermetallic phases. In the present work, we consider the pure elements forming the Laves phase (Fe,Cr,Si)₂(Nb,W) in Crofer 22H in their stable form (e.g. bcc and diamond). Calculations of their total energies in their stable form (and magnetic state) are necessary for further calculations of enthalpy of formation for example, which are then used as input parameters for thermodynamic modeling. In addition, phonon calculations have been done in order to check the reliability of the results and to test some different parameters for further calculation in more complex structures.

1. Energy of formation

The energy of formation of a compound refers to the pure elements in their stable form according to Eq. 43:

$$\Delta_f H(A_x B_y) = E_{A_x B_y} - \frac{x}{x+y} E_A - \frac{y}{x+y} E_B \quad \text{Eq. 43}$$

The calculation of the enthalpy of formation of several compounds in different systems were performed in the present work, their results will be developed in a later chapter and discussed. Here results of the pure elements used as reference state are given. The total energies of the ground state of the pure elements in their stable form were calculated by DFT, and the results were compared with available data from literature. The energy of formation of the pure elements can only be compared with literature data, where the same codes and same basis function have been used as in the present work. The data presented here have been generated using VASP [96], with the PAW potential [103] within the GGA-PBE approximation. Other specific parameters as the k-points and the energy cut-off

are specified in *Table 13-17*. The results were obtained by following three different steps of calculation in order to get accurate values of the total energy. For all calculations, the ground state energies were reached in three steps: 1) the volume and the atomic positions were relaxed; 2) the stresses and forces were relaxed; 3) the tetrahedron method with Blöchl correction was used for accurate calculations of the ground state. For all calculations, an accuracy 10^{-6} was set up in the self-consistency loop. Results are given in *Table 13-17* for each pure element.

Species	Structure	Magnetic state	k-points	Energy cutoff (eV)	E_f (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	V (Å ³)	Ref.
Cr	bcc	AFM	11x11x11	400	-915.229	2.8359	22.81	Present work
		-	-	-	-913.284	2.847	23.08	[107]
		AFM	12x12x12	-	-913.909	2.8375	22.846	[108]
		AFM	16x16x16	400	-931.08			[109]

Table 13 – Energy of pure Cr in the BCC structure compared with literature.

Species	Structure	Magnetic state	k-points	Energy cutoff (eV)	E_f (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	V (Å ³)	Ref.
Fe	bcc	FM	11x11x11	400	-800.306	2.8304	22.79	Present work
					-798.397	2.822	22.5	[107]
		FM		500	-792.603	2.832	22.7	[88]
		FM		400	-792.614	2.832	22.7	[38]
		FM	12x12x12		-788.67	2.8256	22.56	[108]

Table 14 – Energy of pure Fe in BCC with the different parameters used to carry out the calculations.

The results are compared with available literature data.

Species	Structure	Magnetic state	k-points	Energy cutoff (eV)	E_f (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	V (Å ³)	Ref.
Nb	bcc	NM	11x11x11	400	-974.43	3.327	36.83	Present work
		NM			-969.385	3.322	36.7	[107]
		NM		500	-971.04	3.322	36.7	[88]
		NM		400	969.717	3.32	36.6	[38]

Table 15 – Calculated energies of Nb as well with the lattice parameters.

Species	Structure	Magnetic state	k-points	Energy cutoff (eV)	E_f (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	V (Å ³)	Ref.
Si	diamond	NM	11x11x11	400	-523.40	5.4683	163.51	Present work
					-524.06	5.468		[110]

Table 16 – Energies and lattice parameters calculated in diamond structure compared with literature.

Species	Structure	Magnetic state	k-points	Energy cutoff (eV)	E_f (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	V (Å ³)	Ref.
W	bcc	NM	11x11x11	400	-1255.496	3.173	31.915	Present work
					-1232.899	3.19	32.46	[107]

Table 17 – Energies of pure W calculated by DFT after optimization of the structure.

For all the pure elements, there is a good agreement of the present results with literature data. The small difference found is due to different approximations used for the calculations (i.e. k-points, pseudo-potential); and it is within the error range as any other experimental data. For calculations of the energy of formation of the different compounds, calculation of the total energy of the pure elements of the present work as reference state has been used. For compounds where energies of

formation were available these results were compared to check their reliability. This will be discussed in further chapters when necessary.

2. Phonon calculations of the pure elements

To simulate the temperature dependence in the solid, we used phonon calculations based on short displacement methods. In practice, we used the supercell methods with finite displacement also called the direct method as implemented in the phonopy package [111]. The supercells were built from the optimized structures from DFT as previously described and the atoms are displaced from their equilibrium position by 0.01 Å. The accuracy of the phonon calculations can be influenced by different parameters. Of course the primary parameter to get correct phonon calculations is a good minimization of the ground state (e.g. suitable k-points, low residual forces). The other parameter that can influence the accuracy of the results is the size of the supercell.

Harmonic approximation was used to compute the thermodynamic properties at constant volume. The force constants of the different supercells were calculated with VASP, they were then used in phonopy [111] to calculate the phonon frequency and to derive their relative thermodynamic properties. The phonon contribution to the Helmholtz free energy is given by:

$$F_{phonon}(T) = \frac{1}{2} \sum_{q,v} \hbar \omega_{k,v} + k_B T \sum_{q,v} \ln \left[1 - \exp \left(-\frac{\hbar \omega_{k,v}}{k_B T} \right) \right] \quad Eq. 44$$

Where k and $\omega_{q,v}$ are the wave vector and the band index, respectively. The phonon frequency is expressed as ω_q .

In any crystal, thermal expansion influences the thermal properties. The thermal expansion has therefore to be taken into account to give accurate results. The corresponding thermodynamic properties are then calculated at constant pressure for several volumes. Therefore the quasi-harmonic approximation was taken into account. In the quasi-harmonic approximation, the thermodynamic properties are calculated for different volumes close to the equilibrium volume. The volume dependency is then added. It results the heat capacity at constant volume as given in Eq. 41.

In the present chapter, phonon calculations of two pure elements, Fe and W, are given as an example within the harmonic and the quasi-harmonic approximations and compared with literature data.

As the pure elements Fe and W crystallize in BCC structure, they contain only two atoms per unit cell. These calculations have been used to test different set of parameters to get accurate calculation of the thermodynamic properties. The successful parameters were then used to calculate more complex intermetallic phases, nevertheless it was checked that the chosen parameters gave suitable results for more complex structures.

2.1 Iron, Fe

The structure of Fe was carefully optimized taking into account the ferromagnetism. The relaxed structure (energy and lattice parameters are given in *Table 14*) was then used to build a 2x2x2 supercell. The force on the displaced atoms are then computed with VASP [96]. The different forces calculated for each structure are then collected and used to calculate the phonon frequencies, which in turn are used to calculate the temperature dependent thermodynamic properties. The calculated thermodynamic data in the harmonic approximations are given in *Figure 20*.

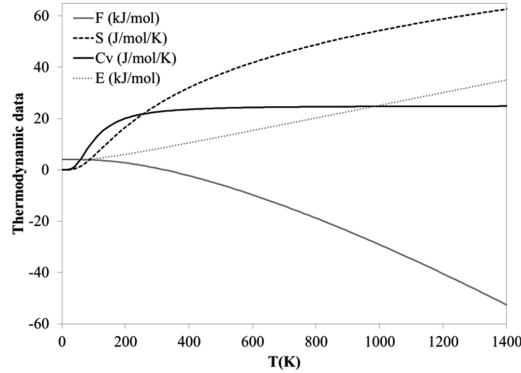


Figure 20 – Thermodynamic properties of Fe obtained the calculated phonon harmonic calculations.

In the quasi-harmonic approximation (QHA) for Fe, nine different volumes with a maximum volume change of about 12 % were used. The energy dependency of the volume at 0 K is shown in *Figure 21*.

The minimum of the Helmholtz free energy vs temperature (*Figure 22*) is obtained by fitting the volume-energy data to the Vinet equation of state [112] (*Eq. 45*).

$$G(T, P) = \min_V [U(V) + F_{ph}(V, T) + PV] \quad \text{Eq. 45}$$

Thermodynamic functions of Fe are then obtained by derivation.

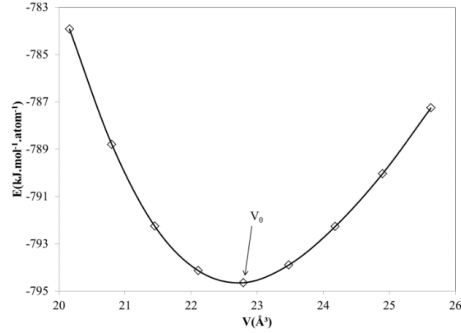


Figure 21 – Energy dependency volume for pure Fe in BCC structure given at 0K.

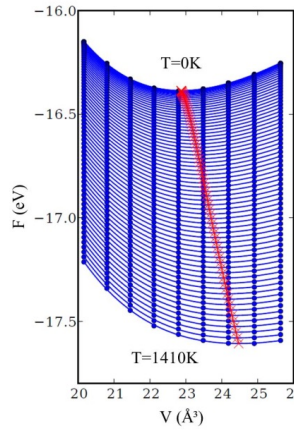


Figure 22 – Helmholtz free energy of Fe as function of volume and temperature for determination of the minimum energy (red line).

For metals, the electronic contribution influences the thermodynamic properties. Depending on the metals, their influence can be negligible or not. The higher the electron valency is, the higher the

influence of electron contribution is. Hence, for iron this influence is not negligible and its value was taken from [82] (see Fig. 8 of the original paper) and added to the heat capacity and entropy curves (Figure 23a).

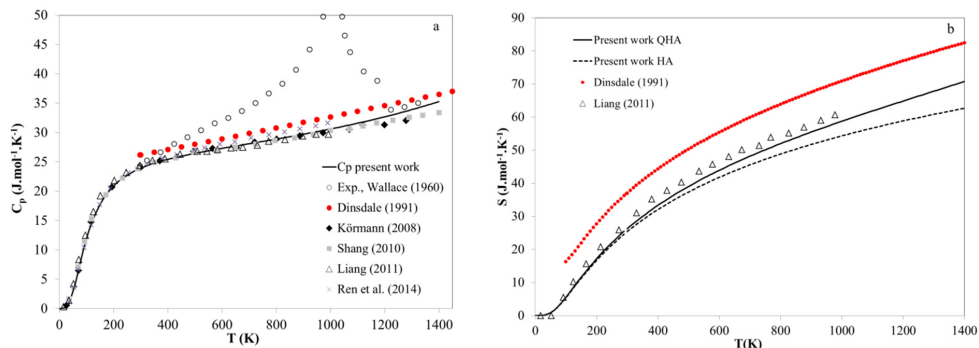


Figure 23 – a) Heat capacity of Fe compared with literature data from experiments [113] and phonon calculated heat capacity [114–117]. The red point from Dinsdale [81] are the commonly used data in Calphad modeling for the description of pure Fe (these data don't take into account the magnetism).

b) Entropy of Fe from QHA and HA phonon calculation compared with phonon QHA calculations of Liang [116] and the Dinsdale polynomial [81].

The experimental results of the heat capacity of Fe cannot be fully reproduced by phonon calculations in QHA approximation due to the magnetic excitation [115]. The magnetic contribution was the subject of several studies in the literature [114,115], and show the difficulty to model Fe. In Calphad type modeling, the magnetism of iron is treated in a separate function. This leads to the difference between the phonon and the Dinsdale polynomial [81] (for example in the entropy curve - Figure 23b). The magnetic contribution to the enthalpy of Fe is about 8 kJ.mol⁻¹ at 300 K (Figure 24), this can result in a non-stability of intermetallic phases (i.e. Laves-Fe₂W and μ -Fe₇W₆ phases in Fe-W system as seen in Chapter IV) whereas experiment and DFT calculations suggest the opposite.

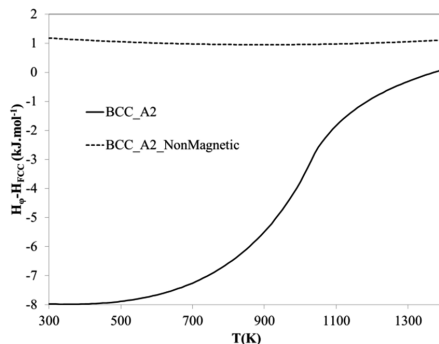


Figure 24 – Enthalpy of Fe calculated from the Dinsdale polynomial [81] and used in the Calphad modeling for the reference state of BCC Fe. The calculation of the enthalpy refers to pure Fe-FCC.

The treatments of the magnetism as an independent function in the description of the pure element [81] have to be taken into account when performing a thermodynamic optimization of a phase diagram. In the systems where Fe is present, the enthalpies of formation of the compound should be written by taking into account the 8 kJ.mol⁻¹ of difference due to the magnetism. In many of the systems studied in the present work Fe is present so the magnetic contribution has to be treated carefully.

2.2 Tungsten, W

In the same way as for Fe, phonon calculations in harmonic and quasi-harmonic approximations have been done for W. Thermodynamic properties were then obtained from them. W does not show magnetic order and its thermodynamic properties are not influenced by it. The volume dependency to the free Gibbs energy is given in Figure 25 and the heat capacity derived from it is given in Figure 26. The electronic contribution to the heat capacity is rather small compared to that of Fe (see Fig. 8 in [82]). Nevertheless, its contribution was added to our calculated heat capacity (given in Figure 26).

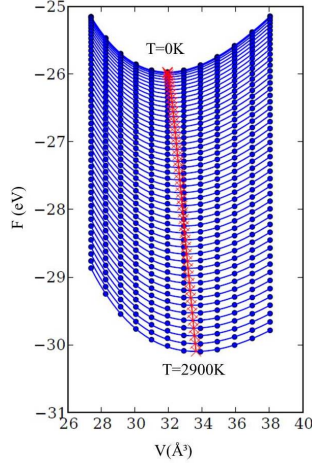


Figure 25 – Helmholtz free energy of *W* in *QHA* approximation as a function of the unit cell and temperature. The minimum values of the fitted equation of state (Vinet equation) are given by the red cross.

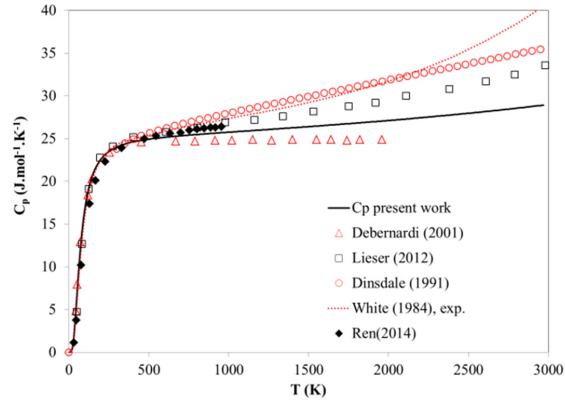


Figure 26 – Heat capacity of *W* calculated in the *QHA* approach compared with experimental data [118] (round dotted red point), SGTE pure element from the Dinsdale polynomial [81] (red empty circles) and other phonon calculations in harmonic approximation [119] (red empty triangles) and *QHA* [117,120] (empty squares and black diamond).

II. Binary boundary systems

1. Calculation in binary systems: Fe-Nb, Fe-Si, Nb-Si, Cr-Nb

Ab initio calculations have been performed using ADF-BAND [98] in this part of the present work in several binary systems where a lot of literature data are available to test some different parameters (i.e. SCF convergence, magnetism, functionals...) and check for consistency with experiments or/and other ab initio calculations. ADF [98] is using an all electron method for the description of the elements. This method could be seen as very accurate, nevertheless it is computationally very expensive and requires a lot of computational time, whereas it has been shown that a pseudo-potential (or PAW) method was also accurate.

The calculations have been done for stable compounds on the different considered systems. For the exchange-correlation energy, we used the general gradient approximation (GGA) [103] of Perdew, Burke and Ernzerhof (PBE) [121]. The magnetism was taken into account for the compounds containing Fe and Cr. The scalar relativistic ZORA effect was implemented in the calculations. The energy of formations were calculated at 0K using an energy convergence of 10^{-6} eV and a TZP basis set with small core was used.

Calculations using ADF-BAND [98] have been done in the Fe-Si, Fe-Nb, Nb-Si and Cr-Nb systems. The calculated enthalpies of formations are shown in *Figure 27-30* and compared with available literature data. The DFT calculations from the literature have been done by using VASP which is a pseudo-potential method whereas ADF-BAND is an all-electron method.

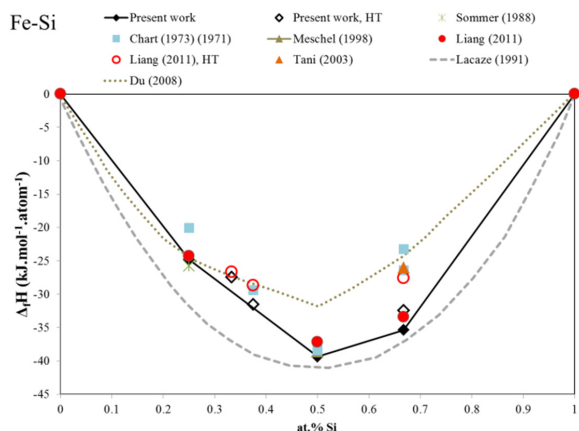


Figure 27 - Enthalpy of formation of Fe-Si of the present work (black diamond) compared with experiments [122–125] (blue square, brown diamond, star) DFT [116,126] (red empty square, orange triangle) and Calphad [127,128] (dotted line, point line). HT=High Temperature.

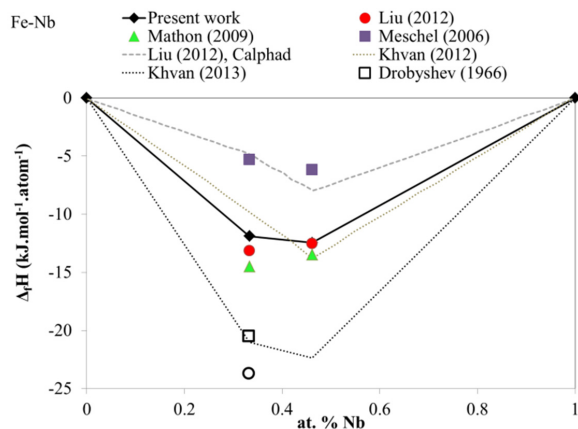
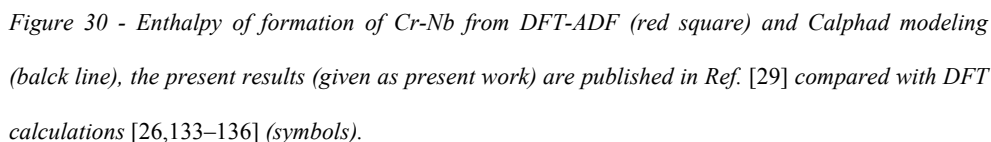
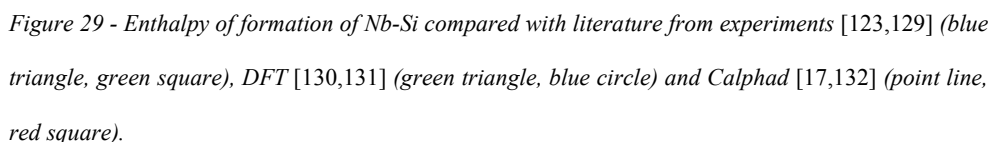


Figure 28 - Enthalpy of formation of Fe-Nb of the present work compared with experiments [36,37] (violet square, empty square), DFT [38] (red circle) and Calphad [38–40] (dotted, dashed, point lines).



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approximations in the DFT calculations (i.e. all-electron for the present work and pseudo-potential for the references) give results in good agreement.

2. Cr-Nb

The results of the enthalpies of formation in the Cr-Nb (i.e. end-members of the C15 laves phase) have been used as input data for the thermodynamic modeling of this system and compared with the available literature (*Table 18*).

End-members	$\Delta_f H$ (kJ.mol ⁻¹ .atom ⁻¹)	Optimized lattice parameters a (Å)	References
Cr ₂ Nb	-1.83 (FM)	6.96	Present work
	-1.72 (NM)	6.93	
	-3.31 (FM)	6.93	[26]
	-2.48 (FM)	6.82	[133]
	-2.31 (FM)	6.93	[135]
	-2.1 (FM)	6.94	[134]
	-2.34 (FM)	-	[136]
Nb ₂ Cr	75.38 (FM)	7.51	Present work
	75.39 (NM)	7.50	
	75.33 (FM)	7.50	[26]
Cr ₂ Cr	30.56 (FM)	6.64	Present work
	33.55 (NM)	6.66	
	27.29 (FM)	6.58	[26]
	26.4 (FM)	-	[137]
Nb ₂ Nb (NM)	15.33 (NM)	7.74	Present work
	16.2 (NM)	7.69	
	16.2 (NM)	-	[137]

Table 18 – Enthalpy of formation of the optimized end-members from DFT calculation compared with literature data (also DFT calculations).

As mentioned in the literature review (Chapter I- Cr-Nb), the re-optimization of this system was necessary as it was shown by Aufrecht et al. [28] that the high temperature Laves phase (C14) was

metastable whereas it was always shown as a stable phase on the existing phase diagrams. Taking into account this new finding and using DFT calculations of this work, thermodynamic optimization of Cr-Nb system was done during the present work and has been published by Schmetterer et al. [29]. The phase diagram resulting from this optimization is shown in *Figure 31*. It is in good agreement with experimental data. A full literature survey and the methods of calculation are given in Schmetterer et al. [29].

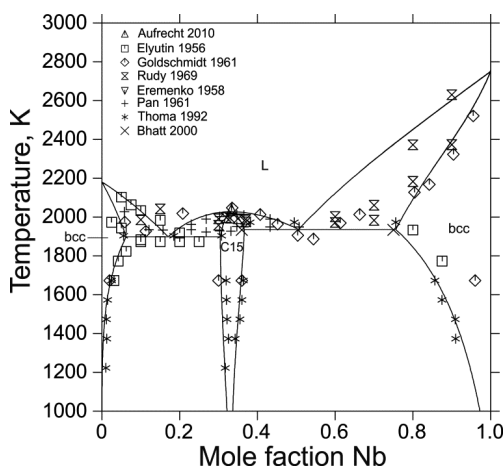


Figure 31 – Optimized phase diagram as presented by Schmetterer et al. [29] using DFT calculations of the present work and compared with available literature data [28,30,51,138–140].

New experimental data for the invariant reactions have been published by Stein et al. [18] after the publication of our assessment of the Cr-Nb system [29]. Comparison of the temperature of the invariant reaction of the calculated and experimental data is given in *Table 19*.

Invariant reaction	Calculated values [29] T(K), x(Nb)	Experimental values	Experimental values [18] T(K), x(Nb)
E ₁ : L → Cr + C15	1902K, x(Nb)=0.1713	1903, x(Nb)=0.185 [140]	1933K, x(Nb)=0.19
C: L → C15	2027K, x(Nb)=0.333	2033K [28]	2000K, x(Nb)=0.331
E ₂ : L → Nb + C15	1936K, x(Nb)=0.5089	1973K, x(Nb)=0.495 [140]	1954K, x(Nb)=0.48

Table 19 – Comparison of the temperature of the invariant reactions using the calculated phase diagram and the experimental values.

The results of our assessment are consistent with the experimental values. The new values from experiments given by Stein et al. [18] are slightly shifted (higher temperature for the two eutectics and lower for the congruent point). As these authors considered the new experimental evidence that C14 is metastable, their data are reliable. A new thermodynamic assessment could be considered necessary in order to take into account these new data for the invariant reactions.

3. DFT calculations in other systems

Some problems have been encountered by using the ADF code [98]. Indeed already by its method, i.e. all electrons, the calculations are computationally expensive. In addition, this software had several bugs that were reported to the software developer. One of the major bugs with this program was to use a specific space group (i.e. space group 166, R-3m). For this space group one of the symmetry of the crystal structure was not correctly implemented. Three atoms of the same Wyckoff position were occupying the same position whereas they should have been shifted by translation using the correct symmetry. Nevertheless, compounds with this space group are present in several of the studied systems.

The use of another software for the computation of the energy of the structures has been considered necessary due to the several bugs as well as the time required for the computation of a structure. The commercial software VASP [96] was considered as a suitable choice for the present work. Using the VASP software, calculations of the enthalpies of formation (and phonon) have been done for end-members in the Fe-W binary system and Fe-Cr-Nb and Fe-Nb-Si ternary systems. The calculations will be developed for each system in their respective part with the details used to carry out the calculations.

Chapter IV – Modeling of Fe-W phase diagram using first-principles and phonons calculations

A part of this chapter is based on a paper accepted for publication in Calphad [141].

Introduction

Thermodynamic calculations using the Calphad method (Calculation of phase diagram) for systems with many components as found in real materials, e.g. steels and Ni-base super alloys, have become firmly established tools in industry and research. Such calculations rely on thermodynamic databases composed from many individual called phase diagram optimizations (thermodynamic assessments), predominantly done for 2-, 3- and 4-component systems and extrapolations for higher systems. While the benefits of modeling are obvious, i.e. for example savings on resource intensive experiments and tests, the accuracy of the calculations relies on the underlying thermodynamic descriptions of binary and ternary systems.

A conventional optimization is based on a set of experimental data (phase diagram data and thermodynamics) and the parameters for the chosen model are obtained in a sophisticated kind of constrained data least-square fit (the “optimization”). This means that the number of parameters that can be used depends on the amount of available input data – a limited amount of data only allows for the use of few model parameters. The limitations of this approach are evident from many optimized datasets where linear functions or even constant values are being used in the functions describing the Gibbs energy. This effectively means that only the enthalpy and entropy of formations can be used for the thermodynamic modeling. At high temperatures this can lead to the appearance of inverted miscibility gaps in the liquid phase as well as other problems.

In the present work, a combination of density functional theory (DFT) and phonon calculations as well as Calphad method was used to establish a new thermodynamic dataset for the binary Fe-W system. This system is relevant for steel research, in particular for the further development of Laves phase reinforced ferritic steels [1,142]. Although this system is not particularly complex it poses experimental difficulties due to the high liquidus temperatures (especially on the W-rich side), slow

diffusion at practical temperatures in experiments and therefore slow equilibration of samples. Hence, this system is a good showcase how results from atomistic simulations can enhance the thermodynamic description when experimental data are missing or ambiguous.

Two thermodynamic datasets were generated, i.e. a) based on the calculation of the enthalpy of formation of the various end members (for the two intermetallic phases, λ -Fe₂W and μ -Fe₇W₆) (see p. 80) at 0K obtained from DFT calculations and b) based on the results of quasi-harmonic phonon calculations (temperature dependent description of the Gibbs energy of formation of the end members). From these datasets, two phase diagrams were calculated. The results from the various modeling techniques, the thermodynamic descriptions and the phase diagram calculations are compared with each other and existing literature data.

I. Literature survey

A summary of the literature survey of the Fe-W binary system was given in the literature survey (Fe-W, p 16). Furthermore, a full literature survey was written for the publication.

The Fe-W binary system contains two intermetallic phases, the Laves λ -Fe₂W and the μ - phases, which form both by peritectic reactions (*Table 20*). The phase diagram was recently reinvestigated experimentally by Antoni et al. [47] due to several controversies. The last assessment of this system was given by Gustafson [48] and is shown in *Figure 6* and compared to experimental data. The invariant reactions of the system are given in *Table 20* with their respective temperatures and compositions.

Invariant reactions	T(K), x(W)	References
P1: Liquid+BCC => μ	1910K x(W, Liq)=0.206 x(W, BCC)=0.974*	Sinha et al.[44]
P2: L+ μ => BCC	1821K x(W, Liq.)=0.112 x(W, BCC)=0.143	Sinha et al.[44]
C: L=>BCC	1802K x(W,Liq)=x(W,BCC)=0.044	Sinha et al.[44]
P3: BCC+ μ =>C14	1333K x(W,C14)=0.3333	Sinha et al. [44]

Table 20 – List of the invariant reactions in Fe-W phase equilibria given in the literature and considered for the thermodynamic modeling. *The phase boundary on the W-rich side is not well defined, Ichise [143] gave a phase boundary at 97.4 %. We used this data for the peritectic temperature given by Sinha et al. [44].

II. Modeling

In the present work, a multiscale modeling approach was chosen in order to describe the thermodynamic properties of the Fe-W system. This included the use of DFT calculations, phonon calculations and Calphad. For the latter, the PARROT module of the software Thermo-Calc [144] was employed which utilizes the sublattice or substitutional solution model for solution phases. The computer simulations preceding the Calphad type assessment therefore had to be based on the same models as imposed by Thermo-Calc. Furthermore, in order to be compatible with existing databases, the same models as in the TCFE database were used:

- a two-sublattice model (**Fe**,W)₂(**Fe**,**W**)₁ for the Laves phase
- a four-sublattice model (**Fe**,W)₁(**W**)₄(**Fe**,**W**)₂(**Fe**,**W**)₆ for the μ -phase
- substitutional solution model (one sublattice model) for the BCC, FCC and liquid phases

Main components on the sublattices are shown in bold print. More details on the use of these sublattice models will be described in section II. 3 (p. 80).

1. Density Functional Theory (DFT)

DFT calculations were carried out using the VASP software package [96] using the GGA-PBE approximation for the exchange energy and projector augmented waves (PAW) [145] as basis set. For

the different structures the k-mesh points were used according to the Monkhorst-Pack scheme; they are reported in *Table 23*, *Table 24* and *Table 25*. For all calculations, the ground state energies were reached in three steps: 1) the volume and the ion positions were relaxed; 2) the stresses and forces were relaxed; 3) the tetrahedron smearing method with Blöchl correction [146] was used for accurate calculations of the ground state. For all calculations, an accuracy of 10^{-6} eV was set up in the self-consistency loop and an energy cutoff of 400 eV was used. The magnetism of the compounds was taken into account due to the presence of Fe. For each Fe containing compound, calculations were made in the ferro- and non-magnetic states in order to determine the more stable one. For the Fe-free end-members, a non-magnetic state was used.

The energies of formations of the end-members were calculated according to *Eq. 46*.

$$\Delta_f H(Fe_xW_y) = E_{Fe_xW_y} - \left[\frac{x}{x+y} E_{Fe} + \frac{y}{x+y} E_W \right] \quad \text{Eq. 46}$$

$E_{Fe_xW_y}$, E_{Fe} and E_W are the computed total energies of the compound Fe_xW_y and of the pure elements in their stable states (e.g. BCC for Fe and W), respectively (*Table 23*). The results of the calculated enthalpies of formation are given in *Table 24* and *Table 25* for each end-member of the Laves and the μ -phase.

2. Density Functional Theory Phonon Calculations

In order to determine the temperature dependence of the Gibbs energy functions, phonon calculations were done for the end-members as well as for the pure elements. The phonon calculations allow to take into account the lattice vibrations in the crystal and to get thermodynamic properties over a range of temperature, whereas DFT calculations are limited to 0 K. These allow to get the temperature dependence of the total Gibbs energy and to be used for the Calphad assessment. Our calculations were carried out using the software package Phonopy [111], which is based on the supercell method with finite displacements [147] (also called the direct or Parlinski-Li-Kawazoe method). The supercell is built from the structures optimized as previously described, and displacements of $0.01 \text{ \AA}$ were used.

A harmonic approximation was used to compute the thermodynamic properties of the various stable and metastable end-members. Thermal properties at constant volume were derived from the phonon density of states. The phonon contribution to the Helmholtz free energy is given by:

$$F_{phonon}(T) = \frac{1}{2} \sum_{q,v} \hbar \omega_{q,v} + k_B T \sum_{q,v} \ln \left[1 - \exp \left(-\frac{\hbar \omega_{q,v}}{k_B T} \right) \right] \quad Eq. 47$$

Where q and v are the wave vector and the band index, respectively. The phonon frequency is expressed as ω_q .

The heat capacity at constant volume is written as:

$$C_V = \sum k_B \left(\frac{\hbar \omega}{k_B T} \right)^2 \frac{\exp \left(\frac{\hbar \omega}{k_B T} \right)}{\left[\exp \left(\frac{\hbar \omega}{k_B T} \right) - 1 \right]^2} \quad Eq. 48$$

VASP [145] was used to calculate the force constants of the different supercells with displaced atoms. The force constants were then used in phonopy [111] to calculate the phonon frequencies and their related thermodynamic functions (Eq. 47 and Eq. 48). Such calculations were carried out for each end-member of the Laves and μ -phases. Different sizes were used for the supercells of the Laves and the μ -phases, 2x2x2 and 2x2x1 respectively. As the c lattice parameter of the μ -phase is large compared to the a parameter, the size of the cell was not increased in this direction. Furthermore, the size of the supercell was kept as small as possible in order to keep the computational effort and time reasonable. Computational details for the various compounds are given with the results in Table 26 and Table 27.

In metallurgy, phase diagrams are usually shown for constant pressure (i.e. at 1 atmosphere) or the influence of the pressure is considered negligible. Therefore, the temperature dependence of the materials properties is not only governed by the harmonic approximation but also by thermal expansion. The corresponding thermodynamic properties should therefore be obtained for constant pressure instead of constant volume. Therefore, quasi-harmonic phonon calculations were carried out for the stable end-members λ -Fe₂W, μ -Fe₆W₇ and μ -Fe₇W₆. This type of calculation is computationally more expensive by a factor of approx. 10 as the phonon properties have to be

calculated for different volumes. The gain in accuracy does not justify using this approximation for all end-members. Hence, we have not used the QHA approximation for the metastable end-members.

In the quasi-harmonic approximation, the Gibbs energy was calculated for different volumes and then fitted to a Vinet equation of state [112] (Eq. 49).

$$G(T, P) = \min_V [U(V) + F_{ph}(V, T) + PV] \quad \text{Eq. 49}$$

Where V and P are volume and pressure, respectively. U(V) expresses the total energy of the electronic structure at constant volume [112]. The Gibbs energy curve in QHA approximation for Fe₂W is given as an example in *Figure 34*. The heat capacity at constant pressure is then calculated from Eq. 50:

$$C_P(V, T) = C_V + VB\alpha^2T \quad \text{Eq. 50}$$

Where B and α are the bulk modulus and the coefficient of thermal expansion, respectively.

In the same way, the entropy in quasi-harmonic approximation (QHA) is obtained from Eq. 51:

$$S_{QHA} = S_{ph} + VB\alpha^2T \quad \text{Eq. 51}$$

The volumes for the calculation of the Gibbs energy in the QHA approach were carefully chosen around the equilibrium. Some volumes too far from the equilibrium volume (> 8%) were discarded as their phonon dispersion curves showed negative frequencies. The number of computed volumes as well as the minimum, equilibrated and maximum volumes are given in *Table 21* for compounds where QHA approximation has been computed.

Compounds - Structure	Number of computed volumes	Min V, Eq. Vol., Max Vol. (Å ³)
Fe ₂ W – C14 Laves phase	7	133.12, 145.86, 159.39
Fe ₇ W ₆ – μ phase	7	500.5, 508.08, 539.18
Fe ₆ W ₇ – μ phase	5	505.16, 520.62, 552.49

Table 21 – Number of volumes and volumes of the crystal structure used to compute the quasi-harmonic calculations.

3. Calphad modeling

The Fe-W phase diagram in this work was modeled according to the Calphad method. In this method the thermodynamic equilibria are calculated from parameterized expressions of the Gibbs energies of the various phases as a function of composition, temperature (and pressure). In the present work, the Thermo-Calc software [144] was used throughout. Basic information on this method can be found in [78] as well as in the previous part Chapter II, Part III, p. 34.

This method consists on modeling the total Gibbs energy of a system as explained in previous part (i.e. Chapter II).

In the CEF, each phase with an appreciable homogeneity range can be divided into a number of sublattices on which several (but not necessarily all) constituting atoms can be placed. For crystalline phases with sufficiently simple crystal structures, the sublattices can be taken or derived from the crystallographic lattices of the crystal structure. Each arrangement of one atom on each sublattice (= “constituent array”) represents a limiting composition of the particular phase and is called an end-member. All end-members have their own energies of formation, most of which are hypothetical and inaccessible by experiments. In the present work, they were obtained from DFT calculations.

In Thermo-Calc [144], the interaction parameter L_{ij} of the Gibbs excess energy is expanded into a Redlich-Kister series:

$$L_{i,j}^{\alpha} = {}^0L_{i,j}^{\alpha} + {}^1L_{i,j}^{\alpha}(x_i - x_j) + {}^2L_{i,j}^{\alpha}(x_i - x_j)^2 + \dots + {}^nL_{i,j}^{\alpha}(x_i - x_j)^n \quad \text{Eq. 52}$$

The $L_{i,j}^{\alpha n}$ can themselves be temperature dependent (Eq. 53). Typically, parameters up to second order are used.

$${}^kL_{i,j}^{\alpha} = A + BT \quad \text{Eq. 53}$$

In a Calphad assessment, the end-member energies (G-terms) and interaction parameters (L-terms) are derived from available experimental and thermodynamic data. Results from atomistic simulations can either be used directly as functions of temperature or in the same way as experimental data.

3.1. Substitutional solutions: Liquid, BCC, FCC

The liquid, BCC and FCC phases are described using a substitutional solution model (one sublattice model). The molar Gibbs energy is given by Eq. 54:

$$G_m^\alpha = x_{Fe} {}^0G_{Fe}^\alpha + x_W {}^0G_W^\alpha + RT(x_{Fe} \ln x_{Fe} + x_W \ln x_W) + x_{Fe}x_W L_{Fe,W}^\alpha \quad Eq. 54$$

Where x_{Fe}, x_W are mole fractions of Fe and W respectively. G_m^α are the Gibbs energies of pure components. $L_{Fe,W}^\alpha$ is the interaction parameter describing the excess Gibbs energy; its constituent $L_{i,j}^\alpha$ are modeled using the Redlich-Kister polynomials as written in Eq. 52 with linear temperature dependence (Eq. 53).

3.1. λ -Fe₂W C14-Laves phase

The C14 AB₂ Laves phase crystallizes in the hexagonal MgZn₂ structure with three different Wyckoff positions (A on 4f, B on 2a and 6h). The Laves phase is shown as a line compound in the experimental Fe-W phase diagram and has been modeled by Gustafson [48] as such. At present, a two-sublattice model (A,B)₂(A,B)₁ is frequently being used and has been implemented in various databases. In the two-sublattice model, the 2a and 6h positions are combined into one sublattice resulting in (Fe,W)₂ (W,Fe)₁ for the Fe₂W phase and the following expression for the Gibbs energy:

$$G^{C14} = y_{Fe}^1 y_W^2 G_{Fe:W}^0 + y_W^1 y_{Fe}^2 G_{W:Fe}^0 + y_W^1 y_W^2 G_{W:W}^0 + y_{Fe}^1 y_{Fe}^2 G_{Fe:Fe}^0 \\ + RT[2(y_{Fe}^1 \ln y_{Fe}^1 + y_W^1 \ln y_W^1) + (y_{Fe}^2 \ln y_{Fe}^2 + y_W^2 \ln y_W^2)] \quad Eq. 55$$

where $y_{W,Fe}^{1,2}$ are the lattice fractions of components W and Fe in sublattices 1 and 2. $G_{Fe:W}^0$ (representing Fe₂W) can be experimentally determined, e.g. enthalpy of formation at 298 K from drop solution calorimetry and C_p-measurements. The energies for the other combinations are hypothetical compounds and can only be estimated or calculated using ab initio methods. No interaction parameters were included in the description of this phase.

Although the physically correct model would be a three sublattice model, in the present work it was decided to use the common two sublattices model for this phase in order to maintain consistency with existing databases. The gain in accuracy by introducing the third sublattice compared to the higher number of parameters (especially when expanding to ternary systems and with respect to the limited data basis in the Fe-W system) has up to now not justified this step.

3.2. μ -phase

The μ -phase shows a small homogeneity range according to the experimental data [44,47] that is shifted towards the Fe-rich side of the system at higher temperatures. Furthermore, this phase has a deficiency of W. The common four sublattice model $(\text{Fe},\text{W})_1(\text{W})_4(\text{Fe},\text{W})_2(\text{Fe},\text{W})_6$ combining two Fe-free $6c$ positions ($x=0.333$ and $x=0.165$) into one has been used. According to experimental and theoretical investigations of this phase [85], this model allows to cover the whole homogeneity range and is compatible with other systems. According to this sublattice model, the atoms are distributed over the different Wyckoff position as showed in *Table 22*.

Space group R-3m, No. 166 Wyckoff position				
3b	6c ₁ , 6c ₂	6c ₃	18h	
(Fe,W)	(W) ₄	(Fe,W) ₂	(Fe,W) ₆	End-member
Fe	W	Fe	Fe	Fe ₉ W ₄
W	W	Fe	Fe	Fe ₈ W ₅
Fe	W	W	Fe	Fe ₇ W ₆
W	W	W	Fe	Fe ₆ W ₇
Fe	W	Fe	W	Fe ₃ W ₁₀
W	W	Fe	W	Fe ₂ W ₁₁
Fe	W	W	W	FeW ₁₂
W	W	W	W	W ₁₃

Table 22 - Atoms placed on the different Wyckoff position constituting the different end-members in the μ phase according to the four sublattice model [85] and name of the different end-members according to this model.

The general expression for the Gibbs energy of the μ -phase is given by *Eq. 56*:

$$\begin{aligned}
G^\mu = & y_{Fe}^1 y_W^2 y_{Fe}^3 y_{Fe}^4 G_{Fe:W:Fe:Fe}^0 + y_{Fe}^1 y_W^2 y_{Fe}^3 y_W^4 G_{Fe:W:Fe:W}^0 \\
& + y_{Fe}^1 y_W^2 y_W^3 y_{Fe}^4 G_{Fe:W:W:Fe}^0 + y_{Fe}^1 y_W^2 y_W^3 y_W^4 G_{Fe:W:W:W}^0 \\
& + y_W^1 y_W^2 y_{Fe}^3 y_{Fe}^4 G_{W:W:Fe:Fe}^0 + y_W^1 y_W^2 y_{Fe}^3 y_W^4 G_{W:W:Fe:W}^0 \\
& + y_W^1 y_W^2 y_W^3 y_{Fe}^4 G_{W:W:W:Fe}^0 + y_W^1 y_W^2 y_W^3 y_W^4 G_{W:W:W:W}^0 \\
& + RT[(y_{Fe}^1 \ln y_{Fe}^1 + y_W^1 \ln y_W^1) + 2(y_{Fe}^3 \ln y_{Fe}^3 + y_W^3 \ln y_W^3) \\
& + 6(y_{Fe}^4 \ln y_{Fe}^4 + y_W^4 \ln y_W^4)]
\end{aligned}
\tag{Eq. 56}$$

No interaction parameters were included in the description of this phase.

4. Optimization method

The optimization of the phase diagram was done using the PARROT module implemented in the Thermo-Calc software [144]. PARROT is based on the least-squares method and allows fitting parameters in the mathematical model chosen for the description of the phases so that they represent the available experimental data.

Each phase of the Fe-W system is modeled separately. For the pure elements Fe and W, we used the unary database from SGTE according to Dinsdale [81] in order to stay in consistency with other work and existing databases. The two intermetallic compounds (i.e. C14 Laves and μ - phases) are modeled using the sublattice models as previously described.

For the modeling of Fe-W, two different optimizations of the phase diagram are proposed; one including the DFT calculations for the description of the energy of the end-members, the second including the use of the phonon results for a description of the temperature dependence of the end-members. For both phase diagrams, the same sublattice models were used.

4.1 BCC, Liquid, FCC

For these phases, we used the values of the excess energy given by Gustafson [48] as starting values for our optimization. We started the assessment procedure with the liquid. In addition to the phase boundary data [44], there are thermodynamic data given on the Fe-rich [148–150] side of the system.

We used these data to obtain the excess energy of the liquid phase using the L_0 parameter with temperature dependence and the L_1 and L_2 parameters without temperature dependence. The optimized liquid phase was the basis to optimize the BCC phase which with the limited experimental knowledge cannot easily be optimized. Hence, the parameters for BCC, were optimized together with those for the liquid. The different data points from Sinha and Hume-Rothery [44] on the Fe-rich side describing the liquid + BCC equilibrium were used. The local minimum given at 1802 K at 4.4 at. % W [44] forming the congruent melting point was taken with a higher weight in order to correctly fix this part of the phase diagram. Additionally, we also used a higher weight on the peritectic reactions P_1 and P_2 (see *Table 20*). While data are well known on the Fe-rich side on the phase diagram, there are less data on the W-side due the experimental difficulties caused by the high temperature and slow diffusivity. In order to model this side of the phase diagram, we used the data from Sinha and Hume-Rothery [44] for the temperature and the composition of the liquid (i.e. 1910 K and 20.6 at. % W). However, the composition of W-BCC was taken from Ichise et al. [143] (i.e. 97.4 at. %), as these data are in agreement with the extrapolation of the data from Antoni-Zdziobek et al. [47] to higher temperature. The thermodynamic data were used with lower weight than the phase boundaries. The data of Sudavstova [148] were only taken with a weight of 20 % of the data from the phase boundary data. A higher weight on these data resulted in a too stable liquid phase which would be inconsistent with the experimental phase diagram data. As previously mentioned in the literature part, the data from [148] are not well documented and can only be used with care. Lowering the weight on the thermodynamic data allowed to converge the optimization of the excess Gibbs energy for the BCC and Liquid and to provide a consistent description in good agreement with experimental data points from the literature [44,47].

At the last step of the optimization of the liquid and BCC phases, we introduced phase boundary data at lower temperature from Takayama et al. [151] and Antoni-Zdziobek et al. [47], in order to check their reliability with the actual description of the BCC phases. As these results immediately were in good agreement, full convergence and agreement between the experiments and the calculated values were quickly reached.

4.2 FCC

The excess Gibbs energy of the FCC-phase was optimized separately from the BCC and Liquid. This phase forms a closed γ -loop within the BCC-phase and thus highly depends on its correct description. Various experimental data of the phase boundaries [45,152,153] were used for the determination of the excess Gibbs energy. For the optimization of the FCC-phase, only the L_0 parameter with linear temperature dependence was used. The use of the L_1 parameter was tested but resulted in a stabilization of the phase around 50 at. % W so that it was finally rejected.

For the Liquid, BCC and FCC phases, the Gibbs energy curves were checked for several temperatures in order to verify the stability of the phases and their correct behavior.

4.3 The intermetallic phases: Laves- and μ -phases

As already mentioned in the previous section, two different approaches were taken for the description of these two phases. They will be described in the following section.

a) Using 0K enthalpies of formation from DFT calculations only

The calculated values of the enthalpy of formation from DFT simulations were used in the description of the different G-parameters of the end-members of the intermetallic phases. The data describing “pure” end-members, i.e. those containing one element only, were taken from the literature [38,137]. For λ -Fe₂Fe (i.e. C14 Laves phase only containing Fe), the value from Liu et al. [38] was used as it had already been used in several other assessments containing an Fe-based Laves phase, whereas the data from Sluiter [137] were taken for the pure λ -W₂W and μ -W₁₃. As shown in *Table 24* and *Table 25*, these values are in good agreement with the calculated end-member energies from the present work.

In the Calphad formalism, the enthalpy of formation in the Gibbs energy descriptions of the end-members typically refers to the pure elements (i.e. Fe, W) as given by the GHSERXX functions from Dinsdale [81]. These functions do not include magnetic contributions, which are added separately (for Fe in this case). In the DFT calculations magnetism is taken into account. If the enthalpies of

formation for the end-members from the DFT calculations are used directly as enthalpy parameters (with GHSERFE as reference) the resulting discrepancy would cause the non-stability of the Laves and μ -phases at low temperature in the calculated phase diagram, while DFT calculations (present work) and experiments [154] suggest the opposite. The enthalpies of formation of the end-members calculated by DFT in the present work were therefore treated as experimental values and the corresponding parameters were optimized referring to the pure element GHSERXX functions as described by Dinsdale [81].

b) Finite temperature DFT

The results from the phonon calculations (harmonic and quasi-harmonic approximations) were used to obtain a temperature dependent description of the end member Gibbs energies. They were treated as mathematical functions following the polynomial conventionally used in the Calphad approach as shown in *Eq. 57*:

$$G = A + BT + CT\ln(T) + D_2T^2 + D_3T^3 + D_{-1}T^{-1} \quad \text{Eq. 57}$$

This polynomial can in principle be extended to a higher power series as a function of temperature, but becomes more and more complex without contributing to the improvement of the fit to experimental or calculated values. It has to be noted that in any case this polynomial cannot be extended to low temperatures ($\sim 100\text{K}$). The heat capacity is derived from this function and is given in *Eq. 58*:

$$C_P(T) = -T \left(\frac{\partial^2 G}{\partial T^2} \right) = -C - 2D_2T - 6D_3T^2 - 2D_{-1}T^{-2} \quad \text{Eq. 58}$$

Close to the melting temperature of the compound, the experimental heat capacity curves often show a nonlinear tendency. The D_3 parameter allows to take into account this effect. In the present phonon results, this non-linear behavior of the heat capacity was not found so that the D_3 parameter was not used except for the description of the Fe_7W_6 end-member.

Our phonon heat capacity data were fitted according to *Eq. 58* from 100 K to their high temperature limits using a nonlinear least squares fit. The B parameter of the Gibbs energy function (*Eq. 57*) was then obtained by fitting the previously obtained parameters (C and D_n) to the entropy curve of the phonon calculations. The resulting parameters (*Table 26* and *Table 27*) were used as starting values for the thermodynamic assessment of the Fe-W system. For the hypothetical end-members, the parameterized Gibbs energy functions were used directly without modification in the assessment. The parameters of the λ -Fe₂W and μ -Fe₇W₆ end-members were optimized as they were found to highly influence the stability of the phases. Nevertheless, no big deviations resulted from the optimization. Additionally, we included a D_3 parameter for the description of the Gibbs energy function of μ -Fe₇W₆ as the minimization over the sublattices showed a nonlinear tendency at high temperature (*Figure 40*). The obtained parameters are given in *Table 27*.

III. Results and discussions

1. DFT calculations

The results of the energies of formation of the pure elements Fe and W as well as their lattice parameters (output from VASP) are given in *Table 23* where they are also compared with available literature data. A lower energy than in the literature [38,88,107] was noticed in the present work. For Fe, this can be explained by the use of the highest numbers of k-mesh points in the present work, while for W there is no information about the k-mesh in the literature [107].

Compound	k-mesh point	Energies (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	References
Fe (FM)	11x11x11	-800.234	2.8304	Present work
	11x11x5	-792.614	2.832	[38]
	11x11x7	-792.603	2.832	[88]
		-798.39686	2.822	[107]
W	11x11x11	-1253.702	3.173	Present work
		-1232.8993	3.19	[107]

Table 23 – Energies and lattice parameters of Fe and W in their pure stable state bcc compared with literature.

For the different end-members present in the system (e.g. Laves and μ -phases), the enthalpies of formation were calculated according to *Eq. 46*. Enthalpy of formation results are presented in *Table 24* and *Table 25* with their respective optimized lattice parameters for ferro- and non-magnetic states and a comparison with available literature data. Good agreement with literature data can be seen.

Compound		k-mesh points	FM				NM				References
			$\Delta_f H$ (kJ.mol ⁻¹ . atom ⁻¹)	Lattice parameter		V (Å ³)	$\Delta_f H$ (kJ.mol ⁻¹ . atom ⁻¹)	Lattice parameter		V (Å ³)	
				a (Å)	c (Å)			a (Å)	c (Å)		
C14 Laves phase	Fe ₂ W	11x11x7	-2.83	4.659	7.757	145.86	-0.966	4.668	7.618	143.76	present work
			-8.93	-	-	-	-	-	-	-	Calphad, present work
		11x11x8	0.68	4.678	7.587	143.79					VASP, [155]
			-2.54								Exp., [154]
			-1.85								Exp., [148]
			-3.07	4.71	7.72	148.3	-0.75	4.699	7.61	145.5	VASP, [117]
	W ₂ Fe	11x11x7	82.29	5.152	8.19	188.29	82.29	5.152	8.19	188.29	present work
		11x11x8	82.6	5.15	8.1865	188.04					VASP, [155]
	Fe ₂ Fe	11x11x7	16.69	4.68	7.63		35.86	4.511	7.278	128.28	present work
		11x11x5	14.71	4.68	7.62	144.3	35.42	4.531	7.288	129.6	VASP, [38]
			38.81	-	-	-					VASP, [156]
		24x24x24	14.7	4.69	7.668	146.1					VASP, [157]
			31.1	-	-	-					VASP, [137]
		11x11x8	38.27	4.53	7.23	128.5					VASP, [155]
	W ₂ W	11x11x7					44.18	5.19	8.58	200.15	present work
		11x11x8					44.4	5.19	8.57	199.92	VASP, [155]
							43.7				VASP, [137]
		10x10x10						-	-	-	VASP, [120]

Table 24 – Enthalpies of formation of end-members in C14 Laves phase compared with literature [38,117,137,148,154–157].

Compound		k-mesh points	FM			NM			References
			$\Delta_f H$ (kJ.mol ⁻¹)	Lattice parameter		$\Delta_f H$ (kJ.mol ⁻¹)	Lattice parameter		
				a (Å)	c (Å)		a (Å)	c (Å)	
μ phase	Fe ₉ W ₄	9x9x5	0.46	4.69	25.35	6.15	4.66	24.87	Present work
			-5.8	-	-	-	-	-	Calphad, present work
	Fe ₈ W ₅	9x9x5	6.01	4.75	25.72	8.02	4.74	25.11	Present work
	Fe ₇ W ₆	4x4x1	-3.24	4.73	25.9	-0.46	4.74	25.56	Present work
			-8.17						Calphad, present work
			-0.71						Exp., [148]
			-2.52	4.75	25.77	-0.71	4.743	25.30	[117]
	Fe ₆ W ₇	9x9x5	-0.975	4.81	26.02	0.44	4.82	25.74	Present work
			-5.2						Calphad, present work
	Fe ₃ W ₁₀	9x9x5	33.086	5.10	26.86	37.45	5.07	26.79	Present work
	Fe ₂ W ₁₁	9x9x5	37.245	5.13	27.098	41.08	5.11	27.30	Present work
	FeW ₁₂	9x9x5	28.89	5.12	27.47	34.02	5.11	27.46	Present work
	W ₁₃	9x9x5				34.53	5.122	28.347	Present work
						36			VASP, [137]

Table 25 – Enthalpies of formation of the end-members in the μ phase in Fe-W system compared with literature [117,137,148].

For the Laves and the μ - phases, the DFT calculations suggest that most of the end-members containing Fe are more stable in their ferromagnetic state so that these data were used in subsequent calculations.

2. Phonon results

In the phonon calculations, the stabilities of the end-members of the Laves and the μ -phase were checked using the phonon dispersion curves. For some (hypothetical) end-members, negative (imaginary) phonon frequencies were found in their respective dispersion curves. They are the result of so-called mechanical instability of the structure at compositions of hypothetical end members that do not exist in reality but are nevertheless needed to compute the homogeneity ranges of the phases. In particular, mechanically unstable end-members were found in the μ -phase for Fe_9W_4 , Fe_8W_5 and FeW_{12} . Several different sets of k-points were tried, but all tested set ups showed negative phonon frequencies. For example, in the Fe_9W_4 end-member Fe is placed on the 6c ($z=0.052$) position (see Table 22), whereas in the stable structure this site is occupied by only W (experimentally proven and thermodynamically stable). Thus, the 6c position is highly unfavorable to be occupied by Fe. Furthermore, this end-member has a stoichiometry close to the composition of the Laves phase which is thermodynamically stable. Due to these reasons, no optimized structure reached a local minimum in the configurational space [83] leading to mechanical instability of the end-member in the phonon calculations. Negative phonon frequencies can be observed in a high amount. If in some cases these negative phonon frequencies can be seen as a residual artifact and a new optimization of the geometry of the crystal structure is necessary it is not the case here. The dispersion curves of $\mu\text{-Fe}_9\text{W}_4$ show that this end-member would transform in a more stable form [83] (i.e. Laves phase in this case). As the thermodynamic properties are not calculated from negative frequencies so the heat capacities of these end-members deviate strongly from the Dulong-Petit rule ($\sim 25 \text{ J}\cdot\text{mol}^{-1}\cdot\text{atom}^{-1}$). As a consequence, we rejected the phonon calculation results for these compounds in the subsequent Calphad modeling. Instead, we only used the enthalpy of formation obtained from DFT to describe these end-members.

In the Laves phase, phonon calculations were performed using the ferromagnetic state for Fe_2Fe and Fe_2W whereas nonmagnetic states were used for W_2Fe and W_2W (no imaginary frequencies on the phonon dispersion curve). The results of the heat capacity at constant volume are shown in Figure 32. In addition, the heat capacity at constant pressure for Fe_2W is given.

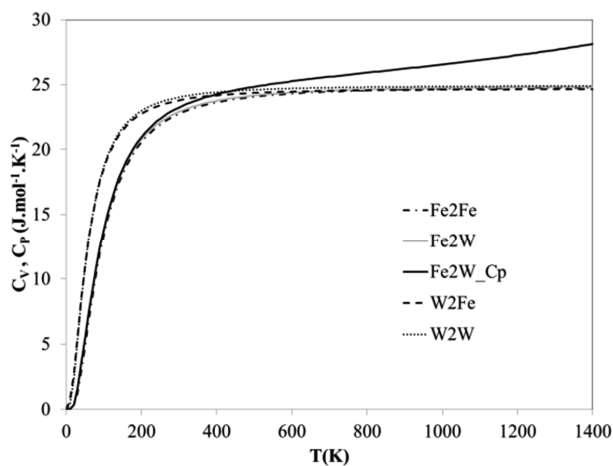


Figure 32 – Calculated heat capacity at constant volume from phonon calculations for the different end-members in the Laves phase. The heat capacity at constant pressure of the stable compound Fe_2W is given as black line obtained from quasi-harmonic approximation phonon calculations.

For Fe_7W_6 , using $9 \times 9 \times 5$ k-mesh points provided higher energies when optimizing the geometry than using a set of $4 \times 4 \times 1$ k-mesh points. For both sets of k-mesh points, we used the optimized structure to compute phonon calculations. Using optimized geometry from $9 \times 9 \times 5$ k-mesh point, negative phonon frequencies were present whereas this was not the case for the structure from $4 \times 4 \times 1$ k-mesh points. To calculate the thermodynamic properties we used the optimized geometry structure from $4 \times 4 \times 1$ k-mesh points as it represents the local minimum ground state for this structure.

The heat capacity curves are given in Figure 32 and Figure 33 for the Laves and μ -phases, respectively. All the computations of the heat capacity at constant volume reach the value of Dulong-Petit of $25 \text{ J.mol}^{-1}.\text{atom}^{-1}$ at high temperature. For the Calphad modeling, they have been described as given in Eq. 57. The obtained parameters are given in Table 26 and Table 27 for the different end-members in the Laves and μ -phases, respectively. Nevertheless, for the thermodynamic modeling, the pure end-members $\lambda\text{-Fe}_2\text{Fe}$, $\lambda\text{-W}_2\text{W}$ and $\mu\text{-W}_{13}$ were described without the phonon data as they were taken from Liu et al. [38] and Sluiter [137] for consistency with other assessments. The parameters

fitted to Eq. 57 to describe the heat capacity curves of the phonon results were then adjusted from optimization when necessary.

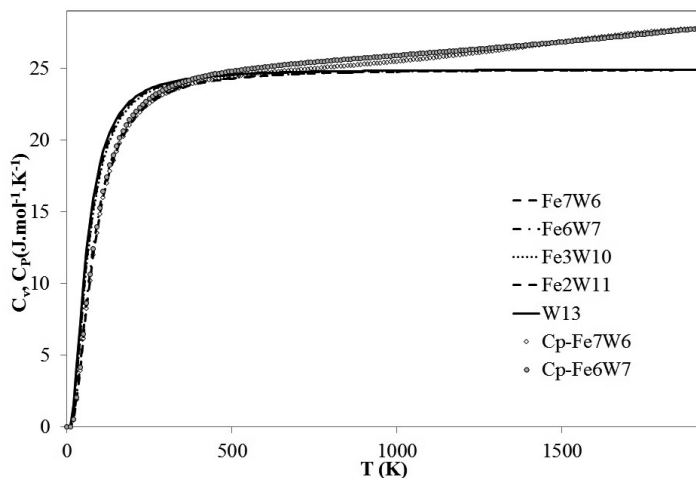


Figure 33 –Heat capacity at constant volume and pressure for the different end-members in the μ phase. The data of the end-members which showed imaginaries frequencies are not shown here.

	Magnetism	SC	A	B	C	D ₂	D ₋₁
Fe ₂ Fe	FM	2x2x2	50070	397.02	-72.02	-1.125e-3	181868.8
Fe ₂ W	FM	2x2x2	-8490	397.1	-72.11	-2.93e-3	190812.9
FeW ₂	NM	3x3x1	246880	368.55	-73.41	-2.93e-4	95255.9
W ₂ W	NM	2x2x2	103590	374.94	-74.15	-2.94e-4	97122.9

Table 26 – Optimized parameters obtained by fitting the phonon results to Eq. 58 for the end-members in the Laves phase and used as starting values for Fe₂W and W₂Fe description on the thermodynamic phonon phase diagram (SC: supercell).

	Magnetism	SC	A	B	C	D ₂	D ₋₁
Fe ₉ W ₄	FM	-	6039.8	-	-	-	-
Fe ₈ W ₅	FM	-	78212	-	-	-	-
Fe ₇ W ₆	FM	2x2x1	-42129.9	1647.7	-306.171	1.43e-2	623344.5
Fe ₆ W ₇	FM	2x2x1	-12680.7	1697.2	-314.1	1.214e-2	647792.1
Fe ₃ W ₁₀	FM	2x2x1	430118	1680.9	-321.69	-8.17e-4	510840.6
Fe ₂ W ₁₁	FM	2x2x1	484188	1675.9	-322.00	-7.198E-4	491025.1
FeW ₁₂	FM	-	375549	-	-	-	-
W ₁₃	NM	2x2x1	448880	1656.4	-322.24	-6.44E-4	454523.77

Table 27 – Fitting of the phonon heat capacities curves for the μ phase according to Eq. 58 and used as starting values for the thermodynamic modeling of the phonon phase diagram.

The stable end-members (i.e. λ -Fe₂W, μ -Fe₇W₆ and μ -Fe₆W₇) were modeled within the quasi-harmonic approximation in order to take into account the thermal expansion. The Gibbs energy of λ -Fe₂W, μ -Fe₇W₆ and μ -Fe₆W₇ within the quasi-harmonic approximation is given in *Figure 34*, *Figure 35* and *Figure 36*, where for each volume point the phonons were calculated. From these calculations, the heat capacity at constant pressure is obtained (Eq. 50). The heat capacities at constant pressure for λ -Fe₂W and μ -Fe₇W₆ are given in *Figure 39* and *Figure 40*. The fitted results (Eq. 57) were then used for the descriptions of these end-members in the phonon phase diagram.

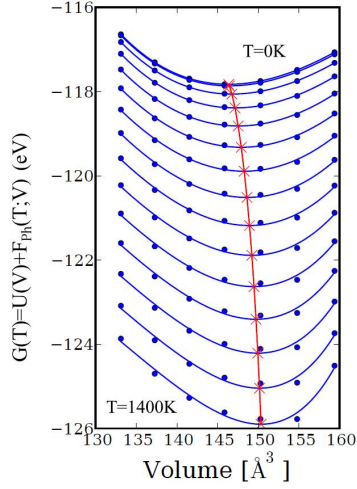


Figure 34 – Gibbs energy of Fe_2W in QHA approximation as a function of the unit cell and temperature. The minimum values of the fitted thermodynamic calculations are given by the red crosses.

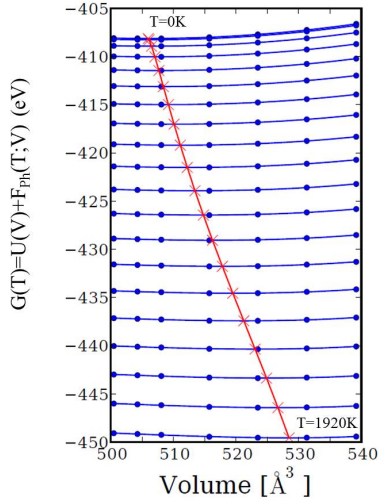


Figure 35 - Gibbs free energy ($G(V, T)$) of $\mu\text{-Fe}_7\text{W}_6$ calculated using the QHA approximation. The point denotes the calculated $G(V, T)$ at different volumes, the red crosses are the minimization of the curve according to the Vinet equation of state.

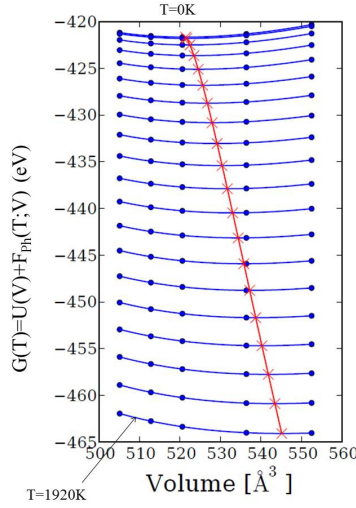


Figure 36 - Gibbs free energy ($G(V, T)$) of $\mu\text{-Fe}_6\text{W}_7$ calculated using the QHA approximation.

3. Thermodynamic optimization

As mentioned in the previous part the liquid, BCC and FCC phases were modeled independently from the DFT and phonon calculations. Nevertheless as last step of the optimization, the use of DFT and/or phonon calculations influence the interaction parameters of the BCC phase. It turns out that the L_0 parameters of the BCC had to be adjusted depending on the use of the DFT or phonon. As the BCC phase is in direct interaction with the Liquid and FCC phases, their L_0 parameters were also adjusted depending on the use of DFT or phonon calculations.

3.1. DFT based phase diagram

The end-members of the intermetallic phases were defined with their respective enthalpies of formation obtained from DFT calculations (Table 24 and Table 25). The enthalpies of formation of several end-members were optimized using the calculated enthalpies of formation as experimental values as the function of pure Fe described by the GHSEFE function [81] which does not include the contribution of magnetism. The obtained optimized enthalpies of formation are given in Table 24 and Table 25 and compared with the DFT values. There is a difference about 8 kJ.mol^{-1} per Fe atom (see Figure 24) arising from the magnetism of Fe which is not included in the Dinsdale polynomial [81].

The phase boundaries and the temperature dependence of the two intermetallic phases (Laves and μ phases) were adjusted by introducing temperature dependent parameters for the stable end-members (i.e. λ -Fe₂W, μ -Fe₇W₆ and μ -Fe₆W₇) and the Fe-rich end-members of the μ -phase (i.e. Fe₉W₄).

The optimization of the Laves phase was done by using a parameter for the temperature dependence of the stable end-member (i.e. Fe₂W) and using the data of Sinha and Hume-Rothery [44] for the peritectoid reaction BCC+ μ =C14 at 1333 K. In addition, we used the phase boundary data from Antoni-Zdziobek et al. [47] to check that the convergence of the optimization was correctly achieved. We did not introduce temperature dependent parameters for the metastable end-members of the Laves phase.

In the same way as for the Laves phase, the μ -phase was optimized by using the higher peritectic reaction given by Sinha and Hume-Rothery [44] (i.e. 1910 K) and phase boundary data from Antoni-Zdziobek et al. [47]. Three end-members were adjusted with their temperature dependence (i.e. Fe₉W₄, Fe₇W₆ and Fe₆W₇). The introduction of the temperature dependence of Fe₉W₄ influences the phase boundary on the Fe-rich side of the μ -phase whereas for the temperature dependence of Fe₇W₆, the peritectic temperature is affected. Temperature dependence of the Fe₆W₇ was introduced to destabilize the μ -phase on the W-rich side.

Nevertheless, we did not use interaction parameters for these phases, to keep the modeling as simple as possible (with modeling of higher systems based on Fe-W in mind) and due to the limited amount of available experimental data.

The optimized excess Gibbs energy parameters are given in *Table 28*. The calculated phase diagram obtained from this optimization is shown in *Figure 37* compared with experimental data. The calculated invariant reactions obtained from this phase diagram are given in *Table 30*.

Phase	Parameters	Remark
Liquid	${}^0G_{\text{FeW}} = -19406.3 + 12.1 \cdot T$	(Opt)
	${}^1G_{\text{FeW}} = 1743.7$	(Opt)
	${}^2G_{\text{FeW}} = 4633.3$	(Opt)
BCC	${}^0G_{\text{FeW}} = 18713.6 + 10.8 \cdot T$	(Opt)
	${}^1G_{\text{FeW}} = -8860$	(Opt)
	${}^2G_{\text{FeW}} = 7494.1$	(Opt)
FCC	${}^0G_{\text{FeW}} = 10944.5 + 5 \cdot T$	(Opt)
C14_Laves	${}^0G_{\text{Fe:Fe}}^{C14} = 3 {}^0G_{\text{Fe}}^{BCC_A2} + 44130$	[38]
	${}^0G_{\text{Fe:W}}^{C14} = 2 {}^0G_{\text{Fe}}^{BCC_A2} + {}^0G_{\text{W}}^{BCC_A2} - 26803.6 + 13.5T$	DFT, Opt
	${}^0G_{\text{W:Fe}}^{C14} = {}^0G_{\text{Fe}}^{BCC_A2} + 2 {}^0G_{\text{W}}^{BCC_A2} + 246880$	DFT
	${}^0G_{\text{W:W}}^{C14} = 3 {}^0G_{\text{W}}^{BCC_A2} + 131400$	[137]
μ	${}^0G_{\text{Fe:W:Fe:Fe}}^{\mu} = 9 {}^0G_{\text{Fe}}^{BCC_A2} + 4 {}^0G_{\text{W}}^{BCC_A2} - 76287.8 + 46.48T$	DFT, Opt
	${}^0G_{\text{W:W:Fe:Fe}}^{\mu} = 8 {}^0G_{\text{Fe}}^{BCC_A2} + 5 {}^0G_{\text{W}}^{BCC_A2} + 78213$	DFT
	${}^0G_{\text{Fe:W:W:Fe}}^{\mu} = 7 {}^0G_{\text{Fe}}^{BCC_A2} + 6 {}^0G_{\text{W}}^{BCC_A2} - 106224.1 + 48.93T$	DFT, Opt
	${}^0G_{\text{W:W:W:Fe}}^{\mu} = 6 {}^0G_{\text{Fe}}^{BCC_A2} + 7 {}^0G_{\text{W}}^{BCC_A2} - 67688.98 + 80T$	DFT, Opt
	${}^0G_{\text{Fe:W:W:W}}^{\mu} = 3 {}^0G_{\text{Fe}}^{BCC_A2} + 10 {}^0G_{\text{W}}^{BCC_A2} + 430118$	DFT
	${}^0G_{\text{W:W:W:W}}^{\mu} = 2 {}^0G_{\text{Fe}}^{BCC_A2} + 11 {}^0G_{\text{W}}^{BCC_A2} + 484188$	DFT
	${}^0G_{\text{Fe:W:W:W}}^{\mu} = 1 {}^0G_{\text{Fe}}^{BCC_A2} + 12 {}^0G_{\text{W}}^{BCC_A2} + 375549$	DFT
	${}^0G_{\text{W:W:W:W}}^{\mu} = 13 {}^0G_{\text{W}}^{BCC_A2} + 469300$	[137]

Table 28 – Thermodynamic parameters for the description of the DFT phase diagram.

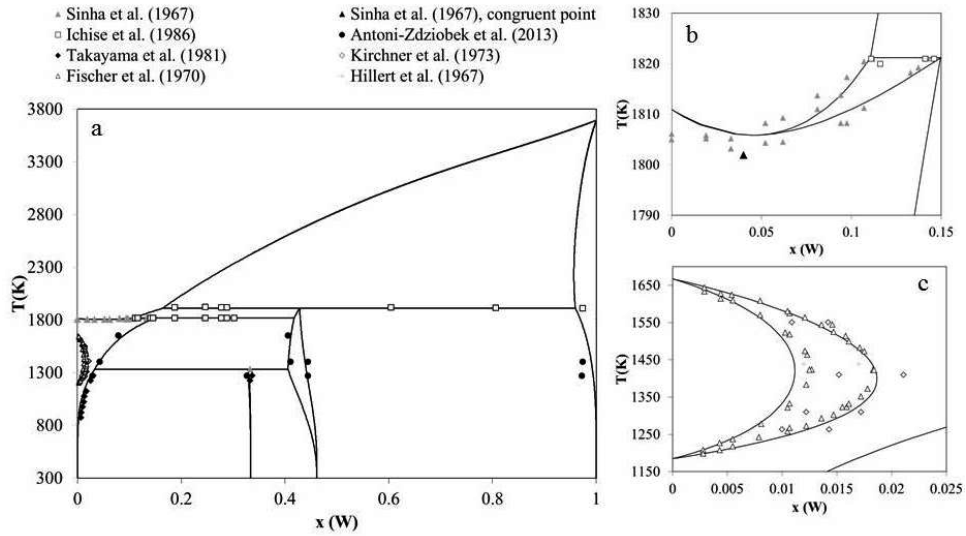


Figure 37 - Calculated phase diagram (solid line) using DFT calculations for the description of the end-members compared with experimental data [44,45,47,143,151–153]. a) Full range of the phase diagram, b) Congruent point on the Fe-rich side, c) Gamma loop.

3.2. Phase diagram based on phonon calculations

The calculated phase diagram obtained using phonon calculation is given in Figure 38 and their respective parameters in Table 29. The particularity of our phonon calculations is that they describe a Gibbs energy function for a fixed site occupation as they are calculated for each end-member whereas the phase itself is described by a mixed occupation, which is modeled by the sublattice model formalism. The principle of the Calphad method is the minimization of the Gibbs energy, the calculation of the phase diagram is done by using the site fractions that minimize it. The use of phonon results can easily be handled for the description of ordered phases as e.g. the Laves phase. However, in the case of the μ -phase e.g. the Fe_7W_6 stoichiometry can be achieved by full ordering on the sublattices or mixed occupancies. As Thermo-Calc minimizes the Gibbs energy of the phase over the sublattices it results in a mixed occupancy for a given composition while the phonon data were generated for the ordered end member. As a consequence, the Gibbs energy for the composition Fe_7W_6 obtained from Thermo-Calc and phonon calculations deviate. In order to make comparisons with phonon data for the

considered end-members in the μ -phase, we defined each end-member where phonon calculations were used for the description of the Gibbs energy as a separate phase (line compound). The same Gibbs energy function is used to describe the end-member in the μ -phase and in the separate phase. As a consequence the heat capacity curves of the end-members obtained from the thermodynamic modeling is in agreement with the phonon calculated heat capacity, but different from the heat capacity of the μ phase with the same composition.

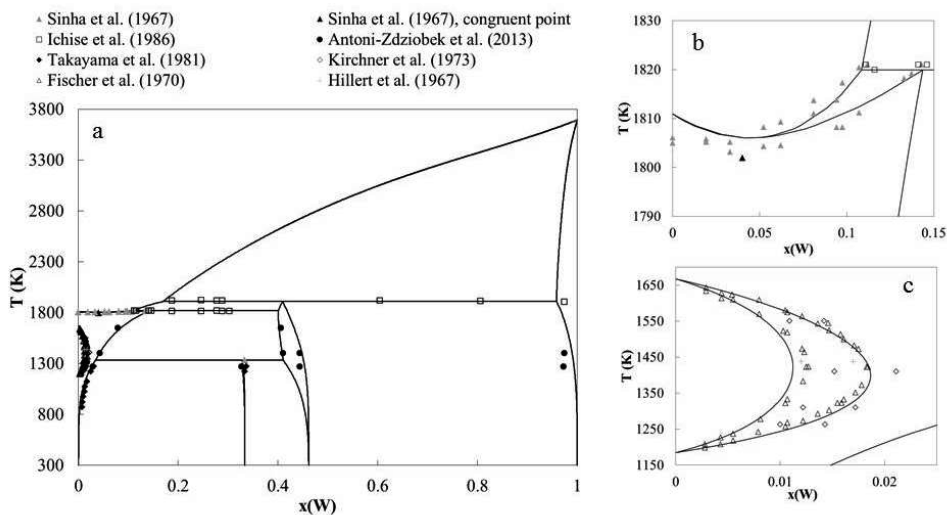


Figure 38 - Calculated phase diagram with phonon calculations for the description of the end-members compared with experimental data [44,45,47,143,151–153]. a) Full range of the phase diagram, b) Congruent point on the Fe-rich side, c) Gamma loop.

As the description of pure Fe does not include the magnetic contribution in the description given by Dinsdale [81], we optimized the enthalpy of formation of the end-members in the same way as in the DFT based phase diagram. The hypothetical end-members that are highly unstable had no need to be adjusted, whereas the stable end-members had to be fitted to the DFT results (Table 24 and Table 25) used as „experimental“ data. The enthalpy of formation of Fe_9W_4 also had to be optimized as it influences the phase boundary of the μ -phase on the Fe-side and also the Laves phase temperature dependence. For the mechanically unstable end-members (no heat capacity description from phonon

calculations) it was necessary to optimize their temperature dependence using phase boundary data [47] and the peritectoid P₁ invariant reaction [44].

Phase	Parameters	Remark
Liquid	${}^0G_{FeW} = -25460.9 + 14.97T$	(Opt)
	${}^1G_{FeW} = 1743.7$	(Opt)
	${}^2G_{FeW} = 4633.3$	(Opt)
BCC	${}^0G_{FeW} = 8707.65 + 15.84T$	(Opt)
	${}^1G_{FeW} = -8860.1$	(Opt)
	${}^2G_{FeW} = 7494.1$	(Opt)
FCC	${}^0G_{FeW} = 980.9 + 10 * T$	(Opt)
C14_Laves	${}^0G_{Fe:Fe}^{C14} = 3 {}^0G_{Fe}^{BCC_A2} + 44130$	[38]
	${}^0G_{Fe:W}^{C14} = -31100.4 + 376.7T - 70.434T \ln T - 4.92E^{-3}T^2 + 166642T^{-1}$	Opt, Phonon
	${}^0G_{W:Fe}^{C14} = +246880 + 368.28T - 73.5T \ln T - 2.98E^{-4}T^2 + 100893T^{-1}$	Phonon
	${}^0G_{W:W}^{C14} = 3 {}^0G_W^{BCC_A2} + 131400$	[137]
μ	${}^0G_{Fe:W:Fe:Fe}^{\mu} = 9 {}^0G_{Fe}^{BCC_A2} + 4 {}^0G_W^{BCC_A2} - 76329 + 39.68T$	Opt
	${}^0G_{W:W:Fe:Fe}^{\mu} = 8 {}^0G_{Fe}^{BCC_A2} + 5 {}^0G_W^{BCC_A2} + 78213$	DFT
	${}^0G_{Fe:W:W:Fe}^{\mu} = -141384.5 + 1685T - 315.63T \ln T - 5.1419E^{-3}T^2 - 1.3199E^{-6}T^3 + 694229.8T^{-1}$	Opt, phonon
	${}^0G_{W:W:W:Fe}^{\mu} = -111740.6 + 1697.2T - 314.1T \ln T - 0.01214T^2 + 647792.1T^{-1}$	Opt, phonon
	${}^0G_{Fe:W:Fe:W}^{\mu} = +430118 + 1680.9T - 321.69T \ln T - 8.17E^{-4}T^2 + 510840.6T^{-1}$	Opt, Phonon
	${}^0G_{W:W:Fe:W}^{\mu} = +484185 + 1675.9T - 322.00T \ln T + 7.198E^{-4}T^2 + 491025.1T^{-1}$	Opt, phonon
	${}^0G_{Fe:W:W:W}^{\mu} = 1 {}^0G_{Fe}^{BCC_A2} + 12 {}^0G_W^{BCC_A2} + 375549$	DFT
	${}^0G_{W:W:W:W}^{\mu} = 13 {}^0G_W^{BCC_A2} + 469300$	[137]

Table 29 – Thermodynamic parameters for the description of the phonon phase diagram.

In Figure 39 and Figure 40 the C_p curves of λ -Fe₂W and μ -Fe₇W₆ from the optimization procedure are compared to the fitting of the results obtained from the phonon calculations. For λ -Fe₂W, the optimized C_p curve follows the results obtained from the phonon calculations, while in case of μ -Fe₇W₆, the C_p curve of the optimized data shows another trend as the one obtained from the phonon calculation. Nevertheless, the C_p curve of Fe₇W₆ defined as a new phase is in full agreement with the calculated C_p phonon data. As Thermo-Calc does not calculate this curve for the “stoichiometric” compound, i.e. μ -Fe₇W₆, but for the site fractions that minimize the Gibbs energy, the result can be

different. Furthermore, the C_P curve obtained from the thermodynamic optimization shows a strong increase at high temperature, which is consistent with experimental heat capacity curves that show a high increase close to the melting point of the phase. The data of the optimized C_P curve of Fe_7W_6 is consistent with the phase diagram behavior.

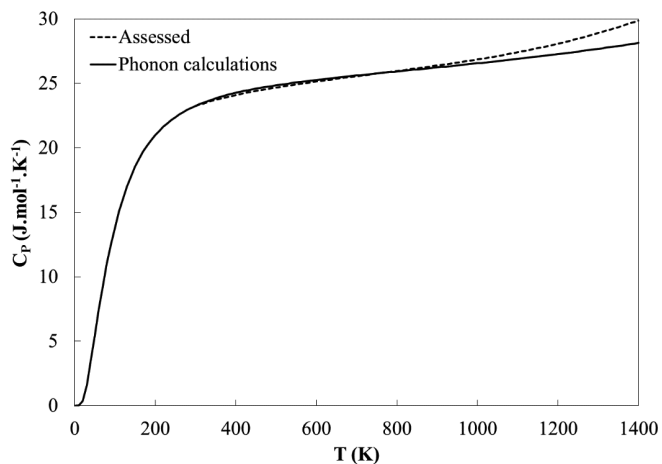


Figure 39 – Heat capacity of Fe_2W Laves phase calculated from phonon quasi-harmonic approximation (C_p) and Calphad.

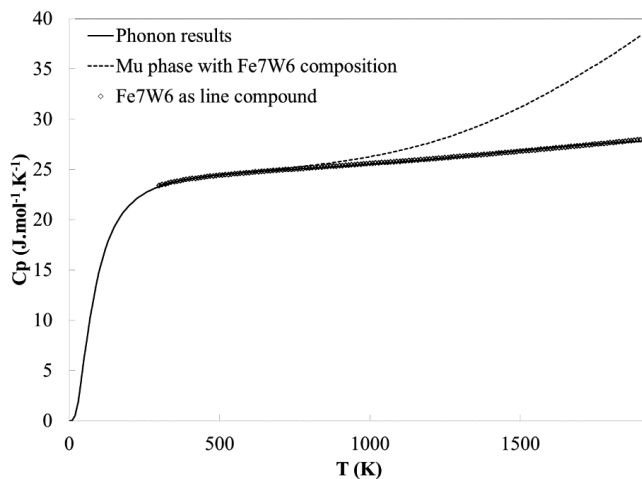


Figure 40 – Heat capacities of $\mu\text{-Fe}_7\text{W}_6$ calculated from the phonon thermodynamic assessment compared with results from quasi-harmonic approximation.

3.3. Comparison of the phase diagrams

The two proposed optimized phase diagrams calculated from the respective datasets are given in

Figure 41. In their general outline, they show a good agreement, and only differ in homogeneity range of the μ -phase. In the description of the phonon-based phase diagram, the homogeneity range of this phase is slightly bigger and has a phase boundaries with more pronounced curvature. In both phase diagrams, the homogeneity range of the μ phase is given with a better accuracy than the description given by Gustafson [48].

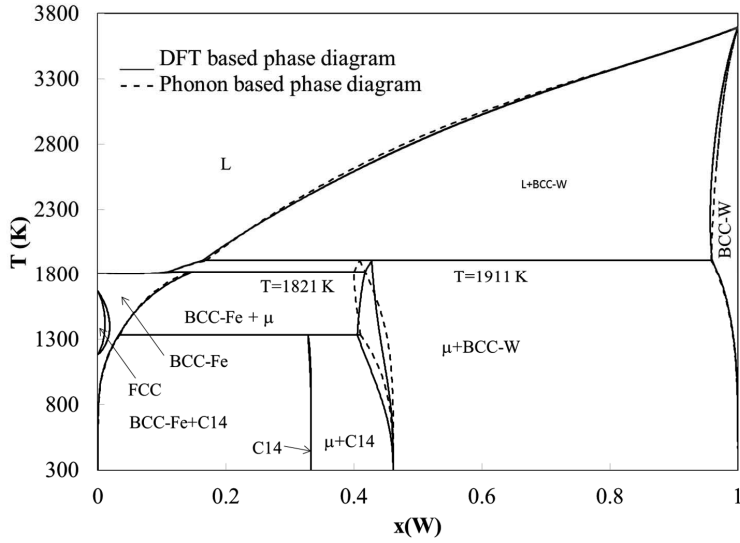


Figure 41 – Comparison of the two optimized phase diagram, with solid lines the calculated phase diagram from DFT and with dotted lines the calculated phase diagram using phonon calculations.

For both phase diagrams, the data of the invariant reactions are listed in *Table 30* with their respective temperatures and phase compositions. The calculated invariant reactions from both phase diagrams are in very good agreement with experimental data (*Table 20*). The largest deviation from the experiments is found for the congruent melting point of the BCC phase given at 1802 K by Sinha and Hume-Rothery [44] and calculated at 1806 K using our thermodynamics descriptions. The difference of 4 K is considered acceptable due to the use of a melting temperature of BCC-Fe by Sinha and Hume-Rothery [44] 5 K below the currently accepted temperature in [45].

	DFT phase diagram	Phonon phase diagram
Invariant reactions	T (K), x(W, at.%)	T (K), x(W, at.%)
P1: Liquid+BCC => μ	T=1910.74 x(BCC,W)=0.9597 x(Liquid,W)=0.1636 x(μ ,W)=0.4280	T=1910.63 x(BCC,W)=0.9585 x(Liquid,W)=0.1691 x(μ ,BCC)=0.4097
P2: L+ μ => BCC	T=1821.16 x(BCC,W)=0.1496 x(Liquid, W)=0.1107 x(μ ,W)=0.4176	T=1819.91 x(BCC,W)=0.1436 x(Liquid, W)=0.1085 x(μ ,W)=0.4003
C: L=>BCC	T=1805.86 x(BCC,W)=x(Liquid,W)=0.04517	T=1806.04 x(BCC,W)=x(Liquid,W)=0.0444
P3: BCC+ μ =>C14	T=1332.97 x(Laves, W)=0.3286 x(BCC,W)=0.0329 x(μ ,W)=0.4055	T=1332.91 x(Laves)=0.3284 x(BCC,W)=0.0341 x(μ ,W)=0.4105

Table 30 – Calculated invariant reactions obtained from the optimized phase diagrams based on DFT and phonon calculations.

The calculated enthalpies of mixing of liquid as a function of composition at 1900 K are shown in Figure 42, where good agreement with the data from Sudavstova [148] can be seen.

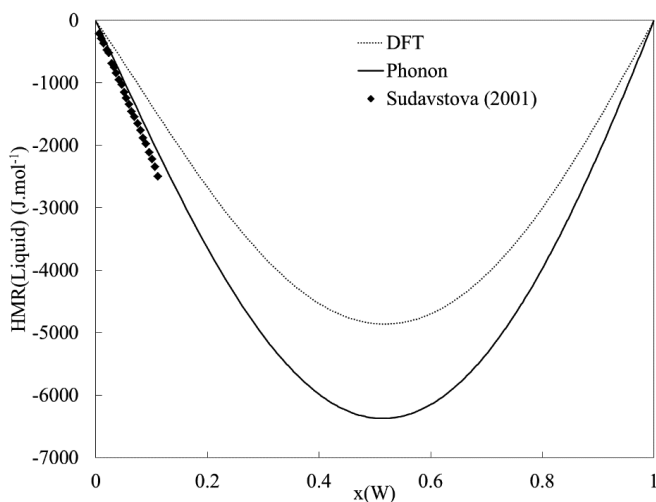


Figure 42 – Enthalpy of mixing of liquid at 1900 K calculated from the DFT and phonon optimized phase diagrams compared with data from Sudavstova [148].

The two optimized phase diagrams show that within the progress of atomistic simulation (i.e. DFT, phonon) allow to access thermodynamic properties of intermetallic phases where experimental

determination can become difficult (slow diffusivity, high temperature). Nevertheless, the use of phonon calculation data for the description of the Gibbs energy of end-members of disordered phases can lead to difficulties due to the global minimization algorithm implemented in thermodynamic calculation software such as Thermo-Calc [144].

Conclusion

In the present work, computational tools (DFT, phonon) were used to describe the two intermetallic phases (Laves and μ -phases) where thermodynamic data were missing or controversial. As any optimization of phase diagram can be better described with better knowledge of the thermodynamic properties of the intermetallic phases, DFT and phonon data are useful tool to obtain missing thermodynamic data.

The use of phonon calculated data could be seen as a great advantage for the description of the thermodynamic data where there are no experimental data. Nevertheless, their use should be taken with great care as they can lead to the creation of an artificial miscibility gap due to the minimization of the Gibbs energy through the sublattice as implemented in the different Calphad software codes.

In conclusion, the phonon data can be used for a better description of “ordered” phases (e.g. the Laves phase in our study). However, for partially disordered phases they should be used with care.

The two proposed phase diagrams have shown to be consistent with the thermodynamics of the system. Two versions of the phase diagram are presented in order to leave the choice for the next user which description to be used in higher order systems.

Chapter V – The Cr-Fe-Nb system

Introduction

In the present study, the Cr-Fe-Nb ternary system was evaluated experimentally (isothermal sections, solidification and thermal behavior) and computationally (DFT and Calphad modeling) in order to have a full understanding of the phase equilibria. The present experimental data were used for the thermodynamic optimization of the system.

The study of the Cr-Fe-Nb ternary system is divided in three different sections dedicated to the different aspects of phase equilibria and methods for their determination as follows:

- Experimental determination of the isothermal sections at 700°C, 1050°C and 1350°C
- Thermal and solidification behavior of the Cr-Fe-Nb system
- Thermodynamic modeling of the Cr-Fe-Nb ternary system

Literature

A literature survey had been given in the literature survey (p. 25). The system has been studied by Kaloev et al. [66,67] at two different temperatures (i.e. 700 and 1000°C), including the determination of phase equilibria using quantitative SEM. Their results are reproduced in the system report of Landolt-Börnstein [69] with a few modifications due to new findings in the limiting binary systems.

To the knowledge of the literature, this system doesn't contain any ternary compound. In contrast a high solubility of a third element is observed in the binary phases. BCC-(Fe,Cr) form a mutual continuous solid solution at high temperatures. The dissolution of Cr in the binary phase C14-Fe₂Nb laves phase is particularly high. This phase is reported to form a quasibinary system with the corresponding Cr₂Nb HT and LT (C14 and C15) phases which have limited solubility of Fe [158]. This would correspond to a local minimum in the ternary system as it is below the binary invariant reaction. This postulation, however, has not been proven until now and needs to be checked.

Mansour et al. [159] have investigated the temperature dependence of the Cr-Fe-Nb system in the Fe-rich corner. In their investigation they never referred to the existing knowledge of the system. They

have measured invariant reactions from as-cast samples without any further annealing and derived temperature dependence. These results are certainly not accurate as some assignments of the DTA measurements given are not consistent with the binary boundary phase diagram.

Although the micrographs in their work show eutectic structures, the assignment of reaction temperatures does not seem very likely, see e.g. the proposed eutectic at 835 °C which is far below any binary reaction temperature involving the liquid phase and as described would be in conflict with the phase equilibria corresponding to the γ -loop in the Fe-Cr system [4]. Similarly doubtful is the participation of the $\text{Fe}_{21}\text{Nb}_{19}$ (μ) phase in a Fe-rich eutectic reaction. This phase was identified by EDS measurements, but was not found in the corresponding XRD measurement. Its presence in such a reaction would be remarkable even if the $\text{Fe}_2\text{Nb} - \text{Cr}_2\text{Nb}$ quasibinary system did not exist.

The associated binary systems have been already studied and are well determined. The Fe-Nb system has been recently studied by Voß et al. [35]. The system contains two intermetallic phases, i.e. Laves and μ phases. In the Cr-Nb system, some controversies have been clarified recently. In former investigations, two different polytypes of Laves phases (C15 at low temperature and C14 in high temperature) have been postulated, but Aufrecht et al. [28] showed the metastability of the C14 high temperature Laves phase. In addition, a recent careful literature survey [29] has shown that this phase was often extrapolated from higher order systems or found in samples containing impurities. Recent investigations of Cr-Nb-Al by Stein et al. [18] also accepted the metastability of the C14 in Cr-Nb phase diagram and new DTA measurements have been done in order to determine the temperature of the invariant reactions. The Fe-Cr binary system is accepted from the Massalski version [24].

The various crystalline phases and solid solutions present in this system have been thoroughly characterized in the literature. Crystallographic data are summarized in *Table 31*.

Phase	Pearson Symbol	Space Group	Strukturbericht designation	Prototype	Lattice parameters [pm]
(Fe)	cI2	Im-3m	A2	W	286.64
γ -Fe	cF4	Fm-3m	A1	Cu	365.99
(Cr)	cI2	Im-3m	A2	W	288.4
(Nb)	cI2	Im-3m	A2	W	330
Cr ₂ Nb	cF24	Fd-3m	C15	MgCu ₂	518.6
Fe ₂ Nb	hP12	P6 ₃ /mmc	C14	MgZn ₂	a=484.14
					c=789.33
Fe ₇ Nb ₆	hR13	R-3m	D8 _s	Fe ₇ W ₆	a=492.6
					c=2680
σ -FeCr	tP30	P4 ₂ /mnm	D6 _b	σ -FeCr	a=879.95
					c=454.42
Metastable					
(Fe,Cr) ₆ Nb ₆	cF96	Fd-3m		NiTi ₂	a=1130

Table 31 - Crystallographic data of all phases involved in the present study of Cr-Fe-Nb.

I. Experimental determination of the isothermal sections at 700°C, 1050°C and 1350°C

1. Experimental

Samples in ternary systems have been prepared from high purity metals Fe (MaTeck, 99.99 %), Nb (99.99 %, MaTeck), Cr (99.99 %, Alfa Aesar) by levitation induction heating method under high purity Ar atmosphere on a home-made device. The levitation chamber was evacuated to 10^{-5} mbar and refilled with Ar before melting the samples. The samples were then melted at least three times and turned upside down after each melting process in order to ensure a good homogenization of the samples. After alloying, the total mass of each samples was checked, the loss was less than 0.7 % from the predicted mass.

In the first step, the samples were crushed into several pieces and checked carefully with respect to inhomogeneity. No samples were found with inhomogeneity.

In the present work, three different isothermal temperatures, 700°C 1050°C and 1350°C were studied. For the two lowest temperatures, the samples were placed in alumina crucibles that were sealed in evacuated quartz tubes (10^{-5} mbar) and placed in a tubular furnace under Ar. Samples to be annealed at

1350°C were wrapped first with Nb foil and then with Ti used as oxygen getter. The inner Nb foil isolated the samples from Ti. They were then heated several days in furnace under pure Ar (6.0) after initial flushing of the furnace with Ar/H₂. After the required annealing time, the samples were quenched in cold water as quickly as possible in order to preserve the high temperature state.

The samples to be studied by SEM/EDX were embedded into carbon containing, electrically conducting resin and were then grounded and polished. The grinding was done using SiC paper with 240-1200 mesh grain size subsequently. Then they were polished using 6 and 3 µm diamond paste followed by a finish of SiO₂ dispersed in a NaOH solution. Microstructures and composition of the phases were investigated using a Zeiss Supra 50 VP or a Zeiss Merlin SEM (Zeiss, Oberkochen, Germany), both equipped with EDX systems from Oxford Instruments (High Wycombe, UK). The latter SEM was also equipped with an electron backscatter diffraction (EBSD) system from Oxford Instruments. EBSD analysis was carried out for phase identification on selected samples.

X-ray diffraction measurements were carried out with crushed powder samples on a Bruker D8 diffractometer (Bruker AXS, Karlsruhe, Germany) using Cu K α radiation and Sol-X detector. For the samples that could not be crushed due to their hardness, the embedded sample pieces were used. The refinements of the crystal structures were done using Topas 4 software (Bruker AXS, Karlsruhe, Germany) and crystallographic information from the literature and reported in *Table 31*.

2. Results and discussion

Information about the crystal structures present in the system are given in *Table 31*. The selected samples for each isothermal section are given in *Table 33*, *Table 34* and *Table 35* with their respective annealing time as well as their chemical composition determined by EDX and their crystallographic structures obtained from XRD.

From the results obtained by SEM/EDX and XRD as given in *Table 33*, *Table 34* and *Table 35*, the three isothermal sections at 700, 1050 and 1350°C were constructed and are given in *Figure 43*, *Figure 44*, *Figure 45*, respectively.

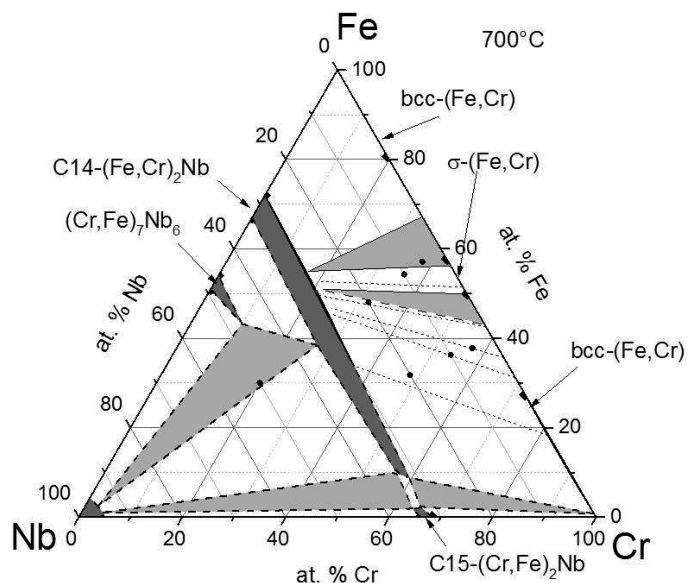


Figure 43 - Isothermal sections of Cr-Fe-Nb given at 700°C. The results showed in dashed line were extrapolated from higher temperature isothermal sections as well from binary data.

In the 700°C isothermal section, some of the results were extrapolated from higher isothermal sections, binary systems and from Kaloev et al. [67] in order to determine the different three phase fields that could not be determined with our selected samples. The results from the extrapolations are indicated by dashed lines.

The main features of this phase diagram are the extension of the two binary Laves phases (C15 on the Cr-Nb side and C14 on the Fe-Nb side) into the ternary, resulting in broad homogeneity ranges (see Figure 43, Figure 44 and Figure 45).

The change of Laves phase polytype is featured by a narrow two phase field region. The $(\text{Fe,Cr})_2\text{Nb}$ laves phase crosses the whole phase diagram and cuts it into two parts. On the Fe-Nb side, there is an extension of the μ phase into the ternary, as well as Fe and Cr dissolve in Nb-BCC. This discussion will be divided in different sections in order to present the different aspects of the phase diagram. The first part is dedicated to the Laves phase phase field, the second to the Fe-Nb part and the third is about the Fe-Cr side of the phase diagram.

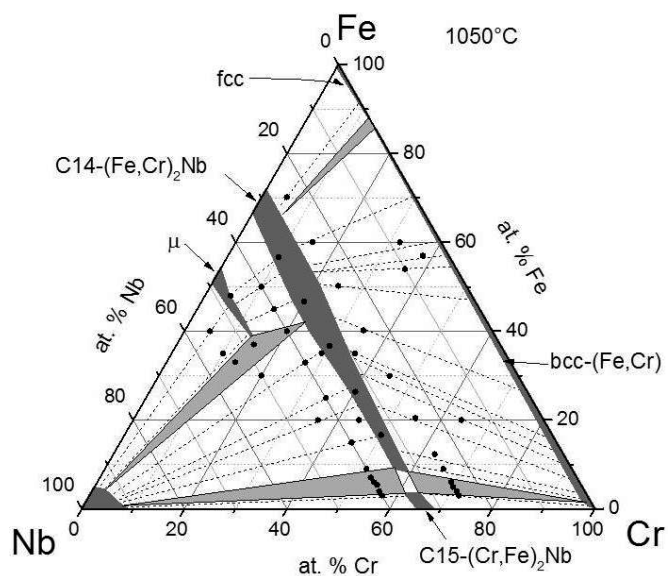


Figure 44 - Isothermal section at 1050°C. Phase equilibria are drawn according to the experimental results given in Table 34.

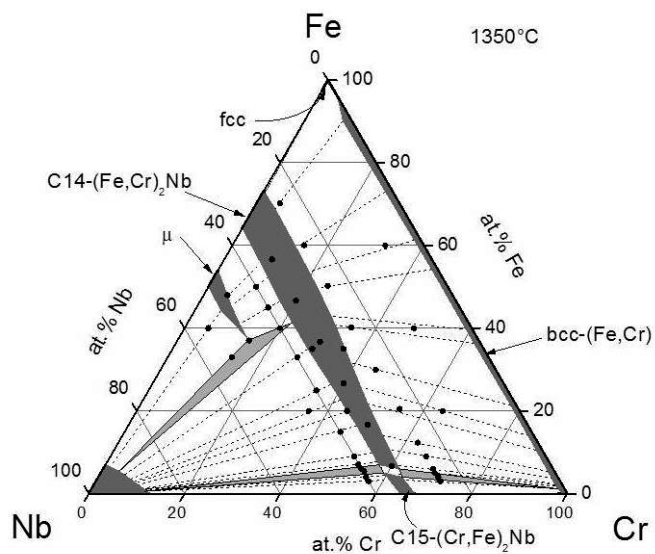


Figure 45 - Isothermal section at 1350°C according to the experimental results from SEM/EDX and XRD given in Table 35.

2.1 Extension of C15 and C14 laves phase (Fe,Cr)₂Nb in the ternary system

The three different isothermal sections (*Figure 43*, *Figure 44* and *Figure 45*) show an extension from Fe₂Nb to Cr₂Nb of the Laves phase with a change of polytype at low Fe content on the Cr₂Nb side. The C15 laves phase is stable in the Cr-Nb binary system over the complete temperature range up to its congruent melting point [18,28,29], whereas the C14 laves phase type is stable in the Fe-Nb binary system. It is a common phenomenon that small additions of a third element result in a polytype change; see for example the effect of Al in Cr-Nb-Al [18,160].

The addition of Fe to Cr₂Nb implies a change of structure polytype from C15 to C14. The two structure polytypes, cubic C15 and hexagonal C14 are closely related [10,18,160]. The change of structure polytype is due to an increase of the electron concentration by adding Fe, more favorable as C14 structure polytype. Furthermore the C14 crystal structure is more flexible with respect to the addition of a third element [160,161], which results in the frequent occurrence of C14 type Laves phases in ternary systems.

The C15 Laves phase can dissolve a certain amount of Fe, estimated up to 7 at. % by Grujicic et al. [68] at 1200°C. The two higher isothermal sections (*Figure 44* and *Figure 45*) of the present work suggest a lower amount of Fe dissolving into the C15 laves phase (3.6 at. % Fe at 1050°C and 5 at.% Fe at. % 1350°C). By increasing the temperature, a higher amount of Fe can be dissolved into the C15. The dissolution of Cr into the C14 is also higher at higher temperature (i.e. 59.2 and 61.9 at. % Cr at 1050 and 1350°C, respectively).

Due to the structural similarities, the two phase field [C15-Cr₂Nb + C14-Fe₂Nb] is difficult to determine and was established from the combined results obtained by SEM, EBSD and XRD. With microstructure evaluations from SEM, it was not possible to distinguish the two different Laves phases (see *Figure 46*). Nevertheless, the XRD evaluation suggested the presence of two different Laves phase polytypes in the relevant samples (21, 22, 48, 49 on the Cr-rich side and samples 24, 25, 45, 46 on the Nb-rich side, see *Table 34*). Electron backscattered diffraction (EBSD) was therefore employed for the evaluation of the two phase field. EBSD phase maps are shown as examples for two different samples 22 (Cr₆₉Fe₆Nb₂₅) and 25 (Cr₅₄Fe₆Nb₄₀) (*Figure 47*) annealed at 1050°C on each side of the

two phase field region, their respective XRD analysis are given in *Figure 47*. The two different polytypes of Laves phase can then be observed as a fine microstructure (“salt and pepper” mixture). From these results, we estimated the two phase region C14-C15 to be about 4.9-5.8 at. % at 1050°C and about 2-2.8 at. % at 1350°C. On the Cr-rich side the two phase field region is narrower.

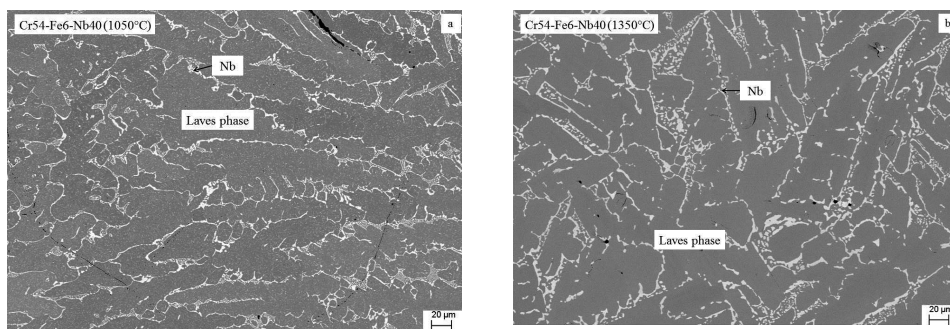


Figure 46 - Metallography investigation of sample 25 (Cr54Fe6Nb40) in the three phase field (C15+C14+Nb) annealed at a) 1050°C, b) 1350°C.

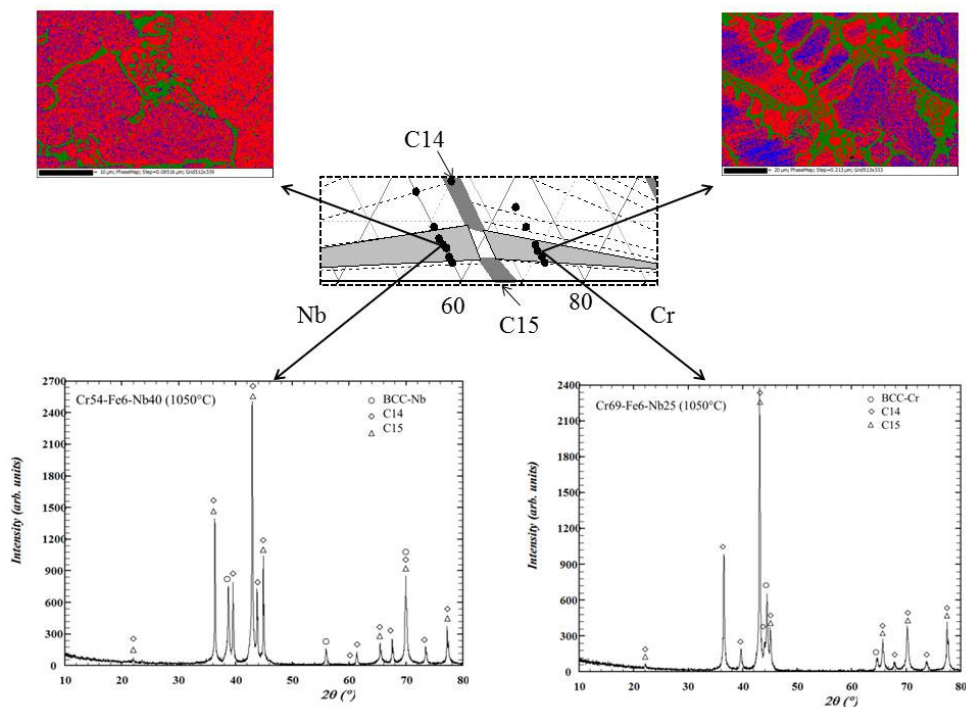


Figure 47 –EBSD phase maps for the 1050°C isothermal section for the determination of the phase field between the C14 and C15 Laves phase together with the evaluation of the structures with XRD.

For the two upper pictures, the different colors represent, red=C14, blue=C15 and green=BCC.

On each side of the Laves phase, the lattice parameters, volumes as well as c/a ratio in C14 were determined from samples placed along 25 and 40 at.% Nb and annealed at 1050°C and 1350°C, respectively (Figure 48 and Figure 49). The lattice parameter in the C14 laves phase follow a linear tendency with the increase of Cr content which is consistent with a continuous solid solution. There is then a discontinuity on the lattice parameters once the system reaches the boundary of the two phase field (C14-C15), this is shown by the dotted line in Figure 48 and Figure 49. The lattice parameters as well as the c/a and volumes show a deviation from the linear tendency observed at higher Cr content. At 1350°C, the two phase field region is reached at 56.1 at. % Cr showed by a break in the increase of the linear tendency of the c/a ratio.

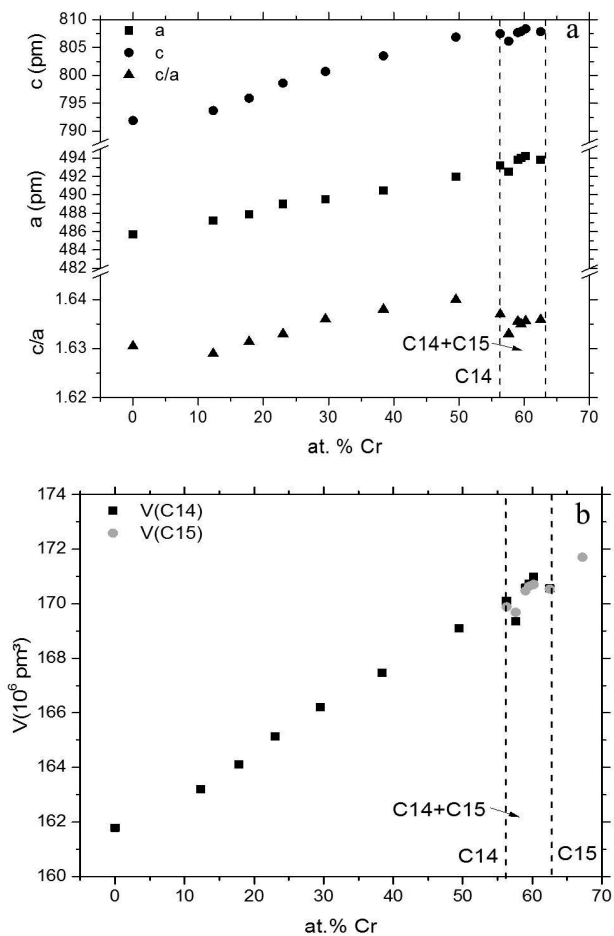


Figure 48 – a) Lattice parameters and b) unit cell volume of the Laves phase determined by XRD for samples with 40 at. % Nb after annealing at 1050 °C. Data for binary Cr_2Nb and Fe_2Nb at 1100 °C were taken from reference [34].

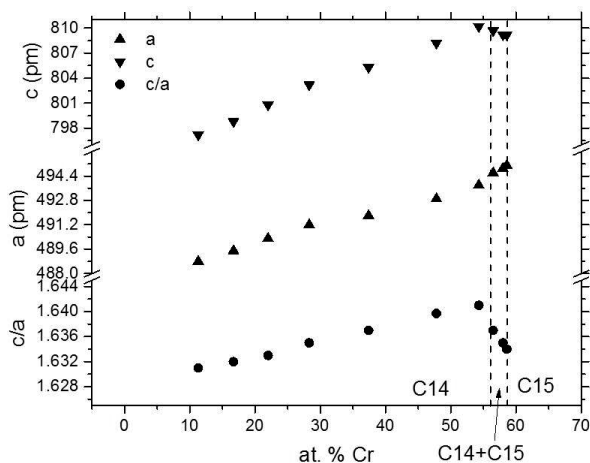


Figure 49 – Lattice parameters and c/a ratio of the C14 phase at 1350°C along the section of 40 at. % Nb.

Some samples show the presence of C36 laves phase structure (Sample 47 at 1350°C for example, see Table 35). Grujicic et al. [68] also found this phase in their investigation on the addition of Fe into Cr_2Nb . This phase is metastable and is an intermediate phase between C14 and C15 [14], its formation was found in samples which are close to the boundary between the two phase field and the C14 phase. The C36 phase forms during quenching, which can be difficult at high temperature and if done too slowly result in the formation of this phase.

2.2 The Nb rich side

With the highest melting point from the three pure compounds, Nb solid solution extends into the ternary system with an increase of solubility of Fe and Cr by increasing the temperature. The Nb solid solution forms two three phase fields:

- A three phase field is formed on the Fe-Nb side, Nb is in equilibrium with the μ phase and the C14 Laves phase (Figure 50);
- On the Cr-Nb side, where Nb, C15 and C14 laves phase are in equilibrium and form a narrow three phase field region. This part was already previously discussed.

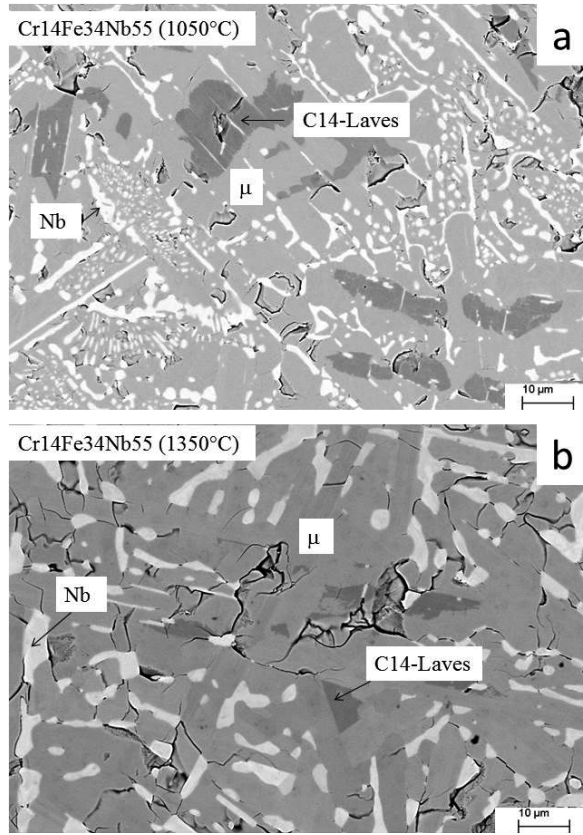


Figure 50 – Backscattered electron SEM images of the microstructure of the sample 28 ($\text{Cr}_{14}\text{Fe}_{34}\text{Nb}_{55}$) annealed at a) 1050°C and b) 1350°C showing the three phase fields $\text{Nb} + \mu + \text{C14}$.

The μ phase extends into the ternary phase diagram at the different annealing temperatures. There is a higher dissolution of Cr by increasing the temperature and its stability field becomes wider which is in consistency with the binary Fe-Nb data [35]. Cr dissolves about 13.8 and 14.6 at. % into $\mu\text{-Fe}_7\text{Nb}_6$ at 1050 and 1350°C, respectively. For the 700°C isothermal section, the dissolution of Cr into the μ phase was estimated to be about 10 at. % by extrapolating from the higher isothermal section and the binary Fe-Nb data. It is slightly higher than the value given by Kaloev et al. [67]. Nevertheless, their three phase field postulations do not agree with the higher temperature investigations performed in this work.

For the two highest isothermal sections (i.e. 1050°C and 1350°C), the evolution of the lattice parameters of the μ phase are given in *Figure 51* as a function of Cr content. As the concentration of Cr increases the lattice parameters (a and c) increase linearly which is consistent with the homogeneity range of the μ phase. The samples annealed at 1350°C have larger lattice parameters than the ones annealed at 1050°C.

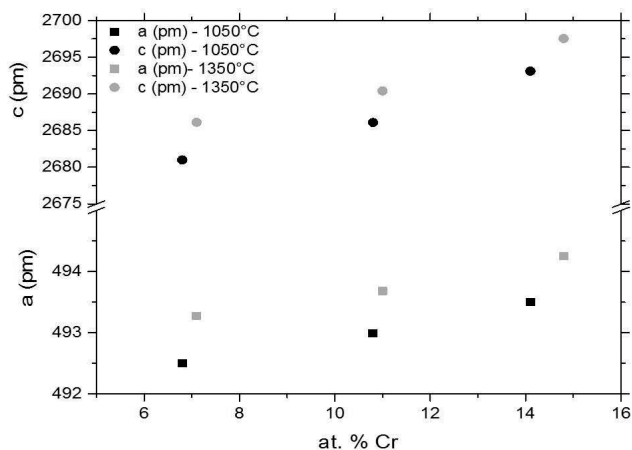


Figure 51 - Lattice parameters of the μ phase as a function of the Cr content after annealing at 1050°C and 1350°C.

In the isothermal section at 1050°C, for several samples surrounding the μ phase, the presence of a new phase named T in the tables (referred to the *Table 34*) with a composition very close to the μ phase was identified. Using TOPAS 4 for the crystallographic refinement, it was found that this phase is cubic with a structure type Ti_2Ni and a lattice parameter of 1130 pm (*Table 32*). This phase is in equilibrium with the μ and Nb phases accordingly to SEM results. This phase has already been found in an earlier version of the phase diagram Fe-Nb [59] and was denoted η . Nevertheless in more recent studies [31,35] of the phase diagram Fe-Nb, the η -phase was not found. Further investigations [162,163] were carried out to know the stability of this η phase. The existence of this phase in the Fe-Nb system was clarified by the references [162,163] and it was concluded that this phase is formed in presence of impurities (O, N and S) but not in the pure binary Fe-Nb system.

Empirical formula	(Fe,Cr) ₆ Nb ₆ O _x
Crystal system, Space Group	Fd-3m
Structure type	NiTi ₂
Cell parameter (pm)	1130
Cell volume (pm ³)	14.434x10 ⁸

Table 32 – Refinement of the crystal structure T (η).

In the ternary Cr-Fe-Nb phase diagram, Kaloev et al. [66] did not mention the presence of this phase. In addition, Malfliet et al. [164] investigated the influence of oxygen on Cr-Fe-Nb (in the Fe-Nb rich side). For the samples without oxygen treatment, their findings agreed well with the results of Kaloev et al. [66]. Nevertheless, they found that only a small amount of oxygen can lead to the formation of a quaternary phase with a structure type NiTi₂. Our evaluation of the crystal structure T shows that Nb occupies the 48f Wyckoff position, Fe and Cr shared the 16d and 32e positions. Refining the atomic position showed that Fe and Cr shared at 1:1 the 16d position whereas Fe fully occupies the 32e Wyckoff position. This finding is in very good agreement with the results of the oxygen treated Cr-Fe-Nb samples of Malfliet et al. [164] (see Table 5 of the original paper). As shown by the literature [162,164] this η -phase forms with a small amount of impurities (O, C, N), it is doubtful that this phase exists in the ternary Cr-Fe-Nb phase diagram as well. Nevertheless, some of our samples have shown its presence as well. It is therefore concluded that this phase is metastable in the ternary system and formed due to some impurities in the samples. This phase is not shown in the presented 1050°C isothermal section.

2.3 The Cr-Fe side

In the binary Cr-Fe, there is one intermetallic phase (σ -phase) in addition to the pure phases Fe (BCC and FCC) and Cr (BCC). According to Kaloev et al. [67] there is a dissolution of Nb of about 10 at. % into σ -(Fe,Cr). The present investigation of the isothermal section at 700°C disagrees with this finding. It was not found that Nb dissolves in the σ phase. The sigma phase σ -(Fe,Cr) is in equilibrium with BCC and C14-Laves phase as seen by the microstructure analysis (Figure 52a) and the XRD evaluation (Figure 52b).

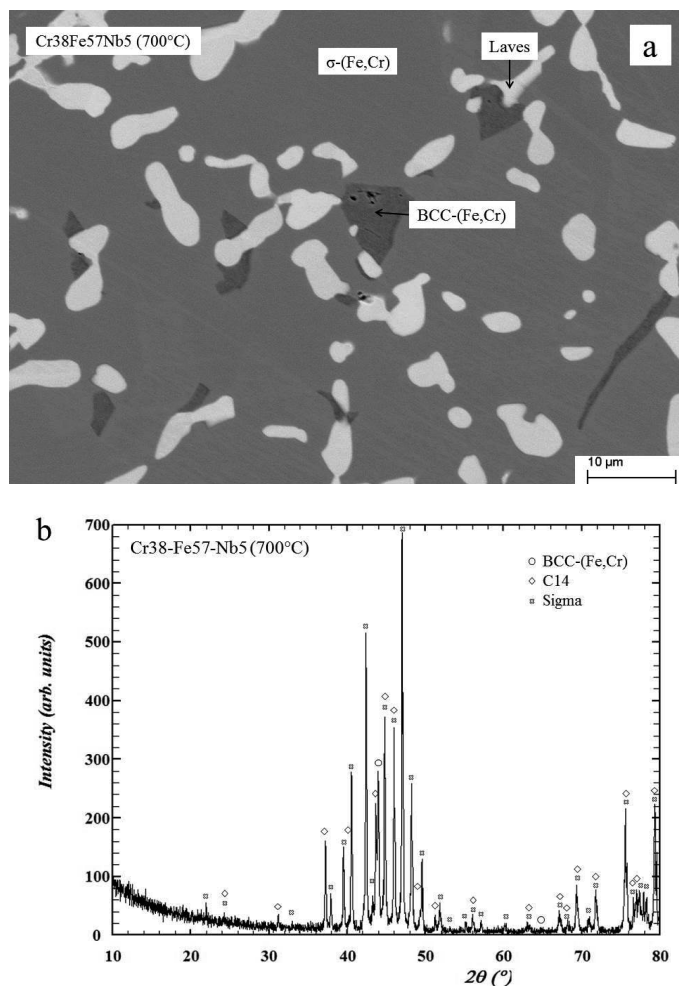


Figure 52 – Microstructure and XRD analysis of the three phase field C14+ σ +BCC on the Fe-Cr-rich side at 700°C. a) SEM image of the microstructure of the sample 15 (Cr38Fe57Nb5) annealed at 700°C showing the three phase field equilibrium between BCC, σ and C14 Laves phase and b) XRD evaluation of the sample.

At high temperature (i.e. 1350°C), the Cr-Fe side of the phase diagram features by a continuous solid solution of BCC-(Cr,Fe) phase. At lower temperature (i.e. 1050°C) this phase is interrupted at low Cr content by the FCC γ -loop where Cr dissolves in γ -Fe. This three phase field was estimated.

Cr forms a three-phase field equilibrium with the Laves phases polytypes (C14 and C15). As explained in the previous part about the Laves phase, this three phase-field is narrower as the temperature increases.

Conclusion

Three isothermal sections of the Cr-Fe-Nb phase equilibria were presented here. This work highlighted the phase field region of the two different Laves phases as these intermetallic phases are of primary importance for the development of high temperature materials. It was not found that Nb dissolved in σ -(Fe,Cr) as suggested in previous evaluation of the phase diagram.

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	Composition (EDX) Cr Fe Nb at. %
Isothermal section 700°C						
15	Cr38Fe57Nb5	700°C, 108 d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=483.0(4) c=787.9(6)	17.5 55.1 27.5
			σ-(Fe,Cr)	P4 ₂ /mnm	a=879.0(1) c=456.1(0)	43.7 56.3 0.1
			bcc-(Fe,Cr)	Im-3m	a=288.4(4)	32.8 67.2 0.1
16	Cr36Fe54Nb10	700°C, 108 d	σ-(Fe,Cr)	P42/mnm	a=878.8(9) c=455.6(1)	48.5 51.5 0.1
			(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=482.9(6) c=787.7(5)	20.0 52.7 27.3
17	Cr32Fe48Nb20	700°C, 108 d	bcc-(Fe,Cr)	Im-3m	a=287.8(7)	57.4 42.6 0.0
			(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=484.3(0) c=790.8(3)	21.6 49.6 28.8
18	Cr57Fe38Nb5	700°C, 108 d	bcc-(Fe,Cr)	Im-3m	a=292.3(6)	64.3 35.7 0.0
			(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=484.5(1) c=796.2(9)	25.5 46.8 27.8
19	Cr54Fe36Nb10	700°C, 108 d	bcc-(Fe,Cr)	Im-3m	a=296.7(5)	68.7 31.2 0.1
			(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=477.9(3) c=789.6(3)	26.5 45.3 28.2
20	Cr48Fe32Nb20	700°C, 108 d	bcc-(Fe,Cr)	Im-3m	a=288.1(4)	84.3 18.9 0.2
			(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=486.5(5) c=795.4(9)	36.2 33.9 29.9
30	Cr20Fe30Nb50	700°C, 108 d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=489.1(4) c=798.8(1)	26.9 38.5 34.7
			μ-Fe ₇ Nb ₆	R-3m	a=493.8(1) c=2697.(8)	17.0 36.3 46.8
			(Nb)	Im-3m	a=329.6(2)	3.5 3.4 93.1

Table 33 - Results of the phase analysis by SEM/EDX and XRD of samples annealed at 700°C.

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	Composition (EDX) Cr Fe Nb at. %
Isothermal section 1050 °C						
1	Cr32Fe60Nb8	1050 °C, 28 d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb	Im-3m P6 ₃ /mmc	a=287.7(2) a=483.4(2) c=788.3(4)	39.4 60.2 0.3 17.1 54.4 27.6
3	Cr64Fe20Nb16	1050 °C, 28 d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb	Im-3m P6 ₃ /mmc	a=288.7(2) a=489.0(3) c=798.8(14)	90.3 9.4 0.2 38.1 30.8 31.1
4	Cr35.75Fe35Nb29.25	1050 °C, 28 d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb	Im-3m P6 ₃ /mmc	a=288.5(8) a=487.1(1) c=796.6(2)	84.5 15.3 0.3 33.7 35.9 30.4
5	Cr44Fe20Nb36	1050 °C, 28 d	(Fe,Cr) ₂ Nb (Nb)	P6 ₃ /mmc Im-3m	a=491.4(1) c=805.6(2) a=329.3(6)	45.4 20.3 34.2 6.1 1.6 92.2
6	Cr29.25Fe35Nb35.75	1050 °C, 28 d	(Fe,Cr) ₂ Nb Phase T (Nb)	P6 ₃ /mmc Fd-3m Im-3m	a=489.4(1) c=800.7(2) a=1132.4(5) a=329.4(4)	30.4 35.2 34.4 17.7 32.4 49.9 4.6 4.5 90.9
7	Cr36Fe20Nb44	1050 °C, 28 d	(Fe,Cr) ₂ Nb (Nb)	P6 ₃ /mmc Im-3m	a=491.3(1) c=804.5(2) a=329.7(2)	42.4 23.3 34.4 6.2 2.0 91.8
8	Cr10Fe35Nb55	1050 °C, 28 d	μ-Fe ₇ Nb ₆ Phase T (Nb)	R-3m Fd-3m Im-3m	a=493.7(3) c=2692(3) a=1130.4(5) a=329.3(2)	12.9 39.9 47.3 9.7 41.9 48.4 1.8 3.3 94.8
15	Cr38Fe57Nb5	1050°C, 34d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P6 ₃ /mmc Im-3m	a=487.3(7) c=792(2) a=483.0 c=787 a=287.8	Not found in EDX 18.2 53.3 28.5 42.6 57.1 0.3
16	Cr36Fe54Nb10	1050°C, 34d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P6 ₃ /mmc Im-3m	a=4.835(1) c=788.6(4) a=287.6(5)	18.5 53.3 28.2 45.2 54.5 0.3
21	Cr72Fe3Nb25	1050°C, 34d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb Cr ₂ Nb	Im-3m P6 ₃ /mmc Fd-3m	a=288.6(2) a=492.2(1) c=807.2(2) a=697.8(1)	98.6 0.7 0.7 Not found in EDX 63.9 3.5 32.6

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	Composition (EDX) Cr Fe Nb at. %	
22	Cr69Fe6Nb25	1050°C, 34d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb Cr ₂ Nb	Im-3m P6 ₃ /mmc Fd-3m	a=288.6(2) a=491.4(2) c=806.2(5) a=696.3(3)	97.6 Not found in EDX 61.2	1.5 6.4 32.4
23	Cr66Fe9Nb25	1050°C, 34d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb	Im-3m P6 ₃ /mmc	a=288.5(4) a=491.5(7) c=805.8(8)	96.1 57.7	2.3 10.1
24	Cr57Fe3Nb40	1050°C, 34d	(Nb) (Fe,Cr) ₂ Nb Cr ₂ Nb	Im-3m P6 ₃ /mmc Fd-3m	a=328.3(1) a=493.8(2) c=807.6(8) a=698.6(8)	7.4 Not found in EDX 62.5	0.3 2.9 34.7
25	Cr54Fe6Nb40	1050°C, 34d	(Nb) (Fe,Cr) ₂ Nb Cr ₂ Nb	Im-3m P6 ₃ /mmc Fd-3m	a=328.9(1) a=493.8(5) c=807.6(1) a=698.6(1)	7.5 Not found in EDX 59.0	0.7 6.3 34.7
26	Cr51Fe9Nb40	1050°C, 34d	(Nb) (Fe,Cr) ₂ Nb Cr ₂ Nb	Im-3m P6 ₃ /mmc Fd-3m	a=328.8(0) a=493.2(1) c=807.4(8) a=697.8(1)	6.8 56.3 Not found in EDX	1.1 9.3 92.7
27	Cr5Fe48Nb47	1050°C, 34d	(Nb) μ-Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	Im-3m R-3m P6 ₃ /mmc	a=329.5(9) a=492.4 c=2679.2 a=486.5(6) c=792.3(9)	1.0 4.7 9.5	6.3 47.9 55.6
28	Cr14Fe34Nb55	1050°C, 34d	(Nb) μ-Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb Phase T	Im-3m R-3m P6 ₃ /mmc Fd-3m	a=329.4(5) a=493.8(4) c=2693.3 a=488.9(9) c=795.9(7) a=1131.1	2.3 14.7 24.8 Not found in EDX	3.4 37.9 40.1 94.3 47.4 35.1
29	Cr5Fe40Nb55	1050°C, 34d	(Nb) μ-Fe ₇ Nb ₆	Im-3m R-3m	a=329.1(6) a=493.5(3) c=2685.2(5)	1.0 5.8	5.3 45.6
30	Cr20Fe30Nb50	1050°C, 37d	(Nb) μ-Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	Im-3m R-3m P6 ₃ /mmc	a=329.4 a=493.7 c=2695(4) a=489.3 c=799.5(4)	2.9 16.2 26.2	2.9 36.3 37.9

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	Composition (EDX) Cr Fe Nb at. %
31	Cr10Fe50Nb40	1050°C, 34d	μ -Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	R-3m P ₆₃ /mmc	a=492.5(8) c=2681.0(4) a=487.2(5) c=793.7(7)	6.8 46.0 47.2 12.3 53.0 34.7
32	Cr15Fe45Nb40	1050°C, 34d	μ -Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	R-3m P ₆₃ /mmc	a=492.99 c=2686.1(0) a=487.9(1) c=795.9(8)	10.8 41.9 47.3 17.8 46.9 35.3
33	Cr20Fe40Nb40	1050°C, 34d	(Nb) μ -Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	Im-3m R-3m P ₆₃ /mmc	a=329.0(6) c=2693.1(5) a=493.5(8) c=798.3(1)	3.2 5.1 91.8 14.1 38.8 47.1 23.0 42.1 35.0
34	Cr15Fe37Nb48	1050°C, 34d	(Nb) μ -Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb Phase T	Im-3m R-3m P ₆₃ /mmc Fd-3m	a=329.2(3) c=2693.2(2) a=493.8(3) c=798.9(11) a=489.1(4) c=798.9(11) a=1130.6(6)	2.4 4.2 93.4 13.6 38.9 47.4 22.7 42.2 35.1 Not found in EDX
35	Cr45Fe15Nb40	1050°C, 35d	(Fe,Cr) ₂ Nb (Nb)	P ₆₃ /mmc Im-3m	a=491.9(9) c=806.8(6) a=329.3(9)	49.5 16.3 34.2 6.5 1.3 92.2
36	Cr35Fe25Nb40	1050°C, 35d	(Fe,Cr) ₂ Nb (Nb)	P ₆₃ /mmc Im-3m	a=490.5(6) c=803.4(9) a=329.4(4)	38.4 27.2 34.4 6.3 2.8 90.9
37	Cr27Fe33Nb40	1050°C, 35d	(Fe,Cr) ₂ Nb (Nb) Phase T	P ₆₃ /mmc Im-3m Fd-3m	a=489.5(8) c=800.7(6) a=329.5(4) a=1132.9(8)	29.5 36.2 34.3 4.2 3.1 92.7 Not found in SEM
38	Cr63Fe12Nb25	1050°C, 35d	(Fe,Cr) ₂ Nb (Nb)	P ₆₃ /mmc Im-3m	a=491.1(4) c=805.6(4) a=288.6(4)	54.7 13.2 32.1 95.3 3.5 1.2
39	Cr55Fe20Nb25	1050°C, 35d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P ₆₃ /mmc Im-3m	a=489.2(5) c=801.5(9) a=288.5(9)	44.8 23.5 31.6 93.1 6.3 0.6
40	Cr45Fe30Nb25	1050°C, 35d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P ₆₃ /mmc Im-3m	a=487.6(5) c=797.5(9) a=288.4(9)	35.0 34.4 30.6 87.4 12.3 0.3

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	Cr	Composition (EDX) Fe at. %	Nb
41	Cr35Fe40Nb25	1050°C, 35d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P6 ₃ /mmc Im-3m	a=485.7(7) c=793.6(4) a=287.8(9)	27.2 73.5	43.3 26.3	29.6 0.3
42	Cr25Fe50Nb25	1050°C, 35d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P6 ₃ /mmc Im-3m	a=484.1(8) c=790.7(5) a=287.8(9)	20.8 52.7	50.6 47.0	28.6 0.3
43	Cr15Fe60Nb25	1050°C, 35d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P6 ₃ /mmc Im-3m	a=482.7(6) c=787.6(3) a=287.6(3)	12.9 29.4	59.3 70.4	27.8 0.4
44	Cr5Fe70Nb25	1050°C 35d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P6 ₃ /mmc Im-3m	a=481.8(7) c=785.7(5) a=287.0(4)	4.5 32.3	67.9 91.3	27.6 0.5
45	Cr56Fe4Nb40	1050°C 35d	Cr ₂ Nb (Fe,Cr) ₂ Nb (Nb)	Fd-3m P6 ₃ /mmc Im-3m	a=698.9(2) c=808.3(6) a=494.2(1) a=329.2(7)	61.7 60.2 15.9	4.2 4.7 1.1	34.1 35.0 83.0
46	Cr55Fe5Nb40	1050°C 35d	Cr ₂ Nb (Fe,Cr) ₂ Nb (Nb)	Fd-3m P6 ₃ /mmc Im-3m	a=698.82(2) c=807.8(8) a=493.9(9) a=329.2(8)	60.4 59.5 2.1	5.6 6.3 0.0	34.1 34.2 97.9
47	Cr53Fe7Nb40	1050°C 35d	Cr ₂ Nb (Fe,Cr) ₂ Nb (Nb)	Fd-3m P6 ₃ /mmc Im-3m	a=697.5(2) c=806.1(4) a=492.5(2) a=288.6(8)	58.8 56.4 2.2	7.1 8.7 0.4	34.2 34.9 97.4
48	Cr71Fe4Nb25	1050°C 35d	Cr ₂ Nb (Fe,Cr) ₂ Nb bcc-(Fe,Cr)	Fd-3m P6 ₃ /mmc Im-3m	a=697.2(6) c=807.2(6) a=493.4(3) a=297.6(9)	63.6 62.6 97.9	4.4 5.9 1.2	32.0 31.5 0.9
49	Cr70Fe5Nb25	1050°C 35d	Cr ₂ Nb (Fe,Cr) ₂ Nb bcc-(Fe,Cr)	Fd-3m P6 ₃ /mmc Im-3m	a=696.8(4) c=804.5(5) a=491.5(8) a=288.4(7)	62.7 60.7 95.1	5.1 7.1 1.6	32.2 32.2 3.2

Table 34 –Experimental results of the phase analysis of samples annealed at 1050°C.

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters (pm)	Composition (EDX) Cr Fe Nb at. %		
Isothermal section 1350°C								
1	Cr32Fe60Nb8	1350 °C, 5 d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb	Im-3m P6 ₃ /mmc	a=288.1(2) a=482.9(2) c=787.6(2)	37.4 19.5	61.3 53.8	1.3 26.7
2	Cr48Fe40Nb12	1350 °C, 5 d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb	Im-3m P6 ₃ /mmc	a=288.3(2) a=485.1(3) c=791.7(3)	59.4 29.5	39.7 42.9	0.9 27.6
3	Cr64Fe20Nb16	1350 °C, 5 d	bcc-(Fe,Cr) (Fe,Cr) ₂ Nb	Im-3m P6 ₃ /mmc	a=288.7(2) a=488.5(2) c=799.1(6)	85.0 44.6	14.0 25.6	1.0 29.8
4	Cr35.75Fe35Nb29.25	1350 °C, 5d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=486.5(1) c=795.3(1)	35.5	35.7	28.8
5	Cr44Fe20Nb36	1350 °C 5 d	(Fe,Cr) ₂ Nb (Nb)	P6 ₃ /mmc Im-3m	a=492.7(1) c=807.2(2) not found in XRD	44.1 8.6	19.9 2.7	36.0 88.7
6	Cr29.25Fe35Nb35.75	1350 °C, 5d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=490.2(1) c=802.2(2)	29.5	34.9	35.5
7	Cr36Fe20Nb44	1350 °C, 5 d	(Fe,Cr) ₂ Nb (Nb)	P6 ₃ /mmc Im-3m	a=492.7(2) c=806.9(5) a=327.2(2)	41.3 7.7	22.7 2.7	36.0 89.6
9	Cr10Fe56.67Nb33.33	1350 °C, 6d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	- - -	10.2	56.2	33.7
10	Cr20Fe46.67Nb33.33	1350 °C, 6d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	- - -	20.2	46.1	33.7
11	Cr30Fe36.67Nb33.33	1350 °C, 6d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	- - -	30.4	36.3	33.3
12	Cr40Fe26.67Nb33.33	1350 °C, 6d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	- - -	39.9	26.5	33.6
13	Cr50Fe16.67Nb33.33	1350 °C, 6d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	- - -	49.7	16.7	33.6
14	Cr60Fe6.67Nb33.33	1350 °C, 6d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	- - -	59.8	6.6	33.6
21	Cr72Fe3Nb25	1350°C, 10d	Cr ₂ Nb (Cr)	Fd-3m Im-3m	a=696.2(9) a=288.5(1)	64.8 97.5	3.3 1.2	32.0 1.4

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters (pm)	Composition (EDX)		
22	Cr69Fe6Nb25	1350°C, 10d	(Fe,Cr) ₂ Nb (Cr)	P6 ₃ /mmc Im-3m	a=491.0(4) c=806.0(1) a=288.8(7)	Cr 61.4	Fe 6.8	Nb 31.8
23	Cr66Fe9Nb25	1350°C, 10d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P6 ₃ /mmc Im-3m	a=491.2(4) c=805.7(9) a=288.9(5)	Cr 58.0	Fe 10.6	Nb 31.4
24	Cr57Fe3Nb40	1350°C, 10d	Cr ₂ Nb (Nb)	Fd-3m Im-3m	a=700.8(1) a=326.8(9)	Cr 60.7	Fe 3.3	Nb 36.1
25	Cr54Fe6Nb40	1350°C, 10d	(Fe,Cr) ₂ Nb Cr ₂ Nb (Nb)	P6 ₃ /mmc Fd-3m Im-3m	a=494.9(4) c=809.2(3) a=700.0(2) a=326.8(5)	Cr 58	Fe 6.3	Nb 35.7
26	Cr51Fe9Nb40	1350°C, 10d	(Fe,Cr) ₂ Nb Nb	P6 ₃ /mmc Im-3m	a=493.8(2) c=810.2(9) a=326.7(8)	Cr 54.3	Fe 9.8	Nb 35.9
27	Cr5Fe48Nb47	1350°C, 10d	μ-Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	R-3m P6 ₃ /mmc	a=492.7(7) c=2681.6(1) a=497.7(9) c=7.912(2)	Cr 5	Fe 47.6	Nb 47.3
28	Cr14Fe34Nb55	1350°C, 10d	μ-Fe ₇ Nb ₆ (Nb)	R-3m Im-3m	a=494.4(9) c=2698.0(0) a=327.2(2) Not found	Cr 15.1	Fe 37.1	Nb 47.9
29	Cr5Fe40Nb55	1350°C, 10d	μ-Fe ₇ Nb ₆ (Nb)	R-3m Im-3m	a=494.4(8) c=2688.9(7) a=327.4(7)	Cr 5.5	Fe 44.3	Nb 50.2
31	Cr10Fe50Nb40	1350°C, 10d	μ-Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	R-3m P6 ₃ /mmc	a=493.2(7) c=2686.1(8) a=488.8(2) c=797.2	Cr 7.1	Fe 45.4	Nb 47.5
32	Cr15Fe45Nb40	1350°C, 10d	μ-Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	R-3m P6 ₃ /mmc	a=493.6(8) c=2690.4(6) a=489.5(2) c=798.8(9)	Cr 11	Fe 41	Nb 48
33	Cr20Fe40Nb40	1350°C, 10d	μ-Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb	R-3m P6 ₃ /mmc	a=494.2(5) c=2697.5(4) a=490.3(5) c=800.8(4)	Cr 14.8	Fe 37.1	Nb 48.1
						Cr 22	Fe 41.3	Nb 36.8

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters (pm)	Composition (EDX)		
						Cr	Fe at. %	Nb
34	Cr15Fe37Nb48	1350°C, 10d	μ -Fe ₇ Nb ₆ (Fe,Cr) ₂ Nb (Nb)	R-3m P ₆₃ /mmc Im-3m	a=494.4(6) c=2697.4(5) a=490.4(7) c=801.1(3) a=327.7	14.6 21.7 3.6	37 41.2 5.5	48.4 37.1 91.1
35	Cr45Fe15Nb40	1350°C, 13d	(Fe,Cr) ₂ Nb (Nb)	P ₆₃ /mmc Im-3m	a=492.9(3) c=808.2(7) a=326.7(7)	47.8 9.1	16.4 2.1	35.7 88.8
36	Cr35Fe25Nb40	1350°C, 13d	(Fe,Cr) ₂ Nb (Nb)	P ₆₃ /mmc Im-3m	a=491.8(5) c=805.3(5) a=327.1(7)	37.4 6.9	26.4 3.4	36.2 89.6
37	Cr27Fe33Nb40	1350°C, 13d	(Fe,Cr) ₂ Nb (Nb)	P ₆₃ /mmc Im-3m	a=491.2(4) c=803.2(7) a=327.3	28.3 5.1	35.0 4.5	36.7 90.4
38	Cr63Fe12Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb (Cr)	P ₆₃ /mmc Im-3m	a=490.4(2) c=803.9(7) a=288.7(7)	54.4 93.3	14.1 5.4	31.5 1.3
39	Cr55Fe20Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P ₆₃ /mmc Im-3m	a=489.0(9) c=801.2(9) a=288.9(5)	47.4 88.3	21.8 10.5	30.8 1.2
40	Cr45Fe30Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P ₆₃ /mmc Im-3m	a=487.1 c=797.1(3) a=288.5(8)	38.9 77.7	31.7 21.3	29.4 1.1
41	Cr35Fe40Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P ₆₃ /mmc Im-3m	a=485.5(8) c=792.4(7) a=288.6(6)	31.1 62.3	40.8 36.4	28.1 1.3
42	Cr25Fe50Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P ₆₃ /mmc Im-3m	a=483.3(6) c=788.6(2) a=295.6(2)	23.0 44.6	49.7 54.2	27.3 1.2
43	Cr15Fe60Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P ₆₃ /mmc Im-3m	a=482.1(6) c=786.4(1) a=292.6(3)	13.7 25.5	59.4 73.0	26.9 1.5
44	Cr5Fe70Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb	P ₆₃ /mmc	a=482.0(6) c=786.2(6) Too small to be quantify	4.5 8.1	68.8 90.1	26.6 1.8
45	Cr56Fe4Nb40	1350°C, 13d	Cr ₂ Nb (Nb)	Fd-3m Im-3m	a=700.3(6) a=326.9(1)	59.7 11.4	4.4 0.6	35.9 87.9

Sample No.	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters (pm)	Composition (EDX)		
46	Cr55Fe5Nb40	1350°C, 13d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=495.1	Cr	Fe	Nb
			Cr ₂ Nb	Fd-3m	a=700.1(7)	58.3	6.0	35.7
			(Nb)	Im-3m	a=326.7(8)	11.2	0.9	87.9
47	Cr53Fe7Nb40	1350°C, 13d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=494.6(5)	56.5	7.7	35.8
			C36	P6 ₃ /mmc	a=497.7(8)	11.1	1.1	87.8
			(Nb)	Im-3m	a=326.8(6)			
48	Cr71Fe4Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=493.2(3)	62.4	4.8	32.9
			Cr ₂ Nb	Fd-3m	a=697.5(2)	96.6	1.8	1.6
			(Cr)	Im-3m	a=289.1(6)			
49	Cr70Fe5Nb25	1350°C, 13d	(Fe,Cr) ₂ Nb	P6 ₃ /mmc	a=492.6(7)	62.0	5.8	32.1
			Cr ₂ Nb	Fd-3m	a=697.1(1)	96.4	2.2	1.5
			(Cr)	Im-3m	a=289.0(9)			

Table 35 – Results of the analysis by SEM/EDX and XRD of the samples annealed at 1350°C.

II. Thermal and solidification behavior of the Cr-Fe-Nb system

Thermal analysis and determination of characteristic temperatures have not been carried out so far in this system and no liquidus projection is available in the literature. The invariant reactions of the limiting binary systems are given in *Table 36*. Nevertheless the liquidus projection of phase equilibria is rather important for the understanding of the liquid-solid phase changes. In addition, in the thermodynamic optimization of phase diagram (i.e. Calphad method), the invariant reactions data are taken with a higher priority for the determination of the thermodynamic parameters of the considered system.

Reactions	Type	Temperature (°C)	References
Nb-Cr			
$L = NbCr_2$	C	1727 ± 2	[18]
$L = C15 + Nb$	e1	1681 ± 2	[18]
$L = C15 + Cr$	e2	1660 ± 2	[18]
Fe-Nb			
$L + C14 = \mu$	p1	1523 ± 2	[35]
$L = Nb + \mu$	e4	1508 ± 2	[35]
$L = bcc\text{-}Fe + C14$	e3	1373 ± 2	[35]
Fe-Cr			
$L = bcc$	C	1515.7	[24]
$fcc = bcc$	C	853.3	[24]
$bcc = \sigma$	C	831.5	[24]
$\sigma = bcc' + bcc''$	Eutectoid	511.9	[24]

Table 36 – Invariant reactions in the binary boundary systems of the Cr-Fe-Nb phase diagram.

The present work aims to investigate the solidification behavior of the system as well as the liquidus projection.

The samples have been prepared as explained in the previous part (see p. 109). For the study of the solidification behavior, the microstructures of the as-cast samples were studied by SEM.

The DTA measurements have been done on a Setsys Evolution using alumina crucibles with a heating rate of 5°C/min. However, some problems have been encountered using alumina crucibles as they are not stable at high temperature. New crucibles to carry out the DTA measurements were therefore necessary for the investigation of the temperature dependence of the samples.

1. Results

The nominal compositions as well as the results of the phase analysis after alloying of the as-cast samples are summarized in *Table 38*. The composition and the primary crystallization of the as-cast samples are given in *Figure 53*.

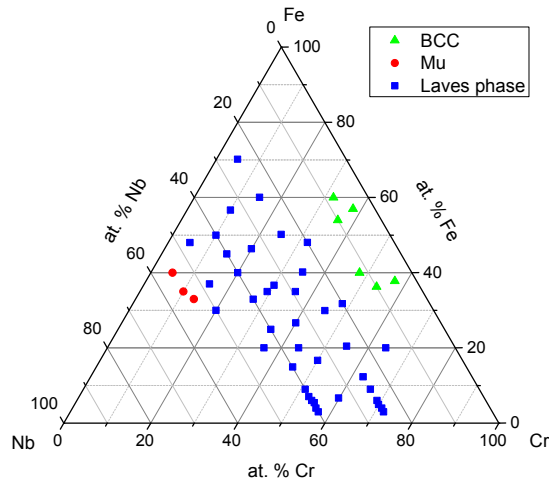


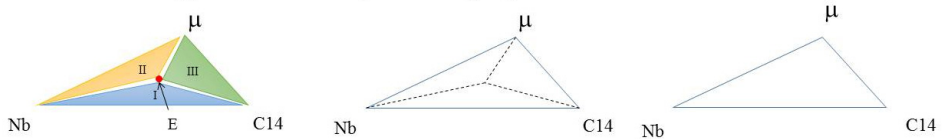
Figure 53 – Compositions and primary crystallization of the as-cast samples.

As shown by the isothermal sections presented in the previous chapter, the Cr-Fe-Nb phase diagram does not contain ternary compounds but instead a high solubility of a third element in the different binary compounds (except for σ phase which has no perceptible solubility of Nb according to the results obtained at 700°C).

The presented primary crystallizations of the as-cast samples as well as the binary invariant reactions (from literature, see *Table 36*) were used to construct the liquidus projection (*Figure 55*). The compositions of the ternary invariant reactions and their types (i.e. type I-Eutectic, type II-U, type III-Peritectic) have to be consistent with the data obtained for the isothermal sections:

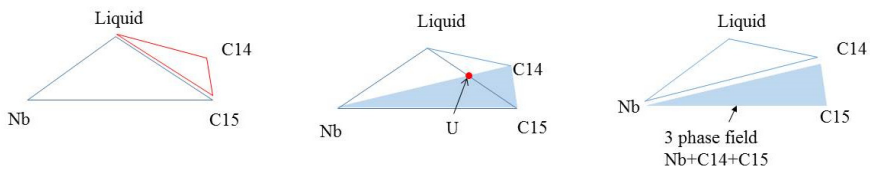
- At the eutectic point which is a local minimum, the liquid is in equilibrium with three solid phases, C14 + μ + Nb. The eutectic point must lie within the three phase field as shown in the scheme below;

Sketch of a E type reaction: $L \leftrightarrow \mu + C14 + (Nb)$



- In a U type reaction, liquid and one solid phase are in equilibrium with two other solids, so the composition of this point cannot be in the three phase equilibrium as seen as sketched below.

Sketch of a U type reaction: $L + C15 \leftrightarrow C14 + (Nb)$



The invariant reaction types were determined from the study of the microstructure of the as-cast samples by SEM. Following the sketches of the different types of the invariant reactions, the three phase field ($C14 + \mu + Nb$) of the different isothermal sections have been superposed for the determination of the eutectic point. In the same way, following the sketch of the U-type reaction, their respective places were determined.

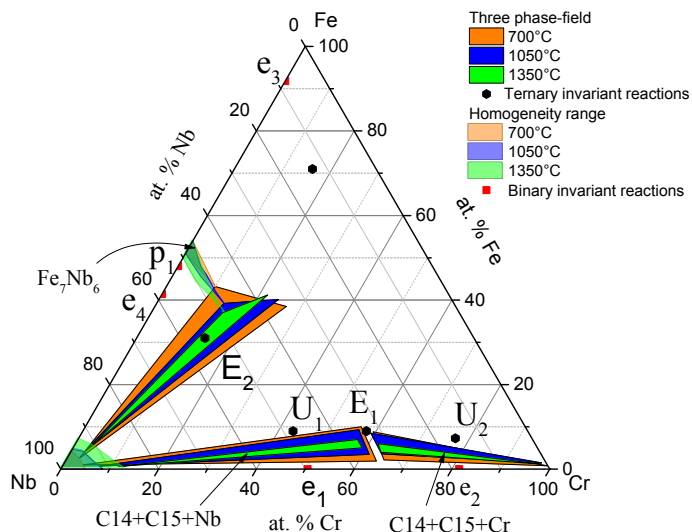


Figure 54 - Superposition of the three phase fields obtained from the studied of the isothermal sections as given in Figure 43, Figure 44 and Figure 45 as well as the binary invariant reactions as a guide to place the ternary invariant reactions in consistency with the isothermal sections.

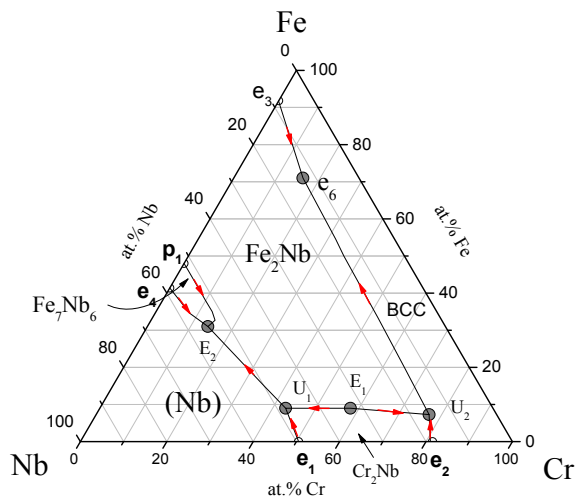


Figure 55 – Liquidus projection of the Cr-Fe-Nb system.

The invariant reactions in the ternary system are summarized in *Table 37*, where preliminary DTA measurements allowed the determination of their respective temperature. Nevertheless, further measurements are necessary for a better accuracy of these data.

Reactions	Type	Temperature (°C)
$L \leftrightarrow C14 + C15$	E1	unknown
$L + C15 \leftrightarrow C14 + Nb$	U1	1637
$L + C15 \leftrightarrow C14 + Cr$	U2	1590
$L \leftrightarrow \mu + C14 + BCC$	E2	unknown
$L \leftrightarrow BCC + C14$	e6	unknown

Table 37 – Invariant reaction in the ternary Cr-Fe-Nb system.

2. Discussion

2.1 Cr-Nb side

The Cr-Nb phase diagram exhibits two eutectic invariant reactions as shown by Stein et al. [18] (see Fig. 1 of the original paper) around the C15 Laves phase. This phase extends into the ternary phase diagram with a dissolution of Fe into it until a change of Laves phase polytype to C14 occurs. The change of Laves phase polytypes is characterized by the presence of a two phase field region which becomes narrower as the temperature is high (see isothermal section at 1050 and 1350°C, *Figure 44* and *Figure 45*). The microstructures of as-cast samples on each side of this two phase field region containing 9 at. % of Fe are shown in *Figure 56* and *Figure 57*. These two samples are composed of primary crystallized C14 Laves phase surrounded by a fine microstructure (Laves phase + BCC) characteristic of an eutectic.

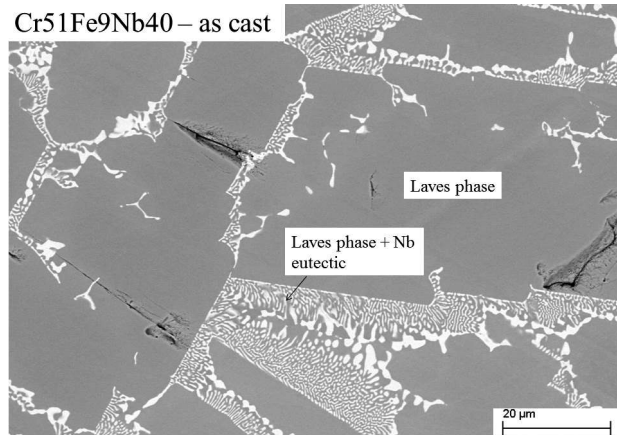


Figure 56 - As cast microstructure of the sample 26 - Cr51Fe9Nb40 with primary crystallized the Laves phase (dark) and (Nb)+C14 eutectic.

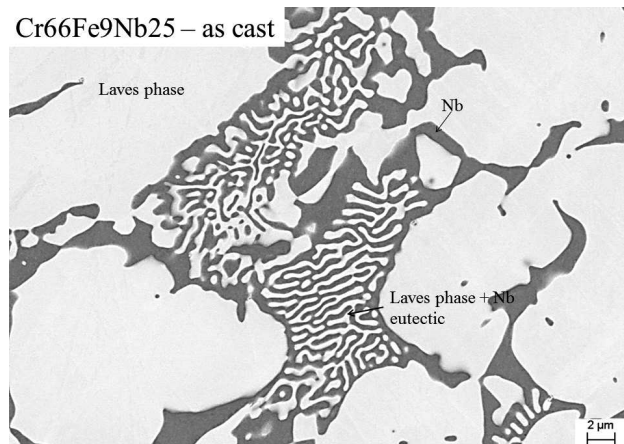


Figure 57 - As cast microstructure of the sample 23 – Cr66Fe9Nb25 with the primary crystallized Laves phase (light) and C14+ (Cr) in eutectic.

The ternary system is characterized by two U type reactions on each side of the C14-C15 Laves phase. The two invariant reaction points are symbolized as U_1 and U_2 on the liquidus projection (Figure 55), U_1 ($L + C15 \leftrightarrow C14 + (Nb)$) and U_2 ($L + C15 \leftrightarrow C14 + (Cr)$).

1.1. Fe-Nb side

In the Fe-Nb system, there are two invariant reactions surrounding the μ phase, a peritectic reaction (i.e. $L + C14 \leftrightarrow \mu$ at $T=1523^\circ\text{C}$) and an eutectic reaction on the Nb rich side of the μ phase (i.e. $L \leftrightarrow \mu + \text{BCC}$ at $T=1508^\circ\text{C}$) [35]. The microstructure investigations of alloys containing a small amount of Cr in this part of the phase diagram reveal an eutectic invariant reaction (*Figure 58*) named E_2 in the liquidus projection (*Figure 55*), forming a local minimum on this part of the phase diagram.

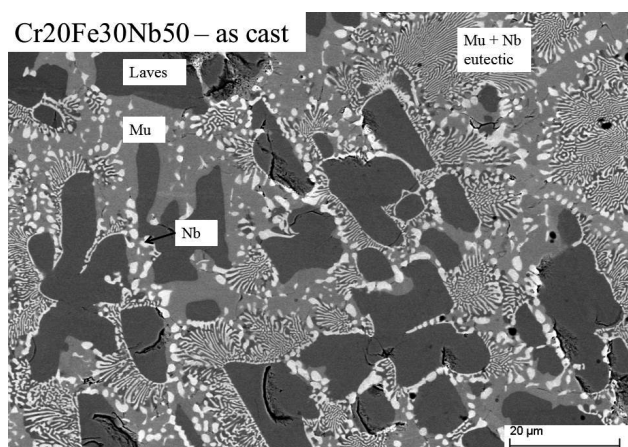


Figure 58 – SEM micrograph of cast sample 30 (Cr20Fe30Nb50), C14 Laves phase being the primary crystallization surrounded by an eutectic microstructure formed by μ and Nb.

1.2. Cr-Fe side

The Cr-Fe side is characterized by BCC primary crystallization (see *Figure 53*). The positions of our different samples allow the determination of a liquidus valley in this region (around 17 at. % Nb). The evaluation of the microstructure of the as-cast sample 15 revealed a fine eutectic microstructure ($C14 + \text{BCC}$) with BCC as primary crystallization (*Figure 59*). A local minimum exists in the liquidus valley, which is named e_6 on the liquidus projection (*Figure 55*).

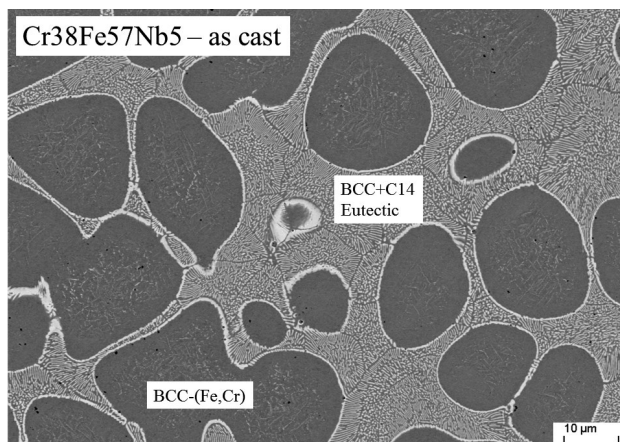


Figure 59 – SEM image of the microstructure of the sample 15 (Cr38Fe57Nb5) as cast showing BCC as primary crystallization and a eutectic microstructure (C14+BCC).

1.3. Ternary invariant reaction

Along two different isopleths with 25 and 40 at. % Nb, several samples have been placed in order to construct vertical sections. The DTA measurements of these samples are ongoing and will allow to obtain the temperature of solidus and liquidus as function of the composition. With this, temperatures and compositions of the invariant reactions will be determined with better accuracy. Nevertheless, DTA measurements have been done for a couple of samples allowing an estimation of the temperature of the invariant reactions (given in *italic*) in the reaction scheme *Figure 60*. These data should be considered preliminary results and are subject of changes with the investigation of the isopleths.

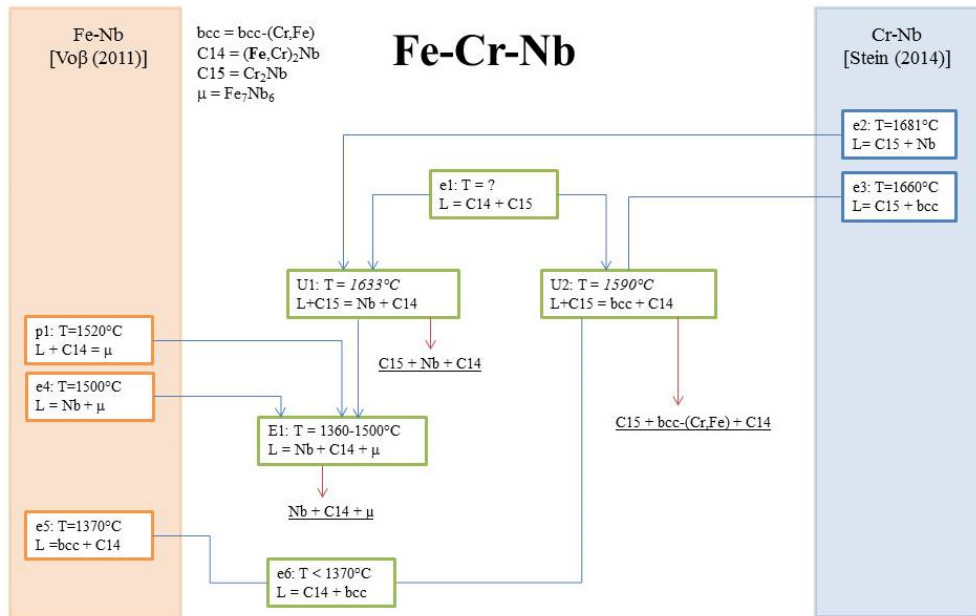


Figure 60 - Reaction scheme of the Cr-Fe-Nb phase equilibria without taking into account the Cr-Fe side. The data for the binary Cr-Nb and Fe-Nb and were taken from Stein et al. [18] and Voss et al. [35], respectively.

Conclusion

Using data on primary crystallization and the knowledge of the invariant reactions in the binary system, the liquidus surface of the Cr-Fe-Nb system has been constructed. Further DTA measurements are necessary for the determination of the temperature of the invariant reactions and for the determination of the U type compositions (obtained from the isopleth).

Sample	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	EDX results Cr Fe Nb at. %
as cast series; * = primary crystallization						
1	Cr32Fe60Nb8	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr) unknown phase	* P6 ₃ /mmc Im-3m	a=483.4(3) c=788.3(7) a=288.0(2)	35.9 36.4 27.7 61.2 37.7 1.1 38.6 44.3 17.1
2	Cr48Fe40Nb12	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	P6 ₃ /mmc Im-3m	a=485.1(3) c=792.9(6) a=288.2(2)	21.3 52.3 26.4 37.6 61.6 0.8
3	Cr64Fe20Nb16	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P6 ₃ /mmc Im-3m	a=488.6(4) c=800.7(6) a=288.9(2)	50.0 20.8 29.3 80.3 17.9 1.8
4	Cr35.75Fe35Nb29.25	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P6 ₃ /mmc Im-3m	a=487.2(2) c=797.1(3)	33.9 35.8 30.2 58.7 40.2 1.0
5	Cr44Fe20Nb36	as cast	(Fe,Cr) ₂ Nb (Nb)	* P6 ₃ /mmc Im-3m	a=491.9(2) c=806.1(3) a=328.5(7)	11.7 3.2 85.0 45.1 19.8 35.1
6	Cr29.25Fe35Nb35.75	as cast	(Fe,Cr) ₂ Nb (Nb)	* P6 ₃ /mmc Im-3m	a=490.1(2) c=801.0(4) a=328.5(7)	30.1 35.0 34.8 5.8 5.2 89.1
7	Cr36Fe20Nb44	as cast	(Fe,Cr) ₂ Nb (Fe,Cr) ₂ Nb (Nb)	* P6 ₃ /mmc P6 ₃ /mmc Im-3m	a=493.9(2) c=809.0(3) In eutectic a=326.9(2)	42.5 20.8 36.8 34.2 27.6 38.2 9.2 6.1 84.7
8	Cr10Fe35Nb55	as cast	Phase T (η) μ Fe ₇ Nb ₆ (Nb)	Fd-3m R-3m Im-3m	a=1130.4 a=495.5(4) c=2699(2) a=329.3(4)	12.8 37.9 49.4 8.5 40.9 50.5 2.5 8.9 88.6
9	Cr10Fe56.67Nb33.33	as cast	(Fe,Cr) ₂ Nb μ Fe ₇ Nb ₆	* P6 ₃ /mmc R-3m	a=485.7(1) c=792.5(2) not found	10.4 56.3 33.3 5.8 36.6 42.7

Sample	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	EDX results Cr Fe Nb at. %
10	Cr20Fe46.67Nb33.33	as cast	(Fe,Cr) ₂ Nb μ-Fe ₇ Nb ₆	* P _{6₃/mmc}	a=487.2(1) c=795.8(2) not found	20.4 46.0 33.6 9.6 32.3 44.0
11	Cr30Fe36.67Nb33.33	as cast	(Fe,Cr) ₂ Nb Nb-oxide traces	* P _{6₃/mmc}	a=488.2(1) c=798.7(2) not found	30.2 36.6 33.1
12	Cr40Fe26.67Nb33.33	as cast	(Fe,Cr) ₂ Nb Nb-oxide traces	* P _{6₃/mmc}	a=490.2(1) c=802.8(2) not found	40.0 26.2 33.8
13	Cr50Fe16.67Nb33.33	as cast	(Fe,Cr) ₂ Nb Nb-oxide traces	* P _{6₃/mmc}	a=491.3(1) c=805.9(2) not found	50.2 16.2 33.6
14	Cr60Fe6.67Nb33.33	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P _{6₃/mmc}	a=493.0(1) c=809.2(2) not found	59.7 6.7 33.7 83.6 8.5 8.0
15	Cr38Fe57Nb5	as cast	bcc-(Fe,Cr)	* Im-3m	a=288.1(2)	41.9 57.3 0.8
16	Cr36Fe54Nb10	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P _{6₃/mmc}	a=483.5(3) c=797.2(40) a=484.1(3) c=790.0(7) a=288.0(2)	Too small to be quantified 25.4 47.4 27.2 41.3 58.0 0.8
17	Cr32Fe48Nb20	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P _{6₃/mmc}	a=485.0(3) c=792.4(7) a=288.0	26.3 41.6 32.1 45.7 53.8 0.5
18	Cr57Fe38Nb5	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P _{6₃/mmc}	a=484.7 c=793.1 a=288.6	33.4 41.4 25.2 62.7 36.2 1.2
19	Cr54Fe36Nb10	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P _{6₃/mmc}	a=487.1 c=797.8 a=288.5	36.7 34.3 29.1 68.2 30.2 1.7
20	Cr48Fe32Nb20	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P _{6₃/mmc}	a=486.2 c=794.6 a=288.5	40.7 28.8 30.5 62.1 37.1 0.8
21	Cr72Fe3Nb25	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P _{6₃/mmc}	a=492.6(2) c=808.6(3) a=289.4(2)	65.6 2.5 31.8 93.7 2.6 3.6
22	Cr69Fe6Nb25	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P _{6₃/mmc}	a=492.1(2) c=807.5(3) a=289.3(2)	62.7 5.5 31.8 91.8 5.7 2.5

Sample	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	EDX results Cr Fe Nb at. %
23	Cr66Fe9Nb25	as cast	(Fe, Cr) ₂ Nb bcc-(Fe, Cr)	* P ₆₃ /mmc Im-3m	a=491.4(4) c=806.1(4) a=289.4(2)	60.0 8.5 31.5 90.0 7.4 2.5
24	Cr57Fe3Nb40	as cast	C36-Cr ₂ Nb (Nb)	* P ₆₃ /mmc Im-3m	a=496.0(2) c=1623.8(10) a=326.5(4)	61.2 2.9 35.9 18.6 0.9 80.5
25	Cr54Fe6Nb40	as cast	(Fe, Cr) ₂ Nb (Nb)	* P ₆₃ /mmc Im-3m	a=495.3(3) c=812.7(4) a=326.3(3)	58.5 5.7 35.7 17.2 1.6 81.3
26	Cr51Fe9Nb40	as cast	(Fe, Cr) ₂ Nb (Nb)	* P ₆₃ /mmc Im-3m	a=494.7(2) c=808.6(3) a=326.7(4)	55.6 8.8 35.6 14.7 2.7 82.7
27	Cr5Fe48Nb47	as cast	μ-Fe ₇ Nb ₆ (Fe, Cr) ₂ Nb (Nb)	R-3m * P ₆₃ /mmc Im-3m	a=494.8(2) c=2692(2) a=489.6(2) c=797.9(7) a=329.6(6)	4.2 46.1 49.7 7.5 55.3 37.2 1.2 13.0 85.3
28	Cr14Fe34Nb55	as cast	μ-Fe ₇ Nb ₆ Phase T (η) (Fe, Cr) ₂ Nb (Nb)	* R-3m Fd-3m P ₆₃ /mmc Im-3m	a=495.5(3) c=2705(2) a=492.4(7) c=804(2) a=327.4(3)	10.5 39.9 49.7 16.0 34.8 49.3 22.4 38.8 38.8 4.7 7.9 87.4
29	Cr5Fe40Nb55	as cast	μ-Fe ₇ Nb ₆ μ-Fe ₇ Nb ₆ ^a (Nb)	* R-3m R-3m Im-3m	a=495.5(3) c=2694.8(16) --- a=327.8(3)	6.3 43.5 50.2 3.9 45.2 50.9 1.6 10.4 88.1
30	Cr20Fe30Nb50	as cast	μ Fe ₇ Nb ₆ (Fe, Cr) ₂ Nb (Nb)	R-3m * P ₆₃ /mmc Im-3m	a=495.5 c=2707 a=491.8 c=804.9 a=327.8	15.5 35.2 49.4 28.4 33.3 38.2 6.0 7.2 86.8
31	Cr10Fe50Nb40	as cast	μ-Fe ₇ Nb ₆ (Fe, Cr) ₂ Nb	R-3m * P ₆₃ /mmc	a=494.0(2) c=2690(2) a=489.2(2) c=797.4(4)	6.9 44.6 48.5 11.6 52.2 36.2
32	Cr15Fe45Nb40	as cast	μ-Fe ₇ Nb ₆ (Fe, Cr) ₂ Nb (Nb)	R-3m * P ₆₃ /mmc Im-3m	a=494.7 c=2696 a=489.9 c=799.3 a=330.5	9.6 37.6 52.9 16.5 44.3 39.2 6.0 28.6 65.4

Sample	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	EDX results Cr Fe Nb at. %
33	Cr20Fe40Nb40	as cast	μ -Fe ₇ Nb ₆ (Fe, Cr) ₂ Nb (Nb)	R-3m * P _{6₃} /mmc Im-3m	a=492.2 c=2702 a=490.7 c=801.5 a=330.4	13.3 34.2 52.6 23.1 38.7 38.3 6.1 17.0 76.9
34	Cr15Fe37Nb48	as cast	μ -Fe ₇ Nb ₆ (Fe, Cr) ₂ Nb (Nb)	R-3m * P _{6₃} /mmc Im-3m	a=495.4 c=2702.5 a=491.5 c=802.6 a=327.9	13.3 33.8 52.9 20.4 38.7 41.0 4.4 10.4 85.2
35	Cr45Fe15Nb40	as cast	(Fe, Cr) ₂ Nb (Nb)	P _{6₃} /mmc Im-3m	a=493.7 c=809.5 a=326.4	47.9 15.9 36.3 13.3 4.5 82.2
36	Cr35Fe25Nb40	as cast	(Fe, Cr) ₂ Nb (Nb)	P _{6₃} /mmc Im-3m	a=492.4 c=806.1 a=326.8	37.5 25.6 37.2 9.1 6.4 84.6
37	Cr27Fe33Nb40	as cast	(Fe, Cr) ₂ Nb μ -Fe ₇ Nb ₆ (Nb)	P _{6₃} /mmc R-3m Im-3m	a=491.6(8) c=804.2 a=496.3 c=2711.3 a=327.5(1)	28.9 34.1 37.0 15.8 35.3 48.9 6.4 6.8 86.9
38	Cr63Fe12Nb25	as cast	(Fe, Cr) ₂ Nb bcc-(Fe, Cr)	P _{6₃} /mmc Im-3m	a=490.4 c=803.9 a=288.9	56.7 11.4 31.8 86.6 11.4 2.0
39	Cr55Fe20Nb25	as cast	(Fe, Cr) ₂ Nb bcc-(Fe, Cr)	P _{6₃} /mmc Im-3m	a=489.1 c=801.3 a=288.7	49.8 19.3 30.9 80.7 17.2 2.1
40	Cr45Fe30Nb25	as cast	(Fe, Cr) ₂ Nb bcc-(Fe, Cr)	P _{6₃} /mmc Im-3m	a=487.2 c=797.2 a=288.1	40.9 28.2 30.9 66.0 33.0 1.0
41	Cr35Fe40Nb25	as cast	(Fe, Cr) ₂ Nb bcc-(Fe, Cr)	P _{6₃} /mmc Im-3m	a=486.3 c=795.7 a=288.3	31.3 38.9 29.8 52.7 46.1 1.3
42	Cr25Fe50Nb25	as cast	(Fe, Cr) ₂ Nb bcc-(Fe, Cr)	P _{6₃} /mmc Im-3m	a=484.9 c=789.8 a=287.4	22.4 46.5 31.1 39.2 60.2 0.6
43	Cr15Fe60Nb25	as cast	(Fe, Cr) ₂ Nb bcc-(Fe, Cr)	P _{6₃} /mmc Im-3m	a=482.6 c=790.2 a=287.4	13.0 57.0 30.0 23.3 75.2 1.4
44	Cr5Fe70Nb25	as cast	(Fe, Cr) ₂ Nb bcc-(Fe, Cr)	P _{6₃} /mmc Im-3m	a=482.2 c=786.2 a=2.872	7.0 83.7 9.3 4.4 65.6 30.0

Sample	nominal composition at. %	annealing	Phases	Space Group	XRD lattice parameters pm	EDX results		
						Cr	Fe	Nb
45	Cr56Fe4Nb40	as cast	(Fe,Cr) ₂ Nb Cr ₂ Nb (Nb)	* P ₆₃ /mmc Fd-3m	a=495.4	59.4	4.0	36.6
					a=701.6			
					a=326.4	19.1	1.2	79.7
46	Cr55Fe5Nb40	as cast	(Fe,Cr) ₂ Nb (Nb)	* P ₆₃ /mmc Im-3m	a=494.9	58.9	5.2	35.9
					a=326.1	17.3	1.7	81.2
47	Cr53Fe7Nb40	as cast	(Fe,Cr) ₂ Nb (Nb)	* P ₆₃ /mmc Im-3m	a=494.8	56.6	7.1	36.4
					a=325.9	16.5	1.9	81.6
48	Cr71Fe4Nb25	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P ₆₃ /mmc Im-3m	a=492.2	64.3	3.8	32.0
					a=289.3	92.4	4.2	3.4
49	Cr70Fe5Nb25	as cast	(Fe,Cr) ₂ Nb bcc-(Fe,Cr)	* P ₆₃ /mmc Im-3m	a=492.2	63.8	4.9	31.3
					a=289.4	91.8	4.1	4.1

Table 38 – Composition of the different alloys with their nominal composition. The identified phases, their respective lattice parameters and the chemical composition of each phase is also given. * denotes the primary crystallization of the samples.

a_e in eutectic

III. Thermodynamic modeling of the Cr-Fe-Nb ternary system

The commercial databases TCFE6 shows a deficiency in predicting Laves phase formation in the alloy Crofer 22H [165]. The present work aims to provide a better description of the calculated Cr-Fe-Nb phase diagram. The optimization of phase diagrams can only be successfully done if there are enough experimental data of the phase diagram to be studied (e.g. experimental phase diagram and thermodynamic data). The experimental data obtained and previously described were used as basis for the optimization of the phase diagram. In the present work, we propose a computation of the Cr-Fe-Nb phase diagram using the Calculation of Phase Diagram (Calphad) method via an optimization of the Gibbs energy. In addition to experimental data, DFT calculations were done to describe the energies of the end-members.

1. Literature review

1.1. Fe-Nb

A literature survey was given in the Introduction (p. 13). Several optimizations of the Fe-Nb system are available in the literature [38–40]. For the present study we used the latest assessment, which was proposed by Khvan et al. [40]. For their assessment they have used DFT calculations for the description of the end-members as given by Liu et al. [38].

1.2. Fe-Cr

A lot of studies are devoted to the Fe-Cr system due to its importance in many functional and structural materials, e.g. stainless steel. A review of the system has been done by [73]. This review has shown the lack of the previous assessment to describe the magnetic order. Based on this conclusion, a new reassessment down to 0K has been done including experimental data and first principles calculations [73]. They have used a two-sublattice model to describe the σ phase.

In the present work, the assessment of Anderson and Sundman [166] was used with the modification of the liquid phase done by Lee [167] as the description of the phase diagram given by Xiong et al. [73] is not compatible with the present commercial database.

1.3. Cr-Nb

An extensive literature review was provided by Schmetterer et al. [29] for this system. They have shown the necessity of reassessing this system as it was proved by Aufrecht et al. [28] that the high temperature laves phase C14 is not stable in Cr-Nb. In their assessment, Schmetterer et al. [29] have used a two-sublattices model for the description of the C15, the energy of formation of which was obtained from DFT calculations (see p. 71 for details and *Figure 30*). For the calculation in the ternary system, we used their description of Cr-Nb [29].

1.4. Cr-Fe-Nb

Regarding its importance in industrial application, the literature on the Cr-Fe-Nb phase diagram is scarce. There is no thermodynamic optimization of the phase diagram available in the literature. The obtained calculated phase diagram from a thermodynamic database (TCFE6) is an extrapolation from the lower order boundary systems.

Due to the restricted literature data on this system and the new findings of the Cr-Nb boundary phase diagram, the phase diagram was experimentally reinvestigated providing three different isothermal sections at 700, 1050 and 1350°C, respectively. The results of the study of the isothermal sections have been described previously in part I of this chapter.

The Cr-Fe-Nb phase diagram is characterized by a solid solution of the C14 and C15 Laves phases cutting the phase diagram in two. In addition Cr dissolves in the binary Fe-Nb μ phase. Kaloev et al. [67] had found that the σ -(Fe,Cr) phase dissolves about 10 at. % Nb but the present investigation of the isothermal section at 700°C (*Figure 43*) reveals that this phase does not dissolve Nb.

The various crystalline phases and solid solutions present in this system have been thoroughly characterized in the literature and are given in *Table 31*. Thermal analysis and determination of characteristic temperatures has not yet been carried out in this system and no liquidus projection is available in the literature. At the present time, DTA measurements are ongoing for its determination. The invariant reactions from the limiting binary systems are given in *Table 37*.

2. Thermodynamic modeling

The CALculation of PHase Diagrams (CALPHAD) method was used for the optimization of the thermodynamic modeling of Cr-Fe-Nb phase diagram. Each phase present in the system is described by its Gibbs energy which is described by an appropriate mathematical model. In the present work, the calculations were carried out with Thermo-Calc [144]. The Gibbs energies of the pure elements were defined according to the Dinsdale polynomial [81].

2.1. Solution phases: BCC, FCC and Liquid

The solution phases were modeled with a substitutional model (one-sublattice model). The molar Gibbs energy is given according to equation:

$$\begin{aligned} G_m^\alpha & \quad \text{Eq. 59} \\ &= x_{Cr} {}^0G_{Cr}^\alpha x_{Cr} + x_{Fe} {}^0G_{Fe}^\alpha \\ &+ x_{Nb} {}^0G_{Nb}^\alpha + RT(x_{Cr} \ln x_{Cr} + x_{Fe} \ln x_{Fe} + x_{Nb} \ln x_{Nb}) + x_{Cr}x_{Fe}x_{Nb}L_{Cr,Fe,Nb}^\alpha \end{aligned}$$

The excess Gibbs energy was only used for the liquid phase.

2.2. σ phase

The σ phase is a non-stoichiometric compound which crystallizes in the $P4_2/mnm$ space group where the 30 atoms in the unit cell share five different Wyckoff positions. It can be in general written as $A_2^{12}B_4^{15}C_8^{14}D_8^{12}E_8^{14}$ where the bottom index indicates the number of atoms per position and the top index indicates the coordination number [87] (see *Table 11* in the Calphad chapter).

Many studies [87,168,169] reported the modeling of the five different positions independently in order to model correctly the non-stoichiometric behavior of this phase. For a binary system as Fe-Cr, it means to calculate $2^5=32$ different configuration and for a ternary $3^5=243$ configurations. For computational reasons and due to the complexity of the five sublattices, this will not be used.

With respect to the crystallographic structure, a three-sublattices model $(A,B)_{10}(B)_4(A,B)_{16}$ is the most convenient [13,86]. The first sublattice is occupied by $A_2^{12}D_8^{12}$, the second by B_4^{15} and the third by $C_8^{14}E_8^{14}$. By extension this model can be written as $(A,B)_{10}(B,C)_4(A,B,C)_{16}$ (e.g. $(Fe,Nb)_{10}(Cr,Nb)_4(Fe,Nb,Cr)_{16}$). This model was recently used for the ternary Fe-Nb-V [88].

The total Gibbs energy according to this sublattice model is written as:

$$G^\sigma = \sum_i \sum_j \sum_z y_i^1 y_j^2 y_z^3 G_{i,j,z}^\sigma \quad \text{Eq. 60}$$

$$+ RT \left(10 \sum_{i,j,z} y_{i,j,z}^1 \ln y_{i,j,z}^1 + 4 \sum_{i,j,z} y_{i,j,z}^2 \ln y_{i,j,z}^2 \right. \\ \left. + 16 \sum_{i,j,z} y_{i,j,z}^3 \ln y_{i,j,z}^3 \right)$$

Where y_i^k is the site fraction of i on the k sublattice, $G_{i,j,z}^\sigma$ is the Gibbs energy of the compound.

2.3. Laves phases C14 and C15

The Laves phases are the largest group of intermetallic phases with an ideal stoichiometry AB_2 . The C14 and C15 are closely related by their structure. Although C14 contain three different Wyckoff positions, it is common to model it as a two sublattices model $(A,B)(A,B)_2$, first due to the occupancy of each atom in one sublattice and secondly to be consistent with the C15 Laves phase which contains only two independent positions. The two-sublattice model can be as well used by extension into the ternary system $(A,B,C)(A,B,C)_2$.

The total Gibbs energy of C14 and C15 Laves phases according to the two-sublattice models is written:

$$G^\lambda = \sum_i \sum_j y_i^1 y_j^2 G_{i,j}^\lambda + RT \left(2 \sum_i y_i^1 \ln y_i^1 + \sum_i y_i^2 \ln y_i^2 \right) + \sum_i \sum_j \sum_k y_i^1 y_j^2 y_k^3 L_{i,j,k}^\lambda \quad \text{Eq. 61}$$

Where λ denotes C14 or C15 phases, y_i^1 is the site fraction of i on the first sublattice and y_j^2 the site fraction of j on the second sublattice. $G_{i,j}^\lambda$ is the Gibbs energy of the end-members i_2j .

2.4. μ phase

The μ phase has a rhombohedral structure (space group R-3m). The atoms are located on five different positions. The three different 6c Wyckoff positions are occupied by Nb in the binary Fe-Nb, the 18h by Fe whereas the 3b is shared between each other.

Different models exist but as demonstrated by Joubert and Dupin [85], the best way to model this phase in order to respect the overall occupancy is to use the (A,B)(A)₄(A,B)₂(A,B)₆ model. In Calphad type modeling, different models exist for the μ phase. There are four atomic environment type [170] that suggest a four sublattices model [13,170]. The two highest-coordinated atoms (6c' and 6c'') go on the same sublattice where only Nb goes. The third 6c position is occupied by Fe and Nb whereas the two other Wyckoff positions are shared between the two atoms. Furthermore as demonstrated by Joubert and Dupin [85], this model is the most suitable to cover the overall range composition of the μ phase. Two of these end-members have an ideal composition (AA₃A₂B₆ and BA₄A₂B₆) [85]. With this model it is possible to model the phase from 46% to 53.8 % of B (Fe) atom which cover largely the composition range in the Fe-Nb system [35]. By extension, in the ternary system, the sublattice model becomes (Fe,Nb,Cr)(Nb)₄(Fe,Nb,Cr)₂(Fe,Cr,Nb)₆.

This model has been successfully used recently in the ternary systems Fe-Nb-Mn [38] and Fe-Nb-V [88].

3. DFT calculations

The DFT calculations were carried out using the VASP software package [96] using the GGA:PBE for the description of the exchange and correlation energy and the projector augmented waves (PAW) [145] as basis set. For all calculations, the ground state energies were reached in three steps: 1) the volume and the ion positions were relaxed; 2) the stresses and forces were relaxed; 3) the tetrahedron method was used for accurate calculations of the ground state energy using a k-mesh point for the description of the periodicity of the crystal structure according to Monkhorst-Pack (i.e. 9x9x5 for σ , 11x11x7 for C14, 17x17x17 for C15 and 9x9x5 for μ). For all calculations, an accuracy of 10^{-6} was set up in the self-consistency loop and an energy cutoff of 400 eV was used. The magnetism of the compounds was taken into account due to the presence of Cr and Fe. For each compound containing chromium and iron, calculations were made in ferro- and non-magnetic state in order to determine the most stable state. For the (Fe,Cr)-free end-members, a non-magnetic state was used. The energy of the pure Cr was obtained for the anti-ferromagnetic state.

The energies of formation of the end-members for each phase were calculated from *Eq. 25* according to the chosen sublattice model.

The energies of the pure elements calculated in their stable form are given in *Table 39*, the comparison of these results were compared in Chapter III dedicated to the pure elements.

Phase	k-points	E (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)
α -Fe (FM)	11x11x11	-800.234	2.8304
Cr (AFM)	11x11x11	-915.229	2.8359
Nb (NM)	11x11x11	-974.43	3.327

Table 39 – Total Energy ground state of the pure elements calculated by DFT.

The energy of formation of the end-members which were available in the literature were not recalculated, they are given in *Table 40-43* with their respective references. For the C14 Laves phase, the calculated enthalpies of formation in the Fe-Nb system were taken from Liu et al. [38]. The end-members of C15

Laves phase in Cr-Nb were taken from Schmetterer et al. [29]. Details of these calculations are given in a previous chapter (p. 71).

4. Results and discussions

4.1 DFT calculations

The energies of formation of the different end-members calculated for each phase are reported in *Table 40-43*. For the different phases, some tendency on the stability of the end-members can be seen according to these results.

a. σ phase

The energies of the end-members according to the three sublattices model $(\text{Fe,Nb})_{10}(\text{Cr,Nb})_4(\text{Fe,Nb,Cr})_6$ are given in *Table 40*. The present results are compared with available literature data.

A ₂ ¹² D ₈ ¹² ; 2a, 8i	B ₄ ¹⁵ , 4f	C ₈ ¹⁴ E ₈ ¹⁴	Compound	FM			NM			Ref.
				$\Delta_f H$ (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	c (Å)	$\Delta_f H$ (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	c (Å)	
Fe	Nb	Fe	Fe ₂₆ Nb ₄	1.90	9.05	4.66	15.61	8.74	4.49	[88]
Fe	Nb	Nb	Fe ₁₀ Nb ₂₀	16.60	9.20	5.82	19.81	9.15	5.77	[88]
Fe	Nb	Cr	Fe ₁₀ Nb ₄ Cr ₁₆	5.82	8.81	4.66	6.04	8.79	4.69	Present work
Fe	Cr	Fe	Fe ₂₆ Cr ₄	29.15	8.87	4.40	32.25	8.87	4.41	Present work
				16.49	8.45	4.46				[157]
Fe	Cr	Cr	Fe ₁₀ Cr ₂₀	5.62	8.70	4.43	5.70	9.14	6.04	Present work
Fe	Cr	Nb	Fe ₁₀ Cr ₄ Nb ₁₆	22.54	9.06	5.69	24.62	9.05	5.54	Present work
Nb	Nb	Fe	Nb ₁₄ Fe ₁₆	18.42	9.40	4.92	29.37	9.24	5.00	Present work
Nb	Nb	Nb	Nb ₃₀				12.30	-	-	[137]
Nb	Nb	Cr	Nb ₁₄ Cr ₁₆	26.48	9.57	4.93	26.49	9.24	4.93	Present work
Nb	Cr	Fe	Nb ₁₀ Cr ₄ Fe ₁₆	21.27	9.15	4.91	25.05	8.95	5.02	Present work
Nb	Cr	Nb	Nb ₂₆ Cr ₄	14.11	10.44	5.28	14.11	10.04	5.28	Present work
Nb	Cr	Cr	Nb ₁₀ Cr ₂₀	35.69	9.36	4.86	35.69	9.36	4.86	Present work

Table 40 – Energies of formation of the different end-members calculated in the σ phase.

The results of the enthalpies of formation in the σ phase are plotted as an iso-contour plot in *Figure 61* as a function of composition.

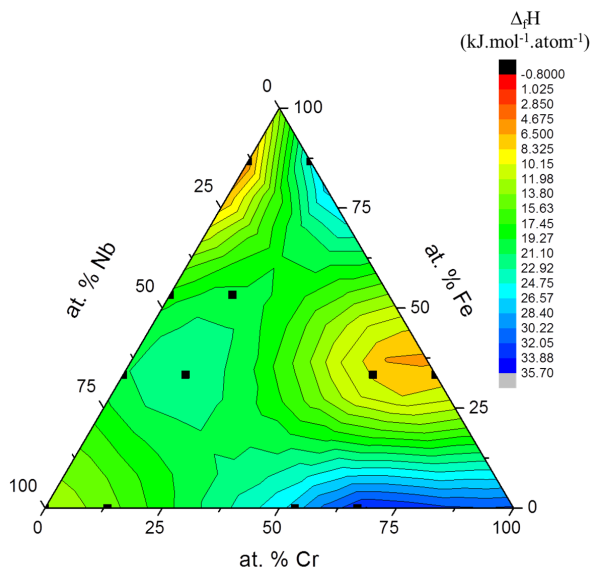


Figure 61 – Ternary iso-contour of the relaxed structure (given in $\text{kJ.mol}^{-1}.\text{atom}^{-1}$) of the hypothetical end-members calculated by DFT in the σ phase according to a three sublattices model.

According to the chosen sublattice model to describe the σ phase, none of the end-members are stables (*Table 40* and *Figure 61*) which are consistent with experimental data (see isothermal section at 700°C – *Figure 43*– indicating that no Nb dissolves in the σ phase).

b. Laves phase C14 and C15

The total energies of the end-members in the C14 and C15 Laves phase are given in *Table 41* and *Table 42*, respectively.

Compound		FM			NM			References
		$\Delta_f H$ (kJ.mol ⁻¹ .atom ⁻¹)	Lattice parameters		$\Delta_f H$ (kJ.mol ⁻¹ .atom ⁻¹)	Lattice parameters		
			a (Å)	c (Å)		a (Å)	c (Å)	
C14 Laves phase	Fe ₂ Nb	-11.88	4.82	7.88	-10.06	4.77	7.8	present work
		-13.16	4.822	7.839	-12.99	4.766	7.811	[38]
		-14.5	-	-				[156]
		-14.5	4.768	7.875				[157]
	Nb ₂ Fe	60.39	5.44	8.29	81.29	5.36	8.30	present work
		60.01	5.42	8.59	79.03	5.23	8.587	[38]
		2.1	-	-				[137]
		2.03	5.248	8.198				[157]
	Fe ₂ Fe	25.6	4.68	7.75	37.3	4.49	7.46	present work
		14.71	4.68	7.62	35.42	4.531	7.29	[38]
		38.81	-	-				[156]
		14.7	4.69	7.668				[137]
		31.1	-	-				[157]
		38.27	4.53	7.23				[155]
	Nb ₂ Nb				14.23	5.59	8.54	present work
					16.54	5.47	8.7	[38]
					15.602	-	-	[156]
					15.6	5.492	8.642	[157]
					15.4			[137]
					16.2	5.5	8.68	[26]
	Cr ₂ Cr	27.48	4.64	7.66	27.48	4.64	7.66	Present work
		28	4.67	7.691				[157]
		28.63	4.64	7.66				[26]
		25.4	-	-				[137]
	Cr ₂ Nb	-6.58	4.88	8.06	-6.58	4.88	8.06	Present work
		-3.38	4.9	8083				[157]
		-1.81	4.88	8.06				[26]
		-0.61						[133]
	Nb ₂ Cr	68.99	5.34	8.32	70.32	5.47	8.01	Present work
		71.7	5.482	8.033				[157]
		76.52	5.38	8.36				[26]
	Cr ₂ Fe	29.8	4.61	7.42	30.84	4.61	7.42	Present work
		31.1	4.634	7.459				[157]
	Fe ₂ Cr	23.96	4.64	7.56	22.95	4.56	7.27	Present work
		16.8	4.65	7.47				[157]

Table 41 – Energies of the end-members of the C14 Laves phase compared with available literature data.

Compound		FM			NM			References
		$\Delta_f H$ (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	V (Å ³)	$\Delta_f H$ (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	V (Å ³)	
C15 Laves phase	Cr ₂ Nb	-1.83	6.96	337.2	-1.72	6.93	332.8	Present work (ADF) [29]
		-3.31	6.93	333.1				[26]
		-2.48	6.82	317.2				[133]
		-2.31	6.93	332.8				[135]
		-2.1	6.94	334.3				[134]
		2.34						[136]
		-4.63	6.949	335.6				[157]
	Nb ₂ Cr	75.38	7.51	423.6	75.39	7.50	422.38	Present work [29]
		75.33	7.5	421.2				[26]
		60.4	7456	414.5				[157]
	Cr ₂ Cr	30.56	6.64	292.75	33.55	6.66	287.5	Present work [29]
		27.29	6.58	284.9				[26]
		26.4	-					[137]
		26.6	6.616	289.6				[157]
	Nb ₂ Nb				15.33	7.74	463.7	Present work [29]
					16.2	7.69	455.5	[26]
					16.2	-		[137]
					15.7	7.678	452.6	[157]
	Cr ₂ Fe	27.72	6.7	287.5	27.72	6.88	325.7	present work
		28.7	6.521	277.3				[157]
	Fe ₂ Cr	18.6	6.82	317.2	29.5	6.8	314.4	Present work
		25.4	6.44	267.1				[157]
	Fe ₂ Fe	39.43	6.8	314.4	52.04	6.75	307.5	Present work
		34.6	6.44	267.6				[157]
	Nb ₂ Fe	50.23	7.94	428.6	80.0155	7.4388	411.6	Present work
		75.3	7.393	404.1				[157]
	Fe ₂ Nb	-14.4	6.82	317.9	-9.88	7.13	362.4	Present work
		-14.3	6.799	314.3				[157]

Table 42 – Enthalpies of formation of the different end-members of the C15 Laves phase. The similar end-members as for the Cr-Nb system were calculated using ADF as described in p. 72 and in reference [29].

The enthalpies of formation of the end-members of C14 and C15 (Table 41, Table 42) are compared in Figure 62. The assessed enthalpies of formation of C14-Fe₂Nb and C15-Cr₂Nb are also given for comparison.

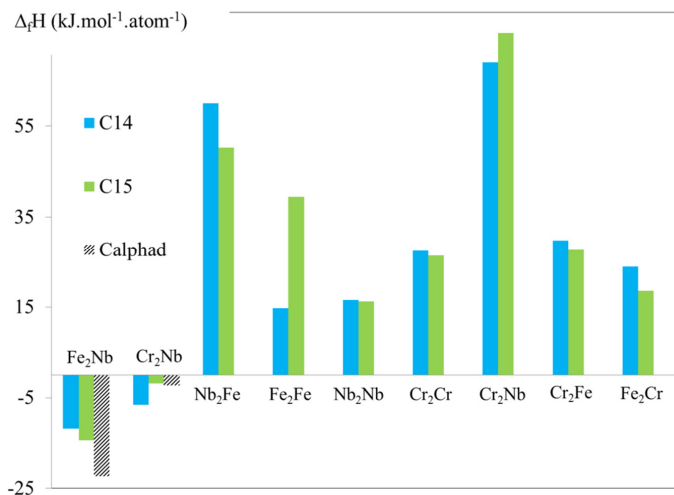


Figure 62 – Comparison of the enthalpies of formation of C14 and C15 Laves phase. For the stable end-members, i.e. C14-Fe₂Nb and C15-Cr₂Nb the Calphad optimized values have been added for comparison from references [29,40] for Fe-Nb and Cr-Nb, respectively.

The enthalpies of formation of the two different polytypes of Laves phase are very close. For the two stable end-members, the respective metastable end-members are more stable energetically according to the DFT calculations. As the two polytypes are closely related, these results are not surprising, furthermore within the approximation done to do the DFT calculations the difference of energies between the two polytypes is in the range of the accepted error. For C14-Fe₂Nb and C15-Cr₂Nb, the Calphad optimized value is lower (i.e. more stable) than its respective value obtained from DFT. For both thermodynamic assessments the optimization of the energy of formation of the stable end-members was obtained by taking into account experimental data.

c. μ phase

According to the DFT results (Table 43), the plot of the energies of the end-members of the μ phase in the Cr-Fe-Nb is given in Figure 63.

(Fe,Cr,Nb)	(Nb) ₄	(Fe,Nb,Cr) ₂	(Fe,Cr,Nb) ₆	Compound	FM			NM			Ref.
					ΔH (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	c (Å)	ΔH (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	c (Å)	
Fe	Nb	Fe	Fe	Fe ₉ Nb ₄	-7.71	4.84	25.43	0.32	4.71	25.79	[38]
Fe	Nb	Fe	Cr	Fe ₃ Nb ₄ Cr ₆	-1.87	4.74	28.23	3.19	4.73	28.05	Present work
Fe	Nb	Fe	Nb	Fe ₃ Nb ₁₀	32.81	5.42	27.53	34.69	5.44	27.37	[38]
Fe	Nb	Nb	Fe	Fe ₇ Nb ₆	-12.55	4.86	27.00	-9.05	4.88	26.25	[38]
					-13.50	4.87	26.64	-	-	-	[157]
Fe	Nb	Nb	Cr	FeNb ₆ Cr ₆	-6.60	4.85	29.07	-6.60	4.85	27.82	Present work
Fe	Nb	Nb	Nb	FeNb ₁₂	13.54	5.42	28.14	13.71	5.48	27.44	[38]
Fe	Nb	Cr	Fe	Fe ₇ Nb ₄ Cr ₂	-12.06	4.76	27.01	-9.71	4.77	26.60	Present work
Fe	Nb	Cr	Cr	FeNb ₄ Cr ₈	-4.18	4.76	28.11	-4.16	4.76	28.11	Present work
Fe	Nb	Cr	Nb	FeNb ₁₀ Cr ₂	14.38	5.43	28.01	16.13	5.43	27.99	Present work
Cr	Nb	Fe	Fe	CrNb ₄ Fe ₈	-6.46	4.78	26.76	-4.88	4.79	26.39	Present work
Cr	Nb	Fe	Cr	Cr ₇ Nb ₄ Fe ₂	-0.23	4.77	28.22	1.45	4.75	28.13	Present work
Cr	Nb	Fe	Nb	CrNb ₁₀ Fe ₂	21.08	5.29	29.00	21.08	5.29	29.00	Present work
Cr	Nb	Nb	Fe	CrNb ₆ Fe ₆	-19.35	4.88	27.85	-16.39	4.92	27.17	Present work
Cr	Nb	Nb	Nb	CrNb ₁₂	4.03	5.44	29.31	4.05	5.44	29.29	Present work
Cr	Nb	Nb	Cr	Cr ₇ Nb ₆	-6.93	4.90	28.85	-7.30	4.97	29.17	Present work
					-0.77	4.92	27.64	-	-	-	[157]
Cr	Nb	Cr	Fe	Cr ₃ Nb ₄ Fe ₆	-8.45	4.80	8.92	-6.62	4.82	26.32	Present work
Cr	Nb	Cr	Cr	Cr ₉ Nb ₄	-0.68	4.79	28.03	-0.68	4.79	26.77	Present work
Cr	Nb	Cr	Nb	Cr ₃ Nb ₁₀	16.42	5.45	28.07	17.86	5.44	28.07	Present work
Nb	Nb	Fe	Fe	Nb ₃ Fe ₈	-2.11	4.85	26.66				[38]
Nb	Nb	Fe	Cr	Nb ₅ Fe ₂ Cr ₆	5.34	4.82	28.84	7.03	4.79	28.79	Present work
Nb	Nb	Fe	Nb	Nb ₁₁ Fe ₂	38.40	5.45	28.49	38.44	5.45	28.52	[38]
Nb	Nb	Nb	Fe	Nb ₇ Fe ₆	-10.11	4.97	27.03	-7.60	5.02	26.10	[38]
					67.98	5.25	25.82	-	-	-	[157]
Nb	Nb	Nb	Cr	Nb ₇ Cr ₆	-2.75	4.97	29.17	-2.75	4.97	29.17	Present work
					66.11	5.30	26.22				[157]
Nb	Nb	Nb	Nb	Nb ₁₃	-	-	-	17.50	5.46	28.83	[38]
					-	-	-	20.70	-	-	[137]
					-	-	-	16.10	4.43	9.52	[157]
Nb	Nb	Cr	Fe	Nb ₅ Cr ₂ Fe ₆	-8.41	4.86	27.30	-6.52	4.90	26.69	Present work
Nb	Nb	Cr	Cr	Nb ₅ Cr ₈	3.27	4.84	28.70	3.27	4.84	28.70	Present work
Nb	Nb	Cr	Nb	Nb ₁₁ Cr ₂	15.21	5.41	29.18	17.03	5.44	28.89	Present work

Table 43 – Enthalpies of formation of the end-members of the μ phase obtained from DFT calculations.

According to the DFT results, Cr dissolving in the binary Fe-Nb μ phase is more favorable on the 3b Wyckoff position as it can be seen by comparing the enthalpies of formation of Fe:Nb:Nb:Cr (FeNb₆Cr₆, $\Delta_f H = -6.60$ kJ.mol⁻¹.atom⁻¹) and Cr:Nb:Nb:Fe (CrNb₆Fe₆, $\Delta_f H = -19.35$ kJ.mol⁻¹.atom⁻¹).

The contour map of the results of the enthalpies of formation is given in *Figure 63*.

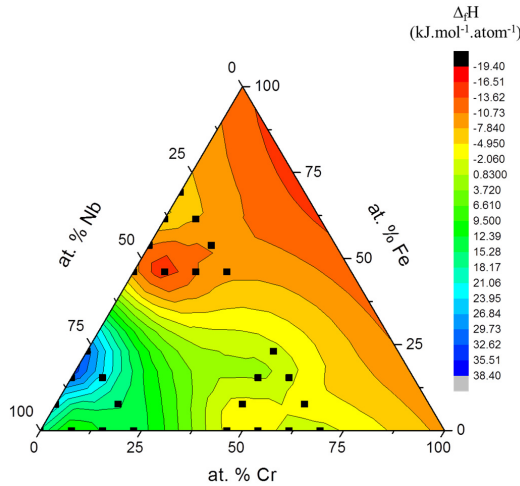


Figure 63 – Ternary iso-contour plot of the ground state energies of the hypothetical end-members (black point) calculated by DFT in the μ phase according to a four sublattices model (Fe,Cr,Nb)(Nb)₄(Fe,Cr,Nb)₂(Fe,Cr,Nb)₆. The enthalpies of formation are given in kJ.mol⁻¹.atom⁻¹.

The results of the enthalpies of formation of the end-members show that this phase is stable throughout a large part of phase diagram, which is inconsistent with the stability of the μ phase determined experimentally. In the Nb-rich corner, the enthalpies of formation of the end-members are positive and therefore unstable, which is consistent with experimental evidence. Nevertheless, on the Cr-Nb side, the end-members are rather too stable, stabilizing the μ phase on this side of the phase diagram. Because this phase should not be stable in the Cr-Nb side, the end-members on this side of the phase diagram will have to be optimized in order to fit to the experimental evidence.

4.2 Optimization of the phase diagram

The optimization of the phase diagram was obtained according to the Calphad formalism using ThermoCalc software [8]. For the description of the different phases, the energies of formation of the end-members obtained by DFT were used.

At first the phase diagram was calculated using the energies of the hypothetical end-members obtained from DFT without any modification. The calculated Cr-Fe-Nb phase equilibria are given in *Figure 64*.

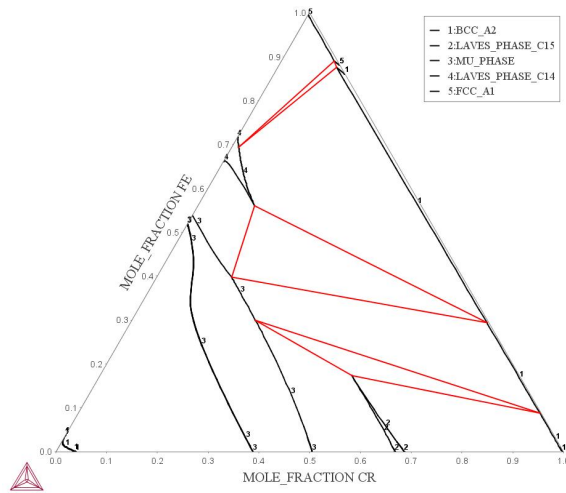


Figure 64 – Calculated isothermal section at 1050°C of the Cr-Fe-Nb phase diagram using the DFT values for the description of the end-members.

As the μ phase is rather too stable according to the DFT calculations, this phase appears to be quite stabilized on the Cr-Nb side which is inconsistent with experimental evidence. In addition the description of the C14 Laves phase is not stable enough. According to this result, the σ phase does not appear which is consistent with the experimental results.

a. BCC, FCC

These two phases are well represented by using the binary interaction parameters. In the present work, no ternary interaction was used to describe these phases.

b. Liquid

The liquidus values are not yet available for the present optimization of the phase diagram. The ternary interaction parameters cannot be accurately optimized. Nevertheless, the liquid phase appeared in the highest calculated isothermal section (i.e. 1350°C) which is in contradiction to experimental evidence (Figure 45).

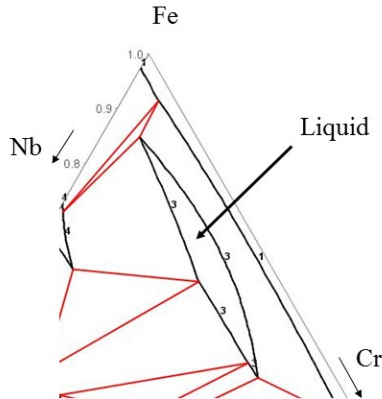


Figure 65 – Calculated iron rich corner of the Cr-Fe-Nb phase diagram at 1350°C showing the presence of the liquid phase.

As the liquid should not be present at this temperature, interaction parameters were guessed in such way that the liquid is not present at this temperature. To describe the liquid phase, ${}^0L_{Cr,Fe,Nb} = {}^1L_{Cr,Fe,Nb} = {}^2L_{Cr,Fe,Nb} = 30000$ were used. In this way, the excess Gibbs energy is desymmetrized and the liquid is not stable at 1350°C. Nevertheless, further data are necessary for an accurate description of the liquid phase.

c. Intermetallic phases

As the DFT calculations of the end-members of the different phases are considered accurate, most of the enthalpies of formation are kept as obtained by this calculation. In order to modify the stability of the phases, the temperature dependence parameters of specific key end-members were optimized instead.

Nevertheless, as the C14 and C15 are closely related by their crystal structure, by their energy and symmetry in this system, the enthalpies of formation of the end-members have to be consistent with the stability of the Laves phases polytypes.

C14 and C15 Laves phase

In the present Cr-Fe-Nb system, the two Laves phase are symmetric to each other and cross the phase diagram from one binary side (i.e. Fe-Nb) to the other (i.e. Cr-Nb). This feature has to be taken into account for the optimization of the phase diagram. The parameters of one side of the phase diagram were fixed according to the other side. For example the parameters of the C15-CrNb binary system from Schmetterer et al. [29] were used to describe the interactions parameters of the C14-CrNb. It results:

$${}^0L_{Cr,Nb:Cr}^{C15} = {}^0L_{Cr,Nb:Nb}^{C15} = {}^0L_{Cr,Nb:Cr}^{C14} = {}^0L_{Cr,Nb:Nb}^{C14} = 35896 \quad Eq. 62$$

$${}^0L_{Cr:Cr,Nb}^{C15} = {}^0L_{Nb:Cr,Nb}^{C15} = {}^0L_{Cr:Cr,Nb}^{C14} = {}^0L_{Nb:Cr,Nb}^{C14} = -55035 \quad Eq. 63$$

As C14-Cr₂Nb is not stable, the enthalpy of formation of this end-member was first set up to 500 J.mol⁻¹ lower than for C15-Cr₂Nb and keeping the same temperature dependence. The two parameters were then optimized by taking into account the two experimental three phase fields (i.e. Nb+C14+C15 and Cr+C14+C15). For C15 Fe₂Nb, the enthalpy of formation by DFT was taken as it is lower than the Calphad optimized value of C14-Fe₂Nb value. Its temperature dependence was optimized.

Interactions parameters ${}^0L_{Cr:Fe,Nb}^{C14}$ were used to describe the width of the C14 Laves phase.

μ phase

As the μ phase is rather too stable using the DFT enthalpies of formation, it is necessary to destabilize this phase especially on the Cr-Nb side. The Cr:Nb:Nb:Cr and Fe:Nb:Nb:Cr end-members were destabilized by using a positive temperature dependence (see *Table 44*, +80 and +20, respectively).

On the Fe-Nb side, the Cr:Nb:Nb:Fe end-member is rather too stable which makes the μ phase extending far into the ternary. Positive temperature dependence was therefore added to this end-member.

A negative temperature dependence has been added to Nb:Nb:Cr:Fe end-member to stabilize the μ phase at higher temperature.

4.3 Results of the optimization of the phase diagram

The obtained calculated isothermal sections are presented in *Figure 66* and *Figure 67* and compared with experimental data. The proposed optimized parameters lead – in general – to a good description of the phase diagram.

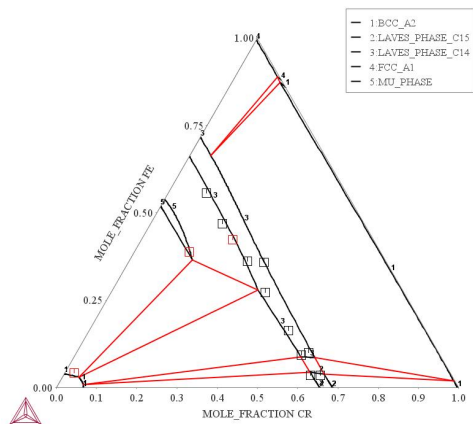


Figure 66 – Calculated isothermal sections at 1050°C obtained from the present optimized parameters as given in Table 44 and compared with experimental data (black squares, phase boundary of the Laves phase and red squares, the three phase field C14+ μ +BCC).

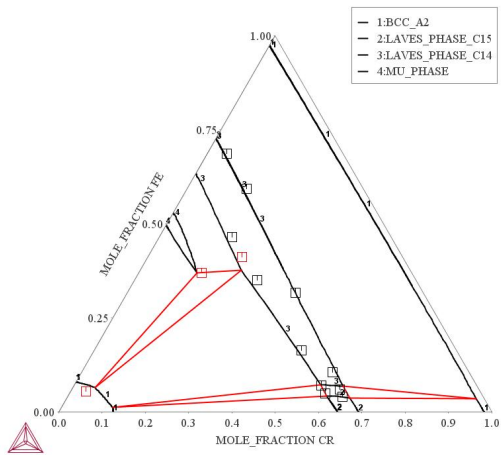


Figure 67 – Calculated isothermal sections at 1350°C from the present optimization compared with experimental data (black squares phase boundary of the Laves phases and red squares the three phase field μ +C14+(Nb)).

Without any ternary interaction parameters to describe BCC and FCC, these two phases appear to be well described.

The optimized parameters as given in *Table 44* provide a good accuracy of the description of the different phases for the highest isothermal sections (i.e. 1350°C, see *Figure 67*). Nevertheless, at lower temperatures, the μ phase extends too much into the phase diagram which is inconsistent with the experimental data.

Several attempts were performed to destabilize the μ phase at lower temperatures and to get better agreement with the three phase field (i.e. μ + BCC + C14). This results in a destabilization also at high temperature, which is inconsistent with experimental results. In order to improve the present results it is necessary to optimize the liquid phase according to the liquidus surface. As these data are not yet available, it is not possible to optimize the liquid phase. As there is an eutectic invariant reaction in this region, the liquid is in direct equilibrium with C14, μ and BCC, its knowledge (temperature and composition) will be of great importance to fix the liquid and to describe of the μ phase in the ternary system.

Phase	Thermodynamic parameters	Remark
Cr-Fe-Nb		
Liquid	${}^0L_{Cr,Fe,Nb}^L = {}^1L_{Cr,Fe,Nb}^L = {}^2L_{Cr,Fe,Nb}^L = 30000$	Opt.
C14 (Cr,Fe) ₂ Nb	${}^0G_{Cr:Nb}^{C14} = -3704.2 - 11.687 \times T$	Opt.
	${}^0L_{Cr,Nb:Cr}^{C14} = {}^0L_{Cr,Nb:Nb}^{C14} = {}^0L_{Cr,Nb:Cr}^{C15} = {}^0L_{Cr,Nb:Nb}^{C15} = 35896$	Present work
	${}^0L_{Cr:Cr,Nb}^{C14} = {}^0L_{Nb:Cr,Nb}^{C14} = {}^0L_{Cr:Cr,Nb}^{C15} = {}^0L_{Nb:Cr,Nb}^{C15} = -55035$	Present work
	${}^0L_{Cr:Fe,Nb}^{C14} = -100000$	Opt.
C15	${}^0G_{Fe:Nb}^{C15} = -43200 + 7.347 \times T$	Opt.
μ	${}^0G_{Cr:Nb:Nb:Cr}^{\mu} = -90090 + 80 \times T$	Opt.
	${}^0G_{Fe:Nb:Nb:Cr}^{\mu} = -85761 + 30 \times T$	Opt.
	${}^0G_{Nb:Nb:Cr:Fe}^{\mu} = -109330 - 60 \times T$	Opt.
	${}^0G_{Cr:Nb:Nb:Fe}^{\mu} = -251550 + 80 \times T$	Opt.

Table 44 – Optimized parameters for the description of the calculated Cr-Fe-Nb phase diagram.

Chapter VI – Thermodynamic modeling of the Fe-Nb-Si system

Introduction

The Fe-Nb-Si ternary system is important for industrial applications. As explained in the introduction, silicon plays an important role on the stabilization and promotion of the Laves phases. The reason that Froitzheim [3] proposed to add Si in addition to Nb and W in the interconnector steel called Crofer 22 APU was to prevent Si-diffusion at the Cr_2O_3 /alloy interface by forming a dense isolating SiO_2 by oxidation and to promote the formation of Laves phase. This modified steel is known under the name Crofer 22H [1].

Several studies outlined the importance of this system because of:

- the extension of the Laves phase into the ternary phase diagram [15];
- the precipitation of Laves phase in ferritic steel which improve the mechanical properties of structural materials [2];
- the ternary compound $\text{Nb}_4\text{Fe}_4\text{Si}_7$ with its high oxidation resistance can be used as coating for Nb-silicide applications [171].

Furthermore the calculated phase diagram would be of great importance for a better understanding of the chemical driving forces between the phases especially in the middle of the phase diagram (~30 at. % Si) [171], where several ternary compounds surround each other. The different ternary compounds have closely related structures and their formation would be well understood by knowledge of their Gibbs energy function obtained from a thermodynamic assessment of this system.

I. Literature

Only few studies of the Fe-Nb-Si ternary system are available in the literature. An intensive discussion about the literature of this system has been given in the Literature survey (p. 20).

As the proposed phase diagram given by Goldschmidt [59] was doubtful at several points (i.e. huge solubility of Si in the Laves phase) and not in agreement with the new knowledge of the binary boundary phase diagrams (i.e. the Fe-Nb side given by Goldschmidt is incompatible with the binary Fe-Nb system [35]), new experimental investigations have been recently provided by Wang et al. [15].

The only available liquidus data for this system are reported by Goldschmidt [59]. They were determined during arc-melting of the samples. These data are inconsistent with the phase diagram data. Furthermore, liquidus data have to be measured on equilibrated samples and not during the casting process.

The only determination of thermodynamic properties was done by Sudavtsova et al. [172], who measured the partial and integral enthalpies of mixing of pure Nb in FeSi at 1887 K. Their temperature measurement is incompatible with the reported liquidus temperatures from [59].

The solidus temperature of the ternary Laves phase compound $\text{Nb}_4\text{Fe}_3\text{Si}_5$ has been determined by Yu et al. [173] at $T=1359^\circ\text{C}$.

The different crystal structures of the ternary compounds in the Fe-Nb-Si phase equilibria are well documented in the literature [55,62,63,65,174] and are summarized in *Table 7*.

II. Thermodynamic modeling

The thermodynamic modeling of Fe-Nb-Si is not available in the literature. In commercial thermodynamic databases (i.e. TCFE6), the representation of this system does not include the different ternary compounds. Furthermore as silicon is not included for the description of Laves phase, this phase appears as a mutual solid solution crossing the complete phase diagram from the Fe-Nb to the Nb-Si side (see *Figure 13*).

The experimental and calculated phase diagrams (See *Figures 8-13* in the literature survey, p. 20) are in disagreement to each other. The thermodynamic modeling is relevant for the understanding of the phase

stability in specific condition and cannot be used for this system as it has yet not been optimized, nevertheless it would be relevant to have a correct thermodynamic modeling of Fe-Nb-Si phase equilibria.

The thermodynamic modeling of Fe-Nb-Si was done using the Calphad method [78]. This method has been explained in Chapter III (p. 34). For the computation of thermodynamic modeling of the ternary system Fe-Nb-Si, binary boundary phase diagrams from the literature were used. A schematic representation of the three binary phase diagram is given in *Figure 68* with their respective references.

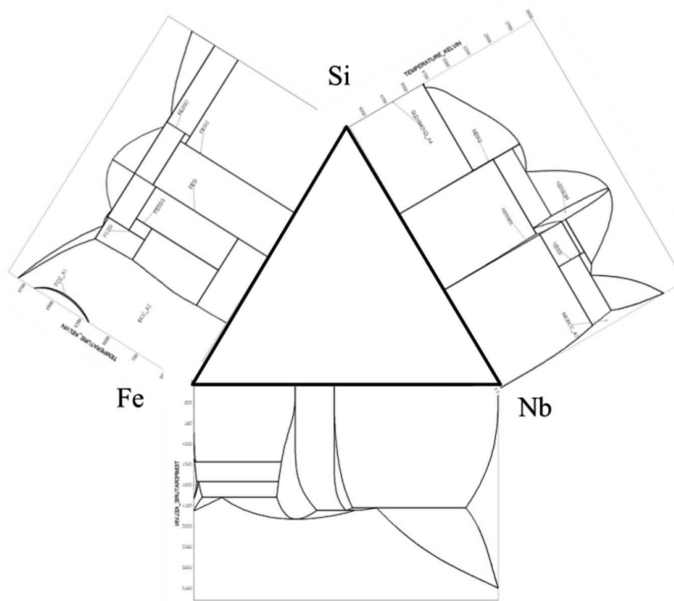


Figure 68 – Binary boundary phase diagrams as given in the literature: Fe-Nb [40], Nb-Si [75] and Fe-Si [175].

1. Fe-Nb

The Fe-Nb system has been assessed several times during the last couple of years [38–40,156] and is still subject of new studies due to some difficulties to use the binary calculated phase equilibria for higher

order systems. In the present work, we used the most recent assessment available done by Khvan et al. [40]. They used a two sublattice model for the Laves phase $(\text{Fe},\text{Nb})_2(\text{Fe},\text{Nb})$ and a four sublattice model for the description of the μ phase $(\text{Fe},\text{Nb})(\text{Nb})_4(\text{Fe},\text{Nb})_2(\text{Fe},\text{Nb})_6$.

2. Nb-Si

There are several assessments of this system available in the literature. In the TCFE6 database, the optimization of this system has been taken from Fernandes et al. [176]. In their assessment, they described the high temperature and the low temperature Nb_5Si_3 phases as two different phases with different sublattice models. Whereas $\alpha\text{-Nb}_5\text{Si}_3$ is described as a line compound, a three-sublattice model is used for the high temperature phase. The other intermetallic phases are described as line compounds.

In the present work, data were taken from Geng et al. [75] based on the previous assessment made by the same authors [177]. They used new available experimental data from David et al. [17]. Whereas in a previous assessment (i.e. [176]), the intermetallic phases (NbSi_2 , $\alpha\text{-Nb}_5\text{Si}_3$) were considered as line compounds, they used sublattice models to describe these phases with homogeneity range instead. This assessment has been used for the higher order system Nb-Si-Mo by the same author [75].

3. Fe-Si

The Fe-Si system was recently subject of discussion as the description of Lacaze and Sundman [127], used in commercial databases and in higher order systems, contains a high temperature miscibility gap and it is not recommended to use it for higher order system. This system is rather important for industrial materials such as steel. It is so necessary to have a correct representation of the calculated phase diagram. One of the major difficulties for the calculated description of the Fe-Si phase diagram is the DO_3 -B2 phase transition. In the assessment of Lacaze and Sundman [127] the DO_3 phase is not described.

Ohnuma et al. [74] made experimental work and used diffusion couples to study the $\text{DO}_3/\text{B2}$ transition. Then they proposed a thermodynamic assessment with ordered/disordered phases (A2, B2 and DO_3

phases). This makes the thermodynamic description very complex, which therefore will not be used in the present work.

The calculated phase diagram proposed by Lacaze and Sundman [127] is used in the TCFE database but due to its inverse miscibility gap, Yuan et al. [175] have reconsidered the optimization of the system to avoid this. They did not take into account the recent investigation of Ohnuma [74]. Yuan et al. [175] proposed a new version of the thermodynamic description of this system. In their description, they made the choice to avoid too complex models for calculation in higher order systems, for instance they did not take into account the disorder DO₃ phase. In the present work, we used the Yuan et al. [175] description.

4. Fe-Nb-Si

There is no thermodynamic modeling of the Fe-Nb-Si system in the literature. In addition to the dissolution of a third element into the binary phases, there are six ternary compounds as mentioned in the literature survey. Their crystallographic structures have been investigated and are available in the literature (given in *Table 7*, p. 20). For the thermodynamic modeling of the Fe-Nb-Si ternary system, we take into account the dissolution of a third element into the binary phases (e.g. Si in Laves and μ phases) as well as the ternary phases. The ternary phases have no perceptible homogeneity range, they were therefore defined as stoichiometric compound for the Calphad modeling.

The optimization of the phase diagram was done using Thermo-Calc [8] based on experimental data [15]. In addition, enthalpies of formation of the different end-members have been calculated using the DFT method. The thermodynamic modeling of this system can only be done partially as there are no reliable liquidus data, which are nevertheless of great importance for the optimization of a phase diagram.

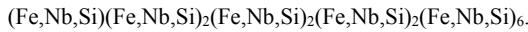
4.1 Laves phase

The C14 Laves phase polytype crystallizes in the space group P6₃/mmc with three different occupied Wyckoff positions. Within its usual formula AB₂, A occupies the 4f position and B the 2a and 6h

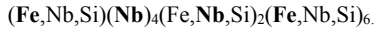
positions (see Chapter introduction). Silicon as a third element occupies the position of B [20]. The homogeneity range can be described using the compound energy formalism using a two-sublattice model. The two-sublattice model in the ternary is written (Fe,Nb,Si)₂(Fe,Nb,Si) allowing silicon to occupy positions of Fe and Nb.

4.2 μ phase

The μ phase crystallizes in space group $R\bar{3}m$. The atoms are allocated into five different Wyckoff positions. In the ternary Fe-Nb-Si system, significant amounts of Si dissolve in this phase. Following the principle of the sublattice model where each atom may occupy each different Wyckoff position would yield to 243 different end-members according to:



This would be rather computationally expensive. In order to reduce the number of end-members, some sublattices were combined. Joubert and Dupin [85] suggested a four-sublattice model where the second and the third sublattices are combined and only contain Nb:



This model has been described in the Calphad part (p. 37).

4.3 Ternary compounds

The ternary compounds have been modelled as line compounds as they do not show homogeneity range.

The Gibbs energy of the considered phases are written:

FeNbSi

$$\begin{aligned} G^{FeNbSi} - \sum x_i^0 H_i(298.15K) & \quad \text{Eq. 64} \\ & = a + bT + 0.333 {}^0G_{Fe}^{BCC} + 0.333 {}^0G_{Nb}^{BCC} + 0.333 {}^0G_{Si}^{Diamond} \end{aligned}$$

FeNbSi₂

$$G^{FeNbSi_2} - \sum x_i^0 H_i(298.15K) = a + 0.25 {}^0G_{Fe}^{BCC} + 0.25 {}^0G_{Nb}^{BCC} + 0.5 {}^0G_{Si}^{Diamond} \quad Eq. 65$$

Fe₄Nb₄Si₇

$$G^{Fe_4Nb_4Si_7} - \sum x_i^0 H_i(298.15K) \quad Eq. 66$$

$$= a + bT + 0.267 {}^0G_{Fe}^{BCC} + 0.267 {}^0G_{Nb}^{BCC} + 0.467 {}^0G_{Si}^{Diamond}$$

Fe₃Nb₄Si₅

$$G^{Fe_3Nb_4Si_5} - \sum x_i^0 H_i(298.15K) \quad Eq. 67$$

$$= a + 0.25 {}^0G_{Fe}^{BCC} + 0.333 {}^0G_{Nb}^{BCC} + 0.4167 {}^0G_{Si}^{Diamond}$$

FeNb₄Si

$$G^{FeNb_4Si} - \sum x_i^0 H_i(298.15K) \quad Eq. 68$$

$$= a + bT + 0.167 {}^0G_{Fe}^{BCC} + 0.667 {}^0G_{Nb}^{BCC} + 0.167 {}^0G_{Si}^{Diamond}$$

With a is expressing the enthalpy of formation of the considered compound and b is the temperature contribution term to the Gibbs energy. The a terms were obtained from DFT calculation for the ternary compounds with no mixed occupation in their Wyckoff position (i.e. Nb₄FeSi, FeNbSi, Fe₄Nb₄Si₇ and FeNbSi₂) in order to have a primary overview of their thermodynamic stability.

III. DFT calculations

The enthalpy of formation at 0 K of the end-members was obtained according to the formula (Eq. 25) taking into account the energy of the pure elements in their stable structure (Pure elements chapter, p. 59).

The DFT calculations were done using the VASP software package [96] within the framework of the projector augmented wave method [103] using the PBE exchange correlation-potential [121]. An energy cutoff of 400 eV was chosen for all calculations. The Brillouin zone was chosen carefully according to the

crystal structure to obtain the minimum energy during test calculation convergences (especially for the ternary compounds). For the Laves and the μ phases, we used the same set of k-points as Liu et al. [38], i.e. $9 \times 9 \times 5$ and $11 \times 11 \times 5$, respectively. For the ternary compounds, the chosen set of k-points is given in *Table 47* with the results of their respective enthalpy of formations and lattice parameters. For each structure, the minimization of the energy to reach the ground state was obtained in three steps: 1) Relaxation of the ionic positions and volume, 2) Forces, stresses and volume were relaxed, 3) Tetrahedron method was used for an accurate calculation of the total energy. As the studied system contains Fe, we took into account the magnetism. Calculations were done for non-magnetic (NM) and ferromagnetic (FM) compounds in order to check the most stable state.

IV. Results and discussion

1. DFT calculations

As previously mentioned, the enthalpies of formation of the different end-members were calculated using VASP. Some of the different end-members (i.e. Laves and μ phases) were already available in the literature and have not been recomputed. We used the calculations from Liu et al. [38] for the end-members in Fe-Nb (see *Table 45* and *Table 46*).

1.1 C14 Laves phase

The results of the enthalpy of formation and the lattice parameters according to the two sublattice model are presented in *Table 45* for the Laves phase.

Compound		FM				NM				References
		ΔH (kJ.mol ⁻¹ .atom ⁻¹)	Lattice parameters		V (Å ³)	ΔH (kJ.mol ⁻¹ .atom ⁻¹)	Lattice parameters			
			a (Å)	c (Å)			a (Å)	c (Å)	V (Å ³)	
C14 Laves phase	Fe ₂ Nb	-11.88	4.82	7.88	160.2	-10.06	4.77	7.8	153.7	present work
		-13.16	4.822	7.839	157.9	-12.99	4.766	7.811	157.9	[38]
		-14.5	-	-	-					[156]
		-14.5	4.768	7.875	155.04					[157]
	FeNb ₂	60.39	5.44	8.29	212.46	81.29	5.36	8.3	206.51	present work
		60.01	5.42	8.59	218.54	79.03	5.23	8.587	218.7	[38]
		2.1	-	-						[137]
		2.03	5.248	8.198	195.54					[157]
	FeFe ₂	37.2	4.52	7.23	127.92					present work
		14.71	4.68	7.62	144.54	35.42	4.531	7.288	129.6	[38]
		38.81	-	-						[156]
		14.7	4.69	7.668	146.07					[157]
		31.1	-	-						[137]
		38.27	4.53	7.23	128.49					[155]
	NbNb ₂					14.23	5.59	8.54	230.9	present work
						16.54	5.47	8.7	229.6	[38]
						15.602	-	-		[156]
						15.6	5.492	8.642	225.74	[157]
						15.4				[137]
						16.2	5.5	8.68	227.39	[26]
	SiSi ₂					47.46	5.6	6.92	187.94	present work
						2.5				[137]
						56.52	5.6	6.91	187.67	[155]
	NbSi ₂					-6.17	4.99	7.93	171.00	present work
	SiNb ₂					40.38	5.42	8.58	218.94	present work
	FeSi ₂	37.03	5.28	6.79	163.93					present work
48.92		5.11	6.93	156.71					[155]	
SiFe ₂	8.59	4.66	7.67	144.24					present work	
	19.91	4.55	7.56	135.54					[155]	

Table 45 – Calculated enthalpies of formation of the C14 Laves phase end-members in the Fe-Nb-Si ternary system. In red, the enthalpies of formation which have been used for the assessment.

1.2 μ phase

Enthalpies of formation computed by DFT for the μ phase are given in Table 46 according to the four sublattice model.

End-members					FM			NM			Ref.
(Fe,Si,Nb)	(Nb) ₄	(Fe,Nb,Si) ₂	(Fe,Si,Nb) ₆	Compound	ΔH (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	c (Å)	ΔH (kJ.mol ⁻¹ .atom ⁻¹)	a (Å)	c (Å)	
Fe	Nb	Fe	Fe	Fe ₉ Nb ₄	-7.71	4.84	25.43	0.32	4.71	25.79	[38]
Nb	Nb	Fe	Fe	Fe ₈ Nb ₅	-2.11	4.85	26.66				[38]
Fe	Nb	Nb	Fe	Fe ₇ Nb ₆	-13.04	4.88	26.74				Present work
					-12.55	4.86	27.00	-9.05	4.88	26.25	[38]
					-13.50	4.87	26.64	-	-	-	[157]
Nb	Nb	Nb	Fe	Fe ₆ Nb ₇	-10.11			-7.60	5.02	26.10	[38]
Fe	Nb	Fe	Nb	Fe ₃ Nb ₁₀	32.81	5.42	27.53	34.69	5.44	27.37	[38]
Nb	Nb	Fe	Nb	Fe ₂ Nb ₁₁	38.44	5.45	28.49	38.40	5.45	28.52	[38]
Fe	Nb	Nb	Nb	FeNb ₁₂	13.54	5.42	28.14	13.71	5.48	27.44	[38]
Nb	Nb	Nb	Nb	Nb ₁₃	-	-	-	17.50	5.46	28.83	[38]
					-	-	-	20.70	-	-	[137]
					-	-	-	16.10	5.43	9.52	[157]
Si	Nb	Nb	Nb	Nb ₁₂ Si ₁	-	-	-	4.72	5.31	29.47	Present work
Nb	Nb	Nb	Si	Nb ₇ Si ₆	-	-	-	-32.62	5.11	26.80	Present work
Si	Nb	Nb	Si	Nb ₆ Si ₇	-	-	-	-22.36	5.08	26.24	Present work
Fe	Nb	Nb	Si	FeNb ₆ Si ₆	-43.38	5.03	26.01	-43.38	5.03	26.01	Present work
Si	Nb	Fe	Nb	Fe ₂ Nb ₁₀ Si	21.01	5.23	28.52	21.00	5.22	28.62	Present work
Nb	Nb	Fe	Si	Fe ₂ Nb ₅ Si ₆	-19.79	4.99	26.26	-19.79	4.99	26.26	Present work
Si	Nb	Fe	Si	Fe ₂ Nb ₄ Si ₇	-7.77	4.92	25.80	-7.77	4.92	25.80	Present work
Fe	Nb	Fe	Si	Fe ₃ Nb ₄ Si ₆	-32.59	4.93	25.07	-28.97	4.93	25.07	Present work
Si	Nb	Nb	Fe	Fe ₆ Nb ₆ Si	-25.21	4.90	26.60	-19.41	4.93	25.85	Present work
Si	Nb	Fe	Fe	Fe ₈ Nb ₄ Si	-16.24	4.77	25.78	-11.85	4.79	25.13	Present work
Fe	Nb	Si	Fe	Fe ₇ Nb ₄ Si ₂	-20.73	4.78	27.44	-13.34	4.75	27.09	Present work
Fe	Nb	Si	Si	FeNb ₄ Si ₈	-24.99	5.16	25.87	-24.79	5.13	25.80	Present work
Fe	Nb	Si	Nb	FeNb ₁₀ Si ₂	8.02	5.34	29.28	8.02	5.34	29.27	Present work
Si	Nb	Si	Fe	Fe ₆ Nb ₄ Si ₃	-37.62	4.77	27.41	-28.94	4.79	26.78	Present work
Si	Nb	Si	Si	Nb ₄ Si ₉	-	-	-	-2.08	4.94	29.35	Present work
Si	Nb	Si	Nb	Nb ₁₀ Si ₃	-	-	-	-1.28	5.30	29.90	Present work
Nb	Nb	Si	Fe	Fe ₆ Nb ₅ Si ₂	-14.21	4.86	28.03	-8.69	4.88	27.26	Present work
Nb	Nb	Si	Si	Nb ₅ Si ₈	-	-	-	-16.48	5.57	25.74	Present work
Nb	Nb	Si	Nb	Nb ₁₁ Si ₂	-	-	-	8.68	5.35	30.22	Present work

Table 46 – Enthalpies of formation and lattice parameters of the end-members in the μ phase of the Fe-Nb-Si system calculated with VASP and compared with available literature data. In red, the data which have been used for the thermodynamic optimization.

The projection of the results of the enthalpy of formation at 0K as function of the composition in the μ phase has been plotted in *Figure 69* for a better overview of the stability of the end-members.

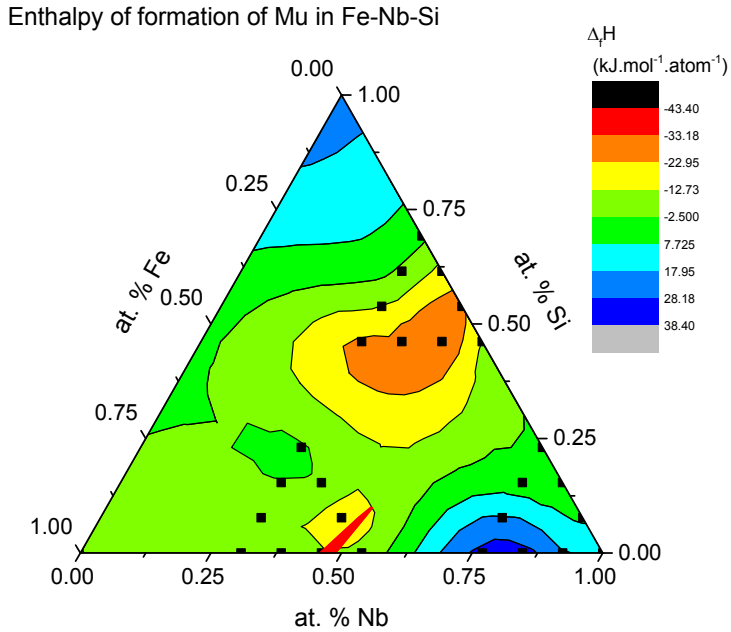


Figure 69 – Mapping of the stability of the end-members of the μ phase calculated by DFT at 0K. The different end-members are represented by the black square. The homogeneity range of the μ phase at 1000°C from Wang et al. [15] is given in red for comparison.

In general, the end-members with ideal compositions on the Nb-Si side are favorable (e.g. Nb₇Si₆ and Nb₆Si₇). The addition of Fe stabilizes these end-members. As an example FeNb₆Si₆ (Fe:Nb:Nb:Si) which is highly favorable, this end-member has the Nb in the 6c positions (favorable). The SiNb₆Fe₆ end-member is less negative, so Si has a site preference on the 18h Wyckoff position.

Fe is not favorable in the 6c position. Si is more favorable at this position (see Fe:Nb:Si:Fe and Fe:Nb:Fe:Si).

1.3 Ternary phases

The enthalpies of formation of the ternary compounds have been computed by DFT in order to get a first idea of the stability of the compounds and to avoid to have to guess them. The compounds with mixed occupation on certain Wyckoff positions have not been calculated by DFT (e.g. $\text{Fe}_3\text{Nb}_4\text{Si}_5$ and FeNb_2Si_2). The data of the energy of the compounds and their lattice parameters are reported in *Table 47*. The lattice parameters are compared with the experimental data available from the literature (given in brackets). Only a small deviation of the calculated/experimental lattice parameters can be noticed. Three of the ternary compounds (i.e. FeNbSi , $\text{Fe}_4\text{Nb}_4\text{Si}_7$ and FeNbSi_2) have similar enthalpies of formation, which is consistent with their respective position in the phase diagram.

Phase	k-mesh points	$\Delta_f H$ ($\text{kJ.mol}^{-1}.\text{atom}^{-1}$), FM	$\Delta_f H$ ($\text{kJ.mol}^{-1}.\text{atom}^{-1}$), NM	a (Å)	b (Å)	c (Å)
Nb_4FeSi	11x11x11	-28.67	-28.67	6.19 (6.19 [65])	6.19 (6.19 [65])	5.099 (5.056 [65])
FeNbSi	11x11x11	-59.56	-59.56	6.23 (6.231 [63])	3.73 (3.677 [63])	7.00 (7.19 [63])
$\text{Fe}_4\text{Nb}_4\text{Si}_7$	9x9x3	-59.60	-59.60	12.56 (12.65[178])	12.56 (12.65 [178])	5.03 (4.981 [178])
FeNbSi_2	9x8x9	-58.04	-58.03	8.69 (8.69 [55])	7.56 (7.57 [55])	9.71 (9.73 [55])

Table 47– Enthalpy of formation of the ternary compound in Fe-Nb-Si phase diagram. The lattice parameters on brackets are the experimental data from literature for comparison.

2. Thermodynamic optimization of the phase diagram

Using the binary phase boundary systems and the calculated enthalpies of formation of the end-members the calculated phase diagram is shown in *Figure 70*.

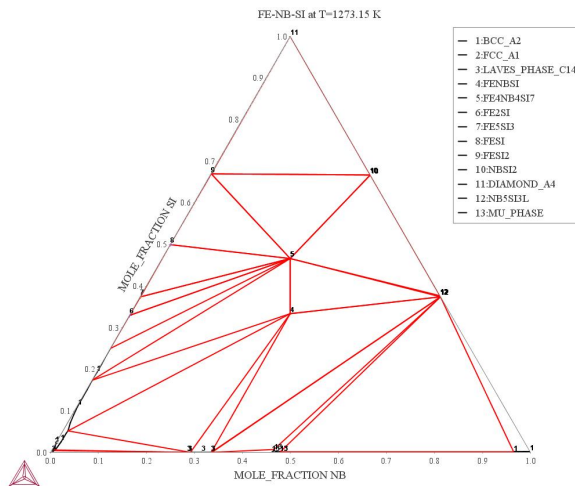


Figure 70 – Calculated isothermal section of the Fe-Nb-Si phase diagram at 1000°C without optimization.

Using only the DFT calculations, the phase diagram was calculated at 1000°C (Figure 70), four of ternary compounds are not shown in this calculation. In addition, the dissolution of Si in the Laves and μ phases on the Fe-Nb rich side is not shown.

At first the DFT calculations of the ternary compounds were used without any modification. The enthalpies of formation of the two ternary compounds which were not calculated by DFT (i.e. $\text{Fe}_3\text{Nb}_4\text{Si}_5$ and FeNb_2Si_2) were estimated to be similar to the other phases in the surrounding (i.e. NbFeSi_2 , $\text{Nb}_4\text{Fe}_4\text{Si}_7$ and NbFeSi). The starting values were taken around $60 \text{ kJ.mol}^{-1}.\text{atom}^{-1}$. Temperature dependences to the enthalpies of formation were introduced to stabilize or destabilize certain phases. Nevertheless no temperature dependences were used for the description of $\text{Fe}_3\text{Nb}_4\text{Si}_5$ and FeNb_2Si_2 .

In the Nb-rich corner, FeNb_4Si is not shown in the non-optimized phase diagram. This phase was stabilized by introducing negative temperature dependence to its enthalpy of formation.

2.1 Dissolution of Si in the Laves phase

As Si should dissolve into the ternary system, therefore ternary interactions parameters were introduced. The interaction parameter ${}^0L^{C14}(\text{Fe,Si:Nb})$ was introduced allowing dissolution of Si into the Laves phase. A second ternary interaction parameter ${}^0L^{C14}(\text{Fe,Si:Fe})$ was then introduced to simulate the widest range of the homogeneity of the Laves phase. For both interaction parameters, a linear temperature dependence was used. Their respective values are given in *Table 48*.

2.2 Si in the μ phase

The dissolution of Si is not featured in the calculated phase diagram obtained from the DFT calculations. In order to dissolve Si in the μ phase, an optimization of the parameters for this phase is required. A temperature dependence term has been added to the Si:Nb:Nb:Fe end-member allowing the dissolution of Si into the μ phase.

In order to improve the description of the μ phase an interaction parameter has been added.

2.3 Results of the thermodynamic assessment

For the first time, a thermodynamic assessment is given for the Fe-Nb-Si phase diagram. This assessment is only tentative as further experimental data would be required for a full assessment of this system. For example, liquidus and invariant reaction data would be of great importance in order to fix the liquid phase and the temperature dependence of the different phases.

Parameters obtained for the optimization of the phase diagram are given in *Table 48*.

Phase	Thermodynamic parameters	Remark
Fe-Nb	-	[40]
Fe-Si	-	[175]
Nb-Si	-	[75]
C14 (Fe,Si)₂Nb	${}^0L_{FeSi:Fe}^{C14} = -410000 - 10 \times T$	Optimized
	${}^0L_{NbSi:Nb}^{C14} = -320000$	Optimized
	${}^0L_{FeSi:Nb}^{C14} = -481000 - 35 \times T$	Optimized
μ	${}^0G_{Si:Nb:Nb:Fe}^{\mu} = -327728 - 100 \times T$	Optimized
	${}^0L_{Si:Nb:Fe:Fe}^{\mu} = -120000$	Optimized
FeNbSi	${}^0G_{FeNbSi} = -178680 - 7 \times T$	Optimized
FeNbSi₂	${}^0G_{FeNbSi_2} = -232160$	DFT
FeNb₂Si₂	${}^0G_{FeNb_2Si_2} = -320000$	Optimized
Fe₃Nb₄Si₅	${}^0G_{Fe_3Nb_4Si_5} = -743500$	Optimized
Fe₄Nb₄Si₇	${}^0G_{Fe_4Nb_4Si_7} = -894000 + 60 \times T$	Optimized
FeNb₄Si	${}^0G_{FeNb_4Si} = -172020 - 45 \times T$	Optimized

Table 48 – Optimized parameters for the thermodynamic description of the Fe-Nb-Si system.

It results the present phase diagrams given at 1000 and 1200°C (Figure 71 and Figure 72, respectively) which are compared with some literature data.

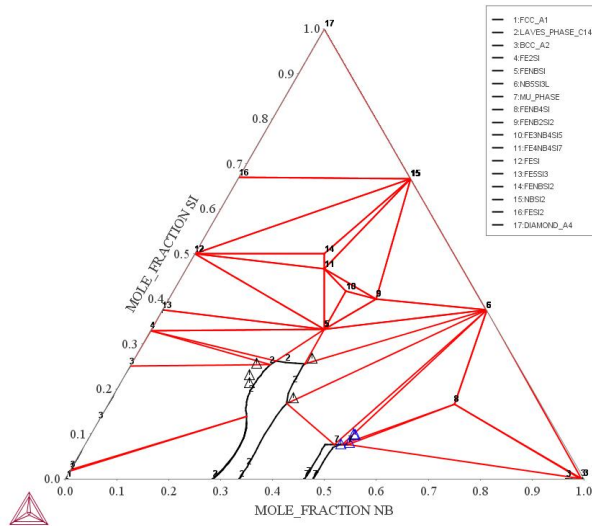


Figure 71 - Calculated isothermal section at 1000°C in comparison to experimental data given by Wang et al. [15], triangle symbols in black for the Laves phase and in blue for the μ phase.

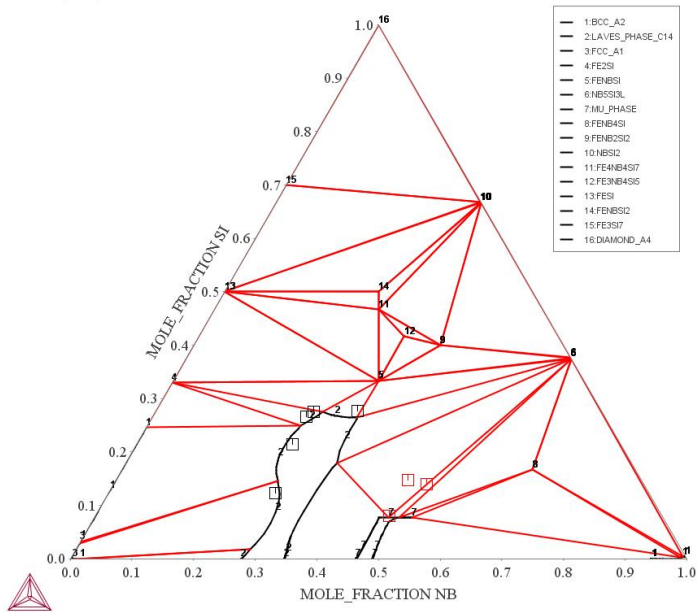


Figure 72 – Calculated isothermal section given at 1200°C in comparison with data from Wang et al., black squares (Laves phase), red squares (μ phase).

There is an overall good agreement of the calculated phase diagram with experimental data (Figure 71 and Figure 72). The dissolution of Si into the μ phase is lower in the calculated phase diagram. Further optimization of this phase would be required for a better description of the μ phase.

The calculated phase diagram shows an equilibrium $C14+Fe_2Si+FeNbSi$ which is not shown by Wang et al. [15], this is due to the description of Fe_2Si given by Yuan et al. [175] in the description of Fe-Si system.

In the highest isothermal section the laves phase and the $FeNbSi$ ternary phase are experimentally in equilibrium with the liquid [15], but this is not shown in the calculated phase diagram.

Conclusion

A tentative optimization of the Fe-Nb-Si phase diagram has been given here. There is an overall good description of the phase equilibria. Nevertheless, for an improvement of the thermodynamic optimization of this phase diagram further experimental data are necessary. At first, the liquidus projection of the present phase diagram is required as well as the temperature dependence of the thermodynamic properties of the ternary compounds.

Chapter VII – Microstructure of alloys and phase equilibria of sub-system Laves phase containing systems

In the present work, phase equilibria of binary and ternary systems containing Laves phase have been determined by experiment and/or modeling. The knowledge of phase equilibria is of great importance when developing materials with specific properties.

Many technically relevant materials are composed of about 10 to 15 elements, whereas the study of phase equilibria is often restricted to a maximum of four elements (as it is not easy to visualize with more elements). Nevertheless, it is often possible to predict the formation of certain phases by studying the binary, ternary or quaternary phases as each individual phase will be formed with less than 5 elements.

The studies of sub-systems like Cr-Nb, Fe-W, Cr-Fe-Nb and Fe-Nb-Si in the present study are compared with the development and studies of model alloys which can be then in successful cases being produced at an industrial scale.

During the development of new alloys, different compositions of elements are tried or the additions of new elements are studied.

I. Development of laves phase strengthened ferritic steel

During the development of the high chromium ferritic steel Crofer 22H, different additions of elements have been tested in order to check their effect on the properties of new steels, especially oxidation behavior and creep strength. Froitzheim et al. [142] have proposed different additions of element in order to promote the formation of strengthening Laves phase particles. In this study, the direct influence of sub-system phase diagram was pointed out. In the alloy reference Crofer 22 APU, where the ferritic matrix is composed of α -(Fe,Cr), the addition of Nb to Crofer 22 APU results in the precipitation of Nb with Fe and Cr and form Laves phase particles as can be seen on the Cr-Fe-Nb phase diagram (*Figure 73*).

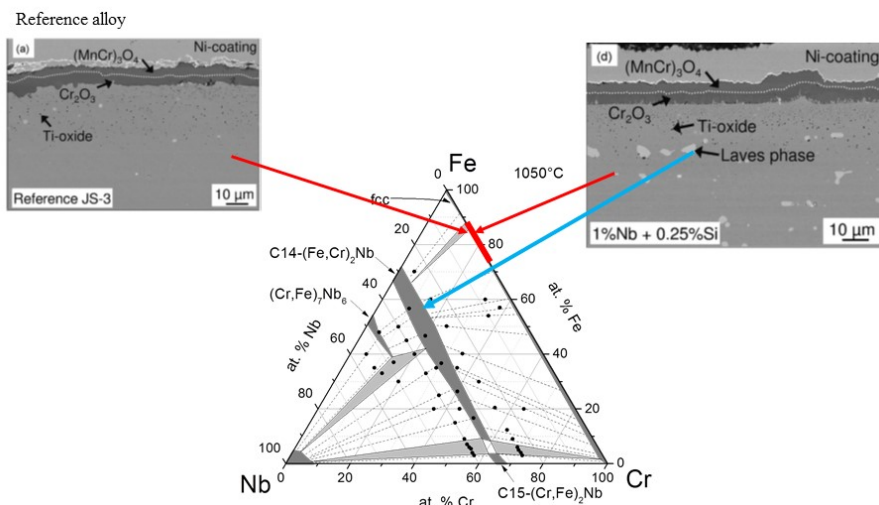


Figure 73 – Comparison of the microstructure of model alloy [142] with the Cr-Fe-Nb phase equilibria (present work). The two microstructure images are taken from Froitzheim et al. [142], the left microstructure is the reference alloy (model alloy with a similar composition as Crofer APU) and on the right microstructure of the model alloy with the addition of Nb and Si.

The Laves phase in the newly developed alloy as given by Froitzheim et al. [142] has a similar composition as expected from the Cr-Fe-Nb phase equilibria. Si dissolves in the Laves phase. The formation of the Laves phase is promoted by Si, the comparison with the ternary Cr-Fe-Nb system from the present work is done in a lower order system. As mentioned by the authors [142] “There are limited thermodynamic data available for the Fe-Cr-Nb-Si system” which limits the understanding of the chemical driving forces behind the formation of the different phases. The only article related to the quaternary phase diagram is given by Vilasi et al. [179] but they do not provide a lot of information about Si dissolving in the ternary Laves phase. Nevertheless, according to several studies on the Laves phase, Si promotes the formation of the Laves phase, stabilized it and occupies the position of Fe and Cr in the crystal structure. The overall Laves phase formula is written as $(\text{Fe,Cr,Si})_2\text{Nb}$.

In a similar way, Laves phase strengthening particles can be formed with tungsten (W). Abe et al. [180] have studied the effect of Fe_2W on the creep properties of ferritic-martensitic steel. Furthermore in long term application, the effect of W forming a Fe_2W Laves phase allows an improvement of solid solution strengthening of ferritic-martensitic steel [2].

II. Development of Nb-silicide

Even if the goal of the present study is an improvement of the thermodynamic database for steel, this database could also be used for the development of other materials (i.e. Ni-superalloys, Nb-silicides) as soon as they contain the same elements and the phases of interest as in the present study.

For the replacement of Ni-superalloys, Nb is one of the materials which is attractive for high temperature structural materials but has a poor oxidation resistance.

In order to develop further Nb alloys, several authors have tested different alloy composition as well as their casting process. Zhang et al. [181] have studied the addition of Fe on such alloys in order to improve the oxidation resistance. The addition of Fe into Nb-silicides has promoted the formation of Nb_4FeSi compound. This phase was observed in the Fe-Nb-Si phase diagram [15] (*Figure 74*).

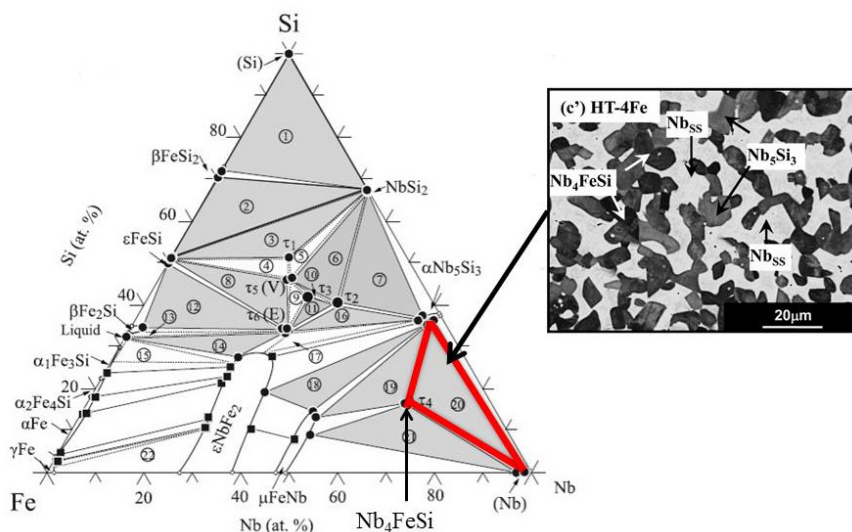


Figure 74 –Comparison of the Fe-Nb-Si phase diagram (reproduced from Wang et al. [15]) with Fe+Nb-silicide microstructure (micrograph reproduced from [181]). The three phase field region $Nb+Nb_4FeSi+Nb_5Si_3$ is observed on the micrograph.

III. Phase diagram vs ferritic steel alloy

The present comparison between model alloys and sub-systems of binary, ternary and quaternary alloys is of great importance for the development of new materials. Nevertheless, in addition to the different systems studied in the present work, other systems containing Laves phases are of great importance but are not well studied in the literature. For more clarity, the different sub-systems of Laves phase containing systems are reported in Figure 75. A color code is used to show which systems have been already studied and which systems would need to be studied for better understanding of the Laves phase containing systems in Crofer 22H and similar ferritic steels.

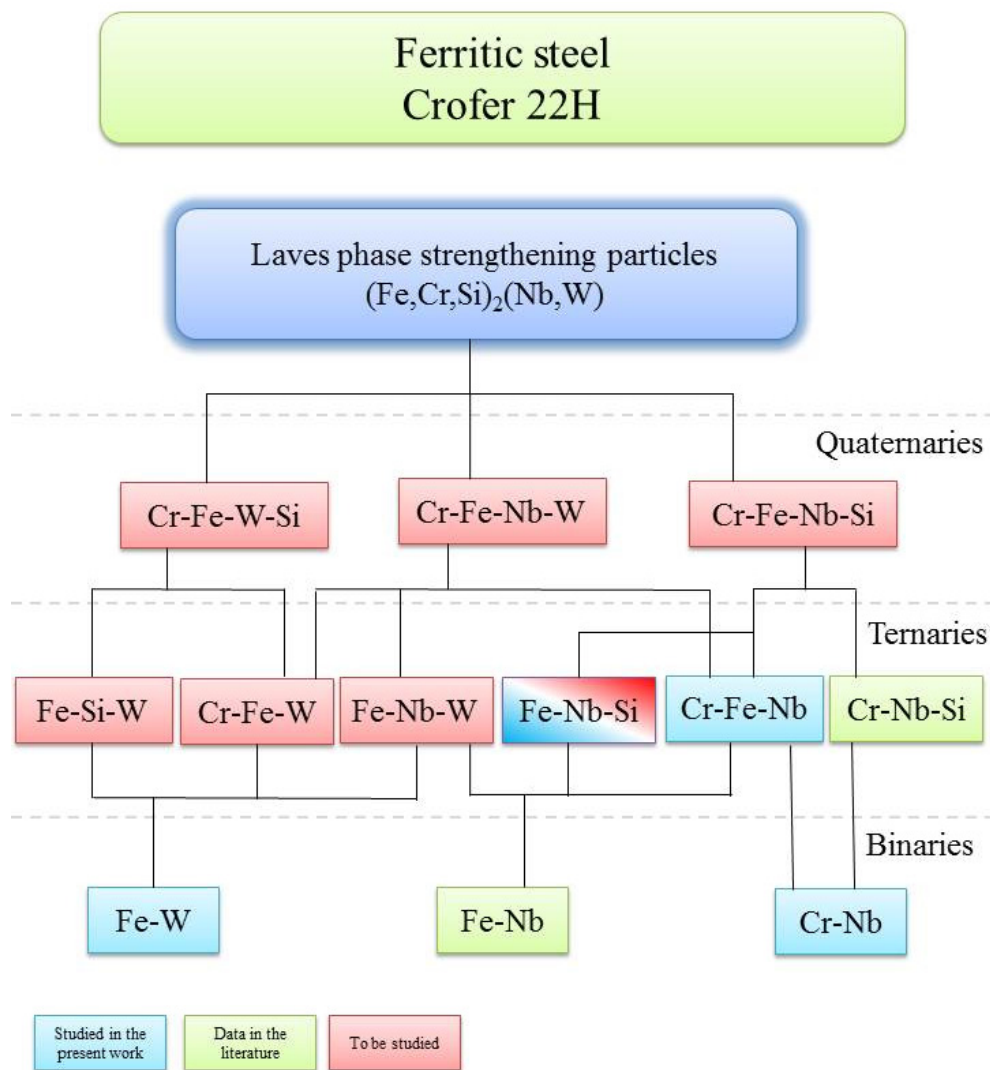


Figure 75 - Schematic representation of the different sub-systems containing Laves phase in Crofer 22H.

As can be seen on Figure 75, several ternaries and quaternaries systems would require further investigations. A careful literature survey would reveal where there is a need for investigation (i.e. experimental and/or modelisation). At first, it would be relevant to study the ternary Fe-W-Si and Cr-Fe-W.

Conclusion

In the present work, several methods have been used for the thermodynamic assessment of the phase diagrams related to ferritic steels containing Laves phases e.g. Crofer 22H. For the different systems several approaches were used to obtain suitable input data for the thermodynamic modeling.

In the two binary systems Cr-Nb and Fe-W, the available experimental data were sufficient and a computational approach based on DFT was used for the determination of the energies of formation of the compounds present in the system. In the Fe-W system, a new approach was tried by using phonon calculations to model the different end-members according to the chosen sublattice models. It was found that to model a disordered phase, the use of phonon calculation could lead to the creation of an unrealistic miscibility gap. So these data should be treated with care in thermodynamic modeling. In this system, two optimizations of the phase diagrams have been proposed which can be used for higher order systems.

In ternary systems Cr-Fe-Nb and Fe-Nb-Si, the literature survey revealed a lack of experimental data. It was therefore decided to reevaluate these phase diagrams experimentally. In the Fe-Nb-Si phase diagram, new experimental data from the literature became available during the time of the present work. So these data were used for the thermodynamic modeling. In addition DFT calculations were performed for the determination of the energy of formation of the different compounds. These data were of great help for the ternary compounds. Using these data (i.e. experimental and DFT), a partial thermodynamic modeling is given in the present work. Nevertheless, further experimental investigations are required for the determination of the liquidus surface and of the temperature dependence of the thermodynamic properties of the different phases for a suitable thermodynamic modeling of the Fe-Nb-Si system. In addition further thermodynamic experiments (e.g. calorimetry, KEMS measurements) would be of great help for a successful thermodynamic optimization.

In the Cr-Fe-Nb ternary system, experimental work was done to obtain different isothermal sections to have a suitable picture of the system. DFT calculations were done for the determination of the energy of

formation of the compounds. Using these data it was possible to give a primary overview of the thermodynamic modeling of the system. Some difficulties were encountered to model the μ phase. Data on the liquidus surface will be of great help to model the ternary interaction of the liquid phase and to give a better representation of the μ phase.

Further studies of Laves phase containing systems are necessary for the development of a suitable thermodynamic database to be used in the development of new ferritic steel strengthened by Laves phase particles. First, additional experimental data are necessary for several systems (i.e. liquidus data for Cr-Fe-Nb and Fe-Nb-Si as well as for other systems) due to a lack of knowledge in the literature. Second, DFT has shown to be a versatile method to complement experimental data in the thermodynamic optimization of phase diagram. For the creation of the thermodynamic database for steel systems, combined studies (experiments and computational data) are shown to be successful for thermodynamic optimization of the respective phase diagrams.

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