A Market-Based Optimization Approach for Domestic Thermal and Electricity Energy Management System: Formulation and Assessment

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Received: 29 March 2018; Accepted: 7 May 2018; Published: 15 May 2018

Abstract: The increase of domestic electrical and thermal controllable devices and the emergence of dynamic electrical pricing leads to the opportunity to integrate and optimize electrical and thermal energy at a house level using a home energy management system (HEMS) in order to minimize the energy costs. In the literature, optimization-based algorithms yielding 24-h schedules are used in spite of their growing complexity with the number of controllable devices and their sensitivity to forecast errors which leads, in most of the cases, to suboptimal schedules. To overcome this weakness, this paper introduces a domestic thermal and electrical control based on a market approach. In contrast with the optimization-based HEMS, the proposed market-based approach targets a scalable and reactive optimal control. This paper first formulates the market-based optimization problem with generality and discusses its optimality conditions with regards to the microeconomic theory. Secondly, this paper compares its optimality to an optimization-based approach and a rule-based approach under forecast errors using Monte Carlo simulations. Finally, this paper quantifies and identifies the effectiveness boundaries of the different approaches.

Keywords: demand response; dynamic pricing; energy management; market-based optimization; multi-agent system

1. Introduction

The electricity grid is facing a major change due to the increase of renewable energy sources, which decreases on one side the greenhouse gas emission but may threaten on the other side the security supply due to its volatility and its near zero marginal cost.

Consequently, the electrical grid should change from a consumption driven to generation driven paradigm. For this purpose, home energy management systems (HEMS) jointly optimizing electrical and thermal energy use at house level play a crucial role in integrating the flexibility of controllable electrical loads and thermal or electrical buffers.

In accordance with literature, the implemented HEMS approaches minimize the device operation costs according to feed-in or demand response (DR) tariffs. HEMS objective function may be formulated differently depending on whether it leverages forecast uncertainty information i.e., stochastic optimization [1–3] or not i.e., deterministic optimization [4–11]. Among them, the most common HEMS optimization approaches formulate this problem as a linear [6,7], mixed integer linear (MILP) [4,5,8,9], quadratic [10,11] or dynamic programming [1] yielding typically 24-h schedule for the appliances.

The main HEMS challenge is to deal with forecast errors leading to suboptimal schedules. Two promising approaches address this shortcoming: stochastic optimization and reactive control. The former does so, by incorporating forecast uncertainty in the formulation, thus increasing the...
problem complexity whereas the latter, with fewer studies in the literature, relies on an algorithm with low complexity which leads on the one hand to a less optimal solution and on the other hand allows a fast reaction to forecast deviations.

Specifically, this work studies a reactive control approach by introducing the market-based optimization approach for thermal and electrical control at a household level, which stands out for its reactivity because of its lack of complexity compared to optimization approach. In contrast to the literature, this work integrates specific domestic thermal flexibilities [12], e.g., the space heating or domestic hot water demand and formulates the market optimization problem and the objective function followed by each flexible device in a general way [13,14]. Specifically, the conditions that lead to an optimal market, i.e., minimizing the operation costs, are discussed and applied to derive the mathematical formulation of the optimal bidding strategies of interruptible loads such as heat pump and energy storages, e.g., water tank or battery.

This work compares the optimality of different HEMS approaches under different forecast error: an optimization-based, a market-based and a rule-based control. Authors [4,7,9,10,13,14] assess the HEMS under specific seasonal conditions and over few days with perfect forecasts, leading to conclusions that lack generality. Instead, the results presented here consider different forecast errors and a broad set of scenarios with Monte Carlo simulations to extract more general conclusions than in the literature, i.e., independent of the user behavior and the weather conditions.

Finally, this study quantifies and identifies the HEMS approaches that lead to the minimum cost under (i) two different pricings: Feed in Tariff and Time Of Use Tariffs, (ii) a broad set of operating conditions and (iii) in presence of forecast errors.

The contribution of this paper can be summarized as follows:

- Formalization of the market-based optimization problem for optimal domestic thermal and electrical management, in a general way and based on microeconomic theory.
- Optimality comparison of the market-based with the optimization-based and the rule-based approaches under a broad set of scenarios with Monte Carlo simulations.
- Identification of the effectiveness boundaries of the compared approaches under different forecast errors.

2. The Market-Based Approach for Optimal Domestic Thermal and Electrical Management

The approach presented here is based on the energy market mechanism, where each flexible device is a market player bidding its flexibility. This decentralized optimization takes place in the house and controls the domestic electrical or thermo-electrical devices with flexibility to minimize the energy costs according to given electricity price scheme i.e., feed-in or DR tariffs (Equation (26)).

It is based on a bottom up approach: every controllable device in the house is a market participant which negotiates its demand or offer of electricity and/or thermal energy (heat or cooling) on a virtual domestic market in the house to fulfill economically its need. The market participants interact among them (Figure 1) via a bidding process on this domestic market to balance energetically the system i.e., the house, through an internal market equilibrium price. This in-house market price steers the devices and differs from the residential electricity price scheme such as DR tariffs.

This market approach extends the well-known Powermatcher approach [12] by introducing in addition to the electricity market, one or more local heat and/or cooling markets which accounts for the thermal flexibilities in the subsystem, formed by thermal sinks and sources e.g., a room and its associated space heating system (Figure 1). Equilibrium thermal and electricity prices have to be determined to ensure a thermal and electrical power balance in the house.

This section presents first the market-based optimization and its theoretical background. Based on this, the bidding strategies for interruptible loads and energy storages are then derived. Finally, its possible implementation as a multi-agent system is described.
2.1. Microeconomic Theory

In contrast with a centralized optimization-based approach [1,3,4,6–11], the market-based is a decentralized approach where each participant bids according to its local information. This section presents and discusses the market-based optimization conditions which lead to system cost minimization.

Economy is stated as a constrained optimization where the resources are scarce [15]. On one hand, the consumer’s objective is to maximize its utility subjected to a budget constraint whereas on the other hand, the producer’s objective is to optimize its production in order to maximize its profit. Finally, the global objective of economy is to maximize the consumer and producer surplus, called global welfare, although the two objectives are competing. Therefore, economy can be described as an optimization problem with a local goal achieved at consumer/producer level and a global goal achieved at the market level. The maximization of the consumer and producer surplus is achieved if the market is competitive. A competitive market is defined by four hypotheses, which drive the market design [15], and are formulated here in terms of the HEMS problem addressed in this work:

- Products homogeneity: electricity and heat do not take various forms and are well defined worldwide according to standard energy unity
- Free entrance in the market, i.e., no barrier to enter in the market. In this work, each controllable device in the house can participate in the market.
• Transparency: everyone has the same information from the market. In this work, each participant has access to the current and past market prices.

• Actors are price takers, meaning that they consider in their bidding strategy that they cannot influence the market price, even if it is the case. This is considered in the bidding strategy, as shown in the following.

The first economy theorem states that a competitive market leads to a Pareto optimal solution, corresponding to solution which does not allow to make a participant better off without making any another one worse off. Furthermore, a competitive equilibrium leads to a solution which maximizes the consumer and producer surplus [15]. In addition to this, the consumer’s objective, maximize utility, leads to a minimization of its utility costs (dual problem) whereas the producer’s objective, maximize profit, leads to a minimization of its production cost [16]. Based on these results and the first economy theorem, it can be stated that a competitive market leads to a minimization of the global costs, defined by the utility costs and the production costs.

In summary and according to competitive market theory, the minimization of global costs is achieved if each producer bids at its marginal costs and each consumer bids for minimizing its utility costs.

In the following, the bidding strategies for interruptible loads and energy storage are derived according to the derived optimality conditions.

2.2. Interruptible Loads

The interruptible load can be interrupted and resumed at a later time with associated constraints or penalties linked to the user’s comfort. The interruptible loads bidding strategies presented in this work are appropriated for thermo-electrical devices such as heat pump (HP) or electrical heater (EH), characterized by an electrical consumption and a thermal production. For this reason, its associated bid has to tender an electrical power demand on the electrical market and a heat power supply on the heat market, both of them depending on the electricity and heat price [13, 14].

2.2.1. Heat Pump

Based on the microeconomic theory, the heat pump has to maximize its profit on the markets whereas considering it is a price taker.

$$\max \underbrace{p_h \cdot Q - C(Q)}_{\text{Profit \ [\}}$$

s.t.

$$\forall t, \quad P = \frac{Q}{COP}$$

where the costs can be expressed as

$$C(Q) = p_e \cdot P + C_{\text{fixed}} = p_e \cdot \frac{Q}{COP} + C_{\text{fixed}}$$

where

- $Q$: produced thermal power [kW]
- $P$: consumed electrical power [kW]
- $p_h$: heat price [€/kWh]
- $p_e$: electrical price [€/kWh]
- $C_{\text{fixed}}$: fixed costs, e.g., maintenance [€/h]
- $COP$: coefficient of performance of the HP [-]
- $\eta_{EH}$: EH efficiency [-]
In the case of a competitive market, the market participant is a price taker \( \left( \frac{dp}{dQ} = 0 \right) \), meaning that it does not consider in its bidding strategy that it can potentially influence the market price, even it could. Indeed, if a large consumer/producer considers in its bidding strategy that it influences the market price to maximize its profit, the market is no more competitive but oligopolistic and does not lead to an operating cost minimization. Considering a competitive market, the maximization (Equation (1)) leads to:

\[
\begin{align*}
Q \frac{dp}{dQ} + p \frac{dQ}{dQ} - \frac{dC(Q)}{dQ} &= 0 \quad (4) \\
p = \frac{dC(Q)}{dQ} \quad (5)
\end{align*}
\]

From there, the optimal HP bidding strategy is to bid its thermal power on the thermal market and its associated electrical consumption on the electrical market if:

\[ p_h = \frac{p_e}{\text{COP}} \quad (6) \]

2.2.2. Electrical Heater

Identically, the EH bids can be derived by considering a slightly different cost function.

\[ C(Q) = p_e \cdot P + C_{\text{fixed}} = p_e \cdot \frac{Q}{\eta_{EH}} + C_{\text{fixed}} \quad (7) \]

Based on Equation (7), the optimal EH bidding strategy is to bid its thermal power on the thermal market and its associated electrical consumption if:

\[ p_h = \frac{p_e}{\eta_{EH}} \quad (8) \]

As stated in Equations (6) and (8), the bidding strategy of thermo-electrical devices is depending on the electrical and the thermal prices. Given that they produce heat and consume electricity, they have to bid in two markets: thermal and electrical. Note that if a HP and EH compete in the same thermal market, the HP will be more likely used because of its smaller heat price (COP is in the range of 3 while EH efficiency is around 1). Furthermore, additional constraints can influence the bidding strategy, e.g., a minimum must-run time, which avoids too many unit switch on/switch off.

2.3. Energy Storage

The energy storage units can be used to store and dispense energy when needed, e.g., water tank or battery. In a market, they maximize their profit by buying energy when prices are cheap and by selling energy when prices are high, while being constrained by their state of charge. The optimal bidding strategy for storage unit is formulated in Equation (9), while considering that the storage unit is a price taker to achieve a competitive market.

\[
\begin{align*}
\max \sum_{t=1}^{T} \left( \text{Sold energy} - \text{Bought energy} \right) \\
\text{s.t.} \\
\forall t, \quad \text{SOC}_{\text{min}} \leq \text{SOC}(t) \leq \text{SOC}_{\text{max}} \quad (10) \\
\forall t, \quad T_{\text{min}} \leq T(t) \leq T_{\text{max}} \quad (11)
\end{align*}
\]
The state of charge of the battery and water tank energy storage is defined as:

\[ SOC(t) = \frac{E(t)}{E_{\text{max}}} \in [0,1] \]  

(12)

- \( E_{\text{discharge}} \) optimal discharging energy at the considered time step \( t \) [kWh]
- \( E_{\text{charge}} \) optimal charging energy at the considered time step \( t \) [kWh]
- \( SOC \) state of charge [-]
- \( T(t) \) internal temperature of the house [°C]
- \( E \) stored energy [kWh]
- \( p \) internal market price [€/kWh]
- \( \Delta t \) time step duration between decisions [h]
- \( T \) number of scheduling time interval [-]
- \( E_{\text{max}} \) maximum stored energy [kWh]

### 2.3.1. Battery

In this work, a lithium ion battery model is considered (Table 1). The energy balance constraint is formulated as:

\[ E(t + \Delta t) = E(t) + \left( \eta_{\text{batt}} E_{\text{charge}}(t) - \frac{E_{\text{discharge}}(t)}{\eta_{\text{batt}}} \right) \]  

(13)

The evolution of the stored energy in the battery is defined in Equation (13) and depends on its previous energy state, the charge/discharge energy and the combined battery-inverter system efficiency. The state of the art of the charging strategy is the Constant Current Constant Voltage (CCCV) which implies a maximum power charge or discharge depending on the current battery voltage, i.e., the state of charge [17]. In this work, the battery power is considered constant all along the time interval between two decisions.

\[ 0 \leq P_{\text{charge}}(t) \leq P_{\text{charge, max}}(SOC(t)) \]  

(14)

\[ 0 \leq P_{\text{discharge}}(t) \leq P_{\text{discharge, max}}(SOC(t)) \]  

(15)

- \( P_{\text{charge, max}} \) maximum admissible charging power [kW]
- \( P_{\text{discharge, max}} \) maximum admissible discharging power [kW]
- \( \eta_{\text{batt}} \) system efficiency (battery and inverter) [-]

### 2.3.2. Water Tank

Water tank (WT) storages are used for storing thermal energy, e.g., domestic hot water (DHW) or space heating (SH) demand. As any energy storage, WT is constrained by its energy balance formulated as:

\[ E(t + \Delta t) = E(t) + \left( E_{\text{charge}}(t) - E_{\text{discharge}}(t) - Q_{\text{loss}}(t) \cdot \Delta t \right) \]  

(16)

The stored energy in a water tank storage is function of the averaged inside temperature \( T(t) \).

\[ E(t) = \rho(T(t)) \cdot V \cdot c_p(T(t)) \cdot T(t) \]  

(17)

The water parameters \( \rho \) and \( c_p \) describe respectively the density and the heat capacity of water, in function of its temperature. \( V \) denotes the volume of water in the WT. The storage is empty (\( SOC = 0 \)) when the inside temperature equals to \( T_{\text{min}} \) whereas it is full (\( SOC = 1 \)) when it equals to \( T_{\text{max}} \).

\[ SOC(t) = \frac{E(t) - E(T_{\text{min}})}{E(T_{\text{max}}) - E(T_{\text{min}})} \in [0,1] \]  

(18)
The heat losses are function of the water tank temperature and is typically formalized as a linear function of the stored energy.

\[ Q_{\text{loss}}(t) = q_{\text{loss}} \cdot E(t) \]  

(19)

- \( \rho \): density of water [kg/m³]
- \( c_p \): heat capacity of water [kW/K/kg]
- \( V \): water tank volume [m³]
- \( q_{\text{loss}} \): thermal loss of the WT [kW/kWh]

### 2.3.3. Thermal Wall Mass

The thermal wall mass of a house can also be used as a thermal storage. In contrast with the battery or the water tank, the thermal wall mass cannot intentionally be discharged. The considered ISO standard house model [18] is characterized by a one-zone model i.e., parameters and variables are averaged and represent the overall house behaviour. The house model can be expressed as:

\[
T(t + \Delta t) = T(t) + \frac{1}{C_{\text{house}}} \left( E_{\text{charge}}(t) + \eta_{\text{h,gn}} \cdot \Delta t \cdot (Q_{\text{sol}}(t) + Q_{\text{int}}(t)) - \Delta t \cdot Q_{\text{loss}}(t) \right)
\]  

(20)

- \( C_{\text{house}} \): thermal house capacitance [kWh/°C]
- \( \eta_{\text{h,gn}} \): gain utilization factor [-]
- \( Q_{\text{sol}} \): solar heat gains [kW]
- \( Q_{\text{int}} \): internal heat gains [kW]
- \( Q_{\text{loss}} \): heat losses by transmission and ventilation [kW]

Standard values are used for the operating conditions (room temperature, air exchange rate, internal heat sources) and for the solar radiation reduction factors, e.g., shading. The calculation of this different parameters are based on a harmonized approach in the framework of the european project DATAMINE [19].

The maximization problem formulated in Equation (9) requires knowing upfront the price to determine the optimal charge and discharge power in function of the current market price. Given the challenge of forecasting the market price, Klaassen et al. [20] proposes to use a naive forecast with a heuristic approach for optimizing the storage units bidding strategy: buy at periods of low prices and resell it in periods of high prices in function of the SOC and the market price knowledge. Where the maximum and the minimum market price is determined according to the considered forecast. This paper follows this approach.

### 2.4. Market Price Determination

Each flexible device optimizes locally its tender and bids it on the associated market so that an equilibrium price can be determined. This work considers the electrical and thermal consumers/producers as source of flexibility. For this reason, thermal and electrical equilibrium market prices, ensuring the power balance, have to be determined subsequently at each time step.

#### 2.4.1. Thermal Market Price

The thermal flow in the house are physically constrained, e.g., the electrical heater installed in a given room cannot supply the space heating demand in another room. Note that heat transfer between rooms is assumed to be negligible and is not considered from a market exchange point of view. The elements that can exchange physically thermal energy, have to trade in the same local thermal market, annotated \( i \). According to microeconomic theory, the local thermal market \( i \) determines a thermal equilibrium price \( p_{h,i}^{\text{th}} \) according to a merit order, for each electrical price (Equation (21)). The resulting electrical bid is then considered in the electrical market price determination.
\[ \forall p_e, \min_{p_h} |Q_{\text{supply},i}(p_e, p_h) - Q_{\text{demand},i}(p_e, p_h)| \]  

(21)

Subject to  

\[ p_e, p_h > 0 \]  

(22)

where \( Q_{\text{supply},i} \) and \( Q_{\text{demand},i} \) are the sum of all the bids respectively supplying and consuming thermal energy in the local heat market \( i \). The Equation (21) minimizes the difference between the demand and the supply and leads for this reason, to an equilibrium between thermal demand and supply.

### 2.4.2. Electrical Market Price

An electrical equilibrium market price has to be then determined according to a merit order approach, formalized as [16]:

\[ \min_{p_e} |P_{\text{supply}}(p_e) - P_{\text{demand}}(p_e)| \]  

(23)

Subject to  

\[ p_e > 0 \]  

(24)

\[ P_{\text{tot}}(p_e) = 0, \text{ if } p_e > p_{\text{grid}}(t) \]  

(25)

where \( P_{\text{supply}} \) and \( P_{\text{demand}} \) are the sum of all the electrical bids respectively supplying and consuming electrical energy in the house. The bids from EH or HP are considered in the \( P_{\text{demand}}(p_e) \) term. Therefore, the market price, formulated as a minimization problem in Equation (23), leads to an equilibrium between electrical demand and supply. The grid constraints in Equation (25) expresses that the grid can provide all the required power if the internal electrical market price is larger or equal to the current grid pricing \( p_{\text{grid}} \), e.g., TOU.

The optimal thermal and electrical prices are determined thanks to a merit order approach which leads to a global minimization of the thermal and electrical demand and supply mismatch, for given bids. The merit order has a low complexity and does not suffer from convergence issue (the bidding and pricing process takes around 1 s, Table 3).

The major advantages of the presented market-based approach is:

- the low complexity of the algorithm which allows a reactive control to an unexpected event or a wrong forecast.
- its scalability: any kind of thermal or electrical flexibility can participate without increasing the problem complexity. Each participant must be able to communicate and produce an optimal bid in accordance with the market design guidelines (Section 2.1).

On the other hand, because of the considered naive price forecasts and the heuristic nature of energy storage bidding strategies, the market-based approach leads to suboptimal solutions. One of the objective of the work is to determine if its reactive control feature can counterbalance the suboptimality of its solution (Section 7).

### 2.5. Multi-Agent System Implementation

A multi-agent system (MAS) is characterized by several physical or virtual entities, which communicate, interact, sense and act. Each agent has a local objective whereas the group of entities forming the MAS have a global objective. In a market case, the local objective is to minimize the utility costs for consumers or to bid at marginal costs for producers whereas the global market objective is to maximize the global welfare or in other words minimize the total costs while ensuring the power balance.

In the frame of this work, a MAS platform developed in Python [21] according to the IEEE Foundation for Intelligent Physical Agents (FIPA) [22] was implemented. FIPA releases number of standards specifying how agents can communicate between each others. Python was chosen given that it enables an easy integration into low power hardware with basic python interpreter, e.g., Raspberry
Pi Zero or Arietta G25. In contrast with the existing MAS software e.g., JADE [23] or osBrain [24], the developed python library is compatible with different domestic communication technologies, e.g., Wifi, Zigbee, Bluetooth LE, Z-Wave, Thread or DigiMesh.

The objective of the MAS implementation is to minimize the hardware energy usage while keeping a robust, reliable and scalable system. For these reasons, the MAS is designed with [12]:

- **tree topology** which stands out because of its low number of sent messages per market cycle, equals to the number of agents and by its tractable and intuitive tree creation that follows the physical layout of the network.
- **resilience against hardware failure** thanks to the self-organized election of the aggregator, Section 2.5.2.
- **plug and play** thanks to the dynamic tree topology creation, Section 2.5.2.
- **low energy usage** given (i) the limited number of messages sent per market cycle and (ii) the idle mode of every agents between each market cycle.

2.5.1. Agents

For achieving this, three different types of agents are defined (Figure 1):

- **The device agent** acts on behalf of a physical device on the market and computes the optimal bid with respect to its constraints and the microeconomic theory.

- **The local aggregator agent** extends specifically the Powermatcher approach. It calculates the thermal market equilibrium price according to the merit order formulated in Equation (21) in function of the electrical price. In doing so, a local equilibrium heat/cooling price ensures a thermal balance of the subsystem considered, as demonstrated in [13,14].

- **The global aggregator agent** aggregates the different electrical bids and determines the electricity market price according to the merit order formulated in Equation (23). The electrical market price ensures an electrical power balance in the system [12]. This entity is also the link with outside such as a larger coordination algorithm which could optimize the residential energy usage or provide grid services e.g., balancing or congestion management. This agent can indeed offer the current domestic flexibility using bids to an upper layer of the residential coordination algorithm, as in [25,26].

2.5.2. Tree Topology and Self-Organized Aggregator Election Process

The global aggregator agent is elected with a bully algorithm. After its election, it broadcasts a proposal for a parent-child relationship to its neighbors to build the tree topology. This proposal will be accepted by its neighbors if it has no parent yet. Once accepted, this node proposes as well to its neighbors and the process continues until that each node is the child of another one. This process enables a dynamic integration or removal of any agents in a plug and play way. At the beginning of a market cycle, all the agents wait for a proposal of the aggregator of the last market cycle. If it proposes then, no election is carried out and the network is considered as operative. Once elected, the aggregator broadcasts a market initiation request to its neighbors that they forward to their neighbors as well. Based on this request, each node aggregates the bids from its children and sends it to its parent. A market cycle takes place every minute.

This work does not investigate in detail the MAS features and the presented results consider a perfect MAS operation.

3. Optimization-Based Approach: Mixed Integer Linear Programming

This section presents the optimization-based HEMS approach, minimizing explicitly the operating cost to which the market-based approach is compared. The presented approach does not integrate in the objective function, the deterioration costs due to the battery or the heat pump usage. Although the optimization and the market-based approaches have the same objective function, the former is
implemented in a centralized way and the latter in a decentralized way. Each approach considers the same controllable devices and is presented according to its (1) objective function formulation and its (2) flexible load modeling.

In the literature [4,5,8,9,27], the HEMS optimization-based formulation as a MILP problem has been widely studied because of its mature solving algorithm which ensures a global optimal solution. This work is based on these different works. In the following, the objective function and the modeling of different flexible devices are briefly presented.

3.1. Objective Function

The optimization minimizes the operating costs of the different flexibility sources according to the considered electricity price scheme and/or the feed-in tariff, embedded in $C_{imp}(t)$. The typical HEMS objective function is formulated as:

$$\text{Minimize} \sum_t \left( \frac{E_{imp}(x(t))C_{imp}(t) - E_{exp}(x(t))C_{exp}(t) + C_{discomfort}(t)\cdot \Delta t}{\text{import cost over } \Delta t} \cdot \text{export revenue over } \Delta t \cdot \text{discomfort costs over } \Delta t \right)$$

$$\text{Subject to } \forall t, \begin{cases} E_{imp}(x(t)) + E_{PV}(t) & \text{electrical production over } \Delta t \\ E_{exp}(x(t)) + \sum_{d=0}^{D} P_d(x(t)) \cdot \Delta t + E_{dem}(t) & \text{electrical consumption over } \Delta t \end{cases}$$

$x(t)$ vector of decision variables for the flexible devices in $t$
$\Delta t$ time step duration between decisions [h]
$E_{imp}(x(t))$ imported energy from the grid at time step $t$ [kWh]
$E_{exp}(x(t))$ exported energy to the grid at time step $t$ [kWh]
$C_{imp}(t)$ importing cost depending on the considered tariff, e.g., TOU [€/kWh]
$C_{exp}(t)$ exporting revenue depending on the considered tariff [€/kWh]
$C_{discomfort}(t)$ discomfort cost associated to a constraint violation [€/h]
$E_{PV}(t)$ forecasted photovoltaic energy (PV) production at time step $t$ [kWh]
$E_{dem}(t)$ forecasted uncontrollable electrical demand at time step $t$ [kWh]

The considered flexible load models or constraints have to be linear, given that a MILP problem is formulated. The following presents the linear models or constraints for interruptible load and energy storage and their associated comfort constraint formulation.

3.2. Flexible Load Modeling

Based on the considered house set up (Figure 4), the following presents the heat pump, the electrical heater, the battery, the water tank and the thermal wall mass models.

3.2.1. Heat Pump

The heat pump is a very efficient thermo-electrical device which extracts heat power from a low temperature source such as air or water, using an inverse fridge cycle. The heat pump supplies typically a water tank storage which can supply DHW and/or space heating (SH) needs. The heat power output of a heat pump is governed by the coefficient of performance (COP) which is a function of ambient temperature and the flow temperature for hot water supply. The MILP model of a HP is formulated as:

$$Q_{HP}(t) = \text{COP} \cdot P_{HP}(t)$$

$$P_{HP}(t) = x_{HP}(t) \cdot p_{HP}^{max}(t)$$
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\[ P_{HP} \quad \text{electrical power of HP [kW]} \]
\[ Q_{HP} \quad \text{thermal power of HP [kW]} \]
\[ COP \quad \text{coefficient of performance [-]} \]
\[ x_{HP} \quad \text{binary element of the decision variable vector } x(t) \text{ describing the HP operation} \]

3.2.2. Electrical Heater

The electrical heater is a fast heating device constituted by an electrical resistance which can be installed in rooms to provide SH or in the water tank to supply DHW demand. The heat power output of electrical heater is governed by the efficiency of the electrical heater according to

\[ Q_{EH}(t) = \eta_{EH} \cdot P_{EH}(t) \]  
(30)

\[ P_{EH}(t) = x_{EH}(t) \cdot P_{max}^{EH}(t) \]  
(31)

\[ P_{EH} \quad \text{electrical power of EH [kW]} \]
\[ Q_{EH} \quad \text{thermal power of EH [kW]} \]
\[ \eta_{EH} \quad \text{EH efficiency [-]} \]
\[ x_{EH} \quad \text{binary element of the decision variable vector } x(t) \text{ describing the EH operation} \]

In this work, the HP and EH operation is restricted to on-off operation.

3.3. Energy Storage Model

An energy storage is characterized by the possibility to store and restore energy when required. From a modeling point of view, an energy storage is constrained by its physical constraints, i.e., its maximum power and state of charge boundaries. The considered models are identical to the ones presented in the Section 2.3. The only difference is that the MILP formulation solves the problem centrally, considering these models as a constraint. With the market-based approach, the identical models are considered but each device optimizes locally its bidding strategy according to Equation (9).

3.3.1. Battery

The MILP battery model is described in Section 2.3.1. \( P_{charge} \) and \( P_{discharge} \) are the continuous elements of the decision variable vector \( x(t) \) describing respectively the charging and discharging battery power.

3.3.2. Water Tank

The MILP water tank model is described in Section 2.3.2. \( P_{charge} \) and \( P_{discharge} \) are the continuous variables describing respectively the charging power, i.e., heat production from the EH, and discharging power, i.e., the DHW demand.

3.3.3. Building Wall Mass Storage

The first-order thermal model of the house is described in Section 2.3.3. \( P_{charge} \) is a continuous variable describing the charging power, i.e., the heat production from the HP.

4. Conventional Control

This section presents the benchmark control approach. The conventional control is a rule-based control based on the current system states, e.g., temperature, state of charge. It is a robust approach because it does not require communication infrastructure between the different devices given that each of them maximize independently the user comfort. This is the less optimal but the most commonly implemented control in house.
4.1. Objective Function

The objective function is local at the device level and is to maximize the user comfort and the house self-consumption if there are PV panels and domestic battery.

4.2. Interruptible Load Control

The interruptible loads are controlled to maximize the user comfort. The space heating device, e.g., HP, is controlled with a PID controller based on the room temperature and keeps it at a given reference value, independently of the other system states.

4.3. Energy Storage

The domestic battery is controlled independently such as to maximize the self-consumption of the house. If the house is exporting electricity because of a high PV production, then the battery is charged whereas when the house is importing electricity, the battery is discharged until to reach its minimum state of charge.

5. Case Study

Simulations are used here to compare the optimality of the different HEMS approaches under a broad set of scenarios with forecast errors. In the literature, most authors [7,9,10,13,14] consider perfect forecasts and specific evaluation scenarios over a short time period, neglecting the results dependency on seasonal and user profiles. Instead, this paper uses Monte Carlo simulations to assess the optimality of the different HEMS approaches and extract conclusion as general as possible. In this paper, the considered uncertainty sources (Figure 2) are:

- scenarios: irradiation (I), temperature (T), uncontrollable electrical (E) and domestic hot water (DHW) demand.
- their associated forecast errors

Figure 2. HEMS cost evaluation approach for a given scenario $s$ and its associated forecasts $f$ with a given forecast error.

This section presents first the scenarios and the forecasts used in the assessment of the different HEMS approaches. Then, the considered house configuration and the demand response tariffs are introduced.
5.1. Scenarios Generation

As shown on Figure 2, the HEMS assessment requires irradiation (I), temperature (T), uncontrolable electrical (E) and domestic hot water (DHW) demand and their associated forecasts. In this work, the electrical and DHW demand is based on a stochastic model using the user occupancy and stochastic behavior models. In this way, a dependent electrical demand [28] and DHW profiles [29] are generated, based on an identical occupancy profiles. In the frame of this work, 10,000 different weekly profiles of electrical and DHW demand are generated with a granularity of 1 min. Finally, each scenario is rescaled according to the average consumption in Germany [30] (Table 1). While the temperature and irradiation are based on 5 years of historical data from the German National Meteorological Service (Deutscher Wetterdienst) [31].

A forecast error associated to each scenario is generated according to the state-of-the-art forecasting method Auto Regressive Moving Average (ARMA).

5.2. Forecast Generation

The Autoregressive Moving-Average (ARMA) is a very popular approach for short-term forecasting [32] in many different fields such as econometrics, electrical load demand [33], irradiation or DHW [27]. ARMA models are characterized by a Gaussian distribution of error (Figure 3) and a linear dependence to previous real value [32].

![Figure 3. Forecast generation process based on the ARMA forecasting method and the average MAPE forecast error.](image)

The forecast distribution, a Gaussian $f(\cdot)$ defined by the real value $x$ and the standard deviation $\sigma$, is derived from the Equation (32). In accordance with literature [27,33], this work considers the Mean Average Percentage Error (MAPE) as forecast error metric. For each bins of the forecast, the MAPE is calculated and weighted accordingly to its probability as defined by:

$$\text{MAPE}(x, \sigma) = \int_{-\infty}^{\infty} \text{probability of forecast } y \cdot \text{MAPE of forecast } y \cdot \frac{|y - x|}{x} dy \quad (32)$$

In this way, the generic forecast:
- corresponds to the ARMA forecasts error distribution: Gaussian
- captures the dependence with previous value because forecasted values are based on the real value, embedding implicitly this dependence
- unbiased: the expected value of the generated forecast corresponds to the real value
- compatible with methods requiring probabilistic forecasts

In the following, different forecast errors are considered in the optimization approach. The reference forecast error is based on the state-of-the-art forecast error (SOTAFE) derived from literature, i.e., MAPE of electrical and DHW demand: 60% and MAPE of irradiation: 30% [27]. For example, the case with a SOTAFE-50% corresponds to a case where the forecast error is decreased by 50% for all the considered forecasts.

5.3. House Configuration

This study considers the most representative house in Germany according to the national statistical data. According to [34], the most typical German family houses are occupied by an average number of 3.57 persons, rounded to 4. The thermal model and electrical consumption of the considered household (Table 1) is based on average value according to [30]:

- House type: Single Family House -SFH- (57% of building stocks) with 2 floors (56% of the SFH)
- Construction period: 1958–1968 with usual refurbishment (15.3% of SFH were built during this period)

The first order house model and the house parameters are derived from seasonal average value based on [30] and according to the EN ISO 13790 standard [18]. In addition, the considered heating units in the house (Figure 4) are based on the most typical installation in Germany [30] which provides electrical flexibility, i.e., gas or oil based units are not considered as a possible configuration. Based on this, the space heating (SH) demand is provided by a heat pump with 2 levels of output power to fulfill the peak demand. Whereas the DHW is produced by an electrical boiler supplying a water tank (WT) [30]. The storage capacity of the water tank is dimensioned according to the number of inhabitants and follows typically a rule of thumb: 40–50 L./pers. Based on an economical study, the photovoltaic (PV) system and the domestic battery are dimensioned according to the yearly electrical consumption. The most typical PV installation in Germany is considered: 6.2 kWp [30]. Based on this PV installation and the domestic yearly consumption, the most cost-efficient solution according to [35] was derived: 4 kWh.

![Figure 4. Thermal (red) and electrical (blue) representation of the considered house set up with a heat pump (HP) supplying the space heating (SH), an electrical heater (EH) supplying a water tank (WT) for domestic hot water demand (DHW), photovoltaic panels (PV) and domestic battery.](image)
Table 1. parameters used in the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of inhabitants</td>
<td>4 persons</td>
</tr>
<tr>
<td>Location</td>
<td>North Germany</td>
</tr>
<tr>
<td>Grid electricity price</td>
<td>30 c€/kWh</td>
</tr>
<tr>
<td>Feed-in tariff</td>
<td>12 c€/kWh</td>
</tr>
<tr>
<td>Yearly uncontrollable electrical demand [36]</td>
<td>4200 kWh_e/y</td>
</tr>
<tr>
<td>Yearly space heating demand (norm VDI4656) [30]</td>
<td>16,120 kWh_e/y</td>
</tr>
<tr>
<td>Yearly DHW demand [36]</td>
<td>1815 kWh_e/y</td>
</tr>
<tr>
<td>Heat power production of EH</td>
<td>5 kW</td>
</tr>
<tr>
<td>EH efficiency</td>
<td>0.98</td>
</tr>
<tr>
<td>Water tank for DHW</td>
<td>200 L</td>
</tr>
<tr>
<td>Heat power production of HP</td>
<td>[0, 7.5, 15] kW</td>
</tr>
<tr>
<td>HP Coefficient of Performance</td>
<td>3</td>
</tr>
<tr>
<td>House temperature constraints</td>
<td>[17.5, 19.5] °C</td>
</tr>
<tr>
<td>Photovoltaic installation [30]</td>
<td>6.2 kWp</td>
</tr>
<tr>
<td>Battery installed [35]</td>
<td>4 kWh</td>
</tr>
<tr>
<td>Maximum forecast error for electrical demand [27,33]</td>
<td>MAPE 60%</td>
</tr>
<tr>
<td>Maximum forecast error for DHW demand [27]</td>
<td>MAPE 60%</td>
</tr>
<tr>
<td>Maximum forecast error for irradiation [27]</td>
<td>MAPE 30%</td>
</tr>
<tr>
<td>MILP time interval &amp; scheduling horizon</td>
<td>15 min over 24 h</td>
</tr>
<tr>
<td>MILP rescheduling time</td>
<td>12 h</td>
</tr>
<tr>
<td>Conventional control frequency</td>
<td>15 min</td>
</tr>
<tr>
<td>Market-based control frequency</td>
<td>1 min</td>
</tr>
</tbody>
</table>

5.4. Demand Response Tariffs Considered

Feed-in tariff (FiT) is based on the current tariff in Germany in 2017: the importing electricity costs is 30 c€/kWh and exporting price is 12 c€/kWh. The FiT should continue to decrease in the future to incentivize the reduction of the technology cost.

Time of use (TOU) is divided into different unit prices for usage during different blocks of time in order to encourage customers to shift consumption when demand is low. The multiple TOU tariffs in the literature does not enable to extract a typical TOU. Therefore, the considered TOU tariff in this work (Table 2) is based on actual TOU tariffs which implement a three period tariff [37]. Based on the price ratio in the literature [37–40], the peak demand and the off peak prices are adapted according to the current tariff in Germany (30 c€/kWh), whereas feed-in tariff is still considered.

Table 2. TOU tariff according to the literature [37–40].

<table>
<thead>
<tr>
<th>Period</th>
<th>Time</th>
<th>Ratio</th>
<th>Cost (c€/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off peak</td>
<td>01:00–07:00</td>
<td>0.6</td>
<td>18</td>
</tr>
<tr>
<td>Shoulder</td>
<td>07:00–13:00 &amp; 23:00–01:00</td>
<td>1.0</td>
<td>30</td>
</tr>
<tr>
<td>Peak</td>
<td>13:00–23:00</td>
<td>1.4</td>
<td>42</td>
</tr>
</tbody>
</table>

6. Illustrative Control

This section illustrates the different control achieved with the different HEMS approaches.

6.1. Optimization-Based

Figure 5 presents the results of the MILP control with perfect forecast under TOU tariff. This figure highlights that

- EH and HP never consume in peak price period and concentrate their consumption in off-peak price period.
- when EH and HP consume in shoulder price period, it is because of the PV production or because of the constrains, e.g., the WT state of charge is too low or the house temperature reaches the temperature limit.
- the battery maximizes first the self-consumption given that the feed-in tariff (12 c€/kWh) is cheaper than the off-peak price (17 c€/kWh). When there is PV production (day 0 to day 5), the battery is only charged by the PV production in spite of the shoulder price period. When there is no PV production, it charges in period of off-peak price while it discharges only in period of peak price period.

Figure 5. Illustration of MILP control with TOU tariff and perfect forecast.

MILP controls every 15 min the different devices and calculates every 12 h a new optimal schedule. The presented results are with perfect forecast anticipates perfectly the PV production, the electrical and DHW demand. For this reason, the MILP can perfectly adapt its control in function of the situation. With forecast error, the control is based on wrong information. It will capture then less PV production or will be forced to turn on the EH or the HP during the peak price period because of constraints violation, leading to higher total costs.
6.2. Market-Based

Figure 6 presents the results of the market-based control. The market-based control is based on the current system states and naive forecasts. It highlights that

- EH and HP consume mainly during off peak price period. Nevertheless, they consume in shoulder price period only if the constraints are violated.
- The battery charges only during off peak price period and discharges during the peak price. In contrast with FiT, the battery under TOU tariff stores a very small amount of PV production.

![Diagram of market-based control with TOU tariff](image)

**Figure 6.** Illustration of market-based control with TOU tariff.

The market-based control occurs every minute and allows well capturing fast PV variation with FiT, for example. On the other hand, it leads to a more dynamic control leading to more switch on/switch off of the HP or the EH. It anticipates less the future than the optimization-based control but is able to react very fast to a unexpected event, in contrast with the optimization. All in all, the market-based control outperforms the conventional control and is outperformed by optimization-based approaches regardless of the forecast errors (Figure 7).
6.3. Conventional Control

The conventional control is only based on the current system states. The conventional control is the most commonly implemented control in house given that there is no need of communication and that it optimizes locally the user comfort. Its control highlights that

- EH and HP consume regardless of the electricity price or the PV production. Nevertheless, one can observe that the HP is mainly switched on in off-peak price period which occurs during the night and corresponds, by chance, to the high heat demand because of the lower temperature.
- the battery is controlled to maximize the self-consumption. It is discharged during the consumption period which follows the PV production and corresponds, again by chance, to peak price period.

In spite of the conventional control follows a local objective, maximize the user comfort and the self-consumption, one can observe that its leads to a meaningful control with TOU. The battery charges during off peak price or PV production period and discharge during the peak period.

Note that the market-based and the conventional control approaches do not consider implicitly forecasts. The market-based approach considers implicitly naive forecasts \([13,20,41]\) whereas rule-based does not base its decision on forecasted value. The presented cost results are averaged value defined according to Equation (33).

7. Optimality Comparison of HEMS Approaches

This section presents first a comparison of optimality performance between the three different HEMS approaches based on their total weekly costs. Then, the specific device costs associated to each HEMS approach is discussed.

7.1. Total Cost

Based on the Monte Carlo simulations, the total expected costs \(\hat{C}_{\text{HEMS}}\) are derived for given operating condition scenarios \(s\) and its associated forecasts \(f\). \(S\) and \(F\) are respectively the number of considered operating condition scenarios and their associated forecasts. \(\text{Costs}_{s,f}\) are the derived costs for one specific evaluation scenario (Figure 2).

\[
\hat{C}_{\text{HEMS}} = \frac{1}{S} \sum_{s=1}^{S} \left( \frac{1}{F} \sum_{f=1}^{F} \text{Costs}_{s,f} \right) \tag{33}
\]
7.1.1. Feed-In Tariff

Based on the Figure 7, the optimization-based with perfect forecasts saves about 255 \( \text{c€/w} \) compared to the reference case for the considered house with FiT. In addition, the same optimization-based control with the state-of-the-art forecast (SOTA) still saves 75 \( \text{c€/w} \), compared to the conventional control case. Whereas the market-based control saves 65 \( \text{c€/w} \). Based on these results, the optimization-based control is competitive with forecasts improved by 50% or more (SOTA-50%), compared to the conventional or the market-based control. With the SOTA, the market-based and the optimization approaches perform identically.

7.1.2. Time of Use Tariff

The presented TOU results are achieved with the same assessment conditions than the previous results with FiT. Only the electricity pricing policy is different. In addition, according to literature, the TOU tariff is perfectly known in advance and is not affected by forecast error.

The results on Figure 8 highlight a saving potential of 1000 \( \text{c€/w} \) with an optimal HEMS. This is about 4 times the saving potential achieved with an optimization-based approach under FiT. This can be explained by the presence of period of time with very cheap electricity price (off peak), offering more potential to shift the consumption. In spite of forecast error (SOTA), the optimization-based approach still outperforms largely the conventional control. Nevertheless, the forecast error increases the costs by 240 \( \text{c€/w} \) compared to the perfect forecast case. The market-based control performs well and leads to costs savings of 430 \( \text{c€/w} \) compared to the conventional control.

![Figure 8. TOU tariff: total weekly costs for the different approaches with different forecast errors.](image)

The two studied cases, respectively FiT and TOU, highlight the saving potential brought by the optimization-based control regardless of the forecast error and the good performance of the market-based approach. The TOU allows larger savings because of the presence of well-known period of time with very cheap electricity price. Finally, this study highlights also the impact of the tariff on a 4 persons house without advanced HEMS, i.e., with a conventional control, they will pay 300 \( \text{c€/w} \) more with the TOU tariff than with the FiT.

7.2. Specific Device Costs Comparison

Because of the final state of the storing devices such as battery or water tank, the comparison of different control approaches based on the total cost can lead to wrong conclusion. For example, the total costs of a battery with a full SOC at the end of the simulation is much higher than an empty one, in spite of a better control.
Instead, this work uses the specific device cost to compare the different control approaches. The specific device cost, in \( \text{c€/kWh} \), is the total cost associated to a controllable device normalized by its total energy consumption. Thanks to the energy normalization, these costs are no more directly depending on the final states of the storing elements and allows a better comparison of the control approach performance.

Table 3 gives insights about the performance of the different HEMS approaches:

- **Heat pump control** with market-based control under FiT does not bring additional saving potential compared to the conventional or the optimization-based control. The market-based under TOU reduces by 8% its average specific costs compared to the conventional control. The small saving potential of the HP (30 c€/kWh) compared to EH (20–26 c€/kWh) can be explained by (i) its good efficiency decreasing its electrical consumption and thus its consumption flexibility (ii) the seasonal effect: space heating is required one third of the year only.

- **Electrical heater (EH) control** with the market-based reduces by 10 to 20% its average specific costs compared to the conventional control. The optimization control outperforms slightly the market-based control.

- **Battery control** with the market-based brings 35 to 60% of additional gains compared to the optimization-based control. This can be explained by the frequency control. Optimization-based control takes a control decision every 15 min and does not change it during this time interval. While the market-based control takes a decision every minute and can better capture the feed-in power by charging the battery with a good accuracy.

Table 3. Specific device costs (c€/kWh) and average running time (s) associated to the optimization approach, the conventional control and the market-based.

<table>
<thead>
<tr>
<th>Feed-In Tariff</th>
<th>Conv. Control (c€/kWh)</th>
<th>Optimization (c€/kWh)</th>
<th>Market-Based (c€/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Running time</td>
<td>0.02 s</td>
<td>112 s</td>
<td>1 s</td>
</tr>
<tr>
<td>Heat pump</td>
<td>29.8</td>
<td>29.63 (−0.5%)</td>
<td>29.65 (−0.4%)</td>
</tr>
<tr>
<td>Electrical heater</td>
<td>29</td>
<td>26.2 (−9.5%)</td>
<td>26.5 (−8.5%)</td>
</tr>
<tr>
<td>Battery gain</td>
<td>5.2</td>
<td>8.1 (+55%)</td>
<td>10.9 (+110%)</td>
</tr>
<tr>
<td>TOU Tariff</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat pump</td>
<td>30.0</td>
<td>19.8 (−34%)</td>
<td>27.7 (−8%)</td>
</tr>
<tr>
<td>Electrical heater</td>
<td>24.9</td>
<td>17.7 (−29%)</td>
<td>20.0 (−20%)</td>
</tr>
<tr>
<td>Battery gain</td>
<td>7.3</td>
<td>7.8 (+7%)</td>
<td>12.7 (+73%)</td>
</tr>
</tbody>
</table>

The better performance of the MILP can be mainly explained by the explicit formulation of the cost minimization and the optimality of its solution which allows a better anticipation, in spite of the forecast errors. The market-based approach highlights the benefit of a smaller time interval between control action which allows a better capture of the feed-in power by charging the battery with a good accuracy. On the one hand, the heuristic nature of the market-based approach leads to a problem with a small complexity and thus to a reactive control. On the other hand, it leads to a suboptimal solution. The presented results seem to show that the reactive control feature cannot counterbalance the suboptimal solution.

Finally, the reduction of the time interval of the optimization-based approach could be an option but it will lead to a more complex problem because of the increase of the number of decision variables. In the literature, the typical control interval for a scheduling problem is about 15 min to one hour.
7.3. Study Limitation

The results presented in this work compare the optimality of different HEMS approaches while it includes generality in the operating conditions and the forecast scenarios. Nevertheless, the results are only valid for

- a typical German house according to statistics [30]
- a house equipped with battery, electrical water heater and heat pump
- the current German FiT and the most typical TOU tariff according to literature.

The presented simulation considers a perfect signal transmission between the different devices and the HEMS unit as well as hardware failure is not considered. So, the loss of control of a device is not taken into account in this work.

More specifically, the presented results with the optimization-based approach are the upper bound of the cost savings given that perfect models of the house, heat pump, battery and electrical heater are considered in the problem formulation. More information about the impact of model error can be found in [27].

Future work could consider the deterioration costs due to the battery or the heat pump usage.

8. Conclusions

This paper formulates in a general way the market-based optimization problem to control domestic thermal and electrical flexibilities. Specifically, it discusses the conditions leading to operation cost minimization and derives the optimal bidding strategies for interruptible loads and energy storages. Second, this paper presents the optimization and the rule-based approaches to which the market-based is compared. Third, the presented approaches are compared with Monte Carlo simulations to extract results generality, in contrast with literature. For this reason, the evaluation method incorporates (i) generic forecast errors in the evaluated scenarios and (ii) representative operating conditions for different users and over different seasons.

Two different price schemes are considered in the frame of this work. With feed-in tariff (FiT), the HEMS saving potential is mainly depending on the PV production. While with Time Of Use (TOU) tariff, the HEMS saving is mainly depending on its capacity to shift consumption during period of time with cheap electricity price and PV production.

From the results analysis, the main conclusions are:

- the added value of a HEMS is highly dependent on the adopted tariff. Results shows that the optimal saving potential brought by a HEMS compared to a conventional control is in average 130 €/y with FiT, and 500 €/y with TOU tariff. The larger saving potential with TOU tariff can be explained by the presence of well-known period of time with very cheap electricity price.
- the market-based control outperforms the conventional control and leads to average costs saving of 35 €/y with FiT and 220 €/y under TOU.
- the forecast error decreases the optimization-based potential by 100 and 130 €/y in average for both tariffs with the state-of-the-art forecaster (SOTAFe). Nevertheless, the optimization-based with forecast errors still outperforms the conventional-control and the market-based control.
- the market-based control of the battery outperforms the optimization-based control because of its control frequency allowed by the reactive control: every minute vs. every 15 min with the MILP which allows the battery to better capture the feed-in or the consumption of the house.

In spite of its reactive control, the market-based approach is slightly (FiT) to largely (TOU) outperformed by the optimization-based approach with state-of-the-art forecast error. This can be mainly explained by the explicit formulation of the cost minimization and the optimality of its solution which allows a better anticipation, in spite of the forecast errors.

On the one hand, the heuristic nature of the bidding strategies leads to a small complexity and thus to a reactive control. This feature leads to a better control of the battery because of a smaller time
interval between control action which allows a better capture of the feed-in power. On the other hand, the market-based approach leads to a suboptimal solution because of the heuristic nature of energy storage bidding strategies and the considered naive price forecasts. The presented results seem to show that the reactive control feature cannot counterbalance the suboptimality of the solution. Given this, future work should focus on improving the optimality of the bidding strategy by integrating accurate price forecasts.

Author Contributions: Conceptualization, B.F. and A.M.; Methodology, B.F.; Software, B.F.; Validation, B.F.; Formal Analysis, B.F.; Investigation, B.F.; Resources, A.M.; Data Curation, B.F.; Writing—Original Draft Preparation, B.F.; Writing—Review & Editing, B.F. and A.M.; Visualization, B.F.; Supervision, A.M.; Project Administration, A.M.; Funding Acquisition, A.M.

Funding: This research was funded by Bundesministerium für Bildung und Forschung (German Federal Ministry of Education and Research) grant number 13N13297.

Conflicts of Interest: The authors declare no conflict of interest.

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