

Reliability Evaluation of Hydraulic Pump Based on Performance Degradation

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Aiming at the disadvantage of long time and high cost in the reliability evaluation of the hydraulic pump based on the life test or accelerated life test, a reliability evaluation method for hydraulic pump based on performance degradation is proposed. The stochastic volatility, individual differences and measurement errors have been considered in the creation of the degradation model. Compared with the method based on degradation path model and Wiener process model, it is necessary to consider the stochastic volatility with time, individual differences and measurement errors of performance degradation in the reliability evaluation of hydraulic pump based on degradation.

Keywords: Hydraulic pump, reliability evaluation, performance degradation, Wiener process

Target audience: Hydraulic Pump Manufacturer, Hydraulic Pump Customer, Aircraft Industry

1 Introduction

The hydraulic pump is the core component in the hydraulic system, and its reliability and remaining useful life have a vital influence on the reliability and health of the equipment. Therefore, the accurate evaluation of the reliability and the remaining useful life of the hydraulic pump has great significance to the failure prediction and health management (PHM) of the equipment [1-3]. At present, the reliability evaluation method based on life test or accelerated life test is common. In America, the standard of life test for military hydraulic pump has undergone a series of development such as MIL-P-19692C, MIL-P-19692D, MILP-19692E, SAE-AS19692A and so on. Russia has issued a series of guidelines for accelerated life test: OCT 100228-77, OCT 100389-80, OCT 100149-82, M4-73 and M35-84[4]. In the literature [5], the sensitive stress of hydraulic pump was analysed and the multiple stress accelerated model was given. In the literature [6], double – constant - stress sequential test was carried out by increasing the load, and the method of determining the sequential accelerated life test of the aviation gear pump was obtained. However, the reliability evaluation method based on life test or accelerated life test needs to observe the failure time of the product, which takes long time. The hydraulic pump has large power and long life. Carrying out the long life test not only consumes a lot of manpower and material resources, but also is not consistent with the rapid development demand of the hydraulic pump. Therefore, short time, high efficiency and accurate reliability estimation method is urgently needed.

The technology of reliability evaluation and life prediction based on performance degradation can obtain reliability information without failure, it has gradually become the development trend of reliability evaluation and remaining useful life prediction of highly reliable products with long lifetime[7-8], and has been widely utilized in the models of degradation and reliability evaluation, such as LED[9], bridge beam [10], crack length [11] and lithium-ion battery[12], etc. It also provided a method of reliability evaluation and remaining useful life predicting of hydraulic pump. The hydraulic pump is a typical product with performance degradation. Affected by wear, temperature, load

and other environmental factors, the characteristic parameters of its components will degrade and this phenomenon will result in the degradation of output flow the main performance index of piston pump. And soft failure will occur when performance degradation exceeds the specified threshold [13]. Based on this characteristic, literature [13] took piston pump flow signal as performance degradation parameter, based on considering randomness of degradation, combined with path tracing method and graphic method, reliability evaluation of piston pump is realized. In literature [14], the signal of high temperature and low flow were taken as performance degradation parameter, and the change rule of reliability was obtained by using degradation path fitting method to analyse reliability of an aviation piston pump under constant stress accelerated degradation test (CSADT). Literature [15] took the signal of high temperature and low flow of piston pump as the degradation characteristic parameter. Under the CSADT, the least square method is used to fit the degradation path and multiple regression method is used to get the degradation model, and the life of the piston pump is successfully predicted. In literature [16], volumetric efficiency was taken as the signal of performance degradation and obtained the characteristic life of the axial piston pump by applying nonlinear least squares method and optimization theory to study the degradation path of performance parameters of hydraulic pump under accelerated stress.

It can be seen that the reliability evaluation of hydraulic pump based on performance degradation is relatively few, and smooth curves is used to fit the degradation path frequently. Because of the factors such as wear, environment, load and other uncertainties, the performance degradation path of the hydraulic pump has randomness. The impact of environment, instruments and manufacturing errors in manufacturing process result in the inconsistency of product degradation, that is, individual differences. The interference of the external environment or the inaccuracy of measuring instruments will result in the measurement errors. These factors have a certain influence on the reliability evaluation of the hydraulic pump. Therefore, in this article, the volumetric efficiency was taken as the characteristic parameter of performance degradation. Under the consideration of the individual differences and the measurement errors, we established the performance degradation process with stochastic characteristics of hydraulic pump based on linear Wiener process. The effectiveness and accuracy of the hydraulic pump reliability evaluation method proposed are demonstrated through case studies.

2 Degradation Model of Hydraulic Pump Based on Linear Wiener Process

2.1 Degradation model Based on Linear Wiener Process

Because of the uncertainty of the factors such as wear, environment, load and so on, the performance degradation path of the hydraulic pump has randomness. The Wiener process driven by Brown motion can characterize the dynamic characteristics of the degradation. So it can build the model of the hydraulic pump performance degradation path effectively. Suppose the real degradation process of the hydraulic pump can be interpreted by linear Wiener process

$$x(t) = x(0) + \lambda t + \sigma_b B(t) \quad (1)$$

Where, $x(t)$ denotes the real value of degradation of product's performance at time t , $x(0)$ denotes the initial value of degradation, λ is the drift parameter representing the degradation rate and σ_b is the volatility parameter, $B(t)$ denotes a standard Wiener process representing the degradation dynamic characteristics.

2.2 Degradation Model Considering Individual Differences and Measurement errors

However, the real degradation data is difficult to be measured precisely because of the random errors caused by the instrument or environment. Similar to the degradation model proposed in literature [17], based on wiener process with measurement errors under constant stress, measurement value of performance degradation of hydraulic pumps can be described as follow

$$y(t) = x(t) + \varepsilon \quad (2)$$

Where, $y(t)$ represents the value of degradation of product's performance characteristic at time t and the measurement errors term ε follows a normal distribution as $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

The degradation rate of hydraulic pumps is different affected by the random effects such as environment, instrument, manufacturing errors in manufacturing process. So it cannot be neglected in the reliability evaluation and the remaining useful life test. Therefore, refer to the literature Peng and Tseng^[17] and Tang^[18], drift parameter λ is set as a random parameter to represent the individual differences, which follows a normal distribution as $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$. λ and $B(t)$ are mutually independent.

The hydraulic pump is deemed to be soft-failed when its degradation first exceeds a predefined failure threshold ω . Therefore, the hydraulic pump's lifetime T at first hitting time can be defined as follow

$$T = \inf \{t : y(t) \geq \omega \mid y(0) < \omega\} \quad (3)$$

The probability density function and the reliability function of the life distribution can be expressed as^[19]

$$f(t) = \left[\frac{\omega \sigma_B^2 + \mu_\lambda \sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + \sigma_B^2 t) \sqrt{\sigma_\varepsilon^2 + \sigma_B^2 t + \sigma_\lambda^2 t^2}} - \frac{\sigma_\lambda^2 \sigma_\varepsilon^2 t (\omega - \mu_\lambda t)}{(\sigma_\varepsilon^2 + \sigma_B^2 t) (\sigma_\varepsilon^2 + \sigma_B^2 t + \sigma_\lambda^2 t^2)^{3/2}} \right] \phi \left(\frac{\omega - \mu_\lambda t}{\sqrt{\sigma_\varepsilon^2 + \sigma_B^2 t + \sigma_\lambda^2 t^2}} \right) \quad (4)$$

$$R(t) = \Phi \left(\frac{\omega - \mu_\lambda t}{\sqrt{\sigma_\varepsilon^2 + \sigma_B^2 t + \sigma_\lambda^2 t^2}} \right) - \frac{\sigma_B^2}{\sqrt{\sigma_\varepsilon^4 - 4\sigma_\lambda^2 \sigma_\varepsilon^2}} \exp \left(\frac{2\mu_\lambda \omega}{\sigma_B^2} + \frac{2\mu_\lambda^2 \sigma_\varepsilon^2}{\sigma_B^4} + \frac{2\sigma_\lambda^2 (\omega \sigma_B^2 + 2\mu_\lambda \sigma_\varepsilon^2)}{\sigma_B^4 (\sigma_\varepsilon^4 - 4\sigma_\lambda^2 \sigma_\varepsilon^2)} \right) \times \Phi \left(- \frac{\omega (\sigma_B^2 + 2\sigma_\lambda^2 t) + \mu_\lambda (\sigma_B^2 t + 2\sigma_\varepsilon^2)}{\sqrt{(\sigma_\varepsilon^2 + \sigma_B^2 t + \sigma_\lambda^2 t^2) (\sigma_B^4 - 4\sigma_\lambda^2 \sigma_\varepsilon^2)}} \right) \quad (5)$$

Where, ϕ and Φ are the probability density function and the distribution function of the standard normal distribution.

Similarly, the remaining useful life at first hitting time can be defined as $L_t = \inf \{l : T - t \mid T > t\}$, and the probability density function and the expectation value of the remaining useful life can be respectively written as

$$f_{L_t}(l_t) = f(t + l_t) / R(t) \quad (6)$$

$$E_{L_t}(l_t) = \int_0^\infty l_t f_{L_t}(l_t) dl_t = \int_t^\infty (s - t) f(s) ds / R(t) \quad (7)$$

3 Parameter Assessment

Assume that n units are to be tested. Each unit is measured once every Δt hours and is measured m times totally. For 2.1 section, the degradation process without consideration of individual differences and measurement errors, the increment of the degradation of the i -th unit under the j -th measurement $\Delta x_{ij} = \lambda \Delta t + \sigma_B \Delta B(t_j)$, $\Delta x_{ij} = x_i(t_j) - x_i(t_{j-1})$, $\Delta B(t_j) = B(t_j) - B(t_{j-1})$, $\Delta t = t_j - t_{j-1}$, t_j is the time of measurement, $t_0 = 0$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$. Known from the stationary independent increment of Wiener process, $\Delta x_{ij} \sim N(\lambda \Delta t, \sigma_B^2 \Delta t)$. According to the maximum likelihood estimation (MLE) method, we can obtain

$$\hat{\lambda} = \frac{\sum_{j=1}^m \sum_{i=1}^n \Delta x_{ij}}{mn \Delta t} \quad (8)$$

$$\hat{\sigma}_B^2 = \frac{\sum_{j=1}^m \sum_{i=1}^n (\Delta x_{ij} - \hat{\lambda} \Delta t)^2}{mn \Delta t} \quad (9)$$

For the 2.2 section, the hydraulic pump performance degradation process considering the individual differences and measurement errors, the increment of performance degradation is not independent. In this section, the two-step MLE method^[12] is used to estimate the unknown parameters of the degradation model. According to the Equation (2), the performance degradation of the i -th unit under the j -th measurement can be expressed as

$$y_i(t_j) = \lambda_i t_j + \sigma_B B(t_j) + \varepsilon \quad (10)$$

Let $\mathbf{Y}_i = (y_i(t_1), \dots, y_i(t_m))^T$ represent the vector of the performance degradation of the i -th unit, $\mathbf{Y} = (\mathbf{Y}_1', \dots, \mathbf{Y}_n')$ represent the performance degradation matrix of the sample, $\mathbf{t} = (t_1, \dots, t_m)^T$, from the Equation (10), it can be seen that $\mathbf{Y}_i$ obeys a multivariate normal distribution whose mean value is $\bar{\mu}_i = \lambda_i \mathbf{t}$ and the covariance is Σ

$$\Sigma = \sigma_B^2 \mathbf{Q} + \sigma_\varepsilon^2 \mathbf{I}_m \quad (11)$$

Where, $\mathbf{I}_m$ is a unit matrix with order m , and $\mathbf{Q} = \begin{bmatrix} t_1 & \cdots & t_1 \\ & t_2 & \cdots & t_2 \\ \vdots & \vdots & \ddots & \vdots \\ t_1 & t_2 & \cdots & t_m \end{bmatrix}$.

The degradation inspections in different units are mutually independent, and then the log-likelihood function (Log-LF) can be expressed as

$$L(\theta | \mathbf{Y}) = -\frac{mn}{2} \ln(2\pi) - \frac{n}{2} \ln |\Sigma| - \frac{1}{2} \sum_{i=1}^n (\mathbf{Y}_i - \lambda_i \mathbf{t})^T \Sigma^{-1} (\mathbf{Y}_i - \lambda_i \mathbf{t}) \quad (12)$$

Where, $\theta = (\lambda_1, \dots, \lambda_n, \sigma_B, \sigma_\varepsilon)$ denotes the vector of unknown parameters.

Equation (13) can be derived from the first derivation of λ_i in Equation (12), which can be written as

$$\frac{\partial L(\theta | \mathbf{Y})}{\partial \lambda_i} = \mathbf{t}^T \Sigma^{-1} \mathbf{Y}_i - \lambda_i \mathbf{t}^T \Sigma^{-1} \mathbf{t} \quad (13)$$

Let Equation (13) equal to zero, the restricted MLE of λ_i can be obtained by the following equation

$$\hat{\lambda}_i(\sigma_B, \sigma_\varepsilon) = \frac{\mathbf{t}^T \Sigma^{-1} \mathbf{Y}_i}{\mathbf{t}^T \Sigma^{-1} \mathbf{t}} \quad (14)$$

Substitute Equation (14) into Equation (12), the profile Log-LF of the unknown parameter can be expressed as the following equation

$$L(\sigma_B, \sigma_\varepsilon | \mathbf{Y}) = -\frac{mn}{2} \ln(2\pi) - \frac{n}{2} \ln |\Sigma| - \frac{1}{2} \sum_{i=1}^n (\mathbf{Y}_i - \hat{\lambda}_i \mathbf{t})^T \Sigma^{-1} (\mathbf{Y}_i - \hat{\lambda}_i \mathbf{t}) \quad (15)$$

The MLE of the model parameters, σ_B and σ_ε can be obtained by maximizing the profile Log-LF (15). The MLE of λ_i can be obtained by substituting $\hat{\sigma}_B$ and $\hat{\sigma}_\varepsilon$ into Equation (14). Finally, the estimated values of μ_λ and σ_λ^2 can be obtained by Equations (16) and (17)

$$\hat{\mu}_\lambda = \frac{1}{n} \sum_{j=1}^n \hat{\lambda}_j \quad (16)$$

$$\hat{\sigma}_\lambda^2 = \frac{1}{n} \sum_{j=1}^n (\hat{\lambda}_j - \hat{\mu}_\lambda)^2 \quad (17)$$

4 Application Example

The A4VS series axial piston pumps produced by a company were used as the test object to demonstrate the validity and accuracy of the reliability evaluation and the remaining useful life prediction method. The rated pressure of this series of pumps is 350 bar, and the peak pressure is 400 bar. Four hydraulic pumps were randomly selected for the test, and the measurement interval was set as $\Delta t=12\text{h}$, test pressure was set as rated pressure. The volumetric efficiency ($z_i(t_j)$, $j=1,2,\dots,85$) was calculated. The performance degradation value $d_i(t_j)$ of hydraulic pump's volumetric efficiency can be expressed as $d_i(t_j)=z(0)-z_i(t_j)$, $z(0)=0.955$ is the mean of initial volumetric efficiency of the axial piston pump under the rated pressure, and the performance degradation trend of the tested pumps are shown in Figure 1.

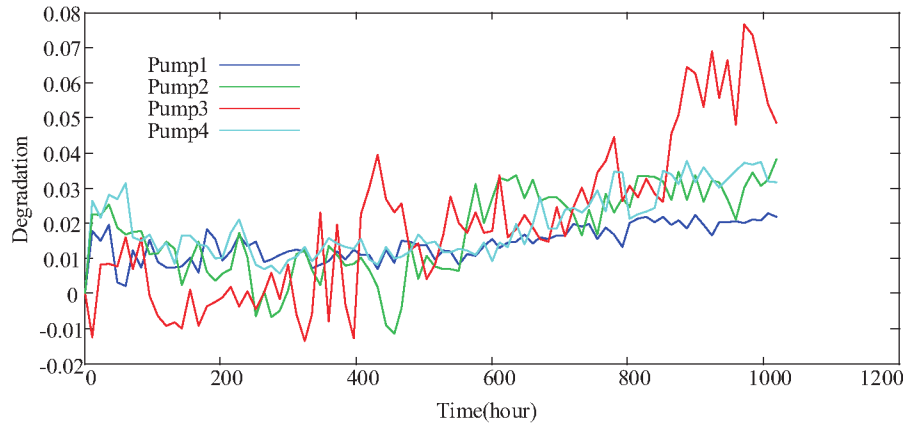


Figure 1: A trend Diagram of the Performance Degradation of the Hydraulic Pump.

In order to facilitate subsequent calculation and analysis, the degradation data shown in Figure 1 is standardized by Equation (18)

$$y_i(t_j) = d_i(t_j) / d_{\max} \quad (18)$$

Where, $i=1,2,\dots,n$, $j=1,2,\dots,m$, $d_{\max}$ is the maximum value of the performance degradation $d_i(t_j)$.

In order to demonstrate the validity and accuracy of the proposed method, the model defined in 2.1 section is denoted as M1, the model in Section 2.2 is denoted as M2 and the method of curve fitting based on [16] is defined as M3.

For the method M1, using the autocorrelation function to inspect the independence of the increment of standardized performance degradation firstly. Figure 2 is an autocorrelation function chart of the standardized performance degradation increment of 4 hydraulic pumps. From Figure 2, it can be observed that the first order autocorrelation coefficients are close to or exceed the two horizontal lines, that is, approximate or exceed the positive and negative two times of the sample autocorrelation coefficient approximate standard deviation ($\pm 2/\sqrt{m}$). It means if standardized degradation obeys Wiener process model, the standardized incremental sequence of degradation did not pass the inspection of independence. We should not choose M1 method but M2 to model the hydraulic pump performance degradation process.

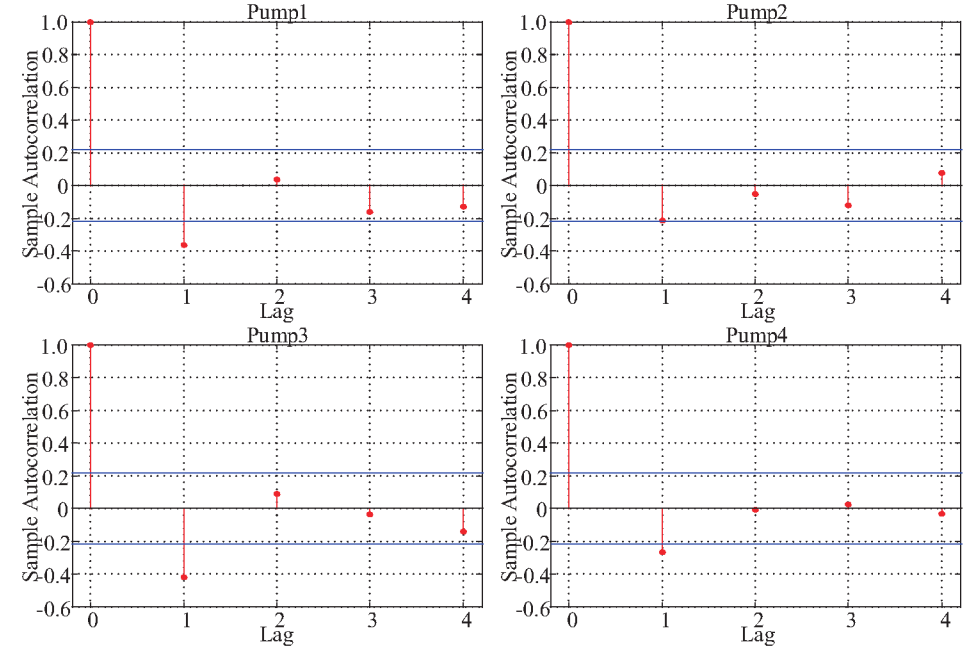


Figure 2: Independence Test of Performance Increment.

Similar to literature [16], we think that the soft failure occurred on the hydraulic pump when the volumetric efficiency of the hydraulic pump drops to 87%. The initial volumetric efficiency of the series axial piston pump is 95.5%, so we think that the hydraulic pump failed when the performance degradation (Figure 1) reaches the failure threshold of 0.085. Aimed at the standardized data, failure threshold ω is $0.085 / y_{\max}$. According to Equations.

(8) and (9), the parameter estimation of M1 can be obtained, $\hat{\lambda}=4.4798 \times 10^{-4}$, $\hat{\sigma}_\lambda^2=8.1470 \times 10^{-4}$. Let $\hat{\sigma}_\lambda^2=0$ and $\hat{\sigma}_\varepsilon^2=0$, it can be calculated that the average life of the hydraulic pump is 2.4771×10^3 h by substituting the parameters into the Equation (7).

For the method M2, according to the parameter estimation method proposed in the section 3, we can obtain $\hat{\lambda}_1=2.8078 \times 10^{-4}$, $\hat{\lambda}_2=4.6211 \times 10^{-4}$, $\hat{\lambda}_3=6.7492 \times 10^{-4}$, $\hat{\lambda}_4=4.1791 \times 10^{-4}$, $\hat{\mu}_\lambda=4.5893 \times 10^{-4}$, $\hat{\sigma}_\lambda^2=2.0021 \times 10^{-8}$, $\hat{\sigma}_\varepsilon^2=2.7833 \times 10^{-4}$ and $\hat{\sigma}_\varepsilon^2=3.3039 \times 10^{-3}$. It can be calculated that the average life of the hydraulic pump is 3.7706×10^3 h by substituting the parameters into the Equation (7).

Comparing the parameter estimation results of M1 and M2, it can be seen that the estimated value of drift coefficient $\hat{\lambda}$ is close to $\hat{\mu}_\lambda$, which demonstrated the correctness of the two models fitting for degradation trend. The result of diffusion coefficient estimation $\hat{\sigma}_\varepsilon^2$ is quite different. This is because M1 does not consider the measurement errors. The fluctuation of measurement errors is reflected by diffusion coefficient. Meanwhile, it demonstrated that the measurement errors cannot be ignored.

The results of method M3 are given by literature [16]. The life distribution of the hydraulic pump obeys the two-parameter Weibull distribution, its characteristic value is $m=2.5$, $\eta=4482.4$, and the average value of life is 4.1275×10^3 h.

Comparing the average value of life obtained by the three methods, it can be seen, the average values of M1 and M2 are less than M3. Because of failure was defined by the first hitting time (Equation (3)), the result tends to be conservative, which has positive significance for the preventive maintenance and health management of hydraulic

pump. Meanwhile, the average life of M2 is close to M3, and the correctness of the method proposed in this paper is also demonstrated. For a more direct comparison to illustrate the characteristics of the three methods, Figure 3 and Figure 4 give the probability density function curves and reliability function curves respectively.

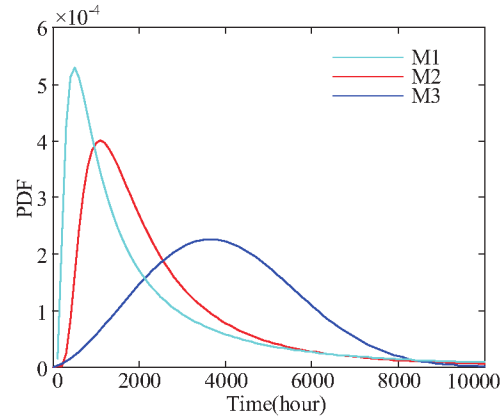


Figure 3: Probability density function graph.

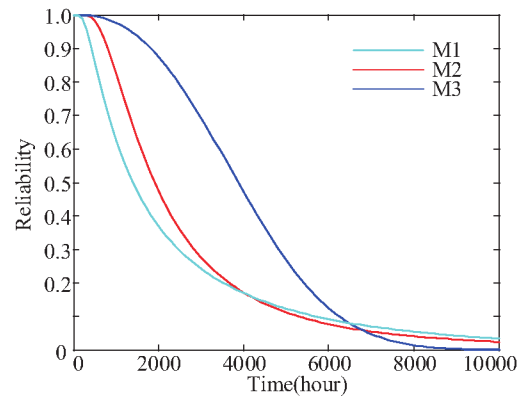


Figure 4: Reliability function graph.

As can be seen from figure 3, the life distribution of M1 is very concentrated, and it is mainly concentrated within 2000h, which is not in conformity with the reality. The life distribution of M3 is very dispersive, which is not very reasonable in the life distribution of marginal. It can be seen from figure 4, the reliability estimation results of M1 and M2 are more conservative than that of M3, which is consistent with the above conclusions. The pump 3 almost failed at the end of the test, indicating that the reliability of these hydraulic pump at time 1000h was lower than that of the M3. Therefore, we think the reliability estimation method proposed in this paper, M2, can be used to estimate the life distribution of the batch hydraulic pump well.

5 Conclusion

Taking volumetric efficiency as the characteristic parameter of performance degradation, a reliability evaluation method for hydraulic pump based on performance degradation is proposed. And the stochastic volatility, individual differences and measurement errors have been considered in the creation of the degradation model. Aiming at the performance degradation data of the 4 hydraulic pumps, the two-step MLE method is used to estimate the unknown parameters, and the average life and the life probability density function of the batch of pump are obtained. Compared with the reliability evaluation method of degradation path model and Wiener process model, it is

necessary to consider the stochastic volatility, individual differences and measurement errors in the reliability evaluation of the hydraulic pump based on degradation. Using the first hitting time to define the degradation failure of the hydraulic pump will lead to the results tend to be conservative. The reliability estimation method based on performance degradation does not need to observe the failure of hydraulic pump, compared with the method based on lifetime, it can reduce the test time and the cost.

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Nomenclature

Variable	Description	Unit
$x(t)$	The real value of product performance degradation at time t	[-]
λ	Drift coefficient	[-]
σ_B	Diffusion coefficient	[-]
$B(t)$	Standard Brownian motion	[-]
$y(t)$	Degradation path of product	[-]
ε	Measurement errors	[-]
T	First-hitting-time of product	[hour]
$f(t)$	The probability density function of the product life	[-]
$R(t)$	Reliability function	[-]
$\phi(\cdot)$	The probability density function obeying the standard normal distribution	[-]
$\Phi(\cdot)$	Standard normal distribution function	[-]
L_i	Remaining useful life of hydraulic pump	[hour]
$f_{L_i}(l_i)$	Probability density function of remaining useful life	[-]
$E_{L_i}(l_i)$	Remaining useful life expectation	[hour]
n	Total sample number	[-]
m	The measurement number of each sample during the test	[-]
Δt	The interval of test time	[hour]
Δx_{ij}	The increment of the degradation of the i -th unit under the j -th measurement	[-]
Y_i	The vector of the performance degradation of the i -th unit	[-]
Y	The performance degradation matrix of the sample	[-]
I_m	Unit matrix with order m	[-]
Σ	Covariance matrix	[-]

t	Vector of measurement time	[hour]
θ	Vector of unknown parameters	[-]
$z_i(t_j)$	The volumetric efficiency of the i -th unit under the j -th measurement	[-]
$d_i(t_j)$	The performance degradation of the i -th unit under the j -th measurement	[-]
$y_i(t_j)$	The normalized performance degradation of the i -th pattern under the j -th measurement	[-]

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