Long-Term Open-Pit Planning by Ant Colony Optimization

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ZUSAMMENFASSUNG

Die Aufgabenstellung einer langfristigen Planung von Festgesteinstagebauen mit diskontinuierlicher Gewinnung ist eine große kombinatorische Herausforderung, die nicht durch mathematische Programmierung in angemessener Zeit gelöst werden kann. Diese Dissertation stellt einen neuentwickelten metaheuristischen Algorithmus vor, der auf den Theorien des Ameisenalgorithmus (Ant Colony Optimization, ACO) basiert. Darüber hinaus wird die Anwendung des entwickelten Modells anhand einer langfristigen Planung eines zwei-dimensionalen hypothetischen Block-Modells untersucht verifiziert.

ACO beschreibt das natürliche Verhalten von Ameisen bei der Futtersuche, das die kürzeste Strecke zwischen Kolonie und Nahrungsquelle zum Ziel und bereits mehrfach erfolgreich zur Lösung anderer kombinatorischer Probleme beigetragen hat. In der Natur wird das Problem der optimalen Routenfindung mittels Pheromonen, die eine Nachricht von einer Ameise an die nächste übertragen, gelöst. Die Pheromone steuern die Wegfindung der Ameisen, so dass sie nicht nach dem Zufallsprinzip wandern, sondern den Pheromonspuren folgen. Mit der Zeit verdunsten die Pheromone von der Spur, die selten oder gar nicht mehr genutzt wird, währenddessen die Route mit der kürzesten Strecke erhalten bleibt.

Um mit der ACO-Theorie eine langfristige Planung eines Festgesteins-tagebaus zu simulieren, wird die Anzahl der Pheromonspuren jedes Blocks mit der Anzahl der Planungsperioden gleichgesetzt. Die Pheromonspuren, die einem Block zugeordnet werden können, stellen die maximale Abbauteufe einer jeden Blockspalte pro Abbauperiode dar.

Die Form eines bestimmten Tagebaus kann, unter Beachtung der Böschungswinkel, durch ein einfaches Datenfeld von ganzen Zahlen dargestellt werden. Dabei stellt jedes Element dieses Datenfeldes die Tiefe des Tagebaus in einer einzelnen Spalte des Block-Modells dar. Wenn dieses Konzept zu einer langfristigen Produktionsplanung erweitert wird, wird jeder Produktionsplan durch ein Datenfeld dargestellt, dass mehrere Abbauteufen für jede Spalte des Blockmodells in Relation zu den verschiedenen Produktionsperioden aufweist.

Am Anfang wird eine initiale Tagebauplanung anhand des Lerchs-Grossmann Algorithmuses und den von Wang & Sevim entwickelten Algorithmus „Alternative zur Parametrisierung Algorithmus“ erstellt und die Werte der Pheromon Werte entsprechend initialisiert. Basierend auf der Tagebauplanung werden den Blöcken, die in direkter Nähe zu den Blöcken des tiefsten Punkts liegen, während der Initialisierung höhere Pheromonwerte zugeordnet.

Diese Vorgehensweise erzeugt eine Reihe von zufälligen Zeitplänen, die nicht weit von der ursprünglichen Lösung sind.

In jeder ACO-Iteration werden basierend auf den aktuellen Pheromonmengen zuerst mehrere Tagebaupläne erstellt. Dieser Prozess wird als “Bestimmung der Teufe“ gekennzeichnet und implementiert. Während des Prozesses wird die Teufe in jeder Periode für jede Spalte des Blockmodells bestimmt. Je höher der Wert der Pheromonspur eines Blocks ausfällt, desto größer ist die Möglichkeit, dass der Block als maximale Abbauteufe für die jeweilige Periode gewählt wird. Anschließend werden die Pheromonwerte aller Blöcke um einen gewissen Betrag durch Evaporation verringert. Im nächsten Schritt werden die Pheromonwerte der Blöcke, die den Abbaustand zur jeweiligen Periode begrenzen, je nach Qualität der Lösung des folgenden Abbaustands erhöht. Durch wiederholte Iterationen werden die Pheromonwerte der Blöcke, die die Form der optimalen Lösung definieren erhöht, während die Werte der anderen Blöcke signifikant verkleinert werden.

Die ACO Optimierung Iterationen können auf verschiedene Arten implementiert werden. In der ersten und einfachsten Methode, Ant System (AS), dürfen alle konstruierten Tagebaustände zur Pheromonablagerung beitragen. Die zweite Methode, elitäres Ant-System (EAS) zeichnet sich dadurch aus, dass der optimale Plan zusätzlich Pheromone in jeder Iteration ablegt. AS_{rank} ist die dritte Methode in der nur eine geringe Anzahl von guten Tagebauplänen Pheromon hinzufügen kann. Die weiteren Varianten, Max-Min Ant System (MMAS) und Ant Colony System (ACS), erlauben nur den bis zu diesem Zeitpunkt besten Abbauplanungen Pheromone abzulegen und nutzen zusätzlich spezielle Pheromoneinschränkungen, die eine Stagnation im lokalen Optimum verhindern.

Um die Effizienz des Algorithmus zu überprüfen wurde ein Computerprogramm entwickelt, dass auf Visual Basic 2005 als Programmiersprache aufbaut. In einer Fallstudie wurde ein Blockmodell einer hypothetischen Eisenerzlagerstätte mit 1000 Blöcken erstellt. Anhand des Blockmodells wurden die verschiedenen Varianten der ACO analysiert, um die beste Kombination der ACO-Parameter zu identifizieren. Die Analyse zeigte, dass die ACO den Wert der ersten Tagebauplanungen bis zu 34 % in einer akzeptablen Rechenzeit verbessern kann. Diese Verbesserung ist vor allem der Berücksichtigung von evtl. Einbußen zuzuschreiben, die aus einer Überschreitung von Kapazitätsgrenzen oder Produktqualitäten resultiert. Es konnte bewiesen werden, dass die MMAS Variante, die Variante mit der größten Exploartion von Lösungen ist, währenddessen die AVS Variante die schnellste Methode ist. Diese beiden Varianten sind die Einzigen, die sich aufgrund des Speicherbedarfs von Rechnern auf große Blockmodelle anwenden lassen.

ABSTRACT

The problem of long-term planning of a hard rock open pit mine (discontinuous exploitation operation) is a large combinatorial problem which cannot be solved in a reasonable amount of time through mathematical programming models because of its large size. In this thesis, a new metaheuristic algorithm has been developed based on the Ant Colony Optimization (ACO) and its application in long-term scheduling of a two dimensional hypothetical block model has been analysed.

ACO is inspired by the foraging behaviour of ants (i.e. finding the shortest way from the colony to the food source), and has been successfully implemented in several combinatorial optimization problems. In nature, ants transmit a message to other members by laying down a chemical trail called pheromones. Instead of travelling in a random manner, the pheromone trail allows the ants to trace the path. Over time, the pheromones layed over longer paths evaporate, whereas those over shorter routes continue to be marched over.

In order to simulate the ACO process for long-term planning of a hard rock open-pit mine, various programming variables have been considered for each block as the pheromone trails. The number of these variables is equal to the number of planning periods. In fact these pheromone trails represent the desirability of the block for being the deepest point of the mine in that column for the given mining period.

The shape of any given pit (in respect to the slope angles) can be represented by means of a simple array of integer numbers. Each element in this array shows the depth of the pit in an individual column of block model. Extending this concept to a long-term production planning, a mine schedule would be represented by an array that has several mine depths at each column of block model related to different production periods.

At the beginning, the values of the pheromone trails are initialized according to a mine schedule generated by Lerchs-Grossmann's algorithm and the alternative to parameterization algorithm of Wang & Sevim. During initialization, relatively higher values of pheromones are assigned to those blocks that are close to the deepest points of the push backs in the initial mine schedule. This leads the procedure to construct a series of random schedules which are not far from the initial solution.

In each ACO iteration, several mine schedules are constructed based on current pheromone trails. This is implemented through a process called "depth determination". In this process the depth of a mine in each period is determined for each column of the block model. The

higher the value of the pheromone trail of a particular block, the higher the possibility of selecting that block as the pit depth in that period. Subsequently the pheromone values of all blocks are reduced to a certain percentage (evaporation) and additionally the pheromone value of the participating blocks used in defining the constructed schedules are increased according to the quality of the generated solutions. Through repeated iterations, the pheromone values of the blocks which define the shape of the optimum solution are increased whereas those of the others have been significantly evaporated.

The ACO optimization iterations could be implemented in a variety of ways. The Ant System (AS) is the first and simplest method, whereby all of the constructed schedules are allowed to contribute in the pheromone deposition. In each iteration of the second method, the Elitist Ant System (EAS), the best schedule found up to that iteration (the best-so-far schedule) is also allowed to deposit pheromones. AS_{rank} is the third method in which only a few good schedules are able to add pheromones. The other variants are the Max-Min Ant System (MMAS) and the Ant Colony System (ACS), which allow only the best-so-far schedule to deposit pheromones and utilise special pheromone limitations in order to prevent the stagnation in local optimums.

To test the efficiency of the algorithm, a computer program has been developed in Visual Basic 2005 programming language. As a case study, the block model of a hypothetical iron ore deposit with 1000 blocks was considered and different variants of ACO had been analysed in order to find the best combination of ACO parameters. The analysis revealed that the ACO is able to improve the value of the initial mining schedule by up to 34% in a reasonable computational time. This is mainly contributed to the consideration of the penalties to the deviations of the capacities and the production qualities from their permitted limits. It had also been proved that the MMAS is the most explorative variant, while ACS is the fastest method. These two variants also count as the only variants which could be applied to a large block model in respect to the amount of memory needed.

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LIST OF ABBREVIATIONS

ACO	Ant Colony Optimization
ACS	Ant Colony System
ANTS	Approximate nondeterministic Tree Search
AS	Ant System
DP	Dynamic Programming
EAS	Elitist Ant System
GA	Genetic Algorithm
IP	Integer Programming
LG	Lerchs & Grossmann's algorithm
LP	Linear Programming
MIP	Mixed Integer Programming
MMAS	Max-Min Ant System
NCF	Net Cash Flow
NPV	Net Present Value
PSO	Particle Swarm Optimization
SA	Simulated Annealing
SP	Stochastic Programming
TS	Tabu Search

1 INTRODUCTION

Modern societies require a supply of raw material for its growth and sustenance. Most of these materials are attained by means of surface and underground mining operations. As compared to the underground mining, surface mining account for a significant proportion of the produced minerals currently having many advantages in terms of large production equipment size, short preproduction development period, high ore recovery and less labour requirements. It is categorized into open pit, strip, alluvial and in-situ mining methods (Hartman, 1987). Hard rock open pit mining is a mineral exploitation method by which the deposit is accessed by digging a large opening in the ground surface, called pit, to uncover the ore to air. The initial mining phase starts with a small pit, and then develops to a larger pit which encloses it. The process proceeds until a final shape of the mine called "ultimate pit limit" is reached. These sequences of pits are known as mining sequences or push backs. Mining operations in each push back starts from the most upper part and proceed towards its bottom (Sevim & Lei, 1998). The long-term mining sequence is obtained from a series of nested pits. The objective of pit optimization is to find the sequence that will maximize the economic rewards. The results of these calculations are used as a guide for short-term production planning which may be for a quarter, month or week.

The last 30 years have seen a widely-publicized revolution in the application of numerical methods in the mining industry. With the application of geostatistics, 3-D modelling, Lerchs-Grossmann algorithm and many other computer-based procedures, open-pit mining operations are routinely producing better mine plans on ever more complicated and often lower grade deposits, and with staffing levels that would have been unthinkable prior to the early 1980s. Recent studies in the field of open-pit optimization have been focused on finding new algorithms which are:

- less complex methods in terms of comprehensibility and programming; and
- require shorter computing times in order to be applicable to the large deposits; and
- allow the incorporation of real mining complexities such as variable slopes, working slopes, time value of money, quality and quantity of planned material, related uncertainties, etc. (Dowd & Onur, 1993).

Almost all computerized hard rock open-pit mine planning methods are based on block models. A block model divides the whole ore body and surrounding waste rocks into 3D blocks adjacent to each other (as shown in Figure 1-1). The model may have millions of blocks depending on the size of deposit and the size of blocks. The average ore grade of each

block is estimated using geostatistical approaches or conditional simulation methods (Sevim & Lei, 1998).

1.1 COMPLEXITY OF PIT OPTIMIZATION

The variables involved in production planning of a hard rock open-pit mine interact in a circular manner. Without the knowledge of one variable, the next variable in the cycle cannot be determined, Figure 1-2. The time taken to mine all the pits in the sequence will represent the mine life, while the outline of the last pit in the sequence will define the ultimate pit limits (UPL). To differentiate ore from waste, a cut-off grade must be determined which is a function of final commodity price, mining and processing costs. The annual mining rate and consequently the life of the project are unknown at the beginning of the planning. It can be observed from Figure 1-2 that the costs and revenues and consequently cut-off grade must be defined first in order to determine the ore body extension and calculate the economic values of the blocks. After that the ultimate pit limit is determined and then used to develop a production schedule, including the annual production and mining sequence. Subsequently, the selected annual production and mining phases are used to revise the revenues and costs. Clearly the value of any given variable in this cycle cannot be calculated if the value of the previous variable is unknown. Assuming the fixed values for one or more variables along the cycle would lead to an inferior planning. In fact, this is a multi-variable optimization problem that requires simultaneous solutions. Unfortunately, such a solution is not easy to achieve and after three decades of continuing efforts, the long-term production planning of an open-pit mine as a whole is still an unanswered issue. (Sevim & Lei, 1998).

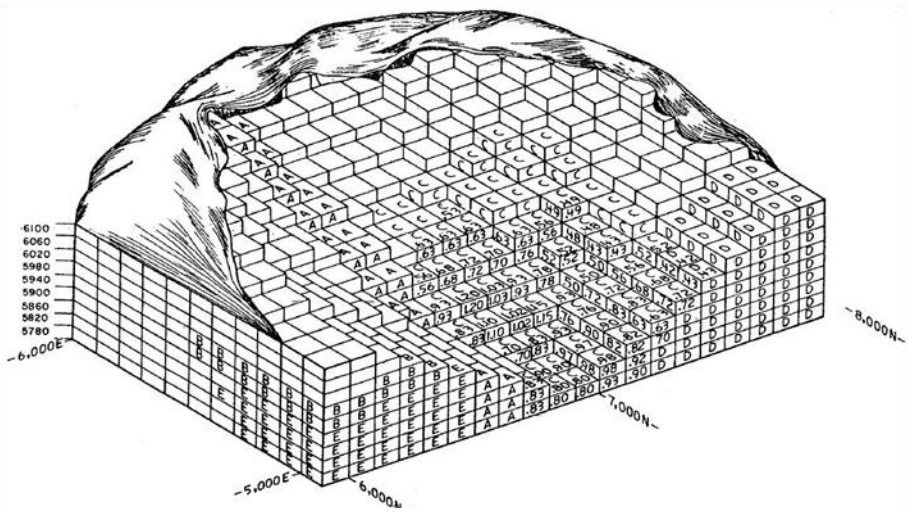


FIGURE 1-1 BLOCK MODEL
HUSTRULID & KUCHTA 1995

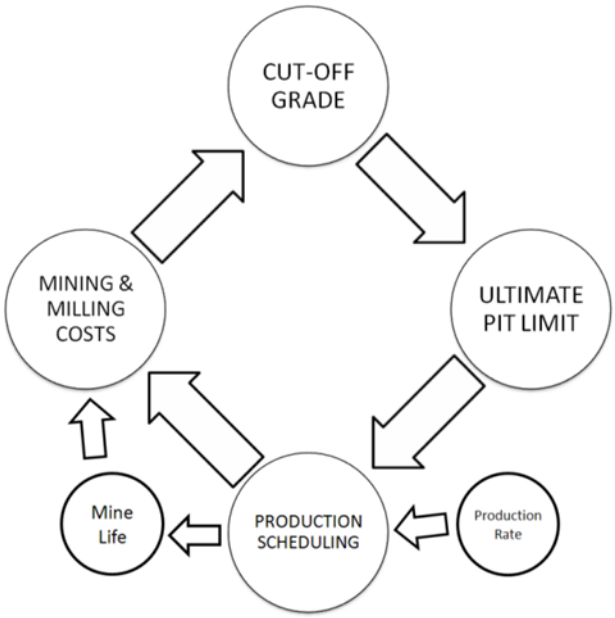


FIGURE 1-2 CIRCULAR FASHION OF OPEN PIT OPTIMIZATION
SEVIM & LEI 1996

In order to overcome this complexity, conventionally partial solutions of the problem are found for one or two parameters by firstly fixing the values of the other parameters. Typically, revenues and costs are estimated initially and the corresponding cut-off grade is calculated and subsequently an economic value is assigned to each block of the model. The process is followed to determine the maximum value pit (UPL) by using a graph theory based optimization algorithm. Once the UPL is defined, different mining push backs are fit within its. Then a trial-and-error approach is used to determine the final schedule with the highest possible economic value subject to the operational constraints. One of the most popular methods consists of generating a series of nested pits within the pre-determined UPL by using of the parameterization technique. This series are then searched for a mining sequence that would satisfy the operational constraints and targets. (Sevim & Lei, 1998).

However, there are several fundamental problems with this method. For example, the mining cost is a function of the production capacity. Therefore, the mining cost of a specific block would be different when the mine is planned to produce dissimilar amounts of the ore. Thus, an annual production rate, which is obviously not optimized, must be assumed primarily in order to calculate the economic values of the blocks. Another problem is in determination of the UPL. The UPL, which is the final shape of the mine, has to be the natural result of the mine sequence optimization. But in this approach, however, mine sequences are enforced to fit into the pre-determined ultimate pit. In fact, the UPL and the

mining schedule should not be calculated individually if correct optimization is desired. (Sevim & Lei, 1998).

1.2 NUMERICAL EXAMPLE

Figure 1-3a illustrates a simple numerical example of the problem. It is a two dimensional block model with 50 block columns and 10 rows. The blocks have been assumed to be square and the slope angle has been considered as $45°$. The blocks have been classified as ore and waste in a random manner and the economic values of ore blocks have been randomly generated. The value of waste blocks has been set to -6.

The ultimate pit limit (UPL) of the model has been determined by the graph algorithm of Lerchs-Grossmann. As is shown in Figure 1-3b, there are 151 ore blocks and 66 waste blocks inside this pit and the aggregate value of the blocks fallen inside the pit (marked by the number 1) is as 321.

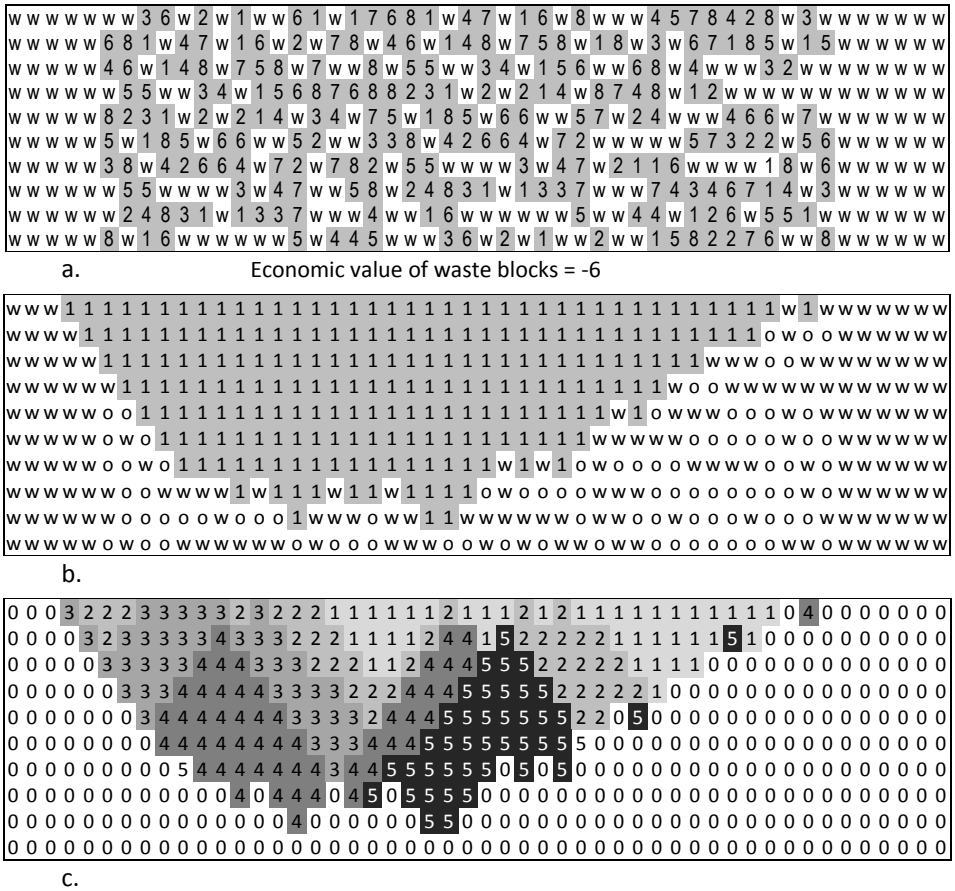


FIGURE 1-3 NUMERICAL EXAMPLE

TABLE 1-1 CHARACTERISTICS OF THE PUSHBACKS OF THE NUMERICAL EXAMPLE DESIGNED BY LG AND PARAMETERIZATION

Pushback No.	Ore number	Waste number	Total number	Average of ore value	Pushback Value (Undiscounted)	Completion of push back*	Pushback Value (Discounted**)
1	30	10	40	5.93	118	3.97	80.80
2	26	14	40	5.12	49	7.41	24.16
3	26	14	40	4.65	37	10.86	13.14
4	39	15	54	4.18	73	16.02	15.84
5	30	13	43	4.07	44	20	6.54
Total	151	66	217	4.75	321	20	140.49

* Mining for 20 years in 5 uniform period ** Annual interest rate : 10%

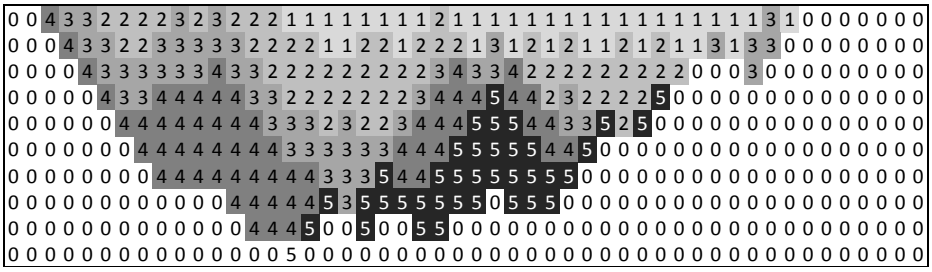


FIGURE 1-4 IMPROVED SCHEDULE OF THE NUMERICAL EXAMPLE

An initial long-term plan had been created for the model using the heuristic method of Wang & Sevim (1995), Figure 1-3c. The desired size of push backs in this case was set to 40 blocks. It could be observed that the designed push backs are almost uniform in size, with a descending average from 5.93 to 4.07. Assuming an interest rate of ten percent and the mine life of 20 years, the value of the generated schedule decreases from the attractive 321 units (in the undiscounted case) to 140.49 units considering the time value of money (discounted value), Table 1-1.

The other schedule shown in Figure 1-4 clearly reveals that the independent determination of the UPL and mining push backs do not always lead to an optimum mine schedule. This schedule produces a higher net present value (NPV), despite having a smaller undiscounted value, Table 1-2 .

TABLE 1-2 CHARACTERISTICS OF THE NUMERICAL EXAMPLE'S PUSHBACKS IMPROVED BY ACO

Pushback No.	Ore number	Waste number	Total number	Average of ore value	Pushback Value (Undiscounted)	Completion of push back*	Pushback Value (Discounted**)
1	32	6	38	5.44	138.08	3.86	95.56
2	39	19	58	5.13	86.07	8.55	35.06
3	33	18	51	4.30	33.90	12.53	10.30
4	39	23	62	3.92	14.88	17.23	2.90
5	23	15	38	1.04	-66.08	20	1.04
Total	166	81	247	19.83	206.85	20	147.86

* Mining for 20 years in 5 uniform period ** Annual interest rate : 10%

In the last few decades, several tricks have been used traditionally to overcome this problem. In the oldest attempt, mine design (UPL and push backs) process repeated several times by using a series of discounted block values obtained in current design instead of undiscounted block values. This method does not lead to an optimum schedule for all cases. The mostly used method, especially in commercial packages, is suggested by Whittle in 80's. He uses two extreme methods of mining operations called Best Case and Worst Case mining scenarios to determine the final boundary of mining. Nevertheless, the ideal algorithm is one that solves two UPL and push back design problems simultaneously. Such a capability has only ever been provided in a genetic algorithm method so far. The new ACO mine scheduling approach is another method possessing this ability.

1.3 A BRIEF REVIEW OF THE LITERATURE

The manual method of open-pit design required the extensive work of a planning team to define boundaries of ore in vertical sections and designing of the mine configurations in these sections according to the requirements and the available economic and technical information. Clearly this method was very time/labour intensive and would only be applicable for very small mineral deposits. The emergence of computers in the field of open-pit planning has engaged researchers to develop better and faster approaches. However even after about 40 years, the field still needs to develop more powerful tools.

In early studies (Pana, 1965; Williams, 1974; Lemieux, 1979) the moving cone algorithm was used to design the outline of the final pit shape. The algorithm starts from the surface and searches for ore blocks with positive economic value. Thereafter, it constructs a minimum removal cone on such a block. All blocks inside this cone are considered as removed if the sum of the economic values of these blocks is positive. The algorithm continues the search until all the ore blocks in the model have been examined. Despite the 3D nature of the method and the ability to consider variable slopes, it was proved very soon that the moving cone algorithm was not able to find the best solution in all cases.

After development of various 2D mathematical methods that were able to find the optimum UPL on vertical sections, several studies were carried out to combine 2D sections and provide 3D pits (Johnson & Sharp; 1971, Wright;1987). Later Koenigsberg (1982) and Wilke & Wright (1984) succeeded in directly applying dynamic programming to solve the 3D pit design problem. The Lerchs and Grossmann's algorithm might be one of the most utilized algorithms in open-pit optimization field. Authors used the graph theory to formulate the model optimum pit limit, see chapter 2.3.1. Afterwards several researchers attempted to develop more efficient algorithms for the UPL problem (Huttagosol and Cameron, 1992; Yegulalp and Arias, 1992; Zhao and Kim, 1992; Hochbaum, 2001).

Subsequent studies focused on a more general problem rather than the UPL. It was the production planning problem. This challenging problem tried to answer the following questions (Dagdelen & Johnson, 1986):

- Should a given block be mined by the end of mine life or not? (UPL problem)
  If yes:
- When should it be mined? (Mine sequencing problem), and
- Where should it be send? E.g. processing plant, leach pad or waste dump? (Cut-off grade problem)

As Chapter 2 explains, this is a huge mathematical programming model that could not be solved by available computer software and hardware.

An early optimization attempt in production scheduling reverts back to the studies done by Wilke and Reimer (1977). Authors proposed a linear programming model for the short-term production scheduling of an open-pit iron ore mining operation. Later, Jordi and Currin (1979) proposed a goal programming model to optimize net present value, the total net profit and the total gold output. Zhang et al. (1986) described a new Interactive Dynamic Optimization Method (IDOM) that combined inventory theory, dynamic programming,

computer simulation and interactive technique to formulate a production scheduling problem of open-pit mines.

One of the major developed concepts was the application of lagrangian parameterization for optimization of production planning problem by Dagdelen and Johnson (1986). This concept uses the UPL algorithm applied to block models with varied block values to produce production schedules. The concept of lagrangian parameterization utilized later by Whittle to develop the most known commercial package in the field of open-pit optimization. Whittle's method is a heuristic approach that uses the different block values to produce a series of nested pits and selects the UPL and mining push backs corresponding to the maximum NPV. Sevim and Lei (1996) described methodology comprising a group of heuristic algorithms and unconventional dynamic programming. This method had the capability to determine the cut-off grade, mining and milling production rates, mining sequence, mine life and UPL. In the recent studies conducted by Ramazan & Dagdelen (1998), a new algorithm which could develop push backs of minimum stripping ratio was presented. Ramazan & Dagdelen and Jonson (2005) proposed a new production scheduling optimization technique based on the fundamental tree algorithm to decrease the number of integer variables and to solve the problem as a mathematical programming model.

Debny & Schofield (1996) attempted to use metaheuristic algorithms for first time in pit optimization; however, their developed genetic algorithm model was not able to be applied in real mining cases because of long computational times. In another study, Kumral and Dowd (2005) proposed another metaheuristic algorithm based on simulated annealing to improve the value of a given sub-optimal mine schedule.

The non-deterministic view to the open-pit optimization is another research field which has received a lot of attraction in recent years. Osanloo & Gholamnejad (2008) modelled the long-term production scheduling problem by chance-constrained binary integer programming in a stochastic environment. Dimitrakopoulos (1998) outlined a general framework for modelling uncertainty and assessing geological risk. Conditional simulation is a class of Monte Carlo techniques that can be used to equally generated representatives of the in-situ ore body variability. Achireko & Frimpong (1996) proposed a new algorithm called MCS/MFNN which had the capability to address the random field properties associated with the ore grades, reserve and commodity prices. After modelling the block characteristics by conditional simulation, they used artificial neural networks to classify the blocks into classes based on their conditioned values. The error back propagation algorithm is then used to optimize the UPL by minimizing the desired and actual output errors in a multilayer

perception under pit wall slope constraints. Frimpong et al. (1998) developed an intelligent pit optimizer (IPOP) to deal with the random properties of optimized pit layouts. It combines stochastic models of ore reserves and commodity prices to generate economic block and target values.

Recently, designing integrated intelligent systems for decision making on mineral resource exploitation is becoming of increasing interest. They provide analysis with intelligent design options to deal with structural, hydrological and tectonics problems of mine design. Frimpong and Szymanski (2002) have discussed current state-of-art technology and research in intelligent modelling, and have also addressed the current and future research frontiers in intelligent modelling.

1.4 STRUCTURE OF THE RESEARCH

The main objective of the current study was to develop a mathematical and computer modelling background of a new metaheuristic algorithm based on Ant Colony Optimization (ACO) for optimization of long-term open-pit designs.

To achieve this, a computer program has been provided to test the proposed algorithm. As a case study, the block model of a hypothetical iron ore deposit is generated and the values of grades have been randomly assigned. The application of the newly developed algorithm is tested several times to achieve the best range of the parameters and proper variants of the ACO method.

Chapter 2 elucidates the background, concepts and problems associated with the existing open-pit optimization techniques, such as mathematical modelling, heuristic and metaheuristic algorithms. This chapter contains the analytical survey of the literature on open-pit optimization algorithms and discusses their limitations. Chapter 3 deals with the basic fundamentals of the ant colony optimization. The principals of ACO have been explained by using the well-known travelling salesman problem in Chapter 3. In Chapter 4 the major solution steps for a long-term open-pit planning problem by means of ACO is presented. The user interface of the programmed software and the results of its application on the case study have been included in this chapter. Chapter 5 contains all conclusions and recommendations for further research works arising from this research study.

2 THE PROBLEM OF LONG-TERM OPEN-PIT PRODUCTION PLANNING

The open-pit mine production scheduling problem can be defined as discovering the sequence in which rock blocks should be removed from the deposit as a certain material type in order to maximise the total discounted profit from the mine subject to a variety of physical and economic constraints. The size and the complexity of the problem cause that the currently available tools and methods are either yield suboptimal answers or not suitable for application to reasonable-sized deposits. This part discusses the long-term open-pit production planning problem from a mathematical programming point of view.

2.1 MATHEMATICAL PROGRAMMING MODELS

2.1.1 INTEGER LINEAR PROGRAMMING (IP)

Integer linear programming (IP) can be effectively used to model the production scheduling problem of an open-pit mine (Caccetta et al., 1998). It can be defined as following:

Objective function

The objective function could be expressed as the maximization of net present return by mining and processing of blocks.

$$Maximize \quad \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} C_n^{tm}.x_n^{tm}$$

Where

x_n^{tm} : The binary decision variables of the model ($x_n^{tm} = 1$ if block n is mined as type m in time period t and $x_n^{tm} = 0$ if otherwise).

C_n^{tm} : The objective function coefficients, representing the return from (or the cost of) mining of block n as type m in time period t.

n : The index of the blocks in the ore body, $n = 1,2, \dots, N$.

m : The index of different possible types that a block may be mined as (for instance $m = 1$ if the block is mined as waste, $m = 2$ if the block is mined as processing ore and $m = 3$ if the block is mined as leaching ore and etc.), $m = 1,2, \dots, M$.

t and r: The index of periods over which the mine is being scheduled, $t = 1,2, \dots, T$.

The model consists of $M \times N \times T$ binary variables.

The target will be subject to the following constraints:

Mining capacity constraints

Total tonnage of extracted material should be between a pre-determined upper and lower limit.

$$\begin{cases} \sum_{m=1}^{M}\sum_{n=1}^{N} W_n . x_n^{tm} \le MC_{max}^{t} \\ \sum_{m=1}^{M}\sum_{n=1}^{N} W_n . x_n^{tm} \ge MC_{min}^{t} \end{cases} , \text{for } \forall t$$

Where

W_n : Total tonnage of block n.

MC_{max}^{t} , MC_{min}^{t} : The maximum and minimum allowed capacity of mining operation for the period of t.

The model has $2 \times T$ number of mining capacity constraints.

Processing capacity constraints

Quantity of each material type should also be between the defined boundaries:

$$\begin{cases} \sum_{n=1}^{N} W_n . x_n^{tm} \le PC_{max}^{tm} \\ \sum_{n=1}^{N} W_n . x_n^{tm} \ge PC_{min}^{tm} \end{cases} , \text{for } \forall t \text{ and } \forall m$$

Where

PC_{max}^{tm} , PC_{max}^{tm} : The maximum and minimum allowed capacity of processing of material type m for the period of .

The number of processing capacity constraints will be $2 \times M \times T$.

Constraints for the average grade of products

Average grade of each production element should be between pre-determined limits:

$$\begin{cases} \sum_{n=1}^{N} (g_n^k - G_{max}^{ktm}).W_n.x_n^{tm} \leq 0 \\ \sum_{n=1}^{N} (g_n^k - G_{min}^{ktm}).W_n.x_n^{tm} \geq 0 \end{cases} \text{, for } \forall m \text{ , } \forall k \text{ and } \forall t$$

Where

k : The index of valuable elements in blocks (such as copper, silver and gold), $k = 1,2, \dots, K$

g_n^k : The grade of element k in block n

G_{max}^{ktm} , G_{min}^{ktm} : The maximum and minimum limits of the average grade defined for the element k of the material type m in the period t .

Based on this formula there will be $2 \times M \times T \times K$ number of average grade constraints required in the model. Normally no constraint required at least for the waste, therefore, the maximum number of constraints could be considered as $2 \times (M-1) \times T \times K$.

Reserve constraints

The reserve constraints are mathematically necessary to ensure that a block is mined only once.

$$\sum_{m=1}^{M} \sum_{t=1}^{T} x_n^{tm} \leq 1 \quad , \quad \text{for } \forall n$$

The number of constraints required in this case equals to the number of blocks, N.

The main point in this formula is that when $\sum_{m=1}^{M} \sum_{t=1}^{T} x_n^{tm} = 0$, it means that the block n will not be mined at all. In other words the problem of ultimate pit limit (UPL) has also been enclosed in this formulation.

Sequencing constraints

The sequencing constraints ensure that a block can only be removed if all overlaying blocks have been removed in the previous or current periods.

$$\left(\sum_{m=1}^{M} x_n^{tm} \right) - \left(\sum_{m=1}^{M} \sum_{r=1}^{t} x_l^{rm} \right) \leq 0 \quad , \quad \text{for } \forall t, \forall n \text{ and } \forall l$$

Where

l : The index for the set of overlying restricting blocks that should be removed earlier for mining of block , Γ_n , $l = 1,2, \dots, L$

The model should have $T \times N \times L$ number of this constraint type. This constraint could be written in a compressed version with less number of constraints:

$$L.\left(\sum_{m=1}^{M} x_n^{tm}\right) - \sum_{l=1}^{L}\left(\sum_{m=1}^{M}\sum_{r=1}^{t} x_l^{rm}\right) \leq 0 \quad , \text{ for } \forall t \text{ and } \forall n$$

Ramazan et al. (2004) showed that first case is faster at the run time.

Binary variables

Finally the variables of the model should be binary. $x_{nt}^{m} = 0 \ or \ 1 \ \ \forall n, \forall m, \forall t$

Through simply stated, an integer linear programming formulation of the open pit scheduling problem usually involves a large number of variables and constraints. For example, a small copper-molybdenum deposit containing 10,000 blocks and 10 planning periods would require the solution of an integer programming problem with 300,000 variables, 200 mining and milling constraints, 900,000 sequencing constraints and 10,000 reserve constraints. Clearly this is beyond the capacity of current integer programming packages.

2.1.2 The linear programming (LP) formulization of the model

Johnson (1969) has discussed the LP modelling of the long-term open-pit production planning problem. The major benefit in this model is that the fractional block extraction becomes possible.

The LP model could be easily achieved by new definitions for the decision variables, coefficients of the objective function and discarding the binary nature of the variables in MIP formulation as following:

x_n^{tm} : The proportion of block n to be mined in period t as a processing type .

C_n^{tm} : The NPV resulting from mining a unit weight of material in block n during period t if it is considered as processing type m.

Johnson (1969) proposed to solve this problem by decomposing of the large multi-period production planning model into a master problem and a set of sub-problems that are exactly similar to UPL problem. After solving all sub-problems by well-known UPL algorithms such as Lerchs-Grossmann's algorithm, solving the master problem would be relatively simple.

Although this method produces optimum solutions for each period individually, however, it does not optimize the problem totally. It also encounters situations in which some portion of a block could be extracted while all the overlying blocks have not been fully mined. In other words some percentages of overlying blocks are suspended in air. These downsides and high number of constraints cause the approach not to be practical. (Osanloo et al., 2008)

2.2 MATHEMATICAL APPROACHES FOR SOLUTION OF THE MODEL

Several approaches have been proposed in literature to solve this model. Dagdelen and Johnson (1986) and Caccetta et al. (1998) used lagrangian parameterization in order to relax mining and milling constraints into objective function. Consequently the problem could be handled by repetition of any UPL algorithm such as Lerchs-Grossmann (1965) graph theory based algorithm. Caccetta et al. (1998) utilized lagrange multipliers to omit the mining and milling constraints and solved the model using subgradient optimization method. Dowd & Onur (1992) and Onur & Dowd (1993) formulated the problem as a dynamic programming model. Later Ramazan et al. (2005) described the application of fundamental tree algorithm to reconstruct the mining blocks and decrease the number of variables in scheduling problem without reducing the resolution of the model or optimality of the results. A fundamental tree is defined as any combination of blocks such that the blocks can be profitably mined respecting slope constraints. Following comprehensively reviews the most important literatures.

2.2.1 LAGRANGIAN PARAMETERIZATION

The idea of lagrangian parameterization originates from the fact that the mining and processing constraints are relatively few in number but complicate the underlying structure of the whole problem. Using Lagrangian multipliers, the complex multi-period problem of open-pit production planning could be decomposed into smaller single-period problems that could be solved using optimum pit limit (UPL) design algorithms. Considering very efficient UPL algorithms such as Learchs-Grossman (1965) and Zhao-Kim (1992) and etc., this makes possible to solve a relatively large long-term open-pit scheduling problem.

Before going into the formulization of this powerful concept, it is needed to define the UPL problem.

Ultimate Pit Limit (UPL) Problem

When formulated as a mathematical program, the objective in solving UPL problem is to find all the available ore material in the deposit which will maximize the profits and when mined, will satisfy the pit slope requirements. This problem can be formulated as:

$$Max \sum_{i} C_i X_i$$

subject to : $X_i \in \Gamma X_i \quad$ for all i

Where C_i is the economic value of the block, ΓX_i is the set of the blocks which must be removed earlier in order to reach the block X_i and X_i is one if the block i is mined and is zero otherwise. (Dagdelen & Johnson, 1986). Details of the well-known Lerchs-Grossmann algorithm of UPL calculation has been explained in chapter 2.3.1.

Understanding Lagrangian parameterization

The mining and milling constraints could be simply relaxed by multiplication of constraints in Lagrange multipliers and subtracting of resulted phrase from the objective function. Therefore the model can be re- written as:

$$Maximize \quad \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{t=1}^{T} D_n^{tm} . x_n^{tm}$$

Subject to : Sequencing and Reserve constraints

Where D_n^{tm} are the new coefficients that have been obtained by subtracting Lagrange multipliers from the original C_n^{tm} coefficients.

Now the Lagrange multipliers should be adjusted using the sub-gradient method until the optimum schedule is obtained. At each step, only a problem similar to an ultimate pit limit problem needs to be solved. In cases that there are no multipliers that can result in a feasible solution for the constraints, this method may not converge to an optimum solution. This leads to a problem known as the gap problem.

Caccetta et al. (1998) applied the method on a real ore body with 20,979 blocks and six time periods and the schedule obtained was within 5% of the theoretical optimum. One year later Akaike & Dagdelen (1999) proposed a 4D-network relaxation method which was capable to consider dynamic cut-off grade concept during the scheduling process and handle the stockpile option.

2.2.2 Clustering approach

One of the recent mathematical approaches to solve an IP model of production planning of an open-pit mine is the clustering method proposed by Ramazan et al. (2005). Clustering is defined as the classification of a large amount of data into a relatively few number of similar classes. The reason is to reduce complexity in the considered application in order to obtain

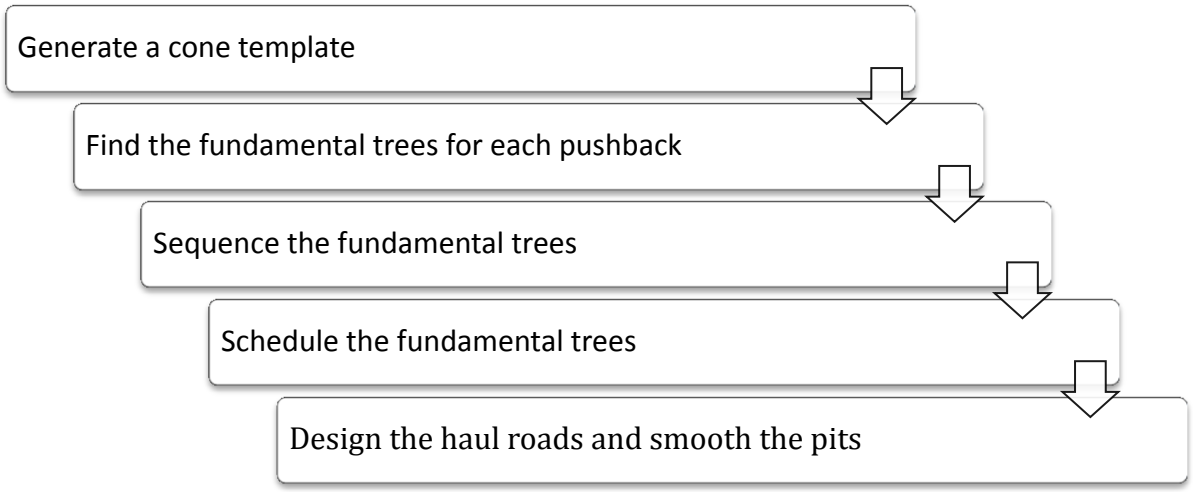


FIGURE 2-1 STEPS OF THE MINE PLANNING METHOD BASED ON THE FUNDAMENTAL TREE ALGORITHM

improved decisions based on the available information. Authors combined ore and waste blocks together to decrease the number of binary variables in the linear programming model. They defined the fundamental tree as any combination of blocks within the push backs, such that can be profitability mined and satisfy the slope constraints and no chosen sub-set of a fundamental tree meeting these requirements could be found. The clustering process is done using an LP formulation in a way that no available information of any individual block to be lost. Steps of algorithm have been shown in Figure 2-1.

Figure 2-2 illustrates a 2D block model on which three fundamental trees are created by a linear programming formulation. Tree I can be mined first after which trees II and III become accessible for mining. After determination of the fundamental trees, their precedency is calculated by means of a cone template. In this stage, each fundamental tree is treated as an individual mining block containing a certain ore quantity, metal content and quality characteristics. Now an IP model could be generated by assigning a binary variable to each fundamental tree and each production period. This model is then solved by CPLEX software and contents of the UPL are allocated to 3 to 5 smaller volumes (push backs). Finally, fundamental trees are scheduled by an IP formulation including all mining and milling operational constraints and tree sequence requirements. (Osanloo et al. 2008, Ramazan et al. 2005)

The main advantages of this method are:

- The number of model's binary variables is directly proportional to the number of trees and periods. Therefore, it can result in reducing the size of the model and, hence,

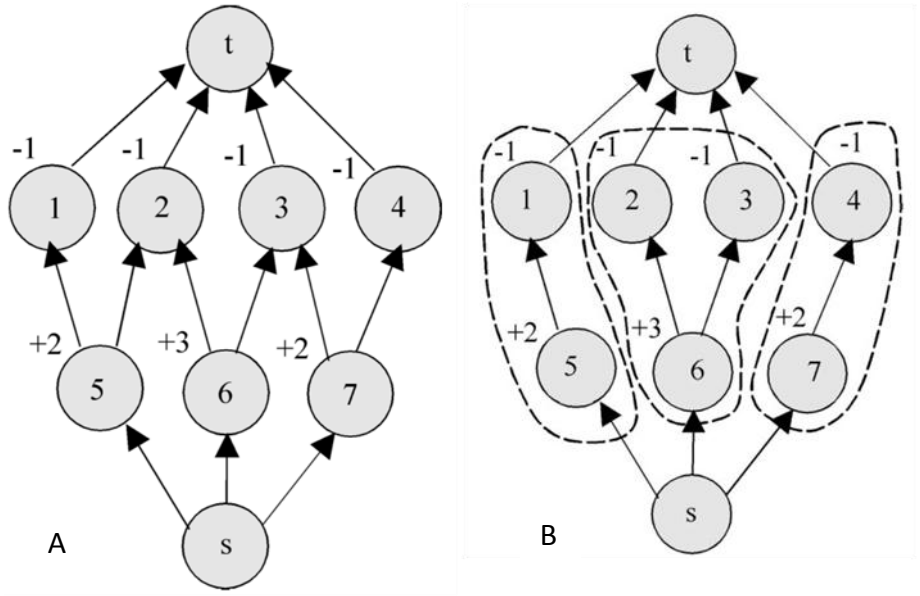


FIGURE 2-2 NETWORK REPRESENTATION OF A 2-D BLOCK MODEL,
(A) THE NETWORK REPRESENTATION OF THE SOLUTION OF THE LP FORMULATION AS THREE FUNDAMENTAL TREES SURROUNDED BY DASHED LINES (B) (RAMAZAN ET. AL. 2005)

bigger block models can be solved by this technique. Authors indicated that by using this method the number of binary variable can be decreased from 38,457 to 5,512 in a case. (Ramazan et al. 2005).

- The gap problem could be eliminated.
- Further studies on this algorithm revealed that it gives a schedule with a 6% higher NPV than those predicted by the use of other software such as Mintec Scheduler, NPV Scheduler and Millawa algorithm of Whittle. (Bernabe and Dagdelen, 2002).

The drawbacks of the method are as follows.

- In very large deposits, the number of trees to be scheduled and corresponding binary variables in the model would be still high enough to make the model impractical.
- Since the fundamental trees are defined inside push backs, the optimality of this method will be up to the optimality of the push back determination routine.
- Probably more than one iteration of the LP formulation is necessary for identifying optimal fundamental.
- The complexity of the implementation of the steps of this method highly affects its public acceptance. (Osanloo et al. 2008, Ramazan et al. 2005)

2.2.3 Branch and cut technique

Branch-and-cut technique is an efficient method for solution of combinatorial optimization problems that are formulated as mixed integer linear programming problems. It is an exact algorithm which combines a cutting plane and the branch-and-bound algorithm. It works by solving a sequence of linear programming relaxations of the IP problem. The cutting plane improves the relaxation of the problem to more closely approximation. Branch-and-bound algorithm carries out by a sophisticated divide and conquer tactic to solve problems. Efficiently solution of a general IP problem is usually not possible by means of a cutting plane approach; and normally branching is also necessary, which consequences in a branch-and-cut approach (Mitchell 1999). Perhaps the best known branch-and-cut algorithms are those that have been used to solve the travelling salesman problem (TSP). (Appelegate et al. 1995).

Application of Branch-and-cut procedure for solution of IP model of the long-term open-pit scheduling problem is effectively outlined by Caccetta and Hill (2003). Explicitly incorporation of most operational constraints such as maximum vertical depth, minimum pit bottom width and stockpile option in the optimization procedure could be accounted as the major advantages of their method. The process is able to provide acceptable solutions for production planning problems of medium sized mines. Nevertheless, for large problems it is relatively difficult to obtain an optimal solution. Authors illustrated that on a case study with about 209,600 blocks and ten planning periods, a solution with precision of 2.5% could be found within four hours. The other disadvantage of their process is that it does not consider cutoff grade optimization. The authors recommended that by conjunction of their branch-and-cut methods with heuristics or meta-heuristics a good (possibly optimal) solution can be obtained. This would be also suitable to show how far from optimal this solution may be obtained. (Caccetta and Hill 2003).

Defining all variables of the periods as binary values would prevent this algorithm to generate even a feasible solution for the long-term open-pit scheduling model; consequently, the number of binary variables should be reduced by setting of some variables as real numbers. For instance, setting of the variables corresponding to the positive blocks as binary and the rest of variables as real could significantly decrease the solution time. By this action the IP model transforms to an MIP model. This strategy is usually applied to all of the above-mentioned IP models. (Osanloo et al. 2008).

2.2.4 Dynamic programming (DP) formulation

This method consists of dividing of the prime problem into smaller problems for which an optimal solution could be easily found. To do this, it searches all possibilities and chooses the

optimum one. Unlike other operations research methods, there is not standard mathematical approach for dynamic programming models solution. (Osanloo et al. 2008).

In this regard, the long-term production planning problem is modelled as a graph whose nodes represent the state of the system and arcs are related to the action that takes the system from one state to another. Solution of the production planning problem could be considered as finding the path with the highest value. (Osanloo et al. 2008).

In the first attempts, dynamic programming was applied on the open-pit production planning problem by Roman (1974). His formulation simultaneously calculates the pit limit and block sequencing problems. In this formulation, the location of the block that must be mined in the last period is determined at starting stage of the sequencing process. Accordingly, all possible ways to schedule blocks above the particular block respecting to the slope constraints are checked and then the optimum sequence is determined by a NPV calculation routine. The sequence related to the highest NPV is chosen and its value is assigned to the initial pit outline. The blocks near the pit boundary have to be examined to define if they contribute to a positive NPV or not. Thus the blocks that do not correspond to a positive NPV are detached from the pit and a new pit sequence and NPV are attained. This procedure continues until no block is required to be detached from the pit. The advantage of this method is that it is based on the time value of the money and considers scheduling in determination of the ultimate pit limit. The drawbacks of this technique are as follows:

- It cannot be applied on large block models.
- There is no guarantee that mining and milling constraints will be met in each period.
- The effect of the mine size on the unit cost is not considered. (Osanloo et al. 2008).

Another DP formulation for the long-term planning problem expressed by Dowd and Onur (1992 & 1993). They indicated that in the previously proposed DP model, the number of alternatives which have to be considered is very high and beyond the memory of current PCs. They showed that this number could be reduced by means of elimination of the unattractive alternatives. Authors proposed algorithm could take all kinds of constraints, mobility and equipment access constraints into consideration and eliminate unattractive sequences as soon as they appear. The long calculation time was the major drawback of this method.

The other attempt was a method proposed by Tolwinski and Underwood (1992) by combining DP, stochastic optimization, artificial intelligence and heuristic rules to solve the long-term production planning problem. In their DP model the problem was equivalent to

finding a path with the largest value in a graph$G = (S, E, W)$, where S denotes the set of nodes related to the states of the model as the sequence of the pits, E is the set of block removal edges and W is the weights (rewards) associated with E elements. Supposing S_0 to be the node related to the initial state of the mine, the problem of optimal production scheduling would be equivalent to finding a path $S_0, S_1, \dots, S_T$ through the graph G that minimizes the total reward. For a practical case with numerous nodes, solution of DP model all of the nodes requires to be taken into account (blind search) and this leads to NP hard condition. To avoid it, Tolwinski and Underwood suggested an informed search method, based on AI and heuristic rules. Despite practicality of this technique for big models, it still suffers from the lack of guarantee to provide mathematically proven optimal solution or even sometimes a feasible solution. (Osanloo et al. 2008).

Later Tolwinski (1998) and Tolwinski and Golosinski (1995) developed a routine based on the depth first search method for the DP formulation. Again in spite of the capability of application on large block models, however, the gaining the highest NPV was still infeasible.

In another study Erarslan and Celebi (2001) utilized a simulative optimization model to find the optimum pit limit and production plan. In their DP formulation each extracted block was defined as a state of a DP stage. Despite very big advantages of this method such as simultaneous calculation of UPL and scheduling, ability to estimate unit costs for each new pit scenario and considering all types of constraints; again like the other DP approaches, finding the optimal solution was not practical for large deposits. (Osanloo et al. 2008).

2.3 HEURISTIC ALGORITHMS

In the last three decades lots of heuristic algorithms have been suggested in the field of open-pit production planning, but mostly cover only an individual part of the global optimization and needs be integrated with others to provide a general solution for the problem. Among them Whittle's method and Sevim's techniques are relatively integrated approaches that could cover all parts of the open-pit scheduling problem.

2.3.1 WHITTLE'S OPTIMIZATION PROCESS

Whittle's process is based on the fast implementation of a series of Lerchs-Grossmann (LG) algorithm. This algorithm produces the mathematically optimum final pit shell, but only if maximum undiscounted cash flow is the criterion for optimization. The process tries to assist selection of the optimum final pit by providing a best and worst case mining schedules and associated NPV curves. This normally produces a very wide range of possible pits among

which the engineer must pick a single optimal pit up, usually by guesswork, experience or rules-of-thumb.

It should be noted that the only used optimization criterion in Whittle's method is to maximize the Net Present Value (NPV) of the cash flows gaining from the sales of metal or concentrate obtained from the pit. Therefore if the company derives revenues from downstream activities or if it looks for the maximum utilization of the mineral resource, or uses some measure other than NPV, the engineer must adjust the criteria accordingly.

In this part, firstly the steps of well-known Lerchs-Grossmann (LG) algorithm have been explained. Then the concepts of the best and worst mining cases on the nested pits created by LG algorithm have been described. Finally the suggested method of whittle and milawa algorithm for providing of mine schedules have been stated.

Definition of UPL problem

During an open-pit mining operation blocks of the rock are extracted from the earth and surface of the land is being continuously excavated towards forming a deeper and deeper pit until its ultimate shape and termination of the operation. In order to design the optimal outline of a pit, one that maximizes the profit, the entire area is divided into a 3-dimensional grid of blocks and the metal content of each block is estimated based on the available geological information gotten from drill cores. Then each block gets an economic weight, representing the value of the ore in it, minus the costs involved in removing and processing of that block. While trying to maximize the total economic weight of the blocks to be extracted, there are also contouring constraints that have to be respected. These constraints correspond to the slope stability requirements of the open pit mining and the precedence constraints that prevent blocks from being mined before the others in higher layers of them. Subject to these constraints, the objective of the optimization problem is to find the most profitable set of the blocks.

The UPL problem can be modelled as a directed graph $G = (V, A)$. Each block i corresponds to a node with a weight b_i representing the net economic value of the individual block. There is a directed arc from node i to node j if block i can not be extracted before block j, which is normally in a layer right above block i. Now the objective is to find the set of blocks to be extracted while maximizes the profit. This is equivalent to finding a maximum-weight closed set of nodes, where a set of nodes is closed if it contains all successors of the nodes in the set. Such a set is called a maximum closure of G.

Lerchs-Grossmann algorithm

The algorithm works by flagging certain blocks as "strong", meaning that they are planned to be mined. Blocks that are not strong are labelled as "weak", represents that there is no current plan to mine them. A block is considered to be strong if it belongs to a group of linked blocks, known as a branch, with a total positive value. Initially, each block is a separate branch and thus only the blocks with a positive economic value are strong.

The algorithm checks for the arcs that run from a strong block to a weak block. Such pairs of blocks indicate a precedence conflict, and the algorithm tries to resolve this conflict by changing the links between blocks. As these changes are established, a tree structure is built up in which the blocks are linked together in branches. Lerchs and Grossmann indicated that when a check through all the arcs does not detect any possible strong to weak connection, then those blocks which are labelled as strong, constitute the optimal pit. They also demonstrated that this situation would be reached after a finite, but unknown, number of iterations. In practice the number of required checks would be anything from ten to a few hundred times of the arcs number to achieve optimality.

Before the algorithm is explained, the following terms should be defined (Stuart, 2008):

Slope Graph: A directed graph whose vertices represent the blocks of rock in the model. Directed edges of this graph point upwards to other vertices which must be mined earlier so that acceptable pit slopes to be left. This graph does not change during the algorithm process. The object of the LG algorithm is to find the maximum closure on this graph.

Tree Graph: A rooted tree whose edges always coincide with the edges of the slope graph (regardless of direction). This graph changes in the course of the algorithm.

Dummy root: An extra non-existent vertex assumed to lie below the rest of the blocks, and is considered to be the root of the tree graph at all times.

Edge support: All edges and vertices in the tree graph support a mass. The mass is supported by an edge which is the sum of economic value of the vertices on the leafward side branch. In other words, by removing an edge, the tree graph is divided into two separate branches and the support of this edge refers to the sum of economic values of the branch which does not include the dummy root.

Vertex support: Is equivalent to the sum of economic value of the vertex itself and all vertices on its leafward side branch.

P and M edges: A P (Plus) edge on the tree graph is defined as the one which has the same direction as the slope graph in the direction away from the root. In other words, the P edges go up and the M edges go down. Up and down concepts are defined by spatial location.

S and W edges: An S (strong) edge is either a P edge with a positive mass support or an M edge with a null or negative mass supports. In contrast, a W edge could be an M edge supporting a positive mass or an S edge which supports a zero or negative mass.

S and W vertices: A vertex defined as S (strong) if at least one strong edge exists on the path between that vertex and the dummy root. Otherwise it will be W (weak).

Normalized tree*:* A tree is called normalized if all its strong edges originate from the dummy root. Namely, only dummy edges can be strong and all others are weak in a normalized tree. In order to normalize a tree, all strong edges that do not originate from the root should be removed and replaced with a dummy edge connecting the severed branch to the root.

Steps of the algorithm

The four major steps of the LG algorithm are as follows:

Step1. Initialize the tree graph by connecting all vertices to the dummy root. This is obviously a normalized graph. All edges that point to a positive block are strong and vice versa. Support of all vertices is also equal to its economic value.

Step 2. Find a directed edge (A, B) in the slope graph, such that A is a strong vertex but B is not. If there is no edge with this condition, the algorithm is completed and the maximum closure (solution) is the set of all strong vertices.

Step 3. Add the edge found in Step 2 to the tree graph and remove the dummy edge supporting the (former) strong branch.

Step 4. Normalize the tree graph and looping back to the Step 1. (Stuart, 2008).

A closer look at Step 2:

After finding an unconnected edge from a strong vertex S_n to a weak vertex W_n on the slope graph in Step 1, a new tree T_s will be created during Step 2, from the previous normalized one T_t, Figure 2-3A. This is done by adding a new edge (S_n,W_n) and chopping off the dummy edge from the strong branch (D,S_1), (Figure 2-3B). The existing (S_n,W_n) arrow in the slope graph reveals that S_n is always physically below W_n . (Stuart, 2008).

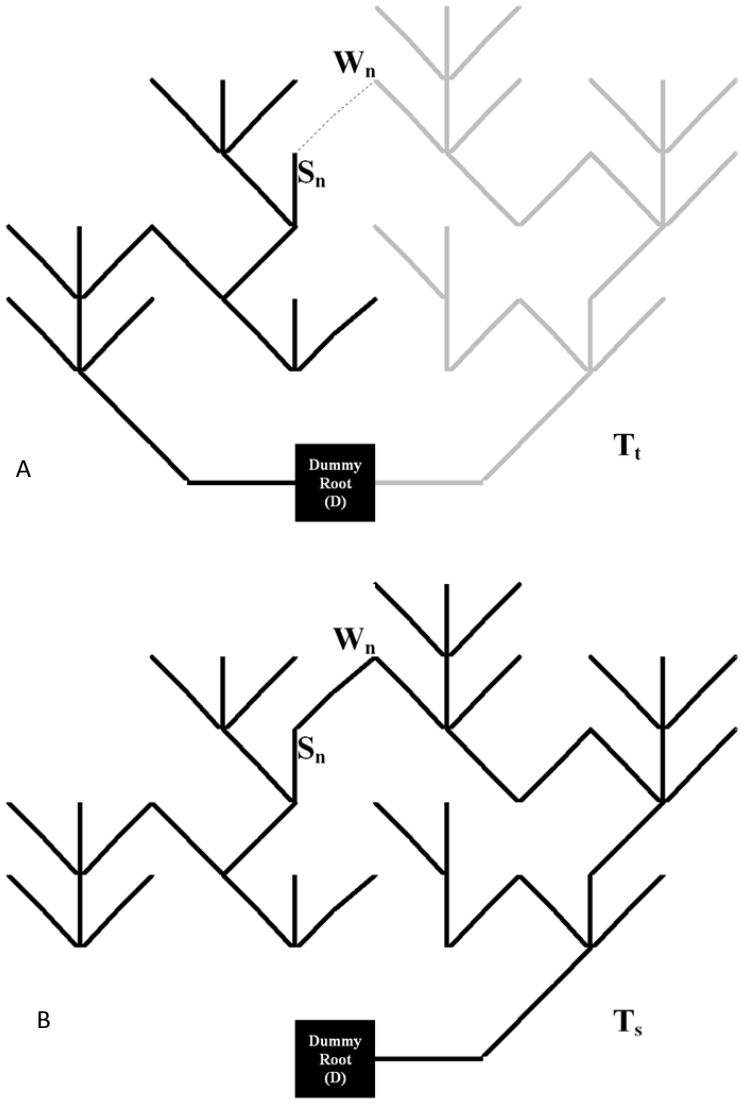


FIGURE 2-3 SECOND STEP OF LERCHS AND GROSSMANN ALGORITHM
FINDING A NEW EDGE FROM STRONG TO WEAK (A), CUTTING THE ROOT CONNECTION OF STRONG BRANCH (B)

According to the definition of a normalized tree and the strong and weak branches, it could be consequent that (Stuart, 2008):

The deleted edge (D, S_1) was a strong edge and was supporting a positive mass (support of S_1). This mass equals to the sum of economic values of all blocks in this branch. The rest of the edges in this branch were weak edges, thus they are either leafwards supporting a negative mass or rootwards supporting a positive mass.

All edges in the weak branch were weak; therefore they are either leafwards supporting a negative mass or rootwards supporting a positive mass.

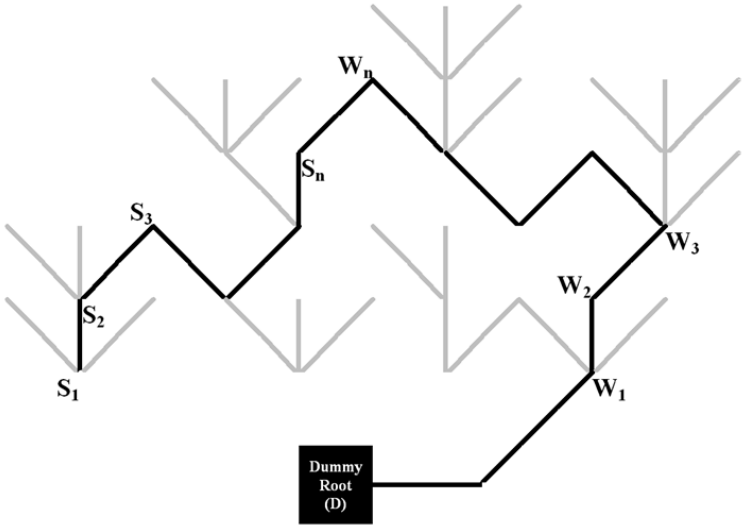


FIGURE 2-4 CRITICAL PATH

After Step 2, the only parts of the tree graph that will be altered is the vertices and edges on a path from S_1 to the root which could be shown as $\{S_1,S_2...S_n, W_n, W_{n-1}....W_2,W_1,D\}$ and termed the critical path (black path on Figure 2-4).

Support of vertices on critical path could be updated as following:

By definition the supported mass is the sum of all individual masses on the branch side of a vertex. After step 2, S_1 loses the contribution of everything that S_2 supports. If the new and old masses supported by S_1 are shown in $M_{new}S_1$ and $M_{old}S_1$ then it could be written:

$$M_{new}S_1 = M_{old}S_1 - M_{old}S_2$$

Notice that $M_{old}S_1$ is the mass of the whole strong branch prior to Step 2.

Following the path from S_1 to S_n after Step 2, the mass supported by any vertex S_x on this path equals to the mass of the old strong branch $M_{old}S_1$ minus the mass that was supported by the next vertex on the path $M_{old}S_{x+1}$. Therefore generally, one could write:

$$M_{new}S_x = M_{old}S_1 - M_{old}S_{x+1}$$

The mass supported by S_n, $M_{new}S_n$, is equal to the mass of the entire strong branch before Step 2, which was equal to $M_{old}S_1$.

The masses supported by the all vertices on the critical path from W_n to D will be increased by the weight of the entire strong branch. So that

$$M_{new}W_x = M_{old}W_x + M_{old}S_1$$

It could be easily seen that edges between S_1 and S_n turn from P to M edges or from M to P (leafwards to rootwards and vice versa) whereas edges between W_n and the root are unchanged. The connection from S_n to W_n is always an M edge. Consequently the following cases could happen for the edges on the critical path (Stuart, 2008):

On the S part of the path, a former P edge (that is now transformed into an M edge) must have supported a negative or zero mass, based on the definition of normalised tree. The new supported mass is found by subtracting this negative value from $M_{old}S_1$, which is the mass of the whole strong branch. So the new supported mass must be greater than or equal to $M_{old}S_1$ (always positive). Consequently a P edge on the S part of the path will always become a weak edge after executing Step 2.

The support of new (S_n,W_n) edge will be always equal to $M_{old}S_1$, the mass of entire strong branch. Considering this positive number and the fact that a new edge is always a rootward edge, this means that this edge will always be weak.

An M edge on the W part of the chain (that was M and is still an M edge after Step 2), should support a positive mass before by definition of a normalized tree. Therefore by adding $M_{old}S_1$ (the mass of the strong branch) to its support, the new support will always be positive and accordingly, the edge will always remain weak (Stuart, 2008).

In contrast, it is not possible to establish a general rule for P edges in both W and S parts. This means that whether a P edge is strong or weak (that were previously M in S side and P in W side) should be identified by calculation of supported masses after operation of Step 2.

Numerical example

Suppose a very small numerical example considering a two dimensional block model containing 6 blocks as shown in Figure 2-5. The numbers represent the economic value of the blocks.

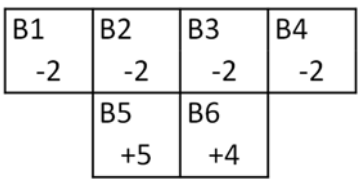


FIGURE 2-5 SMALL NUMERICAL EXAMPLE BLOCK MODEL TO ILLUSTRATE LG

To illustrate the algorithm, the problem has been presented in a graph figure. In the following figures the strong nodes and edges are shown in dark black and the weak ones

have been drawn in gray. Steps using the Lerchs-Grossmann algorithm in order to obtain the solution are as follows:

A dummy root is added to the graph and initially all blocks are connected to the root. The supported mass of all blocks are equal to their economic value and all edges are leafwards. Therefore by the definition of strong and weak edges, all edges which point towards positive blocks are strong. Hence the positive blocks will be strong as they possess a strong edge. In contrast all negative blocks and their corresponding edges will be weak.

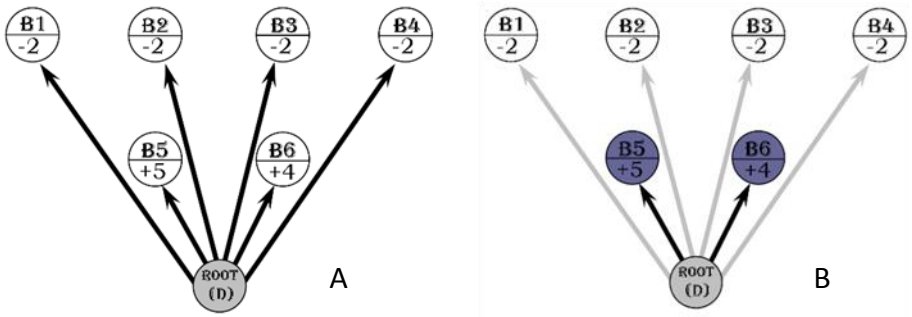


FIGURE 2-6 ADDING DUMMY ROOT AND INITIAL STRONG AND WEAK NODES AND EDGES

The first possible connection from a strong vertex to a weak one would be considered between B5 to B1, Figure 2-7. By adding of this connection and removing the edge between the root and B5, the supported mass of B1 will change to +5-2=+3, resulting in a strong D-B1 connection. In spite of the positive support of the B5-B1 edge, it is weak because of its rootward direction. This graph does not need to be normalized because all its strong edges start from the root.

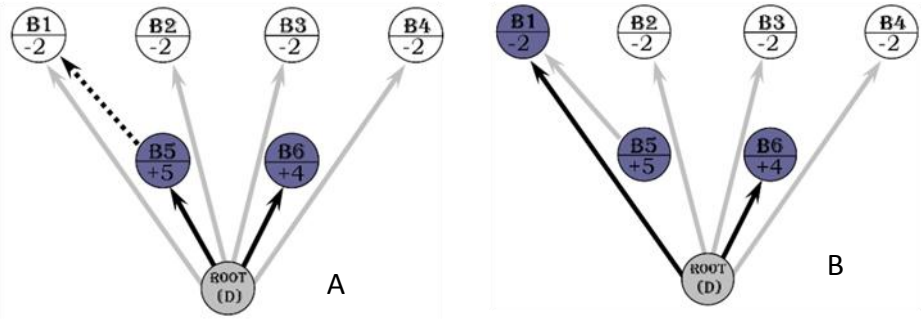


FIGURE 2-7 FIRST ITERATION

A second possible connection would be from B5 to B2, Figure 2-8. Adding B5-B2 and chopping D-B1 will change the support of B1, B2 and B5 vertices to -2, +3 and +1 respectively. This means that the D-B2 leafward edge is strong and the B5-B2 and B5-B1 connections are weak. All these three blocks are strong because of having a strong

connection to the root. Again, all strong edges originate from the root and no normalization is needed.

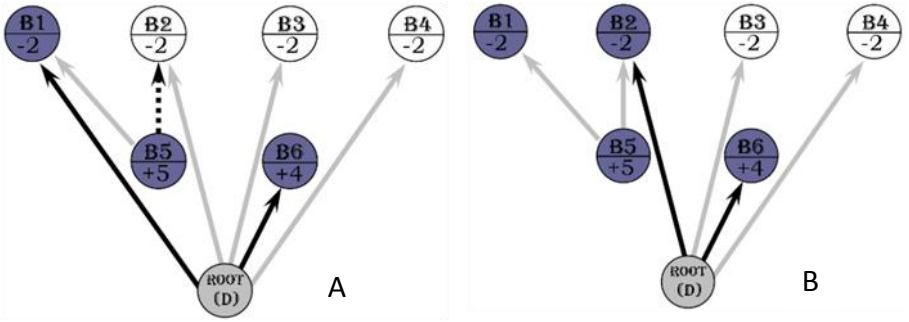


FIGURE 2-8 SECOND ITERATION

The other possible connection could be found between B5 and B3. Inserting this connection will change the support of B3, B5 and B2 edges (path from W1 to S1) to -1, +1 and -2 respectively. This reveals that all B5-B1, B5-B2, B5-B3 and D-B3 edges are weak. At this moment the tree has only one strong edge which originates from the root and there is no need for normalization.

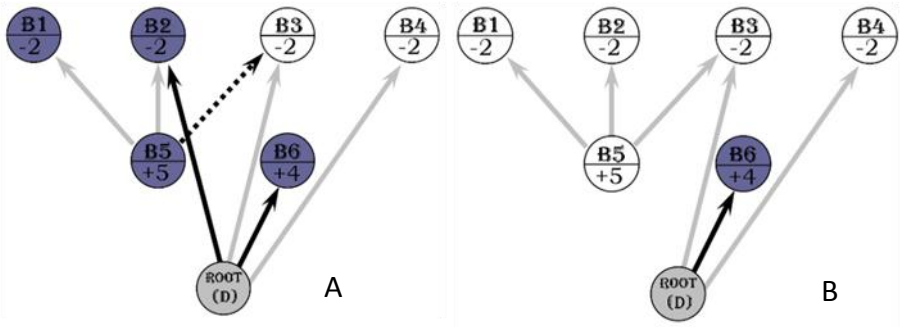


FIGURE 2-9 THIRD ITERATION

The next possible edge would be from B6 to B2. After adding this connection and cutting of the D-B6 edge, the support of B6, B2, B5 and B3 blocks will be altered to +4, +2, +5 and +3. Thus the D-B3 and B5-B2 edges will be strong edges by definition.

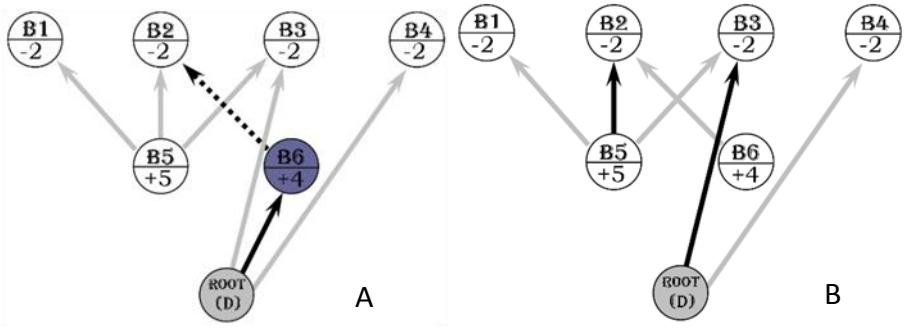


FIGURE 2-10 FOURTH ITERATION

Now the B5-B2 connection is a non-dummy strong edge that should be removed and replaced by a new connection from the root to B2. Eventually the support of B1, B2, B3, B5 and B6 vertices will be changed to -2, +2, +1, +3 and +4. All of these blocks are strong as they have a strong edge in their connection to the root.

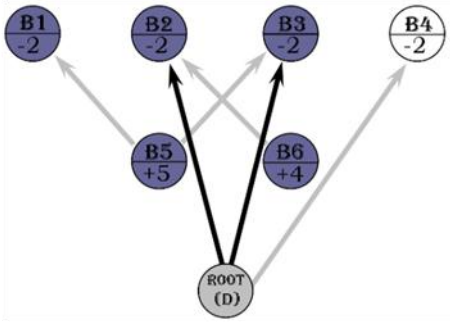


FIGURE 2-11 NORMALIZATION

The next connection from strong to weak blocks could be found again from B6 to B4. By drawing this edge and deleting D-B2 the supported mass of B2, B6 and B4 will be altered as -2, +2 and 0 respectively. It results in all B6-B2, B6-B4 and D-B4 edges as well as B2, B4 and B6 vertices to become weak. No normalization is required at this stage.

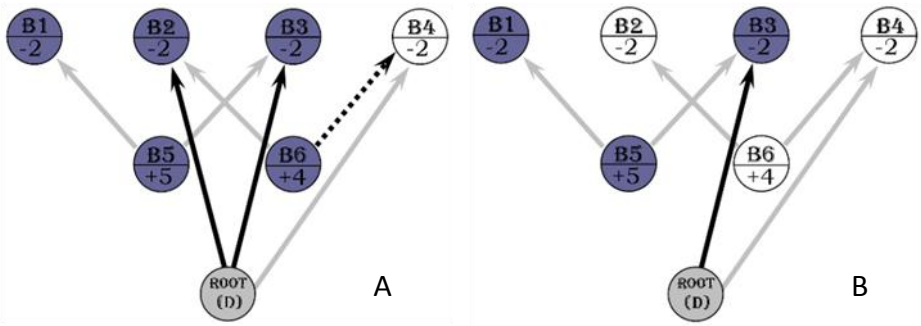


FIGURE 2-12 FIFTH CONNECTION AND CUT

The last possible connection appears now from B5 to B2. After adding this edge and removing the D-B3 connection, the supported mass of B3, B5, B2, B6 and B4 will be changed to -2, +1, -1,+3 and +1 . The result is that, except for D-B4, all other edges of the tree are weak. The resultant tree does not require any normalization and there is no other possible connection from strong to weak blocks. It means that the algorithm has terminated and the solution (blocks that are inside the ultimate pit) is the set of all strong blocks. In this case, all blocks of the model fall within the solution set.

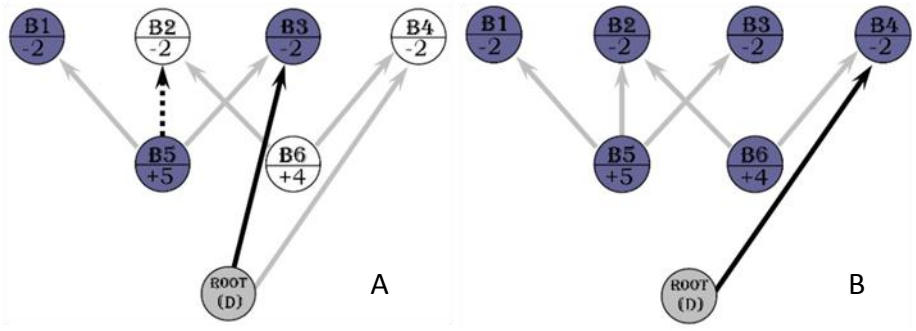


FIGURE 2-13 THE LAST CONNECTION AND CUT

Whittle found that it is faster to start at the bottom of the model than at the top. Additionally he established that the tree structure is much less tangled and easier to resolve if the arcs toward a block be checked rather than the arcs way from a block. Finally, when the arcs toward a block reveal more than one conflict, he found it advantageous to carefully choose which conflict to resolve first. As a whole, these ideas sped up the running of the Lerchs-Grossmann algorithm by about a factor of ten. (Whittle 1999).

Construction of nested pits

Whittle process starts by carrying out fifty to one hundred LG optimizations, for a list of different metal prices. In this approach, a series of pits with different sizes are obtained where each of the pits has the highest undiscounted dollar value for the considered pit size. The obvious way to do this is to optimize for the lowest price first (finding the smallest pit) and then to remove the mined blocks before optimizing with the next higher price. Whittle found that it is very much faster to start with the highest price, then the lowest price, and then to do a "binary chop" in which repeatedly a price from top and bottom of the prices list that has not been optimized yet is chosen and that is likely to split the largest group of blocks remaining. In this way, each optimization involves fewer and fewer blocks and arcs than the last, and the process goes faster and faster. He also found that it is faster not to

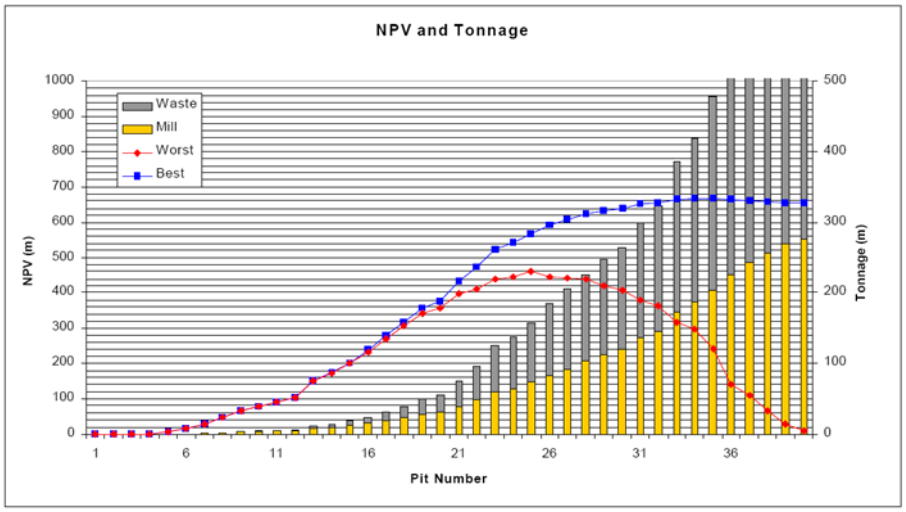


FIGURE 2-14 TYPICAL NPV-TONNAGE GRAPH IN WHITTLE'S METHOD
WHARTON 2000

start each optimization from scratch and keep the existing tree structure as starting point for the next run with only adjusted block values. (Whittle 1999).

The best and worst case mining scenarios

The best case scheduling involves mining with many small cutbacks and indicates the highest possible NPV. It assumes that the waste associated with the ore required in any one year is mined in that same year. This is rarely the case in reality and so produces an optimistic view of the project NPV. (Wharton 2000).

The worst case schedule assumes all benches are mined in sequence from top to down and in their entirety, this is a very pessimistic scenario rarely seen in practice. (Wharton 2000).

UPL selection based on the NPV-Tonnage graph

Figure 2-14 shows a typical NPV-Tonnage graph by which Whittles process tries to find a good mine schedule. It represents the NPV and ore and waste tonnage for each nested pit. The best and worst NPV curves show an upper and lower limit on the value that can be achieved. (Wharton 2000).

Now it is up to the mine designer to select one of the nested pits as UPL based on their corporate objectives of his/her company. Unsophisticated users may use highest pit on the best case curve. More advanced users often use the maximum value pit estimated by taking the average NPV of the best and worst case curves. Some users vary this technique and use pits that are 60 or 70 percent of the difference between best and worst case values. For example for the case shown in Figure 2-14, although the pit 35 have the maximum NPV, pit

32 would be selected as UPL because it provides almost the same NPV with 60 m tones less mining. (Whittle 1999, Wharton 2000).

Mine scheduling

Figure 2-15 shows the yearly production of the mine for the best and worst case mining scenarios considering a mining capacity of 35 million tons in the first year and 40 million tons for the rest of mine life and a processing capacity of 5 million tons for first two years, 8 million tons for third year and 10 for the rest of mine life. (Wharton 2000).

The worst case sequence suffers from having a high stripping ratio at the starting years of the mine and in this case it has led to a mill process that is starved of material in the early years of the mine. On the other hand the best case sequence which is based on mining each pit-shell one after the other, is not practical in a mining sense, however, it provides an upper limit on the NPV value of the mine. In this case it has a value of $ 368 m. (Wharton 2000).

The next step in Whittle's process is to provide a set of practical push backs and try to maximize the value of the mine. After that the Milawa algorithm is used to generate the mine schedules. It can operate in either the NPV mode where it will seek to maximize the NPV or a balancing mode where it will seek to maximize the use of production facilities early

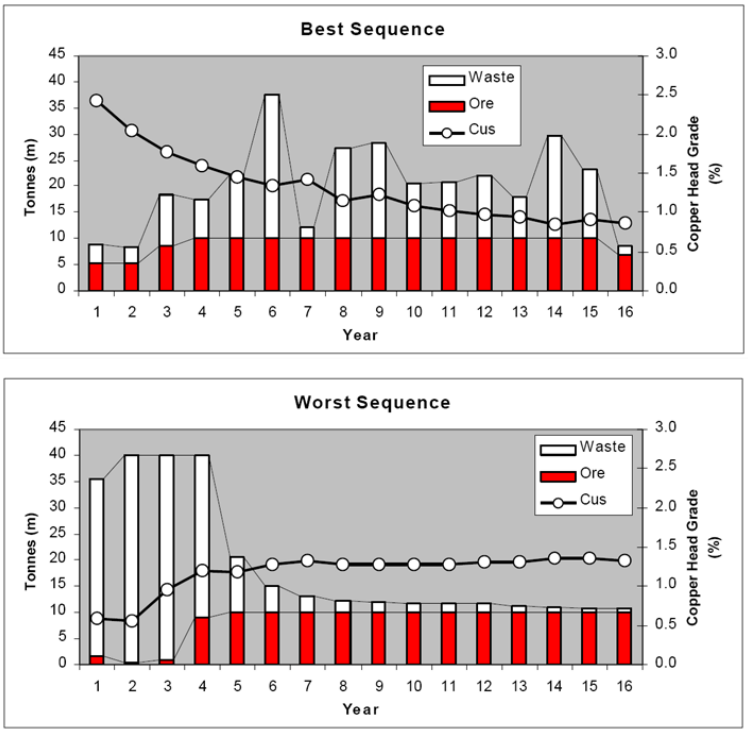


FIGURE 2-15 YEARLY PRODUCTION OF MINE FOR THE BEST AND WORST CASE MINING SCENARIOS WHARTON 2000

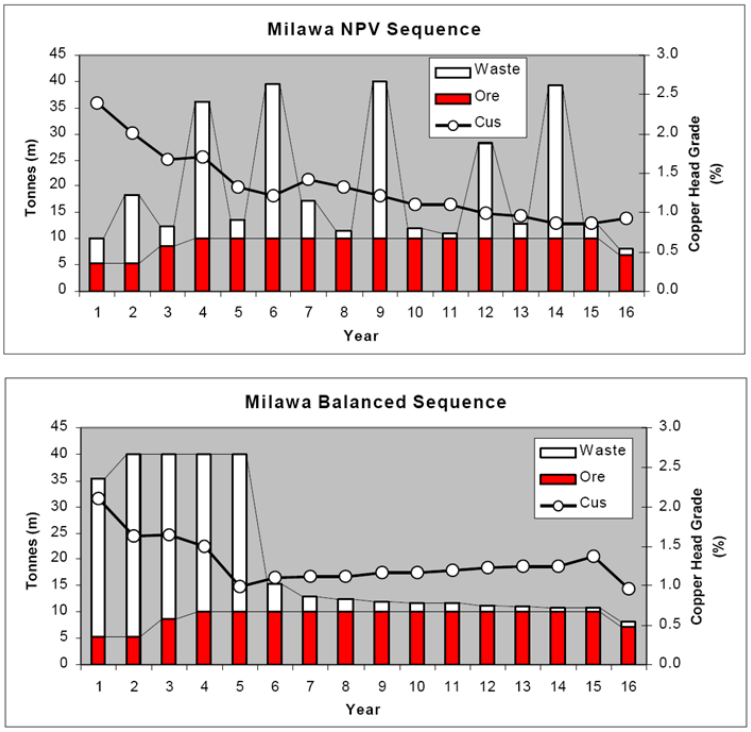


FIGURE 2-16 MILAWA ALGORITHM, NPV MODE (A), BALANCE MODE (B)
WHARTON 2000

in the life of the mine. In this case the chosen intermediate push backs consist of the pits 11, 14, 19, 22, 25, 28 and 30. The resulting sequence by Milawa NPV has a value of $ 354m which is slightly lower than the expected maximum limit. The mill is fully utilised during the life of the project, however, the mining rate suffers from peaks and drops, Figure 2-16A. This may be acceptable considering the good NPV; however, better balanced equipment utilization would lead to lower investment, better employment and less use of mining contractors during the production peaks. (Wharton 2000).

The balancing Milawa algorithm improves the steadiness of the mine production and yields the following graph, Figure 2-16B, and a NPV of $249m which is too poor to be considered. The problem arises from the fact that the algorithm has not been given a sensible target mining rate to use. It has used the maximum mining rate specified. (Wharton 2000).

Plotting cumulative period tonnages of waste and ore for the best, worst and Milawa NPV cases, Figure 2-17A, shows that such a huge stripping is not necessary at the start of the project. In this case a relatively balanced production could be achieved by smoothing the Milawa NPV curve, Figure 2-17B. (Wharton 2000).

The shown schedule (dotted line in Figure 2-17B) has a quite different production in periods 1 to 5 (17.3 m) to those in period 6 and 7 (37.5m) and from period 8 onwards the

requirements are slightly higher than the first period (23.1m). The requirement for pre-stripping drops in the final years of the project as the final pushback is mined, Figure 2-18. This planning provides an NPV of $341m, which has a great improvement on the previous value of $249m and is very close to the Milawa NPV mode value of $ 354m. (Wharton 2000).

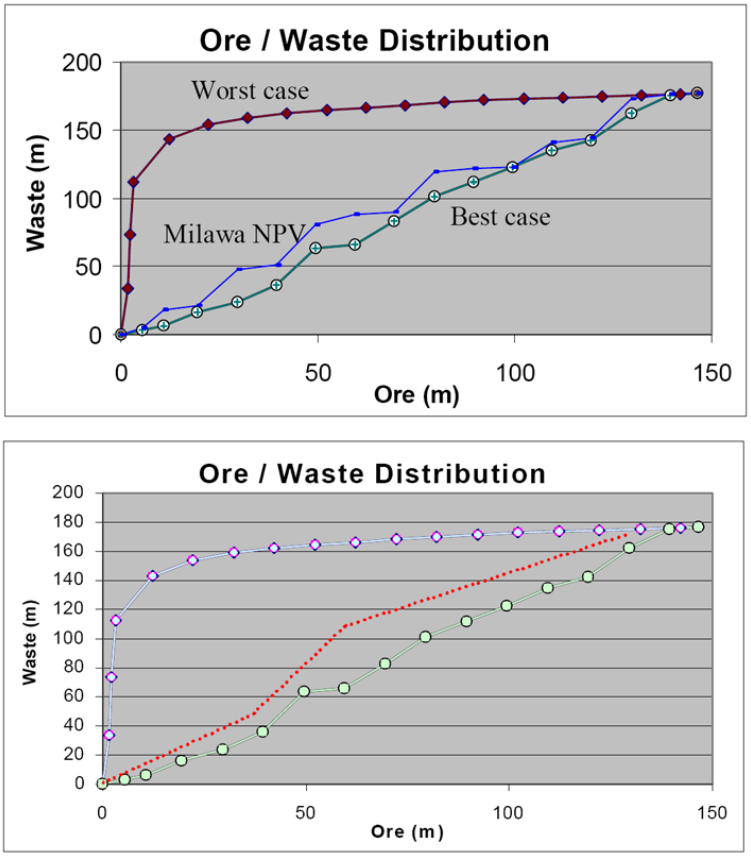


FIGURE 2-17 ORE-WASTE GRAPH OF THE PERIODS
(A)FOR THE BEST, WORST AND MILAWA NPV CASES , (B) SMOOTHED PRODUCTION GRAPH(B), WHARTON 2000

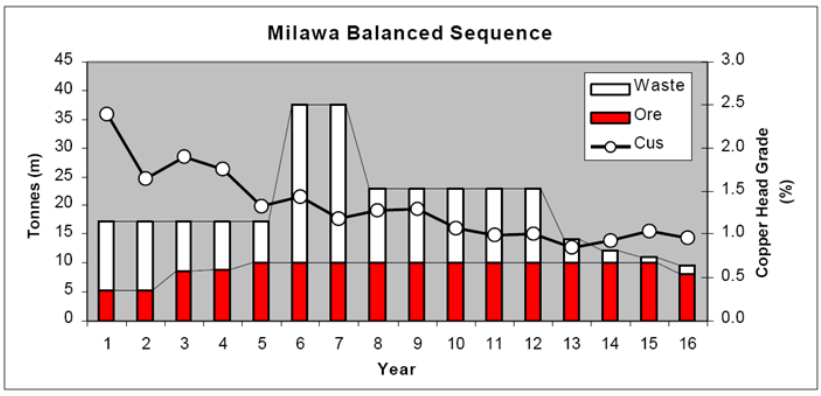


FIGURE 2-18 FINAL SCHEDULE
WHARTON 2000

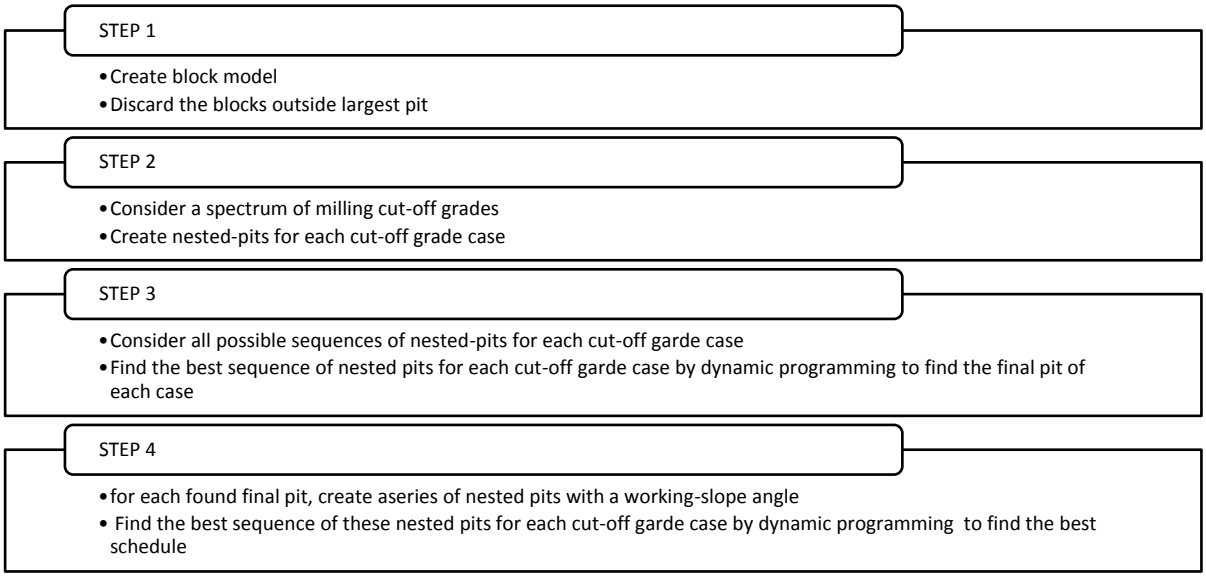


FIGURE 2-19 MAIN STAGES OF SEVIM'S SUGGESTED PROCESS

2.3.2 SEVIM'S SUGGESTED APPROACH

Another heuristic process of open-pit production planning has been developed by H. Sevim at Department of Mining Engineering, Southern Illinois University, U.S.A. With this method the best mining sequence, the ore and waste production rates, the ultimate pit limits and the mine life can be obtained simultaneously. It also allows the best milling cut-off grade to be determined by a systematic search.

The process consists of four main steps. Firstly a block model is created based on the boundaries of the ore body and then a bounding algorithm discards those blocks which fall outside the largest feasible pit. In second step a series of milling cut-off grades are assumed, and for each one a sequence of nested pits is created using a heuristic algorithm. Considering of several cut-off grades enables the procedure to systematically search for the best cut-off grade without making any assumption about the other variables of the planning circle. The heuristic algorithm for generation of nested pits tries to design each pit to contain the highest quantity of metal among all possible pits with the same size. The sequence of these pits, called as 'maximum-metal' pits, is the best mining schedule since they could produce the maximum possible returns comparing to the other pits with the same sizes. Third stage is the phase of forming of all feasible mining push backs based on these pits and then economic evaluation of these push backs by the NPV method in order to find the best push back sequence. Fourth step consists of searching for the best sequences of the working-slope pits which end up to a final-slope pit by repeating the action of step II and III. In other words, after generation of a series of working-slope maximum-metal pits inside a

number of large final-slope pits, the networks of all feasible sequences are formed and then the best sequence (highest NPV) is searched. These steps are repeated for all assumed cut-off grades and the best simultaneous solution for the working-slope pit sequences among all sequences and all cut-off grades is found. Natural outcomes of this evaluation are the answers for the best sequence, cut-off grade, production rates, ultimate pit limits and mine life. (Sevim & Lei, 1996).

Nested-pits creation algorithm

Q. Wang and H. Sevim (1995) proposed a new heuristic algorithm that although does not guarantee to generate the optimum sequence of nested pits in the absolute sense, however, their numerical experiments showed that the obtained results are very near to the optimum pits generated by the parameterization method. The fact is that the algorithm looks somehow superior to the parameterization because of eliminating the gaps in the pit sequence and in generating a sequence that is closer (but not always exact) to the desired increment. Furthermore their required time of the calculation was only a fraction of the time needed by the parameterization method. (Wang & Sevim 1995).

The nested pits creation algorithm is actually developed based on the idea that the finding of a maximum-metal pit of M blocks out of M+N Blocks is equivalent to the removal of N least-metal blocks. A downward cone template, Figure 2-20, is utilized to find the set of least-metal blocks. It explores the block model and finds the cones comprising N blocks or less. The found cones are sorted then in an ascending order of their average metal grade. Finally a pit is constructed by unification of the first K cones (lower metal content) to form the N least-metal blocks set. These blocks are eliminated from the block model for the next step and a new iteration starts on the remained M blocks, Figure 2-21. The process continues until designation of all blocks of the model to the different pit shells. (Wang & Sevim, 1995).

The steps of the algorithm are demonstrated through a two-dimensional model (Table 2-1) containing 15 square blocks (numbered from 1 to 15 and indicated in upper-left corner) with hypothetical random grades (shown by the lower number). Pit slopes in both directions are

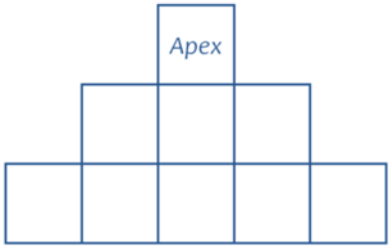


FIGURE 2-20 CONE TEMPLATE
WANG & SEVIM (1995)

assumed to be simply 45°. Now, the goal is to find a sequence of max-metal pits with a size increment of 3 blocks and at most 4 blocks for the smallest pit. Evaluations start from the lower-left block and sequentially move to the upper and right blocks. (Wang & Sevim, 1995)

a. STEPS OF THE ALGORITHM

The steps of the algorithm are as following: (Wang & Sevim, 1995)

By putting the apex of the cone template on the most lower-left block (Block no. 1), three blocks (Blocks 1, 8 and 13 with the average grade of *0.567 g/t*) fall inside the cone. The cone is acceptable (because the number of blocks is not greater than the increment size, 3); therefore, it can be stored in sorting array as the first cone.

First column has no more blocks and algorithm considers the bottom block of the next column (Block 8). Placing the apex of cone shell on Block 8, Blocks 8 and 13 with the average grade of *0.65 g/t* will fall inside. This cone is also acceptable and is inserted in the sorting array as the second place because of its average grade which is greater than that of the previous cone.

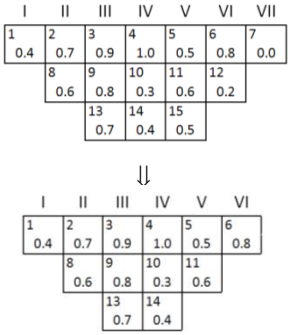
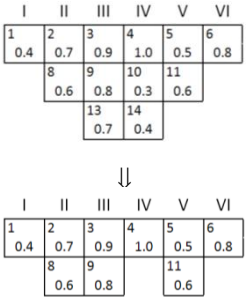
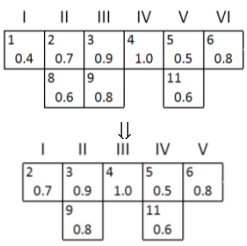
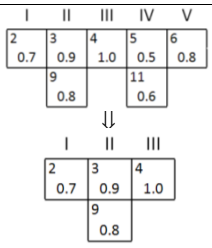
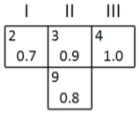
When the next upper block in column II (Block 2) comes to be the apex of the cone shell, it will contain five blocks (2, 8, 9, 13 and 14), which is more than 3. Consequently, the cone is ignored and the algorithm continues by the bottom block of the third column.

Setting the cone apex on the Block number 13 shows that it is the only block of the cone. The average grade of 0.7 *g/t*, which is greater than the average grade of previous cones, locates it in the third place of the array.

Block 9 is the next one that should gain the apex of the cone template. This time the cone consists of three blocks (9, 13 and 14) with an average grade of 0.633 *g/t*. this makes it acceptable and inserts it in between Cone 1 and Cone 2 in the array. In other words, the new cone becomes Cone 2 and the former Cones 2 and 3 turn into Cones 3 and 4 respectively.

The procedure continues until all seven columns are considered and an array of cones with ascending average grade order is obtained. The first cone of the array (lowest average grade) contains three blocks (7, 12 and 15) which is equal to the desired nested pit size. Hence these three blocks are disregarded from the block model, the block model is updated and algorithm moves to the next iteration. During the second iteration remaining 12 blocks of maximum-metal pit (shown below the original in table), will be altered in the same way.

TABLE 2-1 STEPS OF WANG&SEVIM'S SUGGESTED METHOD FOR NESTED PITS GENERATION

		If	Included blocks	Mean Grade	Sorted Cone No	Removed Cones
Iteration 1		APEX=1 ⇒	13,8,1	0.567	6	
		APEX=8 ⇒	13,8	0.650	8	
		APEX=2 ⇒	Block No>3	-	-	
		APEX=13 ⇒	13	0.700	9	
		APEX=9 ⇒	14,13,9	0.633	7	
		APEX=3 ⇒	Block No>3	-	-	
		APEX=14 ⇒	14	0.400	3	
		APEX=10 ⇒	Block No>3	-	-	
		APEX=4 ⇒	Block No>3	-	-	
		APEX=15 ⇒	15	0.500	4	
		APEX=11 ⇒	14,15,11	0.500	5	
		APEX=5 ⇒	Block No>3	-	-	
		APEX=12 ⇒	12,15	0.350	2	
		APEX=6 ⇒	Block No>3	-	-	
		APEX=7 ⇒	15,12,7	0.233	1	*
Iteration 2		APEX=1 ⇒	1,8,13	4	0.567	
		APEX=8 ⇒	8,13	7	0.650	
		APEX=2 ⇒	Block No>3	-	-	
		APEX=13 ⇒	13	-	0.700	
		APEX=9 ⇒	9,13,14	6	0.633	
		APEX=3 ⇒	Block No>3	-	-	
		APEX=14 ⇒	14	1	0.400	*
		APEX=10 ⇒	10,13,14	2	0.467	*
		APEX=4 ⇒	Block No>3	-	-	
		APEX=11 ⇒	11,14	3	0.500	
		APEX=5 ⇒	Block No>3	-	-	
		APEX=6 ⇒	6,11,14	5	0.600	
Iteration 3		APEX=1 ⇒	1,8	1	0.500	*
		APEX=8 ⇒	8	4	0.600	
		APEX=2 ⇒	2,8,9	5	0.700	
		APEX=9 ⇒	9	8	0.800	
		APEX=3 ⇒	3,9,8	7	0.766	
		APEX=4 ⇒	4,9,11	9	0.800	
		APEX=11 ⇒	11	3	0.600	
		APEX=5 ⇒	5,11	2	0.550	
		APEX=6 ⇒	6,11	6	0.700	
Iteration 4		APEX=2 ⇒	2,9	4	0.750	
		APEX=9 ⇒	9	5	0.800	
		APEX=3 ⇒	3,9	7	0.850	
		APEX=4 ⇒	4,11,9	6	0.800	
		APEX=11 ⇒	11	2	0.600	*
		APEX=5 ⇒	5,11	1	0.550	*
		APEX=6 ⇒	6,11	3	0.700	*
Iter. 5		APEX=2 ⇒	2,9	1	0.750	*
		APEX=9 ⇒	9	2	0.800	*
		APEX=3 ⇒	3,9	3	0.850	*
		APEX=4 ⇒	4,9	4	0.900	*

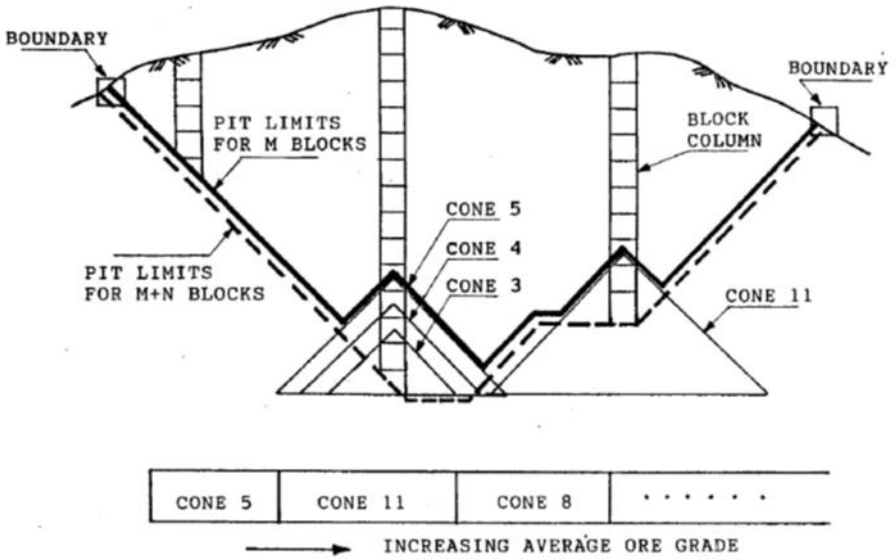


FIGURE 2-21 CONSTRUCTION AND SORTING OF THE CONES
SEVIM &LEI (1998)

By applying the same steps on the updated block model with 12 blocks results in another array of cones. Accordingly, the other least-metal pit shell could be distinguished by merging of the first two cones of this array (Blocks 10, 13 and 14). By disregarding of these blocks, a new maximum-metal pit containing nine blocks is achieved. By repeating the procedure for two more iterations, the number of blocks in remaining maximum-metal pit progressively decreases and finally by reaching to four the algorithm ends. (Wang & Sevim, 1995).

It should be noted that in the third iteration of the algorithm, the first cone has two blocks which is one block less than desired pit size and the combination of the first two cones has four blocks which is again one block more than the proposed size. Since both alternatives have the same difference from the requested increment (three blocks), the first option (Blocks 1 and 8), is selected for elimination because of the lower average grade. Eventually a pit of 7 blocks is remained for the next iteration. (Wang & Sevim, 1995).

Another point that is very important in implementation of the algorithm is that usually only the first few cones of the array need to be stored because the objective of the procedure is only to find a least-metal union containing three blocks. In the extreme case, where each cone contains only one block, a maximum of three cones is needed to make a union of three blocks. So, after building of three first cones, the average grade of the newly built cone is compared with last item of the array. If it is bigger than that of the third cone, it will be rejected. Otherwise, it will be stored in the array according to its average grade and the former last cone will be discarded. Authors proposed that for the increment size of N blocks,

N/2 will be a sufficient number for the size of sorting array. This attitude significantly reduces the amount of required memory and decreases the computing time. The other argument that is very vital in dealing with the large block models is that if the number of blocks for any cone exceeds from the required pit increment size, the size of the cones constructed by putting the apex on upper blocks of this column, which enclose the lower one, will be also bigger than the required increment number and; consequently, they do not need to be evaluated. (Wang & Sevim, 1995).

Five maximum-metal pits with the sizes of 4, 7, 9, 12 and 15 blocks are the final outcomes of this numerical example. Visual analysis of the block model shows that these pits are indeed the maximum-metal pits. In other words, each one of the pits in this series contains more metal than any other pits with the same size. The reason is that by elimination of the lowest average grade cones from a block set, the remaining pit would expect to have the highest probability of being the maximum-metal pit of its size. Authors' tests on artificial random block models indicated that the deviation from real maximum-metal pits, if there is any, is very small. (Wang & Sevim, 1995).

Dynamic programming based search algorithm

The process of searching for the sequence of nested pits corresponding to the highest NPV can be simply formulated as a conventional dynamic programming (DP) problem. In this model, the years are considered as stages and the pits are assumed as the states. Clearly the traditional characteristics of the states in the classic dynamic programing modelling should be satisfied and calculated value of the arcs in one state must be independent of the decisions made about the previous pits. In this network the arcs represent the mining of a pushback and their values are defined as the Net Cash Flows (NCF) created by mining of that pushback. Unfortunately in this formulation the capital investment at an arc depends on the capital that has already been invested in earlier years, and capital investment is directly connected to the production decision. (Sevim & Lei, 1998).

Another improved DP modelling was proposed by Sevim & Lei (1996) by a modification in traditional DP formulation. They defined the states of the network by two variables including the maximum-metal pit and the equipment configuration. Authors used a record containing the number of equipment units and their ages as representative of the equipment configuration. For determination of the required equipment of an arc, all the changes in the status of the equipment such as buying the new equipment, substituting the used equipment, and storing the extra capacity are recorded in the equipment configuration

along its path. Sometimes, several paths of the network reach to a similar equipment configuration which is used by the DP technique as an advantage. (Sevim & Lei, 1998).

A set of 5 nested pits is considered to illustrate the above mentioned process. Having 3, 6 and 10 paths in the first, second and third stages respectively, there will be 19 arcs in total in the network, Figure 2-22. It is assumed that 1, 2, 3, 4 and 5 unit of equipment is required to mine Pit P1, P2, P3, P4 and P5 respectively. Hence, when P1 is mined out, the Path 01 has one unit of one year old equipment and the calculated NPV of $-23.3. The next arc corresponds to a push back from P1 to P3 which needs 2 equipment units. In other words one more unit is required for this push back and there will be a two years old unit plus a one year old unit at the end of the second year with the resulting NPV of $9.6. Only one equipment unit is required along the next path (0134) for removing of the material between P3 to P4. At the end of third year the NPV of $24.4 is reached and the equipment configuration has a three years old unit plus a one year old unit (the unit bought in the second period is has been stored in this year). Following a similar routine for arc 0234, identical equipment configuration is received at the end of pit P4. This means that these two paths direct to P4 with the similar equipment configuration. Comparing the generated NPV of two paths, $24.4 and $27.6 for the paths 0134 and 0234 respectively, reveals that the

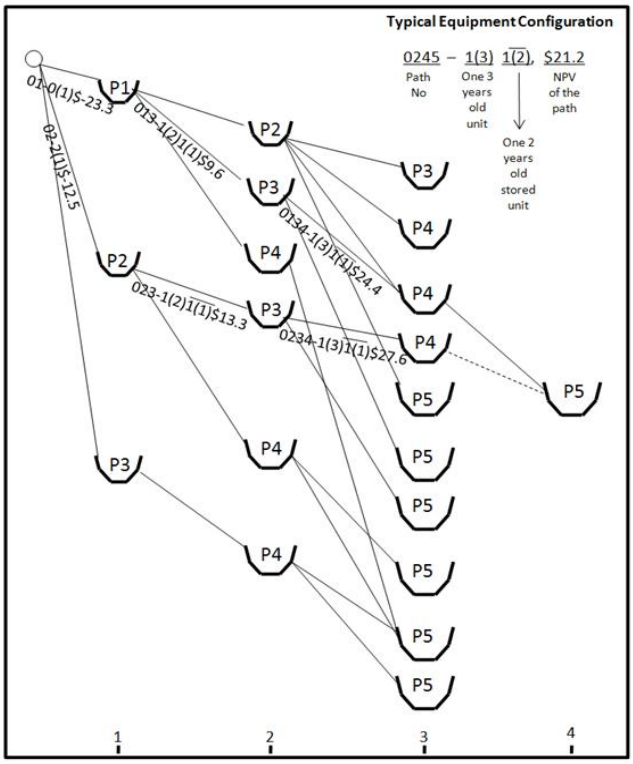


FIGURE 2-22 ILLUSTRATION OF THE DYNAMIC PROGRAMMING APPROACH FOR THE BEST PUSH BACKS SELECTION SEVIM & LEI (1998)

next extensions of the path towards P5 should be made from 0234 instead of 0134. Hence, the decision made about the extension of P4 is independent from the route which has been passed up to this point. The proposed DP algorithm eliminates all the economically inferior Paths (such as 0134) and carries forward the information on superior Paths (like 0234). There are two other similar states in Stage 3 that decreases the number of distinct states at Stage 3 from 10 to 7. This considerably reduces the size of the network. (Sevim & Lei, 1998).

After evaluation of all paths, their NPV is compared by the algorithm to find the highest NPV. The state corresponding to the highest NPV defines the UPL. Then, tracing back from the UPL, the algorithm generates the optimal mine sequence. It should be noted that each state of the network has a related NPV which may decline from that state to the next one if the mining activity along the arc connecting those states does not provide a positive cash flow. (Sevim & Lei 1998).

Considering working-slope angles

A simple hypothetical network has been utilized to demonstrate the process of considering working-slope angles. For this example spectrum of three cut-off grades is considered and for each cut-off grade a series of final-slope maximum-metal pits is determined. Figure 2-23 shows the network of all possible mining sequences that could be formed from these pits. For instance, for the third cut-off grade (X3, drawn in the foreground) five maximum-metal pits are generated from P1 (the innermost and the smallest pit) to P5 (the outermost and the largest pit). It is also assumed that the incremental block numbers between two successive pits to be uniform and equal to the lowest possible production rate throughout the series. (Sevim & Lei 1996).

The network shown in Figure 2-23 demonstrates that if P5 is mined in the first year, the mine will be ended in one year. Instead, by mining pit P1 in the first year, one of the pits P2, P3, P4 or P5 would be considered for the next year. The other connections of the network have the same explanation. Paths of the network could be defined as the sequences of pits which begin at time zero and ends in any of the pits and corresponds to a feasible production schedule. A net present cash flow (NPV) could be calculated for each path by summing up the discounted net cash flows of the push backs along the path to time zero. The best path of each network can be chosen then, as the path corresponding to the highest NPV. Consequently the final best path could be found among the best paths of all networks by comparison of their NPV. (Sevim & Lei 1996).

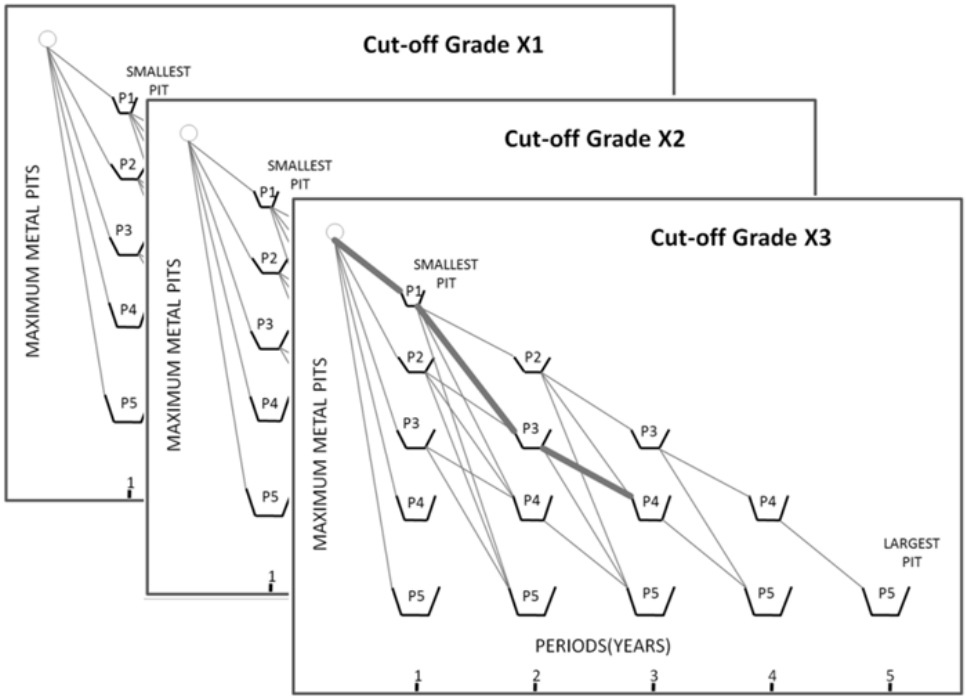


FIGURE 2-23 GENERATED NESTED PITS FOR DIFFERENT CUT-OFF GRADES
SEVIM & LEI (1996)

Suppose that the best path of the foreground network in Figure 2-23 to be as the marked thick line (0-P1-P3-P4). It reveals that the mining ends at P4 and the mining of the pushback from P4 to P5 has a negative cash flow. Therefore P5 is not included in the sequence. However, the best mining sequence needs to be made by a series of working-slope pits nested inside a large final-slope pit which is normally found in previous phase. As shown in Figure 2-24, two series of six working-slope maximum-metal pits are created ending at the pits P3 and P4. Then two new networks have been generated based on these series, and their best sequences have been found. There are two key points which have to be explained here. Firstly, the series of working-slope pits are constructed not only within the best final pit of the phase III but also within some smaller pits, like P3 in this case. The reason is that by applying the working-slope pits, the economics of the project changes too and the found final pit in phase III may no longer be the real UPL. Secondly, dissimilar to phase III, the best paths on the network of the working-slope pits have to be always end up at the last pit (the final-slope pit within which the series of working-slope pits have been generated). (Sevim & Lei 1996).

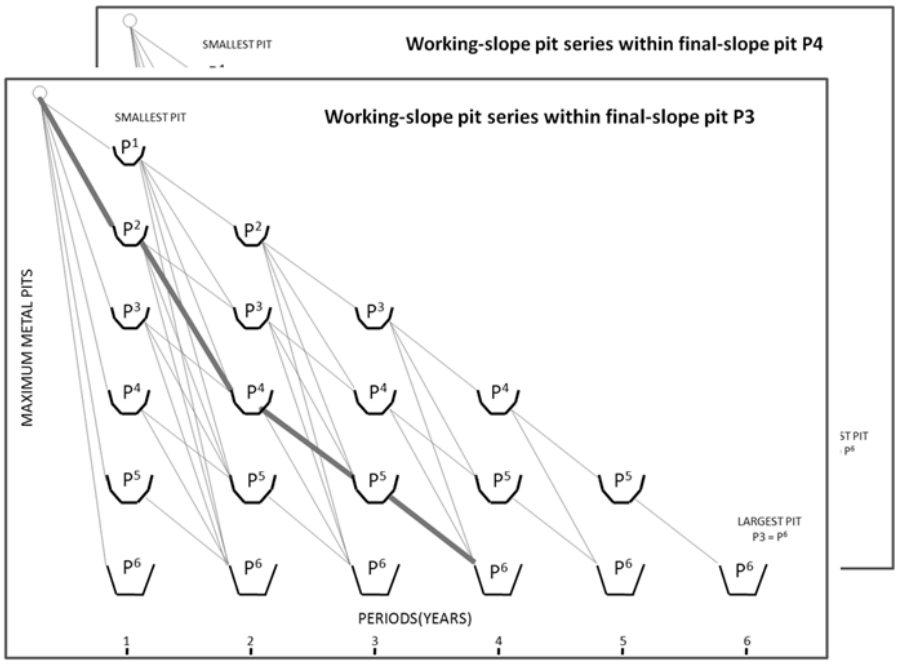


FIGURE 2-24 WORKING-SLOPE PIT SERIES
SEVIM & LEI (1996)

For the best final pit of every cut-off grade in the spectrum a network similar to that presented in Figure 2-24 has to be constructed. Suppose that the best working-slope sequence has been found to be like the thick line marked in the foreground network of Figure 2-24 that (0-P^2-P^4-P^5-P^6). Superscripted numbers represent that the pits have been constructed based on the working-slope angle. The shown path means that the optimum mine-life is four years and the optimum shape of the mine is defined by pit P3. The optimum mining sequence of 0-P^2-P^4-P^5-P^6 reveals that P^2 should be mined in year one. Then P^4 have to be reached by a pushback from P^2 in next year. In year three P^5 needs to be mined; and finally, P3 has to be reached in the fourth year, expressing the ultimate pit. Accordingly, the optimum rates of ore and waste production could be calculated based on the known quantity of ore and waste in each pushback for the selected cut-off grade. This simplified example shows that the proposed process does not determine the mining sequence and the UPL independently. The other advantage is the values of production rates and the cut-off grade are the natural outcomes of the process and no pre-judgement is applied on these values. (Sevim & Lei 1996).

2.4 METAHEURISTIC METHODS

2.4.1 INTRODUCTION

Metaheuristic optimization methods are a higher class of heuristic searching algorithms that are widely used for solving many of NP-hard combinatorial optimization problems.

Combinatorial optimization

A combinatorial optimization problem could be defined as any maximization or minimization problem that is normally easy to state but very difficult to solve. Fundamentally a combinatorial optimization problem involves in finding the values of discrete variables in a way that the optimal value of a certain objective function to be reached subject to some problem constraints. The large Travelling Salesman Problems (TSP), Shortest-Path Problem, Assignment Problem, Sequential Ordering Problem and Knapsack Problem are typical examples of the combinatorial optimization problems. (Dorigo & Stützle, 2004).

Combinatorial optimization problems are basically associated with a set of *problem instances*. The term *problem* denotes the overall query which has to be answered, generally having several parameters (variables) with undetermined values. A problem with particular values for its parameters called as *instance*. In other words an instance of a combinatorial problem Π could be defined as a triple (*S,f,Ω*), where *S* refers to the set of candidate solutions, *f* denotes the objective function having a value *f(s)* for each candidate solution $s \in S$, and *Ω* represents the set of problem constraints. Solutions that satisfy the problem constraints *Ω* define the set of feasible solutions $\tilde{S} \subseteq S$. Finding the globally optimal feasible solution s^* is the aim of any optimization problem. For instance in a minimization problems the target is to find a solution $s^* \in \tilde{S}$ in a way that $f(s^*) \leq f(s)$ for all $s \in \tilde{S}$. (Dorigo & Stützle, 2004)

Metaheuristics

In engineering applications, most of the combinatorial problems are NP-hard. Normally the optimal solution of such a problem cannot be obtained within an acceptable computation time. Therefore approximation methods have to be utilized in order to practically answering of the large instances of the problem. An approximation method returns a near-optimal solution in a comparatively shorter time. Sometimes the approximation methods are colloquially called *heuristics.* They normally act by building new solutions or improving the available solutions by using a set of problem-specific knowledge. (Dorigo & Stützle, 2004).

During last decades the new class of heuristic procedures, known as metaheuristics, has got a strong research attraction. A metaheuristics could be defined as a series of algorithmic

ideas that improve the heuristic methods and make them to be applicable to an extensive range of challenging problems. They are usually inspired by the biology and the nature and their application has expressively improved the capability of the algorithms in finding high quality solutions to very hard combinatorial problems, particularly for large and poorly understood problems. The family of the metaheuristics includes, but not limited to, Genetic Algorithm, Simulated Annealing, Tabu Search, Ant Colony Optimization, and Particle Swarm Optimization. (Dorigo & Stützle, 2004).

2.4.2 Genetic Algorithm (GA)

Genetic algorithm is a search procedure that mimics the operation of genetics and natural selection. It begins the search with a population of random solutions and evolves this population over a series of generations by applying probability techniques and reproduction operators to each member of the population. Reproduction operation consists of two main steps known as crossover and mutation. The crossover operator combines the selected pairs of the solutions to produce new and potentially better solutions whereas the mutation operator provides the potential diversity in the population. The higher the quality of solutions (fitness values), the higher the possibility of their survival in next generations.

The only study on the application of genetic algorithm in optimization of an open-pit mine production planning carried out by Denby and Schofield (1994). Figure 2-25 compares their proposed algorithm with the conventional long-term mine planning process.

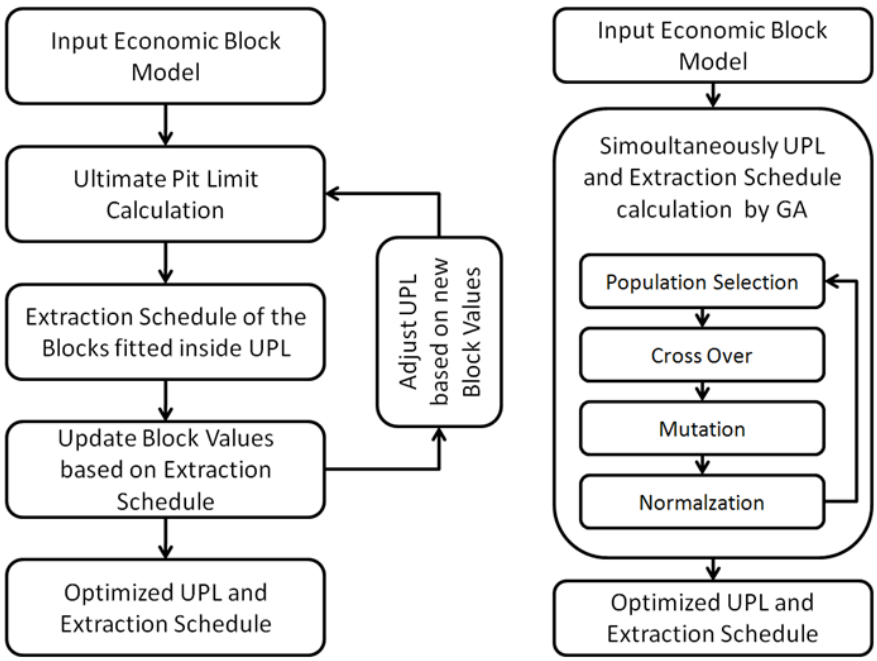


Figure 2-25 Genetic algorithm versus conventional mine planning (Simultaneous calculation of the UPL and production planning)

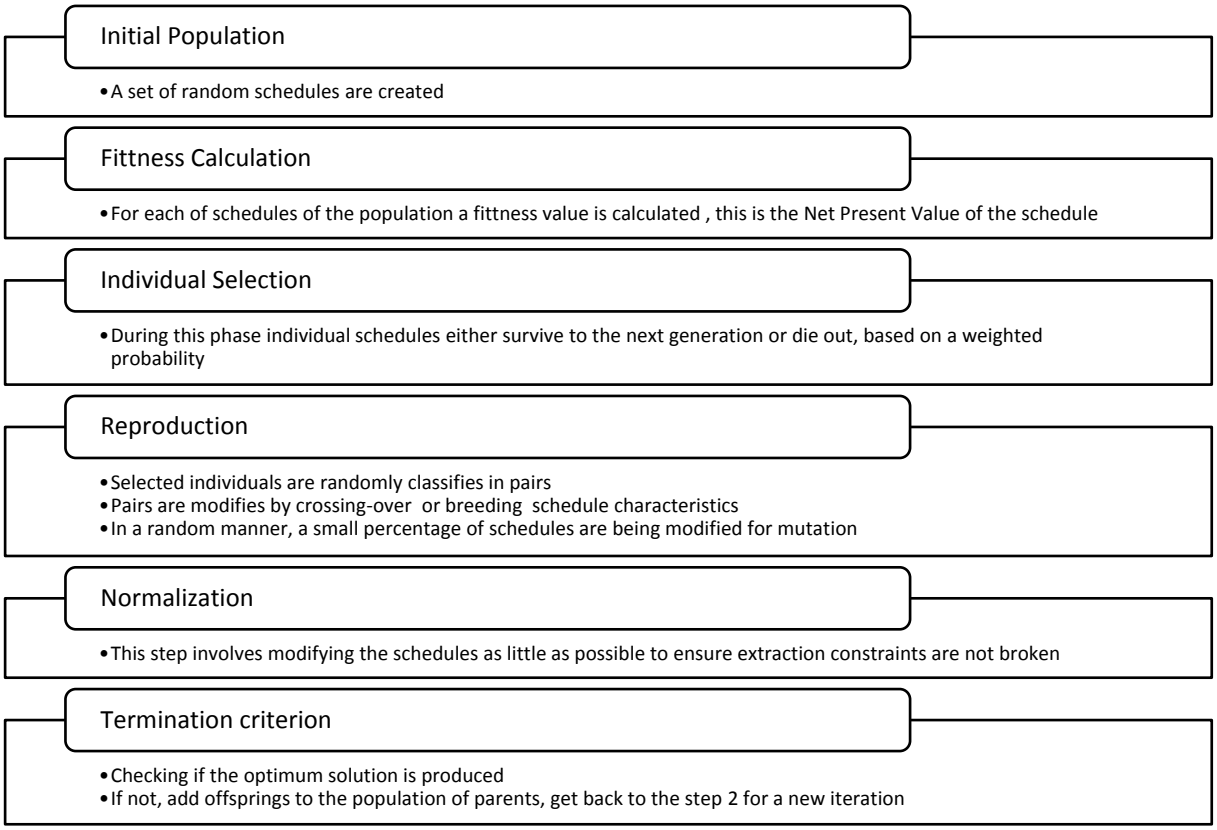


FIGURE 2-26 THE MAIN STEPS OF OPEN PIT PRODUCTION PLANNING BY GENETIC ALGORITHM

The main advantage of this method was its ability to solve UPL and long-term planning problems simultaneously. By choosing proper values for genetic parameters, the method was capable to get good results in an acceptable time. On the other hand, the method was suffering from the fact that the results were not reproducible because of the stochastic nature of the algorithm. The major steps of the procedure have been summarized in Figure 2-26.

Chromosome representation of the pits

Denby and Schofield (1994) developed a special multi-dimensional genetic algorithm representation. This structure was capable to model the three dimensional spatial data and the time elements of the problem. They encoded the long-term mine schedules as a set of matrix cells where the values assigned to the cells denoted to their period of extraction. (Denby and Schofield, 1994).

Initial population

Procedure starts with generation of a random population of feasible schedules. Every schedule could be considered as a combination of a series of nested surfaces of pits. Each

surface is constructed as a 2D array of random elevations ranging in value between the previous larger pit and the ground surface. At the beginning, the generation of these surfaces are unconstrained from size and permitted slopes angles point of view. Therefore they have to be normalized in the next step to ensure that a practical pit surfaces containing a proper volume of material are produced. The population size is one of the controllable parameters in the genetic algorithm systems. In this case it is set to 50 on the basis of experience in other fields. (Denby and Schofield, 1994).

Although Denby and Schofield generated the individual schedules by a random mechanism, however they envisaged for the future systems that the efficiency of the technique may improve by intelligent selection of feasible schedules. (Denby and Schofield, 1994).

Assessment of pit fitness

Defining a proper function for assessing of the quality of solutions is another fundamental stage in success of a genetic algorithm. Fitness value for each of the schedules in the population is calculated as the net present value (NPV) of the schedule. Authors have reported that the introduction of highly complex fitness functions has little or no effect on the overall efficiency of the system. (Denby and Schofield, 1994).

Reproduction of pit population

Reproduction is a critical stage during which a new generation is produced and individual schedules either survive to the next generation or are removed altogether. In this process schedules with high fitness values have more chance for surviving than those with lower fitness values. In this stage, it must ensure that a sufficient genetic diversity is maintained from generation to generation as well as convergence to an optimum result is sufficiently rapid by permitting the good schedules to reproduce faster than the bad schedules. (Denby and Schofield, 1994).

a. Crossover

Approximately 70% of schedules are randomly combined in pairs on a probabilistic basis during crossover. This will result in the crossed pairs having modified schedule characteristics. The operator increases the fitness values of some schedules and improves their chances of survival into future generations, but some others will possibly have lower fitness values, reducing their chances of survival. (Denby and Schofield, 1994).

b. MUTATION

Mutation is the other important operator in reproduction. It randomly acts on a probabilistic basis on approximately 0.1% of the cells in the schedule to maintain genetic diversity and prevents the system from stagnation in an incorrect optimum. It is done by randomly modification of the elevations of the selected cells. (Denby and Schofield, 1994).

c. NORMALIZING OF THE PITS

The action of crossover and mutation operators normally does not care to the shape of the pits in generated mine schedules. This leads the resulting pits to violate constraints and a normalization process to be needed after each action. Normalization procedure involves in modification of the schedule, as little as possible, to ensure that the extraction constraints, such as the number of cells in each scheduling period or the geomechanical sequencing constraints (slope angles) are not exceeded. The normalization process consists of two stages. In first pass surface points are gradually brought closer together till the slope angle and geotechnical constraints to be satisfied. Then, during the size constraint pass, surface points are either raised or lowered within geotechnically accepted limits until the size of pit becomes acceptable. These can sometimes result in significant alterations to the schedules. (Denby and Schofield, 1994).

d. LOCAL OPTIMIZATION

Authors discovered that the addition of a local optimizer greatly improves the optimization performance. They employed a programmed logic rule to swap blocks of high value that are scheduled for the late extraction periods with blocks of low value that are scheduled for early extractions. The approach utilizes pure logic and is not influenced by probability. Once the surface has been checked by local optimizer, it is necessary to normalize the surface again. Authors reported a 35% improvement in the speed of algorithm by use of the local optimizer. (Denby and Schofield, 1994).

Termination condition of the algorithm

The number of generations required to reach the optimum schedule varies depending on the complexity and the scale of the problem. Authors informed that for a problem consisting 200 blocks, approximately 95% of final optimum could be reached after 50 to 120 generations, whereas to reach 98% of the optimum requires as many as 380 generations. However, the way in which the genetic algorithm is formulated has a significant effect on the efficiency of the system. (Denby and Schofield, 1994).

2.4.3 Simulated annealing (SA)

Annealing is the process in which a metallic or glass solid is heated up sufficiently to allow its atoms and molecules to reach in a stress-free state (but not so much that cause melting) and then cooled gradually down to rearrange in a new configuration. In simulated annealing method, the value of the function under optimization is equivalent to the energy of the solid in reality. It begins with a random solution and then perturbs that solution slightly to create another potentially better solution. If the new solution satisfies the constraints and corresponds to a better objective function value than the existing solution, it is accepted without question. However, if the fitness value of new solution is less than that of previous, then a decision on its acceptance has to be made based on the current temperature of the system. This lets the procedure to jump out of potentially sub-optimal solutions. During the iterations of the algorithm, the temperature of the system is gradually lowered until the approximately optimum solution has been found. (Thomas 1996).

Initial temperature and the cooling rate are the critical factors in the success of simulated annealing process. Excessively low starting temperature makes the process to converge too quickly and a sub-optimal solution might be produced. In contrast, extremely high initial temperature would cause spending a long time on poor initial solutions. Similarly, rapidly cooling of the system potentially gets locked around a local-optimum solution and produces a sub-optimal consequence. On the other hand, disproportionately slow cooling rate unnecessarily rises the computation time. (Thomas 1996).

Kumral and Dowd (2005) investigated the solution of the open-pit mine production scheduling problem by using of SA metaheuristic. Figure 2-27 shows the major steps of this process. The idea behind this research was that any sub-optimal schedule can be improved by using SA. Therefore, they constructed a sub-optimal schedule by a conventional production planning algorithm and submitted it to the SA to improve its fitness value. The main supersede of this routine is that it employs a multi-objective function (comprised three minimization components in this study). On the other hand, independent determination of UPL and production schedule would be counted as a disadvantageous for this method. (Kumral & Dowd 2005).

Objective function

The objective function of the problem is expressed as minimization of a multi-objective function comprised three cost components including:

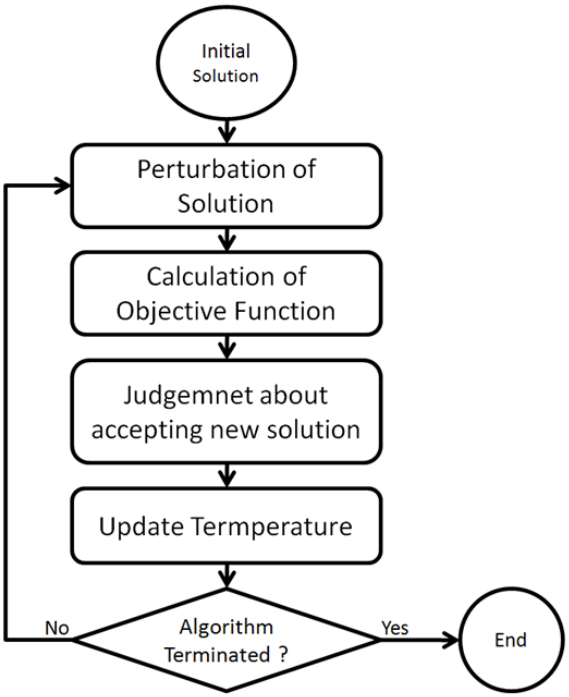


FIGURE 2-27 STEPS OF OPEN PIT SCHEDULE OPTIMIZATION BY SIMULATED ANNEALING

- $Cost_1$: the cost of deviation from required tonnage
- $Cost_2$: the penalty and opportunity cost for each content variable
- $Cost_3$: the cost of the content variability for each content variable (Kumral & Dowd 2005).

The first component arises from the fact that the quantity of extracted ore in each period should be in a specified plant capacity limits. Excessive quantity of ore mining leads to a stock holding cost. Inversely inadequate extraction tonnage would cause contractual costs due to unsatisfied capacity. In order to calculate the first element of the objective function, the total deviation from required tonnage is calculated during the course of the mine life. If the amount of scheduled production mass falls between the specified lower and upper boundary of tonnage tolerance (±5% of the Nominal Mining tonnage in period t, $Mining_t$), no cost will be incurred. Otherwise:

$$cost_1 = \sum_{t=1}^{T} cst1_t = \begin{cases} \lambda_{11} \times gs_t & \text{if} \quad gs_t < 0.95\ Mining_t \\ \lambda_{12} \times gs_t & \text{if} \quad gs_t > 1.05\ Mining_t \\ 0 & \text{if} \quad 0.95\ Mining_t \leq gs_t \leq 1.05\ Mining_t \end{cases}$$

where

$$gs_t = ABS\left[\left(\sum_{n=1}^{N} v_n^t \times x_n^t \times W_n\right) - Mining_t\right]$$

$t = 1, \dots, T$

$$x_n^t = \begin{cases} 1 & \text{if the block } n \text{ is mined in period } t \\ 0 & otherwise \end{cases}$$

$$v_n^t = \begin{cases} 1 & \text{if the block } n \text{ is considered to be mined as ore block in period } t \\ 0 & otherwise \end{cases}$$

where x_n^t is a binary variable which is equal to one if the block n is considered to be mined in period t, v_n^t is a binary variable which is equal to one if the block n is considered to be extracted as an ore block in period t, N is the number of blocks in the block model, $cst1_t$ is the value of first element of objective function in period t, gs_t is the absolute deviation of the scheduled tonnage from the required tonnage in period t, λ_{11} is the cost coefficient for the mining rates lesser than desired capacity and λ_{12} is the cost coefficient for the mining rates further than the planned tonnage, T is the number of planning periods considered for scheduling, W_n is the weight of block n, $Mining_t$ is the nominal required tonnage in period t. (Kumral & Dowd 2005).

Secondly the average content of any considered parameter in the mined ore have to be between the stated limits (±10% of the specified grade). For less/more contents than the nominal content, a penalty/opportunity cost is deserved. Industrial, operational, qualitative or environmental reasons leads to the penalty cost for low quality ore production. Whereas excessively high-quality ore production schema in early phases may cause the content constraints in subsequent years not to be reached and consequently an opportunity cost to happen. (Kumral & Dowd, 2005).

$$cost_2 = \sum_{t=1}^{T}\sum_{p=1}^{P} cst2_t^p = \begin{cases} \lambda_{21}.dev_t^p & \text{if} \quad dev_t^p \le 0.9.Grade_t^p \\ \lambda_{22}.dev_t^p & \text{if} \quad dev_t^p \ge 1.1.Grade_t^p \\ 0 & \text{if} \quad 0.9Grade_t^p \le dev_t^p \le 1.1Grade_t^p \end{cases}$$

where

$$dev_t^p = ABS\left[\frac{\sum_{n=1}^{N} v_n^t \times x_n^t \times g_n \times W_n}{WO_t} - Grade_t^p\right]$$

where dev_t^p is the deviation of the grade of the parameter p in period t from designed value, P is the number of parameters, g_n is the grade of the parameter p in block n , $cst2_t^p$ is the cost of second element of the objective function related to the deviation of the grade of

the parameter p in the period t, $Grade_t^p$ is the designed grade of the parameter p in period t and WO_t is tonnage of the ore in period t in the current schedule, λ_{21} and λ_{22} are the cost coefficients for the low and high grade mining respectively. (Kumral & Dowd, 2005).

Kumral & Dowd (2005) considered the last element of the objective function to minimize the content variance of the variable under consideration. They expressed that by sending the extracted ore to stockpiling or processing operation, the variance could has a direct influence on the mill efficiency or parameters of the stacking and reclaiming. Moreover, the fluctuations of the content may result in disturbing the quality of the process or the finished product. (Kumral & Dowd 2005).

$$cost_3 = \sum_{p=1}^{P} \sum_{t=1}^{T} \lambda_{31} \times vb_m^p$$

where

$$vb_m^p = \frac{\sum_{n=1}^{N} v_n^t \times x_n^t \times (g_n)^2 \times W_n}{WO_t} \text{-} Grade_t^p$$

where vb_m^p is the content variance of the parameter p in period t and λ_{31} is the cost coefficient .

Using a weighting summation, three mentioned components are converted into a single objective function. Magnitude of the weights (priorities of the objective function components) seems to depend on the ore body, sales contract, structure of the ore market and the plant characteristics. (Kumral & Dowd 2005).

$$Minimize\ cost = \sum_{n=1}^{nobj} cost_n \times weight_n \qquad \text{(Kumral \& Dowd, 2005)}$$

where $weight_n$ is the priority coefficient of the objective component n and $nobj$ is the number of objectives components (three in this case). Considering an unbiased combination of the components, the sum of weights needs to be equal to one. (Kumral & Dowd 2005).

$$\sum_{n=1}^{nobj} weight_n = 1 \qquad \text{(Kumral \& Dowd, 2005)}$$

Initial solution

It has been experienced that the computational time to converge towards a good solution could be very long utilizing a random initial solution. Kumral and Dowd (2005) used lagrangian parameterization method proposed by Dagdelen and Johnson (1986) to obtain a favourable initial solution.

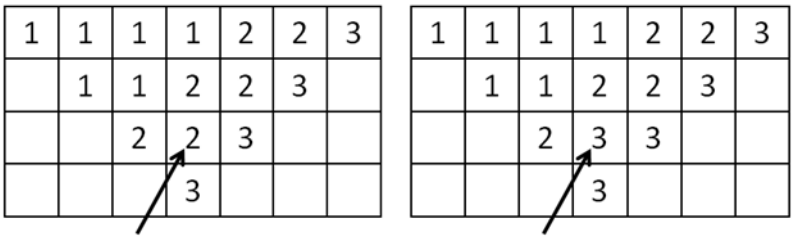


FIGURE 2-28 THE MECHANISM OF BLOCK PERTURBATION
KUMRAL & DOWD 2005

Constraints

Kumral and Dowd (2005) implemented their optimization formulation subject to a series of constraints. The first constraint implies that the number of periods should be equal to or higher than the minimum acceptable number of phases.

Furthermore, authors considered the access constraint to guarantee that the required working space of the loading equipment has been provided and a safe working slope has maintained. For example, with regular cubic blocks and the slope angle of 45° in all directions, a block is only minable in a given period if all of the nine blocks on its upper level have been extracted in previous periods or simultaneously in current period. In other words, in order to extract a mining block, all blocks within an extraction cone of the block have to be removed earlier or at the same time. Allowing the blocks to transfer from a period to another, during the perturbation mechanism, depends on satisfaction of this constraint. The walls of the mining cone are usually designed based on the slope angles in four principal directions. (Kumral & Dowd, 2005).

Perturbation mechanism

Perturbation mechanism accomplishes by shifting a certain number of blocks of a solution to the next or previous scheduling phases to produce a new solution. Transferring blocks are randomly selected and are reassigned to the neighbouring periods. Direction of the alteration to either the next or the previous period is also chosen in a random manner. Switching of the blocks will be rejected if it causes that the ore to waste ratio in any period to be violated. Mechanism has been demonstrated on a vertical two dimensional section in Figure 2-28. (Kumral & Dowd, 2005).

Perturbation is permitted only when it does not exceed the access constraint. To do that, Kumral and Dowd (2005) proposed a special checking method using an upward-downward cone template shown in Figure 2-29. When the blocks supposed to be transferred from a period to the next period, all the blocks inside cone A need to be considered for earlier

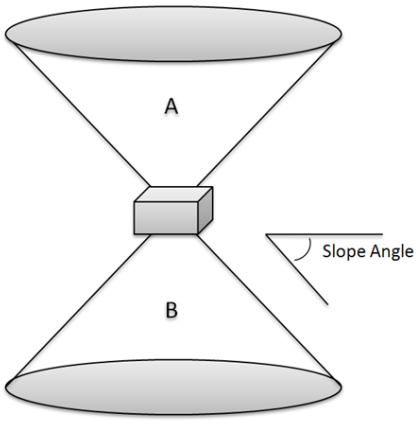


FIGURE 2-29 UPWARD-DOWNWARD CONES TO DETERMINE TRANSFERABILITY OF BLOCKS
KUMRAL & DOWD 2005

periods or the same period and all the blocks inside cone B have to be designed for later periods. On the other hand, when the blocks supposed to be transferred from a period to the previous period, all the blocks inside cone A need to be considered for earlier periods and all the blocks inside cone B have to be designed for later periods or the same period. (Kumral & Dowd, 2005).

Acceptance criterion

The possibility of accepting a perturbed solution at temperature of T, $P_{xy}(T)$, could be expressed as:,

$$P_{xy}(T) = min\left\{1, e^{-\sum_{i=1}^{nobj}(\omega_i d_i)}\right\} \qquad \text{(Kumral \& Dowd, 2005)}$$

where

$$d_i = \left(\frac{(cost_i(y) - cost_i(x))/cost_i(x)}{T}\right), \text{ and } \sum_{i=1}^{nobj} \omega_i = 1 \qquad \text{(Kumral \& Dowd, 2005)}$$

where x is the current solution, y is the new solution and ω_i is the priority weight of the i^{th} elements of the objective function $(Cost_i)$. The relative deviations d_i are determined separately for each component of the objective function, $cost_i, i = 1, \dots, nobj$. This would let the algorithm not being dependent on only one objective.

Authors recommended to use the $(1 - \Delta f/T)$ approximation instead of the exponentiation $e^{-\Delta f/T}$, because of the shorter calculation time. They also found that using a discrete approximation represented by a look-up table could be even faster. Considering the acceptance probabilities of 0.995 and 0.0067 as the boundary limits, the value of Δf, will be equal to $T/200$ and $5T$ respectively. In other words:

$T/200 \leq \Delta f \leq 5T$ or $1 \leq 200\,\Delta f/T \leq 1000$ (Kumral & Dowd, 2005)

Therefore the value of $P_{xy}(T)$ for different values of an integer Index Number (from 1 to 1000) could be previously computed and saved in a table. Then by rounding the value of $200\Delta f/T$ to the nearest integer and using the pre-computed table, the acceptance possibility could be easily obtained. (Kumral & Dowd, 2005).

Cooling schedule and termination rule

Kumral and Dowd (2005) used a fluctuating cooling process. They proposed to cool the system after any accepted solution and heat it up after any rejected solution. They used $T \leftarrow T/(1+\beta T)$ and $T \leftarrow T/(1-\alpha T)$ functions for cooling and heating the system respectively. Temperature of the system will be balanced by having $\beta/\alpha = k$ heating iteration against each cooling one. In other words, system heats up when the proportion of the rejected to the accepted moves is higher than k, which will subsequently decrease the number of rejects against acceptances. In contrast, when the proportion of the rejected to the accepted moves is less than k, system will be cooled and this will increase the number of rejects against acceptances. Hence, the schedule tends to converge theoretically to a point that the ratio of rejected to accepted solutions to be around k. Authors used this fact as the termination criterion too. They found that using $\beta = 0.001$ and $\alpha = 0.000012$ the value of objective function decreases slowly at the beginning of the procedure and reaches a stable state at the end. (Kumral & Dowd, 2005).

Initial temperature

Another substantial parameter in the performance of the SA is the initial temperature. Too high initial temperature makes the procedure to scatter for a long time on poor solutions. Excessively low initial temperatures could also lead the algorithm to be trapped in a local optima. The conducted experiments by the authors revealed that by taking the ratio of the number of the rejected moves to the accepted moves as 150 yields good solutions in shorter running time in comparison with the other initial temperatures. Initial temperature was selected as 1.67 to produce this ratio. (Kumral & Dowd, 2005).

3 ANT COLONY OPTIMIZATION (ACO)

Ants first evolved around 120 million years ago, take form in over 11,400 different species and are considered one of the most successful insects due to their highly organised colonies, sometimes consisting of millions of ants. The field of “ant algorithms” derived from the observation of the behaviour of real ants, in order to inspire the basic idea of designing of the innovative algorithms for answering the optimization problems. One of the most effective models of ant algorithms known as Ant Colony Optimization (ACO) has been magnificently applied on several combinatorial optimization problems such as travelling salesman, sequential ordering, general assignment, multiple knapsack and network routing problems to produce high quality approximate solutions. It has been inspired by the foraging behaviour of the ants. (Dorigo & Stützle, 2004).

In the real world, ants (initially) wander randomly, and return to their colony after finding food while laying down pheromone trails. A *pheromone* is any chemical or set of chemicals produced by living organisms to transmit a message to other members of the species. Ants tend likely not to travel at random, but to instead follow the pheromone trails and reinforcing it. Over time, however, the pheromone trails evaporate and lose their attraction strength. The more time that takes for an ant to travel along a path (longer paths), the higher evaporation of the pheromones. In contrast, shorter paths get more attracted and their pheromone density increases up to be balanced with the evaporation rate. In fact, the pheromone evaporation avoids the system to convergence to a local optimal solution. In other words, the first paths chosen by the ants would be followed by the other ones, if there were no evaporation. By finding a new path with shorter distance from colony to food source, other ants are also promoted to follow that path, and eventually all the ants follow a single path. (Wikipedia).

Probably, the best way of illustrating how the ACO metaheuristic functions is by explaining how it has been utilized to solve the Travelling Salesman Problems (TSP). TSP is a comprehensively investigated problem in the literature and for a long time has appealed a significant amount of study efforts. The main reasons for the selection of TSP as the base problem to describe the operational procedure of ACO are: (Dorigo & Stützle, 2004)

- TSP is a *NP*-hard optimization model that frequently arises in engineering applications;
- TSP is a typical problem to which ACO algorithm has been originally applied;
- TSP is an comprehensible problem, therefore, the behaviour of the algorithm is not complicated by unnecessary details;

- TSP has been known as a typical examination bed for new solution approaches and a good performance on the TSP is normally considered as evidence of their practicality;
- The history of ACO indicates that the most efficient ACO algorithms for TSP were also effective for a wide range of other problems. (Dorigo & Stützle, 2004).

3.1 TSP DEFINITION

TSP is the problem of a salesman who wants to start from his hometown and travel to a certain number of customer cities (visiting each city exactly once) and finally get back home through the shortest path. Mathematically it could be denoted as a complete weighted graph $\mathrm{G} = (\mathrm{N}, \mathrm{A})$ where N is the set of nodes (cities), and A is the set of arcs (roads). Each arc $(i, j) \in A$ has a value (length) d_{ij}, which reveals the distance from city i to city j, and $i, j \in N$. The goal of the problem is to discover the shortest Hamiltonian circuit of the graph. Hamiltonian circuit is defined as a closed tour that visits each of the nodes exactly once. $d_{ij} = d_{ji}$ for every pair of the nodes in symmetric TSPs while in asymmetric TSPs at least for one pair of the cities $d_{ij} \neq d_{ji}$. (Dorigo & Stützle, 2004).

The problem has shown an NP-hard behaviour even by removing the condition of one time visiting of each city. Traditionally the following approaches have been applied to solve TSP:

- Analytical algorithms: They find the exact solutions and work fast only for relatively small problem sizes. Examples are linear programming, brute force search, dynamic programming and branch-and-bound.
- Heuristic algorithms: They deliver either seemingly or probably good solutions, but they could not be proved to be optimal. Numerous approximation algorithms like nearest neighbour and greedy algorithm are included in this class.
- Metaheuristic algorithms: They could yield better solutions and high approximation in reasonable time for large problems. Examples are genetic algorithm and ant colony optimization. (Wikipedia).

3.2 BASIC ELEMENTS IN SOLUTION OF TSP BY ACO

3.2.1 CONSTRUCTION GRAPH

The problem is represented as a mathematical graph structure called construction graph. It is identical to the problem arrangement, i.e. a set of nodes Ccorrespond to the cities and the set of arcs correspond to the roads. A weight is assigned to each arc which represents the distance d_{ij} between cities i and j. The set of all possible Hamiltonian walks are the states of the problem. (Dorigo & Stützle, 2004).

Generally it favours to work on complete graphs in which there is at least one route between any two nodes. For incomplete graphs, it is possible to add new arcs to convert it to complete graph. Assigning large weights to the additional arcs guarantees that they will not be used in the optimal solution and final answer will not be affected.

3.2.2 CONSTRAINTS

All cities must be visited in the TSP and the visit has to be at most once. This is the single constraint of TSP. In order to satisfy this constraint, at each step of the algorithm, ants are only allowed to choose their next destination (the feasible neighbourhood set N_i^k of an ant k located in city) among those cities that have not been visited yet. (Dorigo & Stützle, 2004).

3.2.3 PHEROMONE TRAILS AND HEURISTIC INFORMATION

Each arc i, j of the graph has been assigned a pheromone τ_{ij} value representing the desirability of the city j to be visited after city i. A heuristic information value η_{ij} is also allocated to each arc which is usually defined as the inverse of the distance from city i to j, i.e. $\eta_{ij} = 1/d_{ij}$. (Dorigo & Stützle, 2004).

3.3 VARIANTS OF ACO ALGORITHM FOR TSP

The early types of ACO consisted of updating the pheromone trails immediately after moving from a city to another; but later studies showed that it would be more effective if the pheromone values to be updated after construction of all tours. Generally the quantity of deposited pheromone by each ant is a function of its tour length. Nowadays the preliminary variants have been abandoned due to their lower performance. (Dorigo & Stützle, 2004).

3.3.1 ANT SYSTEM (AS)

Ant system (AS) is the simplest version of ACO. It was initially proposed by Dorigo et al. (1991); and developed later by Dorigo, Maniezzo & Colorni (1991); Dorigo (1992). The algorithm starts by using *initial pheromone* values on graph edges which is usually determined heuristically. It is followed then by two main steps of the ACO algorithm known as the *solution constructions* and the *pheromone update*. (Dorigo & Stützle, 2004).

Pheromone Initialization

The value of initial pheromone is one of the key controllable parameters in AS. By too low initial pheromone values the exploration is biased by the first tours and generally results in trapping inside inferior zones of the search domain. Oppositely, the extremely high pheromone values can lose many of the primary iterations until the evaporation reduces the trails adequately so that the deposited pheromone of the ants to be able to affect the

search. Dorigo & Stützle (2004) suggested setting the initial pheromone trails to a value slightly higher than the expected pheromone deposition by the ants in one iteration. They proposed to use a rough estimate of this value as m/C^{nn}, where m represents the number of ants, and C^{nn} denotes the length of any initial tour generated by any tour construction procedure such as nearest-neighbourhood. (Dorigo & Stützle, 2004).

Solution construction

In the course of each ACO iteration, a series of solutions (tours) are constructed by m artificial ants. They concurrently build a series of tours by starting from a randomly chosen city and step by step travelling through all other cities. At each step, ant k utilizes a probabilistic choice rule, named *random proportional rule*, to decide about the next travelling city. The probability of choosing $j$as the next city by ant k, when it is presently located at city i, is equal to:

$$P_{ij}^{k} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k}[\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}} \quad \text{if} \quad j \in N_i^k$$ (Dorigo & Stützle, 2004)

Where $\eta_{ij} = 1/d_{ij}$ are the heuristic information of the system, α and β are the relative prominence of the pheromone values and the heuristic information, and N_i^k is the set of feasible neighbourhood cities of ant k when being at city i (the set of not visited cities).

The random proportional rule implies that the higher the value of the pheromone trail and heuristic information of a certain arc, the higher the chance of choosing that arc. The relative values of the parameters α and β defines the performance of the algorithm from pure greedy search ($\alpha = 0$) to completely pheromone based action ($\beta = 0$) either which lead to rather poor results or rapid stagnation. Relevant values of involved parameters for different variants of ACO algorithm have been indicated in Table 3-1. (Dorigo & Stützle, 2004).

In practise, each ant has to preserve a memory to save the list of cities that already visited, in the order that they were visited. It is quite convenient in defining the list of feasible neighbourhoods during tour construction as well as in calculation of the tour length and retracing of the path while pheromone deposition. (Dorigo & Stützle, 2004).

Solution construction could be implemented in parallel or sequentially. The parallel method consists of letting all ants to make a move to next city at each construction step, while the sequential approach allows an ant to build a complete tour before starting the next ant. However, both alternatives are equivalent in AS. (Dorigo & Stützle, 2004)

TABLE 3-1 PARAMETER SETTINGS FOR ACO ALGORITHMS WITHOUT LOCAL SEARCH
DORIGO &STÜTZLE 2004

ACO algorithm	α	β	ρ	m	τ_o
AS	1	2 to 5	0.5	n	m/C^{nn}
EAS	1	2 to 5	0.5	n	$(e+m)/ \rho C^{nn}$
AS_{rank}	1	2 to 5	0.1	n	$0.5w(w-1)/\rho C^{nn}$
MMAS	1	2 to 5	0.02	n	$1/ \rho C^{nn}$
ACS	1	2 to 5	0.1	10	$1/nC^{nn}$

n: the number of cities in a TSP instance.
EAS: parameter e should set to $e = n$.
AS_{rank}: number ofranked ants is $w = 6$.
MMAS: maximum pheromone trail limit is $\tau_{max} = 1/\rho C^{bs}$ and $\tau_{min} = \tau_{max}(1 - \sqrt[n]{0.05})/((avg - 1).\sqrt[n]{0.05})$, where avg is the average number of different choices available to an ant at each step while constructing a solution
TSP instances with up to 200 cities, should use always the iteration best pheromone update rule,
In larger instances both the iteration-best and the best-so-far pheromone update rules should be used alternately.
ACS: In local pheromone trail update rule: $\xi = 0.1$.
In pseudorandom proportional action choice rule: $q_0 = 0.9$.

Update of Pheromone Trails

The pheromone trails need to be updated after construction of all tours. In general the update process has two major stages called evaporation and deposition. Evaporation lowers the pheromone value of all arcs by a constant factor. Then deposition adds extra pheromone on the arcs that the ants have crossed in their tours. Pheromone evaporation and deposition could be mathematically expressed as:

$$\tau_{ij} \leftarrow (1 - \rho)\tau_{ij} \qquad \forall (i,j) \in L \qquad \text{(Dorigo \& Stützle, 2004)}$$

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} , \quad \forall (i,j) \in L \qquad \text{(Dorigo \& Stützle, 2004)}$$

$$\Delta\tau_{ij}^{k} = \begin{cases} 1/C^{k} & \text{if} \quad (i,j) \in T^{k} \\ 0 & \text{otherwise} \end{cases} \qquad \text{(Dorigo \& Stützle, 2004)}$$

Where ρ is the evaporation rate $(0 < \rho \leq 1)$, $\Delta\tau_{ij}^{k}$ is the amount of pheromone that ant k deposits on the arcs that it has visited and C^{k} is the length of the tour T^{k} built by k-th ant and is computed as the sum of the lengths of the arcs belong to T^{k}. (Dorigo & Stützle, 2004).

In fact the evaporation avoids the algorithm to accumulate unlimited magnitude of the pheromone on arcs. I other words, it enables ACO to disremember the poor solutions that formerly found. Indeed, the pheromone value of an arc will be decreases exponentially in a few number of iterations if it is not passed by the ants often. (Dorigo & Stützle, 2004).

It should be noted that during pheromone deposition, the amount of deposited pheromone by any ant is directly proportional to the quality of its tour. Consequently, arcs that are passed by numerous high quality (short tour) ants will collect further pheromone. This increases their attraction to be taken by the next ants in upcoming iterations of the algorithm. (Dorigo & Stützle, 2004).

3.3.2 Elitist Ant System (EAS)

The elitist strategy of Ant System was one of the primary improvements on the initial AS presented by Dorigo (1992) and Dorigo et al., (1991) and (1996). The main enhancement of EAS comes from a special attention which has been given to the best tour that found since the start of the algorithm (it will be indicated as T^{bs}, the best-so-far tour, in the following). In other words, EAS utilizes a supplementary ant to deposit further pheromone to the arcs of the best-so-far tour. This is a typical example of a daemon action in ACO. Pheromone evaporation in EAS is applied similar to that was in AS. (Dorigo & Stützle, 2004).

To implement the extra strengthening of tour T^{bs}, an amount of e/C^{bs} is deposited to its arcs in each iteration. Where C^{bs} is the length of the T^{bs} tour and e is a coefficient that expresses the relative significance given to the best-so-far tour T^{bs}. Accordingly, the pheromone deposit equation can be rewritten as:

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} + e\Delta\tau_{ij}^{bs} \ , \qquad \text{(Dorigo \& Stützle, 2004)}$$

$$\Delta\tau_{ij}^{bs} = \begin{cases} 1/C^{bs} & \text{if} \quad (i,j) \in T^{bs} \\ 0 & \text{otherwise} \end{cases} \qquad \text{(Dorigo \& Stützle, 2004)}$$

Experiments of Dorigo (1992) revealed that the better tours in a lower number of iterations could be found using the elitist strategy with an appropriate value of e. (Dorigo & Stützle, 2004).

3.3.3 Rank-Based Ant System (AS_{RANK})

Rank Based Ant System (AS_{rank}) was the other significant enhancement over the AS, proposed by Bullnheimer et al. (1999). In AS_{rank} the ants are sorted based on their tour length and a rank r is assigned to each ant accordingly. Then, only the $(w-1)$ best ranked ants and the best-so-far ant are allowed to deposit pheromone. Deposited pheromone of

each ant is also proportional to its rank. Hence, the best-so-far ant always deposits the largest amount of pheromone and gives the strongest feedback, with a weight of w. Thus, the pheromone update rule of AS_{rank} could be written as:

$\tau_{ij} \leftarrow \tau_{ij} + \sum_{r=1}^{w-1}(w-r)\Delta\tau_{ij}^{r} + w\Delta\tau_{ij}^{bs}$, (Dorigo & Stützle, 2004)

Where $\Delta\tau_{ij}^{r} = 1/C^{r}$ and $\Delta\tau_{ij}^{bs} = 1/C^{bs}$.

Bullnheimer et al. (1999) disclosed that AS_{rank} performs slightly better than EAS and considerably better than AS. (Dorigo & Stützle, 2004).

3.3.4 *MAX–MIN* Ant System (*MMAS*)

MAX–MIN Ant System (MMAS) is one of the most efficient and detailed studied ACO algorithms, Stützle & Hoos (1997) and (2000); Stützle (1999). MMAS presents four main modifications to AS. Firstly, it only allows the iteration-best ant or the best-so-far ant to deposit pheromone. This would usually lead to a rapid stagnation situation due to the extreme growth in pheromone amount of initially constructed good but suboptimal tour. MMAS applies three other adaptations to prevent stagnation. Second modification of MMAS involves in limiting the pheromone values to the range of $[\tau_{min}, \tau_{max}]$. Thirdly, it initializes the pheromone trails to the upper limit and uses a quite low evaporation rate. As a final adjustment, algorithm reinitializes the pheromone trails each time that the system seems to approach stagnation. The process of evaporation is as same as in AS. The deposition of new pheromone can be written as below:

$\tau_{ij} \leftarrow \tau_{ij} + \Delta\tau_{ij}^{best}$, $\Delta\tau_{ij}^{best} = 1/C^{best}$ (Dorigo & Stützle, 2004)

$\Delta\tau_{ij}^{best} = 1/C^{bs}$, or $\Delta\tau_{ij}^{best} = 1/C^{ib}$ (Dorigo & Stützle, 2004)

where C^{ib} is the length of the iteration-best tour.

MMAS alternatively utilizes the iteration-best and the best-so-far tours to update pheromone trails. In fact, using the best-so-far ant for pheromone update makes the search to concentrate quickly around T^{bs}, while application of the iteration-best ant is less focused. Stützle (1999) showed that the frequency of using best-so-far instead of iteration-best could be determined according to the size of the TSP instance. He proposed to use only iteration-best pheromone updates for small TSP instances. But instead, for large TSPs with hundreds of cities, the best performance obtains by progressively increasing of the frequency of using the best-so-far tour. (Dorigo & Stützle, 2004).

The Limits of Pheromone Trails

MMAS is known as one of the most explorative ACO algorithms. This power comes from the effect of imposed lower and upper boundaries on pheromone values (τ_{min} and τ_{max}). The upper limit prevents the primarily found arcs to become predominant and lead to stagnation. The lower limit also protects the poorly visited arcs to get out of calculation. In fact, the imposed pheromone boundaries limit the probability P_{ij} of selecting a path ij to the interval of $[P_{min}, P_{max}]$, with $0 < P_{min} \leq P_{ij} \leq P_{max} \leq 1$. (Dorigo & Stützle, 2004).

Assuming the upper pheromone limit in long run equal to $1/\rho C^*$ (C^* is the length of the optimal tour) the maximum permitted value of pheromone τ_{max} could be set to its preliminary estimation as $1/\rho C^{bs}$. Obviously, the value of τ_{max} should be updated after finding each new best-so-far tour. Evaluations of Stützle (1999) revealed that the lower pheromone limit plays even more important role in preventing stagnation. He suggested that the lower pheromone limit to be set as a fraction of upper limit ($\tau_{min} = \tau_{max}/a$). (Dorigo & Stützle, 2004).

Pheromone Trail Initialization and Re-initialization

In MMAS the pheromone trails are initially set to τ_{max} and the pheromone evaporation rate is set to a quite low level. This action generates a gradual growth in the relative difference of the pheromone trails of the arcs which makes the primary phases of the MMAS to be very explorative. (Dorigo & Stützle, 2004).

As another development, MMAS occasionally re-initializes the pheromone trails to raise the exploration of the less attractive arcs. This is usually triggered when the algorithm approaches stagnation. The stagnation would be distinguished when no improved tour is found after a given number of iterations. (Dorigo & Stützle, 2004).

3.3.5 Ant Colony System (ACS)

Ant Colony System (ACS) is another innovative ACO algorithm proposed by Dorigo & Gambardella (1997a,b) by application of three major alterations in AS. Firstly, it uses an aggressive action choice rule called *pseudorandom proportional rule* to more strongly use of the system experience. Furthermore, the pheromone update occurs only on the arcs of the best-so-far tour. Finally, besides the general pheromone update, ACS removes some pheromone from the arcs which have been passed through during tour constructions. This is applied immediately after a move and improves the exploration of the other paths. (Dorigo & Stützle, 2004).

Tour Construction

The routine that ACS follows to move from a city i to another city j entirely differs from that of previous variants of ACO. It is based on a so called *pseudorandom proportional rule*. The rule could be expressed as following:

$$j = \begin{cases} \arg max_{l \in N_i^k} \{\tau_{il}[\eta_{il}]^\beta\}, & \text{if} \quad q \leq q_0 \\ J, & \text{otherwise} \end{cases}$$ (Dorigo & Stützle, 2004)

where q is a random variable with normal distribution, q_0 is a parameter valued between zero and one and J is a random variable selected similar to the main AS equation ($\alpha = 1$). In plain words, with probability of q_0 the ant will choose the city which has the highest learned knowledge i.e. $\tau_{il}[\eta_{il}]^\beta$; whereas with probability of $(1 - q_0)$ it will utilize a probabilistic approach similar to AS. The explorative behaviour of the algorithm could be controlled by tuning of the parameter q_0. (Dorigo & Stützle, 2004).

Global Pheromone Trail Update

Similar to MMAS, the only ant that is permitted to deposit pheromone here is the best-so-far ant. But the major difference in ACS compared to the previous versions is that the evaporation of pheromone is also applied to the arcs of T^{bs} only. Thus, the update in ACS is implemented as the following equation:

$\tau_{ij} \leftarrow (1 - \rho)\tau_{ij} + \rho\Delta\tau_{ij}^{bs}$, $\forall (i,j) \in T^{bs}$ and $\Delta\tau_{ij}^{bs} = 1/C^{bs}$ (Dorigo & Stützle, 2004)

Limiting the process of evaporation to only the arcs of the best-so-far ant reduces the computational complexity of the problem. In other words, the deposition and evaporation could be combined in a single step by using a discounted pheromone magnitude which runs into a weighted averaging between the old and deposited pheromone values.

Experiments indicated that in small TSP instances the iteration-best tour could also be considered for the pheromone update. But the best-so-far ant generates better solutions for large instances with more than 100 cities. (Dorigo & Stützle, 2004).

Local Pheromone Trail Update

Another big difference of ACS with the former ACO algorithms is the considering of a local pheromone update rule, additional to the global pheromone trail updating. The ants apply this local pheromone update rule during their tour construction and immediately after having crossed an arc (i,j):

$\tau_{ij} \leftarrow (1 - \xi)\tau_{ij} + \tau_0$ (Dorigo & Stützle, 2004)

Where ξ $(0 < \xi < 1)$ and τ_0 are the parameters of the local pheromone update. Investigates showed that the value of 0.1 would be a proper estimate for the value of ξ. It is also found that the value of τ_0 can be set as equal to the initial pheromone trail values. Consequently, a good initial estimate for τ_0 could be $1/nC^{nn}$, where n is the number of cities and C^{nn} is the length of a possible tour constructed by any heuristic such as nearest-neighbour method. (Dorigo & Stützle, 2004).

In the earlier discussed AS variants it did not matter if the tour construction to be done in parallel or sequential way. But it is important to note that, because of the local pheromone update rule in ACS, this makes a big difference. The idea behind the local pheromone update is to makes the arcs which have been passed by any ant, less desirable for the next ants. This could prevent the ants not to converge to the generation of a common path; i.e., not to show a stagnation behaviour. In order to generate such an improved exploration power, the pheromone trail τ_{ij} of arc (i, j) is reduced to some percentage, immediately after passing over of an ant. In fact the local pheromone update provides an escalation in the exploration of arcs that have not been visited yet. Consequently, to benefit from the information of each ant by the others, during each iteration, all the ants have to move in parallel. (Dorigo & Stützle, 2004).

Additional Remarks

Ant Colony System is based on a former algorithm proposed by Dorigo & Gambardella (1996) known as Ant-Q. The main practical difference between ACS and Ant-Q is in the formula of the calculation of the parameter τ_0, which in ACS is set to $1/nC^{nn}$ but in Ant-Q is equal to $\tau_0 = \gamma max_{j \in N_i^k}\{\tau_{ij}\}$, where γ is a parameter and $max_{j \in N_i^k}\{\tau_{ij}\}$ is the maximum of pheromone trails among all the cities that the ant has not visited yet when ant k is positioned at the city i (the neighbourhood cities N_i^k). (Dorigo & Stützle, 2004).

The individual idea of calculating τ_0 was originally inspired by an equivalent well-known reinforcement learning algorithm (Sutton & Barto, 1998) and a related method used in Q-learning (Watkins & Dayan, 1992). Later experiments showed that the setting of τ_0 to a small fixed value leads to approximately the same performance while causing major simplification in the algorithm; subsequently, the Ant-Q was substituted by ACS. (Dorigo & Stützle, 2004).

A remarkable similarity exists between *MMAS* and *ACS* algorithms. MMAS explicitly confines the pheromone trails to the defined maximum and minimum limits. A quite similar action can be distinguished in ACS which performs implicitly. The fact is that the pheromone trails can never fall under τ_0 in *ACS* executions due to the initial values of the pheromone trails and

their both global and local update rules; because, as it could be read from their formulas, the initial values of the pheromone trails are set to the value of τ_0 and the amount of deposited pheromone are always more than or equal to τ_0. On the other hand, the pheromone trails can never exceed the value of the $1/C^{bs}$ which can easily be substantiated from the global pheromone update formula. In other words, it is implicitly assured in *ACS* formulation that the pheromone trails to be limited to the boundary of $\forall(i,j)\colon \tau_0 \leq \tau_{ij} \leq 1/C^{bs}$. (Dorigo & Stützle, 2004).

As the last point, it should be point out that *ACS* is the only *ACO* algorithm that limits the number of choices that need to be considered during each tour construction stage by using a candidate lists. Generally, the list includes a series of the best-ranked alternatives, defined based on the heuristic norms. For example in the *TSP* case, the candidate list of each city *i* would comprise the cities *j*, which are in a short distance. The list of candidates could be defined in several ways. Ordinarily, ACS sorts the neighbours of the city i in an ascending order of distances firstly and then selects a few number of the closest cities to be inserted in the i's candidate list. Therefore, the candidate lists could be constructed before beginning of the solution and remain stable during the entire calculation procedure. The ant k when situated at city i, decides on the subsequent city j only among those cities in the candidate list that are not passed yet. In case all the cities in the list of candidates are already visited, then one of the remaining cities is considered to be evaluated. Experimental evaluations have revealed that the quality of solution obtained by the algorithm can be improved by use of candidate lists. However, the significantly increase in the speed of the solution procedure is more important benefit of using the candidate lists. (Dorigo & Stützle, 2004).

3.3.6 Approximate Nondeterministic Tree Search (ANTS)

The Approximate nondeterministic tree search (ANTS) proposed by Maniezzo (1999) is another ACO algorithm that gets some concepts from mathematical programming. In fact it can be frankly extended to the branch & bound procedure. Hence, the name ANTS originates from the fact that this algorithm could be interpreted as an approximate nondeterministic tree search. Precisely, ANTS calculates the lower bounds by a partial solution in order to find the heuristic information to be used by each ant during the construction of solutions. Actually, the algorithm can be extended to an exact mathematical programming algorithm; however, ACO part of the algorithm is presented here. (Dorigo & Stützle, 2004).

Besides the presentation of the lower bounds estimation technique by mathematical programming in ANTS, it offers also two further adjustments to AS. The first is the use of a

novel action choice rule and the second is the *modified pheromone trail update rule,* described in following. (Dorigo & Stützle, 2004).

Use of Lower Bounds

In order to calculate the heuristic information related to desirability of adding an arc (i, j), the ANTS algorithm uses lower bounds by discovering of a partial solution. Algorithm adds the arc to current partial solution in a trial manner and estimates the cost of a complete tour including this arc through a lower bound. The value of heuristic information η_{ij} is then calculated based on the estimated cost value, to enter in the probabilistic procedure of decision making by the ants in the tour construction stages. As a result, desirability of adding a particular arc increases by decreasing the estimated cost. (Dorigo & Stützle, 2004).

The advantage of using lower bounds in calculation of the heuristic information is that it prevents discarding of the feasible moves which lead to partial solutions and their estimated costs are larger than that of the best-so-far solution. However, it has a drawback that at each single construction step of an ant a lower bound needs to be calculated and therefore computational time would be significantly increased. Hence the lower bound has to be calculated efficiently to compensate the disadvantage as much as possible. (Dorigo & Stützle, 2004).

Solution Construction

Unlike the most other ACO algorithms, during the solution construction by the ants, ANTS uses a quite different rule for calculation of the probabilities of the ant k situated at city i to choose the next city j. The utilized rule could be expressed as following:

$$P_{ij}^{k} = \frac{\xi\tau_{ij}+(1-\xi)\eta_{ij}}{\sum_{l\in N_i^k}\xi\tau_{il}+(1-\xi)\eta_{il}}, \quad \text{if } j \in N_i^k \qquad \text{(Dorigo \& Stützle, 2004)}$$

where ξ is a parameter, $0 \leq \xi \leq 1$, and N_i^k is the feasible neighbourhood set as before. Similarly the probability of moving to a city which does not belong to this set is zero.

This formula, compared to that of AS, has the advantage of using only one parameter ξ rather than two ($\propto$ and β). Furthermore, it has a simpler mathematical structure to combine pheromone trails and heuristic information (only summation instead of multiplications and powering) and consequently it is faster to compute. (Dorigo & Stützle, 2004).

Pheromone Trail Update

One of the other particular characteristics of the ANTS is that it does not evaporate the pheromone trails explicitly. The procedure of pheromone updates in ANTS could be expressed as following:

$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$, (Dorigo & Stützle, 2004)

Where $\Delta\tau_{ij}^{k}$ is defined by:

$$\Delta\tau_{ij}^{k} = \begin{cases} \vartheta\left(1 - \frac{C_k - LB}{L_{avg} - LB}\right) & if \ \text{arc } (i,j) \text{belongs to } T^k \\ 0, & \text{otherwise} \end{cases}$$ (Dorigo & Stützle, 2004)

where ϑ is a constant, LB is the value of a lower bound on the optimal solution value computed at the beginning of the algorithm ($LB \leq C^*$ where C^* is the length of the optimal tour), and L_{avg} is the moving average of the last l tours constructed by the ants, i.e., the mean length of the l most recent solutions that produced during the process (l is the constant coefficient of the algorithm). (Dorigo & Stützle, 2004).

In other words, if the length of any constructed tour is longer than the current moving average, the pheromone trail of the belonging arcs will be decreased; whereas, the pheromone trail of the arcs of a better ant's solution (with shorter length) will be increased. The dynamic scaling of the objective function differences is the other outcome of this formulation. It is particularly beneficial during the last iterations of the algorithm when the absolute difference between the solution qualities gets smaller and, accordingly, C^k becomes equal to L_{avg}. The algorithm could be stopped once a solution with an objective function value equal to LB is obtained, since LB has been considered as an estimate of the optimal solution. (Dorigo & Stützle, 2004).

It should be noted that up to now ANTS has not been applied to the TSP. However, very good results have been reported for the application of ANTS on a quadratic assignment problem. (Dorigo & Stützle, 2004).

3.3.7 Hyper-Cube Framework ACO

Blum, Roli, & Dorigo (2001) introduced the hyper-cube framework of ACO. The main characteristic of the hyper-cube framework is that it automatically converts the pheromone values to fall them always in the interval [0, 1]. The idea was inspired by the mathematical programming formulation of many combinatorial optimization problems, in which the problem solutions could be effectively encoded by the binary vectors. The decision variables

of a binary optimization model can only accept the values {0, 1} which are classically related to the solution elements such as those are used during solution construction by the ants. In other words, each solution of the problem corresponds to a corner of an *n*-dimensional hyper-cube (*n* is the number of decision variables in the problem). Problem relaxation is one the leading techniques for generation of the lower bounds for the model. This lets the decision variables to take values from the distance of [0, 1]. Therefore, the set of possible solutions S_{rx} could be considered as the set of all vectors $\vec{v} \in R^n$ that are convex combinations of binary vectors $\vec{x} \in B^n$:

$$\vec{v} \in S_{rx} \Leftrightarrow \vec{v} = \sum_{\vec{x}_i \in B^n} \gamma_i \vec{x}_i \,, \quad \gamma_i \in [0,1], \sum \gamma_i = 1 \qquad \text{(Dorigo \& Stützle, 2004)}$$

Accordingly, the pheromone trail values of an ACO problem are normalized to fall in the interval [0, 1] and the vector of pheromones $\vec{\tau}_i = (\tau_1, \dots, \tau_n)$ will be corresponded to a point in problem domain, $\tilde{S}$. Clearly, any solution of the problem could be represented by a binary $\vec{\tau}$ vector. (Dorigo & Stützle, 2004).

Similarly, a series of decision variable x_{ij} could be considered for each arc (i, j) of a TSP problem. The value of decision variable will be set to $x_{ij} = 1$ when the arc (i, j) has been contributed in construction of the tour, and to $x_{ij} = 0$ otherwise. In this regard, each decision variable will be associated with a pheromone value. In fact, this is the standard approach in solution of TSPs by means of ACO algorithms, which previously described. (Dorigo & Stützle, 2004).

Pheromone Trail Update Rules

As mentioned, the pheromone trails need to be in the interval $[0, 1]$ in the hyper-cube framework. This could be simply realised by slightly adjusting of the standard pheromone update rules as following:

$$\tau_{ij} \leftarrow (1 - \rho)\tau_{ij} + \rho \sum_{k=1}^{m} \Delta\tau_{ij}^k \qquad \text{(Dorigo \& Stützle, 2004)}$$

Where $\Delta\tau_{ij}^k$ is defined as:

$$\Delta\tau_{ij}^k = \begin{cases} \frac{1/C^k}{\sum_{h=1}^{m}(1/C^h)}, & \text{if arc}(i,j) \text{ is used by ant } k \\ 0, & \text{otherwise,} \end{cases} \qquad \text{(Dorigo \& Stützle, 2004)}$$

This formulation assures that the pheromone trails remain less than one after update. In other words, the new pheromone is a move of the old pheromone vector towards the vector of the weighted average of the solutions. (Dorigo & Stützle, 2004).

3.4 ADDING LOCAL SEARCH TO ACO

It is nearly an accepted rule in all metaheuristics that by combining a local search algorithm and using a better initial solution the quality of obtaining solutions as well as the calculation speed could be significantly improved. Similarly there is a considerable potential of improvement in ACO to use local search. Experiments revealed that the iterated local search algorithm is one of the best-performing algorithms for TSP. (Dorigo & Stützle, 2004).

To apply the local search on ACO, the solutions should be converted to their local optimum after that the ants have completed their solution construction. It should be noted that the pheromone trails are updated on the arcs of locally optimized solutions and clearly after application of the local search routine. The combination of ACO tour construction with local search could lead to better solution in most of the cases; because the neighbourhood set that they uses is quite different and there is a quite high potential of improving a solution constructed by ACO by means of local search. Obviously, the local search is not able to obtain high quality solutions as standalone and usually requires a good starting solution to only improve it. Such a solution could be delivered by ACO. (Dorigo & Stützle, 2004).

3.5 IMPLEMENTING ACO ALGORITHMS FOR TSP

3.5.1 DATA STRUCTURES

The series of mandatory data structures is required to store: TSP instances, pheromone trails and artificial ants. Following describes an overall summary of the key data structures that are necessary for execution of an ACO algorithm.

Intercity distances

For a TSP with n number of cities the easiest way to save all pre-computed intercity distances is to use a two dimensional $n \times n$ matrix. However, it is usually impossible (or too expensive) to store the full distance matrix in the main memory for very large instances of ACO. Alternatively, the distances between a city and the cities of its nearest-neighbour list could be calculated and stored in the memory. This can significantly reduce the necessary volume of required memory and computation. Another tip which could be used to speed up the algorithm is to store distances as integers, since the operations on integers are generally done considerably faster than the operations on real numbers. (Dorigo & Stützle, 2004).

Nearest-Neighbour Lists

As mentioned, using a list of nearest neighbours for each city could be suitable to speed up the algorithm. To do so, a sorting routine is applied for each city of problem instance. The

major speedup arises by cutting of the list after a certain number of cities in the list. (Dorigo & Stützle, 2004).

Pheromone Trails

There is a value corresponding to each arc of the construction graph that needs to be saved, related to the amount of its pheromone trail. For an asymmetric TSP instance, a number of $n \times n$ distinct pheromone values needs to be stored. It can be accomplished by utilizing a simple $n \times n$ matrix. Despite the fact that the required number of variables for a symmetric ACO instance equals to $n(n-1)/2$, similar to the distance matrix, using a symmetric $n \times n$ matrix to store the pheromones would be appropriate in this case too. (Dorigo & Stützle, 2004).

Combining Pheromone and Heuristic Information

A vast number of probability calculations are required to combine the values of pheromone trails and heuristic information based on the formula $P_{ij}^{k} = [\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta} / \sum_{l \in N_i^k}[\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}$ during each stage of the tour constructing, when an ant k located on a city i chooses the next city j. These are very close values that have to be calculated in each iteration by all of the ants on each locating city. Experiments showed that the calculation times may be considerably decreased by means of a supplementary matrix to store the values of the $[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}$. Once again, like the pheromone values and the distance matrices, it is convenient to store the values of $[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}$ in a $n \times n$ matrix for a symmetric TSP instance. Furthermore, considering the fact that the $[\eta_{ij}]^{\beta}$ values are constant during entire process, its values could be stored in another supplementary matrix to avoid re-computing of these values in each iteration. (Dorigo & Stützle, 2004).

Pheromone Update

Limiting the calculation of the values in the matrix of nearest-neighbour list of a city would be another optimization technique in speeding up the algorithm. In case of large TSP instances with thousands of cities, this could reduce the computation time significantly. (Dorigo & Stützle, 2004).

Representing the Ants

During each tour construction, the ant needs to store its partial solution which has built up to that point. It is also required to define the feasible neighbourhoods of each city as well as calculating and saving the objective function value of the generated solution.

Storing of the so far generated solution can be easily implemented by saving the partial tour in an adequately big array. This could be also used to check whether a city has not been visited yet and belongs to the feasible neighbourhood. Inappropriately, the computation time dramatically intensifies by application of this simple feasible neighbourhood determination method. The best trick to round this problem could be by simply designation of an extra binary array to each ant and setting the values to 1 if the city has already been visited and 0 if not. After each move, the binary array has to be updated. Finally, the length of the tours can simply computed by summing the arc lengths in the tour. In conclusion, representation of any ant requires a data structure that consists of one variable for storing of the tour length, one $n + 1$ dimensional array for storing the tour and a n dimensional binary array for saving the past nodes. (Dorigo & Stützle, 2004).

Overall Memory Requirement

In summary, a TSP problem with n number of cities requires four $n \times n$ dimensional matrices for saving of distance matrix, pheromone matrix, heuristic information matrix and pre-computed probabilities matrix. Additionally it needs another $n \times nn$ dimensional matrix for the lists of the nearest-neighbours, where nn is the maximum number of the nearest-neighbours for a city. Furthermore, two one dimensional arrays with the size of $n + 1$ and n are needed for every ant to save the tour and the visited cities respectively. A single integer variable will also store the length of the constructed tour. Information of the all individual ants has to be memorized by the end of iteration too (except in MMAS and ACS in which saving of the iteration-best ant is enough). Moreover the intermediate results, such as the best-so-far solution, and some statistical information about the performance of the algorithm have to be also saved; however, the later data structures occupies a very little memory compared to earlier arrays. To conclude, approximately $32n^2$ bytes of memory will be needed for a TSP instance with n cities (except for MMAS and ACS which is much less). (Dorigo & Stützle, 2004).

3.5.2 Algorithm steps

Data Initialization

The following steps have to be executed respectively during data initialization of the program:

- Reading the instance,
- Computing the distance matrix,
- Determining the nearest-neighbour lists for all cities and

- Initializing the pheromone matrix and the pre-computed probabilities matrix.

It is also necessary to update the parameters of the algorithm and the information of the ants after each iteration. Variables such as CPU usage, number of iterations, the best-so-far solution that keep the track of the statistical information need to be revised after each iteration too. (Dorigo & Stützle, 2004).

Termination Condition

Calculation can be ended when at least one of the following termination conditions is reached:

- Finding a solution within a certain distance from estimated optimal solution;
- Exceeding the maximum number of tour constructions or algorithm iterations;
- After a definite time of CPU execution;
- Stagnation of the algorithm. (Dorigo & Stützle, 2004).

Solution Construction

The process of each solution construction consists of the following steps:

- All cities should be marked as unvisited to clear the memory of previous ants;
- A random initial city needs to be designated to each ant;
- Letting the ants to move from city to city (based on the AS choice rule) and generate their tours;
- Computing the length of the constructed tour.

All of ants need to follow steps above in a parallel or sequential order (for ACS only in parallel). The number of construction steps is the same for all of the ants because they all have to visit exactly n cities. (Dorigo & Stützle, 2004).

Local Search

Small adjustments could be applied on the constructed tours using a local search procedure to improve their qualities. (Dorigo & Stützle, 2004).

Pheromone Update

The pheromone update procedure applies at the end of each iteration, and involves in two major stages: pheromone evaporation and pheromone deposition. Evaporation diminishes the pheromone value of all arcs by a constant evaporation factor while deposition adds some extra values to the pheromone of the arcs belonging to the constructed tours. Only one or very few number of ants are permitted to deposit pheromone (except in the

conventional AS and EAS). Hence, the procedure of pheromone trail deposition is not very long and complex excepting in AS and EAS. Thus, the speeding up tricks are usually unnecessary for the pheromone trail update procedures, particularly for ACS where both the pheromone evaporation and deposition actions are applied only on the arcs that are crossed by the best-so-far ant. (Dorigo & Stützle, 2004).

3.5.3 CHANGES FOR IMPLEMENTING OTHER VARIANTS OF ACO

Most of the above-mentioned points are common for all of ACO variants; however, there are individual essential adjustments corresponding to each ACO algorithm. Some of these variations are as following:

- Deposited pheromone in EAS and AS_{rank} is applied proportional to the quality of solutions. It could be implemented by adding some weight factors to the standard pheromone deposition.
- To control the pheromone limits in MMAS it would rather to integrate it into the procedure of pheromone update. (Dorigo & Stützle, 2004).

ACS is the particular variant of ACO whose execution involves in more individual cares, some of which are summarized below:

- *The pseudorandom proportional action choice rule* is used during tour construction in ACS. Accordingly, a random number q should be generated for the each move of the ants, to decide between the next best and the AS decision rule.
- A special procedure should be programmed to be applied immediately after moving an ant to a new city in order to consider the local pheromone update.
- The global pheromone trail update is applied at the end of each iteration only on arcs belonging to the best-so-far tour. Its implementation is similar to that of local update.
- In ACS, the pre-computed probabilities matrix does not need to be updated in the course of the algorithm (except during initialization) due to the special formulation of the local and global pheromone trail update rules. (Dorigo & Stützle, 2004).

4 ACO APPROACH FOR THE LONG-TERM SCHEDULING OF OPEN-PIT MINES

Figure 4-1 shows the proposed process of long-term open-pit production planning in this research. The algorithm consists of saving P number of variables for each block of the model,τ_{ip} , which represents the pheromone value related to mining of block i in p^{th} push back. P is the number of scheduling periods. The magnitude of saved pheromones represents the desirability of a block to be the deepest point of the mine in that pushback.

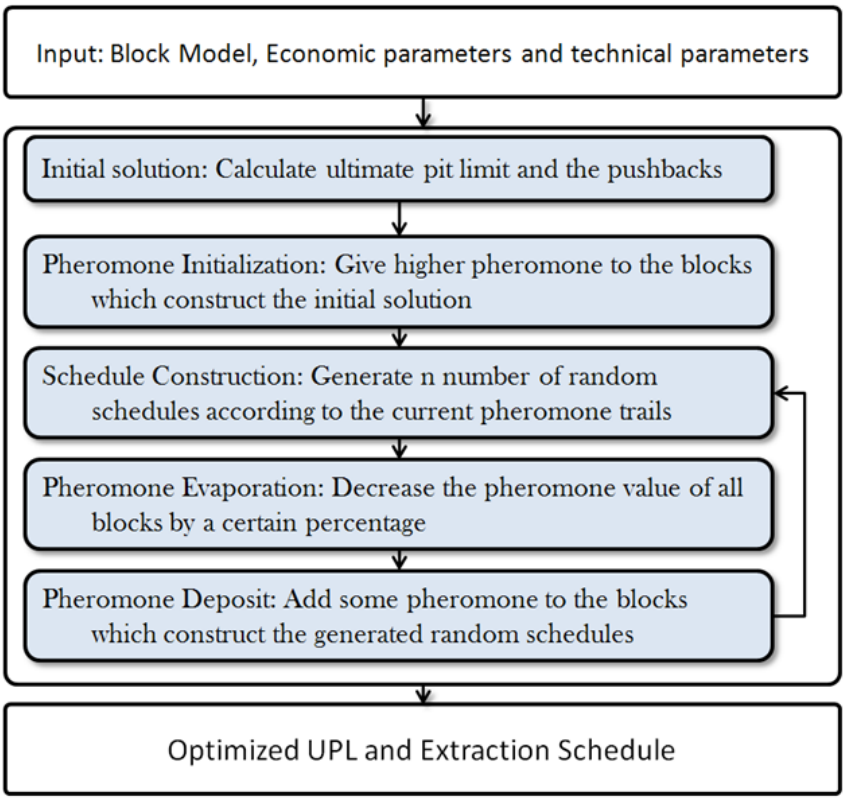


FIGURE 4-1 MAIN STEPS OF ACO FOR LONG-TERM PRODUCTION PLANNING OF OPEN PIT MINES

The initial value of these variables are assigned based on a sub-optimal mine schedule generated by Lerchs-Grossmann and Wang-Sevim algorithms. Then the random mining schedules are constructed according to the initial pheromones. These schedules deposit an extra pheromone proportional to their economic quality. This action along with pheromone evaporation lead the algorithm towards the optimum boundary of mining push backs.

4.1 PHEROMONE INITIALIZATION

Experiments showed that the calculation time increased dramatically using the uniform initial pheromone pattern. Therefore in this study, a sub-optimal solution for the problem of

long-term open-pit scheduling is firstly determined by means of Lerchs-Grossmann's algorithm of UPL design and the Wang-Sevim's nested pits design algorithm. Then, initial pheromone trails are assigned to the blocks according to this sub-optimal solution.

Normally the shape of a desired pushback does not change drastically from a sub-optimal solution to the optimal one. Thus assigning of higher pheromones to a few numbers of blocks around the sub-optimal pit depth could be enough to lead the algorithm towards the optimal solution. This process has been illustrated in Figure 4-2. Consider the pit shape shown in Figure 4-2a to be the outline of the mine in p^{th} extraction period. During the process of pheromone initialization, the pheromone value of the highlighted blocks in Figure 4-2b related to the period, τ_{ip}, are set to relatively high values.

4.2 CONSTRUCTION OF SCHEDULES

In order to construct a mine scheduling solution, a series of feasible nested pits related to the different mining push backs should be created. Each one of these pits consisted of a series of block columns. The shape of any pit could be defined by determining of the pit depth in its block columns.

4.2.1 THE PROCESS OF DEPTH DETERMINATION

Depth determination for a column of blocks requires the following information for each block:

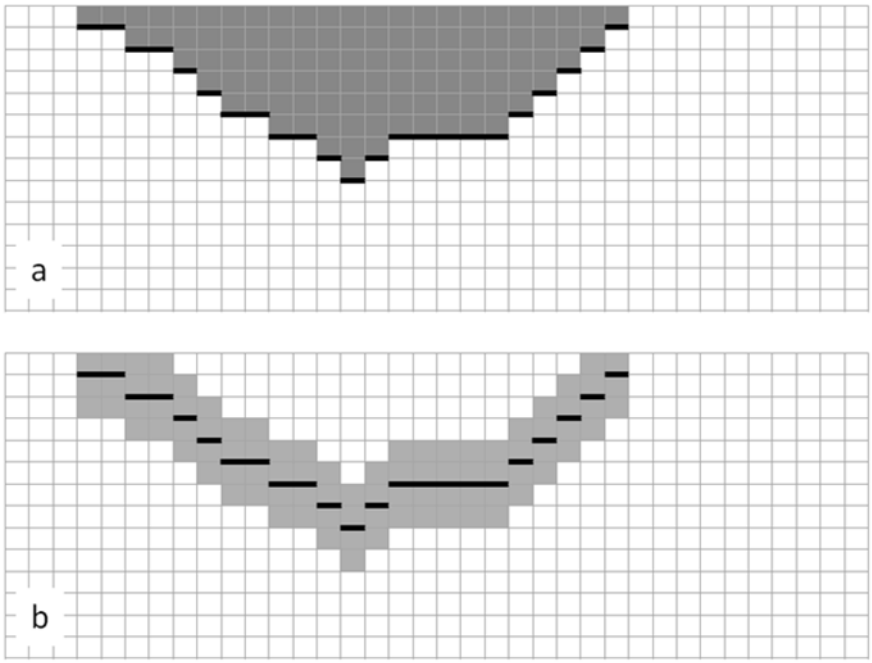


FIGURE 4-2 PHEROMONE INITIALIZATION OF THE BLOCKS.

- Pheromone values of each block
- Tonnage and average grade of valuable minerals and troublemaker elements in the blocks
- Direct costs and revenues associated with mining and milling of the blocks

The upper and lower boundary of the permitted pit depth should also be available for the column. The calculated pit depth should also fall between these maximum and minimum allowed depths. The maximum allowed depth (dark lines in Figure 4-3) defines the deepest possible mining depth and could be calculated based on the maximum slope angle and distance from the borders of the block model. On the other hand, minimum depth of each column is determined according to the shape of the mine in earlier push back (dark dotted lines in Figure 4-3). Clearly, there is no minimum depth for the first pushback. The process of depth determination for a hypothetical block column has been illustrated in Table 4-1.

It should be noted that in this research the process of depth finding is done only for the columns containing at least one ore block. The depth of the pit in totally waste columns will be defined in the next step of pit generation algorithm, called normalization, from the neighbouring selected depths.

Another important point is that the initial pheromones are assigned only to the ore blocks. Therefore, the selected depth will always coincide on an ore block. The reason is that there is no benefit in adding a waste block to the set of blocks considered to be inside the pit.

Similarly, there will be no pheromone update (evaporation or deposition) for waste blocks. If the optimum depth lies on a waste block and the depth finding process defines an upper ore block instead, the optimum position will be generated automatically in the next step (pit generation from selected depths). On the other hand if the pit depths go deeper than the optimum level, the fitness of its generated schedule would be low and the schedule will die out in ACO process.

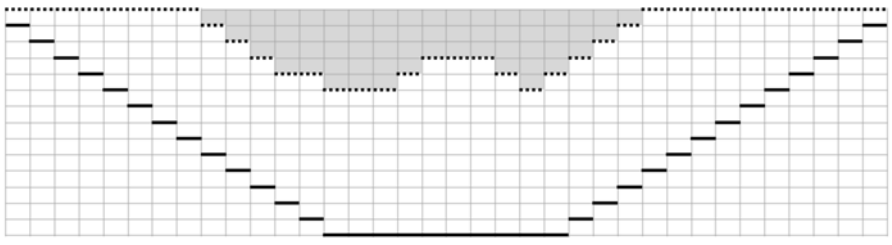


FIGURE 4-3 MAXIMUM AND MINIMUM DEPTH DEFINITION IN DEPTH DETERMINATION PROCESS

TABLE 4-1 THE PROCESS OF DEPTH DETERMINATION

Block Column	Pheromone*	Heuristic Information*	Selection possibility**	Cumulative Possibility
	0	0	0.0000	0.0000
	280	8	0.0285	0.0285
Min Depth	0	0	0.0000	0.0285
W	0	0	0.0000	0.0285
O	330	6	0.0297	0.0583
O	540	7	0.0930	0.1514
W	0	0	0.0000	0.1514
O	670	6	0.1227	0.27424
O Selected Depth	890	8	0.2889	0.5631***
O	750	9	0.2308	0.7939
W	0	0	0.0000	0.7939
O Max Depth	870	5	0.1725	0.9664
	350	6	0.0335	1.0000
	0	0	0.0000	1.0000

* without unit

**based on $P_{ij}^{k} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k}[\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}}$ formula ($\propto = 1$ and $\beta = 1$)

***selected depth according to the random number (0.6328)

4.2.2 PIT GENERATION ACCORDING TO THE SELECTED DEPTHS (NORMALIZATION)

Usually the set of selected depths (the bold red lines in Figure 4-4a) do not result in a feasible pit shape. In fact it is the consequence of independent depth determination in each column. Thus a feasible pit generation process called normalization is required after determination of depths for each pit related to the mining periods. It is made in a manner that the obtained pit shape covers all the determined depths as well as the outline of earlier push backs, Figure 4-4b.

Supposing square blocks and a slope angle of 45 degrees, the process could be explained in the following steps:

- Start from the deepest level of the block model and check all the blocks from left to right. If the calculated depth of any column is equal to 1, then flag the block as an In-Pit block.
- Move to the upper level and check all blocks from left to right. If the calculated or minimum depth of any column is equal to or lower than the current level, flag the block as an In-Pit block. Moreover, if at least one of the three underlying blocks of any

block is flagged as In-Pit, then flag the corresponding block of the column as an In-Pit block as well.

- Repeat the previous actions up to the uppermost level.

After normalization of the pit, its size should be validated. In case of very big or very small generated pits, the algorithm reproduces this pit again from the beginning by determining the pit depths for block columns. Sometimes the generation of an abnormal (but valid) pit for the earlier push backs makes it impossible for the process to continue to the next push backs. Therefore if the pit generation process was not successful after a certain number of trials (for example 100 times), the algorithm leaves this set of pits and begins constructing another set from the first push back.

4.2.3 Mine schedule construction from generated pits

In the last step of the solution construction, individual pits which have been created for the different mining push backs are combined to produce a mine schedule, Figure 4-5.

4.3 Pheromone update

Results of the constructed mine schedules are transferred to the ACO optimization model as a series of decreases and increases in pheromone values of the blocks.

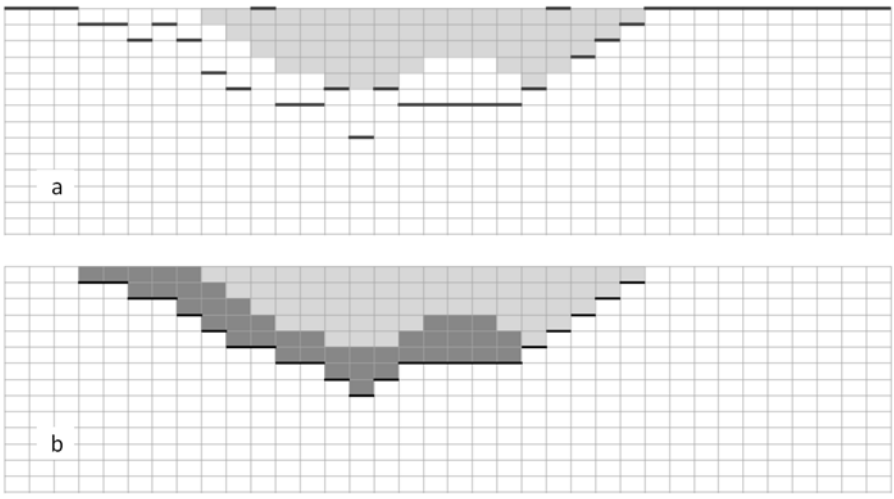


Figure 4-4 Generation of a new pit based on the selected depths and previous pit

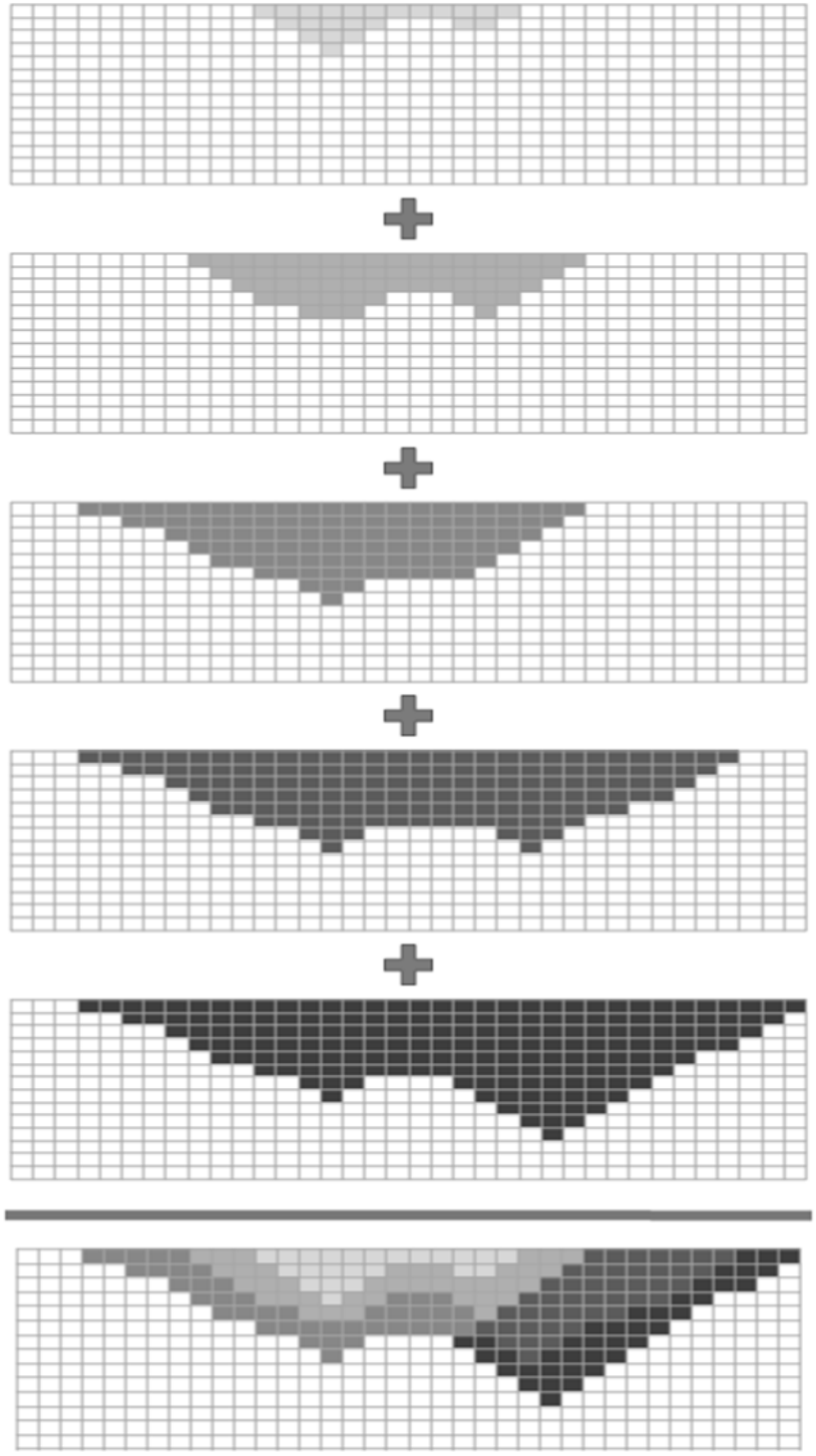


FIGURE 4-5 COMBINATION OF GENERATED PITS TO PRODUCE A MINE SCHEDULE

4.3.1 PHEROMONE EVAPORATION

The first step of the pheromone update process consists of a uniform reduction in the value of all pheromones in order to help the ACO optimization model disregard the bad solutions.

In this stage, the pheromone value of all blocks corresponding to all production schedules should be decreased by a certain percentage.

4.3.2 Pheromone Deposition

Consider B_{ip}^{k} to be the deepest block of p^{th} push back of k^{th} constructed mine schedule in the i^{th} column of the block model. The pheromone value of this block grows during the deposition phase by adding an additional pheromone value. The magnitude of the added pheromones differs for different variants of ACO algorithm.

It should be noted that the deposition action is applied only to the ore blocks. In cases where the pit depth lies on a waste block, the additional pheromone is assigned to an imaginary block on the ground surface. This causes that the desirability of other ore blocks of the column not to increase because of the lack of pheromone deposition in the optimal depth.

4.4 Implementation tool

To evaluate the applicability of the proposed ACO algorithm for long-term planning of open-pit mines, a computer program has been developed in Visual Studio 2005 programming environment for the implementation of calculations. The program interface consists of four different graphical user interface windows, including the input block model, input parameters, initial solution and ACO optimizer tabs, Figure 4-6.

4.4.1 Input block model tab

Implementation of the algorithm starts with importing a uniform block model to the program. The block model should be prepared as a text (ASCII) file in which the information of each block should be written in a separate line called records. These information fields could include coordinates, metal grades and troublous elements.

In order to import the block model file into the program, the user can type the exact address into (1) or could browse through folders by clicking button (2). Clicking the preview button (3) will show the first 100 lines of the inputed file in the preview text box (12). This helps the user recognize the structure of the input file and assists in filling in the following parameters. Then, the number of blocks in X, Y and Z directions must be entered in (4).

The program has the ability to use both index coordinates, (i,j,k), and real world coordinates, (x,y,z), which is selectable in (5). In fact the program does not need the real coordinates for optimization and the i, j and k indexes are used in all calculations. Therefore if the real world coordinates are imported into the program, the origin of the block model and size of blocks will be calculated using the given information.

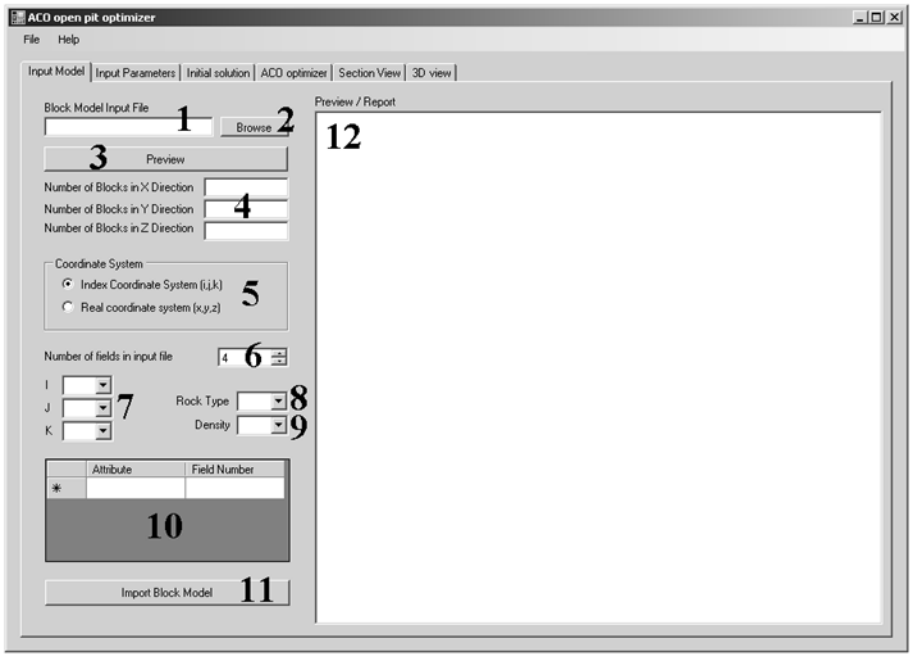


FIGURE 4-6 THE INPUT BLOCK MODEL TAB

The number of information fields in each record should be set in (6), after which the list boxes of (7), (8), (9) and (10) will be updated. The user must then choose the proper place of information related coordinates, rock type, density and production elements in these lists. Finally by clicking button (11) the model is imported and a short report regarding the number of blocks, dimensions, origin of the model and average grade of elements will be shown in the preview/report text box (12).

4.4.2 INPUT PARAMETERS TAB

In this tab, Figure 4-7, the economic and technical parameters of the mine are defined. Firstly, the user should define the units in (1). Then the dimension of blocks, the number of blocks in each direction and the origin of the block model should be entered in (2). The program calculates the block dimensions and the origin of the model if the (x,y,z) coordinate system has been utilized in the imported block model.

Product prices are another important economic parameter that should be defined in (3) for each production element such as copper, gold, etc. Finally the properties of the material for each rock type should be entered in (4). Required information includes:

- Type of material (ore or waste),
- Mining cost per ton,
- Modification of mining cost by elevation,
- Mining reference elevation,

- Modification factor per bench,

Additionally, the following information regarding the ore blocks should be provided:

- Mining recovery rate,
- Mining dilution percentage,
- Processing costs,
- Processing recovery rate

Having this information, the program can calculate the economic value of the blocks by clicking button (5).

4.4.3 INITIAL SOLUTION TAB

The first part of this tab, Figure 4-8, is designated to the parameters of the initial solution. The user has to define the final and working angles of the mining slopes in different directions in section (1). Then by clicking the UPL calculation button (2), a short report regarding the number of ore and waste blocks inside the UPL and the average grades will be given in the report box (7). Then the number of blocks for each push back can be estimated in order to reach the desired number of push backs based on the obtained UPL report. By entering the estimated number of blocks for each push back in (3) and by clicking the 'Create Push Backs' button (4), the program will create an initial solution for the ACO optimization process. A brief report will be displayed again in (7) about the number of blocks and average grade of elements for the created initial solution.

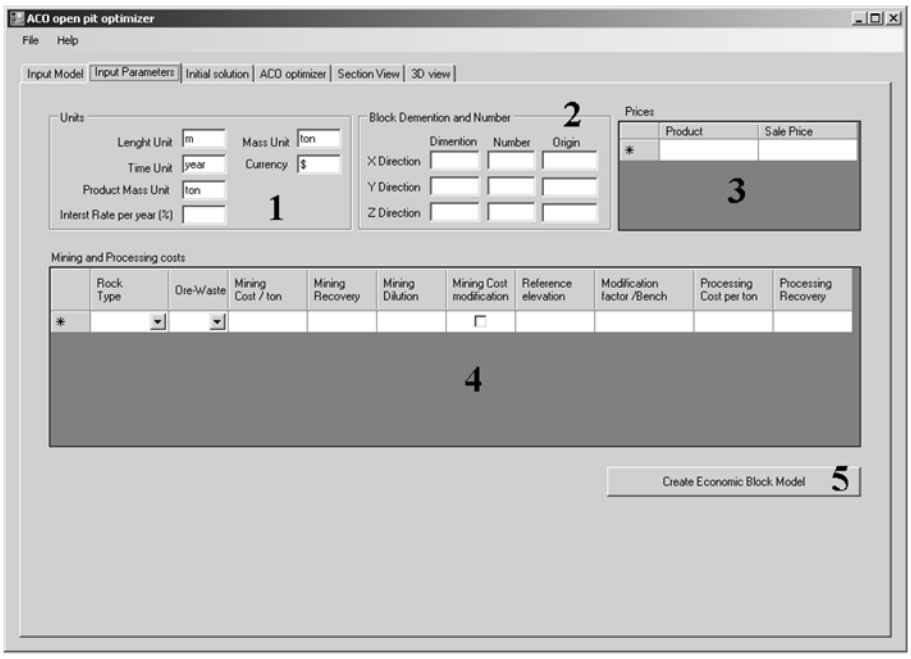


FIGURE 4-7 INPUT PARAMETERS TAB

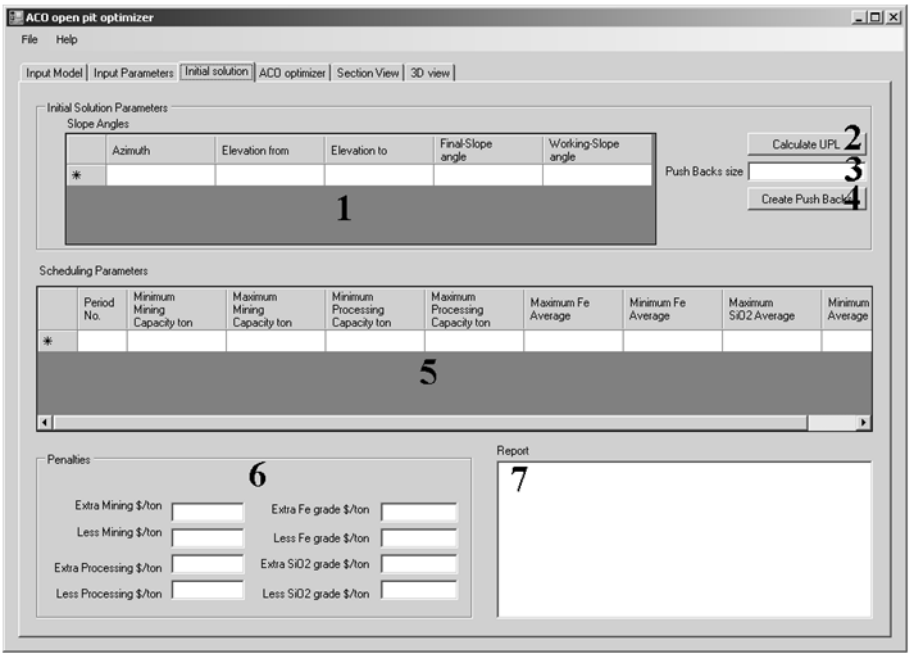


FIGURE 4-8 INITIAL SOLUTION TAB

Prior to ACO optimization, it is necessary to define the mine scheduling parameters and the penalty coefficients in section (5) and (6). Required information includes the maximum and minimum mining and milling capacities, the maximum and minimum limit of average grade for each element (Fe and SiO2 in this case) and the penalty cost related to each of these items.

4.4.4 ACO OPTIMIZER TAB

The ACO optimizer tab contains the tools and parameters required for the improvement of initial solutions through implementation of ACO iterations. It consists of two groups of parameters including general and ACO variants parameters as well as a graph display tool to show the variations in parameters during the run of the program, Figure 4-9 .

General ACO parameters are as following:

- Initial pheromone value, (1)
- Number of upper initialized blocks, (2)
- Number of lower initialized blocks, (3)
- Priority coefficient of pheromone value, (4)
- Priority coefficient of heuristic information value, (5)
- Coefficient of evaporation rate, (6)
- Number of ants in each iteration, (7)

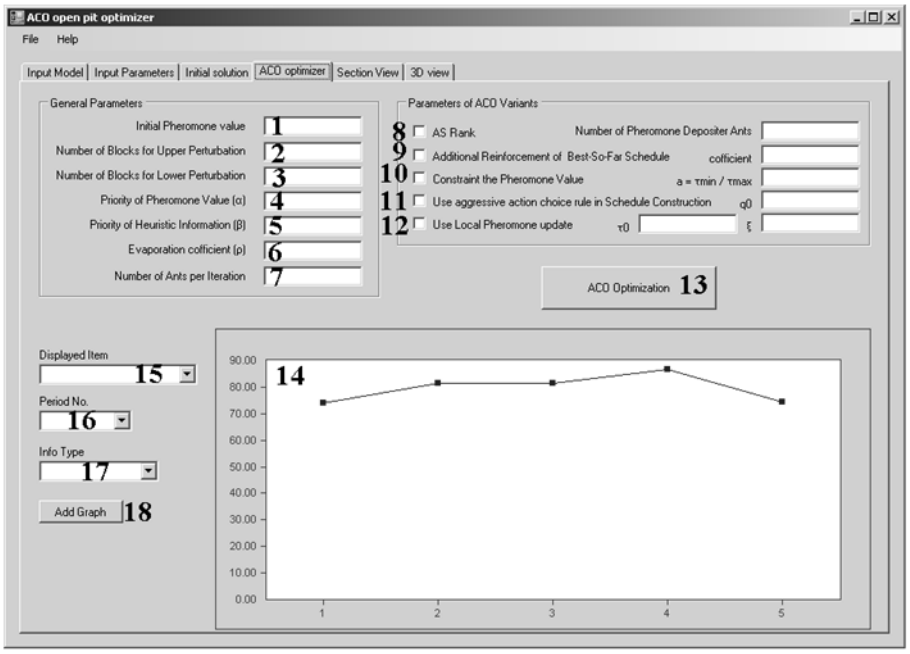


FIGURE 4-9 ACO OPTIMIZER TAB

Parameters of ACO variants include:

- For the AS_{rank} condition : the number of pheromone depositor (ranked) ants, (8)
- For the elitist ant system (EAS): the coefficient of additional reinforcement for the best-so-far schedule, (9)
- For max-min ant system (MMAS): the ratio of upper pheromone limit to the lower pheromone limit, (10)
- For ant colony system (ACS): the probability under which the tour construction process is carried out according to the aggressive action choice, (11), and the ξ and τ_0 coefficients of the local pheromone update, (12).

Eventually by each clicking of the ACO optimization button, an optimization iteration is implemented and the results are shown in the chart (14). The horizontal axis of this chart represents the ACO iteration and the vertical axis reveals the information that could be defined from the left hand side boxes. The ACO optimization process is repeated until the graph stabilizes and the optimum solution is found.

The graph (14) has the ability of drawing several items which should be selected from the box (15). These items include:

- Economic value of the schedules
- Penalty of the schedules
- Push back information

For the last case, the number of push backs and required information must be chosen from (16) and (17) respectively. Selected information types could be one of the Fe or SiO2 average, the number of blocks (ore, waste or total) or the life of the periods. Then the defined chart would be added to the graph by clicking the button (18). The chart is able to display up to 10 different information graphs simultaneously.

4.5 CASE STUDY

A hypothetical block model of an iron ore deposit containing 1000 blocks was created and the grades of Fe and SiO2 were randomly assigned to all ore blocks. The grades of Fe and SiO2 varied from 45 to 65 and from 5 to 15 percent respectively. According to these grades the net economic values of the blocks were calculated in the distance of 1 to 9. A constant value of -6 was assigned to the waste blocks.

At the beginning the outline of the ultimate pit is determined by the Lerchs-Grossmann's graph algorithm. The calculated UPL contains 455 ore and 161 waste blocks which led to 681 units of undiscounted economic value. Then mining push backs were generated by the alternative to parameterization algorithm of Wang-Sevim. Through this, 9 uniform push backs with the size of 70 blocks were constructed. Considering an annual interest rate of 10% and the mine life of 20 years, the discounted economic value of the constructed initial schedule was calculated as 323 units. Table 4-2 shows the block numbers and the average grades of the push backs in the initial solution.

As a simple scheduling condition, the following restrictions were considered for each period of this case study:

- Mining rate: from 59 to 64 blocks per period
- Processing rate: from 47 to 53 blocks per period
- Average allowed grade of Fe: from 54 to 56 percent
- Average allowed grade of SiO2: from 9 to 11

Anything exceeding these limits has been considered to have 1 currency unit of penalty cost for each of the extra or fewer blocks. Consequently the value of the constructed initial scheduling solution received 79 currency units of penalty costs and its economic value dropped to 244 units.

TABLE 4-2 CHARACTERISTICS OF THE INITIAL PUSH BACKS

Pushback No.	Ore number	Waste number	Total number	Push back life (cumulative)*	Average Fe grade	Average SiO2 grade
1	49	14	63	2.1	53.2	8.3
2	54	12	66	4.5	53.1	9.1
3	46	22	68	6.5	54.3	8.4
4	44	13	57	8.4	53.9	8.3
5	49	21	70	10.6	53.7	8.8
6	44	24	68	12.5	53.5	8.1
7	52	18	70	14.8	53.4	8.7
8	65	21	86	17.7	53.1	8.9
9	52	16	68	20	51.9	9.2
Total	455	161	616			

* Mining in 20 years and 9 uniform period

4.6 ACO VARIANTS AND SETTING OF PARAMETERS

In order to analyse the efficiency of different ACO variants in optimizing the long-term planning of open-pit mines and finding the best values of the ACO parameters, the program was utilized to be run using the following alternatives.

4.6.1 ANT SYSTEM (AS)

As described in previous chapter, this is the simplest ACO system in which all ants have the ability to deposit pheromone proportional to the quality of their constructed tour. In the basic run of the ant system, the following values have been considered as key parameters:

- The number of ants (number of tours in each iteration, m) is considered to be equal to the number of block columns in the model. In this case study it was 100. All of these ants were allowed to deposit pheromone.
- Principally it is possible that a negative value schedule be constructed by some of the ants. On the other hand, as mentioned in Chapter 3.3.1, the deposited pheromone by each ant is proportional to the quality of its tour. Because a negative pheromone deposition is meaningless, therefore the value of the worst schedule is added to the fitness value of all schedules to ensure that all of them are above zero. Consequently the pheromone value of different iterations might not be comparable. In order to

make a balance between the deposited pheromone of different iterations, all the fitness values were divided by the highest fitness value. Eventually all the fitness values fell in the distance of zero and one. This is almost similar to the hyper-cube framework ACO and has been applied in all cases of this research.

- As described earlier, a good heuristic procedure to initialize the pheromone trails in the AS could be to set them to a value slightly higher than the expected amount of pheromones deposited by the ants in one iteration. A rough estimate of this value can be obtained as $m \times C^{nn}$, where m is the number of ants, and C^{nn} is the discounted value of the initial schedule. Accordingly the initial values of the pheromone trails were set to 100. This number was assigned only to the ore blocks close to the outline of push backs.
- Similar to the application of ACO for solution of TSP, the value of the evaporation coefficient, , had been set to 0.5 in this case as well.
- The upper and lower perturbation distance is considered as zero. In other words, relatively high pheromone values were assigned only to blocks which constructed the mining push backs.
- Equal priority was considered for the pheromone trails and heuristic information in the basic case, ($\propto = 1$ and $\beta = 1$).
- According to the justified fitness values of the constructed schedules the amount of deposited pheromone by each ant is considered to be equal to its fitness value which is always between 0 and 1.

The efficiency of the basic ant system has been shown in Figure 4-10. The main point in this graph is that it proves the ACO has the ability of improving the quality of initial solutions generated by Lerchs-Grossmann algorithm and parameterization. The graph reveals that right from the first iteration, ACO algorithm improves the value of the mine schedule and after 14 iterations it reaches its best solution at 265.1309. In comparison to the initial solution which had a value of 244.0635, this meant more than an 8 percent improvement in the value solution. After the 12^{th} iteration, the algorithm scatters around the level of 263.

The value of any schedule has two major components which are the revenue and penalty costs. Actually the utilized algorithm for the construction of the initial solution (Lerchs-Grossmann plus parameterization) takes only the first component (revenue) into account. Consequently the total combination could not be optimized. In fact ACO searches for the solutions which have a higher total value despite containing a lower revenue. The variation of the revenue and penalty cost values for the basic AS is shown in Figure 4-11. The original

combination of revenues and penalty costs has been changed from 323.2117 and 79.15385 to 321.6918 and 56.56096. In other words, the found obtained solution has about two units less revenue but 13 units of lower penalty costs which led to some 11 units of improvement. The following solutions after the 12th iteration have less total value despite having fewer penalties.

The standard deviations of solutions for the size of mining and processing operations, as well as for Fe content, are shown in Figure 4-12. The graph reveals that ACO has decreased the deviation of push backs from the planned values.

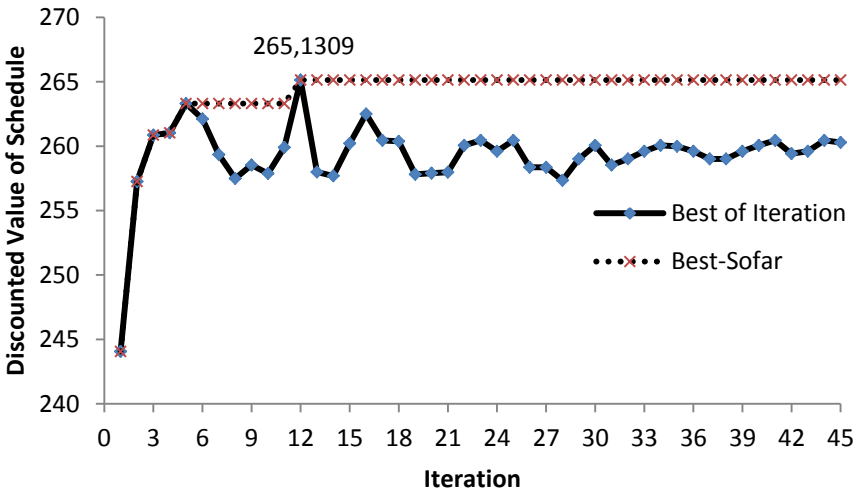


FIGURE 4-10 IMPROVEMENT OF SCHEDULING VALUE BY BASIC AS

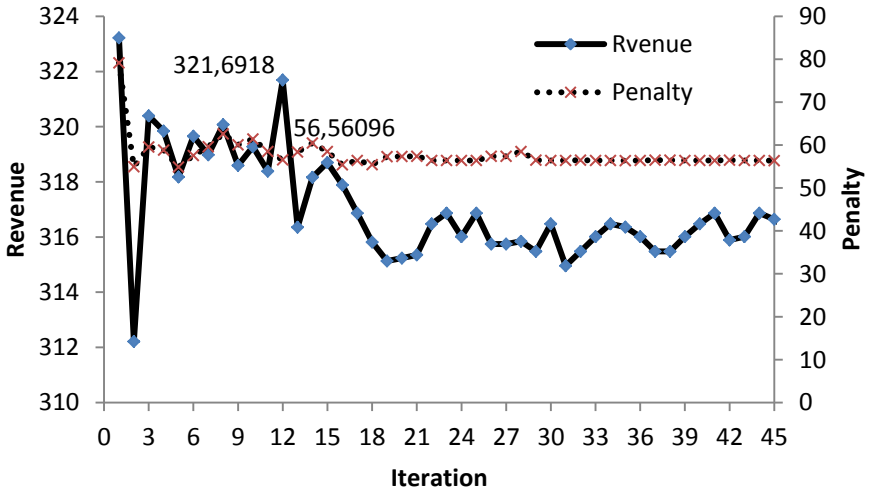


FIGURE 4-11 VARIATION OF REVENUES AND PENALTY COSTS DURING BASIC AS

It should be noted that because of the randomized nature of ACO, the calculation processes in different runs of the program were not similar and the value of the best found solution varied from 260 to 270. But in a correct routine, the final solution should be almost the same apart from the transitional answers in previous iterations. This means that there would be a possibility of further improvements by adjusting the parameters of the algorithm.

In the following, the effect of changes in different ACO parameters on the efficiency of basic AS variant has been analysed.

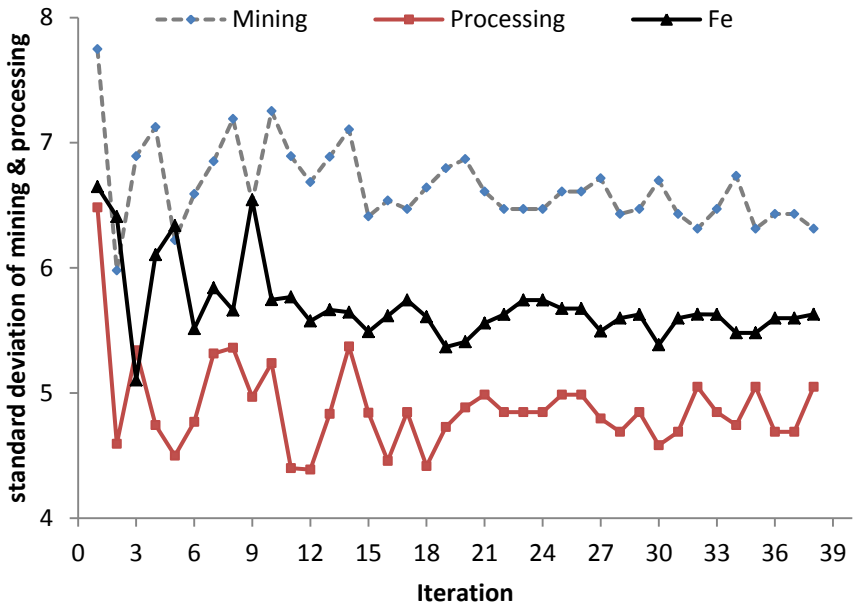


FIGURE 4-12 DECREASING EFFECT OF THE VARIANCE IN BASIC AS OPTIMIZATION FOR MINING AND PROCESSING CAPACITIES AND FE CONTENT

TABLE 4-3 EFFECT OF ANT NUMBER ON THE SOLUTION QUALITY AND CALCULATION TIME

Ant number	Value of the best found schedule	Required time to get the best solution (milliseconds)	Required iterations to get the best solution
50	262.3471	2984	9
75	266.9035	10469	15
100	265.1309	11188	12
125	266.0709	8719	9
150	266.0709	9688	8
200	268.5578	42281	24
300	267.5038	57094	21

Number of ants in each iteration

The number of ants is one of the major parameters of the ACO. The importance of the ant numbers comes from its impact on the running time of each iteration. It initially was set equal to the number of block columns in the model which is 100. Effect of using higher or lower number of ants in each iteration has been shown in Table 4-3 .

Although the judgment about the best number of ants according to this table is not easy, however, the table shows that not only do very high numbers not improve the solution noticeably, but they also drastically increase the calculation time. Very low ant numbers also led to fast stagnation. Consequently it seemed that the selected number was relatively appropriate and could vary from 20% fewer or more without any big effect on the algorithm.

Initial pheromone value

The result of changes in initial pheromone values is shown in Table 4-4. The lower initial pheromone lets the program search more among remote solutions, and away from initial schedule which led to a slightly better solution. But it increases the calculation time by around 50 percent. On the other hand, a high initial pheromone value made the algorithm stagnate to solutions around the initial answer and consequently led to poor results. Again it seemed that the selected initial pheromone value was in the proper range.

Priority factors of pheromone and heuristic information

Table 4-5 represents the efficiency of the algorithm with different combinations of pheromone values and heuristic information priority factors. The outcome revealed that the heuristic information is either unimportant in the process or it has not corresponded to an appropriate property. It is suggested to use relatively lower values for the priority factor of heuristic information (β).

TABLE 4-4 EFFECT OF THE INITIAL PHEROMONE ON THE SOLUTION QUALITY AND CALCULATION TIME

Initial Pher.	Value of the best found schedule	Required time to get the best solution (milliseconds)	Required iterations to get the best solution
50	266.4478	15312	24
100	265.1309	11188	12
200	263.3134	2844	6
500	262.4578	2859	6

TABLE 4-5 EFFECT OF PRIORITY FACTORS OF PHEROMONE AND HEURISTIC INFORMATION ON THE SOLUTION QUALITY AND CALCULATION TIME

Initial Pher.	Value of the best found schedule	Required time to get the best solution (milliseconds)	Required iterations to get the best solution
α=1 β=0	273.5045	51859	76
α=1 β=0.5	272.2923	34672	49
α=1 β=1	265.1309	11188	12
α=1 β=2	260.0672	15312	23
α=1 β=5	252.4027	734	2

4.6.2 ELITIST ANT SYSTEM (EAS)

The concept of EAS is to consider a strong emphasis to the best-so-far solution in the pheromone update step. In other words, the best-so-far ant deposits as much pheromone as that of e normal ants. Considering $= 100$, efficiency of EAS is shown in Figure 4-13. As it is shown in Table 3-1, the value of the initial pheromone is different from that of AS and the e and ρ parameters should be considered in the initial pheromone values formula as $(e + m) \times C^{nn} / \rho$ (400 in this case).

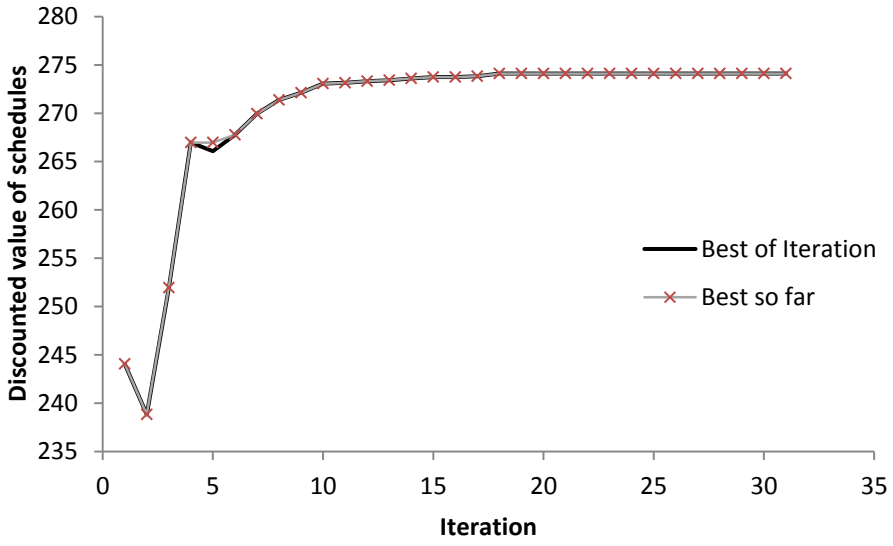


FIGURE 4-13 EFFICIENCY OF EAS WITH $e = 10$

TABLE 4-6 EFFECT OF THE REINFORCEMENT TO THE BEST SO FAR ANT ON THE SOLUTION QUALITY AND CALCULATION TIME

e	Value of the best found schedule	Required time to get the best solution (milliseconds)	Required iterations to get the best solution
25	274.9540	14312	22
50	274.7419	13000	20
75	274.7166	13812	21
100	274.1010	11641	18
125	274.0421	8281	13

The most noticeable thing about this graph is that the elitist ant strategy has eliminated the scattering behaviour of the algorithm (compared to the AS which was never led to a firm solution). Unlike in Figure 4-10, in the EAS the best solutions of the iterations are almost close to the best so far schedule.

Based on the graph, the quality of the solutions decreased in the first iteration after which it continuously increased up to the 18^{th} iteration that corresponded to the ever-best schedule which had a value of 274.1010.

The results obtained from the analysis of different values of e have been compared in Table 4-6. The table provides a highly positive correlation between the application of EAS strategy and the basic AS for all values of e. Although no significant difference is apparent, however a value between 75 to 100 percent of normal ant numbers seem to be relevant for the coefficient of e. Higher values than this would prevent the algorithm from adequately discovering far domains from the initial solution.

4.6.3 RANK BASED ANT SYSTEM (AS_{RANK})

In AS_{rank} each ant deposits an amount of pheromone that decreases with its rank. In addition, As in EAS, the best-so-far ant always deposits the largest amount of pheromones. In other words, in each iteration of AS_{rank} only the $(w-1)$ best ranked ants and the ant that produced the best-so-far tour (this ant does not necessarily belong to the set of ants of the current iteration) are allowed to deposit pheromones. The best-so-far tour gives the strongest feedback (with weight w) and the r-th best ant of the current iteration contributes to pheromone update with the weight of $w-r$. The initial pheromones are also assigned based on the formula $0.5w(w-1) \times C^{nn}/\rho$ which equates to 400 for the current case study supposing a value of $r = 10$ and $\rho = 0.1$. The efficiency of the algorithm and the calculation time for this variant is displayed in Figure 4-14.

Figure 4-14 reveals that the ranking strategy allows the algorithm to be continued to more than a hundred iterations. Therefore the program would be able to improve the quality of the solution. In this case study a value of 279.0614 has been obtained for the best found solution in 107th iteration. Although this value is slightly higher than the 277.20928 in the 40th iteration, it is up to the planning engineer to decide on spending almost triple calculation time in order to improve the solution for less than 1 percent.

The quality of solutions and calculation times for different numbers of ranked ants, w, are compared in Table 4-7. In each case the value of the initial pheromone has been chosen based on the $0.5w(w-1) \times C^{nn}/\rho$ formula. Table 4-7 shows that increasing w not only increases the calculation time but also decreases the quality of the solution. Hence a value between 5 to 15 percent of the number of normal ants is suggested for the ranked ants' number (w).

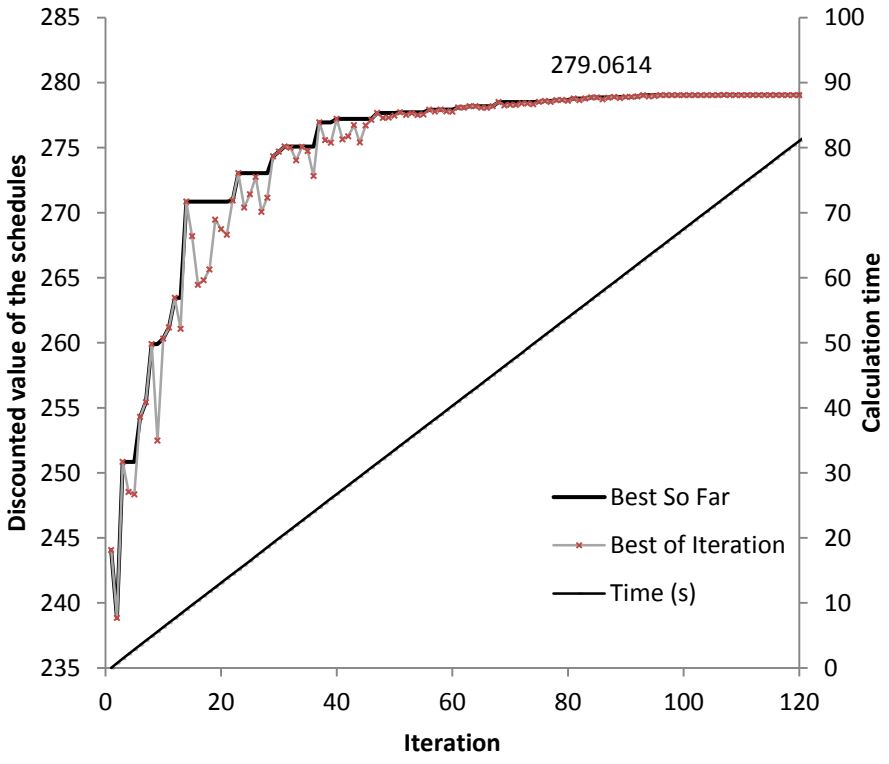


FIGURE 4-14 EFFICIENCY OF AS_{RANK} WITH $w = 10$

TABLE 4-7 EFFECT OF REINFORCEMENT TO THE BEST SO FAR ANT ON THE SOLUTION QUALITY AND CALCULATION TIME

w	Value of the best found schedule	Required time to get the best solution (milliseconds)	Required iterations to get the best solution
5	279.0302	70781	106
10	279.0614	72234	107
20	278.7891	86078	130
30	278.6068	115016	173
50	278.2272	111828	166

4.6.4 MAX-MIN ANT SYSTEM (MMAS)

The main characteristics of MMAS are:

- Only the iteration-best ant or the best-so-far ant is allowed to deposit pheromones.
- Pheromone trail values are limited to the interval $[\tau_{min}, \tau_{max}]$.
- Pheromone trails are initialized to the upper pheromone trail limit.
- Pheromone evaporation rate is very small.
- Pheromone trails are reinitialized each time the system approaches stagnation or when no improved tour has been generated for a certain number of consecutive iterations.

As described in AS, it is necessary to make a justification for the values of the constructed mine schedules in order to abate the effect of negative schedules. This converts the discounted value of the solutions to the distance of [0,1] in each iteration. Therefore there will not be any difference between the iteration best solution and best so far solution (both being 1). Eventually the value of the initial pheromone (and also the τ_{max}) could be calculated as $1/\rho$ which becomes, in this case, study equal to 50 considering $\rho = 0.02$. Our experiments showed that using slightly higher values of ρ (0.03 to 0.05) with $2/\rho$ to $3/\rho$ (i.e. 40 to 80) initial pheromones could reach better solutions in less iteration. Unlike in the TSP application the value of τ_{max} is constant here and will not change during the iterations.

In the solution of TSP, the ratio of τ_{min}/τ_{max} is suggested by Stützle (1999) to be as $(1 - \sqrt[n]{0.05})/((avg - 1).\sqrt[n]{0.05})$, where avg is the average number of different choices available to an ant at each step while constructing a solution. A fixed value of 5 is used for the τ_{min} in our case study. The pheromone trails are re-initialized when no improvement occurs after 10 iterations.

Figure 4-15 shows that the MMAS is able to get out of stagnation situations and improve the quality of solution to 284.5371 which is slightly higher than that of previous variants. In this case the evaporation rate and initial pheromone rate have been supposed as $\rho = 0.04$ and $\tau_{initial} = 60$.

The main power of MMAS comes from its explorative nature which lets the program use higher perturbation distances which may lead to better solutions. However this will take more calculation time and higher scattering iterations before improvements are noticed, Figure 4-16.

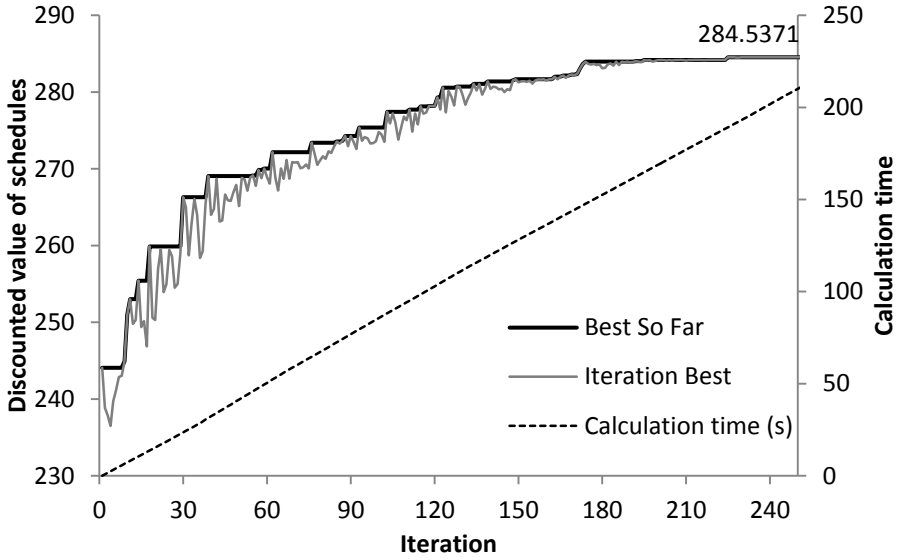


FIGURE 4-15 EFFICIENCY OF MMAS WITH $\rho = 0.04$ AND $\tau_{initial} = 60$

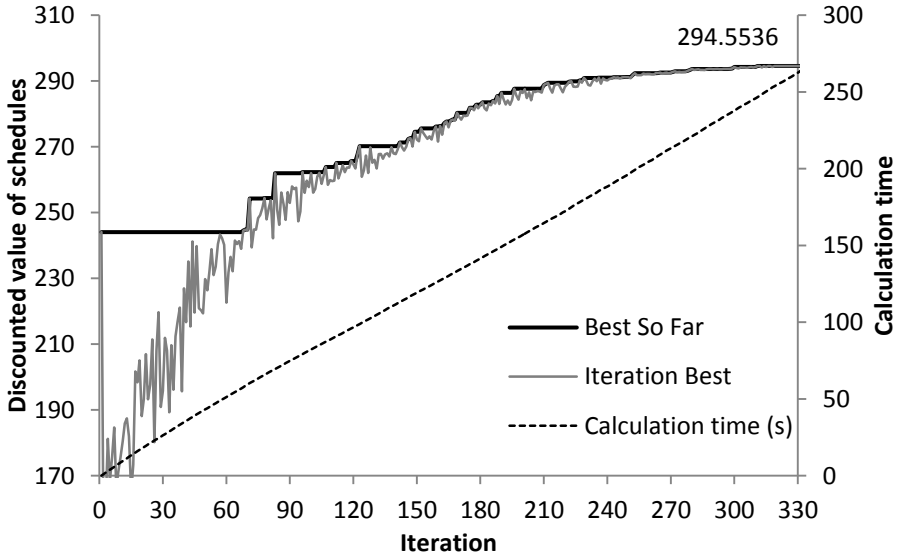


FIGURE 4-16 POSSIBILITY OF USING HIGHER PERTURBATION DISTANCE AND APPLICATION OF MMAS TO GET BETTER SOLUTION

4.6.5 Ant Colony System (ACS)

The ant colony system differs from the previous variants from the following points of view:

- **Pseudorandom proportional action choice rule:** with the probability of q_0 the ants makes its destination to the node which has the highest $[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}$. Whereas it has the $1-q_0$ probability of using the same routine as AS for the selection. A value of $q_0 = 0.9$ is suggested for the TSP solution.
- **Global pheromone update:** in ACS only one ant (the best-so-far ant) is allowed to add pheromones after each iteration. Additionally, unlike AS, the evaporation process only applies to the arcs of the best-so-far tour, not to all the arcs.
- **Local pheromone update:** the ants use a local pheromone update rule that they apply immediately after having crossed an arc during the tour construction of ACS.

In order to evaluate the efficiency of ACS on our case study, the best values of the parameters were found as below:

- Number of ants: 10
- Evaporation rate: 0.1
- Initial pheromone value: 0.01
- Local pheromone update factor: 0.1
- Pseudorandom choice probability: 0.9

Obtained results of using ACS have been shown in Figure 4-17. The main noticeable point in ACS is that the number of ants has been drastically reduced which has direct effect on the calculation time of each iteration. For instance, the run time of iterations have been reduced from 700 to 1000 milliseconds in previous variants of ACO, to less than 100 ms in ACS. Another factor that helps the speed of the ACS algorithm is the fact that pheromone evaporation and deposition happen only on the arcs of the best so far solution. Consequently, when compared to the other variants of ACO, ACS could reach much better solutions in a given time of calculation. This might be very beneficial for the big block models.

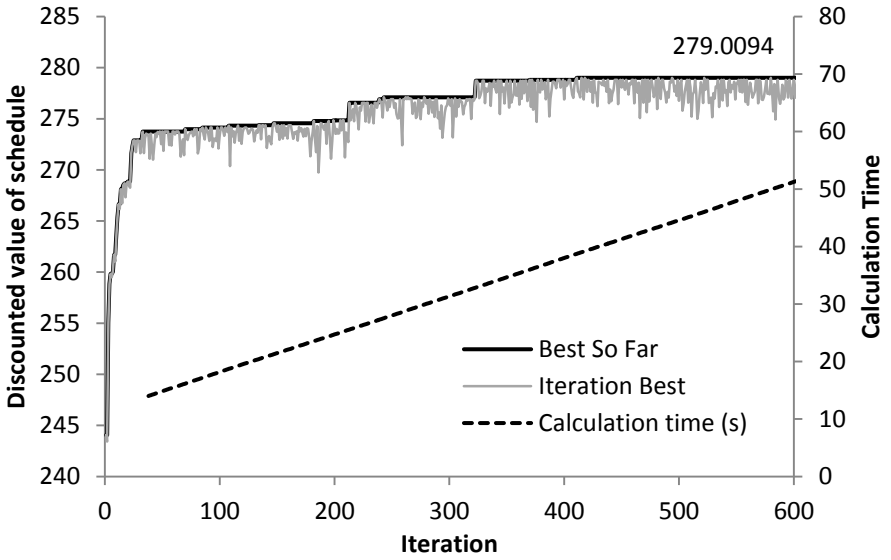


FIGURE 4-17 EFFICIENCY OF ACS WITH $\rho = 0.1$ AND $\tau_{initial} = 0.01$

5 CONCLUSION

A new algorithm for open-pit optimization using ant colony optimization has been developed and used to optimize the long-term scheduling of open-pit for a two dimensional randomly simulated block model. The algorithm is inspired by foraging behaviour of ants.

The shapes of the pits (respective of the slope angles) are represented by means of a simple array of integer numbers. Each element of this array shows the depth of a pit in an individual column of block model. Extending this concept to the long-term production planning, any mine schedule is represented by an array that has several mine depths at each column of block model related to different production periods.

In order to simulate the process, some programming variables are considered for each block as the pheromone trails. The numbers of these variables are equal to the number of planning periods. In fact these pheromone trails represent the desirability of the block for being the deepest point of the mine in that column for the given mining period.

At the beginning, the values of the pheromone trails are initialized according to a mine schedule generated by Lerchs-Grossmann's algorithm and the alternative to parameterization algorithm of Wang & Sevim. During initialization, relatively higher values of pheromone are assigned to those blocks that are close to the deepest points of the push backs in the initial mine schedule. This leads the procedure to construct a series of random schedules which are not far from the initial solution.

In each ACO iteration, several mine schedules are generated based on the current pheromone trails. This is implemented by a process called "depth determination". In this process the depth of a mine in each period is determined for each column of the block model. The higher the value of the pheromone trail of a certain block, the higher the possibility of selecting that block as the pit depth in that period. Then the pheromone values of all blocks are reduced to a certain percentage (evaporation) and additionally the pheromone value of those blocks that participated in defining the constructed schedules are increased according to the quality of the generated solutions. Via repeated iterations, the pheromone values of the blocks which define the shape of the optimum solution are increased whereas those of the others have been significantly evaporated.

The analysis carried out on the case study revealed that the ACO can improve the value of the mining schedule by up to 34%. This is mainly contributed to the fact that penalties can be considered in relation to their deviation from the permitted limits.

5.1 DISCUSSION

The major benefits of using of the proposed algorithm compared to the former methods are:

- Most of the available methods and algorithms follow a certain target (such as highest NPV, lowest stripping ratio etc.) in their solution strategy; therefore, later changes in the desired target would be very difficult or impossible with them. The ACO approach is able to consider any kind of objective functions in the optimization process. Even very complex objective functions do not have a significant influence on the efficiency of the method. This is because of the fact that all schedules are generated based on the pheromone values in the ACO, regardless of the objective function. Then the values of the generated schedules are calculated in the next step according to the defined target.
- Variable slope angles can be modelled with ease in the generated schedules. The method even has the ability of applying very complex slope differentiations. The only change in the program would only be required in the normalization routine. It is also possible to consider working slope angles by supposing different values for the slopes of the inner periods and the most outer phase.
- During the ACO optimization iterations thousands of mine schedules are randomly created according to the pheromone values. In order to model uncertainty related to the characteristics of the blocks, these schedules can be constructed based on a series of the random variables instead of deterministic values.
- In each iteration of the ACO, n number of mine schedules are being constructed. The calculation time of the algorithm is highly dependent on the value of n which is usually (except for ACS) considered equal to the number of block columns in the model. In other words the calculation time of a double sized block model in each direction (8 times more blocks) is expected to be only 4 times more. Therefore the required calculation time of a block model with one million blocks is expected to be around 100 times that of the case study used in this research.
- In the Ant System (AS) and Elitist Ant System (EAS) variants of the ACO algorithm, a large number of mining schedules (all constructed schedules) have to be saved in the memory during each iteration to be used in the pheromone update stage. This makes the application of these variants very difficult, or even impossible, for large block models (because of being heavily memory intensive). Memory usage relatively decreases in the AS_{rank} and reaches an acceptable range in the Max-Min Ant System (MMAS) and Ant Colony System (ACS) where only the best schedule needs to be saved

during the iteration. For a block model with one million blocks, the capacity of 4MB will be sufficient for the MMAS and ACS variants, for example.

On the other hand disadvantages of the method which have to be considered are:

- The process is not mathematically proven to always reach the best schedule.
- The ACO algorithm needs to save numbers of variables in the memory for each block representing the desirability of the block for being the pit depth in different mining periods. In fact, the number of these variables is equal to the number of planning phases. In addition, it might be essential for large block models to provide another module to exclude unnecessary blocks and to manage the required memory.
- The efficiency of ACO algorithm is highly dependent on the parameters like number of ants, evaporation rates, deposited pheromones in each iteration, etc. The found combinations of these parameters for this case study are not essentially the best combination for all deposits and block models. Hence a trial and error process might be necessary at the beginning to set the relevant combination of parameters for each individual case.
- In addition to the initial solution's primary function of leading the algorithm towards a relatively good solution, it is also necessary to control the size of generated pits. Without an initial solution, the program might scatter among unacceptable sized solutions. As described in Chapter 4.1, relatively higher values are assigned to the blocks close to the initial solution depths in order to initialize the pheromone values. However, adding high pheromone values only to the small number of blocks does not let the algorithm to deviate from the primary schedule. The distance that the schedules are allowed to be constructed is set by the perturbation number during the initialization. The bigger this value, the higher the possibility of finding better solutions. For the studied case, only the max-min ant system (MMAS) was able to accept a higher perturbation distance. Hence obtaining the optimum solution is not always reachable by the other variants of ACO if it is far from the initial solution.
- The required memory for a large block model is high for the AS, EAS and AS_{rank} variants which make them impractical for a real deposit in practice.
- Calculation time is around two hours for a block model with one million blocks except the ACS variant which is fast enough even for large models. The only problem which we faced with the ACS in the studied case was that it was not explorative enough to approach the optimum solution.

5.2 PERSPECTIVE RESEARCH

The current research comprised a background study for the application of the new metaheuristic methods in the optimization of the long-term open-pit planning. Further investigations are suggested in the following fields.

- The elaborated program has allowed for the implementation of two dimensional cases and a 1:1 slopes. However, its application in a real mining case has not been tested yet. Supplementary programming is suggested to be done in a 3D extension of the algorithm, and should consider different angles and working slopes. Additionally it is also suggested that a faster programming environment such as c++ to be used instead of the currently used VB language.
- The studied case shows that the ACS is comparatively fast and MMAS provides a relatively explorative approach. Application of a combination of these two alternatives is suggested to be studied.
- The family of metaheuristics is not limited to the studied algorithms explained in this thesis. Application of other methods such as particle swarm optimization (PSO) and Tabu search (TS) are certainly additional future research subjects.

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