Computational framework to evaluate pressure distribution on rail track using Hertzian approach

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Summary

Wearing out of the rail profile is one of the major reasons for train derailment. Wear of profile and service life of rail largely depend on the contact conditions between rail and wheel. An accurate prediction of contact pressure distribution on rail thus plays an important role in forecasting rail life. In this paper, FE analysis is carried out to estimate contact pressure distribution on the top of rail surface using Abaqus®. Hertzian pressure distribution is also determined analytically for variation of rail head profile. The simulation results from the FEA are compared with Hertzian pressure distribution for two contacting elastic surfaces and considerable agreement is obtained within range of the compared value.

Keyworts: Rail/wheel contact, Hertzain contact pressure, Finite element modeling, Simulation

1 Introduction

Derailment is the major concern for the Indian Railways. It is responsible for fatal accidents and huge monetary losses from time to time [1-4]. Wear and rolling contact fatigue are the primary reasons for cracks on rail surface which eventually lead to failure of rails and derailment [5,6]. Subsequently, an accurate wear analysis of rail surface requires pre-processing of contact stresses due to rail-wheel interaction [7]. Service life of a rail is severely influenced by the contact conditions, which in itself is largely dependant upon the geometries of the contacting surfaces. Figure.1 clearly shows the distinction between a new profiel and a worn out profile of an UIC60 rail. Alteration in contact profile may have a significant impact on contact pressure and contact patch size. In Indian Railways, the problem aggravates to a greater extent because the bodies in contact are acting under higher axle load and braking frequency [8] than the rest of the world.

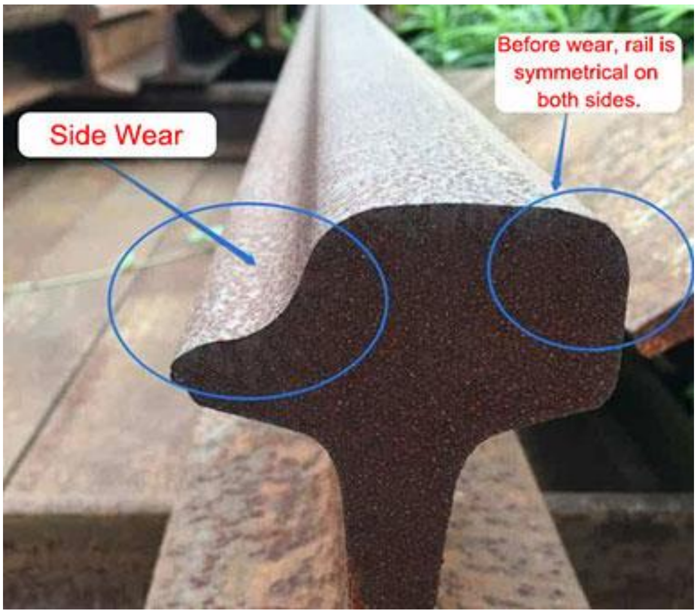

Figure 1. Profile of an UIC 60 rail before and after the wear

Problem of contact between two surfaces can be customarily classified into normal and tangential contact. Normal contact was first aimed by Hertz [9] and assumed the contacting surfaces to be elastic. Furthermore, Hertzian assumption of contacting surfaces to be elastic half spaces isolates the normal contact from the tangential contact problem. Subsequently, various researchers [10, 11] have carried out their research towards modifying the normal pressure distribution and developed non-Hertzian, semi-Hertzian or multi-Hertzian approaches. Monfared et. al [12] have compared Hertzian, non-Hertzian and Finite Element approach to assess contact characteristics and concluded that contact pressure distribution from Hertzian method confirms with Finite Element model for linear elastic material behavior. Literature further suggests that, although new methods have been proposed, Hertzian approach is one of the simplest and sufficiently accurate to determine contact pressure on rail.

Paper is organized into three sections beginning with a discussion on the basics of Hertzian contact model in the first section, continuing to finite element methodology used and ending with results and comparative discussion of the analytical and computational approach. The main focus of this paper is to develop a modeling framework for evaluation of contact characteristics on the rail owing to the variation in rail profile. A computational analysis alongside an analaytical estimation using Hertzian approach are performed by using Abaqus® to determine contact characteristics between rail and wheel. Standard

UIC60 rail profile and S920 wheel profile used predominantly in Indian Railways is considered for the purpose of analysis.

2 Hertzian Contact Model

Hertzian theory forms the basis of contact between two steady state bodies. Hertzian contact model was developed considering two cylindrical elastic bodies of revolution under the following assumptions [9]-:

1. Bodies in contact have perfectly smooth surfaces.
2. Size of the contact area is considerably smaller than the radius of curvature of bodies in contact.
3. Contact between the surfaces is frictionless so that only normal force, and no tangential force, is responsible for the contact pressure.
4. Material properties are linearly elastic, isotropic and homogenous in nature.

When bodies in contact are pressed together with a vertical force F, it results in the initial point contact transformed to surface contact. Contact area will be elliptical in shape having semi-major and semi-minor axis as *a* and *b* respectively. Hertzian theory suggests that normal pressure, Z in the contact area is given by

$$Z\,(x,y) = \frac{3F}{2\pi ab}\sqrt{1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2} \tag{1}$$

Here, *a* and *b* can be obtained from

$$a = m\sqrt[3]{\left[\frac{3\pi F(K_1+K_2)}{4(K_3+K_4)}\right]} \tag{2}$$

$$b = n\sqrt[3]{\left[\frac{3\pi F(K_1+K_2)}{4(K_3+K_4)}\right]} \tag{3}$$

m and *n* are Hertzian coefficients as function of auxiliary angle expressed in terms of elliptic integrals, F is the vertical force on the wheel due to mass of the loaded vehicle *a* and *b* are length of semi-major and semi-minor axis of contact ellipse respectively. K_1, K_2, K_3 and K_4 are Hertzian constants such that

$$K_1 = \frac{1-\vartheta_{wheel}^2}{\pi E_{wheel}} \tag{4}$$

$$K_2 = \frac{1-\vartheta_{rail}^2}{\pi E_{rail}} \tag{5}$$

$$K_3 = \frac{1}{2}\left[\frac{1}{R_1}+\frac{1}{R_1'}+\frac{1}{R_2}+\frac{1}{R_2'}\right] \tag{6}$$

$$K_4 = \frac{1}{2}\left[\left(\frac{1}{R_1}+\frac{1}{R_1'}\right)^2+\left(\frac{1}{R_2}+\frac{1}{R_2'}\right)^2+2\left(\frac{1}{R_1}-\frac{1}{R_1'}\right)\left(\frac{1}{R_2}-\frac{1}{R_2'}\right)\cos 2\varphi\right] \quad (7)$$

where R_1, R_2 are principal rolling radius of the wheel and rail respectievely and R_1`, $R_2$` are principal transverse radius of curvature of wheel and rail profile respectievely. φ is the angle between the normal plames that contain the curvatures $\frac{1}{R_1}$ and $\frac{1}{R_2}$ [13].

3 Methodology

For analysis of the problem, a benchmark model is first prepared in *Abaqus-CAE* to understand the complexities of contact modeling. CAD models of standard wheel and rail profile templates employed by Indian railways are used for FE analysis. Mesh density in the contact region is found to impact stress distribution on the rail significantly. Therefore, a mesh size of 3 mm in the contact region is used to standardize the results as shown in Figure 2. FE analysis is conducted using explicit commercial code Abaqus/Explicit®. Rail and wheel geometry was meshed as shown in figure 1(a) using C3D8R 8-node 48,897 linear brick reduced integration elements with hourglass control from *Explicit* element library. Inertial mass of 7000 kg was applied on the wheel to account for the weight of empty vehicle on a single wheel. The wheel is simulated to rotate corresponding to a translational velocity of 26 kmph for 2m.

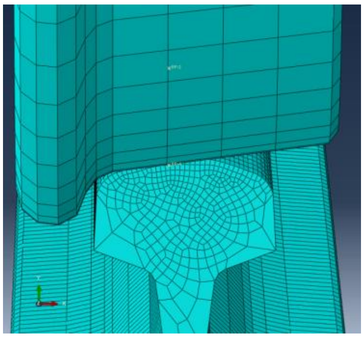

Figure 2: Rail/wheel contact mesh

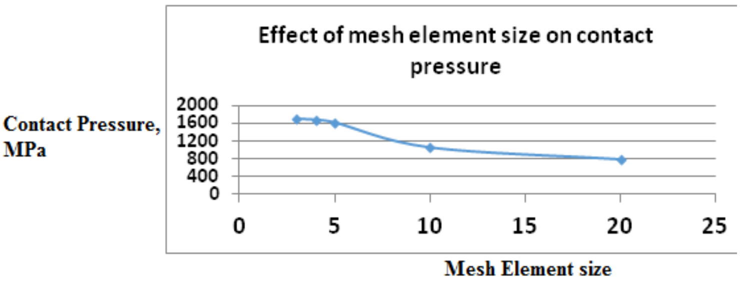

Figure 3: Mesh convergence study for maximum contact pressure on rail of profile radius 300 mm

Surface to Surface contact discretization formulation available in Abaqus *Explicit* reduces sensitivity with respect to choice of *master* and *slave* surfaces. It enforces contact over the *Slave* surface unlike *Node to surface* technique where contact is maintained at discrete *Slave* points. *Finite sliding* formulation though computationally expansive, allows larger sliding and rotation between the contacting surfaces than the *small sliding* formulation. *Penalty contact* constraint formulation allows a balanced *master-slave* approach, while simultaneously redcing the probability of penetration. *Normal* contact is depicted using default *Hard* contact behaviour. Mesh convergence study as shown in Figure 3 is performed to minimize the effect of mesh size on the contact pressure distribution on the rail surface. The results are evaluated in the region of 0.985-1.0 m on the rail surface. Hertzian pressure distribution is determined analytically for radius of curvature of rail in the plane of cross section from 220 mm (worn profile) to 300 mm (new profile). FE model for all the configurations are processed successfully to obtain the results.

4 Results and Discussions

A rail profile typically consists of three sections namely- Rail Gauge, Rail Shoulder and Rail Crown as shown in Figure 4. Currently, axles in Indian Railway are designed to carry a maximum load of 20.3 tonnes. Therefore, a vertical load of 70 kN was applied on the wheel having a radius of 460 mm. Principal radius of curvature of rail is taken to be infinity.

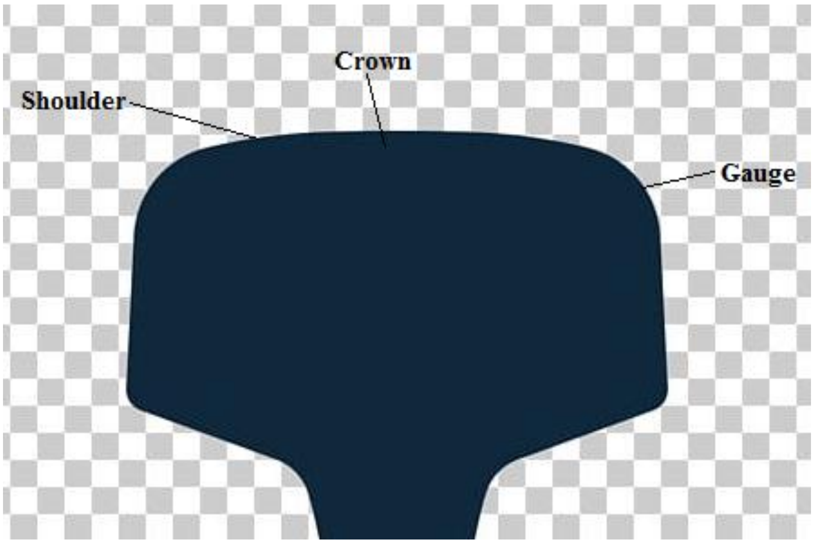

Figure 4: Rail profile is classified into- Rail shoulder, rail crown and rail gauge

Contact pressure on the rail due to the vertical load was evaluated both analytically and using FEA for varying rail transverse profile radius. FEA results are found to be slightly higher than the Hertzian pressure distribution as shown in Figure 5. The reason for percentage error in the solution can be attributed to the fact that the geometrical parameters (shape and size) of contact patch defined by Hertz does not changes during the course of the motion between the contacting surfaces, which makes Hertzian predictions far from reality.

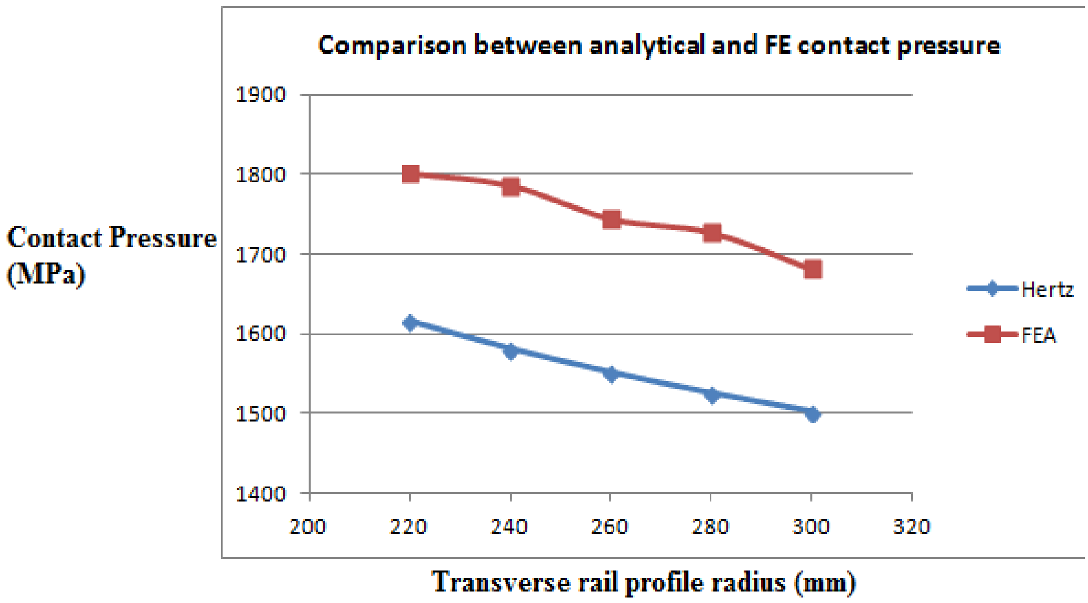

Figure 5: Comparison between Hertzian and FEA contact pressure on rail due to variation in rail profile radius

It can also be seen in Table 1 that contact pressure increases as the rail profile starts to wear out. Similar increase in contact pressure is expected rail is loaded by a worn wheel. Numerical approach available [14] suggests that FEA results are more accurate than the traditional contact approaches. Figure 6 shows the contact patch on a rail profile of 300 mm radius in the rail crown region under 70 kN vertical load.

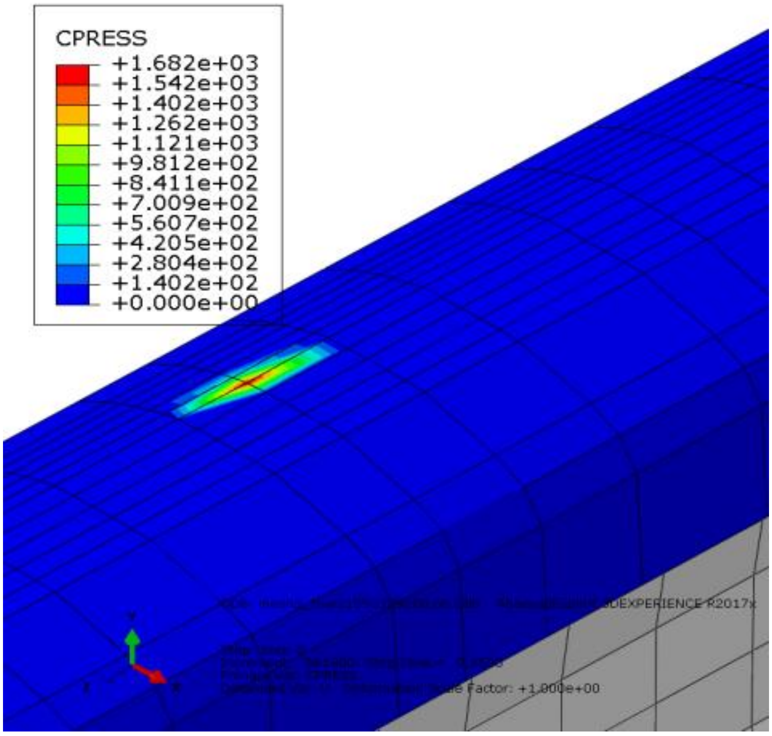

Figure 6: Maximum contact pressure on rail of profile radius 300 mm is 1682 MPa

Table 5: Comparison of effect of rail profile radius on contact pressure using Hertzian distribution and FEA

Principle rail profile radius (mm)	300	280	260	240	220
Analytical Contact Pressure (MPa)	1502	1525	1552	1582	1616
FEA contact pressure (MPa)	1682	1728	1745	1787	1802

Gaussian Integration methodology is implemented for calculating the contact patch pressure and area but while employing second order element, oscillations in contact pressure result is observed. Therefore, Simpson’s rule is put into application to obtain more realistic output. Simpson’s rule uses fixed intervals and has therefore suitable accuracy to evaluate stiffness matrix of high order elements [15]. The oscillating behavior is called chattering effect. Contact analysis is highly sensitive to mesh size as can be seen from figure 7 where contact pressure changes significantly with an increasing number of mesh elements.

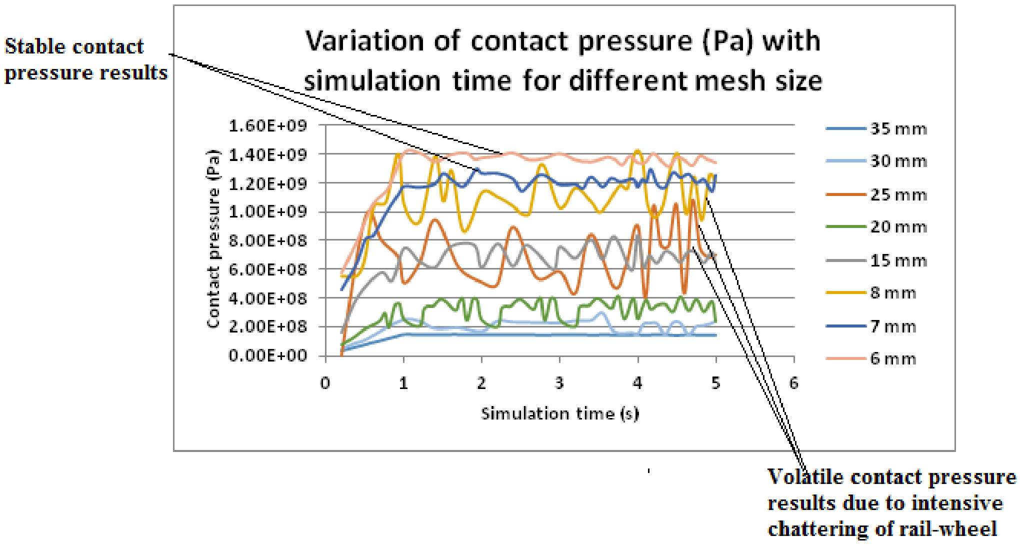

Figure 7: Contact pressure (Pa) distribution over the rail for different mesh element sizes

Chattering effect is clearly visible in coarse mesh contact simulation. A coarse mesh may produce huge variation in contact pressure over the simulation time. As the mesh element size reduces, variation of contact pressure becomes less volatile and achieves a constant output over the simulation duration. Variation of contact pressure with simulation time at a mesh size of 6 mm is also shown separately in Figure 8. The volatility in contact pressure decreases as finer mesh sizes are computed. Mesh element size is further not reduced due to the requirement of an intensive computational effort. Effect of mesh size on the maximum contact pressure can also be seen from Figure 7. Furthermore, study indicates that contact pressure on a new rail profile is 1682 MPa which leap up to 1802 MPa as the rail wears out.

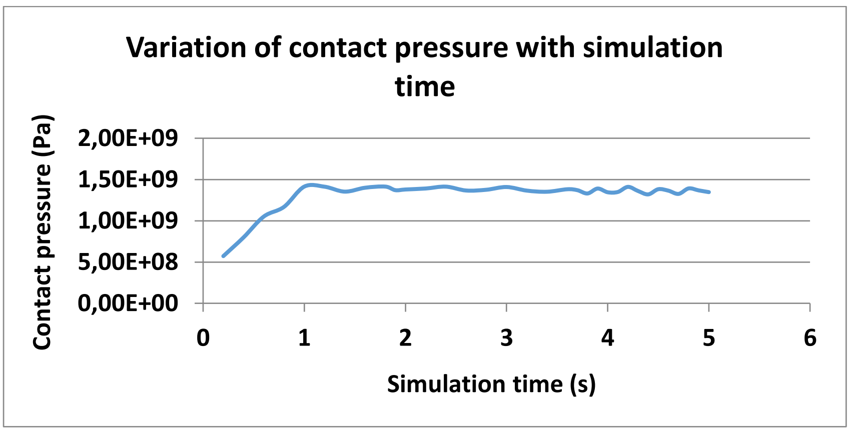

Figure 8: Contact pressure (Pa) distribution over the rail for a mesh size of 6 mm

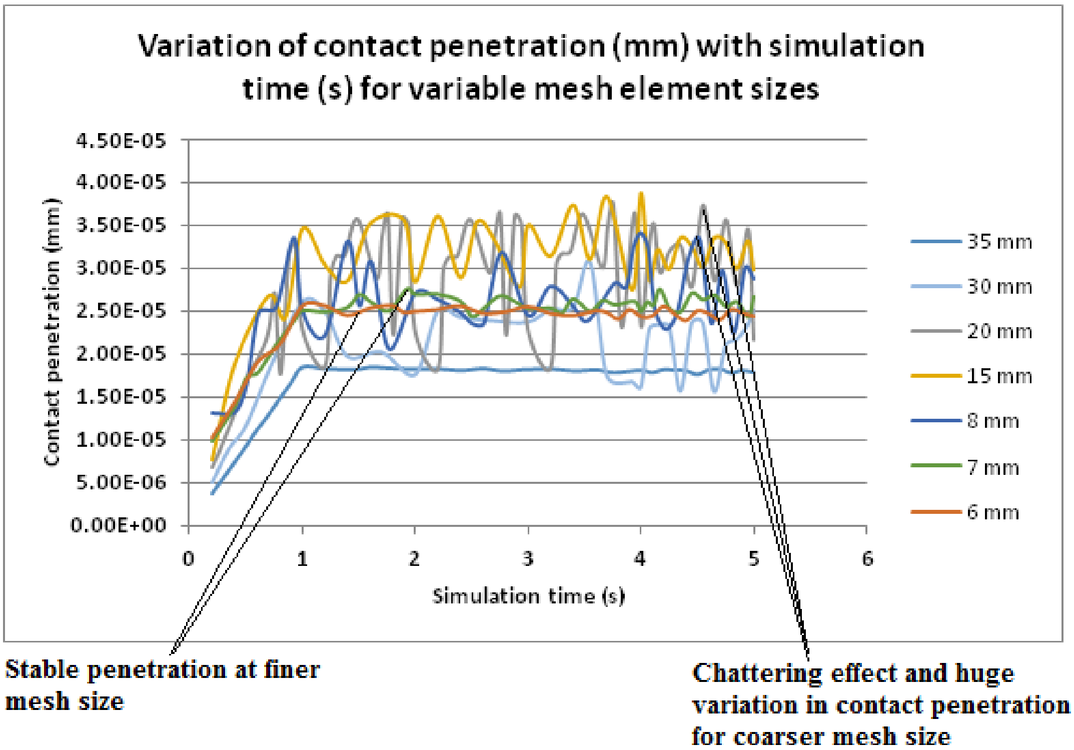

Figure 9: Contact penetration (mm) distribution over the rail for different mesh element sizes

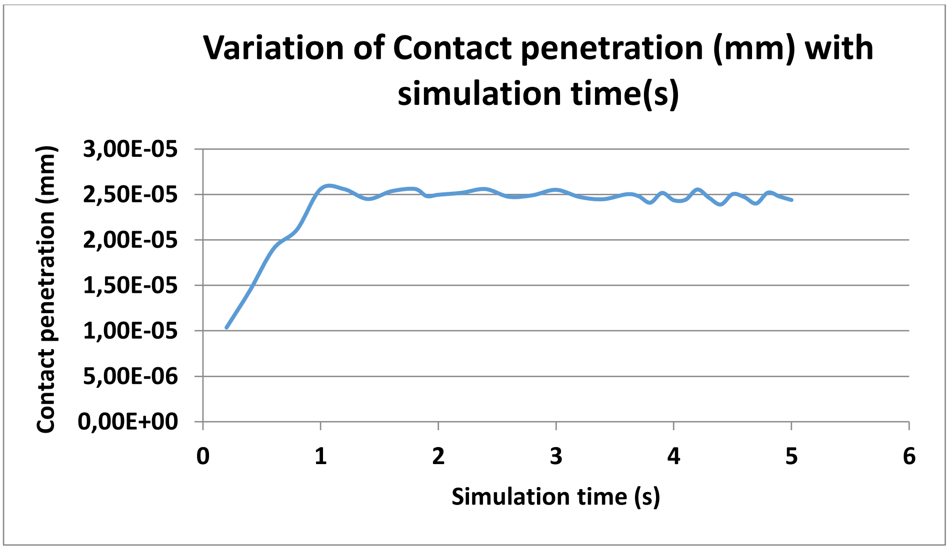

Figure 10: Contact penetration (mm) between rail-wheel for the simulation duration for a mesh size of 6 mm

Similar trends are observed for variation in contact penetration over the simulation time as shown in figure 9. A finite value of contact penetration can be noticed due to the application of *penalty* constraint formulation between rail and the wheel. Penalty constraint seeks to resolve contact penetrations that exist by providing a small residual penetration at the beginning of each time increment. It is a well known fact that smaller and stable value of contact penetration is an indication of a good contact throughout the rail-wheel motion.

5 Conclusion

Wear has a significant impact on the contact characteristics. The present study concludes that a worn out rail profile is more prone to higher contact pressure. Difference in contact pressure between a new and worn rail profile is more than 10% as confirmed by both Hertzian and FEA methodologies. Increase in contact pressure is responsible for further aggravating the problem of wear of rail profile. This study also helps in understanding the significance of mesh size in a numerical contact model and states the importance of selecting the critical step time period for an explicit analysis. Future work may include a comparative study of new and worn wheel profile on contact stress and patch size on rail. Furthermore, experimental investigation can be done to validate the accuracy of numerical model and analytical Hertzian approach.

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