Received: 6 September 2022 | Accepted: 1 November 2022

DOI: 10.1002/pamm.202200007

A parametric study of high velocity impact prediction using the Mie-Grüneisen equation

Stephan Kalapis[1,*], **Moritz Kärtner**[1], and **Kai-Uwe Schröder**[1]

[1] Institute of Structural Mechanics and Lightweight Design, RWTH Aachen University, Wüllnerstraße 7, 52062 Aachen

Space debris is a serious threat to space travel. Even small fragments are enough to damage or destroy satellites. To protect against space debris, satellites have protective shields, which come in various forms. These shields must be optimized not only for impact protection but also for secondary factors such as weight and size. New shield systems can be tested experimentally with high-speed guns. This requires a lot of time, energy, and cost. Here, numerical solvers can be used to evaluate these new shield systems. For example, the smoothed-particle-hydrodynamics (SPH) method is commonly used to simulate the large deformations resulting from an impact. Special equations of state are used to capture the effects that occur. One such model is the Mie-Grüneisen equation of state. It is widely used for this application but does not account for the physical effects that can occur at very high speeds. Other, more complex equations of state can cover these physical effects, but they are often not implemented in commercial numerical solvers such as ABAQUS. Here we show that the Mie-Grüneisen equation of state can lead to quantitatively reasonable results compared to experimental tests of hypervelocity impacts in ranges up to 10 km/s, even though the physical effects are not covered. A parametric study shows that the Mie-Grüneisen equation of state can be used to predict plate collapse as a function of impactor diameter and velocity for various target thicknesses. Initial designs of new isotropic protective shields for space debris can thus be numerically tested with low effort and cost.

1 Introduction and state of the art

Innovations in research have strongly promoted space travel in recent years and increased the number of satellites in orbit. One example is the US company SpaceX, which has reduced the cost of transporting satellites into space with a reusable launch vehicle. New technologies that make electric rocket motors more efficient, for example, will lead to cost savings in the future for flying satellites into their target orbit. A non-linear increase in the satellite population is therefore to be expected. In 2010, there were 958 operational satellites in Earth orbit. By New Year's Eve 2020, there were already 3,371 operational satellites - an increase of 252 % in just 10 years. The lifetime of satellites has also increased in recent years, so that lifetimes of over 15 years are already being realised today. [1]

With a growing population in the various orbits, it is only a matter of time before satellites collide with each other unintentionally. This is what happened on 10 February 2009, when the communications satellites Kosmos 2251 and Iridium 33 collided. The debris clouds that result from a crash consist of various-sized debris from the satellites, which can only be partially tracked and catalogued from the ground. Especially the small debris pieces that cannot be detected pose a danger to other satellites or space stations, as they cannot be avoided. The debris can reach a speed of up to 15 km/s relative to the hitting object. Any type of such space debris poses significant dangers. Impacts with space debris can lead to the complete fragmentation of the target object. The resulting new debris clouds can in turn hit other satellites and fragment them. This can result in a cascade, so that an orbit becomes completely contaminated with space debris, making it extremely difficult or impossible to fly through. This scenario is called the Kessler syndrome, which was described by Donald J. Kessler as early as 1978 [2].

To prevent spacecrafts from being damaged by space debris, they are equipped with special shields. The solely purpose of those shields is to stop the debris, while the degradation of this component is accepted. Normally, the shields of a spacecraft have only the function to keep off damage - which leads to additional weight. New shielding systems can be designed to fulfill more purposes than just keeping space debris away like bearing starting loads, transmitting data or enhancing the temperature balance for instance. Those new shield designs may be fabricated with Additive Manufacturing tools, leading to truss or lattice core shields. Those new shields may be tested with gas guns if they withhold an impact of a given specimen at a given speed, but these tests are costly, time-consuming and only a few institutions worldwide are able to perform such tests. [3]

Nowadays, the shield consists of multiple walls, where the debris hits the first wall and is broken up into a cloud, such that smaller fragments with lower kinetic energy hit the subsequent walls. Based on the extensive tests of NASA, the researcher E. Christiansen put a lot of effort to describe the behavior of the shields empirical. For various shielding systems he formulated equations, which give a critical diameter of a projectile for a given speed which strikes a given plate. For a single monolithic plate the equation follows

$$d_c = \left(\frac{t}{k} \frac{BHN^{0.25} (\rho_t/\rho_p)^{0.5}}{5.24 (V cos\theta / C_t)^{2/3}} \right)^{\frac{18}{19}} \tag{1}$$

---

[*] Corresponding author: e-mail stephan.kalapis@sla.rwth-aachen.de, phone +49 241 80-96838

where t is the plate thickness, BHN the Brinell Hardness, k the damage scenario, V the initial Speed, θ the striking angle and C_t the speed of sound, ρ_t and ρ_p are the density of target and projectile. Three damage scenarios are calculable: perforation ($k = 1.8$), detached spall ($k = 2.2$) or incipient attached spall ($k = 3.0$) [4]. This equation is known as Ballistic Limit Equation (BLE), which serves as a handy tool for rapid designing of conventional shielding systems. Such BLEs are available for different systems, but not yet for truss or lattice core shields.

Furthermore, the research community for high and hypervelocity impacts was concerned with improving the numerical codes such that they mimic the reality in better quality. In particular, the crater diameter of plates were of recent interest [5,6]. For these calculation, the Mie-Grüneisen equation of state (EOS), which is described in more detail in section 2.1, was often used. Until now and to the best knowledge of the authors, it has not been shown whether the Mie-Grüneisen EOS is suitable for the velocity range of 0.5 - 10 km/s.

Thus, it is beneficial to perform numerical tests beforehand in order to ensure a fast, robust and cheap developing process of new space debris shielding systems. To establish such a virtual testbed, a numerical method, material laws and relevant geometries must be identified. As seen later, compared to the numerical method, finding the law to describe the material behaviour of such high and hypervelocity impacts is not a trivial task. This paper focuses on two goals: first to show that high and hypervelocity impacts are calculable with a commercial numerical solver, e.g. ABAQUS by Dassault Systèmes and second, that given material laws do not need to cover every physical effect which occurs during an impact. These two goals lead to the opportunity of rapid designing and evaluating of new shield systems. Within this paper only the integrity of the shield is studied, so only whether the shield withstands the impact or not.

This paper is structured as follows: in section 2 is given the theory of how to calculate high and hypervelocity impacts. Furthermore, the usage of ABAQUS and the implementation of the SPH-Method is briefly explained. In section 3 the parametric study is shown. First, the model and the iteration procedure is explained, second the results are discussed. The paper closes in section 4 with the conclusion.

2 Theory of calculating high and hyper velocity impacts

The typical velocity range of high and hyper velocity impacts are roughly between 0.5 km/s and 10 km/s, which result into high strain rates. While one body approaches and intersects another one with this extreme velocity, very high pressures of up to 90 GPa are expected. These pressures are not calculable neither by standard nor linear approaches. Thus, a new calculation method must be derived in order to calculate the impact event correctly. First, the theory behind calculating hyper velocity impacts is given. After that, the implementation in ABAQUS is explained.

2.1 Calculating impacts with hydrocodes

Within high and hyper velocity impacts the acting bodies will see stresses in the amount of gigapascal. Common calculation procedures fail to accurately predict the material behaviour. Thus, a new material law is needed. For this purpose, the very governing physical principles within such an impact must be studied in detail. In general, the acting stress can be divided into the hydrostatic and the deviatoric stress. Whilst the hydrostatic stress acts simultaneously in all three dimensions and only produces change in the volume of the body, the deviatoric stress can act in less dimensions and can change the appearance of the body. If very high stresses are acting, it is difficult to distinguish the deviatoric stresses from the hydrostatic stresses [7].

To describe the material behaviour within an high and hyper velocity impact, a one-dimensional space is considered. This one-dimensional space is filled with the target material. One side of this domain is bounded, wherever the other side is infinite long. Now, the bounded side is instantaneously accelerated, such that a shock wave is initiated in the material. This shock wave compresses the material. The state in front of the shock wave is denoted with index 0, the side behind the shock wave is denoted with 1 as seen in Figure 1. The material properties are characterized by density ρ, particle velocity u, stress p and internal energy e. Furthermore, u_p is the velocity of the shockwave influenced particle and U_s the shockwave velocity. This

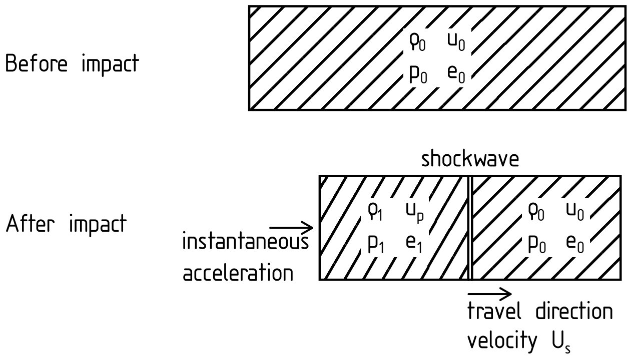

Fig. 1: Schematic drawing of uniaxial strain due to instantanious acceleration.

setup enables the usage of the uniaxial strain, which is expected to occur in hyper velocity impacts. Over the shock wave, different equations of conservation are set [8]:

$$\text{Conservation of mass:} \qquad \rho_1 = \rho_0 \frac{U_\mathrm{s} - u_0}{U_\mathrm{s} - u_p} \tag{2}$$

$$\text{Conservation of impulse:} \qquad p_1 - p_0 = \rho_0 \, U_\mathrm{s} \, u_p \tag{3}$$

$$\text{Conservation of energy:} \qquad p_1 u_p = \frac{1}{2} \, \rho_0 \, U_s \, u_p^2 + \rho_0 \, U_s \, (e_1 - e_0) \tag{4}$$

In order to link the density, internal energy and the governing pressure of the compressed material, an EOS is needed. For solid materials under very high pressures, as they occur in high and hyper velocity impacts, the Mie-Grüneisen EOS is suitable. This EOS has the form

$$p = p_H \left(1 - \frac{\Gamma_0 \eta}{2}\right) + \Gamma_0 \rho_0 e_0 \tag{5}$$

with

$$p_H = \frac{\rho_0 c_0^2 \eta}{(1 - s_s \eta)^2}. \tag{6}$$

The variable Γ_0 represents the Grüneisen-Coefficient, p_H the Hugoniot, c_0 the bulk speed of sound, s_s a linear coefficient and $\eta = 1 - \rho_0/\rho$, respectively. The Mie-Grüneisen EOS treats the solid like a fluid, but it cannot represent phase changes of the material. As aluminum is often used for the shields, the sublimation of aluminum shall be considered. As a rule of thumb one can say, that for impact velocities above 2 km/s the aluminum changes its phase to gas [8]. Strictly speaking, velocities over 2 km/s are not calculable by the Mie-Grüneisen EOS. However, the rest of the chapter will show that it will also deliver reasonable results at higher speeds.

With the eqs. 2-4, eq. 5 and a given boundary equation, the unknown variables of the uniaxial strain due to high and hyper velocity impacts are calculable.

2.2 Implementation of SPH within ABAQUS

Smoothed Particle Hydrodynamics (SPH) is a numerical method, which is capable to calculate the behaviour of solids and fluids. Its main purpose was to help Astrophysicists to represent and predict the movements of galaxies [9]. In contrast to the common used Lagrange-Method for numerical calculations, the SPH-Method is meshless, thus it is advantageous if high strain rates and large deformations or even multiple separations of the material appear. In hypervelocity impacts, all of these extreme states occur simultaneously, though the SPH-Method is predestined for further usage. The SPH is built on the assumption, that free particles interact through their density over a finite distance. Particles, which are closer to each other are weighted higher through a kernel function. For further mathematical background of the SPH-Method the interested reader is referred to [9].

ABAQUS, Dassault Systèmes offers the usage of SPH in different ways. All of them have in common, that Lagrange elements are converted into particles, either by the solver or by hand. In the first case, within the element selection of the Complete ABAQUS Environment (CAE) a specific time stamp can be chosen when the element shall be converted into a selectable amount of particles for the usage of SPH-Method. In the latter case, the input file must be manipulated by hand such that the solver uses the SPH-Method.

Unlike normal SPH codes, particles in ABAQUS cannot be assigned boundary conditions, but they can be bound to regular Lagrange elements. To apply a clamped boundary condition on a plate out of particles, the edge particles are tied to Lagrange elements, which in turn are assigned the boundary conditions.

Beforehand of the parametric study, the SPH-Method in ABAQUS was tested. To do so, high velocity tests with Lagrange-Method and SPH-Method were performed and compared, while all other parameters were held constant. As can be exemplarily seen in Figure 2, the calculations of both methods show very similar results. Furthermore, the Taylor Bar Impact test was performed and compared with literature data [10]. In sake of brevity, these tests are not explained in detail here. We want to stress, that these tests led to good agreements and the SPH-Method in ABAQUS is working correctly according to the conducted tests.

3 Parametric Study

In this section, the parametric study for high and hypervelocity impacts is performed. First, the underlying model and general procedure is explained. Second, the results of the implemented model are discussed.

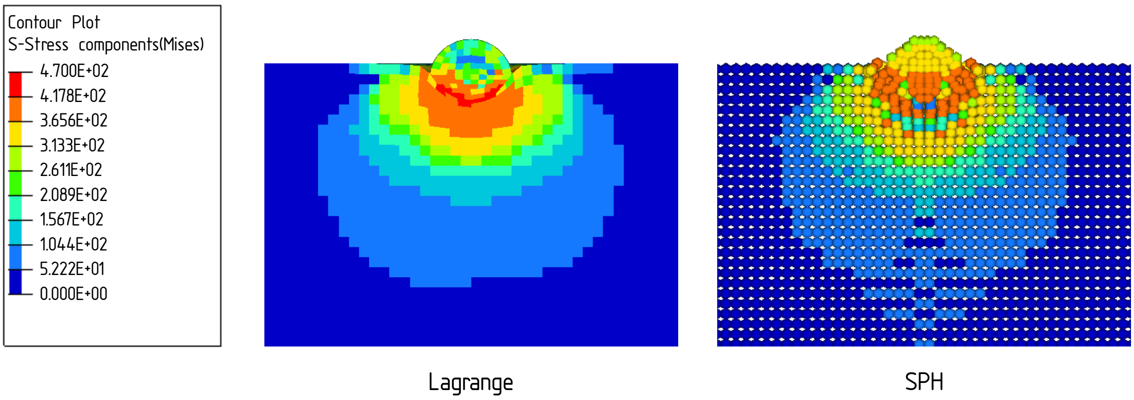

Fig. 2: Stress distribution during impact of a 5 mm sphere at 0.5 km/s on semi-infinite plate. Lagrangian model (left) and SPH model (right) during the impact at the time of maximum penetration depth. Stress values are in MPa.

3.1 Model

The model consists of a single target plate which is impacted by a sphere, see figure 3. The lengths a and b of the overall plate and a_{SPH} and b_{SPH} of the SPH area are calculated automatically on predefined experienced values, such that the impact is not influenced by the fixation. The plate thickness t is iterated from 2 mm to 10 mm with a step of 1 mm. Since only an integer number of particles with the same distance to each other can be generated in the plate and impactor, this number is automatically calculated to fit best the given geometries. In this model, the widely used Johnson-Cook strength model is chosen to map the deviatoric stresses [11]. The constants for this model are given in Table 1. The hydrostatic stresses are mapped with the aforementioned Mie-Grüneisen EOS [8]. Their constants are given in Table 2.
For each plate thickness, an impact condition of the BLE from eq. 1 can be defined, which returns for a given impactor velocity the minimum impactor diameter which results in penetration of the plate. These two parameters are varied throughout the studies.

Table 1: Constants for Johnson-Cook strength model for aluminum [12]

	Density [t/mm^3]	shear modulus [MPa]	A [MPa]	B [MPa]	n	m	θ_0 [K]	θ_{melt} [K]
Aluminum	2.7E-09	25,500.0	262.0	162.1	0.2783	1.34	293.2	925.0

Table 2: Constants for Mie-Grüneisen equation of state for aluminum [12]

	Grüneisen-Coefficient Γ_0	speed of sound c_s [m/s]	slope s_s	specific heat [J/(kg K)]
Aluminum	1.97	5,240.0	1.4	885.0

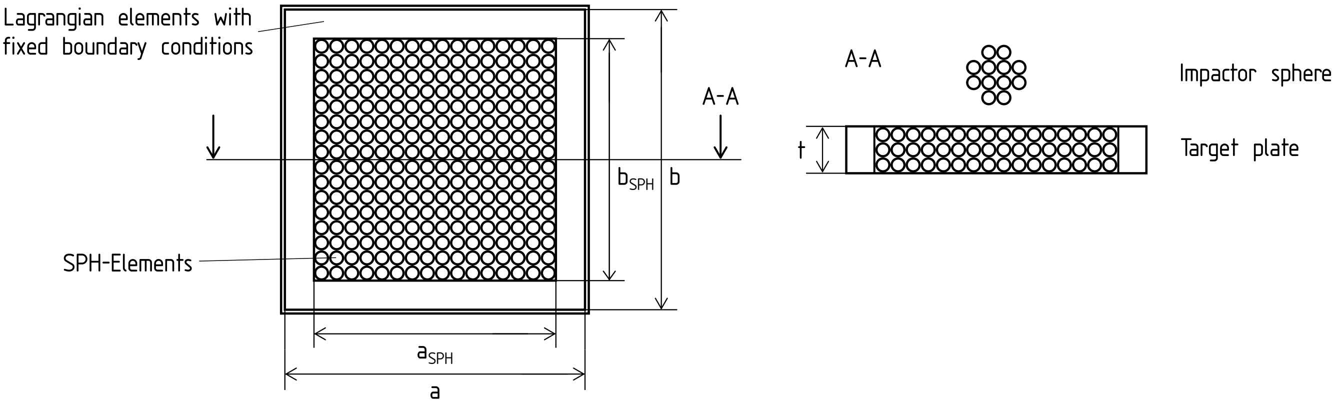

Fig. 3: Geometry of plate and impactor used in the parametric study.

During the parametric study, the predictions of the BLE from eq. 1 and the numerical model whether the plate is penetrated or not are compared with each other. Starting from a given velocity, the diameter resulting in penetration according to the BLE is calculated. This diameter is used for the sphere which impacts the plate in the ABAQUS SPH model. If the model calaculates a penetration of the plate, the kinetic energy of the impactor is increased by increasing impactor diameter and velocity, otherwise impactor diameter and velocity are decreased. This is repeated until a change in failure mode is detected.

As seen in Figure 4, the impactor velocity and diameter are chosen along a line which lies perpendicular to the tangent of BLE in the initially evaluated point. The iteration along the perpendicular is repeated several times along the BLE. The velocity increment, both during the iteration in one point as well as during iterating over the whole BLE, is scaled according to the incline of the BLE at a given velocity to ensure even spacing of the tested diameter-velocity combinations.

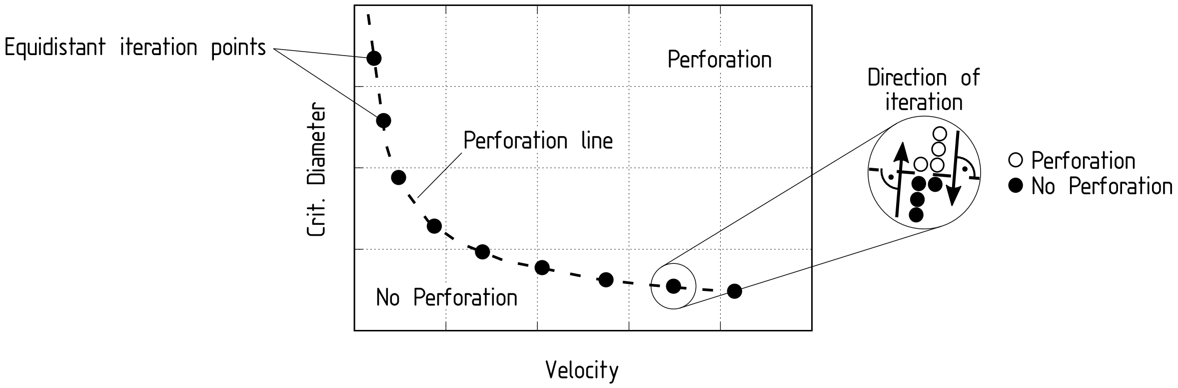

Fig. 4: The perforation line distinguishes the area of perforation or no perforation. With iteration on selected points the perforation line is searched for.

Whether perforation has taken place or not is decided based on the number of sphere particles behind the plate. It was established in advance that it is possible for a few particles - out of several hundred - to slip through the plate on impact. A perforation of the plate is therefore declared if at least 5 sphere particles are counted on the back of the plate after the impact, which is due to the double-axis symmetry of the plate.

The entire procedure for iterating over each plate is implemented with Python. The geometries of the plate and impactor and its initial velocity are specified in Python and written into an input-file and passed to ABAQUS for solving. Post-processing is done again in Python to decide whether a perforation has occurred or not. Depending on the result just calculated, another simulation with increased or decreased energy is performed or the next iteration point is calculated on the BLE. Inside this code, several functions to prevent abortions due to numerical errors or timeout are incorporated. A maximum of 3 hours constrains excessive durations und accelerates the general time needed.

3.2 Results

With the material law from section 2.1, geometry and variables from section 3 the parametric study is performed. In total 9 plates with thicknesses between 2 mm and 10 mm were calculated. On average, each plate is examined with 11 equidistant iteration points, each requiring 8 attempts to detect the perforation line. There are some deviations in the above mentioned numbers between the different sheet thicknesses, so that in the end 740 initial jobs have to be calculated. Out of these, 33 timed out and 27 were aborted due to numerical errors, which leaves the study with 680 succesful jobs. Since only the change from perforation to no perforation or vice versa is of interest, the surplus jobs are deleted. Thus, 212 jobs are left to predict the perforation line of all plates.

Figure 5 shows four exemplary results of the study. It is worth noting that the other five plate thicknesses of the study show very similar results. Here, the corresponding empirical BLE and the first no perforation event of each valid iteration is presented. As one can see, the numerical results and the BLE have good agreement. Especially for velocities over 2 km/s, where the used Mie-Grüneisen EOS does not consider every ocurring physical effect, the found results are in good agreement with empirical literature data [4], for thicker plates better than for thinner ones. Surprisingly, the more significant deviations are found for velocities lower than 2 km/s. This circumstance might be attributed to the implemented SPH method within ABAQUS, where unfavourable ratios of number of particles in plate and impactor might affect the solving process.

If the estimated and calculated velocities are compared, $R^2 > 95\,\%$ yields for all plates and even $R^2 > 97\,\%$ for thicker plates. Thus, it is possible to calculate high and hypervelocity impacts of aluminum impactors on aluminum plates with the Mie-Grüneisen EOS.

4 Conclusion

Rising traffic to orbits around the earth raises the threat of impacts due to space debris for spacecrafts. To prevent further damage, spacecrafts are equipped with shields, which secures internal systems and electronics from fatalities. With the upcoming improvements in Additive Manufacturing it is possible to design new shield concepts, which not only serve the safety of the spacecraft. As the testing of these shields is quite costly, a virtual testbed is needed. Furthermore, this testbed shall be implemented within commercial software like ABAQUS, as no additional effort for deriving and implementing code is needed.

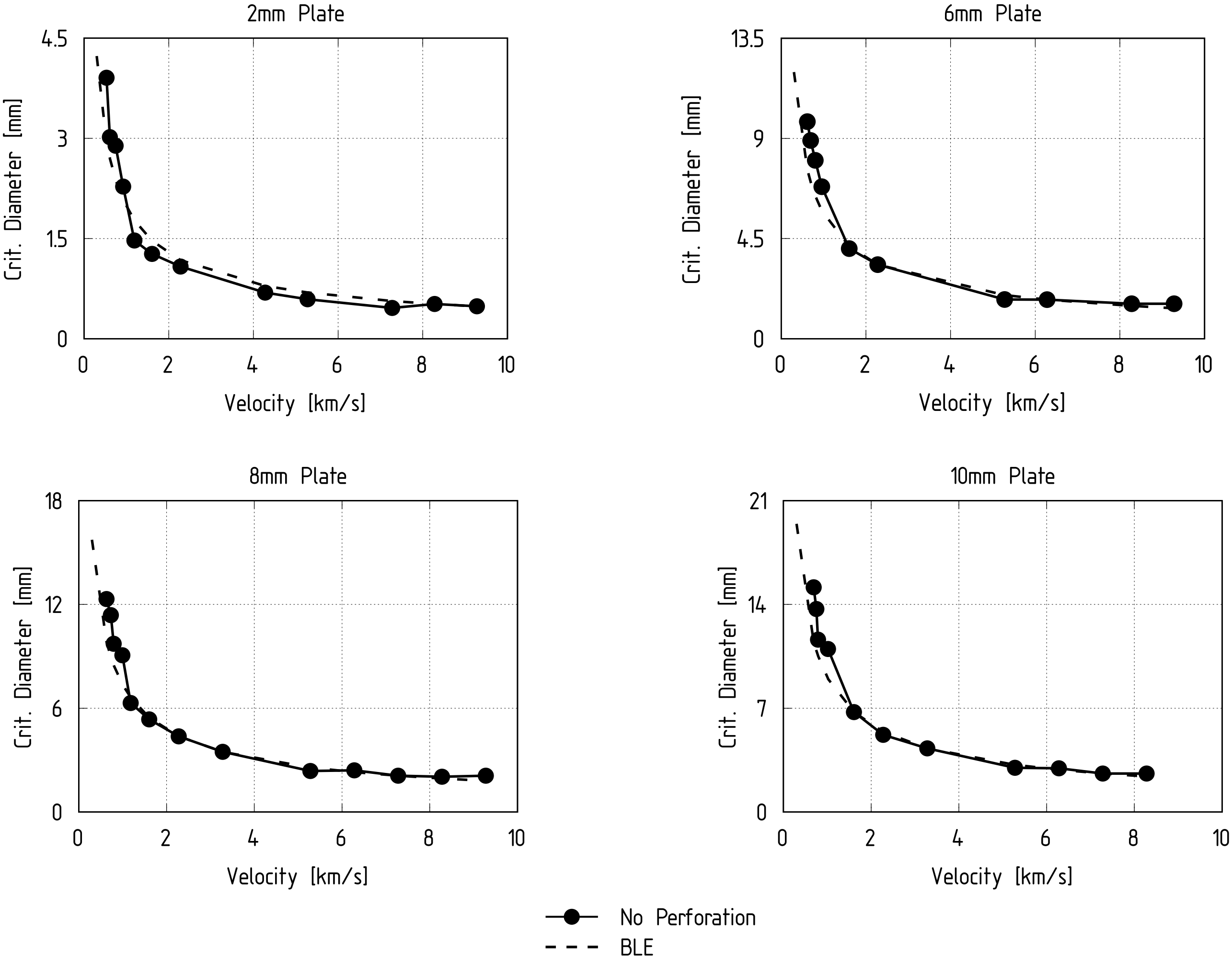

Fig. 5: Selected results of the parametric study. $R^2 > 95\,\%$ for all results regarding estimated and calculated velocity.

In this paper, we showed that high and hypervelocity impacts are calculable within ABAQUS using the Mie-Grüneisen EOS, the Johnson-Cook strengthening law and the SPH method. The obtained results are in very good agreement with the corresponding empirical observations. As the Mie-Grüneisen EOS does not consider phase changes but treats the metal like a fluid, the energy for the phase change may be neglected in terms of observation for perforation or no perforation. It should be noted that in this study the focus was on the damage scenario alone and not on the crater diameter, particle distribution in the debris cloud or similar.

For the future work the optimal design of an additive manufactured shield is planned. This shield may be used for other usages like bearing loads during the start besides just shielding against space debris.

Acknowledgements Open access funding enabled and organized by Projekt DEAL.

References

[1] SIA - Satellite Industry Association, State of the Satellite Industry Report, June 2021, https://brycetech.com/reports/report-documents/SIA_SSIR_2021.pdf, last visited 18.08.2022
[2] D.J. Kessler, B.G. Cour-Palais, J. Geophys. Res **83**, p. 2637–2646. (1978)
[3] M. Schubert, A. Dafnis, MATEC Web Conf., 304 (2019) 07003
[4] NASA Johnson Space Center, Handbook for Designing MMOD Protection, JSC-64399 Version A
[5] Z.L. Zhang et al., Int. J. Comput. Methods **17** (2020)
[6] B.M. Corbett, Int. J. Impact Eng. **33**, p. 431-440 (2006)
[7] J.A. Zukas (ed.), High velocity impact dynamics (Wiley, New York, 1990)
[8] J.A. Zukas (ed.), Introduction to hydrocodes (Elsevier, Amsterdam, 2004)
[9] R. Gingold, J. Monaghan, Mon. Notices Royal Astron. Soc. **181**, p. 375-389 (1977)
[10] J.W. House, Taylor Impact Testing, Air Force Armament Laboratory, AD-A215 018, (1989)
[11] G.R. Johnson, W.H. Cook, Eng. Fract. Mech. **21**, pp. 31-48 (1985)
[12] Abaqus Technology Brief, Simulation of the ballistic perforation of aluminum plates with Abaqus/Explicit, TB-12-BP-1 (2012)