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High-Frequency Flow Rate Determination—A Pressure-Based Measurement Approach

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Abstract: Accurate flow measurement is critical for hydraulic systems because it represents a crucial parameter in the control of fluid power systems and enables the calculation of hydraulic power when combined with pressure data, which is valuable for applications such as predictive maintenance. Existing flow sensors in fluid power systems typically operate invasively, disturbing the flow and providing inaccurate results, especially under transient conditions. A conventional method involves calculating the flow rate using the pressure difference along a pipe via the Hagen–Poiseuille law, which is limited to steady, laminar, incompressible flow. This paper presents a novel soft sensor with an analytical model for transient pipe flow based on two pressure signals, thus eliminating the need for an actual volumetric flow sensor. The soft sensor was derived in previous research and validated with a distributed parameter simulation. This work uses a constructed test rig to validate the soft sensor with real-world experiments. The results highlight the potential of the soft sensor to accurately and computationally efficiently measure transient pipe volumetric flow based on two pressure signals.

Keywords: soft sensor; high-frequency compressible volumetric flow rate measurement; pressure-based sensor; pressure pulsation; pulsation generation



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1. Introduction

Volumetric flow measurement is critical to the longevity and performance of hydraulic systems. Accurate flow rate measurement is required for condition monitoring, predictive maintenance, and control engineering in mobile and stationary hydraulic systems. Knowing the volumetric flow rate in combination with pressure allows the calculation of hydraulic power, one of the most important parameters to characterize a fluid power system in terms of efficiency and potential losses. Furthermore, the volumetric flow rate represents a relevant fluid power system control variable.

Current volumetric flow sensors face two significant challenges. First, many must be physically installed inside the pipe, often using turbines or rotors that disrupt the flow and add complexity to the system. Second, these sensors contain mechanical parts with inertia that limit their effectiveness at high frequencies. For example, measuring pump pulsation becomes problematic because the pulsation frequency depends on the number of displacement units and the speed of the pump. In addition, pumps generate harmonic frequencies beyond the range of most state-of-the-art sensors. These limitations highlight the need for alternative methods of flow measurement. Ideally, a sensor would be minimally invasive, capable of detecting transient flow conditions, and operate as a soft sensor using pressure signals commonly found in hydraulic systems.

The concept of deriving flow rates from pressure data has been introduced previously. Analytical methods for determining volumetric flow rates date back to the 19th century. One widely known method is the Hagen–Poiseuille law (HP) [1], which relates flow to the frictional pressure drop in a pipe. However, this law is limited to steady, laminar flow and does not account for high-frequency effects. The so-called Richardson effect [2] alters the velocity profile at higher frequencies. By incorporating a dynamic term into the Hagen–Poiseuille equation, it becomes possible to estimate transient flow rates. This paper presents an equation that led to developing a soft sensor for calculating transient flow rates based on pressure signals.

The following sections will cover the essential theoretical background and the development of the soft sensor. First, a brief overview of volumetric flow rate measurement techniques and models is provided in Section 2. Building on this foundation, the analytical model for the soft sensor is derived and examined in Section 2.3. The requirements for a suitable test rig are discussed, and the chosen concept is described. The soft sensor is then validated through multiple test cases presented in Section 2.5, followed by the results in Section 3. Finally, the paper concludes by discussing the findings and their implications.

2. Materials and Methods

2.1. Volumetric Flow Rate Measurement Methods

Accurate knowledge of volumetric flow is crucial to maintaining performance in fluid systems, leading to various sensors' development. These sensors can generally be divided into two types: invasive and non-invasive sensors.

Invasive sensors, such as positive displacement meters and turbine flow meters, determine volumetric flow by monitoring the movement of a given volume over time. However, due to the inertia of their components, they struggle to measure transient flows [3] accurately. A commonly used method in this category is to measure the pressure drop across a component designed to create flow resistance, such as an orifice [4]. The flow rate is derived from the following equation:

$$Q_O = \alpha_D A_D \sqrt{\frac{2\Delta p}{\rho}} \quad (1)$$

This equation is mainly independent of the fluid's viscosity and is only slightly affected by temperature-induced variations in the density ρ . The flow rate depends on factors such as the geometric parameter A_D , the discharge coefficient α_D , the fluid density ρ , and the pressure difference Δp . However, the intrusive installation of the orifice plate can significantly disrupt the flow pattern. A key challenge with differential pressure meters is their limited ability to handle unsteady flow, as the underlying Bernoulli principle assumes steady flow conditions. Research by Wiklund et al. [5] examining unsteady flow through differential pressure meters between 0.01 and 10 Hz showed that these meters become unreliable for flows above 2 Hz.

Another invasive technique is using vortex flow meters, which base their measurements on the Kármán vortex street. Although this method results in a relatively low-pressure drop through the vortex body, its accuracy is compromised in laminar and low-turbulent flows due to its dependence on the Reynolds number (Re) [6]. Hot-wire anemometry (HWA) is another invasive technique used to measure flow velocity by detecting the cooling rate of a heated wire placed in the flow [7]. Despite its high sampling rate (up to 500 kHz [7]), HWA can only provide accurate point measurements and causes minimal flow disturbance. When multiple wires are required to capture a velocity profile, the mounting structure causes additional flow disturbance.

Non-invasive flow meters are the second category and have the advantage of not disturbing the flow or causing pressure drops. Electromagnetic flowmeters use Faraday's law of induction, making them suitable for detecting transient flows without being invasive. However, the fluid must be conductive, with a minimum conductivity of 5×10^{-10} S/cm [4], which is much higher than the conductivity of typical hydraulic oils such as HLP 46, which is around 5×10^{-13} S/cm [8]. Ultrasonic flowmeters are another non-invasive option for measuring transient flows without significant system modifications. These sensors use two transducers mounted on the outside of the pipe to measure the time it takes for a signal to travel between them [9]. However, this method assumes symmetrical flow profiles, making it less accurate for turbulent flows that contain eddies.

An advanced, minimally invasive technique is particle image velocimetry (PIV), which uses laser-illuminated seeding particles in the fluid to map a two-dimensional velocity profile. The particle movements are recorded by cameras and processed to estimate flow velocity and, ultimately, volumetric flow rate [10]. However, PIV requires transparent piping, which is rarely feasible in industrial settings, and the introduction of seeding particles can contaminate the fluid. Another minimally invasive method is the Coriolis flowmeter, which passes the fluid through a vibrating bent tube, generating Coriolis forces that are proportional to the mass flow [4]. The measurement frequency is limited by the tube's resonant frequency, with typical tubes limited to response times of 5 ms for a resonant frequency of 200 Hz. However, their vibrational frequencies can be much higher [11].

A recent development in volumetric flow rate measurement with a soft sensor has been developed by Hucko et al. [12], which determines the flow rate based on the flow forces acting on a valve. One of the main benefits of this approach is that a small inlet regarding the flow development is required.

In summary, current volumetric flow sensors have several limitations. Many need to be more suitable for measuring transient flows due to slow response times [4,5], and those that offer faster response rates often impose requirements that cannot be met in typical industrial fluid systems [8–10]. In contrast, the presented soft sensor exhibits multiple advantages. These include the minimal invasiveness of the sensor, which, therefore, has no influence on the flow that should be measured. Additionally, the soft sensor can model fast transient responses of the flow to pressure changes [13]. The sensor has no limit regarding the properties of the fluid, such as conductivity or viscosity, which is required in some flow rate sensors [14]. Lastly, the novel soft sensor has a theoretical no-lower limit for flow rate. The lower limit for the flow rate is only caused by the measuring devices and their inherent noise.

2.2. Transient Flow Rate Models

Recent developments in transient flow measurement include significant contributions by Brereton et al. In one of their studies, they presented a method for treating arbitrary transients in laminar pipe flow, starting from an initial steady state, by relating the flow rate to the history of the pressure gradient without requiring assumptions about velocity profiles [15]. Later, in 2008, Brereton et al. introduced an alternative, indirect method that relates the flow rate to the history of the centerline velocity [16]. Sundstrom et al. [17] contributed by improving friction modeling, which significantly reduced errors in flow rate calculations in the pressure-time method commonly used for flow measurements in hydraulic systems. In 2019, Foucault et al. proposed a novel approach for time-resolved transient flow rate estimation based on differential pressure measurements. Their method exploited kinetic energy and relied on only two coefficients, effectively predicting laminar flow conditions in real time [18]. García et al. focused on unsteady turbulent pipe flow, validating their method through experiments involving the transient response of the

velocity field following external perturbations, with a pressure step function as a test case [19]. Further work by García et al. in 2022 explored the transition from turbulent to laminar flow without an increase in bulk velocity, explaining the laminarization process through a newly developed mathematical model [20].

Urbanowicz et al. conducted a thorough review of analytical models for accelerated incompressible Newtonian fluid flow in pipes, comparing various methods based on imposed pressure gradients and flow rates while also analyzing their complexity and applicability to laminar and turbulent flows. Although these models perform well for laminar flows, they encounter challenges in accurately predicting turbulent flow behavior [21].

In 2023, Urbanowicz et al. advanced the modeling of laminar water hammer phenomena by developing an analytical solution validated by numerical simulations and experimental data [21]. Later that year, they proposed new analytical models for wall shear stress during water hammer events, extending the range of validity by incorporating quasi-steady and transient hydraulic resistance assumptions. These models simplified the mathematical representation and provided explicit analytical expressions further validated by numerical simulations [22]. Meanwhile, Bayle et al. investigated wave propagation models in water hammer scenarios. Their first study developed a rheology-based model for viscoelastic pipes validated with experimental data [23]. In a subsequent paper, they formulated a wave propagation model in the Laplace domain that could be applied to various boundary conditions in pipe systems [24].

Current advancements in transient flow measurement include methods for calculating flow rate based on pressure gradient history and centerline velocity, improved friction modeling for pressure-time accuracy, and real-time differential pressure estimation. Developments also cover unsteady turbulent flow analysis, laminarization modeling, analytical solutions for a laminar water hammer, and wave propagation models for viscoelastic pipes, all validated by experimental data and simulations.

2.3. Analytical Soft Sensor Model for Flow Rate Calculation

The description and complete derivation of the analytical model of the soft sensor based on two pressure signals is provided in separate manuscripts [13,14] and is beyond the scope of this work. First, the equation for volumetric flow rate calculation under the assumption of an incompressible fluid was presented in the work of Brumand et al. [25]. In a second work by Brumand et al. [14], this equation was further investigated and expanded so that the compressible effects of the fluid are considered. The present work reiterates the main steps of the derivation to help the reader better understand. For more details regarding the derivation, the equations, and the variables used, please refer to the works of Brumand et al. that are mentioned above. The derivation of the system equations from the general Navier–Stokes equations is skipped in the present work, and the derivation begins from the so-called “two-dimensional viscous compressible model” as found in a literature review paper by Stecki [26]. The associated assumptions can also be found in Stecki’s work. Almondo [27] expanded on this model and proposed the solution for the volumetric flow rate at the outlet of the pipe as:

$$Q_2^* = \frac{1}{Z_c \sinh(\gamma L)} p_1^* - \frac{\coth(\gamma L)}{Z_c} p_2^*. \quad (2)$$

Here, the pressure values p_1^* and p_2^* are constant over the pipe’s cross-section. This assumption can be taken due to the assumed small pipe radius; therefore, $\frac{\partial p_1^*}{\partial r} = \frac{\partial p_2^*}{\partial r} = 0$ over the radial coordinate in the pipe r .

Assuming an infinitely long pipe, i.e., without the reflection of waves at the end of the pipe, the characteristic impedance describes the complex ratio of the pressure to the flow rate for any point in the pipe by [26]:

$$Z_c = \frac{p^*}{Q^*}. \quad (3)$$

In the presented model, the characteristic hydraulic impedance Z_c is defined as:

$$Z_c = \sqrt{\frac{I_0(\sqrt{\zeta})}{I_2(\sqrt{\zeta})}} \frac{R_H}{8Dn}. \quad (4)$$

Furthermore, the propagation operator relates the pressure as well as the flow rate at two points in the pipe by the following relation, assuming that there are only downstream waves and no reflection [26]:

$$e^{-\gamma L} = \frac{p_2^*}{p_1^*} = \frac{Q_2^*}{Q_1^*} \quad (5)$$

Here, the propagation operator γ is defined as:

$$\gamma = \sqrt{\frac{I_0(\sqrt{\zeta})}{I_2(\sqrt{\zeta})}} \frac{\zeta}{L} Dn \quad (6)$$

Inserting both the propagation operator γ and the characteristic impedance Z_c yields:

$$Q_2^* R_H = \frac{8Dn \sqrt{I_2(\sqrt{\zeta})}}{\sinh \left[\sqrt{\frac{I_0(\sqrt{\zeta})}{I_2(\sqrt{\zeta})}} \zeta Dn \right] \sqrt{I_0(\sqrt{\zeta})}} p_1^* - \frac{8Dn \sqrt{I_2(\sqrt{\zeta})}}{\tanh \left[\sqrt{\frac{I_0(\sqrt{\zeta})}{I_2(\sqrt{\zeta})}} \zeta Dn \right] \sqrt{I_0(\sqrt{\zeta})}} p_2^* \quad (7)$$

In the following, the used variables are presented: $I_n(x)$ is the modified Bessel function of the first kind of order n . $\zeta = s \frac{R^2}{\nu}$ is the normalized Laplace variable over the pipe radius R and the kinematic viscosity ν . $Dn = \frac{\nu L}{R^2 a}$ is the dissipation number, with a being the speed of sound within the fluid and L being the length of the pipe. The dissipation number is a ratio that describes the pressure waves' travel time along the length of the pipe to the timespan of the viscous radial diffusion of axial momentum [13].

Also, the hydraulic resistance of the pipe is defined as $R_H = \frac{8\eta L}{\pi R^4}$. The hydraulic resistance comes from the law of Hagen and Poiseuille, where it relates the steady laminar incompressible flow rate to the pressure difference over the length of a round pipe by [1]:

$$Q = \frac{\pi R^4}{8\eta L} (p_2 - p_1) = \frac{1}{R_H} \Delta p \quad (8)$$

To simplify Equation (7), the prefactors of the pressure functions are written as weighting functions as follows:

$$Q_2^* R_H = W_1^{*'}(\zeta) p_1^* - W_2^{*'}(\zeta) p_2^* \quad (9)$$

In the derivation, it proved necessary to expand by a factor of $\frac{\zeta}{\zeta}$ to assure that the weighting functions have an inverse Laplace transformation. Therefore,

$$Q_2^* R_H = W_1^{*'}(\zeta) \frac{\zeta}{\zeta} p_1^* - W_2^{*'}(\zeta) \frac{\zeta}{\zeta} p_2^* = W_1^*(\zeta) \zeta p_1^* - W_2^*(\zeta) \zeta p_2^* \quad (10)$$

The weighting functions need to be inverse Laplace transformed (ILT) to obtain a usable equation in the time domain. When the ILTs are found, the time solution of the volumetric

flow rate is given in Equation (11). Note that the multiplication with the normalized Laplace variable ζ of the pressure in Equation (10) corresponds to the derivative of the pressure with respect to the normalized time in the time domain in Equation (11); therefore, $\zeta p_1 = \frac{\partial p_1(t_n)}{\partial t_n}$. Also note that a multiplication in the Laplace domain of two functions, here $W_1^*(\zeta)$ and ζp_1^* , equals a convolution integral in the time domain [28].

$$Q_2(t_n)R_H = \left[\int_0^{t_n} W_1(t_n - \tau_n) \frac{\partial p_1(t_n - \tau_n)}{\partial t_n} d\tau_n - \int_0^{t_n} W_2(\tau_n) \frac{\partial p_2(t_n - \tau_n)}{\partial t_n} d\tau_n \right]. \quad (11)$$

The derivation of the ILT of the weighting functions will be presented in Appendix A.2.

2.4. Test Rig

A suitable test rig concept had to be developed to validate the derived system equations for the soft sensor. The main task of the test rig is to provide a known transient flow rate through a hydraulically smooth, straight pipe. This measuring pipe must fulfill the assumptions made in the system equations. Furthermore, measuring pressure signals at at least two positions in the measuring line must be possible. The pressure difference for the system equation is formed from these measurement signals. The calculated volumetric flow rate is then compared with the volumetric flow rate provided by the test rig. The test rig should be able to generate different flow conditions in the laminar and turbulent Reynolds number range. The behavior of hydraulic pumps was considered to estimate the achievable flow conditions. These generally provide a non-constant volumetric flow rate Q , with the degree of non-uniformity δ indicating the size of the pulsation that occurs. Depending on the pump type of the hydraulic pump, undesirable degrees of non-uniformity of up to $\delta = 0.3$ may occur [29]. The frequency of these pulsations can be estimated further for pumps that operate according to the positive displacement principle and have an odd number of pistons [30].

$$\delta = \frac{Q_{max} - Q_{min}}{Q_m} \quad (12)$$

$$f = 2zn. \quad (13)$$

There are multiple approaches for generating specific pulsations within fluid power systems. One widely used approach involves placing a valve directly into the fluid stream and controlling the flow rate by adjusting the valve [31]. In this method, the oscillation frequency and amplitude are primarily governed by the valve's dimensions and the system's natural resonance frequency. Another technique employs a side-discharge valve [32], which offers a key advantage because it diverts only a portion of the flow through the valve. This enables the use of a smaller, quicker valve. Despite the differences, both methods focus primarily on creating and analyzing pressure waves within the system. Thus, more than these methods are needed to measure the unsteady flow rates with high precision.

Generating pulsation via a hydraulic cylinder's movement and geometric displacement is more promising. The displaced volumetric flow rate can be approximately calculated via the kinematics of the cylinder using $Q_c = Av$, whereby the movement can be generated in different ways. For example, a cylinder used as a displacer could be activated via a thrust crank or by coupling with another cylinder. When calculating the reference volumetric flow rate in this way, it is essential to note that compressibility effects and effects caused by the propagation of generated pressure waves are not considered. Therefore, measures must be implemented to minimize these effects, as calculating the transient volumetric flow rate would otherwise be highly susceptible to errors. The error resulting from the compressibility effects can be reduced by operating at higher pressures associated with a higher and more steady bulk modulus K . Further errors occurred due to wave reflection in the pipe. This is because pressure waves are reflected when the characteristic impedance

changes, which could significantly change the volumetric flow within the measurement pipe. These changes occur, for example, at points of installations in the pipe or when the pipe cross-section changes [33]. This results in the requirement to reduce cross-sectional changes to a minimum and to terminate the measurement pipe with a low-reflection line terminator (LRLT). Details about the LRLT can be found in Appendix B.1.

Based on the stated requirements and the challenges associated with providing a reference transient volumetric flow rate, a test rig concept was developed and validated in the work of Brumand-Poor et al. [25]. The hydraulic circuit of this test rig is displayed in Figure 1.

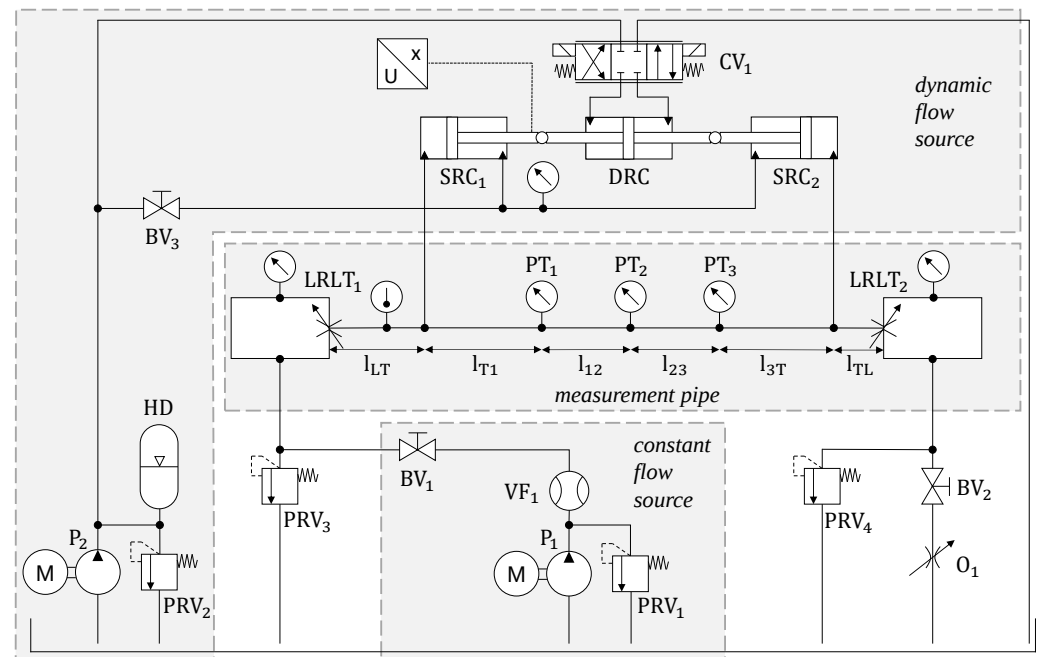


Figure 1. The hydraulic circuit of the test rig.

The core component of the test rig is the measurement pipe with a length of 3.22 m, which includes three designated locations for pressure measurements. The distances between the first and second pressure transducers, PT_1 and PT_2 , and between PT_2 and PT_3 , are $l_{12} = 0.33$ m and $l_{23} = 0.47$ m, respectively. The measurement pipe is terminated on both sides by a $LRLT_{1,2}$ to prevent the reflections of pressure waves, with each LRLT positioned $l_{LT} = l_{TL} = 0.36$ m from the nearest T-fitting. The adjustable orifice O_1 allows for the adjustment of the pressure inside the measurement pipe to reduce compressibility effects. The $LRLT_1$ is also connected to the steady flow supply, which includes a hydraulic pump (P_1), a pressure relief valve (PRV_1), and a volumetric flow rate sensor (VF_1). The measurement pipe is also connected to the dynamic flow source, comprising a double-rod cylinder (DRC) that drives two single-rod cylinders ($SRC_{1,2}$) to generate the dynamic volumetric flow rate. The T-fittings connecting the measurement pipe to the dynamic flow source are distanced $l_{T1} = 1.5$ m from PT_1 and $l_{3T} = 0.2$ m from PT_3 , ensuring sufficient inlet and outlet zones.

The double-rod cylinder is actuated by a control valve CV_1 to precisely control the amplitude and shape of the provided flow, allowing for varying operating points. The velocity of the piston movement is measured with a position sensor connected to one of the rods. Additionally, the dynamic flow source is equipped with a hydraulic pump P_2 , a pressure relief valve PRV_4 , and a switching valve SV_3 . The switching valves allow for the pre-pressurization of the single-rod cylinders to balance the force of the pressure

in the measurement pipe and thus reduce the stress on the connection of the cylinders. Further, the coupling of the single-rod cylinders ensures that the fluid volume within the measurement pipe remains unchanged, as the suction action of the other pulls in the volume displaced by one cylinder. This is important as the LRLT adjustment depends on the volumetric flow rate. Thus, the orifice of both of the LRLTs is adjusted based on the mean volumetric flow rate Q_m provided by the pump P_1 as shown in Equation (14). Any deviations in the volumetric flow rate through the orifice from Q_m lead to a change in characteristic impedance. This introduces an error in the volumetric flow rate calculation, which increases with increasing frequencies [25]. Also, the switching valves $SV_{1,2}$ enable the operation of the test bench in either an oscillating or pulsating mode. At the same time, the pulsating means the overlap of the mean volumetric flow rate Q_m with the dynamic $Q_c = A \cdot v$. For validating the soft sensor, the volumetric flow rate computed from pressure measurements is compared with the test rig's flow rate calculated as $Q = Q_m + Q_c$.

$$\Delta p = \frac{Q_m \cdot \rho \cdot a}{2 \cdot A_{pipe}} \quad (14)$$

As depicted in Figure 2, the realization of the test bench concept is shown. The image highlights critical components: the LRLT_{1,2}, circled in red, connected to both ends of the measurement pipe (circled in orange). Additionally, the coupled cylinders (circled in blue) are crucial in generating the dynamic volumetric flow rate, as they are responsible for the oscillating fluid movement. Details concerning the installation of the pressure transducers into the pipe can be found in Appendix B.2. In short, at each position, three transducers can be installed without interfering with the flow in the pipe.

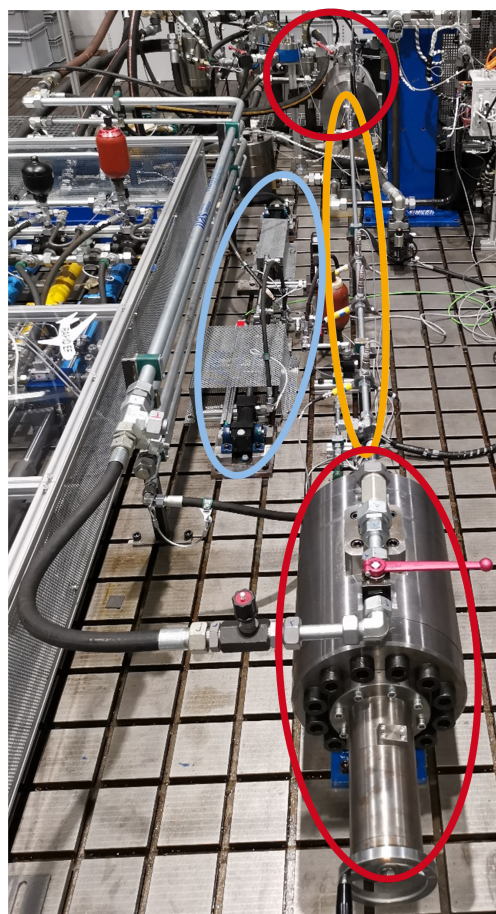


Figure 2. Picture of the constructed test rig.

2.5. Test Cases

Different flow scenarios were tested to validate the novel equation, as shown in Table 1. First, a steady volumetric flow rate Q_{Stat} was chosen. Then, the degree of non-uniformity δ of the flow rate and the frequency for the superimposed sine wave were determined. A mean system pressure of 100 bar guarantees that the compression effect is significantly reduced. The chosen combinations of volumetric flow rates and pressure are suitable so that the pump can deliver a steady laminar flow.

Table 1. Pressure conditions set for each test case.

Test Case	System Pressure p_{Sys} [bar]	Mean Volumetric Flow Rate Q_{Stat} [L/min]	Degree of Non-Uniformity δ [-]	Frequency f [Hz]
Sine (Figures 3–5)	100	40	0.347, 0.495, 0.635	5
Sine (Figures 6–8)	100	40	0.381, 0.614, 0.787	10
Sine (Figures 9–11)	100	50	0.281, 0.381, 0.425	15

3. Results

This section shows the agreement between the measured flow rate, composed of the steady flow rate by the pump and the calculated dynamic flow rates from the movement of the cylinders, and the flow rate calculated by the novel equations using two pressure sensors in the pipe. The presented flow rates are obtained by PT₁ and PT₂, which have the smallest distance regarding the three pressure transducers.

The raw measured signals were filtered using a first-order Butterworth filter [34] with a cutoff frequency that is slightly above the analyzed frequency. For a good comparison, both signals were aligned so that the peaks of the sine waves align. This offset between both volumetric flow rates is due to the spatial distance of both measuring points. The test rig's flow rate is computed by the velocity of the hydraulic cylinder, while the flow rate of the soft sensor is computed at the position of the second pressure transducer. Therefore, a phase offset is caused by the different locations of the measurement. The performance of the soft sensor, characterized by the mean error and the standard deviation of the volumetric flow rate computed by the test rig and the soft sensor, is provided in Table 2. The mean error and mean standard deviation are computed by taking the difference between both flow rates using the discrete sample points and dividing by the maximum flow rate of the test rig. Afterwards, the mean and standard deviation functions of Matlab were used [35]. Furthermore the maximum error is also shown in Table 2. The function for the mean error in percentage is read as follows:

$$Error = \frac{1}{n} \sum \text{abs} \left[\frac{Q_{SoftSensor} - Q_{TestRig}}{Q_{TestRig}} \% \right] \cdot 100 \quad (15)$$

Table 2. Mean errors and standard deviation for the tested frequencies and degrees of non-uniformity.

Frequency f [Hz]	Degree of Non-Uniformity δ [-]	Mean Absolute Error [%]	Standard Deviation [%]	Max Absolute Error [%]
5 (Figures 3–5)	0.347, 0.495, 0.635	2.9, 3.7, 4.9	1.6, 2.4, 4.1	5.8, 8.7, 13
10 (Figures 6–8)	0.381, 0.614, 0.787	4.0, 1.6, 1.0	3.4, 3.1, 3.2	10, 6.5, 6.8
15 (Figures 9–11)	0.281, 0.381, 0.425	4.5, 1.8, 2.8	2.4, 3.8, 1.5	9, 8, 6.3

To provide a better comparison and underline the advancement of the soft sensor, the mean, maximum, and standard deviation of the error is computed by the volumetric flow rate determined by the law of HP. These results are shown in Table 3.

Table 3. Mean errors and standard deviation for the tested frequencies and degrees of non-uniformity.

Frequency f [Hz]	Degree of Non-Uniformity δ [-]	Mean Absolute Error HP [%]	Standard Deviation HP [%]	Max Absolute Error HP [%]
5 (Figures 3–5)	0.347, 0.495, 0.635	39.4, 63.2, 86.9	44, 70.5, 97.2	69.8, 112, 158
10 (Figures 6–8)	0.381, 0.614, 0.787	87.1, 135, 179	97.5, 151, 201	164, 228, 317
15 (Figures 9–11)	0.281, 0.381, 0.425	87.2, 127, 172	97.2, 141, 191	150, 224, 288

For 5 Hz, both signals show good alignment, as can be seen in Figures 3–5. The mean errors were 2.9%, 3.7% and 4.9%, with standard deviations of 1.6%, 2.4% and 4.1%. The maximum error of the soft sensor was 4.9%, while HP produced a maximum error of 86.9%.

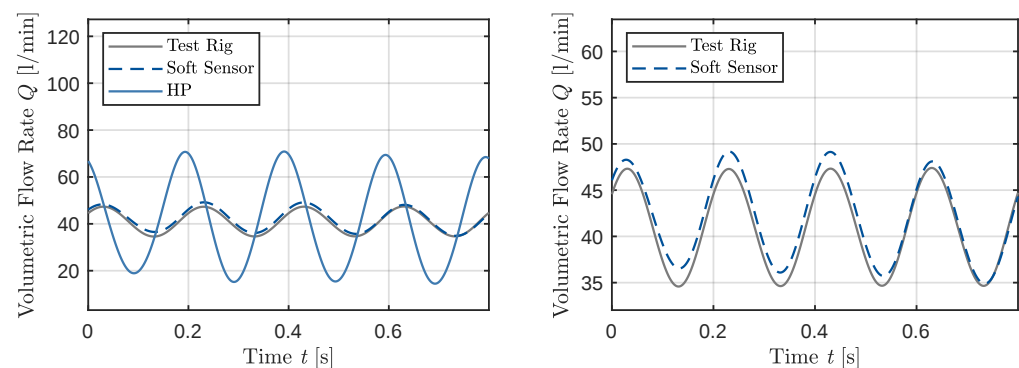


Figure 3. Measurement of the sine wave with a frequency of $f = 5$ Hz, a degree of non-uniformity of $\delta = 0.347$ and a steady flow rate of $Q_{Stat} = 40$ L/min.

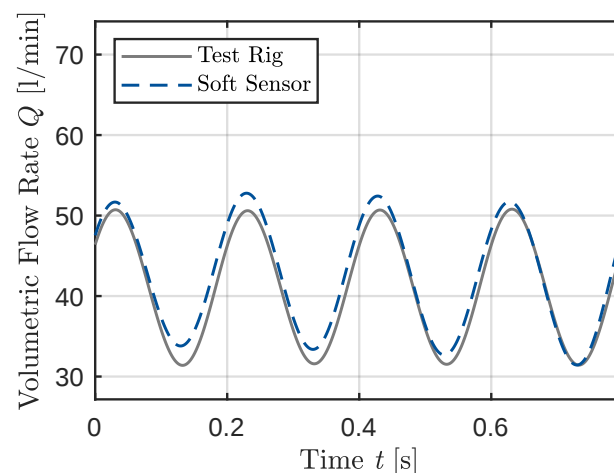


Figure 4. Measurement of the sine wave with a frequency of $f = 5$ Hz, a degree of non-uniformity of $\delta = 0.495$ and a steady flow rate of $Q_{Stat} = 40$ L/min.

In the case of a sine wave with 10 Hz, good agreement was achieved between the novel equation and the measured flow rate as shown in Figures 6–8. The mean errors were 4%, 1.6% and 1%, with standard deviations of 3.4%, 3.1% and 3.2%. The maximum error of the soft sensor was 4%, while HP produced a maximum error of 179%.

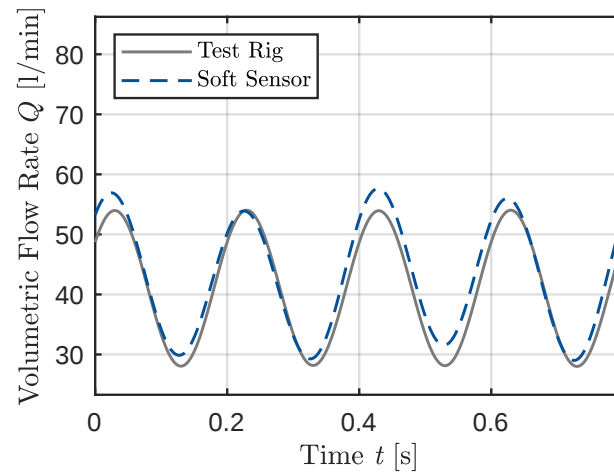


Figure 5. Measurement of the sine wave with a frequency of $f = 5$ Hz, a degree of non-uniformity of $\delta = 0.635$ and a steady flow rate of $Q_{Stat} = 40$ L/min.

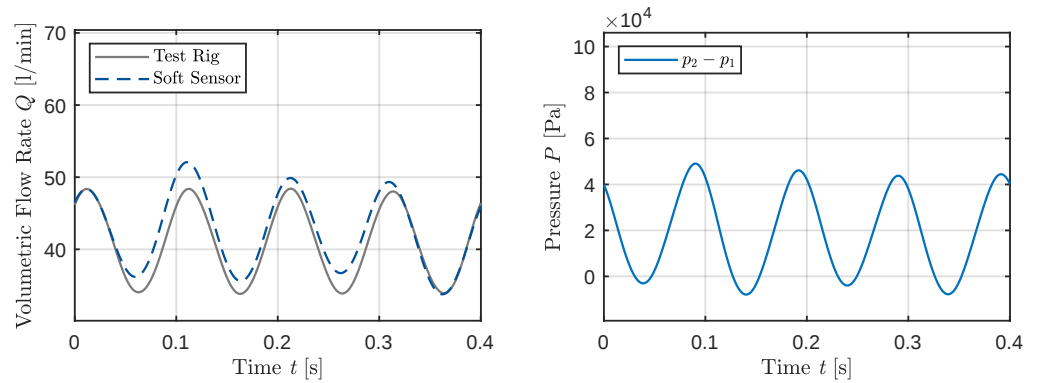


Figure 6. Measurement of sine wave with a frequency of $f = 10$ Hz, a degree of non-uniformity of $\delta = 0.381$ and a steady flow rate of $Q_{Stat} = 40$ L/min. The right figure shows the pressure difference between the pressure transducers.

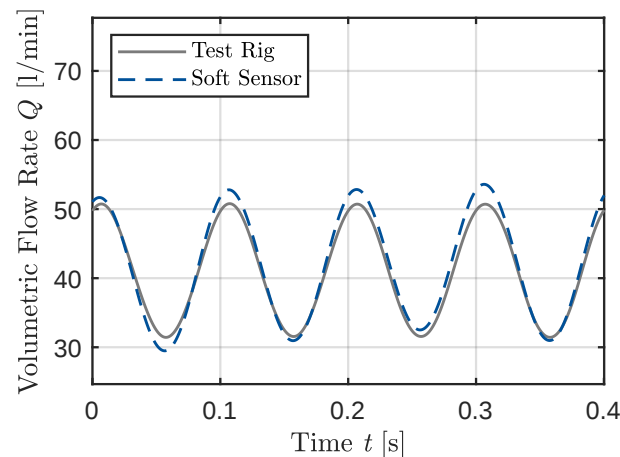


Figure 7. Measurement of sine wave with a frequency of $f = 10$ Hz, a degree of non-uniformity of $\delta = 0.614$ and a steady flow rate of $Q_{Stat} = 40$ L/min.

Lastly, the case displayed in Figures 9–11 of a sine wave with $f = 15$ Hz showed the least agreement, but the novel equation still managed to represent the system's behaviour. The mean errors were: 4.5%, 1.8% and 2.8%, with standard deviations of 2.4%, 3.8% and 1.5%. The maximum error of the soft sensor was 4.5%, while HP produced a maximum error of 172%.

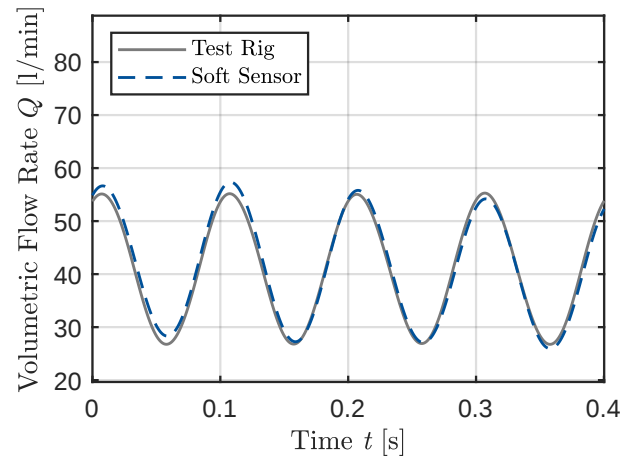


Figure 8. Measurement of sine wave with a frequency of $f = 10$ Hz, a degree of non-uniformity of $\delta = 0.787$ and a steady flow rate of $Q_{Stat} = 40$ L/min.

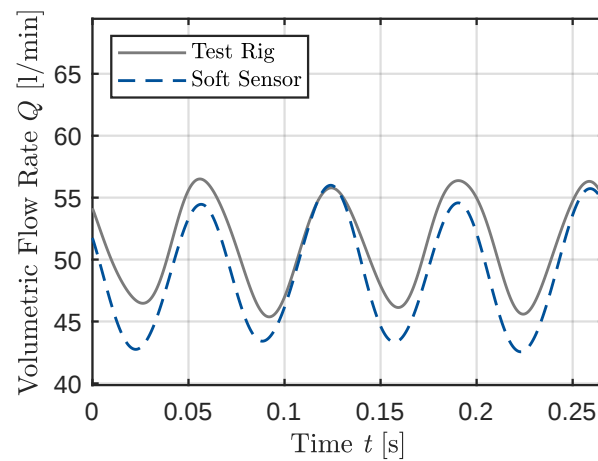


Figure 9. Measurement of the sine wave with a frequency of $f = 15$ Hz, a degree of non-uniformity of $\delta = 0.281$ and a steady flow rate of $Q_{Stat} = 40$ L/min.

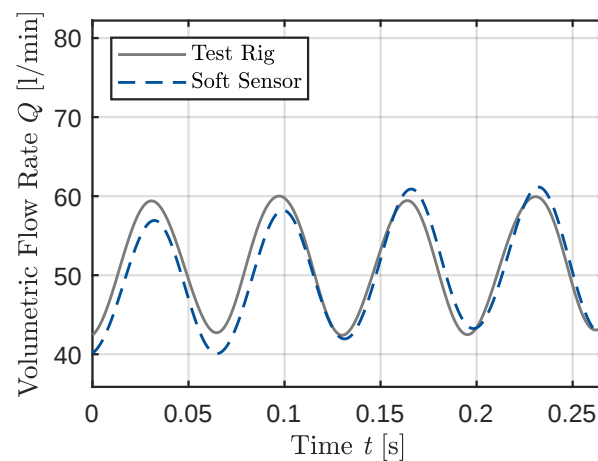


Figure 10. Measurement of the sine wave with a frequency of $f = 15$ Hz, a degree of non-uniformity of $\delta = 0.318$ and a steady flow rate of $Q_{Stat} = 40$ L/min.

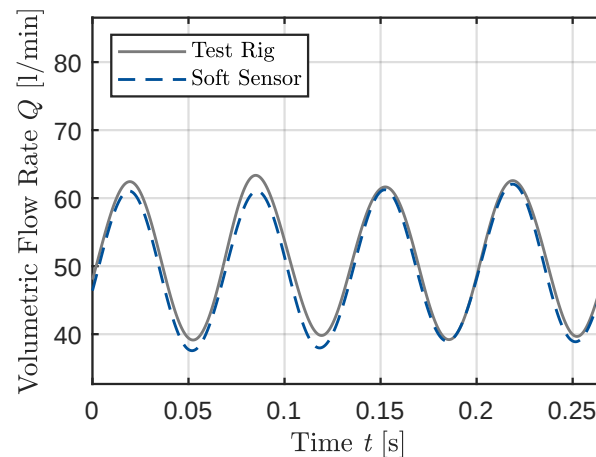


Figure 11. Measurement of the sine wave with a frequency of $f = 15$ Hz, a degree of non-uniformity of $\delta = 0.425$ and a steady flow rate of $Q_{Stat} = 40$ L/min.

In all cases analyzed, the mean error and standard deviation were below 5%, which shows that the novel equation agrees well with reality.

4. Discussion

The presented results show the promising ability of the soft sensor to measure transient volumetric flow rates accurately. The developed sensor utilizes the signals of two pressure transducers in a pipe to compute the transient flow rate efficiently and accurately. The computation is in real time due to the analytical model of the soft sensor. The computed volumetric flow rate matches well with the reference flow rate provided by the test rig. Several test cases are created by the coupled cylinder and pump, which are investigated and compared to the soft sensor. Different sine waves with varying frequencies and degrees of non-uniformity are investigated. The results underline the accuracy and advancement of the soft sensor in comparison to the law of HP since the different error metrics are, in all cases, significantly better.

The soft sensor captures frequencies up to 15 Hz and degrees of non-uniformity up to 0.787 with reasonable accuracy. Notably, the sensor matches the amplitude and phase of the created flow rate. The mean error in all measurements is below 5%, and the overall mean error considering all measurements at once is around 3%. The standard deviation of the error is also below 5%, underlining the good performance of the soft sensor. This mean error represents a good accuracy for the volumetric flow rate determination, especially compared to the HP computation, which displayed the lowest mean error of 39.4% and the highest of up to 179%. Furthermore, the smallest distance between the pressure transducers, $l_{12} = 0.33$ m, was used to obtain the volumetric flow rate, underlining the high accuracy and feasibility of the soft sensor.

An outlier can be seen in Figure 6. Here, the soft sensor is non-periodic, while the data from the test rig is periodic. The right plot of Figure 6 can explain this aperiodicity. It displays the pressure difference from which the flow rate has been calculated. As can be seen, the pressure difference is not periodic, which leads to the conclusion that the aperiodicity in the flow rate is not due to an error in the method but rather to imprecise measurements of the pressure transducer.

The test case shown in Figure 9 yields the highest mean error of 4.5%, the second-highest maximum error of 9% and a rather small error standard deviation of 2.4%. The results suggest that rather a constant offset is causing the error between both flow rates, which is calculated by the law of Hagen and Poiseuille. The newly proposed method is

intended to capture the dynamic flow rate. As can be seen, the general dynamic behaviour of the flow is captured, while the stationary part seems to be inaccurate. This inaccuracy is mainly due to errors in the pressure measurements, which occurred more frequently at higher frequencies. Higher frequencies resulted in the greater acceleration and deceleration of the cylinder coupling to obtain this transient flow, which affected the entire measurement pipe. These vibrations could cause inaccuracies in the pressure sensor, resulting in a higher error from the soft sensor due to the short length between the two pressure transducers. In prior studies, the soft sensor has shown high accuracy with higher frequencies of up to 1000 Hz [14] and pressure step responses [13] provided by distributed parameter simulations, which suggest that the actual dynamic of the investigated flow by the test rig poses not a significant problem for the soft sensor.

One limitation of the soft sensor is the lack of sufficient inlet and outlet pipe lengths to ensure fully developed flow conditions. This length increases linearly with the Reynolds number for laminar flows, which are usually aimed for in fluid power systems. A significantly shortened inlet section can lead to considerable inaccuracies. Future research will focus on integrating inlet flow models to reduce the required pipe length. Another limitation lies in the accuracy of the pressure sensors. The installation of the soft sensor requires a small distance between both pressure transducers. However, this results in a minimal pressure drop between the sensors, particularly in laminar pipe flows, which dominate most fluid power systems. Such minor pressure differences often fall within the range of the sensors' measurement accuracy, potentially impacting the reliability of the results. The calibration of the pressure sensors can tackle this challenge, and simple filtering techniques can further enhance accuracy by eliminating unwanted noise frequencies [36]. The pressure noise defines the minimum measurable volumetric flow rate, as the analytical model theoretically does not impose a lower limit on flow rate computation. The upper limit is determined by the requirement that the flow remains in the laminar regime, typically in fluid power systems. However, recent research has demonstrated the potential for accurate flow measurements even at higher Reynolds numbers in non-laminar conditions [36]. Additionally, the soft sensor requires a detailed fluid model to function correctly. Variations in kinematic viscosity and density due to changes in pressure and temperature must be accurately known. Failure to account for these variations can introduce significant errors. While established fluid models can be used and parameterized through experiments, their accurate implementation is essential for reliable performance. Finally, correct installation of the pressure sensors is critical to their success. External influences, such as vibrations or mechanical disturbances, can distort the measurements. Given the minor pressure differences involved, the pressure data must be exact. Proper sensor mounting and isolation from external disruptions are necessary to ensure accurate results. Despite these limitations, many challenges can be addressed through careful system design, calibration, and modeling; paving the way for broader adoption of the soft sensor in industrial applications.

5. Conclusions

This paper demonstrates the ability of an analytical soft sensor to determine transient volumetric flow rates based on two pressure signals in a pipe. It begins with an introduction to currently established volumetric flow rate measurements and transient flow rate models. It then describes the soft sensor and presents the test rig constructed and the test cases studied for validation.

The analytical soft sensor accurately and efficiently measures the transient volumetric flow rate provided by the test rig for the nine different test cases. The sensor agrees well with the volumetric flow rate for transient flows up to 15 Hz and a degree of non-uniformity up to 0.787.

The results of this research represent a significant advancement in soft sensors for transient flow rate measurement. This work presents a novel method for obtaining accurate and efficient volumetric flow rates in pipes using two pressure transducers. The proposed soft sensor, grounded in an analytical model, combines precision with computational efficiency, enabling the real-time calculation of the volumetric flow rate. Its minimally invasive design relies solely on pressure signal data, distinguishing it from traditional sensors. The results highlight the soft sensor's potential for diverse industrial applications, such as condition monitoring and control, where real-time and accurate measurements are crucial. By tracking pressure and flow rate, the sensor facilitates the calculation of hydraulic power. It supports predictive maintenance strategies, offering more profound insights into system performance. Unlike conventional numerical simulations, which trade-off between real-time computation and high accuracy, the soft sensor delivers both, making it uniquely suited for demanding operational environments. The soft sensor could be utilized to characterize pump performance under varying operational conditions, ensuring optimal functionality even in high-frequency scenarios. Applications like cylinder control and press operations benefit significantly from its real-time flow rate measurements, enabling precise control and improved system efficiency. Furthermore, the sensor's capacity to monitor flow rates aids in performance tracking and anomaly detection. It supports proactive maintenance by identifying potential issues before they escalate, reducing downtime and maintenance costs.

Future research will be conducted on higher frequency flow rates, which have already been investigated with a distributed parameter simulation [13,14]. Furthermore, the influence of reflections and different pressure profiles will be examined.

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Abbreviations

The following abbreviations are used in this manuscript:

BV	Ball Valve
CV	Control Valve
DRC	Double-rod Cylinder
HD	Hydraulic Damper
HP	Hagen and Poiseuille
HWA	Hot-wire Anemometry
ILT	Inverse Laplace transformation
LRLT	Low-reflection line terminator
O	Adjustable Orifice
P	Hydraulic Pump
PIV	Particle Image Velocimetry
PT	Pressure Transducer
Re	Reynolds number
SRC	Single-rod Cylinder
SV	Switching Valve
VF	Volumetric Flow Rate Sensor

Nomenclature

Symbol	Definition	Unit
*	Denotation of a Variable in the Laplace Domain	[m ² /s]
a	Speed of Sound	[m/s]
A	Cross-section of the Cylinder	[m ²]
A_D	Geometric Parameter	[m ²]
A_{pipe}	Cross-section of the Pipe	[m ²]
Dn	Dissipation Number	[-]
f	Frequency	[Hz]
γ	Propagation Operator	[-]
I_i	Modified Bessel function of the first kind of the i 'th order	[-]
K	Isentropic Bulk Modulus	[Pa]
K_1	First part of the convolution integral	[-]
K_2	Second part of the convolution integral	[-]
$K_{1,app}$	Approximation of K_1	[-]
k	A Natural Number	[-]
l	Pipe section length	[m]
L	Length of the Pipe	[m]
m	Order of poles	[-]
m_i	Part of Assumed Weighting Function	[-]
n_i	Part of Assumed Weighting Function	[-]
N	Upper Limit of Residue Sum	[-]
Δp	Pressure Difference	[-]
p_{Sys}	System Pressure	[bar]
p_1	Pressure at Inlet	[bar]
p_2	Pressure at Outlet	[bar]
Q	Volumetric Flow Rate	[m ³ /s]
Q_c	Volumetric Flow Rate from the Cylinder	[m ³ /s]
Q_m	Mean Volumetric Flow Rate	[m ³ /s]
Q_{max}	Maximum Volumetric Flow Rate	[m ³ /s]
Q_{min}	Minimum Volumetric Flow Rate	[m ³ /s]
Q_{Stat}	Stationary Volumetric Flow Rate	[m ³ /s]
Q_i	Volumetric flow rate at Inlet: $i = 1$ and Outlet: $i = 2$	[m ³ /s]
r	Radial Coordinate of the Pipe	[m]

R	Radius of the Pipe	[m]
Re	Reynolds Number	[-]
R_H	Hydraulic Resistance	[Pa/(m ³ /s)]
s	Laplace Variable	[-]
$\sinh_{approx}$	Approximation of the $\sinh(s)$ Function	[-]
t	Time	[s]
t_n	Normalized Time	[-]
v	Velocity of the Cylinder	[m/s]
v_x	Axial Velocity	[m/s]
W_i	Weighting function at End $i \in \{1, 2\}$ of the Pipe	[-]
W'_i	Weighting function at port $i \in \{1, 2\}$	[-]
$W_{i,dyn}$	Negative of $W'_{i,dyn}$	[-]
$W_{1,comp}$	Compressible Weighting Function at port 1	[-]
W_{inc}	Incompressible Weighting Function	[-]
Wo	Womersley Number	[-]
x	Axial Coordinate of the Pipe	[m]
z	Number of Pistons of an Axial Piston Pump	[-]
α_D	Discharge Coefficient	[-]
δ	Degree of Non-uniformity	[-]
ζ	Normalized Laplace Variable	[-]
ζ_k	Poles of the Weighting Function	[Pas]
Z_c	Series impedance	[bar/(m ³ /s)]
η	Dynamic Viscosity	[Pas]
ν	Kinematic Viscosity	[m ² /s]
ω	Pressure Variation Frequency	[1/s]
ρ	Fluid Density	[kg/m ³]
τ_n	Normalized Time	[s]

Appendix A. Derivation

Appendix A.1. Setup of the Problem

The Navier–Stokes equations were presented in a previous work by Brumand et al. [25] and are presented here as a reference for the reader:

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} \right) = -\frac{\partial p}{\partial x} + \eta \left(\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} \right) \quad (A1)$$

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + \rho \frac{\partial v_x}{\partial x} = 0 \quad (A2)$$

Here, ρ is the density of the fluid, v_x is the axial flow rate, t is the time, x is the axial coordinate within the pipe, η is the dynamic viscosity, and r is the radial coordinate within the pipe.

The assumptions that were taken to simplify the Navier–Stokes equations are described in a previous work by Brumand et al. [14] and are presented in the following:

1. The flow is laminar (Reynolds number $Re \leq 2300$), which means that the fluid layers do not mix. Therefore, the pressure gradient in the radial direction is negligible because the pressure remains constant across the pipe's cross-section.
2. There is axisymmetry in the flow and pipe geometry; therefore, the gradient in the angular direction is zero, i.e., $\left(\frac{\partial}{\partial \varphi} = 0 \right)$.

3. The velocity in the axial coordinate v_x is much less than the speed of sound a . Therefore, $v_x \ll a$, and the Mach number $Ma = v_x/a \ll 1$, and supersonic effects can be ignored.
4. The pipe length is big in relation to its radius ($R \ll L$). Therefore, there are no pressure reflections at the pipe walls.
5. The significant viscous effects in the motion equations are limited to those involving the radial distribution of axial velocity [26].
6. The pipe is horizontal, so gravitation is negligible as the forces are constant over the pipes' length.
7. The fluid density is constant over the vertical position within the pipe because the pipe's diameter is small.
8. Heat transfer is ignored since the focus is on liquids, excluding gases [37].

With the thermal equation of state for liquids, with K being the isentropic bulk modulus:

$$\frac{\partial \rho}{\rho} = \frac{\partial p}{K}, \quad (\text{A3})$$

and also considering the assumptions presented above, the Navier–Stokes equations simplify to:

$$0 = \rho \frac{\partial v_x}{\partial t} + \frac{\partial p}{\partial x} - \eta \left(\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} \right), \quad (\text{A4})$$

$$0 = \frac{1}{K} \frac{\partial p}{\partial t} + \frac{\partial v_x}{\partial x}. \quad (\text{A5})$$

For a detailed derivation, please refer to the work of Brumand et al. [14,25].

Appendix A.2. Derivation of the Inverse Laplace Transformed Weighting Functions

This appendix discusses the inverse Laplace transforms of the weighting functions W_1^* and W_2^* , given in Equation (10). The derivation can be split into two parts. Firstly, the limited case of an incompressible fluid is evaluated, as it simplifies the equations. Afterward, the general solution is found, which consists of a part that is the same as the incompressible case in addition to a part that represents compressible effects.

The needed inverse Laplace transforms of the weighting functions are given by the sum of their residues around the poles of the functions at ζ_k [38]:

$$W_{1,2} = \sum_{k=0}^N \text{Res}(W_{1,2}^*)|_{\zeta_k}, \quad (\text{A6})$$

where N is a natural number for the upper bound of the sum. For the ILT, knowing the position of the poles is necessary. First, the poles for the incompressible case are evaluated. Incompressible fluids have a speed of sound a tending to ∞ , which leads to a dissipation number tending to zero: $\lim_{a \rightarrow \infty} Dn \rightarrow 0$. The dissipation number is part of the argument for both harmonic functions in the weighting functions. Therefore, for the incompressible case, it is probable to approximate the harmonic functions by their first terms of the respective Taylor series approximations: $\lim_{x \rightarrow 0} \sinh(x) = x$ and $\lim_{x \rightarrow 0} \tanh(x) = x$. Applying this, both weighting functions result in the same function and the whole equation reduces to:

$$Q_2^* R_H = \frac{8 I_2(\sqrt{\zeta})}{\zeta^2 I_0(\sqrt{\zeta})} \zeta (p_1^* - p_2^*) = W_{\text{inc}}^* \zeta (p_1^* - p_2^*). \quad (\text{A7})$$

Firstly, it can be observed that the function has a double pole at $\zeta = 0$. Additionally, the residues at the remaining simple poles of the weighting function stem from the Bessel function. Bessel functions have an infinite amount of increasing simple poles that can be expressed as:

$$I_0(\sqrt{\zeta_k}) = 0 \quad (\text{A8})$$

The residue of the double pole at $\zeta_0 = 0$ can be calculated by the use of Equation (A9) [38]:

$$\begin{aligned} \text{Res}(W_{\text{inc}}^*)|_{\zeta_0=0} &= \lim_{\zeta \rightarrow \zeta_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{d\zeta^{m-1}} [W_{\text{exp}}^*(\zeta - \zeta_0)^m] \\ &\stackrel{m=2}{=} \lim_{\zeta \rightarrow \zeta_0} \frac{d}{d\zeta} [W_{\text{exp}}^*(\zeta - 0)^2] \\ &= \lim_{\zeta \rightarrow \zeta_0} \frac{d}{d\zeta} [W_{\text{exp}}^* \zeta^2] \\ &= \lim_{\zeta \rightarrow \zeta_0} \frac{d}{d\zeta} \left[\left(\frac{8I_2(\sqrt{\zeta})}{\zeta^2 I_0(\sqrt{\zeta})} \right) \zeta^2 \right] \\ &= \lim_{\zeta \rightarrow \zeta_0} \frac{d}{d\zeta} \left[\frac{8I_2(\sqrt{\zeta})}{I_0(\sqrt{\zeta})} \right] \\ &= 1 \end{aligned} \quad (\text{A9})$$

On the left are the residues at the remaining simple poles of the weighting function at $I_0(\sqrt{\zeta_k}) = 0$. The first eight poles, depending on the Bessel function, lie at:

$$\zeta_k = [-5.78, -30.47, -74.88, -139.04, -139.04, -222.93, -326.56, -449.93] \quad (\text{A10})$$

With the double pole at $\zeta = 0$ in Equation (A9), the ILT of the incompressible weighting function further depends only on simple poles to which the residue can be calculated more directly by [38]:

$$W_{\text{inc}} = \sum_{k=0}^N \text{Res}(W_{\text{inc}}^*)|_{\zeta_k} = 1 + \sum_{k=1}^N \frac{\text{numerator}[W_{\text{inc}}^*(\zeta_k)]}{\frac{\partial}{\partial \zeta} (\text{denominator}[W_{\text{inc}}^*(\zeta_k)])} e^{\zeta_k t_n}. \quad (\text{A11})$$

Inserting the numerator and denominator of the incompressible weighting function into Equation (A11) yields the inverse Laplace transformed function of W_{inc}^* as:

$$W_{\text{inc}}^*(t_n) = 1 + \sum_{k=1}^N \frac{8I_2(\sqrt{\zeta_k})}{1/2\zeta_k^{3/2}I_1(\sqrt{\zeta_k}) + 2\zeta_k I_0(\sqrt{\zeta_k})} e^{\zeta_k t_n}. \quad (\text{A12})$$

Inserting the poles into the sum gives:

$$\begin{aligned} W_{\text{inc}}^*(t_n) &= 1 - 0.956e^{-5.78t_n} - 0.0344e^{-30.4t_n} \\ &\quad - 0.00571e^{-74.9t_n} - 0.00165e^{-139t_n} - 0.000643e^{-223t_n}. \end{aligned} \quad (\text{A13})$$

The poles for the case of $Dn > 0$ (the case of a compressible fluid) are the same as for $Dn \rightarrow 0$ (the case of an incompressible fluid) in addition to the poles caused by the hyperbolic terms. For the ILT, knowledge about the position of the poles and the order is necessary. To gain information about the order of the poles, an approximation is used for the sinh function that maintains the characteristics of the poles [39]:

$$\sinh(x) \approx \sinh_{\text{approx}}(n, x) = x \left[\prod_{k=1}^n \left(1 + \frac{x^2}{k^2 * \pi^2} \right) \right], \quad \text{for } n \in \mathbb{N}. \quad (\text{A14})$$

The poles of both weighting functions are at $\sinh(x) = \tanh(x) = 0$, and by using Equation (A14):

$$1 + \frac{I_0(\sqrt{\zeta_k}) \zeta_k^2 Dn^2}{k^2 I_2(\sqrt{\zeta_k}) \pi^2} = 0, \quad \text{for } k \in \mathbb{N}. \quad (\text{A15})$$

Rearranging for ζ_k , we obtain:

$$\frac{I_0(\sqrt{\zeta_k}) \zeta_k^2}{I_2(\sqrt{\zeta_k})} = \left(\frac{k\pi}{Dn} \right)^2 - 1. \quad (\text{A16})$$

This equation was solved for the poles ζ_k using the software Maple [40] for various dissipation numbers. The dissipation numbers range from 1×10^{-7} to 1×10^{-1} , with 100 samples per order of magnitude. For each sampled dissipation number, the first 20 poles were computed. Poles that are bigger than that have a negligible effect on the flow rate. This matrix functions as a lookup table for which poles to use during the computation of the volumetric flow rate. Rounding the dissipation number to the nearest sampled dissipation number did not have a visible impact on the result.

With knowledge of the poles, the residues of the time domain solution for each dissipation number can be calculated using Equation (A17):

$$\text{Res}(W_1^*)|_{\zeta_k} = \frac{\text{numerator}[W_1^*(\zeta_k)]}{\frac{\partial}{\partial \zeta} (\text{denominator}[W_1^*(\zeta_k)])} e^{\zeta_k t_n}. \quad (\text{A17})$$

The computation of the residues was automated using the Maple software. Now, the weighting function in the time domain (W_1) is given by:

$$W_{1,\text{comp}} = \sum_k^N \text{Res}(W_1^*)|_{\zeta_k} = \frac{\text{numerator}[W_1^*(\zeta_k)]}{\frac{\partial}{\partial \zeta} (\text{denominator}[W_1^*(\zeta_k)])} e^{\zeta_k t_n}. \quad (\text{A18})$$

An exemplary weighting function is presented in the following as a clarification for the reader. Assuming $Dn = 0.0011$, and using Equation (A16), the first five poles are located at:

1. $\zeta_1 = -38.30 \pm 2818.21i$
2. $\zeta_2 = -53.95 \pm 5658.55i$
3. $\zeta_3 = -65.6 \pm 8502.53i$
4. $\zeta_4 = -76.08 \pm 11348.4i$
5. $\zeta_5 = -85.00 \pm 14195.4i$.

For this exemplary dissipation number, the compressible effects are considered by the following weighting function, consisting of the residues caused by the poles of the $\sinh$ function:

$$\begin{aligned} W_{1,\text{comp}}(t) = & -(0.0000180 + 0.00270i)e^{(-38.30-2818.21i)t_n} + (-0.000018 + 0.00270i)e^{(-38.30+2818.21i)t_n} \\ & + (0.0000066 + 0.00140i)e^{(-53.95-5658.55i)t_n} + (0.0000066 - 0.00140i)e^{(-53.95+5658.55i)t_n} \\ & - (0.0000036 + 0.00093i)e^{(-65.96-8502.53i)t_n} + (-0.0000036 + 0.00093i)e^{(-65.96+8502.53i)t_n} \\ & + (0.0000023 + 0.00070i)e^{(-76.08-11348.4i)t_n} + (0.0000023 - 0.00070i)e^{(-76.08+11348.4i)t_n} \\ & - (0.0000016 + 0.00056i)e^{(-85.00-14195.4i)t_n} + (-0.0000016 + 0.00056i)e^{(-85.00+14195.4i)t_n}. \end{aligned} \quad (\text{A19})$$

Together with the incompressible weighting function (Equation (A13)) caused by the incompressible poles, the whole weighting function is given by:

$$W_1(t_n) = W_{1,\text{comp}}(t_n) + W_{\text{inc}}(t_n) \quad (\text{A20})$$

Note that $W_{1,comp}$ depends on the dissipation number Dn . For a treatise of the resulting convolution integral for calculating the volumetric flow rate (Equation (11)), please refer to the manuscript by Brumand et al. [14].

Appendix B. Test Rig Details

Appendix B.1. Low-Reflection Line Terminator

A LRLT generally consists of an orifice, an adjustment mechanism for the variable adjustment of the orifice cross-section, and a volume behind it. By adjusting the orifice's cross-section, the impedance of the orifice can be matched to the pipe's characteristic impedance, and thereby the LRLT imitates an infinitely long pipe. The pressure wave is not reflected and propagates in the volume of the low-reflection pipe termination. The structure of such a low-reflection line termination is illustrated in Figure A1. The effect of the adjustment on the measured pressure waves in a pipe is shown in Figure A2. Suppose that the orifice cross-section is not adjusted appropriately, and pressure transducers are installed at three locations of a connected measurement pipe. In that case, incoming pressure waves are reflected at the low-reflection line termination, and the pressure histories show a noticeably smaller pressure amplitude and a phase shift (correct figure) compared to the correctly adjusted LRLT (left figure). As in this example, a harmonic volumetric flow rate was induced. The LRLT cannot be adjusted by viewing the measured pressure histories, and instead, an adjustment based on a calculated pressure drop across the orifice was carried out [33].

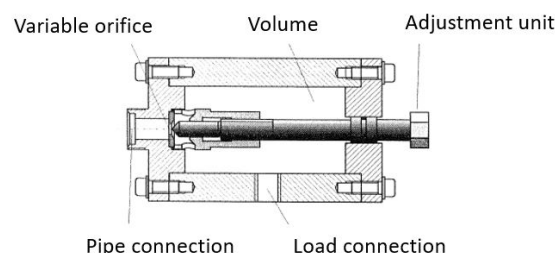


Figure A1. Illustration of an LRLT [25].

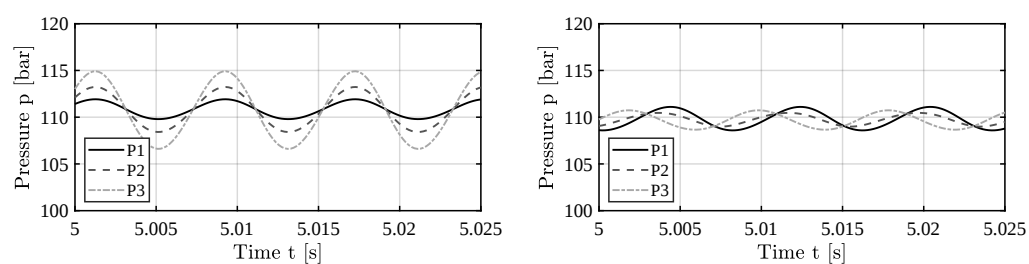


Figure A2. Pressure history: (left) without reflection, (right) with reflection [25].

Appendix B.2. Pressure Transducer Installation

One of the critical challenges in this setup is ensuring that the pressure transducers are integrated to provide highly accurate pressure measurements, as these are crucial for the soft sensor's calculation of the volumetric flow rate. To achieve this, a custom cutting ring fitting was developed. This specialized fitting connects two pipe segments without altering the internal diameter from the measurement pipe to the fitting. It allows the installation of up to three pressure transducers at a single point along the pipe. A cross-sectional view of the fitting is provided in Figure A3.

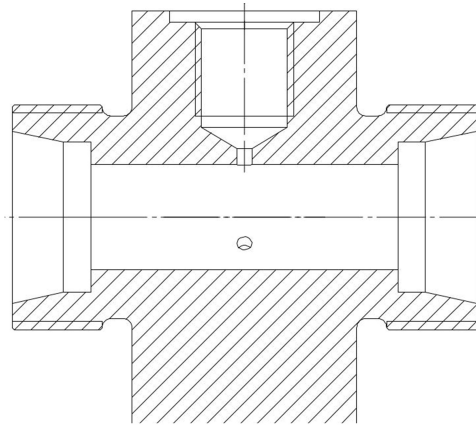


Figure A3. Cross-section of custom cutting ring fitting.

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