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Postbuckling Responses of the Panels Subjected to Combined Compression and Shear Stresses

N. Aswini[a*], H.-G. Reimerdes[b], N.K. Gupta[a]

[a] *Department of Applied Mechanics, IIT Delhi, India *Aluminus*

[b]*RWTH Aachen University, Institut für Strukturmechanik und Leichtbau, Germany*

Abstract

In general, many numerical techniques are in place for the study of buckling and post-buckling responses of the composite structures, but these are computationally expensive. Relatively few equivalent analytical techniques are available, especially for evaluation of post-buckling responses. In this paper an analytical approach is presented for the determination of post-buckling responses of rectangular panels made of symmetric orthotropic laminates and supported by stiffeners (especially used in the monocoque and semi-monocoque aircraft structures), and subjected to combined compressive and shear loads, majorly based on the effective width concept. They are used to calculate the loss in the stiffness of the panels and the compressive stresses induced in the stiffening members, under a combined effect of shear and compressive stresses. For illustration, a series of parametric curves that are characteristic of the post-buckling behavior of the orthotropic panels (of aspect ratio 2.5) with transverse stiffeners have been reported.

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Keywords: Postbuckling analysis; Effective width method; Principle of Diagonal Tension; Rectangular panels with stiffners; combined compressive and shear loads

1. Introduction

The present study is about establishing an analytical approach for the study of the post buckling behavior of the panels whose skin is made of composite laminates and supported by struts and frames as shown in figure [1]. Several studies have been done in this direction (refer [2] – [11]). The current work is based on Levy's work [10,11], for the case of

* Corresponding author
Email address: aswinimanu@gmail.com

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stiffened panels made of symmetric special orthotropic laminates, that are subjected to combined compression and shear loads exceeding the buckling limit.

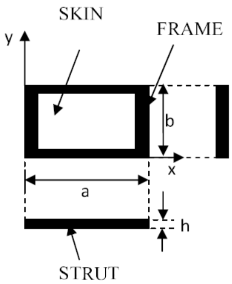


Figure 1: Panel Configuration

Nomenclature	
ER	Effective Width Ratio
s_{ij}	Components of inverse of in-plane stiffness matrix
AR	Aspect Ratio

The panels are considered to be simply supported on all four edges which can be shown as

$$w(x, 0) = 0 \quad w(x, b) = 0 \quad w(0, y) = 0 \quad w(a, y) = 0$$

$$M_y(x, 0) = 0 \quad M_y(x, b) = 0 \quad M_x(0, y) = 0 \quad M_x(a, y) = 0$$

where M_x and M_y are the bending moments about x-axis and y-axis respectively. Referring to figure (1), the frames prevent the displacement parallel to the edges, but allow the rotation about the edges. Hence the neutral strain in y-direction at these edges must be zero and so,

$$(\varepsilon_x)_{x=0,a} = 0 \quad (1)$$

The longer edges at edges y=0 and y=b are simply supported by struts that allow both, the rotations about the edges and the displacement parallel to the edges. The latter corresponds to the shortening of the strut under load. Hence we have,

$$(\varepsilon_x)_{y=0,b} = \frac{P' s_{11}}{A_s} \quad (2)$$

where P' is the compressive force in the strut and A_s is the cross sectional area of the strut.

1.1. Effective Width Concept

According to the effective width concept, the effective width ratio is characteristic of the loss in extensional stiffness encountered beyond the buckling limit. When a panel structure is subjected to external loads, initially the stress distribution is uniform over both the sheet and the stringers. But when buckling limit is exceeded, there is a non-linear stress distribution over the sheet. The true non-uniform stress distribution in the actual width (b) of the sheet under study is compared against a sheet of width be (also referred as effective width) that will be experiencing a uniform stress. Accordingly,

$$ER = N_m/N_e \quad (3)$$

where N_m is the mean stress and N_e is the edge stress, that are given by

$$\begin{aligned} N_m &= \frac{1}{b}\int_0^b N_x dy \\ N_e &= A_{11}\,\epsilon_x \end{aligned} \quad (4)$$

Similarly for panels subjected to shear stresses exceeding the buckling limit, there is a loss in the shear stiffness of the sheet. Accordingly the effective width ratio in this case is defined as, [10]

$$ER = \frac{Q}{-\bar{N}_{xy}a} \quad (5)$$

where Q is the actual shear load carried and N_{xy} is the external shear stress applied. For combined compression and shear case, there is a drop in both the extensional stiffness as well as the shear stiffness. Let these losses be denoted as L_e and L_s respectively and these are determined from Eq. (3) and Eq. (5) as,

$$\begin{aligned} L_e' &= 1 - L_e = N_m/N_e \\ L_s' &= 1 - L_s = -Q/N_{xy}a \end{aligned} \quad (6)$$

2. Analytical Formulation

Based on the choice of the coordinate system as shown in figure (1), a double Fourier sine series given by Eq. (7) is used to represent the laminate midplane displacement field.

$$w = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} w_{m,n} \sin\left(\frac{m\pi\, x}{a}\right) \sin\left(\frac{n\pi\, y}{b}\right) \quad (7)$$

The buckling mode shapes of a panel, subjected to a combination of compression and shear can be determined by the Galerkin method. Complete details about this methodology are available in Ref. [17] and [18]. Accordingly this problem can be formulated into an eigenvalue problem given by,

$$[C]\{X\} = \lambda[E]\{X\} \quad (8)$$

which can be rewritten as

$$[CE] = ([C] - \lambda[E])\,\{X\} = 0 \quad (9)$$

where [C], [E] and [CE] are all square matrices of order m × n where m and n refers to the user-specified number of terms in Eq. (7). The standard Maple 14.0 library routine ([19]) has been used to solve the generalized Eigen value problem. The large deflection equations developed by Von-Karman for a thin orthotropic panel [1] are given as

$$\begin{aligned} s_{11}\,\Phi_{y,y,y,y} + (2\,s_{12} + s_{66})\,\Phi_{x,x,y,y} + s_{22}\,\Phi_{x,x,x,x} &= \\ w_{x,y}{}^2 - w_{x,x}\,w_{y,y} \end{aligned} \quad (10)$$

$$\begin{aligned} D_{11}\,w_{x,x,x,x} + 2\,(D_{12} + 2\,D_{66})\,w_{x,x,y,y} + D_{22}\,w_{y,y,y,y} &= \\ p + \Phi_{y,y}\,(w_{x,x}) + \Phi_{x,x}\,(w_{y,y}) - 2\,\Phi_{x,y}\,(w_{x,y}) \end{aligned}$$

where

- Φ: Airy Stress Function
- w: displacement of points of the middle surface normal and relative to a plane through the corners of the panel
- p: Normal Pressure
- s_{ij}: Components of the inverse of the in-plane stiffness matrix

Then the Airy stress function is chosen so as to satisfy Eq (8), which is given by,

$$\Phi = \frac{1}{2}\bar{N_x}y^2 + \frac{1}{2}\bar{N_y}x^2 - \bar{N_{xy}}xy + \sum_{n=1}^{6}\sum_{m=1}^{10} b_{m,n}\cos\left(\frac{m\pi\, x}{a}\right)\cos\left(\frac{n\pi\, y}{b}\right) \qquad (11)$$

where N_x and N_y are the average stress resultants in the plate in x-direction and y-direction respectively and N_{xy} is the shear stress resultants at the corner of the plate. The term Nx includes both, the applied compressive stress in the strut direction (N_{xe}) and the average stress developed in the plate in the x-direction due to the applied shear stress (N_{xy}). The values of the constant $b_{m,n}$ are derived based on the work reported in [11], but there are also some additional terms other than N_x, N_y and N_{xy} used in the stress function, that are considered negligible [12].

Up to the buckling point, the skin carries all the shearing forces and the frame is unaffected. But beyond this limit, the skin carries only a portion of the shearing forces and develops some diagonal tension field that tends to draw the two frames together. Due to this fact, some additional compressive stresses (P) are induced in the struts beyond the buckling point which is given by particular integral derived as in [11], where

$$P = -\int_0^b N_x dy \qquad (12)$$

The total compressive force (P') on the strut can be given as

$$P' = \int_0^b N_x dy + \frac{N_{xe}\, fl}{h} \qquad (13)$$

The total shearing force Q acting on the panel is given by the sum of the shearing forces acting on the skin and on the upper and lower frames, which is given by

$$Q = Q_w + Q_u + Q_l \qquad (15)$$

The bending moment in the frames is same at each strut point, and so the shearing forces in the upper (Q_u) and lower (Q_l) frames can be given by,

$$\begin{aligned} Q_u &= \int_y^b (N_x)_{x=a}\, dy - \frac{1}{b}\int_0^b (N_x)_{x=a} y dy \\ Q_l &= \frac{1}{b}\int_0^b (N_x)_{x=0} y dy - \int_y^b (N_x)_{x=0} dy \end{aligned} \qquad (16)$$

The shear load carried by the skin is given by

$$Q_w = -\int_0^a (N_{xy}) dx \qquad (17)$$

Then the loss in the extensional stiffness and the shear stiffness of the panel can be determined from the effective width ratios as discussed in section (1.1).

3. Numerical Results

The present approach has been coded using Maple 14.0 and the numerical results are presented mostly for stresses up to 3 × the buckling limit. The results obtained are verified against that available in literature or with non-linear analysis done with ABAQUS. Two cases (refer to table (2)) are considered for illustration, where load cases of k=0, 1 are given (k is the ratio of the applied compressive stress to the shear stress)

Table 1: Material properties data

	(1)	(2)
Material	AS4 fiber and Epoxy resin	Aluminium
Nature	Orthotropic	Isotropic
E_1(N/mm^2)	128000	70000
E_2(N/mm^2)	11300	70000
G_{12}(N/mm^2)	6000	26595.744

Table 2: Different Cases of Panel configuration for AR=2.5 and b=100 mm, h=1 mm

Cases	I	II
Material (Table (1))	(2)	Laminae (1) $[90/0]_s$
b/a	2.5	2.5
As (mm2)	25	0.025
Load case	k=0	k=1

While modeling the problem in ABAQUS [20], the 3-D panel is modeled with 2-D plate elements. Then to realize the boundary conditions properly, instead of one panel a set of 9 panels were created. Prior to the non-linear analysis, some imperfections are introduced by adding the lowest buckling mode.

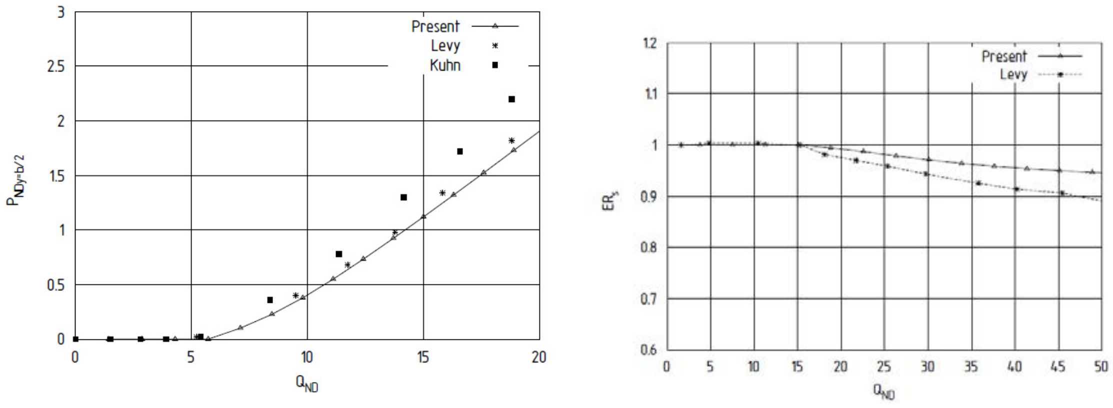
For deriving these characteristic curves by present approach, the related forces are non-dimensionalized as

$$Q_{ND} = \frac{Qb^2}{Eh^3a} \qquad P_{ND} = \frac{Pb}{Eh^3} \tag{18}$$

$\bar{\gamma}$ is the shear deformation of the beam, which is non-dimensionalised as

$$S_{ND} = \frac{-\bar{\gamma}b^2}{h^2} \tag{19}$$

In figure (2a), the results obtained are plotted against those obtained by Levy and Kuhn [11]. As observed, a satisfactory agreement is reported and hence neglection of some boundary conditions related terms in Eq (11) is justified.

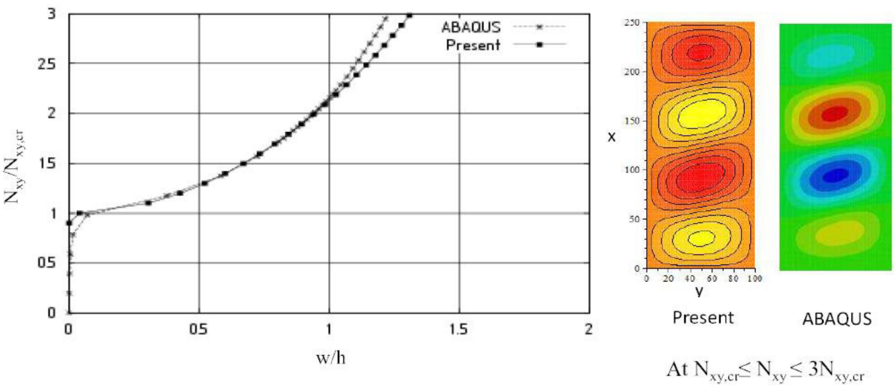


(b) Strut Force Vs Shear Load Curve

(a) Effective Width Ratio Vs Shear Deformation Load Curve

Figure 2: Characteristics of Curves of Panels under Case I

Similar curves have been reported for the other case below.



(a) Out of Plane Deflection Curve

(b) Buckle Mode Shape

Figure 3: Characteristics of Curves of Panels under Case II

4. Parametric Study

In order to account for orthotropic laminates of various configuration, non-dimensional parameters namely μ_c and θ_c are introduced that allow the postbuckling results to be presented as a series of curves on a single plot as illustrated in Figure 4.

$$\mu_c = \sqrt[4]{D_{22}/D_{11}} \qquad \theta_c = \frac{D_{12} + 2\,D_{66}}{\sqrt{D_{11}\,D_{22}}} \tag{20}$$

The present study is restricted to $0.4 \leq \mu_c \leq 2$ and $0.1 \leq \theta_c \leq 10$ AR= 2.5 and A_s of 25% of the panel area. This can include panels of almost all the practical orthotropic configurations. Then L'_s and L'_e computed by effective width concept are also plotted against normalized shear stresses.

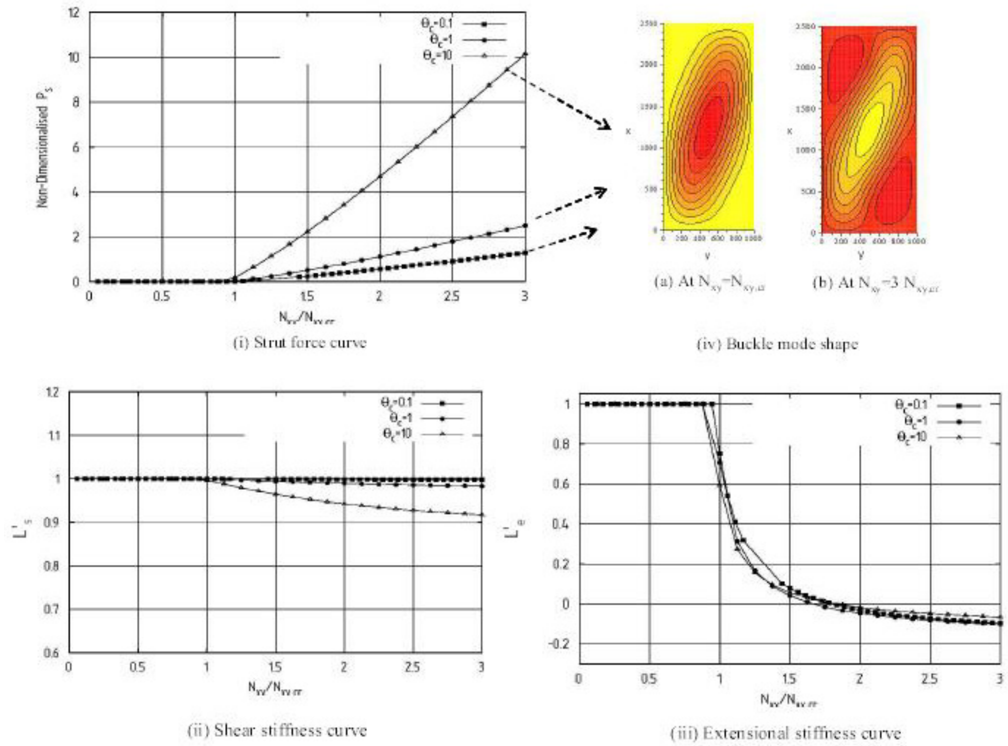


Figure 4: Characteristic curves for stiffened panels of μ_c = 0.4 under load case k=0.5

Some observations based on these characteristic curves were,

- Larger secondary compressive forces are developed in the struts for panels with low value of μ_c (μ_c=0.4) and large value of θ_c (θ_c=10) and vice versa.
- In the presence of shear, diagonal tension stresses provide stiffening of the entire panel, whereas for k=10 the supporting action is provided by the skin due to the prevention of buckling deformation in the neighborhood of the struts. Also frames provide some support by virtue of its periodically varying lateral stresses.
- Drop in extensional stiffness is relatively higher, mainly because, due to shear stress a deformation is exerted in the strut direction and hence the influence of N_{xy} on L'_e cannot be neglected.
- For k=0.5, L'_e can even reach negative values, mainly due to the tension component developed in the strut direction due to the external shear stresses. To establish equilibrium some additional compressive stresses are developed and hence N_x becomes negative and so the extensional stiffness can also become negative. Similar conclusions are also reported in [12].

The parametric curves presented, enable the selection of suitable configuration for the individual panels, to maximize its load bearing capacity in the postbuckling region.

5. Conclusion

Compared to conventional FEM, the present analytical approach is not only computationally efficient but also devoid of convergence issues. However, it should be noted that the deflection coefficients are determined by solving some non-linear equations, wherein the computation time is quite large. Some key characteristic curves have been reported to illustrate the behaviour of the panels beyond buckling, which enable a designer to choose the right configuration for the given application.

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