Bayesian Distribution System State Estimation in presence of non-Gaussian Pseudo-Measurements

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Abstract—Distribution System State Estimation (DSSE) is nowadays essential to enable the smart management of medium and low voltage grids. Due to the lack of a suitable measurement infrastructure, DSSE usually relies on the use of power injection pseudo-measurements derived from the knowledge of the historical and statistical behaviour of loads and generators. The uncertainty of these pseudo-measurements could not fit with the normal distribution typically considered in DSSE. For this reason, suitable approaches have to be designed both to model the pseudo-measurements uncertainty and to duly consider it in the DSSE process. This paper proposes a DSSE algorithm based on the Bayesian theory able to properly handle pseudo-measurements with any uncertainty distribution. The procedure used to cluster different categories of prosumers and to generate the pseudo-measurement parameters provided as input to the DSSE is also presented. Tests on a low voltage network show the applicability of the proposed approach and the associated benefits.

Index Terms—State Estimation, Bayesian Theory, Non-Gaussian uncertainty, pseudo-measurements, Distribution Grids.

I. INTRODUCTION

With the ongoing transformation of the distribution grids in active and complex systems, the need of more advanced tools for the management and control of Medium Voltage (MV) and Low Voltage (LV) grids is now well recognized [1], [2]. Similarly to the transmission systems, State Estimation (SE) will be the core of the Distribution Management Systems (DMS), providing the needed information on the network operating conditions to the different control applications. In last years, many research efforts have been dedicated to the problem of Distribution System State Estimation (DSSE). The main focus has been on the design of efficient SE algorithms able to handle the unbalanced nature and the radial/weakly meshed configuration of distribution grids [3]–[6].

Nevertheless, the development of DSSE still presents many challenges [7]. One of the main problems is the lack of suitable measurement infrastructures in the distribution systems. Such deficiency is commonly faced by considering the so-called pseudo-measurements, related to the load consumption or power generation at the nodes. Pseudo-measurements can be

derived from statistical or historical data associated to specific categories of customers and from forecasts about the expected level of Distributed Generation (DG). In [8], for instance, an Artificial Neural Network (ANN) trained with historical data is used to generate the pseudo-measurements for real-time DSSE.

A problem associated to the use of pseudo-measurements is that their reliability is usually low. As a consequence, such information has a large uncertainty that can lead to poor accuracy in the DSSE results. Another important issue concerns the correct modeling of this uncertainty. Most of the estimators proposed in the literature are based on the Weighted Least Squares (WLS) approach and use a Gaussian distribution assumption to model the uncertainty of all the input measurements. However, several studies show that the Gaussian distribution is generally unsuitable to represent the pseudo measurement uncertainties. In [9], for example, hourly data of load demand were statistically analyzed, bringing to the conclusion that lognormal or beta distributions are more appropriate to describe the uncertainty of load pseudo-measurements. Gaussian Mixture Models (GMM) have been also proposed for representing pseudo-measurements that do not fit with any specific distribution [10], [11]. Such kind of solution can be particularly useful since prosumers can have power profiles totally different from the standard ones [12].

In consideration of the fact that a non-Gaussian uncertainty distribution can be a realistic assumption, appropriate solutions need to be found for the suitable design of DSSE. In [13], pseudo-measurements are modelled through the GMM and an ANN is used to detect the GMM components to be considered as input to the DSSE algorithm. GMM is also adopted in [14], where DSSE is performed through multiple WLS runs combining the different GMM components.

In this paper, instead, a Bayesian estimator is proposed to deal with the non-Gaussian uncertainty of pseudo-measurements. The presented approach allows considering different kinds of probability density functions, even including those that cannot be easily represented through standard distribution models. Following this introduction, in Section II, the bases of the Bayesian theory relevant to the design of the conceived estimator are presented. Section III shows instead the procedure adopted to define values and probability distributions for the pseudo-measurements used in the DSSE

process. Results of tests performed on a small LV network are discussed in Section IV, highlighting the performance of the proposed estimator. Finally, conclusions are reported in Section V.

II. PROPOSED DSSE BAYESIAN ESTIMATOR

The classical measurement model for DSSE is:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e} \quad (1)$$

where $\mathbf{z}$ is the measurement vector, $\mathbf{x}$ is the generic vector of state variables (for instance voltage amplitudes and phase angles), $\mathbf{h}$ is the vector of measurement functions and $\mathbf{e}$ the random vector of measurement errors. The generic measurement z_i has its own probability density function $f_{z_i}(\cdot)$ that depends on the probability distribution that can be attributed to z_i . If normality is assumed, the following holds:

$$f_{z_i}(a) = \frac{1}{\sqrt{2\pi}\sigma_{z_i}} e^{-\frac{(a-m_{z_i})^2}{2\sigma_{z_i}^2}} \quad (2)$$

where m_{z_i} and σ_{z_i} are, respectively, the mean and standard deviation of z_i .

A. Bayes approach for DSSE

For a Bayesian estimation, the underlying idea is to estimate the state $\hat{\mathbf{x}}$ as the expectation of the posterior distribution:

$$\hat{\mathbf{x}} = \mathbb{E}[\mathbf{x}|\mathbf{z}] = \iint \mathbf{a} f_{\mathbf{x}|\mathbf{z}}(\mathbf{a}|\mathbf{z}) d\mathbf{a} \quad (3)$$

where the multidimensional integral is performed in the whole state space. Following the Bayes paradigm, the posterior is built as follows:

$$f_{\mathbf{x}|\mathbf{z}} = \frac{f_{\mathbf{z}|\mathbf{x}} f_{\mathbf{x}}}{f_{\mathbf{z}}} \quad (4)$$

where $f_{\mathbf{x}}$ represents the prior distribution of the state variables and $f_{\mathbf{z}|\mathbf{x}}$ is the likelihood function representing the conditional probability of the measurements given a certain state. In the context of DSSE, the measurements are represented by voltage, current or power measurements and can be obtained from both synchronized and traditional measurement devices. The real measurements are, as aforementioned, integrated with prior information on loads and generators power (see Section III). In particular, the statistical description of the absorbed (or generated) power can be obtained. The underlying probability distribution is usually non-Gaussian. This perfectly fits with the Bayes theory, since the aim becomes to estimate the state from the measurements along with their distribution with the aid of the prior information on the absorbed and generated power.

The proposed DSSE approach directly translates the Bayes equation (4), by considering as state vector the vector of power injections. To complete the state vector, it is necessary to include also a voltage node phasor in $\mathbf{x}$. Then it is possible to use the following state vector (in rectangular coordinates):

$$\mathbf{x} = \begin{bmatrix} V_s^r \\ V_s^x \\ \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \quad (5)$$

where $\mathbf{P}$ and $\mathbf{Q}$ are the active and reactive injection power (node power balance) vectors, while V_s^r and V_s^x are the real and imaginary parts of the voltage at the reference node (for instance the slack bus). It is important to underline that, when no synchronized measurements are available, only the module $V_s = |v_s|$ of the reference voltage v_s is needed. With such a state, and assuming as negligible the shunt admittance of the branches, the Bayes paradigm can be immediately translated.

Considering normally distributed measurements $\mathbf{z}$ the following holds true:

$$f_{\mathbf{z}|\mathbf{x}}(\mathbf{a}|\mathbf{b}) = \frac{1}{\sqrt{(2\pi)^M |\Sigma_{\mathbf{z}}|}} e^{-\frac{1}{2}(\mathbf{a}-\mathbf{h}(\mathbf{b}))^T \Sigma_{\mathbf{z}}^{-1} (\mathbf{a}-\mathbf{h}(\mathbf{b}))} \quad (6)$$

where, for M independent measurements, $\Sigma_{\mathbf{z}}$ is diagonal and $\Sigma_{i,i} = \sigma_{z_i}^2 = \sigma_i^2$:

$$f_{\mathbf{z}|\mathbf{x}}(\mathbf{a}|\mathbf{b}) = \prod_{i=1}^M \frac{1}{\sqrt{2\pi}\sigma_{z_i}} e^{-\frac{(a_i-h_i(\mathbf{b}))^2}{2\sigma_{z_i}^2}} \quad (7)$$

As aforementioned, $h_i(\mathbf{x})$, for a given state, depends on the available measurements. Power injection or flow measurements, for instance, are easily obtained from the state since power injections are available. As a matter of course, it is possible to use an equivalent set of measurements $\mathbf{z}_{eq}$, if helpful to obtain simpler measurement functions. It is important to recall that equivalent measurements can alter the conditional density $f_{\mathbf{z}|\mathbf{x}}$, but for real measurements (small uncertainties) normal approximation, with error propagation, can still hold. Besides, when considering equivalent measurements, the covariance matrix changes into $\Sigma_{\mathbf{z}_{eq}}$.

Once $\mathbf{h}_{\mathbf{z}}$ is clearly defined, it is easy to compute the first member of the numerator in (4), given a point in the state space, as it is necessary in numeric integration (3).

The prior $f_{\mathbf{x}}$ can be easily deduced by the statistical knowledge of $\mathbf{P}$ and $\mathbf{Q}$, either by a model matching with known distributions or in an empirical way, by histograms or empirical distribution fitting (kernel based for instance). For the statistical load or generator data, an analytical expression or an empirical distribution is obtained (and pre-stored) and the prior can thus be numerically computed for each injection/absorption. Some assumptions can be made, depending on the real network operation. For instance, if node prior independence can be assumed, the overall prior probability density of the state can be factorized. Otherwise, a proper modeling of correlation should be considered, as in [15], and included in the overall prior evaluation. The prior for v_s can be easily assumed as uniform in a given interval (± 0.1 p.u., for instance).

The posterior is thus obtained, up to a multiplicative factor represented by the denominator in (4), which, as it will be described better in the following, is not relevant for the numerical algorithm used for the solution. Intuitively, the denominator, given a set of measurements, is a constant and does not depend on the state and thus is not relevant for the space exploration necessary to achieve the estimation.

B. Numerical integration of Bayes estimation

The computation of the average in (3) requires a numerical evaluation. In this paper, a Markov Chain Monte Carlo (MCMC) approach is adopted and, in particular, a Metropolis-Hastings (M-H) algorithm was chosen [16]. Under this paradigm, a sequence of points $\mathbf{x}$ is obtained numerically, as a sample chain, from a defined probability distribution $\Gamma(\mathbf{x})$. The algorithm allows extracting a set of points that has the desired statistical properties, as if the points were directly extracted from the probability distribution.

The method applies to the DSSE Bayesian paradigm of this paper because each point is a state in the multidimensional state-space and the target distribution $\Gamma(\mathbf{x})$ corresponds to the posterior $f_{\mathbf{x}|\mathbf{z}}$ in (4) and needs to be known only up to a normalizing constant, thus avoiding the calculation of the so-called evidence value $f_{\mathbf{z}}$.

The M-H algorithm works starting from a given starting point $\mathbf{x}_{start}$. In this paper, it is represented by the set of the nominal values: nominal voltage at slack bus and nominal active and reactive powers at each node. For each iteration k , a new proposal state point $\mathbf{y}$ is extracted from a given proposal transition distribution $\Pi(\mathbf{x}, \mathbf{y})$, starting from the actual point $\mathbf{x}_k$. The new point $\mathbf{y}$ is retained with an acceptance probability:

$$A_k = \min\left\{1, \frac{\Gamma(\mathbf{y})\Pi(\mathbf{x}_k, \mathbf{y})}{\Gamma(\mathbf{x}_k)\Pi(\mathbf{y}, \mathbf{x}_k)}\right\} \quad (8)$$

The next point in the chain thus becomes $\mathbf{x}_{k+1} = \mathbf{y}$ if accepted, otherwise $\mathbf{x}_{k+1} = \mathbf{x}_k$ is kept. Under a few constraints, the obtained Markov chain converges to $\Gamma(\mathbf{x})$, regardless of the initial point, and is ergodic. For this reason, the sequence of state points can be used to compute different moments and statistics of the desired posterior distribution. From (8) it is clear that, when the proposal distribution P is symmetric ($\Pi(\mathbf{y}, \mathbf{x}) = \Pi(\mathbf{x}, \mathbf{y})$) the acceptance probability is independent from the distribution itself. In this paper a multivariate normal distribution centered on the current state point $\mathbf{x}_i$ is used. The covariance matrix of such multivariate distribution is tuned to allow a faster and more complete exploration of the state space. There are different approaches to have an adaptive proposal extraction and in this paper a custom tailored version of the algorithm proposed in [17] has been adopted to speed up the procedure while limiting the computational load.

Once n points in the state space are obtained, the DSSE estimation is given by the average:

$$\hat{\mathbf{x}} = \frac{1}{n - i_0 + 1} \sum_{i=i_0}^n \mathbf{x}_i \quad (9)$$

where i_0 is the number of points in the so-called initial burn-in period. Due to the property of the extracted samples, the obtained average is the numerical computation of (3) and thus represents the estimation of the expected state given the prior and measurement information.

Fig. 1. Smart Meter connection schemes

III. STATISTICAL ANALYSIS OF POWER PROFILES

State estimation, in distribution grids, is expected to exploit, besides SCADA measurements at primary and secondary substations, mainly active and reactive power measurements at the nodes.

Smart Meters (SMs), connected at the terminals of industrial and residential users, provide active and reactive power injection measurements; or at least periodic energy measurements, from which, knowing the integration time, it is possible to infer the average active power.

In case also active resources, such as photovoltaic (PV) units, are installed, an additional SM (here called SM-PV) is connected at the inverter terminals. Therefore, the SM at the Point of Dispatch (SM-PoD) of the user measures the total power consumption. Thus, the passive load can be obtained as the difference between the generated power at the SM-PV and the total power consumption from the network given by the SM-PoD. The SMs connection schemes for the previous cases are shown in Fig. 1.

When power measurements are not directly measured by devices, statistical knowledge of the electrical users, may be exploited to build the pseudo-measurements, which are expected power values (in watt for active and VAR for reactive) over time.

A possible approach, exploited in the test and results section of this paper, is to install SMs in a subset of the electricity users [12]. Therefore, it would be possible to gather the statistical information on the expected power consumptions/generations and their uncertainty for the remaining users.

Power profiles in distribution grids represent the combination of power consumption by residential and industrial customers and power generation from distributed resources of PV and wind farms. In this paper the focus will be on residential customers with similar electrical appliances and nominal power. Together with the knowledge of the expected value of active and reactive power it is important to evaluate correctly the expected uncertainty, especially when applying DSSE methods. The justification of this study lies in the possible non-Gaussian behavior of power consumption and generation and the resultant power injection at the node. This is more relevant for LV cases, in which a node represents a single electrical user, rather than for MV cases, where many consumers yield to a total power consumption which is statistically closer to a Gaussian distribution [12].

Fig. 2. Comparison of PV power production histograms at 14:00 between summer and winter days

Comparing different daily power profiles suggested that, in each time instant the power generation/consumption (or alternatively the difference of the two) may have a probability distribution, whose pattern should be evaluated for proper integration into the SE as pseudo-measurement. The time resolution chosen for this study is 15 minutes, since the large uncertainty of power consumption and generation associated to pseudo-measurements, higher rates will not bring any advantage. The patterns are differentiated for seasons (winter, middle season, summer), weekdays (working days, Saturdays and Sundays) and, of course, time of the day. In the following paragraphs, the histogram of load and PV active powers (reactive powers are here skipped for the sake of brevity), extracted from SM measurements in the UNARETI (Distribution System Operator in the provinces of Brescia and Milan, Italy) distribution grid, over 2015 and first two months of 2016, will be exploited in order to verify their statistical features and apply the pseudo-measurements modeling.

In Fig. 2 the histogram for PV power production in winter and summer at 14:00 are compared. Similarly, the load histograms are presented in Fig. 3. In Fig. 4 the comparison of the load histograms between summer working day and Sunday at 14:00 is presented. The histograms of two different hours of the day are compared in Fig. 5 with regards to 02:00 and 14:00 of a summer working day.

Some observations could be made based on the previous figures. The load profile changes with day of the week, time of the day and slightly for the season, but maintains a typical exponential distribution pattern. For the case of PV power profile, the season impacts drastically also the pattern of the distribution; similar effects are brought by the time of the day, even if not shown in this paper. The histograms shown are then exploited as probability distribution functions for the pseudo-measurements. For this paper, for instance, the kernel distribution fitting with Epanechnikov kernel was used. The comparison between the histogram and the probability density functions (pdf) is shown for load, PV for a typical summer working day at 14:00 in Figs. 6 and 7.

Fig. 3. Comparison of load power consumption histograms at 14:00 between summer and winter days

Fig. 4. Comparison of load power consumption histograms at 14:00 working day and Sunday of summer

Fig. 5. Comparison of load power consumption histograms at 02:00 and 14:00 of a summer working day

IV. TESTS AND RESULTS

The network tested is a LV grid of UNARETI. The model, in terms of topology (Fig. 8) and lines' parameters (resistance, reactances and capacitances) is replicated accurately in Matlab environment. However, the simulation is run only on phase A, therefore it will consider only phase A cables and customers / PV units. 15 of the 16 buses of the LV grid are electrical cabinets where the residential users are connected.

The users and the PV units of this network have been

Fig. 6. Comparison between load power consumption histogram and pdf at 14:00 of a summer working day

Fig. 7. Comparison between PV power production histogram and pdf at 14:00 of a summer working day

Fig. 8. Topology of UNARETI LV grid

provided with SMs, as shown in 1, therefore the observations collected over 2015 and the first months of 2016, may be used to create realistic simulation conditions to test the state estimator based on Bayesian approach.

In the following, the reported results correspond to a specific configuration that considers more customers and PV units connected to each network node. In particular, the users' configuration and the monitored nodes and branches reported in Table I are adopted to illustrate the outcomes of the proposed method. The customers have a nominal power contract equal

TABLE I
NUMBER OF CUSTOMERS AND PV UNITS FOR EACH NODE AND MEASUREMENT CONFIGURATION

Node	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Customers	3	1	3	1	4	4	2	3	1	3	3	2	1	2	1
PV units	1	2	2	2	2	2	1	2	3	2	1	2	1	2	2
Measurements	V mag: node 1. Active/Reactive power flows: branches 1, 5, 7, 10

a)

b)

Fig. 9. Comparison between prior and estimated posterior of active power injections: a) node 15 b) node 9

to 4.95 kW; the PV generation units have a nominal power equal to 1.29 kVA.

The measurements have been chosen normally distributed with a standard deviation equal to one third of the accuracy. The adopted accuracies are 1 % for voltage magnitude and 3 % for active and reactive power flow. Other measurement types, and among the others synchronized measurements [18], [19], can be considered as well. All the results in the following refer to 14:00 of the working days during summer.

In Figs. 9(a) and 9(b) the results of the MH algorithm execution are reported in terms of active power injection for nodes 15 and 9 respectively. The figures compare the pdf of the prior with the pdf of the posterior obtained by the markov chain points for each node. Fig. 9(a), which refers to a node in an unmonitored feeder, shows that the posterior matches the prior as expected, whereas Fig. 9(b) shows how the posterior can include the information due to the measurements and lead to an estimated power injection that is different from the mean of the prior, that is from the pseudo-measurement usually chosen for DSSE. The non-Gaussianity of the posterior confirms how the proposed method allows having a better description of the estimation uncertainty.

Fig. 10 reports the branch active power flow estimates along with the associated error bar computed by means the 0.025 and

Fig. 10. Active branch power flows estimates

the 0.975 quantiles of the corresponding posterior distributions (to reach a 95% confidence interval). It is clear how the reference conditions are always included in the uncertainty intervals that vary with the characteristics of the probability distribution. In fact, the uncertainty is very low for monitored branches, while it increases for branches that are far from the measurement points. The uncertainty interval is noticeably asymmetric with respect to the estimation, reflecting the non-Gaussian behavior of the estimates. For this reason, the proposed method is particularly well-suited for the estimation of the DSSE output uncertainties. For comparative purposes, the corresponding results of the WLS estimator in [6] are reported. The WLS algorithm is fed with the averages of the priors as pseudo-measurements and it relies on the assumption of Gaussian pseudo-measurements for the evaluation of uncertainty. As it can be observed, the WLS algorithm estimates always have symmetric uncertainty intervals, which in several branches are larger than those provided by the presented Bayesian solution.

Finally, Fig. 11 shows the voltage profile estimates along with the confidence intervals obtained as above. Voltages, which show quite a flat profile due to the low network load, have uncertainties that are strictly linked to the number of voltage measurements, as already demonstrated in [20].

Fig. 11. Voltage profile estimate

V. CONCLUSIONS

The paper presents a Bayesian state estimator for distribution networks that relies on non-Gaussian prior information

on the power absorption/consumption of customers/generators. Statistical data derived from real measurements obtained by SMs actually monitoring some electrical users are used. The validity of the estimator is tested on an example of LV network. The presented results highlight, in particular, the estimator usefulness in the evaluation of the uncertainty associated to the estimated quantities.

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