A Semi-Empirical Lumped Parameter Model of a Pressure Compensated Vane Pump

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This paper presents an experimentally validated semi-empirical lumped parameter model developed for analyzing the dynamic stability and performance limitations of a pressure compensated vane pump system. The model calculates continuous displacement chamber pressure profiles for the determination of the internal forces acting on the vane pump’s pivoting cam. Extensive measurements conducted on a custom test stand were used to define a nonlinearly progressive bias spring model and a transfer function model of the pump control system valves for realistic system characteristics. Analysis of the complete model reveals the performance limitations imposed by the control system valves in terms of system stability and achievable controller bandwidth are the most restrictive.

**Keywords:** Analysis, Control, Simulation, Pressure Compensation, Vane Pump

**Target audience:** Design Process, Systems, Components

### 1 Introduction

The popularity of light on-highway vehicles with automatic transmissions in the US and the nearly constant push for higher system efficiencies has created a demand for research into methods for improving the transmission performance. Currently, these automatic transmission systems often use pressure compensated pump architectures. This paper presents the results of a research study investigating the causes of instability in a pressure compensated vane pump utilized in automotive applications. To investigate the performance limitations in the case of a pressure compensation control system equipped with low cost valves, the authors developed a semi-empirical lumped parameter model of the pressure compensated pump system and verified it with measurements conducted on a specially built test rig. Figure 1 illustrates the subsystems of the developed model.

\[
\text{Figure 1: Illustration of the various subsystems comprising the pressure compensated vane pump model.}
\]

### 2 Displacement Chamber Module

The kernel of the VDVP subsystem is a lumped parameter model calculating the instantaneous pressure profile of each individual DC. As presented in [1], this is accomplished by solving the pressure build up equation for the i-th DC, Equation (1), derived from the conservation of mass law at each time step using instantaneous flow rates exchanged with the ports via turbulent orifice equations, Equations (2) and (3).

\[
p_{DCi} = \frac{1}{V_i} \int \left( Q_{HIPI} + Q_{LOPI} - Q_{HIPO} - Q_{LOPO} \right) \, dt
\]

\[
Q_{HIPI} = \alpha_{H} \lambda_{HPI} \frac{(p_{HIPI} - p_{DCi})}{\mu \lambda_{HPI}}
\]

\[
Q_{LOPI} = \alpha_{L} \lambda_{LOPI} \frac{(p_{LOPI} - p_{DCi})}{\mu \lambda_{LOPI}}
\]

The calculation of the instantaneous absolute pressure \( p_{DCi} \) in Equation (1) depends heavily on an accurate representation of the DC control volume size \( V_i \) and of rate of change that is accomplished by an analytical function of the DC’s angular span \( 2\alpha_i \), angular position \( \theta \), and the eccentricity angle \( \beta \). The \( V_i \) function also depends on the radius of the rotor body \( r \) and the inner surface of the cam \( R \) along with the distances between the centres of these cylindrical surfaces and the pivot centreline, \( l \) and \( L \) respectively, as illustrated in Figure 3.

Equation (1) considers a lumped external leakage flow term \( Q_{HIPI} \), which can be determined from steady state measurements or from a very complex pump model considering the fluid structure behaviour of the pump’s various tribological interfaces. The authors determined a value for this lumped external leakage from measurements for a
few points of operation. As Equation (4) illustrates, this was accomplished by subtracting the compression losses and internal leakage due to cross-port flow and pump kinematics from the measured volumetric flow losses \( Q_e \) in order to determine the portion of the \( Q_e \) attributed to external leakages \( Q_{ex} \). The terms \( Q_{flux} \) and \( Q_{sh} \), referring to the net compression losses and unmeasurable internal leakages respectively, in Equation (4) are accurately calculated using the model represented by Equations (1-3) when the \( Q_{flux} \) term is neglected. As Equation (5) indicates, \( Q_h \) is the difference between the theoretical flow \( Q_t \) and the effective flow measured \( Q_e \).

\[
Q_{sh} = Q_h - Q_{flux} - Q_{ex} 
\]

(4)

\[
Q_t = Q_{sh} - Q_{sh} 
\]

(5)

Figure 4 shows the difference in the calculated instantaneous DC pressures when the external leakage term \( Q_{flux} \) is both considered and neglected in Equation (1). This term, \( Q_{flux} \), represents the portion of the net external leakage \( Q_{ex} \) contributed by each of the seven DC while in the delivery stroke and is distributed over that portion of the shaft revolution. As can be seen in Figure 4, the impact of these external leakages is negligible for the purposes of this research in calculating representative internal forces for the determination of the system stability. In fact, \( Q_{sh} \) was 27% of \( Q_h \) (0.8 [L/min] or 2% of \( Q_{sh} \) for the operating conditions depicted in Figure 4.

The instantaneous flow rates from the suction port into the \( i \)th DC, \( Q_{In}\), and from the \( i \)th DC into the delivery port, \( Q_{Out}\), are characterized by the instantaneous orifice areas \( A_{or}\) and \( A_{in}\). To resolve the minimum cross-sectional area of the flow path perpendicular to the streamlines connecting the \( i \)th DC with the respective port at a given \( \theta \) and \( \phi \) (see /1/ for more details). Having a separate set of \( A_{or}\) and \( A_{in}\) for each uniquely sized DC builds into the model timing effects, cross-port leakage flow (similar to the work in /2/), and a more realistic frequency spread of the port pressure ripple and internal pressure forces. These areas, along with their alignment with the volume function, is what gives the characteristic DC pressure profile seen in /6/ and Figure 5.

![Unwrapped Cam Surface Area Between Consecutive Vanes](image)

**Figure 3:** Geometric parameters used in the calculation of the DC volume and pressure forces acting on the cam inner surface.

![Simulated Pressure Profile Comparison](image)

**Figure 4:** Comparison of the simulated instantaneous DC pressures with and without external leakages considered.

The DC module described by Equations (1-3) was validated using dynamic pressure transducers installed in the modified pump as described in /11/. Figure 5 shows the validation study highlighting the representative nature of the simulated pressure profile. Discrepancies between the simulated and measured profiles shown in Figure 5 are attributed primarily to differences between the pump geometry from 3D CAD data used in the model and the real geometry of the modified test pump used in the experimental investigation. These variations occur in both the area files and in the \( V_i \) terms and reflect the impact of manufacturing tolerances on the profile.

**DC Pressure Profile Validation**

![Operating Conditions](image)

**Figure 5:** Selected DC pressure profile validation figures comparing simulation model results and in-house measurements shown over a single rotation of the shaft.

Realistic internal forces acting on the cam are calculated with Equations (6) and (7) using this validated module to generate the realistic DC pressure profiles. The first step in these calculations is the definition of a function giving the length \( \rho \) depicted in Figure 3, or the distance from the rotor centre to any point \( P \) on the cam inner surface. The second step involves using \( \rho \) to define the vectors \( \tau_{in} \) and \( \tau_{ex} \) in Figure 3. The third step requires the numerical integration of Equation (6) for a grid of \( \theta \) that spans the accepted values at a sufficient level of discretization for small increments (e.g. 0.5°) of \( \phi \) over a complete revolution of the shaft. This third step produces a matrix of influence factors \( \tau_{in} \) for the \( i \)th DC. Internal pressure forces are then calculated at each time step using Equation (7) for the total DC pressure induced pivotal moment acting on the cam \( M_{DC} \), which is the primary moment for the adjustment system to overcome. This approach differs from that of previous researchers /3-5/ and provides the cam dynamics subsystem in Figure 2 with more realistic internal forces.

\[
\tau_{in}(\theta, \phi) = \int_{A} \left[ \frac{P_{in}}{P_{ref}} \right] \left[ \frac{\rho}{\rho_{ref}} \right] \frac{dA}{d\theta} d\phi
\]

(6)

\[
M_{DC} = \sum_{i=1}^{6} \tau_{in} P_{in}
\]

(7)

As mentioned previously, the lumped parameter model presented in this section considers realistic oil properties. The working fluid’s bulk modulus \( K_i \) in Equation (1) and density \( \rho \) in Equations (2) and (3) are calculated from Equations (8) and (9), respectively. In these equations, the variable \( e_{ref} \) refers to the percentage by volume of entrained air bubbles in the working fluid. This can be as high as 9% in certain automotive applications of pressure compensated vane type pumps (1/1 and 5/5). Meanwhile, \( K_{ref} \) and \( \rho_{ref} \) are calculated using an empirically derived hydraulic oil model and \( \rho_{ref} \), the density of air at STP.

\[
K_i = \frac{e_{ref}}{P_{ref}}
\]

(8)

\[
\rho = \left( 1 - e_{ref} \right) \rho_{oil} + e_{ref} \rho_{air}
\]

(9)
3 Semi-Empirical Cam Dynamics Model

A characteristic feature of any simulation tool designed to aid the engineer in an analysis of pump dynamics or in the evaluation and design of pump control systems is the ability to simulate a realistic motion of the pump adjustment system, in this case the pivoting cam of the VDVP. While in theory this is simple enough to accomplish by solving the equation of motion of the cam, implementation can be rather complicated. The engineer must decide what level of model complexity to adopt to achieve their research goals. Typically, model complexity is increased proportional to the number and types of friction terms included (see 6a for a representative list for a pivoting-cam VDVP). To simplify the model used in this study, a single speed-dependent damping term $C_s$ encompassing all friction effects was included in the equation of motion given by Equation (10) derived as illustrated in Figure 6.

$$l_0^2 = k_0 (l_0 - (l_0 \sin(\beta) + l_0)) - C_s \dot{\beta} - M_{DC} - F_{RCP} + F_{SCP \text{ atm}}$$

(10)

In the final two terms on the right-hand side of Equation (10), $M_{DC}$ and $F_{RCP}$ are influence factors similar to those calculated by Equation (6) that convert the control pressure $p_c$ and atmospheric pressure $p_{atm}$ into moments acting on the cam. These influence factors were generated using projected areas from a 3D CAD model of the reference pump. The resulting cam dynamics model is “semi-empirical” due to the definitions of the damping coefficient $C_s$ and spring rate $k$ from experimental results.

3.1 Damping Coefficient $C_s$

Analyzing measured cam motion using the modified VDVP described in 1/ showed an apparently first-order response to a step command. The damping coefficient $C_s$ was then calculated using the measured rise time to result in an overdamped second-order dynamics for Equation (10) with an equivalent response time when considering a nominal bias spring rate. As an added verification that the selected value is reasonable, the power associated with the sum of the friction terms represented by $C_s$ was calculated using the root-mean-squared rotational velocity of the cam $\dot{\theta}_{RMS}$ from the same measurement data using Equation (11).

$$P_{fs} = C_s \dot{\theta}_{RMS}$$

(11)

$$P_{friction} = P_{pump} - P_{sim}$$

(12)

Given a measured shaft power $P_{pump}$ and the simulated shaft power $P_{sim}$ for the same operating conditions, the total friction power $P_{friction}$ is found using Equation (12). Equation (12) is valid here because the lumped parameter DC module calculates the shaft torque neglecting friction while still accounting for the torque losses attributed to pump design features and oil compressibility. Comparing $P_{fs}$ and $P_{friction}$ reveals that the friction power associated with the cam is about 20% of the total friction power, which matches results in 6a for a similar oil temperature.

3.2 Nonlinear Spring Model

A common feature of every pressure compensated pump is the inclusion of a spring to bias the pump displacement to maximum $\beta_0$. This bias spring force, arising from the spring rate $k$ and the compression of the spring between a surface on the cam and a surface on the pump case, is one of the primary forces acting on the cam as illustrated in Figure 6. The first term on the right-hand side of the equality in Equation (10) highlights that the spring force in this example depends on the distance of the spring centreline from the pivot line $l_0$, the free length $l_0$ of the spring, and the compressed length $l_0$ for an eccentricity angle of zero. Note that the compression of the spring at a maximum eccentricity angle gives the spring pre-load.

While measurements of the spring force as a function of the compression of the spring on a separate test stand indicate a constant linear spring rate (the nominal spring rate), various distinct values for $k$ were required for the simulated cam dynamics defined by Equation (10) to match measurements acquired using the modified VDVP. Two possible conclusions to explain this observation are that the description of other internal forces is inaccurate for some operating conditions or that the compression spring is behaving in a nonlinear fashion. Without changing modelling approaches for the VDVP’s displacement chambers to improve the estimation of the DC pressure induced pivot moment and using the measured control pressure $p_c$ in Equation (10), a closer look at the spring force is warranted.

Due to the pump geometry and the pivoting motion of the cam, the spring in Figure 6 does not experience compression between two parallel surfaces. Instead, the spring is subjected to both compression and bending as the angle between the surfaces varies linearly with the eccentricity. Under these conditions, the internal shear and torsional stresses in the spring would be different for a given distance between the surface centres than for the same distance between the centres of two parallel surfaces (see 6a for the internal stresses in this case). Since it is common to model the spring force as acting at a point (in this case the centre of the cam surface in contact with the spring), the use of a constant spring rate may not be appropriate to capture the real response of the spring.

Simulation results improved, for each measurement in the study, when a progressive spring rate as a function of $\beta$ given by Equation (13) was used to calculate the value of $k$ in Equation (10). Equation (13) results in a spring rate that increases from the nominal value to a maximum almost sixty percent greater at the minimum compressed length. Because Equation (13) was derived in an iterative fashion comparing simulation results to various measurements, this $k$ complies the semi-empirical cam dynamics model given by Equation (10).

$$k = 21074\beta^3 - 2993.4\beta^2 - 80.447\beta + 47.104$$

(13)

3.3 Experimental Validation

Figure 7 gives the hydraulic circuit for the experimental setup used to measure the cam motion for various operating conditions. As indicated in the figure, three pressures (inlet $p_i$, outlet $p_o$, and control $p_c$) were measured along with the pump speed $n$ and cam displacement via a LVDV as explained in 1/4. Using this setup, the cam eccentricity model was validated by comparing simulation results with measurements at the same operating conditions. Two example comparisons are included in Figure 8.
5 Pump Control System Model

One automotive application for a pressure compensated vane pump is the simultaneous lubrication, cooling, and supply of hydraulic power to an automatic transmission system. Figure 9 illustrates the baseline pressure compensated pump control system for this type of application studied in this research. In it, an electrical command to the solenoid operated pressure-reducing valve (V3) sets the desired outlet pressure of the pressure compensated vane pump. This desired pressure increases for each clutch-shifting event and returns to a lower value when no between events. In summary, two important requirements of the pump’s pressure compensated control system are to modulate the pump displacement effectively to maintain a constant low pressure regardless of input speed variations and to respond quickly to step commands when required to meet system performance specifications.

Instead of developing a more complex physically based model for the involved pump control system valves, the authors decided to characterize them in a black box approach through measurements of the valve system. This black box model was derived in three parts as illustrated in Figure 10 to be able to facilitate the derivation and to identify the contribution of each aspect of the pressure compensated control system to the overall system response.

As Figure 10 illustrates, a custom valve block containing actual control circuit valves from a transmission was instrumented with various dynamic pressure transducers and LVDT on the custom test rig presented in /1/. These transducers were sampled at a rate of 2 kHz to generate signals that for determining the transfer functions $G_2$, $G_{1a}$, $G_{1b}$, $G_1$, and $G_2$ and the gain $K_s$. These transfer functions were each assumed to represent second-order dynamics with a steady-state gain. The natural frequency, damping ratio, and steady state gain $K$ of each transfer function.
was then tuned in an iterative fashion until a good agreement with the measured output was reached when the measured input was passed through the transfer function. Equations (19-23) give the resulting transfer functions.

\[
G_2 = \frac{K_2 (2.20 \times 10^5)}{s^4 + 0.835s^3 + 2.28 \times 10^3}
\]  
(19)

\[
G_{2A} = \frac{K_{2A} (2.22 \times 10^5)}{s^4 + 0.848s^3 + 2.22 \times 10^3}
\]  
(20)

\[
G_{1B} = \frac{K_{1B} (246.7)}{s^3 + 0.835s^2 + 246.7}
\]  
(21)

\[
G_1 = \frac{K_1 (201.7)}{s^2 + 0.803s^2 + 201.7}
\]  
(22)

\[
G_1 = \frac{1.12 \times 10^5}{s^4 + 0.835s^3 + 1.023 \times 10^5}
\]  
(23)

When the outlet pressure \( P_{\text{o}} \) was held constant at different levels, different steady state gains \( K_2, K_{2A}, K_{1B}, K_0, \) and \( K_1 \) were required for model agreement. Therefore, each of these are represented in the final model as look-up tables with linear interpolation between the values. As Equation (21) reveals, a pure time delay was added to the transfer function between the command signal \( CMD \) and the regulation setting pressure \( P_c \). Equation (22) shows that \( G_1 \) was reduced to a first-order low-pass filter while Equation (23) shows that an additional fast pole was added to the lightly damped poles of \( G_1 \). These transfer functions and gains combine according to the block diagram given in Figure 10, the result is a two-input single-output linear model as represented by Equation (24) giving the control flow \( Q_b \). The control pressure \( P_0 \) is then calculated in the adjustment system dynamics model according to Equation (25) where the partial derivative is the “control piston” area and \( V_0 \) is the control chamber volume.

\[
Q_b = [G_A \quad G_{\text{CMD}}] \begin{bmatrix} P_0 \\ \text{CMD} \end{bmatrix}
\]  
(24)

\[
P_0 = \frac{K_0}{V_0} \int \left( Q_b - \frac{\partial V}{\partial \phi} \right) \, d\phi
\]  
(25)

One result of this block diagram reduction is that the new transfer function \( G_A \) contains a non-minimum phase zero at lower pressures while \( G_{\text{CMD}} \) contains the pure time delay from \( G_{\text{CMD}} \). Therefore, both components of the final transfer function matrix present limitations to the dynamic stability and achievable bandwidth of the system as a whole. The Bode diagram shown in Figure 11 illustrates these limitations.

6 Analysis of the System Model

As stated in Section 1, the model described in the previous sections is useful for realistically analysing the stability and performance limitations of a pressure compensated vane pump system, as shown in Figure 9, designed for use in an automatic transmission application. Upon inspection of Figure 1 and Figure 9, it is clear that the system performance depends on both the dynamics of the pump adjustment system (the pivoting cam and bias spring) and the pump control system (regulation and solenoid valves). While Figure 11 contains ample information regarding the dynamics of the pump control system, the dynamics of the pump adjustment system described in Section 3 may still be unclear and deserve additional discussion.

Critical Frequencies

Figure 12 illustrates a few of the interesting frequencies associated with the cam dynamics model. The dashed red and blue lines in the figure show the range of cam natural frequencies (78Hz to 97Hz) when Equation (13) defines the bias spring. The solid black lines represent the principal harmonic frequencies of the shaft frequency. Since the dominant frequency of the internal pressure forces is typically the second harmonic of the shaft frequency, it is unlikely that the internal forces acting on the cam from the DC pressure variations will excite resonant behaviour in the cam motion, especially considering the high damping of the cam system.

Figure 12 also illustrates the critical frequencies \( f_c \) of the bias spring model (solid red and blue lines) as calculated using Equation (26) taken from \( \beta \). For the spring parameters in Equation (26), \( d_s \) is the nominal wire diameter, \( D_s \) is the mean diameter of the spring, \( N_s \) is the number of active coils, \( \rho_s \) is the density, and \( k \) is given by Equation (13). While \( \beta \) states that Equation (26) is valid for a helical compression spring between two flat parallel plates and the bias spring’s environment in the VDPV does not fit this description, the resulting \( f_c \) can still give the approximate excitation frequencies that may induce other destabilizing effects such as spring surge. An evaluation, and subsequent simulation, of this kind of nonlinear behaviour would require additional research. Suffice it to say, un-modelled dynamics at higher frequencies may contribute to instabilities in the pump adjustment system.

\[
f_c = \frac{1}{2 \pi \sqrt{k \rho_s N_s d_s^3}}
\]  
(26)

Nevertheless, the limiting factor to the system performance from a control perspective is definitely the control system valves as revealed by Equation (24) and Figure 11. Even when viewing the control system valves as a passive element that merely responds as a typical pressure regulator valve the limiting element is not the pump considering the natural frequency of the cam is roughly five times the bandwidth of \( G_1 \) in Equation (24). Even operating at low oil temperatures with higher viscous damping forces would the pump response time approach that of \( G_1 \) although the lower temperature would have a similar effect on the regulator valve speed and increase its response time as well.
7 Conclusion

In conclusion, a brief analysis of the semi-empirical lumped parameter model of a pressure compensated vane pump developed in this paper leads to the following key observations.

- Complex DC leakage path models are not required for the calculation of representative DC pressure profiles, and subsequent internal forces, to a sufficiently accurate level for analysing pump adjustment system dynamics provided an accurate representation of the DC geometry and orifice areas used in Equations (1-3) is available. This modelling approach has been validated with measurement data and provides a best-case scenario for the analysis of the control system performance. This strategy is equally valid for translating-cam VDVP considering their close resemblance to the pivoting-cam VDVP modelled here.

- Due to suboptimal pump geometry and low-cost helical compression springs, the VDVP bias spring may behave nonlinearly as shown in Section 3.3. Additional research of this observed behaviour would better answer whether or not the main issue is the compression between non-parallel surfaces or large variations in the spring characteristics due to manufacturing tolerances and variations in material properties. Additional research into the bias spring in this environment may also reveal that the observed nonlinear behaviour is a function of a variable not yet considered which may necessitate more complex simulation tools to characterize the various tribological interfaces.

- The primary source of performance limitations in a pressure compensated vane pump system is the pump control system. In this example, the baseline system suffers from limitations due to the time delay associated with the solenoid valve V3 in Figure 9 that restrict the achievable bandwidth to below 5 Hz. In fact, measurements indicated that the actual bandwidth of the custom valve block installed on the test apparatus explained in Section 3 might be closer to 1.5 Hz. This is realistic because the actual bandwidth achieved also depends on the non-minimum phase zero generated by the control system architecture.

It is clear from the list of these observations that the most important contributor to the overall system performance of a pressure compensated vane pump is the control system valves and their arrangement. Engineers designing pressure compensated vane pump systems must pay closer attention to this aspect of the control system design and be willing to incur a higher cost to implement better architectures and achieve improved system performance. As illustrated in this paper, the models used in this analysis and design process need not be exhaustive to be useful.

Nomenclature

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Surface area of cam between two consecutive vanes</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$A_{H1}$</td>
<td>Equivalent orifice area representing the resistance of the inlet filter</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$A_{d}$</td>
<td>Orifice area connecting the delivery port and the line</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$A_{H1P}$</td>
<td>Instantaneous orifice area connecting the $i^{th}$ DC with the delivery port</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$A_{H1P}$</td>
<td>Instantaneous orifice area connecting the $i^{th}$ DC with the suction port</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Damping coefficient for the cam dynamics equation of motion</td>
<td>[kgm$^2$/s]</td>
</tr>
<tr>
<td>$CMD$</td>
<td>Electrical command signal</td>
<td>[A]</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Bias spring wire diameter</td>
<td>[mm]</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Differential arc length along the inner surface of the cam between vanes</td>
<td>[mm]</td>
</tr>
<tr>
<td>$D_s$</td>
<td>Bias spring mean diameter</td>
<td>[mm]</td>
</tr>
</tbody>
</table>
\( \rho_{in} \) Instantaneous absolute pressure of the pump delivery port [bar]

\( p_{in} \) Instantaneous absolute pressure of the pump suction port [bar]

\( p_{in,act} \) Absolute pressure at the opening of the pump inlet filter [bar]

\( P_{LF} \) Mean power loss resulting from cam friction and viscous damping [W]

\( R_{friction} \) Total pump power loss resulting from all friction sources [W]

\( P_{pump} \) Measured pump input power [W]

\( P_{in,act} \) Simulated pump input power neglecting friction effects [W]

\( Q_{C,net} \) Net volumetric pump flow lost to oil compressibility effects \([m^3/s]\)

\( Q_{V} \) Volumetric flowrate through the regulation valve or the control flow \([m^3/s]\)

\( Q_{bl} \) Volumetric flowrate through the pump inlet filter \([m^3/s]\)

\( Q_{Io} \) Volumetric flowrate through the load orifice \([m^3/s]\)

\( Q_{HPI} \) Volumetric flowrate from the \( i^{th} \) DC into the delivery port \([m^3/s]\)

\( Q_{UL} \) Volumetric flowrate from the suction port into the \( i^{th} \) DC \([m^3/s]\)

\( Q_{SE} \) Total measured flow losses of the pump under steady state conditions \([m^3/s]\)

\( Q_{SE,DCi} \) External leakage corresponding to the \( i^{th} \) DC \([m^3/s]\)

\( Q_{sl} \) Net internal leakages, unmeasurable, of the pump under steady state conditions \([m^3/s]\)

\( Q_{ch} \) Theoretical flow rate of the pump for the given displacement level and speed \([m^3/s]\)

\( r \) Radius of the rotor body \([mm]\)

\( r_{air} \) Percent entrained air by volume in the fluid/air mixture \([-\%]\)

\( R \) Radius of the inner surface of the cam \([mm]\)

\( R_{gauge} \) Equivalent resistance of the experimental test circuit load \([\Omega/m^2]\)

\( T \) Temperature of the fluid/air mixture measured at the pump inlet \([^\circ C]\)

\( u \) Transducer electrical signal \([V]\)

\( V_{ch} \) Volume of the pump control chamber \([m^3]\)

\( V_{pl} \) Volume of the pump delivery port \([m^3]\)

\( V_{Io} \) Volume of the \( i^{th} \) DC \([m^3]\)

\( V_{Sl} \) Volume of the pump suction port \([m^3]\)

\( x \) Measured LVDT linear displacement in experimental study \([mm]\)

\( \sigma_{Io} \) Turbulent orifice flow equation discharge coefficient [-]

\( a_{Io} \) Half-sector angle giving the size of the \( i^{th} \) DC \([rad]\)

\( \beta \) Cam eccentricity angle (in [rad] or [degrees]) or pump displacement (in [%]) \([rad]\)

\( \beta_{mean} \) Root-mean-squared angular velocity of the cam from measurement data \([rad/s]\)

\( \beta_{ramp} \) Constant describing the change in control chamber volume with eccentricity \([m^3/rad]\)

\( \beta_{p} \) Vector from the pivot centre to a point \( P \) on the cam inner surface \([mm]\)

\( \beta_{p} \) Vector from the centre of the cam inner surface to a point \( P \) on that surface \([mm]\)

\( \phi \) Angular position of a DC as illustrated in Figure 3 \([rad]\)

\( \rho \) Distance from the rotor centre to a point \( P \) on the cam inner surface \([mm]\)

\( \rho_{air} \) Density of air at STP \([kg/m^3]\)

\( \rho_{f} \) Density of the fluid/air mixture \([kg/m^3]\)

\( \rho_{d} \) Density of the pure working fluid \([kg/m^3]\)

\( \rho_{s} \) Density of the bias spring material \([kg/m^3]\)

\( \tau_{i} \) Influence factor converting the \( i^{th} \) DC pressure to a pivoting moment \([m^3]\)

\( \tau_{RC} \) Influence factor converting the control pressure to a pivoting moment \([m^3]\)

\( \tau_{SC} \) Factor converting the pressure in the spring chamber to a pivoting moment \([m^3]\)

\( \omega_{s} \) Natural frequency of the cam \([rad/s]\)

References


