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Double-layer multi-criteria group decision-making approach using neutralized possibility degree-based decision matrix with fuzzy information

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ABSTRACT

Multi-criteria group decision-making (MCGDM) problems are fundamentally characterized by uncertainty and ambiguity. The quantification of such data is vital for determining the most optimal course of action. In this paper, a double-layer possibility degree-based interval-valued q-rung orthopair fuzzy sets (IVq-ROFSs) Vlsekriterijumska Optimizacija i Kompromisno Resenje (VIKOR) approach is presented. First, the possibility degrees of IVq-ROFSs are calculated and processed to establish a new neutralized decision matrix. Second, the components of the new decision matrix are taken into account for obtaining a new pair of positive and negative ideal solutions. The utility and regret measures in the VIKOR approach are ultimately determined by combining criteria weights and cumulative possibility degrees to rank the alternatives. The results of the study show that the proposed approach contributes to acquiring accurate alternative rankings in situations where traditional approaches have had difficulty generating ranks. Also, by eliminating the early normalization stage, which has a direct impact on the final ranking, the proposed approach concludes a unique ranking for each decision-making problem. Several comparative and sensitivity analysis are also conducted to exhibit robustness and applicability of the proposed approach in real world.

1. Introduction

A common phrase used to describe decision-making when there are numerous criteria is “multi-criteria decision making” (MCDM). Multi-Attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM) are two subgroups that fall under this category (Sabaei et al., 2015). Making judgments based on many criteria or attributes is referred to MADM. It entails assessing and contrasting options based on several criteria or attributes, and then picking the best choice. MADM is applicable to a number of industries, including engineering, management, and healthcare. A multi-attribute group decision making (MAGDM) is a decision-making procedure that involves a group of decision makers (DMs) with numerous schemes and multiple characteristics (Li et al., 2021). Data from both quantitative and qualitative sources can be included in MADM. It can also combine subjective evaluations with more established scientific data in the same application. An MADM technique incorporates a value judgment and takes into account the DM’s preference structure. To assist the alternative selection, the DM’s preferences will be included into the decision model, allowing for the simultaneous analysis of several factors. As a result, subjectivity is unavoidable MADM. The DM’s preferences are the result of the DM’s individualized assessment of the options in light of the criteria (Khoshabi et al., 2020).

To handle subjectivity in MADM, fuzzy sets are used as a mathematical technique (Wang, 2015). In MADM, DMs are obliged to assess options using a variety of criteria, some of which may be subjective and vague. Fuzzy sets enable the mathematical representation of ambiguous or unclear information, such as linguistic phrases or qualitative evaluations. This makes it possible for DMs to communicate their preferences in a form that is more flexible and intuitive, minimizing the influence of subjectivity on the decision-making process (Dimova et al., 2006). Numerous fuzzy-based models have been developed to handle subjectivity in decision-making, and fuzzy sets have been frequently used in MADM. Yager proposed the notion of q-rung orthopair fuzzy sets (q-ROFS) to provide a framework that may be used to express and manage membership and non-membership data more successfully (Yager, 2017). Interval-valued q-rung orthopair fuzzy sets (IVq-ROFSs) offer an extension of the q-ROFS by utilizing intervals to handle the uncertainty more thoroughly. This interval-based representation acknowledges that the fuzzy features might not be precisely known and

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might vary within a given range, offering a more flexible way to capture uncertainty.

The VIKOR method was created in 1998 as one of the first MCDM strategies for the multi-criteria planning of complex systems, or systems with many criteria and decision alternatives. This method concentrates on rating and selecting from a variety of viable solutions by presenting a feasible compromise in situations when there are conflicting requirements (Opricovic and Tzeng, 2004). While other state-of-the-art MCDM approaches such as "multi-attributive border approximation area comparison" (MABAC) (Pamučar and Ćirović, 2015), "multi-attribute ideal-real comparative assessment" (MAIRCA) (Pamucar et al., 2018), "technique for order preference by similarity to ideal solution" (TOPSIS) (Yoon and Hwang, 1995) or "measurement of alternatives and ranking according to compromise solution" (MARCOS) (Stević et al., 2020) bring significant advantages in ranking alternatives, this study chose the VIKOR approach due to several reasons including 1) VIKOR offers a compromise approach that integrates competing factors, which makes it appropriate for difficult decision-making situations where compromises could be required. 2) Sensitivity analysis is possible with VIKOR, allowing evaluation of the robustness of the approach to modifications in criterion weights or input data. 3) The unique features of the VIKOR, such as using the decision matrix and normalization method that are improved in this study, align well with the objectives of our decision-making framework.

Despite having advantages, the VIKOR approach also contains several drawbacks. First of all, in situations where the distinctions between alternatives are small, VIKOR might not have enough discriminating ability. Second, the type of normalization approach can change the final ranking of alternatives obtained by the VIKOR approach, since it is highly sensitive to the normalization process. Last but not least, processing the fuzzy data and calculation of the complex fuzzy sets such as IV*q*-ROFSs, may be a challenge in real-world problems, especially in cases with a large number of criteria and alternatives, despite the advantages of IV*q*-ROFSs in handling a huge amount of uncertainty in decision making.

In this paper a novel double-layer possibility degree-based VIKOR is introduced that eliminates the aforementioned drawbacks by establishing a neutralized decision matrix which is normalized by using the concept of IV *q*-ROFSs possibility degrees. The proposed approach provides the following contributions: 1) Introduces a novel possibility degree-based decision matrix which is neutralized according to benefit and cost criteria and eliminates the normalization stage from the conventional VIKOR processes. 2) Integrates the concept of IV *q*-ROFSs possibility degrees into MAGDM problems and provides accurate rankings for alternatives even in cases where the previous approaches were unable to rank because of alternatives' similarities. 3) Reduces the computational complexities and enables DMs to add/remove any number of alternatives or criteria at any time during the calculation process. 4) Provides insights and validations by solving two material selection cases which elucidate the calculation steps for real world problems.

The rest of this study is structured as follows: In **Section 2**, some literature about IV *q*-ROFSs and MCDM techniques are reviewed. The proposed double-layer IV*q*-ROFS possibility degree-based VIKOR is detailed in **Section 3** step by step. **Section 4** shows the applications of the proposed approach on two cases together with the validation through comparative analysis. In **Section 5**, the proposed approach is applied on an illustrative case to simulate the application of the proposed method on real-world problems. In **Section 6** the theoretical, practical and policy implications are stated. The study's concluding observations and findings are presented in **Section 7**.

2. Related works

In this section, a review of related works is presented to discuss the state-of-the-art publications concerning IV*q*-ROFSs and MCDM applications.

2.1. Background and developments of IVq-ROFSs

In traditional fuzzy sets, each element has a degree of membership in the set that ranges from 0 to 1, representing the membership degree and its complement. Hesitation or uncertainty connected to items that are not part of the set, however, cannot be captured by this approach. Therefore, traditional fuzzy sets were expanded upon by Krassimir T. Atanassov in the early 1980s to create intuitionistic fuzzy sets (IFS) (Atanassov, 1986), that were intended to solve some of the drawbacks of traditional fuzzy sets when addressing ambiguity and vagueness. The hesitation degree must not be negative in IFSs, and the summation of the membership and non-membership degrees must not exceed 1. Yager introduced the Pythagorean fuzzy sets (PFS) as a further development of the IFS in 2013 (Yager and Abbasov, 2013). PFS also incorporates the idea of membership and non-membership degrees with a wider solution area to offer a more adaptable and expressive framework for addressing ambiguity. In PFSs, the sum of the squares of the membership and non-membership degrees does not exceed 1, which corresponds to the total uncertainty. Yager also presented *q*-ROFS, a relatively new extension of fuzzy sets, in 2017 (Yager, 2017). The purpose of *q*-ROFS is to offer a framework for more effectively representing and managing membership and non-membership information. The usage of the *q*-levels to indicate the level of information granularity is the defining characteristic of *q*-ROFS. With the help of these *q*-levels, uncertainty and non-membership may be represented more precisely. The accuracy of the information representation grows as *q* rises. The *q*-levels are integers higher than or equal to 2. By using intervals to more thoroughly address the uncertainty, IV *q*-ROFSs provide a further development of the *q*-ROFS. Each element in IV *q*-ROFS has a range of potential values for its membership and non-membership degrees. This interval-based representation provides a more flexible method for capturing uncertainty as it recognizes that these characteristics may not be exactly known and might change within a specific range (Joshi et al., 2018).

Based on the concepts of *q*-ROFSs and certain operational principles of q-rung orthopair fuzzy numbers (*q*-ROFNs), Ju et al. (2019) provided the notion of interval-valued q-rung orthopair fuzzy set (IV*q*-ROFS). Afterwards, given the operational rules of *q*-ROFNs, certain interval-valued q-rung orthopair weighted averaging operators were introduced to develop a novel multi attribute decision-making method. Gao and Xu (2019) introduced addition, multiplication, and inverse for interval-valued q-rung orthopair fuzzy values (IV*q*-ROFVs). Furthermore, they proved and examined the aggregation operators and operating features of IV*q*-ROFVs in more details. Finally, they presented the interval-valued q-rung orthopair fuzzy functions (IV*q*-ROFFs) and also the continuity, derivatives, and differentials of IV*q*-ROFFs. In order to resolve conflicts in Dempster-Shafer theory, Limboo and Dutta (2022) presented a framework for q-rung evidence sets, using q-rung fuzzy numbers for enhanced flexibility. A novel association coefficient measure was presented to handle conflicts and changing belief degrees using a weighted average mass method.

Dong et al. (2023) introduced the complex interval-valued q-rung orthopair fuzzy set (CIV*q*-ROFS) as a more substantial tool for covering imprecision in decision-making. They presented distance measures, Yager operational laws, and their comparison method. Additionally, the paper develops CIV*q*-ROF Yager operators and explores their properties. Ranjan et al. (2023) introduced the Probabilistic Linguistic q-Rung Ortho-Pair Fuzzy Set (PL*q*-ROFS) as an innovative approach for taking uncertainties into account in decision-making. They proposed an integrated framework that combined the Power Average Operator, Archimedean operator, and Full Consistency Method to enhance group decision-making.

2.2. Development of MCDM methods by using IV q-ROFSs

Several studies have recently employed *q*-ROFSs and IV *q*-ROFSs to develop various MCDM techniques further. In order to address uncertain knowledge in MAGDM, Xu et al. (2022) presented a method known as interval-valued probabilistic linguistic q-rung orthopair fuzzy sets (IVPL q-ROFS). In order to calculate attribute weights and rank alternatives, it provides a two-stage TOPSIS technique utilizing IVPL q-ROFS. Ali et al. (2022) developed the complex IV q-ROF Hamy mean (CIVq-ROFHM) and complex IV q-ROF weighted Hamy mean (Cq-ROFWHM) operators, which allow the combination of alternatives into a singleton set. These operators are very helpful when processing complicated data in asymmetric information decision-making contexts. In addition, the study suggests a method for integrating CIV q-ROF data in decision-making procedures. With the help of several illustrative examples, Garg (2021a) highlighted the benefits of his technique over currently used possibility measures for determining the possibility degree of comparison between two IV q-ROFSs. In order to further highlight the excellence and adaptability of the MAGDM approach, Garg (2021a) augmented it with numerical examples to rank alternatives. Khan et al. (2021) introduced linguistic interval-valued q-rung orthopair fuzzy sets (LIVq-ROFS) as a generalization of linguistic q-rung orthopair fuzzy sets. They developed basic operations and aggregation methods for comparing LIVq-ROF values, along with a TOPSIS method for MCDM under LIVq-ROFS environment. An innovative strategy for selecting green suppliers on a big scale is presented by Liu et al. (2019). The supplier evaluations are represented as IV q-ROFSs, DMs are grouped using clustering, and criteria weights are calculated. The best green supplier is then found by the procedure utilizing a particular optimization technique. Examples from the real world show how successful it is in selecting green suppliers on a broad scale.

Interval-valued q-rung orthopair fuzzy Dombi operators (IV q-ROFDWA) were introduced by Wan et al. (2022), the IV q-ROFDWA operator was developed with important features, and an expert weighting approach was suggested. Additionally, a novel distance measure was presented in the research, which was then used in a decision analysis technique termed IV q-ROFSs QUALIFLEX for complicated MAGDM using IV q-ROFSs. The efficiency of the approach was confirmed by a case study on managing the health of hypertension patients. The IV q-ROFSs VIKOR model was introduced by Gao et al. (2020) as a new decision-making technique. By including complicated and unpredictable scenarios that standard models are unable to handle, this approach contributed to decision-making improvements. Zulqarnain et al. (2024) introduced a novel TOPSIS approach using correlation coefficients and weighted correlation coefficients under IV q-ROFSs environment. Through a numerical case about the selection of Cloud Service Providers (CSPs) in cloud service management, the suggested TOPSIS approach is shown to be a robust MCDM tool. Garg et al. (2021b) proposed Muirhead mean and dual Muirhead mean operators under the CIVq-ROF environment. These operators capture multi-attribute correlations and provide versatility through a parameter vector. The paper discusses the operators' advantages, properties, and exceptional cases, presenting a novel CIVq-ROF MAGDM method. Xu (2023a) introduced a novel two-stage decision-making approach to handle environmental concerns related to discarded bike-sharing systems. The approach was using IVq-ROF Einstein operators in the first stage, and a MCDM approach based on TOPSIS under an IVq-ROF environment in the second stage.

Additionally, a lot of studies generalized the concept of IV q-ROFSs by introducing new distance measures (Deveci et al., 2022; Garg et al., 2021a; Gong, 2023; Kamacı and Petchimuthu, 2022; Zeng et al., 2021), aggregation operators (Farid and Riaz, 2023; Garg, 2021b; Hayat et al., 2023; Khan et al., 2023; Peng, 2023; Qiyas et al., 2023; Sarkar et al., 2023; Wan and Lu, 2022), and combination with MCDM techniques (Akram et al., 2023; Jin et al., 2021; Naz et al., 2023; Peng et al., 2023; Qahtan et al., 2023; Siddiqui and Haroon, 2023; Wan et al., 2022; Xu, 2023b; Xu et al., 2022). Table (1) represents the literature of most relevant studies that utilized IVq-ROFSs and possibility degrees.

2.3. Research gaps

There are still gaps that need further research despite all the advances and effectiveness that prior approaches brought to the state-of-the-art literature for the VIKOR approach. The difficulty of fuzzy processing in VIKOR is influenced by the design of fuzzy membership functions, the selection of fuzzy sets, and the aggregation of fuzzy information. Since the complexity increases with higher number of options and criteria, large-scale decision-making problems need rigorous study and efficient algorithms. The following motivations are derived from the literature analysis:

1) One of the main disadvantages of the VIKOR approach is its significant dependence on the normalization process and large impact of it on the final rankings. The VIKOR method mandates that the criteria

Table (1)
Literature of most relevant studies that utilized IVq-ROFSs and possibility degrees.

Reference	MCDM approach	Case study	Type of fuzzy set	Type of calculation
Chen (2014)	QUALIFLEX	Selection of a suitable bridge construction	IVIFSs	Possibility degrees
Wang et al. (2015)	QUALIFLEX	Selection of medical treatment options	Interval type-2 fuzzy sets (IT2FSs)	Possibility degrees
Chen (2015)	PROMETHEE	Landfill site selection and car evaluation problem	IT2FSs	Possibility degrees
Yumin et al. (2017)	MABAC	Selecting Hotels on a Tourism Website	IT2FSs	Possibility degrees
Liu et al. (2019)	MULTIMOORA	Green supplier selection	IVq-ROFSs	Fuzzy calculation
Narayanamoorthy et al. (2019)	VIKOR	Industrial robots' selection	Interval-valued intuitionistic hesitant fuzzy sets (IVIHFSs)	Fuzzy calculation
Gao et al. (2020)	VIKOR	Supplier selection of medical consumption products	IVq-ROFSs	Fuzzy calculation
Arya and Kumar (2021)	TODIM-VIKOR	Medical consumption products	q-ROFSs	Fuzzy calculation
Salimian et al. (2022)	VIKOR-MARCOS	Healthcare Devices	IVIFSs	Fuzzy calculation
Seker et al. (2023)	COPRAS	Analyzing risk factors to overcome pandemic	IVq-ROFSs	Fuzzy calculation
Quek et al. (2023)	VIKOR-TOPSIS	COVID-19 pandemic	Interval-valued neutrosophic soft set (IVNSSs)	Fuzzy calculation
Hendiani and Walther (2023)	TOPSIS	Bridge risk assessment	IVIFSs	Possibility degrees
Singh and Kumar (2023)	VIKOR-TOPSIS	Wastewater case	Picture fuzzy	Fuzzy calculation
Al-Quran et al. (2023)	VIKOR-ELECTRE II	Freight transportation	Cubic bipolar fuzzy	Fuzzy calculation
Deveci et al. (2023)	VIKOR	Transportation system	Type-2 neutrosophic fuzzy	Fuzzy calculation
Luqman and Shahzadi (2023)	Superiority and inferiority	Green supplier selection	IVq-ROFSs	Fuzzy calculation

values be standardized to a common scale before the analysis. Different normalization techniques can lead to different rankings and assessments, which can have an impact on the outcomes (Wieckowski and Salabun, 2020). For example, applying the vector normalization strategy may produce a final ranking of alternatives with respect to a defined set of criteria, while employing the min-max normalization method might result in a different ranking of the same alternatives with the same criteria.

2) Most of the studies that employ the fuzzy VIKOR for solving a case study process the data in the form of complex fuzzy sets (Al-Quran et al., 2023; Deveci et al., 2023; Narayanamoorthy et al., 2019; Quek et al., 2023). Processing fuzzy set into the VIKOR approach can be difficult in practice for a number of reasons. First of all, especially for large datasets, the complexity of fuzzy set operations, such as fuzzy arithmetic and aggregation, can significantly raise computational requirements. Second, the understanding and transmission of the fuzzy results are complicated in practice. DMs may misunderstand or misinterpret information if more work and experience is required to transform fuzzy set-based rankings and suggestions into practical judgments.
3) Another problem is that the other existing MCDM techniques might not be able to produce a distinct ranking of alternatives when the alternatives are performing similarly in terms of some criteria (Garg, 2021a). The information in the form of fuzzy sets, which is gathered for the alternatives in response to the criteria, might be difficult to interpret in certain situations when there is insignificant difference between the alternatives. The existing decision matrices indicate similar results for some alternatives, making them unable to obtain a final ranking order. Accurate evaluation of alternatives is important because judgments made on the basis of faulty rankings may result in suboptimal results or missed opportunities. It's critical to develop new MCDM techniques that can correctly rank alternatives, even when there aren't many distinctions between them, in order to solve these issues.

The mentioned gaps from the literature open the room for studies that can address the existing drawbacks to develop more effective methods. First, as it is obvious from the literature, only a few studies considered the advantages of IVq-ROPFSs for enhancing the VIKOR approach (Arya and Kumar, 2021; Gao et al., 2020). The combination of IVq-ROPFSs possibility degrees and the VIKOR approach presents a reinforced MCDM tool, which overtakes previous methods in addressing decision making problems with any number of criteria and alternatives. Second, the proposed study introduces a novel possibility degree-based VIKOR with a neutralized decision matrix, which eliminates the fuzzy normalization stage of previous VIKOR approaches. Unlike the other existing studies which normalized the criteria using different methods in the early stages of the VIKOR approach, this study proposes a unique neutralized decision matrix which is normalized based on the concept of possibility degrees. Third, determination of positive and negative ideal solutions, which were carried out with fuzzy relations in previous methods (Al-Quran et al., 2023; Narayanamoorthy et al., 2019; Salimian et al., 2022; Singh and Kumar, 2023), are reduced to crisp values that are obtained by possibility degrees in the new decision matrix. Finally, the proposed method is much more accurate in terms of alternative ranking, since it no longer depends on IVq-ROPFSs in their fuzzy form. The fuzzy information for addressing the performance of the alternatives in response to the criteria may be similar in some cases (Garg, 2021a), but the proposed approach calculates unique possibility degrees and increases the ability of VIKOR approach to rank alternatives even in cases where the difference between alternatives is minor.

3. The proposed IVq-ROFS possibility degree based VIKOR approach

In this section, the proposed IVq-ROFS possibility degree based VIKOR approach is modeled in different steps.

3.1. Preliminaries of IVq-ROFS

Before describing the implementation steps, some basic concepts about IFS, PFS and q-ROFS are mentioned that are required for the modeling process.

Consider X as a fixed set. A q-ROFS $\widetilde{Q}$ in X is a component with the form $\widetilde{Q} = \{\langle x, \tau_{\widetilde{Q}}(x), \varphi_{\widetilde{Q}}(x)\rangle | x \in X\}$ in which $\tau_{\widetilde{Q}}(x)$ and $\varphi_{\widetilde{Q}}(x)$ are the degrees of membership and non-membership, respectively, of the element $x \in X$ to $\widetilde{Q}$, which satisifies the following term (Yager, 2017):

$$0 \leq (\tau_{\widetilde{Q}}(x))^q + (\varphi_{\widetilde{Q}}(x))^q \leq 1 \tag{1}$$

The hesitation degree $\pi_{\widetilde{Q}}(x)$ of the element $x \in X$ to $\widetilde{Q}$ is also defined as follows:

$$\pi_{\widetilde{Q}}(x) = \sqrt[q]{1 - (\tau_{\widetilde{Q}}(x))^q - (\varphi_{\widetilde{Q}}(x))^q} \tag{2}$$

If $q = 1$ the q-ROFS reduces to IFS. Similarly, if $q = 2$ the q-ROFS reduces to PFS. Figure (1) displays a graphical description of IFS, PFS, and q-ROFS.

Suppose X is a typical finite non-empty set. An IV q-ROFSs $\widetilde{I}$ in X is a component with the form $\widetilde{I} = \{\langle x, \tau_{\widetilde{I}}(x), \varphi_{\widetilde{I}}(x)\rangle | x \in X\}$. The membership function maps a set of values to satisfy $\tau_{\widetilde{I}}(x) = [\tau^-_{\widetilde{I}}(x), \tau^+_{\widetilde{I}}(x)] \subseteq [0, 1]$ and the non-membership function maps a set of values to satisfy $\varphi_{\widetilde{I}}(x) = [\varphi^-_{\widetilde{I}}(x), \varphi^+_{\widetilde{I}}(x)] \subseteq [0, 1]$.

The hesitation degree $\pi_{\widetilde{I}}(x)$ of the element $x \in X$ to $\widetilde{I}$ is also defined as follows:

$$\pi_{\widetilde{I}}(x) = [\pi^-_{\widetilde{I}}(x), \pi^+_{\widetilde{I}}(x)] = \left[\sqrt[q]{1 - (\tau^+_{\widetilde{I}}(x))^q - (\varphi^+_{\widetilde{I}}(x))^q}, \sqrt[q]{1 - (\tau^-_{\widetilde{I}}(x))^q - (\varphi^-_{\widetilde{I}}(x))^q}\right] \tag{3}$$

Let $\widetilde{I}_1 = ([\tau^-_1, \tau^+_1], [\varphi^-_1, \varphi^+_1])$, $\widetilde{I}_2 = ([\tau^-_2, \tau^+_2], [\varphi^-_2, \varphi^+_2])$ and $\widetilde{I} = ([\tau^-, \tau^+], [\varphi^-, \varphi^+])$ be three IVq-ROFSs. The mathematical operations over these IVq-ROFSs are defined as follows (Wang et al., 2019):

Addition:

$$\widetilde{I}_1 \oplus \widetilde{I}_2 = \left(\left[\sqrt[q]{(\tau^-_1)^q + (\tau^-_2)^q - (\tau^-_1)^q(\tau^-_2)^q}, \sqrt[q]{(\tau^+_1)^q + (\tau^+_2)^q - (\tau^+_1)^q(\tau^+_2)^q}\right], [\varphi^-_1 \times \varphi^-_2, \varphi^+_1 \times \varphi^+_2]\right) \tag{4}$$

Multiplication:

$$\widetilde{I}_1 \otimes \widetilde{I}_2 = \left([\tau^-_1 \times \tau^-_2, \tau^+_1 \times \tau^+_2], \left[\sqrt[q]{(\varphi^-_1)^q + (\varphi^-_2)^q - (\varphi^-_1)^q(\varphi^-_2)^q}, \sqrt[q]{(\varphi^+_1)^q + (\varphi^+_2)^q - (\varphi^+_1)^q(\varphi^+_2)^q}\right]\right) \tag{5}$$

Constant multiplication:

$$\lambda\widetilde{I} = \left(\left[\sqrt[q]{1 - (1 - (\tau^-)^q)^\lambda}, \sqrt[q]{1 - (1 - (\tau^+)^q)^\lambda}\right], \left[(\varphi^-)^\lambda, (\varphi^+)^\lambda\right]\right) \tag{6}$$

3.2. The steps of methodology

Fig. 2 demonstrates the structural representation of the proposed approach with the step-by-step guide for implementing both layers. The first layer shows the data collection and establishment of the first decision matrix based on experts' aggregated judgements. The second layer also exhibits the calculation of possibilities creation of the possibility degree-based decision matrix. The calculations of the new VIKOR indicators are also included in the second layer.

The steps of the proposed approach are listed in detail below:

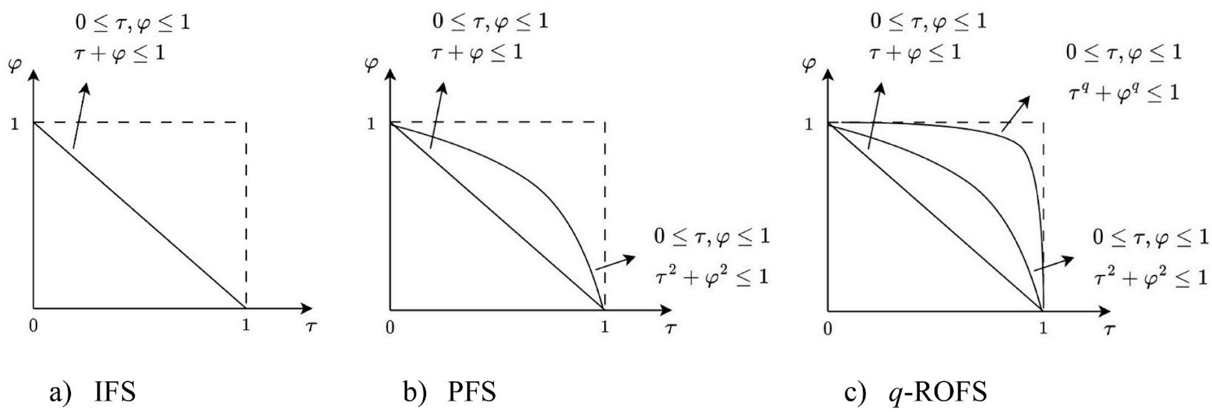


Fig. (1). Graphical representation of IFS, PFS, and q-ROFS.

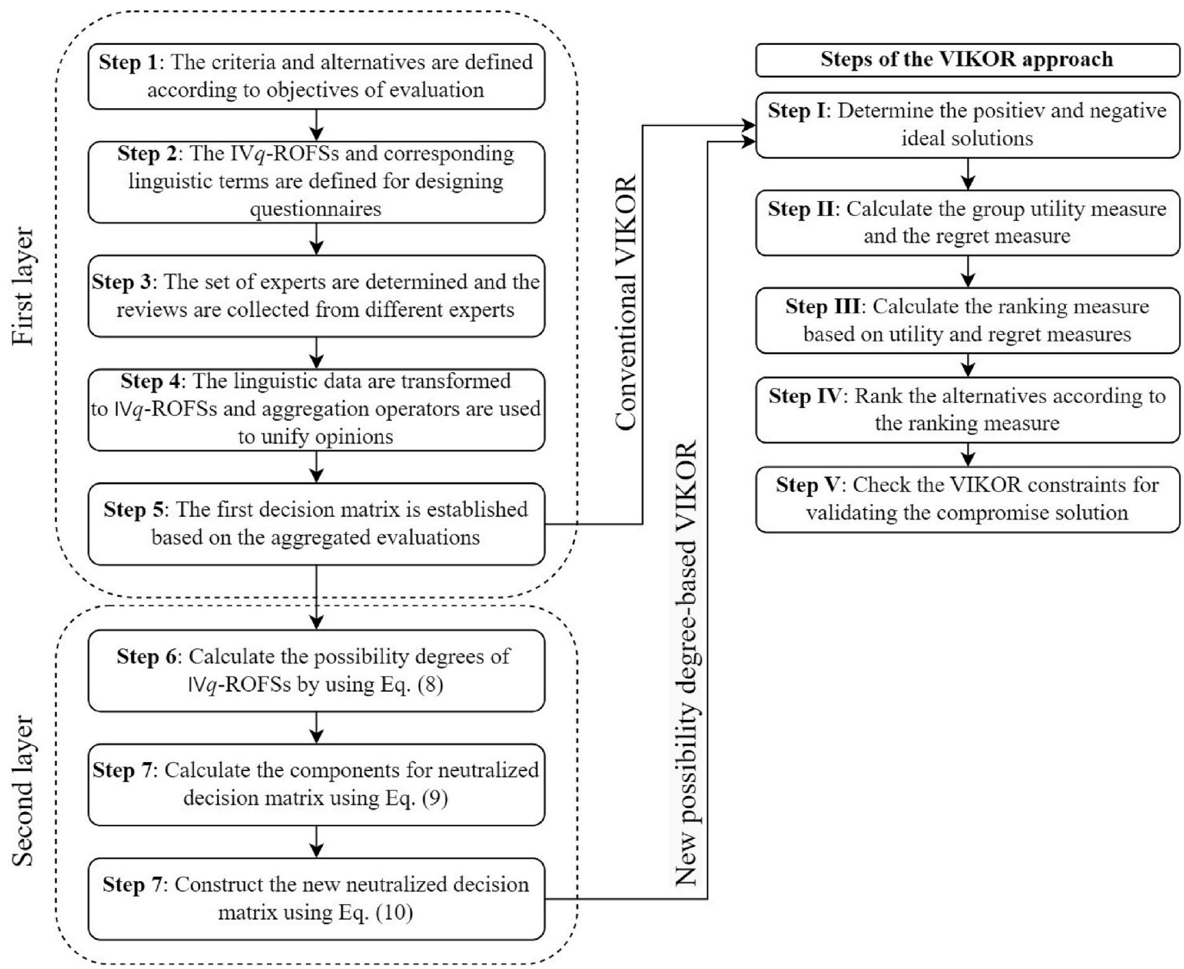


Fig. (2). The framework of the proposed approach.

Step 1. Collection of reviews

The first step towards the implementation of the proposed approach is to collect the relevant data for the mathematical process. Two types of data are required for this methodology which are the criteria weights and performances of alternatives in response to the criteria. The weights of the criteria are usually determined by DMs themselves according to their aims and objectives of decision-making. The performances of alternatives, however, are usually collected from different experts who are specialists in the field. The experts will be provided with appropriate questionnaires and surveys which are designed in a way that require linguistic answers for every question. The linguistic terms are then transformed into IV q-ROFS for the calculation process. A few linguistic-to- IV q-ROFS tables can be found in (Jin et al., 2021; Liu et al., 2019; Seker et al., 2023).

Step 2. Aggregation of reviews

In this step, the different reviews from experts are merged to provide a single value that incorporates all uncertainties relating to the subjective opinions of various experts.

Let $\widetilde{I_l} = ([\tau_l^-, \tau_l^+], [\varphi_l^-, \varphi_l^+])$, $(l = 1, 2, \ldots, k)$ be a set of expert judgements in the form of IV q-ROFSs whose weight vector is $\theta = (\theta_1, \theta_2, \ldots, \theta_k)$ and $\sum_{j=1}^{k} w_j = 1$. The IV q-ROFSs weighted arithmetic mean is defined as follows (Wang et al., 2019):

$$\mathrm{IV}q-\mathrm{ROFWAM}\ (\widetilde{I_1}, \widetilde{I_2}, \ldots, \widetilde{I_k}) = \left(\left[\sqrt[q]{1 - \prod_{l=1}^{k} (1 - (\tau_l^-)^q)^{\theta_j}}, \sqrt[q]{1 - \prod_{l=1}^{k} (1 - (\tau_l^+)^q)^{\theta_j}} \right], \left[\prod_{l=1}^{k} (\varphi_l^-)^{\theta_j}, \prod_{l=1}^{k} (\varphi_l^+)^{\theta_j} \right] \right) \tag{7}$$

Step 3. Calculation of possibility degrees

For two IVq-ROFSs $\widetilde{I_1} = ([\tau_1^-, \tau_1^+], [\varphi_1^-, \varphi_1^+])$, $\widetilde{I_2} = ([\tau_2^-, \tau_2^+], [\varphi_2^-, \varphi_2^+])$, the possibility degree $P(\widetilde{I_1} \geq \widetilde{I_2})$ is calculated as follows:

$$P(\widetilde{I_1} \geq \widetilde{I_2}) = \frac{\max\left(0, 1 - (\tau_2^-)^q \left(1 - (\varphi_2^+)^q\right) - (\varphi_1^-)^q \left(1 - (\tau_1^+)^q\right)\right) - \max\left(0, (\tau_1^-)^q \left(1 - (\varphi_1^+)^q\right) + (\varphi_2^-)^q \left(1 - (\tau_2^+)^q\right) - 1\right)}{2 - (\tau_2^-)^q (1 - (\varphi_2^+)^q) - (\varphi_1^-)^q (1 - (\tau_1^+)^q) - (\tau_1^-)^q (1 - (\varphi_1^+)^q) - (\varphi_2^-)^q (1 - (\tau_2^+)^q)} \tag{8}$$

Using the equation above, the possibility degrees of each pair of alternatives in response to every criterion is calculated. The possibility degree equation $P(\widetilde{I_1} \geq \widetilde{I_2})$ between two IVq-ROFSs $\widetilde{I_1} = ([\tau_1^-, \tau_1^+], [\varphi_1^-, \varphi_1^+])$, $\widetilde{I_2} = ([\tau_2^-, \tau_2^+], [\varphi_2^-, \varphi_2^+])$ satisfies the following circumstances:

1. $0 \leq P(\widetilde{I_1} \geq \widetilde{I_2}) \leq 1$.
2. $P(\widetilde{I_1} \geq \widetilde{I_2}) = 0.5$ if $\widetilde{I_1} = \widetilde{I_2}$.
3. $P(\widetilde{I_1} \geq \widetilde{I_2}) + P(\widetilde{I_2} \geq \widetilde{I_1}) = 1$.

Step 4. Construction of the new possibility degree-based decision matrix

The alternative ψ_i outperforms in terms of a benefit criterion $c_j \in C^+$ if $\widetilde{I}_{i,j}$ has a high possibility of being greater than or equal to $\widetilde{I}_{\acute{i},j}$ for other $n-1$ alternatives, for $\acute{i} = 1, 2, \ldots, n-1$ and $i \neq \acute{i}$. In contrast, alternative ψ_i performs better in a cost criterion $c_j \in C^-$, if $\widetilde{I}_{i,j}$ has a high possibility of being less than or equal to $\widetilde{I}_{\acute{i},j}$ for other $n-1$ alternatives. The possibility degree-based decision matrix component $P_{\widetilde{I}_{i,j}}$ for benefit and cost criteria will be obtained by [3]:

$$P_{\widetilde{I}_{i,j}} = \begin{cases} \sum_{\acute{i}=1, \acute{i}\neq i}^{n} P\left(\widetilde{I}_{i,j} \geq \widetilde{I}_{\acute{i},j}\right) & \text{if } c_j \in C^+ \\ \sum_{\acute{i}=1, \acute{i}\neq i}^{n} P\left(\widetilde{I}_{\acute{i},j} \geq \widetilde{I}_{i,j}\right) & \text{if } c_j \in C^- \end{cases} \tag{9}$$

Information on alternative ratings with respect to criteria is contained in the decision matrix, which has n rows and m columns. The number of different alternatives is represented by the rows, while the number of criteria is represented by the columns. Assume that the possibility degree-based decision matrix component for alternative i in response to criterion j is $P_{\widetilde{I}_{i,j}}$. Following is the structure of the new decision matrix:

$$D_{n,m} = \begin{bmatrix} P_{\widetilde{I}_{1,1}} & P_{\widetilde{I}_{1,2}} & \cdots & P_{\widetilde{I}_{1,m}} \\ P_{\widetilde{I}_{2,1}} & P_{\widetilde{I}_{2,2}} & \cdots & P_{\widetilde{I}_{2,m}} \\ \vdots & \vdots & \vdots & \vdots \\ P_{\widetilde{I}_{n,1}} & P_{\widetilde{I}_{n,2}} & \cdots & P_{\widetilde{I}_{n,m}} \end{bmatrix} \tag{10}$$

Step 5. Determination of ideal solutions

The highest value in each column, which reflects the value that responds to criterion j the best, will be picked to determine the positive vector P^+ as follows:

$$P^+ = \left(P_{\widetilde{I}_1}^+, P_{\widetilde{I}_2}^+, \ldots, P_{\widetilde{I}_m}^+\right) = \left(\max\left[P_{\widetilde{I}_{1,1}}, P_{\widetilde{I}_{2,1}}, \ldots, P_{\widetilde{I}_{n,1}}\right], \max\left[P_{\widetilde{I}_{1,2}}, P_{\widetilde{I}_{2,2}}, \ldots, P_{\widetilde{I}_{n,2}}\right], \ldots, \max\left[P_{\widetilde{I}_{1,m}}, P_{\widetilde{I}_{2,m}}, \ldots, P_{\widetilde{I}_{n,m}}\right]\right) \tag{11}$$

Similarly, the lowest value in each column, which reflects the value that responds to criterion j the worst, will be picked to determine the positive vector P^- as follows:

$$P^- = \left(P_{\widetilde{I}_1}^-, P_{\widetilde{I}_2}^-, \ldots, P_{\widetilde{I}_m}^-\right) = \left(\min\left[P_{\widetilde{I}_{1,1}}, P_{\widetilde{I}_{2,1}}, \ldots, P_{\widetilde{I}_{n,1}}\right], \min\left[P_{\widetilde{I}_{1,2}}, P_{\widetilde{I}_{2,2}}, \ldots, P_{\widetilde{I}_{n,2}}\right], \ldots, \min\left[P_{\widetilde{I}_{1,m}}, P_{\widetilde{I}_{2,m}}, \ldots, P_{\widetilde{I}_{n,m}}\right]\right) \tag{12}$$

The next goal of the VIKOR approach is to use normalized Euclidean distance combined with the weights of criteria to determine the group utility measure S_i and the regret measure R_i for each of the potential alternatives:

$$S_i = \sum_{j=1}^{t} w_j \frac{d\left(P^+, P_{\widetilde{I}_{i,j}}\right)}{d(P^+, P^-)} \tag{13}$$

$$R_i = \max_j w_j \frac{d\left(P^+, P_{\widetilde{I}_{i,j}}\right)}{d(P^+, P^-)} \tag{14}$$

The maximum and minimum values of S_i and R_i are computed as follows:

$$S^+ = \max_i S_i \tag{15}$$

$$S^- = \min_i S_i \tag{16}$$

$$R^+ = \max_i R_i \tag{17}$$

$$R^- = \min_i R_i \tag{18}$$

The characteristics of the group utility S_i and individual regret R_i are integrated as follows in order to evaluate the ranking measure χ_i for the alternative A_i:

$$\chi_i = \lambda \left(\frac{S_i - S^-}{S^+ - S^-}\right) + (1 - \lambda) \left(\frac{R_i - R^-}{R^+ - R^-}\right) \tag{19}$$

When evaluating the compromise solution, the parameter λ, which represents the weight of strategy for the majority of the criterion (the

Input:
- Weights of experts: θ_l;
- Criteria weights: w_j;
- Performance of alternatives in responding to the criteria: $\tilde{I}_{i,j}$;

Output:
- Possibilities of pairwise comparisons: $P(\tilde{I}_{i,j} \geq \tilde{I}_{i\prime,j})$;
- New possibility degree-based decision matrix: $D_{n\times m}$;
- Ranking of alternatives

Begin
1 Change the linguistic data to IVq-ROFSs for weights and performances
2 Aggregate several judgements using IVq-ROFWAM $(\tilde{I}_1, \tilde{I}_2, \ldots, \tilde{I}_n)$
3 Establish the first decision matrix
4 Calculate the possibilities of pairwise comparisons $P(\tilde{I}_{i,j} \geq \tilde{I}_{i\prime,j})$
5 **if** the possibilities satisfy the rules, then:
  Establish the possibility degree-based $D_{n\times m}$ by computing the components $P_{\tilde{I}_{i,j}}$
 elseif the possibilities don't satisfy the rules then:
  Recalculate the possibilities $P(\tilde{I}_{i,j} \geq \tilde{I}_{i\prime,j})$
 end if
6 Regulate the positive P^{+} and negative P^{-} ideal solutions
7 Calculate VIKOR indicators S_i and R_i
8 Obtain max and min values S^{+}, S^{-}, R^{+}, and R^{-}
9 Calculate the ranking measure χ_i
10 Rank the alternatives according to χ_i
11 **if** $\chi_{A^{(2)}} - \chi_{A^{(1)}} \geq 1/(n-1)$ and $A^{(1)}$ is also ranked first according to S or R values
  The compromise alternative contains the alternative $A^{(1)}$
 elseif $\chi_{A^{(2)}} - \chi_{A^{(1)}} < 1/(n-1)$ or $A^{(1)}$ is not ranked first according to S or R values
  The compromise alternative does not contain the alternative $A^{(1)}$
 end if

End

largest group utility), is crucial.

$A^{(1)}$ and $A^{(2)}$ stand for the alternatives that, in terms of χ, are ranked first and second, respectively.

The compromise alternative contains the alternative $A^{(1)}$ if the following conditions are satisfied:

1. $\chi_{A^{(2)}} - \chi_{A^{(1)}} \geq DQ$, where $DQ = \frac{1}{n-1}$ and n defines the total number of alternatives.
2. The alternative $A^{(1)}$ is also ranked first in the ranking sequence according to S or R values.

The detailed framework of the proposed approach is given as the algorithm below:

Algorithm. The new IV q-ROFSs possibility degree-based VIKOR approach

4. Implementation and validation of the method

In this section, the developed method is first implemented and evaluated by applying it to a real-world case on the selection of a cement company. Herein, the cement company selection is elaborated step-by-step to clearly demonstrate the implementation steps (4.1). Afterwards, the results ae compared to results of other methods, and a sensitivity analysis is performed to evaluate the developed method (4.2). Finally, the developed method is validated more thoroughly by demonstrating applicability and superiority of the proposed approach compared to other methods based on four numerical cases from literature. While two cases validate the approach as the same results can be obtained as with other methods, two other examples show that the method dominates previous approaches as it is able to generate results where other methods are not successful (4.3).

4.1. Implementation steps of the cement company selection case

Choosing the appropriate cement is crucial since it has a direct impact on the strength and durability of concrete structures. Concrete's primary component, cement, provides the binding qualities that keep the mixture together. A crucial element in guaranteeing the security and dependability of built structures, including buildings, bridges, roads, and other infrastructure, is the type of cement used, which determines the strength, longevity, and overall performance of the concrete. Strongness, setting time, resilience to environmental conditions, and other criteria vary depending on the kind of building project. By using the proper cement type suited to these particular requirements, the danger of early cracking, structural failure, or deterioration is reduced and the final concrete meets or exceeds the required criteria. Additionally, the characteristics of the cement used have a direct impact on how long a construction will last. The durability of the building may be

Table (2)
IV q-ROFSs decision matrix for the cement company selection.

	C_1	C_2	C_3	C_4
ψ_1	([0.31, 0.31], [0.89, 0.89])	([0.32, 0.32], [0.88, 0.88])	([0.30, 0.30], [0.86, 0.86])	([0.28, 0.28], [0.85, 0.85])
ψ_2	([0.29, 0.29], [0.78, 0.78])	([0.77, 0.77], [0.28, 0.28])	([0.28, 0.28], [0.76, 0.76])	([0.26, 0.26], [0.71, 0.71])
ψ_3	([0.56, 0.56], [0.88, 0.88])	([0.87, 0.87], [0.57, 0.57])	([0.57, 0.57], [0.86, 0.86])	([0.55, 0.55], [0.88, 0.88])
ψ_4	([0.33, 0.33], [0.95, 0.95])	([0.94, 0.94], [0.30, 0.30])	([0.30, 0.30], [0.92, 0.92])	([0.28, 0.28], [0.89, 0.89])
ψ_5	([0.96, 0.96], [0.50, 0.50])	([0.95, 0.95], [0.34, 0.34])	([0.93, 0.93], [0.32, 0.32])	([0.90, 0.90], [0.21, 0.21])

increased by carefully selecting a cement and adding the right additives to boost resistance to elements like chemical exposure, wear and tear, and freeze-thaw cycles.

Given that cement is one of the key components of construction materials, this article addresses the case study proposed by Khan et al. (2022) for choosing a cement company from a list of companies. There is high competition between cement producers around the world. Companies are investigating research and development to produce cutting-edge, environmentally friendly cement formulas with enhanced strength and durability in order to gain an advantage. The intense competition they face forces them to improve their manufacturing procedures, client interactions, and sustainability initiatives while also adjusting to changing building styles and environmental issues. It might be difficult to choose the best cement manufacturer, particularly under ambiguous circumstances. Multiple factors are taken into account and experts are involved in MAGDM. Finding criteria, weighting them, grading, aggregation, and expert insights are some of the steps. By methodically evaluating available alternatives and taking uncertainties into consideration, MAGDM aids in the making of decisions.

Consider that we are trying to rank five alternatives ψ_i $(i = 1, 2, .., 5)$ and choosing one of them to be the future cement manufacturer according to Khan et al. (2022). We have taken into account the following criteria, including: C_1: "The life of the cement", C_2: "The fineness of the cement", C_3: "The handling storage of cement", and C_4: "The effect of climate on cement" with the weight vector (0.34, 0.26, 0.24, 0.16) for the four criteria C_j $(j = 1, 2, 3, 4)$ (Khan et al., 2022). It is important to note that the parameter $q = 3$ is taken into account for IV q-ROFSs which represent the performance of each alternative in response to the criteria. Table (2) displays the decision matrix collected from experts in the form of IV q-ROFSs from Khan et al. (2022).

The steps of implementing the proposed approach for the cement company selection are elaborated as below:

Step 1. Data preparation

After the problem has been identified, the criteria must first be screened to determine whether they are all relevant for the review. To avoid any impurity of the gathered data, the most pertinent criteria should be filtered out. To rank alternatives, the novel VIKOR technique combines the weights of the criteria with their performances. Thus, two types of data are required to initiate the process. First, the data concerning weights of criteria are required which usually are determined by internal DMs who are evaluating the alternatives to select the best performing one. This responsibility is given to DMs because the choice must fit with the DMs' aims and objectives. The decision is more likely to represent their intended results if they are given the opportunity to prioritize the criterion. Second, information gathered about the performance of alternatives from external experienced experts. For the aforementioned case, both the weights of the criteria and the performance of alternatives are collected from Khan et al. (2022).

Table (3)
The calculated possibility degrees of alternatives for the cement selection case.

	C_1	C_2	C_3	C_4
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{2j})$	0.365	0.000	0.397	0.378
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{3j})$	0.377	0.000	0.399	0.450
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{4j})$	0.654	0.000	0.617	0.565
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{5j})$	0.000	0.000	0.000	0.000
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{1j})$	0.635	1.000	0.603	0.622
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{3j})$	0.531	0.479	0.517	0.586
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{4j})$	0.768	0.247	0.711	0.682
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{5j})$	0.000	0.231	0.000	0.000
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{1j})$	0.623	1.000	0.601	0.55
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{2j})$	0.469	0.521	0.483	0.414
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{4j})$	0.787	0.219	0.731	0.623
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{5j})$	0.000	0.198	0.000	0.000
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{1j})$	0.346	1.000	0.383	0.435
$P(\widetilde{\psi}_{4j} \geq \psi_{2j})$	0.232	0.753	0.289	0.318
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{3j})$	0.213	0.781	0.269	0.377
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{5j})$	0.000	0.480	0.000	0.000
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{1j})$	1.000	1.000	1.000	1.000
$P(\widetilde{\psi}_{5j} \geq \psi_{2j})$	1.000	0.769	1.000	1.000
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{3j})$	1.000	0.802	1.000	1.000
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{4j})$	1.000	0.520	1.000	1.000

Step 2. Calculation of possibility degrees

The possibility degrees for the five potential alternatives ψ_i $(i = 1, 2, .., 5)$ in response to the four criteria C_j $(j = 1, 2, 3, 4)$ are calculated by using Eq. (8) and are gathered in Table (3). These possibilities indicate how well each alternative performs in terms of a criterion in comparison with the other alternatives. For instance, for the criterion C_1, the possibility value $P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{4j})$ equals to 0.768, indicating that there is a possibility of 0.768 that alternative 2 performs better than alternative 4 in response to criterion 1.

Figure (3) displays the relational graphs of the alternatives based on each criterion. The nodes show the number of alternatives, and the edges represent the connection between two alternatives. An arrow starting from alternative i and ending in alternative $\acute{i}$ indicates the possibility of alternative i being better or equal as alternative $\acute{i}$ in responding to the corresponding criterion $P(\widetilde{\psi}_{ij} \geq \widetilde{\psi}_{\acute{i}j})$. For instance, in **Fig (3) a)** the arrow starting from alternative 2 and ending in alternative 1 shows the possibility $P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{1j})$, which equals to 0.635 according to Table (3). These graphs are useful to display the superiority of each alternative compared to other alternatives in responding to a particular criterion.

Step 3. Establishing the new decision matrix

After calculating the possibility degrees of alternatives, the components of the new possibility degree-based decision matrix are calculated by using Eq. (9) and are shown in Table (4).

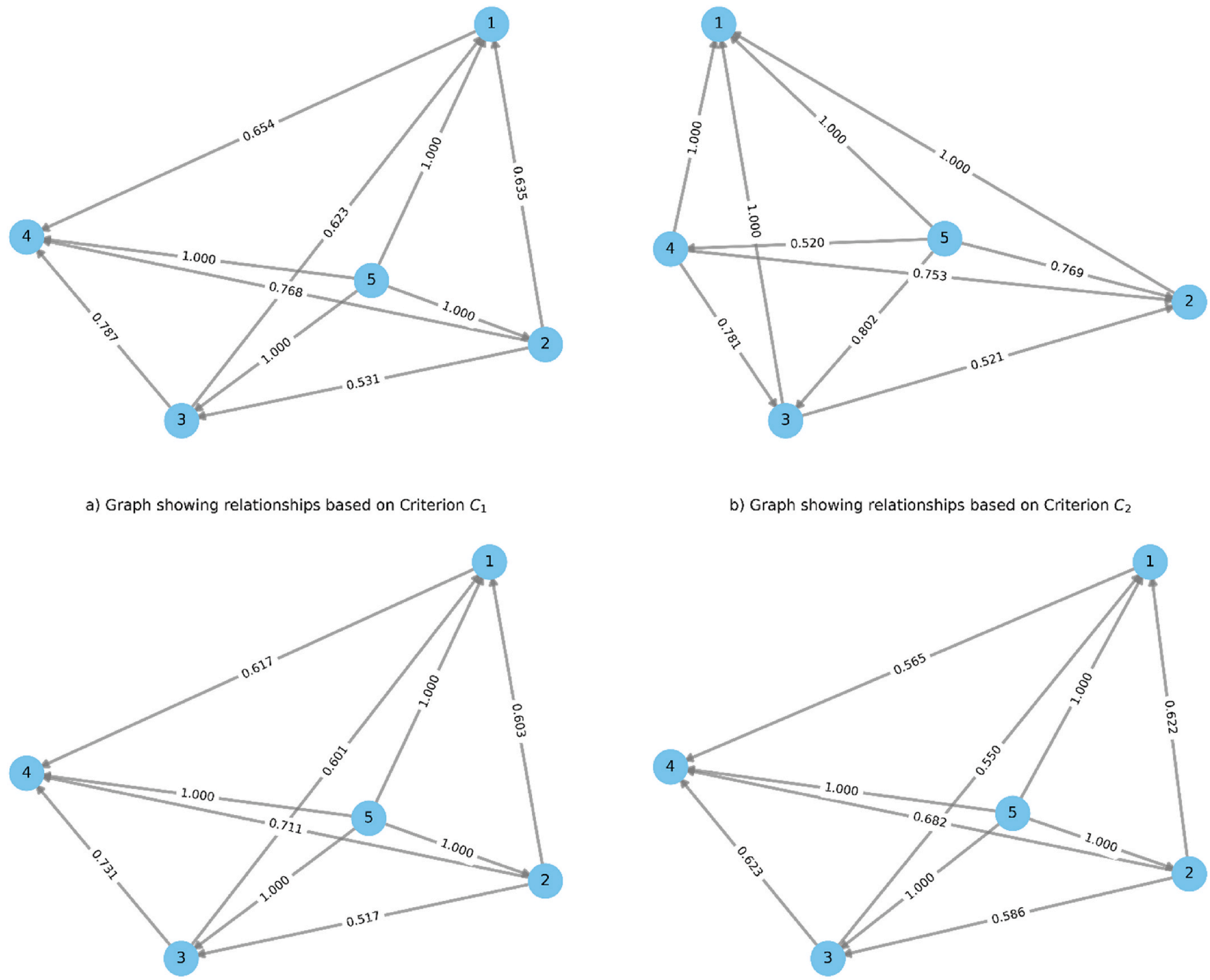
Step 4. Calculating the VIKOR indicators

Once the new decision matrix is established, the positive and negative ideal solutions are determined. The positive ideal solution is obtained by the highest value among the alternatives in responding to each criterion according to Eq. (11). Similarly, the negative ideal solution is determined by the lowest value among the alternatives in responding to each criterion according to Eq. (12). The group utility measure S_i and the regret measure R_i for each of the potential alternatives are also calculated in Table (5).

The final ranking obtained for the four alternatives with the proposed approach according to $\chi_4 > \chi_1 > \chi_3 > \chi_2 > \chi_5$ is equal to $\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$. **Figure (4)** exhibits a comparison of alternatives with positive and negative ideal solutions. This form of contrast shows the state of each alternative between the best and worst possible alternatives. The dashed line represents the alternative, the blue and red lines represent the positive and negative ideal solutions respectively. As it is obvious, alternative 5 is a complete match with the positive ideal solution line, indicating that ψ_5 is the best ideal solution in terms of all the criteria. Between the blue and red lines, other alternatives are positioned to highlight each one's strengths and weaknesses. For instance, alternative 4 contains three weaknesses in C_1, C_3 and C_4 while showing strength in C_2.

4.2. Comparative and sensitivity analysis for the cement company selection case

The proposed cement company selection case study has been resolved by using different approaches. As shown in Table (6), most of the approaches result the same ranking $\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$ for the



a) Graph showing relationships based on Criterion C_1

b) Graph showing relationships based on Criterion C_2

c) Graph showing relationships based on Criterion C_3

d) Graph showing relationships based on Criterion C_4

Fig. (3). The relational graphs of the alternatives based on each criterion.

Table (4)
The new possibility degree-based decision matrix for the cement company selection case.

	C_1	C_2	C_3	C_4
$\Sigma(\widetilde{\psi}_{1j})$	1.396	0	1.413	1.393
$\Sigma(\widetilde{\psi}_{2j})$	1.934	1.957	1.831	1.89
$\Sigma(\widetilde{\psi}_{3j})$	1.879	1.938	1.815	1.587
$\Sigma(\widetilde{\psi}_{4j})$	0.791	3.014	0.941	1.13
$\Sigma(\widetilde{\psi}_{5j})$	4	3.091	4	4

Table (5)
Calculation of the VIKOR indicators for the cement company selection case.

	C_1	C_2	C_3	C_4	S_i	R_i	χ_i
$\Sigma(\widetilde{\psi}_{1j})$	1.396	0	1.413	1.393	0.884	0.276	0.906
$\Sigma(\widetilde{\psi}_{2j})$	1.934	1.957	1.831	1.89	0.602	0.219	0.662
$\Sigma(\widetilde{\psi}_{3j})$	1.879	1.938	1.815	1.587	0.628	0.225	0.685
$\Sigma(\widetilde{\psi}_{4j})$	0.791	3.014	0.941	1.13	0.746	0.340	0.922
$\Sigma(\widetilde{\psi}_{5j})$	4	3.091	4	4	0.000	0.000	0.000
P^+	4	3.091	4	4			
P^-	0.791	0	0.941	1.13			

five cement alternatives. The characteristics of the approaches and the unique features of the case study are only two of the reasons why different approaches provide different rankings. Listed below are a few typical explanations for why various approaches may result in varying rankings:

- Criteria scalability: The sensitivity of various methodologies to a given set of criteria may vary. The rankings may be affected by some approaches that ask you to equalize the criteria or convert them to a standard scale.
- Treatment of uncertainty: Different MCDM approaches may use different measures to account for ambiguity or imprecision. The final rankings may vary depending on how different types of uncertainty are handled.
- Data completeness and quality: The findings of the MCDM analysis may also be influenced by the completeness and quality of the data

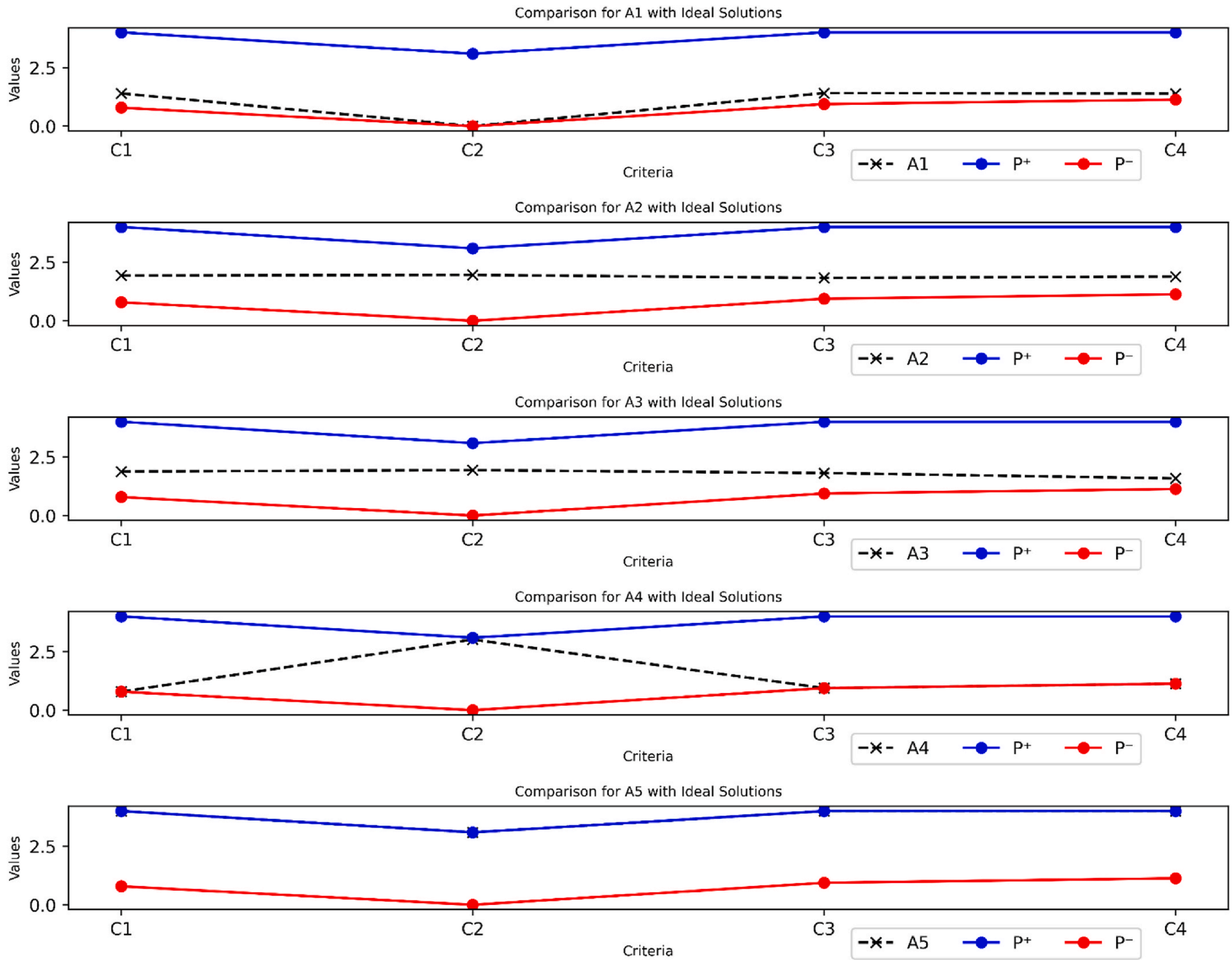


Fig. (4). Comparison of alternatives with positive and negative ideal solutions.

Table (6)
Comparative analysis for the cement selection case.

Reference and method	Ranking
Akram and Shahzadi (2021) using *q*-ROFYHWA	$\psi_5 > \psi_4 > \psi_3 > \psi_1 > \psi_2$
Akram and Shahzadi (2021) using *q*-ROFYHWG	$\psi_2 > \psi_1 > \psi_5 > \psi_3 > \psi_4$
Jana et al. (2019) using Dombi WA	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
Jana et al. (2019) using Dombi WG	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
Liu et al. (2019) using *q*-ROFWA	$\psi_5 > \psi_2 > \psi_1 > \psi_3 > \psi_4$
Liu et al. (2019) using *q*-ROFWG	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
Khan et al. (2022) using *q*-ROFAAWA	$\psi_4 > \psi_5 > \psi_2 > \psi_3 > \psi_1$
Khan et al. (2022) using *q*-ROFAAWG	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
Khan et al. (2022) using *q*-ROFAAWG with $q = 3$	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
The proposed approach	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$

utilized. Different approaches could be more or less responsive to changes in the data.

- Sensitivity to parameter values: Particular processes could have parameters that need to be calibrated or adjusted. Different rankings may be produced by minor adjustments to these parameter values.

As shown in Table (6), different fuzzy MCDM approaches generate different rankings for the five alternatives. There are various reasons for these differences in results, some of which are mentioned as follows: 1) Type of membership function selected: each one of the fuzzy MCDM approaches may employ a unique membership function to define fuzzy sets for variables. The choice of these membership functions definitely influences the final rankings. 2) Aggregation operators: Although all of the approaches deal with fuzzy sets and imprecision of data, they use different aggregation operators to combine the opinions of the different experts. The choice of the aggregation operator also impacts the final ranking. 3) Weighting method: Almost all of the MCDM approaches include weighting of criteria in their implementation steps. However, the type of weighting method and the stages in which the weights of the criteria are integrated into the model differ in each method. The variation of weighting methodologies also impacts the final rankings. 4) Amount of uncertainty handled: The handling of uncertainty differs in each fuzzy MCDM method. Some approaches are stricter in covering uncertainty while other methods are more permissive. This differentiation of handling uncertainty results different rankings.

Figure (5) also presents a graphical representation of the comparative analysis of the differences between the final rankings obtained by different methodologies. The dashed yellow line shows the ranking obtained by using the proposed VIKOR approach. As shown in **Figure (5)**, most of the approaches resulted the same ranking for the five

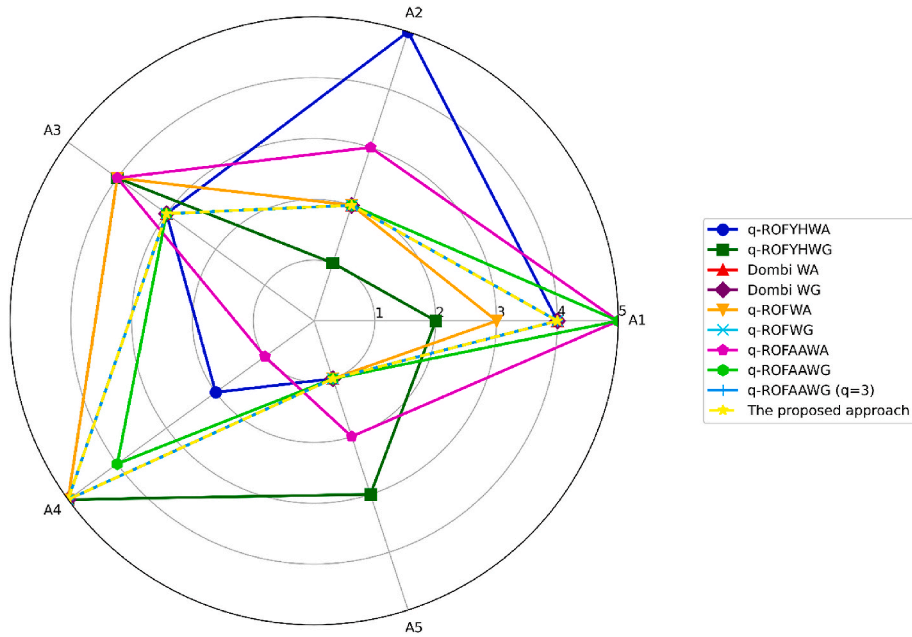


Fig. (5). Graphical representation of the comparative analysis for the cement selection case.

Table (7)
Sensitivity analysis for the cement case while q is changing between 1 and 50.

q	χ_i					Ranking
	A_1	A_2	A_3	A_4	A_5	
1	0.979	0.783	0.656	0.882	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
2	0.931	0.686	0.648	0.907	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
3	0.906	0.662	0.685	0.922	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
4	0.900	0.667	0.727	0.927	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
5	0.900	0.679	0.755	0.927	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
6	0.902	0.690	0.781	0.930	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
7	0.902	0.683	0.793	0.934	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
8	0.903	0.679	0.803	0.942	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
9	0.905	0.681	0.813	0.949	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
10	0.899	0.678	0.805	0.955	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
15	0.882	0.677	0.779	0.989	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
20	0.867	0.714	0.771	1.000	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
30	0.839	0.760	0.775	1.000	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
40	0.829	0.787	0.807	1.000	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
50	0.813	0.805	0.814	1.000	0.000	$\psi_5 > \psi_2 > \psi_1 > \psi_3 > \psi_4$

cement company alternatives $\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$, which is also aligned with the ranking that the proposed VIKOR approach obtained.

In this subsection, a sensitivity analysis is also carried out to account for changes in the final ranking of alternatives in the event that the q and λ values change. The λ parameter enables DMs to alter the ratio of the regret measure and the group utility measure in the final ranking. Table (7) displays the final rankings for each alternative after varying the value of q between 1 and 50.

Figure (6) shows the sensitivity analysis's findings when the q value fluctuates between 1 and 50. Different line slopes demonstrate how each alternative reacts to variations in q. The ranking of the alternatives $\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$ varies from the main ranking $\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$ for the q values below 3. This means that the type of uncertainty with IVIFS ($q = 1$) and IVPFSs ($q = 2$) differs from the type of uncertainty with IV q-ROFSs ($q > 2$) for this particular case. There is also a slight change in the positions of alternative 1 and 3 for $q = 50$. However, most of the values that are considered for q result the same ranking $\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$ for the alternatives, which shows the robustness of the approach.

In order to investigate the robustness of the approach further,

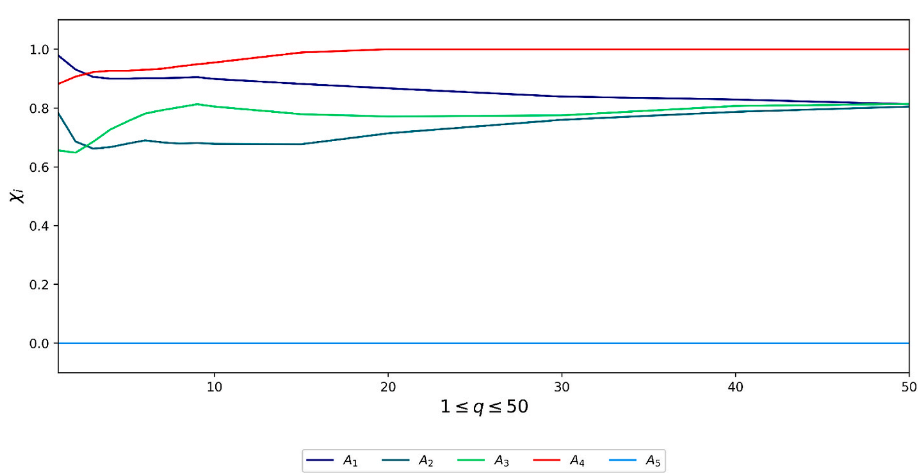


Fig. (6). The sensitivity analysis of alternative rankings when $1 \le q \le 50$.

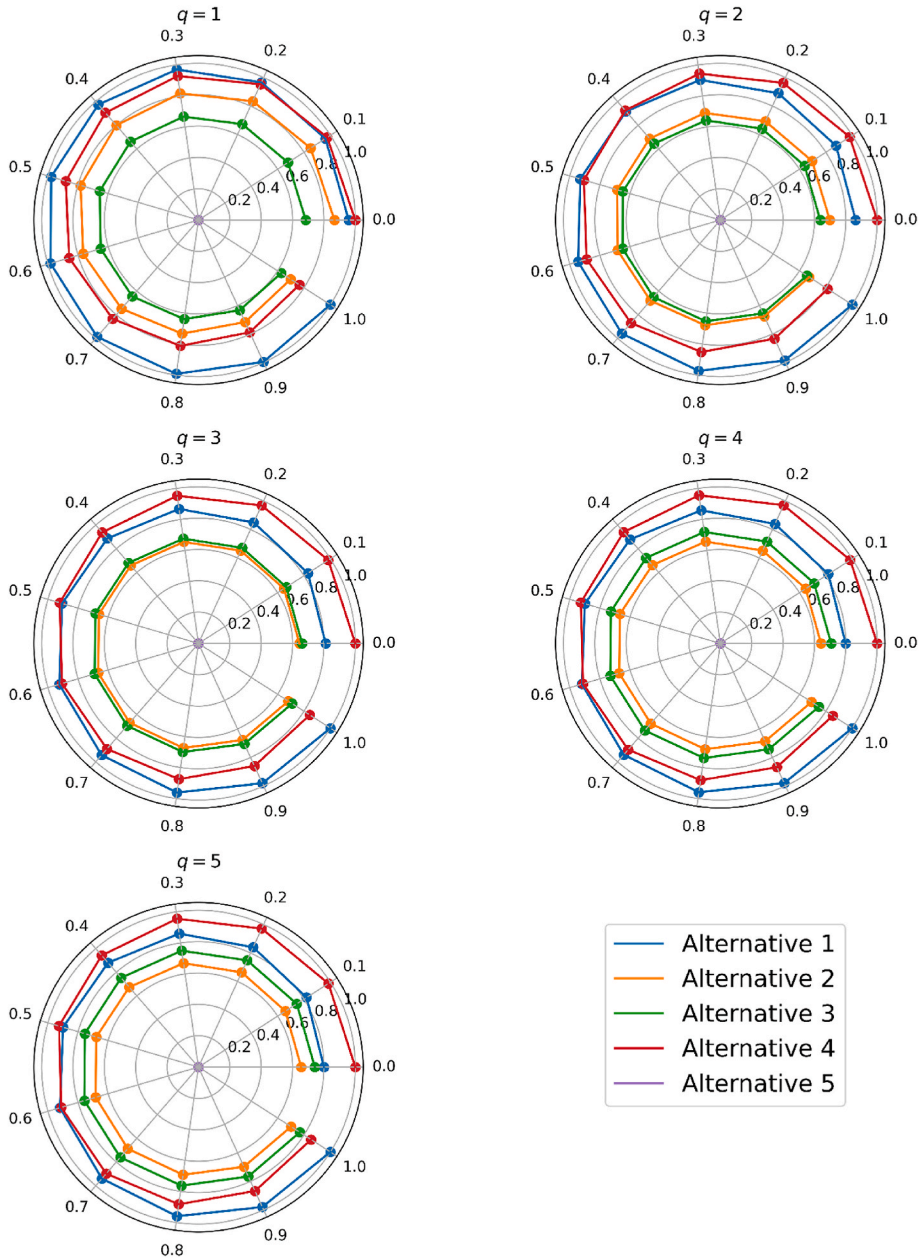


Fig. (7). Sensitivity analysis when both q and λ values change.

another sensitivity analysis is conducted which shows the final rankings by varying both q and λ values. Table (A.1) in the **Appendix** indicates the final rankings obtained when λ changes between 0 and 1 while the q value changes from 1 to 5.

Figure (7) shows the findings of sensitivity analysis when both q and λ values change. As mentioned earlier, this figure shows that if the q value is below 3, the ranking of the alternatives $\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$ varies from the main ranking $\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$. However, for $q = 1$, the final rankings slightly differ in the positions of alternative 1 and 4, while λ values are below 0.2. In other words, the main ranking obtained for five alternatives while $q = 1$ and $\lambda > 0.2$ is $\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$, if the λ value is considered below 0.2, the final ranking changes to $\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$. Similarly, for $q = 2$, the final rankings again slightly differ in the positions of alternative 1 and 4, while λ values are below 0.4. The main ranking obtained for five alternatives while $q = 2$ and $\lambda > 0.4$ is $\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$, if the λ value is considered below 0.4, the final ranking changes to $\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$.

On the other hand, for $q \geq 3$, the final rankings are obtained $\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$ while $\lambda \leq 0.5$. If the λ values are considered higher than 0.5, the final rankings slightly change to $\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$. These slight changes show the sensitivity of alternatives 1 and 4 to higher values for λ. By increasing the λ value, the balance between the group utility S_i and individual regret R_i indicators cause the change in the final values of χ_i which finally changes the ranking of the alternatives. However, in order to make general balance between S_i and R_i, the lambda value is usually considered as 0.5 ($\lambda = 0.5$).

In addition to the aforementioned insights, the following information can also be inferred from the proposed sensitivity analysis:

- Optimal λ value: Sensitivity analysis can occasionally be used to find the λ value that best captures the decision-maker's preferences and goals.
- Trade-off analysis: Sensitivity analysis aids in the understanding of trade-offs between conflicting criteria. For instance, if you see that raising the λ value causes a noticeable change in the order of

alternatives, this indicates that the DMs are prepared to make more significant compromises in order to accomplish their goals.

- Sensitivity to criteria weights: The weights of the criteria have a direct impact on the λ value. The sensitivity of the VIKOR results to changes in the criteria weights may be ascertained with the aid of sensitivity analysis. DMs can use this information to help them decide what weights should be given to the criteria.
- Finding preferred solutions: By analyzing how various λ values affect the rankings of alternatives, DMs may find the solutions that are consistently chosen across a range of decision-maker preferences.

4.3. Validation and superiority of the developed method

In this section, our target is to show the validity of the results. Therefore, we resolve two case studies from the literature using the proposed approach.

4.3.1. Case I: “Green supplier selection for the manufacturing company"

Case 1 is derived from Liu et al. (2019) in which a manufacturing company intends to select the most green supplier in order to reduce CO_2 emissions. In order to increase the accuracy of the results, 20 experts in the field were chosen to express their evaluation about five potential suppliers regarding four different criteria. Each supplier was evaluated according to its performance in responding to four different criteria including: C_1: “The quality of product”, C_2: “The environmental influence”, C_3: “The delivery indicator”, and C_4: “The price”. Among these four, C_4 is considered as a cost criterion. The weight vector for these four criteria was also calculated as (0.241, 0.223, 0.220, 0.316). The IV q-ROFSs with $q = 3$ were taken into account to replace the linguistic judgements of the experts for this case. The possibilities for the aggregated decision matrix are calculated in Fig. (8) as heatmaps for each alternative (see Table (A.2) in the **Appendix)** and the new decision matrix is calculated in **Table (8)**.

Table (8) shows the new possibility degree-based decision matrix for each alternative and also the VIKOR indicators which are calculated by using the new decision matrix. As it is obvious from the table, $\chi_5 > \chi_3 > \chi_4 > \chi_1 > \chi_2$ which results the final ranking $\psi_2 > \psi_1 > \psi_4 > \psi_3 > \psi_5$ for the alternatives.

Table (9) shows the final rankings which are calculated for the same case by using different approaches. There are some differences in rankings between some approaches, which result from rather small difference between some alternatives. For instance, ψ_1 and ψ_2 are so close in terms of responding to criteria. However, the proposed approach indicates that the alternative number 2 is performing better in criteria which have higher weight, thus, it ranks the first in the final ranking order.

Figure (9) also demonstrates a graphical representation of rankings by using different approaches. As it is obvious from **Figure (9)**, the proposed approach ranks the alternatives similar to most of the previous approaches which proves the validity of the approach.

Table (8)
The new decision matrix and the VIKOR indicators for the first case.

	C_1	C_2	C_3	C_4	S_i	R_i	χ_i
$\Sigma(\tilde{\psi}_{1j})$	2.243	1.937	2.353	2.032	0.430	0.203	0.126
$\Sigma(\tilde{\psi}_{2j})$	1.85	1.908	2.318	2.818	0.371	0.184	0.000
$\Sigma(\tilde{\psi}_{3j})$	1.662	2.278	1.894	1.597	0.704	0.316	0.789
$\Sigma(\tilde{\psi}_{4j})$	2.451	2.066	1.771	1.904	0.524	0.237	0.332
$\Sigma(\tilde{\psi}_{5j})$	1.794	1.811	1.664	1.649	0.946	0.303	0.949
P^+	2.451	2.278	2.353	2.818			
P^-	1.662	1.811	1.664	1.597			

Table (9)
The final rankings obtained by different approaches for the first case.

Reference and method	Ranking
Wang et al. (2019) using q-RIVOFWHM	$\psi_1 > \psi_2 > \psi_4 > \psi_3 > \psi_5$
Wang et al. (2019) using q-RIVOFWDHM	$\psi_1 > \psi_2 > \psi_4 > \psi_5 > \psi_3$
Cao et al. (2015) using IF-TOPSIS	$\psi_2 > \psi_1 > \psi_4 > \psi_3 > \psi_5$
Liu et al. (2019) using q-RIVOF-MULTIMOORA	$\psi_2 > \psi_1 > \psi_4 > \psi_3 > \psi_5$
Liu et al. (2019) using q-RIVOFFMF	$\psi_2 > \psi_1 > \psi_4 > \psi_3 > \psi_5$
Liu et al. (2019) using q-RIVOFRP	$\psi_1 > \psi_2 > \psi_5 > \psi_3 > \psi_4$
Liu et al. (2019) using q-RIVOFRS	$\psi_2 > \psi_1 > \psi_4 > \psi_3 > \psi_5$
The proposed approach	$\psi_2 > \psi_1 > \psi_4 > \psi_3 > \psi_5$

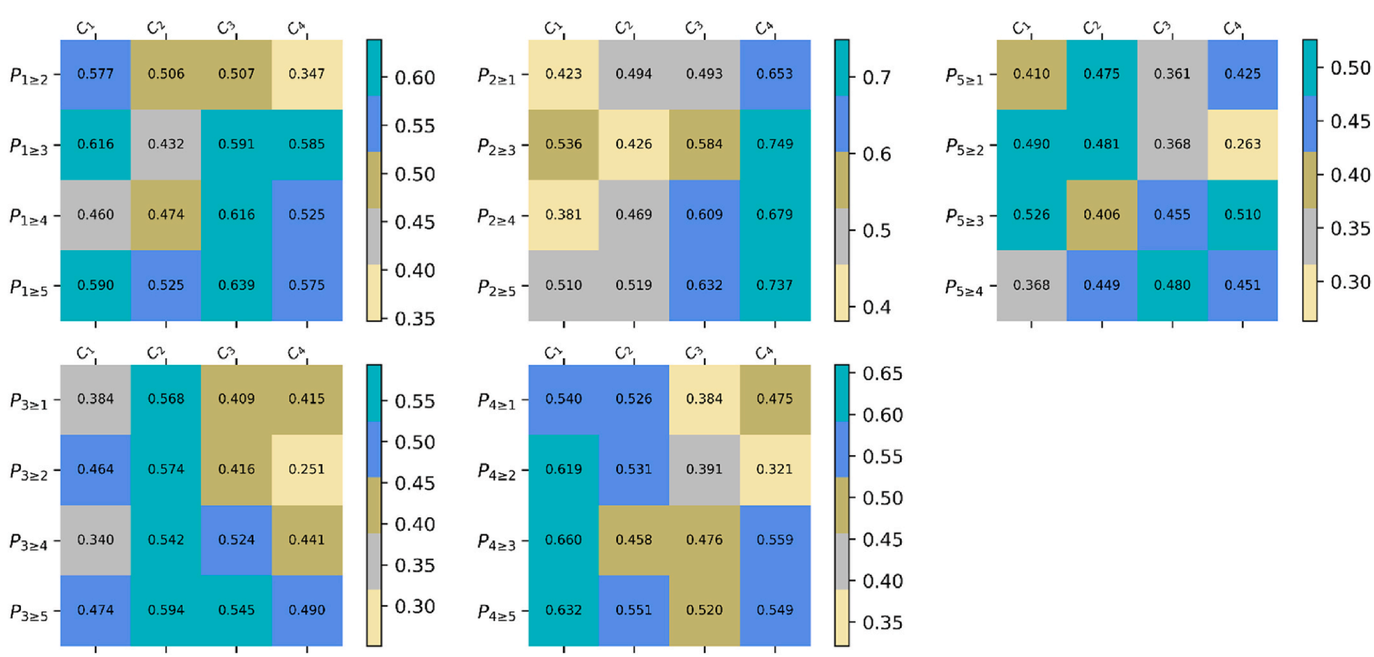


Fig (8). Heatmaps indicating the possibilities of each alternative performing better than other alternatives in response to criteria for the first case.

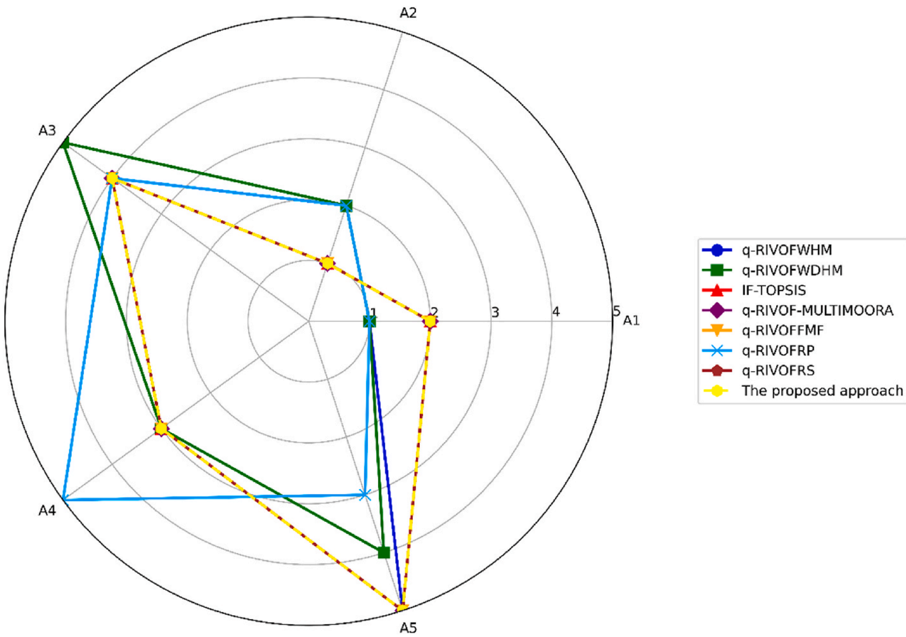


Fig. (9). Graphical representation of different rankings for case 1.

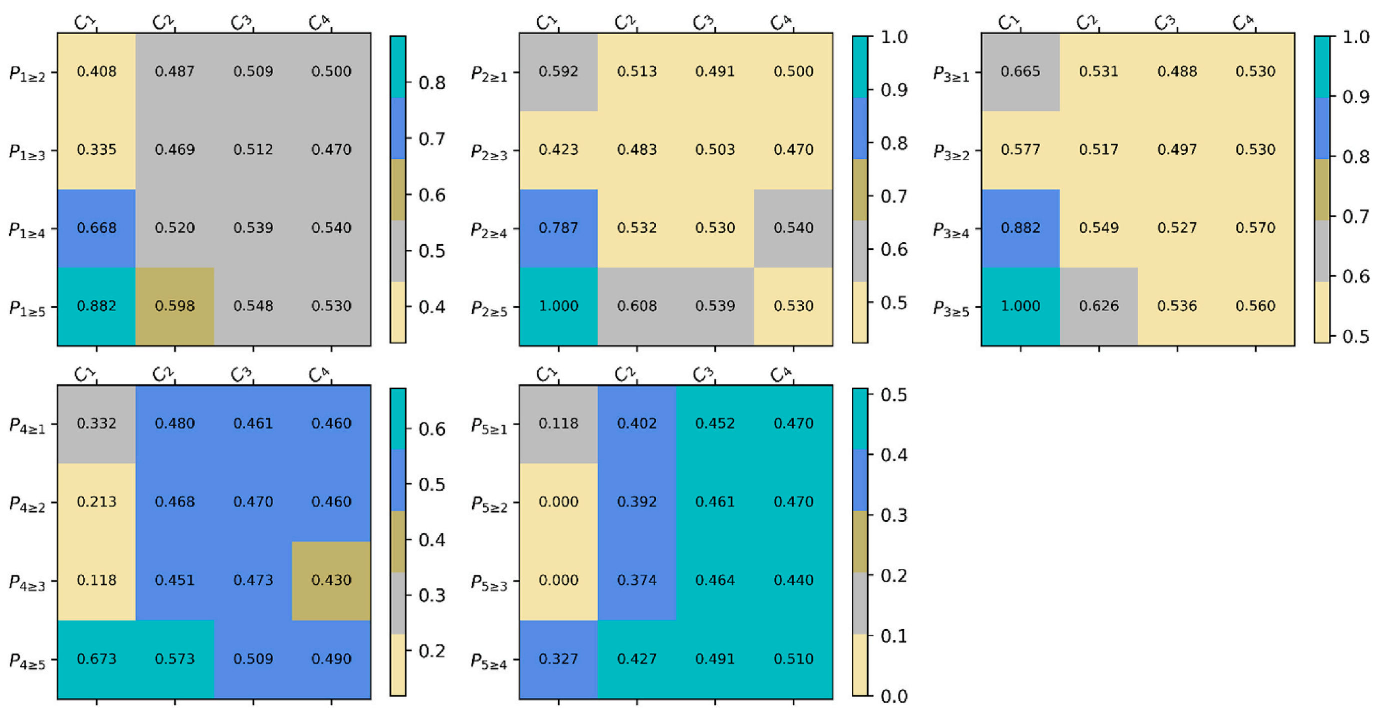


Fig (10). Heatmaps indicating the possibilities of each alternative performing better than other alternatives in response to criteria for the second case.

Table (10)
The new decision matrix and the VIKOR indicators for the second case.

	C_1	C_2	C_3	C_4	S_i	R_i	χ_i
$\Sigma(\widetilde{\psi}_{1j})$	2.293	2.074	2.108	2.04	0.284	0.217	0.277
$\Sigma(\widetilde{\psi}_{2j})$	2.802	2.136	2.063	2.04	0.160	0.084	0.114
$\Sigma(\widetilde{\psi}_{3j})$	3.124	2.223	2.048	2.19	0.025	0.025	0.000
$\Sigma(\widetilde{\psi}_{4j})$	1.336	1.972	1.913	1.84	0.688	0.467	0.673
$\Sigma(\widetilde{\psi}_{5j})$	0.445	1.595	1.868	1.89	0.986	0.700	1.000
P^{+}	3.124	2.223	2.108	2.19			
P^{-}	0.445	1.595	1.868	1.84			

4.3.2. *Case II: "Department of hypertension daily follow-up early warning management"*

The second case study is about building the "Department of Hypertension Daily Follow-up Early Warning Management" to enhance the management effectiveness of rural doctors proposed by Wan et al. (2022). Five different warning color alternatives are evaluated based on the four criteria including C_1: "Blood Pressure Measurement", C_2: "Cardiovascular Disease", C_3: "Hypertension in Family History", and C_4: "Obesity". The evaluation alternatives are: ψ_1: "Red indicates urgent handling and warning inhabitants to seek medical attention and treatment right away", ψ_2: "Orange indicates that immediate medical

Table (11)
The final rankings obtained by different approaches for the second case.

Reference and method	Ranking
Wan et al. (2022) using IVq-ROF QUALIFLEX	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Rahman and Abdullah (2019) using IVPFPWA	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Rahman and Abdullah (2019) using IVPFPWG	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Joshi et al. (2018) using IVq-ROFWA	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Rahman et al. (2018) using IVPFPOWA	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Rahman et al. (2018) using IVPFPOWG	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Wan and Lu (2022) using IVq-ROFPA	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Wan and Lu (2022) using IVq-ROFPWA	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Ashraf et al. (2019) using PFWG	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Ashraf et al. (2019) using PFHWG	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
Wan et al. (2022) using IVq-ROFDWA	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$
The proposed approach	$\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$

attention is necessary and advises locals to visit a doctor as soon as possible", ψ_3: "indicates that prompt follow-up is required to advance management, as illustrated in yellow", ψ_4: "signifies the urgent requirement to market services, as illustrated in blue" and ψ_5:"indicates that management services are not now in need of promotion, displayed in green". The weight vector for the four evaluation criteria was also calculated as (0.7, 0.1, 0.1, 0.1). The IVq-ROFSs with $q = 3$ were also considered to replace the linguistic judgements of the experts for this case. The possibilities for the aggregated decision matrix are calculated in Fig. (10) as heatmaps for each alternative (see **Table (A.3)** in the **Appendix)** and the new decision matrix is calculated in Table (10).

For each alternative, the new possibility degree-based decision matrix is shown in Table (10), along with the VIKOR indicators that are computed using the new decision matrix. As it is obvious from Table (10), $\chi_5 > \chi_4 > \chi_1 > \chi_2 > \chi_3$ which results the final ranking $\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$ for alternatives.

Table (11) displays the final rankings that were determined for the same case study using various methods. As it is shown, all the approaches resulted the same final ranking for these five alternatives. The suggested approach also yields the same ranking of $\psi_3 > \psi_2 > \psi_1 > \psi_4 > \psi_5$, indicating that there are considerable differences between the alternatives, leading to a consistent ultimate rating for all methods.

4.3.3. *Case III: The third numerical case*

To investigate the benefits of the proposed VIKOR approach across the current literature in the IV q-ROFS environment, we provide two counter-intuitive examples (Case III and Case IV) from Garg (2021a) in this section.

For the third example, consider a decision-making problem consisting of four alternatives that are evaluated by a specialist according to the three different criteria. Assume that the weight vector of these criteria is (0.3, 0.4, 0.3) respectively. The information decision of these four alternatives in response to the three criteria can be found in Garg (2021a).

By taking $q = 1$, the final rankings obtained by (Chen, 2014; Wan and Dong, 2020; Zhang et al., 2009) are invalid since they are unable to obtain a unique ranking for the alternatives. Garg (2021a) overcame the shortcoming of the previous approaches and obtained the final ranking $\psi_2 > \psi_1 > \psi_4 > \psi_3$ for the alternatives. However, the proposed VIKOR approach is superior in considering imprecision of IV q-ROFS data by calculating the possibility degrees. Hence, the ranking obtained by the proposed approach $\psi_2 > \psi_1 > \psi_3 > \psi_4$ is slightly different from Garg (2021a). The reason behind this difference is that both ψ_3 and ψ_4 are performing almost similar in response to the criteria, however, ψ_3 is closer to the positive ideal solution, which is calculated based on the cumulative possibilities, since it's performing the best in response to criterion 1.

The steps of implementing the proposed approach are listed below:

Step 1. Calculation of possibility degrees in Table (12).

Step 2. Determination of the new possibility degree-based decision matrix in Table (13).

Step 3. Calculation of the new VIKOR indicators in Table (14).

Step 4. Determination of final rankings according to χ_i values

The final ranking obtained for the four alternatives with the proposed approach according to $\chi_4 > \chi_3 > \chi_1 > \chi_2$ is equal to $\psi_2 > \psi_1 > \psi_3 > \psi_4$ which is shown as a comparative analysis in Table (15).

4.3.4. *Case IV: The fourth numerical case*

For the fourth example, a decision-making problem consisting of four alternatives is considered in which the alternatives are evaluated by an expert according to three different criteria. The weight vector of these criteria is (0.3, 0.4, 0.3) respectively. The information decision of these four alternatives in response to the three criteria can also be found in Garg (2021a). By taking $q = 2$, the final rankings obtained by previous approaches are almost inacceptable since they acquire the same rank for alternatives 2, 3, and 4, despite ranking alternative 1 the first. Although it might be adequate to determine the first rank in some decision-making problems, most of the real-world cases require a sequence of ranking for the potential alternatives to accurately plan the decision.

Garg (2021a) obtained the final ranking $\psi_3 > \psi_1 > \psi_4 > \psi_2$ for the alternatives indicating that the previous approaches were also unable to correctly rank the alternatives as the ψ_3 is ranked the first. By solving the problem with the proposed VIKOR approach, the final ranking $\psi_2 > \psi_1 > \psi_4 > \psi_3$ is obtained which is slightly different with the ranking obtained by Garg (2021a) in terms of the first rank. It might be weird that the first and last ranked alternatives are replaced by using the proposed approach, but it all refers to the input information for these four alternatives. It would be understandable that various cutting-edge methods produce different rankings for these kinds of comparable alternatives since these four alternatives are performing so similarly in response to three criteria that the prior approaches are unable to even determine a

Table (12)
The possibility degrees obtained for the four alternatives for the first case.

	C_1	C_2	C_3
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{2j})$	0.462	0.497	0.660
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{3j})$	0.408	0.501	0.739
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{4j})$	0.477	0.500	0.660
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{1j})$	0.538	0.503	0.34
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{3j})$	0.453	0.504	0.563
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{4j})$	0.517	0.504	0.492
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{1j})$	0.592	0.499	0.261
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{2j})$	0.547	0.496	0.437
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{4j})$	0.570	0.500	0.426
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{1j})$	0.523	0.5	0.34
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{2j})$	0.483	0.496	0.508
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{3j})$	0.43	0.5	0.574

Table (13)
The new decision matrix for the first case.

	C_1	C_2	C_3
$\Sigma(\widetilde{\psi}_{1j})$	1.347	1.498	2.059
$\Sigma(\widetilde{\psi}_{2j})$	1.508	1.511	1.395
$\Sigma(\widetilde{\psi}_{3j})$	1.709	1.495	1.124
$\Sigma(\widetilde{\psi}_{4j})$	1.436	1.496	1.422

Table (14)
The VIKOR indicators obtained for the first case.

	C_1	C_2	C_3	S_i	R_i	χ_i
$\Sigma(\widetilde{\psi}_{1j})$	1.347	1.498	2.059	0.625	0.325	0.587
$\Sigma(\widetilde{\psi}_{2j})$	1.508	1.511	1.395	0.380	0.213	0.000
$\Sigma(\widetilde{\psi}_{3j})$	1.709	1.495	1.124	0.700	0.400	0.876
$\Sigma(\widetilde{\psi}_{4j})$	1.436	1.496	1.422	0.806	0.375	0.933
P^+	1.709	1.511	2.059			
P^-	1.347	1.495	1.124			

Table (15)
Comparative analysis for the first case.

Reference and method	Ranking
Zhang et al. (2009) using PDIVIFSs	Unable to rank
Wan and Dong (2020) using PDM-IVIFSs	Unable to rank
Chen (2014) using IVIF QUALIFLEX	Unable to rank
Garg (2021a) using PDM-IVq-ROFSs	$\psi_2 > \psi_1 > \psi_4 > \psi_3$
The proposed method	$\psi_2 > \psi_1 > \psi_3 > \psi_4$

Table (16)
The possibility degrees obtained for the four alternatives for the second case.

	C_1	C_2	C_3
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{2j})$	0.480	0.477	0.533
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{3j})$	0.521	0.502	0.470
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{4j})$	0.497	0.501	0.503
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{1j})$	0.52	0.523	0.467
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{3j})$	0.541	0.525	0.437
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{4j})$	0.518	0.524	0.471
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{1j})$	0.479	0.498	0.53
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{2j})$	0.459	0.475	0.563
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{4j})$	0.476	0.499	0.533
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{1j})$	0.503	0.499	0.497
$P(\widetilde{\psi}_{4j} \geq \psi_{2j})$	0.459	0.475	0.563
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{3j})$	0.524	0.501	0.467

Table (17)
The new decision matrix for the second case.

	C_1	C_2	C_3
$\Sigma(\widetilde{\psi}_{1j})$	1.498	1.48	1.506
$\Sigma(\widetilde{\psi}_{2j})$	1.579	1.572	1.375
$\Sigma(\widetilde{\psi}_{3j})$	1.414	1.472	1.626
$\Sigma(\widetilde{\psi}_{4j})$	1.486	1.475	1.527

Table (18)
The VIKOR indicators obtained for the first case.

	C_1	C_2	C_3	S_i	R_i	χ_i
$\Sigma(\widetilde{\psi}_{1j})$	1.498	1.48	1.506	0.702	0.460	0.872
$\Sigma(\widetilde{\psi}_{2j})$	1.579	1.572	1.375	0.250	0.250	0.000
$\Sigma(\widetilde{\psi}_{3j})$	1.414	1.472	1.626	0.750	0.500	1.000
$\Sigma(\widetilde{\psi}_{4j})$	1.486	1.475	1.527	0.725	0.485	0.945
P^+	1.579	1.572	1.626			
P^-	1.414	1.472	1.375			

rating for them. On top of this, the proposed approach determines that ψ_2 is placed the first because it matches the positive ideal solution the most. Also, the reason behind this difference is the more amount of imprecision that IV q-ROFS possibility degrees cover. The steps of implementing the proposed approach are listed below:

Step 1. Calculation of possibility degrees in Table (16).

Table (19)
Comparative analysis for the first case.

Reference and method	Ranking
Previous methods	ψ_1 is ranked the first and $\psi_2 = \psi_3 = \psi_4$
Peng and Yang (2016) using IVPFSO	ψ_1 is ranked the first and $\psi_2 = \psi_3 = \psi_4$
Garg (2021a) using PDM-IVq-ROFSs	$\psi_3 > \psi_1 > \psi_4 > \psi_2$
The proposed approach	$\psi_2 > \psi_1 > \psi_4 > \psi_3$

Step 2. Determination of the new possibility degree-based decision matrix in Table (17).

Step 3. Calculation of the new VIKOR indicators in Table (18).

Step 4. Determination of final rankings according to χ_i values

The final ranking obtained for the four alternatives with the proposed approach according to $\chi_3 > \chi_4 > \chi_1 > \chi_2$ is equal to $\psi_2 > \psi_1 > \psi_4 > \psi_3$ which is shown as a comparative analysis in Table (19).

5. Real-world application

In this section, a case study has been presented to show the application of the proposed method in real-world cases with a large number of criteria and alternatives. The case study is providing decision support on the sustainable material selection in the construction industry.

Any construction project must carefully consider the choice of building materials as the built environment's resilience, safety, and sustainability may be impacted by the material selection. The following justifies the significance of material selection:

- Safety: The building's safety may be affected by the material choice. For instance, choosing inferior materials may result in structural failures that put residents in danger. Therefore, it's critical to select materials that adhere to safety regulations and have passed strength and durability tests (Aghazadeh et al., 2019).
- Resilience: The materials utilized also affect the built environment's resilience. The capacity of a structure to survive natural calamities like floods, earthquakes, and hurricanes is referred to as resilience. Resilient materials can prevent deterioration and the need for repairs and reconstruction (Watson et al., 2018).
- Sustainability: The built environment's sustainability may be impacted by the materials chosen. Sustainable materials are recyclable or reusable and have little influence on the environment. Utilizing eco-friendly materials may lower the building's carbon footprint and encourage environmental responsibility (PlanRadar, 2023).
- Cost: When planning a building project, the cost of the materials is crucial. In the long term, choosing the correct materials can help cut operational expenditures. For instance, adopting sturdy materials might lessen the necessity for repairs and maintenance, which can eventually result in financial savings (World Wildlife Fund (WWF) Nepal, 2016).

In the context of sustainable material selection, the integration of experts' evaluations in the proposed VIKOR approach is crucial in addressing complex MCDM problems with a high number of criteria and alternatives. Experts with different backgrounds and perspectives can bring insightful evaluations into the MCDM problem to enhance the quality of the generated results. Hence, a set of experts with different experiences should be chosen to provide information about the most relevant evaluation criteria, the weights of each criterion and also the performance ratings of material alternatives in response to the criteria. In the first stage, a set of criteria were collected for addressing sustainability in material selection. After consultation with experts, a set of 10

Table (20)
The selected criteria for sustainable material selection (Akadiri et al., 2013; Govindan et al., 2016; Zhang et al., 2017).

Sustainability aspect	Sustainability criteria	Definition	Impact
Economic	Initial cost (C_1)	The amount that must be allocated for manufacturing or purchasing of material.	Cost
	Maintenance cost (C_2)	The amount that must be allocated for maintenance over the course of its useful life.	Cost
	Cost of disposal (C_3)	The amount that must be paid for the material's end-of-life disposal.	Cost
Social	Health and safety (C_4)	The material needs to be resilient to all kinds of disruption and provide the user safety and well-being till the very end of its life.	Benefit
	Resilience to decay (C_5)	The capacity to endure erosion, corrosion, etc.	Benefit
	Fire resistance (C_6)	Enduring blasts from fire	Benefit
Environment	Water usage (C_7)	Water consumption associated with the material's life cycle	Cost
	CO_2 emission (C_8)	CO2 emissions during the material's useful life	Cost
	Energy saving (C_9)	Net energy that the substance saves	Benefit
	Reusability and potential for recycling (C_{10})	The material's capacity for recycling and reuse	Benefit

criteria were chosen, which address the three aspects of sustainability in material selection problems. After that, the impact of these criteria was determined. The impact indicates how the corresponding criterion influences the overall sustainability. A cost criterion decreases the overall sustainability level as it increases. Vice versa, a benefit criterion increases the level of sustainability as it increases. Once the criteria are determined, the experts have to answer questions regarding how alternatives perform in responding to each criterion. The questions can be asked in the form of a questionnaire and the aforementioned experts can answer with linguistic terms such as "Extremely High", "Very High", and "Extremely Low" to express their evaluations about the performance rating of each alternative. In order to prevent any misunderstanding in questions about the criteria, a short description should be added to explain what each criterion exactly means. The experts must also decide about the importance of the criteria by using some linguistic terms such as "High Importance", "Medium Importance" and "Low Importance". These linguistic terms help in calculating the weight vector of the criteria for the proposed VIKOR approach. The set of criteria, which were identified for sustainable material selection case, are shown in Table (20).

Consider that we are aiming to rank twelve material alternatives ψ_i $(i = 1, 2, .., 12)$ and choosing one of them as the most sustainable material. We have taken into account ten criteria from Table (20), with the weight vector (0.1, 0.1, 0.1, 0.15, 0.1, 0.1, 0.1, 0.15, 0.05, 0.05) for the ten criteria C_i $(i = 1, 2, .., 10)$. The parameter $q = 3$ is taken into account for IV q-ROFSs which represent the performance of each alternative in response to the criteria in Table (A.4) in the **Appendix**. The possibility degrees for the twelve potential alternatives ψ_i $(i = 1, 2, .., 12)$ in response to the ten criteria C_j $(j = 1, 2, ..., 10)$ are calculated by using Eq. (8) and are gathered in Table (A.5) in the **Appendix.** After calculating the possibility degrees of alternatives, the components of the new possibility degree-based decision matrix are calculated by using Eq. (9) and are shown in Table (21)**.**

The ideal solutions, both positive and negative, are identified after the new decision matrix is constructed. Equation (11) states that the alternative with the highest value that satisfies each requirement is the positive ideal solution. Similar to this, Eq. (12) states that the alternative with the lowest value that satisfies each requirement is the negative ideal solution. For every possible alternative, Table (22) calculates the group utility measure S_i and the regret measure R_i.

The final ranking obtained for the twelve alternatives with the proposed approach is equal to $\psi_4 > \psi_{12} > \psi_1 > \psi_5 > \psi_7 > \psi_{10} > \psi_8 > \psi_9 > \psi_6 > \psi_2 > \psi_{11} > \psi_3$.

6. Theoretical, practical and policy implications

The proposed approach concludes theoretical, practical and policy implications which are described in this section.

6.1. Theoretical implications

The theoretical implications include: 1) Development of decision-making methods: This work advances decision-making techniques by presenting an innovative possibility degree-based VIKOR approach. By adding IVq-ROFSs, VIKOR's usefulness is expanded in circumstances where standard approaches may fail owing to imprecise data or closely matched alternatives. This contributes to the theoretical understanding of multi-criteria decision-making in the face of uncertainty. 2) Handling data imprecision: Including IVq-ROFSs in decision-making provides a theoretical framework for dealing with data imprecision. Traditional techniques frequently struggle with verbal imprecision; however, the suggested approach provides a way to efficiently manage this issue. This work may pique the interest of others interested in imprecise data management. 3) Flexible decision foundation: The suggested VIKOR method's flexibility in dealing with different numbers of alternatives and criteria improves the theoretical foundation for decision-making. It enables DMs to adjust the method to various scenarios and changing conditions, fostering the creation of adaptive decision models.

Table (21)
The new possibility degree-based decision matrix for the sustainable material selection case.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
$\Sigma(\widetilde{\psi}_{1j})$	4.712	5.487	5.21	7.86	7.979	5.895	5.86	5.256	5.753	4.368
$\Sigma(\widetilde{\psi}_{2j})$	4.034	4.555	10.222	2.672	5.288	7.877	5.137	3.325	7.674	5.16
$\Sigma(\widetilde{\psi}_{3j})$	7.682	5.487	4.324	2.672	7.106	1.974	5.137	4.241	1.816	7.016
$\Sigma(\widetilde{\psi}_{4j})$	4.034	4.555	5.713	7.86	6.455	7.877	4.128	9.126	7.674	6.095
$\Sigma(\widetilde{\psi}_{5j})$	4.034	5.941	5.713	7.86	6.028	7.877	4.128	5.256	1.816	7.016
$\Sigma(\widetilde{\psi}_{6j})$	7.682	7.156	4.324	2.672	2.218	6.989	4.728	6.004	1.816	7.016
$\Sigma(\widetilde{\psi}_{7j})$	3.592	5.487	5.713	7.86	6.455	6.989	3.213	5.256	6.824	5.569
$\Sigma(\widetilde{\psi}_{8j})$	7.682	5.941	5.21	2.672	2.218	6.334	5.86	9.126	5.753	5.16
$\Sigma(\widetilde{\psi}_{9j})$	3.592	6.34	5.713	5.888	6.028	1.974	5.137	4.241	6.188	5.16
$\Sigma(\widetilde{\psi}_{10j})$	7.682	4.555	4.324	8.685	2.218	5.12	4.728	4.84	6.824	1.264
$\Sigma(\widetilde{\psi}_{11j})$	3.592	4.555	5.21	2.672	6.028	5.12	8.972	3.325	7.674	7.016
$\Sigma(\widetilde{\psi}_{12j})$	7.682	5.941	4.324	6.627	7.979	1.974	8.972	6.004	6.188	5.16

Table (22)
Calculation of the VIKOR indicators for the sustainable material selection case.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	S_i	R_i	χ_i
$\Sigma(\widetilde{\psi}_{1j})$	4.712	5.487	5.21	7.86	7.979	5.895	5.86	5.256	5.753	4.368	0.469	0.100	0.122
$\Sigma(\widetilde{\psi}_{2j})$	4.034	4.555	10.222	2.672	5.288	7.877	5.137	3.325	7.674	5.16	0.619	0.150	0.900
$\Sigma(\widetilde{\psi}_{3j})$	7.682	5.487	4.324	2.672	7.106	1.974	5.137	4.241	1.816	7.016	0.672	0.150	1.000
$\Sigma(\widetilde{\psi}_{4j})$	4.034	4.555	5.713	7.86	6.455	7.877	4.128	9.126	7.674	6.095	0.405	0.100	0.000
$\Sigma(\widetilde{\psi}_{5j})$	4.034	5.941	5.713	7.86	6.028	7.877	4.128	5.256	1.816	7.016	0.501	0.100	0.181
$\Sigma(\widetilde{\psi}_{6j})$	7.682	7.156	4.324	2.672	2.218	6.989	4.728	6.004	1.816	7.016	0.569	0.150	0.808
$\Sigma(\widetilde{\psi}_{7j})$	3.592	5.487	5.713	7.86	6.455	6.989	3.213	5.256	6.824	5.569	0.523	0.100	0.221
$\Sigma(\widetilde{\psi}_{8j})$	7.682	5.941	5.21	2.672	2.218	6.334	5.86	9.126	5.753	5.16	0.494	0.150	0.668
$\Sigma(\widetilde{\psi}_{9j})$	3.592	6.34	5.713	5.888	6.028	1.974	5.137	4.241	6.188	5.16	0.633	0.126	0.690
$\Sigma(\widetilde{\psi}_{10j})$	7.682	4.555	4.324	8.685	2.218	5.12	4.728	4.84	6.824	1.264	0.588	0.111	0.452
$\Sigma(\widetilde{\psi}_{11j})$	3.592	4.555	5.21	2.672	6.028	5.12	8.972	3.325	7.674	7.016	0.666	0.150	0.988
$\Sigma(\widetilde{\psi}_{12j})$	7.682	5.941	4.324	6.627	7.979	1.974	8.972	6.004	6.188	5.16	0.408	0.100	0.005
P^+	7.682	7.156	10.222	8.685	7.979	7.877	8.972	9.126	7.674	7.016			
P^-	3.592	4.555	4.324	2.672	2.218	1.974	3.213	3.325	1.816	1.264			

6.2. *Practical implications*

Also, there are some practical implications of the proposed approach that are stated as follows: 1) Enhanced decision-making in complex industries: The suggested VIKOR technique's application to a cement industry selection problem proves its practical value in real-world decision-making scenarios. Construction and manufacturing industries, for example, might benefit from this technique to make better informed and strong judgments when faced with multidimensional issues including various criteria and alternatives. 2) Enhanced decision-making accuracy: The study's practical application is its capacity to produce more accurate and dependable rankings of alternatives. This method may be used by DMs in a variety of fields to increase their knowledge of the strengths and weaknesses of alternative options, resulting in better-informed judgments and perhaps lowering the chance of making suboptimal choices. 3) Reduced computational complexity: The study's practical use is further demonstrated by its computational efficiency. When compared to some current approaches, the suggested VIKOR approach needs less processing work when dealing with a large number of criteria and alternatives. In actual decision-making procedures, this efficiency may save time and resources. 4) Adaptability to Changing Conditions: Decision-making criteria and alternatives may develop over time in practice. The capacity of the suggested approach to absorb changes in criteria and adapt to changing situations is a key practical benefit. Adjustments may be made during the decision-making process without significantly interfering with the model's functionality.

6.3. *Policy implications*

Applying the proposed method to the case studies from construction allows to take sustainability into account when choosing materials. This fits with a worldwide trend in which governments are putting more emphasis on sustainable practices. One potential policy outcome is the encouragement of more sustainable business operations by means of incentives or regulations. Also, by using the proposed VIKOR approach, businesses may make more sustainable decisions, which might provide them with a competitive advantage. By rewarding businesses that use sustainable methods with certificates and prizes, policymakers might promote the adoption of such approaches. Moreover, policymakers should find ways to invest in educational initiatives to guarantee that experts in related fields are aware of using cutting-edge MCDM techniques. This might entail cooperation with academic institutions or professional development programs. Finally, the outcomes of the applied method on the cement company selection case could influence the creation of regulations unique to the sector. For instance, in the cement sector, policies might be developed to support energy efficiency, cleaner manufacturing techniques, or lower carbon emissions.

In summary, this work not only adds to the theoretical knowledge of MCDM, but it also has practical and policy implications in terms of more accurate rankings, flexibility, and efficient computing. Its use in the cement industry and sustainable material selection demonstrates its ability to improve decision-making in complicated and dynamic real-world circumstances. Also, in order to make sure that the proposed approach is in line with larger social objectives, it is crucial to interact with stakeholders and policymakers.

7. Conclusion and future studies

In this paper we have developed a double-layer new possibility degree-based VIKOR approach by using IV q-ROFSs. First, we have generalized a new neutralized decision matrix by using possibility degrees of IV q-ROFSs. Second, we recalculated the positive and negative ideal solution for the VIKOR indicators based on the new decision matrix. The choice of the normalization method might have an impact on the final rankings because the traditional VIKOR relies on converting the criteria values to a standard scale. Instead of using the IV q-ROFSs normalizing approach, the recommended possibility degree-based VIKOR improves the results with a distinctive alternative ranking. The suggested VIKOR method excels in situations where the performance of alternatives is so comparable that it cannot be handled by existing approaches.

Comparing the proposed method to some previous methods, the contributions and innovation of the approach may be inferred as follows: 1) The possibilities of IV q-ROFSs have improved the recommended VIKOR's efficiency, enabling it to account for the imprecision of linguistic expert assessments. The suggested approach ranks the alternatives when the earlier approaches are unable to do so because of the amount of imprecision that the possibilities of IVq-ROFSs encompass. 2) The possibility degree-based decision matrix, which is produced by the suggested approach, carries out an internal pairwise evaluation of the alternatives. This internal comparison assists in choosing the best alternative despite the minor differences among the alternatives. 3) DMs may simply raise or decrease the number of alternatives using the suggested VIKOR approach. Due to the approach's lower computational complexity as compared to earlier methods, it is more robust when faced with a large number of criteria or alternatives. Any adjustments to the criteria during the execution of the recommended method are also possible.

The recommended approach has drawbacks even though it greatly contributes to solving decision-making problems. While the suggested method aids in ranking the alternatives, it introduces possibility calculations, making it potentially challenging for practitioners. As a result, expanding on the suggested approach and employing machine learning techniques to simplify computations might be a worthwhile advice in the future. In addition, the proposed VIKOR approach is sensitive to parameters q and λ. Since it uses the IVq-ROFSs, the determination of the exact q value may bring difficulties to DMs. The λ value that balances the utility and regret measures in the VIKOR approach is also determined by DMs by using trial and error or sensitivity analysis and may influence the final ranking of alternatives.

For future studies, different MCDM approaches such as PROMETHEE and TODIM can be developed by using the notion of possibility degrees. Apart from that, other fuzzy types such as cubic fuzzy sets (CFS) or picture fuzzy sets (PFS) may benefit from the concept of possibility degrees to develop MCDM approaches. The proposed approach can also be integrated with weighting methods such as best-worst method (BWM) or ordinal priority approach (OPA) to address MCDM problems more precisely. Since MCDM methods like VIKOR and TOPSIS are more flexible in terms of adding/removing alternatives and criteria when employing possibility degrees, it is also possible to study the effect of possibility degrees on the rank reversal problem of these approaches.

CRediT authorship contribution statement

Sepehr Hendiani: Writing – original draft, Visualization, Validation, Methodology, Conceptualization. **Grit Walther:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix

This appendix contains the information about new decision matrix and possibility degrees of the sustainable material selection case, the calculated possibility degrees of alternatives for the first and second validation cases and also sensitivity analysis for cement company selection case.

Table (A.1)
Sensitivity analysis when both q and λ values vary for the cement company selection case

q	λ	χ_i					Final ranking
		A_1	A_2	A_3	A_4	A_5	
1	0.0	0.958	0.867	0.684	1.000	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$
	0.1	0.962	0.850	0.679	0.976	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$
	0.2	0.966	0.834	0.673	0.953	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.3	0.971	0.817	0.667	0.929	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.4	0.975	0.800	0.662	0.906	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.5	0.979	0.783	0.656	0.882	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.6	0.983	0.766	0.650	0.859	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.7	0.987	0.749	0.645	0.835	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.8	0.992	0.733	0.639	0.812	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.9	0.996	0.716	0.633	0.788	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	1.0	1.000	0.699	0.627	0.765	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
2	0.0	0.863	0.698	0.637	1.000	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$
	0.1	0.877	0.696	0.639	0.981	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$
	0.2	0.890	0.693	0.641	0.963	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$
	0.3	0.904	0.691	0.643	0.944	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$
	0.4	0.918	0.688	0.645	0.926	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_1 > \psi_4$
	0.5	0.931	0.686	0.648	0.907	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.6	0.945	0.683	0.649	0.888	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.7	0.959	0.681	0.651	0.870	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.8	0.973	0.679	0.653	0.851	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	0.9	0.986	0.676	0.655	0.833	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
	1.0	1.000	0.674	0.657	0.814	0.000	$\psi_5 > \psi_3 > \psi_2 > \psi_4 > \psi_1$
3	0.0	0.811	0.644	0.661	1.000	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.1	0.830	0.648	0.666	0.984	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.2	0.849	0.651	0.671	0.969	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.3	0.868	0.655	0.676	0.953	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.4	0.887	0.659	0.681	0.938	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.5	0.906	0.662	0.685	0.922	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.6	0.925	0.666	0.690	0.907	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	0.7	0.943	0.670	0.695	0.891	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	0.8	0.962	0.674	0.700	0.875	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	0.9	0.981	0.677	0.705	0.860	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	1.0	1.000	0.681	0.710	0.844	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
4	0.0	0.799	0.642	0.707	1.000	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.1	0.819	0.647	0.711	0.985	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.2	0.839	0.652	0.715	0.971	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.3	0.859	0.657	0.719	0.956	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.4	0.879	0.662	0.723	0.941	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.5	0.900	0.667	0.727	0.927	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.6	0.920	0.672	0.731	0.912	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$

(continued on next page)

Table (A.1) (*continued*)

q	λ	χ_i					Final ranking
		A_1	A_2	A_3	A_4	A_5	
	0.7	0.940	0.677	0.735	0.897	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	0.8	0.960	0.682	0.739	0.882	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	0.9	0.980	0.687	0.743	0.868	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	1.0	1.000	0.691	0.747	0.853	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
5	0.0	0.800	0.655	0.742	1.000	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.1	0.820	0.659	0.745	0.985	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.2	0.840	0.664	0.748	0.971	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.3	0.860	0.669	0.750	0.956	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.4	0.880	0.674	0.753	0.942	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.5	0.900	0.679	0.755	0.927	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_1 > \psi_4$
	0.6	0.920	0.683	0.758	0.912	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	0.7	0.940	0.688	0.761	0.898	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	0.8	0.960	0.693	0.763	0.883	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	0.9	0.980	0.698	0.766	0.868	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$
	1.0	1.000	0.702	0.768	0.854	0.000	$\psi_5 > \psi_2 > \psi_3 > \psi_4 > \psi_1$

Table (A.2)
The calculated possibility degrees of alternatives for the first validation case

	C_1	C_2	C_3	C_4
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{2j})$	0.577	0.506	0.507	0.347
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{3j})$	0.616	0.432	0.591	0.585
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{4j})$	0.460	0.474	0.616	0.525
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{5j})$	0.590	0.525	0.639	0.575
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{1j})$	0.423	0.494	0.493	0.653
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{3j})$	0.536	0.426	0.584	0.749
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{4j})$	0.381	0.469	0.609	0.679
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{5j})$	0.510	0.519	0.632	0.737
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{1j})$	0.384	0.568	0.409	0.415
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{2j})$	0.464	0.574	0.416	0.251
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{4j})$	0.340	0.542	0.524	0.441
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{5j})$	0.474	0.594	0.545	0.490
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{1j})$	0.54	0.526	0.384	0.475
$P(\widetilde{\psi}_{4j} \geq \psi_{2j})$	0.619	0.531	0.391	0.321
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{3j})$	0.66	0.458	0.476	0.559
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{5j})$	0.632	0.551	0.520	0.549
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{1j})$	0.41	0.475	0.361	0.425
$P(\widetilde{\psi}_{5j} \geq \psi_{2j})$	0.49	0.481	0.368	0.263
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{3j})$	0.526	0.406	0.455	0.51
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{4j})$	0.368	0.449	0.48	0.451

Table (A.3)
The calculated possibility degrees of alternatives for the second validation case

	C_1	C_2	C_3	C_4
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{2j})$	0.408	0.487	0.509	0.500
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{3j})$	0.335	0.469	0.512	0.470
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{4j})$	0.668	0.520	0.539	0.540
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{5j})$	0.882	0.598	0.548	0.530
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{1j})$	0.592	0.513	0.491	0.5
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{3j})$	0.423	0.483	0.503	0.470
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{4j})$	0.787	0.532	0.530	0.540
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{5j})$	1.000	0.608	0.539	0.530
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{1j})$	0.665	0.531	0.488	0.53
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{2j})$	0.577	0.517	0.497	0.53
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{4j})$	0.882	0.549	0.527	0.570
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{5j})$	1.000	0.626	0.536	0.560
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{1j})$	0.332	0.48	0.461	0.46
$P(\widetilde{\psi}_{4j} \geq \psi_{2j})$	0.213	0.468	0.47	0.46
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{3j})$	0.118	0.451	0.473	0.43
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{5j})$	0.673	0.573	0.509	0.490
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{1j})$	0.118	0.402	0.452	0.47
$P(\widetilde{\psi}_{5j} \geq \psi_{2j})$	0	0.392	0.461	0.47
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{3j})$	0	0.374	0.464	0.44
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{4j})$	0.327	0.427	0.491	0.51

Table (A.4)
IV q-ROFSs decision matrix for the sustainable material selection case

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
ψ_1	([0.15, 0.20], [0.60, 0.75])	([0.60, 0.75], [0.10, 0.20])	([0.60, 0.75], [0.10, 0.20])	([0.60, 0.75], [0.10, 0.20])	([0.75, 0.85], [0.05, 0.15])	([0.35, 0.45], [0.40, 0.55])	([0.15, 0.20], [0.60, 0.75])	([0.35, 0.45], [0.40, 0.55])	([0.35, 0.45], [0.40, 0.55])	([0.15, 0.20], [0.60, 0.75])
ψ_2	([0.35, 0.45], [0.40, 0.55])	([0.75, 0.85], [0.05, 0.15])	([0.10, 0.10], [0.90, 0.90])	([0.10, 0.10], [0.90, 0.90])	([0.15, 0.20], [0.60, 0.75])	([0.75, 0.85], [0.05, 0.15])	([0.35, 0.45], [0.40, 0.55])	([0.75, 0.85], [0.05, 0.15])	([0.75, 0.85], [0.05, 0.15])	([0.35, 0.45], [0.40, 0.55])
ψ_3	([0.10, 0.10], [0.90, 0.90])	([0.60, 0.75], [0.10, 0.20])	([0.75, 0.85], [0.05, 0.15])	([0.10, 0.10], [0.90, 0.90])	([0.60, 0.75], [0.10, 0.20])	([0.10, 0.10], [0.90, 0.90])	([0.35, 0.45], [0.40, 0.55])	([0.60, 0.75], [0.10, 0.20])	([0.10, 0.10], [0.90, 0.90])	([0.75, 0.85], [0.05, 0.15])
ψ_4	([0.35, 0.45], [0.40, 0.55])	([0.75, 0.85], [0.05, 0.15])	([0.45, 0.60], [0.15, 0.25])	([0.60, 0.75], [0.10, 0.20])	([0.45, 0.60], [0.15, 0.25])	([0.75, 0.85], [0.05, 0.15])	([0.60, 0.75], [0.10, 0.20])	([0.10, 0.10], [0.90, 0.90])	([0.75, 0.85], [0.05, 0.15])	([0.60, 0.75], [0.10, 0.20])
ψ_5	([0.35, 0.45], [0.40, 0.55])	([0.45, 0.60], [0.15, 0.25])	([0.45, 0.60], [0.15, 0.25])	([0.60, 0.75], [0.10, 0.20])	([0.35, 0.45], [0.40, 0.55])	([0.75, 0.85], [0.05, 0.15])	([0.60, 0.75], [0.10, 0.20])	([0.35, 0.45], [0.40, 0.55])	([0.10, 0.10], [0.90, 0.90])	([0.75, 0.85], [0.05, 0.15])
ψ_6	([0.10, 0.10], [0.90, 0.90])	([0.15, 0.20], [0.60, 0.75])	([0.75, 0.85], [0.05, 0.15])	([0.10, 0.10], [0.90, 0.90])	([0.10, 0.10], [0.90, 0.90])	([0.60, 0.75], [0.10, 0.20])	([0.45, 0.60], [0.15, 0.25])	([0.15, 0.20], [0.60, 0.75])	([0.10, 0.10], [0.90, 0.90])	([0.75, 0.85], [0.05, 0.15])
ψ_7	([0.45, 0.60], [0.15, 0.25])	([0.60, 0.75], [0.10, 0.20])	([0.45, 0.60], [0.15, 0.25])	([0.60, 0.75], [0.10, 0.20])	([0.45, 0.60], [0.15, 0.25])	([0.60, 0.75], [0.10, 0.20])	([0.75, 0.85], [0.05, 0.15])	([0.35, 0.45], [0.40, 0.55])	([0.60, 0.75], [0.10, 0.20])	([0.45, 0.60], [0.15, 0.25])
ψ_8	([0.10, 0.10], [0.90, 0.90])	([0.45, 0.60], [0.15, 0.25])	([0.60, 0.75], [0.10, 0.20])	([0.10, 0.10], [0.90, 0.90])	([0.10, 0.10], [0.90, 0.90])	([0.45, 0.60], [0.15, 0.25])	([0.15, 0.20], [0.60, 0.75])	([0.10, 0.10], [0.90, 0.90])	([0.35, 0.45], [0.40, 0.55])	([0.35, 0.45], [0.40, 0.55])
ψ_9	([0.45, 0.60], [0.15, 0.25])	([0.35, 0.45], [0.40, 0.55])	([0.45, 0.60], [0.15, 0.25])	([0.15, 0.20], [0.60, 0.75])	([0.35, 0.45], [0.40, 0.55])	([0.10, 0.10], [0.90, 0.90])	([0.35, 0.45], [0.40, 0.55])	([0.60, 0.75], [0.10, 0.20])	([0.45, 0.60], [0.15, 0.25])	([0.35, 0.45], [0.40, 0.55])
ψ_{10}	([0.10, 0.10], [0.90, 0.90])	([0.75, 0.85], [0.05, 0.15])	([0.75, 0.85], [0.05, 0.15])	([0.75, 0.85], [0.05, 0.15])	([0.10, 0.10], [0.90, 0.90])	([0.15, 0.20], [0.60, 0.75])	([0.45, 0.60], [0.15, 0.25])	([0.45, 0.60], [0.15, 0.25])	([0.60, 0.75], [0.10, 0.20])	([0.10, 0.10], [0.90, 0.90])
ψ_{11}	([0.45, 0.60], [0.15, 0.25])	([0.75, 0.85], [0.05, 0.15])	([0.60, 0.75], [0.10, 0.20])	([0.10, 0.10], [0.90, 0.90])	([0.35, 0.45], [0.40, 0.55])	([0.15, 0.20], [0.60, 0.75])	([0.10, 0.10], [0.90, 0.90])	([0.75, 0.85], [0.05, 0.15])	([0.75, 0.85], [0.05, 0.15])	([0.75, 0.85], [0.05, 0.15])
ψ_{12}	([0.10, 0.10], [0.90, 0.90])	([0.45, 0.60], [0.15, 0.25])	([0.75, 0.85], [0.05, 0.15])	([0.35, 0.45], [0.40, 0.55])	([0.75, 0.85], [0.05, 0.15])	([0.10, 0.10], [0.90, 0.90])	([0.10, 0.10], [0.90, 0.90])	([0.15, 0.20], [0.60, 0.75])	([0.45, 0.60], [0.15, 0.25])	([0.35, 0.45], [0.40, 0.55])

Table (A.5)
The calculated possibility degrees of alternatives for the sustainable material selection case

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{2j})$	0.444	0.424	0.946	0.946	0.732	0.351	0.444	0.351	0.351	0.444
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{3j})$	0.744	0.500	0.424	0.946	0.576	0.800	0.444	0.430	0.800	0.268
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{4j})$	0.444	0.424	0.537	0.500	0.612	0.351	0.364	0.800	0.351	0.364
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{5j})$	0.444	0.537	0.537	0.500	0.649	0.351	0.364	0.500	0.800	0.268
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{6j})$	0.744	0.636	0.424	0.946	1.000	0.430	0.412	0.556	0.800	0.268
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{7j})$	0.412	0.500	0.537	0.500	0.612	0.430	0.268	0.500	0.430	0.412
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{8j})$	0.744	0.537	0.500	0.946	1.000	0.470	0.500	0.800	0.500	0.444
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{9j})$	0.412	0.570	0.537	0.636	0.649	0.800	0.444	0.430	0.470	0.444
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{10j})$	0.744	0.424	0.424	0.424	1.000	0.556	0.412	0.470	0.430	0.744
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{11j})$	0.412	0.424	0.500	0.946	0.649	0.556	0.744	0.351	0.351	0.268
$P(\widetilde{\psi}_{1j} \geq \widetilde{\psi}_{12j})$	0.744	0.537	0.424	0.570	0.500	0.800	0.744	0.556	0.470	0.444
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{1j})$	0.556	0.576	0.054	0.054	0.268	0.649	0.556	0.649	0.649	0.556
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{3j})$	0.800	0.576	0.000	0.500	0.364	1.000	0.500	0.576	1.000	0.351
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{4j})$	0.500	0.500	0.154	0.054	0.412	0.500	0.430	1.000	0.500	0.430
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{5j})$	0.500	0.612	0.154	0.054	0.444	0.500	0.430	0.649	1.000	0.351
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{6j})$	0.800	0.732	0.000	0.500	0.744	0.576	0.470	0.732	1.000	0.351
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{7j})$	0.470	0.576	0.154	0.054	0.412	0.576	0.351	0.649	0.576	0.470
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{8j})$	0.800	0.612	0.054	0.500	0.744	0.612	0.556	1.000	0.649	0.500
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{9j})$	0.470	0.649	0.154	0.256	0.444	1.000	0.500	0.576	0.612	0.500
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{10j})$	0.800	0.500	0.000	0.000	0.744	0.732	0.470	0.612	0.576	0.800
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{11j})$	0.470	0.500	0.054	0.500	0.444	0.732	0.800	0.500	0.500	0.351
$P(\widetilde{\psi}_{2j} \geq \widetilde{\psi}_{12j})$	0.800	0.612	0.000	0.200	0.268	1.000	0.800	0.732	0.612	0.500
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{1j})$	0.256	0.500	0.576	0.054	0.424	0.2	0.556	0.570	0.200	0.732
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{2j})$	0.200	0.424	1.000	0.500	0.636	0.000	0.500	0.424	0.000	0.649
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{4j})$	0.200	0.424	0.612	0.054	0.537	0.000	0.430	0.946	0.000	0.576
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{5j})$	0.200	0.537	0.612	0.054	0.570	0.000	0.430	0.570	0.500	0.500
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{6j})$	0.500	0.636	0.500	0.500	0.946	0.054	0.470	0.636	0.500	0.500
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{7j})$	0.154	0.500	0.612	0.054	0.537	0.054	0.351	0.570	0.054	0.612
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{8j})$	0.500	0.537	0.576	0.500	0.946	0.154	0.556	0.946	0.200	0.649
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{9j})$	0.154	0.570	0.612	0.256	0.570	0.500	0.500	0.500	0.154	0.649
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{10j})$	0.500	0.424	0.500	0.000	0.946	0.256	0.470	0.537	0.054	1.000
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{11j})$	0.154	0.424	0.576	0.500	0.570	0.256	0.800	0.424	0.000	0.500
$P(\widetilde{\psi}_{3j} \geq \widetilde{\psi}_{12j})$	0.500	0.537	0.500	0.200	0.424	0.500	0.800	0.636	0.154	0.649
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{1j})$	0.556	0.576	0.463	0.500	0.388	0.649	0.636	0.200	0.649	0.636
$P(\widetilde{\psi}_{4j} \geq \psi_{2j})$	0.500	0.500	0.846	0.946	0.588	0.500	0.570	0.000	0.500	0.570
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{3j})$	0.800	0.576	0.388	0.946	0.463	1.000	0.570	0.054	1.000	0.424
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{5j})$	0.500	0.612	0.500	0.500	0.530	0.500	0.500	0.200	1.000	0.424
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{6j})$	0.800	0.732	0.388	0.946	0.846	0.576	0.537	0.256	1.000	0.424
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{7j})$	0.470	0.576	0.500	0.500	0.500	0.576	0.424	0.200	0.576	0.537

(*continued on next page*)

Table (A.5) (*continued*)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{8j})$	0.800	0.612	0.463	0.946	0.846	0.612	0.636	0.500	0.649	0.570
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{9j})$	0.470	0.649	0.500	0.636	0.530	1.000	0.570	0.054	0.612	0.570
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{10j})$	0.800	0.500	0.388	0.424	0.846	0.732	0.537	0.154	0.576	0.946
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{11j})$	0.470	0.500	0.463	0.946	0.530	0.732	0.946	0.000	0.500	0.424
$P(\widetilde{\psi}_{4j} \geq \widetilde{\psi}_{12j})$	0.800	0.612	0.388	0.570	0.388	1.000	0.946	0.256	0.612	0.570
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{1j})$	0.556	0.463	0.463	0.500	0.351	0.649	0.636	0.500	0.200	0.732
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{2j})$	0.500	0.388	0.846	0.946	0.556	0.500	0.570	0.351	0.000	0.649
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{3j})$	0.800	0.463	0.388	0.946	0.430	1.000	0.570	0.430	0.500	0.500
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{4j})$	0.500	0.388	0.500	0.500	0.470	0.500	0.500	0.800	0.000	0.576
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{6j})$	0.800	0.588	0.388	0.946	0.800	0.576	0.537	0.556	0.500	0.500
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{7j})$	0.470	0.463	0.500	0.500	0.470	0.576	0.424	0.500	0.054	0.612
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{8j})$	0.800	0.500	0.463	0.946	0.800	0.612	0.636	0.800	0.200	0.649
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{9j})$	0.470	0.530	0.500	0.636	0.500	1.000	0.570	0.430	0.154	0.649
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{10j})$	0.800	0.388	0.388	0.424	0.800	0.732	0.537	0.470	0.054	1.000
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{11j})$	0.470	0.388	0.463	0.946	0.500	0.732	0.946	0.351	0.000	0.500
$P(\widetilde{\psi}_{5j} \geq \widetilde{\psi}_{12j})$	0.800	0.500	0.388	0.570	0.351	1.000	0.946	0.556	0.154	0.649
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{1j})$	0.256	0.364	0.576	0.054	0.000	0.570	0.588	0.444	0.200	0.732
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{2j})$	0.200	0.268	1.000	0.500	0.256	0.424	0.530	0.268	0.000	0.649
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{3j})$	0.500	0.364	0.500	0.500	0.054	0.946	0.530	0.364	0.500	0.500
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{4j})$	0.200	0.268	0.612	0.054	0.154	0.424	0.463	0.744	0.000	0.576
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{5j})$	0.200	0.412	0.612	0.054	0.200	0.424	0.463	0.444	0.500	0.500
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{7j})$	0.154	0.364	0.612	0.054	0.154	0.500	0.388	0.444	0.054	0.612
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{8j})$	0.500	0.412	0.576	0.500	0.500	0.537	0.588	0.744	0.200	0.649
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{9j})$	0.154	0.444	0.612	0.256	0.200	0.946	0.530	0.364	0.154	0.649
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{10j})$	0.500	0.268	0.500	0.000	0.500	0.636	0.500	0.412	0.054	1.000
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{11j})$	0.154	0.268	0.576	0.500	0.200	0.636	0.846	0.268	0.000	0.500
$P(\widetilde{\psi}_{6j} \geq \widetilde{\psi}_{12j})$	0.500	0.412	0.500	0.200	0.000	0.946	0.846	0.500	0.154	0.649
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{1j})$	0.588	0.500	0.463	0.500	0.388	0.570	0.732	0.500	0.570	0.588
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{2j})$	0.530	0.424	0.846	0.946	0.588	0.424	0.649	0.351	0.424	0.530
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{3j})$	0.846	0.500	0.388	0.946	0.463	0.946	0.649	0.430	0.946	0.388
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{4j})$	0.530	0.424	0.500	0.500	0.500	0.424	0.576	0.800	0.424	0.463
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{5j})$	0.530	0.537	0.500	0.500	0.530	0.424	0.576	0.500	0.946	0.388
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{6j})$	0.846	0.636	0.388	0.946	0.846	0.500	0.612	0.556	0.946	0.388
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{8j})$	0.846	0.537	0.463	0.946	0.846	0.537	0.732	0.800	0.570	0.530
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{9j})$	0.500	0.570	0.500	0.636	0.530	0.946	0.649	0.430	0.537	0.530
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{10j})$	0.846	0.424	0.388	0.424	0.846	0.636	0.612	0.470	0.500	0.846
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{11j})$	0.500	0.424	0.463	0.946	0.530	0.636	1.000	0.351	0.424	0.388
$P(\widetilde{\psi}_{7j} \geq \widetilde{\psi}_{12j})$	0.846	0.537	0.388	0.570	0.388	0.946	1.000	0.556	0.537	0.530
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{1j})$	0.256	0.463	0.500	0.054	0.000	0.530	0.500	0.200	0.500	0.556
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{2j})$	0.200	0.388	0.946	0.500	0.256	0.388	0.444	0.000	0.351	0.500
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{3j})$	0.500	0.463	0.424	0.500	0.054	0.846	0.444	0.054	0.800	0.351
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{4j})$	0.200	0.388	0.537	0.054	0.154	0.388	0.364	0.500	0.351	0.430
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{5j})$	0.200	0.500	0.537	0.054	0.200	0.388	0.364	0.200	0.800	0.351
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{6j})$	0.500	0.588	0.424	0.500	0.500	0.463	0.412	0.256	0.800	0.351
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{7j})$	0.154	0.463	0.537	0.054	0.154	0.463	0.268	0.200	0.430	0.470
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{9j})$	0.154	0.530	0.537	0.256	0.200	0.846	0.444	0.054	0.470	0.500
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{10j})$	0.500	0.388	0.424	0.000	0.500	0.588	0.412	0.154	0.430	0.800
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{11j})$	0.154	0.388	0.500	0.500	0.200	0.588	0.744	0.000	0.351	0.351
$P(\widetilde{\psi}_{8j} \geq \widetilde{\psi}_{12j})$	0.500	0.500	0.424	0.200	0.000	0.846	0.744	0.256	0.470	0.500
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{1j})$	0.588	0.430	0.463	0.364	0.351	0.200	0.556	0.570	0.530	0.556
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{2j})$	0.530	0.351	0.846	0.744	0.556	0.000	0.500	0.424	0.388	0.500
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{3j})$	0.846	0.430	0.388	0.744	0.430	0.500	0.500	0.500	0.846	0.351
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{4j})$	0.530	0.351	0.500	0.364	0.470	0.000	0.430	0.946	0.388	0.430
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{5j})$	0.530	0.470	0.500	0.364	0.500	0.000	0.430	0.570	0.846	0.351
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{6j})$	0.846	0.556	0.388	0.744	0.800	0.054	0.470	0.636	0.846	0.351
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{7j})$	0.500	0.430	0.500	0.364	0.470	0.054	0.351	0.570	0.463	0.470
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{8j})$	0.846	0.470	0.463	0.744	0.800	0.154	0.556	0.946	0.530	0.500
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{10j})$	0.846	0.351	0.388	0.268	0.800	0.256	0.470	0.537	0.463	0.800
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{11j})$	0.500	0.351	0.463	0.744	0.500	0.256	0.800	0.424	0.388	0.351
$P(\widetilde{\psi}_{9j} \geq \widetilde{\psi}_{12j})$	0.846	0.470	0.388	0.444	0.351	0.500	0.800	0.636	0.500	0.500
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{1j})$	0.256	0.576	0.576	0.576	0.000	0.444	0.588	0.530	0.570	0.256
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{2j})$	0.200	0.500	1.000	1.000	0.256	0.268	0.530	0.388	0.424	0.200
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{3j})$	0.500	0.576	0.500	1.000	0.054	0.744	0.530	0.463	0.946	0.000
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{4j})$	0.200	0.500	0.612	0.576	0.154	0.268	0.463	0.846	0.424	0.054
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{5j})$	0.200	0.612	0.612	0.576	0.200	0.268	0.463	0.530	0.946	0.000
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{6j})$	0.500	0.732	0.500	1.000	0.500	0.364	0.500	0.588	0.946	0.000
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{7j})$	0.154	0.576	0.612	0.576	0.154	0.364	0.388	0.530	0.500	0.154
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{8j})$	0.500	0.612	0.576	1.000	0.500	0.412	0.588	0.846	0.570	0.200
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{9j})$	0.154	0.649	0.612	0.732	0.200	0.744	0.530	0.463	0.537	0.200
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{11j})$	0.154	0.500	0.576	1.000	0.200	0.500	0.846	0.388	0.424	0.000
$P(\widetilde{\psi}_{10j} \geq \widetilde{\psi}_{12j})$	0.500	0.612	0.500	0.649	0.000	0.744	0.846	0.588	0.537	0.200
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{1j})$	0.588	0.576	0.500	0.054	0.351	0.444	0.256	0.649	0.649	0.732

(*continued on next page*)

Table (A.5) (*continued*)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{2j})$	0.530	0.500	0.946	0.500	0.556	0.268	0.200	0.500	0.500	0.649
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{3j})$	0.846	0.576	0.424	0.500	0.430	0.744	0.200	0.576	1.000	0.500
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{4j})$	0.530	0.500	0.537	0.054	0.470	0.268	0.054	1.000	0.500	0.576
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{5j})$	0.530	0.612	0.537	0.054	0.500	0.268	0.054	0.649	1.000	0.500
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{6j})$	0.846	0.732	0.424	0.500	0.800	0.364	0.154	0.732	1.000	0.500
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{7j})$	0.500	0.576	0.537	0.054	0.470	0.364	0.000	0.649	0.576	0.612
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{8j})$	0.846	0.612	0.500	0.500	0.800	0.412	0.256	1.000	0.649	0.649
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{9j})$	0.500	0.649	0.537	0.256	0.500	0.744	0.200	0.576	0.612	0.649
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{10j})$	0.846	0.500	0.424	0.000	0.800	0.500	0.154	0.612	0.576	1.000
$P(\widetilde{\psi}_{11j} \geq \widetilde{\psi}_{12j})$	0.846	0.612	0.424	0.200	0.351	0.744	0.500	0.732	0.612	0.649
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{1j})$	0.256	0.463	0.576	0.430	0.500	0.200	0.256	0.444	0.530	0.556
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{2j})$	0.200	0.388	1.000	0.800	0.732	0.000	0.200	0.268	0.388	0.500
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{3j})$	0.500	0.463	0.500	0.800	0.576	0.500	0.200	0.364	0.846	0.351
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{4j})$	0.200	0.388	0.612	0.430	0.612	0.000	0.054	0.744	0.388	0.430
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{5j})$	0.200	0.500	0.612	0.430	0.649	0.000	0.054	0.444	0.846	0.351
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{6j})$	0.500	0.588	0.500	0.800	1.000	0.054	0.154	0.500	0.846	0.351
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{7j})$	0.154	0.463	0.612	0.430	0.612	0.054	0.000	0.444	0.463	0.470
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{8j})$	0.500	0.500	0.576	0.800	1.000	0.154	0.256	0.744	0.530	0.500
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{9j})$	0.154	0.530	0.612	0.556	0.649	0.500	0.200	0.364	0.500	0.500
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{10j})$	0.500	0.388	0.500	0.351	1.000	0.256	0.154	0.412	0.463	0.800
$P(\widetilde{\psi}_{12j} \geq \widetilde{\psi}_{11j})$	0.154	0.388	0.576	0.800	0.649	0.256	0.500	0.268	0.388	0.351

References

Aghazadeh, E., Yildirim, H., Aliparast, S., 2019. Materials selection in the construction projects: challenges, criteria and patterns. Int. J. Adv. Mech. Civ. Eng. 2394–2827.

Akadiri, P.O., Olomolaiye, P.O., Chinyio, E.A., 2013. Multi-criteria evaluation model for the selection of sustainable materials for building projects. Autom. ConStruct. 30, 113–125. https://doi.org/10.1016/j.autcon.2012.10.004.

Akram, M., Khan, A., Luqman, A., Senapati, T., Pamucar, D., 2023. An extended MARCOS method for MCGDM under 2-tuple linguistic q -rung picture fuzzy environment. Eng. Appl. Artif. Intell. 120, 105892 https://doi.org/10.1016/j.engappai.2023.105892.

Akram, M., Shahzadi, G., 2021. A hybrid decision-making model under q-rung orthopair fuzzy Yager aggregation operators. Granul. Comput. 6, 763–777. https://doi.org/10.1007/s41066-020-00229-z.

Al-Quran, A., Jamil, N., Tehrim, S.T., Riaz, M., 2023. Cubic bipolar fuzzy VIKOR and ELECTRE-II algorithms for efficient freight transportation in Industry 4.0. AIMS Math 8, 24484–24514. https://doi.org/10.3934/math.20231249.

Ali, Z., Mahmood, T., Pamucar, D., Wei, C., 2022. Complex interval-valued q-rung orthopair fuzzy Hamy mean operators and their application in decision-making strategy. Symmetry (Basel) 14. https://doi.org/10.3390/sym14030592.

Arya, V., Kumar, S., 2021. Extended TODIM method based on VIKOR for q-rung orthopair fuzzy information measures and their application in MAGDM problem of medical consumption products. Int. J. Intell. Syst. 36, 6837–6870. https://doi.org/10.1002/int.22571.

Ashraf, S., Abdullah, S., Abdullah, L., 2019. Child development influence environmental factors determined using spherical fuzzy distance measures. Mathematics 7. https://doi.org/10.3390/math7080661.

Atanassov, K.T., 1986. Intuitionistic fuzzy sets. Fuzzy Set Syst. 20, 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3.

Cao, Q., Wu, J., Liangb, C., 2015. An intuitionsitic fuzzy judgement matrix and TOPSIS integrated multi-criteria decision making method for green supplier selection. J. Intell. Fuzzy Syst. 28, 117–126. https://doi.org/10.3233/IFS-141281.

Chen, T.Y., 2015. An interval type-2 fuzzy PROMETHEE method using a likelihood-based outranking comparison approach. Inf. Fusion 25, 105–120. https://doi.org/10.1016/j.inffus.2014.10.002.

Chen, T.Y., 2014. Interval-valued intuitionistic fuzzy QUALIFLEX method with a likelihood-based comparison approach for multiple criteria decision analysis. Inf. Sci. 261, 149–169. https://doi.org/10.1016/j.ins.2013.08.054.

Deveci, M., Gokasar, I., Brito-Parada, P.R., 2022. A comprehensive model for socially responsible rehabilitation of mining sites using Q-rung orthopair fuzzy sets and combinative distance-based assessment. Expert Syst. Appl. 200 https://doi.org/10.1016/j.eswa.2022.117155.

Deveci, M., Gokasar, I., Pamucar, D., Zaidan, A.A., Wen, X., Gupta, B.B., 2023. Evaluation of Cooperative Intelligent Transportation System scenarios for resilience in transportation using type-2 neutrosophic fuzzy VIKOR. Transp. Res. Part A Policy Pract. 172 https://doi.org/10.1016/j.tra.2023.103666.

Dimova, L., Sevastianov, P., Sevastianov, D., 2006. MCDM in a fuzzy setting: investment projects assessment application. Int. J. Prod. Econ. 100, 10–29. https://doi.org/10.1016/j.ijpe.2004.09.014.

Dong, X., Ali, Z., Mahmood, T., Liu, P., 2023. Yager aggregation operators based on complex interval-valued q-rung orthopair fuzzy information and their application in decision making. Complex Intell. Syst. 9, 3185–3210. https://doi.org/10.1007/s40747-022-00901-8.

Farid, H.M.A., Riaz, M., 2023. q-rung orthopair fuzzy Aczel–Alsina aggregation operators with multi-criteria decision-making. Eng. Appl. Artif. Intell. 122 https://doi.org/10.1016/j.engappai.2023.106105.

Gao, H., Ran, L., Wei, G., Wei, C., Wu, J., 2020. VIKOR method for MAGDM based on q-rung interval-valued orthopair fuzzy information and its application to supplier selection of medical consumption products. Int. J. Environ. Res. Publ. Health 17, 525.

Gao, J., Xu, Z., 2019. Differential calculus of interval-valued q-rung orthopair fuzzy functions and their applications. Int. J. Intell. Syst. 34, 3190–3219. https://doi.org/10.1002/int.22190.

Garg, H., 2021a. A new possibility degree measure for interval-valued q-rung orthopair fuzzy sets in decision-making. Int. J. Intell. Syst. 36, 526–557. https://doi.org/10.1002/int.22308.

Garg, H., 2021b. New exponential operation laws and operators for interval-valued q-rung orthopair fuzzy sets in group decision making process. Neural Comput. Appl. 33, 13937–13963. https://doi.org/10.1007/s00521-021-06036-0.

Garg, H., Ali, Z., Mahmood, T., Aljahdali, S., 2021a. Some similarity and distance measures between complex interval-valued q -rung orthopair fuzzy sets based on cosine function and their applications. Math. Probl Eng. 2021 https://doi.org/10.1155/2021/5534915.

Garg, H., Naz, S., Ziaa, F., Shoukat, Z., 2021b. A ranking method based on Muirhead mean operator for group decision making with complex interval-valued q-rung orthopair fuzzy numbers. Soft Comput. 25, 14001–14027. https://doi.org/10.1007/s00500-021-06231-0.

Gong, Z., 2023. Novel entropy and distance measures of linguistic interval-valued q-Rung orthopair fuzzy sets. J. Intell. Fuzzy Syst. 44, 7865–7876. https://doi.org/10.3233/jifs-223729.

Govindan, K., Madan Shankar, K., Kannan, D., 2016. Sustainable material selection for construction industry - a hybrid multi criteria decision making approach. Renew. Sustain. Energy Rev. 55, 1274–1288. https://doi.org/10.1016/j.rser.2015.07.100.

Hayat, K., Raja, M.S., Lughofer, E., Yaqoob, N., 2023. New group-based generalized interval-valued q-rung orthopair fuzzy soft aggregation operators and their applications in sports decision-making problems. Comput. Appl. Math. 42 https://doi.org/10.1007/s40314-022-02130-8.

Hendiani, S., Walther, G., 2023. TOPSISort-L: an extended likelihood-based interval-valued intuitionistic fuzzy TOPSIS-sort method and its application to multi-criteria group decision-making. Expert Syst. Appl. 233 https://doi.org/10.1016/j.eswa.2023.121005.

Jana, C., Muhiuddin, G., Pal, M., 2019. Some Dombi aggregation of Q-rung orthopair fuzzy numbers in multiple-attribute decision making. Int. J. Intell. Syst. 34, 3220–3240. https://doi.org/10.1002/int.22191.

Jin, C., Ran, Y., Zhang, G., 2021. Interval-valued q-rung orthopair fuzzy FMEA application to improve risk evaluation process of tool changing manipulator. Appl. Soft Comput. 104 https://doi.org/10.1016/j.asoc.2021.107192.

Joshi, B.P., Singh, A., Bhatt, P.K., Vaisla, K.S., 2018. Interval valued q -rung orthopair fuzzy sets and their properties. J. Intell. Fuzzy Syst. 35, 5225–5230. https://doi.org/10.3233/JIFS-169806.

Ju, Y., Luo, C., Ma, J., Gao, H., Santibanez Gonzalez, E.D.R., Wang, A., 2019. Some interval-valued q-rung orthopair weighted averaging operators and their applications to multiple-attribute decision making. Int. J. Intell. Syst. 34, 2584–2606. https://doi.org/10.1002/int.22163.

Kamacı, H., Petchimuthu, S., 2022. Some similarity measures for interval-valued bipolar q-rung orthopair fuzzy sets and their application to supplier evaluation and selection

in supply chain management. Environ. Dev. Sustain. https://doi.org/10.1007/s10668-022-02130-y.
Khan, M.R., Ullah, K., Karamti, H., Khan, Q., Mahmood, T., 2023. Multi-attribute group decision-making based on q-rung orthopair fuzzy Aczel–Alsina power aggregation operators. Eng. Appl. Artif. Intell. 126 https://doi.org/10.1016/j.engappai.2023.106629.
Khan, M.R., Wang, H., Ullah, K., Karamti, H., 2022. Construction material selection by using multi-attribute decision making based on q-rung orthopair fuzzy aczel–alsina aggregation operators. Appl. Sci. 12 https://doi.org/10.3390/app12178537.
Khan, M.S.A., Khan, A.S., Khan, I.A., Mashwani, W.K., Hussain, F., 2021. Linguistic interval-valued q-rung orthopair fuzzy TOPSIS method for decision making problem with incomplete weight. J. Intell. Fuzzy Syst. 40, 4223–4235. https://doi.org/10.3233/JIFS-200845.
Khoshabi, P., Nejati, E., Ahmadi, S.F., Chegini, A., Makui, A., Ghousi, R., 2020. Developing a multi-criteria decision making approach to compare types of classroom furniture considering mismatches for anthropometric measures of university students. PLoS One 15. https://doi.org/10.1371/journal.pone.0239297.
Li, Y., Liu, M., Cao, J., Wang, X., Zhang, N., 2021. Multi-attribute group decision-making considering opinion dynamics. Expert Syst. Appl. 184 https://doi.org/10.1016/j.eswa.2021.115479.
Limboo, B., Dutta, P., 2022. A Q-rung orthopair basic probability assignment and its application in medical diagnosis. Decis. Mak. Appl. Manag. Eng. 5, 290–308. https://doi.org/10.31181/dmame191221060l.
Liu, L., Cao, W., Shi, B., Tang, M., 2019. Large-scale green supplier selection approach under a q-rung interval-valued orthopair fuzzy environment. Processes 7. https://doi.org/10.3390/pr7090573.
Luqman, A., Shahzadi, G., 2023. Multi-criteria group decision-making based on the interval-valued q-rung orthopair fuzzy SIR approach for green supply chain evaluation and selection. Granul. Comput. 8, 1937–1954. https://doi.org/10.1007/s41066-023-00411-z.
Narayanamoorthy, S., Geetha, S., Rakkiyappan, R., Joo, Y.H., 2019. Interval-valued intuitionistic hesitant fuzzy entropy based VIKOR method for industrial robots selection. Expert Syst. Appl. 121, 28–37. https://doi.org/10.1016/j.eswa.2018.12.015.
Naz, S., Shafiq, A., Butt, S.A., Ijaz, R., 2023. A new approach to sentiment analysis algorithms: extended SWARA-MABAC method with 2-tuple linguistic q-rung orthopair fuzzy information. Eng. Appl. Artif. Intell. 126 https://doi.org/10.1016/j.engappai.2023.106943.
Opricovic, S., Tzeng, G.H., 2004. Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. Eur. J. Oper. Res. 156, 445–455. https://doi.org/10.1016/S0377-2217(03)00020-1.
Pamučar, D., Ćirović, G., 2015. The selection of transport and handling resources in logistics centers using Multi-Attributive Border Approximation area Comparison (MABAC). Expert Syst. Appl. 42, 3016–3028. https://doi.org/10.1016/j.eswa.2014.11.057.
Pamucar, D.S., Tarle, S.P., Parezanovic, T., 2018. New hybrid multi-criteria decision-making DEMATEL-MAIRCA model: sustainable selection of a location for the development of multimodal logistics centre. Econ. Res. Istraz. 31, 1641–1665. https://doi.org/10.1080/1331677X.2018.1506706.
Peng, X., Garg, H., Luo, Z., 2023. When content-centric networking meets multi-criteria group decision-making: optimal cache placement policy achieved by MARCOS with q-rung orthopair fuzzy set pair analysis. Eng. Appl. Artif. Intell. 123 https://doi.org/10.1016/j.engappai.2023.106231.
Peng, X., Yang, Y., 2016. Fundamental properties of interval-valued pythagorean fuzzy aggregation operators. Int. J. Intell. Syst. 31, 444–487. https://doi.org/10.1002/int.21790.
Peng, Y., 2023. Interval-valued q-rung orthopair fuzzy interactive Dubois–Prade operator and its application in group decision-making. Granul. Comput. https://doi.org/10.1007/s41066-023-00395-w.
PlanRadar, 2023. Green Building Materials for a Sustainable Future - PlanRadar.
Qahtan, S., Alsattar, H.A., Zaidan, A.A., Deveci, M., Pamucar, D., Delen, D., 2023. Performance assessment of sustainable transportation in the shipping industry using a q-rung orthopair fuzzy rough sets-based decision making methodology. Expert Syst. Appl. 223 https://doi.org/10.1016/j.eswa.2023.119958.
Qiyas, M., Abdullah, S., Almagrabi, A.O., Khan, F., 2023. Case study for hospital-based Post-Acute Care-Cerebrovascular Disease using Sine Hyperbolic q-rung orthopair fuzzy Dombi aggregation operators. Expert Syst. Appl. 215 https://doi.org/10.1016/j.eswa.2022.119224.
Quek, S.G., Garg, H., Selvachandran, G., Palanikumar, M., Arulmozhi, K., Smarandache, F., 2023. VIKOR and TOPSIS framework with a truthful-distance measure for the (t, s)-regulated interval-valued neutrosophic soft set. Soft Comput. https://doi.org/10.1007/s00500-023-08338-y.
Rahman, K., Abdullah, S., 2019. Generalized interval-valued Pythagorean fuzzy aggregation operators and their application to group decision-making. Granul. Comput. 4, 15–25. https://doi.org/10.1007/s41066-018-0082-9.
Rahman, K., Ali, A., Khan, M.S.A., 2018. Some interval-valued pythagorean fuzzy weighted averaging aggregation operators and their application to multiple attribute decision making. Punjab Univ. J. Math. 50, 113–129.
Ranjan, M.J., Kumar, B.P., Bhavani, T.D., Padmavathi, A.V., Bakka, V., 2023. Probabilistic linguistic Q-rung orthopair fuzzy archimedean aggregation operators for group decision-making. Decis. Mak. Appl. Manag. Eng. 6, 639–667. https://doi.org/10.31181/dmame622023527.
Sabaei, D., Erkoyuncu, J., Roy, R., 2015. A review of multi-criteria decision making methods for enhanced maintenance delivery. Procedia CIRP 37, 30–35. https://doi.org/10.1016/j.procir.2015.08.086.
Salimian, S., Mousavi, S.M., Antucheviciene, J., 2022. An interval-valued intuitionistic fuzzy model based on extended VIKOR and MARCOS for sustainable supplier selection in organ transplantation networks for healthcare devices. Sustain. Times 14. https://doi.org/10.3390/su14073795.
Sarkar, A., Moslem, S., Esztergár-Kiss, D., Akram, M., Jin, L.S., Senapati, T., 2023. A hybrid approach based on dual hesitant q-rung orthopair fuzzy Frank power partitioned Heronian mean aggregation operators for estimating sustainable urban transport solutions. Eng. Appl. Artif. Intell. 124 https://doi.org/10.1016/j.engappai.2023.106505.
Seker, S., Bağlan, F.B., Aydin, N., Deveci, M., Ding, W., 2023. Risk assessment approach for analyzing risk factors to overcome pandemic using interval-valued q-rung orthopair fuzzy decision making method. Appl. Soft Comput. 132 https://doi.org/10.1016/j.asoc.2022.109891.
Siddiqui, Z.A., Haroon, M., 2023. Research on significant factors affecting adoption of blockchain technology for enterprise distributed applications based on integrated MCDM FCEM-MULTIMOORA-FG method. Eng. Appl. Artif. Intell. 118 https://doi.org/10.1016/j.engappai.2022.105699.
Singh, A., Kumar, S., 2023. Picture fuzzy VIKOR-TOPSIS approach based on knowledge and accuracy measures for suitable adsorbent decision making. Appl. Soft Comput. 147 https://doi.org/10.1016/j.asoc.2023.110807.
Stević, Ž., Pamučar, D., Puška, A., Chatterjee, P., 2020. Sustainable supplier selection in healthcare industries using a new MCDM method: measurement of alternatives and ranking according to COmpromise solution (MARCOS). Comput. Ind. Eng. 140 https://doi.org/10.1016/j.cie.2019.106231.
Wan, B., Lu, R., 2022. Some interval-valued q-rung orthopair power average operators and group decision making method. Procedia Comput. Sci. 204, 385–394. https://doi.org/10.1016/j.procs.2022.08.047.
Wan, B., Zhang, X., Xiong, M., Wang, Z., 2022. Interval-valued q-rung orthopair fuzzy QUALIFLEX decision analysis method with Dombi operators. Discrete Dynam Nat. Soc. 2022 https://doi.org/10.1155/2022/4898098.
Wan, S., Dong, J., 2020. A Possibility Degree Method for Interval-Valued Intuitionistic Fuzzy Multi-Attribute Group Decision Making. Decis. Mak. Theor. Methods Based Interval-Valued Intuitionistic Fuzzy Sets, pp. 1–35. https://doi.org/10.1007/978-981-15-1521-7_1.
Wang, J., Gao, H., Wei, G., Wei, Y., 2019. Methods for multiple-attribute group decision making with q-rung interval-valued orthopair fuzzy information and their applications to the selection of green suppliers. Symmetry (Basel) 11. https://doi.org/10.3390/sym11010056.
Wang, J.C., Tsao, C.Y., Chen, T.Y., 2015. A likelihood-based QUALIFLEX method with interval type-2 fuzzy sets for multiple criteria decision analysis. Soft Comput. 19, 2225–2243. https://doi.org/10.1007/s00500-014-1404-8.
Wang, Y.J., 2015. A fuzzy multi-criteria decision-making model based on simple additive weighting method and relative preference relation. Appl. Soft Comput. J. 30, 412–420. https://doi.org/10.1016/j.asoc.2015.02.002.
Watson, S.S., Ferraris, C.F., Averill, J.D., 2018. Role of materials selection in the resilience of the built environment. Sustain. Resilient Infrastruct. 3, 165–174. https://doi.org/10.1080/23789689.2017.1405656.
Wieckowski, J., Salabun, W., 2020. How the normalization of the decision matrix influences the results in the VIKOR method? Procedia Comput. Sci. 176, 2222–2231. https://doi.org/10.1016/j.procs.2020.09.259.
World Wildlife Fund (WWF) Nepal, H.B.P., 2016. Hariyo Ban Program Building Material Selection and Use an Environmental Guide, pp. 1–19.
Xu, Y., 2023a. A two-stage multi-criteria decision-making method with interval-valued q-Rung Orthopair fuzzy technology for selecting bike-sharing recycling supplier. Eng. Appl. Artif. Intell. 119 https://doi.org/10.1016/j.engappai.2023.105827.
Xu, Y., 2023b. A two-phase multi-criteria decision-making with interval-valued Pythagorean normal fuzzy information and its application for module hospital site selection in combating COVID-19. Eng. Appl. Artif. Intell. 126 https://doi.org/10.1016/j.engappai.2023.107081.
Xu, Y., Liu, S., Wang, J., Shang, X., 2022. A novel two-stage TOPSIS approach based on interval-valued probabilistic linguistic q-rung orthopair fuzzy sets with its application to MAGDM problems. Eng. Appl. Artif. Intell. 116 https://doi.org/10.1016/j.engappai.2022.105413.
Yager, R.R., 2017. Generalized orthopair fuzzy sets. IEEE Trans. Fuzzy Syst. 25, 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005.
Yager, R.R., Abbasov, A.M., 2013. Pythagorean membership grades, complex numbers, and decision making. Int. J. Intell. Syst. 28, 436–452. https://doi.org/10.1002/int.21584.
Yoon, K.P., Hwang, C.L., 1995. Multiple Attribute Decision Making: an Introduction.
Yu, S. min, Wang, Jing, Wang, Jian qiang, 2017. An interval type-2 fuzzy likelihood-based MABAC approach and its application in selecting hotels on a tourism website. Int. J. Fuzzy Syst. 19, 47–61. https://doi.org/10.1007/s40815-016-0217-6.
Zeng, S., Hu, Y., Xie, X., 2021. Q-rung orthopair fuzzy weighted induced logarithmic distance measures and their application in multiple attribute decision making. Eng. Appl. Artif. Intell. 100 https://doi.org/10.1016/j.engappai.2021.104167.
Zhang, H., Peng, Y., Tian, G., Wang, D., Xie, P., 2017. Green material selection for sustainability: a hybrid MCDM approach. PLoS One 12. https://doi.org/10.1371/journal.pone.0177578.
Zhang, X., Yue, G., Teng, Z., 2009. Possibility Degree of Interval-Valued Intuitionistic Fuzzy Numbers and its Application, vol. 2, pp. 33–36.
Zulqarnain, R.M., Garg, H., Ma, W.X., Siddique, I., 2024. Optimal cloud service provider selection: an MADM framework on correlation-based TOPSIS with interval-valued q-rung orthopair fuzzy soft set. Eng. Appl. Artif. Intell. 129 https://doi.org/10.1016/j.engappai.2023.107578.