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Integrated optimization of timetabling and Electric Vehicle Scheduling

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ABSTRACT

We tackle the integrated planning problem of periodic timetabling and electric vehicle scheduling, crucial for cities transitioning to electric bus fleets. Given existing timetables, we allow only minor modifications and propose an iterative solution approach that addresses the Electric Vehicle Scheduling Problem (EVSP) in each iteration. Due to the NP-hard nature of EVSP, we employ well-established heuristics and evaluate the quality of the solutions obtained. Specifically, we establish tight approximation bounds for certain iterative heuristics that first solve the Vehicle Scheduling Problem and subsequently adjust solutions to meet battery constraints. We make several key contributions: We provide general insights into heuristic solution quality, establish theoretical performance bounds, and validate these findings through a case study using real-world data from Aachen, Germany. Additionally, we employ our iterative framework to derive managerial insights for bus operators in Aachen by quantifying potential gains from adjusting the timetable to support the transition to a fully electric bus fleet.

1. Introduction

Many cities globally transition their bus fleet from traditional gasoline buses to a fully electric fleet to meet sustainability goals (Sustainable Bus, 2024). While electric buses induce zero local emissions, they face significant challenges such as longer charging times and a more limited driving range compared to their gasoline counterparts. These limitations make the transition highly complex, requiring either a substantially increased number of electric buses, which is costly, or intricate planning of the charging schedule to ensure sufficient battery capacity for all trips.

In this work, we focus on transit network planning that aims to transform gasoline bus fleets to fully electric ones in cities that employ periodic schedules. Within a periodic schedule, the schedule of a single period is repeated at regular intervals (e.g. every hour). Periodic schedules result in easy to memorize timetables that are favored by passengers, and have been successfully implemented in many cities across Germany, the Netherlands, and Switzerland.

Transit network planning is typically divided into five sequential steps: line planning, frequency setting, timetabling, vehicle scheduling, and crew scheduling/rostering (Ceder and Wilson, 1986). For periodic schedules, the line planning and frequency setting steps involve defining the layouts and frequencies of the lines, while the final periodic timetable is completed in the timetabling step by setting the departure times of the individual bus lines. In the vehicle scheduling step, vehicles are assigned to the trips specified in the timetable. For electric vehicles, this step includes determining the charging schedule to ensure sufficient battery capacity for each trip. This planning problem is referred to as the Electric Vehicle Scheduling Problem (EVSP) and studied extensively in scientific literature. In the final step, bus drivers are assigned to the vehicle and trip pairs during crew scheduling/rostering.

In the context of electrification, it would be advantageous to redesign the entire planning process and integrate the planning steps. But in Germany, the first two steps are heavily regulated (cf. SWA, 2019; The Federal Office of Justice Germany, 2024). Consequently, bus operators are generally reluctant to alter line layouts or frequencies. Personnel planning also involves complex challenges with numerous regulatory constraints. Thus, personnel costs are typically estimated at a flat rate and added to the vehicle and operational costs in the earlier planning steps. However, bus operators might consider minor adjustments in the timetabling step, such as to line departure times, if the benefits are substantial.

However, when examining the potential benefits of such an integrated approach, we observed a significant gap not only in practical estimates but also in scientific research on this subject. Against this background, we study the research question of whether an integrated approach, which permits minor adjustments to an existing periodic

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timetable prior to solving the underlying EVSP, can yield significant benefits for the bus operator. To solve the highly challenging underlying optimization problem, we develop a new solution approach inspired by the Logic-Based Benders Decomposition (LBBD) literature. The proposed framework decomposes the problem into a timetabling master problem and an EVSP subproblem that are solved iteratively. Because the EVSP must be solved repeatedly, which is not computationally tractable with an exact method, we extend existing approximation algorithms from related literature streams and develop new ones to tackle it. We then apply the developed algorithms to a case study for the city of Aachen, Germany, deriving both computational and managerial insights.

We begin by reviewing the scientific literature on timetabling and vehicle scheduling and specifying the research gap in Section 2. We then formally define the integrated planning problem and present the LBBD-based framework in Section 3. The subroutines for the timetabling and EVSP steps are detailed in Sections 4 and 5, respectively. Finally, we address our research question in Section 6, where we conduct a computational study based on realistic data from Aachen. Concluding remarks are given in Section 7.

2. Literature review

2.1. Timetabling and vehicle scheduling

In this section, we recap the scientific literature on the timetabling and vehicle scheduling planning steps. The timetabling step consists of finding departure times for each trip while ensuring transfers between trips. For periodic schedules, we only have to decide on departure times for bus lines instead of singular trips, which reduces the problems size. The vehicle scheduling step assigns trips to buses such that each trip can be served by the assigned bus. While these planning steps are usually solved sequentially, we will also highlight some recent developments that attempt to solve both planning problems in an integrated manner.

The planning problem of finding departure times for all lines with given frequencies that satisfy all transfer constraints is often modeled as a Periodic Event Scheduling Problem (PESP). Serafini and Ukovich (1989) introduce the original formulation of the PESP, prove its NP-completeness via a reduction from the Hamiltonian Cycle problem, and propose an initial combinatorial algorithm. Since then, diverse algorithms have been proposed for the PESP, including MIP models (Liebchen and Möhring, 2007; Liebchen and Peeters, 2009), constraint programming approaches (Gattermann et al., 2016; Großmann et al., 2012), and modulo–simplex heuristics (Borndörfer et al., 2017; Goerigk and Schöbel, 2013). A slightly older computational comparison of various solution approaches for the PESP can be found in Liebchen et al. (2008).

Regarding vehicle scheduling, the EVSP originates from the classical Vehicle Scheduling Problem (VSP), which, given a fixed timetable, finds a feasible vehicle rotation, i.e., an assignment of trips to gasoline buses. While the VSP with a single depot and bus type is solvable in polynomial time, there exist multiple extensions of the VSP, considering multiple bus types and depots. Although these are NP-hard problems, practical efficient exact MIP models (and heuristics) exist. For a comprehensive overview of the topic, we refer to the review by Bunte and Kliewer (2009).

The EVSP extends the VSP to electric buses with their inherent limited range and charging requirements. Due to the capacity constraint, the EVSP is directly NP-hard even for a single bus type and depot (Sassi and Oulamara, 2016). From a practical perspective, multiple additional assumptions are often integrated into the EVSP, such as multiple bus types and depots, only depot recharging over night or opportunity recharging between trips, non-linear (re-)charging curves (Olsen and Kliewer, 2020), detailed scheduling of the charging process itself (Abdelwahed et al., 2020), or uncertainty (An, 2020; Liu and Song, 2017; Li et al., 2021; Shen et al., 2023). We refer to Dirks et al. (2021) for a detailed discussion of the various assumptions and the extent to which they have been considered in the literature.

Focusing on exact approaches for EVSP, Janovec and Koháni (2019) formulate and solve the EVSP with a compact MIP model. Adler and Mirchandani (2017) and Niekerk et al. (2017) propose a branch-and-price approach to solve the EVSP for both linear and concave charging functions. In their models, they use a set covering formulation to select a subset of feasible vehicle rotations to serve all trips. The vehicle rotations are generated by a dynamically solved subproblem that can cope with various charging assumptions, making their approach flexible. As their exact approach struggles to solve even medium-sized instances, they enhance it with heuristics that enables them to solve larger instances. In Li (2014), the author also develops a branch-and-price algorithm, but assumes fast recharging or battery swapping for the recharging process. Recently, De Vos et al. (2024) formulate a new branch-and-price model that considers the limited capacity of charging stations and extend it with heuristics to solve larger instances.

Since exact approaches often need to be complemented with heuristics to tackle real-world applications, many heuristics specifically tailored for EVSP have been proposed. Olsen et al. (2020) focus on repair heuristics that first compute an optimal solution for the VSP, disregarding battery capacities, and then repair the vehicle rotations to be battery-feasible. They conclude that the percentage of feasible vehicle rotations generated by their approach meets the requirements for an initial implementation of electric buses in practice. Other matheuristics (Rogge et al., 2018; Liu et al., 2021) and variable neighborhood search methods (Olsen and Kliewer, 2022) extend the original EVSP by the strategic decision of locating charging stations.

2.2. Integrated approaches

Focusing on integrated approaches, the combined problem of designing a non-periodic timetable and VSP has already been extensively researched in the literature (Ibarra-Rojas et al., 2014; Laporte et al., 2017; Desfontaines and Desaulniers, 2018; Carosi et al., 2019). For integrated approaches combining timetabling with the EVSP, there is a recent literature review by Wang et al. (2025). They list 9 papers that explicitly address the integrated problem. Out of these, 8 develop heuristic methods for the case with non-periodic schedules. For example, Xu et al. (2023) develop a MIP model and extend it with a Lagrangian relaxation heuristic to solve instances of larger size. Gao et al. (2025) propose a multi-objective model for optimizing bus timetabling and vehicle scheduling but focus only on a single bus line. Furthermore, both studies focus on planning the timetable and vehicle rotation from scratch, while we assume that an existing periodic timetable is already given, and only slight modifications are allowed before the EVSP is solved.

However, the assumption of a periodic timetable brings challenges distinct from the non-periodic schedules assumed there. While in non-periodic timetables, the departure time of individual trips can be adjusted to better fit the underlying EVSP, only the departure times of whole lines, but no individual trips, can be adjusted in periodic timetables.

To the best of our knowledge, only three recent studies combine periodic timetabling with the VSP. Van Lieshout and Bouman (2018) study the setting from a theoretical side and provide complexity results for the problem of estimating the cost of a vehicle schedule for a given timetable. Van Lieshout (2021) studies the structural properties of the solution space and develops an efficient MIP formulation for the integrated problem. However, the model relies on specific structural properties of the solution space resulting from the VSP and cannot easily be extended to the EVSP. Specifically, the approach relies on vehicle circulation scheduling, i.e., finding sequences of trips that can be consecutively operated by a single vehicle within the periodic pattern. In EVSP, a single vehicle may not be able to operate such a sequence consecutively due to battery constraints. Hence, we cannot restrict ourself to circulation scheduling within the EVSP setting.

Based on the survey by Wang et al. (2025) and our own search, Quttineh et al. (2023) is the only publication that considers both periodic timetabling and EVSP. The problem they consider includes timetabling, EVSP, and charging facility location for both periodic and non-periodic timetables. They also allow opportunity recharging between trips but allow only scheduling of charging activities of predefined length. However, the developed MIP formulation faces strong computational limitations, as they use it only to solve a small-scale instance with up to 188 trips within a case study.

2.3. Contribution

In the following, we now highlight the three main contributions that this paper provides to the existing literature:

1. We present a MIP-based iterative approach for solving the integrated periodic timetabling and EVSP, while allowing only slight modifications to the existing periodic timetable. The proposed framework can be easily implemented within a MIP solver and supports the use of any EVSP solver as a subroutine. Consequently, our method can be easily adapted to incorporate the various practical EVSP assumptions discussed earlier.
2. We make a direct contribution to the EVSP literature by linking the EVSP with the Bin Packing with Conflicts (BPwC), a theoretical optimization problem closely related to the basic EVSP without recharging (Huang et al., 2023; Jansen and Öhring, 1997). We not only show that the existing approximation ratios from BPwC can be directly transferred to a specific EVSP variant with depot recharging, but also extend these results by presenting new approximation ratios for the EVSP with linear recharging. To the best of our knowledge, we are the first to provide approximation bounds for a variant of the EVSP with recharging. Additionally, our theoretical results offer general insights into the quality of repair heuristics that first solve vehicle scheduling without battery restrictions to optimality and then repair the solution to make it battery-feasible.
3. We conduct a computational study based on real-world data from the bus network in the city of Aachen, Germany. In this study, we demonstrate the flexibility of our framework by comparing the performance of the discussed EVSP heuristics, while also examining the impact of different recharging strategies and battery size assumptions. Furthermore, our results provide valuable managerial insights into the potential benefits of allowing slight modifications to the existing periodic timetable for the bus network.

3. Problem setting and iterative solution approach

We consider the following problem setting with a given periodic timetable $\mathcal{S}$ with a cycle time T for the current gasoline bus fleet. The timetable consists of a set of bus lines $\mathcal{L}$. Each line $l \in \mathcal{L}$ operates in a time interval $[d_l, e_l]$, i.e., the first trip starts at d_l and the last trip at e_l. The line is periodically operated with frequency f_l, resulting in a headway $h_l = T/f_l$. The total number of trips of that line is then given by $z \in \mathbb{N}$ with $e_l = d_l + z \cdot h_l$. A periodic timetable $\mathcal{S}$ is then formally defined as a triple $\mathcal{S} = (\mathcal{L}, \{(d_l, e_l) \mid l \in \mathcal{L}\}, \{h_l \mid l \in \mathcal{L}\})$.

Our objective is now to find a new timetable by selecting new bus line start times $s' = (s'_1, \ldots, s'_n)$, where n is the number of bus lines, that is cost-minimal, i.e., results in the lowest objective value of the subsequent EVSP. This process is dependent on the currently existing timetable. Compared to it, the start times d_l of operations of a bus line l may only change by a small amount. Due to the periodic timetable, this induces a shift in starting times of all trips of the respective bus line, e.g., a shift in 5 min results in all trips of this bus line starting 5 min later. For each bus line l, we define D_l as the set of all feasible starting times that would be acceptable by the operator. In addition to these shift constraints, we also require the resulting schedule to be feasible, i.e, satisfy all required transfer constraints.

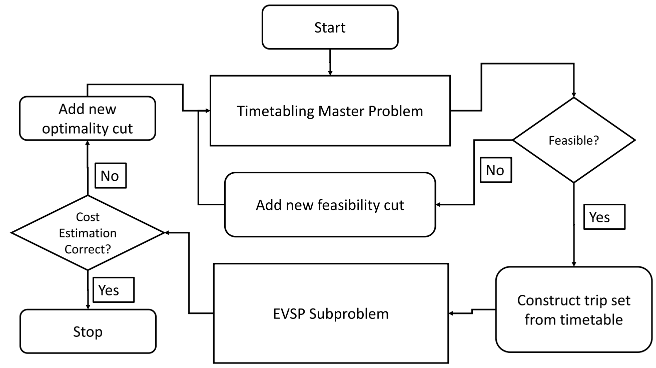


Fig. 1. Iterative Approach for the integrated timetabling and vehicle scheduling problem.

To solve this problem, we draw inspiration from the Logic-Based Benders Decomposition (LBBD) literature (Hooker, 2000). LBBD is a natural extension of the standard Benders Decomposition (Rahmaniani et al., 2017) for two-stage optimization problems with nonconvex second-stage problems and has been successfully applied to a wide range of applications in transportation, scheduling, supply chain, telecommunications, and medicine (Hooker, 2024, 2019). In LBBD, the problem is decomposed into a master and a subproblem, which are solved iteratively.

Fig. 1 illustrates the application of the LBBD framework to our setting. The core of our approach consists of the *Timetabling Master Problem* and *EVSP Subproblem* blocks that correspond to solving the PESP and EVSP, respectively. While we present exemplary implementations of these blocks in Sections 4 and 5, which are used for our case study later, we highlight here that especially the EVSP subroutines can be solved with any EVSP algorithm. This makes our procedure easily adjustable to varying constraints and runtime requirements from practice.

The task of the *Timetabling Master Problem* is to find a good guess for a timetable by selecting start times $s' = (s'_1, \ldots, s'_n)$. As a timetable is subject to some (complex) transfer constraints, not all combinations of feasible starting times result in a feasible timetable. Let AS denote the set of all feasible timetables. Furthermore, let TP be the set of start times that violate the timetabling constraints. If the guessed start times are infeasible, i.e., $s' \in TP$, we add a feasibility cut to exclude s' from being selected again. Otherwise, the *Timetabling Master Problem* assigns a cost estimation to the found feasible timetable, computes the set of trips $\mathcal{T}$ that must be executed to operate this timetable, and passes it to the EVSP subroutine.

The *EVSP Subproblem* finds the assignment of electric vehicles to the trips in $\mathcal{T}$ and computes the cost of the resulting schedule. Afterwards, we compare the found cost with the estimation made by the *Timetabling Master Problem*. If the estimation is wrong, i.e., if the estimated costs were too low, we add an optimality cut ensuring that the schedule is assigned the correct cost. Otherwise, we terminate and have found an optimal timetable if the solution to the EVSP Subproblems are optimal.

Due to the complexity of the subproblem, computing optimal solutions in each iteration is not feasible. To counteract this issue, we propose using approximation algorithms to compute solutions for the EVSP, from which the cuts are generated. This approach yields tractable solution times for the EVSP while preserving a bound on the quality of the obtained solution. To the best of our knowledge, our approach is the first to combine approximation algorithms and LBBD in this fashion.

The main design challenge of our iterative procedure is that the guessed costs have to be a lower bound on the real cost of a timetable.

Therefore, the added optimality cuts can be interpreted as an iterative improvement on the lower bound, and the *Timetabling Master Problem* as a heuristic search for good timetable candidates. Next, we present how we implement the *Timetabling Master Problem* for our case study.

4. Timetabling master problem

We model the Timetabling Master Problem as a MIP with two types of decision variables β and γ_{li}. The binary decisions γ_{li} represent whether line l starts at time $i \in D_l$, and β captures the costs of the vehicle rotation for the selected timetable. For a timetable $S \in AS$ and line $l \in \mathcal{L}$, we denote the times $i_{l,S} \in D_l$ as the starting time of line l in schedule S. TP is then formally defined as the set of all tuples $((l_1, i_1), (l_2, i_2))$, with $l_1, l_2 \in \mathcal{L}$, $i_1 \in D_{l_1}$, $i_2 \in D_{l_2}$ that induce a timetable violation. Specifically, $((l_1, i_1), (l_2, i_2)) \in TP$ if when a line l_1 starts at i_1 and line l_2 starts at i_2, this results in a violation of a transfer constraint in the timetabling problem. The MIP formulation can then be expressed as follows:

$$\min \quad \beta \tag{1a}$$

$$\text{s.t.} \quad \sum_{i \in D_l} \gamma_{li} = 1 \qquad \forall l \in \mathcal{L} \tag{1b}$$

$$\gamma_{l_1 i_1} + \gamma_{l_2 i_2} \leq 1 \qquad \text{if } ((l_1, i_1),(l_2, i_2)) \in TP \tag{1c}$$

$$\beta \geq \beta(S) - \sum_{l \in \mathcal{L}} M_l^S \cdot (1 - \gamma_{l i_{l,S}}) \qquad \forall S \in AS \tag{1d}$$

$$\gamma_{li} \in \{0; 1\} \qquad \forall l \in \mathcal{L}, i \in D_l \tag{1e}$$

$$\beta \geq 0 \tag{1f}$$

The objective (1a) is to minimize the cost of the selected timetable. Constraints (1b) ensure that exactly one starting time is selected for each bus line. Constraints (1c) represent the feasibility constraints. For all violating pairs in TP, the MIP is instructed not to select the violating combination of line start times. Constraints (1d) are the optimality constraints. These constraints ensure that for each feasible schedule S, the cost of the timetable corresponds to the optimal cost derived from the EVSP subproblem, $\beta(S)$. However, if the starting time of a bus line $l \in \mathcal{L}$ is selected to be different from that in S, it is possible to save up to M_l^S cost units with the new schedule. As described in Section 3, both feasibility constraints (1c) and optimality constraints (1d) are initially omitted from the MIP formulation and then iteratively added within our LBBD approach. As a technical remark, note that we always eliminate infeasible schedules using constraints (1c). Consequently, no constraint (1d) is ever generated for a schedule that violates the timetabling constraints.

The choice of the parameter M_l^S has a great influence on the performance of the algorithm. It should be chosen as small as possible, while remaining an upper bound on the costs saved when changing the starting time of line l in schedule S. We set M_l^S to the number of electric buses in S that serve trips from line l. This is an obvious upper bound on the potential savings, because all buses not serving trips from l are still needed when changing the starting time from l. Furthermore, this bound is, in general, tight. For example, consider a scenario where shifting the start time of a line eliminates all temporal constraints with another line, and the buses from that other line have sufficient battery capacity to serve the trips originally scheduled for the shifted line. In that case, all buses serving the original line can potentially be saved in the new solution.

One type of feasible but undesired solution for Model (1) occurs when all bus lines are shifted by the same interval, such as the entire original schedule being shifted by a fixed time. To avoid these unnecessary solutions, we include the following symmetry-breaking constraints in our model

$$\sum_{l \in \mathcal{L}} \gamma_{l i_{l,S}} \leq |\mathcal{L}| - 1$$

for all schedules S that are simply the original schedule shifted by a fixed time frame. Since each line has a unique starting time in schedule S, the above constraint ensures that such a schedule cannot be selected by model (1).

Both feasibility and optimality cuts are based on certain assumptions that warrant discussion. For feasibility cuts, we opted for a rule-based approach where a rule-based method is used to check for violated transfer constraints after a timetable is fixed. This approach is simpler to implement in practice than directly integrating the PESP into the above MIP. Additionally, it offers flexibility to adjust to various additional requirements that may arise in practice. For instance, if some transfer constraints are defined over a triple rather than a pair of bus lines, the rule-based method and the constraints (1c) can be easily adapted to this setting, whereas modifying an integrated model would pose significant challenges.

For optimality cuts, the presented cuts are independent of the EVSP solver used. This independence allows for easy adjustments to the underlying modeling constraints, such as recharging assumptions, without the need to overhaul the framework itself. Furthermore, the constraints partition the overall cost of a schedule $\beta(S)$ into components associated with each bus line's starting time (γ_{li}). As a result, these cuts facilitate a smart enumeration process. Instead of enumerating all feasible schedules, the search direction is guided towards bus lines that significantly impact the overall cost. Additionally, by integrating our iterative framework into a MIP, we obtain the classic benefits of a primal–dual bound, i.e., an optimality gap, at each iteration step.

Note that the quality of the found solution of Model (1) directly depends on the employed EVSP solver. If we solve the EVSP to optimality, i.e., if $\beta(S)$ is the cost of the optimal vehicle schedule for schedule S, then the found solution is also optimal. Additionally, any bound on the quality of $\beta(S)$ directly transfers to that of the integrated problem. If the EVSP is solved heuristically, Model (1) also becomes a heuristic, without providing any indication of the quality of the found solution. This motivates us to focus on approximation algorithms that provide a trade-off between runtime and solution quality for the EVSP in the next section.

5. EVSP subproblem

In this section, we focus on our implementation of an EVSP solver. Because the EVSP is an NP-hard optimization problem (Sassi and Oulamara, 2016) that we have to solve in every iteration of the *Timetabling Master Problem*, we focus on heuristics. We recap some of the basic heuristics from the literature and provide theoretical insights into the quality of the found solutions.

In the following, we focus on a very basic version of the EVSP that includes many simplifying assumptions, such as a homogeneous bus fleet, a linear charging curve, and all trips being round-trips with one centralized hub. While all the presented algorithms can easily be adjusted to model more complex problem settings, our theoretical results — especially the found approximation ratios — depend on these assumptions. As the presented basic setting is a special case of many problem variants with more complex assumptions, we provide general insights into the performance of the discussed heuristics for various EVSP variants.

5.1. Problem setting

The EVSP is formally defined as follows: Given a set of trips $\mathcal{T}$, where each trip t has a start time d_t, end time a_t and energy requirement e_t, the problem consists of finding a feasible vehicle rotation, i.e., assignment of trips to electric buses. As the primary cost driver is the acquisition of additional buses, our objective is to minimize the number of electric buses utilized. We represent each electric bus b by the set of trips assigned to it, i.e., $b \subseteq \mathcal{T}$. We call a bus feasible if the temporal constraints are satisfied, i.e., no two trips in the bus take place at the same time. Thus, we can represent a vehicle rotation by a set of buses $\mathcal{B}$, and finding a feasible vehicle rotation for all trips is then equivalent to finding a partition of $\mathcal{T}$ into electric buses. Each bus has a battery capacity of $D \in \mathbb{R}$ and recharges linearly with a rate r. In some settings, the buses can recharge between two trips. This is called opportunity recharging. The current battery of bus b at time point τ is denoted by $e^b(\tau)$. All buses start fully charged with $e^b(0) = D$. If a bus b serves a trip t, for each time point during the trip's execution $\tau \in [d_t, a_t]$, the current battery capacity is given by

$$e^b(\tau) = e^b(d_t) - e_t \cdot \left(\frac{\tau - d_t}{a_t - d_t}\right).$$

If opportunity recharging between the end of a trip a_t and a time point $\tau \geq a_t$ is possible, the battery capacity is given by

$$e^b(\tau) = e^b(a_t) + r \cdot (\tau - a_t).$$

For our theoretical analysis later, we also make the following assumptions. We assume that all trips are round trips starting and ending at the same bus hub, which has enough charging capacity for all buses. Charging is either performed over night, or as opportunity charging between the trips. The trip's times already include the required changeover times, allowing any time window between two trips to be entirely spent with charging. Additionally, we assume that the charging speed of the battery is at least as fast as the battery consumption through driving. In other words, the time required for a full charge does not exceed the operational time of a bus. Given current advancements in charging technology and the efficiency of electric buses, this assumption holds true for most practical scenarios.

5.2. EVSP without opportunity recharging

The EVSP without recharging consists of finding a feasible vehicle rotation for a given set of trips $\mathcal{T}$. Here, a vehicle rotation is feasible if for each electric bus, the cumulative energy consumption over all trips does not exceed the battery capacity. Thus, no recharging between trips is allowed. This constraint is similar to a Bin Packing Problem, where items are packed into as few bins of fixed size as possible. Actually, there is a variant of the Bin Packing Problem, the so-called Bin Packing with Conflicts (BPwC), which can be directly transformed into the EVSP without recharging. In the following, we will introduce the problem and discuss its connection to the EVSP.

The general bin packing problem asks, given a set of items with sizes and a bin capacity, how many bins are necessary to fit all items. A set of items fits in the same bin, if the sum of the sizes of these items is smaller or equal to the bin capacity. The BPwC extends the bin packing problem to additional constraints ensuring that certain pairs of items are not allowed in the same bin. Thus, in the BPwC, there is an additional conflict graph given with items as vertices and arcs marking pairs of items, which may not be put in the same bin. The items in the same bin have to be an independent set in the conflict graph.

For the special case of the BPwC where the conflict graph is colorable in polynomial time, Jansen and Öhring (1997) introduce a new heuristic. Since the EVSP is part of this special case, we briefly review their proposed heuristic. BPwC is a combination of two well-researched combinatorial problems, the graph coloring problem on the conflict graph and the bin packing problem. The authors propose to solve these two problems sequentially, by first computing a coloring on the conflict graph and then solving the Bin Packing Problem on each color class separately. Since there can be no conflicts between items of the same color class, we only have to solve instances of the regular Bin Packing problem without conflict for each color class. However, since this problem is still NP-hard, the authors test multiple, well-known heuristics for Bin Packing.

Jansen and Öhring (1997) show that the proposed heuristic is an approximation algorithm. Approximation algorithms are heuristics that also provide a quality measure on the found solution called the approximation ratio. The approximation ratio of an algorithm is a measure of how close the solution provided by the algorithm is to the optimal solution. Specifically, for an optimization problem, the approximation ratio R is defined as the maximum ratio between the cost of the solution found by the algorithm and the cost of the optimal solution taken over all possible instances of the problem. Formally, for a minimization problem, it is given by $R = \max_I \left(\frac{A(I)}{OPT(I)}\right)$, where $A(I)$ is the cost of the solution produced by the algorithm for instance I, and $OPT(I)$ is the cost of the optimal solution for instance I. An algorithm with an approximation ratio close to 1 is considered to provide solutions that are close to optimal.

Depending on the Bin Packing heuristics used, the authors show an approximation ratio of 3 for the NextFit Heuristic, 2.7 for the FirstFit Heuristic and 2.5 for the FirstFitDecreasing Heuristic.

There is a direct transformation from BPwC to the EVSP without recharging. We can associate the items with trips and the bins with electric buses. Then, the size of the items become the energy consumption of the trips, and the bin size is equal to the battery capacity. Finally, the temporal constraints can be expressed through the conflict graph. Each trip is then associated with a vertex in the conflict graph, and there is an edge between two trips if the two trips take place at the same time. Then, a bus b satisfies the temporal constraints, if and only if there are no arcs in the conflict graph between trips in b.

Finding the optimal coloring for the transformed EVSP instance is equivalent to solving the VSP. Because the conflict graph does not contain any battery constraints, the VSP boils down to finding a partition of the vertices of the conflict graphs (i.e. the trips) such that no two vertices in the same partition are adjacent to each other, i.e. the trips can be assigned to the same bus. Since the conflict graph represents the temporal constraints, it is an interval graph and the VSP problem can be solved in polynomial time within our setting as shown in Saha (1970) and thus, the coloring problem on the conflict graph is solvable in polynomial time despite the fact that the coloring problem on arbitrary graphs is NP-hard. Thus, we can apply the approximation algorithm proposed by Jansen and Öhring (1997) for the special case where the conflict graph is colorable in polynomial time for the EVSP without opportunity recharging.

5.3. EVSP with opportunity recharging

In this section, we extend the previous approximation results to the EVSP with opportunity recharging. To this end, we recap two basic heuristics often used for EVSP, link them to Bin Packing problem variants, and provide theoretical insights into their solution quality.

We start with an adaptation of the classical First Fit (FF) Heuristic presented in Algorithm 1. Given an ordering of the trips (line 3), the algorithm selects the trips according to this order and places them in the first available bus while respecting all underlying feasibility constraints (line 6–7) Due to its simplicity, this heuristic has often been applied to find some (initial) vehicle rotation, e.g., in Adler and Mirchandani (2017); Dirks et al. (2022).

One advantage of this heuristic is that the point of checking $b_i \cup \{t^i\}$ for feasibility in line 6 is left undefined. The heuristic can be adapted to different assumptions by changing this testing procedure accordingly. In the case of the EVSP with opportunity recharging, we only need to check for two conditions: each bus can only serve one trip at a time and each trip must be battery feasible.

The quality of the solution produced by Algorithm 1 strongly depends on the chosen ordering. Assuming we know the optimal solution to the EVSP, we can choose an ordering based on this optimal solution that groups the trips within a single bus together. Using this ordering, our First Fit (FF) Heuristic would not use more buses than in the optimal solution; hence, solving the problem to optimality. Conversely, finding such an ordering that ensures optimality is NP-hard. Hence,

Algorithm 1 First Fit (FF) Heuristic for EVSP

1: **Input:** A list of trips, each with a starting time, end time and energy consumption, ordering on trips $<^t$
2: **Output:** Assignment of trips to electric buses $\mathcal{B}$
3: $t^1, \ldots, t^n \leftarrow$ Sort all trips according to $<^t$
4: Initialize $b_1 = \{\}$, $k = 2$ and the ordered list $\mathcal{B} = [b_1]$, containing one electric bus with no assigned trips
5: **for** $i = 1, \ldots, n$ **do**
6: find the smallest $j \in \{1, \ldots, k\}$ such that $b_j \cup \{t^i\}$ is feasible
7: $b_j \leftarrow b_j \cup \{t^i\}$
8: **if** no feasible insertion was found **then**
9: Add a new empty electric bus b_k to $\mathcal{B}$.
10: $k \leftarrow k + 1$
11: **end if**
12: goto line 6
13: **end for**

simple orderings based on start time or energy consumption of the trips are common.

Instead of considering all the feasibility requirements at once, a commonly used approach is to first solve the VSP and then apply a repair step that makes the found schedule feasible for electric buses (Olsen et al., 2020). While this approach enables us to employ efficient existing VSP solvers in the first step, the found VSP solution limits the flexibility of the subsequent repair step, resulting in potentially poor solution quality. Furthermore, even the problem of repairing a given VSP solution is still NP-hard. In their NP-hardness proof, Sassi and Oulamara (2016) construct an instance of EVSP with recharging that does not include temporal constraints. Since the VSP solution of this instance would only use one bus, this directly implies NP-hardness of the repairing step. Thus, we also use heuristics for this step.

We study these types of heuristics that first solve the VSP and then repair the solution in detail. Algorithm 2 shows our First Fit Coloring (FFC) Heuristic that first solves the VSP and then applies the FF Heuristic on the vehicle rotation of each bus independently to make the schedule feasible for electric buses. Hence, it follows the same idea as the heuristic from Jansen and Öhring (1997) by first partitioning the trips into color classes by solving the VSP, and ensuring that battery capacities are respected within each color class. In the following, we refer to the buses without resource constraints in the VSP as *gasoline buses*, in order to clearly distinguish them from electric buses that have resource constraints.

Algorithm 2 First Fit Coloring (FFC) Heuristic

Input: A list of trips $\mathcal{T}$, each with a starting time, end time and energy consumption, ordering on trips $<^t$
Output: Assignment of trips to electric buses $\mathcal{B}$.
Compute a solution $\mathcal{G}$ to the VSP with gasoline buses.
for each color gasoline bus $g \in \mathcal{G}$ **do**
Apply the First Fit Heuristic in Algorithm 1 using $<^t$ to solve the EVSP, obtaining a feasible vehicle rotation $\mathcal{B}_g$ for the current gasoline bus.
$\mathcal{B} = \mathcal{B} \cup \mathcal{B}_g$
end for

We proceed by proving approximation bounds for the FFC Heuristic. We then show that the found approximation ratio is tight for all Heuristics using this approach, i.e., any algorithm that first solves the VSP and then operates on the found vehicle rotations independently cannot achieve a better approximation ratio than Algorithm 2.

Our approach to proving the approximation ratio of Algorithm 2 consists of bounding the maximum number of buses required to replace a single gasoline bus. To obtain this bound, we begin by making a structural observation about the partial solutions $\mathcal{B}_g$ for one gasoline bus g: for each pair of electric buses in the partial solution and at each point in time, the sum of their batteries is at least the battery capacity.

Lemma 1. *Let $\mathcal{T}$ be a set of trips, $g \subseteq \mathcal{T}$ a gasoline bus and $b^1, b^2 \subseteq g$ with $b^1 \cup b^2 \subseteq g$, $b^1 \cap b^2 = \emptyset$ be two battery-feasible electric buses serving trips in g. Then, at each point in time τ, the sum of the batteries $e^{b^1}(\tau) + e^{b^2}(\tau) \geq D$.*

Proof. Since all trips can be serviced by one gasoline bus, at each point in time there is at most one trip that needs to be served. Thus, at most one electric bus is driving at any point in time τ and the other electric bus is either charging or already fully charged. If one bus is fully charged, the assumption trivially holds. Let t be the trip in g with latest start time and $d_t \leq \tau$. Since we assume that charging a battery is at least as fast as discharging through driving, the sum over the battery capacities does not get lower if only one bus charges. Thus, $e^{b^1}(\tau) + e^{b^2}(\tau) \geq e^{b^1}(d_t) + e^{b^2}(d_t)$. It only remains to be shown that the assumption holds for the beginning of each trip $t \in g$. We show this by induction over the number of trips $n = |g|$. The assumption holds trivially for $n = 1$. For $n \geq 2$, let $t_n \in g$ be the last trip. By induction, we know that the assumption holds for $g \setminus \{t_n\}$. Additionally, it obviously holds

$$e^{b^1}\left(d_{t_n}\right) + e^{b^2}\left(d_{t_n}\right) \geq e^{b^1}\left(d_{t_{n-1}}\right) + e_{t_{n-1}} + e^{b^2}\left(d_{t_{n-1}}\right) - e_{t_{n-1}} \geq D$$

Thus, the assumption holds for the beginning of each trip. This concludes the proof. □

This Lemma already yields an approximation ratio for instances with large battery capacities, where all trips have an energy requirement of at most half the battery capacity.

Corollary 1. *If each trip t satisfies $e_t \leq \frac{D}{2}$, then the FFC Heuristic is a 2-approximation. This bound is tight.*

Proof. Since the sum of the current battery of two electric buses is always at least the battery capacity, one of the two buses has at least half the battery capacity. Thus, trips smaller than half the battery capacity can always be inserted in one of two electric buses. Thus, the partial solution for each gasoline bus consists of two electric buses. Since the optimal solution of the VSP is also a lower bound on the optimal solution of the EVSP, the solution computed by the heuristic takes at most double the number of buses compared to the optimal solution.

To see that the bound is tight, we consider the following instance with 6 trips. All electric buses have battery capacity $D = 1$, a charge rate of 1 per hour, and require one battery of $D = 1$ to drive one hour. Furthermore, we identify one trip $t_i = (d_i, a_i, e_i)$ by its starting time d_i, end time a_i and energy requirement e_i. The trips are given by

$$\mathcal{T} = \Big\{ t_1 = (0, \frac{1}{2}, \frac{1}{2}), t_2 = (\frac{1}{2}, \frac{3}{8}, \frac{3}{8}), t_3 = (\frac{7}{8}, \frac{3}{8}, \frac{3}{8}),$$
$$t_4 = (\frac{6}{8}, \frac{3}{8}, \frac{3}{8}), t_5 = (\frac{9}{8}, \frac{3}{8}, \frac{3}{8}), t_6 = (0, \frac{1}{2}, \frac{1}{2}) \Big\} .$$

Solutions for this instance with both gasoline and electric buses are visualized in Fig. 2. The time is given on the x-axis with each line representing one bus. Each of the squares represents a trip, where the longer trips with $e_i = \frac{1}{2}$ are colored blue and the shorter trips with $e_i = \frac{3}{8}$ are colored green. The upper half represents an optimal solution using gasoline buses and the lower half an optimal solution using electric buses.

As we can see in the visualization, this instance can be served with two gasoline buses or two electric buses. For bus b^1, the time between

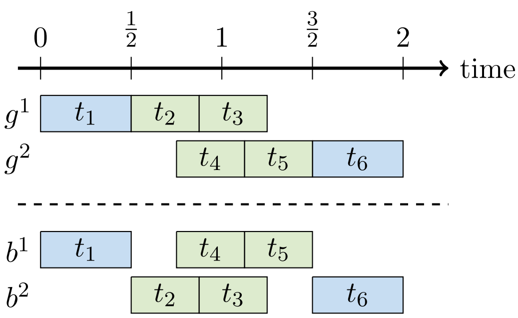


Fig. 2. Visualization of an instance for which the approximation ratio of 2 for FFC is tight. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

trips t_1 and t_4 is exactly sufficient to charge the battery to $\frac{3}{4}$. Thus, the trips t_4 and t_5 can be served. Similarly, the time between trips t_3 and t_6 is enough to charge bus b^2 sufficiently in order to serve trip t_6. It is not possible to use less than two buses because of the temporal constraints between trips t_3 and t_4.

When considering the heuristic FFC, we note that both gasoline buses consist of back-to-back trips for $\frac{5}{4} > 1$ h. With a battery capacity of only 1 h, they cannot be served by one electric bus. Thus, using this solution as a base for the FFC Heuristic leads to 4 electric buses. Consequently, the approximation ratio of 2 is tight. □

For small battery capacities, we obtain a worse approximation ratio.

Theorem 1. *The FFC Heuristic with items sorted by starting time is a 3-approximation.*

Proof. Let $g = \{t_1, \ldots, t_n\} \in \mathcal{G}$ be a gasoline bus from the optimal solution of the VSP with the trips sorted by starting time. We show by induction over the number of trips n that in the result computed by the heuristic, each gasoline bus is replaced by at most three electric buses and at the end time of each trip a_t, one of the three electric buses is fully charged. For $n \leq 2$, the assumption obviously holds. For $n > 2$, the trip t_n would be inserted last by the Heuristic. By induction, we know that the assumption holds for $g \setminus \{t_n\}$. Thus, applying the heuristic to the set of trips $g \setminus \{t_n\}$ computes (at most) three electric buses $b^1, b^2, b^3 \subseteq g$. Additionally, at the end of trip t_{n-1} (resp. at the beginning of trip t_n), at least one of the three buses is fully charged. Thus, t_n can be inserted in that bus. It only remains to be shown that at the end of trip t_n, one bus is again fully charged. If there is a bus which is fully charged at the end of t_{n-1} and t_n is not inserted in this bus, this holds trivially. Thus, we now look at the case of t_n being inserted in the only bus which is fully charged. Here, we have to differentiate between two cases.

Case 1: b^2 or b^3 are fully charged and t_n is inserted in that bus. Since the Heuristic always inserts the trip in the first bus where the trip fits, this can only be the case if t_n does not fit into b^1. Thus, at the beginning of t_n, bus b^1 has battery capacity $e^{b^1}\left(d_{t_n}\right) < e_{t_n}$. With Lemma 1, we get that both buses b^2 and b^3 must have battery $e^{b^j}\left(d_{t_n}\right) > D - e_{t_n}$ for $j \in \{2, 3\}$. Since only one of them serves trip t_n, the other bus charges at least e_{t_n} during the trip and is fully charged by the end of it.

Case 2: b^1 is the only fully charged bus. This can only be the case if t_{n-1} did not fit into b^1, else it would be inserted into b^1, and b^1 could not be the only fully charged bus. We can now use the same argumentation as in case 1 to show that either b^2 or b^3 have to be fully charged by the end of trip t_{n-1}, which contradicts the assumption that b^1 is the only fully charged bus.

Thus, g can be serviced by three electric buses. Since the number of gasoline buses required for the schedule is an upper bound on the number of electric buses required, this concludes the proof. □

Both bounds carry over to the Integrated Timetabling Approach.

Corollary 2. *For the Integrated Timetabling Approach, the solution found by Model (1) when using FFC to solve the EVSP subproblem is at least a 3-approximation. If each trip t satisfies $e_t \leq \frac{D}{2}$, the solution is at least a 2-approximation.*

For Algorithm 1, we discussed that the quality of the found solution is strongly dependent on the chosen ordering. This is not the case for Algorithm 2. In contrast, we now present a counterexample demonstrating that the approximation ratio of Algorithm 2 cannot be improved to better than 3, regardless of the repair algorithm employed. As a direct consequence, we conclude that the approximation ratio of 3 established for FFC in Theorem 1 is tight. These results highlight the inherent limitations of approaches that combine an existing VSP solution with a subsequent repair step.

Theorem 2. *A heuristic for the EVSP which first solves the VSP optimally and then solves the EVSP on each gasoline bus separately cannot be better than a 3-approximation.*

Proof. Let $n \geq 2$, $\varepsilon > 0$ small, and the battery capacity $D = 1$. Furthermore, the buses charge at a rate of 1 per hour and require one battery of $D = 1$ to drive an hour. The time horizon is $\frac{n+1}{2}$ hours.

We begin by looking at a small instance with only three trips t_1, t_2 and t_3. The trips start right after each other in that order and have energy requirement $e_{t_1} = 1, e_{t_2} = \frac{1}{2}$ and $e_{t_3} = \frac{1}{2} + \varepsilon$ respectively. This instance could be served by one gasoline bus; however, even in an optimal solution, three electric buses are required. The first two trips do not fit in the same bus and require two buses. At the end of the second trip, both buses have battery capacity $\frac{1}{2}$, thus the third trip fits in neither of the two buses and requires a third bus. We now present an instance of the EVSP, where the optimal solution to the VSP computed in the heuristic requires n gasoline buses which each include three trips of the form introduced above. We identify one trip $t_i = (d_i, a_i, e_i)$ by its starting time d_i, end time a_i and energy requirement e_i. Then, let the set of all trips be given by

$$\mathcal{T}_j = \{t_1^j = (\frac{j}{2}, 1 + \frac{j}{2}, 1), t_2^j = (1 + \frac{j}{2}, 1 + \frac{j+1}{2}, \frac{1}{2}),$$
$$t_3^j = (\frac{j+1}{2}, \frac{j+2}{2} + \varepsilon, \frac{1}{2} + \varepsilon)\}$$
$$\mathcal{T}_j^* = \{(k, k + \frac{1}{2}, \frac{1}{2}) : k \in \{\left\lceil \frac{j+2}{2} + \varepsilon \right\rceil, \ldots, \left\lfloor \frac{n}{2} + 2 \right\rfloor\}\}$$
$$\mathcal{T} = \bigcup_{j=1}^{n} \mathcal{T}_j \cup \mathcal{T}_j^*$$

The trips in $\mathcal{T}_j$ are the three special trips requiring three buses. The trips in $\mathcal{T}_j^*$ are dummy trips to ensure that for each pair $j \neq j'$, the trips from $\mathcal{T}_j$ and the trips from $\mathcal{T}_{j'}$ are not all served by the same gasoline bus. Both the solution using gasoline buses and the optimal solution using electric buses are visualized in Fig. 3 for even n. Each trip is visualized by a colored rectangle with their position on the x-axis marking start and end times. Each row represent a bus. Trips from t_1^j are drawn in blue, t_2^j in green, t_3^j in violet, and all trips from $\mathcal{T}_j^*$ in yellow. The upper half of the Figure shows a solution of the VSP using gasoline buses. The lower half shows an optimal solution of the EVSP using electric buses. In this instance, an optimal solution with gasoline buses would require at least n buses since there are n yellow trips of the form $(\frac{n}{2} + 2, \frac{n+1}{2} + 2, \frac{1}{2}) \in \mathcal{T}_j^*$ for each $j \in \{1, \ldots, n\}$. Then, an optimal solution using gasoline buses is given by each bus g^j serving all trips $S_j \cup \mathcal{T}_j^*$ for each $j \in \{1, \ldots, n\}$.

The optimal solution of the EVSP using the trips of the gasoline bus g^j requires 3 electric buses due to the trips in $\mathcal{T}_j$. Thus, each Heuristic which separates the gasoline buses requires at least 3 electric buses for each coloring class and thus returns a solution with at least $3n$ electric buses. However, an optimal solution with electric buses only requires $n + 3$ electric buses, given by:

$$b_i = \{t_1^i, t_3^{i+1}\} \cup \mathcal{T}_{i+1}^* \qquad i = 1, \ldots, n-1$$

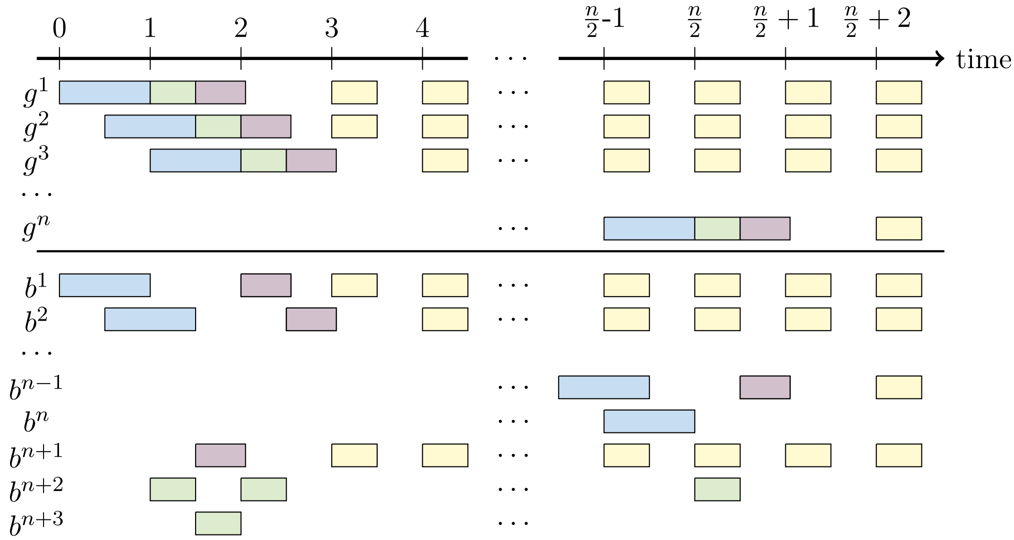


Fig. 3. Visualization of the solutions for the VSP and EVSP for even values of n. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$b_n = \{t_3^1\} \cup \mathcal{T}_1^*$$
$$b_{n+1} = \{t_1^n\}$$
$$b_{n+2} = \{t_2^{2j} : j = 1, \ldots, \frac{n}{2}\}$$
$$b_{n+3} = \{t_2^{2j-1} : j = 1, \ldots, \frac{n}{2}\}$$

Thus, the Heuristic is at best a 3-approximation for large values of n. □

Algorithm 3 First Fit ordered by Coloring (FFobC) Heuristic

Input: A list of trips $\mathcal{T}$, each with a starting time, end time and energy consumption, ordering on gasoline buses $<^{\mathcal{G}}$, ordering on trips $<^t$

Output: Assignment of trips to electric buses $\mathcal{B}$.

Compute a solution $\mathcal{G} = [g^1, \ldots, g^k]$ to the VSP with gasoline buses ordered using $<^{\mathcal{G}}$

Sort the trips in each bus $g^i = [t^{i,1}, t^{i,2}, \ldots, t^{i,|g^i|}]$ according to $<^t$

$\mathcal{B} \leftarrow$ First Fit Heuristic with ordering according to ordered list $g^1 \cup g^2 \cup \cdots \cup g^k$

Theorem 2 clearly shows that heuristics based on first solving the VSP and then repairing the found vehicle rotations are prone to getting stuck in bad local optima after the VSP step. To escape such local optima, we propose combining these iterative heuristics with the global approach of the FF Heuristic. Algorithm 3 presents our combined First Fit ordered by Coloring (FFobC) Heuristic, which is a combination of the FF and FFC Heuristics. We first solve the VSP, but then use this solution to derive an ordering for the FF Heuristic from the found VSP solution. While it is evident that FFobC will always be at least as good as the FFC Heuristic, it also has the chance of escaping bad local optima. Looking at the counter-example from Theorem 2, the new heuristic would return an optimal solution to this instance. Also, the theoretical results from Corollary 1 and Theorem 1 hold for FFobC Heuristic. It is an open question whether a better approximation ratio can be shown. Here, we only provide lower bounds on the approximation ratio.

Lemma 2. *On instances with $e_t \leq \frac{D}{2}$ for all trips $t \in T$, FFobC with items sorted by starting time or energy consumption cannot be better than a $\frac{3}{2}$-approximation. On general instances, FFobC with items sorted by starting time cannot be better than a $\frac{5}{3}$-approximation.*

Proof. Similar to the previous proofs, we consider instances with battery capacity $D = 1$, a charge rate of 1 per hour and battery requirement of 1 to drive one hour. Furthermore, we identify each trip by the triple $t_i = (d_i, a_i, e_i)$ of starting time, end time, and energy consumption.

We begin by showing the bound for instances with $e_t \leq \frac{D}{2}$ for all trips $t \in T$. For $\epsilon > 0$, we consider the following instance with four trips

$$\mathcal{T} = \{ \ t_1 = (0, \frac{1}{2}, \frac{1}{2}), t_2 = (\frac{1}{2}, 1 - \epsilon, \frac{1}{2} - \epsilon), \quad t_3 = (1, \frac{3}{2} - 2\epsilon, \frac{1}{2} - 2\epsilon), t_4 = (1, \frac{3}{2}, \frac{1}{2}) \ \} \ .$$

The parameter $\epsilon > 0$ is chosen very small and is only used to make the sorting by energy consumption unique and equal to the sorting by starting time. Due to the small size of the instance, we can easily find an optimal solution to the VSP using two gasoline buses $g^1 = \{t_1, t_2, t_3\}$ and $g^2 = \{t_4\}$. Similarly, we get an optimal solution to the EVSP using two electric buses $b^1 = \{t_1, t_3\}$ and $b^2 = \{t_2, t_4\}$. However, using FFobC with the given VSP solution as a basis results in a solution with 3 electric buses. For both sortings, we first consider trips t_1 and t_2, which leads to the first electric bus as $b^1 = \{t_1, t_2\}$. Now, the two remaining trips cannot fit in b^1 due to battery constraints. Furthermore, they also do not fit in the same bus due to temporal constraints. Thus, using two more buses $b^2 = \{t_3\}$ and $b^3 = \{t_4\}$ is the only way to serve these trips. As a consequence, the heuristic returns a solution using $\frac{3}{2}$ the number of buses of the optimal solution, which shows the bound on the approximation ratio.

For the general case, we consider the following instance with 8 trips:

$$\mathcal{T} = \{ \ t_1 = (0, \frac{1}{2}, \frac{1}{2}), t_2 = (\frac{1}{2}, 1, \frac{1}{2}), t_3 = (1, \frac{3}{2}, \frac{1}{2}), t_4 = (\frac{3}{2}, \frac{5}{2}, 1), \quad t_5 = (\frac{1}{2}, 1, \frac{1}{2}), t_6 = (\frac{5}{4}, \frac{3}{2}, \frac{1}{4}), t_7 = (\frac{3}{2}, 1, \frac{1}{2}), t_8 = (\frac{3}{2}, 1, \frac{1}{2}) \ \} \ .$$

Fig. 4 visualizes the optimal solutions for both VSP and EVSP with three buses each, as well as the results of FFobC based on the given VSP solution with the trips sorted by starting time. The colors of the trips represent the gasoline bus from the optimal VSP solution they are served by. For example, all trips served by b^1 are colored blue.

When using FFobC based on this VSP solution, the trips are considered in order $t_1, t_2, \ldots, t_8$. The first six trips are inserted into three electric buses as depicted due to battery constraints. Then, at time point $\frac{3}{2}$, the battery of the first two buses b^1 and b^2 is empty and the third bus b^3 starts serving trip t_4. Thus, none of the buses b^1–b^3 can serve the simultaneous trips t_7 and t_8. Consequently, two more buses are required, which leads to the bound of $\frac{5}{3}$ on the approximation ratio. □

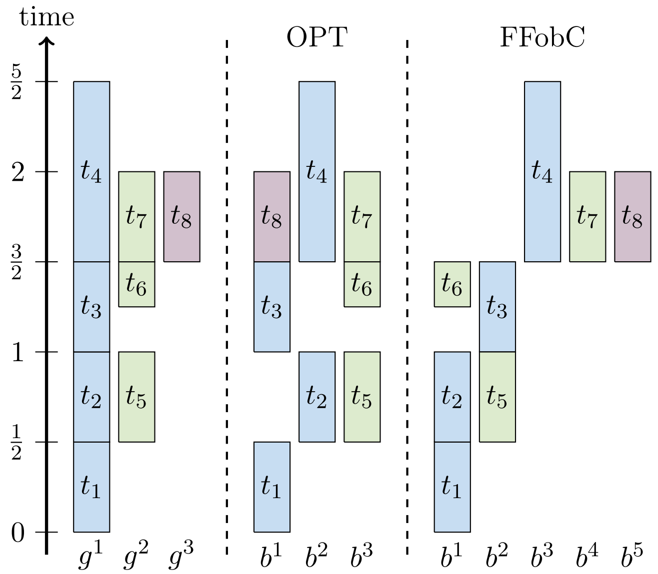


Fig. 4. Visualization of an instance where FFobC has an approximation ratio of $\frac{5}{3}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

6. Case study

We present a computational study to demonstrate the quality and effectiveness of our Integrated Timetabling and Vehicle Scheduling approach. To this end, we used an instance based on real live data from the bus network of the city of Aachen, Germany. The experiments were carried out using a Intel(R) Core(TM) i5-8265U CPU@1.60 GHz and 4 physical cores. The algorithms were implemented using Java 21.0.1 and GUROBI 10.0.3 with Windows 10 as operating system.

6.1. Problem instance and parameters

The city of Aachen, located in western Germany with approximately 250,000 inhabitants, operates a comprehensive public transport system that relies solely on bus services, which is typical for mid-sized cities in Germany.

The bus network of Aachen is structured around a central hub known as the “Bushof”. The Bushof serves as the primary interchange point where most of the bus lines converge, facilitating easy transfers between different lines. This central hub model is designed to optimize connectivity and minimize transfer times for passengers traveling across various parts of the city, and can be found in several cities across Germany.

Our instance includes 8 bus lines with a total of 347 trips and about 280 h total driving time, which could be served by 19 gasoline buses. All data was taken from the public timetable information of the AVV. Four of the lines have a headway of 15 min, two have a headway of 30 min and the remaining two have a headway of one hour. The first trip starts at 5 am and the last trip ends just before 1 am the next day, resulting in an operational time window of 20 h. We slightly adapted the trips to fit our assumption of having only round-trips starting from the same depot by merging two back and forth trips into one round-trip. Considering that the start times match perfectly, this is also implemented in practice. For transfer constraints, we define that if a line l_2 departs within a 5 to 15-min window after the arrival of line l_1, passengers commonly transfer from line l_1 to line l_2. Therefore, in any feasible schedule, l_2 must depart within this time window relative to the arrival of l_1. The complete list of bus lines and transfer constraints used can be found in Appendix.

The electric buses currently in use in Aachen have a battery capacity of 292 kW and a range of 150–200 km (electrive, 2021). Since our formulation expresses battery capacity in terms of maximum driving time, we conservatively estimate a driving time of 3 h by using the speed limit of 50 km/h in German cities. To assess the impact of larger battery capacities, we conducted experiments with driving times of 3 h, 7 h and 16 h. The charging stations currently available in Aachen are used for over-night charging and supply approximately 75 kWh. For opportunity charging between trips, we assume the presence of fast-charging stations, which charge a 300 kW battery to full capacity in 3 h. Additionally, we conducted some tests for the case without opportunity recharging between trips.

For the integrated timetabling approach, we allow a shift in starting times of up to 10 min. We discretize the operational time window into 5 min intervals since shifts of up to 5 min are generally considered acceptable and often do not count as a delay in statistics in Germany. Thus, we permit time shifts of 5 or 10 min in either direction for each line. For the lines with a headway of 15 min, this implies that all starting times permitted by the discretization are feasible and the original timetable only influences the beginning of the first and last trip of the line. Finally, we set M_l^S to the number of buses that serve trips from line l in the heuristic solution of the EVSP for schedule S.

To solve the EVSP subproblem, we tested the three heuristics from Section 5: the (regular) First Fit (FF) heuristic 1, the First Fit Coloring (FFC) heuristic 2, and the First Fit ordered by Coloring (FFobC) heuristic 3. We tested ordering the trips by start time and by energy consumption for each heuristic. We denote the ordering by start time with _start and sort from earliest to latest. The ordering by energy consumption is denoted by _energy and we sort from highest to lowest consumption. This leaves us with a total of 6 heuristics.

For each heuristic, the Integrated Timetabling Approach computed a solution in under two minutes. There were no significant differences in computation time between the heuristics or across different battery capacities. In each computation, the EVSP was solved up to 6000 times, which is a substantial improvement over simple enumeration of all shifted timetables, which would require solving the EVSP for up to 390,000 different schedules ignoring transfer constraints.

6.2. Results without opportunity recharging

We begin by examining the results allowing only overnight charging. While solving this instance, we track all the computed timetables and report our results in the interval form shown in Fig. 5. The range indicated for each heuristic represents the range of EVSP solutions computed during all iterations, while the marked solution indicates the solution obtained from applying the heuristic to the initial schedule currently used in Aachen. The lower bound of the interval is the final result. The interval between best and worst solution represents the potential gains that can be achieved by shifting the start times. It corresponds to the maximum improvement possible through this approach when the current schedule requires many electric buses. Reversely, if this interval has a length of 0, because best and worst solution have the same value, no improvement can be achieved through this approach. The gap between the lower bound of the interval and the initial solution reflects the improvement from the current schedule through the integrated timetabling approach.

Fig. 6 shows the results for all battery capacities and heuristics without opportunity recharging between trips. The results are presented for the different battery capacities. Additionally, the dashed blue line in each graph represents a lower bound on the number of electric buses. In the schedule, trips with a total driving time of ≈280 h have to be served. Thus, even if the trips match perfectly, a schedule requires at least 94 buses with battery capacities of 3 h, 40 buses with battery capacities if 7 h or 18 buses with battery capacities of 16 h. The lower bound for 16 h battery capacities can be raised to 19 buses because the initial schedule can be served by 19 gasoline buses. The number of gasoline

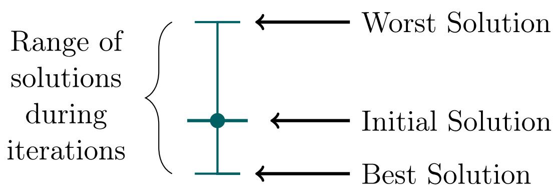


Fig. 5. Legend solution presentation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

buses is also optimal under time shifts, thus, 19 is also a lower bound on the number of electric buses needed.

As can be seen, FF_energy shows no improvement through the integrated approach and computed the same result in each iteration. This may be due to the fact that the ordering of the trips is not affected by shifts in start times. In contrast, all other heuristics either partially sort by start times or are influenced by the gasoline bus plan, which inherently depends on start times. These results highlight the previously discussed high dependency of the selected ordering for the solution quality.

The results show that FFobC generally provides the best results, which confirms our expectations from the theoretical analysis. The only exception to this is FF_start, which computes a solution requiring one less electric bus for the instances with 16 h battery capacity. Compared to FFC, especially for instances with medium and large batteries, the worst solutions computed by FFobC are still better than the best solution computed by FFC. Furthermore, the best solutions computed by FFobC are not far from the lower bounds. Since it is not likely that these lower bounds can be reached for small and medium batteries, this indicates that the computed solutions are close to optimal with an optimality gap of at most 10% on all three instances. The range of solutions during the iterations is small, especially for medium and large battery capacities, which implies less room for improvement when using the integrated approach. Additionally, we note that sorting by start time or energy consumption does not have a large effect on the results. Both sortings provide the same results on instances with medium and large batteries. They only differ by one bus on instances with small batteries, which is not significant compared to the total number of buses needed in these instances. This may be caused by the ordering in the FFobC primarily depending on the gasoline schedule and only secondarily on the start time or energy consumption.

Upon examining the solutions returned by FFC, we observe that it includes many buses with large shares of their battery capacity left unused. This shows that because the VSP solution ignores battery restrictions, the found vehicle rotations cannot easily be executed by a subfleet of electric buses, resulting in the heuristic getting stuck in bad local optima. In contrast, FFobC can escape these parts of the solution space by reassigning buses between the given vehicle rotations from the VSP solution, which explains the difference in solution quality between the two heuristics. Additionally, it is noteworthy that there is a significant difference between the two sorting methods for the instance with small battery capacity. Here, sorting by energy performs much worse than sorting by start time. This is surprising because sorting by decreasing size is usually a good heuristic for Bin Packing. This discrepancy may be due to large trips blocking buses, which are better distributed when sorting by start time.

Finally, FF_start is overall slightly worse than FFobC for small and medium batteries and slightly better for large batteries, where it reaches the lower bound and computes an optimal solution. Notably, the range of solutions becomes larger with increasing battery capacity. This may be because the instance resembles a coloring problem more than a bin-packing problem, especially for medium to large battery capacities. When comparing the two sorting methods, sorting by start time appears to be slightly more effective than sorting by energy consumption across all heuristics.

6.3. Results with opportunity recharging

Next, we examine the influence of opportunity recharging between trips on the integrated approach. In Fig. 7, we see the results for the case that recharging between trips is allowed. In addition to the lower bounds of 19 gasoline buses represented by the dashed, blue lines, the dotted, green lines represent upper bounds for the FFC and FFobC heuristics gained from the approximation ratios proven in Section 5. There, we showed that the heuristics require at most 3 electric buses for each gasoline bus (or 2 electric buses if the battery capacity is larger than double the longest trip). The longest trip in the instance takes 130 min. Thus, for 7 and 16 h batteries, we get an upper bound of 38 electric buses for the initial and optimal solutions by Corollary 1. For battery capacities of 3 h, we get an upper bound of 57 buses by Theorem 1.

As can be seen, incorporating opportunity recharging between trips returns results that are generally similar to those observed without recharging. In particular, the performance of the heuristics relative to each other is similar to the previous section, with FFobC yielding the best results. However, for FF_energy with small and medium batteries, we observe different results through integrated timetabling and thus the potential for improvement. While FF_energy does not improve the solution for small batteries, it requires one bus less for medium battery capacities. Notably, FFC_energy performs better on the instance with a small battery compared to the instance without recharging. The option to recharge likely compensates for scheduling errors made early in the timetable. For battery capacities of 16 h, we also note that the best solutions computed by the FFobC and FF heuristics did not change. For FF_start, this is due to the fact that the best solution in the instance without opportunity recharging already reached the lower bound of 19 gasoline buses. Thus, no better solution is possible. For the other heuristics, this indicates that adding opportunity recharging is not very effective for buses with large battery capacities.

On instances with small and medium battery, the best solution of all heuristics still needs 25% more electric buses than the lower bound given by the number of gasoline buses required. However, this does not necessarily indicate bad performance of the heuristics but rather reflects on the quality of the lower bound on instances with smaller batteries. For large battery capacities, the lower bound is again reached by FF_start even in the initial solution. This suggests that with a battery capacity of 16 h and opportunity recharging, the operational differences between electric and gasoline buses become negligible. This also explains the lack of improvement over the initial schedule in this case, as it was already optimized for gasoline buses.

Finally, we note that the upper bounds computed from the approximation ratio is not tight for the FFobC heuristics and can probably be improved. For the FFC heuristics, the upper bound is nearly reached by the initial solution with battery capacities of 7 h, which shows that the bound is tight even in applications with medium battery capacities for the FFC heuristics. In these computations, the upper bound is broken by some solutions during the iterations. The upper bound only holds for schedules that can be served by 19 gasoline buses. This is not necessarily the case for all schedules computed during the iterations, thus, the worst solution computed needing more than 38 electric buses does not invalidate the approximation ratio.

6.4. Parameter analysis

In the following, we further investigate the influence of three additional parameters: battery capacity, length of discretization steps, and

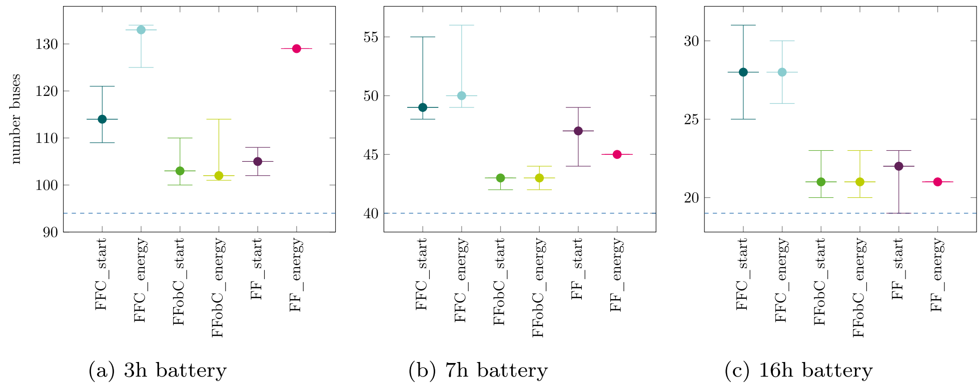


Fig. 6. Start solution and gap between worst and best computed solution to the Integrated Timetabling and Vehicle Scheduling Problem over all iterations, instances without recharging. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

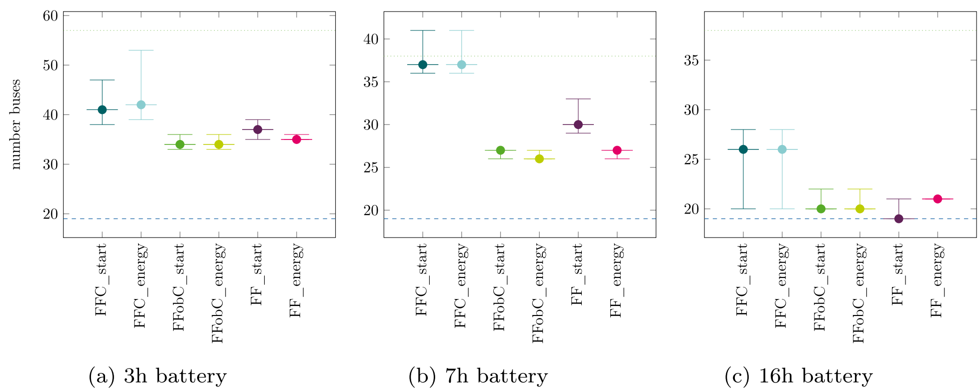


Fig. 7. Start solution and gap between worst and best computed solution to the Integrated Timetabling and Vehicle Scheduling Problem over all iterations, instances with opportunity recharging.

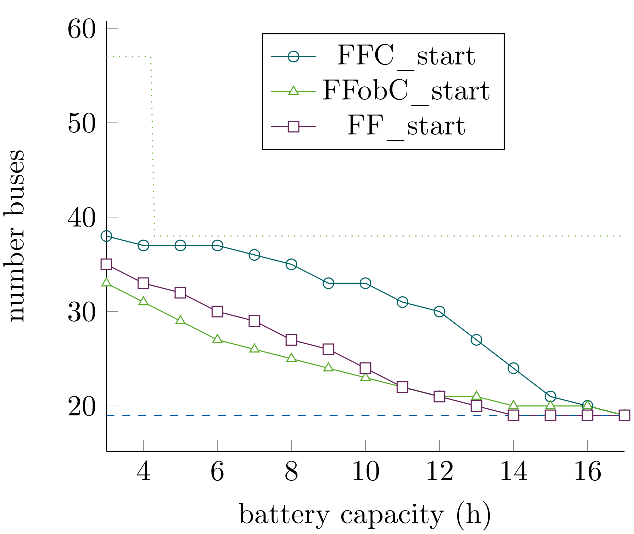


Fig. 8. Best Solutions to the Integrated Timetabling and Vehicle Scheduling Problem with opportunity recharging.

maximum allowed time window in which shifts in starting times are permitted.

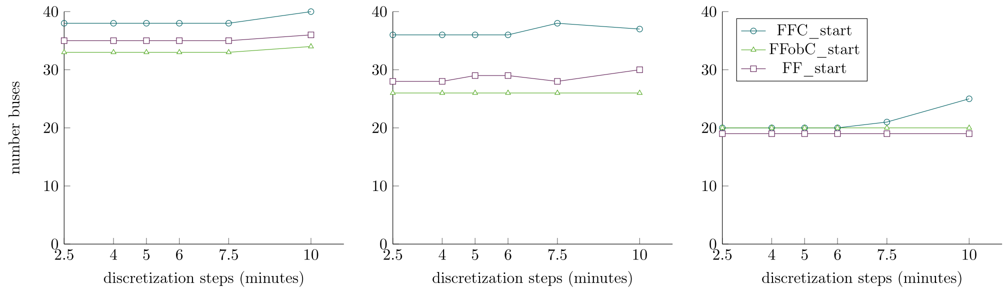
For battery capacities, we compute the solution ranges for all battery capacities from 3 h to 17 h exemplary for FFC_start, FFobC_start and FF_start with opportunity recharging. The results are shown in Fig. 8. The lower bound of 19 gasoline buses is again depicted as a dashed, blue line, and the corresponding upper bound as a dotted, green line.

Comparing the three heuristics, we can see that FFC_start returns the worst results across all battery capacities and nearly reaches the upper bound given by the approximation ratio for battery capacities of 5 h and 6 h. This shows that the approximation ratio found in Corollary 1 is nearly tight on this instance for FFobC_start. The other two heuristics perform better, with FFobC_start performing best for small and medium battery capacities and FF_start performing best on instances with large battery capacities of at least 13 h. Specifically, the lower bound of 19 buses is reached for battery capacities of 14 h when using FF_start, which is earlier than the other two heuristics that only reach the lower bound for battery capacities of 17 h. This confirms our previous results.

Furthermore, for FFC_start, the number of buses barely changes for small batteries of up to 9 h and then reduces fast until reaching the lower bound with a battery capacity of 17 h. The opposite holds for FFobC_start, where the reduction in the number of electric buses needed is faster for small batteries and slows down for battery capacities greater than 12 h. This is due to FFobC_start being more flexible and spreading trips from one electric bus across all other buses when increasing the battery capacity. This is not possible for FFC_start, where the color classes are solved separately with up to 3 electric buses each and trips can only be shifted between these 3 buses when increasing the battery capacity.

Finally, we note that since the lower bound is first reached by FF_start for battery capacities of 14 h, using buses with capacities larger than 14 h when opportunity recharging is possible is unnecessary in this instance. In additional experiments where opportunity recharging is not possible, all heuristics returned at least 20 electric buses for battery capacities of 15 h. Thus, in this case, using battery capacities of up to 16 h can be beneficial.

In all previous experiments, we use a discretization of the operational time window into 5-min discretization steps, resulting in 5



(a) 3h battery (b) 7h battery (c) 16h battery

Fig. 9. Best Solutions to the Integrated Timetabling and Vehicle Scheduling Problem with opportunity recharging for different discretizations.

possible starting times for each line. To evaluate the impact of this choice, we also test how the size of the discretization steps influences the solution quality. To this end, we conduct tests for different granularities for the discretization of the 10 min time window, i.e., using 2.5, 5, and 10 min as step size for the discretization. Additionally, we test new, different-sized discretization steps, in this case 4, 6, and 7.5 min. Considering the maximum deviation of 10 min, this leads to 9 possible starting times for discretization steps of 2.5 min, 5 for discretization steps of 4 and 5 min, and only three for 6, 7.5, and 10 min. For the smallest discretization steps of 2.5 min, the computations did not finish in under one hour, thus we report the best schedule found after one hour computation time. We again restrict ourselves to the case with recharging and ordering of trips by starting time. The results are shown in Fig. 9.

Generally, most algorithms return very similar results for discretizations below 10 min. Results differ only by up to one bus, with most algorithms always finding the same result. Even for larger discretization steps, the results are very similar. In almost all cases, the optimal solutions differ by at most 2 buses between different discretizations. The only exception is FFC_start on instances with large batteries. Here, the solution with using discretization steps of 10 min uses 25 buses, while the solutions for discretization steps of up to 6 min only require 20 buses. These findings indicate that discretization steps of 5 to 6 min offer the best trade-off between better solution through finer granularity and faster computation time. Smaller steps generally do not lead to better results and only increase the computation time, while larger steps may lead to worse results.

We also note that a finer discretization does not always guarantee better solutions. In some cases, using smaller discretization steps leads to one more bus being required in the optimal solution. Examples of this are FFC_start on instances with small batteries when comparing time steps of 10 and 15 min and FF_start on instances with medium batteries. This is due to the previous, optimal solution no longer being feasible under the new discretization steps. However, this cannot happen if one discretization step is a divisor of the other, where the discretization with a finer granularity will yield a comparable or better solution.

Finally, we examine the influence of the length of the time window of allowed deviations. For these tests, we again fix the discretization steps to 5 min. The results for the instances without opportunity recharging are shown in Fig. 10. In addition to the allowed time shifts of up to 10 min we use for all previous tests, we consider bounds of 5, 15, and 30 min. Furthermore, we add the results for the starting schedule for a maximum deviation of 0 min for comparison.

In general, we note that the greatest effect already appears when allowing time shifts of only 5 min. While time shifts of 10 min save an additional bus in most cases, allowing even more deviation does not lead to more savings. This is even more pronounced on instances with opportunity recharging, where nearly all algorithms and instances return the same results for all deviations >0. Longer shifts of ≥ 15 minutes are ineffective as half of the considered lines have a headway of 15 min. Thus, shifting them by 15 min does not change the schedule. Having no further savings on these longer time shifts implies that most of the savings were achieved by shifting the starting times of the lines with 15 min headway.

6.5. Managerial insights

Finally, we look at the potential savings by implementing the changes suggested in this study. The spider plot in Fig. 11 provides a visual representation of the improvements achieved by our proposed changes: Opportunity recharging, using larger batteries and integrated timetabling. Additionally, the plot shows the influence of combining these changes. All heuristics show very similar behavior in this context, thus, we use the results of FFC_start to exemplify this. The plot shows the percentages of electric buses saved for each of the proposed changes. As a basis for the comparisons, we used the initial solution without opportunity recharging between trips and 3 h battery capacity.

Using buses with larger batteries has the largest impact, followed by opportunity recharging that still achieves savings of around 60% of electric buses compared to our comparison scenario. Integrated timetabling achieves the lowest overall savings of only up to 15%. However, integrated timetabling is also the easiest to implement option. While both recharging and buses with larger battery capacity involve significant initial investments into the fleet and infrastructure, adjusting the timetable can be implemented faster and does not require any additional direct investments. Therefore, integrated timetabling can be used as an intermediate solution that enables the (partial) electrification of a larger part of the fleet at an earlier stage. Combining the integrated approach with either larger battery capacities or opportunity recharging results in slight improvements, reinforcing the potential of our integrated timetabling strategy as an economical and efficient solution for enhancing the scheduling of electric buses.

7. Conclusion

In this study, we introduce a novel integrated timetabling approach that facilitates minor adjustments to existing periodic timetables to better implement the transition from gasoline buses to electric buses. We employ an iterative framework that incorporates optimality cuts derived from EVSP solutions into a Timetabling Master MIP model. To effectively address the EVSP, we study three heuristics. The first heuristic is an adaptation of the classical First Fit heuristic for the Bin Packing Problem tailored to the EVSP. The second heuristic involves initially solving the VSP and then repairing the solution. The third heuristic combines the ideas of the previous two. We study the theoretical properties of the discussed heuristics, showing that the second and third heuristic provides an approximation guarantee of at most 3. We also provide general insights into iterative heuristics that first solve the VSP and then adjust the found vehicle rotations to be executed by electric buses, showing that there are instances for which no such algorithm can be better than selecting 3 times more electric buses than the optimal solution.

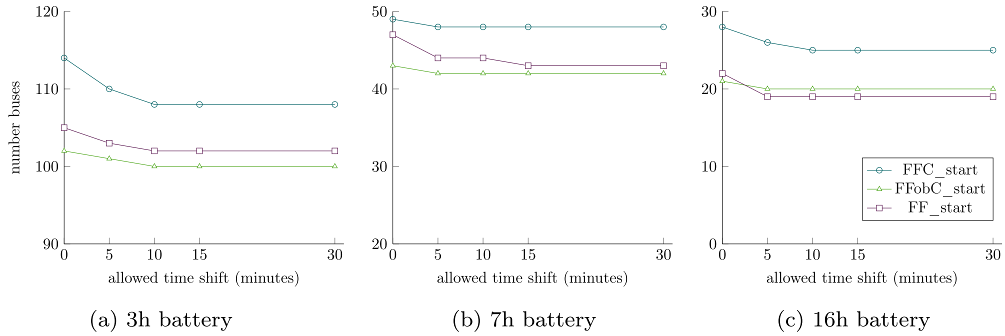


Fig. 10. Best Solutions to the Integrated Timetabling and Vehicle Scheduling Problem without opportunity recharging for different maximum deviations from the starting schedule.

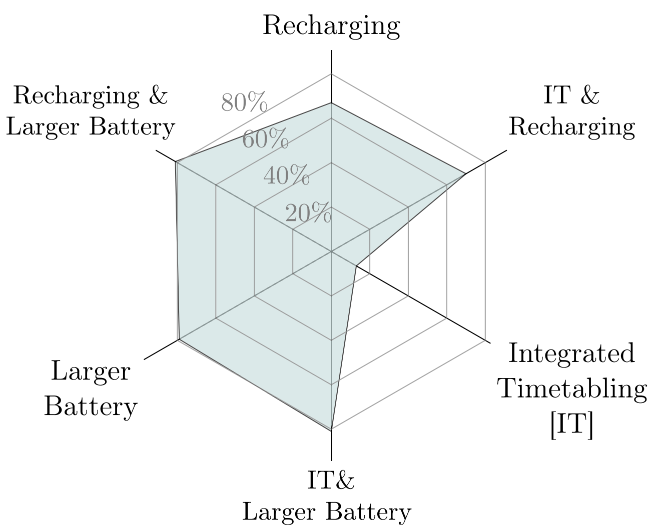


Fig. 11. Improvements through opportunity recharging, larger battery capacity and Integrated Timetabling for FFC_start.

We validate our approach through a case study conducted on the public transportation system in Aachen, Germany. The results confirm our theoretical insights and show worse computational performance of the repair heuristic compared to the other heuristics. We also derive managerial insights based on our analysis.

While our managerial insights indicate only limited benefits from minor adjustments to the periodic timetable, these results are clearly influenced by the highly centralized structure of Aachen's bus network. Future research could extend the analysis to less centralized or decentralized networks to either validate or challenge these findings. Moreover, subsequent studies might examine heterogeneous bus fleets, incorporate non-linear charging profiles, uncertainties arising from variable energy consumption due to traffic conditions or weather, and the costs associated with developing and maintaining charging infrastructure.

CRediT authorship contribution statement

Vladimir Stadnichuk: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Data curation, Conceptualization. **Jenny Segschneider:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Arie M.C.A. Koster:** Writing – review & editing, Supervision, Resources, Methodology, Funding acquisition, Conceptualization. **Grit Walther:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition, Conceptualization.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used ChatGPT to rewrite the original draft with the goal of improving readability and correct spelling/grammatical mistakes. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Vladimir Stadnichuk reports financial support was provided by Ministry of Culture and Science of the German State of North Rhine-Westphalia. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Lines used for the case study

Table A.1 shows a list of all lines used in our case study together with the starting times of the first and last trip as well as the headway and trip-length in minutes. Using these departure and arrival times, we set three transfer constraints: from line 14 to line 24, from line 16 to line 24 and from line 24 to line 14 as described in Section 6. Since the lines 3A, 3B, 13A and 13B have a trip every 15 min, transfer constraints for them would be redundant.

Table A.1
List of all lines used in our case study.

Line	Start time	End time	Headway (min)	Trip-length (min)
3A	05:27	00:42	15	40
3B	05:00	00:15	15	37
13A	07:01	19:31	15	24
13B	06:57	18:57	15	30
14	06:11	23:11	30	95
16	06:02	20:02	60	111
24	7:30	21:00	30	60
46	6:32	20:32	60	130

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