On right conjugacy closed loops and right conjugacy closed loop folders - Data of the small RCC loops

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The data of the non-associative RCC loops of order \( n \) with \( n \in \{ 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30 \} \) is available with the thesis *On right conjugacy closed loops and right conjugacy closed loop folders*. The data is stored in ASCII format and is readable independently of the computer algebra system GAP. But we will illustrate how to read them with GAP. For more information about GAP see [Gap]. For each of the above \( n \) there is one file, named RCCLoops\( n \), i.e. there are the files RCCLoops6, RCCLoops8, ... , RCCLoops30. These files contain encodings of the envelopes of the non-associative RCC loops of order \( n \).

Fix one of the \( n \) above and consider the file RCCLoops\( n \). It contains two lists named grps and classlists. Let \( nr_{RM} \) be the number of groups which are right multiplication groups of a non-associative RCC loop of order \( n \). Then each of the lists grps and classlists contains exactly \( nr_{RM} \) entries; one for each of these groups.

Each entry of the list grps encodes one group \( G \) (as a subgroup of Sym(\( n \))) which is the right multiplication group of a non-associative RCC loop of order \( n \). An entry of the list grps is a list, called conj, of lists. This list conj corresponds to the conjugacy classes of the group \( G \). For example let \( n = 9 \). The second entry of the list grps is:

\[
\text{conj} = \left\{ \\
\begin{array}{l}
(\{ () \}) , \\
(\{ (2,5,8)(3,9,6) , (1,4,7)(2,8,5) , (1,7,4)(3,6,9) \} , \\
(\{ (2,8,5)(3,6,9) , (1,4,7)(3,9,6) , (1,7,4)(2,5,8) \} , \\
(\{ (1,2,3,4,5,6,7,8,9) , (1,5,9,4,8,3,7,2,6) , (1,8,6,4,2,9,7,5,3) \} , \\
(\{ (1,2,6,4,5,9,7,8,3) , (1,5,3,4,8,6,7,2,9) , (1,8,9,4,2,3,7,5,6) \} , \\
(\{ (1,2,9,4,5,3)(7,8,6) , (1,5,6,4,8,9,7,2,3) , (1,8,3,4,2,6,7,5,9) \} , \\
(\{ (1,3,8,7,9,5,4,6,2) , (1,6,5,7,3,2,4,9,8) , (1,9,2,7,6,8,4,3,5) \} , \\
(\{ (1,3,5,7,9,2,4,6,8) , (1,6,2,7,3,8,4,9,5) , (1,9,8,7,6,5,4,3,2) \} , \\
(\{ (1,3,2,7,9,8,4,6,5) , (1,6,8,7,3,5,4,9,2) , (1,9,5,7,6,2,4,3,8) \} , \\
(\{ (1,4,7)(2,5,8)(3,6,9) \} , \\
(\{ (1,7,4)(2,8,5)(3,9,6) \}
\end{array}\right).
\]

This means the group \( G \) has eleven conjugacy classes. Clearly \( G \) as a set is the union of its conjugacy classes.
Corresponding to each group \( G \) encoded in the list `grps` there is an entry in the list `classlists`. This entry is a list, called \( R \), of lists of integers. For example let \( n = 9 \). The second entry of the list `classlists` is

\[
[ [ 1, 4, 7, 10, 11 ], [ 1, 4, 9, 10, 11 ] ]
\]

Each list \( t \) in \( R \) corresponds to a transversal \( T \), such that \( (G, H, T) \), where \( G \) is the group of the corresponding entry in the list `grps` and \( H = \text{Stab}_G(1) \), is the envelope of a non-associative RCC loop of order \( n \). Here \( T \) is the union of the conjugacy classes of \( G \) whose numbers are in \( t \).

We illustrate how to read these files with GAP:

```gap
gap> Read("RCCLoops9");
gap> Length(grps);
3
gap> j:=2;;
gap> conj:=grps[j];
[ [ () ],
  [ (2,5,8)(3,9,6), (1,4,7)(2,8,5), (1,7,4)(3,6,9) ],
  [ (2,8,5)(3,6,9), (1,4,7)(3,9,6), (1,7,4)(2,5,8) ],
  [ (1,2,3,4,5,6,7,8,9), (1,5,9,4,8,3,7,2,6), (1,8,6,4,2,9,7,5,3) ],
  [ (1,2,6,4,5,9,7,8,3), (1,5,3,4,8,6,7,2,9), (1,8,9,4,2,3,7,5,6) ],
  [ (1,2,9,4,5,3,7,8,6), (1,5,6,4,8,9,7,2,3), (1,8,3,4,2,6,7,5,9) ],
  [ (1,3,8,7,9,5,4,6,2), (1,6,5,7,3,2,4,9,8), (1,9,2,7,6,8,4,3,5) ],
  [ (1,3,5,7,9,2,4,6,8), (1,6,2,7,3,8,4,9,5), (1,9,8,7,6,5,4,3,2) ],
  [ (1,3,2,7,9,8,4,6,5), (1,6,8,7,3,5,4,9,2), (1,9,5,7,6,2,4,3,8) ],
  [ (1,6,7)(2,5,8)(3,6,9) ],
  [ (1,7,4)(2,8,5)(3,9,6) ] ]

gap> R:=classlists[j];
[ [ 1, 4, 7, 10, 11 ], [ 1, 4, 9, 10, 11 ] ]
gap> transversals:=List(R, t->Concatenation(grps[j]{t}));;
gap> for T in transversals do Sort(T); od;
gap> transversals;
[ [ (), (1,2,3,4,5,6,7,8,9), (1,3,8,7,9,5,4,6,2), (1,4,7)(2,8,5)(3,6,9), (1,5,9,4,8,3,7,2,6), (1,6,5,7,3,2,4,9,8), (1,7,4)(2,8,5)(3,9,6), (1,8,6,4,2,9,7,5,3), (1,9,2,7,6,8,4,3,5) ],
  [ (), (1,2,3,4,5,6,7,8,9), (1,3,8,7,9,5,4,6,2), (1,4,7)(2,8,5)(3,6,9), (1,5,9,4,8,3,7,2,6), (1,6,8,7,3,5,4,9,2), (1,9,5,7,6,2,4,3,8) ] ]
```

Then each of the triples \( (G, H, T) \) with \( H = \text{Stab}_G(1) \) and \( T \) in `transversals` is the envelope of a non-associative RCC loop of order \( n \). Further, let \( C \) be an isomorphism class of non-associative RCC loops of order \( n \) with right multiplication group \( G \). Then there is exactly one \( T \) in `transversals` such that \( (G, H, T) \) with \( H = \text{Stab}_G(1) \) is the envelope of a representative of \( C \).

References