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Thermomechanical phase-field fracture modeling of fluid-saturated porous media

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In this work, the problem of brittle fracture in a fluid-saturated porous material is extended by considering the non-isothermal states of the sample. The temperature field will affect the problem in two aspects: 1) Temperature-dependent material parameters, such as elasticity modulus (E) and critical energy release rate (Gc). 2) Thermal expansion due to thermo-mechanical volume coupling. In hydraulic fracturing, we further study the effect of the temperature difference between the injected fluid and the surrounding porous media ambient on the crack behavior. The modeling of the porous media domain is based on the macroscopic theory of porous media (TPM), whereas the phase-field method (PFM) is applied to approximate the sharp crack edges by diffusive ones. In the numerical implementation, the coupled system of partial differential equations will be solved using the FEM in order to simulate the heat transition in the crack and non-crack regions.

1 Mathematical modelling

The underlying study considers a fluid-saturated porous medium consisting of immissible solid and fluid constituents φ^α ($\alpha = S$: solid, $\alpha = F$: fluid). Within the framework of theory of porous media (TPM), the saturation condition is defined as $n^S + n^F = 1$ with n^α being the volume fraction, whereas the partial density ρ^α and material density $\rho^{\alpha R}$ are related to each other via $\rho^\alpha = n^\alpha \rho^{\alpha R}$ (references in [1–3]). In this study, the solid material (rock) is considered incompressible $\rho^{SR} = \text{const.}$, while the pore-fluid (water) is treated as a compressible material with the material time derivative defined as $(\rho^{FR})'_S := \mathcal{K}_F (p)'_S$ with $\mathcal{K}_F$ as the fluid compressibility parameter and p as the fluid pressure [4]. Following the kinematics of multiphase materials, a Lagrangian description of the solid matrix via the solid displacement $\mathbf{u}_S$ and velocity $\mathbf{v}_S$ is considered. The pore-fluid flow is expressed either in an Eulerian description using the fluid velocity $\mathbf{v}_F$ or by modified Eulerian settings via the seepage velocity $\mathbf{w}_F := \mathbf{v}_F - \mathbf{v}_S$. Within a geometrically-linear framework, the solid small strain tensor is defined as $\boldsymbol{\varepsilon}_S := \frac{1}{2}(\text{grad}\,\mathbf{u}_S + \text{grad}^T \mathbf{u}_S)$. The onset and propagation of brittle fractures are modeled based on the diffusive interface phase-field modeling (PFM), which uses a scalar phase-field variable d^S to determine the material state, i.e. $d^S = 1$ for the cracked state and $d^S = 0$ for the intact state. Within brittle fracture mechanics, the total potential energy is expressed as the sum of the elastic strain energy and the fracture energy, i.e. $\Psi := \Psi^S_{\text{crack}} + \Psi^S_{\text{el}}$. Following the definition of Ψ^S_{el} in [4] and considering the thermal effects, the total and effective solid stress tensors can, respectively, be expressed as:

$$\begin{aligned} &\boldsymbol{\sigma} = \boldsymbol{\sigma}^S_E - [n^S(\theta^S/\theta^F - 1) + 1]\, p\,\mathbf{I} \qquad \text{with} \\ &\boldsymbol{\sigma}^S_E = \mathcal{G}_d \Big[(\langle \kappa^S\, tr(\boldsymbol{\varepsilon}_S) + m_\theta\, \Delta\theta \rangle_+)\mathbf{I} + 2\,\mu^S\, \boldsymbol{\varepsilon}^D_S \Big] + \big(\langle \kappa^S\, tr(\boldsymbol{\varepsilon}_S) + m_\theta\, \Delta\theta \rangle_- \big)\,\mathbf{I}\,. \end{aligned} \tag{1}$$

The latter formulation proceeds from the assumption of crack degradation under tension and shear (+) and not under compression (−), with $\langle(\,\bullet\,)\rangle_\pm = \frac{1}{2}\big[(\,\bullet\,) \pm |(\,\bullet\,)|\big]$ as the Macauley brackets. Herein, $\mathcal{G}_d$ represents the material degradation function, μ^S and κ^S are the bulk and shear moduli of the porous matrix, respectively, $m_\theta := -3\,\kappa^S\,\alpha^S_\theta$ with α^S_θ as the coefficient of solid thermal expansion, and θ^α is the Kelvin temperature of each constituent. The phase-field evolution is expressed following the standard Ginzburg-Landau approach, see, e.g., [4, 5] for references, as:

$$\frac{\partial \Psi}{\partial d^S} = 0 \quad \Longrightarrow \quad \underbrace{(1 - d^S)\,\tilde{\mathcal{D}}_c}_{\text{driving force}} = \underbrace{\big[d^S - \epsilon^2 \Delta d^S\big]}_{\text{geometric resistance}} \quad \text{with} \quad \tilde{\mathcal{D}}_c = \frac{2\,\Psi^{S+}_{el,max}}{G_c/\epsilon} \quad \text{and} \quad \Psi^{S+}_{el,max} = \max_{\tau\in[0,t]} \Psi^{S+}_{el}\,. \tag{2}$$

In this, ϵ is an internal length scale, G_c is the critical fracture energy of solid and $\Psi^{S+}_{el,max}$ is the maximum positive elastic strain energy density. Assuming quasi-static conditions and neglecting the body forces, the mixture momentum balance reads

$$\mathbf{0} = \text{div}\;\boldsymbol{\sigma}\,. \tag{3}$$

Within the injection of fluid into the notch of the considered sample, the fluid mass balance becomes

$$n^F\,\mathcal{K}_F\,(p)'_S + (1 - d^S)\,\rho^{FR}\,\text{div}\;\mathbf{v}_S - \text{div}\;\Big[\rho^{FR}\frac{K^S}{\mu^{FR}}(\text{grad}\;p - \rho^{FR}\,\mathbf{b})\Big] = \mathcal{Q}(\mathbf{x})\,\hat{\rho}^{Ext}\,. \tag{4}$$

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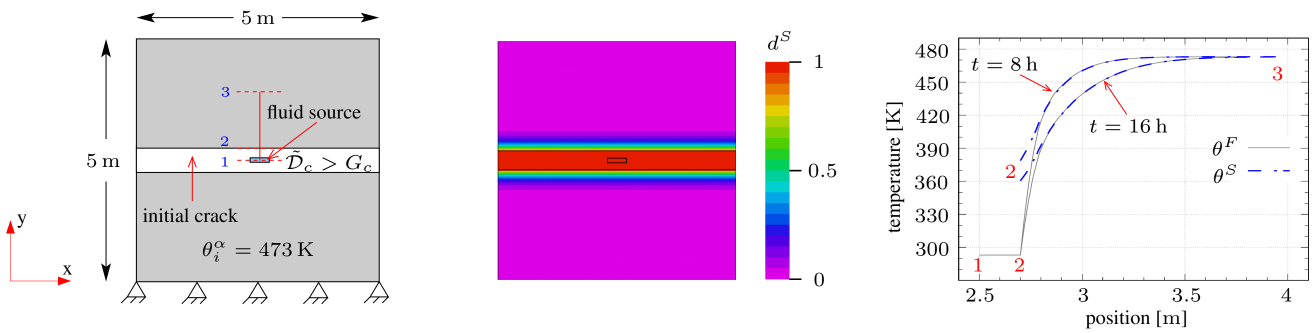
Neglecting the radiation effects, the energy balances of the fluid and solid phases are expressed, respectively, as [2, 6]

$$
\begin{aligned}
&[-n^F\,\rho^{FR}\,C_V^F + n^F\,\theta^F\,p\,(\alpha_\theta^F)^2][(\theta^F)'_S + \operatorname{grad}\theta^F\cdot\mathbf{w}_F] + \alpha_\theta^F\,n^F\,\theta^F\,[(p)'_S + \operatorname{grad}p\cdot\mathbf{w}_F] \\
&+ n^S\,p\operatorname{div}(\mathbf{u}_S)'_S - \operatorname{div}\mathbf{q}^F + (1-d^S)\,\hat{\varepsilon}^F = \hat{\psi}^F\,, \\
&(\operatorname{div}\mathbf{u}_S)\,\sigma_E^S\cdot(\varepsilon_S)'_S - n^S\,\rho^{SR}\,C_V^S(\theta^S)'_S - \frac{\theta^S}{\theta^F}\,n^S\,p\operatorname{div}(\mathbf{u}_S)'_S - \operatorname{div}\mathbf{q}^S - (1-d^S)\,\hat{\varepsilon}^F - \hat{\mathbf{p}}_E^F\cdot\mathbf{w}_F = 0\,.
\end{aligned}
\qquad (5)
$$

Herein, C_V^α is the specific heat capacity, $\mathbf{q}^\alpha := -n^\alpha\,H^{\alpha R}\operatorname{grad}\theta^\alpha$ is the heat influx with $H^{\alpha R}$ is the heat conduction coefficient, $\hat{\varepsilon}^F := \kappa_\theta^\epsilon(\theta^S - \theta^F)$ is the direct energy production with $\kappa_\theta^{\epsilon I}$ is the heat transfer coefficient and $\hat{\mathbf{p}}_E^F$ is the fluid effective momentum production. μ^{FR} is the fluid dynamic viscosity, $\mathcal{Q}(\mathbf{x})$ is the source location function ($\mathcal{Q} = 1$ for the source and 0 otherwise) and $\hat{\rho}^{Ext}$ being the fluid injection (source) term. $\hat{\psi}^F = \hat{\rho}^{Ext}\,C_V^F\,\Delta\theta$ represents the heat production, where $\Delta\theta$ is the temperature difference between the intact material and the injecting fluid.

2 Numerical results and discussion

In the following, numerical simulation of fractured rock with thermal exchange is presented. As illustrated in Fig. 1(a), a hot rock sample with an initial crack of $5\,\mathrm{m}\times 0.4\,\mathrm{m}$ is being injected with a cold fluid in its center through a central notch ($0.4\,\mathrm{m}\times 0.1\,\mathrm{m}$). The initial crack is defined by the high crack driving force on the cracked region. For the parameters, the initial volume fractions are $n_{0S}^S = 0.99$, $n_{0S}^F = 0.01$, the initial permeability $K_0^S = 10^{-14}\,\mathrm{m}^2$, Possion's ratio is $\nu^S = 0.3$, and the effective densities are $\rho^{SR} = 2670\,\mathrm{kg/m}^3$, where other parameters can be found in [4]. Herein, E^S is assumed to decrease linearly from $E^S = 36.9\cdot 10^9\,\mathrm{MPa}$ to $E^S = 30\cdot 10^9\,\mathrm{MPa}$, while G_c decreases from $G_c = 2$ J/m^2 to $G_c = 1.5$ J/m^2 with the decreasing of the temperature. The internal length scale $\epsilon = 0.14\,\mathrm{m}$, whereas the values of $H^{\alpha R}$, C_V^α, and κ_θ^ϵ are adopted from [6]. For the boundary conditions, the bottom of the sample is fixed, while other sides are stress-free and all the boundaries are considered to be adiabatic to the environment. Initially, the temperature is set to $\theta_i^\alpha = 473\,\mathrm{K}$, while the cold water is being injected into the notch at a temperature of $293\,\mathrm{K}$ and a constant rate $q^F = \hat{\rho}^{Ext}\,v^n\,/\rho^{FR} = 0.2\,\mathrm{m}^3/s$ with v^n being the notch volume. Fig. 1(b) shows the distribution of the cracked, the non-cracked, as well as the interface regions based on d^S. The temperature profiles of the solid and fluid components at different positions are shown in Fig. 1(c). It is clear that the cold fluid injection in the cracked region led to a decrease in the temperature of the uncracked solid matrix over time due to the heat transfer process and the thermal interactions between the constituents.



(a) (b) (c)

Fig. 1: a) The geometry, boundary conditions and initial conditions, b) phase-field variable contour plot after 16 hours, and c) temperature evolution profile of the rock sample after 8 hours and 16 hours.

In conclusion, a macroscopic biphasic phase-field-thermo-hydro-mechanical model is presented for the description of the brittle fracture in porous media while considering the accompanied thermal effects and heat transfer process due to cold fluid injection. The problem is simulated using the FEM and showed the thermal transition process in solid and fluid of a porous material between the cracked and the uncracked regions. Moreover, the problem considers the change of material parameters due to the temperature change, such as the elasticity modulus and the critical fracture energy. For future works, the model will be extended to consider crack propagation under non-isothermal conditions, which includes also fluid flow and heat exchange in the fluid and the solid phases.

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