

Design of Kronecker Product Beamformers with Cuboid Microphone Arrays

Xuehan WANG⁽¹⁾, Jacob BENESTY⁽²⁾, Gongping HUANG⁽¹⁾, Jingdong CHEN⁽¹⁾, and Israel COHEN⁽³⁾

⁽¹⁾Center of Intelligent Acoustics and Immersive Communication, Northwestern Polytechnical University, Xi'an, 710072, China.

E-mail: wangxuehan.123@mail.nwpu.edu.cn, gongpinghuang@gmail.com, jingdongchen@ieee.org

⁽²⁾INRS-EMT, University of Quebec, 800 de la Gauchetiere Ouest, Suite 6900, Montreal, QC H5A 1K6, Canada. E-mail:

benesty@emt.inrs.ca

⁽³⁾Department of Electrical Engineering, Technion-Israel Institute of Technology, Haifa 3200003, Israel. E-mail:

icohen@ee.technion.ac.il

Abstract

Microphone array beamforming has been widely used in a wide range of acoustic applications. To make it effective in suppressing noise, yet being able to preserve the fidelity and quality of broadband speech signals of interest, the beamformer needs to be designed with high spatial gain, consistent responses at different frequencies, and high robustness against array imperfections. A great deal of efforts have been devoted in the literature to achieving this goal, among which the Kronecker product beamforming method, developed recently for linear and rectangular microphone arrays, has demonstrated some interesting properties. In this work, we extend this approach to three-dimensional arrays. Focusing on cuboid shape of microphone arrays, we first decompose the global beamforming filter into a Kronecker product of two sub-beamforming filters. Algorithms are then developed to design each sub-beamforming filter so that the global beamformer has a high directivity factor and can be steered flexibly in the three-dimensional space.

Keywords: Microphone arrays, cuboid arrays, superdirective beamforming, Kronecker product.

1 INTRODUCTION

Microphone arrays can be used to solve many important acoustic problems in a wide range of applications. A critical component of a microphone array system is the so-called beamforming, which is basically a spatial filter applied to the array observation signals to estimate the signal of interest while suppressing noise and interferences [1–3]. While beamforming has been studied for many decades in the field of sensor arrays, extending those methods developed for narrowband applications to microphone arrays is not trivial as microphone array beamforming needs to suppress noise and meanwhile preserve the fidelity of the speech and acoustic signals of interest, which are broadband in nature and their frequencies span from 20 Hz to 20 kHz. Among many beamforming methods developed in the literature [5–10], the Kronecker product beamforming developed recently has demonstrated some promising results in terms of performance, robustness, and complexity [2, 11–20]. This method basically decomposes the global beamforming filter into the Kronecker product of two sub-filters. It has been demonstrated, based on linear and rectangular microphone arrays, that by optimizing these sub-filters, the global beamforming filter can be made to achieve high spatial gain and consistent responses over frequencies with high robustness [2, 21].

In this paper, we extend the Kronecker product beamforming from two-dimensional microphone arrays to a particular type of three-dimensional microphone arrays, i.e., uniform cuboid arrays. We first show how to decompose the global beamforming filter into the Kronecker product of two sub-beamforming filters: one corresponds to a linear array and the other corresponds to a rectangular grid array. We then derive algorithms on how to design those two sub-beamforming filters. It is shown that the developed Kronecker product beamformer can

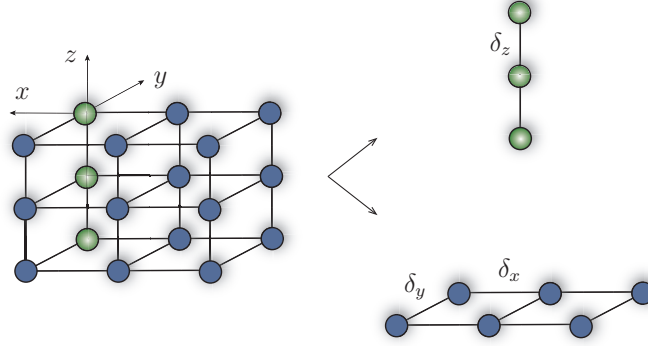


Figure 1. Illustration of the Kronecker product decomposition of a uniform cuboid array into a uniform rectangular subarray and a uniform linear subarray.

achieve a high directivity factor (DF) with a reasonable white noise gain (WNG), and its beampattern has good steering flexibility in the three-dimensional space.

2 SIGNAL MODEL, PROBLEM FORMULATION, AND PERFORMANCE MEASURES

Let us consider a uniform cuboid microphone array as illustrated in Fig. 1. We choose the sensor at the top-left inner corner as the origin of the Cartesian coordinate system. The array consists of M_z parallel uniform rectangular array along z (negative) axis and each rectangular array consists of M_x sensors along the x (negative) axis and M_y sensors along the y (negative) axis. The interelement spacing along each axis is denoted as δ_x , δ_y , and δ_z , respectively.

Assume that, in an anechoic acoustic environment, a far-field source signal (plane-wave) propagates from the direction $\{\theta, \phi\}$ at the speed of sound, i.e., $c = 340$ m/s, and impinges on the cuboid array, where the elevation angle θ ($0 \leq \theta \leq \pi$) is measured downward from the z axis and the azimuth angle ϕ ($0 \leq \phi \leq 2\pi$) is measured counterclockwise from the x axis. The steering matrix of size $M_x \times M_y$ corresponding to the rectangular array can be written as [2, 7]

$$\mathbf{D}_{xy}(\omega, \theta, \phi) = \mathbf{d}_x(\omega, \theta, \phi) \mathbf{d}_y^T(\omega, \theta, \phi), \quad (1)$$

where

$$\mathbf{d}_x(\omega, \theta, \phi) = \begin{bmatrix} 1 & e^{-j\frac{\omega\delta_x}{c}\sin\theta\cos\phi} & \dots & e^{-j\frac{(M_x-1)\omega\delta_x}{c}\sin\theta\cos\phi} \end{bmatrix}^T, \quad (2)$$

$$\mathbf{d}_y(\omega, \theta, \phi) = \begin{bmatrix} 1 & e^{-j\frac{\omega\delta_y}{c}\sin\theta\sin\phi} & \dots & e^{-j\frac{(M_y-1)\omega\delta_y}{c}\sin\theta\sin\phi} \end{bmatrix}^T \quad (3)$$

are the steering vectors associated with the linear subarrays along the x and y axes, respectively, j is the imaginary unit with $j^2 = -1$, $\omega = 2\pi f$ is the angular frequency, and $f > 0$ is the temporal frequency.

Applying the vectorization operation to (1), we can deduce that

$$\mathbf{d}_{xy}(\omega, \theta, \phi) = \text{vec}[\mathbf{D}_{xy}(\omega, \theta, \phi)] = \mathbf{d}_y(\omega, \theta, \phi) \otimes \mathbf{d}_x(\omega, \theta, \phi), \quad (4)$$

where $\otimes$ denotes the Kronecker product operation. Then, the steering matrix of size $(M_x M_y) \times M_z$ correspond-

ing to the cuboid array can be written as

$$\begin{aligned} \mathbf{D}(\omega, \theta, \phi) &= \begin{bmatrix} \mathbf{d}_{xy}(\omega, \theta, \phi) & e^{-j\frac{\omega\delta_z}{c}\cos\theta}\mathbf{d}_{xy}(\omega, \theta, \phi) & \dots & e^{-j\frac{(M_z-1)\omega\delta_z}{c}\cos\theta}\mathbf{d}_{xy}(\omega, \theta, \phi) \end{bmatrix} \\ &= \mathbf{d}_{xy}(\omega, \theta, \phi) \mathbf{d}_z^T(\omega, \theta), \end{aligned} \quad (5)$$

where

$$\mathbf{d}_z(\omega, \theta) = \begin{bmatrix} 1 & e^{-j\frac{\omega\delta_z}{c}\cos\theta} & \dots & e^{-j\frac{(M_z-1)\omega\delta_z}{c}\cos\theta} \end{bmatrix}^T \quad (6)$$

is the steering vector of length M_z associated with the linear subarray along the z axis [Note that $\mathbf{d}_z(\omega, \theta)$ is independent of the azimuth angle ϕ]. Similarly, applying the vectorization operation to (5) yields

$$\mathbf{d}(\omega, \theta, \phi) = \text{vec}[\mathbf{D}(\omega, \theta, \phi)] = \mathbf{d}_z(\omega, \theta) \otimes \mathbf{d}_{xy}(\omega, \theta, \phi). \quad (7)$$

The conventional beamforming is performed by applying a complex-valued filter, $\mathbf{h}(\omega)$ of length $M = M_x M_y M_z$, to all the microphone observations to get an estimate of the desired signal [7]. Assume that the desired source is incident from the direction $\{\theta_d, \phi_d\}$. The distortionless constraint in the desired look direction is required, i.e.,

$$\mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta_d, \phi_d) = 1, \quad (8)$$

where the superscript H is the conjugate-transpose operator.

Three important performance measures are commonly used to evaluate the performance of beamformers: the beampattern, the WNG, and the DF. The beampattern describes the sensitivity of the beamformer to a plane wave; it is defined as [3]

$$\mathcal{B}[\mathbf{h}(\omega), \theta, \phi] = \mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta, \phi). \quad (9)$$

The WNG evaluates the sensitivity of the array to some of its imperfections; it is defined as [4]

$$\mathcal{W}[\mathbf{h}(\omega)] = \frac{|\mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta_d, \phi_d)|^2}{\mathbf{h}^H(\omega) \mathbf{h}(\omega)}. \quad (10)$$

The DF quantifies how the microphone array performs in the presence of spatially diffuse noise; it is defined as [4]

$$\begin{aligned} \mathcal{D}[\mathbf{h}(\omega)] &= \frac{|\mathcal{B}[\mathbf{h}(\omega), \theta_d, \phi_d]|^2}{\frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} |\mathcal{B}[\mathbf{h}(\omega), \theta, \phi]|^2 \sin\theta d\phi d\theta} \\ &= \frac{|\mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta_d, \phi_d)|^2}{\mathbf{h}^H(\omega) \mathbf{\Gamma}(\omega) \mathbf{h}(\omega)}, \end{aligned} \quad (11)$$

where

$$\mathbf{\Gamma}(\omega) = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \mathbf{d}(\omega, \theta, \phi) \mathbf{d}^H(\omega, \theta, \phi) \sin\theta d\phi d\theta. \quad (12)$$

3 KRONECKER PRODUCT BEAMFORMERS

As seen from (7), the steering vector of the cuboid array can be decomposed into the Kronecker product of the counterparts of two subarrays. Consequently, we propose to decompose the global beamforming filter as

$$\mathbf{h}(\omega) = \mathbf{h}_z(\omega) \otimes \mathbf{h}_{xy}(\omega), \quad (13)$$

where $\mathbf{h}_z(\omega)$ is the sub-beamforming filter of length M_z corresponding to the linear subarray along the z axis and $\mathbf{h}_{xy}(\omega)$ is the sub-beamforming filter of length $M_x M_y$ corresponding to the rectangular subarray along the $x - y$ plane.

Now, the beampattern of the Kronecker product beamformer can be written as

$$\begin{aligned}\mathcal{B}[\mathbf{h}(\omega), \theta, \phi] &= [\mathbf{h}_z(\omega) \otimes \mathbf{h}_{xy}(\omega)]^H [\mathbf{d}_z(\omega, \theta) \otimes \mathbf{d}_{xy}(\omega, \theta, \phi)] \\ &= [\mathbf{h}_z^H(\omega) \mathbf{d}_z(\omega, \theta)] [\mathbf{h}_{xy}^H(\omega) \mathbf{d}_{xy}(\omega, \theta, \phi)] \\ &= \mathcal{B}[\mathbf{h}_z(\omega), \theta] \times \mathcal{B}[\mathbf{h}_{xy}(\omega), \theta, \phi],\end{aligned}\quad (14)$$

where $\mathcal{B}[\mathbf{h}_z(\omega), \theta]$ and $\mathcal{B}[\mathbf{h}_{xy}(\omega), \theta, \phi]$ are the beampatterns corresponding to the linear and rectangular subarrays, respectively. It can be easily observed that the global beampattern of the cuboid array is the product of the individual beampatterns of the two sub-beamforming filters.

The distortionless constraint becomes

$$[\mathbf{h}_z^H(\omega) \mathbf{d}_z(\omega, \theta_d)] [\mathbf{h}_{xy}^H(\omega) \mathbf{d}_{xy}(\omega, \theta_d, \phi_d)] = 1. \quad (15)$$

Let us set the distortionless constraint for each sub-beamforming filter as

$$\mathbf{h}_z^H(\omega) \mathbf{d}_z(\omega, \theta_d) = \mathbf{h}_{xy}^H(\omega) \mathbf{d}_{xy}(\omega, \theta_d, \phi_d) = 1, \quad (16)$$

so that the global distortionless constraint in (15) is satisfied.

The WNG of the Kronecker product beamformer is deduced as

$$\begin{aligned}\mathcal{W}[\mathbf{h}(\omega)] &= \frac{|[\mathbf{h}_z^H(\omega) \mathbf{d}_z(\omega, \theta_d)] [\mathbf{h}_{xy}^H(\omega) \mathbf{d}_{xy}(\omega, \theta_d, \phi_d)]|^2}{[\mathbf{h}_z(\omega) \otimes \mathbf{h}_{xy}(\omega)]^H [\mathbf{h}_z(\omega) \otimes \mathbf{h}_{xy}(\omega)]} \\ &= \frac{|\mathbf{h}_z^H(\omega) \mathbf{d}_z(\omega, \theta_d)|^2}{\mathbf{h}_z^H(\omega) \mathbf{h}_z(\omega)} \times \frac{|\mathbf{h}_{xy}^H(\omega) \mathbf{d}_{xy}(\omega, \theta_d, \phi_d)|^2}{\mathbf{h}_{xy}^H(\omega) \mathbf{h}_{xy}(\omega)} \\ &= \mathcal{W}[\mathbf{h}_z(\omega)] \times \mathcal{W}[\mathbf{h}_{xy}(\omega)],\end{aligned}\quad (17)$$

where $\mathcal{W}[\mathbf{h}_z(\omega)]$ and $\mathcal{W}[\mathbf{h}_{xy}(\omega)]$ are the WNGs corresponding to the linear and rectangular subarrays, respectively. It is clear that the WNG of the cuboid array is the product of the WNGs of the two subarrays.

Now, we can design the two sub-beamforming filters individually, and thereby obtaining the global beamformer [22–24].

Apparently, there are different ways to design the sub-beamforming filters. For example, we may design a superdirective beamformer for the rectangular subarray, which is obtained by maximizing the DF while considering the distortionless constraint [2, 25–27]. The result is as follows:

$$\mathbf{h}_{xy, \text{SD}}(\omega) = \frac{\mathbf{\Gamma}_{xy}^{-1}(\omega) \mathbf{d}_{xy}(\omega, \theta_d, \phi_d)}{\mathbf{d}_{xy}^H(\omega, \theta_d, \phi_d) \mathbf{\Gamma}_{xy}^{-1}(\omega) \mathbf{d}_{xy}(\omega, \theta_d, \phi_d)}, \quad (18)$$

where

$$\mathbf{\Gamma}_{xy}(\omega) = \begin{bmatrix} \mathbf{\Gamma}_1(\omega) & \mathbf{\Gamma}_2(\omega) & \cdots & \mathbf{\Gamma}_{M_y}(\omega) \\ \mathbf{\Gamma}_2(\omega) & \mathbf{\Gamma}_1(\omega) & \cdots & \mathbf{\Gamma}_{M_y-1}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Gamma}_{M_y}(\omega) & \mathbf{\Gamma}_{M_y-1}(\omega) & \cdots & \mathbf{\Gamma}_1(\omega) \end{bmatrix} \quad (19)$$

is a symmetric block Toeplitz matrix of size $(M_x M_y) \times (M_x M_y)$ and $\mathbf{\Gamma}_{m_y}(\omega)$, $m_y = 1, 2, \dots, M_y$ are symmetric Toeplitz matrices of size $M_x \times M_x$ whose (i, j) th element is defined as

$$[\mathbf{\Gamma}_{m_y}(\omega)]_{ij} = \text{sinc} \left[\frac{\omega \sqrt{(i-j)^2 \delta_x^2 + (m_y - 1)^2 \delta_y^2}}{c} \right], \quad (20)$$

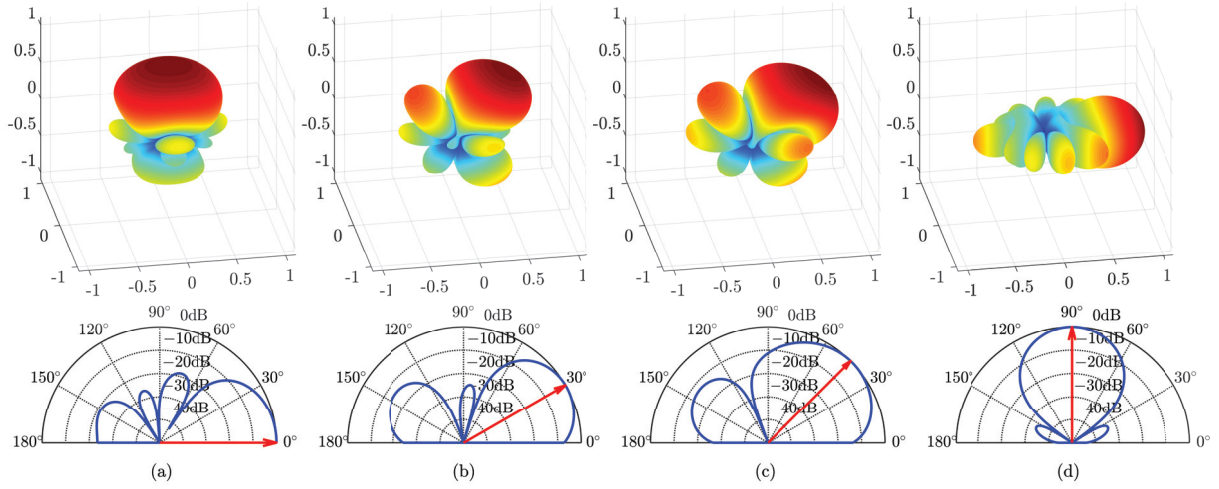


Figure 2. Beam patterns of the Kronecker product beamformer with a cuboid array for different values of θ_d : (a) $\theta_d = 0^\circ$, (b) $\theta_d = 30^\circ$, (c) $\theta_d = 45^\circ$, and (d) $\theta_d = 90^\circ$. The upper subplots show the three-dimensional beam patterns and the lower subplots show the corresponding two-dimensional beam patterns for $\phi = 0^\circ$. Conditions of simulations: $M_x = M_y = 4$, $M_z = 8$, $\delta_x = 1$ cm, $\delta_y = \delta_z = 2$ cm, $\phi_d = 0^\circ$, and $f = 2$ kHz.

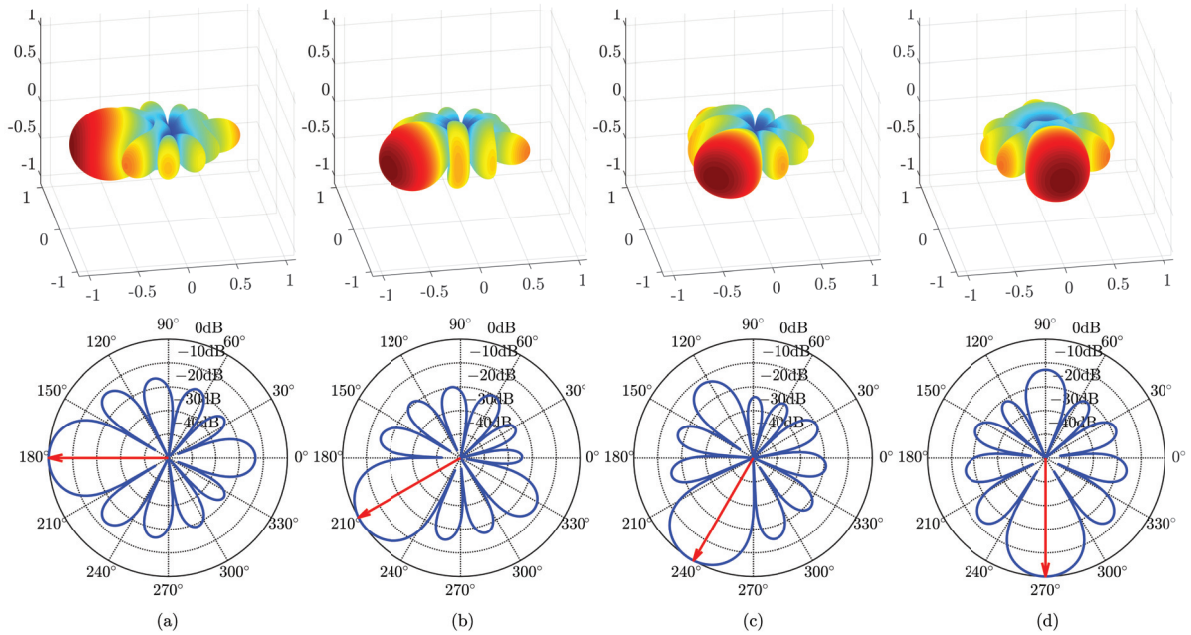


Figure 3. Beam patterns of the Kronecker product beamformer with a cuboid array for different values of ϕ_d : (a) $\phi_d = 180^\circ$, (b) $\phi_d = 210^\circ$, (c) $\phi_d = 240^\circ$, and (d) $\phi_d = 270^\circ$. The upper subplots show the three-dimensional beam patterns and the lower subplots show the two-dimensional beam patterns for $\theta = 90^\circ$. Conditions of simulations: $M_x = M_y = 4$, $M_z = 8$, $\delta_x = 1$ cm, $\delta_y = \delta_z = 2$ cm, $\theta_d = 90^\circ$, and $f = 2$ kHz.

with $\text{sinc}(x) = \sin x/x$, and the subscript SD stands for “superdirective.”

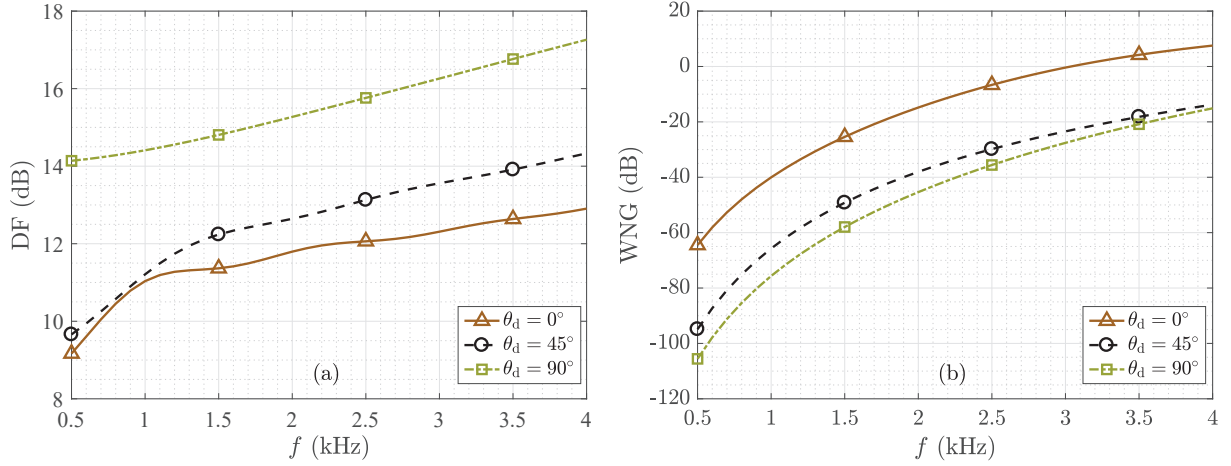


Figure 4. Performance of the Kronecker product beamformer with a cuboid array for different values of θ_d : (a) DF and (b) WNG. Conditions of simulations: $M_x = M_y = 4$, $M_z = 8$, $\delta_x = 1$ cm, $\delta_y = \delta_z = 2$ cm, and $\phi_d = 0^\circ$.

Superdirective beamformers generally have low WNG at low frequencies, leading to the so-called white noise amplification problem. To deal with this problem, let us consider the beamformer that has the largest WNG for the linear subarray, which corresponds to the delay-and-sum beamformer:

$$\mathbf{h}_{z,DS}(\omega) = \frac{1}{M_z} \mathbf{d}_z(\omega, \theta_d). \quad (21)$$

Thus, the global beamformer is obtained as

$$\mathbf{h}(\omega) = \mathbf{h}_{z,DS}(\omega) \otimes \mathbf{h}_{xy,SD}(\omega). \quad (22)$$

This beamformer can achieve a high value of DF while maintaining a reasonable level of WNG for robustness. Meanwhile, it has advantages in computational efficiency since the dimension of the matrix to invert is $M_x M_y$ instead of $M_x M_y M_z$ [2]. Moreover, the designed beampattern can be steered to different desired directions in the three-dimensional space as will be shown in the following section.

4 SIMULATIONS

In this section, we examine the performance of the Kronecker product beamformer through simulations. We consider a uniform cuboid array with $M_x = 4$, $M_y = 4$, $M_z = 8$, $\delta_x = 1$ cm, $\delta_y = 2$ cm, and $\delta_z = 2$ cm.

Figure 2 plots the beampatterns of the proposed beamformer for $\theta_d \in \{0^\circ, 30^\circ, 45^\circ, 90^\circ\}$, $\phi_d = 0^\circ$, and $f = 2$ kHz, where the upper subplots show the three-dimensional beampatterns and the lower subplots show the two-dimensional beampatterns for $\phi = 0^\circ$. Figure 3 plots the beampatterns of the proposed beamformer for $\phi_d \in \{180^\circ, 210^\circ, 240^\circ, 270^\circ\}$, $\theta_d = 90^\circ$, and $f = 2$ kHz, where the lower subplots show the two-dimensional beampatterns for $\theta = 90^\circ$. Clearly, the Kronecker product beamformer can form a main beam to different directions, which implies that the proposed beamforming method has good steering flexibility in the three-dimensional space.

Figure 4 plots the WNG and the DF of the Kronecker product beamformer as a function of frequency, f , for $\theta_d \in \{0^\circ, 45^\circ, 90^\circ\}$, and $\phi_d = 0^\circ$. It is clearly seen that the proposed beamformer can achieve a high DF while maintaining a reasonable level of WNG. It is also observed that the DF increases while the WNG decreases as θ_d increases.

5 CONCLUSIONS

We have proposed a Kronecker product beamforming method for cuboid microphone arrays. This method first decomposes the uniform cuboid array into linear and rectangular subarrays so that the steering vector of the cuboid array is written as a Kronecker product of the steering vectors of the two subarrays. The global beamforming filter is, in a similar manner, also decomposed as a Kronecker product of two sub-beamforming filters. As such, we can design the two sub-beamforming filters individually. In this way, the design of the global beamforming filter becomes very flexible since each sub-beamforming filter can be designed using different methods such as the superdirective beamformer, delay-and-sum beamformer, differential beamformer, and adaptive beamformer. Then we demonstrated a design of a global beamforming filter by combining superdirective and delay-and-sum beamformers. Simulation results verified that the designed beamformer can achieve high DF and a reasonable level of WNG while facilitating good steering flexibility in the three-dimensional space.

ACKNOWLEDGEMENTS

This work was supported in part by the National Science Foundation of China (NSFC) and the Israel Science Foundation (ISF) joint research program (grant No. 61761146001 and 2514/17), the NSFC key program (grant No. 61831019), the NSFC Distinguished Young Scientists Fund (grant No. 61425005), and the ISF Grant 576/16.

REFERENCES

- [1] J. Benesty, J. Chen, and Y. Huang, *Microphone Array Signal Processing*. Berlin, Germany: Springer-Verlag, 2008.
- [2] J. Benesty, I. Cohen, and J. Chen, *Array Processing: Kronecker Product Beamforming*. Berlin, Germany: Springer-Verlag, 2019.
- [3] J. Benesty, I. Cohen, and J. Chen, *Fundamentals of Signal Enhancement and Array Signal Processing*. Wiley-IEEE Press, Singapore, 2018.
- [4] J. Benesty and J. Chen, *Study and Design of Differential Microphone Arrays*. Berlin, Germany: Springer-Verlag, 2012.
- [5] G. Huang, J. Chen, and J. Benesty, "Insights into frequency-invariant beamforming with concentric circular microphone arrays," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 26, pp. 2305–2318, Dec. 2018.
- [6] S. Haykin and K. R. Liu, *Handbook on Array Processing and Sensor Networks*, vol. 63. John Wiley & Sons, 2010.
- [7] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Optimum Array Processing*. John Wiley & Sons, 2004.
- [8] D. H. Johnson and D. E. Dudgeon, *Array Signal Processing: Concepts and Techniques*. Upper Saddle River, NJ: Prentice-Hall, 1993.
- [9] G. Huang, J. Chen, and J. Benesty, "A flexible high directivity beamformer with spherical microphone arrays," *J. Acoust. Soc. Am.*, vol. 143, pp. 3024–3035, May 2018.
- [10] S. Haykin, *Array Signal Processing*. Upper Saddle River, NJ: Prentice-Hall, 1985.
- [11] L. N. Ribeiro, A. L. de Almeida, and J. C. Mota, "Tensor beamforming for multilinear translation invariant arrays," in *Proc. IEEE ICASSP*, 2016, pp. 2966–2970.

- [12] Y. I. Abramovich, G. J. Frazer, and B. A. Johnson, "Iterative adaptive Kronecker MIMO radar beamformer: Description and convergence analysis," *IEEE Trans. Signal Process.*, vol. 58, pp. 3681–3691, Jul. 2010.
- [13] I. Cohen, J. Benesty, and J. Chen, "Differential Kronecker product beamforming," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 27, pp. 892–902, May 2019.
- [14] F. P. Ribeiro and V. H. Nascimento, "Fast transforms for acoustic imaging-part I: Theory," vol. 20, pp. 2229–2240, Aug. 2011.
- [15] B. Masiero and V. H. Nascimento, "Revisiting the Kronecker array transform," *IEEE Signal Process. Lett.*, vol. 24, pp. 525–529, May 2017.
- [16] C. Paleologu, J. Benesty, and S. Ciochina, "Linear system identification based on a Kronecker product decomposition," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 26, pp. 1793–1808, Oct. 2018.
- [17] L. N. Ribeiro, A. L. de Almeida, and J. C. Mota, "Identification of separable systems using trilinear filtering," in *Proc. IEEE CAMSAP*, 2015, pp. 189–192.
- [18] L. N. Ribeiro, A. L. de Almeida, and J. C. M. Mota, "Separable linearly constrained minimum variance beamformers," *Signal Process.*, vol. 158, pp. 15–25, May 2019.
- [19] V. H. Nascimento, M. C. Silva, and B. S. Masiero, "Acoustic image estimation using fast transforms," in *Proc. IEEE SAM*, 2016, pp. 1–5.
- [20] M. Bousse, O. Debals, and L. De Lathauwer, "A tensor-based method for large-scale blind source separation using segmentation," *IEEE Trans. Signal Process.*, vol. 65, pp. 346–358, Jan. 2017.
- [21] W. Yang, G. Huang, J. Benesty, I. Cohen, and J. Chen, "On the design of flexible Kronecker product beamformers with linear microphone arrays," in *Proc. IEEE ICASSP*, 2019.
- [22] G. Huang, J. Benesty, and J. Chen, "Superdirective beamforming based on the Krylov matrix," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 24, pp. 2531–2543, Dec. 2016.
- [23] G. Huang, J. Benesty, and J. Chen, "On the design of frequency-invariant beampatterns with uniform circular microphone arrays," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 25, pp. 1140–1153, May 2017.
- [24] G. Huang, J. Chen, and J. Benesty, "On the design of differential beamformers with arbitrary planar microphone array," *J. Acoust. Soc. Am.*, vol. 144, pp. 3024–3035, Jul. 2018.
- [25] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57, pp. 1408–1418, Aug. 1969.
- [26] H. Cox, R. M. Zeskind, and T. Kooij, "Practical supergain," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34, pp. 393–398, Jun. 1986.
- [27] H. Cox, R. Zeskind, and M. Owen, "Robust adaptive beamforming," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 35, pp. 1365–1376, Oct. 1987.