

## Vibro-acoustic behavior of plates considering static load effect

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### ABSTRACT

In this paper, we present the general form of governing equations of plates considering the thermal/static load effect, and study the effect of thermal/static load on the vibro-acoustic behavior of plates. The static state under thermal/static load is obtained firstly, and then the vibration and acoustic radiation are calculated at the reference static position. Numerical and experimental approaches are also displayed. The results show that both the inner stress and geometry shape will be changed under thermal/static load. With the temperature increasing, the softening effect of thermal stress in the pre-buckling state leads to the decrease of frequencies, while in the post-buckling state the stiffening effect of thermal buckling deformation leads to the increase of frequencies. The variation of vibration and acoustic radiation responses are in accordance with that of the frequencies. Finally, the vibro-acoustic behavior under static load is also discussed in detail.

Keywords: Thermal load effect, Static load effect, Natural characteristics, Vibration response, Acoustic radiation

### 1. INTRODUCTION

With the big progress in hypersonic aircraft, the research on vibro-acoustic characteristics of structures under extreme thermal load, static load, acoustic excitation and mechanical excitation becomes more and more important. The material properties, geometry shape, inner stress will all be changed under the thermal load and static load, which makes the vibro-acoustic analysis much more difficult. Lots of efforts have been made on the related works.

Using the combined FEM/BEM method, Jeyaraj et al detailedly studied the thermal effect on the vibration and acoustic radiation responses of different plates, such as isotropic plate<sup>[1]</sup>, fiber-reinforced composite plate<sup>[2]</sup>, viscoelastic sandwich plate<sup>[3]</sup> and so on. In the research, the critical buckling temperatures and the vibration of the plates are carried out by FEM, while the acoustic radiation is derived by BEM. The method is also utilized in the structures under the water<sup>[4]</sup>. The temperature dependent material properties are considered in the free vibration analysis of functionally graded plates in thermal environment<sup>[5]</sup>. Geng et al<sup>[6-8]</sup> further investigated the effect of thermal environments on the vibro-acoustic responses of plates with different boundary conditions through theoretical, numerical and experimental approaches. The results shown that in the pre-buckling state, the natural frequencies decrease and the response peaks shift to lower frequency range with the temperature increasing. Liu and Li<sup>[9]</sup> studied the vibration and acoustic responses of simply supported sandwich plates considering thermal effect using equivalent non-classical theory. Li and Li<sup>[10]</sup> conducted the vibration and sound radiation analysis of composite plates in thermal environment. Li and Yu<sup>[11, 12]</sup> investigated the vibro-acoustic responses of the sandwich and composite panels in thermal environment using a proposed Piecewise shear deformation theory. Zhao et al<sup>[13]</sup> presented numerical analysis of the vibro-acoustic responses of laminated composite plates in thermal environment based on finite element formulation.

The above research mainly focused on the softening effect of thermal stress in pre-buckling state. However, when the temperature is higher than the critical buckling temperature, the effect of buckling configuration should be paid more attention on. In the experiment about the vibro-acoustic

characteristics of a plate in pre- and post-buckling state, Geng et al<sup>[14]</sup> found that with the temperature increasing, the natural frequencies decrease before the critical temperature and increase after. The reason is that the effect of buckling configuration plays a more important role than the effect of thermal stress, which leads to the stiffening of the bending stiffness. Numerical analysis of thermally-buckled plates also presented the increasing of natural frequencies in the post-buckling state<sup>[15, 16]</sup>. Du et al<sup>[17]</sup> investigated the vibration and acoustic radiation of laminated plates with temperature gradient along thickness. It is shown that the deflection under temperature gradient which results in the increasing of natural frequencies. The same as thermal load, static load will also result in the change of both inner stress state and geometry shape. The difference is that either compressive stress or tensile stress may appear under static load depending on the direction of static load and the initial curvature of the plate. Takabatake studied the effect of dead load on the vibration of beams<sup>[18]</sup> and plates<sup>[19]</sup>. Wang et al<sup>[20]</sup> investigated the effect of static load on vibro-acoustic behavior of clamped plates with different geometric imperfections.

In this paper, we present a review of the research on vibro-acoustic behavior of plates under thermal/static load carried out by our group.

## 2. THEORETICAL APPROACH

As we In the theoretical formulation, considering a plate under thermal load or static load, the displacements can be expressed as the sum of nonlinear static deflections under thermal/static load and the linear dynamic displacements<sup>[20]</sup>. According to the boundary conditions, the displacements are assumed as the superposition of spatial functions. Thus, by applying Hamilton's principle, we can derive two sets of equations in the following form<sup>[17, 20]</sup>:

$$\left[ \mathbf{K} - \mathbf{K}_T + \mathbf{K}_{NL1}(\mathbf{q}^s) + \mathbf{K}_{NL2}(\mathbf{q}^s) \right] \mathbf{q}^s = \mathbf{Q}^s + \mathbf{Q}^T \quad (1)$$

$$\mathbf{M} \ddot{\mathbf{q}}^d + \left[ \mathbf{K} - \mathbf{K}_T + 2\mathbf{K}_{NL1}(\mathbf{q}^0) + 3\mathbf{K}_{NL2}(\mathbf{q}^0) \right] \mathbf{q}^d(t) = \mathbf{Q}^d(t) \quad (2)$$

From Eq.(1), we can derive the nonlinear static deflection under thermal/static load. Then substituting the static deflection into Eq.(2), we can derived the vibration governing equation considering the initial stress and deformation effects under thermal/static load. Introduce Rayleigh integral<sup>[6, 17]</sup>:

$$p(x_p, y_p, z_p, t) = \frac{j\omega\rho_0}{2\pi} e^{j\omega t} \int_{\Omega} \frac{\bar{v}(x, y, t) \cdot e^{-jk_a R}}{R} dA \quad (3)$$

and we can calculate the acoustic radiation pressure at the observation point.

## 3. NUMERICAL APPROACH

Numerical analysis is carried out by FEM/BEM method<sup>[6]</sup>. Firstly, the finite element model of the plate is established in Nastran for the modal analysis. Then the mode results are imported into VA One as shown in Fig. 1. Thus, the vibro-acoustic responses can be calculated by the combined FEM and BEM solver of VA One.

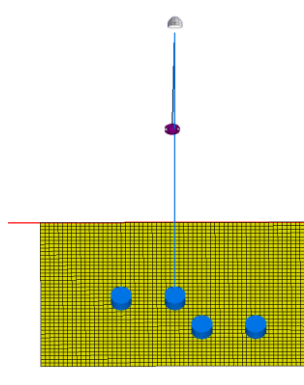


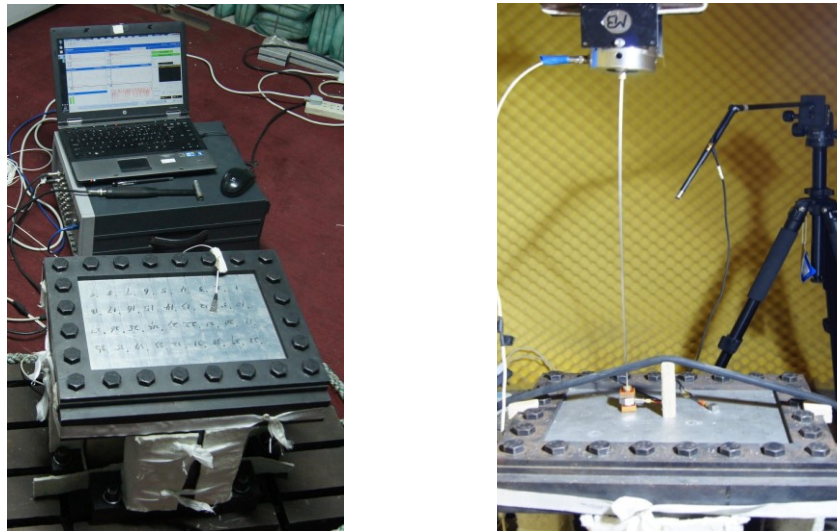
Figure 1 – FEM/BEM model<sup>[6]</sup>

## 4. EXPERIMENT SETUP

As shown in Fig. 2, a series of experiments about the vibro-acoustic characteristics under thermal/static load have been conducted based on the experimental setup.

In the modal test, the single-input single-output hammer (PCB hammer 086C04) based impact method is used. Test points are uniformly distributed on the plate. An accelerometer (PCB 333B32) is located on a proper test point to receive the transient signals when all the test points are impacted by the hammer in sequence. The Impact Testing module of the software package LMS TEST.LAB is used for signal acquisitions and data analysis[8].

In the vibration response measurement, an accelerometer (PCB 333B32) is located at the observation point on the plate to receive the vibration signals under mechanical excitation (MB Exciter Modal 2) or acoustic excitation (Alpine SPS-170A). In addition, a microphone (B&K type 4958) is located above the plate to receive the acoustic radiation pressure from the plate under mechanical excitation.



(a) vibration under thermal load<sup>[8]</sup>

(b) vibration under static load<sup>[20]</sup>

Figure 2 – Experimental setup

## 5. RESULTS DISCUSSION

Based on the above theoretical, numerical and experimental approaches, we have studied the vibro-acoustic characteristics of different plates, revealing the influence mechanism of thermal and static load.

### 5.1 Effect of uniform thermal environment

A clamped plate is shown in Fig. 3. Detailed information about the material properties, geometry dimensions and load cases can be seen in Ref.[6-8, 14].

Fig. 4 gives the first five theoretical natural frequencies of the plate in different thermal environments before critical buckling temperature. It is shown that all the frequencies decrease with the temperature increasing. The vibration and acoustic radiation results in Fig. 5 show the same variation trend with that of the frequencies. The response peaks shift to lower frequency range.

The reason is that in the pre-buckling state, only thermal stress appears in the plate and there's no transverse deformation. Thermal stress results in the softening of bending stiffness and the natural frequencies decrease hereof.

From Fig. 6 we can see that, with the temperature getting higher than critical temperature, the plate is buckled and the buckling deflection gets larger and larger. The stiffening effect of buckling deformation becomes stronger than the softening effect of thermal stress. The frequencies gradually begin to increase. At the same time, the response peaks shift to higher frequency range as shown in Fig. 7. Moreover, the third natural frequency line crosses the fourth natural frequency line.

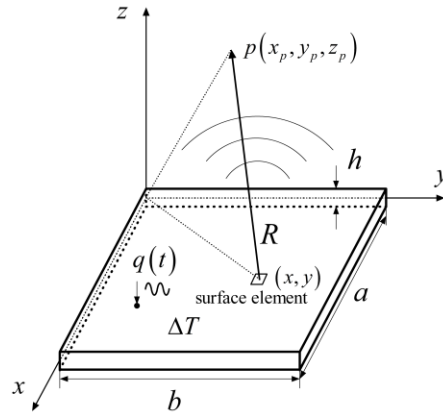


Figure 3 – Scratch of the plate

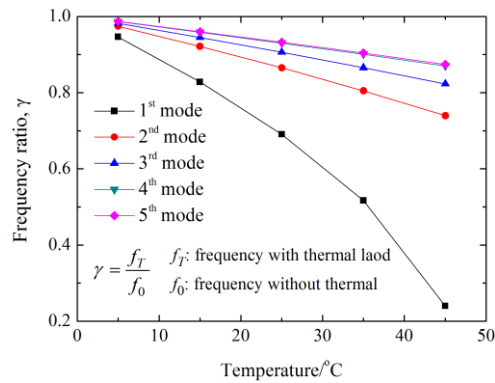


Figure 4 – Natural frequencies before critical buckling temperature<sup>[6]</sup> (theoretical results)

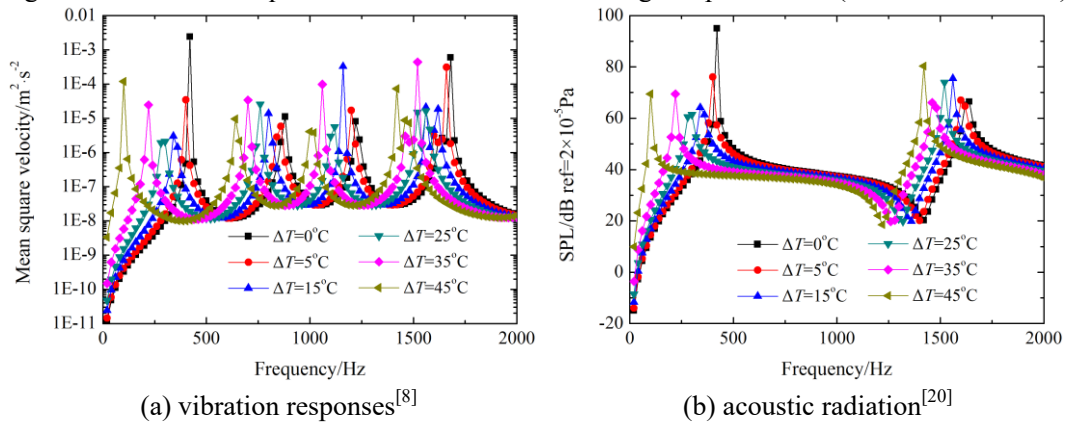


Figure 5 – Vibration and acoustic radiation responses of the plate subjected to point excitation before critical buckling temperature (theoretical results)

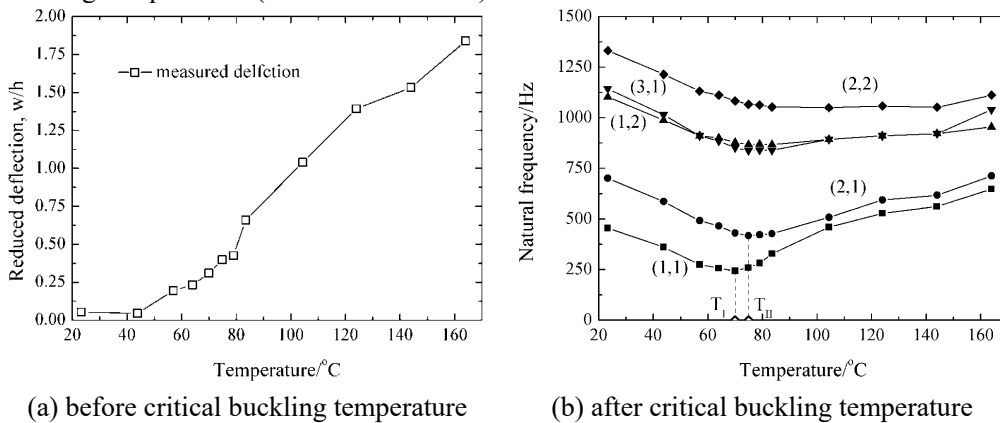


Figure 6 – Center buckling deflection and natural frequencies<sup>[14]</sup> (experimental results)

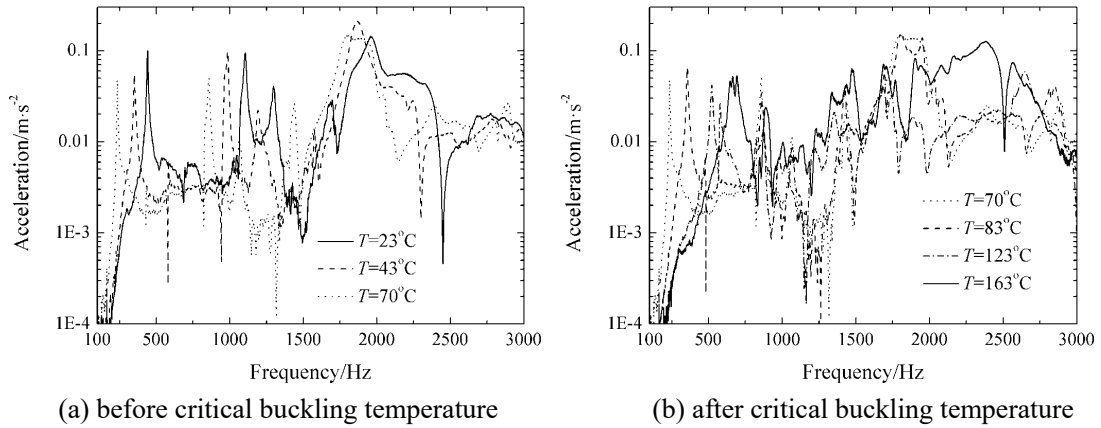


Figure 7 – Vibration responses of the plate subjected to point excitation before critical buckling temperature<sup>[14]</sup> (experimental results)

### 5.2 Effect of thermal gradient

The vibro-acoustic characteristics analysis of laminated plates under temperature gradients in Fig. 8(a)<sup>[17]</sup> shows that the deflection gets larger with the temperature gradient increasing. Fig. 8(b) shows that the resonant frequency increases as the temperature gradient increases. This is because initial deflections caused by temperature gradient intensify the stiffness of the structure. Accordingly, resonant frequency is enhanced.

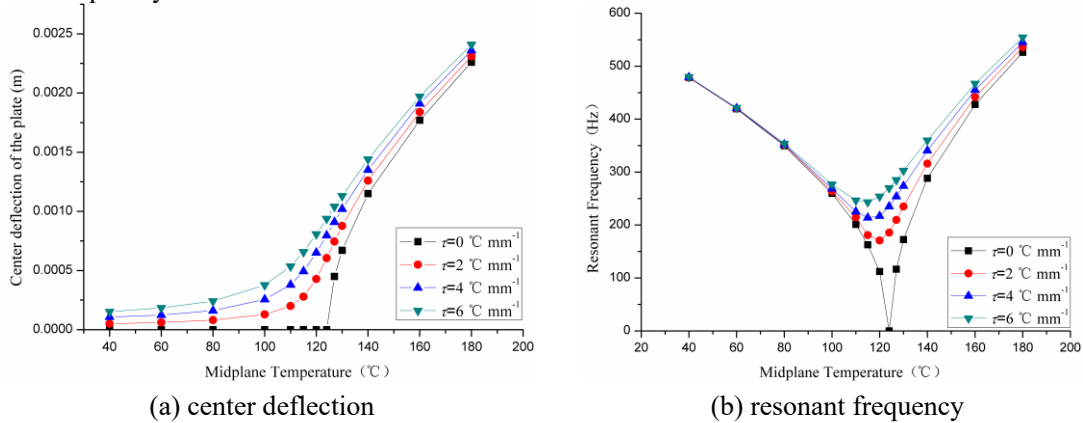


Figure 8 – Center deflection and resonant frequency under different temperature gradients<sup>[17]</sup> (theoretical results)

### 5.3 Effect of static load

Different from the thermal load which leads to compress stress in the plate, the static load may lead to both compress/tensile stress depending on the static load direction and the initial imperfection. In Ref.[20], the vibro-acoustic responses of three plates with different imperfections are studied. The plates are shown in Fig. 9. The deflections and natural frequencies shown in Fig. 10 indicate that when the plate is flat or the directions of the imperfection and the static load are the same, the plate is stiffened and the natural frequencies increase with the static load gets larger. However, when the directions are the opposite as shown in Fig. 9(c), compress stress appears and the curvature becomes small at the beginning. The plate is softened and the natural frequencies decrease. Then the compress stress gradually becomes tensile stress and the curvature turns to increase. The plate is stiffened and the natural frequencies increase.

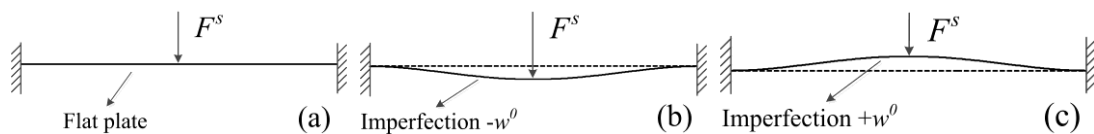


Figure 9 – (a) flat plate, (b) plate with geometric imperfection  $-w^0$  in  $-z$  direction, (c) plate with geometric imperfection  $+w^0$  in  $z$  direction

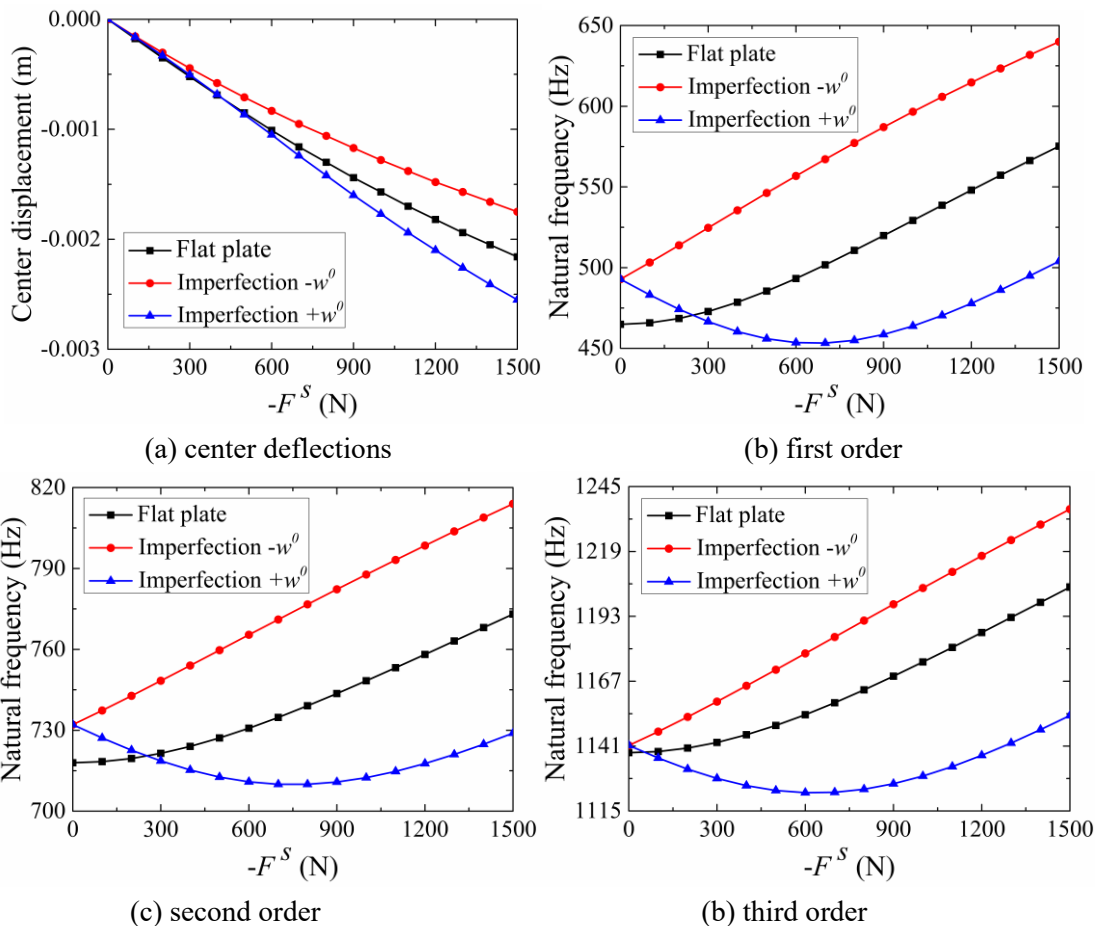


Figure 10 – Center deflection and first three natural frequencies of the three plates versus static load (theoretical results)

## 6. CONCLUSIONS

In this paper, we review the recent works of our group about the vibro-acoustic responses of plates under thermal/static load. Usually, both the inner stress state and the geometry shape may be changed under thermal/static load. Under thermal load, compress stress is the dominating effect in the pre-buckling state, resulting in the softening of the plate and the decreasing of the natural frequencies. While, the effect of buckling deformation is more significant in the post-buckling state and it results in stiffening of the plate and increasing of the natural frequencies. The deflection and natural frequencies increase with the temperature gradient increasing. Different from that under thermal load, both compress and tensile stress will appear under static load. It depends on the static load direction and the initial imperfection. The works reviewed in the paper lay the foundation for the vibro-acoustic analysis of hypersonic aircrafts under extreme thermal load and static load.

## ACKNOWLEDGEMENTS

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