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Principles of transverse flow fractionation of microparticles in superhydrophobic channels

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We propose a concept of fractionation of micron-sized particles in a microfluidic device with a bottom wall decorated by superhydrophobic stripes. The stripes are oriented at an angle α to the direction of a driving force, G , which generally includes an applied pressure gradient and gravity. Separation relies on the initial sedimentation of particles under gravity in the main forward flow, and their subsequent lateral deflection near a superhydrophobic wall due to generation of a secondary flow transverse to G . We provide some theoretical arguments allowing us to quantify the transverse displacement of particles in the microfluidic channel, and confirm the validity of theoretical predictions in test experiments with monodisperse fractions of microparticles. Our results can guide the design of superhydrophobic microfluidic devices for efficient sorting of microparticles with a relatively small difference in size and density.

1 Introduction

Sorting of micron-sized solid particles¹ and polymer molecules,² and isolation of biological cells or viruses from complex mixtures^{3–5} are of immense importance in a variety of materials science and biomedical applications. Many of these technologies belong to a class of separation techniques known as field-flow fractionation (FFF), where suspended particles (in a broad sense, including solid particles, fluid droplets, microcapsules or cells, typically of radius $a = 1\text{--}5\ \mu\text{m}$) migrating in a microchannel are displaced perpendicular to the direction of flow under the action of external (gravity, electric, magnetic, *etc.*) cross fields.^{6,7} In conventional FFF, the external vertical force is balanced by the so-called “hydrodynamic wall-induced lift force”. As a result, particles of different sizes, shapes, densities and/or surface properties migrate forward by remaining at equilibrium distances, δ , from the wall, which leads to their spreading and batch sorting in the flow direction.

In recent years, however, continuous methods in microfluidic devices have been suggested.^{8,9} In these methods, a horizontal external force remains perpendicular to the direction of flow, so that suspended particles spread orthogonally to it. A variety of forces, such as gravity or electric and magnetic fields, have been used to fractionate suspensions. Some recent work suggested the use of (often in addition to an external force) striped or grooved surfaces, oriented at a certain angle to the direction of flow, which could induce deflection of particle trajectories from the mean flow direction. There have been some theoretical¹⁰ and experimental⁵ studies which exploit the difference in interaction of particles with chemically modified stripes. Hydrophilic grooves (Wenzel state) have been used to separate inertial particles in a relatively thin (compared to a texture period) channel.¹¹

In this paper, we propose a new strategy to guide lateral fractionation of particles in microfluidic channels. We suggest the use of highly anisotropic striped superhydrophobic (SH) surfaces in the Cassie state, where air is trapped by the texture. SH surfaces can induce exceptional wetting properties¹² and, due to their super-lubricating potential,^{13–16} are also extremely important in the context of fluid dynamics. The effective hydrodynamic slip¹⁷ of anisotropic SH surfaces is generally tensorial, due to a secondary flow transverse to the direction of the applied force. In the case of striped Cassie surfaces, such a transverse viscous flow^{13,18,19} can be used to enhance the mixing rate^{20,21} in typical for microfluidic devices relatively low Reynolds number flows. Here, we show that generation of shear flow in the transverse direction near the bottom SH wall of a microfluidic channel results in lateral

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displacement of particles, sedimented under gravity, which provides a basis for their efficient fractionation.[†]

2 General theoretical concept

We first present the theoretical basis and physical ideas underlying the approach, which we develop here to separate microparticles. We consider a flow between two parallel surfaces separated by a distance H and inclined at an angle β as sketched in Fig. 1. The lower surface represents a striped SH (Cassie) texture of a period L and the width of the gas sectors is indicated as s , thus the gas area fraction is $\phi = s/L$. We model the SH surface as a flat interface with no meniscus curvature, so that it appears perfectly smooth with patterns of boundary conditions. Local slippage at the hydrophobic solid areas can be of the order of tens of nanometers.^{23–27} Scalar slip lengths at the gas areas are taken as b_1 along stripes and b_2 across stripes, where the gas flux in the gas subphase becomes zero. These local slip lengths are then related to texture parameters as:²⁸

$$\frac{b_1}{L} \approx \phi \frac{\mu}{\mu_g} \frac{\text{erf}(3.1 e/s)}{3.1}, \quad \frac{b_2}{L} \approx \phi \frac{\mu}{\mu_g} \frac{\text{erf}(2.17 e/s)}{8.68}. \quad (1)$$

Here, $\mu/\mu_g \approx 50$ is the ratio of the dynamic viscosities of water and air, and e is the depth of the texture. Note that our results apply to thick channels, $H \gg \max\{L, b_1\}$, but not to thin channels, $H \ll \min\{L, b_2\}$.^{18,29} The vector $G = -\nabla p + \rho g \sin \beta$, where ρ is the density of liquid, is applied in the x -direction at an angle α to the direction of stripes.

The starting point in describing an average flow is the eigenvalues, $b^{\parallel}$ and $b^{\perp}$, of the effective slip length tensor, which correspond to the fastest (greatest forward slip) and slowest (least forward slip) directions.¹⁷ For the striped surface, they correspond to the situations when the direction x of a driving force, G , is aligned at the angle $\alpha = 0^\circ$ and 90° to

the texture. For striped textures, given the area fraction and local slip lengths of the high-slip (gas) and low-slip (solid) regions, the eigenvalues of the slip length tensor can be found numerically.³⁰ However, if we impose no-slip boundary conditions at the solid area, *i.e.* neglecting nanometric slippage of liquid past the smooth hydrophobic surface,^{23–27} which is (partly) justified provided that it is much smaller than b_2 , the expressions for $b^{\parallel}$ and $b^{\perp}$ take the following forms:³¹

$$b^{\parallel} = \frac{L}{\pi} \frac{\ln \left(\sec \left(\frac{\pi \phi}{2} \right) \right)}{1 + \frac{L}{\pi b_1} \ln \left[\sec \left(\frac{\pi \phi}{2} \right) + \tan \left(\frac{\pi \phi}{2} \right) \right]}, \quad (2)$$

$$b^{\perp} = \frac{L}{2\pi} \frac{\ln \left(\sec \left(\frac{\pi \phi}{2} \right) \right)}{1 + \frac{L}{2\pi b_2} \ln \left[\sec \left(\frac{\pi \phi}{2} \right) + \tan \left(\frac{\pi \phi}{2} \right) \right]}. \quad (3)$$

The angles $\alpha = 0^\circ$ and 90° correspond then to special directions, where $b = b^{\parallel}$ and $b = b^{\perp}$, along which fluid forces do not generate a secondary flow transverse to G , *i.e.* in the y -direction. However, in the general case of any direction α , an anisotropic striped texture generates such a transverse flow, thus the flow becomes misaligned with the driving force.¹⁷ If the stripes are inclined at an angle α to G , the forward and transverse effective slip lengths are given by:¹³

$$b^{(x)} = b^{\parallel} \cos^2 \alpha + b^{\perp} \sin^2 \alpha, \quad (4)$$

$$b^{(y)} = \frac{\sin(2\alpha)}{2} (b^{\parallel} - b^{\perp}). \quad (5)$$

We recall that the surface textures which optimize transverse slip differ significantly from those optimizing effective forward slip. It is well known that the effective forward slip of a SH surface is maximized by increasing the gas area fraction, ϕ . In contrast, the transverse flow in SH channels is maximized by stripes with $\phi = 0.5$, where the effective slip is relatively small.^{13,32}

Let us now evaluate the effective (average) slip velocities at the SH wall. We first remark that for a thick channel, the fluid velocity is close to the Poiseuille velocity profile

$$u_x \approx 4U_m(1 - z/H)z/H, \quad (6)$$

where $U_m \approx GH^2/(8\mu)$ is the maximal velocity, and the secondary (transverse) flow is only a small correction. Effective forward (in the x -direction) and transverse (in the y -direction) slip velocities can be then evaluated as:

$$u_{sx} \approx \frac{4U_m}{H} b^{(x)}, \quad (7)$$

$$u_{sy} \approx \frac{4U_m}{H} b^{(y)}. \quad (8)$$

Fig. 1 Sketch of the system: side (a) and top (b) views, with a schematic of transverse flow generation (c, d).

[†] Note that this approach is conceptually different from the recent theoretical idea²² to use hydrodynamic interactions to deflect the trajectory of particles moving along the anisotropic SH wall under an external force.

It immediately follows from the last equation that the velocity u_{sy} can be maximized at $\alpha = 45^\circ$ as it has been predicted before.³² The transverse shear flow due to surface anisotropy is generated only in the vicinity of the SH wall and disappears far from it, which has already been observed in experiment²⁰ and computer simulations.³² Since the flow rate in the y -direction is zero, the backward Poiseuille velocity profile should be generated, as shown in Fig. 1(c):

$$u_y \approx u_{sy}(1 - 4z/H + 3z^2/H^2). \quad (9)$$

The effective velocity profile is then “twisted” close to the SH wall (Fig. 1(d)). The transverse component of the flow is significant only when it is close to the SH wall.

Now consider what happens when we add particles into such a SH channel. If particles enter the separation zone at the mid-plane of the channel, $z \approx H/2$, their initial velocities are close to U_m . Then, particles sediment under gravity (but not really stick to the wall as known from numerous FFF experiments^{6,7,33}), so that they translate forward with lower speeds and simultaneously move in the y -direction. The transverse displacement then depends on their size and density since bigger and heavier particles sediment faster and, therefore, their trajectories should deflect more.

This lateral displacement can be estimated theoretically. Far from the wall, the particle forward velocity is roughly equal to that of a fluid, $v_x(z) \approx u_x(z)$, and its sedimentation rate will be constant and proportional to the Stokes settling velocity, $v_z = U_{St} \cos \beta$, where $U_{St} = 2a^2(\rho_p - \rho)g/(9\mu)$ and β is the inclination angle. Therefore, the instantaneous vertical position of the particle in an inclined channel is $z = z_0 - tU_{St} \cos \beta$ and the sedimentation time is $t_{sed} \approx z_0/(U_{St} \cos \beta)$, where z_0 is the initial position of the particle. For particles distributed around the mid-plane $z_0 = H/2$, the average sedimentation length x_{sed} can be estimated as:

$$x_{sed} \approx \int_0^{t_{sed}} v_x dt = \frac{U_m H}{3U_{St} \cos \beta}, \quad (10)$$

and depends on the particle size. We note that eqn (10) underestimates the sedimentation length, because it does not take into account the hydrodynamic interaction of the particle with the wall. As a result, both the sedimentation rate and the forward particle velocity decrease. More accurate estimates of v_x , v_z and x_{sed} can be obtained if we take into account the particle-wall interaction (see Appendix A). A sedimented particle continues to move with a constant velocity (v_{xw} , v_{yw} , 0), which is much less than U_m . The remaining distance to the end observation window is then $d - x_{sed}$, where d is the length of the separation zone. A time $(d - x_{sed})/v_{xw}$ is required to pass this distance, and therefore we obtain for a lateral displacement

$$\Delta y \approx \frac{v_{yw}}{v_{xw}}(d - x_{sed}). \quad (11)$$

Eqn (11) provides a basis for efficient transverse fractionation. Velocity v_{yw} is expected to be close to the slip velocity u_{sy} , and should only slightly depend on the particle size. For this reason, we control the lateral displacement by choosing d and x_{sed} . Note that depending on their values, two different scenarios occur. When particles have enough time to sediment – this is the situation discussed above. In this regime, the required degree of separation can be attained by choosing the appropriate value of $d > x_{sed}$. However, when $d < x_{sed}$, particles will move far from the SH walls with a large velocity u_x , and there will be no lateral deflection of their trajectory from the x -direction of the main flow. This regime could be used to extract very small or low density particles.

3 Microfluidic device and estimates

To illustrate the concept, we perform an experimental study in a channel with a bottom striped SH wall, as sketched in Fig. 2(a, b). The stripes are tilted at an angle $\alpha = 45^\circ$ to provide a maximal transverse flow. In most of the experiments, we used a horizontal channel. However, it can be inclined up to $\beta = 45^\circ$, which allows for testing of the gravitational influence on the lateral displacement of particles. The channel is fabricated from PDMS using conventional soft lithography.³⁴ The smooth PDMS surfaces are slightly hydrophobic with an advancing angle $\theta_a = 115^\circ$ measured with a drop shape analysis system (DSA100, Krüss, Germany). The details for the preparation of smooth and grooved PDMS surfaces, and the fabrication of the channel structure are given in Appendix B. Here, we only give a brief description of the channel geometry and justify the choice of the experimental parameters.

We use a grooved SH bottom wall with $L = 2 \mu\text{m}$, $s = 0.8 \mu\text{m}$, and $e = 1.9 \mu\text{m}$ (see Fig. 1(a, b)) in the Cassie state, which gives effective longitudinal $\theta_a^\parallel \approx 127^\circ$ and transverse $\theta_a^\perp \approx 149^\circ$ advancing angles,³⁵ both are much larger than θ_a of a flat PDMS surface. Eqn (2) gives for this texture $b_1 \approx 13 \mu\text{m}$ and $b_2 \approx 4.6 \mu\text{m}$. Numerical calculations³⁰ with typical experimental values of local slip (20–50 nm) at the solid areas^{23–27} predict for our SH texture $b^\parallel \approx 170\text{--}230 \text{ nm}$ and $b^\perp \approx 100\text{--}150 \text{ nm}$, and eqn (4) then suggests that with our α , the effective forward slip is $b^{(x)} \approx 140\text{--}190 \text{ nm}$, and the transverse slip is

Fig. 2 (a) Sketch of the microfluidic device (top view), the boxes indicate the regions of interest at the start and end of the active separation zone of length $d = 6 \text{ mm}$; (b) sketch of the active zone of a microchannel (front view); (c) typical micrograph showing $2.5 \mu\text{m}$ particles moving in the vicinity of the SH surface.

$b^{(y)} \approx 3$ nm. Note that eqn (2) and (3) underestimate $b^{\parallel}$ and $b^{\perp}$, which leads to smaller $b^{(x)} \approx 97$ nm. However, the value of the transverse slip, $b^{(y)}$, remains the same, which justifies the use of eqn (2) and (3) for our analysis of lateral displacements.

As a reference system, we use a channel with a homogeneous flat bottom wall made from the same material, so that its (scalar) slip length b is negligible, being much smaller than $b^{(x)}$. The microchannel itself has a rectangular cross-section with a height $H = 102$ μm , which is much larger than our $b^{(x)}$. The width of the separation zone of the SH channel is very large, $w = 800$ μm , to avoid the influence of the side walls on the transverse motion of the particles. The length of the separation zone is $d \approx 6$ mm.

The microfluidic channel is connected to syringe pumps (Harvard Apparatus PHD Ultra, Hamilton glass syringe, volume 5 mL). All suspensions are injected into the channel at an equal flow rate, $Q \approx GH^3w/(12\mu) \approx 2U_mHw/3 \approx 2$ $\mu\text{L min}^{-1}$, which should provide the same $U_m \approx 650$ $\mu\text{m s}^{-1}$ in all experiments and guarantee a small Reynolds number of the channel, $\text{Re} \ll 1$. The forward and transverse effective slip velocities are therefore $u_{sx} \approx 2.5$ $\mu\text{m s}^{-1} \ll U_m$ and $u_{sy} \approx 0.85$ $\mu\text{m s}^{-1}$ as predicted by eqn (7) and (8).

We use a suspension of hydrophilic silica particles with radii ranging from $a = 0.5 \pm 0.03$ to 2.5 ± 0.18 μm and density $\rho_p = 1.8\text{--}8$ g cm^{-3} (Sigma Aldrich) in MilliQ water (with viscosity $\mu = 9.4 \times 10^{-4}$ Pa s) at a mass fraction of 0.5%. The average velocities of the particles in the x -direction at a given (x, y) position are obtained by monitoring their motion in the 700 μm long observation window (see Fig. 2) using an optical microscope (Motic AE2000) equipped with a USB CCD camera (Point Grey Flea3) at a rate of 40 frames per second. It is easy to evaluate that the sedimentation distance of small silica particles ($a = 0.5\text{--}1$ μm) should exceed d , but larger particles are expected to sediment within the channel. Then, the transverse flow will provide significant lateral displacement of large (*i.e.* sedimented) particles of different sizes (see Fig. 2(c)).

Before our main measurements, we verify the applicability of the simplified eqn (6) for the evaluation of v_x at several z -positions in a separate experiment using fluorescently labeled silica particles (fluorescein labeled particles, Kisker Biotech GmbH & Co.) with a radius of 2.5 μm . We measure the z -position of homogeneously distributed particles far

Fig. 3 Measured velocities, v_x , of fluorescently-labeled silica particles (squares) and liquid velocities, u_x , calculated with eqn (6) (solid curve).

from the SH wall (at several x -locations) using confocal microscopy (Leica TCS SP8) and confirm that the experimental values of v_x are consistent with u_x given by eqn (6) as shown in Fig. 3.

4 Results and discussion

We begin by studying the motion of particles in the horizontal channel, $\beta = 0^\circ$. We have first measured the particle velocities at the start of the separation zone and found that particles of all sizes have velocities in the range from 600 to 670 $\mu\text{m s}^{-1}$, which corresponds to the z -position from 35 to 65 μm , thus confirming our assumption that particles enter the channel near the mid-plane $z = H/2$. At the end of the separation zone, small particles keep moving with velocity U_m , but larger particles move with a much slower speed, $v_{xw} \approx 60$ $\mu\text{m s}^{-1} \ll U_m$, being already close to the wall (see Fig. 4(a)). This is in agreement with theoretical estimates. Indeed, eqn (10) suggests that for particles with average radii of 0.5 and 1 μm , the sedimentation length is larger than d , but for larger particles, x_{sed} is smaller than d . For particles of $a = 2.5$ μm , we determine $x_{\text{sed}} \approx 1.7$ mm and for particles of radius $a = 1.5$ μm , we obtain $x_{\text{sed}} \approx 4.8$ mm.

Fig. 4(a) shows that the velocities of sedimented particles near the SH wall are independent of their size. To examine v_{xw} more closely, data from Fig. 4(a) are reproduced (on a different scale) in Fig. 4(b). One conclusion from this plot is that indeed v_{xw} does not really depend on a . Also included in Fig. 4(b) are the velocities of particles measured near the smooth wall. It can be seen that the experimental data shows that the velocities are slightly faster near the SH wall, but this small difference is actually within the accuracy of the experiment. We conclude, therefore, that the forward slip does not really affect v_{xw} . Now, we compare our data with the model, which is usually applied to conventional FFF, where v_{xw} was predicted to increase with a (see Appendix C).⁶ One can see that the predictions of this model (dashed line in Fig. 4(b)) significantly underestimate our results, thus we have to rule out the possible effect of a “hydrodynamic wall-induced lift force” in our system. We also compare our data with predictions of the model, which quantifies the motion of particles

Fig. 4 (a) Forward velocities v_x (open squares) and v_{xw} (filled circles) measured at $x = 6$ mm for particles of different radii. (b) The same values for large particles (filled circles) in comparison with the velocities in the channel with the flat bottom wall (open circles). Dashed and solid lines are the predictions of the models of the near-wall hydrodynamic lift⁶ and of particle motion near a rough wall.³⁶

near a rough wall³⁶ (see Appendix C) shown as a solid curve in Fig. 4(b). This model is more consistent with our data, but it also predicts the increase of v_{xw} with a , which we do not observe in our experiment.

We focus now on transverse particle displacements by monitoring the y -positions of particles at the start and at the end of the separation zone. We have first conducted this experiment in a channel with a smooth bottom wall and, as expected, found no lateral displacement of particles. Fig. 5(a) shows a typical experimental snapshot of suspensions of particles in a channel with a striped SH wall. Fig. 5(b, c and d) display histograms of particle positions y at the start and at the end of the separation zone.[‡] We see that the initial particle distributions are symmetric with an average fixed at zero by choice of the coordinate system and with standard deviations σ varying from 6 to 14 μm for different particles. For small particles of 0.5 μm radius, we did not observe any mean lateral displacement, *i.e.* mean $\Delta y = 0$ as seen in Fig. 5(b). The mean lateral displacements Δy of larger particles are finite and depend strongly on their size. For particles of 1.5 μm radius, we detect the mean $\Delta y \approx 12 \mu\text{m}$, and for particles of 2.5 μm radius, it is $\Delta y \approx 35 \mu\text{m}$ (see Fig. 5(c) and (d)). Note that the distributions of particles spread towards the end of the separation zone, which reflects that at the start, our particles are distributed not only in the y -direction but also in the z -direction.

Thus, we have measured v_{xw} and Δy independently for particles of different sizes. From these data, we can deduce the transverse velocity of particles v_{yw} near the SH wall by using eqn (11) with x_{sed} calculated using eqn (10). The representative results are given in Table 1. Note that for all particles, we

Table 1 Some representative experimental data

a , μm	Δy , μm	v_{xw} , $\mu\text{m s}^{-1}$	x_{sed} , mm	v_{yw} , $\mu\text{m s}^{-1}$
1.5	12	60	4.9	0.65
2.5	35	60	1.9	0.51

found $v_{yw} = 0.6 \pm 0.1 \mu\text{m s}^{-1}$, which is somewhat smaller than the theoretical value of u_{sy} discussed above. This suggests that there is an additional mechanism for dissipation in our system not included in the model (such as, for example, meniscus curvature^{37,38}). We now use experimental v_{yw} to plot in Fig. 6 the top view of the calculated particle trajectories. A general conclusion from this plot is that the predictions of the simplified theory, eqn (11), are in good agreement with numerical results (see Appendix A).

Finally, we study the effect of gravity on the lateral displacements by using two inclination angles $\beta = 25^\circ$ and 45° . The results are plotted in Fig. 7. We see a significant increase in forward velocity for particles of 1.5 μm radius with the inclination angle, but not for 2.5 μm particles, which still move with v_{xw} as in Fig. 4. Calculated using eqn (10), the sedimentation distances in the inclined channels become larger. For particles of $a = 2.5 \mu\text{m}$, we obtain $x_{\text{sed}} \approx 1.9 \text{ mm}$ ($\beta = 25^\circ$) and $x_{\text{sed}} \approx 2.4 \text{ mm}$ ($\beta = 45^\circ$), indicating that they are obviously in the near-wall zone and move forward with v_{xw} . Indeed, we observe experimentally that their distribution remains very close to that shown in Fig. 5(d). However, the sedimentation lengths of particles with $a = 1.5 \mu\text{m}$ become very large, and at $\beta = 45^\circ$, we get $x_{\text{sed}} \approx 6.6 \text{ mm} > d$. In this case, at the end of the separation zone, $x \approx x_{\text{sed}}$, and particles still continue to migrate with relatively high v_x without lateral displacement. Analysis of experimental data for this suspension gives $\Delta y \approx 0$, as expected theoretically. When $\beta = 25^\circ$, for these particles with $a = 1.5 \mu\text{m}$, we estimate that $x_{\text{sed}} \approx 5.2 \text{ mm}$, which is quite close to the outlet of the separation zone. However, it follows from the measured value of v_x that the lateral displacement of these incompletely sedimented particles becomes smaller than that in the horizontal channel (see Fig. 5(c)), and the detected Δy is well below 10 μm . These results demonstrate that the inclination angle of the device can be used to tune the fractionation of microparticles.

Fig. 6 Top view of trajectories of particles of 2.5 μm (black) and 1.5 μm (grey) radii calculated for $v_{yw} = 0.6 \mu\text{m s}^{-1}$. Dashed curve plots the predictions of eqn (11), solid curve shows numerical results and circles show the average experimental Δy .

Fig. 5 Optical micrographs showing particles of 2.5 μm radius at the start and at the end of the separation zone (a). Histograms of Δy measured at $\Delta x = d$ fitted to normal distributions for particles of 0.5 (b), 1.5 (c) and 2.5 (d) μm radii.

[‡] For each histogram, we have analyzed 200 individual particles except for a suspension of particles with radius $a = 2.5 \mu\text{m}$, where only 50 particles have been analyzed.

Fig. 7 Forward velocities of particles of 2.5 μm (filled circles) and 1.5 μm (squares) radii as a function of inclination angle β of the microfluidic device. Filled symbols indicate v_{xw} and open symbols represent v_x .

5 Conclusion

In this paper, we have formulated a concept of transverse flow fractionation of microparticles, which exploits a combined effect of gravity and a transverse hydrodynamic flow near a striped bottom SH surface of a microfluidic channel. We have predicted the transverse displacement of particles theoretically, and generally confirmed the validity of our theory by conducting experimental measurements with mono-disperse suspensions of microparticles. We have to stress, however, that the aim of our experiments was simply to illustrate a theoretical concept. For this purpose, we used the optimal value of the inclination of stripes to the direction of the main flow, α , and the nearly optimal value of the gas fraction, ϕ , which maximize the transverse flow, but note that we had a relatively small value of the texture period, L . Since v_{yw} and Δy scale with L (see eqn (2)–(8)), the lateral displacement can be further enhanced by using textures with larger L . Therefore, our results can guide the design of super-hydrophobic microfluidic devices for efficient sorting of microparticles.

Appendix

A Sedimentation and trajectory of particles near the wall

The simplified model, eqn (11), assumes that particles sediment with the Stokes velocity and their horizontal velocity coincides with that of the fluid. To calculate the sedimentation length more accurately, we have to take into account the hydrodynamic interaction of particles with the wall and use the following expressions:^{39,40}

$$\begin{aligned} v_x &\approx f(z/a - 1)u_x(z), \\ v_z &= U_{\text{St}}/[1 + 1/(z/a - 1)], \\ v_y &\approx u_y, \end{aligned} \quad (12)$$

with the correction factor f given in ref. 33. Note that here, we neglect some possible corrections, which could appear near a SH wall,^{22,41,42} since they become significant only when $z/a - 1 = 0(1)$. We also neglect the forward slip since $u_{sx} \ll v_{xw}$. We assume that the wall does not influence the

transverse velocity of particles, thus $v_y \approx u_y$. The trajectories, $\mathbf{x} = (x, y, z)$, are then $\mathbf{x} = \int \mathbf{v} dt$, where $\mathbf{v} = (v_x, v_y, v_z)$ is given by eqn (12). The sedimentation length is determined from the conditions $v_x(x_{\text{sed}}) = v_{xw}$, where v_{xw} is the experimental value. For $x > x_{\text{sed}}$, we use $v_x = v_{xw}$, $v_y = v_{yw}$, $v_z = 0$.

B Fabrication of microchannels

For fabrication of polydimethylsiloxane (PDMS) microchannels, we used: (1) a master containing the microchannel architecture, (2) a master with rectangular grooves and (3) a silicon wafer as a master for the flat bottom wall. The master with the grooves (2) was purchased from AMO GmbH, Germany. The microchannel master (1) was fabricated by photolithographic patterning of SU-8 (MicroChem Corp., USA) on a silicon wafer.

All the wafers were cleaned and hydrophobized with trichloro(1H,1H,2H,2H-perfluorooctyl)silane (PFOTCS, 7%, Sigma Aldrich) through the gas phase in an evacuated desiccator. PDMS replication was performed by employing a standard procedure using a Sylgard®184 silicone elastomer kit (Dow Corning).³⁵

The geometry of the PDMS pattern was validated by field emission scanning electron microscopy (Hitachi S-4800) and interference profilometry (WYKO NT2000, Veeco, USA).

To construct the closed channel geometries, the microchannel structure (1) was bound to either the flat bottom wall replicated from the unstructured silicon wafer (3) or the PDMS replica of the grooves (2). To facilitate firm binding of the two PDMS layers, the individual layers were subjected to plasma treatment in an evacuated chamber with an oxygen flow of 30 mL min⁻¹ and a plasma power of 200 W for 15 s (PVA TePla 100 plasma system) just before alignment and adhesion of the layers. Access to the closed microchannel structure is facilitated by punching holes through the thicker PDMS layer (Harris Uni-Core punches, Ted Pella Inc.).

C Models of particle motion near the wall

For comparison with experimental data, we use two models for v_{xw} , namely, “the hydrodynamic wall-induced lift” widely accepted in FFF and the model of particle motion near a rough wall.³⁶

The first model⁶ uses a modified equation for particle velocity that takes into account its hydrodynamic interaction with the wall, $v_{xw} = f(\delta/a)u_x(\delta + a)$, where $\delta = z - a$ is the particle-wall distance and f is given in ref. 33. The value of δ is calculated by balancing the external vertical force and the lift given by $F_L = 0.172 \frac{\mu a^3}{\delta} \frac{4U_m}{H}$.

The second model³⁶ predicts $v_{xw} = U_{\text{St}}g(\text{Sh})$, where $\text{Sh} = \frac{4aU_m}{9HU_{\text{St}}}$ is the Shields number and $g(\text{Sh})$ is the fitting function valid in the range $0.04 \leq \text{Sh} \leq 0.8$, which includes our values lying in the interval $0.5 \leq \text{Sh} \leq 0.9$.

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