The influence of buffer time distributions on delay propagation in railway networks

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Abstract

Buffer times are an important element in schedule design preventing the propagation of delays. Current delay prediction models are often based on the assumption of exponentially distributed buffer times. It has been shown that real-world data for buffer times deviates from this behaviour, possibly necessitating more general buffer time distributions. In the present paper the impact of buffer time distributions on the height of knock-on delays is analysed using a simulation approach. The goal is to clarify how model refinements can be expected to influence capacity indicators or the level of service.

Keywords: schedule robustness, delay prediction, buffer times, stochastic simulation
1. Introduction

Maintaining a high quality of operations in case of schedule perturbations is one of the most demanding tasks in railway management. In order to avoid the uncontrolled propagation of delays the schedules are required to be robust [4, 9]. One key measure in ensuring schedule robustness is to incorporate buffer times and running or stopping time margins [8].

Different approaches to determine schedule robustness have been discussed in the literature [2, 10, 13, 6]. The most commonly used indicators are train punctuality or the height of delays. In order to assess different schedule variants delay and punctuality are analysed using stochastic delay prediction techniques (e.g. [1, 15]). In these models, buffer times and running time margins are treated as random variables. Existing approaches usually assume supplements to be a fix share [4] and buffer times to be exponentially distributed [11, 14].

The present paper focuses on the modelling of buffer times. It has been shown that real-world data for timetable buffer times on railway lines are not necessarily exponentially distributed [7]. In [15], a method to assess knock-on delays based on empirical arrival distributions has been proposed and compared to real-world data. Other delay prediction modelling techniques, such as the queueing-based approach described in [11], can be adjusted to cope with more general distributions [14]. However, a detailed analysis of how a more realistic modelling of buffer times affects knock-on delays and punctualities has not been presented, so far.

The goal of this paper is to close this gap by providing a rigorous analysis of knock-on delays on railway lines in case of different buffer time distributions. The analysis is based on a stochastic event-based simulation of train operations. Apart from supporting different buffer time statistics the simulation also allows for train priorities as well as tunable minimum headway times and initial delays, such that different line characteristics can be accounted for.

We are giving insight in the STRELE-framework [11], which is the standard approach used by German infrastructure manager DB Netz AG for delay prognosis and capacity assessment of railway lines, and the developed simulation to assess knock-on delays in section 2. The section is closed by a narrow comparison between STRELE and the simulation. Section 3 gives a detailed overview over the results of this paper. The simulation environment is validated in section 3.1 by comparing the results for exponential
buffer times to those obtained with the STRELE method [11]. A short overview on the complexity of computation for STRELE and simulation is following up in section 3.2, as well as research towards the influence of buffer times in section 3.3, a sensitivity analysis in section 3.4 and a section closing case study.

2. Method

We start to introduce the STRELE framework, which is the German standard formula for the assessment of knock-on delays. The second part describes the rough structure of the programmed simulation tool and gives an idea of the effect of synchronous and asynchronous simulation. In conclusion of the section a short summary of the required input data for STRELE and simulation is given.

2.1. STRELE

We start by briefly reviewing the mathematical framework underlying the STRELE formula. The STRELE formula is a result from the foundations of Schwanhäußer [11]. Though being developed in the 1970s it is still commonly used in Germany to determine railway capacity and has been implemented in various software tools [5, 12]. Given two unidirectional tracks, which is common in the European railway system, both can be treated and assessed as different queues. While conceptualised for double-track railway lines the STRELE-formula can be used for single-track railway lines as well with minor modifications.

The STRELE formula to determine the expected waiting time $t$ for a railway line segment between two overtaking stations is given by:

$$
t = \left( p_{del} - p_{del}^2 \right) \frac{\bar{t}_{del}^2}{\bar{t}_b + \bar{t}_{del}} \cdot \left( 1 - \exp\left( -\frac{\bar{h}}{\bar{t}_{del}} \right) \right) \\
\cdot \left[ p_{eq} \left( 1 - \exp\left( -\frac{\bar{h}_{eq}}{\bar{t}_{del}} \right) \right)^2 + \left( 1 - p_{eq} \right) \frac{\bar{h}_{diff}}{\bar{t}_{del}} \left( 1 - \exp\left( -2\frac{\bar{h}_{diff}}{\bar{t}_{del}} \right) \right) + \frac{\bar{h}}{\bar{t}_b} \left( 1 - \exp\left( -\frac{\bar{h}}{\bar{t}_{del}} \right) \right) \right]
$$

with

- $p_{del}$ – probability of primary delay,
- $\bar{t}_{del}$ – average time of delay,
\( \tilde{b}_t \) – average buffer time,

\( p_{eq} \) – probability between trains with equal rank,

\( \tilde{h} \) – average minimum headway time,

\( \tilde{h}_{eq} \) – average minimum headway time between trains with equal rank,

\( \tilde{h}_{diff} \) – average minimum headway time between trains with different rank.

The computation of expected waiting times follows the principle of calculating the first-order delays and upscaling the result by a factor which incorporates a heavy traffic regime and queuing system results to estimate the delays of higher order. Note that the knock-on delays for a line are calculated as the maximum over all segments between stations where overtaking is possible.

To assess the quality in operation a level of service has to be defined which regulates the maximum permissible knock-on delays in railway operations. The standard for quality in Germany is regulated in directive 405 of DB Netz AG [3]. A more extensive and rigorous discussion of the STRELE-framework can be found in Weik et. al. [14].

### 2.2. Simulation Environment

The Monte-Carlo-Simulation calculates knock-on delays in three steps. A visualisation of the procedure is given in Figure 1.

#### Schedule Generation

By setting a number of trains \( n \) with \( c \) classes, i.e. sets of trains with similar velocity, acceleration, priority and so on, a schedule is generated in the following manner. Since all trains go in one direction and no overtakings are possible the blocking time stairways of a segment can be merged into one block.

Hence the first train starts at time \( t_1 = 0 \) followed by a second train at time \( t_2 = t_1 + h_{1,2} + b_{1,2} \) with minimum headway time \( h_{1,2} \), which is a technical constraint. Additionally buffer time \( b_{1,2} \) is added, which aims to ensure adequate quality in operations by minimizing the transmission of primary delays, e.g. due to bad weather, signal or train malfunction or high
passenger load. The start times of the following trains are calculated in the same manner, i.e. for train $i \in \{2, n\}$: $t_i = t_{i-1} + h_{i-1,i} + b_{i-1,i}$.

Note that minimum headway times are given by train type and infrastructure and the choice of buffer time is within the range of the infrastructure manager. In the simulation framework the effects of different buffer time distributions can be tested in case no schedule is present. On the other hand, given a schedule, a buffer time distribution can be fitted and knock-on delays can be calculated and evaluated.
Delay Implementation

Infrastructure managers have empirical train specific values of delay probability and average delay time of their network, e.g. in Germany [3]. With probability of delay $p_{del,i}$ and the average delay time $t_{del,i}$ the trains $i = 1, \ldots, n$ in the schedule are affected by primary delays. At the end of the delay implementation the schedule usually contains conflicts which have to be solved.

Rescheduling

Solving the induced conflicts can be handled basically in two manners – regarding priorities in service (asynchronous) or resolving the conflicts in ascending order in time without inclusion of ranks (synchronous). In asynchronous simulation trains are scheduled ascending in ranks, whereas in synchronous simulation trains are treated as equal ranked. The first has the advantage of giving precedence for trains which are more important for the infrastructure manager, e.g. long distance trains prior to local trains and freight trains, while the latter consumes in general less time and corresponding capacity of the network. In case of high traffic load the limit the method of operation is mostly First-Come-First-Served, but in lightly or medium perturbed operations priorities should persist. The paper will therefore examine results for the asynchronous rescheduling.

Evaluation

To obtain knock-on delay of a train the planned start time and the primary delay, that even may be 0, are subtracted from the actual start time. Depending on the evaluation one aggregates the knock-on delays for a train class or for all trains by averaging over them.

2.3. Comparison of Input Data in STRELE and Simulation

A brief overview over the similarities and dissimilarities of STRELE and simulation is given in Table 1.

By usage of STRELE many train attributes are lost due to averaging, but the advantages, e.g. accuracy and flexibility, of the simulation are bought by computation time.
Table 1: comparison of input data of STRELE and simulation

<table>
<thead>
<tr>
<th></th>
<th>STRELE</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>buffer time distribution</td>
<td>Exponential Distribution</td>
<td>arbitrary</td>
</tr>
<tr>
<td>delay distribution</td>
<td>mix of Exponential and Degenerate Dist.</td>
<td>arbitrary</td>
</tr>
<tr>
<td>train composition</td>
<td>relevant</td>
<td>relevant</td>
</tr>
<tr>
<td>minimum headway times</td>
<td>average with small variation</td>
<td>class specific</td>
</tr>
<tr>
<td>priorities</td>
<td>arbitrary</td>
<td>arbitrary</td>
</tr>
<tr>
<td>number of trains in schedule</td>
<td>irrelevant</td>
<td>relevant</td>
</tr>
</tbody>
</table>

3. Results

We start by presenting a validation of the simulation for the base case of exponentially distributed buffer times in the schedule. A short discussion of the computational complexity of STRELE and simulation is attached, followed by a demonstration of the main results of this paper – the examination of the influence of buffer time distributions in railway operations.

3.1. Validation of the Simulation for Exponentially Distributed Buffer Times

The first goal is to check that the simulation is able to produce results similar to STRELE for exponentially distributed buffer times. We set the minimum headway time to 4 minutes, delay probability to 0.5 and train delay to 5 minutes for each train to show the goodness of fit by example. We choose 6 model trains to cover two train types per class (long-distance train, local train, freight train). All train types have equal probability and Figure 2 depicts 400 trains which were simulated 500 times for buffer time steps of 0.1 minutes each. Unless stated otherwise, the subsequent results are based on the above input.

In Figure 2 the expected waiting time per train in relation with the corresponding buffer time for simulation and STRELE formula is depicted. It can be seen that for average buffer times starting from 3 minutes, the results of the simulation converge reasonably fast towards STRELE. In case of small buffer times the number of classes chosen is a main factor for the height of the expected waiting times since the probability of shifting a train with lower priority from the beginning to the end of schedule is significantly higher. When
larger buffer times are realised the trains are not packed as tight anymore and the schedule offers more possibilities to store lower ranked trains in earlier positions. In other test cases a similar behaviour for additional cases where the input is in a STRELE compliant manner has been observed.

3.2. Computational Complexity

The implementation of the simulation environment lies within $O(n^2 \cdot \log(n) \cdot s \cdot t)$ with the number of trains $n$, number of runs $s$ per point in time $t$. For example, in case of $n = 400$, $s = 500$ and $t = 70$ the completion of computation takes roughly $300$ s. With more advanced implementation techniques it should be possible to reduce the computation time by another dimension of $n$.

The calculation of STRELE is theoretically done within $O(c^2)$, with $c$ classes/ranks, due to the fact that the preprocessing for the input data of STRELE has to evaluate the matrix of minimum headway times and other train corresponding data. In practice the calculation does not exceed one second unless having a very large number of classes.

3.3. Influence of Buffer Time Distributions

For the influence of buffer time distributions we consider two scenarios. The first one is the generic example we used above in section 3.1. We examine the Exponential distribution as basis as well as Normal distribution with variation $\sigma^2 = 1$, Erlang distribution with shape
parameter $k = 3$ and Degenerate distribution. All of them are scaled to use the same mean buffer time.

**Scenario 1**

In Figure 3 the expected waiting times per train for the above mentioned distributions are depicted over the mean buffer time. Additionally the current Level of Service (LoS) in German Railways [3] is added to demonstrate the differences in the evaluation and can be understood as the expected waiting time $ET_W$ per train that results in an acceptable quality. The LoS is calculated by the following equation:

$$ET_W = 0.257 \cdot \exp(-1.3 \cdot p_{pt}) \cdot t_{sched}$$

with

- $p_{pt}$ – percentage of passenger trains,
- $t_{sched}$ – size of the schedule.

The size of the schedules is in fact distinct for the considered distributions. Since the relative error between mean size of schedules and the original schedules of the distributions is on average less than 1.5% the error made by the usage of the average schedule length seems negligible.

All chosen distributions follow the course of declining mean waiting times with increasing mean buffer time. In contrast to the other chosen distributions the exponential distribution
does not concentrate its probability density in a close range. Therefore it is more likely that extreme values can occur.

The point of intersection is located for Degenerate and Normal Distribution around minute 4, respectively 4.1, for Erlang Distribution around minute 4.5 and for Exponential Distribution at minute 5.4. The evaluation of this result is two-fold. First, it means that for schedules that are constructed with the considered buffer time distributions a difference of around 80 seconds per train may be necessary to retain a reasonable quality standard. Second, when constructing a schedule it is important to take the generated buffer time distribution into account. For two schedules with the same input, but buffer time distribution, different waiting times can evolve and may be different in height of acceptance. Hence it is essential to distinguish between various schedule structures ranging from highly structured phase traffic and nearly random mixed traffic.

**Scenario 2**

![Figure 4: second example for the influence of buffer time distributions](image)

In Figure 4 a second example for the influence of buffer time distributions is given. The input data is given in Appendix A, which is now more versatile. As seen before the results from the exponential distribution are not as stable as from the other distributions, hence afflicted with more fluctuations. The point of intersection for all distributions, but the exponential, is approximately minute 7.8, whereas the exponential distribution intersects the LoS at minute 9. Thence to gain the same Level of Service lines with exponentially distributed schedules require additional circa 70 seconds buffer time per train.
3.4. Sensitivity Analysis

In the following section the influence of parameters on the point of intersection with the LoS regarding the observed buffer time distributions is examined. The reference scenario ($h_{ij} = 4$, $p_{del,i} = 0.5$, $t_{del,i} = 5$, $i = 1, \ldots, 6$) was simulated with 300 trains and 500 runs each for 0.1 minute steps. It gives the following points of intersection with the LoS as measurement of acceptable operation quality:

- Exponential Dist. 5.4
- Normal Dist. 4.1
- Erlang(3) Dist. 4.5
- Degenerate Dist. 4.0

In Table 2 the impact of deviations in the input parameters is depicted for various scenarios. The effects of varying minimum headway time, probability and time of delay are researched individually. It is to note that the LoS is basically alike in the last two cases since it is determined by the length of the schedule which is unaffected by delay parameters. In contrast the change of minimum headway times does alter the schedule in initial state and after rescheduling due to different blocked times by each train. The minimum headway times are altered by a Normal Distribution $\mathcal{N}(\mu, \sigma^2)$.

It is observable that the Exponential Distribution is affected the most by changes in the parameters, produces the highest knock-on delays and therefore requires a high amount of buffer time to perform adequate in operations. Degenerate and Normal Distribution seem to cope best with changing parameters and generate in general smaller knock-on delays and hence earlier points of intersection with the LoS. It is additionally notable that by not only fitting data with exponential distribution, but allowing extra phases towards an Erlang-Distribution, the mass of probability density is not so spread apart and hence produces more stable schedules. To conclude it is advantageous to have centred buffer time distributions to achieve stable operations due to relatively equal distributed buffer times.

3.5. Case Study

In this section we aim to demonstrate applicability of the before-demonstrated simulation. Shown in Figure 5 the probability density functions of the four best-fitting parametric
minimum headway time $h$ | $\mathcal{N}(4,1)$ | $\mathcal{N}(4,2)$ | $\mathcal{N}(10,4)$  
--- | --- | --- | ---  
Exponential Dist. | 5.3/−2% | 5.0/−7% | 10.5/+94%  
Normal Dist. | 4.1/+0% | 4.0/−2% | 7.2/+76%  
Erlang(3) Dist. | 4.5/+0% | 4.3/−4% | 8.1/+80%  
Degenerate Dist. | 4.1/+3% | 4.0/+0% | 7.1/+78%  

delay probability $p_{del}$ | 0.2 | 0.8 | (.1,.2,.4,.6,.8,.9)  
--- | --- | --- | ---  
Exponential Dist. | 3.7/−31% | 6.1/+13% | 5.9/+9%  
Normal Dist. | 3.2/−22% | 4.6/+12% | 4.4/−7%  
Erlang(3) Dist. | 3.3/−27% | 5.0/+11% | 4.9/+9%  
Degenerate Dist. | 3.7/−8% | 4.5/+13% | 4.0/+0%  

time of delay $t_{del}$ | 1 | 8 | (1,2,5,8,12,20)  
--- | --- | --- | ---  
Exponential Dist. | 2.9/−46% | 6.1/+13% | 5.8/+7%  
Normal Dist. | 2.4/−42% | 4.8/+17% | 4.6/+12%  
Erlang(3) Dist. | 2.3/−49% | 5.3/+18% | 5.0/+11%  
Degenerate Dist. | 2.0/−50% | 4.6/+15% | 4.4/+10%  

Table 2: point of intersection with LoS for different parameters

buffer time distributions are given for a railway line in Germany. The corresponding buffer time average out to 6.3 minutes for a total of 224 trains in 32 classes. Exponential and Normal Distribution seem to be good, though not perfect, fits for the distributions we focused on in the paper.

Figure 5: probability density functions of best fitting buffer time distributions

Taking the train specific data from the line and input it in the simulation we can research the impact of the fitting from real data and judge the consequences for the assessment
of railway capacity. In Figure 6 the results of Simulation with Exponential and Normal Distribution as well as STRELE and LoS as reference are depicted. Input for the simulation were 500 trains with 100 runs every 0.1 minutes.

Figure 6: simulation results for real-world input data

It can be observed that regarding all three distributions the LoS of the railway line shows still available capacity for more trains and adequate buffer times for operations. The currently used buffer time is 6.3 minutes whereas the simulation results show that a buffer time of 5.3 minutes is necessary to achieve an adequate operation. Still it can be seen that the point of intersection for STRELE and Normal Distribution is around one minute earlier than for Exponential Distribution. Additionally the Normal Distribution seems to fit STRELE better due to an earlier drop to a lower level.

4. Conclusion

The presented paper gives an impression of the importance of incorporation of real buffer time distributions on railway lines. The influence of buffer time distributions on the expected waiting times and hence capacity seems significant with around 1 minute per train difference in the presented examples. It seems obvious that due to computational time constraints further research should be concentrated on enhancing the STRELE framework or develop a new analytical tool able to involve not only the train characteristics, but additionally the specifics of the railway line.
Acknowledgement This work was supported by DFG grant NI 1594/2-1.

References


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A. Appendix

In the second example we simulated \( n = 300 \) trains with 500 runs in 0.1 minute steps. In Table 3 the minimum headway times for the second example are given for long distance trains (ldt), local trains (lt) and freight trains (ft). Table 4 gives the remaining train parameters for the simulation.

<table>
<thead>
<tr>
<th>headway times</th>
<th>ldt1</th>
<th>ldt2</th>
<th>lt1</th>
<th>lt2</th>
<th>ft1</th>
<th>ft2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ldt1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>ldt2</td>
<td>4.4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>lt1</td>
<td>6</td>
<td>5.5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>lt2</td>
<td>8</td>
<td>7.5</td>
<td>6.2</td>
<td>6.2</td>
<td>5.8</td>
<td>5.8</td>
</tr>
<tr>
<td>ft1</td>
<td>10</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>7.6</td>
<td>7.6</td>
</tr>
<tr>
<td>ft2</td>
<td>13</td>
<td>13</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>9.3</td>
</tr>
</tbody>
</table>

Table 3: minimum headway times for the second example

<table>
<thead>
<tr>
<th></th>
<th>ldt1</th>
<th>ldt2</th>
<th>lt1</th>
<th>lt2</th>
<th>ft1</th>
<th>ft2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{del} )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6</td>
<td>0.25</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>( t_{del} )</td>
<td>5</td>
<td>5</td>
<td>4.5</td>
<td>2</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>probability in %</td>
<td>12.5</td>
<td>12.5</td>
<td>25</td>
<td>25</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 4: train parameters for the second example
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