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Sound propagation in a forest based on 3D multiple scattering theories

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ABSTRACT

This paper overviews recently developed approaches for forest acoustics based on a three-dimensional (3D) multiple scattering theory. The mean sound field is calculated with the effective wavenumber which represents the scattering by trunks, branches, and the canopy. The radiative transfer equation is formulated appropriately to forest acoustics; this result enables calculations of the mean sound intensities transmitted and backscattered from a stand of trees. The correspondence between sound propagation in a turbulent atmosphere and a forest is outlined. This correspondence, and the existing theory of the interference of the direct and ground reflected waves in a turbulent atmosphere, enable analysis of a similar effect in a forest, where these waves are scattered by forest elements. Numerical examples illustrating application of the 3D multiple scattering theory for calculating the mean sound pressure and mean sound intensity are presented. The 3D multiple scattering theory can also be used to study pulse propagation in a forest.

Keywords: Forest acoustics, Multiple scattering, Radiative transfer theory

1. INTRODUCTION

Forest acoustics is important in several applications such as noise reduction by a stand of trees and localization of sound sources, e.g., reference (1). Due to multiple scattering by trunks and branches, attenuation in the canopy, micrometeorology, and interaction with the impedance ground, sound propagation in a forest is a very complicated phenomenon. Despite significant efforts, there were no satisfactory prediction methods based on first principles.

Recently, a 3D multiple scattering theory was applied to forest acoustics and the results obtained were reported in references (2-5). The 3D multiple scattering theory assumes that different scatterers in a forest have random locations. Then, the propagating sound field becomes a random field. This stochastic approach is often used in wave propagation in complex media. Using this approach, closed form equations for the mean sound pressure and mean sound intensity are derived. The 3D multiple scattering theory is a rigorous approach and enables predictions in realistic environments. The theory is related closely to sound propagation in a turbulent atmosphere. The results known in the latter field (6) can be used to advance forest acoustics. The main goal of this paper is to overview the 3D multiple scattering theory as applied to forest acoustics. Note that the theory might not be applicable and needs to be modified for a forest planted in a regular pattern.

The paper is organized as follows. In Sec. II, scattering properties of the canopy and trunk layers in a forest are considered. Section III provides an approach for calculating the mean sound field. In Sec. IV, the radiative transfer equation (RTE) is introduced, which enables calculations of the mean sound intensity. Section V considers the RTE in the high-frequency approximation. The results presented are summarized in Sec. VI.

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2. SCATTERING CROSS SECTIONS IN A FOREST

In a 3D multiple scattering theory, a key quantity is the scattering amplitude $f(\boldsymbol{n}',\boldsymbol{n})$ of a scatterer, which is defined as follows. Consider a plane wave propagating in the direction of the unit vector $\boldsymbol{n}$, which is incident on the scatterer. The scattered wave can be expressed as a product of the Green's function in free space, the amplitude of the incident wave, and the scattering amplitude $f(\boldsymbol{n}',\boldsymbol{n})$, where the unit vector $\boldsymbol{n}'$ is in the direction of the scattered wave. Thus, $f(\boldsymbol{n}',\boldsymbol{n})$ characterizes the amplitude of the wave scattered in the direction of the vector $\boldsymbol{n}'$.

Equations for the statistical moments of the sound field in a forest contain the differential scattering cross section (DSCS), $\sigma_{\mathrm{d}}(\boldsymbol{n}',\boldsymbol{n})$, and the total cross section (TCS), $\sigma(\boldsymbol{n})$. These cross sections may be expressed in terms of the scattering amplitude as

$$\sigma_{\mathrm{d}}(\boldsymbol{n}',\boldsymbol{n}) = \nu|f(\boldsymbol{n}',\boldsymbol{n})|^2, \qquad \sigma(\boldsymbol{n}) = \frac{4\pi\nu}{k}\,\mathrm{Im}\, f(\boldsymbol{n},\boldsymbol{n}). \tag{1}$$

Here, ν is the number of scatterers per unit volume and k is the sound wavenumber in free space. It is often useful to express the TCS as a sum of the scattering cross section (SCS), $\sigma_s(\boldsymbol{n})$, and absorption cross section (ACS), $\sigma_a(\boldsymbol{n})$:

$$\sigma(\boldsymbol{n}) = \sigma_s(\boldsymbol{n}) + \sigma_a(\boldsymbol{n}). \tag{2}$$

The SCS characterizes the loss of energy of a sound wave propagating in the direction of the unit vector $\boldsymbol{n}$ due to sound scattering in all directions. It is obtained by integrating the DSCS over the unit vector $\boldsymbol{n}'$:

$$\sigma_s(\boldsymbol{n}) = \int_{4\pi} \sigma_d(\boldsymbol{n}',\boldsymbol{n})\, d\Omega(\boldsymbol{n}'). \tag{3}$$

Here, $d\Omega(\boldsymbol{n}')$ is the solid angle in the direction of the vector $\boldsymbol{n}'$. The ACS has a similar meaning but is pertinent to sound absorption rather than scattering. The ACS accounts for sound absorption in a forest such as visco-thermal dissipation in foliage.

There are two approaches for determining the DSCS and TCS in a forest: (i) The cross sections can be measured experimentally as explained in references (3,4). (ii) Trunks can be modeled as finite vertical cylinders, branches as finite slanted cylinders, and the canopy layer as diffuse scatterers. Using these scatterers, different realistic forests can be built.

As an example of the second approach, we provide modeling of the trunk and canopy layers, which will be used later in the paper. Trunks are modelled as solid finite vertical cylinders with the scattering amplitude $f(\boldsymbol{n}',\boldsymbol{n})$ from reference (7). The DSCS is then (3):

$$\sigma_d(\boldsymbol{n}',\boldsymbol{n}) = \nu\left|khb\,\mathrm{sinc}[kh(\cos\theta' - \cos\theta)/2]\sum_{m=0}^{\infty} B_m \cos(m(\varphi' - \varphi))\right|^2. \tag{4}$$

Here, h is the length of a cylinder, b is its radius, $\mathrm{sinc}(x)$ is the sinc function, θ and φ are the polar and azimuthal angles of the unit vector $\boldsymbol{n}$, and θ' and φ' are those of the unit vector $\boldsymbol{n}'$. The functions B_m are given by

$$B_m = \frac{\varepsilon_m}{2} J_m'(kb\sin\theta)\left[\sin\theta\, J_m(kb\sin\theta') - \sin\theta' J_m'(kb\sin\theta')\frac{H_m^{(1)}(kb\sin\theta)}{H_m^{(1)'}(kb\sin\theta)}\right], \tag{5}$$

where, ε_m is the Neumann factor, J_m is the Bessel function, $H_m^{(1)}$ is the Hankel function of the first kind, and primes above these functions denote derivates with respect to the argument. Although these formulas appear to be involved, they are relatively easy to implement numerically. The SCS, $\sigma_s(\boldsymbol{n})$, can be obtained with Eq. (3). Since trunks are assumed as solid cylinders, $\sigma_a = 0$ and $\sigma(\boldsymbol{n}) = \sigma_s(\boldsymbol{n})$.

In the considered example, branches can be modeled as solid finite slanted cylinders. The DSCS is still given by Eqs. (4) and (5) with some transformation of the angles. Since the canopy layer usually consists of many small scatterers such as leaves, needles, and twigs, it can be modelled by diffuse scatterers. For such scatterers, the DSCS $\sigma_{\mathrm{d}}(\boldsymbol{n}',\boldsymbol{n})$ does not depend on the vectors $\boldsymbol{n}'$ and $\boldsymbol{n}$. Visco-thermal attenuation and induced vibration in the canopy layer may be modelled by a finite value of the ACS, σ_a.

In a forest, the scattering amplitude and cross sections usually depend on the spatial coordinates $\boldsymbol{R} = (x,y,z)$. In what follows, this dependence is accounted for by including $\boldsymbol{R}$ as an argument of the corresponding quantities, e.g., $f(\boldsymbol{R};\boldsymbol{n}',\boldsymbol{n})$, $\sigma_{\mathrm{d}}(\boldsymbol{R};\boldsymbol{n}',\boldsymbol{n})$, and $\sigma(\boldsymbol{R};\boldsymbol{n})$.

3. MEAN SOUND PRESSURE

In a 3D multiple scattering theory, the sound pressure in a forest can be written as

$$p(\boldsymbol{R}) = \langle p(\boldsymbol{R})\rangle + \tilde{p}(\boldsymbol{R}). \tag{6}$$

Here, the brackets $\langle \quad \rangle$ indicate ensemble averaging, $\langle p(\boldsymbol{R})\rangle$ is the mean sound pressure (also termed the coherent sound field), and $\tilde{p}(\boldsymbol{R})$ is the sound-pressure fluctuation. The mean sound pressure can be calculated using the well-known parabolic equation (PE) in a non-random medium,

$$\left[2ik\frac{\partial}{\partial x} + \Delta_\perp + k^2\left(1 + \frac{c_0^2}{(c_0+c)^2}\right)\right]\langle p\rangle = 0, \tag{7}$$

if the sound wavenumber k is replaced with the effective wavenumber k_{eff} which accounts for sound scattering in a forest. In this equation, the x-axis is in the direction of sound propagation, $\Delta_\perp = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $c(\boldsymbol{R})$ is the sound speed, and c_0 is its reference value. The effective wavenumber is given by

$$k_{\mathrm{eff}}(\boldsymbol{R},\boldsymbol{n}) = k + \frac{2\pi\nu}{k} f(\boldsymbol{R};\boldsymbol{n},\boldsymbol{n}). \tag{8}$$

The effective wavenumber is also used in a 2D multiple scattering theory to predict the mean sound pressure, e.g., references (1,8,9). The effective wavenumbers for 2D and 3D cases coincide for horizontal sound propagation and differ for slanted propagation, see reference (2).

Effective numerical algorithms for solving Eq. (8), which account for atmospheric stratification and sound interaction with the impedance ground, have been developed in atmospheric acoustics, e.g., reference (6). These algorithms can be readily applied to sound propagation in a forest. As an example, Fig. 1 depicts the complex amplitude of the mean sound pressure, $\langle A\rangle = \exp(-ikx)\langle p\rangle$, due to a point unit strength source. In the figure, dashed horizontal lines indicate the trunk and canopy layers, which are 12 m and 25 m high. For this example, the sound frequency is 1 kHz, the source height is 1 m, the number of trees per unit area is $N = \nu h = 0.05$ 1/m^2, and the tree radius is 0.09 m. These parameters and the scattering amplitude for solid finite vertical cylinders (7) enable us to calculate k_{eff} in the trunk layer. In the canopy layer, we assume that $(k_{\mathrm{eff}} - k)$ is that in the trunk layer multiplied by a factor 1.5. The relaxation model (6,10) with parameters pertinent for a fermentation/humus layer in a forest (11) is used to assess sound interaction with the ground, see reference (3) for more details. In Fig.1, four subplots depict gradually increasing complexity of sound propagation. Figure 1(a) corresponds to a source located above an impedance ground in a homogeneous atmosphere. In Fig. 1(b), a 12 m high trunk layer is added to a homogeneous atmosphere. As a result, the complex amplitude in this layer and above is significantly attenuated starting at the range of about 200 m. In Fig. 1(c), a 13 m high canopy layer is added to the previous geometry. This results in significant sound attenuation with height. Finally, in Fig. 1(d), the sound speed has a vertical gradient 0.12 1/s which results in downward refraction.

The mean sound pressure attenuates approximately exponentially with propagation range and is applicable for a relatively small range. Beyond this range, the 3D multiple scattering theory provides approaches for calculating the mean sound intensity as explained in sections 4 and 5.

4. MEAN SOUND INTENSITY

The mean sound intensity is given by $I = \langle p(\boldsymbol{R})p^*(\boldsymbol{R})\rangle$. Substituting with Eq. (6), the mean intensity can be written as

$$I(\boldsymbol{R}) = \langle p(\boldsymbol{R})\rangle\langle p^*(\boldsymbol{R})\rangle + \langle \tilde{p}(\boldsymbol{R})\tilde{p}^*(\boldsymbol{R})\rangle. \tag{9}$$

The first term on the right-hand side $I_c(\boldsymbol{R}) = \langle p(\boldsymbol{R})\rangle\langle p^*(\boldsymbol{R})\rangle$ is termed the coherent sound intensity and the second term $I_d(\boldsymbol{R}) = \langle \tilde{p}(\boldsymbol{R})\tilde{p}^*(\boldsymbol{R})\rangle$ the diffuse sound intensity.

In the radiative transfer theory, the mean intensity can be expressed in terms of the radiance,

$$I(\boldsymbol{R}) = \int_{4\pi} J(\boldsymbol{R},\boldsymbol{n})\, d\Omega(\boldsymbol{n}). \tag{10}$$

The radiance $J(\boldsymbol{R},\boldsymbol{n})$ is defined as the average energy flux within a unit solid angle Ω in the direction of the unit vector $\boldsymbol{n}$. It follows from Eq. (10) that the mean intensity is obtained by integrating the radiance over all directions of the vector $\boldsymbol{n}$. The radiance can be found by solving the RTE:

$$\left(\boldsymbol{n}\cdot\frac{\partial}{\partial \boldsymbol{R}}\right)J(\boldsymbol{R},\boldsymbol{n})+\sigma(\boldsymbol{R};\boldsymbol{n})J(\boldsymbol{R},\boldsymbol{n})=\int_{4\pi}\sigma_{\mathrm{d}}(\boldsymbol{R};\boldsymbol{n}',\boldsymbol{n})J(\boldsymbol{R},\boldsymbol{n}')\,d\Omega(\boldsymbol{n}'). \quad (11)$$

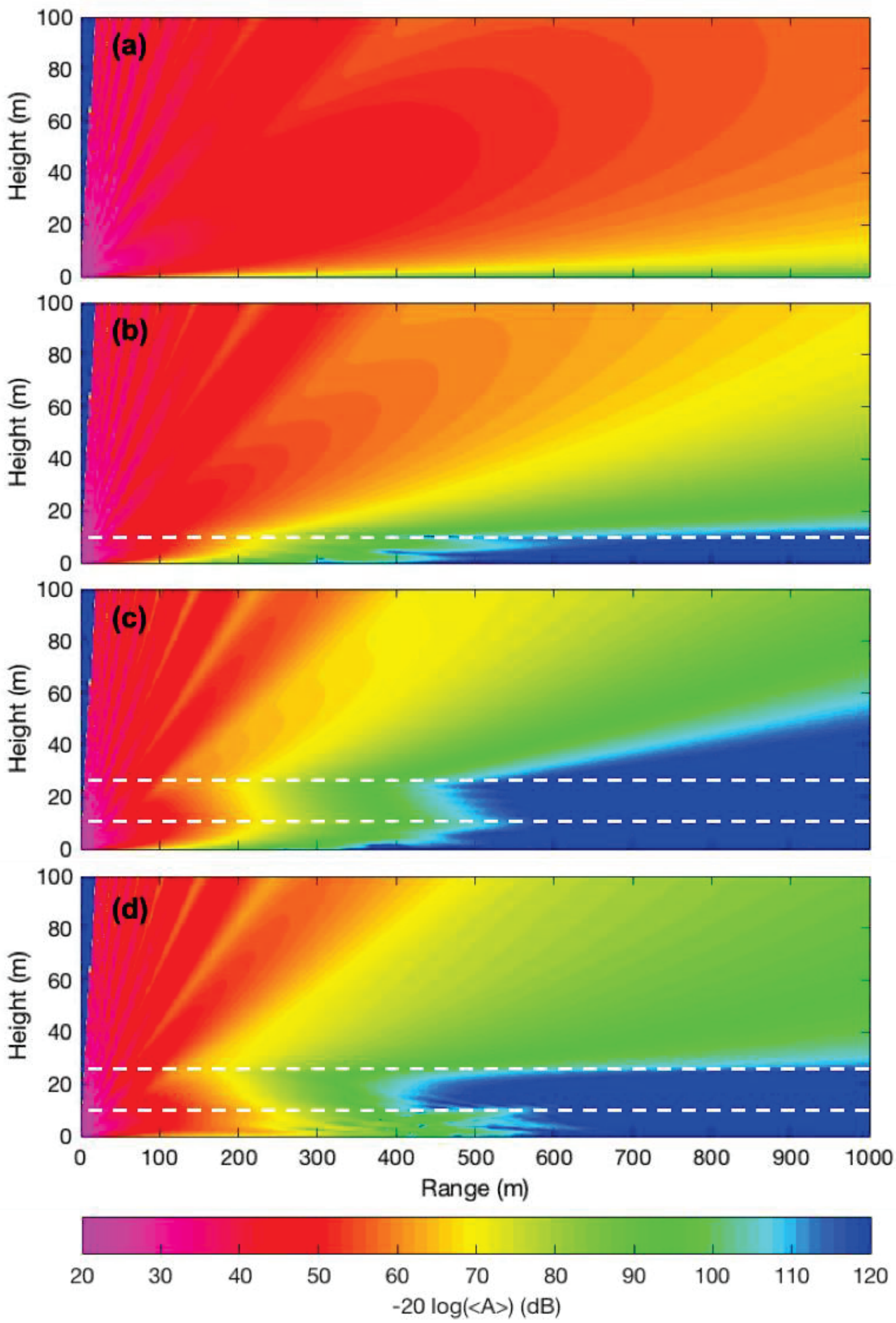

Figure 1 – Complex amplitude of the mean sound pressure versus range and height. The geometries of sound propagation are explained in the text.

Solutions of the RTE are well developed in many fields of physics and can be readily applied to forest acoustics. The RTE accounts for diffraction. Similar to the mean sound intensity, the radiance can be expressed as a sum $J(\boldsymbol{R},\boldsymbol{n})=J_c(\boldsymbol{R},\boldsymbol{n})+J_d(\boldsymbol{R},\boldsymbol{n})$, where $J_c(\boldsymbol{R},\boldsymbol{n})$ is the coherent radiance and $J_d(\boldsymbol{R},\boldsymbol{n})$ is the diffuse radiance. Reference (3) provides RTEs for the coherent and diffuse radiances.

Consider an example of using the RTE in forest acoustics. Let a plane sound wave be normally incident on the forest edge, with a receiver located at the other edge of the forest. The forest is modeled with four layers: ground, trunks, canopy, and open air. This geometry is pertinent to sound attenuation by a stand of trees. Parameters typical for a temperate conifer forest are considered: the height of the canopy layer is 25 m, the height of the trunk layer is 12 m, the number of trees per unit area is 0.05 $1/m^2$, and the tree radius is 0.09 m. Furthermore, the sound frequency is 1 kHz and the height of the receiver is 2 m. Using these parameters, the TCS and DSCS in the trunk layer can be calculated with Eqs. (2)-(5). Currently, scattering and absorption properties of canopy layers are mostly unknown. For concreteness, we assume that $\sigma_a = \sigma_s$ and the TCS $\sigma = \sigma_a + \sigma_s$ is equal to $\sigma(\boldsymbol{n})$ in the trunk layer for the vector $\boldsymbol{n}$ in the direction of the incident wave. Sound interaction with the ground is modelled similar to that in Fig. 1.

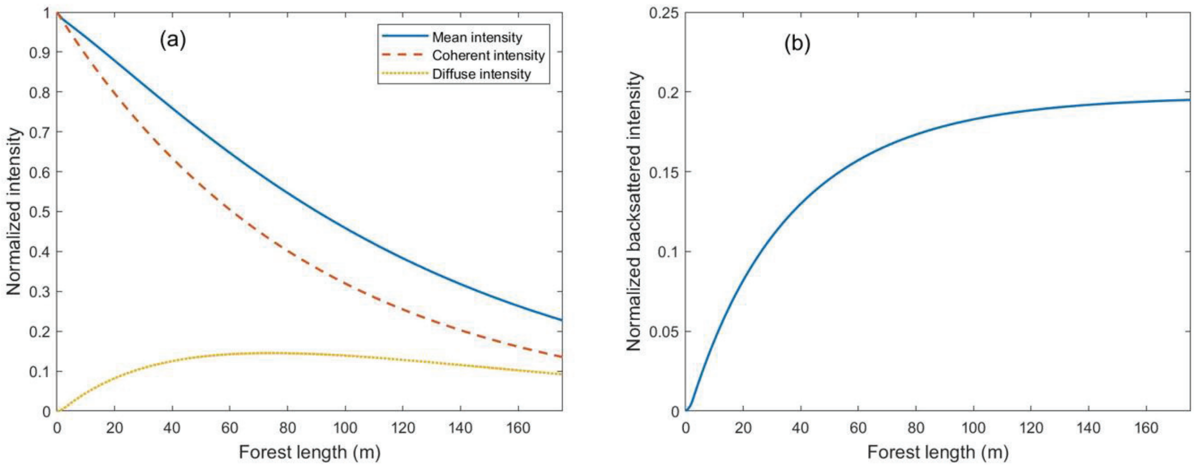

Figure 2 – (a) Normalized sound intensities transmitted through the forest. (b) Normalized sound intensity backscattered from the forest.

For the considered example, the RTE, Eq. (11), is solved in a modified Born approximation. Figure 2(a) depicts the mean, coherent, and diffuse sound intensities, normalized by the incident intensity, transmitted through the forest versus the forest length. The coherent intensity exponentially attenuates with increasing forest length. The diffuse intensity is zero at the forest edge, reaches a maximum at 74.5 m, and then decreases with the forest length. The mean sound intensity is a sum of the coherent and diffuse intensities. Figure 2(b) depicts the mean sound intensity backscattered from the forest. The backscattered intensity monotonically increases with increasing forest length and reaches a plateau at about 140 m.

The RTE is valid for both low and high frequencies. Sound propagation in the atmosphere is usually considered in the high-frequency approximation. In this approximation, the RTE simplifies as explained in the next section.

5. HIGH-FREQUENCY APPROXIMATION

The high-frequency approximation corresponds to narrow-angle sound propagation in a predominant direction, which we align with the x-axis. In this case, the RTE can be reduced to the following PE, see reference (5):

$$\left[\frac{\partial}{\partial x} - \frac{i}{k}\frac{\partial^2}{\partial \boldsymbol{r} \partial \boldsymbol{r}_d} + \sigma_a(x, \boldsymbol{r}) + Q(x; \boldsymbol{r}, \boldsymbol{r}_d)\right] B(x; \boldsymbol{r}, \boldsymbol{r}_d) = 0. \quad (12)$$

Here, $B(x; \boldsymbol{r}, \boldsymbol{r}_d) = \langle p(x, \boldsymbol{r}_1) p^*(x, \boldsymbol{r}_2)\rangle$ is the spatial correlation function of the sound field, $(x, \boldsymbol{r}_1)$ and $(x, \boldsymbol{r}_2)$ are two points of observation located at the same range x, $\boldsymbol{r}_d = \boldsymbol{r}_1 - \boldsymbol{r}_2$ is the vector between the observation points in the plane perpendicular to the x-axis, $\boldsymbol{r} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$ is the geometrical center of these two points, and σ_a corresponds to the ACS in the direction of the x-axis. The mean sound intensity is obtained by setting $\boldsymbol{r}_d = 0$ in the correlation function: $I(x, \boldsymbol{r}) = B(x; \boldsymbol{r}, \boldsymbol{r}_d = 0)$. In Eq. (12), the function $Q(x; \boldsymbol{r}, \boldsymbol{r}_d)$ is given by

$$Q(x;\boldsymbol{r},\boldsymbol{r}_d) = \frac{1}{k^2}\int \sigma_{\rm d}(x,\boldsymbol{r};\boldsymbol{\kappa}/k)\left(1-{\rm e}^{i\boldsymbol{\kappa}\cdot\boldsymbol{r}_d}\right)d^2\kappa, \qquad (13)$$

where $\sigma_{\rm d}(x,\boldsymbol{r};\boldsymbol{\kappa}/k)$ has the following meaning. In the narrow-angle approximation, the DSCS $\sigma_{\rm d}(x,\boldsymbol{r};\boldsymbol{n}',\boldsymbol{n})$ depends only on the difference between the vectors $\boldsymbol{n}'$ and $\boldsymbol{n}$: $\sigma_{\rm d} = \sigma_{\rm d}(x,\boldsymbol{r};\boldsymbol{n}'-\boldsymbol{n})$. In this formula, $\boldsymbol{n}'-\boldsymbol{n} \approx \boldsymbol{n}'_\perp - \boldsymbol{n}_\perp$, where $\boldsymbol{n}_\perp$ is the component of the vector $\boldsymbol{n}$ perpendicular to the x-axis. Then, $\sigma_{\rm d} \approx \sigma_{\rm d}(x,\boldsymbol{r};\boldsymbol{n}'_\perp - \boldsymbol{n}_\perp) = \sigma_{\rm d}(x,\boldsymbol{r};\boldsymbol{\kappa}/k)$, where $\boldsymbol{\kappa} = k(\boldsymbol{n}'_\perp - \boldsymbol{n}_\perp)$. Thus, $\sigma_{\rm d}(x,\boldsymbol{r};\boldsymbol{\kappa}/k)$ is the DSCS in the narrow-angle approximation.

Equation (12), which is termed the second-moment PE, also describes the correlation function of the sound field in a turbulent atmosphere. In this case, the DSCS, $\sigma_{\rm d}(x,\boldsymbol{r};\boldsymbol{\kappa}/k)$, is expressed in terms of the spectra of atmospheric turbulence and $\sigma_{\rm a} = 0$, see reference (5) for details. Effective numerical techniques for solution of the second-moment PE have been developed in atmospheric acoustics (6,12,13) and can be used to study sound propagation in a forest. This approach can be generalized to account for sound refraction in a forest.

Sound propagation in a turbulent atmosphere has been investigated relatively well and is summarized in reference (6). The results from this reference and the correspondence between sound propagation in a turbulent atmosphere and forest can be used to advance forest acoustics. As an example, consider the effect of trees on the interference of the direct and ground-reflected waves. For typical ground impedances of forest floors and certain frequencies and ranges, this interference could result in significant reduction of the sound pressure level. In practice, however, this reduction does not fully occur since scattering by trees diminishes the coherence between the direct and ground-reflected waves. A similar phenomenon exists (and is well studied) in atmospheric acoustics, where turbulence reduces coherence between these waves. Using the similarity between sound propagation in a turbulent atmosphere and forest, we rigorously account for the effect of trees on the interference between the direct and ground-reflected waves, see reference (5). For the mean-squared sound pressure, we have

$$\langle pp^*\rangle = \frac{1}{R_1^2} + \frac{1}{R_2^2} + \frac{2|\mathcal{R}|C_{\rm coh}}{R_1R_2}\cos(k(R_2-R_1)+\beta). \qquad (14)$$

Here, R_1 and R_2 are the path lengths of the direct and ground-reflected waves, $\mathcal{R} = |\mathcal{R}|\exp(i\beta)$ is the spherical-wave reflection coefficient, and $C_{\rm coh}$ is the coherence factor which describes the loss of coherence between the direct and ground-reflected waves due to scattering in a forest. The coherence factor is expressed in terms of DSCS,

$$C_{\rm coh} = \exp\left[-\frac{x}{k^2}\int_0^1 d\eta \int_{-\infty}^{\infty} d\kappa_z\, \sigma_{\rm d}(\kappa_y,\kappa_z)\left(1-e^{i\eta\kappa_z h_{sr}}\right)\right], \qquad (15)$$

where x is the propagation range, $h_{sr} = 2h_sh_r/(h_s+h_r)$, and h_s and h_r are the source and receiver heights. For sound scattering by trunks and $h_{sr} \le h$, Eq. (15) reduces to

$$C_{\rm coh} = \exp\left(-\frac{xNbh_{sr}}{h}\right). \qquad (16)$$

Equations (14-16) enable us to access the effect of trees on the interference of the direct and ground-reflected waves. As an example, the solid line in Fig. 3 depicts the sound pressure level (SPL) relative to that in free space, $10\log(\langle pp^*\rangle R_1^2)$, versus sound frequency f. The propagation range is 100 m, the source and receiver heights are 1 m and 2 m, the height of the trunk layer is 12 m, the number of trees per unit area is 0.05 m^{-2}, and the tree's radius is 0.09 m. The ground impedance is the same as that assumed in Figs. 1 and 2. The other two lines in Fig. 3 correspond to the cases when there is no forest (in Eq. (14), $C_{\rm coh} = 1$) and when the direct and ground-reflected waves are added incoherently ($C_{\rm coh} = 0$). It follows from Fig. 3 that without the forest, the interference results in maxima and minima of the SPL as a function of frequency. With the forest, the interference minima are reduced by several dB due to the coherence loss between the direct and ground-reflected waves, resulting in an apparent increase in the SPL. However, the ground effect interference pattern is predicted not to be completely suppressed since, as shown by several measurements (1), there remains some coherency between direct and ground-reflected paths in a forest. When these waves are incoherent, the maxima and minima are completely suppressed, and the SPL only slightly depends on the frequency (Fig. 3).

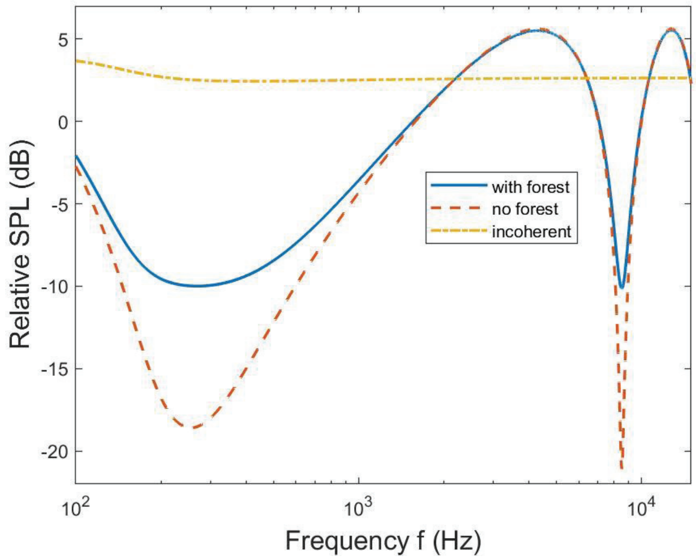

Figure 1 – SPL relative to that in free space for the interference of the direct and ground-reflected waves in a forest.

6. CONCLUSIONS

This paper overviewed application of a 3D multiple scattering theory to forest acoustics (2-5). In this theory, the differential scattering cross section (DSCS), total cross section (TCS), scattering cross section (SCS), and absorbing cross section (ACS) are utilized to characterize different scatterers in a forest such as trunks, branches, and the canopy. Approaches were outlined to access these cross sections.

The mean sound pressure $\langle p(\boldsymbol{R})\rangle$ may be calculated with a standard parabolic equation (PE) if the sound wavenumber in free space, k, is replaced with the 3D effective wavenumber, $k_{\rm eff}$, which accounts for sound scattering in a forest. The mean sound intensity $I(\boldsymbol{R})$ was expressed in terms of the radiance $J(\boldsymbol{R},\boldsymbol{n})$, which satisfies the radiative transfer equation (RTE). In the high-frequency approximation, the RTE reduces to the second-moment PE for the correlation function of the sound field $B(x;\boldsymbol{r},\boldsymbol{r}_d)$; the mean sound intensity is obtained by setting $\boldsymbol{r}_d = 0$ in the correlation function. The correspondence between sound propagation in a turbulent atmosphere and forest was outlined and used to predict the effect of trees on the interference of the direct and ground-reflected waves. The methods for calculating the mean sound pressure and intensity were illustrated by numerical examples.

Propagation of continues sound signals was considered. A 3D multiple scattering theory can also be used to predict pulse propagation in a forest as described in reference (4). 3D multiple scattering theory appears to be a very suitable approach for forest acoustics.

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