Judgments Under

Competition and Uncertainty:

Empirical Evidence From Online-Poker

Von der Fakultät für Wirtschaftswissenschaften der Rheinisch-Westfälischen Technischen Hochschule Aachen zur Erlangung des akademischen Grades eines Doktors der Wirtschafts- und Sozialwissenschaften genehmigte Dissertation

vorgelegt von

Dipl.-Kfm. Jörg Engelbergs

Berichter:
Univ.-Prof. Dr.rer.pol. Rüdiger von Nitzsch
Univ.-Prof. Dr.rer.pol. Wolfgang Breuer

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To my family.
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\( b \) Amount bet
\( c \) Amount called
\( r \) Amount raised
\( \alpha \) Excessive certainty/uncertainty in the probability weighting function
\( \mu^h \) The expected return from a hand \( h \)
\( \mu_i \) Player \( i \)'s expected return
\( \sigma^2_i \) Player \( i \)'s expected variance
\( \sigma^{2,h} \) The expected variance from hand \( h \)
\( \beta \) Event or outcome pessimism in the probability weighting function
\( \gamma \) Exponent in the value function determining curvature
\( \hat{\mu} \) Growth-optimal return given wealth and risk
\( \hat{\Omega} \) Optimal bankroll given risk and return of a game
\( \hat{A} \) Deviation of aggressiveness from average players’ aggressiveness
\( \hat{L} \) Deviation of looseness from average players’ looseness
\( \lambda \) Coefficient of loss aversion in the value function
\( \mu \) Mean of a random variable
\( \mu^* \) Risk-free return
\( \mu_i \) Player \( i \)'s actual return (average amount won per hand)
\( \Omega \) Total wealth of a player, i.e. his bankroll or chips
\( \omega \) Share of wealth invested in a hand
\( \Phi^h \) Normalized relative share of hand \( h \) at showdown
\( \sigma \) Standard deviation of a random variable
\( \sigma^2 \) Variance of a random variable
<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\sigma_i$</td>
<td>Player $i$’s actual standard deviation of amounts won</td>
</tr>
<tr>
<td>$\tilde{\Omega}$</td>
<td>Cutoff bankroll between two gametypes</td>
</tr>
<tr>
<td>$A$</td>
<td>Aggressiveness</td>
</tr>
<tr>
<td>$a$</td>
<td>Minimal time required for a task in the power law of practice</td>
</tr>
<tr>
<td>$act$</td>
<td>Number of actions in a hand</td>
</tr>
<tr>
<td>$b$</td>
<td>Practice effect in the power law of practice</td>
</tr>
<tr>
<td>$bd$</td>
<td>Proxy for boredom, elapsed time over last 10 hands adjusted for actions</td>
</tr>
<tr>
<td>$c$</td>
<td>Rate of learning in the power law of practice</td>
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<tr>
<td>$co$</td>
<td>Proxy for perceived control, bankroll relative to the mean bankroll of the opponents</td>
</tr>
<tr>
<td>$CPT$</td>
<td>Cumulative prospect theory</td>
</tr>
<tr>
<td>$E(X)$</td>
<td>Expected value of a random variable $X$</td>
</tr>
<tr>
<td>$el$</td>
<td>Proxy for elation, rounded cumulated winnings over last five hands</td>
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<tr>
<td>$EU$</td>
<td>Expected utility</td>
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<tr>
<td>$EV$</td>
<td>Expected value</td>
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<tr>
<td>$ex$</td>
<td>Experience measured as groups of 100 hands played so far</td>
</tr>
<tr>
<td>$fl$</td>
<td>The flop</td>
</tr>
<tr>
<td>$H$</td>
<td>Total of hands</td>
</tr>
<tr>
<td>$h$</td>
<td>A subset of poker hands</td>
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<tr>
<td>$i$</td>
<td>Index for players</td>
</tr>
<tr>
<td>$L$</td>
<td>Looseness</td>
</tr>
<tr>
<td>$N$</td>
<td>Total number of cases possible</td>
</tr>
<tr>
<td>$n$</td>
<td>Favorable outcomes to an event</td>
</tr>
<tr>
<td>$nh$</td>
<td>Average number of hands ever played across players in the game</td>
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<tr>
<td>$p$</td>
<td>Probability of an event</td>
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<tr>
<td>$P_i$</td>
<td>Amount bet on horse $i$ to place</td>
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<tr>
<td>$ps$</td>
<td>Part of a player’s session measured as percentile</td>
</tr>
<tr>
<td>$PT$</td>
<td>Prospect theory</td>
</tr>
<tr>
<td>$ri$</td>
<td>The river</td>
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LIST OF SYMBOLS

rp Relative position on the x-axis of the value function
RT Reaction time for processing a task
rt Reaction time in the poker game
$S_i$ Amount bet on horse $i$ to show
$sd$ The showdown
SEM Subjectively expected money
SEU Subjectively expected utility
$T$ Track “take”; transaction costs at racetrack betting
$t$ Elapsed time between hands
$ti$ Proxy for distressed mood (tilt), depth of the player’s bankroll at the beginning of the hand
$tu$ The turn
$u(x)$ Utility function
$v(x)$ Value function
$W$ Win pool
$w(p)$ Probability weighting function
$W_i$ Amount bet on horse $i$ to win
Glossary

A

aggressive  Playing style with dominantly bets and raises.

aggressiveness  Ratio of amounts bet or raised to the total amount staked by a player.

all-in  A player who is betting all his remaining chips is considered all-in.

all-in-equity  The strength of a hand without consideration of possible actions of the players; also called showdown value.

B

bad beat  Losing a hand which is a strong favorite.

bet  If no bet has been made yet a player can stake money according to the limit structure.

big blind  Second position left of the dealer which has a forced bet of the size of one standard betting unit.

bot  A computer program playing poker. An application of Artificial Intelligence.

brick and mortar  Venues for in-person play such as casinos or cardrooms.

button  The dealer’s position; the last position to act.

C

call  Increase one’s stake to the amount of the player who has made the largest bet so far.

calling station  A player with a loose-passive style.

cap  Increase the largest bet so far to the maximum allowed.

CAPM  Capital Asset Pricing Model.

check  Pass the action on to the next player; only allowed if no prior bet has been made.
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<td>coin flip</td>
<td>A situation where the odds are 1 to 1.</td>
</tr>
<tr>
<td>connectors</td>
<td>Hole cards with neighboring ranks.</td>
</tr>
<tr>
<td>CPRG</td>
<td>The Computer Poker Research Group at the University of Alberta.</td>
</tr>
<tr>
<td>dead money</td>
<td>Money put into the pot by players who are not active any more.</td>
</tr>
<tr>
<td>family pot</td>
<td>A pot in which all players participate.</td>
</tr>
<tr>
<td>flop</td>
<td>Second phase of the game in which three community cards are dealt.</td>
</tr>
<tr>
<td>fold</td>
<td>Lay down a hand and give up any interest in the pot.</td>
</tr>
<tr>
<td>hole cards</td>
<td>Cards dealt to a player as private information only; also called pocket cards.</td>
</tr>
<tr>
<td>implied odds</td>
<td>Ratio of amount to pay to current pot plus additional money from bets to come.</td>
</tr>
<tr>
<td>IRC</td>
<td>Internet Relay Chat. A form of real-time Internet text messaging (chat) or synchronous conferencing.</td>
</tr>
<tr>
<td>kicker</td>
<td>Additional card used to break ties for otherwise equal hands.</td>
</tr>
<tr>
<td>limit structures</td>
<td>The minimum/maximum bets allowed in a poker game.</td>
</tr>
<tr>
<td>loose</td>
<td>Few hands are folded; a wide range of hands is played.</td>
</tr>
<tr>
<td>looseness</td>
<td>Share of hands dealt to a player he at least plays to the flop.</td>
</tr>
<tr>
<td>maniac</td>
<td>A player with a loose-aggressive style.</td>
</tr>
<tr>
<td>Monte Carlo sim.</td>
<td>A simulated search algorithm which evaluates a fixed number of random game states.</td>
</tr>
</tbody>
</table>
offsuit  Hole cards with different suits.

passive  Play which is not actively increasing stakes but reacting to others’ bets.

pocket cards  See hole cards.

pocket pairs  Paired hole cards.

pot  Sum of all stakes bet so far by the players.

pot odds  Ratio of amount to pay to size of the pot.

pre-flop  First phase of the game in which every player is dealt two hole cards.

raise  Increase the largest bet so far within the maximum allowed.

rake  Fee taken by poker venues such as casinos or online providers.

river  Fourth phase of the game in which a fifth and final community card is dealt.

rock  A player with a tight-passive style.

semi-bluff  To bluff with a hand which is weak now but can get very strong.

shark  A player with a tight-aggressive style; also called stone killer.

showdown value  See all-in-equity.

slowplaying  Representing a weak hand when in fact one is holding a very strong hand.

small blind  First position left of the dealer which has a forced bet of half the size of a standard unit.

stone killer  See shark.

suited  Hole cards with matching suit.
Glossary

T

tells  Reactions that reveal information about a player’s cards, usually physical.

tight  Play where only a small range of hands is played and most hands are folded.

Tilt  Loss of control and deviation from one’s usual play.

turn  Third phase of the game in which a fourth community card is dealt.

W

WSOP  World Series of Poker.
Part I

Introduction
Chapter 1

Introduction

... omnia, nunc se continet atque duas tantum res anxius optat, panem et circenses. [... everything, now restrains itself and anxiously hopes for just two things: bread and circuses]

Juvenal, Satura, 10.79-81

Why is it worthwhile to study decision-making in an artificial environment such as poker play? The following pages illustrate how the study of games has impacted on various fields in social sciences. The following discussion is extended to shed some light on the characteristics of the poker game which make it a particularly interesting object of study. The contribution and structure of this thesis are outlined at the end of the chapter.

1.1 Games and Social Sciences

1.1.1 Games of Chance and the Development of Probability

Ever since the first humans have used astragali several thousand years ago, chance has been present in games played in societies all around the globe. But not before the sixteenth century the element of chance was linked to mathematical concepts. It was questions such as “Why is a total of 10 more likely when throwing three dice than a total of 9?” or “How many times must one throw two dice to have at least an even chance of throwing two 6s?” that fueled the thoughts of the great thinkers of the time.

---

2Dice made from ankle bones.
3See Epstein (1967, chapter 1).
4See Arnold (1977, pp. 36-38) or Epstein (1967, chapter 3).
Well-known mathematicians like Galileo, Pascal, Fermat, Bernoulli, Bayes or Gauss have worked on problems of games, thus creating a new branch of mathematics, the theory of probability.\footnote{See Levinson (1963, chapter 2).} At the heart of this theory is the fundamental definition of probability which makes chance events a calculable law within sciences. It states that, provided all outcomes are equally likely to occur, the probability of an event $p$ is the number $n$ of cases favorable to that outcome divided by the total number of cases possible $N$.\footnote{See e.g. Arnold (1977, part two).}

$$p = \frac{n}{N} \quad (1.1)$$

This finding demystified events in games of chance such as dice and cards that so far had been considered instruments of religion and magic.\footnote{See e.g. Martinez (1983, pp. 14-18).} On this basis, as Borel (1924)\footnote{Translated in Borel (1953).} writes,

"It was little by little that through the study of simple problems raised by the game of dice, or even the still more elementary game of heads and tails, one was led to conceive of methods by which more complex problems could be treated."

Thanks to games we know about the law of large numbers, combinations and permutations or probability distributions, to name just a few of the further discoveries.\footnote{See Arnold (1977, part two) or Epstein (1967, chapter 2).}

### 1.1.2 Game Theory and Rational Decisions

Once one extends the study to games that involve not only chance but also elements of skill, situations involving conflict and cooperation arise as players have different preferences. Players opt between alternative actions to arrive at their preferred outcome whilst their opponents do likewise.\footnote{See Lucas (1972).} Games serve as simple environments of social interaction with clearly defined rules and goals.

The groundbreaking work as a normative guide on optimal play was the *Theory of Games and Economic Behavior* by von Neumann and Morgenstern (1944). Their analysis of parlor games (like poker) rapidly unfolded at pan-disciplinary level influencing fields such as economics, political science or evolutionary biology.\footnote{See Leonard (1995, pp. 730-731).} The analogy between games and the business world is emphasized by McDonald (1950) as he quotes John Maynard Keynes,

\begin{quote}
See McDonald (1950).
\end{quote}
“Businessman play a mixed game of skill and chance, the average results of which to the players are not known by those who take a hand.”

Many of the early works on game theory have analyzed models of poker (e.g. Kuhn 1950; Nash and Shapely 1950; Nash 1951; Karlin and Restrepo 1957; Goldman and Stone 1960). The game serves as an illustration for topics such as bluffing, (mixed) strategies, signaling, value of information, or updating probabilities and is still instructive to today’s students of rational decision-making.\(^\text{12}\)

As we add another component to the playing of games, the staking of money, we arrive at a further reference of the importance of games on social sciences, namely gambling. The importance of this is already noted by Churchill (1894, pp. 7-11) who observes that the term gambling not only covers playing for money in games but extends to all kinds of betting or speculation in games, business or otherwise.

### 1.1.3 Gambling and Bounded Rationality

At first it was the fundamental properties of games that led to the notion of probability in mathematics. Then it was the search for optimal strategies a rational decision-maker would choose in games that linked the mathematics of game theory to economics. But consequently when comparing the normative principles of game theory with observed decisions in real-life situations, it was evident that they were unsatisfactory as a descriptive model.\(^\text{13}\)

The question for economists and psychologists alike became “How do people actually behave, what are discrepancies to the concepts of rationality and which are the causes?”.\(^\text{14}\) Representative for the discussion is the concept of bounded rationality coined by Herbert A. Simon.\(^\text{15}\) It states that individuals cannot arrive at an optimal solution because they face limitations on available information, cognitive skills, or amount of time to make their decisions. Instead they are seeking a satisfying solution given their restrictions.

While early mathematicians analyzed games of chance to finally develop mathematical probability, early experimenters in psychology like Cohen and Hansel (1956) used elementary gambles to study the nature of subjective probability. But even the simplest gambles present multidimensional stimuli to human cognition.\(^\text{16}\) Central to the analysis

\(^{12}\)See Reiley, Urbancic, and Walker (2005).
\(^{13}\)See e.g. Edwards (1961).
\(^{14}\)See Einhorn and Hogarth (1981).
\(^{15}\)See e.g. Simon (1959).
\(^{16}\)See Payne and Braunstein (1971).
CHAPTER 1. INTRODUCTION

of preferences among these stimuli, e.g. Payne (1975), is the concept of risk, its perception by individuals and the role it plays in determining preferences. A lot of research has been done on risk and quoting Lopes (1983) “The simple, static lottery or gamble is as indispensable to research on risk as is the fruitfly to genetics.”.

It stands to explore how much more can be learned from games once the focus is extended from simplified versions to the full scale of games like poker.

1.2 Qualities of the Game of Poker

From a game-theoretic perspective poker is a n-person, zero-sum, imperfect information, chance, sequential, repeated, non-cooperative game.

- **N-person.** Poker can be played from 2 players up to virtually no limit. Actual participation can easily reach several thousand players in large tournaments. The main event at the World Series of Poker (WSOP) featured at the most 8,773 players in 2006.\(^\text{17}\) Online providers offer tournaments of this size quasi hourly.

- **Zero-sum.** No value is created by playing poker. All money in the game is redistributed, but not necessarily between the players. Depending on the venue there might be a take from the house, the so called *rake*. These transaction costs range from zero, when playing at home with friends, to 1%-5% of the pot size though capped at some fixed amount in online-play.\(^\text{18}\) Costs in casinos can even be higher.

- **Imperfect information.** In contrast to board games where all players have complete knowledge of the entire game state at all times, poker players have to deal with imperfect information. A player’s cards are private information, i.e. they are hidden information for all opposing players.\(^\text{19}\)

- **Chance.** By shuffling the deck of cards randomness is added to the game. The cards that players are dealt are determined by chance. Thus there is an element in poker which cannot be controlled but which is calculable.\(^\text{20}\)

- **Sequential.** The order of play is determined by the players’ position at the table. With every round of play the order is changed so that no player has a continuous advantage from sequential play.

\(^\text{18}\)See e.g. [http://www.pokerstars.com/de/poker/room/rake](http://www.pokerstars.com/de/poker/room/rake).
\(^\text{19}\)See Billings, Davidson, Schaeffer, and Szafron (2002, p. 201).
1.2. QUALITIES OF THE GAME OF POKER

- Repeated. A game consists of a series of hands being dealt. Thus players can adjust their play over the course of a game. Repeated play also serves to allow for cards being dealt to balance over time and change the sequence of play.

- Non-cooperative. Poker is a competitive game where each individual player tries to win as much as he can. Nevertheless he might try to achieve this by implicitly colluding with an opponent against other opponents.\(^{21}\)

Taken from Koller and Pfeffer (1997) table 1.1 gives some examples on games comparing the influence of chance and information. They argue from a game-theoretic perspective that it is the presence of imperfect information which substantially increases the complexity of the game.\(^{22}\) The importance of imperfect information and the resulting uncertainty is also evident from research in Artificial Intelligence. Whereas games of perfect information with or without chance elements are solved by methods like deep search, progress on strong programs on games of imperfect information is limited.\(^{23}\)

<table>
<thead>
<tr>
<th></th>
<th>Perfect information</th>
<th>Imperfect information</th>
</tr>
</thead>
<tbody>
<tr>
<td>No chance</td>
<td>Chess</td>
<td>Inspection game</td>
</tr>
<tr>
<td></td>
<td>Go</td>
<td>Battleships</td>
</tr>
<tr>
<td>Chance</td>
<td>Backgammon</td>
<td>OPEC game</td>
</tr>
<tr>
<td></td>
<td>Monopoly</td>
<td>Poker</td>
</tr>
</tbody>
</table>

Source: Koller and Pfeffer (1997).

Even if a theoretical solution to the game is found, the strategic complexity still overstrains currently available computational power. Fairly simple variants such as 2-player limit Texas Hold’em poker or five-card draw poker have about \(10^{18}\) respectively \(10^{25}\) different possible states in the game.\(^{24}\) The strategic environment of poker is so complex that March and Shapira (1987, p. 1413) state

“[...] the choice of a particular business strategy depends on the same general consideration as the choice of a betting strategy in a game of poker.”

Dreef, Borm, and van der Genugten (2003) have formally proved that even in simple variants of poker some skill is involved. But what constitutes the necessary skill is not

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\(^{21}\) Explicit collusion is not allowed and seriously opposed in card rooms.

\(^{22}\) See Koller and Pfeffer (1997, p. 169).

\(^{23}\) See Billings, Davidson, Schaeffer, and Szafron (2002, p. 201).

easily pinpointed. Martinez (1983) argues that one of the poker game’s qualities is that it reflects many of society’s values such as personal competition or the opportunity to show strong character. For him a consistent winner makes his bets based upon rational, rather than social, considerations. With an additional inclination Borel (1953) points out that an excellent player is not only skillful at combinations but also a good psychologist who varies his play according to his adversary’s manner of play.

Maybe the most prominent quality that makes poker interesting to study from a business student’s point of view is its objective. The one observable fact that distinguishes a winner from a loser at poker is how much money he has made. Trying to maximize wins and minimize losses is quintessential to score as the game is fair. Over the long run everybody is dealt the same share of strong and weak hands.\textsuperscript{25}

\section{1.3 Contribution and Structure of This Thesis}

The central question of this thesis is which heuristics and biases are present in decision-making at poker play. With reference to the discussion above under 1.1 and 1.2 the question can be refined accentuating four core themes. Is there empirical evidence for behavioral patterns in decision-making in a competitive game with uncertainty? Benefits to three areas, namely academic research, poker play, and other decision-making might arise.

Contributions to academic research in behavioral decision-making along the above mentioned four themes are as follows:

- We use empirical data from unsolicited play. As Croson and Sundali (2005, p. 206) suggest this provides a control on results mostly based on lab data. Subjects act self-motivatedly and any bias due to deliberately compliant behavior can be ruled out. As a drawback we cannot isolate specific influences but have to deal with interaction between several factors.

- Further evidence for heuristics and distortions of cognitive processes and the influence of emotions on decisions is presented and discussed. This adds to the literature on behavioral decision-making as it analyzes observable irrationality in decisions.

- In contrast to the major share of the literature a dynamic and competitive environment is studied.\textsuperscript{26} So we can test whether additional motivation arising from competition will reduce or even avoid some biases. Repeated play allows players to

\textsuperscript{25}See Sklansky (1987, pp. 3-6).
\textsuperscript{26}Cf. Lopes (1983, p. 137).
1.3. CONTRIBUTION AND STRUCTURE OF THIS THESIS

learn and improve on their following decisions. More specifically, not only learning from own experience but also learning from competitors is relevant.

- Compared to many studies made under conditions of risk here decisions under uncertainty are analyzed.27 A factor common in real-life decisions where agents might well know possible outcomes but rarely have a way to attach probabilities to them.

To further illustrate the nature of this work we would like to point out four studies which are closely related to the core question of this thesis. First, Smith, Levere, and Kurtzman (2009) study risk-taking behavior of professional poker players in high-stakes games dependent on recent big wins and losses. This work is discussed further in sections 6.6.3 and 10.2. Second, Looijmans, Wiersema, and van Wijngaarden (2008) use situations in Texas Hold’em Poker to test for distorted judgments of probabilities, sunk cost effects, and framing effects in an online experiment. Thereby, participants had to respond to particular instances rather than actually play a repeated game. Third, di Zazzo and Tjäderborn (2006) use a small sample to compare differences in decision-making strategies between more and less experienced poker players. They discuss their findings in light of psychological influences but results are short of significance due to the limited scale and scope of the work. Fourth, Keren and Wagenaar (1985) investigate decision processes of blackjack players in a natural setting. After introducing a normative approach to the game, they find that the observed behavior can best be described by the concept of bounded rationality.

Outside the academic field other authors have touched upon the link between poker and decision-making in business contexts. Their illustrations are instructive to see the ample analogies between playing poker and real life decisions. Brown (2006) shows how gambling and risk-taking concepts link the poker table to an options trader’s trading desk or more generally to investment banking and finance. Müller and Cmiel (2008) go into more detail, pointing out how specific poker hands pose problems similar to setups in trading on the stock market. From the perspective of a wall street attorney Apostolico (2007) demonstrates the usefulness of poker strategies in negotiations and other business contexts.

For those mainly interested in playing poker this work is relevant due to the following aspects:

- It presents data from a broad third-party basis, contrary to most of the poker literature which uses data individually collected by the authors.28

27Ibid. pp. 138-141.
28See e.g. Grudzien and Herzog (2007).
CHAPTER 1. INTRODUCTION

- Psychology which as in Caro (2003) has been mostly linked to physical reactions that reveal information about a player’s cards, so called tells, is extended to psychological influences which are detectable in online-play.

Finally every decision-maker in business, politics or other fields might find this work interesting as:

- Decisions in poker are made under uncertainty, a situation common to real-life decisions. The poker environment offers an easy to access analogy from which influences on one’s own decisions can be recognized.

- The competitive poker environment with its goal of profit maximization closely resembles the situation business managers are facing on a daily basis. Aspects of this environment such as capital management, using competitive advantages or selection of strategies are instructive to improve on decision processes.

An overview of the outline of this thesis is presented in figure 1.1. The structure is divided into five distinct parts.

The introductory first part gives arguments for the relevance of games to academic research and the particular properties of poker which distinguish it from other games. The contribution and structure of this thesis conclude this part.

The second part “The Poker Environment” describes characteristics of the game. Historical developments of poker, its cultural implications, the basic rules and a short mathematical treatise of the currently most popular variety Texas Hold’em are delineated in chapter 2. To establish a basis for further illustrations and to show how decisions in poker can be analyzed, the data set is presented in chapter 3. Chapter 4 installs the psychological dimensions of playing behavior and introduces different types of players. In the final chapter 5 of this part the financial perspective of the poker environment is taken and the discussion revolves around analogies to business and financial markets.

Part three with the sole chapter 6 introduces normative considerations on decision-making. After discussing how concepts of rational decisions can be applied in situations of risk and uncertainty in the game, hypotheses on psychological influences on actual behavior are deduced with reference to behavioral finance. Therefore, relevant literature is presented, briefly reviewed and put into the poker context. The structure of this part matches the empirical tests in the following part.

Tests of the hypotheses established in part three are conducted in part four of this thesis which consists of chapters 7 to 11. Every chapter in this part focuses on a distinct area
of systematic patterns in decision-making. Within each chapter the basic recurring structure is the test method, results and discussion or conclusion.\textsuperscript{29} Chapter 7 demonstrates biases in values due to the reference point effect and the overweighting of low probabilities in line with prospect theory.\textsuperscript{30} As the game progresses, players have to process information. Their heuristic use of information on grounds of availability and representativeness is tested in chapter 8. How players generally adjust their play over time is analyzed in chapter 9. Whether players’ emotions are influenced by observable factors in online-play and subsequently make systematically different decisions is examined in chapter 10. Poker being a game of chance and skill, chapter 11 explores what players’ decisions reveal about their confidence in their skills.

The final part five concludes this thesis. It exhibits perspectives on how to use the empirical findings to advance one’s poker play and indicates further opportunities for research from the study of poker.

\textsuperscript{29}Literature and hypotheses are introduced in part III.

\textsuperscript{30}See Kahneman and Tversky (1979).
| Part I: Introduction | • Why is it worthwhile to study games?  
• What makes poker particularly interesting? |
| --- | --- |

**What characterizes the poker environment?**

| Part II: The Poker Environment | Chapter 2  
*What are social norms of the game?*  
History, tradition, rules ...  
Chapter 3  
*How can decisions be captured?*  
Different layers in the data set and characteristics of the data points  
Chapter 4  
*How can psychology be observed?*  
Measurement of behavior  
Chapter 5  
*Is poker a financial market?*  
Competition and parallels to finance |
| --- | --- |

**How should decisions be made in this environment?**  
*Which behavioral biases can be expected?*

| Part III: Decision-Making in Poker | • How can normative concepts be applied to poker play?  
• What are the implications of uncertainty?  
• Which are hypotheses on actual behavior? |
| --- | --- |

**Which systematic patterns in decision making can be found?**

| Part IV: Testing for Psychological Biases in Poker | Chapter 7  
*Cumulative prospect theory:*  
Biases in values and probabilities  
Chapter 8  
*Information heuristics:*  
Distortions due to biased evaluation of information  
Chapter 9  
*Improving play:*  
Predictability of adjustments over time  
Chapter 10  
*Charged with emotions:*  
Factors influencing emotions $\rightarrow$ effects on play  
Chapter 11  
*Judgments of skill:*  
Players’ confidence in their relative abilities |
| --- | --- |

**Where to go based on the empirical findings in poker?**

| Part V: Conclusion | • What are ways to react to the existence of the patterns?  
• Which further questions are open to discussion? |
| --- | --- |

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*Figure 1.1: Outline of This Thesis*
Part II

The Poker Environment
Chapter 2

Rules of the Game

But for its costliness and dangers, no better education for life among men could be devised than the gambling table—especially the poker game.


2.1 A Brief History of Poker

2.1.1 Genealogy

There is no definite account of how poker evolved through the ages. One of the first games bearing similarities to poker was played by the Chinese emperor Mu-Tsung around 969 with domino cards.\(^1\) But as it remains obscure how this game evolved into poker, more likely influences can be found in games of the same class as poker.

Poker classifies as a vying game where, instead of playing their cards out, the players bet on who holds the best hand by progressively raising the stakes. The game ends either with a showdown when the best hand wins all the stakes (“the pot”) or if only one player has not given up betting so that he wins the pot without showdown. Early vying games appeared from the fourteenth century onwards in Europe. Among them are the German Pochen or Pochspiel (15th century) which was played in France first under the name of Glic and subsequently was called Poque, the English Brag (18th century) or the French game of Bouillotte (late 18th century). The influences of the Persian game As-Nas are disputed since it may as well be that As-Nas was derived from a European vying game. In table 2.1 characteristics of these games are compared to an early form of poker which

\(^1\)See [http://www.poker.com/history-of-poker.htm](http://www.poker.com/history-of-poker.htm).
was played with 20 cards and the game with 52 cards which developed later. The French Poque is also the likely origin of the word “Poker” as it crossed the Atlantic ocean with French settlers to New Orleans where it first became “pokuh” and then picked up its current pronunciation and spelling.\(^2\)\(^3\)

The earliest written references to Poker date back to the third decade of the 18th century. A well attested account of twenty-card poker is from 1847 when Jonathan H. Green mentions a game he first baptized “the cheating game” being played on Mississippi riverboats around 1834.\(^4\) The game was soon challenged by the 52-card game which allowed more players to participate and which ensured that there were enough cards for the recently introduced draw which supported the play for the new flush as well.\(^5\)

### Table 2.1: Relatives and Ancestors of Poker

<table>
<thead>
<tr>
<th></th>
<th>Bouillotte</th>
<th>Poque</th>
<th>As-nas</th>
<th>Poker I</th>
<th>Brag</th>
<th>Poker II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Players</td>
<td>4 (3, 5, 6)</td>
<td>4 (3, 5, 6)</td>
<td>4</td>
<td>4</td>
<td>3-6</td>
<td>3-6</td>
</tr>
<tr>
<td>Cards</td>
<td>20 (28)</td>
<td>32 (36)</td>
<td>20</td>
<td>20</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>Deal</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Turn-up</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Draw</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Fours</td>
<td>Fours</td>
<td>Fours</td>
<td>Fours</td>
<td>—</td>
<td>Fours</td>
<td>—</td>
</tr>
<tr>
<td>Threes</td>
<td>Threes</td>
<td>Threes</td>
<td>Threes</td>
<td>Threes</td>
<td>Threes</td>
<td>Threes</td>
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<td>—</td>
<td>—</td>
<td>—</td>
<td>Threes</td>
<td>Threes</td>
<td>Threes</td>
<td>Threes</td>
</tr>
<tr>
<td>2 Pair</td>
<td>2 Pair</td>
<td>2 Pair</td>
<td>—</td>
<td>—</td>
<td>2 Pair</td>
<td>—</td>
</tr>
<tr>
<td>Pair</td>
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<td>Pair</td>
<td>Pair</td>
<td>Pair</td>
<td>Pair</td>
<td>Pair</td>
</tr>
</tbody>
</table>


### 2.1.2 Conquering the West

With developing commerce on the waterways, gambling spread from New Orleans up the Mississippi and Ohio Rivers. From there it moved westward in the days of the frontier west and poker quickly became a favorite in saloons as pictured in figure 2.1. For the pioneering miners, railroad workers, cowboys and other fortune seekers with a preference for high risk, it provided an entertaining pastime.\(^6\)

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\(^2\)See McManus (2007d).

\(^3\)On this paragraph see Parlett (2005) and also McManus (2007a).


\(^5\)See Parlett (2005).

\(^6\)See Weiser (2005, p. 1) and McManus (2007c).
2.1. A BRIEF HISTORY OF POKER

By the end of the 19th century, gambling, and with it poker, had spread across the West. At about this time states enacted laws against gambling, culminating in the prohibition during the 1920s. In the 1930s Nevada was the first state to relax gambling laws and by the end of the decade Las Vegas had established its prominent role for the gambling scene.\(^7\)

This brief account of the history of the game shows how poker is inherently linked to Western culture. We have to bear this in mind when analyzing the decisions made in the game. The importance of cultural influences is highlighted by Henrich and McElreath (2002) who studied the decisions of small-scale farmers, the Mapuche of Chile and the Sangu of Tanzania, in risky monetary situations. They conclude that the behavior of these groups is substantially different to Westerners as they have not acquired the same rules and preferences for dealing with these situations via social learning. A similar study of economic behavior in fifteen small-scale societies on five continents by Henrich, Boyd, Bowles, Camerer, Fehr, Gintis, and McElreath (2001) also finds large variations across the different cultural groups. Social institutions or cultural fairness norms stemming from social interaction and modes of livelihood coin the preferences within a culture.\(^8\)

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\(^7\)See Weiser (2005, p. 2). An account of the growth of the casino gambling industry in the United States in later years is given by Eadington (1999).

\(^8\)It is not only society influencing gambling but also gambling which influences society. The repercussions gambling can have on societies are discussed, for example, by Churchill (1894) or Bloch (1951).
2.1.3 Modern Times and Online Play

Together with the spread of poker came the development of new varieties. Five-card stud was played as early as 1860. The seven-card version developed a little later toward the end of the century. The currently most popular variant Texas Hold’em was first nationally mentioned as “Texas Hold Me” in 1968. Three years later, in 1971, the year after the first World Series of Poker (WSOP) was held at The Horseshoe Casino where Benny Binion invited six of the best known poker players, Texas Hold’em became the game of choice to determine the champion player.\footnote{See http://en.wikipedia.org/wiki/World_Series_of_Poker and McManus (2008).}

From the 1970s on poker got sporadic coverage on television which gradually was extended and so helped to promote the game’s popularity. A major uplift in the attractiveness of the broadcast was achieved when in 2003 cameras were installed so that viewers could see the hole cards of the players. In 2009 the 40th Annual WSOP featured 57 events and was broadly covered on international sports channels. Simultaneously, the game experienced its second boost from technology with the rise of games on the internet. The industry experienced rapid growth in revenues from $82.7 million in 2001 to $2.4 billion in 2005 and expects a further growth rate of about 14%.\footnote{See http://www.newsweek.com/id/56438 and Ahlberg and Karlsson (2006, p. 2).} The market leader\footnote{The market is fragmented as entry barriers are low. In 2009 there are well over 600 poker sites online, see http://www.pokerscout.com/PokerSites.aspx.} Pokerstars reports having the most players simultaneously playing online when 151,758 players were active on December 30 2007.\footnote{See http://www.pokerstars.com/about.} In 2009 traffic for the major providers usually averages more than 60,000 players.\footnote{See http://www.pokerscout.com/IndustryOverview.aspx and http://www.pokerlistings.com/market-pulse/online-traffic.}

Playing online is different to conventional in-person play.\footnote{Conventional play is located in traditional venues such as casinos and poker rooms, also referred to as brick and mortar or live play.} The ability to observe others’ reactions and body language is removed. Instead, online players focus on betting patterns, reaction time, speed of play, shares of hands played and other non-physical tells. Additionally, the rate of play is increased. With instant shuffling, dealing, counting chips etc. players can easily play one hundred hands per hour at an online table compared to around thirty hands per hour in offline play. An example of an online table interface is shown in figure 2.2 at the center of which are the playing cards, the topic of the next section.
2.2. THE DECK OF CARDS

2.2.1 The French Pattern

Christian Crusaders and Venetian merchants first brought so called “Saracen cards” to Medieval Europe in the second half of the 14th century. Playing cards quickly became fashionable and numerous designs appeared all over Europe. But eventually the economical decks made from cheaply stenciled patterns from France were used most widely. So around 1470 the modern suit signs were established. The four suits are the red hearts (coeurs, ♥) representing the church, red diamonds (carreaux, ♦) for the merchant class, black spades (piques, ♠) signifying the state and black clubs (trefles, ♣) as symbols for the farmers. Examples of these cards can be seen in figure 2.3.\textsuperscript{15}

The pictures on the cards were a balanced collection of legendary heroes and heroines from Jewish, Greek, Roman and Christian history.\textsuperscript{16} As time progressed most of the characteristic features disappeared from the cards. Table 2.2 lists the group as it appeared on

\textsuperscript{15}On this paragraph see McManus (2007b) and International Playing-Card Society (2006).
\textsuperscript{16}This historical fact got perverted in April 2003 when the United States issued a list of the most wanted members of the Iraqi regime with their pictures on playing cards. See http://www.defenselink.mil/news/Apr2003/pipc10042003.html.
the Parisian pattern. In the 19th century a series of improvements changed the style of the cards. Cards got reversible (double-ended) figures and round corners. Indices, usually small markings in diagonally opposite corners, were added and a new card, the Joker, was introduced. Today the most durable playing cards are made from plastic and some have increased indices for better readability as in figure 2.4.\footnote{On this paragraph see International Playing-Card Society (2006).}

Table 2.2: Historical Persons in the Parisian Pattern

<table>
<thead>
<tr>
<th></th>
<th>Hearts</th>
<th>Spades</th>
<th>Diamonds</th>
<th>Clubs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kings</td>
<td>Charles</td>
<td>David</td>
<td>Caesar</td>
<td>Alexander</td>
</tr>
<tr>
<td>Queens</td>
<td>Judith</td>
<td>Pallas</td>
<td>Rachel</td>
<td>Argine</td>
</tr>
<tr>
<td>Jacks</td>
<td>La Hire</td>
<td>Ogier</td>
<td>Hector</td>
<td>Judas</td>
</tr>
</tbody>
</table>

2.2. Sinister Ploys

Any reader of the previous section has participated in a test similar to the one of Bruner and Postman (1949). Figure 2.4 contains an experiment on the perception of incongruous information. Like in Bruner and Postman (1949) one of the cards is printed in reversed color. The reader is asked to verify that there is a red ten of spades. As perception of a stimulus depends upon the expectancy of the observer about 90% of the subjects could not correctly recognize the sharped cards within one second of exposure. Instead four kinds of reaction appeared.\(^\text{18}\)

- Dominance. Subjects effectively denied the incongruous element. One of the characteristics dominates so the red ten of spades might be recognized as the ten of hearts or the ten of spades.

- Compromise. Two or more expectations are mixed creating something like a purple ten of spades or hearts.

- Disruption. None of the subject’s expectancies is matched and no conclusion is drawn as to the nature of the card. Quoting from Bruner and Postman (1949, p. 214) “I don’t know what the hell it is now, not even for sure whether it’s a playing card.”

- Recognition. The observer notices the incongruity either as spades being the wrong color or the red card being marked with the wrong suit.

\(^{18}\)Also see von Nitzsch and Goldberg (2004, pp. 59-62).
However, there is evidence that previous experience with incongruity reduces misperceptions.\textsuperscript{19} Furthermore if the information was blatantly contradictory, say, a blue ten of circles, this would result in easy recognition and also increase the resistance to other misinformation.\textsuperscript{20} An option to improve perception generally offered by internet poker software is the use of four-colored playing cards. In a typical four-color deck, hearts are red and spades are black as usual, but clubs are green and diamonds are blue which makes it easier to recognize a flush.

Poker has been used for ploys since its earliest days. Cardsharps used a wealth of devices to trick the unsuspicous. Quinn (1912/1969, 29-44) discloses many of the methods such as prepared cards, stocking techniques, holding out or shiners.\textsuperscript{21} Despite these cunning practices cheating is most of all a game of social psychology. For example, sharps in the steamboat era went to great length to appear as convincing gentleman, well-spoken, with courtly manners and well-tailored suits.\textsuperscript{22} As Cummins (1999) states, it are dominant individuals who monitor the behavior of those in lower-ranking positions to detect cheating and deception. By pretending a higher social position sharps reduce the likelihood of being detected.

Of course, cheating is also present in online play. Some online players accuse operators of the games of non-random card dealing although the algorithms are regularly certified. There have been instances of insider cheating where employees of the poker room used software backdoors to gain access to their opponents’ hole cards.\textsuperscript{23} Collusion is an issue with several players exchanging information about their cards or single players simultaneously using multiple accounts to gain an unfair advantage. Also a wide range of software which offers players an illegitimate edge has been prohibited.\textsuperscript{24}

### 2.2.3 Sinistral Plays

To conclude the section on playing cards we ask the reader to participate in a second psychological experiment. Supposing that everybody has had reasonable contact with playing cards, one kind or the other, it should be an easy task to draw a simple representation of, say, the king of hearts. Or is it not?

\textsuperscript{19}Ibid. p. 213.
\textsuperscript{20}See Loftus (1979). On the suggestibility of memory see Weingardt, Loftus, and Lindsay (1995); a suggestion such as “without the ten of hearts it would not be a straight” may induce responses favoring the red ten of spades to be stated a ten of hearts. Failures of recognition during sequences are discussed by Simons (1996).
\textsuperscript{21}Also see McManus (2007e).
\textsuperscript{22}See McManus (2007f).
\textsuperscript{23}See \url{http://www.msnbc.msn.com/id/21381022} and \url{http://www.msnbc.msn.com/id/26563848}.
\textsuperscript{24}See e.g. \url{http://www.pokerstars.com/poker/room/prohibited}. 
Nickerson and Adams (1979) asked their subjects to draw a comparably common object, a United States penny. What they got was usually an abundance of errors. Reproducing a penny from memory resulted in omissions, incorrect locations and wrong siding for most of the eight critical features. The median number of objects recalled and located correctly was three. Only one of the 20 subjects got all eight properties right, an active penny collector. Facing their subjects incomplete and imprecise memory Nickerson and Adams conclude that there is no need for them to be any better. For the usual task, distinguishing a penny from other coins, no detailed representation of a penny is needed. Visual details of an object are typically available from memory only to the extent that they are useful in everyday life.

Now we would not expect it to be useful for you to know precisely in which direction kings, queens or jacks are facing (mostly left), which objects they are holding (mostly swords and flowers) and in which hand (left-handedness is overrepresented) or who is wearing which kind of beard (no queen is).

2.3 Texas Hold’em

This section describes the rules of Texas Hold’em poker which is the most strategically complex variant of poker and is used for determining the world champion.\(^{25}\) Of the 57 events at the 2009 WSOP no less than 33 are of the Hold’em kind.\(^{26}\)

2.3.1 Cards Dealt and Rounds Played

Beginning with the player left of the dealer each player is dealt two private hole cards, also called *pocket cards*, face down. This phase is called *pre-flop* and offers the first opportunity for betting. On the completion of the betting three community cards shared by all players are dealt face up on the table in the next phase. They are known as the *flop*. On the flop a second round of betting occurs. When finished a fourth community card, the *turn* or also called *fourth street* is dealt face up and the third round of betting begins. The final face up community card, the *river* or *fifth street*, is dealt and the last round of betting follows. If after betting on the river more than one active player remains, the hole cards are exposed and the winner of the pot is determined by the best five-card poker hand, using any combination of the two private cards and the five community cards. In the event of identical hands, the pot will be divided equally. After the pot is awarded all

\(^{25}\)See Davidson, Billings, Schaeffer, and Szafron (2000).

\(^{26}\)See [http://www.worldseriesofpoker.com](http://www.worldseriesofpoker.com).
cards are collected, the position of the dealer, the button, moves to the next player by one position clockwise, cards are shuffled and new hands are dealt.

2.3.2 Game Varieties

Texas Hold’em is further distinguished in four varieties which influence the amounts players can stake. The four limit structures Limit, No Limit, Pot Limit and Mixed Texas Hold’em are discussed in turn.

Betting in Limit Hold’em follows predetermined, structured amounts. For example, in 10/20 Limit Hold’em the size of every bet and raise pre-flop and on the flop is $10.\textsuperscript{27} On the turn and river bets and raises are $20. During each phase every player might bet at most four times, limiting the maximum stake put into the pot per player to $40 pre-flop and on the flop and to $80 on turn and river.

In No Limit Hold’em there is a defined minimum bet but players can always bet as much as they want up to all of their chips. Betting all chips is called an all-in. There is also no restriction on the number of rounds a player might bet during any of the phases.

There is also a given minimum bet in Pot Limit Hold’em above which players can always bet up to the size of the pot. The maximum bet is defined as the total of the active pot plus all bets on the table plus the amount the active player must first call before raising.

In Mixed Hold’em the game switches between hands of Limit Hold’em and No Limit Hold’em.

2.3.3 Betting and Raising

For every hand played two compulsory bets have to be made. The first position clockwise from the dealer usually has to post a forced bet of half the size of a standard unit, the small blind. The second position, immediately clockwise from the small blind, posts another forced bet, the big blind which is twice the size of the small blind. Thus in a 10/20 Limit Hold’em game the small blind posts $5 and the big blind $10 before play begins.

Betting begins with the player left of the big blind.\textsuperscript{28} Generally available actions for each player are fold, check, bet, call or raise. Exactly which betting options are available

\textsuperscript{27}In stating limit varieties $ is the usual denomination and $-signs are omitted throughout this thesis.
\textsuperscript{28}In case of a two-player game this would be the small blind who also is the button.
depends on the action taken by the previous player. Each player has always the option to *fold*, to discard his hole cards and give up any interest in the pot. Any player who has folded is inactive for the remainder of the hand and only becomes active again when the next hand is dealt. If no bet has been made yet a player may *check*, which is to pass the action on to the next player, or *bet* in line with the requirements on minimum and maximum allowed by the variety. If a player has bet, subsequent players can fold, call or raise. To *call* is to stake the amount of the player who has made the largest bet so far. To *raise* is, in addition of equalizing the largest bet so far, also to increase it within the maximum amount allowed.

Vying in poker follows the equalization method as illustrated in table 2.3. The number of rounds the action might go to a player is determined by the variety played. As discussed above in Limit Hold’em players are only allowed to stake up to four bets per phase. The four bets are (1) bet, (2) raise, (3) re-raise, and (4) cap (the final raise). In other varieties there is no cap and several re-raise are allowed. The best introduction on how to build strategies based on the available options is Sklansky (1987).

<table>
<thead>
<tr>
<th>Player</th>
<th>Action</th>
<th>Total staked</th>
<th>Total pot</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Bet 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>Raise to 2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>Call 2</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>Call 2</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Second</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Re-raise to 3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>B</td>
<td>Call 1</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>Fold</td>
<td>(2)</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>Cap at 4</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>Third</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Call 1</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>B</td>
<td>Fold</td>
<td>(3)</td>
<td>13</td>
</tr>
<tr>
<td>D</td>
<td>No further action</td>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

Source: Own example, compare Parlett (2005).

### 2.3.4 Hands at Showdown

In poker, cards are ranked, from best to worst, ace (A), king (K), queen (Q), jack (J), 10 (T), 9, 8, 7, 6, 5, 4, 3, 2, A (low). Aces only appear low when they are part of a straight or straight flush. All suits are equally valued and are only relevant to determine whether a hand is of the flush or straight flush type. Hands are ranked, first by category, then by the ranks of the individual cards. The order of cards is not important, though for the
ease of recognition hands are usually presented with higher ranking cards on the left.

The standard ranking of five-card poker hands is as follows from best to worst. Examples are given in table 2.4.

- Straight flush. A straight flush contains five cards in sequence, all of the same suit. Two such hands are compared by their highest card; since suits have no relative value, two otherwise identical straight flushes tie. An ace-high straight flush is know as a Royal Flush the best possible poker hand.

- Four of a kind. Four cards of one rank, and an unmatched card of another rank are also known as quads. As several players may show quads via community cards, the unmatched card serves as kicker identifying the best hand.

- Full house. Also known as a full boat or boat contains three matching cards of one rank, and two matching cards of another rank where the one with the higher ranking set of three is the better hand.

- Flush. A flush contains five cards of the same suit, not in rank sequence. Two flushes are compared by their highest cards and consecutively in order of the lower cards.

- Straight. Five cards of sequential rank but in more than one suit form a straight. Straights are compared by their highest card.

- Three of a kind. Also called trips or set is made of three cards of the same rank, plus two unmatched cards. Trips of the same rank are compared using the two kickers.

- Two pair. Any two cards of the same rank, plus a different pair of cards of another rank and one unmatched card, is called two pair. Comparison is first by the higher pair, then by the lower pair, finally by the kicker.

- One pair. This is a hand with two cards of the same rank plus three other unmatched cards. The non-paired cards are compared in descending order to determine the winner if several hands have the same pair.

- High-card. If no two cards have the same rank, the five cards are not in sequence, and the five cards are not all the same suit, a player is said to have “nothing”. His hand is described as "king high", "ace-queen high", or by as many cards as are necessary to break a tie.

All the properties of the game discussed in this section provide extensive width for mathematical analysis, some of which will be summarized in the following section.
Table 2.4: Examples of Standard Poker Hands in Descending Order

<table>
<thead>
<tr>
<th>Hand</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight flush</td>
<td>K♦ Q♦ J♦ T♦ 9♦</td>
</tr>
<tr>
<td>Four of a kind</td>
<td>9♥ 9♠ 8♠ 9♦ A♥</td>
</tr>
<tr>
<td>Full house</td>
<td>J♥ J♣ J♦ 4♥ 4♠</td>
</tr>
<tr>
<td>Flush</td>
<td>J♣ 9♠ 8♠ 7♠ 6♠</td>
</tr>
<tr>
<td>Straight</td>
<td>A♥ 2♦ 3♥ 4♠ 5♠</td>
</tr>
<tr>
<td>Three of a kind</td>
<td>Q♥ Q♦ Q♠ A♣ 3♠</td>
</tr>
<tr>
<td>Two pair</td>
<td>6♠ 6♦ 5♥ 5♦ 9♣</td>
</tr>
<tr>
<td>One pair</td>
<td>T♦ T♣ A♣ 8♣ 2♣</td>
</tr>
<tr>
<td>High-card</td>
<td>J♣ 9♠ 8♠ 6♠ 3♣</td>
</tr>
<tr>
<td>Best hand:</td>
<td>Highest straight flush</td>
</tr>
<tr>
<td>Royal flush</td>
<td>♠ A♣ K♣ Q♣ J♣ T♣</td>
</tr>
<tr>
<td>Worst hand:</td>
<td>Lowest high-card, kicker</td>
</tr>
<tr>
<td>7-5-high</td>
<td>7♦ 5♦ 4♥ 3♠ 2♣</td>
</tr>
</tbody>
</table>

2.4 Some Probabilities

For decades varieties of poker have interested mathematically inclined researchers; to mention just a few, see Newman (1959), Joseph (1973), Zadeh (1977), Mazalov, Panova, and Piskuric (1999) or Haigh (2002). At the heart of every analysis lies the calculation of probabilities as in equation (1.1).

In more complex situations the use of extensions from combinatorial analysis is advisable. A combination is a selection of a number of elements from a population considered without regard to their order. This is exactly what is done in poker problems when $r$ cards are taken from a population of $n$ cards. The solution to this is,

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!}
\]  

(2.1)

where \(\binom{n}{r}\) is the symbol used for binomial coefficients, i.e. it is the \((r+1)\)st coefficient of the expansion of \((a + b)^n\).\(^{29}\)

In addition to probabilities we will generally state frequencies as well. It has been shown by Gigerenzer and Hoffrage (1995) that it is easier to evaluate an information stated in frequency format than in probability format.\(^{30}\) The difference in interpreting a 0.0311% probability of getting a straight flush on a 7-card hand to the fact that of

\(^{29}\)See Epstein (1967, pp. 17-20) including the derivation from permutations.

\(^{30}\)Precisely it is the combination of both modes that reduces biases. Frequentistic judgments alone are no more sophisticated or accurate (Griffin and Buehler (1999)).
133,784,560 possible 7-card hands 41,584 are straight flushes is evident. The same can also be stated as a chance to get a straight flush in about 1 of 3,217 7-card hands or odds of 1 to 3,216.\(^{31}\)

2.4.1 Starting Hands

In Texas Hold’em every player is dealt two hole cards from the deck of 52 cards. As their sequence is of no relevance we directly apply equation (2.1) to get the number of possible starting hands,

\[
\binom{52}{2} = \frac{52!}{2!(50)!} = \frac{52 \times 51}{1 \times 2} = \frac{2,652}{2} = 1,326
\] (2.2)

As suits are valued equally many of these hands have the same value before the flop. The 1,326 combinations can be further reduced into 169 distinct starting hands. These split into the following groups which influence the strength of the hand.

- 13 pocket pairs. Hole cards can pair on each of the 13 ranks. There are \(\binom{4}{2} = 6\) suit combinations for each pocket pair which adds to \(13 \times 6 = 78\) or about 5.88% of the 1,326 starting hands. We denote hands of this kind in the form \((XX)\), for example, pocket aces are written as \((AA)\).

- 78 suited cards. There are \(\frac{13 \times 12}{2} = 78\) instances where ranks are not paired regardless of order. With any of the four suits we get \(78 \times 4 = 312\) hands where cards share the same suit. These are approximately 23.52% or odds of 1 to 3.25. We abbreviate this group by \((XYs)\) (s for suited) where usually \(X\) is the higher ranking card; examples are \((AKs)\), \((Q9s)\) or \((86s)\).

- 78 unsuited non paired cards. Again we find \(\frac{13 \times 12}{2} = 78\) non-paired hands in any order. The first suit being any of the four and the second suit any of the remaining three gives \(\binom{4}{1}\binom{3}{1} = 12\) suit combinations for each hand and a total of \(78 \times 12 = 936\) of the 1,326 starting hands are of this group. We indicate these about 70.59% of hands by \((XYo)\) where \(o\) stands for offsuit. Examples being \((AQo)\), \((K7o)\) or \((54o)\).

A further characteristic worth mentioning are hands with directly consecutive ranking cards, called connectors or connected. Ranging from \((A2)\) to \((AK)\) there are 13 different ranks of connectors. As they can be suited or offsuit we find \(13 \times (\binom{4}{1} + \binom{3}{1}) = 208\) hands with connected cards.

\(^{31}\)Odds are defined as the ratio \((\frac{1}{p} - 1) : 1\) where \(p\) is the probability.
2.4. SOME PROBABILITIES

Knowing the identity of two of the cards the number of possible hands opponents can have is reduced. With only 50 cards remaining there are only \( \frac{50 \times 49}{2} = 1,225 \) hands that a single opponent can have before the flop compared to the 1,326 starting hands a player can be dealt. Consequently, though cards can be dealt \( \binom{52}{2} \binom{50}{2} \div 2 = 812,175 \) ways in a head-to-head match in Hold’em the situation reduces to only \( 169 \times 1,225 = 207,025 \) distinct match ups.\(^{32}\)

By adding more opponents the number of possible combinations increases substantially. With a second opponent a player faces \( \binom{50}{2} \binom{48}{2} \div 2! = 690,900 \) combinations.\(^{33}\) And the richness of the game is seen when \( n \) opponents can hold

\[
\prod_{k=1}^{n} \binom{52 - 2k}{2} \div n!
\]

combinations. For 9 opponents these are about \( 6.2211 \times 10^{20} \).

2.4.2 Complete Hands

During play the value of hands can change as community cards are seen on flop, turn and river. For any player \( \binom{50}{3} = \frac{117,600}{6} = 19,600 \) different flops are possible. By the turn \( \binom{50}{4} = 230,300 \) and the river \( \binom{50}{5} = 2,118,760 \) possible boards can be seen.

On the flop a player can evaluate his hand with regard to the ranking presented in 2.3.4 for the first time. Of these 5-card hands \( \binom{52}{5} = \frac{311,875,200}{120} = 2,598,960 \) combinations are possible. Their frequency and probability are shown in table 2.5.

As players get two extra cards in any 7-card poker game like on turn and river in Texas Hold’em the hand can be improved and higher valued hands become more probable. Although there are \( \binom{52}{6} = 20,358,520 \) combinations of 6-card hands and \( \binom{52}{7} = 133,784,560 \) of 7-card hands, the number of distinct 5-card hands made from these is reduced from 7,462 to 6,075 in the 6-card case and even further to 4,824 in the 7-card case. This is because some 5-card hands are impossible with the best 5-card hand from more than five cards. For example, there is no 7-high with seven cards.\(^{34}\) Frequency and probability of 7-card hands are shown in table 2.6.

\(^{32}\)169 distinct starting hands for the first player as shown above, but note that for the second player hands do not collapse as suits become relevant to determine a winner.

\(^{33}\)We divide by the number of ways hands can be distributed between \( n \) opponents, \( n! \).

\(^{34}\)See http://www.suffecoool.net/poker/7462.html.

\(^{35}\)Due to complexity the mathematical derivation is not included. Please see http://en.wikipedia.org/wiki/Poker_probability for the computations.
CHAPTER 2. RULES OF THE GAME

Table 2.5: Frequency and Probability of 5-Card Poker Hands
This table presents the frequency of poker hands made from 5-cards and its mathematical
derivation in column 2. Column 3 shows the probability of the given hand. The last column
gives the number of hands of this type which have a distinct value eliminating hands which are
identical but for suits.

<table>
<thead>
<tr>
<th>Hand</th>
<th>Frequency</th>
<th>Probability</th>
<th>Distinct values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Royal flush</td>
<td>4</td>
<td>0.000154%</td>
<td>1</td>
</tr>
<tr>
<td>Straight flush</td>
<td>36</td>
<td>0.00139%</td>
<td>9</td>
</tr>
<tr>
<td>Four of a kind</td>
<td>624</td>
<td>0.0240%</td>
<td>156</td>
</tr>
<tr>
<td>Full house</td>
<td>3,744</td>
<td>0.144%</td>
<td>156</td>
</tr>
<tr>
<td>Flush</td>
<td>5,108</td>
<td>0.197%</td>
<td>1,277</td>
</tr>
<tr>
<td>Straight</td>
<td>10,200</td>
<td>0.392%</td>
<td>10</td>
</tr>
<tr>
<td>Three of a kind</td>
<td>54,912</td>
<td>2.11%</td>
<td>858</td>
</tr>
<tr>
<td>Two pair</td>
<td>123,552</td>
<td>4.75%</td>
<td>858</td>
</tr>
<tr>
<td>One pair</td>
<td>1,098,240</td>
<td>42.3%</td>
<td>2,860</td>
</tr>
<tr>
<td>High-card</td>
<td>1,302,540</td>
<td>50.1%</td>
<td>1,277</td>
</tr>
<tr>
<td>Total</td>
<td>2,598,960</td>
<td>100.0%</td>
<td>7,462</td>
</tr>
</tbody>
</table>


Table 2.6: Frequency and Probability of 7-Card Poker Hands
This table presents the frequency of poker hands made from 7-cards in column 2. Column 3 shows the probability of the given hand. The last column gives the number of hands of this type which have a distinct value eliminating hands which are identical but for suits.35

<table>
<thead>
<tr>
<th>Hand</th>
<th>Frequency</th>
<th>Probability</th>
<th>Distinct values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Royal flush</td>
<td>1,081</td>
<td>0.000808%</td>
<td>1</td>
</tr>
<tr>
<td>Straight flush</td>
<td>40,502</td>
<td>0.0303%</td>
<td>9</td>
</tr>
<tr>
<td>Four of a kind</td>
<td>224,848</td>
<td>0.168%</td>
<td>156</td>
</tr>
<tr>
<td>Full house</td>
<td>3,473,184</td>
<td>2.60%</td>
<td>156</td>
</tr>
<tr>
<td>Flush</td>
<td>4,047,644</td>
<td>3.03%</td>
<td>1,277</td>
</tr>
<tr>
<td>Straight</td>
<td>6,180,020</td>
<td>4.62%</td>
<td>10</td>
</tr>
<tr>
<td>Three of a kind</td>
<td>6,461,020</td>
<td>4.83%</td>
<td>575</td>
</tr>
<tr>
<td>Two pair</td>
<td>31,433,400</td>
<td>23.5%</td>
<td>763</td>
</tr>
<tr>
<td>One pair</td>
<td>58,627,800</td>
<td>43.8%</td>
<td>1,470</td>
</tr>
<tr>
<td>High-card</td>
<td>23,294,460</td>
<td>17.4%</td>
<td>407</td>
</tr>
<tr>
<td>Total</td>
<td>133,784,560</td>
<td>100.0%</td>
<td>4,824</td>
</tr>
</tbody>
</table>

Chapter 3

Construction of the Data Set

The Internet is the world’s largest library. It’s just that all the books are on the floor.

John Allen Paulos

3.1 Sample – the IRC Poker Database

3.1.1 Background

Although it would be possible to collect empirical data on decisions in poker play by observing in-person games, only few data points could be acquired at substantial costs.\footnote{On alternative approaches to investigate decisional behavior see Payne, Braunstein, and Carroll (1978).}

With the advent of poker play on the Internet information has necessarily been present in a computerized form which allows direct access to larger amounts of data.\footnote{Cf. 2.1.3. On the use of software to gather data on gambling behavior see Dixon, MacLin, and Hayes (1999a,b).}

Empirical research in this thesis is based upon data from the earliest games played online. Before online casinos offered real-money games, poker was played on the Internet Relay Chat (IRC) which is a form of real-time Internet text messaging (chat) or synchronous conferencing. The server was available from 1995 to 2001. At those times user interfaces were in an early phase of development. In order to participate players had to log-on to \texttt{irc.poker.net} and type their commands or use macros to do so.\footnote{See \url{http://www.rgpfaq.com/ircpoker.html}.}

Participants of these games were poker enthusiasts and although the game was for free-money competition was strong. This was due to the fact that to access higher tiered games
Chapter 3. Construction of the Data Set

Players had to accumulate a larger bankroll which was tracked between games. Some of the players were regular casino players trying to improve their skills, and some such as Chris “Jesus” Ferguson have won at the WSOP.\(^4\) It has been reported by Kachelmeier and Shehata (1991) that even under high monetary incentives behavioral influences on decisions are present. Therefore, the limitations of using a sample based on free-money play should be noted, but as players participate voluntarily and invest their time, it can be postulated that motivation is sufficiently warranted to allow for proper conclusions.

A program written by Michael Maurer quietly recorded the games played on the IRC channel. The collection of these records, the IRC Poker Database, is the basis of the empirical analysis in this thesis. The complete database is hosted on a server of the Computer Poker Research Group (CPRG) at the University of Alberta, accessible under http://games.cs.ualberta.ca/poker/IRC/IRCdata.\(^5\) The following sections present details regarding the database.

So far data from this source has been used by Henstra and van der Zwan (2007), Brown (2004) and Pfund (2007). The former use the data in a data mining project to discover patterns in player actions.\(^6\) Brown (2004) compares players’ actions to game-theoretic optimal values. Pfund (2007) tests the performance of a program playing Hold’em poker (a bot) using hands from the IRC Poker Database.

### 3.1.2 Available Game Varieties

Over the seven years of recorded play several poker varieties were offered. An overview is presented in table 3.1. Anybody could get 1,000 chips for free once a day and join any but two channels. To access channels on higher limit play in Texas Hold’em, players first had to win at smaller limit tables to gather 2,000 chips for the 20/40 Limit and 5,000 chips for the 50/100 Limit.

Texas Hold’em has been detailed in 2.3. In 7-card stud every player is dealt three private cards, two hidden, one face up. In following phases one further face up card is dealt to each player on fourth street, fifth street and sixth street. On the final seventh street, players still in the game receive another card, this one dealt face down. There are no community cards. Every player is dealt four hidden hole cards in Omaha. Community cards are as in Texas Hold’em (three on the flop, one each on turn and river). Players win


\(^5\)The CPRG focuses on research in Artificial Intelligence.

\(^6\)A similar idea is presented by Cattral and Oppacher (2007).
Table 3.1: Game Varieties on IRC Poker Channels

This table gives an overview on games played on the IRC poker server. Variety and limit structure are stated in columns 1 and 2, respectively. Column 3 quotes the channel name for further reference. Additional notes are given in column 4. The use in this thesis is summarized in the final column; “none” indicating that data is not used at all, games marked with “partial” are only included in basic statistics, for games on which extended analysis is based the column includes the abbreviation for reference throughout this thesis.

<table>
<thead>
<tr>
<th>Variety</th>
<th>Structure</th>
<th>Channel</th>
<th>Comment</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Texas</td>
<td>hold’em</td>
<td>10/20 Limit</td>
<td>#botonly</td>
<td>Partial</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 Limit</td>
<td>#h1-nobots</td>
<td>Yes; “10/20 I”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 Limit</td>
<td>#holdem</td>
<td>Yes; “10/20 I”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 Limit</td>
<td>#holdemii</td>
<td>Partial</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20/40 Limit</td>
<td>#holdem2</td>
<td>Yes; “10/20 III”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50/100 Limit</td>
<td>#holdem3</td>
<td>Yes; “20/40”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 Pot-Limit</td>
<td>#holdempot</td>
<td>Partial</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 No-Limit</td>
<td>#nolimit</td>
<td>Partial</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X/Y No-Limit</td>
<td>#tourney</td>
<td>Partial</td>
</tr>
<tr>
<td>7-card</td>
<td>stud</td>
<td>10/20 Limit</td>
<td>#7stud</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 Limit</td>
<td>#7studhi</td>
<td>None</td>
</tr>
<tr>
<td>Omaha</td>
<td></td>
<td>10/20 Limit</td>
<td>#omaha</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 Limit</td>
<td>#omahahi</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 Pot-Limit</td>
<td>#ohlpot</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/20 Pot-Limit</td>
<td>#omahapot</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X/Y Pot-Limit</td>
<td>#ptourney</td>
<td>None</td>
</tr>
</tbody>
</table>


on the best 5-card poker hand from any two of their four private cards and any three of the five community cards. It is not possible, as in Hold’em, to compose a best hand only from community cards. Due to these fundamental differences to Texas Hold’em, analysis in this thesis focuses on data from Hold’em games.

An additional restriction occurs as utilization of the ten channels with Hold’em games has to depend on the amount of data available as well as suitability of the information from the observed parameters included in the database for the intended tests. The following sections discuss these limitations.

### 3.1.3 Structure and Extent

Information in the IRC Poker Database is stored on a monthly basis and structured in three types of files. Each database contains a file that records the list of all players who
are involved in a game, a roster file called **hroster**. An additional file, a game file called **hdb**, includes information on a particular game which is common to all players. Finally there is a single file for each player, such as **pdb.smith**, in which player specific data is stored. How these files stack up is summarized in table 3.2.

### Table 3.2: Scale and Scope of the IRC Poker Database

This table summarizes the extent of the IRC Poker Database. Included are all types of Texas Hold’em as introduced in table 3.1. Column 1 adds a one-digit number which will be used to short-reference the gametypes. Column 2 states the IRC channel. The months with available data are summarized in column 3 in the form month/year. The total number of roster and game-files in column 4, the number of files with information on players in column 5 and the storage space required in megabytes (MB) in the final column conclude the table. Note that the number of player files does not relate to the actual number of players as the database is structured on a monthly basis.

<table>
<thead>
<tr>
<th>No.</th>
<th>Channel</th>
<th>Timespan</th>
<th>Main-files</th>
<th>Player-files</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>#botsonly</td>
<td>10/98; 03/99-10/99</td>
<td>18</td>
<td>113</td>
<td>6 MB</td>
</tr>
<tr>
<td>1</td>
<td>#h1-nobots</td>
<td>02/98-12/00</td>
<td>70</td>
<td>31,746</td>
<td>431 MB</td>
</tr>
<tr>
<td>2</td>
<td>#holdem</td>
<td>04/95-05/97; 07/97-08/97; 02/98-12/00; 02/01-10/01</td>
<td>144</td>
<td>68,637</td>
<td>1,390 MB</td>
</tr>
<tr>
<td>3</td>
<td>#holdemii</td>
<td>09/96-03/97</td>
<td>14</td>
<td>1,290</td>
<td>14 MB</td>
</tr>
<tr>
<td>4</td>
<td>#holdem1</td>
<td>08/98-10/99</td>
<td>30</td>
<td>15,322</td>
<td>261 MB</td>
</tr>
<tr>
<td>5</td>
<td>#holdem2</td>
<td>04/95-05/97; 07/97-08/97; 02/01-10/01</td>
<td>144</td>
<td>26,634</td>
<td>878 MB</td>
</tr>
<tr>
<td>6</td>
<td>#holdem3</td>
<td>05/95-08/96; 10/96-05/97; 07/97-08/97; 02/98-12/00; 02/01-10/01</td>
<td>140</td>
<td>4,595</td>
<td>151 MB</td>
</tr>
<tr>
<td>7</td>
<td>#holdempot</td>
<td>11/95-05/97; 07/97-08/97; 02/98-12/00; 02/01-10/01</td>
<td>130</td>
<td>15,547</td>
<td>787 MB</td>
</tr>
<tr>
<td>8</td>
<td>#nolimit</td>
<td>05/95-01/96; 12/96-01/97</td>
<td>22</td>
<td>128</td>
<td>3 MB</td>
</tr>
<tr>
<td>9</td>
<td>#tourney</td>
<td>05/95-05/97; 07/97-08/97; 02/98-12/00; 02/01-10/01</td>
<td>142</td>
<td>47,799</td>
<td>1,720 MB</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td><strong>04/95-10/01</strong></td>
<td><strong>854</strong></td>
<td><strong>211,811</strong></td>
<td><strong>5,641 MB</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Missing:</strong></td>
<td><strong>06/97; 09/97-01/98; 01/01</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The summary of scale and scope of the database in table 3.2 reveals that not all channels were used extensively. Not suited for in-depth analysis are gametypes 0, 3 and 8 as the data basis is too small. The remaining gametypes spread over a timespan of roughly 6.5 years but there are some gaps in the data with seven months missing. Gaps are due to downtimes of the server or inaccessibility of the games for the observer program so that distortions regarding playing behavior are unlikely.

Even after the exclusion of the three gametypes with lowest utilization there are still well over 200,000 files and more than 5,600 MB of raw data. The next section summarizes
which information is included in the raw data, how it is structured and its usefulness for the intended analysis.

3.1.4 Observed Parameters

To reconstruct the process of play it is necessary to combine information from all three kinds of source files, the game file, the roster file and player files. Any particular game is identified by the timestamp attributed to it. This identifier is a unique integer based on the UNIX timestamp which is a running meter of seconds.\(^7\)

An example of the available information is presented in table 3.3 on page 36. In putting this example together data from ten files is used. The storage of information as it is structured in the IRC Poker Database possesses the following disadvantages for statistical analysis:

- Data is stored across files. This would require to open and close files during analysis.

- Information is not stored in the finest resolution. For example, players’ actions are stored in a column per phase which is a combination of decisions made over rounds of play.

- Some data is redundant. For example, information on the number of players involved in a game is stored in every file increasing the size of the database.

- Information is not complete. The database does not give exact bet amounts per round of play and hole cards are only included if seen at showdown.

Whereas the first three issues can be dealt with by restructuring the database as will be shown in the following section, the last argument produces additional constraints. Because players’ stakes are only given as a total for the hand, it is not feasible to unambiguously reconstruct how betting proceeded in games where bet amounts can be varied. This is the case in gametypes 7, 8 and 9 which are of either pot-limit or no-limit structure. Consequently these gametypes are excluded from in-depth analyses. The fact that hole cards are only available if seen at showdown is of less concern. Lack of knowledge about folded hole cards matches the information available to players participating in the game. Therefore, any conclusions drawn from this data are equivalent to reasoning which is available to the active players.

CHAPTER 3. CONSTRUCTION OF THE DATA SET

Table 3.3: Format of Files in the IRC Poker Database

This table gives an example of a particular game from the 95/100 Limit category (#holdem3). Panel A shows the information on the game recorded in the hdb file. Panel B presents information on the roster from hroster. Finally, player-specific data and their actions are seen in panel C constructed from the eight relevant player files, such as pdb.Marzon. A column-by-column description of the format is presented in table 3.4 on page 37.

**Panel A: Game information**

<table>
<thead>
<tr>
<th>Timestamp</th>
<th>Dealer</th>
<th>Hand</th>
<th>Players</th>
<th>Flop</th>
<th>Turn</th>
<th>River</th>
<th>Showdown</th>
<th>Board</th>
</tr>
</thead>
<tbody>
<tr>
<td>766309976</td>
<td>6</td>
<td>4</td>
<td>11</td>
<td>3,400</td>
<td>3,400</td>
<td>3,400</td>
<td>3,400</td>
<td>4</td>
</tr>
<tr>
<td>766309976</td>
<td>7</td>
<td>4</td>
<td>12</td>
<td>3,400</td>
<td>3,400</td>
<td>3,400</td>
<td>3,400</td>
<td>4</td>
</tr>
<tr>
<td>766309976</td>
<td>6</td>
<td>4</td>
<td>13</td>
<td>3,400</td>
<td>3,400</td>
<td>3,400</td>
<td>3,400</td>
<td>4</td>
</tr>
<tr>
<td>766309976</td>
<td>7</td>
<td>4</td>
<td>14</td>
<td>3,400</td>
<td>3,400</td>
<td>3,400</td>
<td>3,400</td>
<td>4</td>
</tr>
</tbody>
</table>

**Panel B: Roster information**

<table>
<thead>
<tr>
<th>Timestamp</th>
<th>Players</th>
<th>Name 1</th>
<th>Name 2</th>
<th>Name 3</th>
<th>Name 4</th>
<th>Name 5</th>
<th>Name 6</th>
<th>Name 7</th>
<th>Name 8</th>
<th>Name 9</th>
<th>Name 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>766309976</td>
<td>8</td>
<td>Marzon</td>
<td>spiney</td>
<td>doublebag</td>
<td>neoncap</td>
<td>maurer</td>
<td>andrea</td>
<td>zorak</td>
<td>justin</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

**Panel C: Player information**

<table>
<thead>
<tr>
<th>Player</th>
<th>Timestamp</th>
<th>Players</th>
<th>Position</th>
<th>Pre-Flop</th>
<th>Flop</th>
<th>Turn</th>
<th>River</th>
<th>Showdown</th>
<th>Bankroll</th>
<th>Action</th>
<th>Winnings</th>
<th>Cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marzon</td>
<td>766309976</td>
<td>8</td>
<td>1</td>
<td>Bc</td>
<td>bc</td>
<td>kc</td>
<td>kf</td>
<td>12,653</td>
<td>300</td>
<td>0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>spiney</td>
<td>766309976</td>
<td>8</td>
<td>2</td>
<td>Bc</td>
<td>cc</td>
<td>kc</td>
<td>f</td>
<td>10,237</td>
<td>300</td>
<td>0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>doublebag</td>
<td>766309976</td>
<td>8</td>
<td>3</td>
<td>cc</td>
<td>r</td>
<td>b</td>
<td>bc</td>
<td>7,842</td>
<td>500</td>
<td>0</td>
<td>Jh</td>
<td>Qh</td>
</tr>
<tr>
<td>neoncap</td>
<td>766309976</td>
<td>8</td>
<td>4</td>
<td>f</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>7,857</td>
<td>0</td>
<td>0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>maurer</td>
<td>766309976</td>
<td>8</td>
<td>5</td>
<td>f</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>12,711</td>
<td>0</td>
<td>0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>andrea</td>
<td>766309976</td>
<td>8</td>
<td>6</td>
<td>cc</td>
<td>c</td>
<td>c</td>
<td>f</td>
<td>7,190</td>
<td>300</td>
<td>0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>zorak</td>
<td>766309976</td>
<td>8</td>
<td>7</td>
<td>r</td>
<td>c</td>
<td>c</td>
<td>cc</td>
<td>4,460</td>
<td>500</td>
<td>0</td>
<td>As</td>
<td>Kc</td>
</tr>
<tr>
<td>justin</td>
<td>766309976</td>
<td>8</td>
<td>8</td>
<td>c</td>
<td>c</td>
<td>c</td>
<td>r</td>
<td>4,304</td>
<td>500</td>
<td>2,400</td>
<td>Ad</td>
<td>Qs</td>
</tr>
</tbody>
</table>

Source: http://games.cs.ualberta.ca/poker/IRC.
### Table 3.4: Description of Columns of Files in the IRC Poker Database

<table>
<thead>
<tr>
<th>Column</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Timestamp (unique integer)</td>
</tr>
<tr>
<td>2</td>
<td>Number of game set (incremented when column 3 resets)</td>
</tr>
<tr>
<td>3</td>
<td>Number of game reported by dealer program</td>
</tr>
<tr>
<td>4</td>
<td>Number of players dealt cards</td>
</tr>
<tr>
<td>5</td>
<td>Number of players who see the flop / pot size at beginning of flop</td>
</tr>
<tr>
<td>6</td>
<td>Number of players who see the turn / pot size at beginning of turn</td>
</tr>
<tr>
<td>7</td>
<td>Number of players who see the river / pot size at beginning of river</td>
</tr>
<tr>
<td>8</td>
<td>Number of players at showdown / pot size at showdown</td>
</tr>
<tr>
<td>9</td>
<td>First card on flop (format: rank/suit)</td>
</tr>
<tr>
<td>10</td>
<td>Second card on flop (format: rank/suit)</td>
</tr>
<tr>
<td>11</td>
<td>Third card on flop (format: rank/suit)</td>
</tr>
<tr>
<td>12</td>
<td>Card on turn (format: rank/suit)</td>
</tr>
<tr>
<td>13</td>
<td>Card on river (format: rank/suit)</td>
</tr>
</tbody>
</table>

As to panel B in table 3.3 on page 36

<table>
<thead>
<tr>
<th>Column</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Timestamp</td>
</tr>
<tr>
<td>2</td>
<td>Number of players dealt cards</td>
</tr>
<tr>
<td>3+</td>
<td>Player nicknames</td>
</tr>
</tbody>
</table>

As to panel C in table 3.3 on page 36

<table>
<thead>
<tr>
<th>Column</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Player nickname</td>
</tr>
<tr>
<td>2</td>
<td>Timestamp of the hand</td>
</tr>
<tr>
<td>3</td>
<td>Number of players dealt cards</td>
</tr>
<tr>
<td>4</td>
<td>Position of player (starting at 1 for the small blind)</td>
</tr>
<tr>
<td>5</td>
<td>Betting action pre-flop</td>
</tr>
<tr>
<td>6</td>
<td>Betting action on flop</td>
</tr>
<tr>
<td>7</td>
<td>Betting action on turn</td>
</tr>
<tr>
<td>8</td>
<td>Betting action on river</td>
</tr>
<tr>
<td>9</td>
<td>Player’s bankroll at the beginning of the hand</td>
</tr>
<tr>
<td>10</td>
<td>Total amount of player staked during hand</td>
</tr>
<tr>
<td>11</td>
<td>Amount won by player</td>
</tr>
<tr>
<td>12+</td>
<td>Player’s hole cards (if revealed at showdown)</td>
</tr>
</tbody>
</table>

Encoding of betting action

<table>
<thead>
<tr>
<th>Encoding</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>—</td>
<td>No action; player is no longer active</td>
</tr>
<tr>
<td>B</td>
<td>Blind bet (small or big blind)</td>
</tr>
<tr>
<td>f</td>
<td>Fold</td>
</tr>
<tr>
<td>k</td>
<td>Check</td>
</tr>
<tr>
<td>b</td>
<td>Bet</td>
</tr>
<tr>
<td>c</td>
<td>Call</td>
</tr>
<tr>
<td>r</td>
<td>Raise</td>
</tr>
<tr>
<td>A</td>
<td>All-in</td>
</tr>
<tr>
<td>Q</td>
<td>Quits game</td>
</tr>
<tr>
<td>K</td>
<td>Kicked from game</td>
</tr>
</tbody>
</table>


3.2 Database – Decision-Oriented Setup

To perform the restructuring of the database an algorithm runs through all gametypes and months and reconstructs the process of play. The program is written in the Python programming language and converts the database to a space-delimited ASCII structure.  

3.2.1 Hierarchical Structure

Raw data from the IRC Poker Database is reorganized in a hierarchical structure consisting of four layers as depicted in figure 3.1. Data concerning players are summarized in a single database. At the same level a separate database stores data on games played. It contains information that is common to all players involved in a game like the number of players dealt cards or the cards seen on the board. Data is linked to the next lower level via the timestamp of the game. At this level all properties which are constant for a player’s hand are captured. A separate database is created for every gametype due to size constraints. The lowest level files contain the specifics for any discrete action of a player. Connection to the higher level, the player’s hand, is made using both timestamp and player nickname.

In addition to the variables described in table 3.4 the algorithm extracts several other parameters into the databases describing the decision environment. These databases

---

8See http://www.python.org.
are imported to *Stata* for statistical analysis.\(^9\) Whenever new variables will be defined throughout this thesis they will be added to one of the four database-layers. The definition of variables as generated from the raw data is presented in table A.1 in the appendix.

### 3.2.2 Determining Hand Strength

As determining the relative value of one’s hand is one of the core tasks of a poker player this section briefly discusses how hand strength is computed while analyzing the IRC Poker Database.

The 5-card poker hands introduced in section 2.4.2 are all unaffected by the order of the cards. Therefore we know that any permutation of the 5-cards results in the same relative value which can be measured on a scale ranging from 1 for the best hand (a royal flush) to 7,462 for the worst hand (a 7-5-high).

Suffecool (2005) describes an ingenious method to attribute these values to 5-card poker hands. By assigning a prime number to every rank of the cards a simple multiplication of the prime number for all five cards results in a unique product for each hand.\(^10\) For example, a King-high straight always generates a product value of 14,535,931 which is \(37 \times 31 \times 29 \times 23 \times 19\).\(^11\) In addition, it only has to be verified whether we have a flush hand or not. Once this is known a simple look up will either return that the player has the 1,601st-best hand, a King-high straight, or the 2nd-best hand possible, a King-high straight flush.\(^12\)

So, to determine a player’s hand strength on the flop the algorithm first compares all suits and decides whether it is a flush hand or not. Then the product value of the primes attached to the ranks is computed. This value is looked up in either of two dictionaries (flush, no-flush) and translated to the hand strength. On the turn, this procedure is repeated \(\binom{6}{5} = 6\) times for every combination of 5-card hands possible. The resulting hand values are compared and the highest value is the player’s hand strength on the turn. On the river, there are \(\binom{7}{5} = 21\) combinations to be evaluated returning the final hand strength. Of course, these considerations are only possible if a player’s hole cards are revealed.

---

\(^9\)See [www.stata.com](http://www.stata.com).

\(^{10}\)Prime numbers are from 2 for a deuce to 41 for an ace.

\(^{11}\)“King \(\times\) Queen \(\times\) Jack \(\times\) Ten \(\times\) Nine”.

3.2.3 Exclusion of Observations

During the reorganization of the IRC Poker Database to the final database structure the database was also monitored for consistency. This test encountered several types of exceptions which lead to an exclusion of observations. These exceptions will be discussed subsequently.

A number of erroneous records was caused by neighboring columns that had merged. For example, a player’s name merged with the next cell, the timestamp, would result in a new player, say “Marzon766303976”, for whom no player file could be found. Errors like this also affect the computation of other variables. Consequently all records where the read-out of a cell did not match the expected variable type (string, integer, etc.) and length were excluded.

A second type of exceptions was due to excessive observations. In some instances records were encountered although the player could perform no further action (fold, quit, kicked, all-in). These observations were dropped.

The IRC Poker Database was recorded on a system running a Unix environment. Unix allows filenames including special characters such as ||, \ or * which may not be used in a Windows operating system. Consequently, files with data on players whose nicknames included any of these characters could not be opened. Hence data of 77 players could not be used.

Some data, not causing any of the three exceptions mentioned above, has been recorded twice. As these would distort the analysis a search for duplicates was performed and multiple records of the same observation were deleted.

Finally, as discussed in section 2.2.2, cheating players are an integral part of the poker game. So too on the IRC poker channels, where players have the opportunity to create fake accounts which they can use to “harvest” chips with their main account. Thus a routine was run to look for conspicuous play and mark the involved players. To do this, measures of behavior which will be discussed in detail in section 4.3.1 were used to identify suspicious players.

Table 3.5 summarizes the extent to which observations had to be excluded. Most of the false records and duplicates are in channels which are not used for in-depth analysis. 

\footnote{Observations are not deleted from the database as they are not erroneous per se. Instead a filter is used to exclude the marked players from analysis whenever appropriate.}
anyway. The large number of duplicates for the #holdem channel could have been caused by prototyping errors as this is the original channel. With regard to the presence of conspicuous play it is obvious that the lower limit channels are more affected, especially the tournament games are easily manipulated. Conspicuous play in the channel reserved for bots is mainly caused by bots operating on simplified rules such as always fold, always call or always raise.

Table 3.5: Exclusion of Observations by Type
This table states the number of records which were excluded for each gametype (column 1). Three types of exclusions are distinguished. In column 2 and 3 the total of exclusions due to erroneous records and excessive observations is reported for hands and actions, respectively. Columns 4 and 5 show the number of duplicates deleted. The final column reports the number of players marked as suspicious due to conspicuous play; their relative share is given in parentheses. The last row summarizes the impact of exclusions on the database on games.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>False records</th>
<th>Duplicates</th>
<th>Suspicious</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hands</td>
<td>Actions</td>
<td>Hands</td>
</tr>
<tr>
<td>#botsonly</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>#h1-nobots</td>
<td>10</td>
<td>30</td>
<td>218</td>
</tr>
<tr>
<td>#holdem</td>
<td>117</td>
<td>425</td>
<td>305,750</td>
</tr>
<tr>
<td>#holdemii</td>
<td>0</td>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>#holdem1</td>
<td>6</td>
<td>13</td>
<td>113</td>
</tr>
<tr>
<td>#holdem2</td>
<td>17</td>
<td>31</td>
<td>57</td>
</tr>
<tr>
<td>#holdem3</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>#holdempot</td>
<td>35</td>
<td>127</td>
<td>197,090</td>
</tr>
<tr>
<td>#nolimit</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>#tourney</td>
<td>127,671</td>
<td>234,393</td>
<td>1,669,017</td>
</tr>
<tr>
<td>Games</td>
<td>59,527</td>
<td>656,037</td>
<td></td>
</tr>
</tbody>
</table>

3.3 Sample Characteristics

This section describes basic characteristics of the poker environment found in the sample after the structural redesign of the database and the exclusion of observations as discussed in sections 3.2.1 and 3.2.3.

3.3.1 Sample Size

In order to determine breadth and depth of the sample variables tracking play for every player are added. In addition to the actions, hands and gametypes played, sessions are evaluated for each player. We arbitrarily define a session as a series of hands between each of which not more than 10 minutes pass. Therefore, the time lag between hands is
CHAPTER 3. CONSTRUCTION OF THE DATA SET

This figure shows a box-plot of the total hands played by the players in each gametype. The upper bounds of the box are the 3rd quartile, the line in the middle is the median, the lower bounds are the 1st quartile. The upper/lower whiskers represent values which are smaller/larger or equal to the quartile + 1.5-interquartile ranges.

Figure 3.2: Distribution of Hands Played Over Players by Gametype

calculated and whenever this measure is above 600 seconds the session counter is increased by one.

Table 3.6 summarizes the sample size. Confirming the account under 3.1.3 gametypes 0, 3 and 8 are of the smallest size, limiting potential analysis. The different natures of limit vs. pot-limit or no-limit play are also visible. On average there are about three actions per hand played in limit games. Compared to this, pot-limit games only show 2.6 and no-limit games in tournaments even less than two actions per hand.

It is also evident that the sample is highly skewed. Some players have a deep record of play whereas others are only active sporadically. The skewed distribution of hands played is illustrated in figure 3.2. Although there is a reasonable number of players with more than 1,000 recorded hands, these numbers are small compared to games played on online casinos. Nowadays frequent players who can play several tables simultaneously easily accumulate 10,000s of hands per year.
### Table 3.6: Breadth and Depth of the Sample

This table summarizes the sample size included in the data set. Whereas the breadth of the data can be seen from the totals given, the depth is captured by mean and median per player. Note that the total regarding players is not the sum of players over gametypes as some players are present in several gametypes (see table 3.7).

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Players</th>
<th>Games</th>
<th>Total</th>
<th>Mean</th>
<th>Median</th>
<th>Gametype</th>
<th>Hands</th>
<th>Total</th>
<th>Mean</th>
<th>Median</th>
<th>Gametype</th>
<th>Actions</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90</td>
<td>24,366</td>
<td>252</td>
<td>2.8</td>
<td>1</td>
<td></td>
<td></td>
<td>53,090</td>
<td>589.9</td>
<td>23</td>
<td></td>
<td>143,218</td>
<td>1,591.3</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>11,775</td>
<td>1,005,470</td>
<td>136,661</td>
<td>11.6</td>
<td>2</td>
<td></td>
<td></td>
<td>4,813,344</td>
<td>408.8</td>
<td>67</td>
<td></td>
<td>15,216,362</td>
<td>1,292.3</td>
<td>240</td>
</tr>
<tr>
<td>2</td>
<td>23,422</td>
<td>2,672,088</td>
<td>506,027</td>
<td>21.6</td>
<td>3</td>
<td></td>
<td></td>
<td>16,634,678</td>
<td>710.2</td>
<td>78</td>
<td></td>
<td>48,702,812</td>
<td>2,078.3</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>805</td>
<td>47,213</td>
<td>3,282</td>
<td>4.1</td>
<td>2</td>
<td></td>
<td></td>
<td>146,910</td>
<td>182.5</td>
<td>49</td>
<td></td>
<td>483,064</td>
<td>585.0</td>
<td>159</td>
</tr>
<tr>
<td>4</td>
<td>6,443</td>
<td>496,598</td>
<td>82,354</td>
<td>12.8</td>
<td>3</td>
<td></td>
<td></td>
<td>3,056,285</td>
<td>474.4</td>
<td>88</td>
<td></td>
<td>8,909,236</td>
<td>1,382.8</td>
<td>294</td>
</tr>
<tr>
<td>5</td>
<td>7,506</td>
<td>1,930,280</td>
<td>168,932</td>
<td>22.5</td>
<td>5</td>
<td></td>
<td></td>
<td>10,027,363</td>
<td>1,335.9</td>
<td>283</td>
<td></td>
<td>26,776,988</td>
<td>3,563.8</td>
<td>811</td>
</tr>
<tr>
<td>6</td>
<td>1,718</td>
<td>494,360</td>
<td>13,682</td>
<td>8.0</td>
<td>3</td>
<td></td>
<td></td>
<td>1,580,182</td>
<td>919.8</td>
<td>220</td>
<td></td>
<td>4,511,431</td>
<td>2,624.7</td>
<td>643</td>
</tr>
<tr>
<td>7</td>
<td>4,887</td>
<td>1,823,452</td>
<td>108,722</td>
<td>22.2</td>
<td>4</td>
<td></td>
<td></td>
<td>8,906,200</td>
<td>1,822.4</td>
<td>230</td>
<td></td>
<td>23,118,423</td>
<td>4,730.6</td>
<td>601</td>
</tr>
<tr>
<td>8</td>
<td>88</td>
<td>10,174</td>
<td>287</td>
<td>3.3</td>
<td>2</td>
<td></td>
<td></td>
<td>27,318</td>
<td>310.4</td>
<td>124</td>
<td></td>
<td>61,629</td>
<td>700.3</td>
<td>281</td>
</tr>
<tr>
<td>9</td>
<td>15,966</td>
<td>3,261,244</td>
<td>433,985</td>
<td>27.2</td>
<td>3</td>
<td></td>
<td></td>
<td>18,347,143</td>
<td>1,149.1</td>
<td>107</td>
<td></td>
<td>36,075,253</td>
<td>2,259.5</td>
<td>210</td>
</tr>
<tr>
<td>Total</td>
<td>32,159</td>
<td>11,765,245</td>
<td>1,454,184</td>
<td>—</td>
<td>—</td>
<td></td>
<td></td>
<td>63,592,513</td>
<td>—</td>
<td>—</td>
<td></td>
<td>163,998,416</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Table 3.7: Number of Gametypes Played
This table shows the extent to which players participate in multiple gametypes. Column 1 gives the number of gametypes ever played. Column 2 states the number of players and column 3 their relative share.

<table>
<thead>
<tr>
<th>Number</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15,460</td>
<td>48.1</td>
</tr>
<tr>
<td>2</td>
<td>6,501</td>
<td>20.2</td>
</tr>
<tr>
<td>3</td>
<td>3,652</td>
<td>11.4</td>
</tr>
<tr>
<td>4</td>
<td>2,412</td>
<td>7.5</td>
</tr>
<tr>
<td>5</td>
<td>1,861</td>
<td>5.8</td>
</tr>
<tr>
<td>6</td>
<td>1,406</td>
<td>4.4</td>
</tr>
<tr>
<td>7</td>
<td>708</td>
<td>2.2</td>
</tr>
<tr>
<td>8</td>
<td>98</td>
<td>0.3</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>44</td>
<td>0.1</td>
</tr>
<tr>
<td>Total</td>
<td>32,159</td>
<td>100.0</td>
</tr>
</tbody>
</table>

3.3.2 Players

With no personal data recorded in the IRC Poker Database not much can be concluded on the population characteristics of the sample. The little information available is discussed in this section.

The only personalized data included in the database is a player’s nickname. Players may use any name of up to nine characters to create an account on IRC poker.\textsuperscript{14} Even this space is not fully used by most players with 17% of the names consisting of four or less characters and 84% of eight or less.

There is an indication that players might be using multiple accounts.\textsuperscript{15} Of the 32,159 players about 4,000 used a nickname ending in a number. This could be on purpose, like in “Lucky7”, as a name was already taken, e.g. “nguyen70”, or for conveniently naming multiple accounts, for example nicknames from “aces001” to “aces007”.

A case-insensitive search on phrases used in names gives an additional impression of the player population. First of all, as could be expected, they are poker players. References to terms of the game are abundant, see table 3.8 for some examples.

Second, gender is an issue with 260 names comprising “boy”, 106 “guy”, 55 “girl”, 27

\textsuperscript{14}On the consequences of special characters in this file see 3.2.3.
\textsuperscript{15}Cf. 3.2.3.
3.3. SAMPLE CHARACTERISTICS

Table 3.8: Names Referring to Poker

This table shows the results of a search for some exemplary strings with reference to poker contained in the 32,159 player nicknames. Phrases are grouped in topics, e.g. “allin”, “raise”, “call” and “fold” are all actions.

<table>
<thead>
<tr>
<th>String</th>
<th>Frequency</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ace</td>
<td>187</td>
<td>Not counting any “face” like “Pokerface”</td>
</tr>
<tr>
<td>Full</td>
<td>36</td>
<td>E.g. “AcesFull”</td>
</tr>
<tr>
<td>Flush</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Straight</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Allin</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Raise</td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>Call</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>Fold</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>Shark</td>
<td>51</td>
<td>A winning type of player</td>
</tr>
<tr>
<td>Winner</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Loser</td>
<td>37</td>
<td>Some externally focused, e.g. “UrLosers”</td>
</tr>
<tr>
<td>Luck</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>Flop</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Turn</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>River</td>
<td>84</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>811</td>
<td></td>
</tr>
</tbody>
</table>

“lady”, 31 “babe” and 37 “baby”. Thus we might conclude that the majority of players is male which is usual for poker. So as has been discussed in 2.1.2 with regard to culture this sample is furthermore distorted toward male decision-making. The existence of gender specific risk taking has empirically been found for example by Levin, Snyder, and Chapman (1988) or Eckel and Grossman (2002). Byrnes, Miller, and Schafer (1999) perform a meta-analysis and provide ample references. The use of female names by male players in their accounts is also said to be a successful mode of deception. Players utilizing this seem to agree with Wilson and Daly (1985, p. 66) who expect “an evolved inclination toward the social display of one’s competitive risk-taking skills, and [...] this should be especially a masculine trait.”. Male players might be seduced by fake female players to excessively increase their risk-taking, to their likely detriment. As the German professional player Sandra Naujoks notes “Many men are testosterone loaded ego-players who do not want to lose a hand against a woman”.\footnote{Translated from German “Viele Männer sind testosterongeladene Ego-Spieler, die keine Hand gegen eine Frau verlieren wollen.” in FAZ No. 102, p. 9, 4th May 2009.} Female pros like Jennifer Harman or Jennifer Tilly might use the typical male reactions by looking especially vulnerable or seductive; for their typical appearance see figure 3.3.

In contrast to the frequent use of poker or gender terms we do not find much evidence
for the relevance of nationality. There are only seven player names stating USA, four records related to Canada, one for Russia and England. No mention is made regarding France or Germany.\footnote{Search terms are “USA”, “Canad”, “Russia”, “Engl”, “Franc”, and “Germ”.
}

Remarkably, the casts from Star Wars and Star Trek are fully represented by their fan base in the sample. We find Yoda (5), Skywalker (2), Han Solo (1), Darth Vader (1), Princess Leia (1) and the IRC Poker favorites from Star Wars, Obiwan (11) and Chewbacca (10). From Star Trek there are Kirk (8), Spock (5), McCoy (3), Sulu (2), Scotty (2) and Chekov (2). The sole female character from the bridge of the Enterprise, Uhura, is not present.

A final search resulting in 275 matches has been made for “bot”. A bot being a computer program playing poker as an application of Artificial Intelligence. IRC Poker has been a testbed for bots and one channel (#botsonly) has been even dedicated solely to this purpose. Though poker has been studied in machine learning since Findler, Klein, Johnson, Kowal, Levine, and Menig (1974) and Findler (1977) respectively, it has been the CPRG at the University of Alberta who advanced the topic over the last years. Among other things the programs are now able to generate models of their opponents (Billings, Peña, Schaeffer, and Szafron 1998), dynamically adjust probabilities (Billings, Peña, Schaeffer, and Szafron 1999; Billings, Papp, Peña, Schaeffer, and Szafron 1999;
Korb, Nicholson, and Jitnah 1999; Nicholson, Korb, and Boulton 2006; Gilpin and Sandholm 2007a) or to avoid deterministic play (Schaeffer, Billings, Peña, and Szafra 2001).\textsuperscript{18} We find bots in all gametypes and mark all players whose name contains “bot” so they can easily be excluded from any analysis.\textsuperscript{19} As the channel reserved for bots will not be part of in-depth analysis, and as even this channel is small compared to those with mostly human players, not many hands from artificial intelligence will remain in the data used for tests. Furthermore, as long as bots are not programmed to act on any of the independent variables we use, e.g. they would react to boredom, their presence works against behavioral patterns in the data.\textsuperscript{20}

3.3.3 Time

The database covers a period from April 1995 to October 2001 with only seven months missing.\textsuperscript{21} Concerning play on different days of the week no particular day sticks out. Games played are spread evenly from 14.0% of the games on the day of lowest activity to 14.5% at the day with the highest activity.

However, over the course of a day activity changes substantially with twice as many games played during the peak compared to low traffic. The hourly pattern which is shown in figure 3.4 is also typical for today’s online traffic as tracked by, among others, \url{http://www.pokerlistings.com/market-pulse/online-traffic}. Peak usage is for US and European afternoons, supporting that playing poker on the internet is still more hobby than work.

A second indication of poker being a leisure-time activity is the typical duration of a session. Summarized in table 3.9 we see that mean time played in a session is about 30 minutes depending on the gametype. Sessions in lower limits tend to be shorter, providing evidence for players’ increased dedication at higher limits. The length of sessions played online is also significantly shorter than what McGlothlin (1954, p. 146) found for women playing in poker clubs who averaged about five hours per session.

Summing time played per player over all gametypes, we find that about 10,000 players participated for more than 5 hours and of these 7,500 players even for more than 10 hours. With several hundreds of hours invested by some players it should be noted that gambling

\textsuperscript{18}Introductions to the topic can be found at Papp (1998) or Peña (1999).

\textsuperscript{19}This might lead to false-positives which is acceptable and leave out some bots named otherwise which is inevitable as they cannot be found out.

\textsuperscript{20}So far, work from the CPRG or others has, to our knowledge, not included any of the independent variables we use for testing in programming a bot.

\textsuperscript{21}See table 3.2 in section 3.1.3.
This figure graphs the relative frequency of games played on IRC poker channels per hour. Time is converted to Greenwich Mean Time (GMT).

Figure 3.4: Relative Frequency of Games per Hour of Day

Table 3.9: Duration of Sessions
This table summarizes the duration of sessions for each gametype. In column 2 mean values are given in minutes of play. Column 3 states the median.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27.6</td>
<td>5.4</td>
</tr>
<tr>
<td>1</td>
<td>25.9</td>
<td>16.3</td>
</tr>
<tr>
<td>2</td>
<td>29.8</td>
<td>18.2</td>
</tr>
<tr>
<td>3</td>
<td>24.0</td>
<td>14.4</td>
</tr>
<tr>
<td>4</td>
<td>29.8</td>
<td>17.9</td>
</tr>
<tr>
<td>5</td>
<td>36.9</td>
<td>23.8</td>
</tr>
<tr>
<td>6</td>
<td>40.6</td>
<td>26.2</td>
</tr>
<tr>
<td>7</td>
<td>46.1</td>
<td>29.9</td>
</tr>
<tr>
<td>8</td>
<td>34.9</td>
<td>20.6</td>
</tr>
<tr>
<td>9</td>
<td>31.1</td>
<td>26.8</td>
</tr>
</tbody>
</table>
3.3. SAMPLE CHARACTERISTICS

This figure presents a box-plot of the time (in seconds) players take on average for an action in each gametype. The upper bounds of the box are the 3rd quartile, the line in the middle is the median, the lower bounds are the 1st quartile. The upper/lower whiskers represent values which are smaller/larger or equal to the quartile + 1.5-interquartile ranges. Any value outside these ranges is represented by a dot. Only players with at least 1,000 hands played and who are not marked suspicious are evaluated.

Figure 3.5: Average Time per Action

can be a source of addiction. Becker and Murphy (1988) demonstrate how individuals who discount the future heavily are more likely to rationally develop addictions as their present utility from consumption is lowered by tolerance due to past consumption. Among others, variable reinforcement schedules of gambling, entrapment and irrational beliefs, are usually stated as sources of compulsive gambling.\textsuperscript{22} The process is self-enforcing. Control over gambling is reduced with increased duration or frequency of play.\textsuperscript{23}

How fast are decisions made in online poker? Figure 3.5 provides the answer for IRC poker. Not taking gametypes 0, 3 and 8 into account where less than 50 players are included in the analysis speed of play significantly depends on the limit played. On the lower levels of limit play (10/20 limit; gametypes 1, 2 and 4) the mean player takes about 15 to 19 seconds on average to decide upon an action. At the higher limit games decisions are made faster, 14 seconds at 20/40 limit and less than 8 seconds on 50/100 limit.

\textsuperscript{22}See e.g. Wenger, McKechnie, and Wiebe (1999, pp. 13-14).
\textsuperscript{23}See Scannell, Quirk, Smith, Maddern, and Dickerson (2000, p. 426).
We hypothesize that this effect is due to increased experience and learning when players become familiar with certain situations and adopt standardized decision rules. The role of experience and learning will be discussed in depth in section 9.

The speed of play has been increased even further by online casinos. Now there are time allowances of usually only 15 seconds per action on sites such as www.fulltiltpoker.net. This time can only be extended upon request and the extra time is granted from a limited “reserve”. The use of tick-boxes which allow to preselect a choice helps to accelerate play.

However, which action a player choses is still based on his own decisions. How his decisional behavior can be measured and which types of players can be found in online poker will be discussed in the next chapter.
Chapter 4

Measures of Behavior at Poker

Tum sumus incauti, studioque aperimur in ipso, nudaque per lusus pectora nostra patent [Sometimes, when we are not properly on our guard, when we are carried away by the heat of the game, we forget ourselves and let our inmost nature stand revealed.]

Ovid, Ars Amatoria 3.371-372

4.1 Two Behavioral Dimensions

Poker players just like the managers interviewed by March and Shapira (1987) make a sharp distinction between risk and gambling. Risk is seen as manageable and risks are readily taken whenever the uncertainty surrounding the decision can be reduced by skill or information. In contrast to this, situations where chances are externally given and uncontrollable are referred to as gambling. A person betting all his chips without looking at his cards will frequently and nonchalantly be called a gambler rather than a player by his opponents.

Perceived risk is a core characteristic of any kind of gamble, risky or uncertain, and a descriptive theory of decision-making has necessarily to include this concept. Risk as well as attractiveness of gambles are judged based on the stimuli a gamble provides. At this point, heuristics and biases in the decision-makers’ cognition enter the process.

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3See Nygren (1977, p. 578).
5See Arrow (1982).
Although the exact form of the functional relationship between stimuli and perceived risk may vary (Coombs and Meyer 1969; Keller, Sarin, and Weber 1986), its multidimensional nature is evident (Coombs and Bowen 1971; Nygren 1977). Of the many possible dimensions two have empirically been found to be prevalent: the amount of potential loss and the probability of occurrence of loss.\(^6\) Therefore, we aim for measures of behavior at poker including these fundamental dimensions.

The same stimuli dimensions can generally be framed in several alternative ways. Thereby, it is not only framing the issue from a personal or group perspective (Vaughan and Seifert 1992) but also the content domain (Slovic, Fischhoff, and Lichtenstein 1984; Levin, Johnson, Russo, and Deldin 1985; Weber, Blais, and Betz 2002) that determine the mental representation of the risk involved. In a gambling environment the most natural strategy to deal with information is using a numerical representation (Rettinger and Hastie 2001). But even within a specific domain like gambling the variables do not need to be the same. Shapira and Venezia (1992) find that the demand for lotteries is primarily determined by the size of the first prize and the number of small prizes, measures which do not translate to poker. In the poker environment the two characteristic dimensions are the probability of winning with a given hand (cf. section 2.4) and the stakes to be won and lost, respectively.\(^7\) How these dimensions can be measured will be discussed in the following two sections.

### 4.1.1 Looseness

Due to the random dealing process every player may expect to get the same share of good and bad starting hands over the long-run. Consequently, given that players are able to rank-order starting hands correctly and rather play a good hand than a bad one, the more hands a player plays the lower the average strength of the hands he plays.\(^8\) Ranking starting hands’ strength on a scale from 0 to 100, playing every hand would result in an average strength of 50. Only playing the best hand gives an average strength of 100.

Not knowing the actual hands a player is dealt, we can nevertheless conclude that, ceteris paribus, the more hands he plays the worse his probability of winning the hands he plays in the long-run. We define a player’s looseness as measure to account for the

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\(^7\)The amount of potential loss and potential gain are invariably linked through the equalization method; see 2.3.3.

\(^8\)Both assumptions seem reasonable. For example Spetzler and von Holstein (1975, p. 350) state that subjects can easily identify with probability of events relative to poker hands. The second argument would have to be extended to be game-theoretically precise to: rather play the best hands and an appropriate share of the worst hands (to bluff) than mediocre hands; e.g. Chen and Ankenman (2006).
4.1. TWO BEHAVIORAL DIMENSIONS

probability dimension of risk as follows:

**Definition 1** A player’s looseness is the share of hands dealt to him which he at least plays to the flop

or formally

\[ L_i = \frac{h_{fi}^i}{H_i} \]  

(4.1)

where \( L \) is the looseness of player \( i \), \( H \) the total of hands dealt and \( h \) the number of hands played on the flop (\( fl \)). Note that \( L_i \in [0; 1] \).

It is obvious that alternative definitions could be developed easily. Andersson, Karlsteen, and Andersson (2005, p. 3) define looseness as how many times a player voluntarily put money in the pot (without being forced to pay the blind fee). This measure is not bounded and causes additional computational difficulties as it is conditional (blind/no blind). Therefore, it is not considered further. The measure could be defined on other phases of the game (turn (\( tu \)), river (\( ri \)) or showdown (\( sd \)). As \( H_i \geq h_{fi}^i \geq h_{iu}^i \geq h_{ri}^i \geq h_{sd}^i \) evaluating later phases of the game pushes the measure toward the lower bound. To keep skewness as low as possible play to the flop is evaluated. This also avoids distortions due to hands where players should stay in the game as they are paying according to the odds for drawing.

The poker jargon distinguishes players on the looseness dimension from tight where only a small range of hands is played and most hands are folded (\( L \rightarrow 0 \)) to loose with few folds and a wide range played (\( L \rightarrow 1 \)).

From an economic perspective looseness relates to the rate or relative frequency of investments. A loose player could be seen as an optimistic investor who takes many opportunities. Tight players act more conservatively and only selectively invest for value.

### 4.1.2 Aggressiveness

The vying property of the game (cf. 2.3.3) implies that a player influences the amount of potential loss and potential game by his actions. On the one hand he can keep the amounts staked as low as possible by checking or calling (or eventually folding). On the other hand he can actively increase the stakes by betting or raising. The dimension covering the differences between these kinds of actions is a player’s aggressiveness.

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\(^9\)Which follows easily from \( h_{fi}^i \geq 0 \lor H_i > 0 \lor H_i \geq h_{fi}^i \).

\(^{10}\)For example Schoonmaker (2000, pp. 84-87).
Aggressiveness is defined by Andersson, Karlsteen, and Andersson (2005, p. 3) as a player’s raise percentage plus his bet percentage divided by his call percentage. This measure has a lower bound of 0 but is not bounded upwards. Another definition is given by Schoonmaker (2000, pp. 83) who writes that “Aggression should be measured, not by the total number of raises, but by the ratios of raises to calls and bets to checks.” Here the measure is composed of two ratios and no advise is given on how to combine both. In both definitions no account is taken of the fact that in limit games a bet or raise on turn or river involves twice the amount required pre-flop and on the flop. The definition of aggressiveness used in this thesis is:

**Definition 2** A player’s aggressiveness is the ratio of amounts bet or raised to the total amount staked

or formally

\[
A_i = \frac{S^b_i + S^r_i}{S^b_i + S^r_i + S^c_i} \tag{4.2}
\]

where \(A\) is the aggressiveness of player \(i\), \(S^b\) the amount bet, \(S^r\) the amount raised and \(S^c\) the amount called.\(^{11}\) With \(S^b_i, S^r_i, S^c_i \geq 0\) it is easy to see that \(A_i \in [0; 1]\).\(^{12}\) In order to avoid additional complexity this measure leaves some aspects of aggression untouched such as check-raising or betting/raising from an early position, actions which are particularly aggressive.

The aggressiveness dimension is a continuum from **passive** to **aggressive**. A player is passive if he mostly reacts by calling others’ bets and raises (\(A \to 0\)). An aggressive player actively and predominantly increases the stakes by betting and raising (\(A \to 1\)).\(^{13}\)

Picking up the analogy to business, aggressiveness is an indicator of the agent’s typical investment volume. He has the choice whether to keep capital expenditure low by investing the minimum required to match the competition or to expand the scale and go big business. A bet or raise can be seen as an investment in additional machinery or the entry in more markets and thus is an aggressive signal or threat, respectively.

### 4.2 Four Archetypical Players

The two dimensions, looseness and aggressiveness, cannot only be used to analyze playing behavior but also offer a way to classify players according to their playing style. Since

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\(^{11}\)Note that in IRC poker no actual dollars are used. Players can also invest chips by an all-in action. As every all-in action can be either classified as a bet, raise, or call it is not mentioned separately. Here total amounts are used, e.g. \(S_i = \sum_{h=1}^{H} S_i^{b,h}\) where \(S_i^{b,h}\) is the amount bet in hand \(h\).

\(^{12}\)The measure is not defined for a player who has only paid the blinds.

\(^{13}\)For example Schoonmaker (2000, pp. 87-90).
4.2. FOUR ARCHETYPICAL PLAYERS

The four most widely used playing style classifications are shown in a matrix based on the two dimensions used for distinction, looseness and aggressiveness.

Figure 4.1: Two-Dimensional Scheme of Playing Styles

Blake and Mouton (1964) introduced their managerial grid the use of two-dimensional schemes to rate behavioral styles has been implemented manifold. This method is also widely used in the poker environment (e.g. Schoonmaker 2000; Burns 2004, pp. 71-80). Typically four different styles are distinguished which will be discussed in turn. A comprehensive account of playing styles and additional strategic advise is presented by Schoonmaker (2000, 2007). Here we focus on the behavioral characteristics. Figure 4.1 illustrates how the four archetypical playing styles relate to the two dimensions of behavior.

4.2.1 Rocks

A player who acts extremely tight-passive is called a rock. Players of this kind fold many hands and prefer winning small amounts rather than enduring large swings between winning and losing.

Schoonmaker (2000, pp. 207-209) notes several behaviors that are characteristic of rocks. The three most prominent are

- **Control.** They are focused on avoiding risks and the extreme conservatism marks their behavior. Neat unremarkable clothing, few gestures and expressions, or efforts to avoid attention can be observed.

- **Well-organized.** Chips are well organized and cards are held or folded accurately. Only investing the minimum required to play at a table is also characteristic of
- *Indifference.* Rocks do not take much interest in the other players except for their actions. They do not chat much and do not show a lot of response to the action at the table.

The most common rocks are retired people looking for entertainment on limited expenses. To summarize, rocks are risk-averse players who invest only conservatively in the best hands.

### 4.2.2 Calling Stations

A *calling station* stays in many hands but seldom bets or raises. His playing style is extremely loose-passive. Their primary objective is not to win but to socialize and pass time.\(^{14}\)

According to Schoonmaker (2000, pp. 159-160) three of their outstanding qualities are

- *Friendliness.* An accommodating, non-demanding manner is characteristic as calling stations are afraid of intruding or offending others.

- *Relaxation.* As they are more interested in socializing than winning, little attention is paid to the cards or the action. Consequently, loose-passive players often lack card-reading skills and have less confidence in their decisions.

- *Timidity.* Calling stations tend to fear conflict and competition. They are afraid of being bluffed or feeling foolish for folding a winning hand, so they often pay to see.

In short, calling stations try to be everybody’s darling. In business, they would be followers offering me-too products in an undisputed market. They tend to herd together with others when making their decisions.

### 4.2.3 Maniacs

The *maniac* plays a wide range of hands and bets and raises frequently, a loose-aggressive playing style. He is literally addicted to action. His bankroll varies significantly as the maniac stakes more chips and does so more frequently than any other kind of player.\(^{15}\)

Schoonmaker (2000, pp. 109-112) lists several behavioral cues which indicate a maniac.

\(^{14}\)See Schoonmaker (2000, pp. 76-77).

\(^{15}\)See Schoonmaker (2000, pp. 77).
4.2. FOUR ARCHETYPICAL PLAYERS

- **Machismo.** They are at the center of attention, participating in many hands and urging others to play. Men are much more likely to be of this kind, trying to show off, for example with excessively large buy-ins.

- **Extroversion.** Maniacs are loud, offensive and use extensive gesture and mimic. Their appearance is eye-catching or even flashy.

- **Lack of control.** Due to the high level of adrenaline these players try to maintain, they are in a state of constant nervous tension. Maniacs arrange their chips negligently and often tinker with their cards.

To sum up, maniacs are the most impatient and active players. Their poker play resembles the activity level of day-traders or some entrepreneurs. Substantial risks are taken frequently and with high confidence.

4.2.4 Sharks

The fourth playing style is the *shark*, also called *stone killer*. His main goal is to win. Most professionals are of this kind. Quoting Schoonmaker (2000, pp. 77) “...they are neither afraid of, nor addicted to taking risks. Risks are just a part of the game that should be calculated and controlled”

Drawing on Schoonmaker (2000, pp. 250-251) here are three major characteristics of sharks

- **Discipline.** Sharks control the desires present in other styles. They do not gamble, socialize or relax but are playing for the challenge. Therefore, they are constantly alert and follow the action in a concentrated manner.

- **Profit-orientation.** They stay alert and try to identify situations where the cards give them an edge. In these selected cases they invest aggressively.

- **Methodical.** Play is systematic and deliberate. Sharks are rule focused and concentrate on proper conduct. They try to learn and optimize their style whenever possible.

Players of this kind are the venture capitalists or private equity firms at the poker table. They invest selectively but do not shun taking large risks. The market is constantly screened for useful information and new opportunities.
4.3 Observed Behavior

In this section method and results of a classification of players based on records of past play are presented. For each player in the player database and each gametype, the actions are analyzed to calculate looseness and aggressiveness (cf. equations (4.2) and (4.1)). Furthermore, the average number of players in the games of a specific player is added to the player database. With more players in a game looseness and aggressiveness are lower as hands have to be especially strong to be played against multiple opponents and fewer opportunities to bet/raise arise as opponents might already have taken the action.

In a second step players with suspicious behavior are identified (cf. 3.2.3).

4.3.1 Borderline and Regular Behavior

Four criteria are defined which indicate behavior that is not normal.

1. Some players almost never win. It can be assumed that they are losing on purpose as the random dealing process guarantees that they will eventually hold a winning hand. A very low percentage of hands won can only be achieved by intentionally folding before showdown. The percentage of hands lost is calculated for each gametype and all players with more than 95% of hands lost or who are in the top percentile for this measure are marked.

2. Some players win too often. In analogy to what has been said under 1. under normal circumstances players cannot win a share of close to 100% of their hands. Thus the percentage of hands won is calculated and all players are marked who win more than 95% of their hands or are in the top percentile for this measure. This extraordinary success indicates that they are playing against opponents who lose on purpose, e.g. by using multiple accounts to harvest chips.

3. Some players play almost every hand or no hand at all. In section 4.1.1 it has been shown that $L_i \in [0; 1]$, now we exclude borderline values and mark all players for whom $L_i \notin [0.01; 0.99]$. Either they are losing on purpose or they do not take the game seriously, both conditions are not desired while studying decision-making behavior.

4. Some players raise every hand and some never bet or raise. Raising every hand favors very volatile and chance dependent outcomes. Never betting and raising also significantly reduces the skill involved in the game and is suspicious. Therefore,

---

$^{16}$In a similar approach Herbert (2008) uses rule-learning algorithms to identify playing style from observed play.
4.3. OBSERVED BEHAVIOR

analogous to 3., given that $A_i \in [0;1]$ (see 4.1.2), players with $A_i \not\in [0.01;0.99]$ are tagged.

The results of the analysis of suspicious behavior are presented in table 4.1. For each criterion players are found. Most conspicuous behavior is due to an unusually high percentage of lost hands. Substantially fewer observations of an extraordinarily high percentage of hands won are found. This supports the assumption that multiple accounts are used to transfer chips to a target account. Comparing the results of the looseness criterion to the one on aggressiveness it is apparent that more tags are caused by the frequency of play for all gametypes except the tournament variety. As the tournaments are no-limit games it is feasible to move all-in early in the game trying to “double up” or “gambling up” a very aggressive strategy which significantly reduces the skill involved in the game.

Table 4.1: Suspicious Players by Gametype and Criterion

In this table the results of the search for suspicious behavior are presented by gametype (column 1) and criterion. The number of players marked due to extremely high percentage of hands lost is found in column 2. The further criteria, extremely high percentage of hands won (column 3), borderline looseness (column 4) and borderline aggressiveness (column 5), and the total number of players marked as suspicious (column 6) are given (cf. table 3.5). Note that the total does not equal the sum of columns 2 to 5 as some players satisfy multiple criteria. For direct comparison the total of players in a gametype is included in column 7.

<table>
<thead>
<tr>
<th>#</th>
<th>Loss</th>
<th>Win</th>
<th>$L_i$</th>
<th>$A_i$</th>
<th>Suspicious</th>
<th>All players</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25</td>
<td>3</td>
<td>32</td>
<td>7</td>
<td>38</td>
<td>90</td>
</tr>
<tr>
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<td>922</td>
<td>469</td>
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<td>269</td>
<td>75</td>
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<td>148</td>
<td>424</td>
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<td>48</td>
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<td>115</td>
<td>404</td>
<td>4,887</td>
</tr>
<tr>
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<td>5</td>
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<td>9</td>
<td>3,044</td>
<td>156</td>
<td>936</td>
<td>3,667</td>
<td>5,600</td>
<td>15,966</td>
</tr>
</tbody>
</table>

Excluding all players tagged suspicious the results of the evaluation of looseness and aggressiveness are tabulated in 4.2. With regard to looseness it is found that nearly the whole spectrum is populated. The highest diversity is in the original channel (gametype 2) where players exhibit looseness from .018 to .990. The distribution is almost symmetrical around a mean of about .5 depending upon the variety. This is a nice property and confirms the chosen definition (cf. 4.1.1). Exceptions are the no-limit and no-limit tournament games (gametypes 8 and 9) which show a considerably lower looseness with means of approximately .3. This can be attributed to the fact that in no-limit games less
hands can be played based on drawing to a better hand if players with made hands raise their bets sufficiently. Consequently less hands are played overall. Aggressiveness is also distributed around means slightly above .5 (.51 to .57) if the channel reserved for bots is excluded (gametype 0). The distribution is symmetrical, too, with the broadest range in gametype 2 (.011 to .988). Note that the number of observations is lower by up to nine observations (gametype 2) which occurs if players who have only played a few hands stay in the game by checking the big blind, i.e. $b_i + r_i + c_i = 0$ so that $A_i$ is not defined.

Table 4.2: Regular Behavior by Gametype

The behavior of players who are not marked suspicious is summarized in this table as follows. The distribution of looseness in the player population is in panel A and the same regarding aggressiveness in panel B. Statistics include the minimum value observed for any player (column 2), the 25th percentile (column 3), mean and median (column 4 and 5), the 75th percentile (column 6), the maximum (column 7) and the total number of players evaluated (column 8). No values for aggressiveness in gametypes 7 to 9 are given as amounts called, bet or raised in these games cannot be gathered from the IRC Poker Database. Values are rounded to three decimals.

<table>
<thead>
<tr>
<th>#</th>
<th>Min</th>
<th>p25</th>
<th>Mean</th>
<th>p50</th>
<th>p75</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.116</td>
<td>.389</td>
<td>.491</td>
<td>.508</td>
<td>.600</td>
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Panel B: Aggressiveness

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</tbody>
</table>

Comparing the entry-level 10/20 limits (gametypes 0 to 4) to the advanced levels of 20/40 and 50/100 it is noticeable that looseness decreases with higher limits but aggres-
siveness tends to increase with the exception of gametypes 0 and 3. Behavior becomes tight-aggressive as players advance further. This pattern could well be caused by changes in the game environment which require strategic adjustments. Therefore, the number of players in the games has to be analyzed (table 4.3) since with more players in the game less hands should be played and more calls are unavoidable as the betting/raising action is shared. With a mean of 2.84 players there is the least competition in gametype 0 which explains the very high aggressiveness. For the other well populated varieties there are between 6.4 and 7.5 players on average on the low limits. On the higher limits games are less frequented with a mean of 6.6 at 20/40 and 4.8 at 50/100. As the presence of fewer players would favor strategies with more hands being played, the observed values of looseness imply actual differences in behavior. No such conclusion can be drawn regarding aggressiveness which is in line with the expected upward shift due to the presence of fewer players.

Table 4.3: Average Number of Participants per Player by Gametype
For every player the average number of players in games he participates in has been calculated. Here summary statistics across the player population as in table 4.2 are given to illustrate how the different gametypes are frequented. In the 20/40 Limit (gametype 5), for example, the player who participated in the most highly frequented games (column Max) was playing at a table with 11.2 players on average (him and 10.2 opponents), whereas the median player for this gametype engaged in games with an average of 6.74 players. Values are rounded to two decimals.

<table>
<thead>
<tr>
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<th>p25</th>
<th>Mean</th>
<th>p50</th>
<th>p75</th>
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<td>8.15</td>
<td>9.74</td>
<td>23.00</td>
<td>10,366</td>
</tr>
</tbody>
</table>

4.3.2 Playing Style Classification
Based on the distribution of looseness and aggressiveness in the player population, playing style can be classified according to the types discussed in section 4.2. No absolute thresholds are used but playing style is evaluated relative to the behavior found in the population. This creates natural benchmarks and avoids selecting arbitrary values. For

\[ \text{All unpaired mean-comparisons (t-tests) are significant beyond the .0001 level as sample sizes are sufficiently large.} \]
both dimensions, looseness and aggressiveness, the distribution is split into deciles and every player assigned according values from 1 to 10.

All players with regular behavior and for whom both dimensions are defined are classified as follows\textsuperscript{18}

- **Calling Station.** Player ranks in three lowest deciles (1-3) for aggressiveness and three highest deciles (8-10) for looseness.

- **Rock.** Player ranks in three lowest deciles (1-3) for aggressiveness and three lowest deciles (1-3) for looseness.

- **Maniac.** Player ranks in three highest deciles (8-10) for aggressiveness and three highest deciles (8-10) for looseness.

- **Shark.** Player ranks in three highest deciles (8-10) for aggressiveness and three lowest deciles (1-3) for looseness.

- **Other.** Player is neither of the above four types.

Note that not all classes have to be populated equally if distributions of looseness or aggressiveness are sufficiently skewed or narrow. An example of the resulting grid is presented in figure 4.2. It shows that the whole range of playing styles is practiced but that for the lion’s share of players playing style is less pronounced.

The results of the classification for all varieties are put together in table 4.4. The four archetypical playing styles each account for approximately 10% of the players in every gametype with a somewhat lesser share of rocks and maniacs represented slightly more often. How successful the different playing styles are, is part of the discussion in the following chapter.

\textsuperscript{18}See Schoonmaker (2000, pp. 71-80) who uses a grid from 1-9.
The classification of players in the 50/100 Limit games according to playing style is graphed in this figure. For every individual player $L_i$ and $A_i$ give a point in the grid and the extreme types are marked according to their archetype.

Figure 4.2: Classification of Players in Styles Grid
Table 4.4: Population of Playing Styles by Gametype
For each variety the number of players and the relative percentage of the playing styles are tabulated. Values are rounded to one decimal of a percent.

<table>
<thead>
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<th>Calling Station</th>
<th>Rock</th>
<th>Maniac</th>
<th>Shark</th>
<th>Other</th>
</tr>
</thead>
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<td>11.5%</td>
<td>7</td>
<td>13.5%</td>
</tr>
<tr>
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<td>819 8.5%</td>
<td>773</td>
<td>8.0%</td>
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<td>12.9%</td>
</tr>
<tr>
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<td>1,970 10.5%</td>
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<td>6.4%</td>
<td>1,892</td>
<td>10.1%</td>
</tr>
<tr>
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<td>54 8.9%</td>
<td>63</td>
<td>10.4%</td>
<td>62</td>
<td>10.2%</td>
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<tr>
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<td>7.5%</td>
<td>626</td>
<td>11.6%</td>
</tr>
<tr>
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<td>753 10.6%</td>
<td>529</td>
<td>7.5%</td>
<td>683</td>
<td>9.6%</td>
</tr>
<tr>
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<td>159 9.8%</td>
<td>144</td>
<td>8.9%</td>
<td>161</td>
<td>10.0%</td>
</tr>
<tr>
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<td>429 9.6%</td>
<td>393</td>
<td>8.8%</td>
<td>400</td>
<td>8.9%</td>
</tr>
<tr>
<td>8</td>
<td>11 13.9%</td>
<td>6</td>
<td>7.6%</td>
<td>3</td>
<td>3.8%</td>
</tr>
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<td>1,234 11.9%</td>
<td>856</td>
<td>8.3%</td>
<td>563</td>
<td>5.4%</td>
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</table>
Chapter 5

Poker Finance

It is not whether you are right or wrong that is important, but how much money you make when you are right and how much you lose when you are wrong.

George Soros

This chapter introduces first basic parallels between poker and finance. Some mathematics are used to underline the relevant principles but for simplicity no rigorous proofs are included. In financial terms the discussed topics are the valuation of risky assets, portfolio selection and the value of information as competitive advantage. In the poker environment these relate to the strength of hands played, playing style and positional play.

5.1 Investing into Hands

Playing Texas Hold’em follows clear-cut life-cycles. An entrepreneur might think of them as the seed stage, early stage and expansion/growth phase of a startup. In the seed stage the entrepreneur somehow comes up with a rough idea of a project but does not know the market value yet. The poker player is dealt two hole cards pre-flop, has an approximate impression of their strength but has not got a 5-card hand yet, neither does he know anything about the market environment, i.e. competition. Product development is mostly done in the early stage and the product can be launched. Still commercial success can only be estimated vaguely. With three cards on the flop a 5-card hand is seen but only preliminarily so as turn and river still offer additional potential. Also somewhat more is known about competition as can be seen who remains in the game. In the later stages the product has been put to the market and feedback is known. Now the investments required for further expansion (e.g. international roll-out) increase substantially and most
entrepreneurs or venture capitalist who financed the business so far are looking for an exit strategy. Yet in analogy, the poker player can now assess the relative strength of his hand fairly well (based on his best 5-card hand out of 6 or 7-cards available). Simultaneously, on turn and river in limit games the bets are twice the amount compared to bets pre-flop and on the flop, so staying in the game gets expensive. The player now has to consider intensely whether to fold (liquidate the investment), bet/raise (try to crowd out the competition) or any action leading to showdown (which might be putting the business to the test of an initial public offering).

As this analogy underlines playing poker has a lot in common with investing into business. In the following sections three of the governing principles will be discussed in detail. First, influences on the fundamental value of hands and the calculation of this value are presented. Then in section 5.1.2 the relation between risk and return is examined. Finally, in 5.1.3, differences in investment behavior by playing style are explored.

5.1.1 Equity-Valuation of Hands

There are 169 distinct starting hands a player can be dealt in Texas Hold’em (cf. 2.4.1). If we want to determine their relative strength a method is required which excludes any influences due to the players’ actions. This is generally done by assuming that all players move all-in pre-flop and compare their hands at showdown. Consequently, the showdown value or all-in-equity can be calculated. To do so two approaches are usually applied.\(^1\) In a search by brute force all possible combinations are evaluated to get the percentage of pots won (including partial pots for splits). For a two player game 20,975,724,400 states\(^2\) have to be evaluated which takes some time but can be done.\(^3\) However, this method reaches its limitations if games with more players have to be evaluated. Alternatively, a Monte Carlo simulation can be performed where, for example, 1,000,000 random games are simulated for each of the 169 possible starting hands.\(^4\) The results of such a simulation are presented in table 5.1.

By inspecting table 5.1 the first three properties of the starting hands influencing all-in-equity can be seen; a fourth will be discussed further down. First, pocket cards stick out among cards for each rank, which directly gives the second feature, the rank of the cards. Consulting, say, the column from (49s) to (92o) the showdown value decreases monotonically with lower ranking cards except for the pocket pair (99) where 72.1 is even

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\(^1\)See Sylvestro (2006) who combines both methods.
\(^2\)((50)\(^5\) \text{ opponent hands} \text{ and } (48)\(^5\) \text{ possible boards}.
\(^3\)See e.g. Shackleford (2004).
### Table 5.1: All-In-Equity in a Two-Player Game

Each entry in the following table shows the percentage of pots won (including partial pots for splits) for a hand in a game with one opponent who is holding a random hand. Values are based on a Monte Carlo simulation of 1,000,000 games for each entry. The player’s hand is given by the combination of the rank taken from column and row. The diagonal consisting of paired pocket cards from (AA) to (22) separates suited hands in the upper triangle (e.g. (AKs), (32s)) from offsuit hands in the lower left half of the table.

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<th></th>
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<th>Q</th>
<th>J</th>
<th>T</th>
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<th>8</th>
<th>7</th>
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<td>58.9</td>
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<td>38.0</td>
<td>37.9</td>
<td>57.0</td>
<td>38.0</td>
<td>36.3</td>
</tr>
<tr>
<td>3</td>
<td>55.6</td>
<td>51.2</td>
<td>47.9</td>
<td>45.0</td>
<td>42.4</td>
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<td>37.5</td>
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<td>34.4</td>
<td>53.7</td>
<td>35.1</td>
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<td>34.0</td>
<td>33.9</td>
<td>32.5</td>
<td>31.2</td>
<td>50.3</td>
</tr>
</tbody>
</table>

Source: Brecher and Churilla (1999)

Higher than 63.0 for (A9s). Third, suited hands have an advantage compared to their offsuit counterparts between 1.6 ((AKs) vs. (AKo)) and 3.9 ((32s) vs. (32o)).

All-in-equity does not tell us how much money a player will make on a particular hand but it helps to grasp the expected value which can be extracted from a hand. Mathematically the expected value $E(X)$ is the weighted average of values $x_1, x_2, ..., x_n$ a random variable $X$ can assume, weighted by the respective probabilities $p_1, p_2, ..., p_n$, as expressed by \(^5\)

$$E(X) = p_1x_1 + p_2x_2 + ... + p_nx_n = \sum_{i=1}^{n} p_ix_i \quad (5.1)$$

For example, a player holding a pocket pair of kings (KK) who is all-in for an investment of $100 against one opponent holding a random hand can expect to end up with $0.824 \times ($100 + $100) + (1 \mathbf{-} 0.824) \times 0 = $164.8, a return of 64.8%. Note that this is an aggregate figure of all situations in which the player can expect to win, draw or lose. It represents the long-run average result of the simulation performed an arbitrarily

\(^5\)See e.g. Epstein (1967, p. 23).
All-in-equity for all 169 starting hands in Texas Hold’em is shown. Hands are ordered by the rank of the all-in-equity in a 2-player game. Different shadings of gray are used to distinguish the all-in-equity for \( n \)-person games, where \( n \) is 2, 4 or 10. Filled circles are used for hands with suited and/or connected cards; circles are empty otherwise.

Figure 5.1: All-In-Equity Depending on Number of Players in the Game

large number of times. This value is also known as the mean which is represented by the symbol \( \mu \), i.e. \( \mu = E(X) \).\(^6\)

If the player’s (\( KK \)) would be against (\( AA \)) the expectation changes dramatically. With a tool like Pokerstove\(^7\) it is easy to analyze that he will now only win 17.82%, tie 0.46% and lose 81.72% of the possible games. Accordingly, the expected value changes to \( 0.1782\times$200+0.0046\times$100+0.8172\times$0 = $36.1 \), a loss of 63.9% of the initial investment.

The showdown value depends significantly on the number of opponents since with an increasing number of competing hands, the winning 5-card hands tend to be stronger. Therefore, starting hands which can improve a lot by drawing to, say, a straight or flush are relatively stronger in multi-player games. As a fourth property besides pairs, ranks, suited/offsuit, connected pocket cards provide an additional advantage especially in games with many players. While in a 2-player game (\( QTo \)) is a better hand than (\( JTo \)) with an all-in-equity of 57.4 compared to 55.4 the order changes in a 10-player game. There (\( QTo \))

\(^6\)See Epstein (1967, p. 25).
\(^7\)www.pokerstove.com.
\(^8\)Here the value from split pots is shown explicitly in the second addend.
5.1. INVESTING INTO HANDS

For games with 2, 4 and 10 players (different shadings of gray) the normalized relative share of hands at showdown ($\Phi_h$, cf. equation (5.2)) is graphed. The example is from the 10/20 I variety (the original channel).

Figure 5.2: Hands at Showdown Depending on Number of Players in the Game

has an all-in-equity of 12.9 and ($JT\cdot$o) with 13.1 becomes the slightly better hand. Figure 5.1 illustrates the relationship between the number of players in the game and the card fundamentals. The rank order of starting hands in the 2-player game ranging from (AA) to (32o), as it can be deduced from table 5.1, serves as scale in figures throughout this thesis. The reader may verify that suited or connected cards improve their rank in games with several opponents whereas small pocket pairs lose much of their initial strength.

Are players aware of the hands’ strength based on equity-valuation? In figure 5.2 the normalized relative frequency of the 169 starting hands at showdown is graphed.\(^9\) To compare between different hands frequencies have to be normalized as follows. First note that for each pocket pair there are $\binom{4}{2} = 6$ combinations, for each suited hand $\binom{4}{1} = 4$ and for each offsuit hand $\binom{4}{1}\binom{3}{1} = 12$. To compare these types frequency is doubled for pairs and tripled for suited hands. The resulting normalized frequency $f_{hn}$ is summed over all hands to give the total normalized frequency $F^n = \sum_{h=1}^{169} f_{hn}$. We take

$$\Phi_h = \frac{f_{hn}}{F^n/169} \quad (5.2)$$

\(^9\)Only the frequency of hands at showdown can be observed (cf. 3.1.4).
to denote the normalized relative share of hands at showdown. Thus we account for the base rates and if all hands were played to showdown with the same propensity then
\[ \Phi^h = 1 \quad \forall \quad h = (32o), ..., (AA). \]

As the curves in figure 5.2 are upward sloping players evidently realize the rank order of starting hands. Slopes get steeper and spikes are more pronounced as the number of players is increased. This implies that hands are played to showdown more selectively in multi-player games with additional emphasis on the best hands (at the far right) and hands which can improve substantially (spikes).

5.1.2 Hands as Securities

In the discussion of showdown value above, the uncertainty players are facing is not relevant as their opponents are assumed to hold a random hand. The long-run approach ascertains that a hand like (QTs) will be a winner (at least in terms of share of hands won). But the situation players actually have to deal with is a bit more complicated. First, they can be sure that their opponent will be dealt random hands in the long-run but for the immediate action it is just one hidden hand they are up to. Second, and more important for the discussion now, is that they are making decisions with limited resources. Fluctuations caused by wins and losses could cause their capital to run dry so that they will not be there to acknowledge the long-run. The risk they are running has to be accounted for.

The first moment of the distribution of wins/losses, the mean, has been introduced in equation (5.1). In addition, another parameter is generally used to measure the spread or variability of a distribution of values, the variance. Variance \( \sigma^2 \) is defined by

\[ \sigma^2(X) = E[(X - \mu)^2] = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i) \]  

and to recover the original units of the distribution, the standard deviation \( \sigma \) is defined as

\[ \sigma(X) = \sqrt{\sigma^2(X)} \]  

In a finance classic Lintner (1965, pp. 15-19) considers the situation of an investor who can invest any part of his capital in certain risk-free assets and any fraction of his capital

\[ \text{See e.g. Epstein (1967, pp. 24-26).} \]
5.1. INVESTING INTO HANDS

in any or all of a given finite set of risky securities.\footnote{He further assumes that market prices are given. This cannot be said for poker in a given hand but we assume that the market is competitive in the long-run with many players. Also note that in poker all assets are marketable, i.e. no non-marketable assets like human capital exist (cf. Mayers 1973).} In poker the risk-free alternative for a player is to surrender his hand immediately, eventually forfeiting the blinds or any interest in the pot. Therefore, the risk-free alternative bears a negative interest rate which in play on an official venue is even worse as rake has to be paid to the operators. Although players cannot invest an arbitrary amount into a particular hand they can nevertheless minimize or maximize their exposure by the actions they choose. Using all instances when they are dealt the same hand, the relative size of the investment compared to other investments can be measured.

We can adopt the findings of Lintner (1965) that an investor’s expected rate of return is related linearly to the risk of return on his investment as measured by the standard deviation of his return in the following way\footnote{The model assumes that borrowing and lending of the riskless asset is unrestricted, which does not apply to poker. However, Black (1972, p. 455) writes that the expected return is still a linear function of risk even if no riskless borrowing or lending is allowed, though intercept and slope may be different. Although psychological risk is influenced by many dimension (variance, semivariance, skewness, range, probability of losing, amount to lose, ...) preferences are single-peaked so individuals seem to integrate risk into a one-dimensional construct which in our case is represented by the standard deviation of return (Coombs, Donnell, and Kirk (1978)).}:

\begin{align*}
\bar{\mu}_i &= \mu^* + \omega(\bar{\mu}^h - \mu^*) \tag{5.5} \\
\bar{\sigma}_i^2 &= \omega^2 \bar{\sigma}_{2,h}^2 \tag{5.6}
\end{align*}

where \(\bar{\mu}_i\) is player \(i\)’s expected return, \(\mu^*\) the risk-free return, \(\bar{\mu}^h\) the expected return from hand \(h\), \(\omega\) the share of wealth invested in a hand, \(\bar{\sigma}_i^2\) player \(i\)’s expected variance and \(\bar{\sigma}_{2,h}^2\) the expected variance from hand \(h\).

We can see that by increasing his investment \(\omega\) a player increases his expected return but also the risk he takes.\footnote{If \(\bar{\mu}^h < \mu^*\) a player would actually expect to have a lower return by investing more, therefore we assume that \(\bar{\mu}^h \geq \mu^*\).} Assuming that players have stable expectations about return and risk of the hands they play, we substitute and rearrange the above equations to get

\[\bar{\mu}^h = \mu^* + \frac{\bar{\mu}_i - \mu^*}{\bar{\sigma}_i} \bar{\sigma}^h \tag{5.7}\]

With \(\bar{\sigma}_i\) and \(\bar{\mu}_i\) constant by assumption a linear relationship should be observed between risk and return for all hands. Figure 5.3 shows that this can be empirically confirmed.
for the showdown market. Hands with high showdown value such as high pocket pairs also bear the highest risk in terms of variability of the results. Overall the linear relationship between return and risk involved is apparent.

This figure shows the mean amount won and the standard deviation of the amount won at showdown for all starting hands. Note that the average of the mean amount won over all hands does not equal zero. Amount won at showdown includes the money from players who quit/folded in earlier stages so it is not a zero-sum distribution between the involved players. The sample is from the 50/100 Limit variety.

Figure 5.3: Risk-Return Relationship for Hands at Showdown

The risk-return relationship of the cards is one of the fundamental characteristics of the game. Players who do not adjust their actions according to these card fundamentals will suffer low returns or even losses. In figure 5.4 the hands seen at showdown are compared for overall winners to losers. On the low and medium limit there is a significant difference in the cards played to showdown. Winners are more selective and hence show a higher proportion of strong hands, both from all-in-equity (at the right of the graphs) and drawing (spikes). At the highest limit the gap from card selection between winners and losers disappears. It seems that players are familiar with the risk-return trade-off based on the fundamental values of the particular hands.
The normalized relative share of hands at showdown (cf. equation (5.2)) is compared between winners and losers (different shadings of gray). The three graphs from top to bottom are for the low 10/20 I Limit, the medium 20/40 Limit and the high 50/100 Limit.

Figure 5.4: Share of Hands at Showdown for Different Limits by Winner/Loser
5.1.3 Influence of Playing Styles

The relationship laid down in equations (5.5) to (5.7) leaves room for a variety of individual preferences regarding risk and return. So far it has only been assumed that for any \( \bar{\mu}_i \) and \( \bar{\sigma}_i \) the relationship may hold. Extending the original model Lintner (1969) proves that there will be an equilibrium in the market even if investors have different judgments. Consequently whatever the actual expectations of the player the rank order of \( \bar{\sigma}^h \) is still linked to the rank order of \( \bar{\mu}^h \), with merely a different slope \( \bar{\mu}_i - \bar{\mu}^* \bar{\sigma}_i \). So to speak, every player assures his own personal equilibrium between expected risk-adjusted returns.

In section 4.2 four types of players were introduced with distinct preferences regarding frequency and volume of investments. We repeat the method on which figure 5.3 is based, this time separating risk-return profiles by playing style. The results can be found in figure 5.5. As expected slopes of the linear regressions of the risk-return relationship are different between playing styles. Conclusions are limited to the risk adjusted excess return \( \bar{\mu}_i^* - \bar{\mu}_i \bar{\sigma}_i^* \) as a whole. It stays obscure, however, whether, say, the steep slope for maniacs is caused by an excessive expected return \( \bar{\mu}_i \) or the expectation of very low risk \( \bar{\sigma}_i \); having met some maniacs at the poker table we suppose that it is rather the former than the latter.

Extrapolating the lines in figure 5.5 it can be seen that the intercepts are not equal for all playing styles. How can differences in the risk-free alternative (the intercept, \( \mu^* \)) be explained? Put simply, the risk-free alternative in poker is not riskless. Bearing in mind that we can only evaluate hands seen at showdown, a lot of action is not incorporated in the diagram. During prior stages money is already invested and by not going to showdown this money is eventually lost. In table 5.2 performance for hands which are not seen at showdown is summarized by style and variety. In every variety all styles are net losers on average if their hand does not reach showdown. On low limits sharks and rocks perform best while on medium and high limits maniacs and sharks accumulate the lowest losses pre-showdown. Considering this, players have to estimate the risk-free rate as it is uncertain how many hands will not be played to showdown. Lintner (1969, section 2) accounts for this case and deduces that this will affect the individual investor’s portfolio but still allow for an equilibrium in the market.

Summing up, it has been seen that cards have fundamental properties determining their expected value. As the poker environment is a competitive market the higher the expected return the higher the expected risk of the hand will be, just like in a security.

\[ \text{\footnotesize Even if some investors do not have complete information about all securities, i.e. players do not know the relative ranking of all 169 hands, Merton (1987) concludes that the market will generally follow the risk-return relationship but less well-known investments tend to have relatively larger expected returns.} \]
Cf. figure 5.3 with additional separation of playing styles (different shadings of gray). The sample is taken from the 10/20 I Limit variety. Results of linear regressions are represented by solid lines.

Figure 5.5: Risk-Return Relationship for Hands at Showdown by Playing Style
Table 5.2: No-Showdown Performance by Playing Style
For all hands not seen at showdown the average absolute amount won/lost (columns Abs) and the average amount won/lost (columns Net) are summarized for each playing style.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Call. Station</th>
<th>Rock</th>
<th>Maniac</th>
<th>Shark</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Abs</td>
<td>Net</td>
<td>Abs</td>
<td>Net</td>
<td>Abs</td>
</tr>
<tr>
<td>10/20 I</td>
<td>22.6</td>
<td>-11.8</td>
<td>8.9</td>
<td>-4.6</td>
<td>35.0</td>
</tr>
<tr>
<td>10/20 II</td>
<td>23.4</td>
<td>-11.7</td>
<td>9.5</td>
<td>-4.8</td>
<td>53.3</td>
</tr>
<tr>
<td>10/20 III</td>
<td>23.4</td>
<td>-12.5</td>
<td>8.7</td>
<td>-4.4</td>
<td>41.0</td>
</tr>
<tr>
<td>20/40</td>
<td>31.0</td>
<td>-11.1</td>
<td>15.2</td>
<td>-6.0</td>
<td>42.1</td>
</tr>
<tr>
<td>50/100</td>
<td>74.5</td>
<td>-12.7</td>
<td>39.8</td>
<td>-13.3</td>
<td>94.6</td>
</tr>
<tr>
<td></td>
<td>Abs</td>
<td>Net</td>
<td>Abs</td>
<td>Net</td>
<td>Abs</td>
</tr>
<tr>
<td></td>
<td>10.1</td>
<td>-2.2</td>
<td>12.8</td>
<td>-4.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13.1</td>
<td>-1.3</td>
<td>15.3</td>
<td>-4.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13.2</td>
<td>-2.2</td>
<td>13.2</td>
<td>-4.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>18.0</td>
<td>-2.7</td>
<td>21.8</td>
<td>-4.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>54.7</td>
<td>-5.5</td>
<td>63.8</td>
<td>-6.0</td>
<td></td>
</tr>
</tbody>
</table>

market. Analysis of hands at showdown reveals that players recognize valuable hands and market them accordingly. However, there are differences in the investment style and expectations that do not only affect the performance in the later stages of the cycle but also the early phases. In the following section these aspects will be put together to give a picture of players’ success.

5.2 Venturing Players
Whereas the previous section was concerned with how risk and return of the cards become related through the competitive mechanism of the game and the expectations of the individual investors, in this section the overall risk and return for the individual is considered.

5.2.1 Risk-Return Profiles
In two of the seminal works that constituted the Capital Asset Pricing Model (CAPM), Markowitz (1952) and Sharpe (1964) established how risk and return combine if investors hold a portfolio of risky assets.\(^\text{15}\) Poker players obviously cannot diversify their investments during a single hand but they can do so over the course of the game. We can interpret the sequential investment as a matter of intertemporal choice which was integrated into the model by Merton (1973). He theorizes that “... in equilibrium, investors are compensated in terms of expected return, for bearing market (systematic) risk, and for bearing the risk of unfavorable (from the point of view of the aggregate) shifts in the investment opportunity set.”\(^\text{16}\) Because hands are dealt randomly it cannot rationally be

\(^{15}\)For recent discussions of the CAPM and further references see Perold (2004) and Fama and French (2004).

\(^{16}\)Ibid. p. 882.
expected that the available alternative will be any different for future hands. Therefore the model is kept as simple as possible and we are back with the original authors. What they pointed out is that given investments of a fraction $\alpha^h$ of the individual’s $(i)$ wealth in assets (hands $h$) the expected return of the portfolio is

$$\bar{\mu}_i = \sum_{h=1}^{169} \alpha^h \bar{\mu}^h$$ (5.8)

the weighted sum of the expected returns of the investments. The standard deviation of the portfolio becomes

$$\bar{\sigma}_i = \sqrt{\sum_{h=1}^{169} \sum_{j=1}^{169} \rho^{hj} \alpha^h \alpha^j \bar{\sigma}^h \bar{\sigma}^j}$$ (5.9)

where $\rho^{hj}$ is the correlation coefficient between the return of hands $h$ and $j$. Its values range from -1 in case of a perfect negative linear relationship to 1 for a precise positive linear link. A value of 0 indicates independence. If for some investments the value lies within this range, the attainable combinations of $\bar{\mu}_i$ and $\bar{\sigma}_i$ will form what is usually called a bullet shape.

Figure 5.6 shows the actual return $\mu_i$ (average amount won per hand) and risk $\sigma_i$ (standard deviation of amounts won) of players in the highest limit. Except for one outlier with nearly $50 won, risk-return combinations pack nicely to the mentioned bullet shape, a pattern also found for the other gametypes. Note that playing more than 1,000 hands and losing on average more than $5 per hand in a game with a buy-in of at least $5,000 is only feasible if additional chips are won at other varieties. We also see that, quite naturally, there is a minimum risk involved in the game.

Whose venture is best? In order to discuss which player performs best some assumptions common in finance have to be made. First, given a specific level of return, we assume that individuals prefer to have the least risk possible. Second, given a specific level of risk, a higher return is preferred. Given these assumptions and, so to speak, by drawing a vertical and horizontal line through every dot in figure 5.6 all players can be excluded for whom another player is found in the upper left quadrant including the bounds (a better/equal return and lower/equal risk). Repeating this method for all players only a small

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17 In section 8.2 irrational beliefs about randomness will be discussed; we assume that the effects are not sufficient to influence the aggregate market equilibrium.

18 There are 169 distinct starting hands. $h = (32o),...,(AA)$.

19 The exact definition is $\rho^{hj} = \frac{cov(h,j)}{\bar{\sigma}^h \bar{\sigma}^j}$ where $cov$ means covariance.

20 As the return on, say, $(T7o)$ will generally not be perfectly linked to $(K9s)$ this seems a reasonable assumption.
For each player who has played at least 1,000 hands in 50/100 Limit, is not marked suspicious and not a bot, $\mu_i$ and $\sigma_i$ are indicated.

Figure 5.6: Risk-Return Profile for Players in 50/100 Limit

number remains at the upper left edge of the bullet. This is what Markowitz (1952) calls the efficient frontier. All these players manage their portfolio efficiently and to determine who is best amongst them becomes a matter of risk tolerance.

At this point we might draw a parallel between a fund manager and a poker player. Taking this analogy a step further the question arises whether there are funds of funds in the poker environment. And, not surprisingly, there are. Players sometimes swap parts of their performance, so that A might participate on B’s risk and return for a certain share. For example, two friends are regulars at a casino where they independently play cash games at different tables. Both are about equally skilled so that they expect the same risk and return from their games. They have established an agreement to swap half of their wins/losses with each other. In doing so they diversify their portfolio of hands further by adding interest in another portfolio which is not perfectly correlated, thus reducing their individual risk.\(^{21}\)

\(^{21}\)See Chen and Ankenman (2006, chapter 25).
5.2. VENTURING PLAYERS

5.2.2 Success by Playing Style

As Schoonmaker (2000, p. 18) claims “One of the best ways to improve your results is to change your style”, he remarks that nearly all successful professionals are both tight and selectively aggressive (sharks). Rocks are said to be able to grind out small profits. Calling stations, however, are heavy losers and maniacs will only have positive returns in some games. We take this as hypothesis and run linear regressions as shown in equations (5.10) and (5.11) for the major varieties,

\[
\mu_i = \alpha_0 + \alpha_1 CS_i + \alpha_2 RO_i + \alpha_3 MA_i + \alpha_4 SK_i \quad (5.10)
\]

\[
\sigma_i = \beta_0 + \beta_1 CS_i + \beta_2 RO_i + \beta_3 MA_i + \beta_4 SK_i \quad (5.11)
\]

where \( CS_i, RO_i, MA_i \) and \( SK_i \) are dummy variables for the four playing styles regarding player \( i \). The resulting coefficients and levels of significance are tabulated in 5.3. Relative to the benchmark of the non-distinct “Other” playing style the coefficients show the effect the pronounced styles have on return and risk. Take as an example a shark in the 20/40 Limit. His average amount won per hand will be $4.4 above any “Other” player, so we can expect the shark to be an overall winner with an expected return of $4.1. Simultaneously, with a value of $83.9, the risk he takes is significantly lower.

Tight-aggressive is indeed the most successful playing style. Sharks are net winners in all gametypes and achieve this even at lower risk compared to “Other” players. Rocks show neither significant wins nor losses, but they are running lower than usual risk, so from a financial perspective their play is efficient. Inefficient are the playing styles of calling stations and maniacs. They accumulate significant losses at higher than normal risks. Interestingly, we see that the loose-aggressive playing style of maniacs performs better at higher limits where their losses are smaller than those of a loose-passive style (calling stations), an indication that this style can perform well in certain types of games.

By revisiting the results obtained in table 5.2 the success of the playing styles can be explained further. Take, for example, the difference between rocks and sharks. Rocks are losing between $2.2 and $3.5 (low limits), $3.3 (20/40) and $7.8 (50/100) more than sharks on hands that do not go to showdown. This explains the differences in performance to a large part. Using their aggressive play sharks manage to win hands by making their opponents fold before showdown. The passive style of rocks lacks this characteristic so that their main way to extract value is from strong hands at showdown. Compare this to maniacs who are losing between $7.0 and $11.8 on low limits and $3.0 on middle limits more than sharks if there is no showdown, but on the high limit they have an advantage
CHAPTER 5. POKER FINANCE

Table 5.3: Success by Playing Style

Results of the linear regression of average amount won per hand depending on playing style (cf. equation (5.10)) are summarized in panel A. For equation (5.11) the results are shown in panel B. Regression coefficients are rounded to one decimal (columns Coef.) and levels of significance to three decimal places (columns $P>|t|$). For sample size see table 4.2 in section 4.3.1.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Call. Station</th>
<th>Rock</th>
<th>Maniac</th>
<th>Shark</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P&gt;</td>
<td>t</td>
<td>$</td>
<td>$P&gt;</td>
<td>t</td>
</tr>
<tr>
<td>Panel A: Return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10/20 I</td>
<td>-6.0</td>
<td>.000</td>
<td>.5</td>
<td>.478</td>
<td>-13.3</td>
</tr>
<tr>
<td>10/20 II</td>
<td>-5.7</td>
<td>.000</td>
<td>.3</td>
<td>.831</td>
<td>-14.6</td>
</tr>
<tr>
<td>10/20 III</td>
<td>-7.0</td>
<td>.001</td>
<td>.9</td>
<td>.687</td>
<td>-15.1</td>
</tr>
<tr>
<td>20/40</td>
<td>-7.2</td>
<td>.000</td>
<td>-1.2</td>
<td>.272</td>
<td>-7.0</td>
</tr>
<tr>
<td>50/100</td>
<td>-4.4</td>
<td>.075</td>
<td>2.7</td>
<td>.302</td>
<td>-2.6</td>
</tr>
<tr>
<td>Panel B: Risk</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10/20 I</td>
<td>5.8</td>
<td>.000</td>
<td>-17.5</td>
<td>.000</td>
<td>52.7</td>
</tr>
<tr>
<td>10/20 II</td>
<td>3.2</td>
<td>.138</td>
<td>-17.6</td>
<td>.000</td>
<td>69.8</td>
</tr>
<tr>
<td>10/20 III</td>
<td>1.1</td>
<td>.788</td>
<td>-25.2</td>
<td>.000</td>
<td>77.1</td>
</tr>
<tr>
<td>20/40</td>
<td>12.2</td>
<td>.000</td>
<td>-20.5</td>
<td>.000</td>
<td>60.7</td>
</tr>
<tr>
<td>50/100</td>
<td>22.5</td>
<td>.000</td>
<td>-33.1</td>
<td>.000</td>
<td>74.4</td>
</tr>
</tbody>
</table>

of $2.4. Detrimental to their return is that they will pay to see hands at showdown that usually are stronger than the hands they are holding. Calling stations have neither an advantage from showdown value nor from aggression.

5.3 Positional Play as Competitive Advantage

Competitive advantages need not be based on card fundamentals or aggression. It is a well-known fact in poker that it is valuable to sit in a later position. Sklansky (1987) dedicates a whole chapter to position and states\(^{22}\)

“[...], in all poker games it is far better to be last to act, primarily because it is generally easier to decide what to do after you have seen what your opponents have done.”

This can also be confirmed empirically. The rules of the game provide for a fair distribution of the advantage from position as the players’ positions change after each hand when the button is moved clockwise by one position. Hence in a sufficiently large sample it can be expected that for a particular position the skill of the players sitting in this position will converge to the mean. So by calculating the average amount won per hand

\(^{22}\)Ibid. p. 155.
for each position the value of position can be measured. The results of this calculation for the 10/20 I Limit can be found in figure 5.7.

The average amount won/lost per hand is graphed for each position in the 10/20 I Limit game. On the x-axis the total number of players in the game is distinguished. Games from 2 to 10 players are analyzed. The bars shaded in gray correspond to positions 1 to 10 from left to right.

Figure 5.7: Amount Won/Lost Depending on Position

Indeed, the competitive advantage of position can be empirically confirmed and is significant. Regardless of the number of players in the game, the later the position the higher the average amount won, with the exception of the blinds. The big blind is losing more in all games than the small blind (except for games with 2 and 3 players where the difference is small). Remarkably only the blinds are losing overall which illustrates the disadvantage of being forced to pay. In this game where the small blind is $5 and the big blind $10, losses range from about 30-50% of the forced investment for the small blind and can be more than 40% of the big blind’s forced payment. Competitive advantage from later position can be more than $2 per hand won. Compared to an expected return of $2.8 for sharks in this gametype (see table 5.3) this is a substantial edge.

What favors players in later positions? First, they gain information. Most of the other players have acted before and in later position the relative strength of a hand can be evaluated much more easily. In addition, the size of the pot might have increased
For games in the 10/20 I Limit with 10 players at the table the actions are evaluated. The x-axis distinguishes between the positions. Actions are graphed from bottom to top (black to light gray) with increasing aggressiveness (fold-check-call-bet-raise). “Other” actions include all-in, quit or being kicked from the game.

Figure 5.8: Choice of Action Depending on Position

already so that a hand that might improve can now pay based on the pot odds. In figure 5.8 the split of actions is shown for each position, and the following patterns can be observed with regard to later positions

- More hands are folded. The share increases from 16% to 22%.
- Players check less. A drop from 42% to 13%.
- Calls are more frequent. 39% calls in position 10 compared to 23% in first position.
- If anything, less bets are made. A small decrease from 13% to 11%.
- Raises are played more often. The share is more than threefold with 14% vs. 4%.

Overall, actions in later positions are more aggressive. Passive play drops from 65% to 52% (check, call) and aggressive actions increase from 17% to 25% (bet, raise). A characteristic which we have seen is common to successful playing style. This leads directly to the second advantage from later position, selecting the right hands to play.

\[\text{\footnotesize\textsuperscript{23}}\text{A drawing hand which has, say, a 1 in 5 chance to make a winning flush, can rationally pay 10\$ to stay in a pot of 50\$, as the pot odds of 10:50 are at least as good as the odds of winning.}\]
Small blind and big blind force players into more hands. The results are described in figure 5.9. Evaluating the hands seen at showdown, a steeper curve indicates more selective play (cf. figure 5.2). It appears that players in position 2 (the big blind) are least chary with the investments they are carrying through to the end. A smaller forced bet (small blind, position 1) reduces the effect. Nevertheless, still a higher proportion of weak hands is seen at showdown compared to players who do not have to enter the game by a down-payment (here graphed for position 5).

For positions 1, 2 and 5 (different shadings of gray) the normalized relative share of hands at showdown (cf. equation (5.2)) is graphed. The example is from the 10/20 I variety.

Figure 5.9: Choice of Hands Depending on Position

To conclude, position changes playing style in two dimensions. First, in later positions actions tend to be more aggressive with more raises and fewer folds. Second, paying blinds causes a looser style as players adhere to weak hands. Put together, style in early positions is more loose-passive, which is costly, and in later positions tight-aggressive, which is successful as seen in part 5.2.2.
Part III
Decision-Making in Poker
Chapter 6

Decision-Making in Poker – Rational and Otherwise

The heart of poker is decision-making.

Chen and Ankenman (2006, p. 47), *The Mathematics of Poker*

Why are people gambling? Wagenaar, Keren, and Pleit-Kuiper (1984) observe that Blackjack players realize that the odds are against them but stay in the game nevertheless, suggesting that gamblers have multiple goals other than making money. Some might play to pass time, some to meet friends, others to get excitement from gambles or to test their skills in competition. How diverse the objectives may be, it is easy to agree that winning more or losing less is desirable for everyone as all other goals are subordinate to this maxim. By winning one can pass more time, meet friends more often, place larger and more thrilling bets and boast in competition versus others.

In a fair, zero-sum game like poker, all differences between making money or losing stem from the players’ decisions. In this chapter we discuss how decisions should be made applying methods from decision analysis and add evidence to the discussion where research has shown that actual behavior differs from the normative approach.\(^1\) Thereby, we first use a simplified decision environment with exposed cards, a restriction we later remove for a more complete discussion.

\(^1\)An overview on what decision analysis does and how it is done is given by Keeney (1982).
6.1 Playing with Cards Face Up – Risky Decision-Making

If everything is known by all players, we can use the laws of probability to assess the consequences of all alternative actions available for a player. Players can then choose the action which is maximally exploitive of the situation.

6.1.1 A Dollar is a Dollar is a Dollar ...

Using equation (5.1) we can calculate what the player can expect to earn on average over potential outcomes and based on this choose the alternative which offers the highest expected value. Take for example the following situation:

Case study 1:
The game is 10/20 Limit Hold’em. Player X is first to act and has \((K\heartsuit Q\spadesuit)\). Player Y has \((9\spadesuit 8\spadesuit)\). There are $200 in the pot and the board is \(K\spadesuit 7\spadesuit 3\spadesuit 2\spadesuit\). X is ahead with a pair of kings and Y can only beat his hand if he catches a \(\spadesuit\) on the river completing his flush. The probability for this is \(\frac{9}{44}\approx 20\%\) with nine \(\spadesuit\)s left in the deck and eight cards exposed. There will be no betting on the river as both players will know who has won. If player X checks, Y will check as well as he only wins 20% of the time. Using equation (5.1) X’s expected value from checking is

\[
E_X(\text{check}) = \frac{35}{44} \times 200 + \frac{9}{44} \times 0 \approx 159
\]

Now consider Y’s options in addition to checking. He will neither bet nor raise as the bet is heavily unfavorable. His expected value from calling a bet is

\[
E_Y(\text{call}) = \frac{9}{44} (240 - 20) + \frac{35}{44} (0 - 20) \approx 29
\]

as he is winning $240 on a flush, otherwise winning nothing and in any case investing $20 for the call. By folding Y’s expectation is simply \(E_Y(\text{fold}) = 0\) As Y gains more by calling than folding \((E_Y(\text{call}) > E_Y(\text{fold}))\) X’s expected value from betting becomes

\[
E_X(\text{bet}) = \frac{35}{44} (240 - 20) + \frac{9}{44} (0 - 20) \approx 171
\]

Consequently the play should go: X bet and Y call.

In this situation player Y rather calls X’s bet than not as he is getting the right pot odds. The concept of pot odds compares what a player has to pay to stay in a pot of a given size, in this case 20 to 220, to the odds of him winning the pot, here 9 to 35. As the odds of winning are shorter the player should take the gamble. Analysis like in case study 1 can be extended to cover multiple rounds of play, several players etc. but they all reduce to comparing the expected value of alternative actions.\(^3\)

\(^2\)Note that \(E_Y(\text{check}) = \frac{9}{44} \times 200 + \frac{35}{44} \times 0 \approx 41\); X gains additional expected value of $12 at the cost of Y as the game is zero-sum.

\(^3\)For further discussion see Chen and Ankenman (2006, chapter 4).
However, already in the early 17th century the limitations of expected value calculations have been discussed by Bernoulli. In what has since then been known as the St. Petersburg game a gambler is offered, against a fixed payment, a series of coin tosses which will continue as long as it comes down to “heads”. The payout depends on the number of “heads” seen, with $2 for the first and doubling the amount for every additional throw, i.e. 2\(^n\) for \(n\) consecutive “heads”. Calculating the expected value gives

\[
E(X) = \sum_{n=1}^{\infty} \frac{1}{2^n} 2^n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 1 + 1 + 1 + \ldots
\]

an arbitrarily large amount; a sum nobody is willing to pay.

In a similar undertaking at high stakes poker, poker professionals like Tom Dwan, Phil Ivey, Gus Hansen or Ilari Sahamies agreed in several instances to play hands for ten thousands of dollars (up to $200,000) and check until showdown. As they would bet on the next random hands to come, they could expect to win 50% of the time (a coin flip situation). Although the expected value is zero, many other players would prefer not to take this gamble, even accounting for all other objectives. To account for these preferences we have to reconsider the value of a $.

### 6.1.2 What is a Dollar to a Millionaire?

Bernoulli (1954, p. 24) already notes that “... the determination of the value of an item must not be based on its price, but rather on the utility it yields.” Contrasting the effect an additional dollar has on a child with $10 in his savings box to an investment banker with $1,000,000 in his investment portfolio, it is easy to see that the marginal utility of an additional dollar becomes smaller as wealth increases. We denote the function relating wealth \(\Omega\) to utility by \(u(\Omega)\). In analogy to equation (5.1) expected utility is defined as

\[
EU = E(u(X)) = p_1u(x_1) + p_2u(x_2) + \ldots + p_nu(x_n) = \sum_{i=1}^{n} p_iu(x_i) \quad (6.1)
\]

If an individual’s valuation of money is best described by a concave utility function,
he is risk-averse and prefers a certain amount to a risky gamble with the expected value of the exact same amount, since $\sum_{i=1}^{n} p_i u(x_i) < u(\sum_{i=1}^{n} p_i x_i)$. The strength of the aversion can be measured by the Pratt-Arrow function of local risk aversion.\(^{10}\)

$$r(x) = -\frac{u''(x)}{u'(x)}$$ \hspace{1cm} (6.2)

If $r(x)$ is positive there is a local propensity to insure and negative values can be interpreted as a propensity to gamble, a relationship illustrated in the following example.

Case study 2:
The game is No-Limit Hold’em. X is rather wealthy having $1,000,000 in chips and Y has his total bankroll in the game, $80,000. Blinds are $2,000/$4,000. X moves all-in with ($\text{A}\heartsuit\text{T}\clubsuit$). Holding ($\text{T}\spadesuit\text{T}\diamondsuit$) Y has to decide whether to call.

Y’s expected value from calling is substantial as his hand is a large favorite\(^{11}\)

$$E_Y(\text{call}) = 0.693(160,000 - 76,000) + 0.307(0 - 76,000) \approx 34,880$$
and compared to $E_Y(\text{fold}) = 0$ he should call based on the expected value. However, staking all his money Y will be concerned with the utility he can expect and not with the mathematical value. Three different valuations of money will be discussed in turn.

Variant I:
Assume that his utility function is $u_Y(x) = \log(x + 1)$. The gamble presented to him is either to double his bankroll to $160,000 or lose everything which yields an expected utility of

$$EU_Y(\text{call}) = 0.693 \log(160,001) + 0.307 \log(1) \approx 3.6$$

With an alternative utility from folding (only the big blind is lost) $EU_Y(\text{fold}) = \log(76,001) \approx 4.9$ he rather folds. As this gamble would bankrupt player Y he rather surrenders the big blind and probably quits playing this opponent. At his current bankroll his risk aversion is still too strong. From equation (6.2) we get

$$r_I(x) = \frac{1}{x+1}.\hspace{1cm} (6.12)$$

Variant II:
Now the utility function may be $u_Y(x) = \sqrt{x}$. Then $EU_Y(\text{call}) = 0.693 \sqrt{160,000} + 0.307 \sqrt{80} \approx 277$, and $EU_Y(\text{fold}) = \sqrt{76,000} \approx 276$. With this utility function, player Y is better off by calling. With an overall steeper curve, risk aversion is smaller which can be seen by calculating the Pratt-Arrow measure $r_{II}(x) = \frac{1}{2x}.\hspace{1cm} (6.13)$

Because $r_I(x) > r_{II}(x)$ if $x > 1$ risk aversion is stronger in variant I.

Variant III:
\(^{10}\)See Pratt (1964); for additional discussion and a measure not based on actuarial considerations see Dyer and Sarin (1982).

\(^{11}\)2.1% ties are split for simplicity.

\(^{12}\) $u'(x) = \frac{1}{x+1}$, $u''(x) = -\frac{1}{(x+1)^2}$.

\(^{13}\) $u'(x) = \frac{1}{2\sqrt{x}}$, $u''(x) = -\frac{1}{4x^{3/2}}$. 
Finally, how does the situation develop if players change hands, i.e. Y is to call with the underdog (A\heartsuit T\spadesuit)? Due to the negative expected value of the gamble, the utility function has to be convex if Y would call, indicating a propensity to gamble. Assume that it is of the general class \( u_Y(x) = x^a \). Then we can solve the inequality

\[
0.307(160,000)^a + 0.693(80)^a \geq (76,000)^a
\]

to find approximately \( a \geq 1.587 \) and consequently \( r_{III}(x) = -\frac{587}{x} \) which is negative for all positive \( x \), reflecting the liking of risk. Hence if Y values more money higher than losing money with at least \( u_Y(x) = x^{1.587} \), then he should call even with the weaker hand.

With these simple utility functions, a risk averse individual will never take a gamble with negative expected value and a risk seeking individual will never buy insurance. However, we frequently witness that people do both, for example, insure their house and buy lottery tickets. Friedman and Savage (1948) were first to propose a utility function with concave and convex intervals so that both behaviors could be explained. Their interpretation that the convex segment represents the transition to a different socioeconomic level whereas the concave part is relevant for small shifts within the current social class, again underlines that considering the utility function as a normative means is advisable for decisions affecting the total wealth or income of an individual. For decisions which affect only a small share of a player’s wealth, i.e. a player with a sufficient bankroll in a poker game, the value of money is approximately linear and in poker advise is usually given on grounds of the mathematical expected value (e.g. Sklansky (1987, pp. 9-18), Chen and Ankenman (2006, pp. 13-19)).

6.2 Holding Cards Face Down – Playing with Uncertainty

6.2.1 Why Poker is not Roulette

In most casino games such as roulette, craps or slot machines, chances are known and risk is calculable. In poker, players have to deal with uncertainty. With the presence of hidden information – opponents’ pocket cards are unknown – probabilities are no longer known. Instead, uncertainties have to be described in terms of probability distributions.\(^{14}\) All information that can be gathered should be used to find the probability distribution of the hands an opponent may hold. Based on this distribution further conclusions can be drawn, but in fact risk is determined by only one hand from the distribution. Among the potential hands the opponent might hold there could be better or worse compared to

\(^{14}\)See Arrow (1951, pp. 416-420) who surveys several ways to describe uncertainty.
the current situation.

As long as probabilities can be calculated, the situation is said to involve risk, which arises under complete information as in section 6.1. If several probabilities are possible, i.e. a probability has to be attached to a probability, which is the case with imperfect information, there is uncertainty. This is the fundamental difference between poker and roulette.

6.2.2 He Could Have That or That or That ...

With cards face down players have to put their opponents on hands to arrive at a probability distribution. As Chen and Ankenman (2006, p. 60) point out

“This is generally a process of reduction and elimination; hands that the opponent would have played in some clearly different way should be reduced in relative probability within the distribution.”

Evidently, then the distribution depends on an opponent’s playing style. A player might use the information he gets from hands seen at showdown, as is in figure 6.1, to build a basic distribution of hands played by style. We can see that the distribution of tight playing styles (rock and shark) is skewed to the right. The probability that they are showing a strong hand at showdown has to be weighted more than the probability of weak hands. For loose playing styles (calling station and maniac) the distribution is flatter. Relative weighting of small hands has to be higher for players with this style. Obviously, this is only a part of the information that players can use to estimate their opponents likely hands. Tells and betting patterns provide additional information.15

Case study 3:
The game is 10/20 Limit Hold’em with 6 players. Player X is in the small blind with $800, the action is folded to him and his cards are hidden. Player Y is the big blind, has $900 and (K♥T♦). Player X raises. Two variants with different styles of X will be discussed.

Variant I:
Suppose X is known to be a complete maniac. So far he has raised every hand from the small blind for dozens of hands. Thus no hand can be eliminated from the distribution, i.e. he can have any hand except those containing K♥ or T♦ as they are in Y’s hand. Against such a random hand Y has 59.7% equity so it is advisable to re-raise.

15See Chen and Ankenman (2006, chapter 5).
The results of a quadratic regression are graphed for each of the four playing styles in the 50/100 Limit. The share of all hands which are expected to be dealt to a player that go to showdown is estimated.

Figure 6.1: Share of Dealt Hands at Showdown by Playing Style

**Variant II:**

X has been at the table for hours but only played about 1 in 5 hands and rarely raised. We classify him as a rock. He might raise in this situation with the following hands (99+), (A7+), (K9+), (QJ+) where (XX+) indicates the pair or any better pair and (XY+) any hand with the higher card and a kicker of at least rank Y. These are about 17% of all hands, a fairly tight range. Against this distribution (K♥T♦) has only 38.4% equity and he should not raise but call. If Y got a tell indicating X is holding an especially strong hand, he might narrow the distribution further to, say, (TT+), (AT+), (KJ+) against which his equity is a mere 28.7% advocating a fold.

Due to the dynamics of the game, the assessment of an opponent’s hand distribution can be revised several times during play. Cards seen on the board have to be removed from the set and additional actions of the opponent take place.

### 6.2.3 Effects of Ambiguous Circumstances

Consider the example in table 6.1 proposed by Ellsberg (1961) and since then known as Ellsberg’s paradox. People are faced with an urn known to contain 30 red balls and 60

---

16Eventually we might also distinguish between suited and offsuit hands, say, (A9s+), (ATo+), if no distinction is made both are meant.
black and yellow balls in unknown proportion. First they are asked whether they prefer to bet on red (I) or bet on black (II). Then, second, they can choose between betting on red or yellow (III) or betting on black and yellow (IV). Ellsberg finds a very frequent pattern of response to be: I preferred to II and IV preferred to III. This is surprising as III and IV can be generated from I and II by simply adding $100 on the drawing of yellow, whereby the probability for this event is the same for both alternatives. Adding a sure-thing to both alternatives, however, should not change the order of preference.

Table 6.1: Ellsberg’s Paradox

<table>
<thead>
<tr>
<th></th>
<th>30</th>
<th>60</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>$100</td>
<td>$0</td>
<td>$100</td>
<td>$0</td>
</tr>
<tr>
<td>Black</td>
<td>$0</td>
<td>$100</td>
<td>$0</td>
<td>$100</td>
</tr>
<tr>
<td>Yellow</td>
<td>$0</td>
<td>$0</td>
<td>$100</td>
<td>$100</td>
</tr>
</tbody>
</table>

Source: Ellsberg (1961, p. 654)

How can the change in preferences be explained? In choice I the subject knows that the probability of winning is one third. Not knowing anything about the proportion of black to yellow balls, the probability of winning by choosing II can be anything between zero and two thirds. Lacking additional information we must conclude that all probabilities are equally likely, so that on average a probability of one third for winning can be expected in this case as well. The difference between I and II is the presence of ambiguity which Weber and Camerer (1987, p. 330) define as follows

**Definition 3** Ambiguity is uncertainty about probability, created by missing information that is relevant and could be known.

Accordingly, in III the probability of winning is ambiguous. It is anywhere from 1/3 to 1, and equal weighting of this range gives two thirds, the same as the probability of drawing black or yellow in choice IV.\(^{17}\) Many experiments show that people prefer betting on events whose likelihoods they know more about, either from additional information or because they feel competent.\(^{18}\) The effect is so strong that it persists in market settings where decisions are influenced by decisions of other participants; discomfort and regret due to hindsight are the likely causes (Sarin and Weber 1993). Presence of feelings of anxiety or

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\(^{17}\)Raiffa (1961, p. 691) acknowledges that this kind of calculation is rarely done by individuals who with stated probabilities, in contrast to more ambiguous situations, readily calculated e.g. expected values.

\(^{18}\)See Weber and Camerer (1987) for a comprehensive survey of the theoretical and empirical literature. In a similar vein Gneezy, List, and Wu (2006) document an effect they baptize uncertainty effect. Their subjects were willing to pay $38 for a restricted $50 gift certificate, yet they were only willing to pay $28 for a lottery with an equal chance of the $50 certificate or one for $100 with the same restrictions.
6.2. HOLDING CARDS FACE DOWN – PLAYING WITH UNCERTAINTY

discomfort due to ambiguity about probabilities can be interpreted in two ways. First, as modifications in the utilities of the outcomes where the same outcome is less preferred under ambiguous circumstances (e.g. Sarin and Winkler (1992), Fellner (1961, esp. pp. 676-677)). Second, probabilities in ambiguous settings are said to be soft and individuals adjust them as new information is received (Einhorn and Hogart 1985; Viscusi and Magat 1992).

Picking up the poker example from case study 3, first consider playing against the known maniac who you experienced playing like this maybe for years. The distribution of the hands he plays is everything but ambiguous as he simply plays every hand. However, there is still uncertainty as you do not know which particular hand is against you. And if you would know this as well, there would nevertheless remain the risk caused by the cards to come on the board. Now a new player is at the table who you have not seen play before. He has raised the first two hands and now, after everyone else has folded, he raises in the small blind before you. Here, as Einhorn and Hogart (1985, p. 435) phrase it, “... ambiguity results from uncertainty associated with specifying which of a set of distributions is appropriate in a given situation.”. The new player could be a complete maniac or a rock who only plays the best hands but has happened to find three strong hands in a row. Therefore, there are many possible distributions for this kind of play and many players would fold in this situation. Before calling such a hand they will try to rule out some distributions to reduce the amount of ambiguity.\(^{19}\)

Of course, only rarely will players go through the burden of precisely weighting opponents’ hand distributions and calculating probabilities. This would slow down the game tremendously, which is, by unwritten poker ethics, only acceptable for important hands where large sums are at stake. Nevertheless, even with merely partial information available regarding probability distributions or utility functions, players can rule out alternatives which are dominated by other actions (Kirkwood and Sarin (1985), Weber (1987), Keppe and Weber (1990)). For example, a player last to act on the flop who is drawing to an ace-high flush and has to pay $20 to a pot of $160 should in no case fold. Calling is the better alternative whatever\(^{20}\) the player’s preferences and the opponents’ hands. In most situations, however, players have to resort to judging the probability of different hands and outcomes to the game. This is where psychology enters the game.

In their experiments on decision-making psychologists found increasing evidence that human cognition is limited with regard to the application of mathematical concepts (e.g.

\(^{19}\)Ibid. p. 435.
\(^{20}\)Of course, in realistic boundaries.
Simon 1959; Holyoak and Spellman 1993). A plethora of heuristics and biases has been identified in human judgment and behavior (Birnbaum 1992; Rabin 1998). Some of them are especially relevant in the poker environment and in part IV we will look for empirical evidence attesting their presence. Beforehand the respective theory and literature will be laid out in the following sections.

6.3 Evaluation of Probabilities and Outcomes

6.3.1 From Normative to Descriptive Models of Decision-Making

Soon after the concept of expected utility had been proposed, critics argued that though it is appealing from a normative point of view it does not succeed in describing actually observed behavior. At the forefront of the critics was Allais (1953) who designed two simple choices which are summarized in table 6.2.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Gain $100m with certainty</td>
</tr>
<tr>
<td>B</td>
<td>10 in 100 chance to gain $500m</td>
</tr>
<tr>
<td></td>
<td>89 in 100 chance to gain $100m</td>
</tr>
<tr>
<td></td>
<td>1 in 100 chance to gain nothing</td>
</tr>
<tr>
<td>C</td>
<td>11 in 100 chance to gain $100m</td>
</tr>
<tr>
<td></td>
<td>89 in 100 chance to gain nothing</td>
</tr>
<tr>
<td>D</td>
<td>10 in 100 chance to gain $500m</td>
</tr>
<tr>
<td></td>
<td>90 in 100 chance to gain nothing</td>
</tr>
</tbody>
</table>

Source: Allais (1953, p. 527)

Allais (1953) found that the majority of people preferred situation A to B and D to C, a pattern of choice which is not consistent with maximizing expected utility. We can express the preference of D to C in terms of expected utility by

\[
E(u(C)) < E(u(D))
\]

\[
\Leftrightarrow \quad 11\% u($100m) + 89\% u($0) < 10\% u($500m) + 90\% u($0)
\]

and by two simple transformations

\[
\Leftrightarrow \quad 11\% u($100m) < 10\% u($500m) + 1\% u($0)
\]

\[
\Leftrightarrow \quad 100\% u($100m) < 10\% u($500m) + 89\% u($100m) + 1\% u($0)
\]
which is nothing else but

\[ E(u(A)) < E(u(B)) \]

This shows that the pattern of preferences observed by Allais (1953) is in contradiction to the so-called independence axiom or sure-thing principle of expected utility. It asserts that preferences between two alternatives should be independent of outcomes they have in common. Violations are so strong that they even persist after comprehensive explanations of the situation (Slovic and Tversky 1974). The paradox is caused by the psychological valuation of probabilities. In this case, as Allais (1953, p. 529) writes, we evidence the psychological effect certainty has compared to lower probabilities. People generally value certainty subjectively higher than what is mathematically correct relative to other probabilities. Subsequently, psychologists developed an increasing interest in the study of subjective probability as it transforms the mathematical or objective probability (e.g. Edwards 1954b; Cohen and Hansel 1956). Adding this train of thoughts to the concepts of decision-making used so far, four models of decision-making as summarized in table 6.3 can be stated (Edwards 1955; Pruitt 1962; Payne 1973). In the models subjectively expected money (SEM) and subjectively expected utility (SEU) objective probabilities are replaced by a function \( w(p) \) which transforms them into subjective probability. At the loss of the normative strength of expected value and expected utility models, this generalizes the decision-making model tremendously as every subject is treated separately. Hence SEU stands out among the four models as the one which best describes observed behavior (Coombs, Bezeminder, and Goode 1967).

Table 6.3: Four Models Of Decision-Making

<table>
<thead>
<tr>
<th>Concept of Probability</th>
<th>Concept of Outcomes</th>
<th>Objective</th>
<th>Subjective</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Objective</td>
<td>( EV = \sum_{i=1}^{N} p_i x_i )</td>
<td>( EU = \sum_{i=1}^{N} p_i u(x_i) )</td>
</tr>
<tr>
<td></td>
<td>Subjective</td>
<td>( SEM = \sum_{i=1}^{N} w(p_i) x_i )</td>
<td>( SEU = \sum_{i=1}^{N} w(p_i) u(x_i) )</td>
</tr>
</tbody>
</table>

\( EV = \) expected value \( SEU = \) subjectively expected utility

Source: e.g. Edwards (1955)

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21 Camerer and Ho (1994) prove that even the more general axiom of betweenness (if X is preferred to Y, then probability mixtures of X and Y must lie between them in preference) is violated.

22 Another simple example where the pattern of preferences contradicts the expected utility model is designed and tested by Paterson and Diekmann (1988).

23 For a formal discussion of the definition of subjective probability see Anscombe and Aumann (1963).

24 Evidence of distinct individual gambling behavior at roulette is given by Hochauer (1970).
CHAPTER 6. DECISION-MAKING IN POKER

One of the first experiments on subjective probability was conducted by Preston and Baratta (1948) who presented subjects with bets of varying amounts to be won and probability of winning. Subjects were then asked to bid competitively on the wagers. Bids exceeded mathematical expectation for small probabilities and were less than mathematical expectation for large probabilities with the point of indifference slightly below 25%. Other experimenters followed suit providing supportive evidence on the psychological aspects of probability judgment (e.g. Edwards 1953, 1954a; Cohen and Hansel 1958).

6.3.2 Subjective Probability at Racetrack Betting

How does the psychological weighting of probabilities affect decision-making? In this section we will have a brief look at racetrack betting which like poker is a gambling market where participants wager on uncertain outcomes.

The racetrack market functions via the parimutuel system where the payoffs per dollar bet are jointly determined by all participants and transaction costs. Take for example the bet on a horse to win. With $W_i$ the total amount bet on horse $i$ to win, bets are summed across all horses to form the win pool $W = \sum_i W_i$. Payoffs per dollar bet if, and only if, the horse wins are $\frac{(1-T)W_i}{W}$ where $T$ is the track “take”, transaction costs deducted as revenue for the track and for taxes, which is usually between 10% and 15%.25 Hence the winning odds given by $\frac{(1-T)W-W_i}{W_i}$ are determined by the joint judgment of all bettors.26 This allows, a posteriori, to compare the objective probability for winners from a particular group of horses to the subjective probability as it is expressed by the winning odds.27 Several other types of bets have developed, among which the place bet $P_i$ (the horse is to finish first or second) and the show bet $S_i$ (the horse finishes in the top three) are the most popular. Corresponding odds are $\frac{(1-T)P_i-P_j}{2P_i}$ if horse $i$ places together with horse $j$ and $\frac{(1-T)S_i-S_j-S_k}{3S_i}$ if horse $i$ is in the top three with horses $j$ and $k$.28 Naturally, these odds are shorter, i.e. the probability of a payoff is higher than the respective win bets.

At the other end of the spectrum are exacta and trifecta bets which win if the first two (three for trifectas) horses of the race are bet on in the correct order.29 Obviously, odds on these wagers are longer compared to simple win bets.

To our knowledge Griffith (1949) was first to empirically compare the chances of horses to win with the odds quoted at the track. He finds that below probabilities of approximately 20% the psychological weighting becomes too large. Above it is too small. McGlothlin (1956, p. 606).

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26 See e.g. McGlothlin (1956).
27 Subjective probability is the reciprocal of the odds plus one. See Griffith (1949, pp. 290-291).
Glothlin (1956) extends the analysis to more than 9,000 races and calculates expected values for odds from 1-20 to just below 26-1 in nine groups. In his sample odds shorter than 3.5-1 and 5.5-1, or probability values of 15% to 22%, resulted in positive expected value after correcting for transaction costs. Expected value on longer odds diminishes significantly. This disparity is generally called the favorite-longshot-bias as betting on favorites is not sufficient or longshots are overbet, respectively. Although subjective probabilities are disproportionately weighted relative to objective probabilities, the horses rated by the public as most likely to win, do in fact win most often. Generally, the correlation between subjective and objective probabilities is very high attesting the expertise of the market participants.\(^{30}\)

During a day at the races usually eight different races take place. McGlothlin (1956) uses this fact to analyze behavior over a sequence of bets. Interestingly, betting behavior is not stable over the course of a day. At later races horses with a high probability of winning, but with accompanying low payoffs, become increasingly unpopular. Simultaneously the proportion of the amount bet in the win pool rises relative to all bets in win, place, and show pools, indicating a growing preference for bets with longer odds as betting proceeds.\(^{31}\) The effect that the favorite-longshot-bias is most pronounced for the last race of a racing day is referred to as Gluck’s Second Law.\(^{32}\) Despite the early evidence given by McGlothlin (1956) the effect is disputed and, for example, Winter and Kukuk (2006) find no track or sequential effects.

Since the earliest investigations the favorite-longshot-bias has been found in race betting in many countries including the U.S., the UK, Australia, Japan, and Germany (see e.g. Winter and Kukuk (2006) with further references) and besides horse-racing for dog-racing, too (Terrell and Farmer 1996; Terrell 1998). It has also not only been found for the win pool but for the place and show pools as well as for the exacta or trifecta bets (e.g. Winter and Kukuk 2006; Gramm, McKinney, and Owens 2007). In these pools the effect is so pronounced that it has been possible to design systems which overcome the track take and result in positive expected value (Hausch, Ziemba, and Rubinstein 1981; Lo, Bacon-Shone, and Busche 1995). A number of factors influencing the presence of the bias have been put forward. Among them the presence of information costs (Terrell and Farmer 1996), adverse selection problems faced by bookmakers, i.e. insider trading, (Williams and Paton 1997) and risk-loving preferences of bettors (Jullien and Salanié 2000).


\(^{31}\)Ibid. pp. 610-613.

Taking the observations from the racetrack to the poker table we can state our first hypotheses on decision-making behavior. As racetrack bettors demonstrate a preference for low probability-high payoff combinations, a similar overweighting of small probabilities might be expected for poker players, so we formulate

**Hypothesis 1** Players will disproportionately prefer situations which offer a large pot at a low probability of winning.

From Gluck’s Second Law we deduce the following

**Hypothesis 2** Toward the end of their session players increase risk-taking.

### 6.3.3 Probability Weighting Function

In the model of subjectively expected utility the function \( w(p) \) has to account for all distortions of probability, including the Allais-Paradox and the favorite-longshot-bias. More generally, probabilities follow the general law from psychophysics that individuals are less sensitive to changes the further these are away from a reference point. For probability judgments there are two natural reference points, certainty and zero. So the function has to allow for diminishing sensitivity the further it is away from 0 and 1.\(^{33}\) One such weight function has been proposed by Karmarkar (1978, 1979) who maps probabilities onto subjective weights by the relation:

\[
\frac{w_i}{1 - w_i} = \left( \frac{p_i}{1 - p_i} \right)^\alpha \quad \text{where} \quad 0 < \alpha < \infty
\]

which is nothing else but

\[ \iff \text{Weighted Odds}_i = (\text{Odds}_i)^\alpha \]

and expressing the relation in odds we can solve for \( w_i \)

\[ \iff w_i = \frac{(\text{Odds}_i)^\alpha}{1 + (\text{Odds}_i)^\alpha}; \quad \text{Odds}_i = \frac{p_i}{1 - p_i} \]

or alternatively as the functional relationship \( w(p_i) \)

\[ \iff w(p_i) = \frac{p_i^\alpha}{p_i^\alpha + (1 - p_i)^\alpha} \]

In this relationship \( \alpha \) can be interpreted as excessive uncertainty if \( \alpha < 1 \) and excessive certainty (\( \alpha > 1 \)), of which the former is the usual case. For all values of \( \alpha \) this function

6.3. EVALUATION OF PROBABILITIES AND OUTCOMES

has three points where \( w(p_i) = p_i \) at 0, \( \frac{1}{2} \), and 1. However, this function is not yet fully satisfactory. It maps the overweighting of small probabilities and underweighting of probabilities near certainty, but the point of indifference is stuck to \( \frac{1}{2} \) which is higher than what we have seen in empirical data so far. Therefore, Lattimore, Baker, and Witte (1992) add a second parameter \( \beta \) to the function which might signify event or outcome pessimism in case \( \beta < 1 \) or an optimistic view of the \( i \)th outcome occurring if \( \beta > 1 \). For \( \beta = 1 \) the model reduces to the one proposed by Karmarkar (1978, 1979). The extended function is

\[
w(p_i) = \frac{\beta p_i^\alpha}{\beta p_i^\alpha + \sum_{k=1}^n p_k^\alpha} \quad \text{for } i, k = 1, 2, \ldots, n, \ k \neq i, \text{ and } \alpha, \beta > 0 \quad (6.3)
\]

Most studies find that for the majority of individuals the parameters fulfill \( \alpha < 1 \) and \( \beta \leq 1 \) (Lattimore, Baker, and Witte 1992; Gonzalez and Wu 1999). Generally, this implies that the function is concave to a point of approximately .4 (e.g. Wu and Gonzalez 1996, p. 1687-1688), a value somewhat higher than the results from studies at the racetrack. It is important to note that although the function maps probabilities onto an interval \([0, 1]\) it does not represent probabilities but probability weights.\(^{36}\) In figure 6.2 the fundamental properties of the function can be seen. There is overweighting of small probabilities, underweighting of large probabilities, diminishing sensitivity further away from certainty and zero, and the point of inflection at around .4 for the median data (right panel). More generally, parameter \( \alpha \) mainly determines the curvature of the function (left panel) whereas \( \beta \) influences the elevation (middle panel). Probability weighting is a very individual cognitive process as the diversity of empirically observed functions reveals. We illustrate this fact by the most optimistic (function with largest weights in the right panel) and pessimistic (lowest weights, right panel) subjects from the study of Gonzalez and Wu (1999).

A further phenomenon, which can be explained by the probability weighting function, is \textit{subadditivity}. In the experiments conducted by Shanteau (1974) “... the judged worth of two-part bets was less than the sum of the worths of the parts.”, the subadditivity effect. Formally, this means that \( w(p_1 + p_2) < w(p_1) + w(p_2) \).\(^{38}\) Consequently, unpacking the description of an event increases the attractiveness. For example, Fox and Tversky (1998) asked subjects about their beliefs who would win the 1996 NBA playoffs. Subjects’

\(^{34}\)Other functional forms used are, for example, \( w(p_i) = \exp(-\beta(-\log(p_i))^\alpha) \) or \( w(p_i) = \frac{\beta p_i^\alpha}{\beta p_i^\alpha + (1-p)^\alpha} \), which are discussed by Gonzalez and Wu (1999), Prelec (1998).

\(^{35}\)For median data of Gonzalez and Wu (1999) \( \alpha = .44, \beta = .77 \) \( w(p_i) = p_i \) at \( p = 0 \wedge p = .385 \wedge p = 1 \).

\(^{36}\)See e.g. Gonzalez and Wu (1999, pp. 131-132).

\(^{37}\)For a comparison between parameter estimates also see Wu and Gonzalez (1996, esp. footnote 14), Rieger and Wang (2006, p. 674).

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Some examples of the probability weighting function as of equation (6.3) with \( n = 2 \) are presented. In the left panel functions for values of \( \alpha \) from .2 to 1.8 with fixed \( \beta \) at .6 are graphed. Functions in the middle graph are for a constant \( \alpha \) of .6 and varying \( \beta \) between .2 and 1.8. The third graph presents results from Gonzalez and Wu (1999). The functions represent the most optimistic \((\alpha = .65, \beta = 1.51)\) and pessimistic \((\alpha = .15, \beta = .21)\) subjects and the median data \((\alpha = .44, \beta = .77)\).\(^{37}\)

Figure 6.2: Some Probability Weighting Functions

judgment that one of the four leading teams of the Eastern conference would win the playoffs ended up to be higher than the judgment of the winner being from the Eastern Conference rather than the Western Conference although the Eastern Conference includes even more than the four specific teams.\(^{39}\) An example how subadditivity affects judgments in poker is presented in the following case study.

Case study 4:
The game is Texas Hold’em. Even before being dealt a hand players will have expectations about the likelihood that their hand will develop to, say, a straight or a flush. The knowledgeable player will know either from table 2.6 or from experience that the probability of getting a straight in a 7-card hand is roughly 4.62% and for a flush 3.03%.\(^{40}\) Most players, however, will not have these figures readily available and base decisions on their subjective judgments.

Now we might ask something like “You can buy a gamble which offers you to receive $1 if your next hand becomes a straight; you will get nothing if there is no straight. How much would you be willing to pay for it?” Assuming that utility is linear for the next $1, the judgment will be completely determined by the probability weighting function. For this we might assume the form \( w(p) = 0.77p^{0.44}(1-p)^{0.44} \).\(^{41}\) Substituting 4.62% for \( p \) we find a weight of about .17. So a payment of $.17 would just be accepted. For the same gamble on a flush instead of the straight the weight is approximately .14.

What if we had offered a gamble to pay in case of a straight or flush. The joint

\(^{37}\) Again median data from Gonzalez and Wu (1999) in equation (6.3).

\(^{39}\) For further evidence regarding subadditivity see Tversky and Koehler (1994), Fox, Rogers, and Tversky (1996).

\(^{40}\) We neglect the joint event of a straight flush which is 0.03%.

\(^{41}\) We use the median data from Gonzalez and Wu (1999) in equation (6.3).
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probability is 7.65% and consequently a price of $.20 is acceptable.\textsuperscript{42} This is obviously less than what we would have received if we offered two separate gambles reflecting the distortion of subadditivity and the value that can be extracted by unpacking side bets at the table. Earning $.31 per $ at an expected cost of $.0765 would be quite a nice margin to extract from biased evaluation of probabilities.

The implications of the psychological weighting of probabilities have been shown to persist in market environments (Camerer 1987), in areas familiar to consumers such as health risks (Viscusi, Magat, and Huber 1987), and regardless of the mode of elicitation (Hurley and Shogren 2005). Furthermore, the same principles we have shown in this section for risk also apply under conditions of uncertainty (e.g. Tversky and Fox 1995; Wu and Gonzalez 1999; Kilka and Weber 2001).

6.3.4 Value Function

The preceding discussion illustrated how probabilities are transformed into subjective weights as they are part of the subjectively expected utility model (SEU). Now we should expect the model to adequately describe observed decision-making behavior, or should we not?

Consider the problems from Kahneman and Tversky (1979) stated in table 6.4. For these choices the SEU model predicts that if B is preferred to A then D should also be preferred to C. Probabilities in both choices are equal so there will be no effect from subjective probability. What about utility? As when viewed in terms of final states the two choices are identical \([A=C=(50\%-\$1,000;50\%-\$2,000)\) and \([B=D=\($1,500\) for sure]\) utility will be the same as well. Evidently, subjects did not follow this reasoning.\textsuperscript{43} Instead

\textsuperscript{42}Here \[w(.0462 + .0303) < w(.0462) + w(.0303).\]

\textsuperscript{43}Empirical evidence on the neglect of final wealth is also presented by Gertner (1993) who analyzes data from the TV-show “Card Sharks” and finds that behavior “... violates any theory in which a person maximizes a utility function whose only argument is final wealth, ...”.

Table 6.4: Decision Problems Drafted by Kahneman and Tversky (1979)
The following two problems of choice between two alternative situations were presented to subjects. The percentages of subjects who chose either alternative are stated in brackets.

You have been given $1,000. Now you have a choice between

<table>
<thead>
<tr>
<th>Situation</th>
<th>Description</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50% chance to win $1,000</td>
<td>16%</td>
</tr>
<tr>
<td>B</td>
<td>$500 with certainty</td>
<td>84%</td>
</tr>
</tbody>
</table>

You have been given $2,000 and may now choose between

<table>
<thead>
<tr>
<th>Situation</th>
<th>Description</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>50% chance to lose $1,000</td>
<td>69%</td>
</tr>
<tr>
<td>D</td>
<td>A sure loss of $500</td>
<td>31%</td>
</tr>
</tbody>
</table>

Source: Kahneman and Tversky (1979, p. 273)
they chose the risky alternative for the problem stated in terms of losses and the certain alternative in the gains situation. Kahneman and Tversky (1984, p. 342) criticize the descriptive worth of the utility function as found in the SEU model because

“This representation appears psychologically unrealistic: People do not normally think of relatively small outcomes in terms of states of wealth but rather in terms of gains, losses, and neutral outcomes [...] the psychophysical analysis of outcomes should be applied to gains and losses rather than to total assets.”

Their proposition takes the model of Friedman and Savage (1948), which we discussed above in section 6.1.2, a step further as outcomes are now framed as gains or losses relative to a reference point following the same psychophysics like probabilities, i.e. diminishing sensitivity further away from the reference point. However, the function still has a concave part indicating risk aversion in the domain of gains and a convex part reflecting risk seeking in the domain of losses. The natural reference point separating these domains is an individual’s initial wealth. Leopard (1978), for example, demonstrates that subjects took more risk when they had fallen behind than when they were ahead in a series of consecutive gambles. Similarly, Morgan (1983) reports an increased preference for riskier decisions after subjects had predominantly lost in the early trials in sets of risky gambles. But the reference point is not static. People adapt to the new found status following gains and also, but somewhat slower, following losses (Arkes, Hirshleifer, Jiang, and Lim 2008). Reference points are also context dependent. If, for example, an individual is confronted with a specific, challenging goal, the motivational processes influence cognition and the goal might serve as a reference point (Payne, Laughhunn, and Crum 1980; Heath, Larrick, and Wu 1999). There may also be multiple reference points when besides the status quo or initial wealth after a decision is made the outcome is compared to an alternative outcome which could have been achieved as well. Such a foregone alternative can induce regret or joy depending on whether the actual outcome is better or worse (Boles and Messick 1995).

At cash games in the poker environment, where players can enter or leave the game at any time with their money, initial wealth is a prominent figure. It is the money brought to the table, the buy-in. Only rarely do players set goals like earning $x over the next 100 hands or within the next hour. Because aspiring to goals is difficult in an environment which is naturally volatile and where even if every decision was right goals could be missed, usually participants mostly care about being ahead or having fallen behind. Thus

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44 Rachlin, Logue, Gibbon, and Frankel (1986) compare the cognitive model of human choice with a behavioral model of animal choice and present ample parallels on the psychophysical properties.
45 Hershey and Schoemaker (1980) also find an inflection point in the utility function over losses, supporting the idea that individuals are not only risk seeking in the domains of losses.
we propose

**Hypothesis 3** The money brought to the poker table serves as reference point during a player’s session.

Once a player has left the table and returns at a later point in time the reference point will change to the new buy-in.

Imagine you happen to find a winning lottery ticket on the sidewalk but lose it again later during the day. How do you feel compared to how you felt prior to stumbling over the ticket? As losses loom larger than gains most people in this situation feel worse than before, a pattern termed *loss aversion*.\(^{46}\) Experimenters observed ratios of about 2:1 to 2.5:1 (Tversky and Kahneman (1991, pp. 1053-1054), Kahneman (1992, p. 298)), so we assume that losses are valued by a factor of 2.25 relative to equivalent gains which is the median reported by Tversky and Kahneman (1992, p. 311).\(^{47}\) Summing up, the value function has to reflect three effects. First, the reference point as inflection between risk aversion and risk seeking. Second, diminishing sensitivity as values move away from the reference point. And third, pain (loss) is more urgent than pleasure (gain). One such function which is generally used is the two-part power function of the form\(^{48}\)

\[
v(x) = \begin{cases} 
  x^\gamma & \text{if } x \geq 0 \\
  -\lambda (-x)^\gamma & \text{if } x < 0 
\end{cases}
\]  

(6.4)

In this function \(\lambda\) is the coefficient representing loss aversion. The exponent \(\gamma\) determines the curvature of the function and thus represents risk aversion, risk seeking and diminishing sensitivity.\(^{49}\) Some examples of value functions are graphed in figure 6.3 for \(\lambda = 2.25\) and \(\gamma = .37, .52, \text{and } .88\). With these parameters the function is concave for gains, convex for losses, and steeper for losses than for gains. The Pratt-Arrow measure of risk (see equation (6.2)) is \(-\frac{1}{(\gamma-1)x}\) over the whole range. This shows that loss aversion does not affect local risk aversion and highest measures are obtained closest to the reference point.\(^{50}\) Comparing the measures at two different points we get \(r(x_2)/r(x_1) = x_2/x_1\) which for a constant \(x_1\) is a linear relationship independent of the parameters \(\gamma\) and \(\lambda\). It implies that risk aversion/seeking at \(x_2\) is proportional to risk aversion/seeking at \(x_1\). Consequently, we state

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\(^{46}\)See e.g. Tversky and Kahneman (1991).

\(^{47}\)Loss aversion also explains why acts leading to the worse outcome are considered worse than omissions leading to the worse outcome (Baron and Ritov 1994).

\(^{48}\)See e.g. Tversky and Kahneman (1992, pp. 309-311).

\(^{49}\)Different exponents for gains and losses could be assumed but usually a single exponent is sufficiently explanatory. See ibid. and Rieger and Wang (2006, esp. p. 666).

\(^{50}\)The measure is not defined for \(x = 0\) and \(\gamma = 1\).
Hypothesis 4  Players will increase risk taking linearly the further they have fallen behind and reduce risk taking linearly the further they are ahead.

Three value functions are graphed based on equation (6.4). $\lambda = 2.25$ in all three cases. $\gamma$ is varied between .37 (Camerer and Ho 1994, solid gray line), .52 (Wu and Gonzalez 1996, solid black), and .88 (Tversky and Kahneman 1992, dashed black).

Figure 6.3: Some Value Functions

6.3.5 Prospect Theory and Related Models

The results discussed in the preceding sections were first collected to form a comprehensive theory by Kahneman and Tversky (1979). As the major difference to SEU is the judgment of outcomes from a reference point, the theory is called prospect theory (PT). Prospect theory distinguishes two phases in decision-making. First, options are edited in an early phase, then evaluation takes place in a subsequent phase. In the editing phase individuals perform the following actions:

- Coding. Outcomes are coded as gains and losses relative to a reference point.
- Combination. Prospects with identical properties are combined.
- Segregation. Riskless components are separated, i.e. prospects are split in sure and risky outcomes.
- Cancellation. Elements shared by all outcomes are excluded.
- Simplification. Probabilities or outcomes are rounded and extremely unlikely alternatives dropped.

52 See Kahneman and Tversky (1979, pp. 274-275).
- Detection of dominance. Alternatives which are worse in all aspects are discarded.

After this reframing of the options the actual judgment is made. In this phase prospects are evaluated based on the probability weighting function and value function of the individual as discussed above. Prospect theory is able to explain observed decision-making behavior to a great deal. In contrast to the classic analysis, however, the presence of both insurance and gambling for the same individual is not explained by the concavity of the utility function (value function, respectively) but through the probability weighting function (Wakker, Thaler, and Tversky 1997, p. 20). The theory was later extended to cumulative prospect theory (CPT) by the same authors, which also accounts for decision-making under uncertain prospects (Tversky and Kahneman 1992; Wakker and Tversky 1993). It also adds the presence of two different probability weighting functions in the domains of gains and losses, respectively. PT and CPT perform well compared to alternative theories in describing observed decision-making behavior under risk (e.g. Weber and Camerer 1987; Camerer 1989) and uncertainty (e.g. Bernstein, Chapman, Christensen, and Elstein 1997; Abdellaoui, Vossman, and Weber 2005). Table 6.5 illustrates how CPT relates to the more classic models of decision-making. It is an extension of table 6.3 and adds the subjective coding of outcomes as prospects in the domain of gains or losses and similarly a domain-dependent weighting of probabilities by the individual. Overall the more we move to the right or bottom of the table the better the descriptive performance of the model and the less its normative appeal. At the same time the information on which decisions are based also shifts from an objective to a subjective interpretation. How individuals process the information that is ultimately evaluated will be discussed in the next section.

Table 6.5: More Models Of Decision-Making

<table>
<thead>
<tr>
<th>Concept of Outcomes</th>
<th>Objective</th>
<th>Subjective</th>
</tr>
</thead>
<tbody>
<tr>
<td>EV</td>
<td>SEM</td>
<td>SEU</td>
</tr>
<tr>
<td>EU</td>
<td>PT</td>
<td>CPT</td>
</tr>
</tbody>
</table>

\(PT = \) prospect theory \(CPT = \) cumulative prospect theory

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53Wakker (2004) decomposes probability weights in a component reflecting risk attitude and another influenced by the properties of uncertainty.

54Usually denoted by \(w^+(p)\) for gains and \(w^-(p)\) for losses.
6.4 Information Heuristics

At a fundamental level all information needed for decision-making in poker can be gathered by one of three main quantification processes. Small numbers of objects can be processed by subitizing, simultaneous perception of all items. Larger quantities are counted and for very large figures we have to resort to estimation (Brown and Siegler 1992, p. 406). By moving his attention back to the game a player distracted beforehand will notice that play is at the turn as there are four cards on the board. No counting is required for this. Then, after being involved in some hands he might have lost track of his chip stack. The remaining money is usually figured out by counting. Trying to answer questions such as how frequently someone decides to bet or raise estimation has to be used. It is for this kind of task that information heuristics become relevant.

Two heuristics have been identified which are generally used to quantify the frequency or probability of an event (Tversky and Kahneman 1973, p. 208). First, a person may assess the ease with which such an event can be remembered or imagined, in other more psychological words, the associative distance or availability heuristic is used to arrive at a judgment. You might have played two hands against the newcomer at the table and he bet and raised right through to showdown, you have not payed much attention to the other hands he played so you come up with a rather high estimate of his bet/raise percentage. Second, comparing the properties of an event to those of the structure from which it originates, an assessment of similarity or connotative distance can be made, a judgment by the representativeness heuristic. There is a new player at the online table, “Andrea”, who beats the game with a loose-aggressive style. Thinking that this style is not what you experienced from female players so far, you conclude the first hands are out of the ordinary and the style will surely become more tight-passive as play proceeds. Only after losing all your chips to a flopped straight on a pocket ($7\spadesuit5\spadesuit$), an unlikely hand for a tight-passive style, you learn that Andrea is a guy from Italy. The influences of both heuristics are pointed out in this section.

6.4.1 Judgment by the Ease of Mind – the Availability Heuristic

Things or events which are frequently or likely experienced are strongly bonded in memory through repetition and consequently come to mind easily. Human reasoning uses the inverse of this relationship to infer judgments of frequency or probability. The availability of an event or object is used to provide the required estimate (e.g. Tversky and Kahneman 1973; Gabrieleck and Fazio 1984). A process helpful for a variety of real life decisions but a possible bias for more logical or probabilistic problems (Pollard 1982). For example, Brown and Siegler (1992) asked for population estimates of the 100 countries with a pop-
ulation of over 4 million. The resulting estimates were higher for well-known countries and those with a lot of attention from the media, e.g. Sweden or Israel. Less prominent countries or those with few coverage were estimated well below their actual population, say, Nigeria or Bangladesh.\textsuperscript{55} Tversky and Kahneman (1973) questioned their subjects regarding each of the letters (K,L,N,R,V) whether the letter is considered more likely in the first position or in the third position in English words. A significant majority judged the first position more likely though the reverse is true for all five letters. Here it appears easier to come up with words starting with the specific letter than to find examples of the letter in a third position.

Before considering further factors influencing availability it is apt to discuss the human capacity for processing information. It was Miller (1956) who established two basic limitations on the amount of information humans can receive, process, and remember. First, he found that absolute judgment has a capacity span which is able to distinguish from 1.6 bits to 3.9 bits of information depending on the kind of stimulus presented. Thereby, one bit is the amount of information needed to make a binary decision, a distinction between two alternatives. Presented, for example, with different tones, listeners were able to correctly identify 2.5 bits or about six different pitches. Second, the span of immediate memory is limited by the number of chunks of information. The usual amount for immediate recall is about seven chunks. The distinction is used extensively by memory masterminds who recode several bits of information into a single chunk so that they are able to reproduce considerable input. A sequence of binary digits like 10101 might be recoded as 21.\textsuperscript{56} In poker, information processing is more important in games where private cards are exposed and discarded like in 7-card stud.\textsuperscript{57} At Texas Hold’em information has to be processed mostly between hands as private cards are seen only at showdown. So it is not surprising that in online play many professionals manage several tables of this game at the same time; multi-tasking which puts considerable strains on the cognitive processing system and can only be sustained as capacity requirements for the individual tasks are low.\textsuperscript{58}

The ease with which instances come to mind does not only depend on the existence of memory traces within the cognitive system, availability, but also on the speed or recall latency of these traces, accessibility (MacLeod and Campbell 1992). Usually, the influence of both factors is meant by referring to the availability heuristic. In their original paper on

\textsuperscript{55}As of 2009 populations are 9.3m Swedish, 7.4m for Israel, 154.7m in Nigeria, and 162.2m in Bangladesh.

\textsuperscript{56}Also see Baddeley (1994) on the distinction between absolute judgment and immediate memory.

\textsuperscript{57}Each player is first dealt two cards face down and one face up, then in three phases an additional open card is dealt to each player, followed by a final private card. The best 5-card hand of the player’s 7-cards is used.

\textsuperscript{58}On the effects of multi-tasking see e.g. Navon and Gopher (1979).
the availability heuristic Tversky and Kahneman (1973, pp. 210-211) presented a study where subjects are asked to estimate within 7 seconds the number of instances they could come up with in 2 minutes for a given category like four-legged animals, or specific city names. As it turned out both the estimate and the actual number are highly correlated. Not only the traces in memory but also the speed of recall matters in estimating class populations.

Many additional factors influence our ability to remember specific occurrences. So not only the actual frequency of repetitions is important but also the spacing between items. Apparent frequency increases with spacing between repetitions (Hintzman 1969; Underwood 1969). Memory is better for objects at the beginning of a series, the primacy effect, and for later positions, the recency effect (e.g. Jahnke 1965). We might also use analogous expectations for explaining events (Read and Cesa 1991), for example, when asked how likely we think a capital is located at a river or coast we might consider big cities at these locations although capitals are not necessarily big cities and vice versa.

If the availability heuristic is used in estimates at the poker table, decision-makers will be influenced by the information they receive during the game. Most importantly, the impact will be concentrated to a limited amount of information with more weight on early or recent experiences as well as events repeated over the game.

Case study 5:
Here we present some examples of the availability heuristic at work in the poker environment in stories about a fictitious player called Bill.

- Primacy effect. When Bill started to learn to play poker his mentor had a nervous quirk. Whenever he had a strong hand he would squint with his left eye. Although this is a very rare tell Bill quotes it as the first he looks for. As this is the first tell he learned it is readily available and comes to Bill's mind easily.

- Recency effect. Throughout his poker career Bill has lost many hands where he was a significant favorite to win, a so-called bad beat. Lately, he was ahead on the flop with top two pair and his opponent improved his lower two pair to win with a full house. Now he has got (J♣6♠) in a No-Limit game and the flop comes J♥6♣3♦. He raises in second position and is called by the other player. The turn comes 3♣ and Bill is confronted with an all-in. As his recent bad beats are readily available he estimates the probability to be against three of a kind or a full house to be quite high. He folds and is shown (K♠J♠) by the happy winner.

- Spaced repetitions. Usually, Bill enters full table cash games with 10 players. There he sees only few hands at showdown and usually a flush or better wins the big pots. Neglecting the fact that most pots and money are won before showdown, he estimates the probability of successfully drawing to a flush as quite high. Therefore he substantially increases the share of hands with suited
Overall the availability of instances will affect players’ estimates used in upcoming decisions. Whenever availability increases, the weighting of the underlying events will also be increased in future choices (e.g. Estes 1976). For example, seen hands at showdown will strengthen associative bonds that these are hands to have at showdown. Consequently, we propose

**Hypothesis 5** *Hands frequently seen at showdown will influence future play.*

Here, the focus is on the frequency of the hands so it is likely that the availability heuristic is used in estimation. In other situations the structure or pattern of events might be more prominent so that the representativeness heuristic is at work.

### 6.4.2 What is Random Should Look Random – the Representativeness Heuristic

In the words of Kahneman and Tversky (1972, pp. 451-452)

“According to the representativeness heuristic, one evaluates subjective probability by the degree of correspondence between the sample and the population, or between an occurrence and a model. This heuristic, therefore, emphasizes the generic features, or the connotation, of the event. According to the availability heuristic, on the other hand, subjective probability is evaluated by the difficulty of retrieval and construction of instances. It focuses, therefore, on the particular instances, or the denotation, of the event.”

Besides the similarity of a sample to the population, representativeness also requires that fundamental characteristics can be found not only in the sample as a whole but also for any subsample. Population characteristics are expected to manifest locally in each part of a sample. Consider births of boys and girls. It is known that both genders are about equally likely for a population. Consequently for a family the *exact* order of births of boys and girls G B G B B G is judged more likely than B G B B B B although, of course, both are equally likely (Kahneman and Tversky 1972, p. 432). A sample with the same proportions as the population is more representative and thus perceived as more frequent. A representative sample will share a set of properties including the number of identical objects, sidedness, range, mean, or variance with the parent distribution (Bar-Hillel 1980b). In contrast to this, sample size, an important statistical measure, is frequently not given the appropriate weight in intuitive judgments (Evans and Dusoir 1977; Bar-Hillel 1979). If asked to judge which sample of coin-toss series is more likely to reflect a biased coin
in the experiment of Evans and Dusoir (1977, pp. 131-132) 22 of 48 subjects granted equal evidence to the majority of samples like 7:3 versus 700:300. Here in the mind of the decision-maker less information is valued as strongly as more information.

What about new information? Take a common example from the medical profession. There is a diagnostic process for a particular disease which properly identifies people to have the condition in 80% of the cases. However, 10% of the time it will wrongly indicate the disease for people who do not have it. On average 5% of the population are affected. Now a (random) person from the population is screened positively. How likely is this particular person to be ill? Many individuals show an inclination to suppress existing information and attribute greater validity to the new, diagnostic information. They will state the person to have the disease with a chance of about 80%, thus neglecting the base-rate information in favor of the indicator, behavior which is called the base-rate fallacy (e.g. Bar-Hillel 1980a; Grether 1980; Bar-Hillel and Fischhoff 1981). The rational way to integrate the newfound evidence is part of any good textbook on statistics and known as Bayes’ rule or Bayes’ theorem. If we let $A$ be a positive result of the diagnostic and $B$ the presence of the disease, the theorem states that

$$p(B|A) = \frac{p(A|B)p(B)}{p(A|B)p(B) + p(A|notB)p(notB)} \quad (6.5)$$

Substituting the known facts we get $p(B|A) = \frac{(0.8)(0.05)}{(0.8)(0.05) + (0.1)(0.95)} \approx 29.63\%$, a much lower probability than 80% due to the diagnostic imperfections. If the indicator would identify the condition all the time, i.e. $p(A|B) = 100\%$, this would improve to about 34.5%, and if it were to give false indications only half as frequently, i.e. $p(A|notB) = 5\%$, an even better performance could be achieved with roughly 45.7%. Updating evidence in this way is called Bayesian inference, a method quite useful at the poker table as Chen and Ankenman (2006, chapter 3) illustrate and on which we base the following case study.

**Case study 6:**

A new player joins the game in position 9. Assume we know from extensive experience that 10% of all players are maniacs who raise 80% of the time being in this position. All other styles will raise only 20% of the hands in the same situation. In the first hand he plays he raises. How likely is he to be a maniac? We use equation (6.5) and get

$$p(maniac|raise) = \frac{p(raise|maniac)p(maniac)}{p(raise|maniac)p(maniac) + p(raise|other)p(other)}$$

$$= \frac{(0.8)(0.1)}{(0.8)(0.1) + (0.2)(0.9)} \approx 30.8\%.$$ 

From a single observation we can infer that this player is a maniac with an about
1 in 3 probability. Assume he will raise again the next time in this seat. Again we update our information. Instead of the population parameters we can use the prior evidence for this player and get

\[
p(\text{maniac}|2\text{nd raise}) = \frac{(0.8)(0.308)}{(0.8)(0.308) + (0.2)(0.692)} \approx 64.0\%.
\]

With the additional evidence we can diagnose this player more likely to be a maniac than not.

By comparing how representative observed patterns are of different underlying schemes, decision-makers can improve on their estimates concerning the likely nature of the process. As it is quite a competitive advantage to discover the style of others in a poker game, Rapoport and Budescu (1997, p. 603) mention that experienced poker players try to mislead their opponents by deliberately randomizing their actions.\textsuperscript{59} The authors go on to study how well individuals perform in randomizing tasks. They find that subjects do rather poorly because local information is excessively considered in making the next choices. Individuals trying to randomize see their responses in a window of memory, and try to balance relative frequencies over the local sample. If they were induced to ignore the local sample, performance increased, but unfortunately it is difficult for decision-makers to ignore information even when it is in their interest to do so. This curse of knowledge is also shown by Camerer, Loewenstein, and Weber (1989). In their experiment agents are unable to neglect the additional information they possess in a competitive market environment although this would be advisable.

The urge to create local samples representative of an overall random process results in significant biases in choices. In search of these flaws Rapoport and Budescu (1992) tested how well subjects randomized their strategy in a game similar to matching pennies. Results suggested two differently biased groups, one displaying positive recency and the other exhibiting negative recency. Andreassen (1987) also discusses that people sometimes predict recent changes to reverse in the future (regression, negative recency) and in other circumstances assume changes will persist (trend, positive recency).\textsuperscript{60} In line with Ajzen (1977) he states that in determining which bias will occur the changes in the underlying cause are the most important factor. If causal information is presented it is used to adjust predictions to the newfound model and there is positive recency. If fundamental causes are lacking, regression is expected as long runs and symmetry are considered non-random characteristics whereas randomness is associated with many alternations (Lopes

\textsuperscript{59}Also see Rapoport, Erev, Abraham, and Olson (1997) on choices in a simplified poker game.

\textsuperscript{60}Lindman and Edwards (1961) describe a situation where the negative recency effect changes over trials. For a review of alternative theories on information-processing in binary choices see Jones (1971).
and Oden 1987). Both biases are visualized in figure 6.4.

Based on a process “S” which generates events “A” and “B” with probabilities $\alpha$ and $\beta$ general characteristics of a repetition bias and an alternation bias are presented (biased transitions are in italics) where $\epsilon > 0$. Cf. Lopes and Oden (1987, Figure 1).

Figure 6.4: State Transition Diagrams for Biased Processes

Negative recency is also known as the gambler’s fallacy. The most common example being the roulette player who thinks that black has to come up with certainty after he has seen 15 times red in sequence. Basically, the gambler’s fallacy suggests that the conditional probability for another similar event decreases as past observations of this event accumulate (Brickman and Pierce 1972). Like Cohen, Boyle, and Shubsachs (1969) we might distinguish negative recency, which, in their terms, is restricted to situations, where predictions of the next outcome are made with or without knowledge of previous results or predictions. They reserve the gambler’s fallacy for settings where the next outcome is not only predicted but also bet on. How this and other effects of the representativeness heuristic affect real-life betting and wagering is discussed in the next section.

6.4.3 Field Studies of Representativeness

Lotteries are a worldwide phenomenon. In more than 100 countries, states have created lotteries and the most popular type, the numbers game where players may choose their own numbers, frequently creates jackpots worth tens of millions of dollars (Clotfelter and Cook 1991, pp. 227-228). These games usually operate on a parimutuel basis, the state takes its share which adds to state revenues and the remainder is equally distributed between those who picked the winning numbers. Therefore, individual payouts are lower if one picks numbers that are public favorites as the pot has to be shared with many others. However, for many players all numbers are not created equal despite this incen-
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tive to choose unpopular numbers.\textsuperscript{61} Chosen numbers often have a meaning to gamblers like birthdays or anniversaries, they might form a nice pattern on the ticket or are simply based on superstition; the most popular combination in the UK National Lottery is \{7, 14, 21, 28, 35, 42\} all multiples of the lucky number seven (Simon 1999, pp. 245-250). As a consequence there are more combinations picked more often than expected by chance and also a greater number of combinations is chosen infrequently than in a random-selection distribution.\textsuperscript{62} The best strategy, therefore, would be to use a random number generator and review whether the numbers look like a popular choice.\textsuperscript{63}

What happens after a number has been drawn? Among others Holtgraves and Skeel (1992), Clotfelter and Cook (1993), Terrell (1994) and Simon (1999, with further references) have researched this topic in lottery games. They find consistent evidence that after particular numbers are drawn, betting on this combination is substantially less in the following weeks. Only later will the amount wagered gradually regress to the original level. Lottery players appear to use the representativeness heuristic and consequently succumb to the gambler’s fallacy. The same numbers winning again just does not look random, though statistically chances are the same for all numbers at the next drawing including the latest winners (if, of course, the device picking the numbers is indeed random).

At the poker table randomization is achieved by shuffling the cards after every hand. We know that chances of being dealt two specific cards irrespective of their order are 1 in 1,326 or about 0.07\%. More simply we might split all possible starting hands by half in good and bad. Then being dealt a good hand has a probability of 50\%. If poker players apply the same heuristics in judging randomness like subjects matching pennies in the laboratory or lottery players in the real world, they will expect that the conditional probability of being dealt another good starting hand after the prior hand has also been good is less than an even chance. Furthermore, this principle will not only govern expectations about one’s own hands but also about the hands of all others. If poker players suffer from the gambler’s fallacy they will expect an increasing probability of an opponent holding a weak (strong) hand the more strong (weak) ones he has shown in prior hands. Given these expectations they might adjust their behavior accordingly, betting/raising more against a player they expect more likely to hold a weak hand and folding regularly if they expect a strong hand. On this basis we can test the following hypothesis.

Hypothesis 6 The more strong (weak) hands a player has shown over the last hands,
the less (more) likely others will fold to his next hand, thus decreasing (increasing) his likelihood of winning.

Whereas the gambler’s fallacy is an alternation bias, the hot hand is a belief in positive autocorrelation, a repetition bias. Although formally both biases are inverse to each other their appearance is quite particular as pinpointed by Croson and Sundali (2005, pp. 195-196) who also find empirical evidence for both biases in roulette games at casinos:

“In particular, the gambler’s fallacy is based on beliefs about outcomes like heads or tails, the hot hand on beliefs of outcomes like wins and losses. Thus someone can believe both in the gambler’s fallacy (that after three coin flips of heads tails is due) and the hot hand (that after three correct guesses they will be more likely to correctly guess the next outcome of the coin toss).”

As we have seen above belief in the hot hand will be present if the underlying process is thought to have causally changed, e.g. by improved skill. For example, whenever an individual has been successful recently he is deemed more likely to win whatever he chooses to bet on next. Gilovich, Vallone, and Tversky (1985) investigated the hot hand belief in basketball games. Although they could not detect significant non-random patterns in shooting baskets, fans and players alike believed in streak shooting, they even bet on it.

A similar notion can be held at the poker table. Individuals who have recently won hands might be thought to be “hot”. Notice that winning a hand is—not like being dealt a good hand—fully dependent on a chance process but has causal characteristics. If the presence of the hot hand is true, players are best advised to avoid playing against someone on a winning streak but should look for others who undergo a loosing streak. Behavior which provides us with a testable hypothesis.

**Hypothesis 7** Players show more and longer streaks of wins (losses) than warranted by their general performance.

The discussion in this section has centered around local representativeness, considering small samples in a large population. For these parts, maybe several hands within a session, it is reasonable to assume that the fundamental structures do not change. However, over a longer series, say, from session to session, we can expect that things change a bit as players gain more experience and have the opportunity to learn. The potential of learning and changes in behavior over time are covered in the next section.
6.5 Improving Play

An essential characteristic common to all types of poker games is that hands are dealt repeatedly as long as any two players are willing to participate and have money left to bet. So over the course of the action individuals get performance feedback on their past decision-making and may adjust actions in subsequent choices accordingly. In this section we will first review theory on how humans reason if new problems are encountered, as it is the case for all players joining the game for the first time. We then go on to discuss the impacts of experience and learning on the decision-making process and choices.

6.5.1 Learning the Hard Way

People confronted with problems in new domains are seldom bare any knowledge about the domain. More frequently, novel situations will evoke declarative knowledge about the domain which can be used to apply a rough-cut problem-solving procedure (Anderson 1987). In our poker environment newcomers will usually have been lectured on the rules, ranking of cards and hands, and have heard that to be a winner one has to bluff occasionally. Based on this they may use strategies playing too many hands with aces or are bluffing excessively. Think about a task like rank-ordering the 169 starting hands in Texas Hold’em. It is unlikely that newcomers are aware of something like the concept of all-in-equity introduced in section 5.1.1. They lack an effective problem representation to gain insight on more critical cues (Kaplan and Simon 1990, pp. 374-375). Kaplan and Simon (1990, pp. 381-382) indicate four sources relevant in changing problem representations. First, properties of the problem itself may reduce difficulties. In our example we could rephrase the task to rank-ordering all the pairs, suited and offsuit starting hands, thus making the fundamental characteristics more salient. Second, hints from others can reduce the time spent in evaluating futile ideas. Third, relevant knowledge in the domain can produce routine approaches. Experience with other poker varieties could be of help. Finally, heuristics can lead problem solvers to restate the problem. As there are 13 different ranks and two cards in a starting hand one might envision the hands in a 13 by 13 matrix like in table 5.1.

Lacking an effective representation and problem-solving process for determining which situations are good and which are bad at the poker table, beginners are at a considerable disadvantage. Luckily, new customers are highly valued by online casinos. Throughout the industry sign-up bonuses are awarded to new players, usually matching the first de-

\[ ^{64}\text{Also familiar instances which are only superficially similar to the problem might be used as an analogy, see Catrambone, Jones, Jonides, and Seifert (1995).} \]

\[ ^{65}\text{Characteristics increasing the difficulty of problem-solving are discussed in Kotovsky, Hayes, and Simon (1985) and Kotovsky and Simon (1990).} \]
posit up to an amount of several hundred dollars. In gambling jargon this is house money for which losing does not hurt as much as if it was one’s own money. Equipped with this windfall profit risk-seeking is facilitated and marginal propensity of consumption increased (e.g. Thaler and Johnson 1990), which is what the industry expects and likes to see as they are taking their share on every gamble made.

Both circumstances taken together, the lack of effective strategies in dealing with the decision problems and the welcoming hand offering an instant reward, endorse new players to participate frequently to gain experience. Or in other words

**Hypothesis 8** Players will exhibit higher looseness during their first session compared to later sessions.

As seen in section 5.1.3 looser play is costly in terms of average amount won. Thus newcomers will have to learn the hard way, first enduring lower average returns than they are able to extract once they have excluded irrelevant approaches from the problem-solving procedures.

### 6.5.2 Practice Makes Perfect

In psychological terms every round of play is a stimulus event for poker players requiring a response. The task is mostly a number processing problem which moves from early visual processing, verbal and arabic comprehension, over phonological representation, to semantic magnitude representation (Dehaene and Akhavein 1995) before the actual decision can be made, whether to fold, check, call, bet or raise (etc.). Most of this is done automatically without much mental effort. However, for these basic cognitive processes a minimum time is required which limits the maximally achievable speed of decision-making.

As more and more domain-specific knowledge is acquired through repeated processing of the same or similar tasks, processing will become automatic, relying on stored instances in a decision-maker’s memory. Thus through practice the amount of information which can and will be retrieved as well as the speed of retrieval increase. In other words, whereas novices have to rely on general problem-solving strategies, experienced individuals will have learned specific solutions they adopt if a similar problem comes up. It has been found that the speed-up resulting from practice follows a simple power function of the form

\[ RT = a + bN^{-c} \]  


67 The phrase house money is also more generally used for any winnings from the house. For poker the narrower use applying to money received from the house without participating in a game seems more appropriate as usually money is won from other players and not from the house.
where \( RT \) is the reaction time for processing a task, \( N \) is the number of times the task has been practiced, \( a \) is the minimal time required at least for perceiving and reacting, \( b \) is the practice effect which can be acquired from experience, and \( c \) is the rate of learning (on this paragraph see Logan 1988, 1992).\(^\text{68}\)

In order to realize practice effects tasks have to be set in a consistent environment so that automaticity can develop. This does not mean that the expert will be thinking back to an earlier episode whenever he performs a task. Ross (1984, pp. 374-375) identified four conditions most likely to affect the probability that earlier instances of a problem will be reminded. First, with increasing similarity of the current problem to those in memory the probability of a reminding is increased.\(^\text{69}\) Second, as the number of experiences grows, information in memory is less concrete but abstract properties will be remembered. Third, for more difficult tasks relevant abstract information is difficult to retrieve so that reminding of more concrete episodes becomes more likely. Fourth, factors might be affecting memory retrieval making past episodes more or less salient and reducing the interference between episodes.\(^\text{70}\) An experienced poker player will not take much time to call based on pot-odds if he is drawing to a reasonable winning hand like a flush or straight, and no particular instance will be brought to mind for this response whereas a novice might still need to count the outs and calculate the odds before making his decision.

Reminded examples will help novices to tackle the current problem and in parallel increase the amount of abstract problem-solving information. As the novice is learning from analogy, generalizations from cuing of earlier examples improve later reasoning (Ross and Kennedy 1990; Novick and Holyoak 1991). While the traces in memory get increasingly abstract there is also a strategy shift in problem-solving. Novices tend to work backward from the goal applying means-ends analysis whereas experts are looking for what can be done with the givens of a problem, working forward which is generally faster (Sweller, Mawer, and Ward 1983). Thereby acquiring expertise is not a question of intelligence; for example, Ceci and Liker (1986) show that handicapping skills at the racetrack are unrelated to IQ and also to years of track experience, but the complexity of the abstract problem-solving model which was used, captured decision-making performance to a significant degree.

The impact of experience on task processing is strikingly illustrated by perception in chess. Chase and Simon (1973) asked players of different skill to reconstruct middle-game

\(^{68}\)For a practical example see e.g. Fitts and Seeger (1953).
\(^{69}\)See Lovett and Anderson (1994) for an example of transfer based on similarity in geometry problem solving.
\(^{70}\)See Evans (1984) on the role of salience in heuristic processes.
positions of the board. They found that chess masters performed significantly better in speed and accuracy than advanced players and the worst performing beginners. Experienced players could encode positions into larger chunks, taking several pieces in a familiar constellation at a time. The effect went as far as that, if the setup of the board was derived randomly rather than based on middle-games from actual play, masters performed poorer than the other two groups, mostly as the configuration on the board tended to be extraordinary compared to usual games. Gobet and Simon (1996) also show the importance of practice on recognition processes in problem-solving in simultaneous chess play. It is not uncommon that chess grand-masters take on challenges to play against four or more opponents at the same time. As this leaves little time for look-ahead searches, their decisions are rather based on recognition stemming from extended experience, achieving a rated skill close to their top-performance at a much increased speed.

Players in online poker not only get accustomed to the software interface but can also group the choices they are facing into more and more abstract groups. For them instances might be attributed to categories like “flopping top pair”, “flush draw and overcards”, etc. with a corresponding strategy whereas beginners are likely to think in more concrete examples like “having (A♥T♠) and a board T♥7♥5♣, thus holding a pair of tens”, limiting the speed of decision-making. Speed of play is though not only determined by a player’s own pace but by the average pace of all players involved in the hand. Nevertheless ceteris paribus, i.e. if players are facing a random mix of opponents in terms of speed of decision-making, we expect the following hypothesis to hold

Hypothesis 9 A player’s average reaction time needed to make a decision will decrease following the power law of practice.

6.5.3 Adjustments of Choices

Why have unfavorable biases in human decision-making not been offset by evolutionary pressures? Arkes (1991) discusses three types of judgment errors (strategy-based, association-based, and psychophysically based errors) and argues that the underlying principles are not only costly but also have beneficial aspects. For example, a quick and dirty strategy although more error-prone is easy to execute so that time and effort are saved. Debiasing these errors creates additional costs which have to be weighted against the net disadvantage from the bias. Therefore, judgment errors will not necessarily be drawn to extinction. In competitive environments like business or poker we would also expect that inefficient judgments will be punished driving biased decision-makers out of

71 “Flopping top pair” refers to pairing the highest card of the flop with one of the hole cards, “flush draw and overcards” are instances where the player has four cards of the same suit and both hole cards are ranked higher than any of the cards on the board.
the game. In line with the arguments given by Arkes (1991) this will hold for some but not all biases and also depend on the relative bias to the decision-making of other participants.

Keren and Wagenaar (1987) show that already by allowing a gamble to be repeated sufficiently often, violations of utility theory can be reduced considerably, underlining the fundamental differences of a unique decision versus dynamic environments. Roth and Erev (1995) model adaptive learning strategies in sequential games and find that very simple models consistent with fundamental laws from learning observed in experimental psychology perform quite well. The particular laws they mention are the law of effect which states that choices that have led to good outcomes in the past are more likely to be repeated in the future, and the power law of practice which we have discussed in the preceding section. Learning processes may be distinguished in two major classes (Mookherjee and Sopher 1994, pp. 63-64). First, routine learning where only information concerning one’s own past choices and payoffs is used. Future choices then will be altered based on past successes and failures, and strategies which led to high payoffs will be chosen more frequently. Second, belief learning includes information about opponents’ choices and payoffs as well. Individuals see their decisions in light of those of others and will adapt responding to their relative performance. Mookherjee and Sopher (1994) test for these two classes of rules in an experimental matching pennies game and find strong evidence in favor of the more comprehensive belief learning. Information regarding other players’ choices obviously affects decision-making.

Learning is furthermore not restricted to information about one’s own and opponents’ decisions but may also derive from co-acting individuals in a group. Blascovich, Veach, and Ginsburg (1973) observed risk-taking behavior in casino blackjack where gamblers are individually playing against the house. Their results indicate that people bet more as they play with other people present. They also found that, as individuals familiarized themselves with the game, bet amounts were increased. Blascovich, Ginsburg, and Howe (1976) obtained similar results and emphasize that the changes of players’ risk-taking develop “... as a function of the emergent normative risk levels of the group in which they become a member.” That familiarity with the game increases total amounts bet was also shown for roulette by Ladouceur, Tourigny, and Mayrand (1986).

For poker we have seen that the most successful risk-taking strategy is tight and aggressive play. In a group of different styles sharks show the highest average returns and loose play is generally punished. Attentive players will perceive this and adjust their

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72 Ibid. p. 171.
73 Ibid. p. 276.
74 See section 5.2.2.
choices accordingly. On the one hand debiasing erroneous judgments they will play more selectively, reducing the rate of investment (looseness). On the other hand familiarity with the game induces larger volumes of investments (aggressiveness). Therefore, we expect to find evidence for the following

**Hypothesis 10** With increasing expertise playing style will become tighter and more aggressive.

### 6.6 Charged With Emotions

Surely not the least known fact about playing successful poker is that it requires the ability to maintain the proverbial pokerface. Trying to show strength where there is none, a skillful bluff has to be part of every expert player’s repertoire.\(^{75}\) However, this is not an easy feat to accomplish. Stone-cold bluffing is based on mastering one’s emotional state and suppressing any signals which might help others to discover the true strength. The ups and downs of the game as well as the competitive environment, where taunts and teases through direct contact or chat interfaces are omnipresent, feed players’ emotions and mood while playing. In this section some of the growing body of literature on the effects of affect, emotions and mood on risk-taking is presented.

#### 6.6.1 Linking Emotions and Cognition

Human feeling and thinking is inevitably linked as Zajonc (1980) discusses. Individuals confronted with a stimulus will usually show an affective reaction without extensive prior thought processes. Indeed, affect is always present where there is thought, though the opposite is not the case. In fact, many decisions are not based on cognitive processes at all but are based on emotional arguments. Evolutionary roots of this influence are so deep that all sorts of judgments are faster and more efficient for pictures than for words, mainly as pictures encourage affective reactions more directly and faster than words. Emotions have been present long before language or the complex models of modern decision-making theory evolved and they have been influential on choices ever since. As long as an individual’s affective condition is not apparently unrelated to the object to be decided, it is difficult to find a clear edge between pre-existing feelings and reactions to the target (Schwarz 2000). Thus the primal question “How do I feel about this?” is usually prejudiced by the current mood. Different moods also influence the decision-making strategy which is likely to be applied. Individuals in a happy mood tend to adopt easy to use, heuristic methods whereas people in a sad mood put more emphasis on systematic, bottom-up

\(^{75}\)Some pokerists also consider slowplaying, which is to represent a weak hand when in fact one is holding a very strong hand, a mode of bluffing.
information processing. Loewenstein and Lerner (2002) distinguish two general types of affective influences. One, immediate emotions at the time of decision-making. These can in turn exert a direct impact on preferences, e.g. someone is feeling disgust at the sight of bowels in the soup du jour, or influence the decision indirectly by altering the evaluation of either probabilities or desirability. Someone who has recently been ill but usually does not mind bowels in the soup, might choose a different menu discounting the likely taste of the soup. Two, predictions about the emotional consequences of decision outcomes, in other words expected emotions, will be integrated in the decision-making process.

**Case study 7:**
Player A is sitting left to B in a six-handed No Limit Hold’em tournament. He has had some lucky experiences during the day and his feelings are best described as cheerful, confident or sanguine. He has raised frequently in prior hands and also stolen the blinds from first position the last two times. The aggressive play is quite unusual for this player and might be an indirect effect of the current happiness. Having had to surrender the big blind the last two times has clearly annoyed player B. Now A has raised from first position again and the action is folded to B. Looking at his cards he is immediately let down seeing (K4) a mediocre hand he likes the least. Without thinking a lot he folds with a sigh. Despite his annoyance the direct affect caused his quick decision.

Showing pocket aces, player A also reaps the expected rise in satisfaction which influenced his decision to provocatively raise again. Either B would have been annoyed to re-raise which would be favorable for A or he would gain the satisfying pleasure of having stolen the blinds once more.

Which emotions are relevant for decision-making? Though no definitive answer may ever be obtained to this question, we might get a first impression from the following classification given by Elster (1998, p. 48)

- Social emotions. Anger, hatred, guilt, shame, pride, pridefulness, admiration, and liking.
- Counterfactual emotions. Regret, rejoicing, disappointment, elation.
- Thought of what may happen. Fear and hope.
- Thought of what has happened. Joy and grief.
- Thought of possessions of others. Envy, malice, indignation, and jealousy.
- Other and controversial cases. Contempt, disgust, romantic love, surprise, boredom, interest, sexual desire, enjoyment, worry, and frustration.

The list is far from exhaustive and all emotions are also depending on the nature of the trigger. Elster (1998) adds that positive emotions are generally not chosen but passively undergone. However, up to a certain point individuals can employ methods to block negative emotions.\(^{76}\) This raises the question whether having emotions can at all be part of

\(^{76}\)Ibid. p. 54.
CHAPTER 6. DECISION-MAKING IN POKER

rational choice. Here, he notices that emotions not only bear costs on the decision process but are also beneficial because they replace more elaborate procedures with higher opportunity costs (they take time). Affective reactions serve as quick substitute for cognitive problem-solving. Fessler, Pillsworth, and Flamson (2004) note the usefulness of emotions in decision-making from an evolutionary perspective. Two emotions with similar tendencies, anger and disgust, have opposite effects on risk-taking. Anger increases risk-taking in men (but not women), in accordance with its function to deter transgression through aggressive rivals. Disgust decreases women's risk-taking (though not for men), relating back to its function to ward off contamination. To influence decisions emotions must be representative of the target and relevant (Pham 1998). A feeling like disgust, for instance, will not matter much in the poker environment. The role of emotions also increases in real-life situations compared to laboratory simulations as shown by Anderson and Brown (1984). Not only are emotions relevant for the current decision but one’s mood is also invariably linked to events in memory and thus becomes relevant for similar future decisions. Consequently, as originally shown by Bower (1981), instances are better reminded if the present mood equals the one when events have been recorded.\footnote{For a review and further references see Eich (1995).}

Having seen that and when emotions are likely to influence decision-making, the direction of the relationship has not been touched upon yet. There are two major theories on the relationship between active emotions and risk-taking (Fessler, Pillsworth, and Flamson 2004, p. 108). First, the mood maintenance hypothesis proposed by Isen and Patrick (1983) suggests that people in a positive mood avoid taking risks to maintain their positive state and risks are taken in the presence of a negative mood in order to change it to the positive. Second, the affective generalization hypothesis of Johnson and Tversky (1983) focuses on the effect of emotions on subjective probabilities. They manipulated subjects’ mood by newspaper reports of tragic events and found that estimates of risk frequencies increased subsequently, regardless of similarity between the report and the estimated risks. An effect replicated widely in the literature. For example, Wright and Bower (1992) or Mayer, Gaschke, Braverman, and Evans (1992) also find evidence that happy people make optimistic estimates and sad people take pessimistic views. DeSteno, Petty, Wegener, and Rucker (2000) even discover distinct effects on likelihood estimates for specific emotions such as sadness and anger.

The influences of affect on risk-taking behavior are best described in a two-dimensional circumplex as shown in figure 6.5 taken from Mano (1994, p. 39). In the following sections three emotions are discussed in more detail as they carry special weight in the poker setting. First, as excitement and the thrills of uncertainty are a major motivation
for many players, any change in the environment which does not match this motive will quickly cause boredom. Second, the game is characterized by repeated stimuli of risk and eventual reward. From the more rewarding episodes, i.e. successful streaks, individuals will feel pleasantly aroused, they are elated. Third, contrary to the positive affectivity after winning, losing one’s chips and getting closer to bankruptcy is likely to cause significant negative affect, distress.

Emotions are ordered based on two-dimensions so that closer distance indicates similar effects. Two different sets of dimensions are graphed. Solid lines indicate Arousal-Pleasantness dimensions, dotted lines are for the dimensions of Positive/Negative Affectivity.

Figure 6.5: The Affect Circumplex

6.6.2 Zzzzzz. Boredom

Searching chat logs of online poker will return a substantial fraction of utterances like “Zzzzzz” which is probably the single most frequently used expression. It is usually found when a player takes an excessively long time to make his decision or is disconnected from the network so that everybody else has to wait for him reconnecting. Those writing it are bored by the current speed of play and as a corollary are impatient for the game to continue. As poker games, especially tournaments, last several hours or even days, coping with boredom is a fundamental skill to investigate.

Boredom has not found much interest in the decision-making literature, though it is a widespread emotion in the modern business world where similar tasks are repeated frequently thanks to specialization. In an early study Slovic, Lichtenstein, and Edwards (1965) designed an especially boring task for their subjects and found the following effect: their bored, unmotivated subjects used very easy strategies and changed them seldom. In contrast, highly motivated subjects made careful choices based on more complex considerations, and tried different strategies. In an area closely related to boredom Zur and
Breznitz (1981) investigate the effects of time pressure on risky choices. They observe that subjects under high time pressure are more risk-averse compared to those who are allowed more time for decisions. Assuming time pressure is negatively correlated to boredom, this means that bored individuals make riskier choices. Zur and Breznitz (1981) attribute the observed pattern to differences in information processing. Under high time pressure (8 seconds) only the most important information is used. With more time (16 or 32 seconds) additional information is processed until time runs out. As under stressful situations like time pressure negative dimensions, like amount to lose or probability of losing, get more attention, more gambles might be rejected based on their then prominent negative attributes. A contrary result is found by Silberberg, Murray, Christiansen, and Asano (1988) who run repeated-gambles with either a 25 or 90 seconds interval between trials. Their subjects demonstrate riskier choices for the shorter interval. However, when they indicated the total number of trials to be conducted, the difference disappeared altogether. Therefore, the increased risk-taking for the short interval might be due to a higher expected number of trials in this group.

A different perspective on boredom is that the longer time between gambling episodes in a slow, boring setting is seen as a delay in potential reward. After having taken a decision the individual prefers to have the reward, i.e. the outcome after the responsive actions of all others who are involved, as soon as possible. It is agreed among psychologists that time elapsing between a behavior and its reward has negative effects on rate of learning, strength of responding, and preference (Ainslie 1975, p. 467). Hence, it is possible that a smaller but sooner reward is preferred over a larger reward attained later. A behavior termed impulsiveness. Individuals can control themselves to suppress this impulse only by an effort of will (Ainslie 1975, p. 483).

The choice between outcomes at different points in time can further be seen as intertemporal decision making. Thereby, it has been found that behavior is best described by a hyperbolic discounting function rather than the rational, exponential discounting function (e.g. Ahlbrecht and Weber 1997). Although a wealth of anomalies has been identified (an overview is given by Loewenstein and Prelec 1992), for the discussion of boredom it suffices to note that later outcomes are discounted altogether. Consequently, individuals will try to minimize delays as to maximize the utility of future outcomes. Alternative to seeing delays as a discounting of outcomes, Rachlin, Castrogiovanni, and Cross (1987); Rachlin, Raineri, and Cross (1991) discuss discounting of probabilities due to delays. For example, a 10% chance to gain an object can be seen as a series of trials where on average one has to endure the time and effort of 10 trials to get the object.

78 For an introduction see Eisenführ and Weber (2003, chapter 11).
Consequently, individuals are willing to trade-off between probabilities and delays. The authors also find a hyperbolic relationship between probabilities and delay.

Summing up the discussion above, individuals dislike delays between decision and reward as long as there is an opportunity for alternative or additional, later decisions. As a consequence they try to avoid lags between gambling episodes and are even willing to substitute lower probabilities of winning or lower winnings for a sooner result. The hypothesis for risk-taking behavior of bored poker players therefore is that

Hypothesis 11 With increasing boredom playing style will become looser and more aggressive.

To put it simply, we expect to see that players in a game, where decisions are made slowly, engage in hands more frequently and opt for more aggressive choices looking for additional excitement. They will be trying to overcome the unpleasant, low affect emotion by inducing higher arousal and affectivity.

6.6.3 If everything goes well. Elation

Over the course of a poker episode gambling stimuli will be a repeated source of arousal (if the game is not slow and boring). Due to the randomization properties of the game all but the worst players will eventually have times when everything goes well. Hence, when pleasantness and arousal combine there is sufficient reason for individuals to feel elated.

Isen and Patrick (1983) induced elation in their subjects by distributing McDonald’s gift certificates. They consequently compared the willingness to take risks under positive affect with a control group not given a gift certificate. Their findings indicate that the elated subjects bet more on low-risk but less on high-risk gambles, thus illustrating a tendency to protect their positive feeling. Elation is seen as increasing the negative utility of a loss relative to the utility of a gain in high-risk situations with large potential losses (Isen and Geva 1987). This increased risk aversion runs counter to the influence of positive affect on the perception of probabilities, because probabilities of negative events occurring are lowered and probabilities of positive events heightened (Johnson and Tversky 1983). Optimism induced by prior results then is counteracted by the dissatisfaction with a salient expected loss (Romanus, Hassing, and Gärling 1996; Romanus and Gärling 1997). Only when a substantial, potential loss is emphasized are elated subjects more conservative risk-takers or, analogously, more likely to buy insurance against the negative event (Arkes, Herren, and Isen 1988).

79 The only exception being long-shot gambles which positively affected subjects also took more willingly.
As elation is induced by prior gains, the position relative to the reference point proposed by the prospect theory value function is affected at the same time. Weber and Zuchel (2005) demonstrate that problems framed as a portfolio decision are more likely to evoke greater risk-taking following losses than following gains, consistent with prospect theory, as opposed to presenting the problem as a two-stage betting game for which risk-taking is greater following gains than following losses.\textsuperscript{80} Thus, in situations where gambles are seen as sequential exposures individuals will tend to, as Nygren, Isen, Taylor, and Dulin (1996) phrase it, \textit{cautious optimism} due to the effects captured by prospect theory and implied by elation. Thereby, on the one hand, optimism describes the perception of probability weights compared to unaffected individuals. For elated individuals there is a divergence of probability weighting functions between gains versus losses. On the other hand caution is due to the conservative, mood maintaining focus on potential outcomes, especially negative ones (Isen, Nygren, and Ashby 1988), which is nothing else but increased loss aversion, i.e. a dilation of the value function for losses.

Affective reactions are not always directly related to recent experiences. Johnson (1986) describes situations where the nonoccurrence of a “near outcome” leads to emotions counter to the actual outcomes. For example, someone in a near fatal traffic accident who is injured but not fatally like everyone else, is usually seen as quite lucky despite the actual bad fortune. In contrast, someone whose lottery ticket missed the jackpot by one number still receives a price but is not perceived very happy. As Johnson (1986) phrases it the knowledge of what might have been creates counterfactual emotions, with individuals mentally constructing a “once-possible but unrealized world”. He finds the following affect continuum from good to worse: positive outcome, near negative outcome, control, near positive outcome, negative outcome. Up to a point near outcomes also serve as reinforcements in gambling episodes; only too high a proportion of near-misses ends any expectation of imminent success (Griffiths 1999). On this account, Wohl and Enzle (2003) showed that people who experienced near losses gambled more thereafter than people who experienced a near win on a slot machine style wheel-of-fortune game.

Although prior gains will cause positively affected arousal (elation, happiness, ...), increasing winnings will not proportionately increase elation. As Brickman, Coates, and Janoff-Bulman (1978, p. 917) write

\begin{quote}
“Adaptation level theory suggests that both contrast and habituation will operate to prevent the winning of a fortune from elevating happiness as much as might be expected. Contrast with the peak experience of winning should
\end{quote}

\textsuperscript{80} They refer to any money won from gambling as “house money” a term which we reserve for money from an external source provided for gambling purposes without additional costs.
lessen the impact of ordinary pleasures, while habituation should eventually reduce the value of new pleasures made possible by winning.”

They also observed a significant difference in assignment of responsibility between lottery winners and accident victims. Whereas lottery winners seldom asked the question “why me?”, this is the rule not the exception for accident victims. Such an asymmetry between positive and negative outcomes is also reported by Gilovich (1983) who finds that gamblers remember losses better and spend a lot more time discussing them while wins are accepted at face value. Discounting or transforming losses into near wins also explains why people continue gambling despite continuous unsuccessfulness. Only larger wins disrupt this pattern, individuals pause longer after a big win before engaging in the next risk (Delfabbro and Winefield 1999). This can be interpreted as a form of gambler’s fallacy or representativeness heuristic where luck is thought to be depleted or to reverse. Additionally, whether gains are due to action or inaction will affect the strength of the emotion. It has been shown that affective responses are stronger after action than inaction (Landman 1987).

Following Smith, Levere, and Kurtzman (2009) we summarize the predicted effects of the various influences after wins and losses in table 6.6. Revised assessment refers to updating of skill assessments based on outcomes. For example, players who are winning should think they are good players and make more confident decisions. Their perceived control has increased after the recent successes. Prospect theory, house money, gambler’s fallacy and the hot hand have been discussed in other sections of this thesis. The notion under moods is changed versus Smith, Levere, and Kurtzman (2009) as they exclusively argue on the basis of optimism/pessimism which we consider only part of the emotional influences as just discussed. However, they find an overall increased looseness and aggressiveness after big losses and tighter/more passive play after big wins. From this we take

**Hypothesis 12** *Elated players will reduce risk-taking, with lower looseness and aggressiveness.*

### 6.6.4 On the Brink of Bankruptcy. Distress

To actively participate in a poker game a sufficiently large bankroll is required. Otherwise placing or raising bets is not possible and no interest can be taken in the pot. As the

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81 This need not be so for high-risk gamblers. Priming effects of early gambling episodes can be a deterministic factor in desire to gamble for this group (Young, Wohl, Matheson, Baumann, and Anisman 2008).

82 Similarly, players who have encountered negative events recently will feel their control lessened and tend to more cautious decisions.
Table 6.6: Predicted Risk-Taking after Wins and Losses

Concepts linked to prior gains and losses are listed. The predicted tendencies to change risk-taking are indicated. More means an increased risk-seeking whereas less stands for higher risk-aversion.

<table>
<thead>
<tr>
<th></th>
<th>Wins</th>
<th>Losses</th>
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<tr>
<td>Revised Assessment/</td>
<td>More</td>
<td>Less</td>
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<tr>
<td>Perceived Control</td>
<td></td>
<td></td>
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<tr>
<td>Prospect Theory</td>
<td>Less</td>
<td>More</td>
</tr>
<tr>
<td>Moods</td>
<td>More/Less</td>
<td>More</td>
</tr>
<tr>
<td>House Money</td>
<td>More</td>
<td>Less</td>
</tr>
<tr>
<td>Gambler’s Fallacy</td>
<td>Less</td>
<td>More</td>
</tr>
<tr>
<td>Hot Hand</td>
<td>More</td>
<td>Less</td>
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blinds force players into bets regularly, in a way that is known from business as recurring costs, diminishing capital pressurizes the individual player. He is constantly confronted with how long he can stay in the game without engaging in any action besides paying the blind fees. Here, we see unpleasant arousal, or in other words distress. In poker parlor the term tilt is frequently used when individuals lose control and deviate from usual play. Playing short of money is one of several tilt-inducing situations where people struggle to retain internal emotional control (Browne 1989).

One might think of two different ways of how players may end up at the brink of bankruptcy. First, starting with an outright too small amount of money, they are “underbankrolled” right away. Second, and more likely, they first started with a stack at least sufficient for some rounds of play but then suffered losses. It has been found that individuals who encounter a loss experience a period of distress as they cope with the situation by confrontation (Wortman and Silver 1989). After experiencing uncontrollable events other people show cognitive, motivational, and emotional deficits, as Peterson and Seligman (1984, p. 347) write, “This learned helplessness phenomenon has parallels with depression in people and has been proposed as a model of this psychopathology”. The major distinguishing factor is whether a person explains a bad event by an internal factor, with a consequent likely loss of self-esteem, or by an external factor, which will not affect self-esteem much. A third perspective on the psychological influences of dim perspectives is fight-or-flight behavior. Individuals, be it humans or animals, show polarized behavior if they are attacked. Either they reduce risks as much as possible by trying to escape or increase risks by attacking the aggressor. At the poker table both behaviors

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83 Although the therapeutic value of this coping strategy is widely maintained, the authors argue that it is not necessary to run through distress to cope with loss.

84 Ibid. p. 348.
appear functional, either taking all remaining money and leaving or changing to a more aggressive playing style.

Mano (1992) discusses the specific impacts on judgments of the two dimensions of distress, pleasantness and arousal. The first is a decisive factor for which decision strategy is used. People under distress, i.e. unpleasantness, employ simpler decision strategies. Additionally, arousal induces restrictions on attentional capacity. Taken together, judgments of distressed individuals are more polarized. They are willing to pay more for gambles and are also more likely than unaffected subjects to choose a certain amount over a gamble with less or equal expected value. Pleasantness and arousal combined impair decision-makers’ self-regulation so that there is less consideration of subjective utility and rational calculation (Leith and Baumeister 1996). The exact consequences depend on the specific mood—even among negative emotions not all share the same effects—and situation. Overall, there is consistent evidence that decision-makers’ motives change and decision contents are distorted and processed differently under affective states (Raghunathan and Pham 1999, pp. 56-58).

The psychological implications of distress on judgments lead us to the following

**Hypothesis 13** Distressed players will increase risk-taking in the game.

We restrict the hypothesis to situations where play is continued. Polarization of decisions suggests that at the other extreme players will leave the game preferring the sure amount to the uncertain prospect. However, as this is a binomial decision compared to the available action if the game is continued and other motivations can also cause a player’s session to end, we exclude this aspect from the hypothesis.

### 6.7 Judgments of Skill

Larkey, Kadane, Austin, and Zamir (1997, p. 596) write that “Differences in players’ skill are important determinants in player success in most real games such as poker, chess, basketball, business, and politics.”. They distinguish between three types of games in terms of skill. First, in pure chance games like lotteries, roulette, or craps players’ success is solely determined by a random device which cannot be influenced. Second, skill-chance games feature both a random device and elements which affect the probability of success requiring skill. Examples for this kind of games are poker, backgammon, or bridge. Third, pure skill games have no randomization at all. Success is determined by competing players’ skills. Chess, checkers, or go are games of pure skill. Sometimes it is possible to

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85Emphasis from the original text.
trade-off skill versus chance elements in games. In no-limit poker, for instance, a player could either bet all-in or fold pre-flop, thus excluding any skill elements based on additional information from actions or cards in later rounds. However, only few people prefer chance over skill where both are substitutes as Cohen and Hansel (1959) found when offering this trade-off to their subjects.86

Many skills are required for successful poker play. Parke, Griffiths, and Parke (2005) list the following transferable skills and abilities needed to be a good poker player: critical evaluative, numerical, pragmatism, interpersonal, problem-solving, goal orientation, learning, higher-order analytic and strategic, flexibility, face management or deception, self-awareness, and self-control skills. Despite this rather long list there is a debate in many states and countries whether playing poker for money involves skill elements and thus is a legal activity (Hannum and Cabot 2009). We assume that there is at least some skill involved in poker if the game is played repeatedly, i.e. repeating the identical randomization procedure of dealing cards in an independent sequence will eliminate unsystematic risks and bring out the systematic differences in skills. Consequently, we expect the presence of psychological biases affecting the players’ judgments of their own and others’ skills. Relevant ideas from the literature are discussed in the following sections.

6.7.1 Illusion of Control

In an often cited paper Langer (1975) defines an illusion of control as an expectancy of a personal success probability inappropriately higher than the objective probability would warrant.87 She finds experimental evidence for four skill-related factors which cause ill-founded confidence in individuals participating in a chance task. The four factors which, however, have not found unanimous support in the literature are choice, familiarity, involvement, and competition. Chau and Phillips (1995); Chau, Phillips, and von Baggo (2000) confirm the results for the influence of choice in computer blackjack. Players were more optimistic and increased bet sizes when they could choose their own strategies and had the ability to request extra cards. Dannewitz and Weatherly (2007) contradictingly report increased bet amounts as control over 5-card draw video poker decreased. For the second factor, familiarity, Ladouceur and Mayrand (1984) could not replicate the findings, as regular gamblers rather underestimated than overestimated future successes. The role of involvement is illustrated by studies of betting on dice where subjects are willing to risk more before the dice are rolled than after the throw (Strickland, Lewicki, and Katz 1966).

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86Sometimes gamblers think to be skilled or that they can develop skills by certain procedures (e.g. studying the program, checking the condition of the track for horse race betting) but do not perform better than a random strategy (Ladouceur, Giroux, and Jacques 1998). Hence in assessing skills one has to be wary of objective criteria.

87On the role of the illusion of control in gambling also see Griffiths (1990).
6.7. JUDGMENTS OF SKILL

Wolfgang, Zenker, and Viscusi (1984). Crapshooters also show the belief that “it is possible to control dice by verbal and non-verbal gestures, by words and actions.” (Henslin 1967, p. 319), a form of irrational involvement. The effect of involvement, however, is not undisputed as Ladouceur, Mayrand, Dussault, Letarte, and Tremblay (1984) could not find differences between subjects throwing dice themselves versus passive participation. With regard to competition early successes induce a skill orientation in the task, signaling whether or not it is controllable and whether or not the individual has that control (Langer and Roth 1975). Ladouceur, Gaboury, Dumont, and Rochette (1988) find that even infrequent wins are sufficient to evoke the illusion of control.

Other researchers of this phenomenon have found that there is no illusionary control if gambles are presented as a series where probabilistic outcomes can be thought of as long-run relative frequencies rather than separate instances where such a representation is less likely (Koehler, Gibbs, and Hogarth 1994). The effect also appears to affect pathological and problem gamblers differently than non-problem gamblers (Goodie 2005). They are less affected by control but show greater overconfidence and bet acceptance.

Investigating the implications of an illusionary control in online-poker two of the factors from Langer (1975), choice and involvement, are less interesting. Choices are symmetric between players, without choices there would be no game. Differences in involvement are unobservable from the play itself. Here, additional information like psychobiological data (heart-rate, sweating, etc.) would be needed. The effects of familiarity have been discussed under the topic of increasing expertise in section 6.5. Competition offers the most promising background because it is an overarching aspect in poker. To assess the effects of competition on the illusion of control Langer (1975, pp. 313-315) let subjects participate in a pure chance game. They were betting on drawing cards against one opponent (the experimenter’s confederate) where the higher card won. Thereby, the competitor either appeared as a confident dapper or as a shy and nervous schnook. It turned out that bets in games with the schnook were significantly larger. One must conclude that subjects thought they had more control while facing a seemingly less competent opponent.

In live poker some players might be using this aspect of the illusion of control to induce behavior in others which is favorable to their playing style. Examples are Billy “the Croc” Argyros who is wearing a hat in form of a crocodile and Marcel Lüske with sunglasses upside down, photographs of both are exhibited in figure 6.6. Of course, these habits may be out of sheer extravagance or for publicity reasons. The effect is also less likely in professional settings where players are well aware of each other’s skills.

\(^{88}\)Players in the dice game “craps”.

Online poker does not feature outward appearances but one could hypothesize that nicknames give some room for presenting a serious or not so serious attitude regarding the game. However, stack sizes are frequently seen as representative of a player’s skills. This is most obvious in cash games where usually a minimum and maximum buy-in exists. There players with a table stake above the maximum buy-in have been winners so far, those with a smaller stack than the minimum buy-in must have been losers in the current session. It is likely that players make inferences based on this information about their own skills relative to the others and between all other players. Hence we propose

**Hypothesis 14**  *Players with a relatively large bankroll are affected by the illusion of control. They act less conservatively.*

Extending the reasoning a bit further, not only the conduct within a game but also the selection of games in which to participate might be guided by the impression of control based on the relative wealth among individuals. However, the resulting effect is less clear. On the one hand, players with a small amount of money to invest would tend to avoid games with much larger bankrolls. On the other hand, those who have a fortune might be looking specifically for games with less prosperous players. Therefore, the overall effect concerning selection of games is undecided as in
Hypothesis 15 Players deliberately choose the games which they enter based on the information about players’ wealth so that participants in a game are not a random selection from the player population.

6.7.2 Taking the Fame, Shunning the Blame

In the previous section it has been discussed that the illusion of control boosts the individual’s perception of his skillfulness in performing a task, i.e. an increase in felt competence. Besides the resulting less conservative risk-taking behavior another effect of perceived competence has been identified in the literature: judged knowledge reduces ambiguity aversion. For events, that people think they know more about, probability weights and valuations of lotteries are higher (Keppe and Weber 1995). Ambiguity aversion is basically a comparative phenomenon. It is only relevant if a less ambiguous alternative or more knowledgeable individuals are available, or in the words of Fox and Tversky (1995, p. 599) “Thus, ambiguity aversion represents a reluctance to act on inferior knowledge, and this inferiority is brought to mind only through a comparison with superior knowledge about other domains or of other people.”. An explanation for the effect of felt competence is offered by Heath and Tversky (1991, p. 8) who argue that knowledgeable people may claim credit if they are right and take failures as an exception, whereas, those lacking competence cannot boast on being correct because they are guessing but are open to blame because they are ignorant. Trautmann, Vieider, and Wakker (2008) demonstrate that it is the fear of negative evaluation of others which increases ambiguity aversion if others are more competent and more knowledgeable.

In the poker setting, comparative ignorance will be relevant for several decisions. Those aware of inferior knowledge will try to get as much information as possible, preferring to play against known competitors and trying to see many hands, in order to reduce ambiguity. Players ignorant of their competence relative to others and those with superior knowledge are less likely to show ambiguity aversion.

How does the perception of skill and competence change if feedback via outcomes is available? Of course, there are also psychological biases in attributional styles. How favorable (wins) and unfavorable (losses) outcomes are seen is first of all a matter of personality. It has been shown that optimists are more resilient when faced with negative events (Corr and Gray 1996) and therefore, are more likely to keep to their decision-making strategies. Pessimists are more likely to make different judgments after a failure. There is also an egocentric bias in availability and attribution. One’s own actions are

89 Fox and Tversky (1995) summarize this explanation in calling their hypothesis comparative ignorance.
90 Cf. the discussion on initial learning under section 6.5.1.
remembered more easily and a greater proportion is attributed to oneself than to others than objectively justified (Ross and Sicoly 1979). At the same time attributions suffer from egotism, the tendency to put oneself in the most favorable light. In a competitive setting, like poker, winners tend to attribute their winnings to skill, while the loser sees the winner as lucky. And just from the other perspective the loser perceives his loss as bad luck, while the winner notes the loser’s inferior skills (Snyder, Stephan, and Rosenfield 1976). Hence, there is a distinct difference in the locus of control in the attributions of positive and negative outcomes (Brewin and Shapiro 1984). Attributions can be trained. Information on the factors determining causality (what is due to ability, what is due to chance) and controllability (by oneself, by others) help individuals to identify the determinants of the events and update their assessments accordingly (Luzzo, James, and Luna 1996). This is an important process as (perceived) skill is directly related to confidence in performing a task (Newman 1959).

6.7.3 Overconfidence

Closely related to both the illusion of control and the attribution bias is the psychological finding that people tend to be overconfident. There are two distinct varieties in which overconfidence can be observed. First, chances of success are systematically overestimated, and second, individuals assess their ability relative to others generally higher than statistically valid. The extent of overconfidence is significantly affected by the source of control, it is more prominent in internal (skill-based) situations than external (environment-based) settings (Howell 1971).\(^{91}\) Confidence per se is predominantly determined by the arguments for and against the decision, with insufficient consideration of the weight of the evidence (Griffin and Tversky 1992). An example is the prediction of success in graduate school on the basis of a letter of recommendation. Confidence in the prediction will mostly be driven by the content of the letter and less by the credibility of the writer, so that a warm letter from casual interaction will cause an overconfident prediction and vice versa.\(^{92}\) Gervais and Odean (1997) demonstrate in a model for traders how overconfidence is created based on the attribution bias in a dynamic environment. Successes are weighted more heavily than they are theoretically indicative of true ability. Stotz and von Nitzsch (2005) discuss both perspectives on overconfidence, overconfidence in one’s own knowledge, and overconfidence in one’s own abilities, and show that it is present in analysts’ perception of their forecast quality.

\(^{91}\)Brown and Bane (1975) attribute at least some of the overconfidence in skill tasks to the nonstationarity of outcomes.

\(^{92}\)Ibid. pp. 413–414.
6.7. JUDGMENTS OF SKILL

as the amount of information increases while it is reduced by increasing the perceived difficulty of the task (Peterson and Pitz 1988). People’s confidence is also subject to a hard-easy effect. In addition to mean confidence generally being higher than the share of correct answers, overconfidence increases with item difficulty (Gigerenzer, Hoffrage, and Kleinböting 1991). The second type of overconfidence is best described by the analysis of Svenson (1981) who asked students in the U.S. and in Sweden to assess their safety and skill in driving relative to the other participants. As it turned out most of the subjects thought they were better than average in both dimensions, with median judgments from the 60th to 90th percentile depending on dimension and nationality.

An extensively discussed effect of overconfidence and biased self-attribution is security market under- and overreaction (Daniel, Hirshleifer, and Subrahmanyam 1998). Investors overweight recent information and do not put enough weight on base rate data so that there are systematic price reversals in stocks (DeBondt and Thaler 1987). A portfolio invested in recent losers and short-selling recent winners has been shown to earn significant excess returns (DeBondt and Thaler 1985). Odean (1999) observes overconfidence as one of the sources for excessive trading volume in discount brokerage accounts. Overestimating the value of their information, investors adjust their portfolios too frequently.

Poker players are likely to exhibit both types of overconfidence. As almost all decisions in the game are based on some kind of prior information and estimates of the chances of success are likewise omnipresent, the effects of overconfidence in the value of information cannot be easily disentangled from other biases. Overconfidence in one’s abilities is a more clear-cut point to discuss. Assuming that skills are symmetrically distributed around the mean and that skill is directly correlated to performance, then, if there are no charges from the house, half of the players will actually be losing on average. Hence, one must have the confidence to belong to the half of the player population with abilities better than average to even start playing. Overconfidence will also manifest in players’ estimates of their expected performance which has to be assessed relative to their peers.

Hypothesis 16 Players will show overconfidence in assessing their abilities, expecting returns which are higher than those of the mean performance across players.

The assessment of one’s abilities relative to others has another direct application in poker games, namely, the selection of the opponents and gametype. More skillful and/or confident players may take on games for larger stakes in which they are likely to face

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93 Overconfidence in knowledge is linked to the topic of calibration in learning and is measured similarly; ibid. pp. 513-514.
94 If there is a rake confidence must be even higher; usual estimates are that only 10-20% of the players in online casinos are overall winners.
players who are better than those present at lower limit games. Of course, some players who are not so good might buy-in into high stakes games from wealth they earned somewhere else than poker, but it is reasonable to assume that the majority of players who are high-rollers have paid their dues on smaller limit games. If players systematically overestimate their abilities they will prefer to participate in games which are the worse choice for them in terms of risk and return.

**Hypothesis 17** *Players will move to higher limits too eagerly.*
Part IV

Testing for Psychological Biases in Poker
Chapter 7

Evaluation of Probabilities and Outcomes

You got to know when to hold’em, know when to fold’em,
Know when to walk away and know when to run.
Kenny Rogers (Singer-Songwriter), The Gambler

7.1 Preference to Play Against Long Odds

7.1.1 Is There a Long-Shot Bias? Method

In a poker game there are manifold situations where players have to evaluate small probabilities. With four cards of the same suit on the flop 9 of 47 unknown cards or roughly 19% will make a flush on the turn. Similarly, drawing to an open-ended straight$^1$ is successful in 8 of 47 cases, odds of 4.88 to 1 or approximately 17% of the time. These are calculable risks and most experienced pokerists handle them casually. Less transparent are the probabilities to actually win with a given hand. Even a straight is beaten by a higher straight, flush or better. Here, probabilities are obscured by uncertainty but a basic relationship can serve as an anchor. Assume that all players are equally skilled so that winning a hand is a completely random process. Then, with two players, each can expect to win half the hands (neglecting split pots). And more generally, with $N$ players in the game the expected fraction of wins becomes $1/N$.$^2$ At the same time the equalization method guarantees that there will be at least the $N$-fold amount of a player’s stake, and more if some players have already forfeited the hand.$^3$ This means that with increasing

$^1$Four cards in a row so that either a card above or below will complete the straight.

$^2$With 2 to 10 players at cash games this gives a basic range from 10% to 50%, not accounting for split pots which would slightly increase the values.

$^3$See section 2.3.3 on the equalization method.
participation in a pot, winning a hand becomes a long-shot situation, similar to betting on the underdog at the racetrack where a large amount can be won with a small probability of success.

All gametypes are searched for games with 4 to 10 attendants. Observations of players marked suspicious or artificial intelligence and with less than 1,000 hands in the database are dropped. For every action we determine how many players are already locked in the pot, i.e. they may not act again if the current player checks or calls and all subsequent players also neither bet nor raise.\(^4\) For example, with 6 players at the table the play on the flop goes check, bet, call, call, fold. Then for position 6 we see 3 players are locked in the pot, the one who bet and two who called. If the player calls, then position 1 (who initially checked) sees 4 players invested in the pot and may act. However, if he raised, the player in position 1 only faces 1 locked player (the raiser) but three may act again. All observations concerning the blind positions in the pre-flop phase are dropped as the decisions are extraordinarily tainted by the forced bets. Thus, the number of players already invested presents an estimate on the basic chances of success the decision-maker faces.\(^5\)

What we are interested in is the propensity of players to participate in games with varying probabilities of success. Therefore, the fraction of all action which forfeits participation, i.e. fold, is calculated for a given number of players invested in the pot. Additionally, the data is split by attendance due to different degrees of ambiguity and style. In a game with two players, holding a hand in the top half of the range is better than the expected hand of the other player, whereas, in a game with three players one has to wait for the top 25% of hands to expect the two other hands to be worse. Consequently, in games with more players play becomes necessarily tighter. This will give us data points, say, of the following form: in a game with 8 players where five players are locked in, the next action is fold X% of the time. In order to establish a sound estimate on the propensity to fold, only data points based on at least 100 observations are included.

Can a player facing a large pot not easily pay to draw to the best hand based on pot odds? Indeed, for drawing to, say, a flush which will be the best hand the more money in the pot the better. If this relationship holds, we will note an increasing average return for players participating in long-shots. However, whenever a player is paying for drawing based on pot odds to what he thinks is the best hand but is actually the second best

\(^4\)Equivalently, take the players who may act if the current player bets or raises and subtract the number of players who may act if he checks or calls.

\(^5\)It is only an estimate as most situations are not only uncertain but also ambiguous. Subsequent actions might change the setting. More players can enter the pot or some might quit after a raise.
hand, or if other players are drawing to an even better hand, this decision gives negative expected value. Therefore, we calculate the average return on participation in hands for any of the data points described in the paragraphs above. So, for example, the average amount won in hands for each action in a game with nine players where four players are already invested is calculated. For any action the final amount won/lost of the hand is used, so the same figure will be used several times for every action in the hand resulting in a weighted average return across the number of players invested. Here we drop all data points with less than 1,000 observations as the average amount won is not based on a binomial value like the fraction of fold but subject to substantial variance.

Case study 8:
The game is 50/100 Limit Hold’em. There are three players seeing the flop. Player Z is last to act and has \((T\spadesuit8\spadesuit)\). The board is \(J\diamondsuit9\diamondsuit6\diamondsuit\) so Z has an open-ended straight draw. X has bet, and Y has called, the pot has now accumulated $275. Z has to pay $50 to stay in the game for $275, pot odds of 5.5 to 1. So far it is reasonable to assume he is beaten at least by X or Y but might win with a straight. Odds of getting a straight on the turn are 4.88 to 1, so this is a good call, or is it not? The odds of getting a straight assume that Z has 8 outs, 4 queens and 4 sevens. But what if the other players are holding hands like \((QJ)\), \((QT)\), \((J7)\), \((87)\) which all include one of the required cards and might also be suited to give an opportunity to make a flush by the river. Additionally, there might be strong hands like \((JJ)\), \((99)\), \((66)\), \((J9)\), \((J6)\) or \((96)\) that could improve to a full house, a hand Z cannot beat. So Z has odds of 4.88 to 1 at best. So we might reason that Z has only about 7 outs to make his hand and his odds are 5.71 to 1 to make it on the turn, which is worse than pot odds and gives negative expected value if called. However, the situation might change if Z can get additional money from X or Y once he has made his hand thus improving the reward; a concept called implied odds. He also might raise now which is to semi-bluff. We will not further investigate these strategic opportunities here but summarize that a player deciding whether to stay in a pot with several invested others, faces considerable uncertainty in excess of the obvious pot odds he gets. Enough room to overweight low probabilities.

7.1.2 Overweighting of Low Probabilities. Results and Discussion

The propensity to give up on long-shot situations and the average return if a long-shot is pursued are summarized in figures 7.1 and 7.2. Inspection of the upper graphs reveals that from at least two locked players onwards, the fraction of fold decreases with an increasing share of players who are already invested. Nothing else can be expected by the typical shape of the probability weighting function. With three or more players the fundamental probability to win is 33% and less. A range for which overweighting has been observed not only at the racetrack but also in laboratory settings. This pattern is present for all limits

\footnote{We neglect opportunities like runner-runner tens and/or eights giving the best hand as these probabilities are faint and also apply to both opponents making their runner-runner hands.}
and all kinds of attendance, though there are few instances with many players in a pot for the high limit games. For games with higher attendance the lines are elevated compared to tables with less players, i.e. hands are folded more frequently than expected. Less hands are good enough to compete while increased ambiguity causes decision-makers to engage in less actions. To fold is more common at the more competitive 20/40 Limit and 50/100 Limit where up to about 40% of the actions are fold for certain instances (games with nine or ten players where less than two players are invested). Most lines end below a level of 10%, indicating, that only in a small fraction of the situations, where most of the other participants have entered a pot, the next player to act, will fold.

Typical as it is there is a name for this kind of behavior at the poker table; whenever all players jointly participate in a pot it is called a family pot. It can be argued that they behave like this in order not to disturb the coherence of the group or to socialize with all others (Schoonmaker (2000)). In brick and mortar games or private rounds this appears to be a valid argument. Online games, however, are far more competitive and socializing is a lower ranking goal.

Could the increased propensity to participate be a form of herding? Devenow and Welch (1996) mention three ways in which herding typically arises. First, there may be negative externalities, i.e. harm done to all who are not in the herd, or positive externalities, like additional information only available for those who participate. In a poker game neither form of externalities is present. Players folding after others have invested do not bear any further costs and have access to the same information as if they participated. Second, principal-agent problems, for example, a desire to protect or signal ones reputation, can induce herding behavior. Usually, there is no principal-agent relationship in poker so that this reason for herding does not arise either. Third, as individuals' actions reflect information about an achievable payoff, information cascades may develop. In contrast to investing at a financial market, the observable actions of other agents in poker do not reflect information about a common value object. Rather, we must conclude that any player who invests is expecting to earn positive expected value from having a hand of above average strength. Therefore, the use of prior information should actually reduce the propensity to participate. It appears that none of the three typical ways in which rational herding occurs applies to poker. Prechter, Jr. (2001) notes that unconscious impulses still might spur non-rational herding behavior. However, as poker is a game of competitive judgments, impulses will not be strong enough as to fully suppress decision-making based

\footnote{See Graham (1999) who discusses this aspect in detail for investment analysts.}

\footnote{If a player was backed by an investor bearing part of or all losses a principal-agent situation appears.}

\footnote{See Bikhchandani, Hirshleifer, and Welch (1998) for an extensive discussion. Nofsinger and Sias (1999) and Wermers (1999) for empirical analysis of investor herding.}
7.1. PREFERENCE TO PLAY AGAINST LONG ODDS

on the available facts.

In light of the low fraction of fold we can also rule out that only drawing hands are responsible for the pattern. Although, of course, more hands can reasonably pay to draw in games with more players and a consequently larger pot, it is highly unlikely that in all but one in ten cases the next action is up to the hand with the draw. To make a case in point, we see that at the 10/20 II Limit in games with 8 players whenever seven are already invested the eighth has never folded. This means that for at least 100 observations the player has to hold a hand which is better than the best one of seven others or drawing to one that is better than the ones of seven others. Obviously, here we have overweighting of low probabilities at the prospect of a large win.

The lower graphs in figures 7.1 and 7.2 show the effects of overweighting of low probabilities on returns. Although curves are similar in shape to those seen for the propensity to stop participation in the hand, two differences may be noted. First, as we restrict analysis to data points based on at least 1,000 observations, more extreme values are not populated, e.g. situations where eight players are already locked in the pot are not included at all. Second, as instances with many locked players occur relatively less frequent, there are fewer observations to even out the variance in returns, so that the ends of the curves flutter. Overall clear patterns can be observed for the 10/20 I, 10/20 II and 20/40 Limit. For the 10/20 III and 50/100 Limit there are outliers and too few games with many participants, respectively, preventing a distinct pattern. On a given graph the average amount won does not necessarily center around zero. Because money contributed to the pot by hands which are folded before showdown (all losses) will eventually be attributed to a player who stayed in the game, this so-called dead money is counted each time the to-be winner acts at later rounds, thus shifting the curve upwards. Hence it is not surprising that even for five invested players staying in the game shows positive average returns. More importantly, we see that decisions to enter this kind of situation result in a lower average return than setups with less participation. Furthermore, as it is more common to encounter a higher number of others already invested, if acting from a later position we would expect advantages from positional play as shown in section 5.3. However, despite the correlation between later position and facing decisions involving many opponents the lines are downward sloping from about two locked players onwards.

Summing up, we have seen that in poker games, like at racetracks, long-shots are overplayed. As proposed in hypothesis 1 situations which offer a large pot at a low probability of winning are disproportionately preferred. The overweighting of low probabilities causes increased risk-taking in this kind of situation.
CHAPTER 7. EVALUATION OF PROBABILITIES AND OUTCOMES

Figure 7.1: Propensity and Return on Participation per Number of Locked Players

For each starting limit gametype one of the top three graphs plots the fraction of all actions for a given number of players locked in the pot. Each graph is further split into separate lines for games with different attendance. Obviously, in a game with four players, in no way five players can already be locked in. The three diagrams at the bottom show the average return earned on hands whenever a situation was encountered with the specific number of players locked in. Note that this refers to the complementary actions of the graphs above. The same split by attendance and gametype is applied. Graphs for the advanced limits can be found in figure 7.2.

Average Return on Participation per Number of Locked Players

Propensity to Stop Participation in the Hand

10/20 II Limit

10/20 III Limit

10/20 IV Limit
7.1. PREFERENCE TO PLAY AGAINST LONG ODDS

Cf. figure 7.1. Here the plots for the advanced limits are shown.

Figure 7.2: Propensity and Return on Participation per Number of Locked Players II/II

Propensity to Stop Participation in the Hand

Average Return on Participation

Cf. figure 7.1. Here the plots for the advanced limits are shown.
7.1.3 “Last Hand!”: Risk-Taking Over the Course of a Session

Betting at the racetrack or in a casino will eventually have to end when the venue closes, whereas, online poker offers the opportunity to stay in the game continuously (see section 3.3.3). There, it is up to the individual to decide when to enter or leave the action. Looking for an impact of the upcoming end of a gambling episode, in line with Gluck’s Second Law and hypothesis 2, respectively, we have to take every player’s sessions on its own. From all human players with inconspicuous behavior and at least 1,000 hands we take all sessions with a minimum of 100 hands. Then every session is split into percentiles $p_s$ ($p_s \in [1, 100]$); for example, a session of 300 hands will be split into 100 groups with 3 hands each.

Risk-taking behavior in poker is captured by the two measures, looseness and aggressiveness as introduced in chapter 4. We collapse the data by percentile of session $p_s$, summing all amounts called $c_{p_s}$, bet $b_{p_s}$ or raised $r_{p_s}$, and counting the number of hands at least played to the flop $h_{fl, p_s}$ and the total number of hands $H_{p_s}$. Then we can calculate:

\[
L_{p_s} = \frac{h_{fl, p_s}}{H_{p_s}} \quad \text{and} \quad A_{p_s} = \frac{b_{p_s} + r_{p_s}}{c_{p_s} + b_{p_s} + r_{p_s}} \tag{7.1}
\]

Note that $L_{p_s}$ and $A_{p_s}$ have the same properties as a player’s looseness and aggressiveness. Both measures are influenced by the usual playing style of players who have hands for the particular value of $p_s$. As we are interested in changes of risk-taking behavior we adjust both measures for the mean playing style of the present players. This means that, for example, for the 40th percentile of hands played in a session player A for whom $L_A = .6$ has played 50 hands and player B with $L_B = .3$ played 100 hands, the mean playing style of the present players is $\frac{50}{150} \cdot .6 + \frac{100}{150} \cdot .3 = .4$. Assuming we calculated $L_{p_s}$ to be .3 then the deviation from the average players’ looseness $\hat{L}_{p_s} = -.1$. Similarly, we can calculate the deviation of aggressiveness from the average players’ aggressiveness at a given relative position $\hat{A}_{p_s}$, and both measures $\in (-1, 1)$. Although a player’s hands are included twice by this method, once in calculating the playing style for the player and once in calculating looseness and aggressiveness for a given $p_s$, the overlap is small and acceptable as the range of the independent variable stretches sufficiently.\(^{10}\)

\(^{10}\)As the player population is the same for all values of $p_s$ this shifts all $\hat{L}_{p_s}$ by a constant ($\hat{A}_{p_s}$, as well).

Results for the 10/20 I Limit are presented in figure 7.3. The left panel demonstrates that changes in the frequency of participation are unsystematic except for the last 10% of hands in a session when hands are entered more often. Looseness changes up to 2%points which is a significant but not substantial effect. Data plotted in the right panel tells us
7.1. PREFERENCE TO PLAY AGAINST LONG ODDS

Deviations of looseness (left graph) and aggressiveness (right graph) from average players’ looseness and aggressiveness, $\hat{L}_{ps}$ and $\hat{A}_{ps}$, are scattered for parts of a players’ session from the 1st percentile of hands played to the last (100th) percentile. Solid lines are linear fits. The example is for the 10/20 I Limit.

Figure 7.3: Risk-Taking Depending on Part of the Session

that at the beginning of a session decisions are rather passive but aggressiveness increases over the first 40% of the hands in a session. From then on no clear systematic changes can be seen. A likely explanation is that players joining a table have a good idea which risks to take but they are shy to increase betting amounts. As they are new to the situation and have not gathered much information about the others, the overall situation is comparatively ambiguous and risk-taking is conservative at first. Only when they are about to leave (which they usually know in advance and we know ex post) they take a shot and take risks more frequently. It is simply quite lackluster to announce (at least to yourself) to be playing the last hand and then fold it pre-flop.

Table 7.1: Linear Regressions on Risk-Taking During a Session

The linear regression results for deviations of looseness from average players’ looseness ($\hat{L}_{rp}$) and aggressiveness ($\hat{A}_{rp}$) over the course of a session split into percentiles of hands $ps$ are tabulated. The regression coefficients for $ps$, regression constants, their significance levels, and the $R^2$ of the regression are given. All values are rounded to three digits, only coefficients of $ps$ are rounded to three decimals of a percent as indicated.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Looseness, $L_{ps}$</th>
<th>Aggressiveness, $A_{ps}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>$P&gt;</td>
</tr>
<tr>
<td>10/20 I</td>
<td>.005%</td>
<td>.011</td>
</tr>
<tr>
<td>10/20 II</td>
<td>.007%</td>
<td>.010</td>
</tr>
<tr>
<td>10/20 III</td>
<td>.013%</td>
<td>.000</td>
</tr>
<tr>
<td>20/40</td>
<td>.022%</td>
<td>.000</td>
</tr>
<tr>
<td>50/100</td>
<td>.010%</td>
<td>.001</td>
</tr>
</tbody>
</table>
Graphical results are confirmed by statistical analysis for all game varieties which can be seen in table 7.1. All coefficients on the impact of the part of the session are significantly positive for looseness and aggressiveness. Effects on activity in volume are more pronounced than on frequency of play for all gametypes. The lower $R^2$s on the regressions of $\hat{L}_{ps}$ indicate that the relationship is not best described by a linear equation, the spike of increased looseness at the end of the sessions is evident from figure 7.3. So we cannot reject hypothesis 2, toward the end of their session players increase risk-taking.

### 7.2 All Things Relative ... to a Reference Point

#### 7.2.1 The Reference Point Effect. Method

In section 6.3.4 we proposed hypothesis 3 that the money brought to the poker table serves as reference point during a player’s session. Using data from the hand database we can search for the bankroll at the beginning of the first hand in every session of all players. We do this for all players who are not marked suspicious, are not artificial intelligence, and have at least played 1,000 hands. We do the latter to assure that the players take interest in the game and are motivated. For any calculation regarding a particular player we also need a sufficiently large number of hands to overcome the inherent variability of the game. In poker play 1,000 hands is still a small number, especially in today’s online poker where players easily accumulate several thousand hands in a couple of days or weeks. At the time of IRC poker, however, 1,000 hands is a comparatively broad sample for a single player. In comparison with other empirical research having subjects to play more than a thousand gambles is an effort rarely achieved. We exclude all sessions in which players started on a bankroll of exactly $1,000 as this is the initial endowment granted from the system.\(^\dagger\) Next we calculate the ratio of the bankroll at the beginning of every hand relative to the first hand in the player’s session, the hypothetical reference point. By doing so we get a representation of the relative position on the x-axis of the value function, which we denote by $rp$. For ease of analysis we round to intervals of .005. To illustrate, a relative position of 1.155 signifies that the player’s bankroll at the beginning of the current hand is 15.5% above his bankroll at the beginning of the session. Similarly, a value of 0.740 indicates that the player has fallen behind by 26%. Obviously $rp \in [0, \infty)$.

\(^\dagger\)Due to the buy-in required at higher limits this starting bankroll is only possible for the 10/20 limit games. These sessions will be analyzed later regarding house money effects.

In analogy to the procedure used to arrive at equation (7.1) data is collapsed by interval of the relative position to the reference point $rp$ and the following two measures
are obtained
\[ L_{rp} = h_{rp}^f / H_{rp} \quad \text{and} \quad A_{rp} = \frac{g^b_{rp} + g^c_{rp}}{g^b_{rp} + g^r_{rp} + g^c_{rp}} \] (7.2)

From this we calculate deviations from average players’ looseness \( \hat{L}_{rp} \) and aggressiveness \( \hat{A}_{rp} \) as described in section 7.1.3.\(^{12}\)

To test hypothesis 4 two separate linear regressions are run for \( rp \geq 1 \) and \( rp < 1 \). \( \hat{L}_{rp} \) and \( \hat{A}_{rp} \) are only included in the regression if they are based on more than 1,000 observations, i.e. hands, for the particular \( rp \). This is done to reduce the variability due to chance elements of the game and get more stable measures of risk-taking. The linear regressions also serve to test hypothesis 3. If the bankroll at start of the session serves as reference point then \( rp = 1 \) should be the common point of both regressions where play is normal, i.e. \( \hat{L}_{rp} = \hat{A}_{rp} = 0 \). The described method is a comparison between subjects and has one major drawback. If a single player exhibits the reference point effect and plays many more hands than other players who do not show risk-taking behavior influenced by a reference point, we will regress the behavior of the single player who dominates the sample. To overcome this situational bias a second method is used to increase the reliability of the results.

A dummy variable is created in the hand database which is set to 1 if \( rp > 1 \) and 0 if otherwise. Hands are collapsed by player and by value of the dummy variable. Then looseness and aggressiveness are calculated for each player according to the states of the dummy variable. In doing so, four values are generated \( L_{i,0}, L_{i,1}, A_{i,0}, \) and \( A_{i,1} \). If a player has less than 100 hands in any of the two conditions for either looseness or aggressiveness, he is excluded from further analysis.\(^{13}\) Consequently, we get a distribution of looseness and aggressiveness over players in each condition and can test on equality of means for both distributions. Formally we test whether
\[ \bar{L}_0 = \bar{L}_1 \quad \text{and accordingly} \quad \bar{A}_0 = \bar{A}_1 \]

where \( \bar{a} \) indicates means across players.

Means will be significantly different between the conditions if risk-taking behavior depends on players being ahead or having fallen behind. In this test all players carry weight invariant to the number of hands they have played.

---

\(^{12}\)Here a player’s hands are included twice, too. Once in calculating the playing style for the player and once in calculating looseness and aggressiveness for a given \( rp \). However, the overlap will be small and acceptable as players’ wealth varies, which is usual for poker bankrolls.

\(^{13}\)Here we have to use a smaller sample for each condition compared to the samples for each value in the linear regression as the hand populations are considerably smaller for a single player than for a given relative position for all players.
7.2.2 Risk-Taking in the Domains of Gains and Losses. Results

Figure 7.4 gives an impression on risk-taking behavior as it is related to a player’s current wealth compared to his initial wealth when he joined the game. A linear relationship of both measures of playing style and the relative wealth is evident if players have fallen behind, i.e. \( rp < 1 \). In contrast, no such relationship can be observed for gains. In both directions we see an increasing dispersion of behavior toward more extreme values of \( rp \).

For the 10/20 I Limit which is graphed in the figure both \( \hat{L}_{rp} \) and \( \hat{A}_{rp} \) show non-continuous behavior at \( rp = 1 \). This pattern cannot be observed at any other gametype.\(^{14}\) However, it appears that \( rp = 0 \) is or is close to the point where behavior is normal. Comparing results for looseness and aggressiveness the stronger changes in risk-taking behavior are evident for the latter.

Additional statistics on the other gametypes are summarized in table 7.2. For losses coefficients of \( rp \) are significantly positive at high levels of \( R^2 \) for all gametypes except the high limit indicating increased risk-seeking. Effects are stronger for aggressiveness than for looseness. For the high limit similar results are obtained if the regression is not split at the reference point but a single regression over the whole range is used. This is due to some outliers close to \( rp = 1 \) which cause the split regressions to be flat in both directions. By simply adding the constants of the linear regression and the coefficients of \( rp \) the estimate on decision behavior at the hypothetical reference point can be seen. These estimates are slightly negative but fairly close to zero indicating that behavior consistent with the average behavior is located somewhat lower than \( rp = 1 \). Regarding behavior when ahead, a linear relationship has little explanatory power, all \( R^2 \) are below .2. Coefficients on \( rp \) are not significant or close to zero. Similar to losses, coefficients and constants taken together indicate that non-deviant behavior can be found if players are somewhat below initial wealth.

In table 7.3 the results of the second test are summarized. Mean looseness and aggressiveness are significantly different between losses and gains for all gametypes. This supports the results obtained via the linear regressions. However, differences are small in absolute terms. For example, mean looseness for players who have fallen behind at the 10/20 I Limit is only higher by .008 than the average looseness of players who increased wealth above its initial amount. This underlines that only few observations can be found for the more extreme levels of wealth relative to the bankroll at start of the session and most play happens near the reference point.

\(^{14}\) As we have shown in section 6.3.4 the Arrow-Pratt measure of risk is not defined at the reference point for a typical value function. This is in line with the finding of abnormal behavior at this point.
Deviations of looseness (top panel) and aggressiveness (bottom panel) from average players’ looseness and aggressiveness, $\hat{L}_{rp}$ and $\hat{A}_{rp}$, are scattered for relative values of the current bankroll to the bankroll at start of the player’s session. The example is for the 10/20 I Limit.

Figure 7.4: Risk-Taking Depending on Starting Bankroll as Reference Point
Table 7.2: Linear Regressions on Risk-Taking Relative to the Reference Point
The linear regression results for deviations of looseness from average players’ looseness ($\hat{L}_{rp}$, panel A) and aggressiveness ($\hat{A}_{rp}$, panel B) depending on the bankroll relative to bankroll at start of the session $rp$ are summarized. The table is split in regressions for the domain of losses, $rp < 1$ (columns 2-6), and gains, $rp \geq 1$ (columns 7-11). The regression coefficient for $rp$ and its significance level ($P|t|$) are stated in columns 2, 3 and respectively 7, 8. The regression constant and significance level in columns 4, 5, 9 and 10 are the other statistics given. For the 50/100 Limit the results of the linear regression over the whole range of $rp$ are presented in rows marked 50/100b. All values are rounded to three decimals.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Losses, $rp &lt; 1$</th>
<th>Gains, $rp \geq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef. $P&gt;</td>
<td>t</td>
</tr>
<tr>
<td>Panel A: Looseness, $\hat{L}_{rp}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10/20 I</td>
<td>-.064</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 II</td>
<td>-.131</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 III</td>
<td>-.104</td>
<td>.000</td>
</tr>
<tr>
<td>20/40</td>
<td>-.081</td>
<td>.000</td>
</tr>
<tr>
<td>50/100</td>
<td>-.014</td>
<td>.338</td>
</tr>
<tr>
<td>50/100b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Aggressiveness, $\hat{A}_{rp}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10/20 I</td>
<td>-.164</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 II</td>
<td>-.181</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 III</td>
<td>-.152</td>
<td>.000</td>
</tr>
<tr>
<td>20/40</td>
<td>-.143</td>
<td>.000</td>
</tr>
<tr>
<td>50/100</td>
<td>-.027</td>
<td>.053</td>
</tr>
<tr>
<td>50/100b</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.3: Tests on Equality of Mean Risk-Taking Conditional on Reference Point
The mean values of looseness among players who have fallen behind $\bar{L}_0$ and who are ahead $\bar{L}_1$ are stated by gametype. The level of significance on a test of equality of means under the hypothesis that risk-taking is higher in the domain of losses is given in column 4. Analog figures for aggressiveness are stated in column 5-7. Values are rounded.

|        | $L_0$ | $L_1$ | $Pr(T>|t|)$ | $A_0$ | $A_1$ | $Pr(T>|t|)$ |
|--------|-------|-------|--------------|-------|-------|--------------|
| 10/20 I| .503  | .501  | .0007        | .481  | .477  | .0000        |
| 10/20 II| .523 | .515  | .0000        | .501  | .494  | .0001        |
| 10/20 III| .497 | .488  | .0000        | .484  | .478  | .0009        |
| 20/40   | .449  | .438  | .0000        | .537  | .524  | .0000        |
| 50/100  | .449  | .440  | .0019        | .627  | .607  | .0000        |
7.2.3 Changes in Risk-Taking. Discussion

As put in hypothesis 3 the money players bring to a poker table, their initial wealth, is a natural reference point to which they can easily compare current wealth. Consequently, whether they have won or lost so far during the session is a prominent fact and risk-taking behavior changes relative to the reference point. The finding that the reference point lies somewhat below the initial wealth can have several reasons. First, it might be a statistical artifact caused by the overlap between looseness and aggressiveness for a particular $rp$ and players’ average looseness and aggressiveness into which the same hands have been incorporated. Second, individuals might accept small losses due to variability in the outcomes of the game and first react by tighter and less aggressive actions. Only if losses accumulate does their behavior shift toward risk seeking, implying that the concave part of the value function ranges into losses, or equivalently, that the reference point is not initial wealth but initial wealth less a small amount. As the graphs in figure 7.4 illustrate, deviations are not symmetrical around 0 which supports the idea that the shift of the x-axis intercept to the left is caused by the calculation of the measures. We conclude that it is initial wealth which serves as a reference point for poker players.

In contrast to what has been deduced from a value function like the one in equation (6.3.4) and what constitutes hypothesis 4, the observed relationship between risk-taking and current wealth relative to initial wealth is only linear for losses but not so for gains. A number of reasons can be responsible for this effect. First, the value function could be linear for gains. Then players who accumulate winnings would reason along the linear function and no change in decision-making would occur. Second, as Arkes, Hirshleifer, Jiang, and Lim (2008) have shown, people adapt faster to a new level of wealth following gains than following losses. If such an adaptation happens after a win then we will also not see any change in looseness or aggressiveness. Third, the expected shift toward less risky actions could be offset by an increased propensity to consume. Readiness to gamble with gains will be discussed later in detail in section 10.2.

The fact that deviations of aggressiveness are stronger than changes in looseness can be expected in light of their constituents. On the one hand, looseness as measure of frequency of play is mainly determined by optimism or pessimism which is part of the concept of probability. On the other hand, aggressiveness measures willingness to gamble for more money and thus is primarily related to the concept of money or outcomes. Of course, both measures are not purely determined by either perception. In the present case, decisions are affected by the presence of outcome valuation relative to the reference point. Hence, we can expect larger shifts in the willingness to stake money, i.e. aggressiveness.
Why are poker players acting like this? We offer the following interpretation. Once they have made their first losses, additional losses do not cause as much harm but getting back to even adds a great deal, a relationship well captured by a convex value function for losses. For the other part, having won is good but winning more is still good. After having gained, poker players do not get risk averse. They either stay greedy, or take their winnings and leave, or play more to have more fun. Their value function might only be slightly concave or nearly linear and sometimes even convex for gains. But only rarely will they remain at the table and play less once ahead. In the next section we will investigate whether this pattern is harmful or beneficial to a player’s performance.

7.2.4 Trying to Get Even – Mostly in Vain

Employing the same dataset as in the section above we calculate $\mu_{rp}$ the average amount won per hand for a given relative position to the reference point. And similar to the method in the section above two linear regressions are run, for $rp < 1$ and $rp \geq 1$. In addition, the mean amount won per hand for every player is computed for the two conditions $\bar{\mu}_0$ and $\bar{\mu}_1$ (below or above the reference point) and tests on the equality of means are performed. The same requirements regarding inclusion of observations and number of data points as in section 7.2.1 are applied.

The prevailing effect of behavioral adjustments relative to the reference point is illustrated in figure 7.5. Roughly speaking, the graph is equal to the ones presented in figure 7.4 mirrored at the x-axis. Below the reference point, increased looseness and aggressiveness lead to losses, whereas, performance shows increasing variability with gains. In the vicinity of the reference point, winnings are the usual average. In section 5.3 it has been shown that increasing looseness is generally costly and higher aggressiveness can boost both risk and return. With regard to the effect of evaluation relative to a reference point, the influence of aggressiveness is rather detrimental as it goes parallel to an enhanced frequency of play. These effects are reinforced by the results from other gametypes which are summarized in tables 7.4 and 7.5. Interestingly though, the effects on returns are weaker in the regression results and even disappear in the t-tests for advanced limits. Note that the linear regression coefficient translates directly to the $ amount won, so that relative to the blind payments the effect is substantially reduced at 20/40 and 50/100 Limit. It seems that in these gametypes additional aggression is not necessarily disadvantageous.

7.2.5 Who is Affected Most?

In section 4.2 four distinct playing styles have been introduced. We use this classification and look for differences of the value function between individuals. Therefore, the linear re-
7.2. ALL THINGS RELATIVE ... TO A REFERENCE POINT

Table 7.4: Linear Regressions on Return Relative to the Reference Point
The linear regression results for mean amount won per hand ($\mu_{rp}$) depending on the bankroll relative to bankroll at start of the session $rp$ are summarized. The table is split in regressions for the domain of losses, $rp < 1$ (columns 2-6), and gains, $rp \geq 1$ (columns 7-11). The regression coefficient for $rp$ and its significance level ($P>|t|$) are stated in columns 2, 3 and respectively 7, 8. The regression constant and significance level in columns 4, 5, 9, and 10 and the $R^2$ of the regression in columns 6 and 11 are the other statistics given. All values are rounded to three decimals.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Losses, $rp &lt; 1$</th>
<th>Gains, $rp \geq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>$P&gt;</td>
</tr>
<tr>
<td>10/20 I</td>
<td>6.811</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 II</td>
<td>7.136</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 III</td>
<td>2.802</td>
<td>.006</td>
</tr>
<tr>
<td>20/40</td>
<td>3.478</td>
<td>.000</td>
</tr>
<tr>
<td>50/100</td>
<td>3.032</td>
<td>.548</td>
</tr>
</tbody>
</table>

Table 7.5: Tests on Equality of Mean Return Conditional on Reference Point
The mean values of amount won per hand among players who have fallen behind $\bar{\mu}_0$ and who are ahead $\bar{\mu}_1$ are stated by gametype. The level of significance on a test of equality of means under the hypothesis that return is lower in the domain of losses is given in column 4. Values are rounded.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>$\bar{\mu}_0$</th>
<th>$\bar{\mu}_1$</th>
<th>$\Pr(T&lt;t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 I</td>
<td>.079</td>
<td>.406</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 II</td>
<td>.010</td>
<td>.974</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 III</td>
<td>-.064</td>
<td>.894</td>
<td>.000</td>
</tr>
<tr>
<td>20/40</td>
<td>.092</td>
<td>-.120</td>
<td>.9830</td>
</tr>
<tr>
<td>50/100</td>
<td>.518</td>
<td>.102</td>
<td>.7420</td>
</tr>
</tbody>
</table>
The average amount won per hand for given relative positions to the reference point are plotted. The example is from the 10/20 I Limit variety.

Figure 7.5: Average Amount Won Relative to the Reference Point

Regression as in the two sections above is repeated. As the sample has to be split among the four styles (and the unclassified players) sample size is substantially smaller and we have to allow intervals of $rp$ with more than 100 observations. This necessarily increases the variability and only the results for the gametype with the largest sample (10/20 I Limit) are displayed in figure 7.6. There no clear pattern between the styles can be observed. For either style adjustments in decision-making due to relative judgments are present. Maybe the most pronounced effect can be seen for maniacs, and the least effect is for calling stations’ looseness. It has to be admitted that this interpretation is vague and we refrain from stating statistical results as these do not add much to the discussion.

How can the general changes in risk-taking, regardless of playing style, be explained in light of the value function? Splitting the value function in two branches, above and below the reference point, four distinct patterns of curvature could be distinguished, 1) risk aversion for gains and losses, 2) risk aversion for gains, risk seeking for losses, 3) risk seeking for gains, risk seeking for losses, and 4) risk seeking for gains and losses. But the relative pattern is only twofold. Either risk aversion is equal or more pronounced for gains compared to losses, or risk aversion is weaker for gains than for losses. Results

---

15 Remember that so far we restricted the analysis to values of $rp$ with at least 1,000 observations.
16 Here by risk aversion the whole spectrum is meant from risk seeking to risk aversion.
of the analysis at hand imply that the prevailing relative pattern is less risk aversion for losses than for gains, or equivalently that risk seeking is increased in the domain of losses. This pattern encompasses manifold value function, e.g. risk aversion for gains and risk neutrality for losses, risk seeking for gains and even more risk seeking for losses etc.

7.2.6 Conclusion

In this section we have seen that players care about whether they have won or lost in the ongoing session so far. Initial wealth serves as a reference point to which current wealth is compared. Once they have fallen behind players increase the frequency with which hands are played, but even more so, they show an increased tendency to bet or raise rather than call, underlining the intention to get even. Additional losses do not hurt as much as getting back to the starting bankroll is appreciated. All players, regardless of general playing style, show this behavior which is detrimental to returns on the players’ hands. In doing so poker players are no different to managers who rather invest in projects trying to break-even than in investments which performed well so far or the buyer of stocks who well remembers the initial purchase price and keeps his shares which dropped in value as readily as he bags the gains on stocks which appreciated in value.\footnote{See e.g. von Nitzsch (2002, pp. 110-120), Shefrin and Statman (1985), Odean (1998), Weber and Camerer (1998), Barber, Lee, Liu, and Odean (2007).}
Deviations of looseness (top panel) and aggressiveness (bottom panel) from average players’ looseness and aggressiveness, \( L_{rp} \) and \( A_{rp} \), are scattered for relative values of the current bankroll to the bankroll at start of the player’s session. The analysis is split by playing style. In both panels results for calling stations are shown in the top left graph, maniacs at the top right, rocks at bottom left, and sharks at bottom right. Axis are on the same scale for all graphs in a panel. The example is for the 10/20 I Limit.

Figure 7.6: Reference Point, Risk-Taking, and Playing Style
Chapter 8

Information Heuristics

The world is cruel, and the only morality in a cruel world is chance.
Unbiased. Unprejudiced. Fair.

Aaron Eckhart as Harvey “Two-Face” Dent in *The Dark Knight*

8.1 When Subjective Chances Improve – the Influence of Availability

8.1.1 How Availability Might Influence Play

In the discussion that led to hypothesis 5—hands frequently seen at showdown will influence future play—it was mentioned that hands frequently seen at showdown are an available cue for the players at the table. Building on this we search the history of games at a particular gametype for flushes or straights shown at showdown. We restrict the analysis to these types of hands as they offer particular ways to be overvalued by players. First, selection of starting hands might be biased in favor of any of the two 5-card types. Increasing the proportion of suited starting hands which are played emphasizes flushes, and connected starters similarly relate to straights.\(^1\) Second, in later phases individuals “targeting” at flushes or straights might also draw more often than optimal, thus allowing for potential biases due to availability. Given the presence of past flushes and straights we go on to calculate a moving sum over the last 50 games for each of the two types. For any given game we thus know how many flushes or straights a player will have seen

\(^1\)Pairs increase the probability of having (of course) a pair, two pair, three of a kind, a full house or four of a kind. As less than 6% of dealt hands are pocket pairs they are relatively rare and links appear rather weak.
at show downs) over the prior 50 hands, our independent variable. We do not account for players entering or leaving the game. Therefore, the variable is biased against the availability hypothesis as individuals will have seen, at the most, this number of straights and flushes during the ongoing session.

What we are looking for are changes in the frequency of straights or flushes at showdown depending on recently seen straights or flushes. In doing so it has to be considered that there is a fundamental difference in the probability that at least one player will make any of these hands in a 7-card hand, the more players are present. For example, as stated in table 2.6 a single player will make a flush about 3% of the time. If 10 two-card hands are dealt followed by a random five card board, at least one player will have a flush with a 16.4% probability. Therefore, data is split into series for different attendance (number of players at the table). At the same time only a high base-rate will result in a broad range of values for the independent variable. With many hands that will not go to showdown, only games with high attendance will provide sufficient observations for higher values of the independent variable. Hence, analysis is focused on games with 8, 9 or 10 players. As all other gametypes are mostly short-handed (6 or less players) the analysis is further narrowed to the 10/20 I Limit and 20/40 Limit varieties. An example of what the data looks like is illustrated in table 8.1.

Table 8.1: Excerpt of Basic Data for Tests on Availability
An excerpt of basic data used to test for effects of the availability heuristic at the 10/20 I Limit with ten players is presented. Flush and Straight are columns with binary values which are 1 if at least one flush or straight is seen at showdown, 0 if none is seen, and . if no showdown occurred. Flush50 and Straight50 is the sum of straights and flushes seen over the last 50 games as identified by the timestamp. Vertical dots indicate where games are omitted in the time series.

<table>
<thead>
<tr>
<th>Timestamp</th>
<th>Flush</th>
<th>Flush50</th>
<th>Straight</th>
<th>Straight50</th>
</tr>
</thead>
<tbody>
<tr>
<td>797224802</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>797224892</td>
<td>.</td>
<td>6</td>
<td>.</td>
<td>4</td>
</tr>
<tr>
<td>797224982</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>797226170</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>797226229</td>
<td>.</td>
<td>8</td>
<td>.</td>
<td>4</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>797226786</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>


\(^2\)For details on the derivation see Alspach (2000).
8.1. WHEN SUBJECTIVE CHANCES IMPROVE – THE INFLUENCE OF AVAILABILITY

8.1.2 Statistical Tests and Results

As the dependent variables are binary types, either there is a flush (or straight) or there is not, a logit regression can be used to test that the probability of seeing a flush (or straight) increases, the more these hands have been seen over the prior fifty games. The test results are put together in table 8.2. It is apparent that all except one of the coefficients are in the correct direction but only five are significant beyond the .005-level. Overall, significance is better for the 10/20 I Limit than the 20/40 Limit. Across the board Pseudo $R^2$’s are very low, indicating that not much of the variability in seen flushes or straights can be explained by availability caused by the occurrences in the last fifty games. This is not surprising as there is an underlying randomization due to the dealing of cards.

| Gametype | # Pl. | Coef. | $P>|z|$ | Pseudo $R^2$ | Coef. | $P>|z|$ | Pseudo $R^2$ |
|----------|-------|-------|--------|-------------|-------|--------|-------------|
| 10/20 I  | 8     | .006  | .143   | .000        | .007  | .039   | .000        |
| 10/20 I  | 9     | .005  | .179   | .000        | .011  | .000   | .000        |
| 10/20 I  | 10    | .012  | .000   | .000        | .009  | .000   | .000        |
| 20/40    | 8     | -.001 | .871   | .000        | .010  | .120   | .000        |
| 20/40    | 9     | .001  | .869   | .000        | .003  | .653   | .000        |
| 20/40    | 10    | .010  | .034   | .000        | .007  | .100   | .000        |

In order to even out the effects of randomization dependency is tested using contingency tables. The tables are $n \times 2$-dimensional whereby $n \in [0, 50]$ at most. Summary statistics are exhibited in table 8.3. With tens of thousands of observations from the database in the sample Cramér’s V is used in addition to Pearson’s $\chi^2$ to have a measure of association which accounts for sample size. From the degrees of freedom we can infer that over the last fifty hands at the most 19 flushes and 18 straights were seen at the 10/20 I Limit with ten players at the table, again underlining the low base-rate of the events and the random structure of the game. Of the twelve tests four $\chi^2$’s reach significance. However, Cramér’s V indicates weak association for all tests.

Third, it is tested whether on average more straights or flushes have been seen conditional on a straight or flush seen at the current showdown. To do so data is split condi-
Table 8.3: Contingency Table Statistics on Flush/Straight After Seen Flushes/Straights

Cross tabulation statistics per game variety and number of players at the table (columns 1 and 2) are tabulated. The table is split in statistics for flushes, columns 3-6, and straights, columns 7-10. For each table Pearson’s $\chi^2$, degrees of freedom $d.f.$, significance level $Pr$ and Cramér’s $V$ rounded to three digits are listed.

<table>
<thead>
<tr>
<th>Gametype</th>
<th># Pl.</th>
<th>Flush50 × Flush</th>
<th>Straight50 × Straight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\chi^2$</td>
<td>$d.f.$</td>
</tr>
<tr>
<td>10/20 I</td>
<td>8</td>
<td>19.247</td>
<td>15</td>
</tr>
<tr>
<td>10/20 I</td>
<td>9</td>
<td>10.817</td>
<td>14</td>
</tr>
<tr>
<td>10/20 I</td>
<td>10</td>
<td>38.763</td>
<td>18</td>
</tr>
<tr>
<td>20/40</td>
<td>8</td>
<td>7.436</td>
<td>12</td>
</tr>
<tr>
<td>20/40</td>
<td>10</td>
<td>15.824</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 8.4: T-test Statistics on Flushes/Straights Prior to Flush/Straight

This table shows the results of two-sample t-tests with unequal variances. Statistics relate to game variety and number of players at the table (columns 1 and 2). Differences between the average number of prior seen flushes over fifty games conditional on now seeing no flush at showdown $\text{Flush50}\mid 0$ and conditional on at least one flush appearing at showdown $\text{Flush50}\mid 1$ can be found in column 3; the significance of the t-statistic in column 4. Analog values for the analysis of straights are in columns 5 and 6. All values are rounded to three decimal places.

<table>
<thead>
<tr>
<th>Gametype</th>
<th># Pl.</th>
<th>$\text{Flush50}\mid 0 = \text{Flush50}\mid 1$</th>
<th>$\text{Straight50}\mid 0 = \text{Straight50}\mid 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Diff.</td>
<td>$Pr(T&lt;t)$</td>
</tr>
<tr>
<td>10/20 I</td>
<td>8</td>
<td>-.022</td>
<td>.070</td>
</tr>
<tr>
<td>10/20 I</td>
<td>9</td>
<td>-.019</td>
<td>.089</td>
</tr>
<tr>
<td>10/20 I</td>
<td>10</td>
<td>-.050</td>
<td>.000</td>
</tr>
<tr>
<td>20/40</td>
<td>8</td>
<td>.003</td>
<td>.564</td>
</tr>
<tr>
<td>20/40</td>
<td>9</td>
<td>-.004</td>
<td>.434</td>
</tr>
<tr>
<td>20/40</td>
<td>10</td>
<td>-.032</td>
<td>.017</td>
</tr>
</tbody>
</table>

Finally, we are trying to substantiate a linear relationship between availability and current choice. Therefore, for any given number of prior evidence, e.g. 10 flushes, the fraction of flushes at showdown is calculated (same for straights). Then linear regressions are run...
8.1. WHEN SUBJECTIVE CHANCES IMPROVE – THE INFLUENCE OF AVAILABILITY

with the relative share of flushes/straights as dependent variable and Flush50/Straight50 as regressors. There are significant linear relationships for both types of hands only for the 10/20 I Limit with 10 players. For most other examined settings coefficients are relatively small in contrast to the constant.

Table 8.5: Linear Regressions on Relative Share of Flushes/Straights
Relative share of flushes/straights seen at showdown is linearly regressed as predicted by the number of flushes/straights over the last 50 hands. The regression coefficients and its significance level ($P>|t|$) are stated in columns 3, 4 and respectively 8, 9. The regression constant and significance level in columns 5, 6, 10, and 11 and the $R^2$ of the regression in columns 7 and 12 are the other statistics given. All values are rounded to three decimals.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 I</td>
<td>8</td>
<td>.016</td>
<td>.069</td>
<td>.039</td>
<td>.561</td>
<td>.232</td>
<td>.001</td>
<td>.403</td>
<td>.133</td>
<td>.000</td>
<td>.050</td>
</tr>
<tr>
<td>10/20 I</td>
<td>9</td>
<td>.006</td>
<td>.143</td>
<td>.091</td>
<td>.007</td>
<td>.170</td>
<td>.006</td>
<td>.004</td>
<td>.114</td>
<td>.000</td>
<td>.436</td>
</tr>
<tr>
<td>10/20 I</td>
<td>10</td>
<td>.010</td>
<td>.012</td>
<td>.078</td>
<td>.059</td>
<td>.318</td>
<td>.006</td>
<td>.007</td>
<td>.121</td>
<td>.000</td>
<td>.395</td>
</tr>
<tr>
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<td>.077</td>
<td>.058</td>
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<td>.000</td>
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<td>.128</td>
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<td>.000</td>
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<tr>
<td>20/40 I</td>
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<td>.007</td>
<td>.162</td>
<td>.087</td>
<td>.013</td>
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<td>-.001</td>
<td>.541</td>
<td>.135</td>
<td>.000</td>
<td>.039</td>
</tr>
<tr>
<td>20/40 I</td>
<td>10</td>
<td>.002</td>
<td>.143</td>
<td>.104</td>
<td>.000</td>
<td>.184</td>
<td>-.002</td>
<td>.144</td>
<td>.144</td>
<td>.000</td>
<td>.184</td>
</tr>
</tbody>
</table>

8.1.3 Discussion and Conclusion

What have competitors been doing over recent years? Which products have been brought to market? Which types of projects have been implemented? In an economic context these or similar events are regularly screened and easily accessible for business managers, in other words, they are available. In the poker environment players are confronted with the usual strength of a hand seen at showdown. As estimates or evaluations in future choices are influenced through the availability heuristic, this information impacts on their judgment. At the poker table, increased emphasis is put on hands likely to increase to straights or flushes, and these types are more likely to be carried through to showdown. Building on analogy in the business context, management attention might focus on frequently seen product features or if cost-cutting projects are en vogue, managers’ estimates of the importance of this kind of project might show similar biases as those of poker players.

Although we observe effects stemming from availability in poker play, they are small in magnitude and frequently do not reach statistical significance. Furthermore, biases become smaller the more experienced players are. There are less significant instances at the
higher 20/40 Limit compared to the 10/20 I Limit. With an extended history of events to draw estimates from, it seems that the recency effect and additional traces in memory do not carry much extra weight in forming judgments. The results above should be seen in light of the evanescence of available cues in the randomized environment. Base-rates of the events are fairly low, with a maximum of less than 20 occurrences over the last 50 hands and the majority of data concentrated around frequencies of less than about five triggers. As there is not a constant set of decision-makers but players leave and enter the game, actual histories at an individual level will include sets with less than 50 games, thus additionally reducing the coverage.

We conclude that the availability heuristic is used at the poker table at least by some players in some situations. Its impact is strongest whenever an extraordinary cluster of events has been seen. As only occasionally information about the nature of events, i.e. types of 5-card hands, is revealed, the effect remains small.

8.2 Gambler’s Fallacy in Judging Card Randomization

8.2.1 Chance as a Self-Correcting Process? Method

During a poker session one regularly encounters comments such as “He cannot possibly have good cards again!” or “He has shown weak cards the last hand so he must be strong now!” These prototypical quotes illustrate how individuals perceive chance as a self-correcting process, they are subject to the gambler’s fallacy. However, in contrast to showing this bias in casino games like roulette, it can be costly at the poker table if following decisions are affected. In order to these effects we are looking for changes in the share of hands won conditional on a player’s card history.

First, using the ranking of starting hands derived from the all-in-equity as shown in table 5.1, we establish a distinction between high ranking and low ranking pocket cards. Based on this we split the population about equally in half and, counting from the worst to the best, low ranking starting hands are all hands ranked below 86th, and high rankings are at least the 86th of the 169 distinct 2-card-hands. The split is not exactly in half by base-rate frequency as there are more pocket pairs and suited hands which are relatively less frequent in the high rank group.
8.2. GAMBLER’S FALLACY IN JUDGING CARD RANDOMIZATION

an evaluation over the last ten hands for a medium-run analysis.

Second, data on hands for all unsuspicious, non-AI players with at least 1,000 hands is used. As we are looking for changes in the relative frequency of hands won, we cannot allow games with differing attendance in a single sample as base-rates of winning are dependent on the number of players. Thus we use the attendance for which the most data points are available, which is 10 except for the 50/100 Limit where most hands were played in two-player games.

Third, a dummy variable is used to separate hands won from hands lost. Then data is collapsed by the number of high ranking and low ranking cards seen while calculating the share of wins. By doing so we can compare the mean win rate conditional on a player having shown no cards, high ranking, or low ranking hole cards at the prior hand. For the series over the last 10 hands we use a linear regression to test the effect of the two indicator variables (high and low) on wins. Only if at least 100 observations could be found for a specific combination of values of both variables (e.g. 3 high ranks and 4 low ranks seen) this data point is used in the regression.

8.2.2 Short-Run Effects

The immediate effects of recent information on players’ cards on the probability of winning the current hand are displayed in table 8.6. It is apparent that generally a player who just participated in a showdown is more likely to win the hand. Values are higher for high ranking and low ranking cards in all varieties except the 50/100 Limit. The effect is substantially stronger for low ranking cards where the likelihood of winning increases by about 3.5-4.5% points, compared to roughly 1.5-2.0% for high ranks. Showing strong pocket cards at showdown is the more usual case with frequencies between two and four-times those of weak ranks. Overall, only about one in six to eight hands reaches showdown at all. The effects should be seen in light of the base-rate of wins if all players were equally skilled. This is slightly above 10% for games with ten players due to split pots, which we can also see from the weighted average share of wins in table 8.6, e.g. \[ \frac{0.985 + 2.988 + 1.1964 + 4.35 + 1.4541 + 1.35}{2.988 + 4.35 + 1.35} \approx 10.30\% \] for the 10/20 I Limit.\(^4\)

Is the difference between the 50/100 Limit and the other varieties an indication of an un-learning of the effect through experience? As the analysis for the high limit has been done for 2-player games and for all other limits 10-player games have been used,

\(^4\)There are also effects due to the exclusion of conspicuous or AI-players and players with less than 1,000 hands. For example, the weighted chance of winning in a 2-player game therefore becomes less than 50%. 
Table 8.6: Wins Conditional on Latest Cards Shown

For each gametype the relative frequency of hands won is reported by latest cards shown. For all varieties except the 50/100 Limit where 2-player games are used, samples are restricted to games with 10 players at the table. For these information on the last hand is categorized where either no cards, high ranking (Best 84 of 169 distinct starting hands by all-in-equity), or low ranking ones are shown. In parentheses next to the share of hands won the number of observations on which it is based is given in thousands. Values are rounded to four digits.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>None Shown</th>
<th>High Rank Shown</th>
<th>Low Rank Shown</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 I</td>
<td>.0985 (2,898k)</td>
<td>.1196 (435k)</td>
<td>.1454 (135k)</td>
</tr>
<tr>
<td>10/20 II</td>
<td>.0987 (531k)</td>
<td>.1187 (84k)</td>
<td>.1418 (27k)</td>
</tr>
<tr>
<td>10/20 III</td>
<td>.0981 (451k)</td>
<td>.1179 (67k)</td>
<td>.1435 (20k)</td>
</tr>
<tr>
<td>20/40</td>
<td>.0991 (1,739k)</td>
<td>.1150 (208k)</td>
<td>.1348 (46k)</td>
</tr>
<tr>
<td>50/100</td>
<td>.4970 (312k)</td>
<td>.4966 (42k)</td>
<td>.4935 (19k)</td>
</tr>
</tbody>
</table>

the same procedure is extended to all games from two to ten players at all limits. The results are tabulated in table 8.6. From this it can be concluded that experience is not the causal difference. Regardless of the limit, the same pattern across different levels of attendance is evident. For games with two or three players win-rates are close to the base-rate no matter which cards have been shown. However, the more players are at the table the larger the difference between wins conditional on high ranking or low ranking cards becomes in the prior hand. Thereby, the increase is larger for low ranks. The point of indifference appears to be somewhere between three and four players, or in other words base-rates of winning between about 25-33%. A pattern familiar from the probability weighting function.

What about the directions of the deviations? Bringing hypothesis 6 to mind the impact of recently seen weak starting hands can be well explained. The effect of seen strong hole cards, however, contradicts the hypothesis. So only the part of the hypothesis, the more weak hands a player has shown over the last hands, the more likely others will fold to his next hand, thus increasing his likelihood of winning, holds. This is the alternation bias of the gambler’s fallacy at work, after a weak hand a strong hand is expected so that fewer hands are deemed good against it, and consequently surrendering more hands than optimal. The bias is most pronounced in settings where players’ subjective weighting of probabilities is most distorted, i.e. games with many players where base-rates of winning are low. Take, for example, a game with ten players, here a player holding a hand of random strength will win approximately 10% of the hands. Now the gambler fallaciously expects him to hold a hand of above average strength as he has just shown a weak hand. Such a hand objectively gives him an expected chance of winning of roughly 12.5%. An

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5 Which can be calculated easily using tools like Pokerstove. [http://www.pokerstove.com](http://www.pokerstove.com).
8.2. Gambler’s Fallacy in Judging Card Randomization

Table 8.7: Wins Conditional on Latest Cards Shown by Attendance
See Table 8.6, here with additional samples for other attendances (column 2). Where likelihood of winning is less for either high rank or low rank shown compared to none shown, values are emphasized.

<table>
<thead>
<tr>
<th>Gametype</th>
<th># Pl.</th>
<th>None Shown</th>
<th>High Rank Shown</th>
<th>Low Rank Shown</th>
</tr>
</thead>
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<td>.2770 (59k)</td>
<td></td>
</tr>
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<td>.2127 (135k)</td>
<td>.2370 (56k)</td>
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<td>.1612 (186k)</td>
<td>.1914 (66k)</td>
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<td>.1150 (208k)</td>
<td>.1348 (46k)</td>
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</tr>
<tr>
<td>50/100 2</td>
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<td>.4966 (42k)</td>
<td>.4935 (19k)</td>
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<td>.2096 (2k)</td>
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<td>.1987 (2k)</td>
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<td>.1448 (4k)</td>
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</tr>
<tr>
<td>9</td>
<td>.1155 (19k)</td>
<td>.1279 (2k)</td>
<td>.1364 (&lt;1k)</td>
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</tr>
<tr>
<td>10</td>
<td>.1020 (14k)</td>
<td>.1076 (3k)</td>
<td>.1127 (1k)</td>
<td></td>
</tr>
</tbody>
</table>
increase of 2.5% points or by 25%. Our gambler will now adjust his behavior to account for this change by subjectively weighting, more precisely as discussed in section 6.3 and chapter 7 by subjectively overweighting, the likelihood that now is one of the 12.5% of the winning cases. In a game with only two players where equally skilled players expect to win 50% of the hands each, the same fallacious expectation yields an about 57.8% chance of winning, an increase of 7.8% points or by 15.6%. But every second hand the gambler is also dealt a hand above average. With two hands above average against each other chances of winning are back to 50%. In a game of ten, having a hand above average against one holding also a hand above average and eight holding random hands, each will win about 12.1% of the hands. So, two factors drive the difference across attendance. First, every players’ individual hand has a much larger impact, the less players are present. Therefore, the focus on one’s own cards is increased and the bias becomes less relevant. For example, in a game with two, if dealt a hand above average this is likely to be played regardless of the gambler’s fallacy, and if dealt a hand below average, it should usually be folded (except for bluffing) so that play will be no different after having seen a low ranking card with the other player. Second, in games with more players, changes in expected probabilities of winning will be overweighted.

Table 8.8: Wins Conditional on Current Cards Shown

This table presents the probability of winning conditional on the ranking of current cards. Split pots and hands of non-human, conspicuous players with less than 1,000 hands are excluded. Other definitions are as in table 8.6.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>None Shown</th>
<th>High Rank Shown</th>
<th>Low Rank Shown</th>
</tr>
</thead>
<tbody>
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<td>.3940</td>
</tr>
<tr>
<td>10/20 II</td>
<td>.0331</td>
<td>.4536</td>
<td>.3915</td>
</tr>
<tr>
<td>10/20 III</td>
<td>.0362</td>
<td>.4613</td>
<td>.3984</td>
</tr>
<tr>
<td>20/40</td>
<td>.0478</td>
<td>.4809</td>
<td>.4248</td>
</tr>
<tr>
<td>50/100</td>
<td>.5006</td>
<td>.4984</td>
<td>.4300</td>
</tr>
</tbody>
</table>

Now why is there an increased likelihood of winning after having shown a high ranking 2-card hand? As this is the more usual case, the players’ attention might not be drawn to the ranking of the cards but to whether it was a winning hand or not. Finally, the player has been skillful at selecting the right hand to invest in. The probability of winning (an amount greater zero, thus excluding split pots where no dead money is involved) in the current hand conditional on the showing of cards is summarized in table 8.8 for the same selection of games and attendance as in table 8.6. Not surprisingly, the highest success rate is achieved by high ranking 2-card hands. We preliminarily conclude that for instances where high ranking cards have been seen at showdown, the focus is on winning/losing rather than card randomization. In this case, a repetition bias is present instead of the
alternation bias. Typical quotes might be “He is on a run. I better not challenge him now.”, “Luck is going right his way.”, etc., reasoning indicating a belief in the “Hot Hand”, which we will investigate in detail in section 8.3.

8.2.3 Medium-Run Effects

In the section above the short-run or immediate reactions after the revealing of cards have been shown. In this section we look at the medium-run or latent effect of a hand history over the last ten hands. Other than in the last hand only, this horizon can include both high and low ranking hands. Therefore, a simultaneous regression of both factors is required, and table 8.9 presents the according statistics. All coefficients are highly significant and positive as expected in light of the results from tables 8.6 and 8.7, except in the 2-player game at the high limit. More interestingly, the coefficients are lower than the immediate effects indicating that it is essentially an immediate phenomenon which fades over a few hands. But with coefficients from about half to four-fifths of the short-run changes, two or more instances over the last ten hands will boost the effect above the level observed for the most recent card only. Both negative and positive recency appear to become more urgent as sequences accumulate.

Table 8.9: Wins Conditional on 10 Latest Cards Shown

Fraction of wins is regressed on the number of high and low ranking cards shown over the last 10 hands. The regression coefficients and significance levels ($P>|t|$) are stated in columns 2 to 5. The regression constant and significance level in columns 6 and 7, the adjusted $R^2$ of the regression and the number of observations in columns 8 and 9 conclude the statistics. All values are rounded. Other definitions are as in table 8.6.

| Gametype | Cons. P>|t| | High Coef. P>|t| | Low Coef. P>|t| | Adj. $R^2$ | N |
|----------|-------|---------|---------|---------|-------|------|
| 10/20 I  | .0971 | .000    | .0084   | .000    | .0174 | .000 | .8013 | 49 |
| 10/20 II | .0937 | .000    | .0058   | .016    | .0195 | .000 | .5704 | 37 |
| 10/20 III| .0845 | .000    | .0112   | .000    | .0193 | .000 | .8085 | 36 |
| 20/40    | .0748 | .000    | .0125   | .000    | .0287 | .000 | .9103 | 31 |
| 50/100   | .4973 | .000    | -.0040  | .025    | .0108 | .000 | .5663 | 28 |

8.2.4 Conclusion

In this section we have seen that decision-makers are influenced by the recent history of information they get on the nature of underlying strength of their opponent’s cards. If in an unusual fashion low ranking investments are carried through to the end, they fall for the gambler’s fallacy expecting that a strong hand is due for the following hand. Consequently, precautious actions are taken which result in an increased probability of winning for those having shown the low ranks. In case a strong hand is presented a
positive recency effect can be observed; the phenomenon of winning or losing streaks is further discussed in the following section.

8.3 Winning Streaks – Playing with the “Hot Hand”

Looking for evidence on hypothesis 7—players show more and longer streaks of wins (losses) than warranted by their general performance—randomness in the occurrence of wins and losses has to be analyzed statistically, but as Rapoport and Budescu (1992, p. 353) state

“Because randomness is an unobservable property of a stochastic process, anything less than an infinite series may legitimately not be representative of the long-term output of such a process. It is, therefore, impossible to verify that any given sequence of finite length is, or is not, random.”

or stripped by Scott Adams as shown in figure 8.1 there is no definite way to test for randomness but statistical tests can give an indication on the probability that the process is random.


Figure 8.1: Dilbert on Randomness

Therefore, we employ procedures in the style of Gilovich, Vallone, and Tversky (1985) who were looking for the Hot Hand in Basketball, i.e. analysis of runs, tests on stationarity and series of conditional probabilities. As usual we restrict the analysis to non-suspicious, human players with at least 1,000 hands played in a particular limit.

8.3.1 Run Tests

We use time series data from specific gametypes and separate samples by number of players in the game because of the fundamentally different base-rates of winning. Coding the
events of gaining an amount greater zero as 1 and all other events as 0 each players winning record is converted to a series of binary events. If individuals occasionally have the Hot Hand counting consecutive wins and losses as a run will show that wins (and losses) cluster together so that there are fewer runs in the sequence than expected by chance. For instance a series 011000110001011 contains eight runs, 0 — 11 — 000 — 11 — 000 — 1 — 0 — 11.

For each gametype and attendance a player is tested if his record covers at least 100 observations for the particular kind of setting. We test at most 500 players per setting to keep computation times acceptable. Table 8.10 displays the number of individuals for whom the run test is significant at least at the .05 level. On the basis of the results we cannot reject the null that the series are random. There is no apparent pattern across gametypes or attendance. The number of individuals for whom the run test is significant is not greater than what can be expected given the targeted level of significance. Moreover, we have not even accounted for deviations in contradiction to streaks, therefore, the actual number of significant tests would have to be reduced even further. Thus, on the basis of the number of runs we do not find any evidence in favor of the Hot Hand.

Table 8.10: Run Test Results by Gametype and Attendance
By gametype and attendance the results of run tests on players’ records on wins/losses are tabulated. For how many of them the run test has been found significant better than the .05 level is reported regardless of direction of deviation and how many individuals have been tested is stated as second number (a maximum of 500 is allowed).

<table>
<thead>
<tr>
<th>Gametype</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 I</td>
<td>22/500</td>
<td>30/500</td>
<td>20/500</td>
<td>26/500</td>
<td>32/500</td>
<td>25/500</td>
<td>25/500</td>
<td>24/500</td>
<td>26/500</td>
</tr>
<tr>
<td>10/20 II</td>
<td>20/500</td>
<td>32/500</td>
<td>23/500</td>
<td>33/500</td>
<td>17/500</td>
<td>23/500</td>
<td>22/500</td>
<td>15/500</td>
<td>19/500</td>
</tr>
<tr>
<td>10/20 III</td>
<td>14/226</td>
<td>14/285</td>
<td>15/288</td>
<td>14/334</td>
<td>23/481</td>
<td>22/500</td>
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<td>29/500</td>
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<tr>
<td>20/40</td>
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<td>26/500</td>
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<td>18/500</td>
</tr>
<tr>
<td>50/100</td>
<td>12/283</td>
<td>19/302</td>
<td>21/290</td>
<td>14/267</td>
<td>11/220</td>
<td>7/175</td>
<td>7/132</td>
<td>3/68</td>
<td>—</td>
</tr>
</tbody>
</table>

8.3.2 Conditional Probabilities
As a second test of randomness, probabilities of winning conditional on prior wins and losses are calculated. Records of players with at least 100 hands played in a gametype/attendance combination are used and probabilities up to four prior wins or losses, respectively, are consulted. Picking up on the example from above the probability of winning conditional on one prior win is obtained by those numbers marked with a hat from 011000110001011 and is about 43%. From the overall series we can compute the base-rate of a player winning a hand. In this case, seven of fifteen hands have been won
giving a base-rate of about 47%. If there were streaks in the sequence we would expect the conditional probabilities to be larger the more prior wins have accumulated, which is not the case in the example.

The probability of winning conditional on prior losses (from -4 to -1), unconditional (0), or after wins (from 1 to 4) is graphed for the player population in the sample in a box-plot. The upper bounds of the box are the 3rd quartile, the line in the middle is the median, the lower bounds are the 1st quartile. The upper/lower whiskers represent values which are smaller/larger or equal to the quartile + 1.5-interquartile ranges. Dots are individual players with (conditional) probabilities outside these measures for the population.

Figure 8.2: Conditional Probabilities of Winning in 10/20 I Limit Four-Player Games

An example of the results is pictured in figure 8.2 for four-player games in the 10/20 I Limit. One can easily see that probabilities center around 25% the mean win-rate for this kind of game. Moving to longer streaks, either of wins or losses, conditional probabilities between players show increased variance, most naturally as the number of observations for any player is much smaller for the extremes. If there was positive recency in the data, means would be increasing from left to right, which they are not. Similar results appear for other combinations of gametype and number of players in the game so that we cannot reject the randomness hypothesis based on conditional probabilities.
In section 8.2 the presence of both repetition bias and alternation bias has been shown. If both biases are present in the win/loss records of a particular player, there will be episodes where wins/losses cluster due to the repetition bias but others where streaks are short caused by the alternation bias. Consequently, excessive long and short runs will counterbalance so that testing for the number of runs could miss the point. Therefore, we are looking for a local deviation from randomness by splitting series in chunks of five consecutive hands and counting the number of wins within the chunks. Continuing the example from above we would get three sets of the following kind, 01100 = 2, 01100 = 2, 01011 = 3. If chunks are drawn from a random series, sets will be distributed based on the binomial distribution, i.e. the share of sets should be \( \binom{5}{k} (47\%)^k (1 - 47\%)^{1-k} \) where 47% is the percentage of hands won in the record and the number of wins in a set is \( k = 0, 1, \ldots, 5 \). Using this we can calculate the expected number of chunks for each number of wins within the chunks given the length of the player’s record.

We test all players with at least 50 sets on a \( \chi^2 \)-test of independence between actual and expected distribution of sets up to a maximum of 500 players for a gametype/attendance combination. Results are displayed in table 8.11. As only very few tests reach significance we also cannot reject the randomness hypothesis based on the stationarity procedure.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 III</td>
<td>14/109</td>
<td>1/93</td>
<td>0/90</td>
<td>0/97</td>
<td>1/166</td>
<td>0/247</td>
<td>4/328</td>
<td>3/464</td>
<td>3/500</td>
</tr>
<tr>
<td>20/40</td>
<td>8/500</td>
<td>3/500</td>
<td>0/500</td>
<td>1/500</td>
<td>1/500</td>
<td>6/500</td>
<td>2/500</td>
<td>3/500</td>
<td>1/500</td>
</tr>
<tr>
<td>50/100</td>
<td>3/223</td>
<td>0/240</td>
<td>0/212</td>
<td>0/156</td>
<td>2/108</td>
<td>0/67</td>
<td>0/37</td>
<td>0/16</td>
<td>—</td>
</tr>
</tbody>
</table>

However, as the number of observations for a particular player is fairly small and the probability of observing a set with four or five wins approaches very small probabilities quickly as attendance increases, we might still have an excess of clusters in the population.

\(^6\)Note that the tails of the distribution get flat quickly. With a 50% chance of winning, e.g. in a two-player game, the probability of winning five hands in five is 3.125%. In a three-player game with 33% chance of winning, the same occurs with a probability of about 0.391%.
as a whole. For example, a player with an overall win-rate of 33% and 300 hands played is expected to have won five hands in about one set. If he shows two such sets this will not cause the $\chi^2$-test to become significant. However, if the number of sets with many wins is larger than expected for most players, it might be an observable pattern. Hence for each gametype/attendance combination and by number of wins in a set we count the players who actually have more (or less) instances of the kind than expected. Count data is presented in table 8.12. By close inspection a pattern becomes evident. Moving from games with few players to the right at first too few sets with high numbers of wins are more common, but then, gradually relatively more players show an excess of sets with many wins. From four players onward there are always more players with more than expected sets of four or five wins than less, than expected. As the expected shares for these extremes are marginal only a few instances are sufficient to cause this pattern.

8.3.4 Discussion and Conclusion

Overall series of wins (losses) may very well be generated by a random process, in contradiction to hypothesis 7. But in light of the evidence from the preceding section 8.2 and the pattern seen in table 8.12 there appear to be few instances in the records favoring streak shooting. However, they happen so rarely that they do not disturb the overall randomness.

But why then is the belief in the Hot Hand so popular? Maybe individuals are not observing other players’ streaks the way we do in the above tests. At least one figure is difficult to obtain at the poker table, other players’ base-rates of winning. One might have a rough idea about one’s own share of wins, but for all others, only sample data is available which in most cases will be too small to allow for adequate inferences of the true rate of winning; especially if account is taken for different attendance at the game. Therefore, during play streaks might be most naturally seen in light of average win-rates, i.e. about 50% in two-player games, 33% for three players etc.. But then as some players will be better and some worse than average, the better player is bound to have more streaks than warranted by the average win-rate. Consequently, belief in the Hot Hand might turn out to be a neglect of the base-rate for the individual player. A player winning 60% of his hands will have more streaks than expected if he is thought to be winning 50%, so that indeed he is on streaks though not only occasionally but systematically in line with randomness and his better play. How players improve decision-making over time so that they achieve such a good performance is the topic of the next chapter.
### Table 8.12: Stationarity Test Results by Gametype and Attendance For the Player Population

By gametype and attendance the count of players for whom at a given number $k$ of wins within chunks of five consecutive hands is more (columns marked “>”) or less (“<”) than expected by chance based on their overall win-rate. For better overview the pairwise larger group is marked with an asterisk.

<table>
<thead>
<tr>
<th>Type</th>
<th>$k$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
<th>8</th>
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<tbody>
<tr>
<td>10/20 I</td>
<td>0</td>
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<td><em>550</em> 226</td>
<td><em>586</em> 314</td>
<td><em>599</em> 418</td>
<td><em>722</em> 568</td>
<td><em>833</em> 702</td>
<td><em>959</em> 865</td>
<td>1,045 1,090*</td>
<td>973 1,482*</td>
</tr>
<tr>
<td>1</td>
<td><em>461</em> 173</td>
<td>387 389*</td>
<td>361 539*</td>
<td>439 578*</td>
<td>519 771*</td>
<td>627 908*</td>
<td>790 1,034*</td>
<td>955 1,180*</td>
<td><em>1,256</em> 1,199</td>
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</tr>
<tr>
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<td>306 328*</td>
<td>274 502*</td>
<td>366 534*</td>
<td>433 584*</td>
<td>617 672*</td>
<td>762 771*</td>
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<tr>
<td>3</td>
<td>192 441*</td>
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<td><em>208</em> 205</td>
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</tr>
<tr>
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<td>137 211*</td>
<td>138 200*</td>
<td>172 185*</td>
<td>174 239*</td>
<td>207 261*</td>
<td>229 281*</td>
<td>366 379*</td>
<td></td>
</tr>
<tr>
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<td>173 212*</td>
<td>165 183*</td>
<td><em>170</em> 167</td>
<td><em>191</em> 166</td>
<td>200 211*</td>
<td><em>242</em> 222</td>
<td><em>288</em> 220</td>
<td><em>461</em> 275</td>
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<td></td>
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<tr>
<td>3</td>
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<td>156 209*</td>
<td><em>179</em> 168</td>
<td><em>171</em> 152</td>
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<td><em>208</em> 118</td>
<td><em>230</em> 94</td>
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<td><em>379</em> 118</td>
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<tr>
<td>4</td>
<td>134 252*</td>
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<td><em>184</em> 77</td>
<td><em>139</em> 32</td>
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<td><em>98</em> 8</td>
<td><em>95</em> 5</td>
<td><em>91</em> 1</td>
<td><em>108</em> 4</td>
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<tr>
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<td><em>10</em> 0</td>
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<tr>
<td>10/20 III</td>
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<td><em>67</em> 25</td>
<td><em>59</em> 31</td>
<td><em>58</em> 39</td>
<td><em>106</em> 60</td>
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<td><em>246</em> 218</td>
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<tr>
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<td>60 106*</td>
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<tr>
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<td>37 53*</td>
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<td>120 126*</td>
<td>161 164*</td>
<td><em>249</em> 209</td>
<td><em>364</em> 217</td>
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</tr>
<tr>
<td>3</td>
<td>34 74*</td>
<td>38 55*</td>
<td><em>45</em> 44</td>
<td><em>51</em> 44</td>
<td><em>94</em> 63</td>
<td><em>123</em> 82</td>
<td><em>160</em> 73</td>
<td><em>218</em> 106</td>
<td><em>316</em> 105</td>
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</tr>
<tr>
<td>4</td>
<td>40 68*</td>
<td>44 46*</td>
<td><em>46</em> 23</td>
<td><em>49</em> 8</td>
<td><em>61</em> 10</td>
<td><em>61</em> 7</td>
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<td><em>14</em> 0</td>
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<td><em>435</em> 401</td>
<td><em>417</em> 413</td>
<td>417 480*</td>
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<td>406 452*</td>
<td>408 482*</td>
<td>468 680*</td>
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<td>345 398*</td>
<td><em>358</em> 239</td>
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<td><em>246</em> 14</td>
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</tr>
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<td>5</td>
<td>251 373*</td>
<td><em>257</em> 60</td>
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<td><em>44</em> 0</td>
<td><em>20</em> 0</td>
<td><em>9</em> 0</td>
<td><em>9</em> 0</td>
<td><em>18</em> 0</td>
<td></td>
</tr>
</tbody>
</table>

By gametype and attendance the count of players for whom at a given number $k$ of wins within chunks of five consecutive hands is more (columns marked “>”) or less (“<”) than expected by chance based on their overall win-rate. For better overview the pairwise larger group is marked with an asterisk.
Chapter 9

Improving Play

Listen, here’s the thing. If you can’t spot the sucker in the first half hour at the table, then you are the sucker.

Matt Damon as Mike McDermott in *Rounders*

9.1 All Beginnings Are Difficult

Though we cannot track players’ experience off the record of our database, we can identify the point in time when each individual started playing on a particular channel. This allows us to compare playing styles between early sessions and when players have gained experience with IRC poker.

9.1.1 Method

In order to segregate the effects of early learning episodes from experienced decision-making players’ time series data is used. A dummy variable separates first sessions (dummy set to 1) from all other sessions (0) while the usual restrictions are applied regarding players.\(^1\) Amounts called, bet or raised and the relative frequency the flop is seen give a player’s looseness and aggressiveness under both conditions. So for every player \(i\) four values are generated, \(L_{i,0}, L_{i,1}, A_{i,0},\) and \(A_{i,1}.\) The differences between the two conditions \((\Delta L_i = L_{i,1} - L_{i,0}, \Delta A_i = A_{i,1} - A_{i,0})\) capture changes in risk-taking for each player due to novelty of the environment.\(^2\) Combining the changes in rate and volume of play shows the overall shift in playing style. For example, if both values are negative the individual is more conservative at first (like a rock) and becomes looser/more aggressive in later

---

\(^1\)On the technical definition of a session see section 3.3.1.

\(^2\)As both \(L\) and \(A \in (.01,.99)\) it follows that \(\Delta L\) and \(\Delta A \in (-.98,.98).\)
sessions (shift toward maniac). As style is measured in two dimensions, four categories of
directional shifts in risk-taking behavior may be observed. Additionally, if risk-taking is
dependent conditional on the effective experience, means of the distributions of looseness
and aggressiveness across players will be significantly different, so the null is
\[
\bar{L}_0 = \bar{L}_1 \quad \text{and accordingly} \quad \bar{A}_0 = \bar{A}_1
\]
where bars indicate mean values.³

9.1.2 Results

Test results are summarized in table 9.1. Therein the fundamental difference between the
limits as seen in table 4.2 are reflected. Already in the first sessions that are played on
the advanced limits looseness is lower and aggressiveness (equal or) higher compared to
the low limit varieties. For all limits substantial step-ups in aggressiveness are apparent.
In contrast to this, shifts in looseness are small and mostly insignificant. The majority
of players shifts from a tight/passive or loose/passive to a more aggressive style as more
experience is gained.

Table 9.1: Test Results on Risk-Taking Between First and Later Sessions
Mean values of looseness \( \bar{L}_1 \) in first sessions and later sessions \( \bar{L}_0 \) are stated by gametype.
The level of significance on a test of equality of means under the hypothesis that risk-taking is
increased in later sessions is given in column 4. Analog figures are stated for aggressiveness in
column 5-7. Columns 8-11 list the count of players with a particular shift in playing style, where
↗ indicates an increase in both dimensions, ↖ an increase in aggressiveness and decrease in
looseness, and so on. Values are rounded.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>( \bar{L}_0 )</th>
<th>( \bar{L}_1 )</th>
<th>( \Pr(T&gt;t) )</th>
<th>( \bar{A}_0 )</th>
<th>( \bar{A}_1 )</th>
<th>( \Pr(T&gt;t) )</th>
<th>↗</th>
<th>↖</th>
<th>↙</th>
<th>↘</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 I</td>
<td>.502</td>
<td>.510</td>
<td>.982</td>
<td>.481</td>
<td>.406</td>
<td>.000</td>
<td>751</td>
<td>987</td>
<td>917</td>
<td>525</td>
</tr>
<tr>
<td>10/20 II</td>
<td>.520</td>
<td>.519</td>
<td>.480</td>
<td>.501</td>
<td>.428</td>
<td>.000</td>
<td>260</td>
<td>314</td>
<td>324</td>
<td>172</td>
</tr>
<tr>
<td>10/20 III</td>
<td>.493</td>
<td>.503</td>
<td>.918</td>
<td>.484</td>
<td>.426</td>
<td>.000</td>
<td>162</td>
<td>208</td>
<td>181</td>
<td>125</td>
</tr>
<tr>
<td>20/40</td>
<td>.444</td>
<td>.441</td>
<td>.191</td>
<td>.533</td>
<td>.485</td>
<td>.000</td>
<td>441</td>
<td>484</td>
<td>548</td>
<td>367</td>
</tr>
<tr>
<td>50/100</td>
<td>.446</td>
<td>.428</td>
<td>.009</td>
<td>.620</td>
<td>.592</td>
<td>.000</td>
<td>66</td>
<td>82</td>
<td>92</td>
<td>74</td>
</tr>
</tbody>
</table>

Through graphical analysis as in figure 9.1 the clusters in playing style become clearer.
There is a group of players seen in the lower left quadrant with notably tighter/more
passive play in first sessions before risk-taking is increased later on. No comparable group
is present in the other quadrants although there are a few players in the upper left corner.
The bulk of individuals is at the center of the graph with a slight offset above the origin

³Cf. section 7.2.1.
indicating more frequent play in the first session. Graphs for other limits which are not shown confirm the impression from the 10/20 I Limit.

For every player in the 10/20 I Limit game differences in looseness $\Delta L_i$ and aggressiveness $\Delta A_i$ between first and later sessions are plotted. The number of players in every quadrant is shown in the corners of the diagram.

Figure 9.1: Changes in Playing Style Between First and Later Sessions

### 9.1.3 Concluding Discussion

The results above reveal two rather distinct patterns in risk-taking behavior in novel situations. First, some players take a conservative approach, restricting the frequency and volume of risks taken. Inexperience and insecurity due to a lack of adequate problem-solving skills are likely explanations. Individuals only tentatively accept risks and learn through observing others’ play. As they get more comfortable with the situation they get involved more often and to a larger extent, shifting actions from passivity to activity. The results are probably biased downwards as any prior experience in poker play would reduce the observed effect. Some players might have gained insights in one channel and started anew in a different one, or they might have played at home or in other venues before.

In the second pattern the rate of play is slightly increased for those individuals who are not part of the first group. To understand this behavior a bit better it has to be noted that every player on the IRC games is first equipped with a bankroll of 1,000 chips. If all chips are lost, a new stack of 1,000 chips can be acquired if a certain time has passed.\textsuperscript{4} Thus the virtual money is house money at first and only gradually becomes more valuable as players spend time playing with a stack or earn their own excess chips. So players who enter the

\textsuperscript{4}In todays play money games this is usually a one hour restriction.
game more frequently in their first session than in later sessions might be driven by two rather distinct forces. On the one hand, there is an increased propensity to consume, i.e. to gamble, with the money provided for free, a house money effect. On the other hand, these players are inexperienced and usually have little information about which hands are worth playing. As a consequence, they will have to experiment more to gain a better understanding of the valuation criteria.

9.2 Gambling with House Money

In this section we try to separate the effects from gratuitous chips from those of inexperience. As the tests are analog to those in the section above only aspects which are different will be discussed in detail.

9.2.1 Method

As players are allowed to eventually get another 1,000 chips for free whenever they depleted their resources, there are sessions at later stages of play which will be started with this kind of house money. However, this is only possible at the low limit gametypes as a minimum buy-in in excess of 1,000 chips is required for the higher levels. To identify this kind of session we are first looking for all sessions which are not a player’s first session and start on a bankroll of exactly $1,000. Additionally, we require the bankroll in the last hand prior to this session to be less than $300, thus excluding most of the instances where a bankroll of exactly $1,000 originates from the normal course of play.

To see whether players’ experience is approximately equal between sessions conditional on the house money criterion and all other sessions, the mean number of hands played before under both conditions is calculated and tested for equality.

9.2.2 Results

The procedure is successful in eliminating the influences of experience. As shown in table 9.2 the average number of hands which have been played is close to equal under both conditions. Mean values on looseness and aggressiveness also tell a compelling story. Contrasting these with those in table 9.1 we see that players who lose all their chips and restart generally play more loose/passive than the average player. This calling style is least successful and usually accumulates losses. Furthermore, the effects are larger for this kind of income it has consistently been found that it is spent more readily than other types of assets (Bodkin 1959; Arkes, Joyner, and Pezzo 1994).
in magnitude for both dimensions. Play in house money sessions is even more passive than in first sessions of a player. Although differences in mean values fail to reach significance, mean values of looseness are in the right direction with a higher rate of participation under the house money condition.

Table 9.2: Test Results on Risk-Taking Between House Money and Other Sessions
On columns 1-11 see table 9.1 where the condition marked as one are house money sessions and zero for all other sessions. We expect a higher looseness in house money sessions. The additional columns at the end state the average number of hands played before under both conditions.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>$\bar{L}_0$</th>
<th>$\bar{L}_1$</th>
<th>$\Pr(T&lt;\theta)$</th>
<th>$\bar{A}_0$</th>
<th>$\bar{A}_1$</th>
<th>$\Pr(T&gt;\theta)$</th>
<th>/ \</th>
<th>/ \</th>
<th>/ \</th>
<th>#0</th>
<th>#1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 I</td>
<td>.549</td>
<td>.553</td>
<td>.298</td>
<td>.468</td>
<td>.318</td>
<td>.000</td>
<td>460</td>
<td>447</td>
<td>642</td>
<td>189</td>
<td>2,514</td>
</tr>
<tr>
<td>10/20 II</td>
<td>.567</td>
<td>.595</td>
<td>.038</td>
<td>.501</td>
<td>.378</td>
<td>.000</td>
<td>158</td>
<td>143</td>
<td>146</td>
<td>67</td>
<td>1,789</td>
</tr>
<tr>
<td>10/20 III</td>
<td>.553</td>
<td>.573</td>
<td>.150</td>
<td>.476</td>
<td>.372</td>
<td>.000</td>
<td>96</td>
<td>81</td>
<td>96</td>
<td>38</td>
<td>1,610</td>
</tr>
</tbody>
</table>

The cluster in figure 9.2 are also more pronounced than those seen in figure 9.1 whereby a third group emerges. First, there is the group of players who show tighter/more passive play in house money sessions. A second group is around the origin but slightly shifted to the upper right. Third, the more extreme values of higher looseness in house money sessions are denser populated, i.e. more data points at the top left to right rim. Excluding the first group from the analysis would significantly change the results, rendering differences in looseness significant.

For every player in the 10/20 I Limit game differences in looseness $\Delta L_i$ and aggressiveness $\Delta A_i$ between house money and other sessions are plotted. The number of players in every quadrant is shown in the corners of the diagram.

Figure 9.2: Changes in Playing Style Between House Money and Other Sessions
9.2.3 Concluding Discussion

The presence of three distinct clusters of behavior across players gives an impression of the interacting effects which are at work during repeated decision-making under uncertainty. First, there is the group of tentative decision-makers who are even more shy to make a decision after they have been bankrupted recently, and who consequently become more risk-averse trying to minimize the rate and volume of exposure to risk so that the negative experience will not happen again too soon. Looseness and aggressiveness of these players is lower in house money sessions compared to all other sessions. Then there is a group of individuals enjoying the dynamics of the game, mostly playing for fun. They are consuming what they can get and do not bother much about the risks of a particular hand. This is the group showing the house money effect, i.e. higher looseness and no particular effect on aggressiveness. Finally, there are those players who are at neither extreme. They have accumulated some experience and partaken the first swings in their bankroll, eventually bankrupting due to the underlying volatility of the game. Getting another 1,000 chips to play with might be seen as a continuation of the prior series of decisions by them. Hence they would be in the domain of losses and increase risk-taking as discussed in section 7.2.

Here, the various motivations to participate in a risky game like poker intermingle. We do not see any single interpretation which could explain the diversity of shifts in behavior that is observable from the data. Hence, we conclude that whether an individual will show an increased propensity to consume, the house money effect, insecurity due to recent large losses and therefore conservative decision-making, or neither of both, might very well depend on the individual’s fundamental approach to the game.

9.3 Thinking on One’s Feet

Although every player might be motivated by particular goals reflected in the decision-making behavior, all players have the opportunity to improve their reaction times as they gain more experience. This should happen regardless of the actual choices made. To put it simply, decisions will be made faster whatever they are.

9.3.1 Method

From the database time series data and the number of actions provide the necessary information to calculate reaction times. Data is first sorted by timestamp and the time lag between hands is derived. All games for which more than ten minutes pass before the next game is dealt are excluded. These will be instances where all players leave the table and a new game is only started much later. Then we calculate the total number of actions
taken by all players at the table. From both measures we get the average reaction time per action in a game by simple division.

As the elapsed time is only recorded per game, no individual information is available, but the elapsed time has to be attributed to all players involved in the particular game. Hence we must account for the overall experience of this group. Therefore, the average number of hands played in the gametype is calculated across players involved in the game. To get more robust measures this figure is rounded to the nearest 10 and all values above 2,000 are dropped.

The power law stated in equation 6.6 cannot be regressed directly. Royston and Altman (1997) describe a regressive approximation based on fractional polynomials which is useful here. A fractional polynomial of degree $m$ consists of $m$ integer/fractional powers of the form:\[ fp(x) = a + \sum_{j=1}^{m} b_j x^{c_j} \] (9.1)

In regressing this equation the powers $c_j$ are generally chosen from a restricted set where $\log x$ is used in place of $x^0$. As we are expecting the reaction time to decrease as experience increases we test powers from the set \{-3, -2.99, -2.98, \ldots, -0.03, -0.02, -0.01\} and restrict the fractional polynomial to degree one. So the equation for regressing the reaction time $rt$ on experience as measured by the average number of hands played across players in the game $nh$ becomes

\[ rt(nh) = a + b \times nh^c \quad \text{where} \quad c \in \{-3, -2.99, -2.98, \ldots, -0.02, -0.01\} \] (9.2)

This means 300 models with differing exponents will be tested for goodness of fit.

### 9.3.2 Results
Best fitting values of the regression are summarized in table 9.3. All factors are highly significant and regression fit is also very good as indicated by $R^2$s above .8. The minimum time which is approached asymptotically as repetitions accumulate can be seen from the constant term $a$. It is around 5 to 5.5 seconds per action at the lowest limits and around 4.3 seconds per action at the medium and high limit varieties. The magnitude of improvement $b$ is also dependent on the limit. The very high value of $b$ for the 50/100 Limit has to be seen together with the very low value of $c$, inspection of the graph shows that these are regression artifacts. The graph is very steep close to the origin and then becomes flat quickly, so these values lack a decent interpretation. The other varieties show

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6See Rachet (2007) with further references.
CHAPTER 9. IMPROVING PLAY

a maximum improvement of over 27 seconds in the highly frequented 10/20 I Limit and
rates of learning c range from -.28 for the 10/20 III Limit to -.18 on the 20/40 Limit.

Table 9.3: Test Results on the Power Law of Reaction Time
Regression results for equation 9.2 are presented. The constant a and scaling factor b stem
from the regression with the best fitting power c stated in column 6. The \( R^2 \) of the regression
concludes the table. Values are rounded.

| Gametype | a   | P>|t| | b   | P>|t| | c   | R^2 |
|-----------|-----|------|-----|------|-----|-----|
| 10/20 I   | 5.64| .000 | 27.26| .000 | -.24| .917|
| 10/20 II  | 5.00| .000 | 19.75| .000 | -.19| .903|
| 10/20 III | 5.56| .000 | 25.28| .000 | -.28| .817|
| 20/40     | 4.33| .000 | 16.95| .000 | -.18| .941|
| 50/100    | 4.32| .000 | 60.07| .000 | -.02| .816|

The effect is also clearly evident from the graphical representation of the 10/20 I Limit
in figure 9.3.\(^7\) One easily sees that reaction times are higher at first and then approach
times of around 5 seconds in about 1,000 hands played.

![Figure 9.3: Power Law of Reaction Time](image)

For the 10/20 I Limit the average experience of players in a game as measured by the number
of hands played so far (rounded to 10s) is plotted against the average time needed to take an
action. The solid line represents the best fit from a fractional polynomial regression of degree
one with powers chosen from the set \{-3, -2.99, \ldots, -0.02, -0.01\}; for detailed values see table
9.3.

\(^7\)All other graphs are shaped similarly.
9.3.3 Concluding Discussion

The power law of practice is a well-established relationship. As expected (see hypothesis 9) reaction times improve while playing online poker. At first, players have to become acquainted with the software interface. Thereby, visual representation and required commands are the essential stimuli. Additionally, they process relevant information from the game, ranks and suits of cards, current and potential 5-card hands, and so on.

We have seen that for the overall population of players the power law of practice holds at all levels of the game. However, the information in the database does not allow to draw conclusions regarding the effects of practice at an individual level. As several individuals participate in any game they are responsible for the observed reaction time as a group. Different prior experience and practice with poker or online games will likely affect the initial reaction time and the rate of learning for the individual.

The observed speed of decision-making is remarkably low, approaching four to five seconds with only some practice already. As players have to first notice and process the available information, then make a decision, to finally input it to the interface, the actual decision-making process does not take much time. This speed seems only plausible if reasonable learning of abstract rules is applied. Players will have to categorize game states and according strategies, like it has been found with experienced chess players. From there adjustments in risk-taking behavior can originate from two directions. First, modulations in categories can be made. Instances or game states might be added, dropped or moved between categories, with the consequent change of the related strategy. For example, there might be a category “drawing” first populated with any four-flush hands on the flop and turn with the strategy to call if the pot odds are favorable. Having been beaten by a higher flush several times, a player might decide to exclude any four-flush hand where he is holding weak pocket cards from this category. Second, changes of strategies can be applied. Players could attribute different courses of actions to a category. Continuing the example, the player now might add to the strategy to raise some of the four-flush draws in order to semi-bluff his opponents.

With increasing practice players will make less and less adjustments to both categories and strategies. Consequently, reaction times will fall to a minimum level which is determined by the physical requirements to process the information and perform the adequate actions.
9.4 Adjustments in Playing Style

Analysis in this section focuses on the adjustments players make in risk-taking behavior as they get additional practice. Furthermore, the effects on performance from these adjustments are investigated.

9.4.1 Method

Looking for relative improvements over time the analysis has to center around individual players. A survivorship bias distorts changes in the risk-taking behavior of the population as a whole. Only successful playing styles make up the observations for high levels of practice. All other playing styles will be bankrupt before they accumulate an extensive hand history. Therefore, only regular players with at least 10,000 hands in the database who are not marked suspicious or artificial intelligence form the relevant sample.

For every of these players hands are ordered as time series and only the first 10,000 hands are used. Consecutive groups of 100 hands serve to measure experience $ex$, so there are 100 values for $ex$. Analog to the procedure described in section 7.1.3, especially equation (7.1), the following calculation gives looseness and aggressiveness per level of practice

$$L_{ex} = h_{ex}^{fl} / H_{ex} \quad \text{and} \quad A_{ex} = \frac{S_b^{ex} + S_r^{ex}}{S_{ex} + S_{eex} + S_{cex}}$$ (9.3)

For better comparison we also adjust for the mean playing style of the players, yielding $\hat{L}_{ex}$ and $\hat{A}_{ex}$. This is a constant shift for all values of $ex$ as for all players every value of $ex$ is present precisely once. To test hypothesis 10 linear regressions are run, regressing $\hat{L}_{ex}$ and $\hat{A}_{ex}$ on $ex$. For the same independent variable mean amounts won per hand $\mu_{ex}$ are calculated and regressed.

9.4.2 Results

Regression results are tabulated in table 9.4 and a graphic example from the 10/20 I Limit is presented in figures 9.4 and 9.5. For every but the high limit, there is a significant reduction in looseness with increasing practice. Simultaneously, aggressiveness increases at all limits. The graphic representation also shows that the adjustments continue well beyond the 10,000th hand, as most of the time series graph is found above zero for looseness and below zero for aggressiveness. Due to the calculation arithmetics the graph should balance around zero overall if the time series was extended to all hands ever played.

Regression statistics on the mean amount won per hand are less convincing. Coefficients are only significantly positive for two of the varieties, while the three others fail to
reach significance. $R^2$s are also low with the highest value of .23 for the 10/20 I Limit. Figure 9.5 shows the high volatility in observed average returns over time.

Table 9.4: Regression Results on Risk-Taking Depending on Experience (Hands Played)
The linear regression results for deviations of looseness from average players’ looseness ($\hat{L}_{ex}$, panel A) and aggressiveness ($\hat{A}_{ex}$, panel B) depending on players’ experience $ex$ are summarized. Panel C adds the regression statistics with the mean amount won per hand $\mu_{ex}$ as dependent variable. Values of coefficients are stated as percent which is equivalent to the estimated change from the first to the 10,000th hand. All values are rounded to three decimals.

| Gametype | Coef. | P>|t| | Cons. P>|t| | $R^2$ |
|----------|-------|------|------|-------|
| Panel A: Looseness, $\hat{L}_{ex}$ |
| 10/20 I | -.054 | .000 | .040  | .000  | .855 |
| 10/20 II| -.054 | .000 | .038  | .000  | .536 |
| 10/20 III| -.032 | .002 | .020  | .001  | .092 |
| 20/40 | -.033 | .000 | .024  | .000  | .666 |
| 50/100 | .000  | .914 | -.000 | .995  | .000 |
| Panel B: Aggressiveness, $\hat{A}_{ex}$ |
| 10/20 I | .036  | .000 | -.034 | .000  | .645 |
| 10/20 II| .013  | .034 | -.010 | .000  | .045 |
| 10/20 III| .049  | .000 | -.035 | .000  | .145 |
| 20/40 | .022  | .000 | -.013 | .000  | .436 |
| 50/100 | .033  | .000 | -.023 | .000  | .228 |
| Panel C: Average Returns, $\mu_{ex}$ |
| 10/20 I | .773  | .000 | -.148 | .078  | .230 |
| 10/20 II| .222  | .594 | -.085 | .726  | .003 |
| 10/20 III| -.294 | .564 | .334  | .262  | .003 |
| 20/40 | .080  | .710 | .301  | .017  | .001 |
| 50/100 | 3.292 | .018 | -.290 | .716  | .056 |

9.4.3 Concluding Discussion

Overall there is no evidence to reject hypothesis 10. Increasing expertise affects players’ risk-taking behavior. Their play becomes tighter and more aggressive, applying stricter selection rules to the investments they begin, pursuing these belligerently. Familiarity with the risk environment appears as an important aspect in the chosen strategies. Whereas in an unfamiliar environment risks are taken passively but frequently, gathering information helps decision-makers to discriminate between worthwhile and less promising opportunities and options.

Reward for the adjustments in playing style comes from an increased return on the
Deviations of looseness (left graph) and aggressiveness (right graph) from average players’ looseness and aggressiveness, $L_{ex}$ and $A_{ex}$, are plotted against players’ experience $ex$. Solid lines are linear fits. The example is for the 10/20 I Limit.

Figure 9.4: Risk-Taking Depending on Experience (Hands Played)

The diagram shows the average amount won per hand for given levels of experience at the 10/20 I Limit gametype.

Figure 9.5: Average Returns Depending on Experience (Hands Played)
investments made. However, the effect is small and elusive. Besides the fundamental factor of randomness in the game which is visible here once more, pressure on returns is also generated by the interactive competition between players. Adjusting playing style will not result in increased profits as long as everybody makes the same relative adjustments. Here, as in business, agents have to stay one step ahead of the competition to gain an advantage from general (risk-taking) strategies.

The finding of increasing or about stable mean amounts won per hand with more hands played is an additional finding of interest. With practice players’ reaction times are accelerated as discussed in detail in the preceding section but this does not impair bottom-line profit. Experienced poker players might use quicker speed in judgments to play several games simultaneously which in turn might increase their profits.\(^8\) Here again an analogy to business management appears fit. As managers advance through their career they also gather experience in their industry. At higher positions this is a requisite to handle more projects in the same time, e.g. managing several products, countries or business streams.

\(^8\)The speed-up is a requisite for multi-tasking as human cognition is limited by attentional, processing bottlenecks interfering with simultaneous tasks (Pashler 1992, 1994).
Chapter 10

Charged with Emotions

Paciencia y barajar. [Patience and shuffle the cards.]

Miguel de Cervantes Saavedra, El Ingenioso Hidalgo Don Quixote de la Mancha 2.23.389

In live poker games detecting others’ emotions is a core skill. There are several factors which reveal moods. Facial expression, gesture, or talk are examples of the more prominent tells. How, when and which chips are moved could be other signs. All this is not available in online poker. So how to detect emotions there? Although there is no definite way to ascertain that an online opponent is in a specific mood, some variables can be used as a proxy for the likely emotional state. Throughout this chapter we will assume that the proxies are sufficient predictors of the emotional states and leave this open to further discussion.

10.1 Bored to Risk

10.1.1 Measurement of Boredom and Its Effects

In a game of repeated excitement like poker the absence of new stimuli over a longer period reduces the participant’s stimulation, an unpleasant quietness, boredom. Thus a proxy for likely boredom will be periods when players only take few actions. Sorting hands as time series for each human player who has played at least 1,000 hands and is not marked suspicious, the elapsed time and number of actions taken indicate such a period. As boredom is not a highly affective condition it cannot be expected to appear spontaneously but develop gradually over time. Hence, not only the most recent hand is of interest but a longer series is needed which we arbitrarily set to the last ten hands. Formally the proxy for boredom $bd$ is derived for a player $i$ and hand $h$ based on the
elapsed time between hands \( t \) and the number of actions taken in a hand \( act \)

\[
bd = \sum_{j=h_{i}-10}^{h_{i}} \frac{t_{j}}{act_{j}}
\]

(10.1)

As the elapsed time between hands becomes extremely large in the case a session is interrupted all observations are excluded where \( bd \) is larger than 1,000. Values of \( bd \) are rounded to the nearest integer for grouped evaluation. The usual measures of playing style, looseness and aggressiveness, as well as the elapsed time between hands and the number of actions are dependent on the number of players involved in a game, therefore, observations cannot be compared across different levels of attendance and only data on heads-up games is used where there might be least tolerance for slow playing.

A first simple measure of boredom effects is a player’s activity as dependent variable. For different levels of boredom the mean number of actions is taken adjusted for the usual, i.e. mean, activity of the player. This measure is calculated for all hands played under a certain value of \( bd \). The mean is taken if there are at least 100 observations for a given value of \( bd \) yielding the deviation of activity in a hand from mean players’ activity. All other values of \( bd \) are excluded. Based on this a simple linear regression is run. Also, as established in earlier chapters, for different values of the independent variable looseness and aggressiveness are calculated, formally

\[
L_{bd} = \frac{h_{bd}^{fl}}{H_{bd}} \quad \text{and} \quad A_{bd} = \frac{\$_{bd}^{h} + \$_{bd}^{r}}{\$_{bd}^{h} + \$_{bd}^{r} + \$_{bd}^{c}}
\]

(10.2)

and adjusted for the mean values of players’ overall looseness and aggressiveness, respectively, yielding \( \hat{L}_{bd} \) and \( \hat{A}_{bd} \). Hypothesis 11 is tested using a linear regression of these two variables on \( bd \) as independent variable where only values of \( bd \) are used which are based on at least 100 observations.

In addition to the analysis above of risk-taking behavior across subjects, decision-making is compared within individuals. Therefore, we set a threshold of 300 to the measure of boredom (equation 10.1). We simply assume that a player who has to wait more than 30 seconds for every of his actions over the last 10 hands is bored. A dummy variable is set to one under a bored condition and to zero otherwise. Looseness and aggressiveness are derived for every player who has played at least 100 hands under both

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1 We have found similar but less significant results using only elapsed time or elapsed time adjusted for the total of seen actions as independent variable. For simplicity only the measure with adjustment for players’ own actions is discussed.

2 If the value exceeds 1,000 the observation is excluded as a between session interval is likely to cause values this high.
conditions. The difference of both measures between conditions shows the change in playing style due to boredom.

10.1.2 Results

Table 10.1 summarizes the results of the linear regressions. It is evident that boredom has significant effects on all dimensions of risk-taking. First, the longer players have had to wait for making an action over the last hands the more likely they are to increase their activity in the current hand. Figure 10.1 shows that for a reasonable range of boredom the estimate indicates that players will take one extra action every second hand if they are bored. Second, bored players play more frequently and aggressively. Comparison of the coefficients between the regressions for looseness with those for aggressiveness suggests that changes in the frequency of play are the stronger reaction when facing a low density of stimuli. For all gametypes coefficients on looseness are larger than coefficients on aggressiveness. Bored individuals seem more likely to increase the rate of stimulation than the strength of a single stimulus. The pattern is also evident from figure 10.2, the increase in looseness is steeper than the increase in aggressiveness.

![Figure 10.1: Changes in Activity Due to Boredom](image)

The plot shows the deviation of activity from mean players’ activity under certain states of boredom at the 10/20 I Limit. The solid line is the linear fit.

Results from between subject analysis is confirmed by analysis of playing style within subjects. Measures of playing style under boredom deviate significantly from the absence of the condition, as seen in table 10.2 and figure 10.3. However, the low number of subjects which actually qualified for the within subjects test shows that individuals have only rarely encountered situations we assume to be boring. Although all deviations are
Table 10.1: Regression Results on Risk-Taking Depending on Boredom

The linear regression results for deviations of looseness from average players’ looseness ($\hat{L}_{bd}$, panel B) and aggressiveness ($\hat{A}_{bd}$, panel C) depending on players’ boredom proxy $bd$ are summarized. Panel A adds the regression statistics with the deviations of activity as dependent variable. *Values of coefficients are stated as percent* which is equivalent to the estimated change for waiting an additional 100 seconds. All values are rounded to three decimals.

| Gametype | Coef. | P>|t| | Cons. | P>|t| | $R^2$ |
|----------|-------|---|-----|---|---|-----|
| Panel A: Activity | | | | | | |
| 10/20 I | .103 | .000 | .112 | .000 | .588 |
| 10/20 II | .203 | .000 | -.120 | .000 | .772 |
| 10/20 III | .230 | .000 | -.195 | .000 | .582 |
| 20/40 | .126 | .000 | .112 | .000 | .715 |
| 50/100 | .117 | .000 | .027 | .055 | .305 |
| Panel B: Looseness, $\hat{L}_{bd}$ | | | | | | |
| 10/20 I | .024 | .000 | -.031 | .000 | .587 |
| 10/20 II | .038 | .000 | -.065 | .000 | .734 |
| 10/20 III | .045 | .000 | -.090 | .000 | .454 |
| 20/40 | .026 | .000 | -.021 | .000 | .635 |
| 50/100 | .018 | .000 | -.021 | .000 | .185 |
| Panel C: Aggressiveness, $\hat{A}_{bd}$ | | | | | | |
| 10/20 I | .001 | .000 | .127 | .000 | .128 |
| 10/20 II | .029 | .000 | .086 | .000 | .559 |
| 10/20 III | .024 | .000 | .110 | .000 | .160 |
| 20/40 | .011 | .000 | .076 | .000 | .210 |
| 50/100 | .017 | .000 | .014 | .000 | .197 |

Deviations of looseness (left graph) and aggressiveness (right graph) from average players’ looseness and aggressiveness, $\hat{L}_{bd}$ and $\hat{A}_{bd}$, are plotted against players’ state of boredom $bd$. Solid lines are linear fits. The example is for the 10/20 I Limit.

Figure 10.2: Risk-Taking Depending on Boredom
in the direction as expected, a few statistics fail to reach significance, which might be due to the few subjects in the test.

Table 10.2: Test Results on Risk-Taking Under Boredom and Otherwise
Mean values of looseness under boredom $L_1$ and otherwise $L_0$ are stated by gametype. The level of significance on a test of equality of means under the hypothesis that risk-taking is increased under boredom is given in column 4. Analog figures are stated for aggressiveness in columns 5-7. Columns 8-11 list the count of players with a particular shift in playing style, where $\uparrow$ indicates an increase in both dimensions, $\downarrow$ an increase in aggressiveness and decrease in looseness, and so on. Values are rounded.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>$L_0$</th>
<th>$\bar{L}_1$</th>
<th>$\Pr(T &gt; t)$</th>
<th>$\bar{A}_0$</th>
<th>$\bar{A}_1$</th>
<th>$\Pr(T &gt; t)$</th>
<th>(\uparrow)</th>
<th>(\downarrow)</th>
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<th>(\updownarrow)</th>
</tr>
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<tr>
<td>10/20 II</td>
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<td>.654</td>
<td>.000</td>
<td>.699</td>
<td>.760</td>
<td>.000</td>
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<td>11</td>
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</tr>
<tr>
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<td>.021</td>
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<tr>
<td>20/40</td>
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<td>.000</td>
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<td>.674</td>
<td>.385</td>
<td>28</td>
<td>21</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>50/100</td>
<td>.455</td>
<td>.471</td>
<td>.230</td>
<td>.670</td>
<td>.692</td>
<td>.038</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Changes in looseness and aggressiveness between hands played under supposed boredom and otherwise are scatter-plotted for every player in the 10/20 I Limit game. The number of players in every quadrant is shown in the corners of the diagram.

Figure 10.3: Changes in Playing Style Due to Boredom

10.1.3 Risk as a Means of Diversion
Coping with longer periods of low stimulation appears as an important skill in poker. We have found that risk-taking behavior changes in line with hypothesis 11, bored individuals tend to increase the frequency and volume of risks. Looking for diversion, taking risks is
used as a stimulus whereby a higher rate is preferred over a heightened amplitude.

However, the results should be seen in light of the restrictions the dataset poses on the analysis. Based on the recorded observations no direct control of the emotional state can be devised. Therefore, other factors running counter to boredom cannot be excluded. For example, players in a slow and boring online game might be doing other things in parallel (reading, watching TV, etc.) reducing boredom. Some participants who are used to a more rapid decision-making environment might get bored earlier than other players.

Implications for business and other areas are readily seen. As tasks become monotonic and dull, individuals will eventually get bored, and if some disposition is available for taking risks, their tendency is more frequent investments with higher amounts. Take a manager who is in charge of looking for new markets. For several years he might have proposed only the best opportunities to the board of directors but which they have been declining most of the time. Hence, the bored manager will tend to present new opportunities more frequently including investments he would have excluded before.

10.2 In High Spirits

10.2.1 Measurement of Elation and Its Effects

Recent gambling results are stimuli arousing either pleasant (winning) or unpleasant (losing) emotions. Over the course of a poker game the outcomes of the latest hands represent such stimuli which gives a handy proxy for an individual’s elated mood. For the usual set of players time series data is used to track recent changes in the bankroll. As the number of different amounts that can be won in a limit game is confined to multiples of the blinds, a series over the last five hands gives a broader range of possible outcomes and simultaneously provides a better proxy for the mood which is unlikely to change quickly from hand to hand. Hence, we calculate the absolute amount won/lost over the last five hands rounded to the nearest five chips to assimilate the rare observations of split pots as the independent variable $el$ for elation. The dependent variables describing risk-taking behavior are deviations of looseness and aggressiveness from mean players’ looseness and aggressiveness by elation ($\hat{L}_{el}$ and $\hat{A}_{el}$) in analogy to the description in section 10.1.1 and equation 10.2. As the amount won will be correlated with the number of players in the game the analysis of dependent variables is restricted to observations for those games with the most observations which are ten-player games for all gametypes except the high-limit

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3Non-conspicuous human players with more than 1,000 hands in the database.
4A proxy using the amount won relative to the player’s bankroll yields similar results, so for brevity’s sake only the procedure for the proxy covering the last five hands in absolute values is discussed.
10.2. IN HIGH SPIRITS

where heads-up games are used.

From the discussion in section 6.6.3 it follows that behavior might be discontinuous around break-even. Gambling is likely to increase for profits where there are no substantial losses possible. Therefore, a dummy variable \( \text{win} \) is created set to one if the cumulated winnings over the last five hands are positive capturing dichotomous behavior between gains and losses. Hence, test regressions are as follows

\[
\hat{L}_{el} = \beta_0 + \beta_1 el + \beta_2 \text{win}
\]

(10.3)

\[
\hat{A}_{el} = \beta_0 + \beta_1 el + \beta_2 \text{win}
\]

(10.4)

To account for the variability in the game only values of \( el \) are included in the regression for which at least 100 hand observations are available.

For the within subjects analysis a threshold of a net winning of six big blinds over the last five hands is used, above which the subject is assumed to be elated. As before looseness and aggressiveness are calculated for each player who has played at least 100 hands under both conditions. Mean values of both measures across players are tested for equality under elation and its absence.

10.2.2 Results

Results of the statistical analysis indicate an effect of recent winnings on risk-taking behavior. The coefficients (\( \beta_1 \)) in table 10.3 support a negative relationship between pleasant arousal due to positive outcomes and following risk-taking. Changes are more prominent and explanatory power is greater (cf. Adj. \( R^2 \)s) for volumes of risk rather than frequency of participation. Furthermore, effects per dollar in the game become smaller at higher limits. There is no indication of a discontinuity around break-even. Coefficients on the dummy variable \( \text{win} \) fail to reach significance for all gametypes. The graphical representation in figure 10.4 suggests some additional insights. First, for looseness the effect is driven by losses rather than gains for which the graph is approximately flat. Second, there are two distinct lines for looseness under losses for which we lack an interpretation. Third, there is a minor jump discontinuity at break-even with higher looseness and aggressiveness for small winnings. Fourth, the negative relationship between outcomes and aggressiveness is present for both gains and losses, however, the steeper inclination for losses indicates a weaker effect for players’ recent wins.

Test results of the within subjects analysis, tabulated in table 10.4 and illustrated in figure 10.5, confirm the results for effects on aggressiveness. Players who have won at
Table 10.3: Regression Results on Risk-Taking Depending on Elation

The linear regression results (cf. equations (10.3) and (10.4)) for deviations of looseness from average players’ looseness ($\hat{L}_{el}$, panel A) and aggressiveness ($\hat{A}_{el}$, panel B) depending on players’ elation proxy $el$ are summarized. *Values of coefficients are stated as one-tenth of a percent* which is equivalent to the estimated change for winning $1,000. All values are rounded to three decimals.

| Gametype | $\beta_1$ | $P>|t|$ | $\beta_2$ | $P>|t|$ | $\beta_0$ | $P>|t|$ | Adj. $R^2$ |
|----------|----------|--------|----------|--------|----------|--------|----------|
| Panel A: Looseness, $\hat{L}_{el}$ |
| 10/20 I  | -.091    | .000   | -.005    | .628   | .018     | .001   | .418     |
| 10/20 II | -.092    | .000   | .008     | .486   | -.017    | .003   | .262     |
| 10/20 III| -.080    | .000   | .005     | .628   | -.006    | .289   | .218     |
| 20/40    | -.032    | .000   | -.010    | .204   | -.009    | .034   | .213     |
| 50/100   | -.010    | .149   | -.009    | .336   | .015     | .005   | .004     |
| Panel B: Aggressiveness, $\hat{A}_{el}$ |
| 10/20 I  | -.133    | .000   | -.003    | .717   | -.006    | .209   | .657     |
| 10/20 II | -.135    | .000   | .016     | .058   | -.054    | .000   | .558     |
| 10/20 III| -.094    | .000   | .005     | .589   | -.035    | .000   | .388     |
| 20/40    | -.056    | .000   | -.005    | .588   | -.032    | .000   | .333     |
| 50/100   | -.051    | .000   | -.004    | .577   | .031     | .000   | .815     |

For different levels of elation $el$ the deviations of looseness (left graph) and aggressiveness (right graph) from average players’ looseness and aggressiveness, $\hat{L}_{el}$ and $\hat{A}_{el}$, are graphed. The example is for the 10/20 I Limit.

Figure 10.4: Risk-Taking Depending on Elation
least six big blinds over the last five hands show a higher proportion of bets and raises than players who have won less. However, the results regarding looseness contradict the findings from the between subjects design. Elated players individually show a higher looseness than if they are not elated, and although the effect is small in magnitude it is significant. Hence, we must conclude that some individuals with a quite distinct emotional effect and frequent play are present in the data.

Table 10.4: Test Results on Risk-Taking Under Elation and Otherwise
Mean values of looseness $\bar{L}_1$ under elation and otherwise $\bar{L}_0$ are stated by gametype. The level of significance on a test of equality of means under the hypothesis that risk-taking is reduced under elation is given in column 4. Analog figures are stated for aggressiveness in column 5-7. Columns 8-11 list the count of players with a particular shift in playing style, where ↗ indicates an increase in both dimensions, ↘ ↖ ↙ ↘ an increase in aggressiveness and decrease in looseness, and so on. Values are rounded.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>$\bar{L}_0$</th>
<th>$\bar{L}_1$</th>
<th>Pr(T&gt;t)</th>
<th>$\bar{A}_0$</th>
<th>$\bar{A}_1$</th>
<th>Pr(T&gt;t)</th>
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<th>↖</th>
<th>↙</th>
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<td>636</td>
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<td>10/20 II</td>
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<td>.000</td>
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<td>.000</td>
<td>7</td>
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<td>44</td>
<td>10</td>
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</table>

Changes in looseness and aggressiveness between hands played under supposed elation and otherwise are scatter-plotted for every player in the 10/20 I Limit game. The number of players in every quadrant is shown in the corners of the diagram.

Figure 10.5: Changes in Playing Style Due to Elation
10.2.3 Rather Spend Than Lose

In line with the findings of Smith, Levere, and Kurtzman (2009) and hypothesis 12 the present results also show that elated players take less risky decisions. They participate in fewer hands and bet or raise proportionately less. A readily available interpretation from the literature is that players try to maintain their positive mood by avoiding some risks. And analogously if things are not going well risks are increased to overcome the bad mood.

The finding of a flatter effect for gains than for losses is worth some additional discussion. Here two effects might be at work in parallel. On the one hand there is the desire to reduce risks in order to maintain the good mood. On the other hand recent winnings elicit an increased propensity to spend, a house money effect which might be strengthened by the positive mood. Additionally, as more money accumulates through wins the chance that a loss will be substantial enough to liquidate all gains gets smaller. This might explain the downward sloping relationship between elation and aggressiveness which is relatively less for gains as compared to losses. Individuals who have made substantial winnings are still wary about losing their winnings but they are willing to increase the stakes as they will still be above break-even. At the same time where there is more money it is also more likely that a certain share of it is seen as available for gambling. Torn between not wanting to risk their positive mood but available money for gambling, the rate of participation, as captured by looseness, might not change at all as in the findings above.

Recent outcomes are a good example of the manifold interactions of several psychological effects on risk-taking behavior. Besides emotional states, outcomes also affect the individual position relative to the reference point. Seen as a series they might also contribute to the hot hand or the gambler’s fallacy, although we rather see the gambler’s fallacy due to a random process lacking skill elements like the dealing of cards and the hot hand would not relate to the size of the winnings but clustering. And there might also be effects due to revised assessment, recent winners could perceive an increase in control assessing their skills as relatively better and consequently take riskier decisions. Overall, losses seem to be a less diluted and thus clearer indicator of following risk-taking behavior.

10.3 Cases of Hardship

10.3.1 Measurement of Distress and Its Effects

Poker is essentially a game with and for money. Players lacking capital to invest cannot participate properly in the game. The forced blind bets impose recurring costs which have
to be paid at regular intervals, i.e. every \( n \)-th hand where there are \( n \) players at the table. Hence, a low bankroll creates unpleasant pressure and is a likely indicator of a distressed emotional state. There is a rough guideline among poker players that a bankroll of at least 100 big blinds is required for decent play. This advice underlines that capital is relevant relative to the size of the bets in the game. We use this as our proxy of a distressed mood for all dedicated players (at least 1,000 hands in the database) who are not artificial intelligence and marked for suspicious playing. Precisely, the player’s bankroll at the beginning of the hand as measured in big blind payments is the independent variable \( ti \) for tilt.

We use deviations of looseness and aggressiveness from mean players’ looseness and aggressiveness for different values of \( ti \) as the dependent variables capturing risk-taking. As the rule-of-thumb to have at least 100 big blinds bears an important psychological threshold, two separate linear regressions are run. One estimating the impact of a bankroll above 100 big blinds which is assumedly high enough to avoid distress, the other for the estimate of likely distressing capital resources below 100 big blinds. Thereby, only values of \( ti \) are included in the regressions for which at least 1,000 hand observations are found.\(^5\) For the within subjects design hands are split into two groups for every qualifying player, hands which have been played with a bankroll above 50 big blinds and hands conditional on a smaller bankroll. Then for both conditions looseness and aggressiveness are calculated. Finally, mean values of both measures are tested for significant differences where only players with at least 100 hands for the distressed and not-distressed conditions are included in calculating the means.

### 10.3.2 Results

There are significant effects between a player’s remaining capital and decision-making behavior. The regression results shown in table 10.5 clearly reveal that under high budgetary constraints riskier decisions are taken. More hands are played and betting is more aggressive. The effect on the latter is the more prominent effect with an estimated change from -.095 to -.154 between bankruptcy and resources of 100 big blinds, compared to effects between -.046 and -.082 for looseness. Above bankrolls of 100 big blinds playing style is more stable. Far smaller coefficients and low values for \( R^2 \) indicate that for these situations the absolute capital resources are not a good indicator of risk-taking. The relationship becomes even clearer from the graphical impression in figure 10.6 where an L-shaped pattern of deviations from average risk-taking is evident. The kink being somewhat above 100 big blinds. That it is found below the line of zero deviation does not

\(^5\)We also drop observations with less than four big blinds remaining as there is significant abnormal behavior for bankrolls this low.
contradict that play is “normal” above this bankroll. As the higher risk-taking found at low bankrolls is integrated when calculating average playing style, normal style will be lower than the observed mean style. Only at the high limit no effects of distress appear as there are not sufficient observations of hands with a low bankroll. The regression results are confirmed by the t-tests between conditions which are summarized in table 10.6 and figure 10.7. The majority of players changes to a looser/more aggressive style if they have to act on low bankrolls. However, there are also some who adjust to a tighter/more passive style under capital restraints.

Table 10.5: Regression Results on Risk-Taking Depending on Distress
The linear regression results for deviations of looseness from average players’ looseness (\( \hat{L}_{ti} \), panel A) and aggressiveness (\( \hat{A}_{ti} \), panel B) depending on players’ distress due to a low absolute bankroll measured by \( ti \) are summarized. Values of coefficients are stated as percent which is equivalent to the estimated change of an additional 100 big blinds in capital. All values are rounded to three decimals.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Below 100 big blinds, ( ti &lt; 100 )</th>
<th>At least 100 big blinds, ( ti \geq 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>P&gt;</td>
</tr>
<tr>
<td></td>
<td>Coef.</td>
<td>P&gt;</td>
</tr>
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<td>.000</td>
</tr>
<tr>
<td>10/20 II</td>
<td>-.082</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 III</td>
<td>-.063</td>
<td>.000</td>
</tr>
<tr>
<td>20/40</td>
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<td>.000</td>
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<td>50/100</td>
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<td>.005</td>
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</tr>
</tbody>
</table>

Panel A: Looseness, \( \hat{L}_{ti} \)

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Below 100 big blinds, ( ti &lt; 100 )</th>
<th>At least 100 big blinds, ( ti \geq 100 )</th>
</tr>
</thead>
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Panel B: Aggressiveness, \( \hat{A}_{ti} \)

10.3.3 Costs of Playing on “Tilt”
One could argue that if capital is running out the adjustments in risk-taking are necessary as one cannot wait as long as usual due to the recurring costs. If this was right then performance should be at least as good under the adjustments than otherwise. To test this, mean amounts won are calculated per depth of the bankroll at the beginning of the hand and for the two conditions (below 50 big blinds or not) per player. As seen in tables 10.7 and 10.8, as well as figure 10.8, distress has a significant negative impact on a player’s performance. Hence, the deviations cannot be rational and are far from optimal.
10.3. CASES OF HARDSHIP

For different levels of distress $t_i$ the deviations of looseness (left graph) and aggressiveness (right graph) from average players’ looseness and aggressiveness, $\hat{L}_{ti}$ and $\hat{A}_{ti}$, are graphed. Solid lines are linear regressions which are run separately for bankrolls below 100 big blinds and all else. The example is for the 10/20 I Limit.

Figure 10.6: Risk-Taking Depending on Distress

Table 10.6: Test Results on Risk-Taking Under Distress and Otherwise
Mean values of looseness $\bar{L}_1$ under distress and otherwise $\bar{L}_0$ are stated by gametype. The level of significance on a test of equality of means under the hypothesis that risk-taking is increased under distress is given in column 4. Analog figures are stated for aggressiveness in column 5-7. Columns 8-11 list the count of players with a particular shift in playing style, where ↗ indicates an increase in both dimensions, ↖ an increase in aggressiveness and decrease in looseness, and so on. Values are rounded.

<table>
<thead>
<tr>
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<th>$\bar{L}_1$</th>
<th>Pr($T&gt;t$)</th>
<th>$\bar{A}_0$</th>
<th>$\bar{A}_1$</th>
<th>Pr($T&gt;t$)</th>
<th>↗</th>
<th>↖</th>
<th>↙</th>
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<td>84</td>
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<td>59</td>
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<td>64</td>
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<td>.441</td>
<td>.814</td>
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<td>.644</td>
<td>.141</td>
<td>8</td>
<td>4</td>
<td>10</td>
<td>9</td>
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</table>

Table 10.7: Regression Results on Performance Depending on Distress
The linear regression results for mean amounts won depending on players’ distress due to a low absolute bankroll measured by $t_i$ are summarized. Values of coefficients are stated as percent which is equivalent to the estimated change of an additional 100 big blinds in capital. All values are rounded to three decimals.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Below 100 big blinds, $t_i &lt; 100$</th>
<th>At least 100 big blinds, $t_i \geq 100$</th>
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</thead>
<tbody>
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</tbody>
</table>
Changes in looseness and aggressiveness between hands played under supposed distress and otherwise are scatter-plotted for every player in the 10/20 I Limit game. The number of players in every quadrant is shown in the corners of the diagram.

Figure 10.7: Changes in Playing Style Due to Distress

Table 10.8: Tests on Equality of Mean Amount Won Conditional on Distress

Mean amounts won per hand among players who have a sufficient bankroll not to be distressed $\mu_0$ and who are likely to be distressed due to a low bankroll $\mu_1$ are stated by gametype. The level of significance on a test of equality of means under the hypothesis that performance deteriorates under distress is given in column 4. Values are rounded to three digits.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>$\bar{\mu}_0$</th>
<th>$\bar{\mu}_1$</th>
<th>Pr$(T &gt; t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 I</td>
<td>-.390</td>
<td>-2.369</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 II</td>
<td>.158</td>
<td>-3.984</td>
<td>.000</td>
</tr>
<tr>
<td>10/20 III</td>
<td>-.196</td>
<td>-2.752</td>
<td>.000</td>
</tr>
<tr>
<td>20/40</td>
<td>-1.129</td>
<td>-2.853</td>
<td>.000</td>
</tr>
<tr>
<td>50/100</td>
<td>-.482</td>
<td>-.520</td>
<td>.495</td>
</tr>
</tbody>
</table>
Mean amounts won are plotted against depth of bankroll at the beginning of the hand (in big blinds) for the 10/20 I Limit game. Solid lines are two separate linear regressions, below 100 big blinds and all else.

Figure 10.8: Changes in Performance Due to Distress

10.3.4 Fight-or-Flight

The results above substantiate the implications of felt distress on judgments of risk. As posed in hypothesis 13 we find that distressed players will increase risk-taking. This is in accordance with the well-known fight-or-flight behavior in case one’s back is up against the wall. Players who do not take their chips and run but stay in the game adopt a “fight-them-all” kind of strategy. They take on more challenges (play more frequently) and even more prominently take a more aggressive stance (betting and raising relatively more often). This loose/aggressive playing style is what is known as the maniac playing style whose recklessness is rarely successful.

Interestingly, players at the high limit are rarely found in situations where they have only enough chips to make a few bets. It appears that they possess adequate capital management skills, implying that they will not play if their resources are not sufficient. Finally, there is always the option available to move to a lower limit; 50 big blinds at the high limit are equivalent to 125 big blinds at the medium limit which allows for less stressed play.

This is also a lesson learned for business managers. Capital should be kept at a level facilitating diversified investments. If there is only enough left to make the next investment around the corner, decision-makers should be wary of the risks. Better change the size of the projects so that several can be implemented and recurring costs do not put
considerable restrictions on the strategy.
Chapter 11

Judgments of Skill

It is no good to be the tenth best player in the world if the top nine are in your game.

Poker saying

11.1 Illusion of Control

In this section two consequences of the illusion of control in the poker setting will be analyzed. First, it is investigated how players change their risk-taking behavior depending on the degree of perceived control relative to others present in the game. Second, effects on the meta-game, the selection of which games to enter, are discussed.

11.1.1 Perceived Control and Its Effects

Method In online poker games there is only little information available about opposing players when one is meeting them for the first time. Initial inferences can exclusively be based on a player’s nickname and bankroll. Not before several hands have been played against an opponent an idea about his playing style can be developed. As in IRC poker one always enters the game with the total bankroll, others can use this information to get an indication about past success. It is likely that players rate their competence relative to their opponents based on this information. Hence, though we cannot measure perceived control directly, players’ stacks can serve as a proxy. We define $co$ for measure of perceived control as a player’s bankroll relative to the mean bankroll of the other players in the game. The measure is rounded to the nearest percent. For example, in a game with

\footnote{There are some programs and services available on the net which offer additional information but most are illegal by the standards of the providers.}
three players and stacks of $1,000, $2,000, and $3,000, the measure of perceived control is .4 for the first player, unity for the second player and 2 for the one with the most money.

In preceding chapters other measures related to a player’s bankroll have been used to detect biases in decision-making which have to be discussed in relation to the current analysis. First, the bankroll at the beginning of the session has been shown to be an influential reference point. An increase in the bankroll is a move into the domain of gains causing a more conservative style. This, however, runs counter to the current hypothesis where more money means an increase in perceived control with expected less conservative style. Second, we have used the absolute amount of chips (in terms of big blind bets that can be placed) to measure likely distress in decision-makers. Of course, distress and loss of control, our hypothesis for a bankroll which is relatively small, are closely related concepts. Nevertheless, all observations are dropped where players have less than 100 big blinds remaining, excluding the range for which distress has been shown.

For all human players with at least 1,000 hands and non-suspicious play, the deviations of looseness and aggressiveness from mean looseness and aggressiveness, $\hat{L}_{co}$ and $\hat{A}_{co}$, are calculated by degree of perceived control $co$. Simple linear regressions are run with $\hat{L}_{co}$ and $\hat{A}_{co}$, respectively, as dependent variable and $co$ as independent variable, using all values of $co$ for which at least 1,000 observations are found. In order to assess the quality of $co$ as a predictor of skill, mean amounts won per level of perceived control $\mu_{co}$ is regressed on $co$. As there is a likely interaction between the proxy and the number of players in the game only data with ten players present are used except for the 50/100 Limit for which heads-up games yield the most observations.

In a second analysis players’ performance depending on perceived level of control is examined. Therefore, the analysis above is replicated with mean amount won per hand as dependent variable. All other restrictions are left unchanged.

**Results** The regression results summarized in table 11.1 indicate an unclear effect of the illusion of control at best. Coefficients on both looseness and aggressiveness show no clear direction of the deviations. For some gametypes the effect even fails statistical significance. Regression constants are less interesting and show the pattern which can be expected from the selection of games with the selected attendance. Only two regressions present an overall somewhat satisfactory explanatory power (as indicated by $R^2$). Furthermore, the estimated impact of an increasing level of control is fairly small in absolute terms, noting that a resonable range for the proxy $co$ is between 0 and 5, i.e. the big
11.1. ILLUSION OF CONTROL

The results linking performance to perceived level of control, however, are more clearcut. All coefficients in panel C of table 11.1 are positive though only two are significant beyond the .05 level. As with most other relationships involving mean amounts won per hand only a small fraction of the variance can be explained by one factor, reflecting the high variability and multifactor influences of the game.

Table 11.1: Regression Results on Risk-Taking Depending on Perceived Control
The linear regression results for deviations of looseness from average players’ looseness ($\hat{L}_{co}$, panel A) and aggressiveness ($\hat{A}_{co}$, panel B) depending on players’ perceived level of control co are summarized. Panel C adds the players’ performance, measured by the mean amount won per hand $\mu_{co}$. All values are rounded to three decimals.

| Gametype | Coef. | $P>|t|$ | Cons. | $P>|t|$ | $R^2$ |
|----------|-------|--------|-------|--------|------|
| Panel A: Looseness, $\hat{L}_{co}$ |
| 10/20 I | -.001 | .226 | .042 | .000 | .005 |
| 10/20 II | -.002 | .489 | .057 | .000 | .003 |
| 10/20 III | .001 | .725 | .043 | .000 | .001 |
| 20/40 | .003 | .002 | .046 | .000 | .004 |
| 50/100 | .059 | .000 | .033 | .000 | .366 |
| Panel B: Aggressiveness, $\hat{A}_{co}$ |
| 10/20 I | -.004 | .000 | .054 | .000 | .054 |
| 10/20 II | -.023 | .000 | .061 | .000 | .303 |
| 10/20 III | -.009 | .022 | .052 | .000 | .042 |
| 20/40 | .006 | .000 | .058 | .000 | .078 |
| 50/100 | -.001 | .900 | .029 | .000 | .000 |
| Panel C: Performance, $\mu_{co}$ |
| 10/20 I | .390 | .000 | .215 | .109 | .079 |
| 10/20 II | .574 | .086 | .339 | .253 | .021 |
| 10/20 III | .845 | .041 | .005 | .988 | .034 |
| 20/40 | .235 | .123 | .166 | .444 | .010 |
| 50/100 | 2.716 | .158 | -1.624 | .101 | .024 |

Discussion  It appears that players do not mind much about their own bankroll relative to the bankrolls of the other players, In other words, the relative strength of their capital does not influence risk-taking a lot. This could reflect a negligence of the bankroll factor as a source of information, though it is a valuable source as the analysis of performance shows. Players who have more money than others tend to be more successful so that the stack having five times the bankroll of the average small stack. The regression results are underlined by the graphical representation in figure 11.1, where no particular relationship is evident.
Deviations of looseness (left graph) and aggressiveness (right graph) from average players’ looseness and aggressiveness, $\hat{L}_{co}$ and $\hat{A}_{co}$, are plotted against players’ level of perceived control as measured by their bankroll relative to the average bankroll of all opponents $co$. Solid lines are linear fits. The example is for the 10/20 I Limit.

Figure 11.1: Risk-Taking Depending on Perceived Control

The average amount won per hand is plotted against players’ level of perceived control as measured by their bankroll relative to the average bankroll of all opponents $co$. The example is for the 10/20 I Limit.

Figure 11.2: Performance Depending on Perceived Control
measure is even a measure of actual rather than perceived control. Here we also note the
difference between the analysis in a poker game and the experiment in Langer (1975).
Whereas poker is clearly a game of both skill and chance, where the illusion of control
only applies to a fraction of the game, the laboratory experiments do not involve any skill
elements.

In conjunction with the results of section 10.3 it seems that in poker the loss of control
takes a more important role than the illusion of control. Whereas there is a noteworthy
increase in both aggressiveness and looseness as capital runs dry, a sound reserve of capital
resources relative to the competitors does not lead to less conservative decision-making.

11.1.2 The Meta-Game Decision – Choosing Opponents

Method If players deliberately choose their opponents based on their relative strength
in capital, participants in any game will not be a random selection from the player popu-
lation. To test for non-randomness in the composition of players at a table we take
a random selection of games and note the bankroll of all players present in the game.
Enough games are selected to create a sample of 2,000 hands. This procedure is repeated
in order to give five samples on the distribution of bankrolls across players in a game.

Pairwise t-tests on the equality of means for the bankrolls are run, yielding ten statisti-
cs per gametype. As we have no hypothesis in which direction bankrolls will deviate
the null hypothesis to be rejected is that mean bankrolls are equal across samples. The
procedure is illustrated by the results for the 10/20 I Limit in figure 11.3. For each of the
five samples there is a distribution of bankrolls across the 2,000 hands in the sample. As
samples are not a random draw from the population of hands in the gametype but depend
on players sitting together in a game, there might be deviations between the distributions.

Results Setting a target level of significance of .05 the null hypothesis cannot be re-
jected for any of the tests in the 10/20 I Limit, 2 tests in the 10/20 II Limit, 1 (10/20
III), none (20/40), and none in the 50/100 Limit. Overall, this is not much more than
can be expected by chance and does not present enough evidence to support the idea of
players deliberately choosing their opponents.

Discussion In the IRC poker games, players have obviously not been particular about
whom they want to compete with. Due to the limited offer of games in the IRC era this
might not be what holds for the current situation in online poker. In IRC times there
CHAPTER 11. JUDGMENTS OF SKILL

For random samples of games the distribution of bankrolls across the participating players is graphed. The example is for the 10/20 I Limit.

Figure 11.3: Distribution of Bankrolls in Random Samples

were only three channels for low limit games, and mid and high limit games only offered a single table each. Therefore, selecting opponents was mostly about selecting the time when to play. One might hypothesize that this is a less preferred option for leisure-time players who probably care more about getting any action than no action at all. Finally, someone logging on to the channel probably wants to play and not maybe play.

Nowadays, one can easily choose among thousands of well populated tables with setups of all different kinds in online casinos. Here the effort to look for a game which is favorable regarding opponents might pay off. With the data at hand we leave this issue open for further discussion.

11.2 Overconfidence

How bold should gamblers bet? This is a central question for all those interested in games with elements of chance. It is quite intuitive that the answer depends on the gambler’s available capital and the edge he has got in the gamble at hand. The mathematical optimization answering the question has first been presented by Kelly (1956) and is known as the Kelly Criterion. The underlying concept and recommendation are discussed in the following section before the empirical tests on overconfidence building on this concept are presented.
11.2. AN APPLICATION OF THE KELLY CRITERION

For a gambler it is of interest to know how much to bet given a positive expectation gamble. This question is answered by the Kelly Criterion (Kelly 1956) which targets to maximize the expected value of the logarithm of wealth. To economists and in finance it is known by names like the geometric mean maximizing portfolio strategy, maximizing expected logarithmic utility, the growth-optimal strategy, the capital growth criterion, etc. (Thorp 1997, p. 1). It has been popular in areas like blackjack (e.g. Thorp 1997) or racetrack betting (e.g. Hausch, Ziemba, and Rubinstein 1981; Lo, Bacon-Shone, and Busche 1995; Gramm, McKinney, and Owens 2007), it applies to single, sequential or simultaneous games (Grant, Johnstone, and Kwon 2008), and has recently been proposed for uses in poker (Chin and Ingenoso 200X, 2006; Chen and Ankenman 2006).

The basic assumption is that the gambler’s utility function of wealth is logarithmic, i.e. $U(\Omega) = \ln \Omega$. Now rather than either maximizing the expected value from the gamble which would be to bet the maximum allowed and thus also maximizing the risk of ruin, or minimizing the risk of ruin by betting the minimum and thus also minimizing the expected return, the gambler targets at maximizing the change in logarithmic utility (Thorp 1997, pp. 3-4). Take for example, a bettor who has a bankroll of $10,000 and an edge of 4% in the bets he places, which means he will win 52% of the time and lose 48% of his bets. As he has a positive expectation from betting, maximizing the expected value from the next bet would mean to bet all his money. Then, however, in 48% of the cases he will be bankrupt after the next bet. If he wanted to minimize the risk of ruin he would not bet at all despite his considerable edge. Here, applying Kelly betting, gives the optimal bet size between those two extremes so that the bankroll after any of the outcomes offers the best utility for further activity. Formally, the bettor has to maximize

$$E(U(\Omega)) = 0.52(\ln(10,000 + x)) + 0.48(\ln(10,000 - x))$$

which we differentiate

$$E'(U(\Omega)) = \frac{0.52}{(10,000 + x)} - \frac{0.48}{(10,000 - x)}$$

and setting $E'(U(\Omega))$ equal to zero we find an optimum at

$$x = $400$$

$^2$That a logarithmic utility function of wealth is empirically valid has been shown by Friend and Blume (1975).
In a poker game the choice of bet sizes is narrow as a player only has the opportunity to move between games of different limits. More often it is the question with how much money to enter a game, a perspective for which we may use the Kelly optimization as well. We build on the ideas from Chen and Ankenman (2006, chapter 24) to show that with a distribution of outcomes \( x \) the Kelly Criterion says to maximize

\[
EU = E(\ln(\Omega + x)) - \ln \Omega \\
\Leftrightarrow \quad = E(\ln(1 + x/\Omega))
\]

and using the Taylor series expansion for \( \ln(1 + x/\Omega) \),

\[
\ln \left(1 + \frac{x}{\Omega}\right) = \frac{x}{\Omega} - \frac{x^2}{2!\Omega^2} + \frac{x^3}{3!\Omega^3} - \frac{x^4}{4!\Omega^4} + \ldots
\]

where we ignore the third and higher terms as it is reasonable to assume that a player’s bankroll is substantially larger than any single outcome, gives

\[
EU = E \left( \frac{x}{\Omega} - \frac{x^2}{2!\Omega^2} \right)
\]

As poker players are more concerned with risk and return in a game than outcomes of a single hand, we make two substitutions drawing from the definitions of expected value and variance, \( E(x) = \mu \) and \( E(x^2) = \mu^2 + \sigma^2 \),

\[
\Rightarrow \quad EU \approx \frac{\mu}{\Omega} - \frac{\mu^2 + \sigma^2}{2\Omega^2}
\]

Of this we take the derivative with respect to \( \Omega \)

\[
EU' = -\frac{\mu}{\Omega^2} + \frac{\mu^2 + \sigma^2}{\Omega^3}
\]

From here it is easy to solve for the optimal bankroll given risk and return

\[
\hat{\Omega} = \frac{\mu^2 + \sigma^2}{\mu}
\]

or alternatively, observing risk and bankroll and assuming that players have optimized their bankroll as above, we may consequently deduce the underlying return

\[
\hat{\mu} = \Omega/2 \pm \sqrt{\Omega^2 - 4\sigma^2}
\]
The additive solution may be dropped as return is unlikely to exceed half the player’s bankroll, leaving \( \hat{\mu} = \Omega/2 - \sqrt{\Omega^2 - 4\sigma^2} \). We can also use equation (11.1) to compare between two different gametypes to find the cutoff bankroll \( \hat{\Omega} \) when to move to the higher risk-return games

\[
\frac{\mu_1}{\Omega} - \frac{\mu_1^2 + \sigma_1^2}{2\Omega^2} = \frac{\mu_2}{\Omega} - \frac{\mu_2^2 + \sigma_2^2}{2\Omega^2}
\]

\[\Leftrightarrow \hat{\Omega} = \frac{\mu_1 + \mu_2 + \frac{\sigma_1^2 - \sigma_2^2}{\mu_1 - \mu_2}}{2} \quad (11.4)\]

Equations (11.2), (11.3) and (11.4) present a normative benchmark to the confidence with which players engage in the game. How this applies empirically is discussed in the following sections.

11.2.2 Comparing Expectations to Reality

**Method** Assume that a player knows about the risk-return trade-off he is running in a certain game. Then, if he maximizes his utility based on the Kelly Criterion he will play this game as long as his bankroll is close to the optimum. If his bankroll was larger, he would move to a higher risk-return game, if it was smaller, he would move to lower limits. From the historic data in the records we cannot observe players’ thoughts about their risks and returns but are able to notice their capital resources.

Equation (11.3) presents a framework how to deduct a measure of expected return from known wealth and risk. Wealth is easily obtained as the first bankroll with which players start their game on a specific limit. Chip stacks of $1,000 are excluded as these are likely not to be free choices but rather starting endowments. As for the second factor, risk, we assume that a player knows about the risks he is running in a certain gametype which we can calculate ex post.

So from a player’s first bankroll and overall risk in a gametype the expected return of a Kelly optimising gambler \( \hat{\mu} \) is inferred. To get a measure of overconfidence \( \hat{\mu} \) is

\[\hat{\sigma} = \sqrt{\mu_1 \Omega - \mu_2^2}; \text{ the negative solution to the quadratic expression has been dropped as risk may not be negative.}\]
compared to the actual distributions of returns across players.\textsuperscript{6} If on average players correctly expected their returns and consequently started with the optimal bankroll to maximize utility in line with the Kelly criterion, the measure of overconfidence would have a mean of .5.\textsuperscript{7} If however, they are overconfident, expecting returns better than average, the measure would exceed .5 up to a value of 1 indicating that everybody thinks he will be in the first percentile of players.

\textbf{Results}  In all gametypes players exhibit overconfidence. On average they are expecting to be in a higher ranking percentile of performance than warranted. For the 10/20 I Limit the distribution of expectations measured as actual percentiles of performance is plotted in figure 11.4. To illustrate, the figure reads as follows: about 3.8\% of the players start playing with a bankroll in a game where their ex post risks imply, that to be optimizing capital growth they must be expecting returns which are better than those achieved by 80\% of the player population. As a matter of fact, as only every second player is actually winning, everybody who is participating and thinking he will win must be expecting to have skills and/or luck above average.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure11.4.png}
\caption{Return Expectations in Terms of Actual Returns}
\end{figure}

The x-axis represents the percentiles of actual returns across players from worst to best. Against this the fraction of players is plotted who have been calculated to expect the according return in the 10/20 I Limit.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Gametype & Overconfidence Mean \hline
10/20 I Limit & 0.78 \hline
10/20 II Limit & 0.69 \hline
20/20 I Limit & 0.74 \hline
20/20 II Limit & 0.70 \hline
\end{tabular}
\caption{Distribution of Expectations for different limits}
\end{table}

\textsuperscript{6}Cf. section 5.2 where players’ risk-return profiles are discussed. Players with suspicious behavior and non-human players are excluded from the distribution.

\textsuperscript{7}The measure compares to the one used by Svenson (1981).
11.2. OVERCONFIDENCE

pronounced in the high 50/100 Limit.

Table 11.2: Implied Overconfidence from First Bankrolls
Mean and median values of the percentiles of the actual distribution of returns which is expected by players is shown per gametype.

<table>
<thead>
<tr>
<th>Gametype</th>
<th>Mean</th>
<th>p50</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/20 I</td>
<td>80.0</td>
<td>81</td>
</tr>
<tr>
<td>10/20 II</td>
<td>75.6</td>
<td>76</td>
</tr>
<tr>
<td>10/20 III</td>
<td>78.4</td>
<td>79</td>
</tr>
<tr>
<td>20/40</td>
<td>79.6</td>
<td>80</td>
</tr>
<tr>
<td>50/100</td>
<td>71.8</td>
<td>71</td>
</tr>
</tbody>
</table>

To compare the effects across limits density plots are graphed in figure 11.5. It is apparent that expectations at the low limits are more diverse than at the higher limits. In the middle 20/40 limit there are on the one hand fewer players who start with bankrolls that imply a high return and on the other hand also less players who are less confident in the range of the 50th to 70th percentile. For the high 50/100 limit the distribution is skewed left with a higher fraction of players expecting returns close to an average player.

Density plots around measures of overconfidence (cf. figure 11.4) are graphed by gametype.

Figure 11.5: Overconfidence at Different Limits

**Discussion**

Although the analysis consistently shows overconfidence in online poker in line with hypothesis 16, there are some caveats. First, the derivation assumes that players are optimizing capital growth, and second, that they have an idea about the risks they are facing. It might well be that for some players these assumptions do not hold. Players who
are minimizing the risk of ruin will appear less confident and those maximizing expected value will be seen as overconfident. Misperceptions of risk also dilute the results but can also be seen as a facet of overconfidence. Finally, expecting to run low risks where others incur high risks is also a belief in being better than average.

The grading in percentiles of actual returns across limits reveals that the dynamics of overconfidence are quite complex. From the low to the mid limit we see both an increase and a decrease in overconfidence. There is a larger fraction of players who show a “medium” overconfidence but there are less who exhibit “extreme” overconfidence. An explanation for this behavior might be that individuals participating in the middle limit games have had some successes bolstering their confidence, but at the same time have gained experience in capital management. By reaching the high limit games most players will know that it requires a sufficient bankroll (cf. equation (11.2)) to cushion the swings from the variability in outcomes.

11.2.3 Looking for Big Deals

Method As soon as players have gained the minimum required buy-in for higher stakes games they effectively have a choice between different portfolios or markets in which to invest. On the one hand, they can choose the known market with low stakes where they have experience and are likely to run lower risk. On the other hand, they may trade-up to the higher limit, take on the big boys and eventually gain larger returns though at the cost of increased risks.

In finance it is a well established practice to plot investments in a $\mu$-$\sigma$ chart, so as to compare returns and risks across opportunities. We do exactly the same for all players who are active at different limits, have not shown suspicious behavior at any limit and are human players. The choice of investment is easy whenever one option offers both higher returns and lower risk. Whenever higher returns come together with higher risk the choice is less clear. Equation (11.4) presents a normative solution to this choice, defining the bankroll which is large enough to cover the higher risk of ruin in the gametype with greater variance in outcomes.

To analyze the trade-off, deltas in risk and return are calculated pairwise between gametypes. If returns are higher and risks lower for the higher gametype, it strictly dominates the lower limit. If returns are lower and risks higher, it is obvious that the lower stakes are the better choice. Only if the advanced game either has higher returns and higher risks or—which is less likely but also possible—lower returns and lower risks, the choice problem has to be further analyzed. For these cases we calculate the optimal
cutoff bankroll following equation (11.4) and compare it to the average bankroll per hand with which the player is participating. All decisions to play at the high limit though it is a dominated investment alternative or to play with a bankroll too low are indications of overconfidence. Players who do this must think that they are better in either (or both of) the risk or the return dimension than they actually are.

Results An illustration of the investment comparisons players are facing is presented in figure 11.6. Every player indicated in the graph has (at least) the choice to either play in the 20/40 game or in the 10/20 I Limit. It is evident that only very few players manage to gamble for larger stakes and reduce variance in outcomes. At the same time only a minority of players actually earns additional profits for the increased risk exposure. For all players in the lower right quadrant playing for higher stakes is a dominated alternative. They would do better by staying at the lower limit.

For every non-suspicious human player differences in risk and return between the 50/100 Limit and the 20/40 Limit are scatter plotted. Players located in the top left quadrant have a dominating investment opportunity in the higher limit. Those found in the bottom right quadrant chose the dominated alternative by playing at the higher limit. For all other players optimal choice depends on the player’s wealth.

Figure 11.6: Risk-Return Comparison Between Investments

Counts of players by quadrant of the graphs analog to the one shown in figure 11.6 are stated in table 11.3. The results are similar across different pairs of gametypes. For every pair playing at the higher limit is the dominated alternative for the majority of players. Only for the step from the low to the middle limit does a noteworthy share of players manage to reduce risk.
Table 11.3: Investment Choices Between Limits

The number of players in every quadrant of a plot comparing risk and return between two gametypes are tabulated. There are, for example, 3,847 players who have lower returns but higher risk in the 20/40 Limit compared to their performance in the 10/20 I Limit.

<table>
<thead>
<tr>
<th>Gametypes</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>20/40 vs. 10/20 I</td>
<td>2,026</td>
<td>170</td>
<td>481</td>
<td>3,847</td>
</tr>
<tr>
<td>20/40 vs. 10/20 II</td>
<td>1,406</td>
<td>202</td>
<td>523</td>
<td>2,495</td>
</tr>
<tr>
<td>20/40 vs. 10/20 III</td>
<td>913</td>
<td>102</td>
<td>241</td>
<td>1,462</td>
</tr>
<tr>
<td>50/100 vs. 10/20 I</td>
<td>557</td>
<td>4</td>
<td>9</td>
<td>954</td>
</tr>
<tr>
<td>50/100 vs. 10/20 II</td>
<td>441</td>
<td>7</td>
<td>20</td>
<td>727</td>
</tr>
<tr>
<td>50/100 vs. 10/20 III</td>
<td>294</td>
<td>6</td>
<td>15</td>
<td>500</td>
</tr>
<tr>
<td>50/100 vs. 20/40</td>
<td>602</td>
<td>4</td>
<td>10</td>
<td>939</td>
</tr>
</tbody>
</table>

Figure 11.7 exhibits the analysis of the cutoff bankroll for those players found in quadrant I. For ease of interpretation a solid gray line represents equality between first bankroll or average bankroll, respectively, and the cutoff bankroll as of equation (11.4). A slight elevation in the left graph is seen for the first bankroll which is due to the minimum buy-in required, $5,000 for the 50/100 Limit. Players located below the intersecting line move to the higher limit with a bankroll too low or, as the case may be, stay in this gametype with a chip stack which is too low on average. Results are no different for graphs regarding all other paired comparisons.

For those players where the choice between the 50/100 and 20/40 Limit is not trivial the first bankroll with which play is started (left graph) and the average bankroll over all hands played (right graph) is plotted against the cutoff bankroll (cf. equation (11.4)).

**Figure 11.7: First and Average Bankroll Compared to Cutoff Bankroll**

**Discussion** The preceding analysis has highlighted one of the core skills to play poker successfully: bankroll management. It is essential to choose the right games in terms of risk and reward relative to one’s capital resources. If this is not done, the fundamental
variability in the game increases the risk of ruin, i.e. bankruptcy. Someone who engages in business involving capital stakes that pose substantial strains upon his wealth cannot endure temporal streaks of unfavorable outcomes. However, by deliberately adjusting the size of the wagers this risk is manageable.

With the majority of players having higher risk and lower returns at higher limits, and a noteworthy number of players who participate with too small bankrolls, decision-making behavior regarding bankroll management is far from optimal. There are several motives which might explain this fact. First, playing with the “big boys” can be a satisfying experience. In fact, there are many amateur players who spend large sums of money to play against poker pros although they are quite certain to lose. At least they can claim to have contested with the pros. Second, some of the findings may be due to experimenting and finding one’s boundaries. Risk and return are not known to the players ex ante and once they fulfill the requirements for moving to the higher limit they will have to try to find out about their performance against the selected crowd. After they have gotten their beating they might well move back to the smaller limits, but their poor performance remains in the records. This, however, does not explain why individuals are moving to higher limits with a bankroll too small. This is the third motive and heading for this analysis, overconfidence. Thinking they are better than average, decision-makers overestimate their skill relative to their competitors. They consequently misjudge risk and return potential and move to higher limits too eagerly as posed by hypothesis 17.
Part V

Conclusion
Chapter 12

Perspectives

You have to learn the rules of the game. And then you have to play better than anyone else.

Albert Einstein

12.1 Using Behavioral Psychology as an Edge in Poker

Throughout this thesis behavioral biases and heuristics in decision-making have been shown in the poker environment. The underlying ideas are not new but have repeatedly been shown to be present in risk-taking in the laboratory and other fields. Still, it is instructive to note that individuals show these systematic deviations in a setting which exclusively focuses on making better decisions than others to gain a monetary advantage.

Not much mention has been made of game theory because it is not required as a normative benchmark for the presented analysis. There are no such influences on the game-theoretic optimum as elapsed time between decisions, streaks of hands won, or experience in the game. For these and other factors we have shown systematic deviations in the kind of decisions that are taken. And this is exactly where behavioral psychology can create an edge in poker play and other competitive decision-making settings. Whenever your opponents show systematic shifts in their choices depending on irrelevant factors, this behavior is exploitable. Consequently, there are two ways in which to improve. The first is improving your own play. The second is adjusting play to take advantage of systematic flaws of your opponents.

Nobody is devoid of psychological influences on risk-taking behavior. Some might show biases in many areas, while for others the use of heuristics impacts only few judgments
under uncertainty. Be it as it may, everybody taking decisions under uncertainty should regularly reflect upon potentially distorting psychological factors. Some might become bored easily and make more risky choices in the process than usual. They should develop methods how to avoid or cope with boredom and its effects. Others, for example, who consistently overweight small probabilities could focus on mathematical concepts which help to improve judgments involving this kind of inputs.

The advanced player will not only master his own cognitive biases but know and notice them in opponents. It is advisable to pay attention to simple facts during the game. An example is with how many chips players come to the table, their likely reference point during the session. Winning or losing streaks, series of good or bad starting hands, only few big blinds in chips remaining, etc., all these are hints on the direction of a player’s likely tendency in risk-taking. From this notion one’s own decisions are easily adjusted to exploit the other player’s deviation. For the game theoretically best reaction the reader may refer to Chen and Ankenman (2006) who offer a comprehensive discussion of the topic. A simple rule is whenever someone plays too many hands, to also increase the share of hands played but less than the opponent, and vice versa.

12.2 Using Poker as an Edge in Behavioral Psychology

Poker has become increasingly popular over the last couple of years. While the data used in this thesis stems from one of the first online games ever played, there are now many more providers and thousands of players of the game who continuously take millions of decisions every day. As games are processed electronically data is generally available easily. The most important concern is confidentiality of the information and that providers are usually based in remote legislations.

Although this thesis covers a broad spectrum of topics from behavioral psychology, the full richness of the game has not been reached by far. In the following paragraphs some topics relevant for the game and with potential for further research are discussed briefly.

Disappointment and Regret Under certain circumstances poker players may learn of outcomes that might have been. For example, if they folded and the other player voluntarily shows his cards or if the remaining players go to showdown. In these cases thoughts like “I would have won that hand”, “Good that I folded”, or “I should have known better” readily come to mind. This notion of disappointment or regret if the outcomes
render a decision bad ex post or euphoria if it turns out to have been wise to take a
decision, can cause distortions in decision-making strategies. Some decision-makers may
be willing to pay a premium to avoid potential disappointment (Bell 1982; Loomes and
Sugden 1982; Bell 1985; Loomes and Sugden 1986). At an even more general level players
will be concerned with reducing cognitive dissonance. Information contradicting one’s
decision will be ignored or explained away (Akerlof and Dickens 1982).

**Escalation of Commitment and Sunk Costs** A poker hand involves sequential op-
portunities for decisions. In other words, it offers a dynamic decision-making environment.
Judgments in later rounds or phases might be influenced by an individuals prior actions.
More precisely, with sequential choices commitment to a chosen course of action might
escalate. The so-called sunk cost effect describes situations where people throw good
money after bad once they are invested in a project. The earlier investment, which could
be something like money, time, or another kind of effort, increases their tendency to
continue the undertaking in later stages (Staw 1976, 1981; Arkes and Blumer 1985).

**Mental Accounting** Thaler (1985, pp. 199-200) presents the following example and
discussion from a situation in a poker game

> “Mr. X is up $50 in a monthly poker game. He has a queen high flush and
calls a $10 bet. Mr. Y owns 100 shares of IBM which went up 1/2 today and
is even in the poker game. He has a king high flush but he folds. When X
wins, Y thinks to himself, ‘If I had been up $50 I would have called too.’ [...]”

A player’s behavior in a poker game is altered by his current position in that
evening’s game, but not by either his lifetime winnings or losings nor by some
event allocated to a different account altogether such as a paper gain in the
stock market.”

This is an example of mental accounting. People attribute money to different purposes
so that money in one account is not a perfect substitute for money in a different account.
Thus money loses an important economic criterion: fungibility (Thaler 1980; Henderson
and Peterson 1992; Thaler 1990, 1999). It is likely that mental accounting does not only
segregate money dedicated to poker play from other accounts but individuals might also
set a special account, say, to try out new variants of poker or for tournament play.

**Anchoring, Adjustment, and Framing** Once a stimulus has been presented to an
individual, all following judgments will be anchored on this event. For example, assess-

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1In a poker hand the optimal choice depends on past decisions as prior investments increase the size of
the pot which in turn reduces the share of hands which should be folded following game-theoretic optimal
play. The question is whether players fold even less than this share of hands.
CHAPTER 12. PERSPECTIVES

ing different weights as to their heaviness, the subjective scale is shifted depending on whether a light or heavy weight has been tested at first (Brown 1953). We might expect similar effects for poker players. Once they have been dealt a rather strong hand, like \( AA \), \( KK \), or \( QQ \), judgments in following phases, where the relative strength might change drastically depending on the cards on the board, will be hampered by the pre-flop strength anchor. The anchoring effect operates unintentionally and non consciously so that it is difficult to avoid even if forewarned (Humphrey 1996; Wilson, Houston, Etling, and Brekke 1996). In addition, the framing of the choice problem, for example, by adding (even hypothetical) alternatives or by highlighting different aspects of the gamble, can alter the preferred option (Tversky and Kahneman 1986; Tversky and Simonson 1993; Bazerman, Schroth, Shah, Diekmann, and Tenbrunsel 1994).

**Conjunction and Disjunction Fallacy**  Not so much in Texas Hold’em but more in games like Omaha High/Low, where two out of four private cards and three cards from the board may be used either to form the best or the worst hand to win, decision-makers have to compound several events in judging the prospects. It has been shown that people systematically overestimate the probability of compound events and underestimate disjunctive events (Bar-Hillel 1973; Tversky and Kahneman 1983; Tversky and Shafir 1992; Bar-Hillel and Neter 1993). Hence poker players should be overly willing to play hands which include several opportunities to draw. However, the effect is probably not very pronounced as judgments can be answered by counting favorable outcomes fairly easily, but as Tversky and Kahneman (1983, p. 310) write “Decomposition and calculation provide some protection against conjunction errors and other biases, but the intuitive element cannot be entirely eliminated from probability judgments outside the domain of random sampling.”

**Endowment Effect**  Why are some players, with the most prominent examples the so-called calling stations, reluctant to bet but call quite often? Amongst others one explanation could be a gap between their willingness to pay and willingness to accept. Once endowed with a particular hand, a bet by other players can be seen as an offer to sell, i.e. to give up the hand, whereas betting themselves would mean to pay for the gamble involving the other player’s hand. Procedural invariance between these two perspectives on a barter has been discussed widely in the literature. A recent review is presented by Plott and Zeiler (2005). Various experiments including some involving Las Vegas gamblers can be found in the articles of Slovic and Lichtenstein (1968, 1983); Lichtenstein and Slovic (1971, 1973).2

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2The endowment effect has raised substantial interest in the economic literature as the reversal of preferences due to the method of elicitation violates basic assumptions of economic theory (Grether and Plott 1979; Knetsch and Sinden 1984; Knetsch 1989; Kahneman, Knetsch, and Thaler 1990, 1991).
Status Quo and Omission Bias  Is it worse to call into a better hand or fold to a weaker hand? Do we prefer to act or maintain the status-quo by checking? Literature on the status quo and omission bias indicates that people disproportionately stick with the status quo and that reactions, for example elation or regret, are stronger following action than inaction (Samuelson and Zeckhauser 1988; Spranca, Minsk, and Baron 1991). In general as Ritov and Baron (1992, p. 60) put it “[...] omissions tend to be considered as the norm, and commissions tend to be compared to what would have happened if nothing had been done.”.3

Effort and Risk-Taking Strategy in Tournaments  Tournaments are quite different compared to cash games like the ones discussed in this thesis. In tournaments, payoffs are highly skewed with players eliminated early earning nothing while the bulk of the money is awarded to those finishing in the top places. This incentive structure requires specific adjustments to risk-taking strategy and committed effort (Rosen 1986; Dixit 1987). Lee (2004) and Grund and Gürtler (2005) similarly find that professional players choose the degree of risk-taking depending on the prize structure and that they are more sensitive to losses than to gains in tournaments.

Of course, these examples are not meant to provide an exhaustive list of further behavioral topics in poker, but hopefully, they provide an impression how vastly rich and complex such a basically simple game can be. Poker still offers substantial potential for further insights into decision-making behavior under competition and uncertainty.

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3A notion also discussed in the norm-theory by Kahneman and Miller (1986).
Appendix A

Details on the Database

Table A.1: Description of Variables in the Database

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Player nickname</td>
<td>3.2</td>
</tr>
<tr>
<td>Numactivity#</td>
<td>Total number of actions in gametype #</td>
<td>3.3.1</td>
</tr>
<tr>
<td>Numhands#</td>
<td>Total number of hands in gametype #</td>
<td>3.3.1</td>
</tr>
<tr>
<td>Numsessions#</td>
<td>Total number of sessions in gametype #</td>
<td>3.3.1</td>
</tr>
<tr>
<td>Numgametypes</td>
<td>No. of gametypes ever played</td>
<td>3.3.1</td>
</tr>
<tr>
<td>Isbot</td>
<td>Player is identified as bot</td>
<td>3.3.2</td>
</tr>
<tr>
<td>Amount_call#</td>
<td>Total of amounts called in gametype #</td>
<td>4.3</td>
</tr>
<tr>
<td>Amount_bet#</td>
<td>Total of amounts bet in gametype #</td>
<td>4.3</td>
</tr>
<tr>
<td>Amount_raise#</td>
<td>Total of amounts raised in gametype #</td>
<td>4.3</td>
</tr>
<tr>
<td>Seesflop#</td>
<td>No. of hands played at least to the flop in gametype #</td>
<td>4.3</td>
</tr>
<tr>
<td>Looseness#</td>
<td>Player’s looseness in gametype #</td>
<td>4.3</td>
</tr>
<tr>
<td>Aggress#</td>
<td>Player’s aggressiveness in gametype #</td>
<td>4.3</td>
</tr>
<tr>
<td>Numplayers#</td>
<td>Average number of opponents in gametype #</td>
<td>4.3</td>
</tr>
<tr>
<td>Cheat#</td>
<td>Boolean tag for suspicious behavior in gametype #</td>
<td>4.3.1</td>
</tr>
<tr>
<td>Lossfreq#</td>
<td>Percentage of hands lost (not won) in gametype #</td>
<td>4.3.1</td>
</tr>
<tr>
<td>Winfreq#</td>
<td>Percentage of hands won in gametype #</td>
<td>4.3.1</td>
</tr>
<tr>
<td>Qlooseness#</td>
<td>Decile in the looseness distribution for gametype #</td>
<td>4.3.2</td>
</tr>
<tr>
<td>Qaggress#</td>
<td>Decile in the aggressiveness distribution for gametype #</td>
<td>4.3.2</td>
</tr>
<tr>
<td>Cs#</td>
<td>Dummy for style “calling station” in gametype #</td>
<td>4.3.2</td>
</tr>
<tr>
<td>Ro#</td>
<td>Dummy for style “rock” in gametype #</td>
<td>4.3.2</td>
</tr>
</tbody>
</table>

Continued on next page ...
Table A.1 continued from previous page

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ma#</td>
<td>Dummy for style “maniac” in gametype #</td>
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</tr>
<tr>
<td>Sk#</td>
<td>Dummy for style “shark” in gametype #</td>
<td>4.3.2</td>
</tr>
<tr>
<td>Mu#</td>
<td>Average amount won per hand in gametype #</td>
<td>5.2.1</td>
</tr>
<tr>
<td>Sigma#</td>
<td>Standard deviation of amount won in gametype #</td>
<td>5.2.1</td>
</tr>
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<td>Numruns#</td>
<td>Number of runs of wins and losses (&lt;=0) in gametype #</td>
<td>8.3.1</td>
</tr>
<tr>
<td>Numsoftruns#</td>
<td>Number of runs of wins and losses (&lt;0) in gametype #</td>
<td>8.3.1</td>
</tr>
<tr>
<td>Softlossfreq#</td>
<td>Percentage of hands lost (&lt;0) in gametype #</td>
<td>8.3.1</td>
</tr>
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<td>Pl*win_?win#</td>
<td>Probability of win in a game with * players and ? prior wins in gametype #</td>
<td>8.3.2</td>
</tr>
<tr>
<td>Pl*win_?loss#</td>
<td>Analogous to above</td>
<td>8.3.2</td>
</tr>
<tr>
<td>Pl*loss_?win#</td>
<td>Analogous to above</td>
<td>8.3.2</td>
</tr>
<tr>
<td>Pl*loss_?loss#</td>
<td>Analogous to above</td>
<td>8.3.2</td>
</tr>
<tr>
<td>Pl*winfreq#</td>
<td>Percentage of hands won in games with * players in gametype #</td>
<td>8.3.2</td>
</tr>
<tr>
<td>Pl*softlossfreq#</td>
<td>Analogous to above</td>
<td>8.3.2</td>
</tr>
<tr>
<td>Pl*numhands#</td>
<td>Analogous to above</td>
<td>8.3.2</td>
</tr>
<tr>
<td>Cutoff_#</td>
<td>Optimal wealth for given risk and return in gametype #</td>
<td>11.2</td>
</tr>
<tr>
<td>Cutoff_#1_#2</td>
<td>Optimal cutoff bankroll to move from gametype #1 to #2</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Database on games

| Timestamp | Unix time stamp of the game | 3.2 |
| Gametype  | Reference number of gametype; see table 3.2         | 3.2 |
| Numplayers | No. of players dealt cards                      | 3.2 |
| Flop1rank | Rank of first card on flop                      | 3.2 |
| Flop1suit | Suit of first card on flop                       | 3.2 |
| Flop2rank | Rank of second card on flop                      | 3.2 |
| Flop2suit | Suit of second card on flop                      | 3.2 |
| Flop3rank | Rank of third card on flop                       | 3.2 |
| Flop3suit | Suit of third card on flop                       | 3.2 |
| Turnrank  | Rank of card on turn                            | 3.2 |
| Turnsuit  | Suit of card on turn                             | 3.2 |
| Riverrank | Rank of card on river                            | 3.2 |
| Riversuit | Suit of card on river                            | 3.2 |
| Straight  | Dummy for at least one straight shown at showdown  | 8.1.1 |

Continued on next page ...
<table>
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<tr>
<th>Variable</th>
<th>Description</th>
<th>Reference</th>
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</thead>
<tbody>
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<td>Flush</td>
<td>Dummy for at least one flush shown at showdown</td>
<td>8.1.1</td>
</tr>
<tr>
<td>Database on hands</td>
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<td></td>
</tr>
<tr>
<td>Timestamp</td>
<td>Unix time stamp of the hand</td>
<td>3.2</td>
</tr>
<tr>
<td>Name</td>
<td>Player nickname</td>
<td>3.2</td>
</tr>
<tr>
<td>Position</td>
<td>Position of player (starting at 1 for the small blind)</td>
<td>3.2</td>
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<td>Bankroll_hand</td>
<td>Player’s bankroll at start of hand in chips</td>
<td>3.2</td>
</tr>
<tr>
<td>Bankroll_depth</td>
<td>Player’s bankroll at start of hand in big blinds</td>
<td>3.2</td>
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<tr>
<td>Card1rank</td>
<td>Rank of player’s first hole card</td>
<td>3.2</td>
</tr>
<tr>
<td>Card1suit</td>
<td>Suit of player’s first hole card</td>
<td>3.2</td>
</tr>
<tr>
<td>Card2rank</td>
<td>Rank of player’s second hole card</td>
<td>3.2</td>
</tr>
<tr>
<td>Card2suit</td>
<td>Suit of player’s second hole card</td>
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</tr>
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<td>Amount_won</td>
<td>Absolute amount won by player less his own stake</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_wongross</td>
<td>Absolute amount won by player incl. his own stake</td>
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</tr>
<tr>
<td>Relative_won</td>
<td>Amount won relative to player’s bankroll</td>
<td>3.2</td>
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<tr>
<td>Relative_wongross</td>
<td>Gross amount won relative to player’s bankroll</td>
<td>3.2</td>
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<tr>
<td>Timelag</td>
<td>Time since last hand played in seconds</td>
<td>3.3.1, 10.1.1</td>
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<td>Total number of actions for the hand</td>
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<td>Serial number of player’s hands</td>
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<td>Numsession</td>
<td>Serial number of player’s sessions</td>
<td>3.3.1</td>
</tr>
<tr>
<td>Numhandsession</td>
<td>Serial number of player’s hand in current session</td>
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<td>Bankroll_session</td>
<td>Player’s bankroll at beginning of the session</td>
<td>7.2.1</td>
</tr>
<tr>
<td>Bankroll_reference</td>
<td>Player’s current bankroll relative to session start</td>
<td>7.2.1</td>
</tr>
<tr>
<td>Hand</td>
<td>Player’s hand if seen</td>
<td>8.2.1</td>
</tr>
<tr>
<td>Rk2</td>
<td>Rank of player’s hand from 1 to 169</td>
<td>8.2.1</td>
</tr>
<tr>
<td>Hirank</td>
<td>Dummy for high ranking hands</td>
<td>8.2.1</td>
</tr>
<tr>
<td>Lowrank</td>
<td>Dummy for low ranking hands</td>
<td>8.2.1</td>
</tr>
<tr>
<td>Cumhiranks</td>
<td>All high ranking cards ever shown by player</td>
<td>8.2.1</td>
</tr>
<tr>
<td>Cumlowlranks</td>
<td>All low ranking cards ever shown by player</td>
<td>8.2.1</td>
</tr>
<tr>
<td>Hiranks10</td>
<td>High ranking hands shown over last ten hands</td>
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</tr>
<tr>
<td>Lowranks10</td>
<td>Low ranking hands shown over last ten hands</td>
<td>8.2.1</td>
</tr>
<tr>
<td>Hiranks1</td>
<td>Same as above for last hand</td>
<td>8.2.1</td>
</tr>
<tr>
<td>Lowranks1</td>
<td>Same as above for last hand</td>
<td>8.2.1</td>
</tr>
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Table A.1 continued from previous page

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<tr>
<th>Variable</th>
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</thead>
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<td>Database on actions</td>
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<td></td>
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<td>Timestamp</td>
<td>Unix time stamp of the hand with the action</td>
<td>3.2</td>
</tr>
<tr>
<td>Name</td>
<td>Player nickname</td>
<td>3.2</td>
</tr>
<tr>
<td>Players_active</td>
<td>No. of players still actively contesting the pot</td>
<td>3.2</td>
</tr>
<tr>
<td>Numcheck</td>
<td>No. of players who checked before current player</td>
<td>3.2</td>
</tr>
<tr>
<td>Numcall</td>
<td>No. of players who called before current player</td>
<td>3.2</td>
</tr>
<tr>
<td>Numraise</td>
<td>No. of players who bet or raised before current player</td>
<td>3.2</td>
</tr>
<tr>
<td>Numallin</td>
<td>No. of players who all-in before current player</td>
<td>3.2</td>
</tr>
<tr>
<td>Potsize</td>
<td>Total size of the pot so far</td>
<td>3.2</td>
</tr>
<tr>
<td>Phase</td>
<td>Current phase (Pre-flop 1, Flop 2, Turn 3, River 4)</td>
<td>3.2</td>
</tr>
<tr>
<td>Round</td>
<td>Betting round for the player on a particular phase</td>
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</tr>
<tr>
<td>Last_action</td>
<td>True if this is a player’s last action for the hand</td>
<td>3.2</td>
</tr>
<tr>
<td>Act_if_call</td>
<td>No. of players still to act if the player calls</td>
<td>3.2</td>
</tr>
<tr>
<td>Act_if_raise</td>
<td>No. of players still to act if the player raises</td>
<td>3.2</td>
</tr>
<tr>
<td>Potodds</td>
<td>Amount player has to pay relative to potsize</td>
<td>3.2</td>
</tr>
<tr>
<td>Action_type</td>
<td>Type of action (e.g. small blind, check, bet, ...)</td>
<td>3.2</td>
</tr>
<tr>
<td>Bankroll_before</td>
<td>Player’s bankroll before the action</td>
<td>3.2</td>
</tr>
<tr>
<td>Bankroll_after</td>
<td>Player’s bankroll after the action</td>
<td>3.2</td>
</tr>
<tr>
<td>Handvalue</td>
<td>Value of the 5-card poker hand a player holds</td>
<td>3.2</td>
</tr>
<tr>
<td>Handvaluename</td>
<td>Name of the 5-card poker hand a player holds</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_SB</td>
<td>Amount player has paid for the small blind</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_BB</td>
<td>Amount player has paid for the big blind</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_call</td>
<td>Amount player has paid for a call</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_bet</td>
<td>Amount player has paid for a bet</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_raise</td>
<td>Amount player has paid for a raise</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_allin</td>
<td>Amount player has paid for an all-in</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_hand</td>
<td>Total amount player has so far paid during the hand</td>
<td>3.2</td>
</tr>
<tr>
<td>Amount_phase</td>
<td>Total amount player has so far paid during the phase</td>
<td>3.2</td>
</tr>
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Bibliography

Abdellaoui, M., F. Vossmann, and M. Weber, 2005, “Choice-Based Elicitation and De-
composition of Decision Weights for Gains and Losses Under Uncertainty,” *Management
Science*, 51(9), 1384–1399.


Control,” *Psychological Bulletin*, 82(4), 463–496.

Ajzen, I., 1977, “Intuitive Theories of Events and the Effects of Base Rate Information on


Anderson, G., and R. I. F. Brown, 1984, “Real and Laboratory Gambling, Sensation-


Attributional Effects and the Regressiveness of Prediction,” *Journal of Personality and
Social Psychology*, 53(3), 490–496.


Books.

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BIBLIOGRAPHY


