Autoregressive Modeling of Cycle-to-Cycle Correlations in HCCI Combustion

Jakob Andert¹, Maximilian Wick¹, Bastian Lehrheuer², Christian Sohn¹, Thivaharan Albin³ and Stefan Pischinger²

Abstract
Homogenous Charge Compression Ignition (HCCI) or Gasoline Controlled Auto Ignition (GCAI) combustion is characterized by a strong coupling of consecutive cycles, which is caused by residuals from the predecessor cycle. Closed-loop combustion control is considered a promising technology to actively stabilize the process. Model-based control algorithms require precise prediction models that are calculated in real time. In this article, a new approach for the transient measurement of the autoignition process and the data-driven modeling of combustion phasing and load is presented. GCAI combustion is modeled as an autoregressive process to represent the cycle-to-cycle coupling effects. The process order was estimated by partial autocorrelation analysis of steady-state operation measurements. No significant correlations are found for lags that are greater than one. This observation is consistent with the assumption that cycle coupling is mainly caused by the amount of exhaust gas that is directly transferred to the consecutive combustion. Because steady-state operation results in a hard coupling of actuation and feedback variables, only minor variations of the test data can be achieved. The steady-state tests delivered insufficient data for the generalized modeling of the transient autoregressive effects. A new transient testing and measurement approach is required, which maximizes the variation of the predecessor cycle’s characteristics. Dynamic measurements were performed with the individual actuation of the injection strategy for each combustion cycle. A polynomial model is proposed to predict the combustion phasing and load. The regression analysis shows no overfitting for higher polynomial orders; nevertheless, a first-order polynomial was selected because of the good extrapolation capabilities of extreme outliers. The prediction algorithm was implemented in Matlab/Simulink and transferred to a real-time-capable engine control unit. The verification of the approach was performed by test bench measurements in dynamic operation. The combustion phasing was precisely predicted using the autoregressive model. The combustion phasing prediction error could be reduced by 53% in comparison to a state-of-the-art mean value-based prediction. This work provides the basis for the development of a closed-loop autoregressive model-based control for GCAI combustion.

Keywords
HCCI, GCAI, cycle-to-cycle correlations, transient measurements, autoregressive modeling, design of experiments, test bench

Introduction
Homogenous Charge Compression Ignition (HCCI), which is often referred to as Gasoline Controlled Auto Ignition (GCAI), is characterized by a homogenous fuel/air mixture that is auto-ignited near the top dead center (TDC). In the last decades, numerous researchers have proceeded to work on this combustion system. It was found to have several benefits over conventional gasoline engines, such as lower fuel consumption at part load and minimal NO\textsubscript{x} raw emissions and particulates.¹,²,³

Although GCAI offers significant benefits in comparison to the conventional throttled spark ignition, the particular characteristics cause limitations in the operation range. Schäfflein,⁴ Adomeit,⁵ Kulzer³ and Xie⁶ investigated the possible operation range using different research engines. A comparison of their results is visualized in a load/speed diagram in figure 1. Apparently, the GCAI operation ranges show a considerable limitation in comparison to typical natural aspirated or turbocharged gasoline engines.⁷

GCAI combustion phasing heavily depends on boundary conditions such as the in-cylinder pressure and temperature history, stratification, EGR rate and chemical kinetics of the fuel components. Because of the absence of a direct combustion trigger, the combustion timing can only be indirectly controlled and requires a closed-loop control.²,⁸,⁹,¹⁰,¹¹ In addition to the limited operation range, cyclic variations and combustion instability have been identified as the most

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challenging aspect of GCAI combustion. To better understand the underlying fundamentals, a detailed analysis of the chemical kinetic mechanisms and 3D computational fluid dynamics (CFD) simulations has been performed.

In recent years, characteristic cycle-to-cycle correlations were intensely examined. Deterministic sequences following a stimulus by a step in operation conditions were described by Schäflein. Lundstrom and Seebach applied auto-correlation functions to the indicated load of consecutive cycles and identified linear dependencies at several operation points. The return maps were used to characterize the bifurcative combustion behavior and identify the deterministic dependencies of consecutive cycles.

Recent works show that modeling and prediction of the cycle-to-cycle behavior are the key issues to guarantee a stable HCCI operation under varying conditions. The potential of FPGA based realtime analysis for decoupling of consecutive cycles has been identified.

The bifurcative behavior could successfully be demonstrated on a cycle-to-cycle basis. This approach was extended by applying extreme learning machines by Vaughan. One crucial point of this implementation is the ability to extrapolate from measured data.

For a model-based control of GCAI combustion, a detailed modeling of the underlying effects on a cyclic base is required. The prediction algorithm faces two main challenges. First, a model is required, which accurately represents the nonlinear behavior and can be executed in real-time. Second, for model training, sufficient measurement data must be acquired, which include extreme outliers and unstable cycles.

The conventional approach for test bench measurements is steady-state operation in the time domain and a multidimensional optimization of numerous parameters e.g. by design-of-experiments. The goal of this paper is to demonstrate a different approach by maximizing the variability in the time domain. Measurement data are generated from the strong dynamic excitation of the engine process by varying the injection strategy on a cyclic base. A systematic proceeding for data-driven autoregressive modeling on a cycle-to-cycle base is presented. However, for this feasibility study the number of input or excitation signals is reduced to a minimum. The autoregressive model is implemented in a real-time system and validated by comparing to a state-of-the-art mean value-based prediction.

**Experimental Setup**

The investigations were performed on a single-cylinder engine (SCE) with electromechanical valve train (EMVT), which enabled fully variable valve timing to control the necessary residual gas fraction for the GCAI combustion (see figure 2). The engine was equipped with a piezo-electrically actuated, outwards-opening hollow cone injector in central position which enabled small pilot or post-injection quantities as additional control variables. The geometrical compression ratio of the SCE can be adapted and was set to CR = 12 for all measurements.

The engine was operated at constant boundary conditions on a special test bench, which was used for thermodynamic investigations. Therefore, the intake manifold temperature was conditioned to $T_{int} = 35^\circ\text{C}$ by an electrical air-heating device. The engine temperature (oil and coolant) was constant at $T_{oil} = T_{cool} = 90^\circ\text{C}$. The intake and exhaust pressures were controlled to be $p_{int} = p_{exh} = 1013\text{ mbar}$. The experiments were performed with conventional European RON 96 gasoline, which contained 10 % ethanol.

For the thermodynamic postprocessing, the cylinder pressure must be measured. A piezoelectric pressure transducer (Kistler A6061B), which was flush-mounted in the combustion chamber roof between the intake and the side, was used. The dynamic intake and exhaust gas pressures were measured with piezoresistive pressure transducers.

**Figure 2.** Single-cylinder research engine with electromechanical valve train
transducers (Kistler 4045 A5). Sampling was performed via Kistler charge amplifiers in combination with an 'FEV Combustion Analysis System' (CAS) in an angular resolution of 0.1 °CA. For this investigation, the engine was operated without external EGR at an engine speed of \( n = 2500 \text{ 1/min.} \) Further parameters of the single cylinder research engine are listed in table 1.

### Process Characterization

A good overview of the required steps for nonlinear system identification is provided in Nelles.\(^3\) The following steps were performed according to the proposed method.

#### Model Inputs

HCCI and GCAI do not provide a direct combustion trigger like the spark in conventional engines. In contrast, combustion timing heavily depends on the wall and intake temperatures, intake pressure, exhaust gas recirculation, air/fuel ratio, charge motion and mixture stratification. A detailed physiochemical modeling has to consider all these boundary conditions to precisely predict the phasing of the next combustion cycle. This white-box approach would lead to overly complex models that are not feasible for model-based closed-loop control. A reduced-order model with a limited number of model inputs for combustion prediction is required. Schäfelein\(^4\) rated the effect of relevant actuation variables on combustion phasing (cp. table 2).

All actuation variables in table 2 must be considered model inputs because they have a strong effect on combustion phasing. To reduce the model order, a part of the inputs was set constant. The remaining variable inputs must be carefully selected, because it directly defines the experimental space limits. Furthermore, the variable model inputs define the available outputs of a model predictive combustion controller.

The engine speed and fuel characteristics are considered boundary conditions that cannot be changed for combustion control purposes. The airpath has slow dynamics and is typically not suitable for cyclic actuation. Modification of the engine geometry requires special actuators that are not considered in this study. Evaluation of the actuation variables in table 2 shows that the injection mass, timing and stratification highly affect the mixture, pressure and temperature.\(^5\) Furthermore, the injection strategy can be varied on a cyclic base and enables the direct manipulation of each single combustion cycle. An ignition spark and the EGR rate are not considered control variables to limit the dimensionality of the modeling.

Because of the relevance of gasoline direct injection systems for mass production, the injection strategy was selected as the model input. All remaining input parameters were set to constant values for the following investigations (cp. 3).

#### Excitation signals

The injection strategy offers many degrees of freedom to manipulate the number of injections, timing and fuel quantity. To overcome the ‘curse of dimensionality’, well-known methods for design-of-experiments are commonly used in engine calibration. The exact choice of the excitation signals defines the model input and the output signals of the model-based controller. For a feasibility study of the autoregressive modeling, a reduced dimensionality and complexity of the model and a limited number of parameters is beneficial. However, at least engine load \( \text{imep} \) and combustion phasing \( \alpha_{50} \) must be manipulable using the excitation signals.

The injected fuel mass \( m_B \) directly correlates to the output torque, which is the main output variable of an engine controller. It is used as one excitation for the combustion model and subsequent measurements. At least a second input variable for the direct effect on the mixture homogeneity and combustion phasing is required. When combustion chamber recirculation is used, temperatures of above 1500 K can be reached during recompression.\(^17\) A pilot injection in this phase can be used as a direct trigger for the subsequent combustion cycle. High temperature and residual oxygen may cause the formation of radicals and exothermic pre-reactions, which advance the exceeding of the self-ignition boundaries and the subsequent combustion cycle, respectively. The amount of pre-injection \( X_{pre} \) was successfully used as a control variable to directly affect the combustion phasing for each cylinder individually in multi-cylinder engines.\(^31\)

### Table 1. Single-cylinder research engine parameters

<table>
<thead>
<tr>
<th>Category</th>
<th>Actuation variable</th>
<th>Influence on mixture</th>
<th>Influence on ( p ) and ( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>Engine speed</td>
<td>+</td>
<td>+++</td>
</tr>
<tr>
<td>Fuel</td>
<td>Fuel chemistry</td>
<td>+++</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>Fuel temperature</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>Additives</td>
<td>+++</td>
<td>+</td>
</tr>
<tr>
<td>Airpath</td>
<td>Manifold pressure</td>
<td>+++</td>
<td>+++</td>
</tr>
<tr>
<td></td>
<td>Air temperature</td>
<td>++</td>
<td>+++</td>
</tr>
<tr>
<td>Engine</td>
<td>Compression ratio</td>
<td>+</td>
<td>+++</td>
</tr>
<tr>
<td></td>
<td>Wall temperature</td>
<td>0</td>
<td>+++</td>
</tr>
<tr>
<td></td>
<td>Wall heat losses</td>
<td>+</td>
<td>+++</td>
</tr>
<tr>
<td>Injection strategy</td>
<td>Load/fuel mass</td>
<td>+++</td>
<td>+++</td>
</tr>
<tr>
<td></td>
<td>Air/fuel ratio</td>
<td>+++</td>
<td>+++</td>
</tr>
<tr>
<td></td>
<td>Injection timing</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td></td>
<td>Stratification</td>
<td>++</td>
<td>+++</td>
</tr>
<tr>
<td>Ignition</td>
<td>Ignition spark</td>
<td>0</td>
<td>++</td>
</tr>
<tr>
<td>EGR</td>
<td>Residuals</td>
<td>+++</td>
<td>+++</td>
</tr>
</tbody>
</table>

Table 2. Rating of the effect of actuation variables on the pressure, temperature and mixture (0 no, + low, ++ average, +++ high)\(^4\)
### Table 3. Boundary conditions for the steady-state and dynamic measurements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intake air pressure</td>
<td>( p_{\text{int}} = 1013 \text{ mbar} )</td>
</tr>
<tr>
<td>Exhaust pressure</td>
<td>( p_{\text{exh}} = 1013 \text{ mbar} )</td>
</tr>
<tr>
<td>Oil &amp; coolant temp.</td>
<td>( T_{\text{oil}} = T_{\text{cool}} = 90 \degree \text{C} )</td>
</tr>
<tr>
<td>Engine speed</td>
<td>( n = 2500 \text{ 1/min} )</td>
</tr>
<tr>
<td>Rail pressure</td>
<td>( p_{\text{rail}} = 100 \text{ bar} )</td>
</tr>
<tr>
<td>Intake temperature</td>
<td>( T_{\text{int}} = 35 \degree \text{C} )</td>
</tr>
<tr>
<td>Exhaust valve closing</td>
<td>( EVC = 110 \degree \text{CA bef.TDC} )</td>
</tr>
<tr>
<td>Intake valve opening</td>
<td>( IVO = 110 \degree \text{CA aft.TDC} )</td>
</tr>
<tr>
<td>Pilot injection (end)</td>
<td>( 40 \degree \text{CA bef.TDC} )</td>
</tr>
<tr>
<td>Main injection (end)</td>
<td>( 100 \degree \text{CA aft.TDC} )</td>
</tr>
<tr>
<td>Injected fuel mass</td>
<td>( m_B = \text{variable} )</td>
</tr>
<tr>
<td>Amount of pre-injection</td>
<td>( X_{\text{pre}} = \text{variable} )</td>
</tr>
</tbody>
</table>

In the following steps, two input variables are used for the excitation of the engine process and autoregressive modeling: the injected fuel mass \( m_B \) and amount of pre-injection \( X_{\text{pre}} \). The timing of both injections and the negative valve overlap (NVO) are kept constant (cp. table 3). The valve timing, the injection strategy and the cylinder pressure trace are shown in figure 3 for an exemplary load point of \( \text{imep} = 3 \text{ bar} \) and \( n = 2500 \text{ rpm} \).

![Figure 3. visualization of negative valve overlap and injection timing](image)

**Model Architecture**

The model architecture is an architectural key decision with a major effect on the model predictive controller. The target of this article is a real-time capable implementation; therefore, a data-driven modeling approach is considered. Generally speaking, a linear time-invariant (LTI) discrete system can be described as a system with inputs \( u_1(k) \ldots u_n(k) \), outputs \( y_1(k) \ldots y_m(k) \) and disturbance variables \( s_1(k) \ldots s_p(k) \). The outputs of this simple system only depend on the inputs and disturbances, which also cover stochastic effects.

However, for GCAI combustion, a time-discrete LTI approach is insufficient. Outliers and unstable operation are characterized by the bifurcative behavior, which can successfully be modeled by mathematical models including feedback loops.27,32 The feedback of the previous cycles’ output signals covers the coupling effects that are mainly caused by the exhaust gas recirculation.

For the following implementation, GCAI combustion is modeled as an autoregressive process. Figure 4 shows a first-order implementation model when neglecting disturbances. The output signals \( y_1(k) \ldots y_m(k) \) depend on the inputs \( u_1(k) \ldots u_n(k) \) and feedback of the previous outputs \( y_1(k-1) \ldots y_m(k-1) \).

![Figure 4. AR(1) system with inputs, output and feedback](image)

**Model Order** The definition of the autoregressive model order requires an analysis of the effect of predecessor cycles on the current combustion. Autocorrelation analysis is often used to identify deterministic dependencies in auto ignition combustion.16,17 A typical result for an average load of \( \text{imep} = 3.5 \text{ bar} \) is shown in figure 5. An autocorrelation analysis covers the linear dependencies between \( y_{(k)} \) and \( y_{(k+m)} \), which are also indirectly affected by the cycles \( y_{(k+1)} \ldots y_{(k+m-1)} \). Thus, the order \( p \) of the AR-process cannot be derived from an autocorrelation analysis.

![Figure 5. Normalized autocorrelation \( \rho_{XY}^2(m) \), three measurements at \( n = 2500 \text{ 1/min}, \text{imep}=3.5 \text{ bar} \)](image)

To estimate \( \rho \), a partial autocorrelation analysis is commonly used.33 For an AR(\( p \)) process, the partial autocorrelations \( \rho_{\text{part}}(m) \) are nonzero for \( m \leq p \) and zero for \( m > p \). In other words, the partial autocorrelation function of the AR(\( p \)) process has a cutoff after the lag \( p \).34 Computing the partial autocorrelation of an AR(\( p \)) Gaussian-distributed process from \( T \) subsequent samples, the estimated partial autocorrelations are also Gaussian-distributed. For lags \( m > p \), the mean of these estimators is zero, and the standard deviation is \( \frac{1}{\sqrt{T}} \).35 Thus, the probability for a realization within \( \pm 1.96 \frac{1}{\sqrt{T}} \) is equal to 95%. If the estimated values for the partial autocorrelation are within this confidence interval for \( m > p \), the process is classified as an AR(\( p \)) process. Figure 6 shows the computed partial autocorrelations \( \rho_{\text{part}}(m) \) of combustion phasing for three measurements of different standard deviations \( \sigma_{\text{imep}} \) at a medium load of \( \text{imep} = 3.5 \text{ bar} \). The number of test samples is \( T = 200 \) each; the resulting 95% confidence interval is highlighted by solid lines. For \( m = 1 \), significant dependencies with a negative sign can be observed for all three measurements. For values of \( m \geq 2 \) no significant dependency is observed. Similar results are achieved with a lower load of \( \text{imep} = 1.5 \text{ bar} \) and a high load of \( \text{imep} = 7 \text{ bar} \). The results are consistent with the consideration that the cycle coupling is mainly caused by transferred residuals.
Feedback Signals: The effect on the subsequent cycle is mainly caused by the physical and chemical properties of the recirculated exhaust gas that affect the temperature, stratification, charge motion and amount of unburned fuel residuals. These values can only be calculated indirectly from the pressure trace and require detailed multi-zone models. As mentioned in the model inputs section, a physical-based white box approach will exceed the limitation of a real-time-capable implementation. Therefore, robust surrogate parameters that can be calculated effectively are required to quantitatively model the feedback loop.

Return maps of the combustion phasing have been used to identify the deterministic dependencies of consecutive cycles. Accordingly, a strong effect of the combustion phasing on the subsequent cycle is obvious. Combustion phasing affects the wall heat losses, combustion efficiency and temperature of the residuals. The partial autocorrelation analysis shows a clear effect of the previous combustion phasing on the subsequent cycle is obvious.

Obviously, the combustion phasing is not sufficient for characterization of the state of the recirculated exhaust gas. A second surrogate parameter that correlated with the load is required. The indicated mean effective pressure imep corresponds to the main reference variable of an engine controller and significantly affects the residual gas composition and temperature. The hypothesis of this paper is that a combustion event can be characterized by the phasing $\sigma_{50}$ and indicated mean effective pressure imep. Thus, the feedback signal imep$(k-1)$ is used as a second parameter for the following investigations.

Measurement Methodology

For a stable model-based control of an autoregressive process, the effect of the previous cycles on the current combustion cycle must be predicted.

However, the commonly used methodology of steady-state measurements results in small deviations of the predecessor cycle and insufficient test data. In a steady-state operation, the current and predecessor have identical injection strategies. Except stochastic cyclic fluctuations, the residual gas also has steady-state composition and temperature, which causes a hard coupling of the inputs $u_1(k)\ldots u_n(k)$ and AR feedback $y_1(k-1)\ldots y_m(k-1)$. Modeling of the system reaction to outliers is notably difficult because of the limited test data.

A variation of the past is required to achieve a dynamic representation of the AR(1) process including the feedback loop. A new cycle-based method for the systematic excitation of the predecessor cycles in GCAI combustion is proposed below.

The measurement methodology is divided in two subtasks. First, the boundaries that can be achieved under steady-state conditions are defined. Second, dynamic measurements with cyclic actuation are performed.

Steady-State Measurements

To investigate the steady-state characteristics, a well-known manually tuned operation point ($X_{pre} = 10\%$, $m_B = 9.7$ mg) is used as basis for the measurements. Starting from this injection strategy, a quasi-static two-dimensional variation of the fuel mass and pilot injection fraction was performed. To prevent damage to the test engine, the limitations of the hardware were thoroughly monitored. A maximum peak pressure rise of $(dp/\alpha)_{max} \leq 6.5$ bar/°CA must always be guaranteed (cp. table 1).

The measured operation points are highlighted in figure 8 with crosses. The level curves of max. pressure rise $(dp/\alpha)_{max}$ were calculated using two-dimensional bicubic interpolation. A stability limit is not applied; the low-load limit is characterized by the spontaneous absence of auto-ignition and iteratively proven. Figure 8 shows that the low-load limit is strongly correlated to level curves and follows a minimal level of appr. $(dp/\alpha)_{max} = 1$ bar/°CA.
For each operation point, the indication data and raw emissions were analyzed. Figure 9 shows the exemplary analysis of the nitrogen oxide \( NO_x \) emissions depending on the injection strategy. As expected, a strong correlation of \( NO_x \) emissions to the fuel mass is evident. Furthermore, a larger fraction of pre-injection \( X_{pre} \) causes an earlier combustion phasing and intensified formation of nitrogen oxides.

The operation map reflects the steady-state combustion characteristics depending on the selected degrees of freedom \( m_B \) and \( X_{pre} \) at the given parameters in table 1. Because the GCAI process depends heavily on the predecessor residuals (AR-characteristic), the shown operation map is only valid for the steady-state operation. For dynamic measurements, the available operation map should be significantly different because of the cycle-to-cycle correlation.

**Dynamic Measurements**

The goal of the dynamic measurements is to generate a sufficient data-base for an autoregressive model that precisely predicts the outputs \( y_{(k)} \) as a function of the input \( u_{(k)} \) and feedback \( y_{(k-1)} \). For each injection strategy (input \( u \)), a maximum variance of the predecessor’s cycle characteristics \( imep_{(k-1)} \) and \( \alpha_{SO_{2}(k-1)} \) (feedback \( y \)) is required. The measurements are conducted in a three-step approach; after a definition of the experimental space limits, a real-time-capable implementation is proposed and used for the dynamic measurements in the GCAI operation.

**Experimental Space Limits**

The intended variation of injection strategy and the resulting dynamic composition of the residuals should limit the stability map. Particularly for the low-load operation, a rapid increase in fuel mass is critical. Figure 10 shows the indicated mean effective pressure for cyclic actuation within the defined limits by the steady-state measurements. Cycle 0…8 has a low load with low exhaust gas temperature. The rapid increase of fuel mass at cycle 9 leads to the absence of auto ignition and engine shutdown.

To guarantee a stable operation, a safety margin to the low-load limit should be considered. The defined injection strategies for the dynamic measurements are shown in figure 11. A significant limitation in comparison to the achievable steady-state operation range is obvious.
The set of injection strategies in figure 11 is stored in the RCP unit, and a transition algorithm according to table 4 is implemented. Each injection is triggered by a crank-angle synchronous cycle counter. 540°CA before firing TDC, a set of injection parameters is defined and transferred to the injection power stage.

Before running the transient tests, a steady-state warm-up of the engine is performed until the defined boundary conditions (cp. table 3) are reached. Transient cyclic actuation is manually started by the test bench operator. For an exact allocation during postprocessing, the internal ECU cycle counter and the indication system must be synchronized. An additional trigger signal starts the data acquisition and marks the initial actuated cycle. The indication system CAS is equipped with a cyclic buffer memory. Hence, several cycles before the transient actuation are also captured.

Figure 12. Injection, trigger signal and cylinder pressure at the beginning of the transient measurements

Figure 12 shows the trigger signal, injection and cylinder pressure for the first five cycles of the transient actuation. The two preceding cycles are part of the steady-state operation for engine conditioning.

Dynamic Measurement Results The development of imep and $\alpha_{50}$ during the transient measurement is shown in figure 13. Induced by the cyclic actuation, both characteristic parameters strongly fluctuate as a result of the superposition of the actuation variables $u_k$ and the feedback of the predecessor cycle $y_{k-1}$.

Comparing the state-of-the-art steady-state measurements with constant actuation a) (variation of parameters manually or by DoE) with the automatic approach b), it can be found that for a) the number of measurement points are correlated to the degrees of freedom and for b) the number of cycles is correlated to the degrees of freedom. Accordingly, when following the approach b), an extension of the degrees of freedom can easily be achieved by prolongation of the automatic measurements. It is assumed that for the variation of the boundary conditions well-know methods for design-of-experiments can be utilized.

Modeling

In this section, a real-time-capable autoregressive modeling of the combustion process is analyzed and fit to the dynamic measurements. In the first step, all cycles with a defined injection strategy were extracted to isolate the effects of the feedback signals. A two-dimensional analysis of the effects of the predecessor cycles imep$(k-1)$ and $\alpha_{50}(k-1)$, which represent the AR feedback signals, was performed. After a regression analysis, a polynomial model was implemented in software and validated by engine test bench measurements.

Two-Dimensional Interpolation

For each injection strategy, $2 \cdot r \cdot n = 440$ individual cycles are available per measurement. The first analysis for one specific injection strategy can be performed using the three-dimensional visualization in figure 14. The given example shows the results for injection strategy no. 1. The $x$- and $y$-axes show the feedback parameters imep$(k-1)$ and $\alpha_{50}(k-1)$ of the previous cycle, the $z$-axis represents the actual combustion phasing $\alpha_{50}(k)$. Each measured cycle is marked by a cross.

For better visualization and interpretation, a two-dimensional clustering and interpolation was performed. All measured cycles were clustered with the nearest-neighbor approach in the $xy$-plane, which was defined by the feedback parameters imep$(k-1)$ and $\alpha_{50}(k-1)$. The heights of the $z$-values of the cluster centers were calculated by averaging the $\alpha_{50}(k)$ values of the data points of the corresponding $xy$ cluster. The quantization and averaging results were marked by circles in figures 14 and 15. Between these averaged values, a two-dimensional bicubic interpolation was conducted and visualized as surface plots.
The accuracy of the prediction in figure 16 shows significant differences for the applied injection strategies. For plausibilisation, the best and worst predictions were compared to the initial steady-state measurements. The best accuracy is achieved for strategy no. 3, which is notably close to the manually optimized, stable starting point of the steady-state investigations. The largest deviation occurs for injection strategy no. 21, which implies a very low fuel quantity and a large fraction of pre-injection (cp. figure 11).

Figure 18 shows a projection of the injection strategies no. 3 and 21 in the steady-state operation map of the covariance \( \text{COV} = \sigma_{\text{imep}} / \text{imep} \), which is an accepted standard method to measure combustion stability in internal combustion engines.\(^{36}\) Injection strategy no. 3 has a low covariance of \( \text{COV} < 3\% \) in the steady-state operation and a good predictability of the predecessor’s effect. In

Polynomial Regression Analysis

A polynomial approach was selected to model the AR(1) process. The model was developed with a strong focus on generalization, which indicates the capability to predict new, unknown instances. With increasing model complexity, the generalization error decreases in the first instance. Overfitting increases the computational time and prediction error, which should be avoided. For the provided test data, the optimal polynomial degree was evaluated by cross verification. Half of the test data was used for training, and the other half was used for verification.

Figure 16 shows the mean standard deviation of the prediction error as a function of the injection strategy for different polynomial degrees. The mean value-based approach corresponds to the neglection of predecessor’s effect on the current combustion and has the highest prediction errors of up to 5.5°CA for injection strategy no. 21. Increasing the polynomial’s degree reduces the prediction error for all injection strategies. Overfitting does not occur for the provided test data up to a polynomial’s degree of four. Consequently, a high degree of the model polynomial could be considered.

Figure 14. Combustion phasing \( \alpha_{50}(k) \) as a function of \( \text{imep}(k-1) \) and \( \alpha_{50}(k-1) \) for injection strategy no. 1

Apparently, the combustion phasing in figure 14 depends on the selected feedback variables; otherwise, the result would be a flat horizontal plane. However, for injection strategy no. 1 the early predecessors obviously lead to late combustion and vice versa, whereas a low \( \text{imep} \) of the predecessor has an advancing effect on the combustion.

Similar results were obtained when analyzing the effects of \( \text{imep}(k-1) \) and \( \alpha_{50}(k-1) \) on \( \text{imep}(k) \) (cp. figure 15). A first visual analysis shows that both plots have a comparatively flat surface without distinctive local extrema. This observation leads to the assumption that a low-order approximation is beneficial. Apparently, this analysis is only valid for one injection strategy and must be individually repeated. In the next step, different degrees of polynomial approaches are compared to model the predecessor’s cycle effect.

Figure 15. Indicated mean effective pressure \( \text{imep}(k) \) as a function of \( \text{imep}(k-1) \) and \( \alpha_{20}(k-1) \) for injection strategy no. 1

Figure 16. Expectation of the prediction error of \( \alpha_{50} \) as a function of the injection strategy for different polynomial degrees

Nevertheless, the extrapolation capability is manually analyzed for outliers, which may not be covered by the test data. Figure 17 shows the resulting propagation surfaces for polynomials of the first and second degrees. Apparently, the quadratic approach leads to predictions of heavily delayed combustion for extreme outliers in each direction. For example, an assumed predecessor of \( \text{imep}(k-1) = 4 \) bar and \( \alpha_{50}(k-1) = 25°\text{CA b.TDC} \) results in a predicted combustion phasing of \( \alpha_{50}(k) > 25°\text{CA a.TDC} \). The linear approach appears much more feasible for moderate extrapolation results. Furthermore, the first-order polynomial requires less computational power and offers a significant improvement in comparison to a simple mean value approach. Hence, first-order modeling is used for the prediction algorithm.

The accuracy of the prediction in figure 16 shows significant differences for the applied injection strategies. For plausibilisation, the best and worst predictions were compared to the initial steady-state measurements. The best accuracy is achieved for strategy no. 3, which is notably close to the manually optimized, stable starting point of the steady-state investigations. The largest deviation occurs for injection strategy no. 21, which implies a very low fuel quantity and a large fraction of pre-injection (cp. figure 11).
Figure 17. Measured values $\alpha_{50}$ and interpolated prediction surface plots of the first and second orders for injection strategy no. 22

contrast, injection strategy no. 21 is characterized by a notably high covariance of $COV > 15\%$ in the steady-state measurements.

![Image](image1.png)

Figure 18. Covariance of imep in steady-state conditions, variation of the injection strategy

Injection strategies that guarantee stable steady-state combustion also correlate with good predictability in the dynamic operation.

Software implementation

Based on the regression analysis, the estimators $imep(k)$ and $\alpha_{50}(k)$ for the expectations $\varepsilon(imep(k)|imep(k-1), \alpha_{50}(k-1))$ and $\varepsilon(\alpha_{50}(k)|imep(k-1), \alpha_{50}(k-1))$ were calculated. A set of six coefficients must be calculated, each of which depends on the current injection strategy. The linear approach allows a direct interpretation of the single coefficients; $a_{SS}$ and $a_{LL}$ represent the direct consecutive effect of combustion phasing and load, respectively. Analogous cross-correlations are represented by $a_{LS}$ and $a_{SL}$.

$$
imep(k) = c_L + a_{LL}imep(k-1) + a_{LS}\alpha_{50}(k-1) \\
\alpha_{50}(k) = c_S + a_{SL}imep(k-1) + a_{SS}\alpha_{50}(k-1)
$$

(1)

Figure 19 shows the dependence of $a_{SS}$ on the injection strategy as a two-dimensional map. The 22 discrete injection strategies are marked by circles. A clear distinction between positive and negative values is evident and highlighted by a solid line. Values of $a_{SS} < 0$ lead to alternating combustion behavior and can be found for significant pre-injection amounts. With increased main injection, the autocorrelation factor $a_{SS}$ is greater than zero, and the behavior completely changes. A bicubic interpolation was performed to increase the number of possible predicted injection strategies. An analogous procedure was applied for the remaining coefficients $c_L, a_{LL}, a_{LS}, c_S$ and $a_{SL}$.

![Image](image2.png)

Figure 19. Developing of coefficient $a_{SS}$ as a function of fuel mass $m_B$ and pre-injection fraction $X_{pre}$

The software implementation of the modeling algorithm is displayed in figure 20. The prediction model has four input signals, which represent the feedbacks $y_{(k-1)}$ and the set variables $u_{(k)}$. The six coefficients depend on the current injection strategy and were implemented as two-dimensional lookup tables with interpolation.

![Image](image3.png)

Figure 20. Software implementation of the prediction algorithm in Matlab/Simulink

Verification by Engine Testing

To verify the modeling approach in the time domain, the prediction algorithm was stimulated with measured data from the testbench. As a reference, a prediction based on the averaged results of the dynamic measurements for each injection strategy was used. The measured values, mean value-based prediction and autoregressive prediction are compared in figure 21.

For each combustion cycle, the current injection strategy is known and fed into both prediction algorithms. The
The autoregressive model precisely reflects the alternating behavior of the combustion process. Of particular interest is the phase with three constant injections from cycle 473 to cycle 475. The prediction of the mean value-based approach is a constant value and does not cover the system dynamics. The AR(1) modeling with the feedback variables enables a significant improvement that precisely predicts the alternating combustion behavior.

![Figure 21. Comparison of measurements and prediction of imep and \( \alpha_{50} \) in transient condition for all injection strategies](image1)

Figure 21. Comparison of measurements and prediction of imep und \( \alpha_{50} \) in transient condition for all injection strategies

The prediction algorithm was implemented in Mat-
![Figure 22. Comparison of \( \alpha_{50} \) prediction with mean value and AR(1) model in transient condition](image2)

Figure 22. Comparison of the \( \alpha_{50} \) prediction with mean value and AR(1) model in transient condition

Figure 22 shows the prediction of combustion phasing for both approaches. The estimated values were compared with the measurements by visualization of the scatter bands. The error standard deviation of the mean value-based approach is 3.97 °CA and highlighted by solid lines. The autoregressive model reduces the prediction error by 53.6% to a value of 1.84 °CA. The prediction error is significantly reduced by the autoregressive modeling. The mean value-based approach predicts 22 discrete values according to the number of injection strategies. As a result, 22 horizontal lines appear in figure 22 for the mean value-based prediction.

Notably, similar results were obtained for the combustion load prediction (cp. figure 23). The AR-approach reduces the prediction error by 43.2% in comparison to the mean value-based model.

![Figure 23. Comparison of imep prediction with the mean value and AR(1) model in transient condition](image3)

Figure 23. Comparison of imep prediction with the mean value and AR(1) model in transient condition

The developed approach was calibrated to predict the combustion phasing and engine load but can also be used for other relevant variables such as the peak pressure, acoustic behavior and emissions.

**Conclusions and Outlook**

In this article, a GCAI combustion model based on an autoregressive process is proposed. The process order was estimated by the partial autocorrelation analysis of steady-state operation measurements. No significant correlations were found for lags greater than one. This observation is consistent with the assumption that the coupling effects are mainly caused by the exhaust gas recirculation, which is directly transferred to the consecutive cycle. Instead of implementing a detailed white box model, we decided to elaborate the feasibility of a mainly data-driven black-box model. Data-driven models require sufficient test data for a detailed prediction of the combustion. In particular, variations of the predecessor cycle’s characteristics must be considered. Because measurements in the steady-state operation result in a hard coupling of actuation and feedback variables, only minor variations of the predecessor cycle are achieved.

Hence, a new approach for transient measurement is required. A pair of injection strategies is defined as a dynamic transition with an individual actuation of the injection strategy for each combustion cycle. The cyclic actuation and measurement were synchronized using an offline postprocessing algorithm. A polynomial model is proposed to predict the combustion phasing and load. Regression analysis shows no overfitting for higher polynomial orders, nevertheless a first-order polynomial was selected because of the good extrapolation capabilities for extreme outliers.

The prediction algorithm was implemented in Matlab/Simulink and transferred to a real-time-capable engine management control unit. Verification was performed by test bench measurements in the transient operation. The effects of the recirculated gas is covered by surrogate parameters \( imep_{(k-1)} \) and \( \alpha_{50(k-1)} \) from the previous cycle. The combustion phasing was precisely predicted for any
injection strategy. The combustion phasing prediction error can be reduced by 53% in comparison to a state-of-the-art mean value-based prediction. The presented dynamic measurement method allow the generation of the required test data for the autoregressive model. Accordingly, it served as a prerequisite for the subsequent data driven or grey-box model that allows to predict the influence of the preceding combustion cycle. In this feasibility study the potential of the autoregressive model to predict individual combustion events has been shown.

As a next step, a real-time-capable optimization algorithm must be coupled with the prediction model to close the loop in an actual engine control unit and verify the closed-loop control under realistic conditions.

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Nomenclature

| AR  | Autoregressive  |
| CAS | Combustion Analysis System |
| CFD | Computational Fluid Dynamics |
| CR  | Compression Ratio |
| ECU | Electronic Control Unit |
| EGR | Exhaust Gas Recirculation |
| EMVT | Electromechanical Valve Train |
| GCAI | Gasoline Controlled Auto Ignition |
| HCCI | Homogenous Charge Compression Ignition |
| LTI | Linear Time Invariant |
| RCP | Rapid Control Prototyping |
| RON | Research Octane Number |
| SCE | Single Cylinder Engine |
| TDC | Top Dead Center |

References


