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RESEARCH ARTICLE

An anisotropic damage model for finite strains with full and reduced regularization of the damage tensor

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Abstract
The modeling of damage as an anisotropic phenomenon enables the consideration of arbitrarily oriented microcracks at the material point level. Yet, the incorporation of material softening into structural simulations still requires a regularization of for example, the degrading variable. There exist different possibilities for a regularization in case of anisotropic damage with varying numbers of nonlocal degrees of freedom corresponding to for example, the symmetric integrity tensor , the principal traces of the damage tensor or a scalar damage hardening variable. Here, we propose a finite strain formulation with a symmetric second order damage tensor of which all six independent components are regularized with a corresponding nonlocal degree of freedom. Due to the significant increase in computational cost caused by the full regularization of the damage tensor, alternative approaches for a reduced regularization with fewer nonlocal degrees of freedom are discussed. Thereafter, the results of a numerical example using the model with full regularization are presented.

1 | INTRODUCTION

The incorporation of softening phenomena into structural simulations [1–3] can lead to undesirable occurrences of localization (e.g., [4]), especially when material degradation in the form of damage is considered (e.g., [5]). A remedy for isotropic damage models was proposed by Dimitrijevic et al.[6] and [7] in the form of gradient-extended models that introduce an additional nonlocal field which is coupled to the local damage variables and, thereby, ensures a regularization.

Furthermore, current research is primarily concerned with the modeling of anisotropic material degradation at finite strains (e.g., [8, 9]) and the search for efficient regularization techniques for softening phenomena (e.g., [10, 11]). Here, the presented model utilizes a second order damage tensor and a finite strain formulation and is applied to purely mechanical problems. In future, the anisotropic damage modeling can be incorporated into multiphysical process simulations (e.g., [12, 13]) and medical applications (e.g., [14]).

In Section 2, we present the model's constitutive framework and in Section 3 the corresponding weak forms. Then, we apply the model in a structural simulation in Section 4 and, finally, provide a conclusion in Section 5.

2 | CONSTITUTIVE MODELING

2.1 | Gradient-extension

In this work, damage is described by a symmetric second order damage tensor with six independent components. Inspired by [1], we chose a full regularization of the damage tensor by introducing six additional nonlocal degrees of freedom $\bar{D}_{xx}$, $\bar{D}_{yy}$, $\bar{D}_{zz}$, $\bar{D}_{xy}$, $\bar{D}_{xz}$, $\bar{D}_{yz}$ following the micromorphic approach of [7, 15].

However, this type of regularization requires the consideration of six additional balance equations (cf. Section 3) and affects computational efficiency. Therefore, other regularization methods with three (using the principal traces of the damage tensor, cf. [2]) or just one nonlocal degree of freedom (using the accumulated damage, cf. [3]) will be investigated in future works and examined with respect to their accuracy.

2.2 | Helmholtz free energy

The model is based on a Helmholtz free energy that consists of four terms: the elastic energy ψ_e depending on the right Cauchy-Green tensor $\boldsymbol{C}$ and the damage tensor $\boldsymbol{D}$; the isotropic damage hardening energy ψ_d depending on the accumulated damage ξ_d; the additional damage hardening energy ψ_h depending on the eigenvalues of the damage tensor $\boldsymbol{D}$; the nonlocal energy term depending on the damage tensor $\boldsymbol{D}$, the nonlocal quantities $\bar{D}_{xx}$, $\bar{D}_{yy}$, $\bar{D}_{zz}$, $\bar{D}_{xy}$, $\bar{D}_{xz}$, $\bar{D}_{yz}$ and their corresponding gradients. Thus, it reads

$$\begin{aligned}\psi = \psi_e\left(\boldsymbol{C},\boldsymbol{D}\right) + \psi_d\left(\xi_d\right) + \psi_h\left(\boldsymbol{D}\right) + \psi_{\bar{d}}\Big(&\boldsymbol{D}, \bar{D}_{xx}, \mathrm{Grad}\left(\bar{D}_{xx}\right), \bar{D}_{yy}, \mathrm{Grad}\left(\bar{D}_{yy}\right), \bar{D}_{zz}, \mathrm{Grad}\left(\bar{D}_{zz}\right), \ldots \\ &\bar{D}_{xy}, \mathrm{Grad}\left(\bar{D}_{xy}\right), \bar{D}_{xz}, \mathrm{Grad}\left(\bar{D}_{xz}\right), \bar{D}_{yz}, \mathrm{Grad}\left(\bar{D}_{yz}\right)\Big).\end{aligned} \tag{1}$$

2.3 | Clausius-Duhem inequality

The Clausius-Duhem inequality, including the micromorphic extension (cf. [7, 15]), reads

$$\begin{aligned}-\dot{\psi} + \frac{1}{2}\boldsymbol{S} : \dot{\boldsymbol{C}} + a_{xx}\dot{\bar{D}}_{xx} + \boldsymbol{b}_{xx}\cdot\mathrm{Grad}\left(\dot{\bar{D}}_{xx}\right) + a_{yy}\dot{\bar{D}}_{yy} + \boldsymbol{b}_{yy}\cdot\mathrm{Grad}\left(\dot{\bar{D}}_{yy}\right) + a_{zz}\dot{\bar{D}}_{zz} + \boldsymbol{b}_{zz}\cdot\mathrm{Grad}\left(\dot{\bar{D}}_{zz}\right)\cdots \\ + a_{xy}\dot{\bar{D}}_{xy} + \boldsymbol{b}_{xy}\cdot\mathrm{Grad}\left(\dot{\bar{D}}_{xy}\right) + a_{xz}\dot{\bar{D}}_{xz} + \boldsymbol{b}_{xz}\cdot\mathrm{Grad}\left(\dot{\bar{D}}_{xz}\right) + a_{yz}\dot{\bar{D}}_{yz} + \boldsymbol{b}_{yz}\cdot\mathrm{Grad}\left(\dot{\bar{D}}_{yz}\right) \geq 0.\end{aligned} \tag{2}$$

After inserting the time derivative of Equation (1) into Equation (2), it serves to derive the state laws of the second Piola-Kirchhoff stress

$$\boldsymbol{S} = 2\frac{\partial\psi}{\partial\boldsymbol{C}} \tag{3}$$

and the generalized micromorphic stresses

$$\begin{aligned}a_{xx} &= \frac{\partial\psi}{\partial\bar{D}_{xx}}, \quad \boldsymbol{b}_{xx} = \frac{\partial\psi}{\partial\mathrm{Grad}\left(\bar{D}_{xx}\right)}, \quad a_{yy} = \frac{\partial\psi}{\partial\bar{D}_{yy}}, \quad \boldsymbol{b}_{yy} = \frac{\partial\psi}{\partial\mathrm{Grad}\left(\bar{D}_{yy}\right)}, \\ a_{zz} &= \frac{\partial\psi}{\partial\bar{D}_{zz}}, \quad \boldsymbol{b}_{zz} = \frac{\partial\psi}{\partial\mathrm{Grad}\left(\bar{D}_{zz}\right)}, \quad a_{xy} = \frac{\partial\psi}{\partial\bar{D}_{xy}}, \quad \boldsymbol{b}_{xy} = \frac{\partial\psi}{\partial\mathrm{Grad}\left(\bar{D}_{xy}\right)}, \\ a_{xz} &= \frac{\partial\psi}{\partial\bar{D}_{xz}}, \quad \boldsymbol{b}_{xz} = \frac{\partial\psi}{\partial\mathrm{Grad}\left(\bar{D}_{xz}\right)}, \quad a_{yz} = \frac{\partial\psi}{\partial\bar{D}_{yz}}, \quad \boldsymbol{b}_{yz} = \frac{\partial\psi}{\partial\mathrm{Grad}\left(\bar{D}_{yz}\right)}.\end{aligned} \tag{4}$$

After defining the mechanical and generalized stresses, the reduced dissipation inequality reads

$$\boldsymbol{Y} : \dot{\boldsymbol{D}} + R_d \, \dot{\xi}_d \geq 0 \tag{5}$$

where the definitions $\boldsymbol{Y} = -\partial\psi/\partial\boldsymbol{D}$ and $R_d = -\partial\psi/\partial\xi_d$ hold.

2.4 | Damage onset criterion and evolution equations

The damage onset criterion is defined following [2] as

$$\Phi_d := \sqrt{3}\sqrt{\boldsymbol{Y}_+ : \mathbb{A} : \boldsymbol{Y}_+} - (Y_0 - R_d) \leq 0 \tag{6}$$

where $\boldsymbol{Y}_+$ denotes the positive semi-definite part of $\boldsymbol{Y}$ and $\mathbb{A}$ an interaction tensor with $\mathbb{A}_{ijkl} = (\delta_{ik} - D_{ik})(\delta_{jl} - D_{jl})$ [16].

The evolution equations of the internal variables follow in an associative manner as

$$\dot{\boldsymbol{D}} = \dot{\gamma}_d \frac{\partial \Phi_d}{\partial \boldsymbol{Y}}, \qquad \dot{\xi}_d = \dot{\gamma}_d \frac{\partial \Phi_d}{\partial R_d} \tag{7}$$

where $\dot{\gamma}_d$ is a Lagrange multiplier that is defined according to the Karush-Kuhn-Tucker conditions

$$\dot{\gamma}_d \geq 0, \qquad \Phi_d \leq 0, \qquad \dot{\gamma}_d \Phi_d = 0. \tag{8}$$

2.5 | Specific energies

The elastic energy ψ_e is chosen in line with Reese et al. [17] and is able to fulfill the damage growth criterion of [18] for anisotropic damage at finite strains. The energy consists of a 'classical' Neo-Hookean energy ψ_{NH} that is multiplied by a combined degradation function that can account for isotropic and anisotropic degradation behavior:

$$\psi_e = ((1 - k_{\mathrm{ani}}) f_{\mathrm{iso}} + k_{\mathrm{ani}} f_{\mathrm{ani}}) \psi_{\mathrm{NH}} \tag{9}$$

The material parameter $k_{\mathrm{ani}} \in [0, 1]$ controls the degree of anisotropy and $k_{\mathrm{ani}} = 0$ describes isotropic and $k_{\mathrm{ani}} = 1$ anisotropic damage. The isotropic degradation function f_{iso} is defined as (see ref. [17])

$$f_{\mathrm{iso}} := \left(1 - \frac{\mathrm{tr}(\boldsymbol{D})}{3}\right)^2 \tag{10}$$

and the anisotropic degradation function as

$$f_{\mathrm{ani}} := 1 - \frac{\mathrm{tr}\left(\boldsymbol{C}^2 \boldsymbol{D}\right)}{\mathrm{tr}(\boldsymbol{C}^2)}. \tag{11}$$

The damage hardening energy ψ_d accounts for linear and nonlinear isotropic hardening effects (see e.g., [17, 19]). Moreover, the additional damage hardening energy ψ_h is formulated in terms of the eigenvalues of the damage tensor D_i and ensures that these do not exceed the value of one (see refs. [2, 3]).

The energy of the micromorphic contribution $\psi_{\bar{d}}$ (cf. [7, 15]) reads for six nonlocal degrees of freedom

$$\begin{aligned}\psi_{\bar{d}} = & \frac{H_{xx}}{2}(D_{xx} - \bar{D}_{xx})^2 + \frac{A_{xx}}{2}\mathrm{Grad}\,(\bar{D}_{xx}) \cdot \mathrm{Grad}\,(\bar{D}_{xx}) \\ & + \frac{H_{yy}}{2}\left(D_{yy} - \bar{D}_{yy}\right)^2 + \frac{A_{yy}}{2}\mathrm{Grad}\,\left(\bar{D}_{yy}\right) \cdot \mathrm{Grad}\,\left(\bar{D}_{yy}\right)\end{aligned}$$

$$+\frac{H_{zz}}{2}\left(D_{zz}-\bar{D}_{zz}\right)^2+\frac{A_{zz}}{2}\mathrm{Grad}\left(\bar{D}_{zz}\right)\cdot\mathrm{Grad}\left(\bar{D}_{zz}\right)$$
$$+\frac{H_{xy}}{2}\left(D_{xy}-\bar{D}_{xy}\right)^2+\frac{A_{xy}}{2}\mathrm{Grad}\left(\bar{D}_{xy}\right)\cdot\mathrm{Grad}\left(\bar{D}_{xy}\right)$$
$$+\frac{H_{xz}}{2}\left(D_{xz}-\bar{D}_{xz}\right)^2+\frac{A_{xz}}{2}\mathrm{Grad}\left(\bar{D}_{xz}\right)\cdot\mathrm{Grad}\left(\bar{D}_{xz}\right)$$
$$+\frac{H_{yz}}{2}\left(D_{yz}-\bar{D}_{yz}\right)^2+\frac{A_{yz}}{2}\mathrm{Grad}\left(\bar{D}_{yz}\right)\cdot\mathrm{Grad}\left(\bar{D}_{yz}\right) \tag{12}$$

where H_{xx}, H_{yy}, H_{zz}, H_{xy}, H_{xz}, H_{yz} are numerical penalty parameters and A_{xx}, A_{yy}, A_{zz}, A_{xy}, A_{xz}, A_{yz} gradient material parameters.

3 | WEAK FORMS

Additionally to the weak form of the balance of linear momentum (Equation (13)), the weak forms of six scalar-valued micromorphic balance equations (Equations (14)–(19)) have to be solved at the global level:

$$g_u := \int_{\Omega_0} \boldsymbol{S} : \delta\boldsymbol{E}\,\mathrm{d}V - \int_{\Omega_0} \boldsymbol{f}_0\cdot\delta\boldsymbol{u}\,\mathrm{d}V - \int_{\Gamma_{t0}} \boldsymbol{t}_0\cdot\delta\boldsymbol{u}\,\mathrm{d}A = 0 \tag{13}$$

$$g_{\bar{d}_1} := \int_{\Omega_0} \delta\bar{D}_{xx}H_{xx}\left(D_{xx}-\bar{D}_{xx}\right) - \mathrm{Grad}\left(\delta\bar{D}_{xx}\right)\cdot A_{xx}\mathrm{Grad}\left(\bar{D}_{xx}\right)\mathrm{d}V = 0 \tag{14}$$

$$g_{\bar{d}_2} := \int_{\Omega_0} \delta\bar{D}_{yy}H_{yy}\left(D_{yy}-\bar{D}_{yy}\right) - \mathrm{Grad}\left(\delta\bar{D}_{yy}\right)\cdot A_{yy}\mathrm{Grad}\left(\bar{D}_{yy}\right)\mathrm{d}V = 0 \tag{15}$$

$$g_{\bar{d}_3} := \int_{\Omega_0} \delta\bar{D}_{zz}H_{zz}\left(D_{zz}-\bar{D}_{zz}\right) - \mathrm{Grad}\left(\delta\bar{D}_{zz}\right)\cdot A_{zz}\mathrm{Grad}\left(\bar{D}_{zz}\right)\mathrm{d}V = 0 \tag{16}$$

$$g_{\bar{d}_4} := \int_{\Omega_0} \delta\bar{D}_{xy}H_{xy}\left(D_{xy}-\bar{D}_{xy}\right) - \mathrm{Grad}\left(\delta\bar{D}_{xy}\right)\cdot A_{xy}\mathrm{Grad}\left(\bar{D}_{xy}\right)\mathrm{d}V = 0 \tag{17}$$

$$g_{\bar{d}_5} := \int_{\Omega_0} \delta\bar{D}_{xz}H_{xz}\left(D_{xz}-\bar{D}_{xz}\right) - \mathrm{Grad}\left(\delta\bar{D}_{xz}\right)\cdot A_{xz}\mathrm{Grad}\left(\bar{D}_{xz}\right)\mathrm{d}V = 0 \tag{18}$$

$$g_{\bar{d}_6} := \int_{\Omega_0} \delta\bar{D}_{yz}H_{yz}\left(D_{yz}-\bar{D}_{yz}\right) - \mathrm{Grad}\left(\delta\bar{D}_{yz}\right)\cdot A_{yz}\mathrm{Grad}\left(\bar{D}_{yz}\right)\mathrm{d}V = 0 \tag{19}$$

The set of Equations (13)–(19) emphasizes the increased numerical effort when considering six additional nonlocal degrees of freedom and motivates the investigation of more efficient gradient extensions (cf. Section 2.1).

4 | NUMERICAL EXAMPLE

The structural example considers a symmetrically notched specimen (see Figure 1A) which is clamped at both ends and pulled in vertical direction. Due to symmetry, only one fourth of the specimen is considered in the numerical investigation and symmetry boundary conditions are applied (see Figure 1B).

The dimensions read $\ell = 50$ [mm], $w = 18$ [mm], $r = 5$ [mm] and the thickness is 1 [mm]. Furthermore, plane strain conditions are employed. The elastic material parameters read $\Lambda = 25000$ [MPa], $\mu = 55000$ [MPa], the damage threshold $Y_0 = 30$ [MPa], the degree of anisotropy $k_{\mathrm{ani}} = 1$ [–], the isotropic damage hardening parameters $r_d = 40$ [MPa], $s_d = 10$ [–], $H_d = 10$ [MPa], the additional damage hardening parameters $K_h = 0.1$ [MPa], $n_h = 2$ [–], $a_h = 0.999999$ [–] and the micromorphic penalty and gradient parameters $H_i = 10^5$ [MPa] and $A_i = 5000$ [MPa] for all nonlocal degrees of freedom. Additionally, we consider an artificial viscosity $\eta_v = 10$ [MPa s].

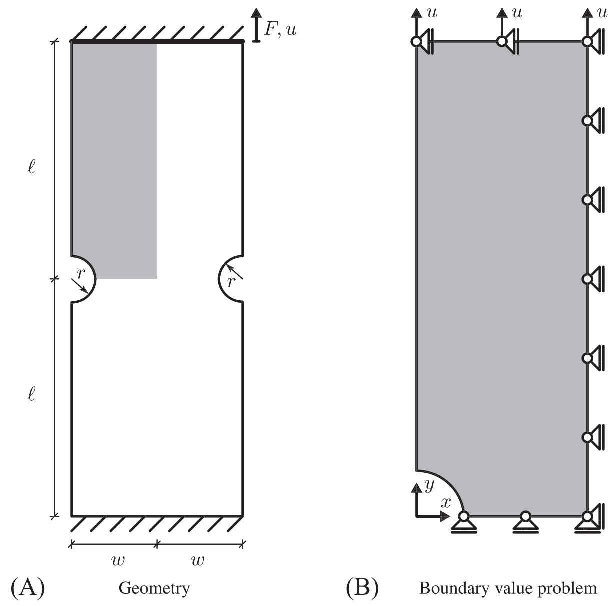

FIGURE 1 Geometry and boundary value problem of the symmetrically notched specimen. Only one quarter of the full specimen is considered in the simulation of the boundary value problem.

Figure 2 shows the plot of the sum of vertical forces F at the top edge over the vertical displacement u of the top edge. The simulations were conducted using seven different meshes with mesh refinement at the position of the crack with 374, 966, 1366, 2201, 2906, 4489, and 10667 elements. Convergence is already obtained with coarse meshes and the difference with respect to the maximum force of the mesh with 374 elements and the mesh with 10667 elements amounts to + 0.19 %.

In Figure 3, the states of crack initiation as well as partial and complete failure are presented for the components D_{xx} (horizontal) and D_{yy} (vertical). The cracks initiate at the notches and propagate horizontally towards the middle of the specimen where they coalesce. Since the specimen is loaded in vertical direction, the vertical component D_{yy} evolves faster than D_{xx} during crack propagation. However, the complete failed state is described by $\boldsymbol{D} = \boldsymbol{I}$ and, thus, $D_{xx} = D_{yy} = 1$.

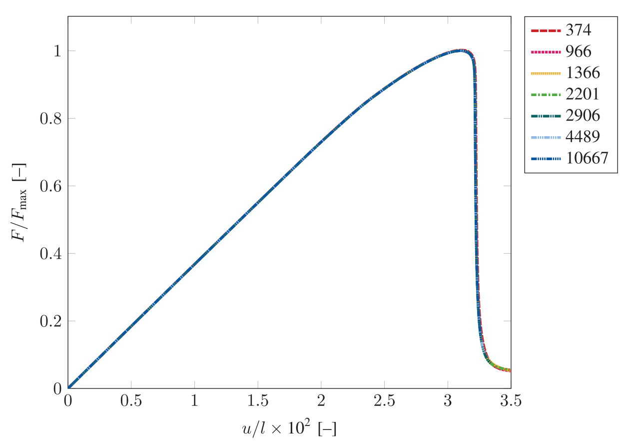

FIGURE 2 Mesh convergence study for the simulations of the symmetrically notched specimen. The forces are normalized with respect to the maximum force of the finest mesh (10667 elements) with $F_{\text{max}} = 5.9678 \times 10^4$ N.

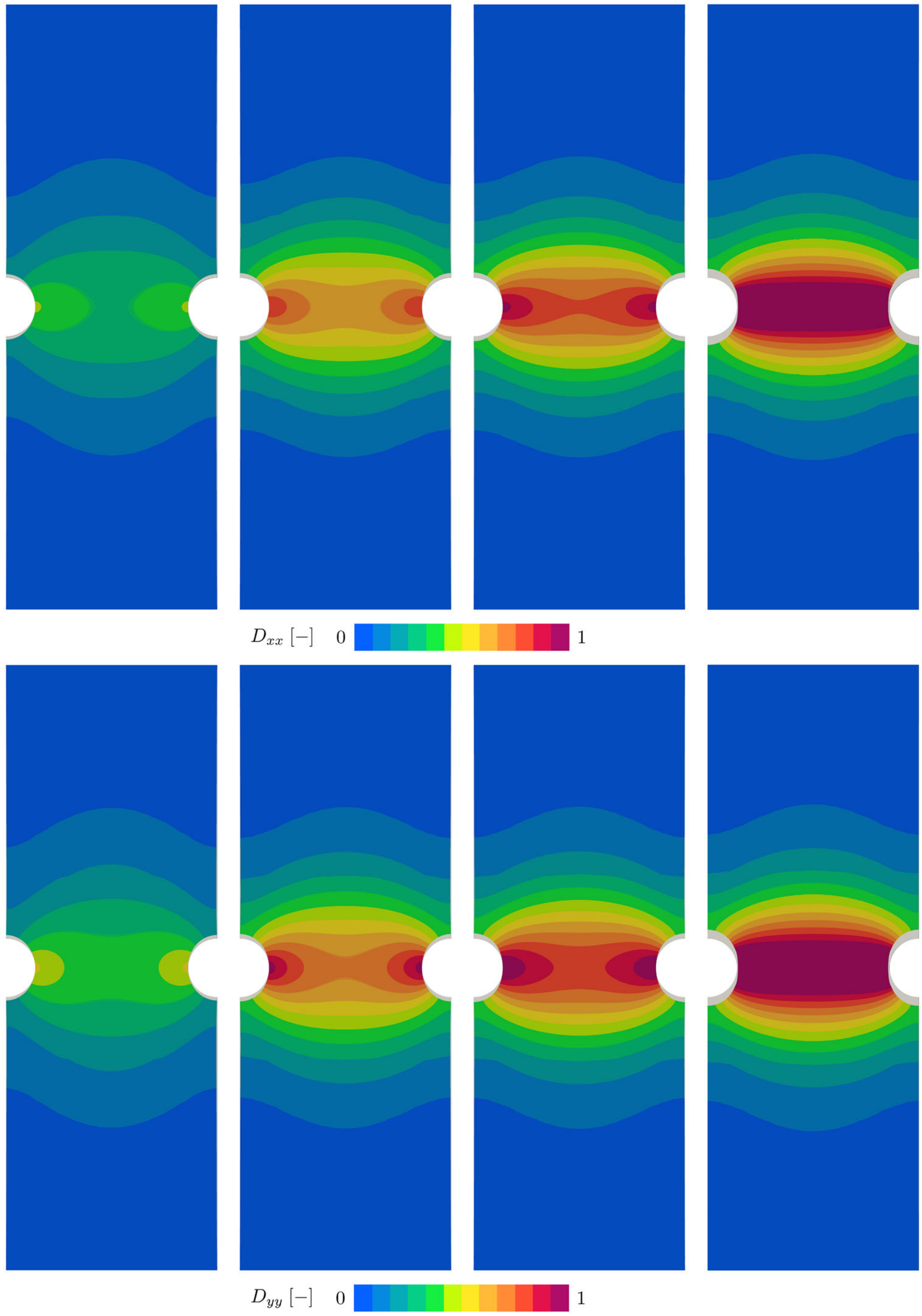

FIGURE 3 Evolution of the horizontal D_{xx} (top row) and vertical D_{yy} (bottom row) component of the damage tensor $\boldsymbol{D}$ for the finest mesh (10667 elements) plotted on the deformed configuration. The undeformed configuration is plotted in the background with solid color.

5 | CONCLUSION

In this paper, we presented a material formulation considering anisotropic damage at finite strains. The model is regularized by six nonlocal degrees of freedom each of which corresponds to one independent component of the symmetric damage tensor. Then, the model is applied to the investigation of a symmetrically notched specimen and yields mesh independent results. In future works, we will investigate model formulations with a reduced number of nonlocal degrees of freedom in order to increase the computational efficiency.

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