

Robot-Based Techniques for Spherical Near-Field Antenna Measurements

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Kurzfassung

Die Einführung roboterbasierter Positioniersysteme in der Antennenmesstechnik beschreibt einen technologischen Umschwung. Das größte Potenzial solcher Systeme ist deren Flexibilität, welche die Kombination der etablierten Messmethoden, beispielsweise planare, zylindrische, sphärische oder auch irreguläre Messungen, innerhalb einer einzigen Antennenmesskammer erlaubt. Andererseits sind roboterbasierte Systeme für das industrielle Umfeld von Fertigungsstraßen optimiert, wodurch sich Herausforderungen bei deren Einsatz in der Antennenmesstechnik ergeben. Hierzu zählt insbesondere die Absolutgenauigkeit, welche durch die systembedingte Verkettung von sechs Achsen größere Toleranzen im Vergleich zu herkömmlichen Systemen aufweist. Ebenso ist das zeitlich präzise Auslösen der Messinstrumentation beim Erreichen der Vielzahl an Messpunkten mit größeren Abweichungen verbunden.

Daher wird in dieser Dissertation untersucht, wie sich die spezifischen Eigenschaften roboterbasierter Systeme auf die sphärische Nahfeldmesstechnik auswirken und welche Möglichkeiten zur Kompensation dieser Einflüsse durch die zugrunde liegende Theorie der sphärischen Wellenexpansion (SWE) bestehen. Hierzu wurde am Institut für Hochfrequenztechnik der RWTH Aachen University ein roboterbasiertes Messsystem für Messungen im Millimeterwellenbereich aufgebaut. Für dieses System wird ein Fehlermodell zur Evaluierung der wichtigsten Einflussfaktoren präsentiert, sowie ein Einmessverfahren vorgestellt, welches die isolierte Identifikation der einzelnen Fehlerterme ermöglicht. Dies erlaubt es mechanische Toleranzen im Aufbau durch die Messbahn des Roboters zu kompensieren und somit zu präziseren Messergebnissen beizutragen. Ebenso wird eine interne Rückkopplungsschleife zur Robotersteuerung verwendet, welche die tatsächliche Position und Orientierung des Roboters an den jeweiligen Messpunkten ausgibt. Dies ermöglicht es die nicht-idealen Messpositionen innerhalb der Transformation ins Fernfeld basierend auf der SWE zu berücksichtigen. Zur Kompensation abweichender Orientierungen wird eine punktwise Korrektur des Modenspektrums der als Messsonde verwendeten Antenne innerhalb der SWE implementiert.

Weitere Analysen befassen sich mit den Vorteilen Spline-basierter Messungen. Diese ermöglichen eine deutlich verbesserte Präzision zur Auslösung der Messinstrumentation bei Erreichen eines Abtastpunktes. Im Vergleich zur konventionellen Implementierung kann der Fehler um das zehnfache reduziert werden. Zusätzlich ermöglichen Spline-Bahnen die effiziente Implementierung irregulärer Abtastgitter. Dies wird am Beispiel einer mathematisch optimierten nicht-redundanten Verteilung von Abtastpunkten demonstriert, welche mit herkömmlichen sphärischen Positionierern nicht oder nur schrittweise und damit nicht effizient implementiert werden kann. Das roboterbasierte System hingegen erlaubt die Vermessung des irregulären Abtastgitters in lediglich 56 % der Dauer einer äquivalenten gleichwinkligen Punktverteilung abzuschließen. Damit erschließt die Kombination der Vielseitigkeit der SWE mit der Flexibilität roboterbasierter Systeme neue Möglichkeiten in der Antennenmesstechnik, welche zuvor hauptsächlich von theoretischem Interesse waren.

Summary

The introduction of robot-based positioning systems in antenna measurements represents a significant technological advancement. The primary advantage of these systems is their flexibility, which enables the integration of various measurement techniques, including planar, cylindrical, spherical and irregular measurements, within a single antenna measurement facility. However, the application of robot-based systems in antenna measurement technology poses challenges due to the optimization for their initial application in the industrial environment of production lines. Specifically, the system-inherent interlinking of six axes results in greater tolerances of the absolute accuracy compared to conventional systems. Additionally, the precise triggering of the measurement instrumentation for a large amount of sampling points is associated with greater deviations.

This dissertation investigates the specific technical aspects of robot-based systems in spherical near-field measurement technology as well as the potentials for compensating these impacts using the underlying theory of the spherical wave expansion (SWE). Therefore, a robot-based measurement system in the millimeter-wave range has been set up at the Institute of High Frequency Technology at the RWTH Aachen University. An error model for the evaluation of the most important impact factors is presented specifically for this system, along with a calibration procedure that enables the isolated identification of the individual error terms. This allows the compensation of mechanical tolerances in the applied setup by adapting the robot's measurement path and, thus, contributes to more precise measurement results. Furthermore, the actual position and orientation at the respective sampling points are provided by an internal feedback loop of the robot controller. Therefore, the non-ideal measurement positions can be considered in the transformation to the far field based on the SWE. In order to compensate for inaccurate orientations, a pointwise correction scheme using the spherical mode spectrum of the probe antenna is implemented within the SWE.

Further analyses focus on the advantages of spline-based measurements. These allow for a significantly improved precision in triggering the measurement instrumentation when reaching a sampling point. In comparison to the conventional trigger implementation, the resulting errors can be reduced by a factor of ten. In addition, spline paths enable the efficient implementation of irregular sampling grids. This is demonstrated by the example of a mathematically optimized, non-redundant distribution of sampling points. Such sampling grids cannot be implemented with most spherical positioners or only in a stepwise and, thus, inefficient scheme. The application of spline-based measurements using robotic systems allow to perform the irregular measurement in only 56 % of the duration of the corresponding equiangular sampling grid.

Therefore, the combination of the versatile applicable principles of the SWE with the flexibility of robot-based systems provides new perspectives in antenna measurement technology which were previously mainly of theoretical interest.

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Introduction

The electromagnetic spectrum is a finite and indispensable resource that supports the proper functioning of modern society and enables various forms of communication and sensing in numerous areas. Its importance is particularly highlighted by the ongoing development of mobile communications and the emergence of transformative technologies such as autonomous driving.

As comprehensively analyzed in [1], the mobile communications sector delivers impressive figures proving that this technology is an integral part of everyday life. The 4G technology with 5.2 billion active customers is currently being phased out by the successor technology 5G. In the third quarter of 2023 alone, 163 million new 5G subscriptions have been added, with a total number of 1.6 billion active 5G contracts expected by the end of the year. In Europe, at least half of the 4G customers are expected to switch to the new 5G standard by 2029. Over the same period, mobile data volumes are expected to approximately double, rising from 21 GB per month and end device in 2023 to more than 56 GB in 2029. Although 5G is still just introduced for end customers and already supports data rates in the order of 1 Gbit s^{-1} , the new 6G mobile communications standard with data rates of up to 100 Gbit s^{-1} is already being developed [2–4].

At the same time, the automotive industry is undergoing a significant shift towards autonomous driving technologies, pushed forward by advances in sensor technology. Although the first radar devices were tested in road traffic back in the 1960s, the technology has reached market maturity today and is, therefore, not focused on the premium segment anymore, but is now also established in the volume market [5, 6]. Today’s radar sensors, which operate in the millimeter-wave range, are crucial for enabling autonomous vehicles to accurately generate an image of the environment in 4D (Cartesian point clouds including velocity estimations) and make real-time decisions for safe navigation [7–10]. With the increasing use of radar-based sensing systems in vehicles, the demand for spectrum resources allocated to radar applications in automotive applications is increasing, underlining the essential role of spectrum availability in the advancement of autonomous driving technology.

In addition to communication and sensing applications, the accurate characterization of antennas is crucial as they serve as the interface between electronic devices and the electromagnetic spectrum in free space. Antenna measurement technology is fundamental to accurately assessing the performance and characteristics of antennas, ensuring the optimal use of the electromagnetic spectrum and, thus, providing the basis for a variety of complex technologies based on it [11, 12]. Even if sophisticated communication technologies apply high-level performance metrics such as data rates or channel qualities and automotive radars based on 3D point cloud information or the angular separation capability of nearby targets, the underlying

technology remains the antenna, enabling the transition of electromagnetic waves into free space. For this reason, antenna measurement technology can be considered as a core component in the development process of such technologies and is often irreplaceable, especially in diagnostic applications [13, 14].

Antenna measurements are basically divided into two categories: Direct far-field measurements as well as near-field measurement techniques. The former offers the advantage that the measurement data collected does not have to be processed and, hence, results are available practically in real time. However, it requires large measurement distances, making either free-field measurement systems or compact antenna test range (CATR) measurement systems the only options. Due to weather dependency, confidentiality problems or external sources of interference, CATR measurements are usually the preferred variant, although the requirements for the size of the measurement chamber and the surface roughness of the employed reflectors in relation to the highest operating frequency significantly affect the costs. This contrasts with near-field measurement techniques, which require much smaller spatial dimensions and, therefore, reduce cost pressure. There are various options for implementing near-field measurements, for example planar near-field (PNF), cylindrical near-field (CNF) or spherical near-field (SNF) measurements. While each of these measurement methods has individual advantages as well as challenges, as stated in [15, 16], another important argument is the mechanical feasibility of the measurement setup. Normally, a dedicated mechanical implementation is used for each measurement technique, such as interlocked linear axes for PNF measurements or a combination of rotational axes for CNF and SNF measurements, resulting in the inability to combine the aforementioned measurement methods and requiring multiple separate measurement chambers.

However, measurement systems based on robot arms allow to dissolve these previously strict boundaries. Robot-based systems are general-purpose positioning systems that usually consist of six (or even more) interlinked rotational axes and, therefore, provide all degrees of freedom within a specific working range to move to a certain position while, in principle, maintaining an arbitrary orientation [17, 18]. Thus, robot-based systems are ideal for performing complex processes and are, hence, originally used in production line automation technology, particularly in electronics manufacturing or the automotive sector, where the number of robot-assisted systems has been steadily rising [19–21]. In 2022 alone, more than 553 k new robot-based systems have been installed worldwide, an increase of 5 % compared to the previous year. As a result, the total number of installed systems is more than 3.9 million, with the strongest growth of more than 31 % in the area of collaborative robots [22]. Nevertheless, the number of robot-based systems used in antenna measurement technology is rather limited and their application is mainly restricted to research institutes or specialized measurement setups. First attempts in this field have been made in [23], in which a robot for microwave imaging measurements has been used for the early detection of breast cancer. A first robot-based antenna measurement system, however, has been presented in 2013 using an industrial robot arm and a rotation axis with attached hexapod [24]. The focus is on the path accuracy of spherical sampling grids, which is evaluated using a laser tracker, resulting for an accuracy requirement of $\lambda/25$ in an estimated maximum operating frequency of 250 GHz for these positioning systems. The first measurement results based this system have been

presented in 2015 [25–28], while another robot-based measurement system specifically designed to measure chip-integrated antennas within the complex environment of a wafer probing station has been established at the same time [29, 30].

Other research institutes adapted the idea of robot-based measurements for special test setups, for example to determine the radiation behavior of a radar sensor using a robot-guided reflector [31] or to evaluate electronically scanning phased arrays [32]. Moreover, novel and sophisticated measurement facilities have been developed based on dual robot positioning systems as exemplarily depicted in Fig. 1.1, which created completely new possibilities for both, conventional antenna measurements and dynamic over-the-air system tests [33–35]. A further state-of-the-art dual robot-based system is presented in [36] together with the development principle of a digital twin, bridging the gap between simulations and measurements for both, electromagnetic and mechanical aspects. Motivated by this system approach, additional studies concerning phase recovery using several piloting signals [37] and an optimized planar bipolar measurement scheme [38] have been investigated.

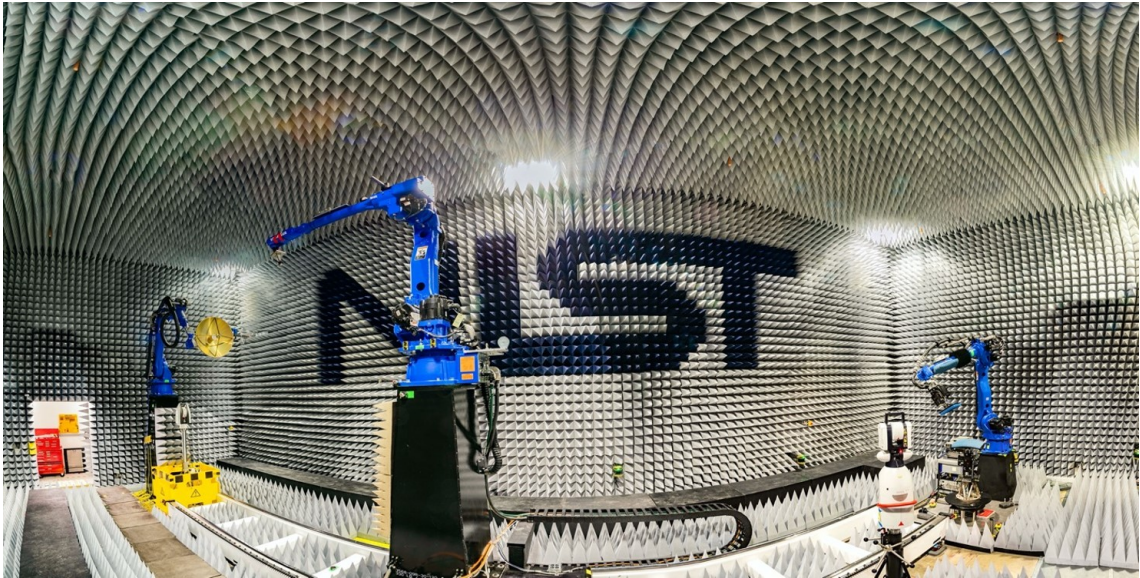


Figure 1.1: Exemplary picture of the NIST multi-purpose measurement facility sourced from [39]. On the left is the dual robot measurement system and on the right the setup based on a single robot including a laser tracker and the additional rotation axis with flange-mounted hexapod [33–35].

Nevertheless, robot-based systems are in the current state primarily used to reproduce conventional measurement systems. Consequently, robots are usually operated by means of canonical path definitions, for example along linear or circular trajectories, i.e. they are used mainly as a combination system for PNF or SNF measurement setups [40]. Nevertheless, it should be noted that more complex approaches also exist, for example to increase the validity range of PNF measurements by sampling rounded side surfaces using the robot [41]. However, even if this results in a non-canonical overall path, the individual linear or circular sections are still defined canonically in this case.

In addition, robot-based systems typically have higher tolerances with regard to their absolute accuracy due to the concatenation of several rotational axes and their

original application in automation technology. For this reason, numerous studies are dedicated to improving the actual path accuracy. For this purpose, the effective path of the robot is determined using external measurement instruments such as laser trackers in order to adjust the positioning of the robots until their actual physical path corresponds sufficiently to the ideally intended path. In addition to intensive modeling and calibration work, this involves tracing the actual measurement path, for example first without correction parameters and, subsequently, by calculating the calibration parameters and including them into the path generation [24, 26, 28, 40].

In contrast, this dissertation pursues a different approach, in which the position uncertainty of the robot is not countered at the mechanical level by means of path corrections, but within the near-field to far-field transformation (NFFFT) based on the spherical wave expansion (SWE). The theory of the SWE enables not only the consideration of non-ideal sampling positions, but also the correction of inaccurate orientations of the probe antenna, individually for each measurement point by the application of a higher-order pointwise probe correction (PPC). The consideration of non-ideal sampling positions and orientations on the electromagnetic level within the SWE, thus, provides the opportunity to avoid tracing multiple motion sequences for path optimization and to use robot-based positioning systems in accordance to their original intended application, but for SNF measurements. This includes, in particular, a more complex motion control by implementing spline-based trajectories, which, to the best of the author's knowledge, have not yet been addressed within the state-of-the-art for SNF measurements. Spline-based motion sequences represent the optimum operating mode of a robot, as no canonical path is enforced and the robot controller is, therefore, granted additional degrees of freedom for path optimization, e.g. regarding higher path speeds or increased smoothness of motion. Furthermore, the irregular and non-redundant maximum determinant sampling grid (MDSG) shifts into focus in contrast to conventional equiangular measurements, as spline-based trajectories enable an efficient implementation, which is difficult or impossible to achieve in comparison to roll-over-azimuth positioners.

In order to validate these research propositions, the robot-based measurement system as depicted in Fig. 1.2 has been developed within this dissertation, which complements the existing measurement capabilities at the Institute of High Frequency Technology (IHF). To process the acquired measurement data, the theoretical principles of the spherical NFFFT are explained in Section 2, focusing on the description of an electromagnetic field based on the SWE, the modeling of an antenna measurement using the spherical transmission formula and, subsequently, the fundamental solution methods. These include the Wacker algorithm, which is the most commonly used solution method, as well as the direct and iterative approach to support irregular sampling grids. Additionally, probe correction techniques required for the application of a higher-order PPC are outlined.

The design of the developed measurement setup is motivated in Section 3, where simulations of the position uncertainty and its effect on the NFFFT are analyzed, the mechanical design is discussed as well as the link budget of the radio frequency (RF) setup is explained in detail. In addition, the control of a robot arm is briefly outlined and a numerical technique for solving the inverse kinematics problem is summarized. The chapter concludes with a consideration of the safety measures and

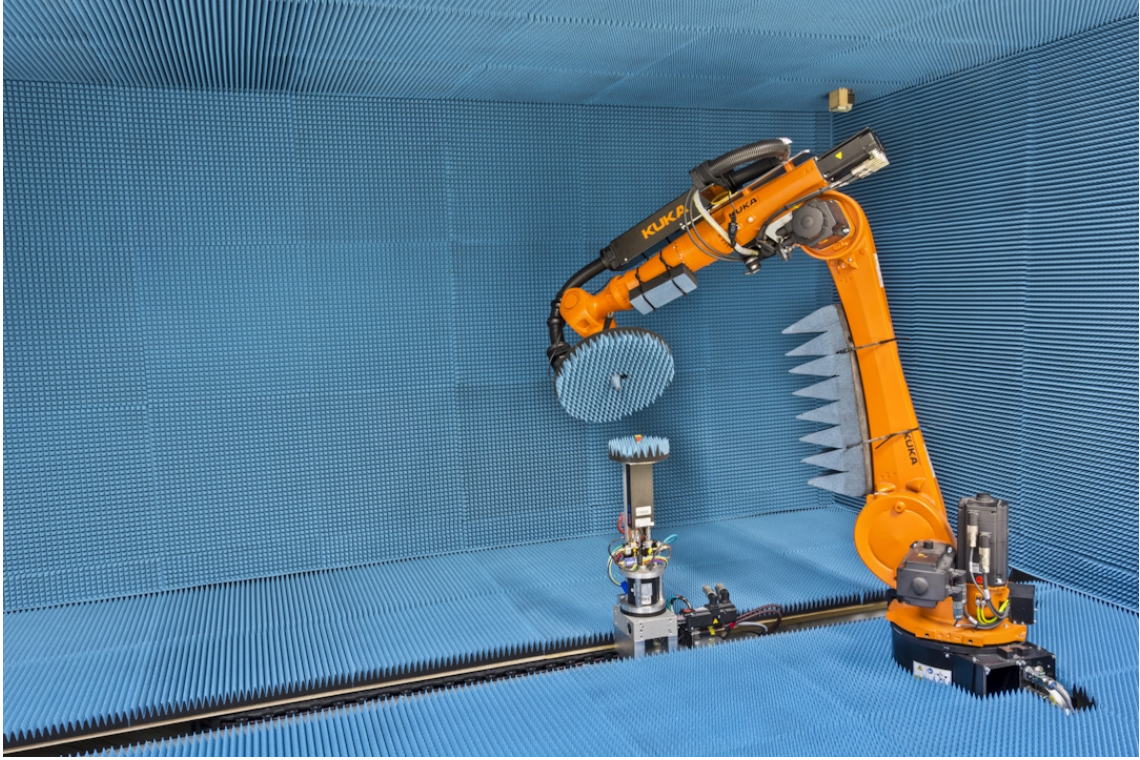


Figure 1.2: The robot-based measurement system developed as part of this dissertation is based on an industrial robot combined with two additional axes within a common kinematics for measurements in the millimeter-wave range from a frequency of 60 GHz and upwards [42].

the overall integration of all components for the realization of complex measurement processes.

Since the alignment of a SNF measurement, especially in the millimeter-wave range, is of utmost importance and as, moreover, robot-based systems provide all degrees of freedom to compensate for mechanical tolerances, a model for the most important error terms is introduced in Section 4. It is supplemented by analyses on the interdependence of individual error terms as well as an alignment technique to identify and isolate these impact factors. In addition, the determined alignment is validated for both, direct far-field measurements and near-field measurements transformed into the far field in comparison with results obtained in the CATR of the IHF.

The theory of the higher-order PPC is explained in Section 5 and its implementation for robot-based systems is analyzed. The application of the PPC is, thereafter, validated by means of a well-aligned and an intended offset measurement, whereby it is demonstrated that the potentials provided by the PPC enable to reconstruct the far-field radiation pattern of the offset measurement in accordance with the well-aligned reference measurement.

Spline-based motion patterns are introduced in Section 6 by a comparison of the actually resulting path with numerical spline implementations. Therefore, it can be assessed if the resulting motion sequences are suitable for the respective measurement project, independent on the availability of the manufacturer's simulation tools. In addition, spline-based measurements provide advanced triggering capabil-

ities, whereby, for example, data acquisition can be triggered on the resulting path between two specified sampling points. However, the position and orientation of the probe antenna cannot be guaranteed in this case as it is pre-determined by the path generation of the robot controller and, thus, the application of the PPC within the NFFFT is mandatory.

The PPC as well as the spline-based motion patterns are ultimately integrated in Section 7 for the implementation of the irregular and non-redundant MDSG. In addition, the numerical stability of the MDSG is investigated and two different path generation algorithms using the sampling points defined by the MDSG are compared in terms of path length, measurement duration and the achievable average path velocity. Furthermore, results of the far-field transformed radiation patterns are presented, taking advantage of the different trigger capabilities of the robot-based system during the measurements. Subsequently, the most important results on the application of robot-based techniques within the spherical NFFFT are summarized in Section 8 and complemented by an outlook on further methods extending beyond the scope of this work.

References to the author's own publications are highlighted in bold numbers (e.g. [42]), whereby a complete list of all authored publications is included at the end of this thesis.

Spherical Near-Field Measurements

The SNF measurement technique is one of the most established methods for determining the radiation pattern of an antenna [43] and is, therefore, the focus of this dissertation. Besides the advantages to the spatial requirements of a measurement chamber, this method is also called the most accurate method in antenna measurement technology [44]. Furthermore, the underlying principle of the SWE provides an exact solution to the problem, which is based on a decomposition of the measured field into a set of complex weighted spherical mode coefficients (SMC). Based on this SMC, for example, antenna parameters such as directivity and gain can be calculated, analyses regarding a possible offset of the antenna under test (AUT) to the measurement coordinate system can be performed and, moreover, the radiation pattern can be determined for arbitrary distances. However, it is based on a sophisticated theoretical foundation, which is comprehensively explained and analyzed in [45]. Thus, only the most important theoretical principles required in the context of this work are summarized hereinafter.

2.1 Spherical Wave Expansion

Spherical waves, commonly referred to as vector spherical harmonics or spherical modes, are characterized as a special solution of the Maxwell equation in a linear, isotropic and homogeneous medium. Since they form an orthogonal basis on a sphere, they are frequently utilized in modeling antenna radiation patterns. The concept of spherical waves has been first theorized in [46], as a solution of the vector wave equation demanding only a coordinate system where the scalar wave equation is separable. Since the first detailed and extensive derivation in [47], the spherical wave theory has been the subject of many scientific investigations [48–52] and still provides the basis for current research topics [53–58].

The spherical wave functions $\mathbf{F}_{smn}^{(c)}$ represent a distinctive solution of Maxwell's equations in spherical coordinates (r, θ, ϕ) with $r \in [0; \infty)$, $\theta \in [0; \pi]$, $\phi \in [0; 2\pi)$ and are defined as

$$\mathbf{F}_{1mn}^{(c)}(r, \theta, \phi) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{n(n+1)}} \left(-\frac{m}{|m|} \right)^m \left(z_n^{(c)}(kr) \frac{im\bar{P}_n^{|m|}(\cos\theta)}{\sin\theta} e^{im\phi} \mathbf{e}_\theta - z_n^{(c)}(kr) \frac{d\bar{P}_n^{|m|}(\cos\theta)}{d\theta} e^{im\phi} \mathbf{e}_\phi \right) \quad (2.1)$$

and

$$\begin{aligned}
\mathbf{F}_{2mn}^{(c)}(r, \theta, \phi) = & \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{n(n+1)}} \left(-\frac{m}{|m|} \right)^m \\
& \left(\frac{n(n+1)}{kr} z_n^{(c)}(kr) \bar{P}_n^{|m|}(\cos \theta) e^{im\phi} \mathbf{e}_r \right. \\
& + \frac{1}{kr} \frac{d}{d(kr)} (kr z_n^{(c)}(kr)) \frac{d\bar{P}_n^{|m|}(\cos \theta)}{d\theta} e^{im\phi} \mathbf{e}_\theta \\
& \left. + \frac{1}{kr} \frac{d}{d(kr)} (kr z_n^{(c)}(kr)) \frac{im\bar{P}_n^{|m|}(\cos \theta)}{\sin \theta} e^{im\phi} \mathbf{e}_\phi \right). \tag{2.2}
\end{aligned}$$

The index s of $\mathbf{F}_{smn}^{(c)}$ indicates the mode of a wave, as can be seen from the comparison of (2.1) and (2.2), since the first term has no radial dependence and, thus, always describes the transverse field component. Assuming a transverse magnetic (TM) wave, the magnetic field is, therefore, described by $\mathbf{F}_{1mn}^{(c)}$ and the electric field by $\mathbf{F}_{2mn}^{(c)}$. Analogously, the reverse case occurs for transverse electric (TE) waves: The electric field is described by $\mathbf{F}_{1mn}^{(c)}$ and the magnetic field by $\mathbf{F}_{2mn}^{(c)}$. The index m ($-n < m < n$) denotes the order of the spherical wave function and determines the oscillation of the field along the ϕ -axis. The degree of a mode is described by the index n ($1 < n < N_n$) and in combination with m describes the field oscillation along the θ -axis. This is illustrated in Fig. 2.1 by increasing the order m while keeping the degree n constant, emphasizing the increased oscillations in ϕ .

The radial dependency is described by the term $z_n^{(c)}$, with the wavenumber $k = 2\pi/\lambda$, the wavelength λ , and the radial function given by the upper index (c) according to

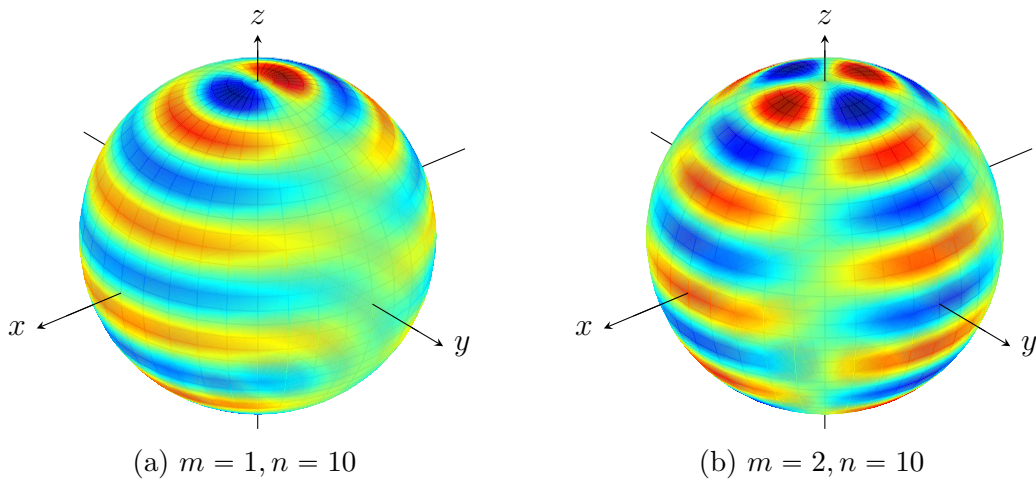


Figure 2.1: Qualitative comparison of the oscillations in θ and ϕ as a function of order m and degree n , illustrating increasing oscillations in ϕ for increasing n and constant m .

$$z_n^{(1)}(kr) = j_n(kr) \quad (2.3a)$$

$$z_n^{(2)}(kr) = n_n(kr) \quad (2.3b)$$

$$z_n^{(3)}(kr) = h_n^{(1)}(kr) = j_n(kr) + in_n(kr) \quad (2.3c)$$

$$z_n^{(4)}(kr) = h_n^{(2)}(kr) = j_n(kr) - in_n(kr) \quad (2.3d)$$

where j_n is the spherical Bessel function, n_n the spherical Neumann function and $h_n^{(l)}$ the spherical Hankel function of l -th kind. Accordingly, a standing wave in radial direction is described by the indices $c \in \{1, 2\}$, a radially outgoing wave by $c = 3$ and a radially incoming wave by $c = 4$. Both, the Bessel and Neumann functions describe radial standing waves, the former being finite $j_n(kr)|_{r=0} = 0$ whereas the latter tends to $\lim_{r \rightarrow 0} n_n(kr) = -\infty$ in the origin. The normalized Legendre polynomials [59] associated to order m and degree n are denoted as $\bar{P}_n^{[m]}$.

The spherical wave functions $\mathbf{F}_{smn}^{(c)}$ allow the calculation of the electric field $\mathbf{E}(r, \theta, \phi)$ for a source-free space $r > r_0$, where r_0 describes a spherical volume enclosing sources as exemplarily illustrated in Fig. 2.2. The physically exact representation is given by the superposition and complex weighting of the theoretically infinite series of wave functions according to

$$\mathbf{E}(r, \theta, \phi) = \frac{k}{\sqrt{\eta}} \sum_{csmn} Q_{smn}^{(c)} \mathbf{F}_{smn}^{(c)} \quad (2.4)$$

where η is the specific wave admittance of the enclosed medium and $Q_{smn}^{(c)}$ are the complex coefficients to weight the power-normalized spherical wave functions, therefore also called SMC. Since the spherical wave functions are dimensionless, the prefactor in front of the summation is required to ensure the correct unit of $\sqrt{W}$ for the SMC. Furthermore, instead of summation over $-N_m \leq m \leq N_m$ (with $N_m \leq n$), $1 \leq n \leq N_n$, $1 \leq s \leq 2$ and $3 \leq c \leq 4$, a single-index notation $j = 2(n(n+1) + m - 1) + s$ with a total number of modes $j \leq J = 2N_n(N_n + 2)$ is often used. Since the SMC are band-limited in relation to the electrical size of the antenna, the representation of the fields also converges using a truncated series following

$$N_m \leq N_n = \lfloor kr_0 \rfloor + n_1 \quad (2.5)$$

where $\lfloor \cdot \rfloor$ indicates the floor function. Typically, although not required, a complete set of SMC in m is obtained by $N_m = N_n$, which is also referred to as $N_{m,n}$ in the following. The constant n_1 is considered as a safety buffer and depends, for example, on the coincidence regarding location and orientation of the AUT and measurement coordinate systems, the radial distance r_1 of the source to the origin of the AUT coordinate system and the source extent r_2 , which are visualized in Fig. 2.2, as well as the required accuracy. At a measurement distance of a few wavelengths from the minimum sphere r_0 , the choice of $n_1 = 10$ is considered as sufficient for most practical applications [45]. However, a more comprehensive analysis on the choice of n_1 with respect to the antenna size, required accuracy as well as the impact of the truncation limit is presented in [60]. Here, the mode truncation constant n_1 is

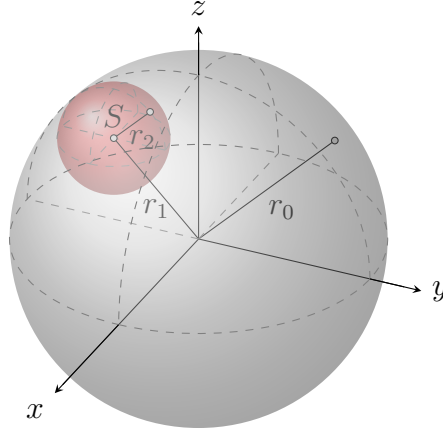


Figure 2.2: Minimum sphere with radius r_0 enclosing a source S with an offset r_1 to the origin of the coordinate system and a source extent of r_2 .

determined corresponding to the power of the truncated modes given by

$$n_1 = 0.045 \sqrt[3]{k r_0} (P_{r_0} - P_{\text{tr}}), \quad (2.6)$$

wherein P_{r_0} is the power of the source within $r < r_0$ relative to the total power and P_{tr} (specified in dB) is the upper limit of the power fraction neglected due to the mode truncation. Furthermore, it allows to estimate threshold values for the truncation constant n_1 as a function of P_{tr} , which are given in [60] by

$$n_1 = \begin{cases} 1.6 \sqrt[3]{k r_0}, & \text{for } P_{\text{tr}} = -40 \text{ dB} \end{cases} \quad (2.7)$$

$$n_1 = \begin{cases} 2.5 \sqrt[3]{k r_0}, & \text{for } P_{\text{tr}} = -60 \text{ dB} \end{cases} \quad (2.8)$$

$$n_1 = \begin{cases} 3.6 \sqrt[3]{k r_0}, & \text{for } P_{\text{tr}} = -80 \text{ dB} \end{cases} \quad (2.9)$$

$$n_1 = \begin{cases} 5.0 \sqrt[3]{k r_0}, & \text{for } P_{\text{tr}} = -120 \text{ dB} \end{cases} \quad (2.10)$$

being valid for all spatially limited source configurations. This is especially important in the case of noise-affected measurements or if there are adverse scattering effects from other objects and, thus, enables a precise filtering based on an adjusted mode truncation. Consequently and under consideration of the Nyquist criterion, it also affects the maximum sampling distance, which is given for a full set of spherical modes $N_n = N_m$ by

$$\Delta\theta = \Delta\phi \leq \frac{180^\circ}{N_n}. \quad (2.11)$$

Similarly, for the case $N_m < N_n$, the angular step size in ϕ may be increased to $\Delta\phi \leq 180^\circ/N_m$ while the angular step size $\Delta\theta$ remains unchanged.

One of the biggest advantages of modeling the electric field of an AUT by means of the SWE is the mathematically exact solution, which is valid in the entire space for $r > r_0$. This avoids problems of spatial truncation as for example with planar or cylindrical wave expansions. The radiation patterns can be calculated at arbitrary distances as well as in the far field ($r \rightarrow \infty$) using the asymptotic formulas of the wave functions available in this case. In addition, the spherical mode spectrum provides further analysis potentials, for example to estimate a possible offset

of the AUT to the measurement coordinate system. In addition, other antenna parameters such as directivity or gain can be calculated directly, but also the modeling of the transmission between two antennas, e.g. between probe antenna and AUT, is facilitated, which is explained in the following.

2.2 Spherical Transmission Formula

The characterization of an AUT requires a transmission between it and a known probe antenna. The type of transmission typically depends on the measurement setup used and is taken into account in the choice of the radial dependency. Accordingly, the probe antenna receives ($c = 4$) when the AUT transmits ($c = 3$), but typically also in the case of standing waves ($c = 1$ or $c = 2$). In general, the procedure is reciprocal, but in the following, in accordance with the measurement setup used as described in Section 3, the case is considered in which the AUT is operated in transmit mode.

The coordinate system of an antenna measurement is described in Fig. 2.3 illustrating that the AUT is located in its (x_a, y_a, z_a) coordinate system, which is stationary in space. The probe antenna is described by the (x_p, y_p, z_p) coordinate system and moves in a radius $r > r_0$ around the AUT, where the minimum spheres of AUT and probe do not overlap. Furthermore, the probe antenna is oriented to the origin of the former coordinate system for each measurement point and the polarization angle to the AUT can be rotated by χ around the z_p -axis. Typically, the polarization angles $\chi = 0^\circ$ and $\chi = 90^\circ$ are chosen, although this is not mandatory.

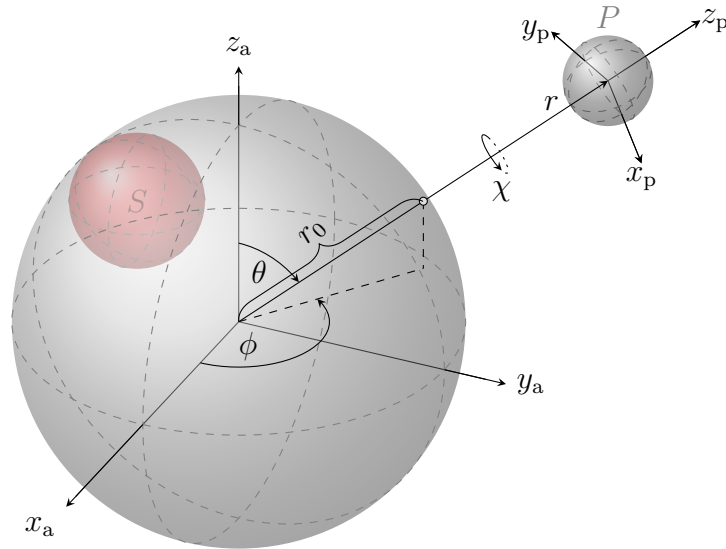


Figure 2.3: Minimum sphere r_0 centered in the AUT coordinate system (x_a, y_a, z_a) containing the source S , and relative position and orientation of the associated probe coordinate system (x_p, y_p, z_p) as a function of (r, ϕ, θ, χ) .

For an AUT in transmit mode and based on (2.4), the electric field $\mathbf{E}_t(r, \theta, \phi)$ of an outward travelling wave is denoted as

$$\mathbf{E}_t(r, \theta, \phi) = \frac{k}{\sqrt{\eta}} \sum_{smn} Q_{smn}^{(3)} \mathbf{F}_{smn}^{(3)}(r, \theta, \phi) \quad (2.12)$$

$$= \frac{k}{\sqrt{\eta}} \sum_{smn} v T_{smn} \mathbf{F}_{smn}^{(3)}(r, \theta, \phi) \quad (2.13)$$

where v is the complex excitation amplitude and T_{smn} are the AUT's transmit coefficients with $Q_{smn}^{(3)} = v T_{smn}$. However, a description of the spherical wave functions in the (x_p, y_p, z_p) coordinate system of the probe antenna is required. Therefore, the relative orientation of the two coordinate systems is described in Euler angles, followed by a translation of r . For the relative orientation the so-called χ -convention is suitable, in which the three successive Euler rotations (α, β, γ) are applied to the axes (z, y', z'') . Since the rotations are consecutive, the prime indicates that the rotations are applied to the already rotated axes. This order of rotations is useful in the case of spherical antenna measurements because the rotations can also be interpreted as rotations around the angles $(\phi, \theta, \chi) = (\phi_0, \theta_0, \chi_0)$ in accordance with Fig. 2.3. In addition, modes are denoted as smn and $\sigma\mu\nu$ with respect to the AUT and probe coordinate systems, respectively, to ensure a clear differentiation.

The stepwise rotations and translations of the spherical wave functions are described by

$$\mathbf{F}_{smn}^{(3)}(r, \theta, \phi) = e^{im\phi_0} \mathbf{F}_{smn}^{(3)}(r', \theta', \phi'), \quad r' > r_0 \quad (2.14a)$$

$$\mathbf{F}_{smn}^{(3)}(r', \theta', \phi') = \sum_{\mu=-n}^n d_{\mu m}^n(\theta_0) \mathbf{F}_{s\mu n}^{(3)}(r'', \theta'', \phi''), \quad r'' > r_0 \quad (2.14b)$$

$$\mathbf{F}_{s\mu n}^{(3)}(r'', \theta'', \phi'') = e^{i\mu\chi_0} \mathbf{F}_{s\mu n}^{(3)}(r''', \theta''', \phi'''), \quad r''' > r_0 \quad (2.14c)$$

$$\mathbf{F}_{s\mu n}^{(3)}(r''', \theta''', \phi''') = \sum_{\sigma=1}^2 \sum_{\substack{\nu=|\mu| \\ \nu \neq 0}}^{\infty} C_{\sigma\mu\nu}^{sn(3)}(kr) \mathbf{F}_{\sigma\mu\nu}^{(1)}(r_p, \theta_p, \phi_p), \quad r_p < (r - r_0) \quad (2.14d)$$

where $e^{im\phi_0}$ in (2.14a) and $e^{i\mu\chi_0}$ in (2.14c) describe the rotation operator along the z -axis and $d_{\mu m}^n(\theta_0)$ in (2.14b) the rotation operator along the y -axis. The translation operator in z -direction in (2.14d) is denoted as $C_{\sigma\mu\nu}^{sn(3)}(kr)$, where r is the radius of the measurement sphere as shown in Fig. 2.3. Since $\mathbf{F}_{s\mu n}^{(3)}(r''', \theta''', \phi''')$ is finite for $r''' > r_0$, the right-hand side of (2.14d) is resembling a standing wave [45]. This is valid for $r > r_p + r_0$, where r_p represents the radial distance of the probe antenna coordinate system (x_p, y_p, z_p) to the coordinate system of the AUT (x_a, y_a, z_a) . For further details on the application and implementation of the translation operator, reference is made to [45, Appendix A3].

The substitution of the standing wave by the superposition of radially inward and outward travelling waves according to

$$\mathbf{F}_{\sigma\mu\nu}^{(1)} = \frac{1}{2} (\mathbf{F}_{\sigma\mu\nu}^{(3)}(r_p, \theta_p, \phi_p) + \mathbf{F}_{\sigma\mu\nu}^{(4)}(r_p, \theta_p, \phi_p)) \quad (2.15)$$

and insertion in (2.13) results in the description of the electric field

$$\begin{aligned} \mathbf{E}_t(r, \theta, \phi) = & \frac{k}{\sqrt{\eta}} \sum_{\substack{smn \\ \sigma\mu\nu}} v T_{smn} e^{im\phi_0} d_{\mu m}^n(\theta_0) e^{i\mu\chi_0} C_{\sigma\mu\nu}^{sn(3)}(kr) \cdot \\ & \frac{1}{2} (\mathbf{F}_{\sigma\mu\nu}^{(3)}(r_p, \theta_p, \phi_p) + \mathbf{F}_{\sigma\mu\nu}^{(4)}(r_p, \theta_p, \phi_p)) \end{aligned} \quad (2.16)$$

depending on the relative position and orientation of AUT and probe.

For a transmission between AUT and probe, the probe is inserted in the origin of the probe coordinate system (x_p, y_p, z_p) . For this purpose, the assumption is made that there is no interaction between AUT and probe, i.e. currents induced on the probe as well as multiple reflections between AUT and probe are considered negligible. Furthermore, the formula describes the transmission behavior in free space, which also excludes room scattering effects, representing a valid assumption for anechoic, shielded measurement chambers. However, this influence can be incorporated into the theory, for example if desired for high-precision measurements or in the case of highly reflective measurement environments. A common approach is to determine the waves incident in the measurement zone, either by simulations or by measurements, and consider them in (2.16) as explained in detail in [61–63]. However, the measurement procedure is time consuming, since it has to be repeated for each required measurement frequency, as well as sensitive to disturbances, for example caused by movements of the positioning system affecting the test-zone field. In addition, mismatch between the probe and the receiver is also considered negligible, since in practical measurement setups the data acquisition is performed relatively and, thus, the influence of this static mismatch has no effect.

Nevertheless, for a general discussion, it is sufficient to consider the modes incident on the probe described by $\mathbf{F}_{\sigma\mu\nu}^{(4)}(r_p, \theta_p, \phi_p)$. Furthermore, if the indices describing the relative orientation between probe and AUT are omitted for a clearer representation, the signal measured by the probe is given by the general transmission formula

$$w(r, \phi, \theta, \chi) = v \sum_{\substack{smn \\ \mu}} T_{smn} e^{im\phi} d_{\mu m}^n(\theta) e^{i\mu\chi} P_{s\mu n}(kr) \quad (2.17)$$

where the introduced probe response constants $P_{s\mu n}(kr)$ are defined as

$$P_{s\mu n}(kr) = \frac{1}{2} \sum_{\sigma\nu} C_{\sigma\mu\nu}^{sn(3)}(kr) R_{\sigma\mu\nu}^p. \quad (2.18)$$

Therefore, the probe response constants $P_{s\mu n}(kr)$ describe the sensitivity of the probe for a mode $(s\mu n)$ of the AUT for a position given by $(r, \phi, \theta, \chi) = (r, 0, 0, 0)$. However, it should be noted that the probe must be known, for example through simulations or measurements. If the probe, depending on the method applied, has been measured or simulated in transmit mode, the conversion to receive coefficients is given by

$$R_{smn} = (-1)^m T_{s,-m,n} \quad (2.19)$$

$$T_{smn} = (-1)^m R_{s,-m,n} \quad (2.20)$$

for reciprocal antennas. In addition, it is important to consider that the probe is oriented in the given model in the negative z_p -direction towards the AUT. Therefore, if necessary, the modes can be rotated accordingly with the known operators, for example by $\phi = -\pi$ and $\theta = -\pi$.

In summary, (2.17) provides the fundamental capability to describe the transmission between two antennas based on their mode spectra, each expressed in the origin of their respective coordinate systems. The radial distance as well as the orientation (r, ϕ, θ, χ) between the coordinate systems is established by rotation and translation operators and allows to quantify the coupling of AUT and probe in a single coordinate system. This provides the basis to determine the AUT's unknown transmission coefficients T_{smn} based on the measured signal $w(r, \phi, \theta, \chi)$ of the probe, as will be explained subsequently in more detail.

2.3 Transformation Algorithms

In the previous chapter, the spherical transmission formula in (2.17) presented the basic relationship for the determination of the SMCs. Besides the theoretical derivation, the numerical calculation of these in order to solve electromagnetic problem poses another challenge. Since the computational complexity depends on the electrical size of the AUT and, depending on the solution method, exhibits a complexity of up to $\mathcal{O}((kr)^6)$, the size of solvable problems is often limited by the available computational capacities. The exponential development of computational capacity described in Moore's Law has alleviated the limitations to a large extent, but an efficient solution procedure is still of great importance today. However, especially at the time of the introduction of the SWE-based spherical NFFFT, an efficient solution approach was, therefore, an absolute exclusion criterion. Around 1975, Wacker [64, 65] and Jensen [66] developed an algorithm which drastically reduced the computational complexity and, thus, enabled the practical applicability of this method. Although this algorithm can only be applied to regular sampling grids and also makes assumptions regarding the usable probes, it is one of the most widely used standard methods even in modern times and is, therefore, explained below. Furthermore, algorithms with higher computational complexity are presented, which are suitable for irregular sampling grids as well as for arbitrary probes and, thus, not only exploit the computational resources available today, but also enable the processing of today's research questions.

2.3.1 Wacker Algorithm

Through Wacker's research, an efficient algorithm with $\mathcal{O}((kr_0)^3)$ for the calculation of the SMCs was found, which enabled the practical application of the NFFFT at those days for the first time. Further contributions were made by Lewis [67] and Larsen [68, 69], who, among other aspects, further simplified the computation of the rotation operator $d_{\mu m}^n(\theta)$ either by symmetry considerations or by transforming the equations to allow the application of a Fourier transform to enable a more efficient computation.

The Wacker algorithm is mainly based on two assumptions: First, only a certain class of probes, namely first-order probes or so-called $\mu = \pm 1$ probes, are considered.

Accordingly, the spherical mode spectrum $R_{\sigma\mu\nu}^p$ of such probes contains only modes with indices $\mu = \pm 1$. Although exact $\mu = \pm 1$ probes do not exist in reality and are a rather theoretical construct, electrically small and symmetric antennas with a small variation along their θ -axis, represent a good approximation. This includes in particular circular open-ended waveguides, rectangular open-ended waveguides, but also rectangular horn antennas. Even if a probe is not completely rotationally symmetric, it may be suitable as a $\mu = \pm 1$ probe if the measurement distance is sufficiently large, since the $\mu = \pm 1$ modes of the probe are significant in contrast to the rest of the mode spectrum. Second, an equiangular sampling grid is required, allowing processing by Fourier transforms. Therefore, the individual summations can be decoupled resulting in an independent calculation of the integrals along the χ -, ϕ -, and θ -axes, which will be explained in the following.

In the first step, the rotation around χ is considered, which is based on the exponential function and, thus, represents an orthogonal basis. Hence, the transmission equation (2.17) can be rewritten to

$$w(r, \phi, \theta, \chi) = \sum_{\mu=-N_\nu}^{N_\nu} w_\mu(r, \phi, \theta) e^{i\mu\chi} \quad \text{with} \quad (2.21)$$

$$w_\mu(r, \phi, \theta) = v \sum_{s=1}^2 \sum_{n=1}^{N_n} \sum_{m=-n}^n T_{smn} e^{im\phi} d_{\mu m}^n(\theta) P_{s\mu n}(kr) \quad (2.22)$$

where $w_\mu(r, \phi, \theta)$ is a coefficient of the finite Fourier series in χ of $w(r, \phi, \theta, \chi)$ and the N_ν represents the maximum degree ν of the probe antenna according to its band-limitation. Next, (2.21) is calculated by multiplying both sides by $e^{-i\mu\chi}$ and, then, integrating over χ in the interval $[0; 2\pi]$. This results in

$$w_\mu(r, \phi, \theta) = \frac{1}{2\pi} \int_{\chi=0}^{2\pi} w(r, \phi, \theta, \chi) e^{-i\mu\chi} d\chi \quad (2.23)$$

and demonstrates that $w_\mu(r, \phi, \theta)$ represents the Fourier transform of the measured signal $w(r, \phi, \theta, \chi)$.

The calculation over ϕ is done by rewriting (2.22) as

$$w_\mu(r, \phi, \theta) = \sum_{m=-N_n}^{N_n} w_{\mu m}(r, \theta) e^{im\phi} \quad \text{with} \quad (2.24)$$

$$w_{\mu m}(r, \theta) = v \sum_{s=1}^2 \sum_{\substack{n=|m| \\ n \neq 0}}^{N_n} T_{smn} d_{\mu m}^n(\theta) P_{s\mu n}(kr) \quad (2.25)$$

which, in analogy to the integral over χ , yields

$$w_{\mu m}(r, \theta) = \frac{1}{2\pi} \int_{\phi=0}^{2\pi} w_\mu(r, \phi, \theta) e^{-im\phi} d\phi \quad (2.26)$$

and, thus, proves that the transmission equation (2.17) is describable as a two-dimensional Fourier transform. Finally, to calculate the interval over θ , (2.25) is transformed into

$$w_{\mu m}(r, \theta) = \sum_{\substack{n=|m| \\ n \neq 0}}^{N_n} w_{\mu m}^n(r) d_{\mu m}^n(\theta) \quad \text{with} \quad (2.27)$$

$$w_{\mu m}^n(r) = v \sum_{s=1}^2 T_{smn} P_{s\mu n}(kr) \quad (2.28)$$

where in contrast to the separation in χ and ϕ , the orthogonality of the rotation operator $d_{\mu m}^n(\theta)$ explained in [45, Appendix A2] and [70] is crucial to enable the integral representation as

$$w_{\mu m}^n(r) = \frac{2n+1}{2} \int_{\theta=0}^{\pi} w_{\mu m}(r, \theta) d_{\mu m}^n(\theta) \sin(\theta) d\theta. \quad (2.29)$$

This demonstrated that the transmission equation can be decomposed into three analytical Fourier transforms. The χ and ϕ integrals are calculated via a periodic, band-limited function and, thus, can be efficiently solved for numerical calculation using a discrete Fourier transform (DFT). However, the numerical calculation of the θ integral in the range $[0; \pi]$ requires the continuation of the rotation operator $d_{\mu m}^n(\theta)$ to an interval of $[0; 2\pi]$. Since $d_{\mu m}^n(\theta)$ is periodic in θ with a period of 2π , the function can be continued depending on whether $(\mu - m)$ is even or odd and, therefore, can be processed via a DFT. However, for the detailed approach to the numerical solution, also for the other integrals, reference is made to [45, Chapter 4.3.3].

With respect to the first assumption of the Wacker algorithm on the $\mu = \pm 1$ probes used, the summation over s in (2.28) is now explicitly expressed as

$$vT_{1mn}P_{1\mu n}(kr) + vT_{2mn}P_{2\mu n}(kr) = w_{\mu m}^n(r) \quad (2.30)$$

and, thus showing, that for each mode pair (m, n) the number of equations in the unknowns vT_{1mn} and vT_{2mn} is equal to the probe's number of modes μ along azimuth. Thereby, in the case of a $\mu = \pm 1$ probe, the system of equations reduces to

$$vT_{1mn}P_{1,+1,n}(kr) + vT_{2mn}P_{2,+1,n}(kr) = w_{+1m}^n(r), \quad (2.31)$$

$$vT_{1mn}P_{1,-1,n}(kr) + vT_{2mn}P_{2,-1,n}(kr) = w_{-1m}^n(r). \quad (2.32)$$

Since the received signal $w(r, \phi, \theta, \chi)$ of a $\mu = \pm 1$ probe only contains two modes

$$w(r, \phi, \theta, \chi) = w_1(r, \phi, \theta) e^{i\chi} + w_{-1}(r, \phi, \theta) e^{-i\chi} \quad (2.33)$$

it follows that

$$w(r, \phi, \theta, \chi) = -w(r, \phi, \theta, \chi + \pi). \quad (2.34)$$

Thus, the number of measurements in χ can be reduced, e.g. to polarization angles $\chi_1 = 0$ and $\chi_2 = \pi/2$, and, based on that, all other polarization angles can

be derived as explained in more detail in [45, Chapter 4.3.3.2]. Therefore, a direct solution is available if $\chi_1 \neq \chi_2$ or $\chi_1 \neq \chi_2 + \pi$ by

$$w_1(r, \phi, \theta) = \frac{1}{2} \left(w(r, \phi, \theta, \chi_1 = 0) - iw(r, \phi, \theta, \chi_2 = \frac{\pi}{2}) \right) \quad (2.35)$$

$$w_{-1}(r, \phi, \theta) = \frac{1}{2} \left(w(r, \phi, \theta, \chi_1 = 0) + iw(r, \phi, \theta, \chi_2 = \frac{\pi}{2}) \right) \quad (2.36)$$

which eliminates the necessity of a DFT over χ and, thus, contributes to an efficient solution using the Wacker algorithm.

2.3.2 Direct Method

Due to the steady increase in processing capabilities, more computationally demanding solution are gaining importance. This shifts the focus of an efficient implementation, for example by the Wacker algorithm, to more flexible methods, which were not solvable in acceptable time or at all in the days of the introduction of spherical near-field measurement technology. This includes, although the approach is the most obvious, the direct solution via a linear system of equations, which can be set up based on the spherical transmission formula. To represent the system of equations, it is convenient to convert the spherical transmission formula into a matrix notation using the single-index j -notation, which has already been introduced in Section 2.1. Therefore, (2.17) is first transformed into a more compact form denoted as

$$w(\vec{p}) = v \sum_{smn} T_{smn} \Gamma_{smn}(\vec{p}) = v \sum_j T_j \Gamma_j(\vec{p}) \quad \text{with} \quad (2.37)$$

$$\Gamma_j(\vec{p}) = \Gamma_{smn}(\vec{p}) = \sum_{\mu} e^{im\phi} d_{\mu n}^m(\theta) e^{i\mu\chi} P_{s\mu n}(kr) \quad \text{and} \quad (2.38)$$

$$\vec{p} = (r, \phi, \theta, \chi) \quad (2.39)$$

which allows to express a data set of L measurements with $L \geq J$ in matrix notation, whereby $\vec{w} \in \mathbb{C}^L$ is defined as

$$\vec{w} = \begin{pmatrix} w(\vec{p}_1) \\ w(\vec{p}_2) \\ \vdots \\ w(\vec{p}_L) \end{pmatrix} = \begin{pmatrix} v \sum_j T_j \Gamma_j(\vec{p}_1) \\ v \sum_j T_j \Gamma_j(\vec{p}_2) \\ \vdots \\ v \sum_j T_j \Gamma_j(\vec{p}_L) \end{pmatrix}, \quad (2.40)$$

noting that the indices of the summation are suppressed for the sake of clarity. However, for a full set of spherical modes with $N_{m,n}$ and taking into account the j -notation, the summation is expressed using the simplified notation from [45] as

$$\sum = \sum_{j=1}^J = \sum_{smn} = \sum_{s=1}^2 \sum_{n=1}^{N_n} \sum_{m=-n}^n \quad (2.41)$$

where the number of modes $J = 2N_n(N_n + 2)$ have been truncated according to the band-limit of the AUT. Representing the summation as a matrix-vector multi-

plication finally allows to separate T_j and Γ_j from (2.40) resulting in

$$\vec{w} = \underbrace{\begin{pmatrix} v\Gamma_1(\vec{p}_1) & v\Gamma_2(\vec{p}_1) & \dots & v\Gamma_J(\vec{p}_1) \\ v\Gamma_1(\vec{p}_2) & v\Gamma_2(\vec{p}_2) & \dots & v\Gamma_J(\vec{p}_2) \\ \vdots & \vdots & \ddots & \vdots \\ v\Gamma_1(\vec{p}_L) & v\Gamma_2(\vec{p}_L) & \dots & v\Gamma_J(\vec{p}_L) \end{pmatrix}}_{\Psi} \underbrace{\begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_J \end{pmatrix}}_{\vec{q}} \quad (2.42)$$

and, thereby, setting up the system of equations to be solved.

The matrix $\Psi \in \mathbb{C}^{L \times J}$, also referred to as the measurement matrix, contains in its columns the basis functions including the effects induced by the probe antenna. They are related to the respective sampling points given in the rows of Ψ . The vector $\vec{q} \in \mathbb{C}^J$ contains the unknown SMC denoted in general as q_j or as T_j in analogy to the derivation of the spherical transmission formula for an AUT in the transmit case presented in Section 2.2. However, the same approach for setting up the linear system of equations is also valid for modes R_j in the receiving case.

Basically, two cases should be considered for solving the system of equations from (2.42). The case $L = J$ results in a quadratic matrix Ψ and, thus, it is in principle possible to solve the problem using matrix inversion resulting in

$$\vec{q} = (\Psi)^{-1} \vec{w}, \quad (2.43)$$

but numerical instabilities cannot be excluded due to an ill-conditioned measurement matrix Ψ . Although numerical optimization approaches are available to ensure a well-conditioned measurement matrix, for example the maximum determinant sampling grid examined in Section 7, no deterministic derivation in the sense of a closed-form solution exists.

This implicitly leads to the second case, where oversampling with $L > J$ is performed to ensure a numerically sufficient condition number κ of the matrix Ψ . However, since Ψ is not in quadratic form anymore, the generalized inverse must be applied, for example in the form of the Moore-Penrose pseudoinverse. For a full row rank of Ψ , i.e. for $\text{rank}(\Psi) = J$, the simplest formulism for denoting the pseudoinverse is the representation by

$$\Psi^+ = \Psi^H (\Psi \Psi^H)^{-1} \quad (2.44)$$

where Ψ^H indicates the Hermitian transposed matrix. Although the given representation is intuitive, the computation of the pseudoinverse using a QR decomposition

$$\Psi = Q \cdot R \quad \text{with} \quad (2.45)$$

$$\Psi^+ = R^+ Q^T \quad (2.46)$$

is preferable in terms of numerical stability [71]. However, once the QR decomposition is calculated, the linear system of equations can be reformulated to

$$\vec{w} = \Psi \vec{q} = QR \vec{q} \quad (2.47)$$

$$Q^T \vec{w} = R \vec{q} \quad (2.48)$$

which can be efficiently solved since R is a triangular matrix and, thus, the solution can be determined by forward- and backward-substitutions without the necessity of a matrix inversion.

Since no assumptions are made regarding the spherical position (ϕ, θ) , radius r , or polarization angle χ in the definition of the system of equations, the direct approach is, therefore, the most general method to calculate the spherical wave spectrum. Although the approach was already proposed in the first publications on the SWE [69, 72], the high computational complexity of $\mathcal{O}((kr)^6)$ caused that this approach has gained importance only due to nowadays available computational capabilities.

An alternative for reducing the computational complexity of the direct method is presented as the FFT/matrix approach in [73]. For this purpose, similar to the Wacker algorithm, first the term $e^{im\phi}$ of the spherical transmission formula given in (2.17) is interpreted as a Fourier series and, therefore, computed using an FFT. Based on the divide-and-conquer principle, $2m + 1$ smaller linear systems of equations are solved in the next step, which allows to reduce the overall complexity of the problem. By using the FFT along the mode dimension m , which represents the variations in ϕ as described in Section 2.1, a constant step size $\Delta\phi$ in this dimension is required. Nevertheless, non-uniform sampling point distributions in θ and r are possible, but due to the FFT in ϕ it is required that these are constant for each ϕ -ring. A popular use case for this solution algorithm is, for example, the thinned equiangular sampling or also called igloo sampling, whereby different radii for the respective ϕ -rings would additionally be possible. Due to the FFT and the partitioning into $2m + 1$ linear systems of equations, the computational complexity is reduced to $\mathcal{O}((kr)^4)$, making this approach an attractive possibility for electrically larger problems as long as the sampling grid requirements are fulfilled.

2.3.3 Iterative Methods

One of the dominant factors of the computational complexity of the direct methods is the size of the system of equations to be solved or, respectively, the number of unknown SMC J to be determined. Thus, the applicability of direct methods depends significantly on the dimensions of the measurement matrix Ψ and the available memory. If the constraints do not allow a direct approach, iterative methods are an alternative to obtain a solution.

Especially for sparse matrices, algorithms exist which can determine a solution in linear runtime and are, therefore, of particular interest. Although the measurement matrix Ψ is not sparse, it represents an orthogonal basis, since the spherical wave functions as the underlying basis are orthogonal on the sphere. Thus, for appropriate sampling grids, the sampling of these basis functions represented in Ψ is also orthogonal, or, in other words, its product is approximately sparse ($\Psi^H \Psi \approx I$). Thus, the system of equations given in (2.42) can be reformulated to

$$\Psi^H \vec{w} = \Psi^H \Psi \vec{q} \approx \vec{q} \quad (2.49)$$

which facilitates the application of iterative approaches. However, additional uncertainties arise regarding the numerical stability, both with respect to the condition of the problem, which is squared due to the matrix multiplication, and the convergence

behavior of the applied solver method. Within the large number of iterative solution methods, the most commonly used in the given context include the conjugate gradient (CG) method, the biconjugate gradient (BiCG) solver, the generalized minimal residual (GMRES) approach and the biconjugate gradient stabilized (BiCGSTAB) algorithm.

The CG method is widely used for solving partial differential equations or optimization problems similar to that stated in (2.49). It was independently proposed in separate elaborations [74, 75] and is particularly useful for solving symmetric, positive definite systems, where the matrix is symmetric and all eigenvalues are positive. Preconditioning of the problem may be implemented to improve its convergence rate. The CG method generates iterates that are orthogonal with respect to the (preconditioned) inner product, which reduces the effects of rounding errors and ensures that the iterates converge to the exact solution in a stable manner. The CG method is generally stable with a computational complexity of $\mathcal{O}(n^2)$, where n is the size of the linear system of equations.

The BiCG method is another iterative method also proposed in [74]. It is designed to solve non-symmetric linear systems, where the matrix is not symmetric and/or the eigenvalues are not all positive. The BiCG method generates two sequences of iterates that are not necessarily orthogonal with respect to any inner product. The numerical stability of the BiCG method can be less reliable than the CG method for some problems, particularly for ill-conditioned matrices or slowly converging problems. However, the method is nevertheless useful because, in contrast to CG, it is not restricted to symmetric and positive definite systems. With a computational complexity of $\mathcal{O}(n^2)$, the method is comparably efficient.

The GMRES method is a popular iterative method for solving non-symmetric linear systems and was proposed in [76]. It generates a sequence of iterates that minimizes the residual in a Krylov subspace. The method is particularly useful for solving large, sparse linear systems with arbitrary eigenvalues. However, the numerical stability of the GMRES method is affected by the same problems as BiCG, resulting in less reliable solutions compared to CG. Its computational complexity is $\mathcal{O}(n^3)$.

BiCGSTAB introduced in [77] is a more recent iterative method that is commonly used to solve non-symmetric linear systems and is an improvement of the BiCG method. The BiCGSTAB method generates two sequences of iterates, similar to the BiCG method, but it includes an additional stabilization step to improve convergence. This stabilization step improves the numerical stability of the BiCGSTAB method, making it for the same computational complexity of $\mathcal{O}(n^2)$ more reliable than the BiCG method.

Another method is described in [78, 79], where the spherical translation operator $C_{\sigma\mu\nu}^{sn(c)}(kr)$ is replaced by a plane wave translation operator. Furthermore, by clustering sampling points and applying a multilevel fast multipole (MLFM) expansion, the computational complexity can be reduced to $\mathcal{O}((kr)^2 \log(kr))$. This approach is very efficient and, thus, suitable in particular for large electromagnetic problems. However, since a spherical wave spectrum can no longer be determined, this method is not included in the following.

If the matrix is symmetric and positive definite, the CG method is usually the best choice. If the matrix is non-symmetric and has arbitrary eigenvalues, the GMRES

method is often preferred. If the matrix is non-symmetric and has a small number of negative eigenvalues, the BiCGSTAB method can be a good choice due to its improved numerical stability over the BiCG method. The efficient approach based on the MLFM expansion is particularly suitable for very large electromagnetic problems and is an alternative if the spherical wave spectrum is not required. However, it is important to note that these comparisons are generalizations, and the optimal choice of method can vary depending on the electrical size, the applied sampling grid or whether a spherical wave spectrum is required.

2.4 Probe Correction

In the case of SNF measurements, the actual measurement data are affected by the probe used, i.e. they do not correspond to the actual present electric fields. To correct the acquired data for the effect of the probe, a technique known as probe correction (PC) is, therefore, required. The working principle of the PC technique involves modeling the probe as a separate entity that interacts with the AUT's electric field and modifies it in a known way. The most significant effect is the receiving or transmitting characteristic of the probe, i.e. the pattern. As a result, by using a probe it is not possible to measure the electric fields present at the exact point, but the measured value is weighted by the aperture of the probe and is, thus, spatially filtered. Especially for PNF measurements, the probe antenna must therefore be taken into account during the transformation into the far field, since the aspect angle of the probe antenna relative to the AUT differs at each sampling point. For CNF measurements, the aspect angle remains constant at least for each measured cylindrical ring, and for SNF measurements even for the whole measurement, allowing a more efficient implementation. The correction process involves subtracting the probe contribution from the measured near-field data, which effectively removes the probe effects. For this purpose, the probe is described analogously to the unknown AUT using the SMC $R_{\sigma\mu\nu}^p$ which are considered in the spherical transmission formula (2.17) using the probe response constants described in (2.18).

In general, probes are divided in two classes, namely first-order probes (FOP) and higher-order probes (HOP). For FOP, the assumption is made that the probe's spherical mode spectrum $R_{\sigma\mu\nu}^p$ contains only modes with indices $\mu = \pm 1$, thus allowing the simplification in the Wacker algorithm stated in (2.31) and (2.32). However, in the sense of the definition, mathematically exact $\mu = \pm 1$ probes do not exist, but electrically small and symmetric probes are a good approximation. The higher the symmetry in azimuth of the probe, the fewer oscillations in the azimuthal pattern occur and, therefore, fewer modes for larger μ are contained in the spectrum $R_{\sigma\mu\nu}^p$. Thus, circular horn antennas or open-ended waveguides are a good approximation for FOP.

Mechanically more complex or larger antennas, e.g. broadband probes, usually contain spherical modes for larger values of μ and are, consequently, referred to as HOP. Analogous to the band-limitation of the AUT's SMC in relation to its electrical size as explained in (2.5), the same applies to the probe antenna. Here, the maximum order and the maximum degree are referred to in the following as N_μ and N_ν , respectively. However, the actual effect of the higher-order modes depends significantly

on the measurement distance r . Since the translation operator $C_{\sigma\mu\nu}^{sn(c)}(kr)$ applied in (2.18) is interpreted as a low-pass filter for higher-order modes in μ , those modes are increasingly attenuated with increasing measurement distance. Accordingly, deviations caused by neglecting higher-order modes can be quantified by evaluating the translated mode spectrum $C_{\sigma\mu\nu}^{sn(c)}(kr)R_{\sigma\mu\nu}^p$ for the actually applied measurement distance r . Moreover, due to this relation, all antennas are regarded as FOP in the far field ($r \rightarrow \infty$), since a planar wavefront is received and, thus, only $\mu = \pm 1$ modes are excited.

In this case, the measured probe signal contains only two modes and can, thus, be represented as already explained in (2.33) leading to the symmetry given in (2.34). Therefore, only two polarization angles, for example $\chi = 0^\circ$ and $\chi = 90^\circ$, are required to calculate the probe signal for arbitrary polarization angles χ , which allows to avoid the χ -integration in the Wacker algorithm. In turn, for the general case, i.e. for HOP at finite measurement distance $r < \infty$, $2\mu + 1$ polarization angles have to be measured. Thus, the Wacker algorithm can be used for HOP correction, but loses its practical applicability due to the large number of polarization angles which must be acquired during a measurement. Nevertheless, the direct approach explained in Section 2.3.2 as well as the iterative method described in Section 2.3.3 bypass this shortcoming and, therefore, represent the more efficient alternative for HOP correction from a practical point of view. Furthermore, it is possible to compensate the effect of the probe antenna, also including HOP, at each measurement point individually, denoted in the following as PPC. It enables the correction of misalignments of the probe antenna at the respective sampling points, if known, or enables the support of multi-probe systems. Such systems may consist of the special case of a single dual-polarized probe antenna [80, 81] or a large number of separately installed probe antennas, as presented for example in [82–86]. Further information on the PPC in the context of robot-based measurement systems is provided in Section 5.

2.5 Summary

In this section, the fundamentals of spherical near-field antenna measurement technology relevant within the scope of this work are outlined based on the extensive research presented in [45, 68, 69, 72, 81]. It is based on the SWE to describe an electric field radiated by a source volume, which is modeled by a superposition of spherical waves (also denoted as spherical modes). Within this superposition, the spherical modes are weighted by complex coefficients, which are referred to as SMC. Consequently, an electric field radiated by an AUT can be modeled by a set of SMC for any distance $r > r_0$ and, thus, also in the far field.

The determination of the SMC is based on the spherical transmission formula (2.17), which establishes a relationship in the mode spectrum between the unknown AUT and a probe antenna assumed to be known. For this purpose, the probe antenna described by its SMC is represented using rotation and translation operators for the respective measurement position in the coordinate system of the AUT. For this purpose, the probe antenna is first rotated around its z -axis to the respective ϕ -coordinate and then around its rotated y' -axis to the required θ -position. Subsequently, it is rotated by the polarization angle χ along the twice rotated z'' -axis

and, thereafter, radially translated by the measurement distance r . Taking into account the radially outwards directed orientation of the probe antenna, the spherical transmission formula allows to set the measurement data acquired with the probe antenna in relation to the unknown SMC of the AUT. Thus, it represents the basis of spherical near-field antenna measurements.

Afterwards, several potential solution methods are discussed, which enable the calculation of the SMC based on the acquired measurement data. The algorithm presented by Wacker [64,65] is still the most efficient way of calculating the SMC using Fourier transforms, if the constraints for the application of this method are met. This includes the restriction to regular equiangular sampling grids, which are required due to the Fourier processing, but also the restriction to FOP. Thereafter, more generalistic approaches, denoted as the direct or iterative method, are presented, which are increasingly moving into focus with the computing capacities available today and, in addition to HOP correction, also support completely irregular sampling grids.

Therefore, all capabilities required for flexible robot-based antenna measurements are provided, for example to compensate for non-ideal measurement positions or deviations in the orientation of the probe antenna, which will be discussed in more detail in Section 5.

Robot-Based Measurement Setup

Based on the introduction in Section 1 on robot-based measurement systems and the various underlying concepts, the measurement setup developed within this dissertation is explained in detail in the following. For this purpose, the requirements for the robot arm regarding the working range and the absolute accuracy are defined. Subsequently, the mechanical setup of the measurement chamber, the RF design, and the control of the overall system are explained. This includes controlling the robot based on its kinematic model, but also the whole measurement system in terms of the interaction between all components used. In addition, the safety concept based on a light fence is explained, as well as commissioning of the system through a multi-stage release procedure to ensure a safe operation.

3.1 Robot Requirements

The robot-based measurement system has been planned to supplement the hybrid CATR already available at the IHF. In this system, it is possible to perform antenna measurements in the near and far field as well as radar cross-section (RCS) measurements. Nevertheless, the last frequency extension to the millimeter-wave range (E-band: 60 GHz to 90 GHz) already confirmed a limited measurement dynamic due to the large mechanical dimensions. In addition, the control of the system for research on irregular sampling grids is also tedious and in many cases only possible in a stepwise scheme via complex correction tables which modify the behavior of each mechanical axis separately. This motivated the construction of a new highly flexible robot-based measurement system to continue the research on all above mentioned measurement techniques of the CATR, but for frequencies beyond 60 GHz. For this purpose, a compact and lightweight receive (Rx) module including the probe antenna is mounted on the robot arm for antenna measurements. For RCS measurements, however, the robot's payload support mounting the transmit and receive (TxRx) module, optionally supplemented by the Rx module to provide a second receive channel, e.g. for polarimetric radar imaging techniques [87]. In addition, in order to take full advantage of the flexibility of the robot arm, a measurement software has been developed in-house to exploit all possibilities provided by the flexible measurement system.

3.1.1 Absolute Accuracy

Since general-purpose positioners as robot arms typically provide a worse absolute accuracy than dedicated positioning systems such as roll-over-azimuth positioners,

the impact of positioning errors on the NFFFT based on the SWE has been investigated in [88] for an upper frequency bound of 300 GHz. A similar approach is applied here for the first planned frequency range of 60 GHz to 90 GHz to assess the results due to positioning errors of robotic arms compared to roll-over-azimuth positioners. For this purpose, a stochastic analysis is performed for N_d characteristic measurement distances, where an amount of N_a AUTs is considered for each measurement distance r_i , separately. To account for different AUTs, these are synthesized from a random set of transmitting modes $Q_{smn}^{(3)}$ (also referred to as q in vectorized form) used to retrieve near-field data E_{NF} for each investigated measurement distance r_i . Hereby, the near-field samples are calculated for N_s equiangular sampling grids denoted as G_i , of which each is subject to a random perturbation with $|\epsilon_i| \leq \epsilon$. Subsequently, the non-ideal samples are used to calculate the disturbed mode set q' as well as the resulting far field E'_{FF} , whereby probe pointing errors caused by the positioning uncertainty are considered using a PPC as explained in more detail in Section 5. Therefore, the impact of probe positioning errors on the NFFFT is assessed independently of the used AUT for different measurement radii. Finally, this process is repeated for both, robot-based and roll-over-azimuth positioners. Here it is assumed that the effects of a Cartesian positioning uncertainty of a robot ϵ_k decrease with an increasing measurement distance r_i , whereas the angular errors of a roll-over-azimuth positioner ϵ_a are independent of r_i . The flowchart shown in Fig. 3.1 illustrates the principle of operation of the stochastic analyses performed with the parameters given in Table 3.1.

Table 3.1: Parameter set of the simulations on the effects of probe positioning errors.

Parameter	Quantity	Value
Frequency	f	90 GHz
Number of measurement distances	N_d	251
Band-limit of AUT	$N_{m,n}$	34
AUT amount	N_a	5
AUT minimum sphere	r_0	≈ 20 mm
Measurement distances	r_i	50 mm ... 300 mm
Number of sampling grid distortions	N_s	24
Error robot	ϵ_k	0.04 mm, 0.50 mm, 1.50 mm
Error roll-over-azimuth positioner	ϵ_a	0.1°

Typical positioning uncertainties derived from data sheets are used to estimate the effect of incorrect probe positions in the NFFFT. In the case of spherical roll-over-azimuth positioners, a worst-case angular positioning uncertainty of $\epsilon_a = 0.1^\circ$ is considered. Robot arms typically operate in Cartesian coordinate systems and can achieve a very high position repeatability of $\epsilon_{k,1} = 0.04$ mm for manually taught positions. However, since this is a very time-consuming and, thus, impractical process for freely configurable sampling grids depending on the AUT, the absolute accuracy is additionally taken into account. Since this metric may differ greatly depending on the manufacturer and the robot model used, two realistic values are considered for this purpose. For commercially available robots without absolute calibration, an upper bound of $\epsilon_{k,3} = 1.5$ mm is assumed. If the robot is calibrated ex works with a

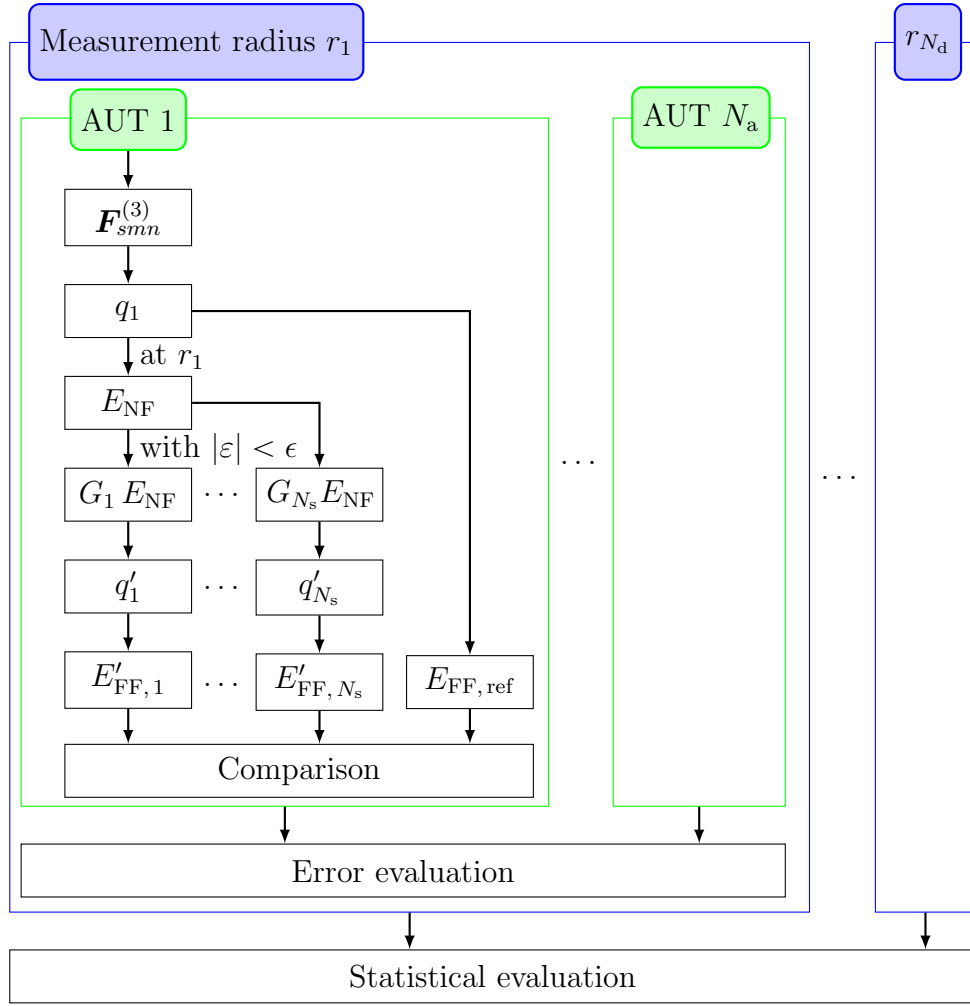


Figure 3.1: Flowchart of the stochastic approach to estimate the effects of probe positioning errors in the NFFFT. The procedure is repeated for Cartesian positioning errors of robot-based measurement systems as well as for angular errors of roll-over-azimuth positioners.

mathematical-physical error model to compensate for mechanical uncertainties such as axis misalignments or axis deflections, an absolute accuracy of $\epsilon_{k,2} = 0.5 \text{ mm}$ can be achieved. The error between the reconstructed radiation pattern using the disturbed and ideal sampling positions is evaluated using the equivalent error signal (EES), which is a commonly used metric in antenna measurement technology to evaluate effects on radiation patterns. It is defined as

$$\text{EES} = 20 \log \left(\left| \left| \frac{f_1(\theta, \phi)}{\max_{\theta, \phi} f_1(\theta, \phi)} \right| - \left| \frac{f_2(\theta, \phi)}{\max_{\theta, \phi} f_2(\theta, \phi)} \right| \right| \right), \quad (3.1)$$

where f_1 and f_2 denote a radiation pattern, respectively. The EES is, unless otherwise specified, used as the error metric throughout this thesis. For all distortions of the sampling grids N_s , the maximum EES including the lower and upper bound for all experiments is given for each positioning system and measurement distance in Fig. 3.2. Since random and independent disturbances are modeled for each in-

dividual sampling point in this numerical analysis, the approach only provides an estimate of the maximum error. Typically, the uncertainties are not completely subject to a random distribution, but exhibit improved repeatability for repetitive trajectories. Nevertheless, the method is suitable for a worst-case estimation, which applies to both, spherical and Cartesian positioning systems and is, thus, ensuring comparability.

The simulation results demonstrate that the reconstruction error for Cartesian positioning systems depends on the measurement radius r_i and decreases for increasing measurement distances. Furthermore, the variability of the occurring deviations depends on the applied positioning uncertainty ϵ_k . In contrast, the reconstruction error for the angular errors of a roll-over-azimuth positioner is independent of the measurement distance.

Based on the aforementioned assumptions, robots employing a factory calibration for increased absolute accuracy with a positioning uncertainty of $\epsilon_{k,2} = 0.5 \text{ mm}$ can achieve for measurement distances of $r_i > 200 \text{ mm}$ a worst-case error in the same order of magnitude as for spherical positioners with $\epsilon_a = 0.1^\circ$. Consequently, this absolute accuracy is defined as a requirement for the robot arm to be used.

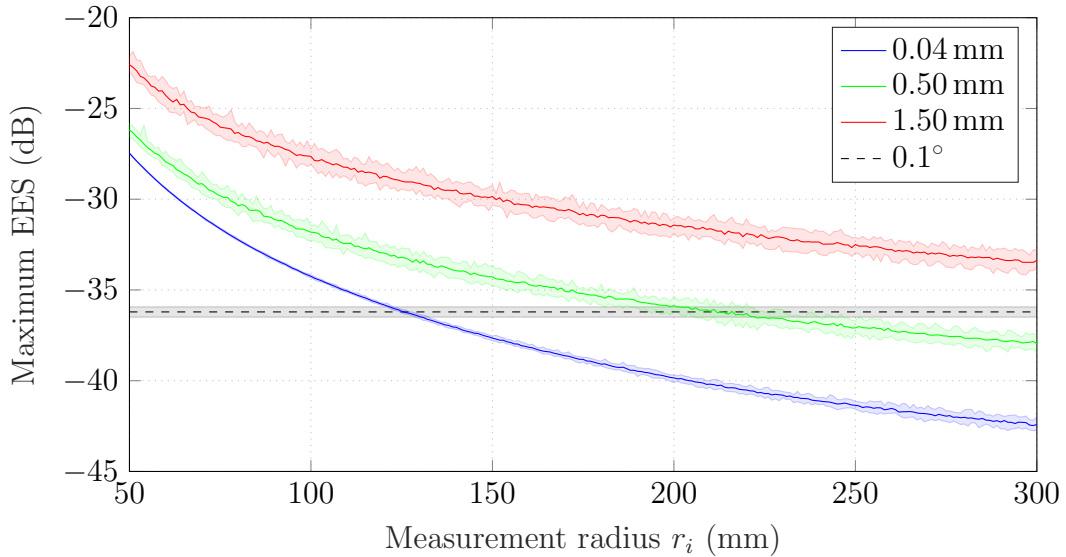


Figure 3.2: Simulative comparison of the maximum error for roll-over-azimuth and robot-based positioners for a frequency of $f = 90 \text{ GHz}$. The deviations caused by angular errors are independent of the measurement radius, whereas deviations caused by Cartesian positioning uncertainties decrease for increasing measurement distances.

3.1.2 Working Range and Payload

With the exception of heavy-duty robot arms, the absolute accuracy is often specified the same for different sized models within the same product series. Therefore, although the measurement radius for SNF measurements in the millimeter-wave range is only a few centimeters, a larger robot model can often be preferred without compromising performance. This allows to reduce the effects of the positioning

errors (see Fig. 3.2), provides additional safety margins with regard to the feasible robot trajectories, but also permits to simplify the setup of the structure mounted on the robot. For example, the probe antenna typically attached to the robot's tool center point (TCP) can be mounted in radial direction, which negatively affects the largest possible measurement radius, but in return facilitates the coordinate transformations. Furthermore, if the robot is also planned for other measurement projects, for example the RCS measurements already mentioned, the setup on the TCP also becomes larger and heavier. While a lightweight and compact receive assembly is normally mounted on the TCP for antenna measurements, the continuation of the three-dimensional tomographic radar imaging research [89, 90] requires mounting a heavier and larger transmit module. Furthermore, since RCS measurements are performed in the far field, the larger distance also requires a larger range of motion of the robot to allow for different angles of incidence on the device under test (DUT). In order to exploit the flexibility of the robot in equally versatile measurement projects, a robot with a working range of 2010 mm and a nominal payload of 8 kg has been selected. Further parameters of the robot are given in Table A.1 and [91]. Moreover, two additional axes, described in Section 3.2, are used. These allow to position the AUT or DUT in the optimum working area of the robot, provide redundancy in terms of the choice of axes for performing measurements, and can reduce any protrusion of the robot arm into the measurement volume.

3.2 Mechanical Design

The measurement system is installed in a chamber covered with VHP-2 absorbers, which provide a typical reflectivity of -55 dB for normal incidence [92]. In the area of the chamber the floor covering has been removed, the concrete floor underneath has been milled smooth and a new floor with increased evenness has been installed according to DIN 18202 [93]. In addition, a sealant of two-component material has been applied to increase the durability of the ground. This reduces possible elasticities of the floor covering so that it cannot result in a, possibly dynamic, positioning error of the robot. In addition, all mechanical components are located on a common solid steel plate. It serves for load distribution and mechanically links all relevant actuators. A picture directly after the integration of the components as well as after completion of the measurement chamber is shown in Fig. 3.3.

3.2.1 Actuators

The main component of the measurement system is the industrial six-axis robot arm with a nominal payload of 8 kg, a maximum working range of 2010 mm and a repeatability of 0.04 mm, as further described in Section 3.1 and Table A.1. The robot has been calibrated ex works using its nominal payload and, thus, has an increased absolute accuracy since axis misalignments or arm deflections are compensated. In addition, the robot has a workspace monitoring system for protection against collisions due to the limited installation space.

Furthermore, a linear axis with an attached rotation axis is used for positioning of the AUT in the robot's optimum working area. It also allows to adjust the distance between the TxRx modules and the DUT in the case of radar measurements. A



(a) Actuators directly after integration on common ground plate and renewed floor. (b) Operational measurement system including setups for AUT and probe.

Figure 3.3: Comparison of the measurement chamber directly after integration of the actuators and in the final state.

double-guided precision linear axis with a repeatability of 0.03 mm is used. It is driven by a ball screw drive with a pitch accuracy of $52 \mu\text{m}/300 \text{ mm}$. To reduce deflection and for precise alignment of the axis, it is supported (apart from being fixed to the common steel plate) on four support structures over its entire length of 3.5 m. This allows to reduce the deflection even at maximum load of more than 100 kg. Further details are provided in [94] and in Table A.2.

Moreover, an additional rotation axis is mounted on the slide of the linear unit. The employed motor and gear ratio of 48:1 provide in combination with a preloaded worm gear low backlash, according to the specifications, to less than 0.1° . Furthermore, the stainless steel turntable is made of a 60 mm hollow shaft mounted on a cross roller bearing, which allows to feed through the required RF, ethernet and power cables. Further details about the rotation axis are provided in [95] or summarized in Table A.3.

A single controller is used which compromises the robot arm and both additional axes in a common kinematics. This enables not only to move all axes synchronously to each other, but also to generate the trigger signals required for performing measurements depending on the movement of the robot or the additional axes.

3.2.2 AUT Setup

The setup placed on the additional rotation axis is intended to mount the AUT or DUT as shown in Fig. 3.4a. It consists of a hollow shaft rotary joint for Gigabit Ethernet and power supply. An additional two-channel RF rotary joint, which enables the operation of the TxRx module, is stacked in its hollow shaft. Since only two RF channels are available, both, a high-frequency and an intermediate-frequency signal are superimposed on each channel. The required signal conditioning by diplexers and subsequent amplifiers is performed directly after the rotary joint. This finally allows the operation of the TxRx module, on whose waveguide interface the AUT is directly mounted and normally operated in transmit mode. However, range reversal can be accomplished by swapping the modules for probe and AUT manually.

The column-like structure above the AUT has mechanical high-precision reference markings for aligning the robot or to mount a DUT during the measurement process. Due to the integrated Gigabit network connection, the measurement setup is designed to be future-proof and can, for example, be used to operate stand-alone sensors like radars as shown in Fig. 3.4b. In addition, further information, e.g. on the temperature of the amplifiers or acceleration feedback of a gyroscope may be integrated into the measurement process via the network interface.

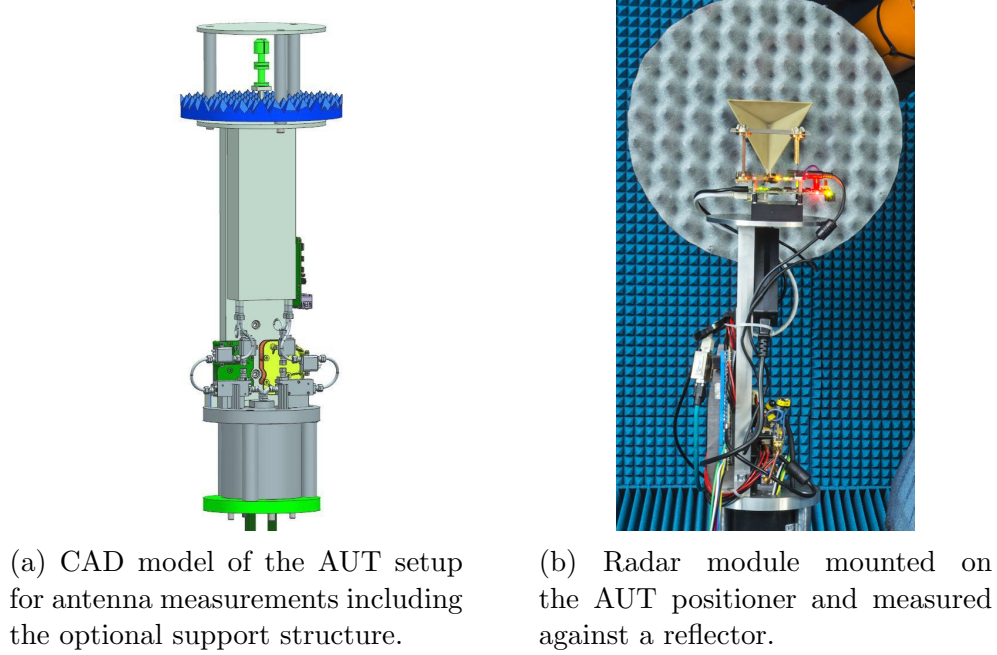


Figure 3.4: CAD model and image of the structure on the additional rotation axis to perform antenna, radar or any other measurements.

3.2.3 Probe Setup

The probe antenna is mounted on the robot's TCP and, thus, poses a particular challenge in terms of cable connections, load data as well as interference contours. The CAD model and a picture of the construction is given in Fig. 3.5. To minimize the setup on the robot in terms of weight and dimensions, the probe is operated with a high-sensitivity Rx-only module. The RF, network and power cables are strain-relieved directly at the end of the robot manufacturer's official cable management to prevent dynamics from causing damage and, thus, increasing durability. Due to the compact design, mini-bend cables are used to connect the Rx module with the RF cables ending directly after the cable management (see Fig. 3.5).

In addition, three different supply voltages, a Gigabit network connection and a USB 3.0 link, normally used for optical measurements, are provided at the TCP. Therefore, this part of the measurement system is also future-proof designed, for example to operate active sensors like radar modules.

The structure shown has a weight of only 475 g and is milled from the solid for precision and stability. However, a weight adapter can be used so that the robot can be operated with its nominal load, for which the factory calibration has been

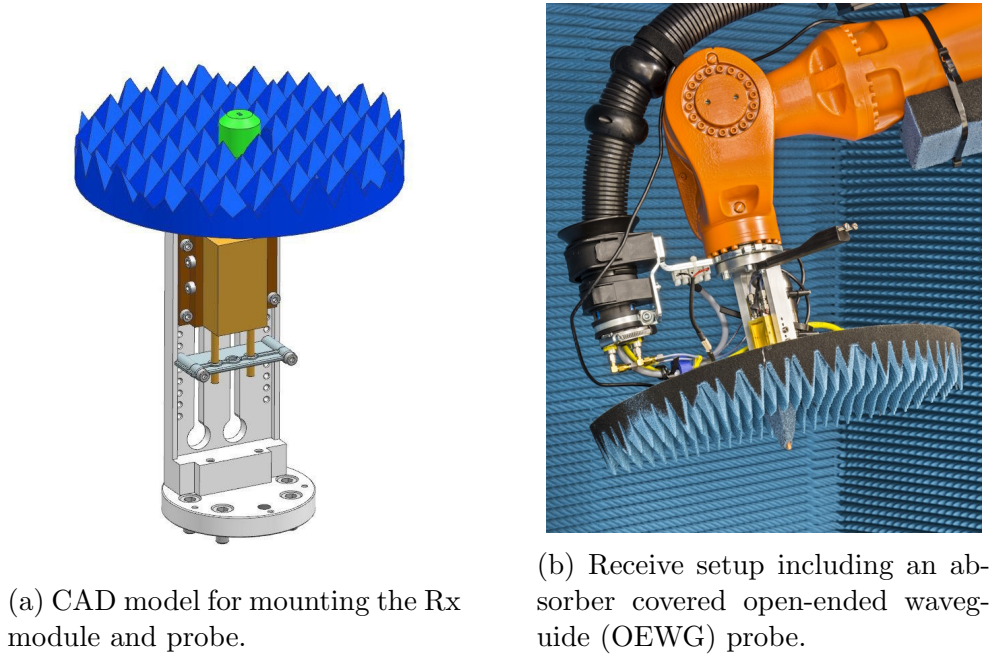


Figure 3.5: CAD model and image of the setup on the robot for antenna measurements. The vertically protruding black 3D print in Fig. 3.5b is required for reference alignment within the context of safe operation.

performed. Furthermore, the weight of each individual component of the construction, down to single screws or plastic parts, has been determined exactly with a laboratory balance and fed into the CAD model. Thus, the load data to operate the robot precisely, e.g. the total mass, the center of gravity, the moments of inertia and the main axes of inertia are known and programmed to the robot controller (RC).

3.3 RF design

The core components for performing antenna measurements are the vector network analyzer (VNA) ZVA67 [96] and the frequency extension modules (FEM) detailed in [97]. The TxRx module (compare Fig. 3.6) requires a local oscillator (LO) and RF signal for operation, the latter being up-multiplied by a factor $N_{\text{FEM}} = 6$ and transmitted via the waveguide interface, hereafter referred to as RF_{Tx} . The RF_{Tx} path has two differently oriented directional couplers, via which both, the transmitted and reflected signal in the RF_{Tx} path are decoupled and mixed down using the multiplied LO signal, resulting in an intermediate frequency (IF) signal. Here, the downmixed transmitted component is referred to as IF_{Ref} and the downmixed reflected component is referred to as IF_{M1} . Analogously, the signal received by the Rx module is also mixed down with the same multiplied LO signal and is referred to as IF_{M2} in the following. The LO and RF signals are generated using a frequency offset of $f_{\text{IF}}/N_{\text{FEM}}$ with $f_{\text{IF}} = 279 \text{ MHz}$, which meets the manufacturer's specifications. Due to this frequency offset it is ensured that the measurement signals IF_{Ref} , IF_{M1} and IF_{M2} are always operated at f_{IF} , independent of the actual frequency in the RF_{Tx} path.

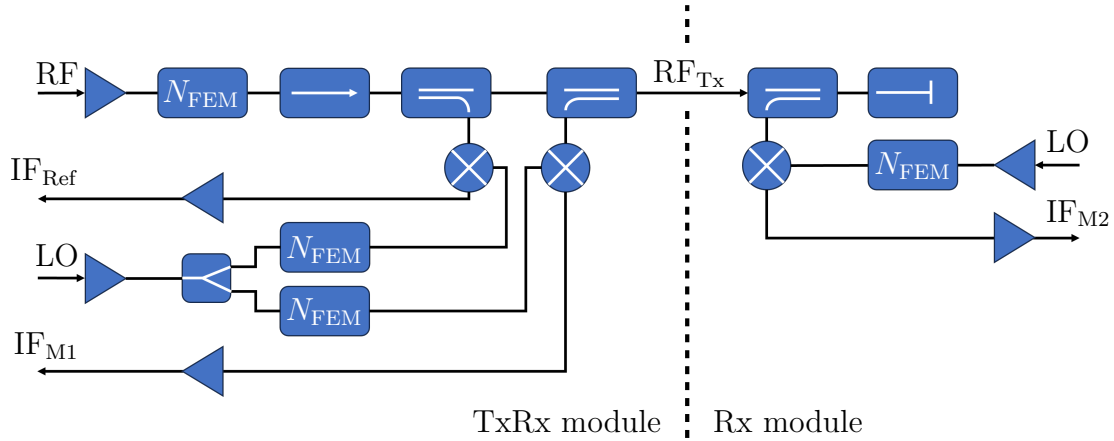


Figure 3.6: Simplified block diagram of the FEM including an indicated transmission path RF_{Tx} , based on [97].

The manufacturer of the FEM states that the multipliers are chosen to keep the LO and RF frequencies lower than 18 GHz, even for higher frequency ranges. To ensure the modular and flexible expansion of the frequency range of the measurement system, this frequency has been, therefore, taken into account as the upper limit throughout the whole RF design, e.g. for cables, connectors, amplifiers or the RF rotary joint. In contrast, components that are exclusively in the IF path have been optimized for f_{IF} only.

To decouple the AUT setup including the measurement equipment from the movement of the rotation axis, a rotary joint is used. It is build into the hollow shaft of the outer rotary joint used for providing power supply and network connectivity, as described in more detail in Section 3.2.2. Since the IF paths operate at a constant frequency and the RF and LO frequencies are only separated by a frequency offset of f_{IF}/N_{FEM} , they cannot be effectively separated, e.g. using diplexers. Therefore, a two-channel rotary joint as specified in [98] is used. Due to the design, the second, eccentric channel exhibits significantly less stable transmission characteristics for a whole revolution. This is shown in Table A.4, which highlights the differences between the two channels in addition to the most relevant parameters concerning antenna measurements. For example, the maximum phase variation for the highest specified frequency in the outer channel amounts up to 25° , whereas the inner channel is specified with a phase variation of 1° only. For this reason, it is essential to route the critical measurement signals through the inner channel. For phase-stable measurements, this is the LO channel, which is used in the TxRx module for downconversion providing the reference channel IF_{Ref} , as well as in the Rx module for downmixing the measurement signal resulting in IF_{M2} . Accordingly, the reference channel IF_{Ref} itself is superimposed with the LO signal on the inner channel, since IF_{M2} is evaluated in relation to it.

In contrast, phase variations in the RF path are less critical because these variations affect both, the IF_{Ref} and the IF_{M1} signals of the TxRx module, as shown in Fig. 3.6. Since wave quantities are usually evaluated relatively to each other, these deviations cancel out. The same applies to the Rx module, where phase variations in the RF

path affect the IF_{M2} signal (via RF_{Tx}) and are, thus, also compensated due to the relative comparison with IF_{Ref} .

Considering the rotary joint as a critical component through which relevant signals are routed, measurements have been conducted in [99] to verify the data sheet [98]. As the phase is most affected by uncertainties, the results for the phase deviations $\Delta\varphi$ and the reproducibility $R_{\Delta\varphi}$ thereof are presented in the following.

For this purpose, the measured phase for one complete revolution is shown in Fig. 3.7 for f_{IF} as well as the lower and upper frequency limits of the LO and RF frequencies. The phase deviation $\Delta\varphi$ is defined as the absolute value of the measured phase compared to a reference value given at a rotation axis position of -180° . As expected, only minor deviations are identified in the IF frequency range, amounting up to 0.01° and 0.14° for the inner and outer channel, respectively. Even in the higher frequency range, the inner channel proves a stable transmission with a maximum deviation of only 0.46° . However, the second eccentric channel performs worse with deviations of 2.47° at 10 GHz up to 17.17° at the highest operating frequency of 18 GHz.

Since the reproducibility $R_{\Delta\varphi}$ is not specified in the data sheet, it has been determined, too, and is analogously defined as the absolute value of the deviations between two consecutive runs. As a high reproducibility has been observed, the results are plotted scaled as $30 R_{\Delta\varphi}$ to improve clarity. Considering this scaling, the reproducibility of the occurring phase deviations is 0.22° for the first and 0.32° for the second channel. Although all measurement results prove better characteristics than specified in the data sheet (channel 1: 0.46° compared to 1° ; channel 2: 17.16° compared to 25°), the results demonstrate that it is crucial to feed the critical LO

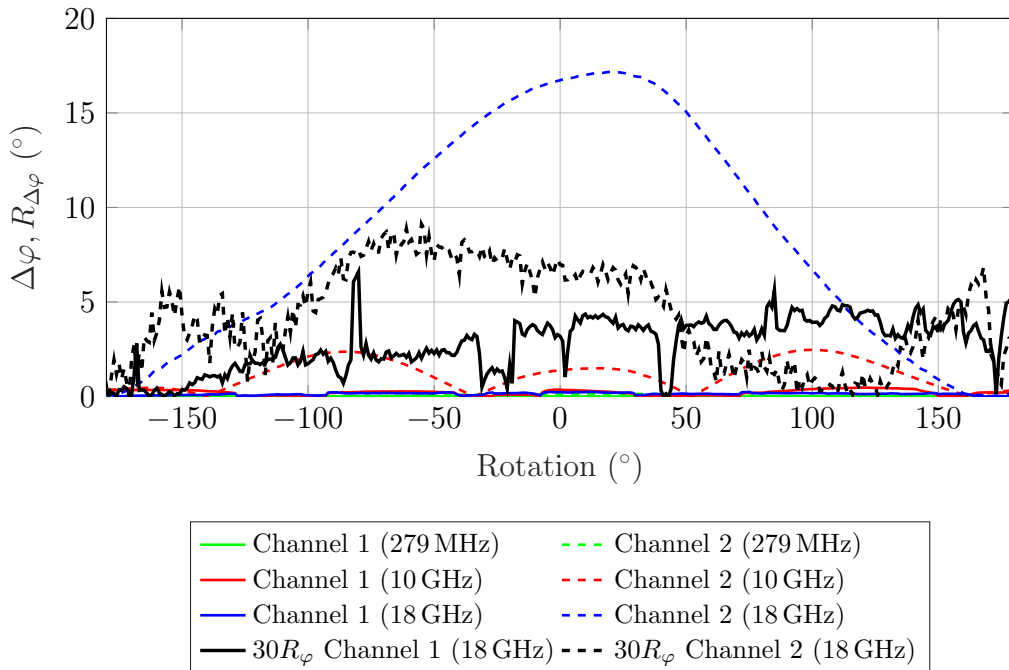


Figure 3.7: Measured phase deviation $\Delta\varphi$ and repeatability $R_{\Delta\varphi}$ for a revolution of the rotary joint specified in Table A.4 and [98]. The measurements have been performed for $f = f_{\text{IF}} = 279$ MHz, as well as for $f = 10$ GHz and $f = 18$ GHz, which represents the lower and upper design frequency.

signal via the inner channel of the rotary joint. The high reproducibility of the measurement series also provides the possibility to compensate for the occurring deviations, especially of the outer channel, using a one-time calibration. However, since the RF path is routed through the outer channel and phase deviations in this path are reflected in all IF channels, especially IF_{Ref} and IF_{M2} , they inherently compensate each other due to the relative evaluation of those wave quantities. Therefore, this calibration is not exploited so far. In addition, the analogous evaluation of the amplitude deviations ΔA and their reproducibility $R_{\Delta A}$ is given in Fig. A.1 for the sake of completeness.

In summary, this results in the RF design shown in Fig. 3.8 for performing antenna measurements: Starting from the VNA, the individual RF paths are led to the LO/IF distribution unit. Here, LO and IF_{Ref} as well as RF and IF_{M1} are superimposed using diplexers and routed to the TxRx module via two separate RF cables. The former signal pair is routed via the centric channel (CH1) of the rotary joint, while the latter uses the outer eccentric channel (CH2). Directly after the rotary joint, the signals are decoupled again using diplexers and amplified. Since the FEM require a constant signal power of $10 \text{ dBm} \pm 3 \text{ dB}$, a VNA-based power calibration is performed before connecting the FEM. This provides the desired signal level over the entire frequency range as well as independent of any cable lengths and, thus, guarantees optimal operation of the FEM without the use of attenuators or equalizers. On the robot side, however, two separate cables are used for LO and IF_{M2} in order to operate the Rx module because, on the one hand, no restrictions with regard to any rotary joint apply and, on the other hand, the compact mechanical dimensions of the probe setup take priority. Therefore, additional components such as diplexers are avoided in this case.

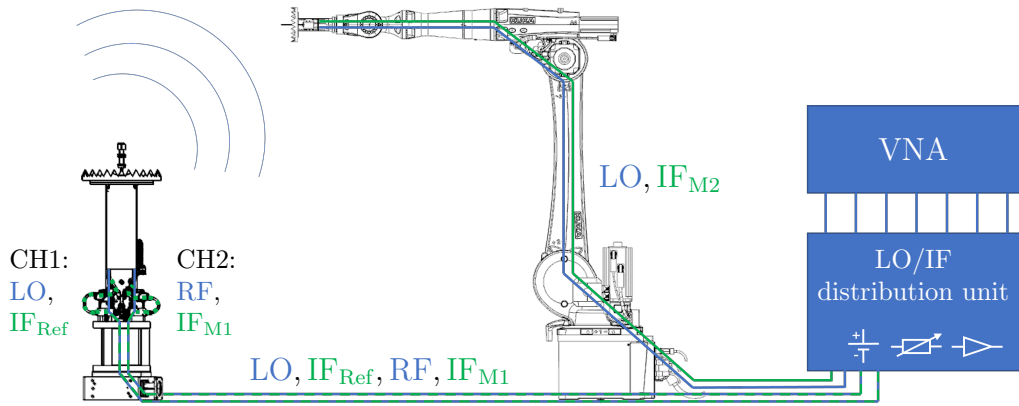


Figure 3.8: Schematic representation of the RF design for performing antenna measurements. Superimposed signals are highlighted using different colors representing the signal's frequency. The LO/IF distribution unit is shown in detail in Fig. 3.9.

In addition to the superposition of the signals required for antenna measurements, further features of the LO/IF distribution unit, which is schematically presented in Fig. 3.9, are explained in the following. The layout of the connectors on the VNA

side allows to connect the VNA using minimum cable lengths and without any cable crossings. Furthermore, the power supply unit (PSU) for all amplifiers and FEM is provided and can be switched via relays. Depending on the requirements, three different operating voltages are currently possible. The variable programmable attenuators (VPA) in the IF paths enable to operate the VNA in the most linear measurement range, regardless of the gain of the currently mounted AUT (or probe). Since the measurement system has been designed for the greatest possible flexibility and, thus, should also allow the operation of the TxRx module on the robot to perform radar measurements, it is possible to feed out all RF signals separately. Therefore, the integrated single-pole double-throw (SPDT) coaxial relays are used to bypass the diplexers (DIP), allowing to reconfigure the RF design without mechanical modifications to the backend. This is intended to reduce the number of mating cycles of the RF components and, thus, increase their durability, but also helps to avoid complexity and human errors during switching the measurement setups. The integrated controller in combination with a hardware extension board manages all functions and provides a network interface so that the LO/IF distribution unit integrates seamlessly with the measurement software and can be controlled remotely through a single user interface.

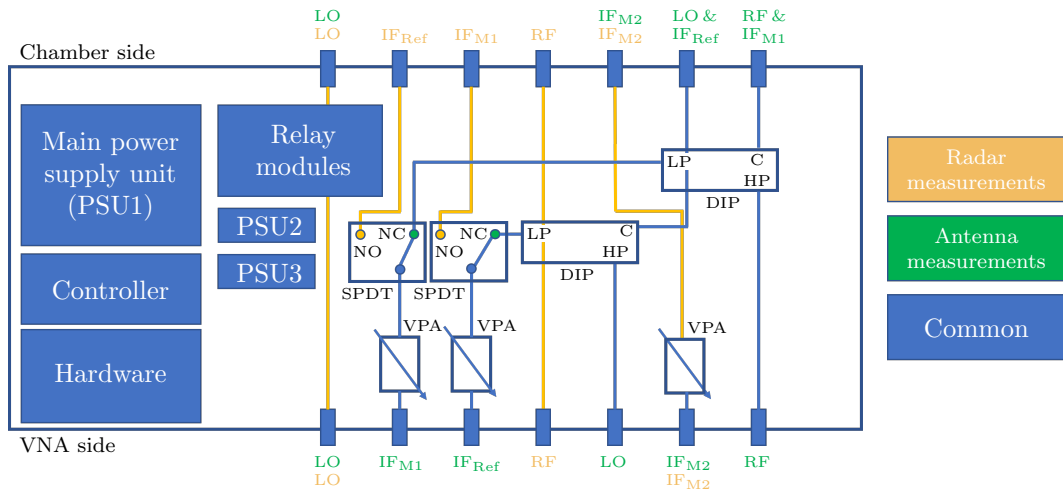


Figure 3.9: Simplified layout of the LO/IF distribution unit which allows to reconfigure the RF design to support antenna as well as radar measurements. The color highlighting indicates the signals required for operating the respective measurement setup.

In addition, the achievable measurement dynamic range is investigated, taking into account the most relevant components, in particular cable losses as well as gain and noise figures of possible amplifiers and their installation position. As a compromise between costs and measurement dynamics, not all amplifiers are installed at their optimum location. Instead of placing the RF amplifiers as close as possible to the VNA, ideally in the LO/IF distribution unit, they are integrated at the robot base or directly behind the diplexer at the rotary joint of the AUT setup. The IF amplifiers are located at the same places and are, thus, closer to their source, which is more advantageous in terms of the noise figure. Taking this component placement

into account, the minimum achievable measurement dynamic range is estimated in Fig. 3.10. For this, a combination using an AUT gain of $G_{\text{AUT}} = 20$ dBi and an open-ended waveguide (OEWG) probe gain of $G_{\text{Probe}} = 8$ dBi has been considered for three different measurement distances.

Due to the modular setup of the test range based on FEM, the design can be easily scaled in frequency. Since the manufacturer of the FEM specifies the same dynamic range for all modules up to 220 GHz [97], this frequency range is considered. Furthermore, a practical verification by measurements at a distance of 0.1 m confirmed an achievable dynamic range of at least 80 dB at a frequency of 60 GHz [100].

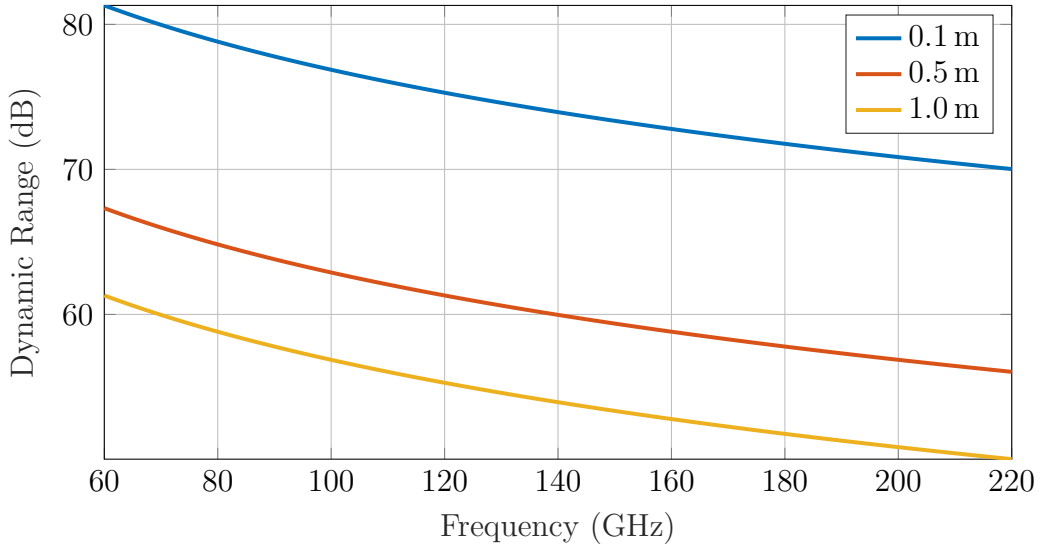


Figure 3.10: Calculated dynamic range for the RF components used, an antenna gain of $G_{\text{AUT}} = 20$ dBi and a probe gain of $G_{\text{Probe}} = 8$ dBi as a function of frequency for three different measurement distances.

3.4 Robot Control

The field of robotics has revolutionized numerous industries by providing efficient, precise, and versatile solutions to complex tasks. The focus of this technological advancement is the understanding of robot kinematics, a fundamental aspect that enables the manipulation and control of robotic systems. Since this technology is employed, but is not itself the subject of research, the main topics such as forward and backward kinematics based on the Denavit-Hartenberg parameters for modeling robot motion will be briefly outlined.

3.4.1 Denavit-Hartenberg Transformation

The Denavit-Hartenberg (DH) transformation [101] is a systematic method for modeling the spatial relationships between consecutive links and joints in a robotic manipulator. It provides a structured framework for defining local coordinate systems, which enables the analysis of kinematic chains. The DH convention standardizes

how rotations and translations are performed, leading to the construction of transformation matrices that characterize the robotic mechanisms.

The DH convention consists of four steps: The first step is to define the reference frames for each joint of the kinematic chain. For this purpose, they are defined that the z_i -axis represents the axis of rotation of the current joint i . The x_i -axis is the common normal of the previous z_{i-1} -axis and the current z_i -axis, where the x_1 -axis of the first joint is taken from the second one. Finally, the y_i -axis is chosen to yield a right-handed coordinate system. Then, in a second step, successive pairs of joints are specified relative to each other in terms of their position and orientation. The DH convention describes this in the following four transformations:

1. The rotation ϑ_i around the z_i -axis describes the joint angle and is chosen so that the direction of the x_i -axis coincides with the direction of the previous x_{i-1} -axis. The corresponding transformation matrix is given by

$$\text{Rot}(z_{i-1}, \vartheta_i) = \begin{pmatrix} \cos \vartheta_i & -\sin \vartheta_i & 0 & 0 \\ \sin \vartheta_i & \cos \vartheta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

2. The joint distance d_i is defined as the distance along the z_{i-1} -axis from the origin of frame $(i-1)$ to the intersection of the x_i and z_{i-1} -axes. This translation is mathematically expressed by

$$\text{Trans}(z_{i-1}, d_i) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

3. The arm element length a_i is defined along the x_i -axis and describes the distance between the intersection of the x_i and z_{i-1} -axes to the origin of frame i and is, thus, aligning the coordinate systems of both frame pairs. It is given by

$$\text{Trans}(x_i, a_i) = \begin{pmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

4. The rotation α describes a twist around the x_i -axis. It is used to transform the z_{i-1} -axis into the z_i -axis or, in other words, it describes the angle of rotation from frame $(i-1)$ to frame i . It is defined as

$$\text{Rot}(x_i, \alpha_i) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha_i & -\sin \alpha_i & 0 \\ 0 & \sin \alpha_i & \cos \alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

In the third step, a homogeneous transformation matrix is constructed for each joint, which describes the translations and rotations required to align frame $(i-1)$ with

frame i , represented by

$$\begin{aligned} {}^{i-1}T_i &= \text{Rot}(z_{i-1}, \vartheta_i) \cdot \text{Trans}(z_{i-1}, d_i) \cdot \text{Trans}(x_i, a_i) \cdot \text{Rot}(x_i, \alpha_i) \\ &= \begin{pmatrix} \cos \vartheta_i & -\sin \vartheta_i \cos \alpha_i & \sin \vartheta_i \sin \alpha_i & \alpha_i \cos \vartheta_i \\ \sin \vartheta_i & \cos \vartheta_i \cos \alpha_i & -\cos \vartheta_i \sin \alpha_i & \alpha_i \sin \vartheta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}, \end{aligned} \quad (3.2)$$

where the joint angle ϑ_i , joint distance d_i , arm element length a_i , and twist α_i are defined as the DH parameters. Applied to a closed kinematic chain, for example a serial robotic arm, it follows that d_i , a_i , and α_i are invariant quantities. They depend only on the specific geometry of the robot or its dimensions and have to be determined only once. The joint angle ϑ_i , on the other hand, is variable in this case. For a graphical illustration of the construction of the DH parameters and more advanced examples, further reading is available in [18].

In a fourth step, these transformations can be concatenated to obtain a complete transformation from a base frame, e.g. $i = 0$, to the frame n of the end-effector, generally represented by

$${}^0T_n = \prod_{i=1}^n {}^{i-1}T_i(\vartheta_i, d_i, a_i, \alpha_i), \quad (3.3)$$

and, therefore, provide the essential information required for both, forward and backward kinematic calculations. Here, forward kinematics, also called direct kinematics, refers to the process of determining the position and orientation of a robot's end-effector or TCP. Through the physical interpretation of the DH parameters as well as its concatenation, this corresponds to the solution of (3.3) for a given set of axis angles (ϑ) as well as the parameters describing each of the robot's axes (d, a, α). The reverse case, backward kinematics or inverse kinematics, works analogously, but is more complex due to the dimension of the solution space and will be discussed in the following chapter.

In summary, the DH transformation method establishes a standardized approach for defining local coordinate systems and capturing the interconnections between consecutive links and joints in a robotic kinematic chain. Nevertheless, based on this, the modified DH transformation has been introduced [18], in which the locations of the frame coordinate systems are in the links instead of the joints, and the sequence of the translation and rotation operators has been adjusted to address practical challenges. This offers advantages in terms of easier parameterization of real existing robots and can help avoid gimbal locks, in which a degree of freedom can be lost due to the alignment of the several rotational axes. However, since the core principle is the same, it will not be addressed in the following. It remains to be stated that due to the construction of transformation matrices and their concatenation, the DH convention facilitates the analysis and modeling of complex robotic motions, and, therefore, contributed significantly to technology advances in the field of robotics.

3.4.2 Inverse Kinematics

Inverse kinematics revolves around the reverse engineering of robotic motion, aiming to find the joint angles and parameters necessary to achieve a specified end-effector

pose. Unlike forward kinematics calculating the spatial configuration based on joint angles, inverse kinematics reconstructs a joint configuration for a certain spatial position. This process is essential for tasks like positioning robotic arms precisely or enabling robots to interact with the environment in a controlled manner.

Also in this case, the DH transformation is crucial and enables to calculate joint angles ϑ which satisfy the trigonometric relationships embedded within the DH transformation and correspond to the specified TCP position and orientation in space. However, this is a challenging process, e.g. due to the existence of multiple solutions, singularities, and complex trigonometric relationships. Since the actual implementation is usually not published by the manufacturers, common strategies reducing the dimensions of the solution space and, thus, reducing the complexity of the problem, are briefly outlined in the following. In addition, the numerical solution is explained in more detail at the end, as it represents a universally applicable and transparent approach for the calculation of inverse kinematics.

Geometric Considerations

This approach describes the use of the robot's inherent structure and the integration of workspace constraints, which allows to reduce the possible range of joint angles. In this way, physically implausible solutions can be excluded and configurations can be selected that are consistent with the mechanical characteristics of the robot as well as its working space. Furthermore, the use of joint constraints and the kinematic structure of the robot helps to identify multiple solutions in order to achieve the most feasible axes configuration.

Analytical Solutions

Certain robotic systems offer the potential for closed-form analytical solutions. These solutions are derived by carefully combining the DH transformation matrices, which encapsulate the transformation between adjacent links and joints. The formulation of equations that connect the DH parameters and the desired end-effector pose leads to a set of equations that can be solved analytically. This elegant approach provides explicit solutions in terms of joint angles ϑ , minimizing computational complexity and offering insights into the relationships between joint movements and spatial configurations. However, the existence of an analytical solution strongly depends on the architecture of the robot and, in principle, cannot be guaranteed.

Task Prioritization

In practical scenarios, the problem of inverse kinematics often involve a hierarchy of tasks or constraints that must be satisfied. Task prioritization techniques aim to address primary objectives, such as achieving the desired end-effector pose, while simultaneously accommodating secondary tasks like avoiding joint limits or maintaining a certain posture. Therefore, such hierarchical approaches align all tasks, ensuring that the most crucial goals are achieved while managing the complexity inherent in multi-objective optimizations.

Singularity Avoidance and Redundancy Utilization

Singularities are critical poses in the robot's workspace where it loses degrees of freedom and arise during inverse kinematics computation. Techniques such as pseudoinverse methods, weighted least squares, and utilizing redundancy are employed to circumvent singularity-induced complications. These strategies help maintain continuous and controlled motion even in the vicinity of singular configurations, ensuring that the robot remains maneuverable and its motion remains well-behaved.

Sampling and Optimization

In intricate cases where the solution space is irregular or complex, techniques involving sampling or optimization algorithms are of importance. Sampling-based methods systematically evaluate the solution space and assess the feasibility of different joint configurations. On the other hand, optimization approaches are used to identify joint angles that meet the desired TCP pose while adhering to mechanical constraints. These algorithms introduce a systematic and a heuristic dimension into solving inverse kinematics problems.

Numerical Methods

Numerical methods are employed for complex robotic structures or in cases where analytical solutions do not exist. These numerical techniques, such as the Newton-Raphson method, gradient descent, or Jacobian-based approaches, iteratively refine joint angles until the end-effector pose closely approximates the desired configuration. Thus, this techniques progressively converge to a combination of axes angles within the high-dimensional solution space which fulfills the specified pose requirements.

The Jacobian inverse technique is a numerical approach to iteratively adjust joint angles towards achieving the specified end-effector pose. The Jacobian matrix relates the joint velocities to the end-effector's Cartesian and angular velocities. It provides a relationship between the movement of joint angles and the TCP's motion. For a typical robot with n joints and an end-effector with six degrees of freedom (DOF), the Jacobian matrix $\mathcal{J}^{6 \times n}$ is given by

$$\mathcal{J} = \begin{pmatrix} \frac{\partial x}{\partial \vartheta_1} & \frac{\partial x}{\partial \vartheta_2} & \cdots & \frac{\partial x}{\partial \vartheta_n} \\ \frac{\partial y}{\partial \vartheta_1} & \frac{\partial y}{\partial \vartheta_2} & \cdots & \frac{\partial y}{\partial \vartheta_n} \\ \frac{\partial z}{\partial \vartheta_1} & \frac{\partial z}{\partial \vartheta_2} & \cdots & \frac{\partial z}{\partial \vartheta_n} \\ \frac{\partial a}{\partial \vartheta_1} & \frac{\partial a}{\partial \vartheta_2} & \cdots & \frac{\partial a}{\partial \vartheta_n} \\ \frac{\partial b}{\partial \vartheta_1} & \frac{\partial b}{\partial \vartheta_2} & \cdots & \frac{\partial b}{\partial \vartheta_n} \\ \frac{\partial c}{\partial \vartheta_1} & \frac{\partial c}{\partial \vartheta_2} & \cdots & \frac{\partial c}{\partial \vartheta_n} \end{pmatrix} \quad (3.4)$$

where x, y, z indicate the Cartesian position of the end-effector and a, b, c its corresponding orientation based on the convention used by the manufacturer, e.g. Euler angles. Given a requested change in the position and orientation of the TCP (Δr_{TCP}), the Jacobian inverse technique determines the associated changes

in the joint angles ($\Delta\vartheta$), which expressed by

$$\Delta r_{\text{TCP}} = \mathcal{J} \Delta\vartheta \quad (3.5)$$

with a graphical illustration provided in Fig. 3.11. It shows a simplified robot setup consisting of three axes ($\vartheta_i, i = 1 \dots 3$) and represents the course of Δr_{TCP} if (only) one of the three joints moves, highlighted by the vector $\Delta r_i = r_{\text{TCP}} - r_{\vartheta_i}$ from each joint to the TCP. The tangent visualizes the respective partial change of the end-effector related to the respective joint positions, i.e. the actual rate of change or, in other words, the respective column $\mathcal{J}_i$ of the Jacobian matrix. Due to the partial derivative, the problem is linearized, i.e. the solution of the Jacobian inverse technique represents the tangential course and not the dashed arc. Although this may be a sufficiently accurate approximation for small changes in the joints, it leads to inaccuracies for larger rotational movements.

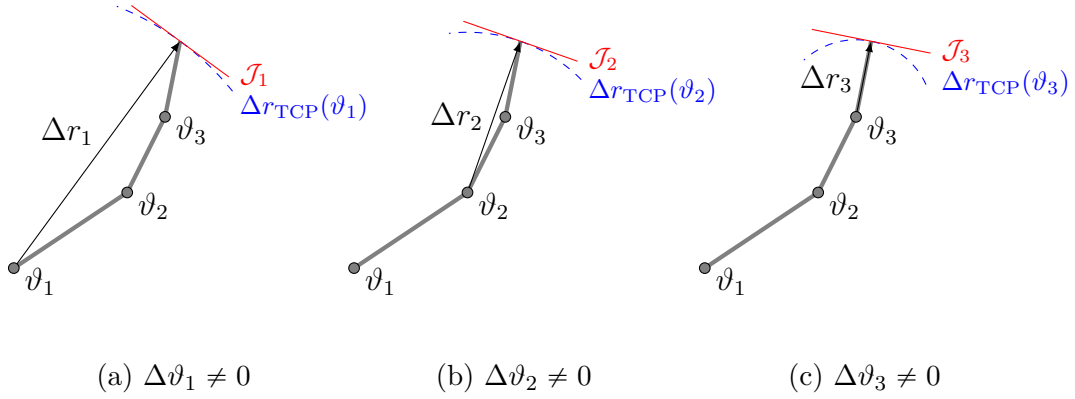


Figure 3.11: Exemplary illustration of individual joint movements for a simplified three-axis robot arm. The blue dashed curve indicates the path of the TCP $\Delta r_{\text{TCP}}(\vartheta_i)$ and the red tangent the rate of change, i.e. the column $\mathcal{J}_i$.

Two approaches exist for the construction of the Jacobian matrix $\mathcal{J}$: On the one hand, the coordinates of the end-effector can be given as a function of the joint angles

$$r_{\text{TCP}} = f(\vartheta) = f(\vartheta_1, \vartheta_2, \dots, \vartheta_n),$$

where the individual entries of the matrix result by the partial derivative. On the other hand, and this is the numerically more advantageous approach, the individual matrix entries can be calculated by the cross product, based on the rotation property known from vector analysis. Essentially, the vectorial rate of change of a point, such as an end-effector, with respect to another point, such as a joint, can be obtained if the positions of these points as well as their axis of rotation are known. This results in the individual entries of the matrix

$$\frac{\partial r_{\text{TCP}}}{\partial \vartheta_i} = e_{z_i} \times \Delta r_i = e_{z_i} \times (r_{\text{TCP}} - r_{\vartheta_i}),$$

where e_{z_i} denotes the normalized vector representing the axis of rotation, here referred to as the z_i -axis in agreement with the convention applied in the DH transformation.

After the matrix is constructed, the joint angles, as the name of the technique implies, result from its inversion. Since the matrix is not necessarily square or cannot be inverted for other reasons, such as singularities caused by $\vartheta_i = 0$, the pseudoinverse is used, resulting in

$$\Delta\vartheta = \mathcal{J}^+ \Delta r_{\text{TCP}},$$

and, thereby, establishing a relation between changes in the position and orientation of the TCP to the required changes in the joint angles. However, as mentioned earlier, this approach provides a linear approximation to the derivation of the position of the end-effector. Therefore, the procedure is executed iteratively, updating the current joint angles with the calculated changes in the joint angles. The Jacobian matrix is then recalculated and the process repeated until the desired accuracy is achieved. The smaller the calculated changes in the joint angles, the more accurate the linear approximation. Although this method may not be successful, e.g. in handling singularities or maintaining joint angle limits, the Jacobian inverse technique is a systematic numerical approach to solve the multidimensional challenges in the context of inverse kinematics problems and provides important potentials for obtaining solutions, especially when combined with the aforementioned strategies.

3.5 Safety Measures

By integrating robots into various industrial processes, these actuators are moving closer to the human environment. Whether they are an integral part of the workplace or human workers interact with these machines only within narrow limits, human safety is always the first priority. Therefore, the selection of the installed safety equipment is justified first and, in a second step, it is explained how to safely put the system into operation, verifying that there are no humans in the hazardous area.

3.5.1 Safety Equipment

One possibility to ensure the safe operation of robotic systems is to enclose them by dividing barriers or provide a special access area through which humans can interact with the machines. The confined dangerous area, thus, designates a spatial region in which people are exposed to hazardous conditions caused by movements or operation of robots. These can be collision risks, entanglements, or other impacts from robotic manipulators that can lead to potential injuries. The size and shape of the hazardous area varies depending on the robot kinematics, structure, workspace, and the intended application.

To mitigate risks in the dangerous area, security equipment is deployed. This equipment includes, for example, physical barriers, interlock systems, door contacts, pressure-sensitive mats, emergency stop buttons, laser scanners and light fences. The primary objective is to prevent unauthorized access and ensure the immediate shutdown of robot operations when humans enter restricted zones.

In commercial environments where robots are integrated into established operations, safety measures are closely aligned with industry regulations. Regulatory bodies such as the international organization for standardization (ISO) and the occupational safety and health administration (OSHA) set guidelines to protect workers.

Conversely, experimental setups can be designed more flexibly as long as they adhere to basic safety principles and document potential hazards within a risk assessment. A light fence, also called a light curtain or light barrier, is used in the developed measurement system and its application is, therefore, described in more detail. Light fences are sophisticated optoelectronic safety devices used in robot-assisted systems that project a field of infrared light beams over the hazardous area. If these beams are interrupted by the presence of a human or an object, the robot's movement is immediately stopped. Light fences are advantageous due to their fast response times and minimal attachment dimensions, making the access physically almost unaffected. However, only a two-dimensional area in space can be monitored, while, for example, laser scanners allow to define complex shaped three-dimensional safety zones.

The mounting distances of photoelectric sensors to the hazardous area are defined in the industry standard ISO 13855 [102]. These standards determine the mounting distances by factors such as the robot's approach speed, the response times of the overall system consisting of safety equipment and robot, and the height of the protective field. A balance is attempted between maximizing safety and maintaining operational efficiency.

The transmitter and receiver of the employed light fence [103] are integrated into the walls of the measurement chamber, thereby reducing electromagnetic reflections or preventing manipulations with the aim of gaining access to the dangerous area. To reduce the dimensions of the chamber due to limited space, a light fence with a length of 1800 mm is used. Therefore, the non-monitored area below and above the protective field is less than 300 mm and 900 mm, respectively. According to [102], this ensures that the safety guard cannot be crawled under or climbed over, reducing the required reach-through (RT) safety distance S_{RT} to the hazardous area. For effective hand and even finger protection, a light fence with a resolution of $d_R = 14$ mm is used, allowing a further reduction of S_{RT} . According to [102], the resulting safety distance is finally calculated by

$$S_{RT}(v_h) = v_H \cdot T_{TD} + C_{RT} \quad \text{with}$$

$$v_H = \begin{cases} 1600 \text{ mm/s}, & \text{if } S_{RT}(2000 \text{ mm/s}) > 500 \text{ mm} \\ 2000 \text{ mm/s}, & \text{otherwise} \end{cases}$$

$$C_{RT} = \begin{cases} 8 \cdot (d_R - 14 \text{ mm}), & \text{if } d_r \leq 40 \text{ mm} \\ d_{SAL}, & \text{otherwise} \end{cases}$$

where T_{TD} is the total delay time, v_H the standard approaching velocity of humans and $d_{SAL} = 850$ mm the human standard arm length. The total delay time T_{TD} takes into account all safety-relevant components and amounts to

$$T_{TD} = T_R + T_{SCU} + T_{LF} + T_{SR} = 319 \text{ ms} \quad \text{with}$$

$$T_R = 240 \text{ ms}$$

$$T_{SCU} = 30 \text{ ms}$$

$$T_{LF} = 39 \text{ ms}$$

$$T_{SR} = 10 \text{ ms}$$

for the developed measurement setup. Based on the data sheets, T_R is the stopping time of the robot for a fully extended arm (slowest case) [91] and T_{SCU} , T_{LF} as well as T_{SR} are the reaction times of the safety control unit (SCU) [104], the light fence [103] and the safety relay [105], respectively. Accordingly, it follows that $S_{RT}(2000 \text{ mm/s}) = 638 \text{ mm} > 500 \text{ mm}$ and, thus, the calculation can be performed with reduced human approaching speed, resulting in the safety distance of

$$S_{RT} = S_{RT}(1600 \text{ mm/s}) = 510.4 \text{ mm}.$$

Since the measurement system is experimental and only operated by trained personnel, this risk assessment refers to the linear axis, i.e. the main working area of the robot. Taking this as a reference, the robot operates in a distance of 940 mm to the light fence and, therefore, complies with the safety distance S_{RT} from this point of view. Nevertheless, depending on the parameterization of a measurement, S_{RT} may not be maintained, hence reference must always be made to the experimental state of the system. For industrially used systems, however, the physically possible working range of the robot must be taken into account, which can be significantly larger.

3.5.2 Release Procedure

To prevent people from accidentally being in a hazardous area, robust release procedures must be implemented, as defined in the general handling of machines in ISO 13849 [106]. These procedures require thorough checks to ensure that people are not in the vicinity of a robot or in a potentially dangerous area, e.g. its working environment, before it is put into operation, as specifically stated for handling robots in ISO 10218 [107]. Release procedures are an important safety measure to prevent the unauthorized or unintended presence of people in the hazardous area of a robotic system. These procedures include a series of steps that personnel must follow before giving the system permission to operate.

For visual inspection of the dangerous area, for example, cameras strategically positioned around the robot's operating environment can be used. They provide remote access and can be monitored by operators prior to system activation. However, because cameras can malfunction or fail to capture certain viewing angles, it is not necessarily safe to rely on them alone. In addition, mirrors can be installed in locations that provide a difficult or obscured view of the hazardous area. They help personnel obtain comprehensive visual information and reduce blind spots that may not be captured by cameras and provide a high level of fail-safety.

However, the installation of such optical systems is disadvantageous for the measurement setup, as they can cause potentially disturbing electromagnetic reflections, which affect the measurement. Also, the brightness in the typically dark absorber chambers is often not sufficient for camera systems or requires the installation of special night vision cameras with residual light amplification. For these reasons, and, since the measurement setup is typically operated by only one person, a minimal invasive method based on release buttons is implemented. Two release buttons are used for this purpose, which must be activated in a specific sequence. The first release button is installed in the entrance area of the measurement chamber. To

activate it, the door of the measurement chamber must be opened, forcing the measurement personnel to check the hazardous or other concealed areas. The second release button is located on the outer wall of the chamber, thus enforcing that the entrance area or the potential danger area must be left by the personnel to actuate the button.

The release buttons feature an indicator lamp which lights up as soon as a release is required. The indicator lamp of the next release button to be actuated flashes to inform the personnel of the correct sequence. As soon as the button has been pressed, the indicator lamp turns off as a feedback to the operator that the button has been pressed successfully. The logic to correctly release the measurement system is implemented in a functionally safe programming language directly in the SCU so that the system cannot be bypassed. The manual process as an integral part of the activation process ensures that operator verification has taken place. Although human error or inattention cannot be ruled out, the enforced verification process is more robust than visual inspection using cameras or mirrors, while requiring fewer installations in the measurement area.

3.6 System Control

After the most important aspects, such as the requirements analysis, the control of the robot or safety aspects as well as the RF design have been considered in detail, the overall system is explained below. In particular, the most important components on the system level are introduced as well as their interaction to achieve complex measurement processes is explained based on the diagram given in Fig. 3.12.

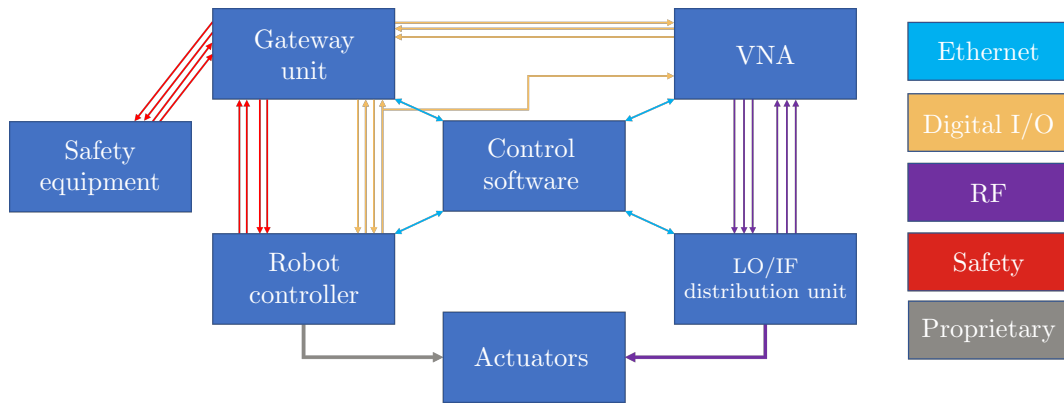


Figure 3.12: Block diagram demonstrating the interaction of the overall system including data flows and control signals.

The control software is the central interface for human-machine interaction and allows to set all relevant parameters. It controls and monitors the interaction between the gateway unit (GU), VNA, LO/IF distribution unit or the RC and handles data flows as well as data storage. The most important feature of the software is the measurement planning component, which guides the operator through the parameterization of the intended measurement. For essential measurement sequences, like PNF or SNF measurements, different implementation strategies are provided,

which define whether the desired measurement paths are realized using the additional axes or the robot or a combination thereof. Hence, the operator is provided access to the degrees of freedom of the measurement system within a reasonable scope and, thus, is able to exploit its flexibility. In addition, completely user-defined measurement sequences, for example irregular sampling grids, are supported by importing the measurement positions using a file-based interface.

In order to avoid incorrect measurement parameters or possible collision risks, a further component of the measurement planning is the integrated 3D visualization. Since the robot is the most flexible and, thus, most critical actuator in terms of collisions, the sampling points acquired by the robot are visualized. An exemplary planned measurement is presented in Fig. 3.13, where the measurement is parameterized on the left-hand side with regard to the spatial coordinates, the cut to be measured, the polarization angle, the radius, but also tilts and offsets as explained in Section 4. The corresponding 3D visualization of this measurement is immediately displayed on the right-hand side. The measurement coordinate system as well as the resulting sampling points including the orientation of the probe antenna are illustrated, whereby the sampling points are coded using different colors. Green sampling points mark the start of a measurement trajectory, while the sampling points given in orange and blue symbolize the various trigger sources explained in Section 6.2, specifically trajectory-defining and trajectory-dependent triggers.

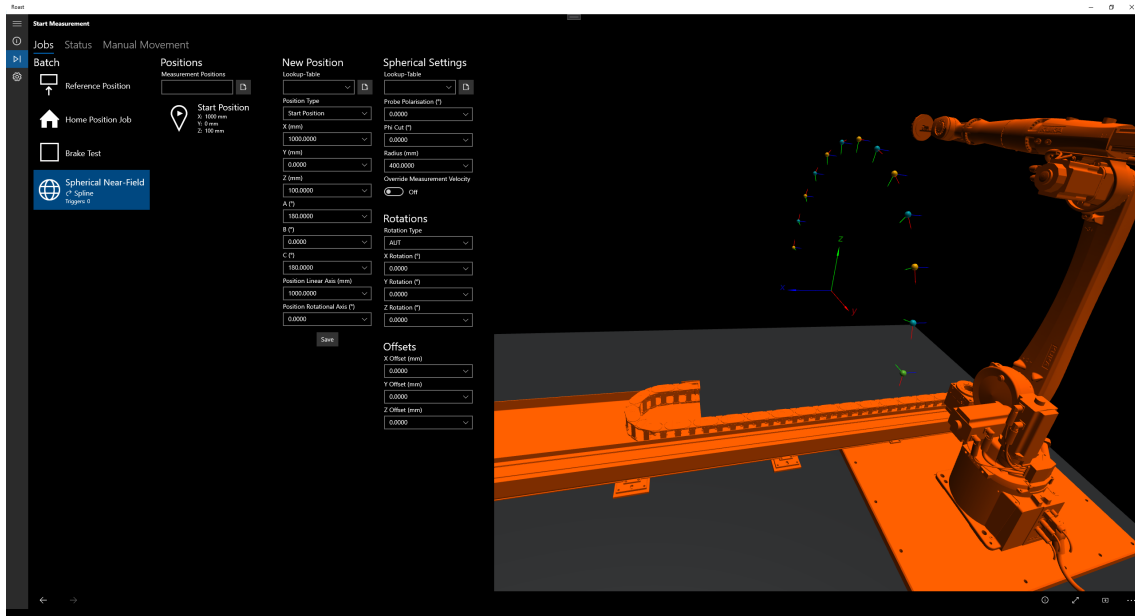


Figure 3.13: Exemplary representation of the parameterization and especially the 3D visualization of a planned spline-based measurement, which is intended to contribute to the identification of potential misconfigurations, i.e. before the start of a measurement.

All measurement and meta data are stored in a relational database, which also serves as the interface for data export and post-processing. Furthermore, a web interface for remotely monitoring the progress of a measurement, tracing the logs or visualizing raw data of a measurement is provided.

Once the planning of a measurement is completed and it is started, the settings are read from the VNA or, depending on the configuration, the VNA is configured. In addition, the RF paths in the LO/IF distribution unit are switched (e.g. for antenna measurements, as explained in Section 3.3), the attenuators are configured and, if not already done, the power supply for all RF components is activated. The desired measurement is divided into one or more measurement jobs, converted into an XML format and transferred to the RC using an ethernet interface. Here, all settings relevant for the movement of the robot are checked for feasibility, e.g. the different velocities or accelerations, and, if no errors occur, the measurement process is started. As soon as the robot reaches a sampling point, a trigger signal is generated in the RC using an I/O module connected via EtherCAT [108–110]. The 24 V signal, which is common for robot- or programmable logic controller (PLC) based systems, is both galvanically decoupled in the GU and converted to the 3.3 V transistor-transistor logic (TTL) voltage levels required for the VNA using high-speed optocouplers [111]. Thus, the time-critical signal is fed with minimum latency to the VNA, which performs the requested measurement for each trigger signal. The 3.3 V trigger signal and other control signals from the VNA, which indicate whether a measurement is continuing or has been completed, are digitized in the GU and communicated to the control software via the network interface.

Depending on the configuration, the robot can either stop at each measurement point and wait for the measurement to complete, or the measurement is performed continuously, with the latter mode resulting in less uncertainties because fewer stops and accelerations are required. However, if a stepwise measurement is required, for example due to long sweep times of the VNA, three additional 24 V signals are used to communicate the VNA-side measurement state between the GU and the RC. These are denoted as trigger acknowledgement (TACK, GU $\rightarrow$ RC), measurement completed (MC, GU $\rightarrow$ RC) and measurement completed acknowledgement (MCACK, RC $\rightarrow$ GU), where the notation with the arrow indicates the direction of each signal, i.e. from the writing to ($\rightarrow$) the reading component. Together with the trigger signal (TS, RC $\rightarrow$ GU) signal already mentioned, this results in four signals that represent a state machine as illustrated in Fig. 3.14. Since all components within the RC operate with a certain system clock (typically 4 ms), this state machine ensures that all control signals have been processed by the respective communication partner before they are reset. For continuous measurements, the feedback overhead in terms of handshaking is reduced to provide the fastest possible measurement speed. Thus, by setting the measurement speed accordingly, it must be ensured that the VNA-side measurement is completed before the next trigger signal is generated in order to avoid data loss.

As explained in detail in Section 3.5, the safety equipment includes an additional emergency stop button to the one on the robot's control pad, as well as a light fence installed in the entrance area of the measurement chamber, which detects intrusion into the dangerous area. In addition, two release buttons as well as a further acknowledgement button for the robot are used to restart the system after an emergency stop has been triggered. The emergency stop and release logic is implemented in a functionally safe programming language in a SCU integrated into the GU. The safety-relevant signals are designed to be redundant and time-clocked for fault detection, also referred to as output signal switching device (OSSD). Since

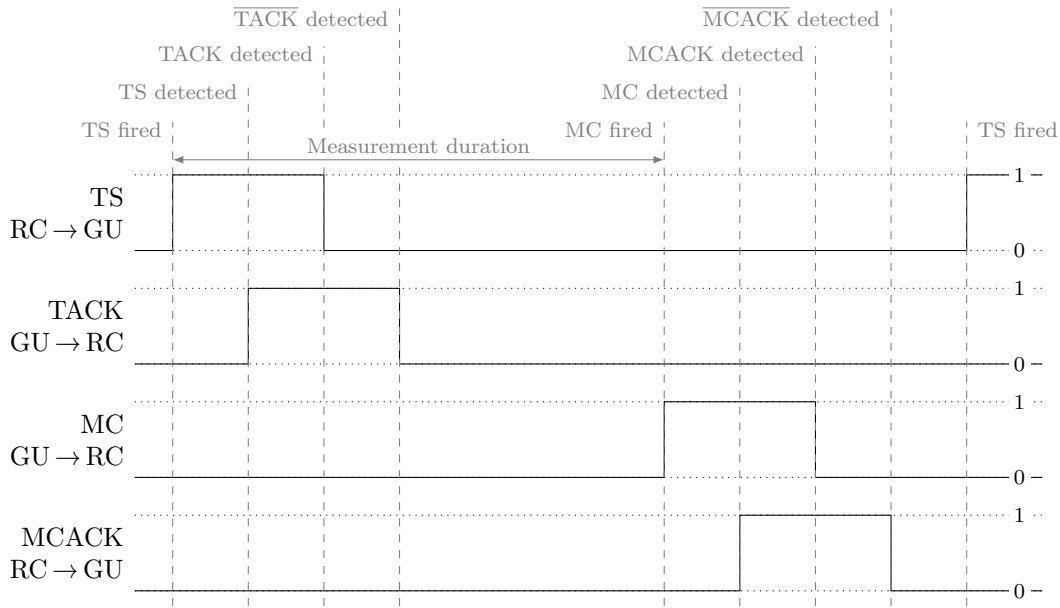


Figure 3.14: State machine represented by control signals to ensure the correct workflow during stepwise measurements.

the RC and the SCU check each other for any emergency stops, both components have safety relays that interrupt the OSSD signals of the other system. In this way, an emergency stop can be triggered across both systems, for example by the light fence in the measurement chamber or the emergency stop button on the robot's control pad. To keep the logic of the SCU robust against program flaws as well as maintainable, only a one-way communication is implemented. For this purpose, three digital signals are used: ready, light fence triggered or emergency stop by an actuated button. These control signals are digitized in the GU and communicated to the control software via ethernet.

Thus, the GU digitizes all hardware signals relevant to both, safety and sequence control. The information is provided via the ethernet interface to the control software, where it is displayed to the operator, aggregated with the data from the VNA and RC, and, subsequently, incorporated into a complex measurement logic. Due to the system-wide awareness of all processes, errors can be detected at an early stage and corrected automatically. For example, if problems occur during data transmission from the VNA or if it is detected that the RC has generated a different number of TS than measurement data having been received from the VNA, the measurement can be repeated automatically. Depending on the type of error and the parameterization of the measurement, complex error recovery strategies can be implemented, e.g. as unidirectional measurements cannot be simply repeated, a secure trajectory to the actual start position has to be identified. The more information the control software possesses, the better and more intelligently it can ensure the reliable operation of the measurement system.

3.7 Summary

The robot-based measurement system developed as part of this dissertation is motivated as an extension of the measurement capabilities of the IHF's CATR, especially for measurements in the millimeter-wave range. Due to the large mechanical dimensions of the CATR, these result in a reduced dynamic range and, therefore, suggesting near-field measurements in this frequency range as a preferable alternative. To continue the research focus of the institute, especially with regard to irregular sampling grids, and to ensure maximum flexibility with regard to PNF, CNF or SNF measurements, a robot-based measurement system is considered promising and future-proof.

Since robot-based systems typically have higher tolerances with regard to the positioning accuracy, simulations are performed to assess the impact thereof on the result of the NFFFT. For this purpose, different orders of magnitude of typical absolute accuracies are used to derive the requirements for the robot on the basis of a stochastic error analysis. As a result, an absolute calibration of the robot, which according to the accuracy data sheet guarantees an absolute accuracy of 0.5 mm and a pose repeatability of 0.04 mm, is required.

Subsequently, the required measurement capabilities are discussed, which, in addition to antenna measurements, should also enable RCS measurements. Therefore, a higher payload of the robot is required, but also additional features for the LO/IF distribution unit to be able to switch between different signal paths depending on the required measurement capability. The implementation thereof is discussed with the remaining RF design of the measurement system in Section 3.3. A theoretical analysis of the achievable dynamic range, taking into account the actually installed components, is consistent with the measurements performed in [100], confirming a dynamic range of more than 80 dB for a measurement radius of 100 mm and AUT and probe gain of 20 dBi and 8 dBi, respectively.

Afterwards, the control of robot-based systems is explained and the common DH transformation is introduced. It provides a mathematical description of the concatenation of several axes, whereby a systematic approach to the definition of the individual coordinate systems for each axis is applied. This allows to determine the position of the TCP for the respective axis positions ϑ_i , taking joint distances d_i , arm element lengths a_i and twists α_i for each axis i into account. It is also the basis for inverse kinematics, which enables the reverse case, i.e. the calculation of the required axis positions ϑ_i for a desired end effector position. Inverse kinematics are challenging due to the multi-dimensional solution space, which may contain singularities such as gimbal locks and, thus, a multitude of solutions exist for the same position of the robot. Besides summarizing several strategies to reduce the complexity of the problem or to avoid singularities, the numerical approach based on the inverse Jacobian matrix is explained. This generalistic approach is independent of the structure of the robot used and is, therefore, demonstrated based on an example of a robot arm consisting of three joints.

Although the developed measurement system is regarded as an experimental setup and is, hence, only operated by trained personnel, safety considerations are made to eliminate any risks for the operating personnel. This includes the safety concept based on a light fence, the calculation of the safety distance based on the reaction

time of all safety-relevant components as well as the release procedure, which ensures a safe commissioning of the system.

The chapter concludes with explanations on controlling the overall system. Block diagrams indicate the interactions between the RC, GU, SCU, LO/IF distribution unit, VNA and the control software, which has also been developed as part of the dissertation. Particular attention is paid to the time-critical trigger signal, which is connected directly to the VNA via high-speed optocouplers. The state machines based on the control signals between GU and RC are explained for both, continuous and the more demanding stepwise measurements, where the RC actively awaits the VNA (or external devices such as radar modules) to complete their measurements. It is also explained that all important trigger or control signals are digitized in the GU and communicated to the control software. As a result, it is not only possible to implement complex measurement sequences in the interaction of all components, but also to detect errors at an early stage, for example if the amount of data transmitted by the VNA does not match the number of digitized trigger signals. Furthermore, recovery strategies are implemented, which automatically repeat individual measurement runs depending on the respective configuration (uni-, bidirectional or spline-based) and, thus, allow the convenient execution of measurement projects.

Error Model

The precise alignment between probe antenna and AUT is essential for ensuring the accuracy of an antenna measurement. In the case of conventional far-field measurements, typically no subsequent transformation relying on the measured phase is performed. In addition, alignment errors only slightly affect the measured radiation pattern since those deviations are typically small compared to the measurement distance which is derived from approximation of the far-field distance. Still, these tolerances are deeply investigated in the typically performed uncertainty analyses and, moreover, separated into several error terms, for example AUT alignment, probe alignment or probe position errors [112]. In contrast, alignment errors constitute a critical error term in near-field measurements, especially at frequencies in the millimeter-wave range. Therefore, numerous studies investigate the impact of the respective sources of error on the measurements. The methods used range from early optical calibration methods based on mirror constructions and theodolites [45] to more sophisticated investigations using state-of-the-art laser trackers [113].

However, the optical attachments required for this purpose, such as mirrors or other reference targets, are often difficult to install in the measurement setup or can contribute to a degradation of the measurement results. Therefore, several methods for the electrical alignment of the measurement setup based on the measured amplitude and phase have been developed [114]. These are based on the calculation of the phase center of an antenna [115] or redundant and theoretically identical measurements, widely known as flip tests [116, 117]. Derived from this, there are also methods to calculate Cartesian offsets from the ideal axis of rotation in roll-over-azimuth positioners using multiple ϕ -cuts of a complete spherical measurement for both polarizations. Here, the phase modulation due to the offset is modeled via a Fourier series expansion and corrected in the post-processing [118]. Other methods, which also require a full spherical measurement, use the SWE and determine the phase center based on the SMC [119, 120].

Correcting the measurement data in post-processing may be the only option for roll-over-azimuth positioners, depending on the type of alignment error, since not all errors can be compensated by the mechanical alignment possibilities of the measurement setup. In contrast, one of the most important advantages of robot-based measurement systems is that the high number of degrees of freedom not only provides the possibility to align the sampling grid to an almost arbitrarily placed AUT, but also allows to compensate all types of alignment errors. The straightforward solution for identifying position and orientation errors are laser tracking systems [26]. However, such metrology instruments are very expensive and, thus, contradict the cost-effective use of industrial robots. In addition, a reference target is required, which is placed near the probe and must not be shadowed, possibly limiting the

flexibility of the robot. Therefore, an electrical alignment procedure adapted to the mechanics of a robot has been developed and automated. It models six error terms, allows their correction and does not require any modifications of the measurement setup.

4.1 Measurement Geometry

In analogy to the electrical alignment procedures for roll-over-azimuth positioners [114], an error model is introduced which defines the most relevant error terms specifically for the developed measurement system [121]. In this setup, the probe antenna is positioned by the robot while the AUT is mounted on a combination of a linear and rotational axis. This is illustrated in Fig. 3.3b, with further details on the setup provided in Section 3. Following the coordinate systems introduced in Fig. 2.3, the most common method are single-cut measurements in $\theta = \theta_m$ performed by the robot in its measurement coordinate system (x_m, y_m, z_m) ¹. The rotation in ϕ is done stepwise using the additional rotation axis ϕ_{axis} , resulting in a complete decoupling of the axes θ_m and ϕ_{axis} . However, in order to exploit the flexibility of the robot, it can also, at least partially, perform measurements in ϕ_m . In this case, the robot scans a certain angular range $\phi_m \in [\phi_{i,\text{min}}; \phi_{i,\text{max}}]$, for example a quarter of the intended measurement sphere. As soon as partial measurement i is completed, the AUT is rotated by $\phi_{i,\text{max}} - \phi_{i,\text{min}}$ using the rotation axis ϕ_{axis} to continue with the next partial measurement $i + 1$ using $\phi_{i+1,\text{min}} = \phi_{i,\text{max}}$. This process is repeated until the data acquisition is completed and, thus, represents only a partial decoupling of the two measurement axes θ_m and ϕ_{axis} . In order to describe an offset as well as a misorientation of these decoupled measurement axes, the first two error terms are denoted as axis intersection offset (AIO) and axis intersection tilt (AIT), respectively.

After quantifying the position of the measurement axes relative to each other, the non-ideal alignment between these additional axes and the AUT mounting is considered. This is described analogously by an offset and an angular error, referred to as AUT mounting offset (AMO) and AUT mounting tilt (AMT). However, these quantities are not related to the AUT itself, but only to its mounting. Tolerances in the mechanical setup attached to the axis of rotation are, therefore, decisive. In addition, the errors are superimposed by the mounting of the TxRx module due to missing fitting elements such as dowel pins or bores. This leads to increased tolerances resulting from the mechanical screw connection. Furthermore, possibly required RF components such as amplifiers or attenuators also contribute to these two error terms, independent of the AUT. The same mechanical tolerances with regard to missing fitting elements or the installed RF equipment, in this case in particular waveguide extensions, also occur at the probe mounting setup. This error term is denoted as the probe mounting offset (PMO). The tilt of the probe is not taken into account here, since typically used probes have a very broad pattern and, thus, angular errors can be neglected in most cases [122]. Furthermore, angular errors of the probe antenna are included in the pre-determined tilts with respect to the axis and the AUT. However, a probe polarization mismatch (PPM) error $\Delta\chi$, i.e.

¹Since measurements along the θ -axis can only be performed using the robot, the index might also be omitted in the following for better comprehensibility.

a rotation of the AUT and probe antenna relative to each other, is considered and, hence, forms the sixth error term.

A sketch of the error model is given in Fig. 4.1 for a cut over the y - z -plane with the x -vector pointing out of the drawing plane. It illustrates the sampling points of a θ -cut for $\phi = 90^\circ$, thus located along the y -axis. In addition, the probe including its coordinate system (x_p, y_p, z_p) is visualized for a single sampling point, pointing to the center of the measurement sphere where the AUT would be ideally positioned. As explained before, the AIO describes an offset to the actual position of the additional axes, which is expressed by Δy_{axis} in this representation. Since the additional axes can also be subject to a non-ideal orientation, a tilting γ_{axis} of the axis coordinate system $(x_{\text{axis}}, y_{\text{axis}}, z_{\text{axis}})$ around its x_{axis} -axis is shown, whereby only the z_{axis} -axis is visualized for the sake of clarity. Based on this coordinate system, the AMO denotes a translation of the AUT coordinate system (x_a, y_a, z_a) by an offset Δy_a . Similarly, the AMT are introduced relative to the axis coordinate system. They are simplified as a rotation around the x_a -axis denoted as γ_a . Again, only the z_a -axis is shown for the AUT coordinate system. While offset and tilt of the axis coordinate system are static, since they refer to the mechanical rotation axis of the additional axes, the offset and tilt of the AUT coordinate system depend on the rotation axis position ϕ_{axis} . Accordingly, the combined angular error γ results from a static component and a dynamic contribution depending on ϕ_{axis} .

Finally, following the same approach, the PMO is considered, which causes the probe coordinate system (x_p, y_p, z_p) to be translated by Δy_p from the rotation axis of the TCP ϑ_6 . For a better visualization, the x_p -axis is omitted in this case as well. As explained earlier, tilts of the probe antenna, AUT and the measurement axes depend on each other and cannot be separated without additional measurements. Since the probe pattern is typically not very directive, the assumption in [122] is followed that tilts of the probe can be neglected. Furthermore, the misalignment between the polarization axis of probe antenna and AUT is indicated as the PPM $\Delta\chi$.

In the following, the determination of the individual error terms is discussed before a technique to separate the different contributions is presented.

4.2 Axis, AUT and Probe Offsets

The measured phase is a sensitive indicator for evaluating path differences between AUT and probe antenna during an antenna measurement. Especially in the millimeter-wave range, even the smallest path differences cause significant disturbances of the measured phase. For example, a path difference of only 0.1 mm at a frequency of 90 GHz causes a phase difference of approximately 10° and, thus, leads to significant deviations of the measured antenna pattern, especially if a NFFT is required.

However, apart from the resulting errors, the measured phase also offers conclusions about the causing misalignments, whereby these are overlaid by the intrinsic phase pattern of the AUT ψ_a . In Fig. 4.1 it is shown that offsets lead to erroneous measurement distances r' for the respective sampling points, for example of a given θ -cut. Following the error model, the non-ideal measurement distance for the respective

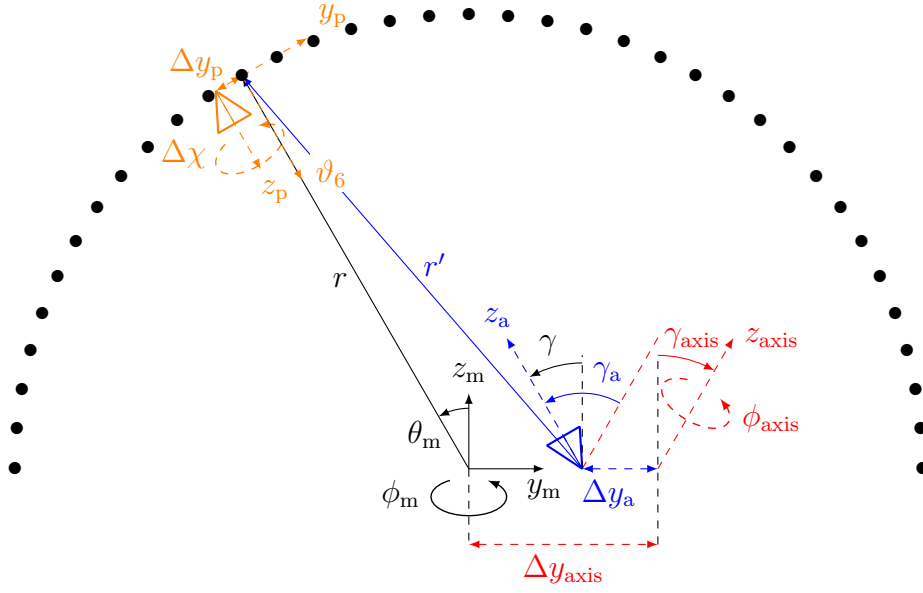


Figure 4.1: Sketch of the error model for compensation of axis, AUT and probe misalignments shown for a cut over the y - z -plane with all x -axes pointing out of the drawing plane.

sampling point is described by

$$r' = \left\| \begin{pmatrix} \Delta x - r \sin \theta \cos \phi \\ \Delta y - r \sin \theta \sin \phi \\ \Delta z - r \cos \theta \end{pmatrix} \right\|, \quad (4.1)$$

where the given offsets Δx , Δy , and Δz are superimposed of all occurring offsets. Hence, the offset shown in Fig. 4.1 results from $\Delta y = \Delta y_{\text{axis}} + \Delta y_a + \Delta y_p$, from which it also follows that the separation of the different error contributions is here not possible yet and will be described later in Section 4.5. To simplify the norm in (4.2), a first-order Taylor series approximation is applied to the root term assuming $\Delta x, \Delta y, \Delta z \ll r$. Thus, the modulation of the measured phase $\Delta\psi$ as a function of the path differences can be described for each sampling point by

$$\begin{aligned} \Delta\psi &= k(r - r') \\ &\approx k[(\Delta x \sin \phi + \Delta y \cos \phi) \sin \theta + \Delta z \cos \theta] \end{aligned} \quad (4.2)$$

$$= k[\delta \sin \theta + \Delta z \cos \theta], \quad (4.3)$$

where (4.2) is due to the Taylor simplification and (4.3) due to neglecting the ϕ -component, since it is constant for the respective θ -cuts and, thus, does not contribute to the modulation. Depending on the measured θ -cut, δ denotes either the offset along the x -axis ($\phi = 0^\circ$) or along the y -axis ($\phi = 90^\circ$) or a combination of both. Therefore, the measured phase response ψ_m results from the phase behavior of the AUT ψ_a as well as the phase modulation $\Delta\psi$ caused by the non-ideal alignment to

$$\psi_m(\theta) = \psi_a(\theta) + \Delta\psi(\theta). \quad (4.4)$$

Although not mandatory, a symmetric AUT pattern is assumed in the following, so that (4.4) yields

$$\begin{aligned}\Delta\psi_m(\theta) &= \psi_m(\theta) - \psi_m(-\theta) \\ &= \delta \cdot 2k \sin(\theta)\end{aligned}\tag{4.5}$$

due to the symmetry of the cosine function. The assumption of a symmetric AUT pattern is made here so that the offset δ can be estimated based on the measurement of a single θ -cut. This requirement is not made for conventional flip tests, since here, based on two redundant measurements $\psi_m(\theta)$ and $\psi_m(\theta + \pi)$, the estimation can also be made for asymmetric antenna patterns. However, this requires two measurements and, thus, twice the measurement time, which causes a considerable additional effort, especially with regard to the alignment method presented in Section 4.5. In this respect, the requirement of a symmetrical radiation behavior of the reference AUT represents a valid compromise at this point. Moreover, it is evident from (4.5) that the estimation of the offset Δz is not possible, since the phase modulation caused therefrom is superimposed by the phase behavior of the AUT ψ_a . If the latter can be assumed as prior knowledge, an estimation of the offset Δz is possible, although this will be omitted at this point for a generally valid approach. Instead, the offset Δz can also be calculated during post-processing of the completed measurement as described in detail in [119, 120].

Based on (4.5), it is possible to estimate the linear offset $\hat{\delta}$ using a least-squares optimization problem given as

$$\min_{\hat{\delta}} \left\| \mathbf{d} - \hat{\delta} \cdot 2\mathbf{k} \odot \mathbf{e} \right\|_2 \tag{4.6}$$

where $\odot$ is a pointwise multiplication and $\mathbf{k}$, $\mathbf{d}$, and $\mathbf{e}$ are the vectorized representations of the wavenumber, $\Delta\psi_m(\theta)$ and the corresponding sine terms of a θ -cut, respectively. The optimization problem stated in (4.6) can be solved simultaneously for multiple frequency points N_f . For this purpose, the vectors $\mathbf{d}$ and $\mathbf{e}$ are constructed such that their length is $N_f \cdot (N_s - 1)/2$, where N_s describes the number of sampling points on a θ -cut. Here, only negligible deviations are to be expected depending on the frequency, which are caused, for example, by measurement errors or the frequency dependence of the AUT's (and probe's) phase center. The advantage that a larger data base provides the possibility to average over a frequency dimension, therefore, prevails.

Several comparison measurements have been performed for verification, whereby three different offsets Δy are presented in Fig. 4.2. For better comparability, the graphs are split at $\theta = 0^\circ$ so that the (unwrapped) phase difference can be directly assessed from the overlaid plots. All measurements show a θ -cut along the y -axis for $\theta = -90^\circ \dots 90^\circ$ with a step size of $\Delta\theta = 0.5^\circ$. However, the gray faded region is excluded for the following analyses, since the acceleration of the robot at the beginning of the measurement leads to deviations in the measured phase, especially in the range $\theta = -90^\circ \dots -88^\circ$ for $\psi_m(\theta \leq 0)$. Since this effect is not entirely caused by the offset, this angular range is ignored so that the estimation of the offset is not affected.

Although the sinusoidal characteristic stated in (4.5) cannot be directly observed in the dotted visualized difference graph due to scaling and angular range, it becomes

clear that $\Delta\psi_m$ scales linearly depending on the offset. The estimated offsets $\hat{\delta}$ based on these data are referred to as test cases 1–3 and are later evaluated along with other measurements in Table 4.1 (see Section 4.4). For the offset measurements, the residual error between the actual misalignment and the estimate obtained by the algorithm is denoted as $|\Delta y - \hat{\delta}|$. For test cases two and three, the residual error is lower than 0.02 mm and 0.01 mm, respectively, which is within the specified repeatability of the robot used and, thus, sufficiently accurate for a precise electrical alignment procedure.

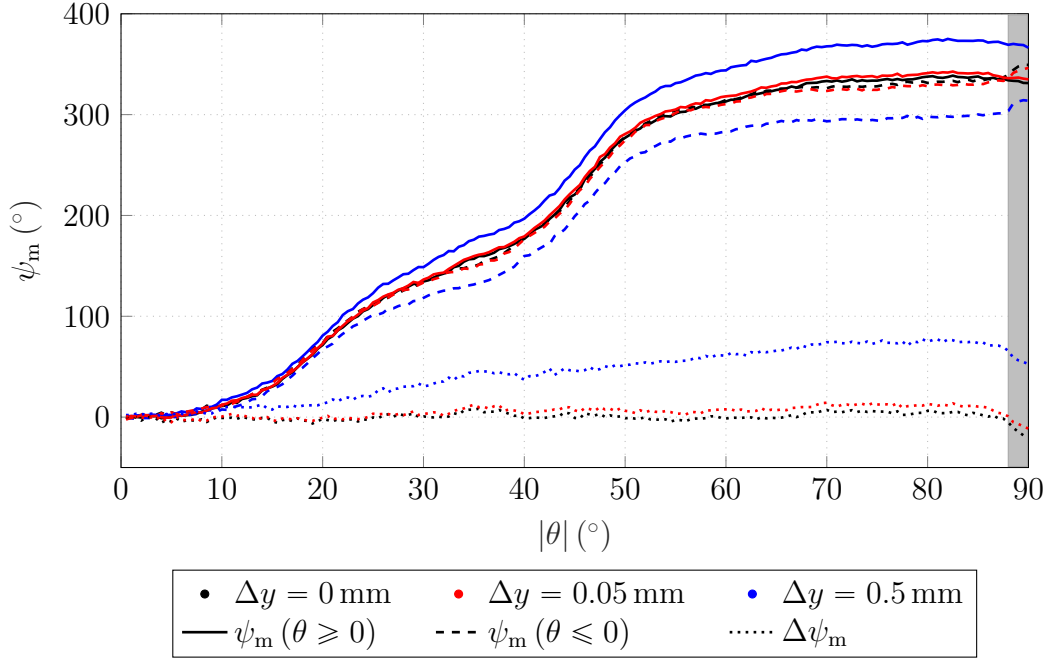


Figure 4.2: Measured phase ψ_m as well as its difference $\Delta\psi_m$ depending on the offset Δy . The color indicates the size of the offset Δy and the line type indicates the angular range ($\theta \leq 0$ or $\theta \geq 0$) as well as the difference $\Delta\psi_m$. The deviations in the gray shaded area are due to the acceleration of the robot at the beginning of a θ -cut.

4.3 Mounting and Axes Intersection Tilts

Another contribution to the misalignment between AUT and probe antenna is described by angular errors. To identify the angular misalignment, the θ -cuts already described in Section 4.2 are used analogously. Instead of an asymmetry as caused by offsets, angular errors lead to an angular offset γ along the θ -axis. There are different possibilities to identify this angular misalignment, for example it can also be done via the measured phase by determining the axis of symmetry. However, since offsets also induce an angular error, albeit a very small one, studies have shown that more reliable and independent estimates of the angular error can be made using the measured amplitude instead of the phase. The measured amplitude

$$A_m(\theta) = A_a(\theta - \gamma) \quad (4.7)$$

is comprised of the amplitude pattern of the AUT A_a shifted by the tilt angle γ . Also in this case the resulting angular error is composed of a superposition of different inclinations as

$$\gamma = \gamma_{\text{axis}} + \gamma_a(\phi_{\text{axis}}) + \gamma_p \approx \gamma_{\text{axis}} + \gamma_a(\phi_{\text{axis}}), \quad (4.8)$$

where γ_p analogously describes the rotation of the probe antenna around its x_p -axis, but is omitted in Fig. 4.1 for clarity. Furthermore, since typically used probe antennas usually have a broad pattern, this contribution represents a possible simplification of the error model [122]. In the following and in accordance to Fig. 4.1, the case is considered that the rotation axis ϕ_{axis} is static and, thus, the angular error γ of the sketched error model refers without restriction of generality to a rotation around the x -axis. For other rotation axis positions, the contributions of the misalignments refer to both, the x -axis as well as the y -axis.

The angular error can be identified, for example, by a simple maximum search in A_m , whereby the maximum may be affected due to noise effects. The less directive the AUT used, the greater the effect of noise on the found maximum. To make the estimation more robust, the maximum is therefore not determined by a single point, but by a symmetry operator. This can be described by an optimization problem denoted as

$$\min_{\hat{\gamma}} \sum_{\theta > 0} \left| A_m(\theta + \hat{\gamma}) - A_m(-\theta + \hat{\gamma}) \right|, \quad (4.9)$$

where $\hat{\gamma}$ is the resulting estimation of the actual tilt γ for the measured θ -cut. Similar to the determination of the offsets, the angular tilt error can be determined over all measured frequencies simultaneously. By averaging over the frequency dimension, noise-induced disturbances can be minimized, making the approach more robust. The results of minimizing the symmetry operator to perform the tilt angle estimation are summarized in Table 4.1, where test cases one and four describe the isolated tilt angle estimations without the presence of offsets. For these isolated test cases, the remaining error is $|\gamma - \hat{\gamma}| \lesssim 0.02^\circ$, indicating that the angle estimation works reliably. The superposition of offsets and tilt errors as well as the interdependence between these terms of the error model will be investigated in more detail in the following Section 4.4.

4.4 Independence of the Error Contributions

In comparison to the idealized consideration of isolated impacts of misalignment, i.e. offsets in Section 4.2 and angular tiltings in Section 4.3, the interactions between these components of misalignment are considered in the following. This corresponds to a typical scenario for measurement systems, in which both error contributions will always occur simultaneously due to non-ideal components or mechanical tolerances in the mounting setups. All of the discussed results summarized in Table 4.1 correspond to a θ -cut for $\phi = 90^\circ$, i.e. along the y -axis. Consequently, $\hat{\delta}$ corresponds to an offset Δy and $\hat{\gamma}$ to a rotation around the x -axis.

The effect of an existing offset on the tilt angle estimation is indicated by test case two. Test cases five and six correspond to the superposition of several components of a misalignment. If an existing offset is not taken into account in the error model, a residual error of the angle estimation results within the range of $|\gamma - \hat{\gamma}| \lesssim 0.3^\circ$.

Table 4.1: Comparison of the estimated and actual alignment errors

Test case		1	2	3	4	5	6
Δy	(mm)	0	0.5	0.05	0	0.5	0.5
γ	($^\circ$)	0	0	0	-1	-1	-1
Δx	(mm)	0	0	0	0	0	0.30
$\hat{\delta}$	(mm)	0.002	0.519	0.058	0.002	0.485	0.484
$ \Delta y - \hat{\delta} $	(mm)	0.002	0.019	0.008	0.002	0.015	0.015
$\hat{\gamma}$	($^\circ$)	-0.02	0.28	0.00	-1.02	-0.80	-0.80
$ \gamma - \hat{\gamma} $	($^\circ$)	0.02	0.28	0.00	0.02	0.20	0.20

More reliable estimations of the angular error ($|\gamma - \hat{\gamma}| \lesssim 0.02^\circ$), represented by test cases one, three and four, are only possible if an existing offset has been corrected in advance. Since an offset between the AUT and the probe antenna also results in an angular misalignment, the angle estimation depends on potentially existing offsets. This angular error in turn leads to a shift of the symmetry axis in the amplitude and phase pattern of the AUT, which in this case adversely affects the angle estimation.

Conversely, however, this dependency is only very weak, i.e. the estimation of the offset works reliably for all test cases considered. The maximum residual error is $|\Delta y - \hat{\delta}| \lesssim 0.02$ mm, which equals less than $\lambda/100$ at a frequency of 90 GHz and can, therefore, be regarded as reasonably precise.

This can also be derived analytically using the error model by taking the angular errors from (4.7) into account when calculating the phase difference in (4.4), resulting in

$$\begin{aligned} \psi_m(\theta - \gamma) - \psi_m(-\theta - \gamma) = \\ \psi_a(\theta - \gamma) - \psi_a(\theta + \gamma) + 2k \sin(\theta) (\delta \cos(\gamma) + \Delta z \sin(\gamma)) \end{aligned} \quad (4.10)$$

with reference to the symmetry requirement motivated for (4.5). As (4.10) confirms, an angular error is reflected in an additional scaling $\cos(\gamma)$ of the estimated offset. However, the gradient of the cosine function exhibits a minimum for $\gamma \approx 0^\circ$, which means that typical angular errors γ only have a negligible impact, for example less than 0.015 % for $\gamma = 1^\circ$.

Nevertheless, the estimated angle is used to shift the measurement data to the assumed axis of symmetry in advance of estimating the offset. Even if the angle estimation is affected by an offset, the additional shift contributes to an improved overall estimation performance regarding the offset. Although the combined estimation is possible in principle, as demonstrated in test cases five and six, the iterative approach generally achieves more reliable results. This is evident in comparison to test case four, which enables roughly a tenfold improvement in the estimation of the angular error and, thus, the iterative approach is considered throughout the following.

4.5 Alignment Technique

The methods presented provide a possibility to estimate offsets and angular errors based on single θ -cut measurements. So far, however, only the superposition of all potential types of offsets $\Delta y = \Delta y_{\text{axis}} + \Delta y_a + \Delta y_p$ and all kinds of angular errors $\gamma = \gamma_{\text{axis}} + \gamma_a(\phi_{\text{axis}}) + \gamma_p$ has been considered. In order to apply the error model described in Section 4.1 and, subsequently, to compensate for these alignment errors using the robot-based measurement system, it is necessary to isolate the contributions of the individual error terms.

For this purpose, the previous approach is extended by taking measurements for different positions of the rotation axis ϕ_{axis} . For each of these positions, a θ -cut is measured along the x - and y -axis, i.e. for $\phi_m = 0^\circ$ and $\phi_m = 90^\circ$, whereby the rotation axis position ϕ_{axis} is kept constant for both measurements. This allows to determine the offset $\hat{\delta}_x$ in the x -direction as well as $\hat{\delta}_y$ in the y -direction, independent of the rotation axis. As the estimation of the offset is robust against angular errors, only the offsets are determined in a first step and the angular errors are estimated separately in a second subsequent step.

The results of the offset estimation are visualized in Fig. 4.3 and illustrate both, the electrical alignment measurements as well as verification results of a camera-based approach. The measurement coordinate system of the robot (x_m, y_m, z_m) is at the origin of the two-dimensional plane, with the axis coordinate system $(x_{\text{axis}}, y_{\text{axis}}, z_{\text{axis}})$ plotted at the center of the measured offsets. The translation between these two coordinate systems is the AIO introduced in Section 4.1. As the measurement is repeated for different rotational axis positions, the estimated offsets follow a circular path. Both, the fitted error model of the electrical and the optical measurement are symbolized by a circular fitting of these offsets. Consequently, the radius from the axis coordinate system to the AUT coordinate system (x_a, y_a, z_a) on the circular path denotes the AMO, i.e. the mounting offset of the AUT from the axis of rotation of the additional axes. The position of the AUT coordinate system in relation to the axis coordinate system is determined via the radial AMO and a reference angle $\phi_{a, \text{ref}}$. The latter provides the link to the axis coordinate system for the reference position of the additional rotation axis specified to $\phi_{\text{axis}} = 0^\circ$.

To avoid affecting the estimation of the offset, electrical alignment measurements with low amplitude, i.e. for cross-polarized measurement constellations, are not taken into account. In addition, the angular step size of the electrical measurements is significantly higher than that of the optical measurement due to the measurement time required for a θ -cut. Therefore, the data basis of the electrical method is severely limited in comparison to the optical approach, but, however, the result of both approaches coincide. The AMO determined using the electrical method is estimated to 0.891 mm, whereas the camera-based approach results in an offset of 0.894 mm. The relative error between the two methods is, thus, less than 0.4 %, which validates the electrical approach. The camera-based approach requires an additional coordinate transformation from the image coordinates to the probe coordinates so that a reference can be established from the image data to the actual position of the probe antenna. Furthermore, a modification of the measurement setup is required so that a reference target can be mounted on the waveguide flange, which can then be tracked using image processing algorithms over one revolution of the rotation axis ϕ_{axis} . Fur-

ther details on the optical alignment process and an exemplary evaluated camera image of the reference target are provided in Section B. However, as both approaches coincide, but the electrical method is less invasive and establishes a direct reference to the coordinate system of the probe antenna, it is the recommended method.

Once the error model for reproducing the AIO and AMO has been determined and the misalignments can be compensated using the robot, a reliable estimation of the angular errors can be achieved. For this purpose, again, two θ -cuts are measured over the x - and y -axis, or $\phi_m = 0^\circ$ and $\phi_m = 90^\circ$, respectively, for different positions of the rotation axis ϕ_{axis} . Since a large angular range is not required for an accurate assessment compared to the estimation of the offsets, the angular range is selected to be larger than the maximum expected tilt in each case. In addition, a fine angular step size is advantageous for an accurate estimation of the angular errors. The measurement results presented below have been acquired over an angular range of $\theta_m \in [-9^\circ; 9^\circ]$ and an angular step size of $\Delta\theta_m = 0.05^\circ$. Based on the measurement data, an error model is fitted which reflects both, the AIT and the AMT as a

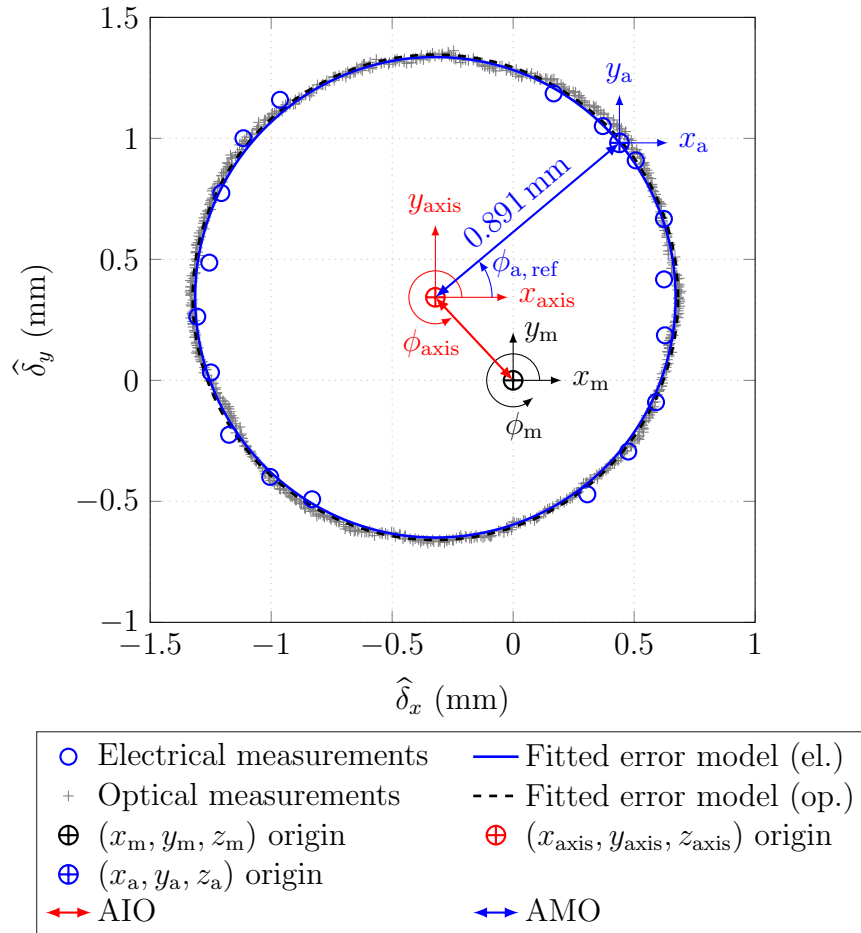


Figure 4.3: Visualization of the AIO between the measurement and axis coordinate system as well as the AMO between the axis and the AUT coordinate system. The electrical approach is validated by camera-based measurements, with the deviations between the two approaches amounting to less than 0.4 %.

function of the rotational axis position ϕ_{axis} , given by

$$\begin{pmatrix} \hat{\gamma} \\ \hat{\beta} \end{pmatrix} = \begin{pmatrix} \hat{\gamma}_{\text{axis}} \\ \hat{\beta}_{\text{axis}} \end{pmatrix} + \begin{pmatrix} \cos(\phi_{\text{axis}}) & -\sin(\phi_{\text{axis}}) \\ \sin(\phi_{\text{axis}}) & \cos(\phi_{\text{axis}}) \end{pmatrix} \cdot \begin{pmatrix} \hat{\gamma}_a \\ \hat{\beta}_a \end{pmatrix}, \quad (4.11)$$

whereby $\hat{\beta}$, $\hat{\beta}_{\text{axis}}$, and $\hat{\beta}_a$ describe the overall rotation around the y -axis, the isolated rotation over the y_{axis} -axis and the isolated rotation contribution around the y_a -axis, respectively. Thus, they correspond to the same scheme as the estimate $\hat{\gamma}$ of γ , which has been introduced in the error model presented in Fig. 4.1, but refer to rotations around the different y -axes.

The model presented in (4.11) describes the rotation of an arbitrarily oriented vector in space around the ϕ_{axis} -axis. The coordinate transformation is comprised of a static component $\hat{\gamma}_{\text{axis}}$ and $\hat{\beta}_{\text{axis}}$, also referred to as AIT, and a dynamic component $\hat{\gamma}_a$ and $\hat{\beta}_a$, also referred to as AMT, where the latter is a function of ϕ_{axis} . An exemplary measurement and the resulting error model are presented in Fig. 4.4, illustrating the overall estimated rotations $\hat{\gamma}$ and $\hat{\beta}$, i.e. around the x - and y -axis respectively, as a function of the rotational axis position ϕ_{axis} . In order to not impair the results, cross-polarized measurements with too low amplitudes are not considered in this evaluation either, so that the error model is not distorted. The model and measurement data provide a reliable correspondence, as evidenced by a root mean square (RMS) model mismatch of 0.02° only. Also the maximum deviation of 0.07° proves that the model reproduces the measurements accurately. This allows to determine the AIT around the x - and y -axis to $\hat{\gamma}_{\text{axis}} = 0.14^\circ$ and $\hat{\beta}_{\text{axis}} = 0.56^\circ$, respectively. For the AMT, the ϕ_{axis} -dependent angular errors are given at the coordinates at which no interaction between the two axes of rotation occurs, yielding $\hat{\gamma}_a = -0.44^\circ$ at $\phi_{\text{axis}} = 0^\circ$ and $\hat{\beta}_a = -0.41^\circ$ at $\phi_{\text{axis}} = 90^\circ$.

The analytical definition for both, the AIT and the AMT, therefore, allows to align the orientation of the measurement coordinate system (x_m, y_m, z_m) relatively to the AUT coordinate system (x_a, y_a, z_a) and, thus, to align the probe orientation in accordance with the AUT orientation for all positions of the additional rotation axis ϕ_{axis} .

In the case of electrical alignment, the measurement setup consisting of the AUT and probe antenna is aligned relatively to each other. This means that not all error contributions can be isolated from each other. The offset of the probe antenna from the mechanical axis of rotation ϑ_6 has already been introduced as the PMO in Section 4.1 and is taken into account as a contribution within the AIO during the alignment of the measurement setup. Consequently, different polarization angles χ of the probe antenna are not taken into account in the AIO. As an example, for a continuous measurement in θ and stepped in ϕ , the AIO has to be calibrated separately for both intended polarization angles, resulting in increased alignment efforts.

Alternatively, the contribution of the PMO can also be modeled if it can be measured independently of the AUT. Therefore, the optical method already known from the estimation of the AMO is an appropriate technique, whereby no dedicated reference target is used in the following, but the circular grooves of a scalar feed horn antenna are detected and tracked using image processing methods.

The evaluation of the resulting PMO is presented in Fig. 4.5 and is, similar to the AMO, defined by the radial distance from the axis of rotation ϑ_6 and a refer-

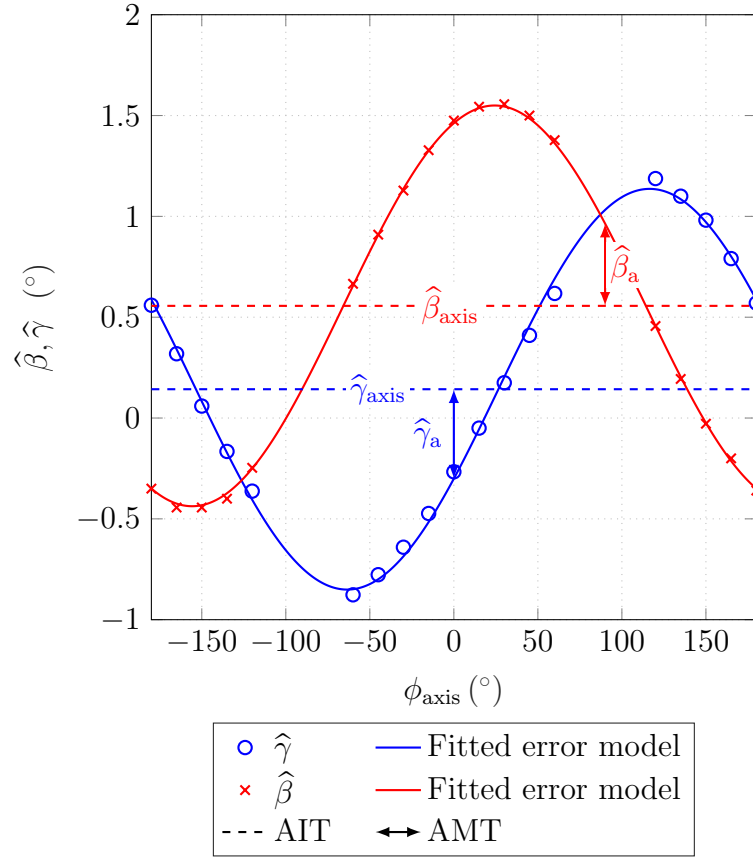


Figure 4.4: Visualization of the AIT between the measurement and axis coordinate system as well as the AMT between the axis and the AUT coordinate system. The RMS and maximum model mismatch error are less than 0.02° and 0.07° , respectively, proving that the model reproduces the measurement data.

ence angle $\phi_{\text{p,ref}}$. The measurement demonstrates that the probe antenna is offsetted approximately 0.11 mm from the axis of rotation, whereby significantly higher variations of the optical measurements are noticeable. On the one hand, this is due to the much smaller offset compared to the AMO, but, on the other hand, also due to the fine structures of the numerous grooves of the probe antenna, for example shown in Fig. B.2. These impede the unambiguous identification of round shapes and therefore lead to greater deviations in the evaluation based on image processing algorithms. For comparison, Fig. B.1 shows an image of the reference target used to estimate the AMO, where the clearly recognizable circular outline of the target allows for more precise processing. Nevertheless, the high amount of measurement data with an angular step size of $\Delta\chi = 0.5^{\circ}$ results in an acceptable average agreement. The RMS error is 0.016 mm only, whereby the maximum error is up to 0.040 mm, which confirms the feasibility of this approach.

By modelling the PMO, multiple realignments to minimize the AIO depending on the required polarization angle can be avoided, while also ensuring a precise alignment of the polarization axis χ between probe and AUT. This means that the robot can compensate for the spatial displacement of the probe during the rotation of the χ -axis, eliminating the effects of the PMO during the alignment of

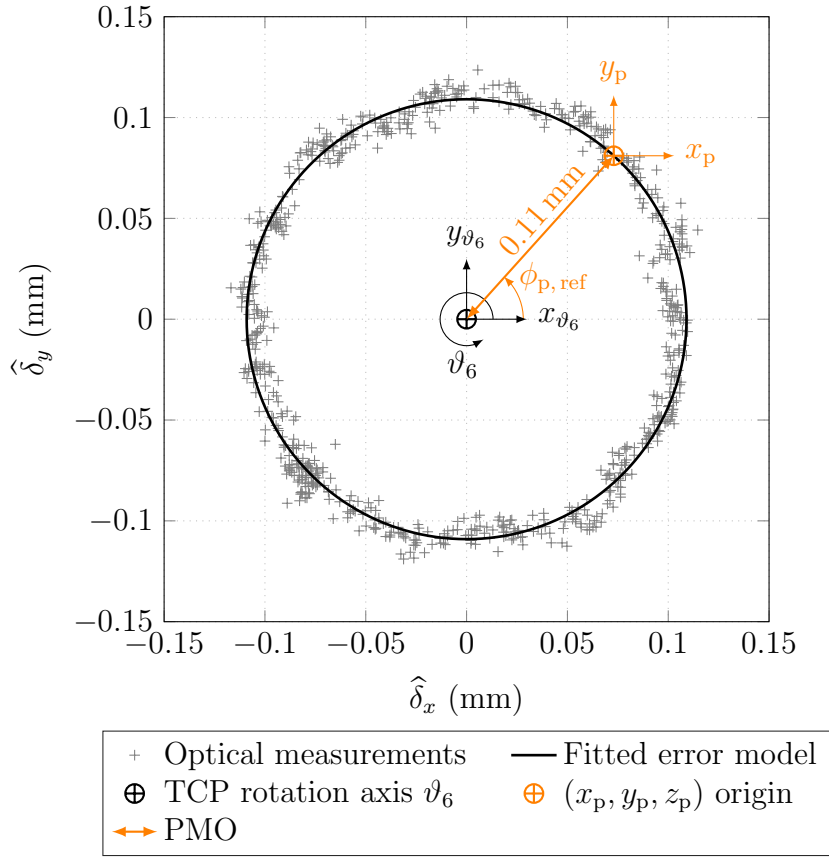


Figure 4.5: PMO between the TCP's axis of rotation ϑ_6 and the probe coordinate system (x_p, y_p, z_p) . The offset has been identified by optical measurements of the grooves of a scalar feed horn antenna used as the probe. The RMS and maximum model mismatch error are approximately 0.016 mm and 0.040 mm, respectively.

the PPM. This is demonstrated in Fig. 4.6 for a minimum search at $\theta = 0^\circ$ of a cross-polarized measurement to align the PPM. Here, the polarization angle has been varied within $\chi \in [-94^\circ; -86^\circ]$ using an angular step of 0.05° . Averaged over the measured frequencies, the minimum is found at $\Delta\chi_{\min} = -89.04^\circ$, representing a PPM of $\Delta\chi = 0.96^\circ$. However, for rotations of $\Delta\chi \lesssim 1^\circ$, the PMO will have a negligible effect. In addition, the displacement of the probe antenna caused by the PMO is perpendicular to the direction of propagation at each sampling point and, therefore, only has a slight effect on the measured phase. For this constellation, a conservatively estimated offset of 0.2 mm causes a phase disturbance of only $\Delta\psi_\perp \leq 0.015^\circ$ at a frequency of 90 GHz. For comparison, the same offset parallel to the direction of propagation would cause a phase error of $\Delta\psi_\parallel > 20^\circ$.

However, it is generally advisable to take the PMO into account, for example for the measurements demonstrated in Section 7 in which the robot samples larger parts of the measurement sphere and, thus, larger rotations of the polarization axis will occur.

In order to evaluate the reproducibility of the alignment procedure, two measurements have been independently aligned at different measurement distances and the results transformed into far field are compared in Fig. 4.7. As part of a provocation

test, an additional inclined waveguide has been mounted before the AUT to induce increased offsets and angular errors. The agreement between the two measurements is evaluated using the EES introduced in (3.1). Despite the provoked offsets and misalignment, the maximum EES between both measurements is less than -40 dB. In average, an EES of -56 dB is achieved, which demonstrates the consistency and, thus, the effectiveness of the alignment method.

4.6 Verification with CATR Results

After the reproducibility of the alignment procedure for different measurement radii has been examined in Fig. 4.7, a further verification of the method and its underlying error model is provided using comparative measurements of a CATR. Although no uncertainty analysis is available for the CATR in the frequency range from 60 GHz to 90 GHz, it is an established measurement facility with well-known alignment, calibration and measurement procedures and is, therefore, suitable to verify the results obtained by the experimental robot-based system. Since the flexibility of the robot system enables direct far-field as well as near-field measurements, these two methods are compared separately with the results achieved in the CATR.

4.6.1 Direct Far-Field Measurements

Depending on the size of the AUT, the robot's range of motion enables direct far-field measurements, especially for measurements in the millimeter-wave range. On the one

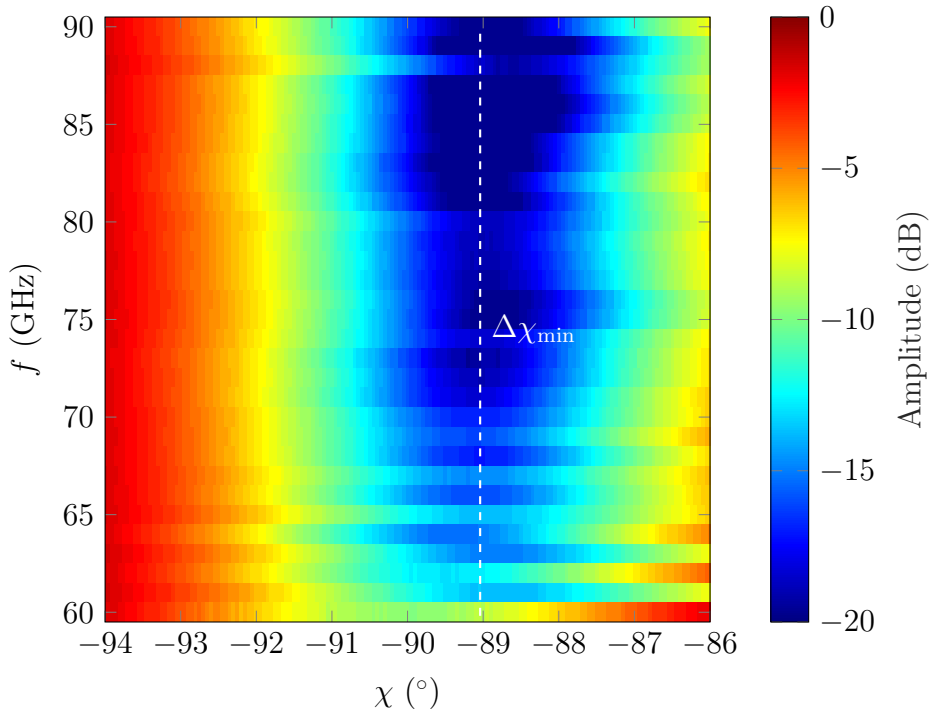


Figure 4.6: Alignment of the polarization-axis based on the minimum amplitude of a cross-polarized measurement. Averaged over the frequency, the minimum is identified at $\chi = \Delta\chi_{\min} = -89.04^\circ$ yielding a PPM of $\Delta\chi = 0.96^\circ$, respectively.

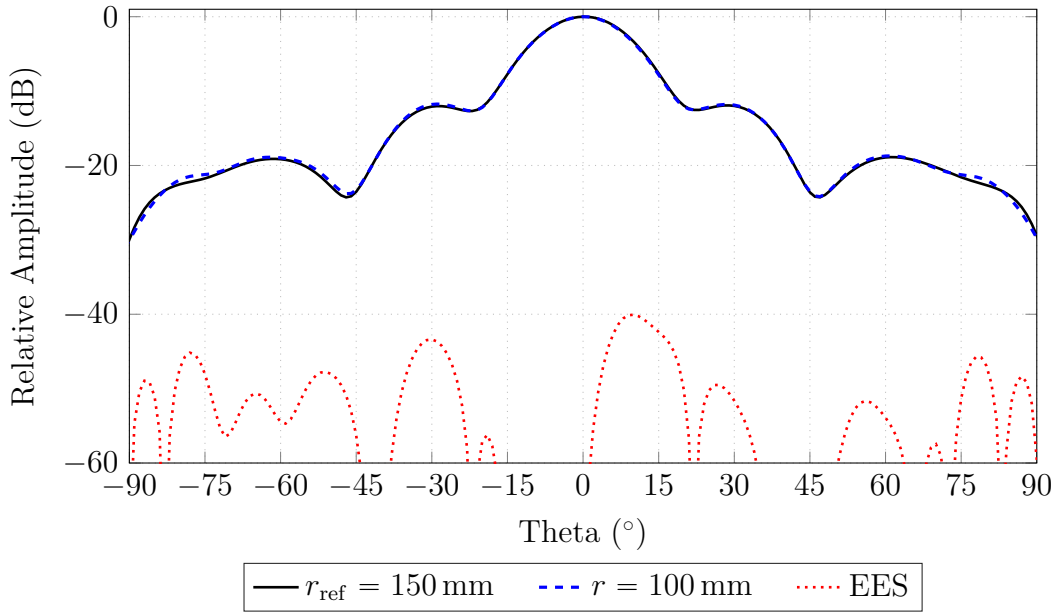


Figure 4.7: Comparison of the results transformed into far field for two separately aligned measurements at a reference measurement distance of $r_{\text{ref}} = 150$ mm and at $r = 100$ mm. As a provocation test, the alignment measurements have been carried out using an additional inclined waveguide before the AUT, which contributes to larger offsets and angular errors.

hand, measurements in the far field are more robust against misalignment due to the increased measurement distance. On the other hand, no transformation into the far field is required, in which the measured phase is highly sensitive to misalignments. For this reason, direct far-field measurements are suitable as an initial verification before near-field measurements are investigated in Section 4.6.2.

Therefore, the AUT described in Section C.1 has been used for this purpose, which exhibits a far-field distance of $d_{\text{FF}} = 294$ mm at a frequency of 60 GHz. In order to comply with the far-field condition, two measurements have been performed using a radius of $r_1 = 400$ mm and $r_2 = 500$ mm, which are compared with the results from the CATR in Fig. 4.8. All measurements show the E -plane of the AUT ($\phi = 0^\circ$), whereby the EES always refers to the difference to the CATR results for both measurement radii. To enable a comparison of the raw data, the data has been acquired with an angular step size of 1° . The comparison proves that the robot-based measurements reproduce the results of the CATR, with an EES of less than -40 dB in a wide angular range. Although both measurements have been conducted in the far field and should, therefore, be identical, slight deviations can be observed, particularly in the nulls. While the deviations in the first null at $\theta \approx 23^\circ$ are barely detectable due to the high signal level, the effects become more apparent for the second null at $\theta \approx 48^\circ$. In this measurement position and for the relatively large measurement radius, the absorber ring mounted on the robot's TCP (cf. Fig. A.2) no longer prevents multipath propagation between AUT, robot and probe antenna, while even the slightest interference become traceable due to the lower signal levels. As the interference in the second zero differs depending on the measurement

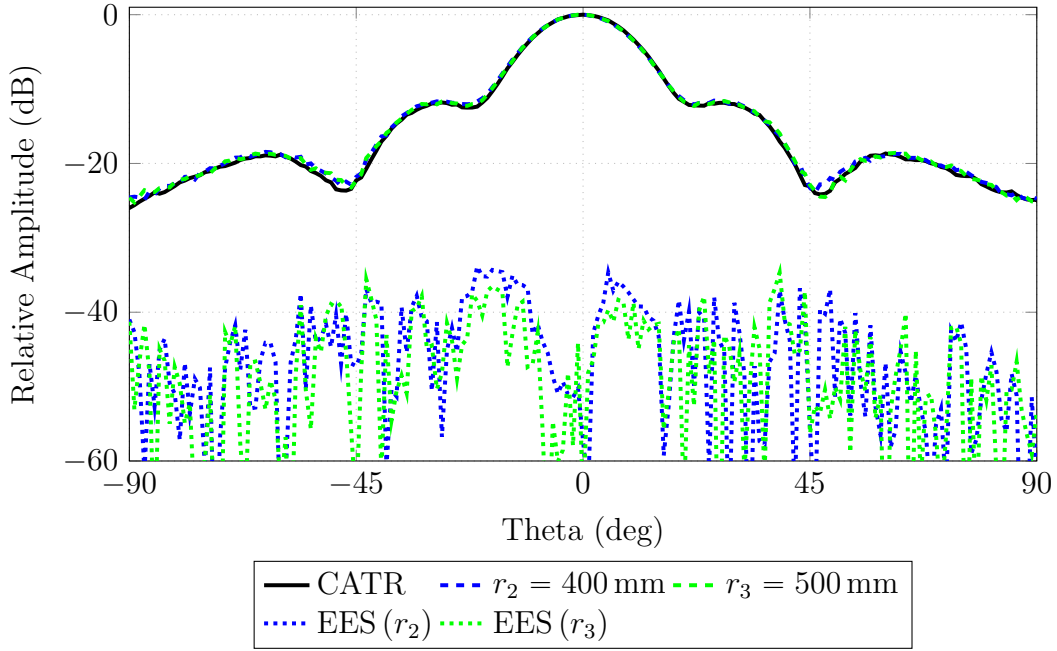


Figure 4.8: The AUT's E -plane of a robot-based direct far-field measurement in comparison with the results from a CATR at a frequency of 60 GHz. For both measurement radii, the EES denotes the difference to the results of the CATR.

radius, room scattering caused by a reflective environment in the immediate vicinity of the measurement volume is a possible reason for the caused deviations. In addition, the EES decreases for increasing measurement radii and, thus, undermines that room scattering possibly affects the second nulls.

Nevertheless, the main lobe is well reproduced and even at the edge region of the measurement no amplitude ripples caused by room scattering are recognizable. However, to further minimize effects caused by room scattering in far-field measurements, an optimized absorber layout or a more directive probe, which acts as a spatial filter and, thus, attenuates room scattering more strongly, is recommended.

4.6.2 Near-Field Measurements

In contrast to far-field measurements, measurements in the near field are significantly more sensitive to misalignments, which are immediately reflected in the measured phase and, thus, impact the result of the NFFT. However, the significantly smaller measurement radius ensures that the absorber ring mounted on the robot's TCP is more effective and the probe antenna, therefore, receives fewer stray signals from the reflective environment. For this reason, near-field measurements transformed into the far field are considered in a second verification step. To ensure comparability, the measurement data acquired in the CATR are also transformed (from far field to far field) and are, thus, subject to the same modal filtering as the robot-based measurements.

The robot-based measurements have been carried out at a measurement radius of $r = 200$ mm. All measurements have been performed with an angular step size of $\Delta\theta = \Delta\phi = 3^\circ$, including mode truncation with $N_{m,n} = 34$, which corresponds to

the minimum sphere of the AUT at the upper frequency bound of 90 GHz. Analogous to the far-field measurements shown in Section 4.6.1, the robot-based near-field measurements have been conducted using an OEWG probe antenna, which has a broad receive characteristic and is, therefore, sensitive to scattering effects caused by a reflective environment. In addition, a HOP correction with $N_{\mu,\nu} = 12$, which corresponds to the minimum sphere analogous to the consideration of the AUT, is applied in order ensure the best possible consideration of the probe's receive characteristics in the NFFFT.

A qualitative comparison of the transformed results of both measurement facilities is given for $\chi = 0^\circ$ in Fig. 4.9 and for $\chi = 90^\circ$ in Fig. 4.10 whereby a frequency of $f = 60$ GHz is considered in the following, so that the mode spectra of both measurements are slightly overdetermined. For $\chi = 0^\circ$, the comparison shows that the side lobes of the robot-based measurement appear more uniform, especially for the θ -cut at $\phi = 0^\circ$ in contrast to $\phi = 180^\circ$. In addition, a better suppression of the cross-polarization is evident for $\theta \approx 0^\circ$ and $\phi = 90^\circ$, which has for the CATR measurement an increased amplitude and also a slightly blurred profile. Also the lower amplitude regions for $\theta \gtrless \pm 45^\circ$ and $45^\circ < \phi < 135^\circ$ show a more distinct pattern with fewer ripples for the robot-based measurement.

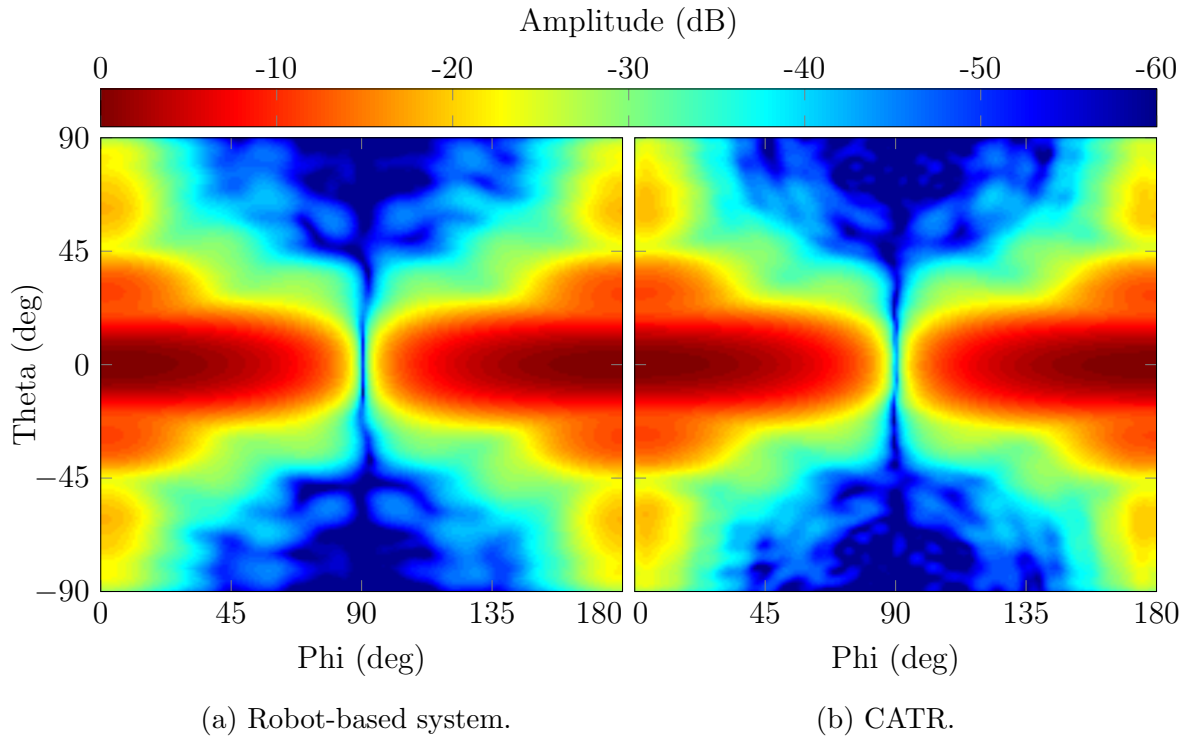


Figure 4.9: Qualitative comparison of the results obtained with the robot-based system and the CATR for $\chi = 0^\circ$ and $f = 60$ GHz. Both transformation results have been obtained with the same mode truncation, whereby a HOP correction has been applied to the near-field measurements of the robot-based system.

For the second polarization angle $\chi = 90^\circ$ given in Fig. 4.10, the differences are more significant. Although the high amplitude regions are comparable between the two measurement systems, there are considerable deviations, particularly for $\theta > 45^\circ$

and $\phi < 90^\circ$, which are emphasized by the elliptical marking. For the results obtained in the CATR, the regions of lower signal level exhibit obvious amplitude ripples, which are asymmetrical and, for example, do not or only weakly occur for $\theta < -45^\circ$. The system is a side-fed single-reflector CATR, which is shown in Fig. B.3, including the measurement setup used. Both, the asymmetrical occurrence and the strongest amplitude ripples for the angular range in which the AUT is oriented towards the feed suggest that these are caused by spill-over effects of the feed. In contrast, no significant amplitude ripples occur in the robot-based measurement. There are also no asymmetries recognizable, which could be caused, for example, by the partially reflective environment or, more specifically, the robot arm. Accordingly, the absorber ring mounted on the robot's TCP provides an effective attenuation of stray signals in near-field measurements, which is further enhanced by the typically higher measurement dynamics due to the significantly smaller measurement distance. Although Fig. 4.9 and Fig. 4.10 are only qualitative comparisons, which are not appropriate for analyzing specific details, they provide a clear overview and, thus, visualize the advantages of the robot-based near-field measurements.

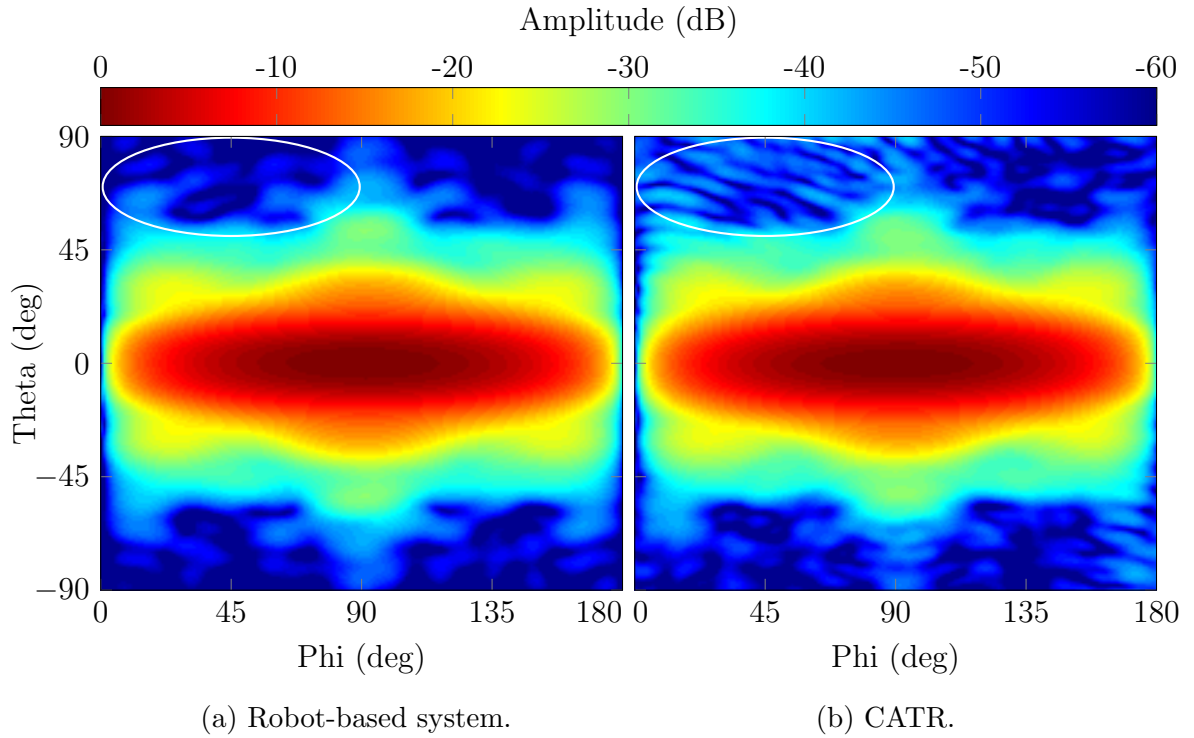


Figure 4.10: Qualitative comparison of the results obtained with the robot-based system and the CATR for $\chi = 90^\circ$ and $f = 60$ GHz. Both transformation results have been obtained with the same mode truncation, whereby a HOP correction has been applied to the near-field measurements of the robot-based system.

For a detailed comparison between the CATR and robot-based results, it is referred to Fig. 4.11. Here, a θ -cut for $\phi = 0^\circ$ is plotted for both polarizations, whereby the difference between the respective measurements can be assessed using the EES. Both measurement systems demonstrate a good agreement for large angular ranges in which the EES is less than -40 dB. Only the second null at $\theta = 39^\circ$

exhibits slightly larger deviations, which are in the range of -38 dB. The deviations for $|\theta| > 85^\circ$ are caused by the truncation error, resulting in invalid and, therefore, incomparable data in this angular range. Nonetheless, the average EES for the presented cut, including the increased deviations due to the truncation error, is approximately -53 dB and, thus, proves the good agreement between the robot-based measurements and the CATR. The cross-polarization suppression of the CATR is limited due to the single reflector layout and is approximately -41 dB in the area of the main beam ($|\theta| < 15^\circ$). Since the robot-based near-field measurements are not subject to this limitation, a significantly improved cross-polarization suppression of -51 dB is determined. Even though this depends on the OEWG probe antenna used, it results in an improvement of approximately 10 dB compared to the CATR.

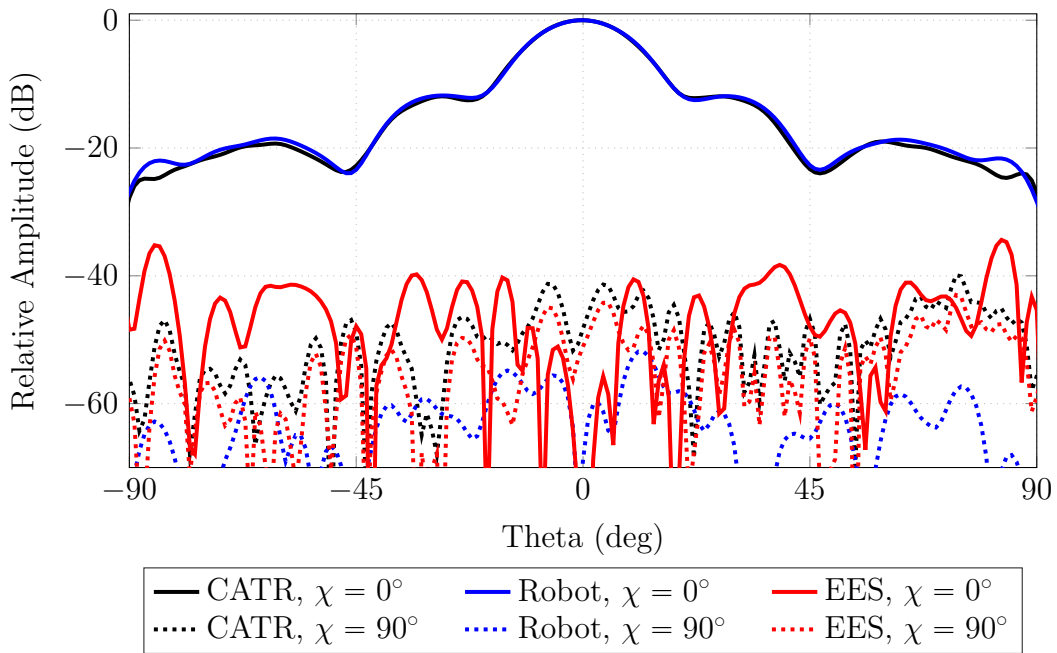


Figure 4.11: Comparison of a θ -cut for $\phi = 0^\circ$ and both polarizations. The presented results have been obtained with the same mode truncation, whereby a HOP correction has been applied to the near-field measurements of the robot-based system.

After the reproducibility of the alignment procedure for different measurement radii has already been proven in Section 4.5, the comparative measurements demonstrate the validity of the analytically derived error model as well as the derivation of the parameters required for its application. The achieved EES in the order of approximately -40 dB compared to the CATR represents an acceptable correspondence, thus validating the measurement capability of the robot-based system for subsequent analyses.

4.7 Summary

For spherical near-field measurements, especially in the millimeter-wave range, the alignment between the AUT and probe antenna is of crucial importance. As the robot provides the necessary flexibility to compensate for certain uncertainties, as

for example mounting tolerances or tiltings, an error model is derived which contains the significant error sources specifically for the developed measurement setup. In particular, the decoupling of the rotation along ϕ performed by the additional rotation axis ϕ_{axis} from the position in θ approached by the robot arm poses a challenge in terms of precise alignment. Therefore, the following six terms have been identified as the most relevant contributions:

- Axis intersection offset (AIO): Describes the offset between the measurement coordinate system of the robot (x_m, y_m, z_m) and the axis coordinate system of the additional axes $(x_{\text{axis}}, y_{\text{axis}}, z_{\text{axis}})$. The AIO is determined along both, the x -axis and the y -axis.
- AUT mounting offset (AMO): Represents the offset of the AUT coordinate system (x_a, y_a, z_a) to the axis coordinate system $(x_{\text{axis}}, y_{\text{axis}}, z_{\text{axis}})$. Due to the additional axis of rotation, the offset is not defined Cartesian, but via a radius and a reference angle for $\phi_{\text{axis}} = 0^\circ$.
- Axis intersection tilt (AIT): The static tilt, i.e. independent of the additional rotation axis ϕ_{axis} , between the measurement coordinate system of the robot (x_m, y_m, z_m) and the axis coordinate system $(x_{\text{axis}}, y_{\text{axis}}, z_{\text{axis}})$. It is expressed by rotations along the x - and y -axis.
- AUT mounting tilt (AMT): Dynamic tilting of the AUT coordinate system (x_a, y_a, z_a) in relation to the robot's measurement coordinate system (x_m, y_m, z_m) depending on the additional rotation axis ϕ_{axis} . The tilts are described by rotations along the x - and y -axis.
- Probe mounting offset (PMO): Offset of the aperture of the probe antenna to the axis of rotation of the TCP ϑ_6 . Analogous to the AMO, it is defined by a radius and a reference angle.
- Probe polarization mismatch (PPM): Twist of the polarization axes of AUT and probe relative to each other. Determined by a minimum search of a cross-polarized measurement and referred to as $\Delta\chi$.

In addition, an efficient measurement method for the determination of offset and tilt is presented, which, based on a symmetry assumption, allows both uncertainties to be estimated based on a single θ -cut measurement and, thus, reduces the measurement time. Since the identification of the individual error contributions requires a large number of measurements, this is of particular importance for the practical applicability of the proposed alignment procedure. In addition, analyses are conducted on the interdependence of the individual error terms, the reproducibility of the measurements for different radii, comparative measurements of the determined offsets using an optical reference measurement as well as a comparison with a CATR.

In this regard, both, direct far-field measurements of the robot-based measurement system as well as near-field measurement data transformed into the far field are compared with the results of the CATR. The compensation of the identified error terms by means of the robot arm allows, depending on the measurement method, to achieve an agreement to the CATR of -40 dB, expressed in terms of the EES. Thus, the result is within the sensitivity analysis presented in Section 3.1 and proves the functionality of the alignment procedure as well as the measurement capability of the developed robot-based test range.

Pointwise Probe Correction

With the steady increase in commercially available computing power and storage capacity, the efficient approach to transforming to the far-field via Fourier transforms is continuously losing momentum. Instead, methods that allow greater flexibility are becoming more important, in particular the possibility for a pointwise probe correction (PPC). For the common case of dual-polarized probes, approaches for the compensation of amplitude and phase variations exist, but it is still assumed that the radiation characteristics are identical for both ports [80]. In reality, this cannot be achieved and, thus, additional sources of errors in the transformation result are introduced. Consequently, these uncertainties increase in the case multi-probe systems, where a large number of probes with their respective tolerances are employed [83, 86, 123]. The PPC proposed in [124] allows not only to consider a certain amount of different probes as for multi-probe systems, but even for each individual sampling point.

In particular the last aspect, the consideration of the probe's actual position and orientation, is important for robot-based measurements. It makes it possible to dispense with complex path corrections of the robot, which involves intensive calibration work and sometimes iterative procedures to optimize the physical path of the robot so that the sampling points can be reached as precisely as possible. Instead, the PPC provides the possibility to correct the non-ideal positions and orientations during post-processing of the measurement, as long as the actual sampling position is known at the end of a measurement.

The PPC is an elegant method to consider this mechanical problem in the theory of the SWE and to solve it without additional measurements or path optimization approaches. In the following, the theoretical basis is explained first and, then, the practical application in actual measurements is presented. Furthermore, the functional principle is verified using two comparative measurements, specifically a well-aligned measurement and a provocation test with intentionally induced misalignments. It is demonstrated that a conventional NFFFT of the offset measurement fails, whereas the application of the PPC enables the accurate reconstruction of the radiation behavior.

5.1 Theory

The consideration of arbitrary probe antenna characteristics in the SWE is based on the extension of the general transmission formula (2.17) corresponding to

$$w^{(l)}(r, \phi, \theta, \chi) = v \sum_{\substack{smn \\ \mu}} T_{smn} e^{im\phi} d_{\mu m}^n(\theta) e^{i\mu\chi} P_{s\mu n}^{(l)}(kr) \quad (5.1)$$

so that it comprises different probe response coefficients

$$P_{\sigma\mu n}^{(l)}(kr) = \frac{1}{2} \sum_{\sigma\nu} C_{\sigma\mu\nu}^{sn(c)}(kr) R_{\sigma\mu\nu}^{(l)p} \quad (5.2)$$

for each measurement point (l) or the l -th probe's spectral representation $R_{\sigma\mu\nu}^{(l)p}$, respectively. In contrast to the derivation, which considers the special case of a transmitting AUT and a receiving probe, the generic radial dependence (c) is assumed here without restriction of generality. However, for multi-probe systems, this allows the SMC associated with the specific probe to be taken into account in the transformation process. In addition, different measurement radii r can be considered using the translation operator $C_{\sigma\mu\nu}^{sn(c)}(kr)$ of (5.2). If the measurement radius is constant, it is possible to calculate the translation only once and apply it to all probe response coefficients in order to reduce the computational costs. The resulting system of equations can be solved by direct or iterative methods, introducing no restrictions on the measurement positions. In sum, arbitrary probe positions, radii, and probe antennas are, thus, made possible, which clearly demonstrates the flexibility of this approach.

Further degrees of freedom exist with respect to the orientation of the probe, which has not been considered in (5.2) so far. Nevertheless, it is possible to represent the actual orientation by additional rotations of the SMCs $R_{\sigma\mu\nu}^{(l)p}$, however, introducing different spectral representations of physically identical probes. Therefore, the physically more explicit formulation also used in [53] of the probe response coefficients

$$P_{\sigma\mu n}^{(l)}(kr, \alpha, \beta, \gamma) = \frac{1}{2} \sum_{\sigma\nu} C_{\sigma\mu\nu}^{sn(c)}(kr) \sum_{-\mu'}^{\mu'} e^{i\mu\alpha} d_{\mu'\mu}^{\nu}(\beta) e^{i\mu'\gamma} R_{\sigma\mu'\nu}^{(l)p} \quad (5.3)$$

is introduced which establishes a direct reference to the respective rotation operators and, thus, separates them from the mode spectra of the physical probes.

Here, the receiving modes of the probe in the non-rotated coordinate system (x_p, y_p, z_p) defined in Fig. 2.3 are given analogously by $R_{\sigma\mu'\nu}^{(l)p}$. The Euler angles (α, β, γ) describe the relative rotation of the coordinate system of the probe, where α describes the rotation around the z_p -axis, β the rotation around the rotated y'_p -axis and γ the rotation around the already twice rotated z''_p -axis. Since rotations of the probe in θ (or $\beta \neq 0$) lead to a shift of the power in the mode spectrum towards higher-order modes, depending on $|\beta|$, these may no longer be negligible. Thus, a HOP correction becomes mandatory even for actual FOP with $\mu = \pm 1$.

A typical application of the PPC are offset measurements. Since offsets lead to an increased minimum sphere, they can conventionally only be transformed by oversampling, which increases the measurement time. However, if the offsets are known, PPC provides a mathematically exact transformation into the far field without any form of oversampling. This is demonstrated for measurements in a CATR in [53], where the orientation variations of the probe are determined by the dimensions of the offset and the measurement geometry due to the employed roll-over-azimuth positionier. However, robot-based measurement setups are able to achieve almost arbitrary orientations at any measurement point, whereby the PPC is the required counterpart to support this capability within the theory of the SWE. For this purpose,

the PPC has been adapted to the robot geometry and the applied coordinate systems in [125]. The implementation has been verified by intentional offset measurements without any form of oversampling. Therefore, the coordinate systems of such offset measurements are summarized again in Fig. 5.1. The measurement coordinate sys-

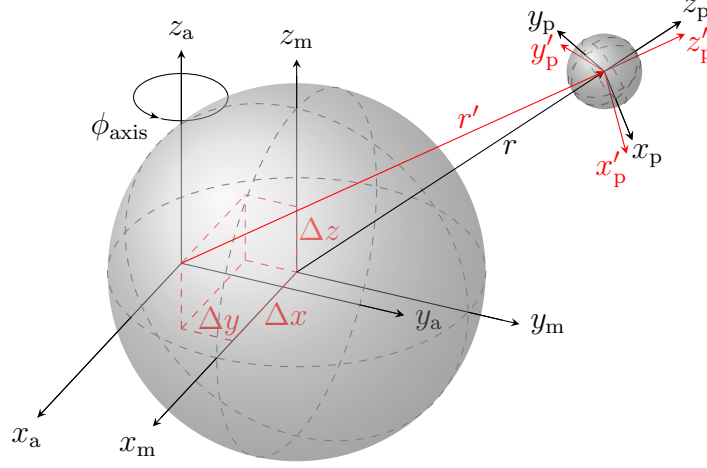


Figure 5.1: Simplified illustration of the translation between AUT (x_a, y_a, z_a) and measurement coordinate system (x_m, y_m, z_m) and its effects on the position and orientation of the probe coordinate system (x'_p, y'_p, z'_p) compared to the coordinate system in the ideal case (x_p, y_p, z_p) .

tem (x_m, y_m, z_m) describes the spherical surface scanned by the robot, which, in this case, is shifted by the offsets $(\Delta x, \Delta y, \Delta z)$ to the physical position of the antenna, represented by the AUT coordinate system (x_a, y_a, z_a) . The translation results in a probe pointing misalignment with respect to the AUT, making it impossible to transform the measurement without errors. In contrast to conventional roll-over-azimuth positioners, the measurement cannot be transformed by oversampling either, since an additional rotation axis denoted as ϕ_{axis} has been used to rotate around ϕ and, hence, results in a AIO to the rotation along θ_m in the measurement coordinate system (x_m, y_m, z_m) of the robot. Thus, the PPC represents the only possibility to transform the resulting irregular sampling grid. Therefore, the actual radius in the measurement coordinate system assumed to be in the origin is calculated according to

$$r' = \sqrt{(\Delta x - x_p)^2 + (\Delta y - y_p)^2 + (\Delta z - z_p)^2} \quad (5.4)$$

and used as the translation distance in (5.3). There are two possibilities to consider the actual orientation: On the one hand, it can be considered in the explicit representation of (5.3) as a kind of pre-distortion, which only describes the change of the disturbed to the undisturbed orientation. On the other hand, however, the actual orientation can also be integrated into the rotation operators of (5.1), which actually represent the ideal orientation of the probe with respect to the sampling grid. Thus, the integration within these rotation operators allows to reduce the computational effort as they are only calculated once. Although this would allow to remove the rotation operators from (5.3), it undermines the purpose of the explicit representation and is, therefore, not considered in the following. Various approaches exist

to determine the misorientation, one of the most intuitive being to rotate a reference orientation R_{ref} by the ideal Euler angles and then rotate back by the actual Euler angles. The representation of the reference orientation by a rotation matrix offers the flexibility to take different rotation conventions into account. For example, depending on the robot system used, this is possible by

$$\tilde{R} = \underbrace{R_z(-\alpha) R_y(-\beta) R_x(-\gamma)}_{R_{\text{actual}}^{-1}} \underbrace{R_x(\chi) R_\theta(\theta) R_\phi(\phi)}_{R_{\text{ideal}}} R_{\text{ref}} \quad (5.5)$$

where the $\phi \theta \chi$ rotations (also called $z y' z''$ or x -convention) are combined with the intrinsic $z y' x''$ rotations (also referred to as roll-nick-gear convention), e.g. used by Kuka robots. The rotation matrices R_{actual} and R_{ideal} summarize the rotations of the actual and ideal probe orientation in relation to a reference orientation R_{ref} , whereby the inversion of R_{actual} is required to compensate for the reversed rotation sequence. Finally, the orientation mismatch $\tilde{R}$ is factorized according to the x -convention by

$$\tilde{R} = R_z(\tilde{\chi}) R_y(\tilde{\theta}) R_z(\tilde{\phi}) \quad (5.6)$$

so that the resulting rotations $\tilde{\phi}$, $\tilde{\theta}$, $\tilde{\chi}$ can be used to pre-distort the probe response coefficients in (5.3) so that they match the actual orientation for each sampling point.

Although the presented approach to determine the rotations for a pre-distortion of the probe response coefficients is intuitive, numerical problems may arise. Especially for larger rotations or for orientations close to a singularity, the representation via Euler rotations can lead to major discrepancies. As an alternative, a solution using quaternions is recommended, which is numerically more stable and is not affected by critical axis constellations such as gimbal locks [18, 126, 127]. Therefore, the ideal and actual orientation are first converted into a representation form based on quaternions

$$Q_r = \mathcal{Q}\{R_{\text{ideal}} R_{\text{ref}}\} \quad (5.7)$$

$$Q'_r = \mathcal{Q}\{R_{\text{actual}} R_{\text{ref}}\} \quad (5.8)$$

where $\mathcal{Q}\{\cdot\}$ denotes the corresponding conversion operator from a rotation matrix to quaternions. Subsequently, the rotation of Q_r to Q'_r is determined as

$$\Delta Q_r = Q'_r \cdot \overline{Q_r} \quad (5.9)$$

with $\overline{Q_r}$ describing the conjugation of the quaternion. The calculated change in orientation is then converted back into a notation based on rotation matrices

$$\tilde{R} = \mathcal{Q}^{-1}\{\Delta Q_r\} \quad (5.10)$$

and transformed into the orientation mismatch corresponding to the rotation operators given in (5.3) using the factorization already described in (5.6).

Irrespective of the method used, based on quaternions or rotation matrices, the individually corrected probe response coefficients are then included in the NFFT. In order to also incorporate the non-ideal position of a sampling point, either the direct approach described in Section 2.3.2 using a matrix inversion or the iterative method explained in Section 2.3.3 is applied to calculate a solution based on PPC.

5.2 Procedure

The procedure for the application of the PPC in the robot-based measurement setup described in Section 3 is explained in the following. It is based on saving both, the ideal and the actual position and orientation for each sampling point during a measurement. Therefore, the actual coordinates (and orientations) are provided by the RC at the exact time of a trigger signal and subsequently communicated to the measurement software. Although this process is interrupt-based and, consequently, involves only minimal time delays, these data are still subject to greater tolerances compared to external measurement systems, such as laser trackers. This is due to the fact that the robot's kinematic model is used to calculate its position and orientation. This model is always subject to error, regardless of whether an absolute calibration is used, and, therefore, represents a fundamental difference to the direct measurement. Nevertheless, this information can be acquired without additional costs or changes to the measurement setup, thus representing an attractive compromise for many applications.

In a first step, the actual measurement coordinates are corrected, i.e. the transformation of the measurement grid performed in Section 4 to compensate for mechanical non-idealities such as offsets or angular errors is reversed. Hereby, the measurement coordinates are referenced back to the AUT coordinate system, whereby the actual position data lead to an irregular sampling grid and the orientation of the probe antenna is also considered non-ideal. In addition, if the measurement has been conducted using the additional rotation axis ϕ_{axis} , its position is taken into account in the measurement coordinates. In other words, the positions as well as orientations are rotated around the z_{axis} -axis by the angle ϕ_{axis} so that the resulting measurement grid ultimately describes the intended hemisphere.

In a second step, a perspective projection of the calculated measurement coordinates is executed. This is necessary because the mounting structure of the probe antenna is not parameterized in the RC, but in the measurement software. The advantage being that the setup on the robot's TCP can be changed flexibly without robot-side modifications to the software or parameterizations. Instead, the probe antenna setup is included within the measurement software, which calculates the measurement coordinates required for the robot, including the particular probe antenna used. For this purpose, the probe extent $(e_{p,x}, e_{p,y}, e_{p,z})$ which describe the position of the aperture of the probe antenna in the flange coordinate system (x_f, y_f, z_f) of the robot's TCP, is projected into the actual probe coordinate system (x'_p, y'_p, z'_p) , taking into account the specific orientation R_{actual} according to

$$\begin{pmatrix} x'_p \\ y'_p \\ z'_p \end{pmatrix} = R_{\text{actual}} R_{\text{ref}} \cdot \begin{pmatrix} e_{p,x} \\ e_{p,y} \\ e_{p,z} \end{pmatrix} + \begin{pmatrix} x_f \\ y_f \\ z_f \end{pmatrix}. \quad (5.11)$$

If preferred, the phase center of the probe antenna can be used in (5.11) instead of the physical extent $(e_{p,x}, e_{p,y}, e_{p,z})$. However, since the phase center is frequency-dependent and may not be known for all probe antennas, this approach is not examined hereinafter. For better visualization, the probe coordinate system (x_p, y_p, z_p) is, here and in the following, oriented inwards, i.e. towards the AUT. In compari-

son, Fig. 5.1 depicts the coordinate system oriented outwards, which is achieved, for example, by rotating the SMC accordingly.

The approach is exemplified in Fig. 5.2, which visualizes the perspective projection for the first four sampling points of an irregular measurement. For the sake of clarity,

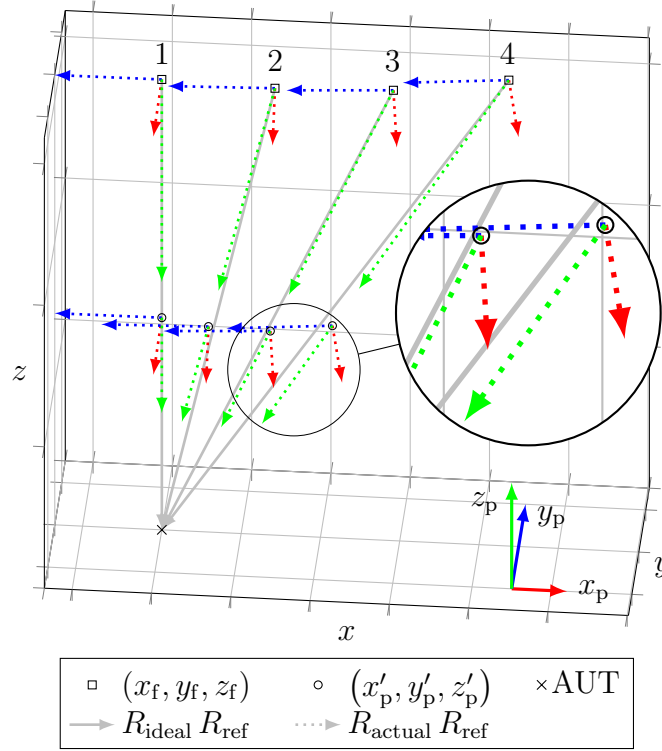


Figure 5.2: Visualization of the perspective projection from the flange coordinate system (x_f, y_f, z_f) to the probe coordinate system (x'_p, y'_p, z'_p) based on the actual orientation R_{actual} . For the fourth sampling point, no orientation has been specified resulting in particularly large deviations, which will be compensated using PPC.

only the z -axis shows the full range of values, which is approximately 320 mm. Compared to the z -axis, the x - and y -axes are scaled 5 and 10 times larger, respectively, to improve comprehensibility. Furthermore, only the z -vector of the ideal orientation is given (gray colored) in order to emphasize the differences to the actual orientation. It can be clearly seen that the actual orientation of the probe antenna must be taken into account for the projection from the flange to the probe coordinate system, as the radial projection $R_{\text{ideal}} R_{\text{ref}}$ does not reflect the effective measurement scenario. The first sampling point marks the start of a measurement and is, therefore, achieved with a precise orientation, as the robot stops briefly at this point. Sampling points two and three are, subsequently, traversed in a continuous path and also show only slight deviations from the ideally planned orientation. The fourth sampling point, in contrast, reveals a clearly deviating orientation, which can be recognized by the non-parallelism of the actual z -vector to the marked ideal orientation given in gray color. In this case, this is due to the fact that this is an additional sampling point (further information on this is provided in Section 6.2) for which no orientation has been specified. Thus, the robot chooses an arbitrary orientation to ensure an optimal

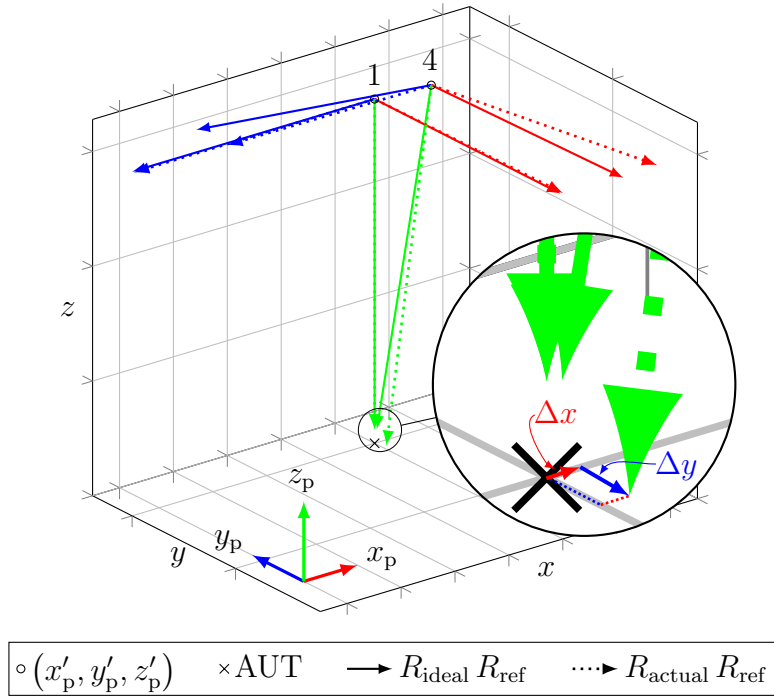


Figure 5.3: Based on the probe antenna coordinate system (x'_p, y'_p, z'_p) , the ideal and actual orientation for sampling points one and four are compared. For better visualization, the z -vector is shown in full length so that the pointing offset to the AUT becomes evident.

path, which results in the relatively large deviation. Therefore, this sampling point is particularly suitable for illustrating the functional principle of the PPC.

To evaluate the impact of this kind of misorientations, Fig. 5.3 provides a direct comparison between sampling point one and four. The perspective projection onto the coordinate system in the aperture of the probe antenna (x'_p, y'_p, z'_p) is presented, with the colored coordinate systems indicating the ideal and actual orientation. As already discussed, there is a good correspondence between the two orientations for the first sampling point. For the fourth scanning point, however, larger deviations around all rotation axes of the coordinate system are visible. In order to better assess this effect, the actual z -vector is visualized in the length of the measurement radius so that it points into the plane of the AUT. It is evident that this results in a considerable probe pointing offset, which is $\Delta x = 1.97 \text{ mm}$, $\Delta y = -3.14 \text{ mm}$ and $\Delta z = -0.41 \text{ mm}$ in this case. As the offset Δz is relatively small, it is not included in the drawing for reasons of clarity, but all offsets correspond to the sketch provided in Fig. 5.1 and can, therefore, be identified. In order to be able to integrate such non-ideal sampling points within the NFFFT without producing significant errors, it is essential to apply a PPC.

The PPC is applied as described in Section 5.1 by assuming a different probe antenna or different probe response coefficients for each sampling point. For this purpose, the additional translation and rotation operators in (5.3) are used to take into account the actual radial position and orientation in the spectral representation of the probe's SMC $R_{\sigma\mu\nu}^{(l)p}$. Although the probe antennas are physically identical, the actual

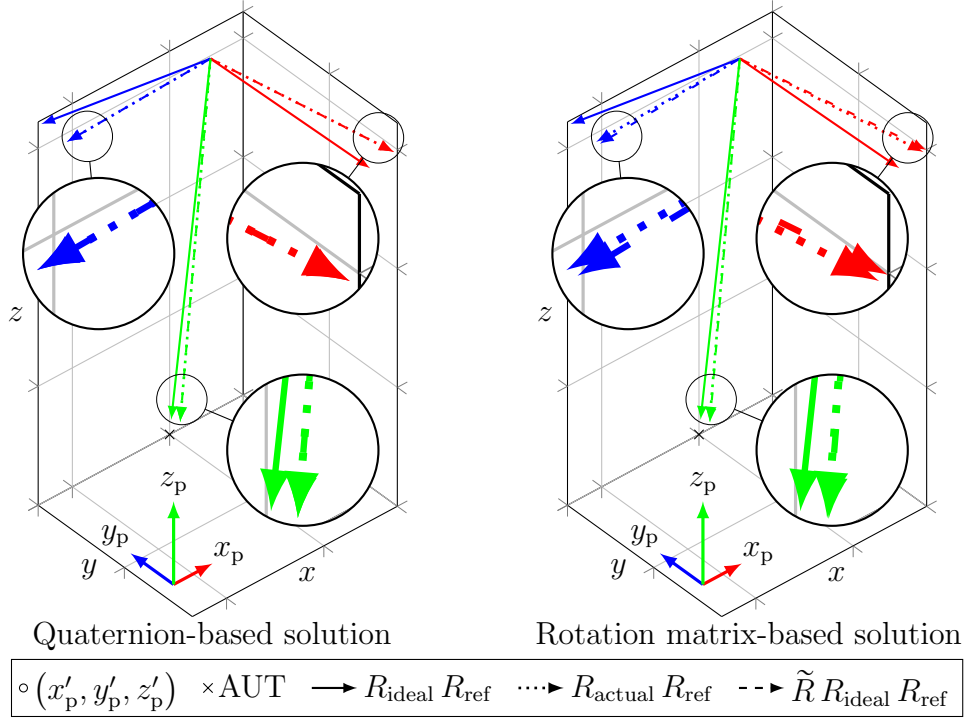


Figure 5.4: Visualization of the ideal, actual, and corrected orientation of the probe antenna. The approach based on rotation matrices shows deviations between the corrected and ideal orientation, which do not occur for the solution based on quaternions.

position and orientation are, however, included due to the spectral pre-distortion into the solution of the NFFFT using a pointwise scheme. While the radial deviation is directly determined based on the actual position as explained in (5.4), there are several approaches for determining the required rotation angles, e.g. using Euler rotations as shown in (5.5) or via a conversion to quaternions as explained in (5.10). Since the fourth sampling point, as already discussed, has a relatively large deviation from the ideal orientation, it is suitable for emphasizing the differences between the two approaches. For this purpose, Fig. 5.4 compares the ideal orientation $R_{\text{ideal}} R_{\text{ref}}$, the actual orientation $R_{\text{actual}} R_{\text{ref}}$ and the resulting correction from the ideal to the actual orientation for both approaches, given as $\tilde{R} R_{\text{ideal}} R_{\text{ref}}$. Accordingly, it is expected that the two latter orientations coincide, which is evident from the superposition of the dotted and dashed lines in the left part of the illustration for the solution based on quaternions. However, this does not apply to the right-hand diagram, which shows the solution using Euler rotations. Here, the corrected and the actual orientation can be clearly distinguished. While the orientation of the z -axis reveals a reasonably acceptable agreement, the x - and y -vectors are significantly disparate. This proves that the mentioned problems of the approach using Euler rotations for larger rotation angles or axis constellations close to singularities, such as in the case of gimbal locks, cannot be neglected. Therefore, it is recommended either to check the numerical stability of the Euler rotations for the actual measurement data set, or to generally use the more stable, albeit possibly more computationally intensive, solution based on quaternions.

5.3 Verification

After explaining the theory as well as the practical challenges associated with the application of the PPC, the verification is carried out in the following using two comparative SNF measurements. For this purpose, a well-aligned measurement is used as a reference case and compared with an intentional offset measurement. It is demonstrated that a conventional NFFFT of the offset measurement fails, while the application of the PPC allows the reconstruction of the radiation pattern in accordance with the reference measurement.

For all measurements, the SMC are calculated in a least-square manner and truncated according to the AUT's minimum sphere of the onset case. Thus, the resulting differences are solely due to the consideration of the actual position and orientation of the probe antenna. The correction data required to perform the PPC are provided by the RC, which outputs the actual position data associated with the respective trigger for each sampling point. The robot utilized for the measurements is factory calibrated, meaning it applies a kinematic model that incorporates uncertainties such as actual axis distances, axis misalignments, assembly tolerances, or gear elasticities. Although the data still have inherently greater tolerances than the direct measurement, it can be obtained without additional hardware and without changes to the measurement setup, making it an efficient compromise for most applications. In the following, the PPC is applied to an onset measurement to obtain a comparative reference case and to investigate the differences to the conventional solution based on the Wacker algorithm. Thereafter, the same procedure is repeated for the offset measurement, proving that the PPC provides a solution to reconstruct the disturbed measurement.

5.3.1 Onset Measurement

The onset measurements have been performed with coincident measurement and AUT coordinate systems ($\Delta x = \Delta y = \Delta z = 0$ mm). This allows a first analysis of the position or trigger accuracy of the robot as well as to determine the impact on the resulting deviations in the measured radiation patterns. For this purpose, the ideally requested sampling positions are compared to the actual trigger positions based on the feedback of the RC. Subsequently, the deviations are calculated and transferred into spherical coordinates.

The deviations for a complete SNF measurement are analyzed in the histogram shown in Fig. 5.5. The probability for the magnitude of an error is plotted on the y -axis and either the magnitude of the radial error Δr_{onset} or the angular errors $\Delta \theta_{\text{onset}}$ and $\Delta \phi_{\text{onset}}$ are plotted on the two x -axes. The first two result from the position and trigger uncertainty of the robot and indicate larger deviations due to the complex concatenation of six independent axes of the robot's kinematics. In contrast, significantly smaller deviations occur for the angular error $\Delta \phi_{\text{onset}}$, as the single dedicated rotation axis ϕ_{axis} is used.

The radial deviations Δr_{onset} are mainly in the range of ± 0.025 mm, with a significant accumulation of 15 % probability of occurrence at a deviation of about 0.003 mm. Thus, the deviations are in the expected order of magnitude and within the specified range of the position repeatability of the robot. The analysis of the same input data

as for the radial deviation, but related to the angular deviation $\Delta\theta_{\text{onset}}$, shows a symmetrical behavior. The highest probability of occurrence is around 11 % at a deviation of approximately 0.14° . In addition, a second smaller accumulation is evident at 0.01° with a probability of 6 %. Due to the external rotation axis denoted as ϕ_{axis} , the angular deviations $\Delta\phi_{\text{onset}}$ are significantly smaller. With a probability of 96 %, the deviations are less than $0.18''$ or 0.00005° , respectively, thereby being within the range of the readout accuracy of the encoders.

After framing the magnitude of the deviations between the ideal and actual positions, their impact on the radiation pattern of the AUT is contrasted in Fig. 5.6 for $\phi = 90^\circ$. For this purpose, the aligned reference measurement is compared under the assumption of ideal sampling positions and under consideration of the actual position using the PPC, emphasizing the differences between the two approaches using the EES.

As already indicated by the magnitude of the occurring position deviations, both transformation results are in close agreement. By taking the actual position into account and applying the PPC, a slight improvement of about -41 dB can be achieved for the co-polarization in the main lobe region. For angular ranges $\theta > 15^\circ$, the resulting differences are less than -50 dB. In the cross-polarization, the differences are insignificant, with the maximum improvement at $-1^\circ \lesssim \theta \lesssim 3^\circ$, showing a maximum variation of only -70 dB.

Accordingly, it is concluded that even radial deviations in the range of the position repeatability or angular errors $\Delta\theta_{\text{onset}} \approx 0.15^\circ$, despite the broad pattern of the OEWG probe used, can lead to traceable deviations in the main beam region. Especially due to the typically higher position tolerances for robot-based measure-

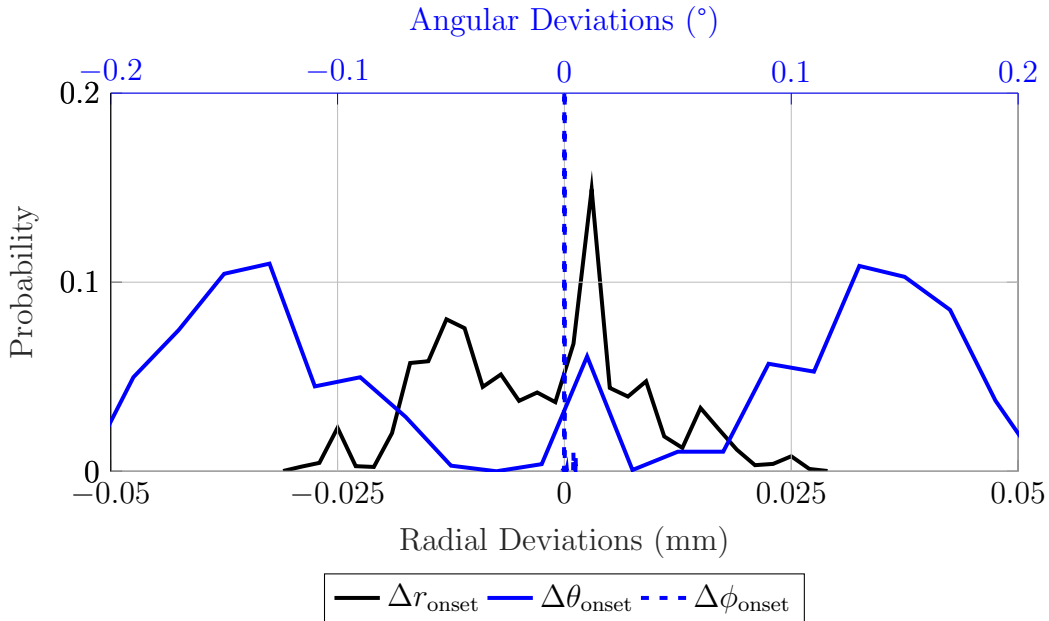


Figure 5.5: Relative probability of the occurring deviations resulting from the ideal and actual position of the probe in the onset reference case for a measurement radius of $r = 200$ mm.

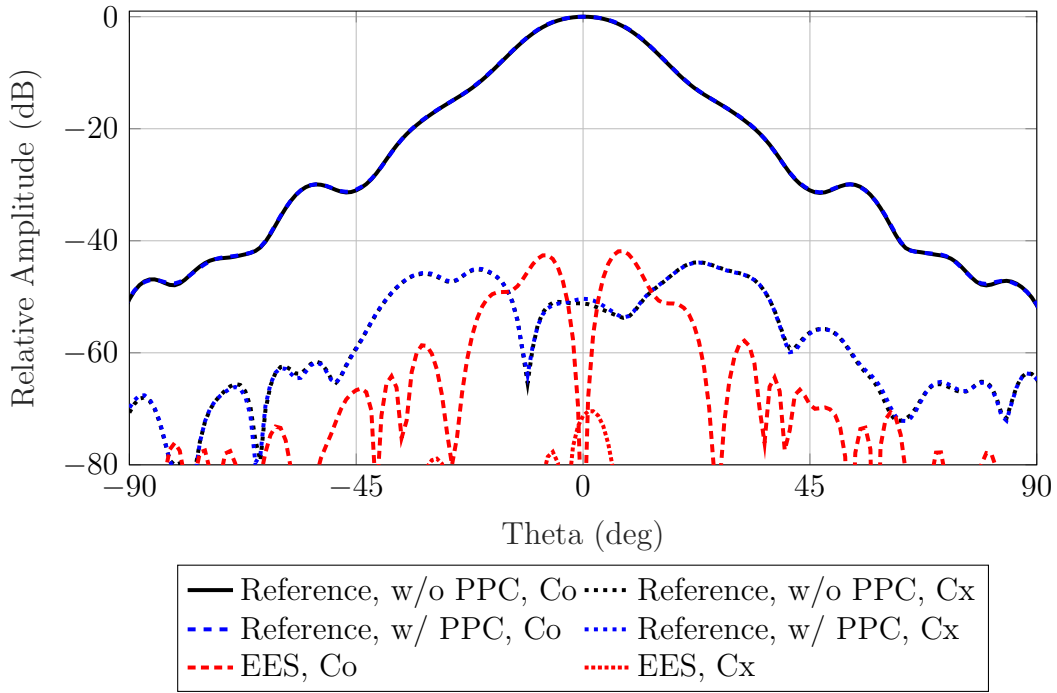


Figure 5.6: Radiation pattern ($\phi = 90^\circ$, $f = 60$ GHz) of the AUT transformed into the far field for the ideally assumed and the actual position of the measured sampling points. The EES denotes the difference between the reference using ideal positions to the results calculated with the PPC.

ment systems, the application of PPC is a convenient method to take those tolerances into account and, thus, reduce uncertainties in the transformation process.

5.3.2 Offset Measurement

The PPC allows to adjust the position and orientation of the probe for each measurement point separately and, thus, provides a possibility to process irregular sampling grids. To demonstrate this, an offset measurement of $\Delta x = 5$ mm between the measurement coordinate system and the AUT coordinate system or the ϕ_{axis} -axis, respectively, is introduced. The offset causes an angular error of $\Delta\theta_{\text{offset}} = 1.4^\circ$ in the main beam ($\theta = 0^\circ$) as well as a radial deviation of $\Delta r_{\text{offset}} = r + 0.0625$ mm. For increasing θ -angles, the angular error decreases and the radial deviation increases until there is no angular error for $\theta = 90^\circ$, but the measurement radius is maximally disturbed with a deviation of $\Delta r_{\text{offset}} = r + \Delta x = 205$ mm. Following this model, the probabilities of occurrence for an entire SNF measurement are given in Fig. 5.7. Here, it is illustrated that $\Delta r_{\text{offset}} > 0.0625$ mm always holds and a strong accumulation of 19% occurs in the order of the offset, which is attributable to $\theta \approx 90^\circ$. The angular errors $\Delta\theta_{\text{offset}}$ are also larger than for the onset measurement, but with weaker accumulations than for the radius since the angular deviation scales depending on the arctan-function. Nevertheless, the strongest accumulation occurs at a deviation of 1.4° with a probability of 11% and is, thus, due to the offset in the main beam of the measurement. Since the ϕ_{axis} -axis is decoupled from the measure-

ment coordinate system, the recorded deviations behave analogously to the aligned measurement and are negligible compared to Δr_{offset} and $\Delta \theta_{\text{offset}}$.

The results of the NFFFT without applying the PPC are shown in Fig. 5.8, where the aligned reference measurement is included for better comparability. It is evident that the transformation fails: Both, the areas around the main lobe and the side lobes show large deviations with a maximum EES of up to -14 dB. In addition, the shape of the main beam is distorted having no distinct peak but an indentation with two small maxima. The deviations in the cross-polarized results have also increased and indicate a maximum EES of -48 dB instead of the previously achieved -70 dB of the aligned measurement.

In contrast, Fig. 5.9 shows the results of the same measurement, but using the higher-order PPC in the NFFFT. The correction data are provided by the RC, i.e. they have been obtained without external measurement equipment such as laser trackers and are, therefore, subject to higher tolerances. Nevertheless, the higher-order PPC shows that both the main beam as well as the side lobes are, despite the displacement of the measurement, well reproduced. The maximum deviation expressed by the EES between the aligned measurement using PPC and the offset measurement using PPC is less than -40 dB in the co-polarization. Averaged over the entire angular range, the resulting EES is -60 dB, demonstrating the very good agreement.

Also in the spectral representation of the calculated radiation pattern given in Fig. 5.10 for the n -spectrum, it is apparent that the onset and compensated offset measurements are in close agreement. Both illustrate that most of the power is concentrated in the lower-order modes and decreases approximately monotonically for increasing order, proving that the AUT is centered and its orientation is aligned with the z_m -axis of the measurement coordinate system. In contrast, the

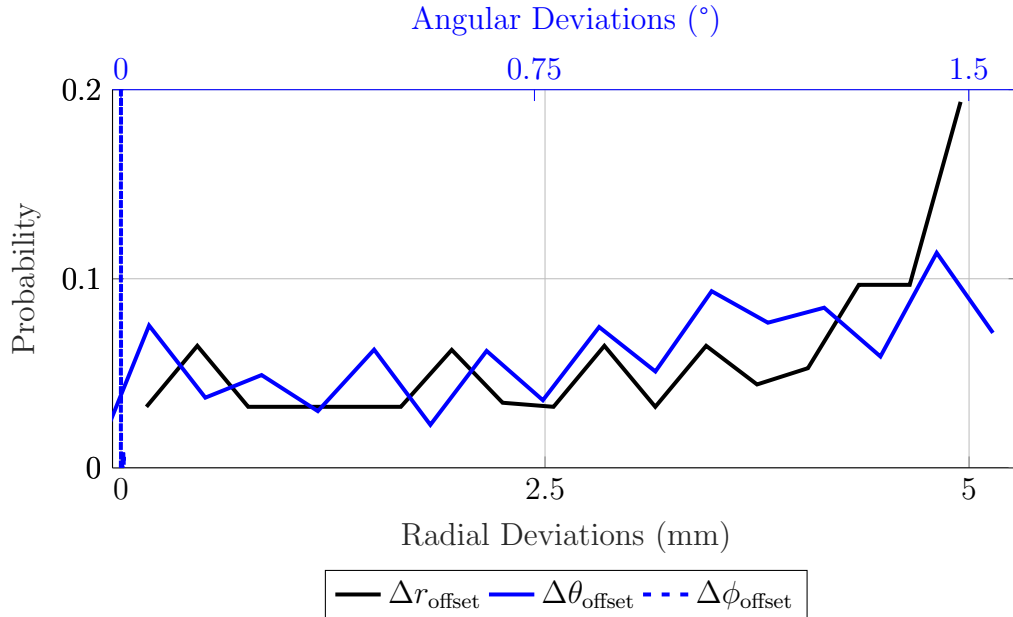


Figure 5.7: Relative probability of the occurring deviations resulting from the ideal and actual position of the probe for an offset measurement with $\Delta x = 5$ mm.

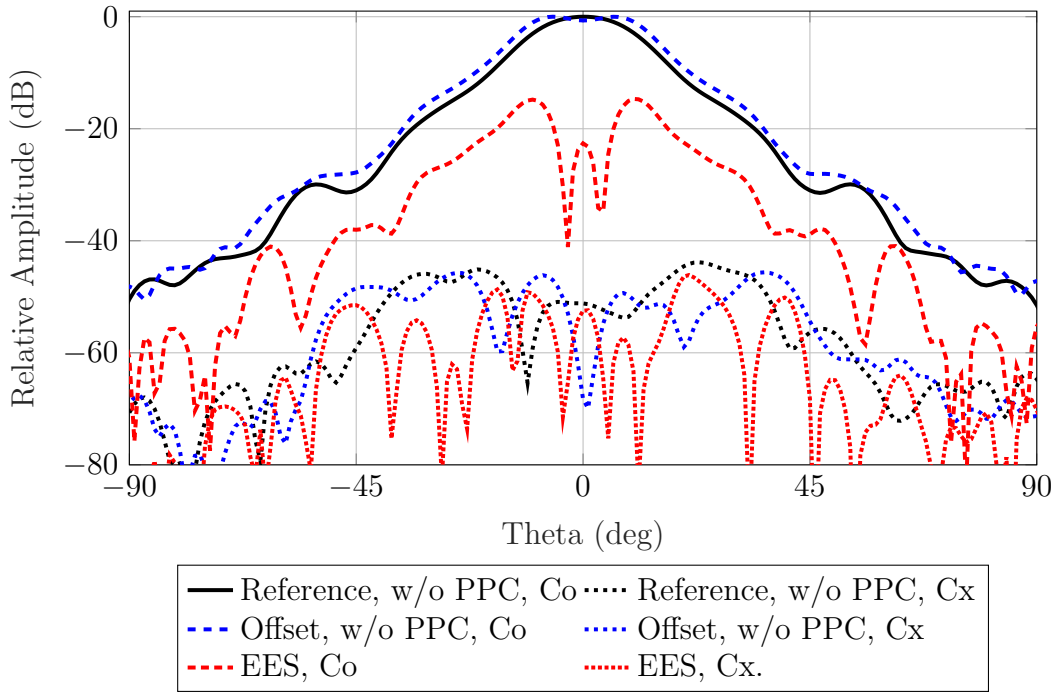


Figure 5.8: Radiation pattern ($\phi = 90^\circ$, $f = 60$ GHz) of the AUT transformed into the far field without application of the PPC for an offset of $\Delta x = 5$ mm. The EES denotes the difference between the onset reference measurement using ideal positions to the results of the offset measurement calculated without PPC.

power for the non-compensated offset measurement increases up to its maximum at $n \approx 5$. The decaying behavior, on the other hand, becomes apparent only after a plateau between $13 \lesssim n \lesssim 23$. Especially in this area, this results in a considerable deviation from the reference and the solution calculated using the PPC as well as a significantly increased power in the higher modes.

The corresponding m -spectrum, which represents the symmetry of the AUT or, more precisely, the oscillations in ϕ , is given in Fig. 5.11 for completeness. Since all measurements, even the non-compensated offset measurement with the incorrectly reconstructed main beam, are symmetric, the differences become apparent only for higher-order modes with lower power. While the onset and offset measurement with PPC indicate a good agreement in the whole spectrum, the deviations of the non-compensated offset measurement become visible only for $m \gtrsim 18$. However, since the normalized power of these modes is lower than -60 dB, the m -spectrum is only of limited relevance within this analysis due to symmetry of the measurements.

Since all measurements are truncated to a hemisphere, the spectrum of the SMC is not band-limited anymore. This effect can be observed in Fig. 5.10, as the power in the highest modes only decays to less than -45 dB (onset, offset w/ PPC) or even to less than -35 dB for the offset measurement without PPC. Consequently, the truncation error is present on the entire radiation pattern. However, for the offset measurement without PPC, this effect is stronger, since the offset inherently leads to an enlarged minimum sphere, which has been deliberately omitted during the design of the measurement. Instead, to focus on the working principle of the PPC,

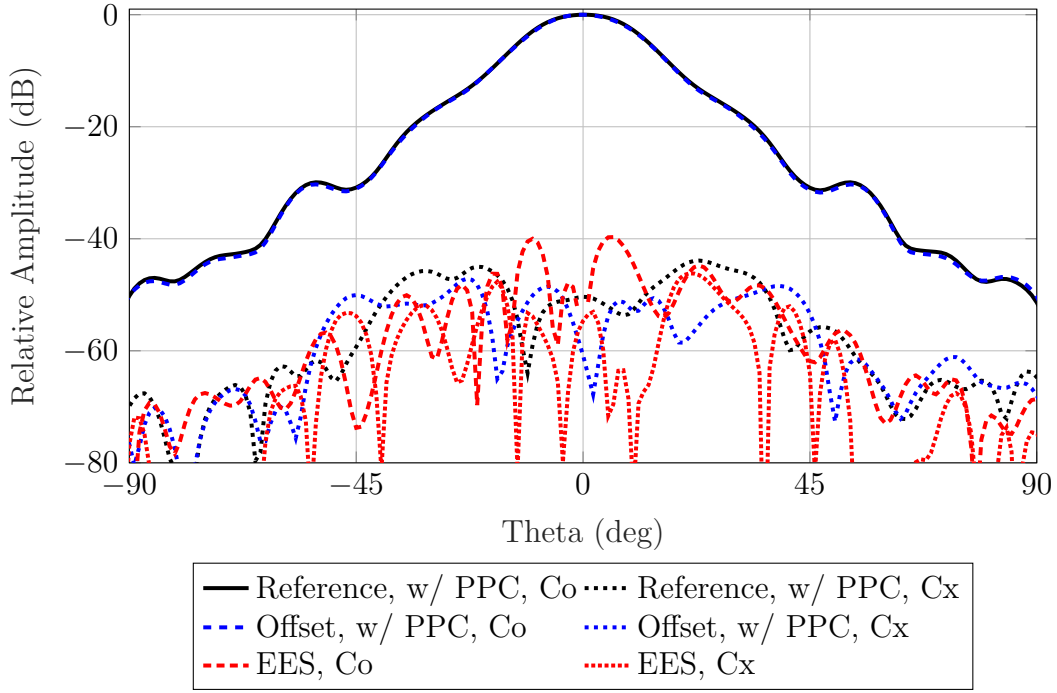


Figure 5.9: Radiation pattern ($\phi = 90^\circ$, $f = 60$ GHz) of the AUT transformed into the far field with application of the PPC for an offset of $\Delta x = 5$ mm. The EES denotes the difference between the reference to the offset measurement, both calculated using the PPC.

the sampling step size has been determined so that even the aligned measurement is close to the band-limitation. For Fig. 5.11, on the other hand, the power decays to the order of magnitude of the noise floor, indicating that the distortion in ϕ is significantly smaller.

5.4 First- and Higher-Order Probes

As already explained in the derivation, the additional rotations of the probe response coefficients require a HOP correction even if a $\mu = \pm 1$ probe or an approximation thereof is used. This is emphasized here for the offset measurement given in Section 5.3.2, where the offset of $\Delta x = 5$ mm leads to a maximum probe pointing error of $\Delta\theta_{\text{offset}} = 1.4^\circ$ in the main beam. Accordingly, the probe antenna is in (5.3) additionally rotated in θ for $\beta = \beta_{\text{max}} = \Delta\theta_{\text{offset}}$, which inevitably invalidates the assumption of an FOP.

To highlight this effect, the mode spectrum in m resulting from the simulation of the applied OEWG probe is presented in Fig. 5.12, which represents a reasonable approximated of an FOP, since the $\mu = \pm 1$ modes contain most of the power and are, therefore, dominant. The comparison to the odd modes with the next highest power already results in a drop of more than 24 dB while the even modes decay to less than 100 dB.

Furthermore, the comparison in Fig. 5.12 additionally demonstrates the resulting mode spectrum for the same probe antenna after the application of the PPC in

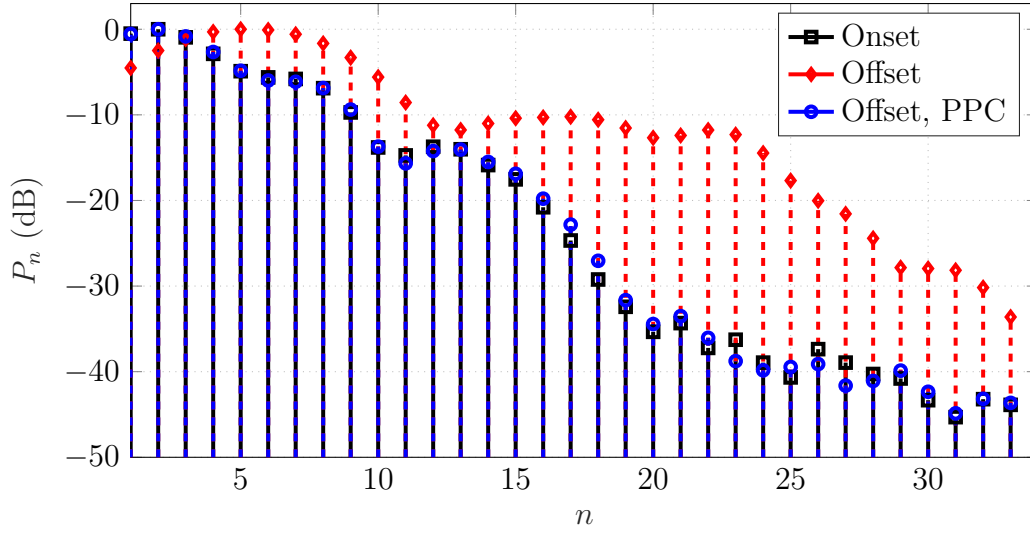


Figure 5.10: Normalized representation of the AUT's radiation pattern in the n -spectrum for onset, offset, and the compensated offset measurement using PPC. The onset and compensated offset measurements show that most of the power is concentrated in the lower-order modes.

case of the offset measurement. Although the rotation of $\beta = 1.4^\circ$ seems almost negligible compared to the very broad radiation pattern of the OEWG probe, large differences occur. The disturbed symmetry with respect to the original z -axis causes a significant power increase in the higher-order modes, especially for the even modes which have been insignificant for the well-aligned FOP. For example, the normalized

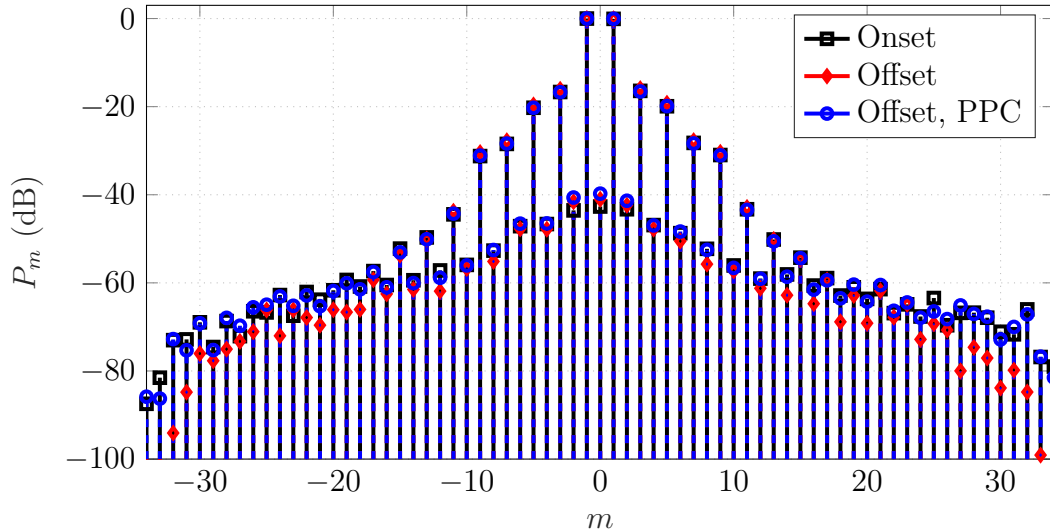


Figure 5.11: Normalized representation of the AUT's radiation pattern in the m -spectrum for onset, offset, and the compensated offset measurement using PPC. Until a power decay of approximately 60 dB ($|m| \lesssim 18$), all measurements show similar oscillations in ϕ , since the patterns are still symmetrical despite the incorrect results in the main beam region for the offset measurement w/o using PPC.

power for $m = 2$ increases from less than -140 dB to approximately -30 dB and is, thus, no longer negligible. Although the PPC allows a pointwise correction of the probe antenna orientation, a HOP correction based on the same band-limitation is typically performed for all sampling points of the measurement. Since the sampling points in the main beam region represent the case with the largest required rotation in θ , they are suitable for determining the required mode truncation. Referring to Fig. 5.12, the band-limitation may be determined, e.g. to $N_\mu = 6$ in order to take all modes up to a power decay of 60 dB into account for the HOP correction. Therefore, a HOP correction is essential even for actual $\mu = \pm 1$ probes in order to compensate for misalignments of the probe antenna.

5.5 Summary

In the conventional NFFFT based on the SWE, the assumption is made that the same probe antenna is used for all measurement points, which is indeed the case for most conventional SNF measurement systems. For systems with dual-polarized [80] or multiple probe antennas [83, 85, 86, 123], calibration approaches exist to compensate for amplitude and phase variations, but the spectral representation of the probe antennas is at least not entirely addressed within the SWE. The PPC, in contrast, not only allows to consider multiple probe antennas during the NFFFT, but for each individual sampling point separately, thus providing the greatest possible degree of flexibility.

This is particularly relevant for robot-based measurement systems: On the one hand, robot-based systems are subject to greater tolerances in terms of positioning accuracy due to the concatenation of six axes. On the other hand, their flexibility allows almost arbitrary orientations of the probe antenna for each individual sampling

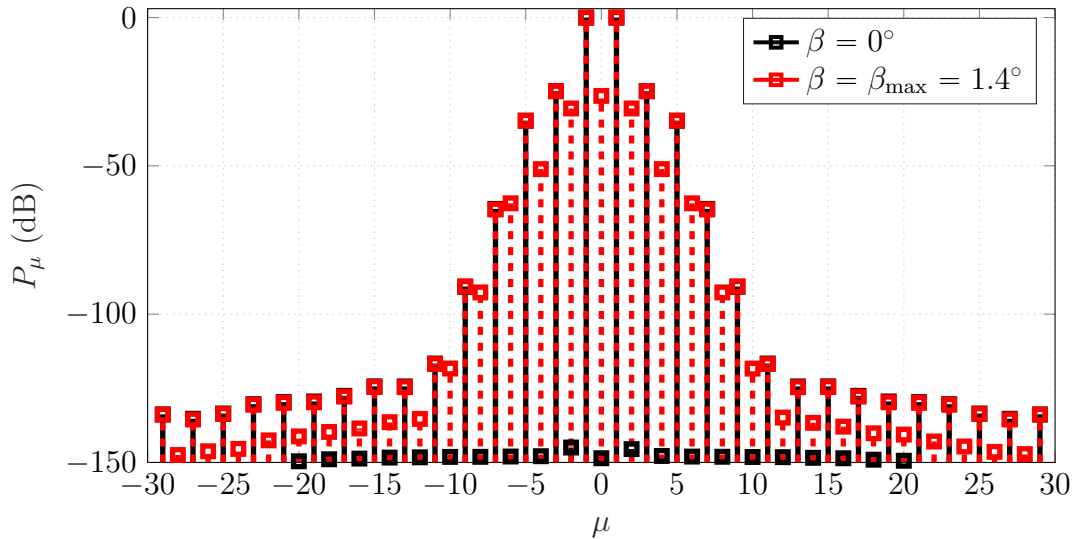


Figure 5.12: Normalized representation of the radiation pattern of a simulated OEWG probe in the μ -spectrum. The additional rotations of the probe to perform the PPC require a HOP correction even for small rotations (here: $\beta_{\max} = 1.4^\circ$) compared to the broad radiation pattern of the probe antenna.

point. Thus, the PPC represents the theoretical complement to the mechanics and provides mathematical support for these degrees of freedom within the SWE.

Therefore, a pointwise formulation of the spherical transmission formula (5.1) is discussed, which is extended by additional rotation operators as shown in (5.3). These are applied to take into account the actual orientation of the probe antenna via a spectral pre-distortion of the probe's SMC $R_{\sigma\mu'\nu}^{(l)p}$. Subsequently, the application in a robot-based measurement system is explained, with a particular focus on stability considerations for Euler rotations and quaternion-based approaches, whereby the arising differences are illustrated using real measurement coordinates.

This is followed by verification of the PPC using two comparative measurements. In a first step, the impact of the actual position and orientation of the probe antenna on the NFFFT is demonstrated. In a second step, a provocation test is conducted using an intended offset measurement, which causes a misalignment of the probe antenna. While the transformation into the far field fails without using the PPC, the radiation pattern of the AUT can be successfully reconstructed if the PPC is applied. Despite the considerable offset of $\Delta x = 5 \text{ mm} \approx \lambda$, the well-aligned reference measurement is reproduced in close agreement as stated by a maximum EES of only -40 dB as illustrated in Fig. 5.9, thus proving the effectiveness of the PPC.

As a result, the PPC describes an elegant solution for taking position uncertainties and measurement point-dependent misalignments of the probe antenna into account within the SWE-based NFFFT and is, therefore, particularly suitable for application in robot-based measurement systems. This is, in general, independent of the system-inherent position (and orientation) tolerances, instead providing the theoretical foundation for new potentials such as the support for additional triggers on the resulting path of spline-based measurements as explained in Section 6.

Spline-Based Measurement Techniques

In antenna measurement technology, the required measurement data is typically recorded at discrete sampling points. Conversely, the actual measurement path through which the sampling points are passed is not decisive. Therefore, using robot-based systems, new potentials are provided for optimizing the underlying path with regard to the mechanics used. Instead of canonical paths, for example linear paths defined by start and stop coordinates, or circular paths in which the circular path is determined by three coordinates, spline paths enable a more optimal operation of a robot system [128]. The fundamental difference to canonical path planning is that no path is defined, but only a set of sampling points, whereas the actual path planning is performed by the RC, taking into account the kinematic and dynamic limitations of the robot used. Accordingly, the physical limits of the robot are already taken into account during path planning, i.e. before the actual motion sequence is started. In contrast, the physical limits of conventional linear or circular paths cannot be checked before or during path planning, which means that they may lead to the measurement project being aborted during the execution of the movement, possibly resulting in an unsafe operating state and, consequently, requiring the operator to reset the system manually.

Compared to conventional types of motion, the programmed path speed can also be better maintained. The set path speed is only reduced if the robot is operating close to a singularity, if there are distinct corners in the path or if large reorientations or large movements of the additional axes are required. However, the last two issues can be avoided by parameterizing the spline path accordingly, for example by changing the orientation guidance on the path or by disabling the synchronous operation with the additional axes. Nevertheless, the robot may come to a halt when using spline paths. This occurs if two consecutive sampling points have the same coordinates or if a composition of linear and circular path elements results in an unsteady course of the velocity vector. However, since this form of mixed operation is not used in the following, the latter aspect is omitted and is only mentioned here for the sake of completeness.

Furthermore, one of the most important advantages of spline paths is their increased reproducibility. In contrast to canonical path definitions, the actually resulting path is independent of the set speed or acceleration profile and even velocity overrides enforced by the operator have no effect on the final path. However, not only the reproducibility of the mechanical path, but also of the entire antenna measurement is improved, as the exact sampling points for a spline path are communicated to the RC and are, therefore, considered during the path planning. This allows to exploit the

native trigger functionalities of the system, which enable a higher accuracy of the generated trigger signals [129], as will be explained in more detail in Section 6.2. Further details on the mechanical-kinematic advantages of spline-based movements and other features, which are not applied in the following, are provided in [130], e.g. path approximation or constant velocity blocks. In the latter, the path velocity of an entire section is adjusted to the point that can be passed through at the lowest possible speed. This facilitates the sampling of irregular grids and contributes to a uniform trigger rate and, thus, reduces smearing effects.

In general, spline-based motion sequences represent a flexible solution for the realization of antenna measurements. Specifically for this application, further advantages of this operational technique are presented besides the increased trigger accuracy. These include the implementation explained in Section 6.1, the available triggering sources discussed in Section 6.2, the migration of conventional measurements to spline-based motion patterns described in Section 6.3 as well as the transition to irregular sampling grids as presented for the MDSG in Section 7.

6.1 Spline Implementation

Manufacturers of robotic arms often consider the exact details of their algorithms for calculating spline movements to be proprietary information. There are various reasons for this practice, either related to the protection of intellectual property and maintaining a competitive advantage in the market or due to the technical embedding in the overall system. The actual implementation of the path generation algorithm for spline trajectories is a core component whose performance depends on the choice of the type of spline (e.g. cubic or quintic splines), but also on the kinematic and dynamic modeling of the robot. Naturally, the spatial position of the spline support points affects the result of the path generation, but other factors are also taken into account, such as the respective axis positions, also in relation to the required pose of the robot, and further information such as the configured load data. The interaction with external systems is also relevant if, for example, external sensors are used to adjust the path in real time. In addition, integration into the safety concept is crucial to ensure that the generated spline path does not generate any collisions or violate defined safety areas.

However, the prediction of the actual path can be simulated in advance using the manufacturer's simulation and planning tools, if these are available. If this is not the case, the resulting path can be determined empirically, whereby according to [130] it can be assumed that it is always reproduced for the same support points regardless of the applied path parameters (velocities, accelerations, user overrides). This approach is used in the following to investigate the effect of different amounts of spline support points on the resulting trajectory. For this purpose, a typical measurement trajectory over a θ -cut of a hemisphere has been chosen, with the actual position of the TCP being recorded every 0.25 s. The measurement has been performed for an angular step size of 5° , whereby the number of support points actually used for the spline trajectory is reduced, hereafter referred to as a thinning factor N_{TF} .

The actually resulting path is shown in Fig. 6.1, where a thinning factor of $N_{TF} = 18$ for a measurement with an angular step size of 5° indicates that only every 18th

sampling point is used as a spline support point. This results in the highest possible thinning of a circular path defined by three points. Using $N_{TF} = 9$ and $N_{TF} = 5$ results in 5 and 9 support points, respectively, whereby for $N_{TF} = 5$ it should be noted that the total number of sampling points cannot be integer divided by N_{TF} , resulting in two support points directly following each other towards the end of the path (left side of the shown cut). The illustration demonstrates that the spline

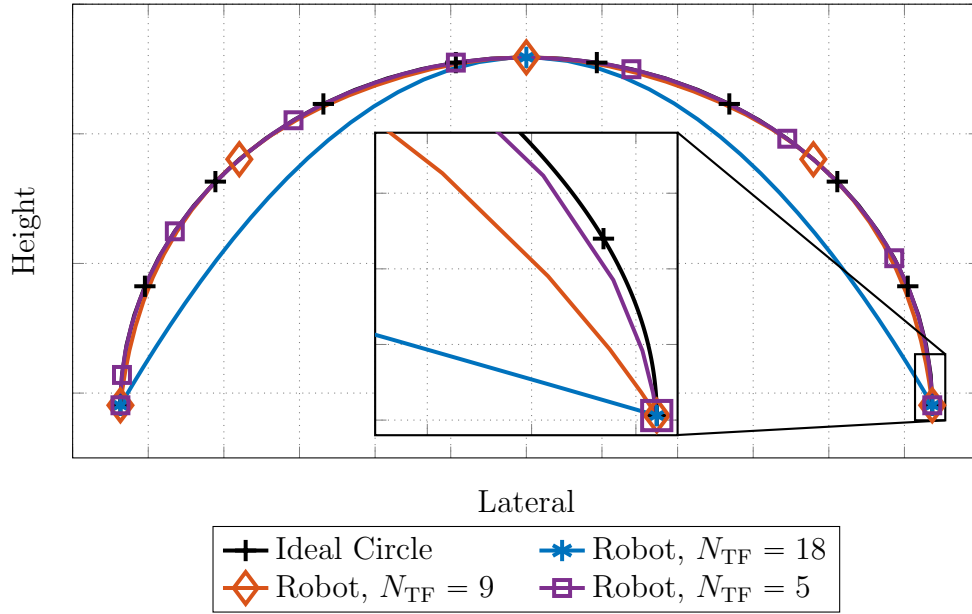


Figure 6.1: Actual spline-based trajectory of the robot for a θ -cut over a hemisphere as a function of the thinning factor N_{TF} compared to an ideal circular path.

paths rapidly converge to the ideal circular path as the number of support points increases. In addition, there is no oscillation or overshoot between the individual support points, which is advantageous in terms of both, the overall length of the path and its acceleration profile, as oscillations would lead to continuous dynamic accelerations of the TCP perpendicular to the direction of the circular path.

To evaluate the applicability of a spline-based movement to perform a measurement, an approximate prediction of the resulting spline path is required if the manufacturer's simulation software is not available. Since the exact implementation is not publicly accessible, two widely used spline variants are compared with the actual path. The first is the commonly used natural or periodically interpolated cubic spline with the boundary condition for endpoints according to Eugene Lee's centripetal scheme [131]. The second variant used is a quintic spline and since higher-order splines tend to oscillate, which can be precluded by the results presented in Fig. 6.1, an implementation according to Akima for non-oscillating spline paths [132–134] is exemplarily used in the following. While the first derivative of cubic splines exists and is continuous, quintic splines provide continuous first and second derivatives. This is important to avoid discontinuities in the respective axis accelerations and, thus, to achieve a more uniform trajectory, as investigated in detail in [135].

The comparison of both spline variants with the actual trajectory is depicted in Fig. 6.2, using the already introduced path parameterization. The path with

the fewest interpolation points ($N_{TF} = 18$) shows a clear deviation between the cubic spline and the actual path. The quintic spline, in contrast, demonstrates an almost exact match with the measured path. However, this trend reverses as the number of support points increases. For $N_{TF} = 9$, the path of the cubic and quintic spline are almost identical and result in approximately the same deviation from the measured path. However, for the path with the most support points, i.e. $N_{TF} = 5$, the correlation is reversed. As the zoomed detail view on the left emphasizes, the cubic spline shows the best agreement, while the deviations of the quintic spline have increased.

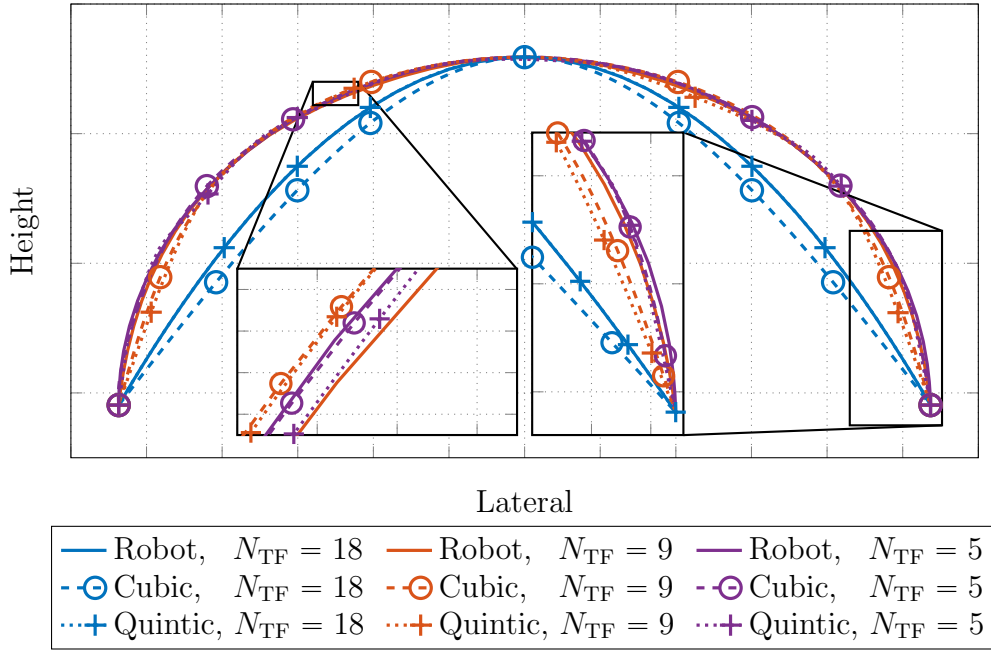


Figure 6.2: Actual spline path of the robot compared to the periodically interpolated cubic spline and the quintic spline according to Akima depending on the spline support point thinning factors N_{TF} .

Apart from the minimum configuration with $N_{TF} = 18$ consisting of only three support points, it can, therefore, be assumed that the periodically interpolated cubic spline provides an acceptable approximation to the actual robot path. In order to verify this for more complex scenarios, a concatenation of two spherical measurement cuts is considered in the following. The spline support point thinning cases $N_{TF} = 18$ and $N_{TF} = 9$ are illustrated Fig. 6.3 and provide a qualitative comparison of the resulting paths, whereby the robot's actual path velocity is marked color-coded. As already discussed, a better estimate of the resulting path is available for an increasing number of support points. This is apparent in Fig. 6.3a for $N_{TF} = 18$, which reveals larger discrepancies for both spline variants. For the case $N_{TF} = 9$ in Fig. 6.3b, the number of support points has been increased evenly, but it is obvious that the discrepancies increase in particular in the direction of the equator of the measurement sphere. It is evident that the spline implementation of the robot avoids overshooting at the equator in the θ -direction and, instead, maximizes the curve radius in the ϕ -direction in order to achieve the highest possible path

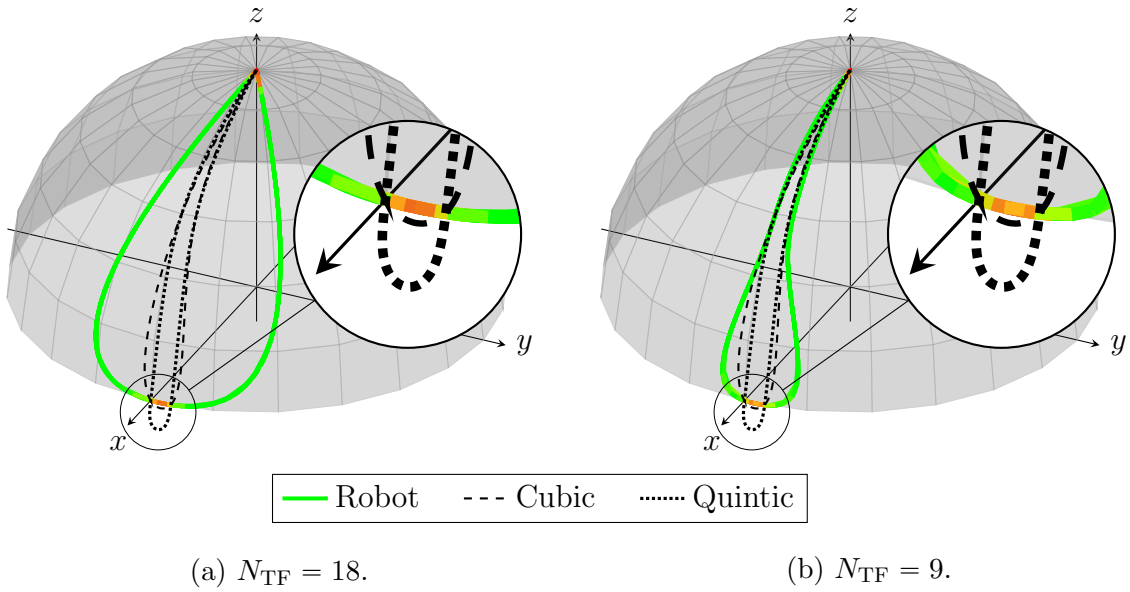


Figure 6.3: Comparison of the actual robot path with cubic and quintic spline (non-oscillating Akima variant) for a more complex measurement scenario based on a concatenation of two θ -cuts for two different thinning factors N_{TF} .

velocity. For both cases, the curve radius is increased significantly more than the estimation of the two spline variants would suggest. Possible reasons for this type of path planning are the working range of the robot, which does not extend far beyond the equator of the measurement sphere, or a form of axis prioritization so that the generated path can be achieved with lower accelerations of larger axes having higher inertias. However, numerous other factors, the exact implementations of which are not disclosed, have an effect on path generation, as summarized for the most important ones in Section 3.4.

Nevertheless, the cubic and quintic spline implementation provide an acceptable agreement with the actual robot trajectory even for complex measurement scenarios if the number of spline support points is sufficiently high or if the degrees of freedom are minimized by sufficiently dense support points. This is illustrated in Fig. 6.4, in which all of the sampling points generated using an angular step size of 5° are used as spline support points (no thinning applied or $N_{TF} = 1$). While both spline implementations reproduce the actual measurement trajectory, it is observed that the turning radius at the equator of the measurement sphere is better reproduced by the cubic spline implementation. The quintic spline, conversely, tends to overshoot in the θ -direction, which is to be expected for a higher-order spline, but does not correspond to the actual robot trajectory.

Nevertheless, a rough prediction of the robot's spline path is possible using common cubic splines if the spline support points are not excessively thinned out, e.g. if a circular path is not only defined with the minimum number of three support points. In this case, a sufficient agreement is achieved for planning the path, for example for designing a measurement or estimating collision risks. To calculate the NFFFT, however, the actual sampling position returned after or during a measurement should

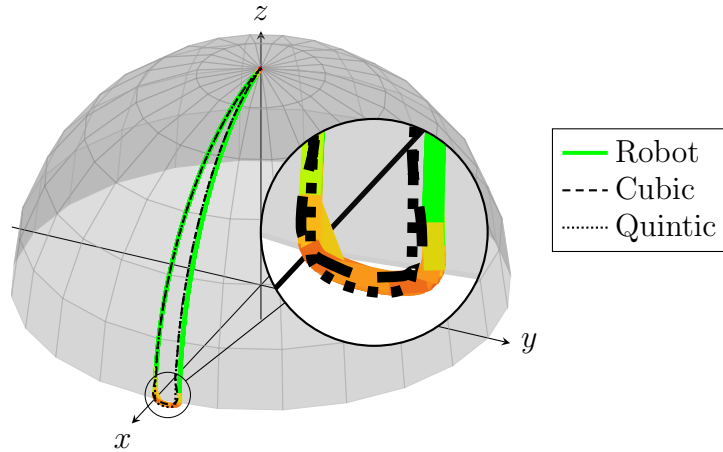


Figure 6.4: Actual robot trajectory overlaid with cubic and quintic spline paths for a complex measurement scenario composed of two θ -cuts without any thinning of spline support points ($N_{TF} = 1$).

always be taken into account using a PPC as described in Section 5, even if the sampling point is reflected by a spline support point.

6.2 Trigger Sources

Triggers are path-related switching actions, for example the switching of a digital output when a sampling point is reached. However, in the original application of a robot, for instance in a production line, the robot does not perform measurements, but welding, milling, painting or pick & place tasks, to name the most common examples. Such tasks typically require only a small amount of triggers, e.g. two to switch a welding device on and off or to signal to external elements of an automation cell that an object has been picked or placed.

In principle, triggers are based on the execution of an interrupt and are limited to a certain amount, e.g. to a total of 32 for the RC used within this thesis [130]. However, similar restrictions also exist for other manufacturers for the reasons mentioned above, thus causing this to be a general problem. The most common approach to overcome this problem is to omit the use of the native trigger functionalities and, instead, execute the trigger generation in a separately executed task. This offers the advantage of implementing a specific, possibly complex logic for firing a trigger and at the same time eliminates most restrictions on their total amount. Depending on the implementation, the remaining limitations are, e.g., the memory available for recording the actual trigger positions opposed to the ideal positions.

This principle is illustrated in Algorithm 1, which outlines a background task for generating the trigger signals. As long as the program is active and a measurement is running, the distance travelled on the axis to be triggered is monitored. As soon as the trigger distance is exceeded, the trigger signal is generated and the distance counter is reset or set to the distance at the time of triggering.

This simple approach is highly flexible, as a custom trigger logic can be directly implemented, as illustrated by the example of the reference axis. It allows to determine

Algorithm 1 Trigger generation based on a cyclic background task

```

1: procedure TRIGGERGENERATION
2:   while State.IsInitialized do ▷ System is initialized
3:     if State.MeasurementActive then ▷ Measurement is running
4:       distance = GETDISTANCE(Job.ReferenceAxis)
5:       ▷ Distance is calculated depending on the trigger axis (robot or additional axes)
6:       if distance  $\geq$  Job.NextTrigger then
7:         FIRETRIGGER() ▷ Interrupt-based trigger generation
8:         UPDATEDISTANCE(distance) ▷ Update distance for next trigger
9:       end if
10:    end if
11:  end while
12: end procedure

```

whether the traversed distance of the linear axis, rotation axis or the robot itself is decisive for the generation of a trigger signal. However, this flexibility is associated with some disadvantages that affect the temporal and spatial precision of the trigger signal. As the task for generating the triggers is a background task, its execution speed depends on the system load and is, therefore, subject to fluctuations. The outlined trigger logic is also not error-tolerant, as time delays in execution lead to a summation of the deviations, as the respective trigger distances are defined relative to each other. Hence, an error propagation occurs, which will steadily become worse as the number of trigger signals increases.

Furthermore, this type of trigger generation is depending on the actual motion sequence. In this case, the movement for generating the trigger signals is defined in the main program by canonical paths. For example, a linear path is defined by start and stop coordinates, while a circular path is defined by start and stop coordinates as well as a third auxiliary position defining the circle. The robot, thus, receives a movement command for a linear or circular path, during the course of which the background task generates the trigger signals. This type of trigger generation is, therefore, also referred to as a path-dependent trigger (PDT).

In order to compensate for the disadvantages of the PDT in terms of its temporal and spatial precision, it is required to employ the native trigger functionalities provided by the manufacturer. Taking into account certain boundary conditions, for example the total amount of triggers limited by the available memory, this can be achieved using spline paths as shown in [129], which reveals a further advantage in addition to their reproducibility and flexibility compared to canonical paths. Therefore, each required sampling point is communicated to the RC and used as a support point for the spline movement to be constructed. Analogously, these triggers are, hence, referred to as path-defining spline support point trigger (SPT).

The underlying concept is explained in Algorithm 2, which defines a spline path in a certain base (or coordinate system), a certain tool (e.g. a probe antenna), and a certain velocity profile. Each support point of this spline path is generated from an array of positions that have been transferred in advance from the control software to the RC. As each sampling point is, therefore, known and also corresponds to a support point of the spline path, certain functions, such as setting digital outputs,

Algorithm 2 Native trigger capabilities using spline-based measurements

```

1: procedure SPLINEMEASUREMENT
2:   SPLINE WITH $Base=Job.Base, $Tool=Job.Tool, $Vel=Job.Velocity
   ▷ Spline block consisting of several spline support points
3:     SPL Job.SplinePosition[0]                                ▷ 1st spline support point
4:     TRIGGER Path=0, Delay=0, Prio=-1 DO FIRETRIGGER()
   ▷ Native trigger (supporting priorities and time or path-dependent delays and lead times)
5:     END SPL
6:     SPL Job.SplinePosition[1]                                ▷ 2nd spline support point
7:     TRIGGER Path=0, Delay=0, Prio=-1 DO FIRETRIGGER()
   ▷ Native trigger (supporting priorities and time or path-dependent delays and lead times)
8:     END SPL
9:     [...]                                                    ▷ Continue scheme for further sampling points
10:  END SPLINE
11: end procedure

```

can be controlled via the native trigger command. The native trigger functionality for the robot used supports different priorities for SPTs as well as spatial or temporal delays and (even) lead times, thus providing advanced features for fine-tuning the precision of the trigger signals.

Nevertheless, in addition to the limited instruction set in the called trigger function, it must be ensured that the simultaneous number of system-wide active interrupts is not exceeded. If a generic measurement routine with a fixed amount of SPT is used, it is possible that this could occur towards the end of the path if the spline path contains more support points than transmitted sampling points. If, in this case, all remaining support points are assigned the same last sampling point position, several SPT will be fired simultaneously, which could violate the above condition. It is, therefore, advisable for generic measurement routines to provide these with different numbers of support points to ensure that the appropriate routine can be selected for the requested number of sampling points.

An examination of the achievable precision of the generated PDT compared to the natively triggered SPT is presented in Fig. 6.5 for different measurement speeds defined by triggers per second (T/s). For this purpose, a planar measurement defined by 861 sampling points has been performed using both methods, first based on canonical trajectories with PDT and, second, using a spline-based measurement with SPT. The Euclidean difference between the ideal and actually measured sampling point position is evaluated and its probability of occurrence is represented by the given probability density function (PDF)s. Despite the relatively limited amount of data, the comparison between PDT and SPT is unambiguous. The deviations of the PDT generated in the cyclic task show a clear dependence on the measurement speed. At a relatively slow measurement rate of $0.5 T/s$, deviations of approximately 0.15 mm occur with an error probability of 34 %. The deviations increase with increasing speed, resulting in a 27 % probability for deviations in the range of 0.55 mm at a measurement rate of $2.0 T/s$, whereby an enlarging spread of the occurring deviations is also evident.

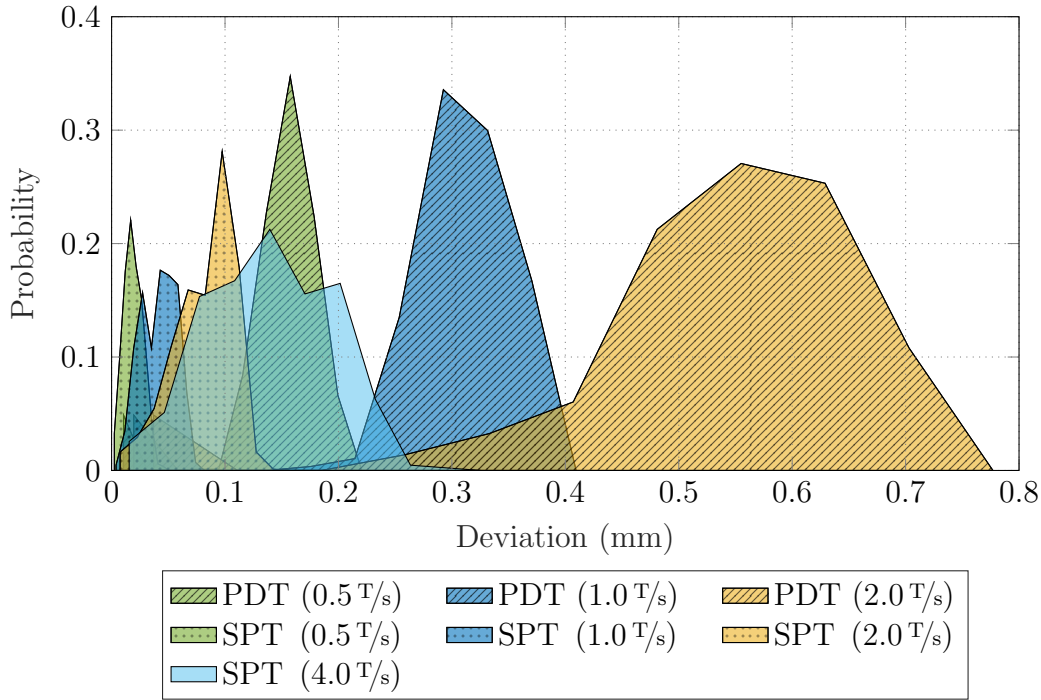


Figure 6.5: Trigger accuracy for canonical path-dependent triggers (PDT) and triggers defined by support points of a spline-based trajectory (SPT), given for different measurement speeds in triggers per second (T/s).

A similar, albeit significantly weaker, trend is also apparent for the SPT implementation. At a measurement rate of $0.5 T/s$, the deviations are only 0.016 mm with a probability of 22% . For a comparable measurement speed of $2.0 T/s$, the deviations rise up to the order of 0.097 mm with a probability of occurrence of 28% . Only at a relatively fast measurement rate of $4.0 T/s$ is a clear spread in the deviations visible, although even in this case the deviations are only 0.201 mm with a probability of occurrence of 16% .

Even at eight times the measurement speed, the deviations of the SPT implementation for $4.0 T/s$ are lower or at least of the same order of magnitude as for the PDT implementation at $0.5 T/s$. At this speed, a direct comparison demonstrates that the deviations of the SPT implementation are approximately by a factor of ten lower than those of the PDT implementation. Hence, it is absolutely recommended to rely on SPT, especially for measurements in the millimeter-wave range.

Apart from the trigger accuracy, however, both trigger sources are able to co-exist and, thus, complement each other. For example, sampling points whose position is particularly critical can be generated using SPT, while less critical or additional sampling points may be generated using PDT. A use case for this hybrid trigger configuration is provided in Section 7.

6.3 Migration of Canonical Measurements

In antenna measurement technology, robot-based systems are often used as an alternative to conventional positioners, such as linear or rotation axes [26, 40]. Since

such dedicated axes realize a measurement path, it is reasonable to operate a robot in a similar way and, thus, predetermining the motion path. The robot is, therefore, often moved along a plane or serves as a substitute for a rotational axis, e.g. as explained in [40] for a θ -axis. In this operating mode with a predetermined motion path, the RC will coordinate all axes to generate a path that is as close as possible to the required linear or circular path.

As an extension of this operating mode, [41] presents the capability of controlling the robot using combinations of linear and circular paths, which can also result in non-canonical sampling surfaces. The desired sampling surface is defined by parameterizing the dimensions of various surfaces as well as transition curve radii and angular ranges. In this way, planar, truncated cylindrical, hemispherical or partial spherical as well as partially oblate and prolate spheroidal measurement volumes can be defined. An additional axis of rotation is also used in this case so that the robot has to move mainly along only one axis or plane to perform the measurements. As a result, planar measurements can be supplemented by rounded side surfaces at the edge of the plane, which increases the range of validity. Simultaneously, such parameterizations also allow an adjustment of the sampling volume in relation to the physical size of the AUT, whereby the requirements for the dimensions of the measurement system can be reduced.

However, this operating mode of a robot over predefined canonical paths represents an unnecessary restriction, as the measurement data is only acquired at discrete sampling positions during antenna measurements. In addition, path-related motion sequences make it difficult for the robot to loop over at the end of one measurement cut to the next one, so that measurements can be continued without halting the robot. As a result, robot-based systems are often only operated uni- or bidirectionally as visualized in Fig. 6.6a, i.e. completely analogous to conventional linear or rotation axes, which significantly limits their potential.

In contrast, the spline-based operation of a robot has several advantages. Since in this case the actually required sampling points are provided to the RC, the path can be pre-calculated and optimized before the measurement starts. This provides the possibility to check whether the required sampling points may be out of reach or to take into account the respective axis accelerations, which depend on the position and pose in space, already during planning the movement path. Therefore, instead of modeling the path for best possible linearity or circularity, it can be optimized for the highest possible path speed or the best possible smoothness of motion. Since the spline-based movement is only specified by a set of given sampling points, the robot can continuously pass over different measurement cuts by incorporating the sampling points of several cuts as support points for a single spline path [128]. This transition process is visualized in Fig. 6.6b at the equator of the measurement sphere as well as for both transitions to the next measurement cut in the illustrations provided in Fig. 6.7. However, this also applies in case the path of the robot contains spatially identical coordinates which differ only in the position of the additional axes ϕ_{axis} , as, for example, in continuous measurements in θ that are stepped in ϕ . This is visualized at the pole of the measurement sphere in Fig. 6.6b. Although the robot will halt in this case, the communication overhead with the RC is reduced, because several measurement cuts are implemented within a single spline-based motion sequence. In this way, the standstill time is reduced compared to canonical paths, which are

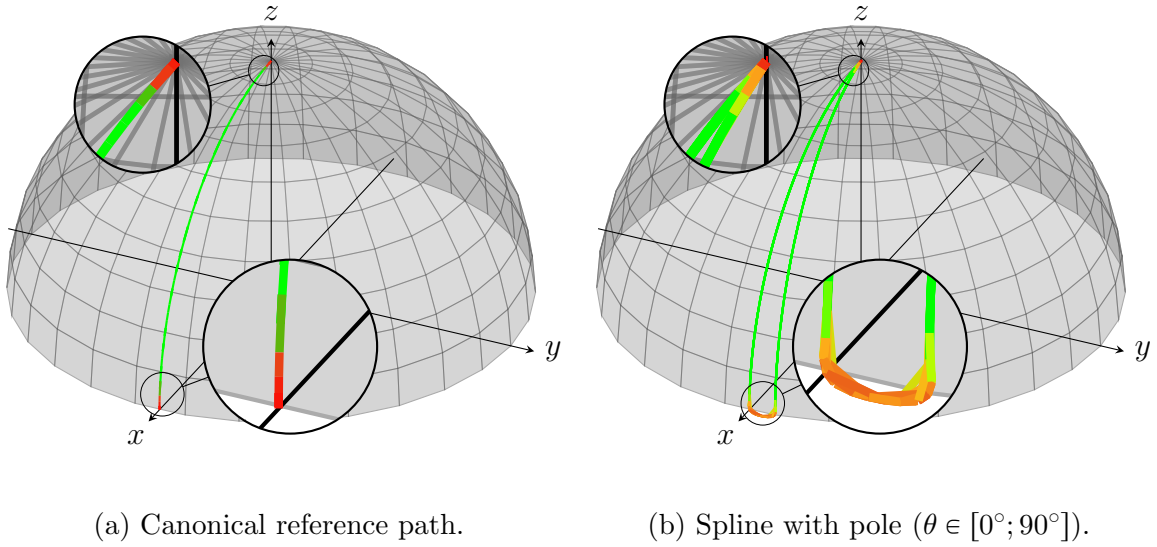
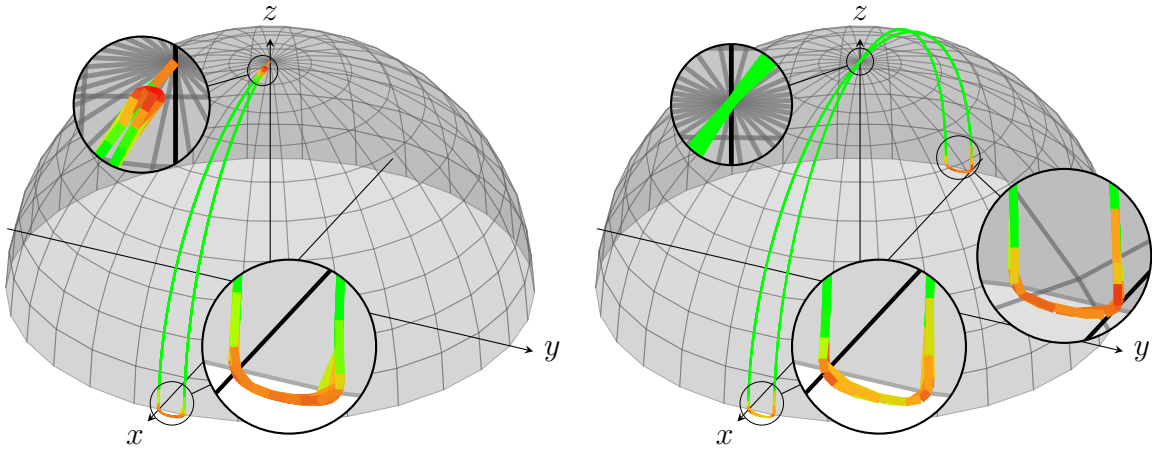


Figure 6.6: Comparison of a canonically defined measurement path and a spline-based variant in which the next measurement cut can be continued directly at the end of the previous one without halting the robot. A qualitative comparison of the robot's path speed is provided by the color-coded trajectory, whereby the same speed and acceleration profile has been used in each case (also in comparison to Fig. 6.7).

communicated individually to the RC for each measurement cut. Although the cases shown are examples of an equiangular measurement, the same procedure can also be used for other sampling grids without restriction of generality.

Nevertheless, a path optimization is presented that avoids stops during the robot trajectory by adapting the measurement sequence so that the robot measures two ϕ -cuts, namely alternating at $\phi = 0^\circ$ and at $\phi = 2\Delta\phi$, where $\Delta\phi$ describes the angular increment along the ϕ -axis. In combination with the additional rotation axis ϕ_{axis} , a complete hemispherical measurement can be performed, whereby unnecessary stops of the robot can be reduced or, depending on the measurement sequence, completely avoided. Furthermore, the additional travel distance in the transition area between two cuts can be used to acquire supplementary measurement data using PDT. These are located directly below the equator and can be used, for example, to minimize the truncation effect. Alternatively, it is also possible to use additional SPT in this area, however, this will affect the resulting path and, consequently, reduce the optimization potential of the RC.

In Fig. 6.6b, the most evident case of the optimization approach is shown, in which a stop of the robot at the equator of the measurement sphere ($\theta = 90^\circ$) can be avoided due to the adapted measurement sequence. This allows to reduce the measurement time, as standstill times are minimized and, simultaneously, the communication overhead with the robot controller is reduced, as several ϕ -cuts can be acquired using a single spline path. For the pole of the measurement sphere at $\theta = 0^\circ$, however, the stop cannot be avoided in this case, as each ϕ -cut samples the pole and, therefore, two identical sampling points follow each other. Nevertheless, the duration of this standstill is shorter than in comparison to the canonical path definition, since the



(a) Spline without pole ($\theta \in [\Delta\theta; 90^\circ]$). (b) Spline over a full θ -cut ($\theta \in [90^\circ; -90^\circ]$).

Figure 6.7: Comparison of two spline-based measurement implementations in which the redundant sampling points at the pole are avoided or full θ -cuts are measured to increase the path velocity. A qualitative comparison of the robot's path speed is provided by the color-coded trajectory, whereby the same speed and acceleration profile has been used in each case (also in comparison to Fig. 6.6).

communication overhead with the robot controller is eliminated, assuming that both sampling points in the pole are contained in the same spline path.

As a further optimization, Fig. 6.7a demonstrates a motion sequence in which the redundant sampling points are avoided as the pole is only sampled once. Accordingly, apart from the first sampling point in the pole, the next ϕ -cut is already started at $\theta = \Delta\theta$, which ensures spatially different sampling points near the pole for each ϕ -cut. The resulting spline path, in combination with the additional rotation axis, allows to perform a complete equiangular measurement without halting the robot. However, since the sampling points near the pole are spatially close to each other and, thus, result in a narrow curve radius, the RC reduces the speed in this case, which is emphasized by the color coding of the path.

Consequently, the next optimization step visualized Fig. 6.7b avoids the turning point at the pole of the measurement sphere. Instead, the measurement is conducted over a full θ -cut, whereby the turning points are at $\theta = \pm 90^\circ$, resulting in a maximum turning radius and, consequently, in a path that can be traversed at maximum speed. Although the path shown in Fig. 6.7b includes the sampling point at the pole for all ϕ -cuts, this type of implementation also allows to sample the pole, in principle, only once. Since the resulting path of the robot, as can be deduced from Fig. 6.1, will only change slightly and the measurement path will be close to the pole anyway, the expected advantage in terms of a reduction in measurement time is negligible and is, therefore, not considered hereinafter.

The possibility to combine several previously canonical measurement paths within one continuous spline path allows to minimize unnecessary halting times of the measurement system. Even though the transition areas between two measurement cuts lead to an increased overall path length, which, moreover, can only be traversed at

reduced speed due to the turning radii, it is shown that this aspect is not dominant in comparison to standstill times. Therefore, the resulting measurement durations of an entire SNF measurement for the various presented implementations are compared relatively to each other. The bidirectional canonical measurement sequence shown in Fig. 6.6a is considered as a reference case and compared with the spline-based variants from Fig. 6.6b and Fig. 6.7 in terms of both, the measurement duration as well as the average path velocity in Fig. 6.8. To ensure comparability, all test cases have been performed with the same speed and acceleration profile. The indicated measurement durations are normalized to the canonical reference profile and the average path velocities are normalized to the specified maximum velocity in order to facilitate a relative comparability.

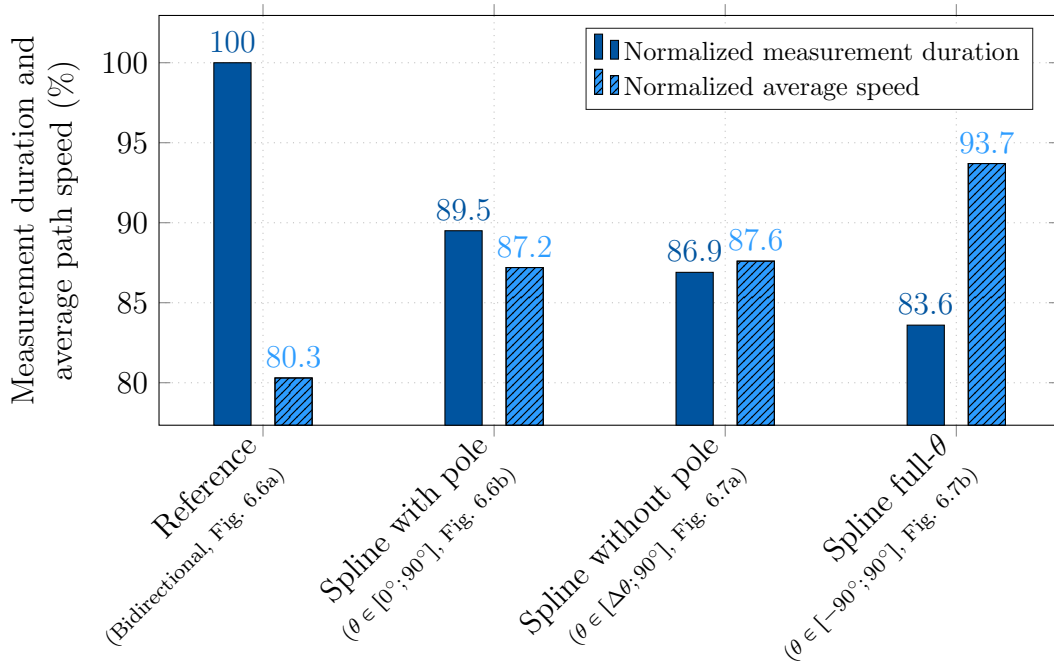


Figure 6.8: Relative comparison of the measurement duration normalized to the reference case and the average path speed normalized to the user-defined maximum speed.

The comparison indicates that the duration of a SNF measurement can be reduced by at least 10 % if spline-based trajectories as shown in Fig. 6.6b are used. In addition, the average path speed can be improved from 80 % of the maximum speed to approximately 87 % through the continuous transition to the next cut of a measurement, resulting in reduced halting times. However, for the subsequent optimization step illustrated in Fig. 6.7a, in which the stopping point at the pole of the measurement sphere is avoided, the resulting difference is only marginal: The time gain is less than 3 % and the average path speed is improved by 0.4 % only, as the narrow curve radius at the pole requires a significant deceleration of the measurement system. Additionally, this measurement sequence inherently requires a more complex NFFT, as the measurement data is acquired on an irregular sampling grid. In contrast, the spline-based implementation shown in Fig. 6.7b provides the greatest time savings, as the turning points of the measurement path at the equator have a

maximum turning radius. In this case, the measurement duration is reduced by 16 % while the average path velocity is raised to more than 93 % of the specified maximum velocity. This measurement sequence, thus, achieves significant time savings compared to the reference case and the other spline-based variants, although effects on the results transformed into far field cannot be completely excluded, as will be discussed in the following.

For this purpose, comparative measurements have been carried out using the probe antenna and AUT described in Appendix C at a frequency of 60 GHz for all the measurement sequences presented. The measurement radius has been set to $r = 200$ mm and sampling grids based on an angular step size of $\Delta\theta = \Delta\phi = 3^\circ$ have been used. The measurement sequence explained in Fig. 6.7a avoids the redundant sampling points in the pole and, thus, results in an irregular sampling grid. To ensure comparability, the SMC of the AUT are calculated for all measurement sequences using the direct method presented in Section 2.3.2 and are, consequently, truncated based on the AUT's minimum sphere. In addition, the higher-order PPC described in Section 5 is applied. The correction data required for the actual position and orientation of the probe antenna are provided by the RC for each sampling point. Although this data is, therefore, subject to higher tolerances, it is available without the need for an external measurement system, such as a laser tracker. This avoids additional costs, but also any necessary modifications to the measurement setup.

The results of the NFFFT are compared in Fig. 6.9, whereby the bidirectional canonical measurement sequence as illustrated in Fig. 6.6a is used as the reference. The radiation pattern for $\phi = 0^\circ$ is presented, highlighting the differences between the various test cases using the EES always referring to the canonical reference.

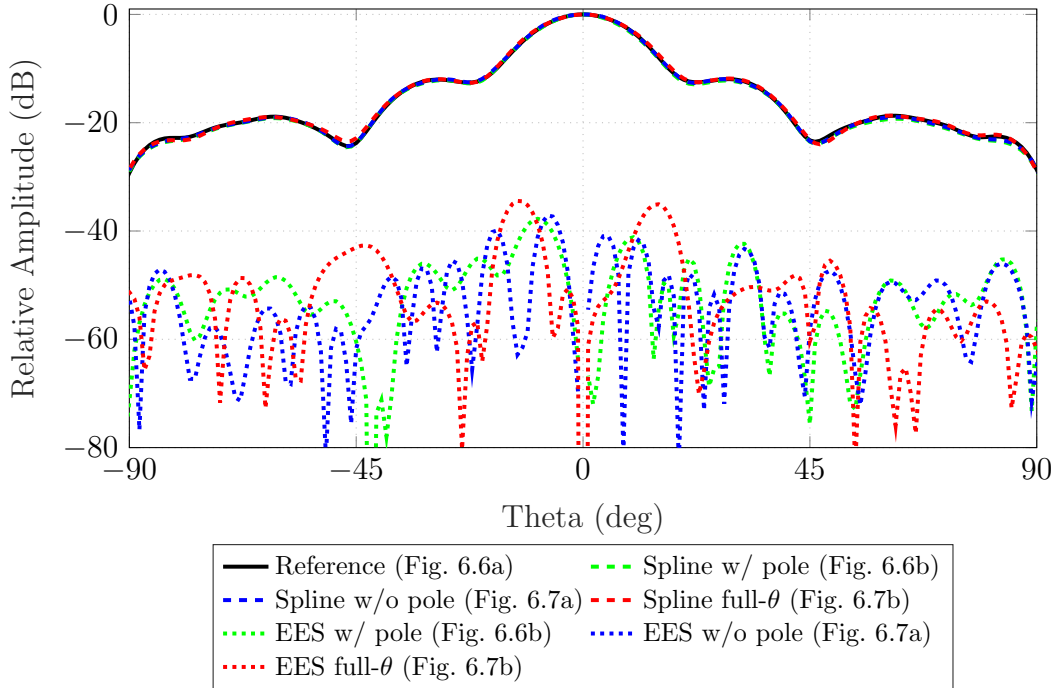


Figure 6.9: Transformed far-field radiation patterns ($\phi = 0^\circ$) for the spline-based measurements presented as well as the corresponding EES with respect to the reference measurement for a frequency of 60 GHz.

The maximum EES of the two spline-based measurements with and without the redundant sampling points in the pole (Fig. 6.6b and Fig. 6.7a) is -37.7 dB and -37.1 dB, respectively, and occurs in the area of the main lobe. This insignificant difference proves that sampling the pole of the measurement sphere once is an acceptable procedure and only negligibly affects the transformation results, since this area is highly oversampled in equiangular measurements. Nevertheless, the time advantage is marginal (approximately 3%), but requires a more complex NFFT due to the resulting irregular sampling grid.

The maximum EES of the spline-based measurement sequence performed over a full θ -cut (Fig. 6.7b) is -34.4 dB and also occurs symmetrically around the main lobe region. In addition, a further distinctive deviation in the range of -42.6 dB occurs in the area of the null at $\theta = -45^\circ$. This asymmetrically occurring deviation is attributable, at least in part, to the sampling over a full θ -cut, causing this measurement series to be susceptible to different room scattering effects compared to the two previously mentioned implementations. In addition, the angular range sampled by the robot is twice as large, which not only results in increased positional uncertainties, but also in considerable changes in the orientation of the TCP, especially for $\theta \approx \pm 90^\circ$. This induces increased cable bending effects, which may impair the entire measurement and, thus, explain the increased deviations in relative comparison with the other measurement sequences. However, if these differences are within an acceptable order of magnitude for the intended application, this implementation provides the greatest acceleration of equiangular measurements, which is more than 17% faster than the canonical reference measurement and raise the average path velocity from approximately 80% to more than 93% of the set maximum speed. This acceleration is achieved by the robot's spline-based motion control, while the adapted measurement sequences further reduce standstill times and ensure smooth transitions between the respective measurement cuts.

6.4 Summary

Spline-based motion sequences represent the optimum operating mode for robot-based systems. Instead of forcing a canonical path, comparable to dedicated axes, e.g. linear or circular paths, only the relevant sampling points or spline support points, respectively, are specified. This provides the RC with degrees of freedom to optimize the path, so that the focus is not, e.g. on the highest possible linearity or circularity of the path, but on the highest possible path speed or smoothness of motion.

Since the actual spline implementation of a robot manufacturer is proprietary, the resulting spline path can only be calculated using the manufacturer's simulation tools, if available. If these are inaccessible, Fig. 6.1 and Fig. 6.2 provide an indication to estimate the resulting spline trajectory. For the robot used within this dissertation, there is, in case of complex paths, an acceptable agreement for the periodically interpolated cubic spline as illustrated in Fig. 6.4, although the actual implementation probably realizes a variant of a quintic spline. Nevertheless, this provides a possibility to estimate the actual spline path and, thus, to estimate if it is suitable for the intended measurements.

Furthermore, spline paths enable to exploit the native trigger functionality of the RC, as the spline support points are known before the start of a measurement. As demonstrated in Fig. 6.5, even for relatively low measurement speeds of 0.5 T/s , the trigger accuracy can be improved by a factor of approximately ten compared to the conventional implementations based on a background task. Moreover, even at a trajectory speed eight times higher, the native trigger functionality of a spline trajectory is better or of a comparable order of magnitude to the conventional implementation. The advantages of spline-based motion sequences also include the implementation of SNF measurements. For example, they allow to combine several measurement runs within one spline path, thus reducing standstill times and minimizing the communication overhead with the RC. In addition, their continuous motion provides an inherent possibility to realize irregular sampling grids, as illustrated in Fig. 6.7a. Depending on the implementation of a spline-based measurement, equiangular measurements can be accelerated by up to 17 %, whereby the average path speed can be increased from 80 % in the case of canonical measurement paths to more than 93 %. Therefore, the application of spline-based motion sequences enables increasing the trigger accuracy, while simultaneously reducing the measurement duration compared to canonical path implementations, even at the same specified maximum path speed. As a result, spline-based measurements represent the recommended operating mode of a robot and facilitate the utilization of the degrees of freedom offered by such positioning systems. Consequently, they also provide efficient implementations for irregular sampling grids, which were previously of rather theoretical interest, as will be demonstrated in Section 7.

Maximum Determinant Sampling Grids

The most established distribution of sampling points in antenna measurement technology is described by the equiangular sampling grid. It combines the advantages of a practical application using roll-over-azimuth positioners as well as an efficient and stable calculation of the SMC based on the SWE [45]. In particular, the simplifications of the Wacker algorithm explained in Section 2.3.1 motivated equiangular sampling grids, which were the only feasible option for calculating a solution of the NFFFT at the time of the introduction of SNF measurement technology due to severely limited computing capacities. With the computing capacities currently available, however, the focus has shifted from the general solution capability to more flexible and faster sampling grids, which reduce the duration of an antenna measurement and, thus, also the costs while simultaneously increasing the throughput of a single antenna measurement facility.

Since equiangular sampling grids are highly oversampled at the pole of the measurement sphere, the straightforward approach is to reduce the redundant data points in this area. One implementation is given by the thinned equiangular sampling grid (also called Igloo sampling) [45, 136, 137], in which the various rings along ϕ are thinned as a function of the θ -coordinate. This allows to reduce the density of the sampling points at the pole, while at the same time maintaining the Nyquist criterion at the equator of the measurement sphere and, moreover, ensuring an equiangular sampling in ϕ . The latter permits to interpolate the sampling points by means of a Fourier transformation to a regular equiangular grid, although this is not mandatory. Alternatively, the direct solution approach described in Section 2.3.2 can be used for a stable calculation of the SMC directly based on the irregular sampling grid.

Another approach, originally used to reduce the number of sampling points in planar measurements [138], describes spiral sampling grids in spherical near-field measurements, first validated using numerical simulations in [139]. Also in this case, the complexity of the NFFFT is reduced by interpolating to a regular equiangular grid, which requires oversampling, especially in the area of the pole to ensure an appropriate accuracy. This, as well as the actual reduction in the number of required sampling points, is confirmed by the experimental validation using a conventional roll-over-azimuth positioner in [140, 141], however, with no statements being formulated regarding the resulting measurement duration. Further challenges with respect to a time-efficient implementation, i.e. the continuous movement of the positioning system along the spiral path, are added in [142], although no comparative figures are given in this case either. Further investigations for three different definitions of

a spiral path according to [143–145] are compared with regard to the numerical stability, resulting path length and estimated measurement durations in stepwise and continuous acquisition mode in [146].

A completely different approach, in which the sampling grid is not predicated by the available positioning system, is given by the MDSG. In this case, the sampling points are given by a mathematically optimized point distribution on a sphere in three-dimensional space $\mathbb{S}^2$, so that the determinant of this point distribution is maximized [147]. For this purpose, the determinant of a basis $b_1, \dots, b_{(N_{\text{MDSG}}+1)^2}$ from spherical polynomials of degree N_{MDSG} for the space $\mathbb{P}(\mathbb{S}^2)$ is maximized according to

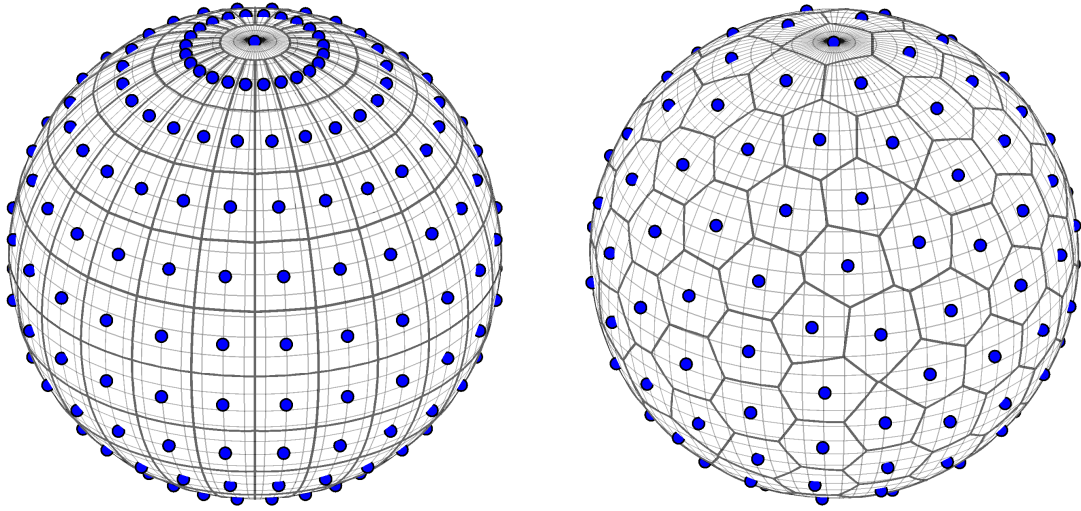
$$\max \det (b_i(x_j))_{i,j=1}^{(N_{\text{MDSG}}+1)^2}, \quad (7.1)$$

where x are denoted as the MDSG nodes on the sphere $\mathbb{S}^2 = \{x \in \mathbb{R}^3 : |x| = r\}$ with radius r . The maximization of (7.1) is a nonlinear optimization problem for which solutions up to degree $1 < N_{\text{MDSG}} < 165$ are provided in [148], which serve as the basis for the following investigations.

The advantage of the MDSG compared to conventional (thinned) equiangular or spiral sampling grids is that (practically) no oversampling is required to avoid an ill-conditioned system of equations for calculating the SMC. Instead, the point distribution of the MDSG is optimized for a maximally large determinant, which conversely leads to a well-conditioned solution matrix. However, the efficient implementation regarding the mechanical feasibility with respect to the positioning system has not yet been addressed or, apart from [129, 149], been implemented using continuous paths.

The solutions calculated in [148] refer to scalar spherical harmonics for which modes with index $n = 0$ exist. To apply the SWE based on an MDSG, two orthogonal polarizations are recorded at each sampling point, where modes $n = 0$ do not exist for vector spherical harmonics. For this reason, an oversampling of two sampling points also occurs with the MDSG, which is, however, negligible compared to conventional sampling grids. In addition, combinatorial analyses in [146] have shown that even omitting the sampling point with the least impact on the solution system leads to a deterioration in stability, although the condition number remains within an acceptable range. Nevertheless, an oversampling of two sampling points is, therefore, the preferred method, as the increase in measurement time due to an oversampling of $\lesssim 1\%$ is not significant. Due to the low density of sampling points (exemplified in Fig. 7.1b), no interpolation to a regular sampling grid is performed, which requires a certain degree of oversampling similar to spiral sampling (especially near the pole). Instead, the direct approach described in Section 2.3.2 is applied to calculate the SMC.

In addition, it is stated in [146] that the MDSG is not sensitive to rounding of the sampling point coordinates to typical accuracies of a positioning system and is, therefore, an appropriate sampling grid for antenna measurements. Furthermore, a measurement strategy for conventional roll-over-azimuth positioners is also proposed in [146], which, however, can only be implemented stepwise for most systems due to the discontinuous θ - and ϕ -coordinates as can be derived from Fig. 7.1. The comparison of an equiangular sampling grid (step size of $\Delta\theta = \Delta\phi = 15^\circ$) with the corresponding MDSG₁₁ ($N_{\text{MDSG}} = 11$ according to (2.5)) shows that the amount of



(a) Equiangular sampling grid with a step size of $\Delta\theta = \Delta\phi = 15^\circ$ and a total amount of sampling points $N_s = 576$. (b) MDSG_{11} corresponding to Fig. 7.1a resulting in a total amount of sampling points $N_s = 288$.

Figure 7.1: Comparison of an equiangular sampling grid and the corresponding MDSG_{11} ($N_{\text{MDSG}} = 11$) including the Voronoi cell tessellation indicated by the thicker gray lines. It is emphasized that all MDSG require only half of the sampling points of the equiangular counterpart.

sampling points is significantly reduced, specifically by a factor of two, and that no oversampling in the vicinity of the pole is required. This is also emphasized by the Voronoi cell tessellation, proving that in the MDSG_{11} each sampling point covers approximately the same spherical surface while the size of the Voronoi cells of the equiangular grid decrease considerably towards the pole. Furthermore, it is evident that the seemingly arbitrary positions of the sampling points pose a challenge for conventional positioning systems and, hence, stepwise measurement schemes are the natural approach. Therefore, especially for higher frequencies or large numbers of sampling points, the MDSG loses practical relevance, as only a continuous data acquisition can be considered as an efficient implementation. Robot-based measurement techniques, however, allow an efficient and continuous data acquisition using an MDSG by taking the sampling points as the support points for a spline path. Thus, the flexibility of a robot-based system allows this previously rather theoretically interesting sampling grid to be transferred into a convenient and relevant practical implementation for SNF measurement techniques.

7.1 Numerical Stability

Performing the NFFFT based on the SWE of an electromagnetic field formulates a linear system of equations. As previously explained in (2.42), this system contains a number of measurement points L , the measurement matrix Ψ and a number J of unknown SMC describing the AUT. To evaluate the stability of the NFFFT, the determinant or, as used in this analysis, the condition number κ of the measurement matrix Ψ can be referred to, which provides a direct sensitivity analysis on the error

propagation in the calculated solution for a perturbation of the input data. The measurement matrix Ψ includes the coordinates of the sampling points in θ and ϕ as well as the probe response constants $P_{s\mu n}(kr)$ for a certain measurement distance r and frequency f , with the size of the matrix being defined by the amount of modes, in this case $N_n = N_m$, and the total number of sampling points L .

Thus, it is possible to analyze the discussed sampling grids with respect to their numerical stability depending on the required oversampling, which is presented in Fig. 7.2. For this purpose, an AUT with $N_{m,n} = 27$ modes, which corresponds to the normally used standard gain horn (SGH) antenna at a frequency of $f = 60$ GHz, is used. In this case, a simulated dipole probe antenna is considered for the general example of a far-field-to-far-field transformation, while a specific analysis for near-field measurements is provided in Section 7.3.1. As shown, the condition number κ of

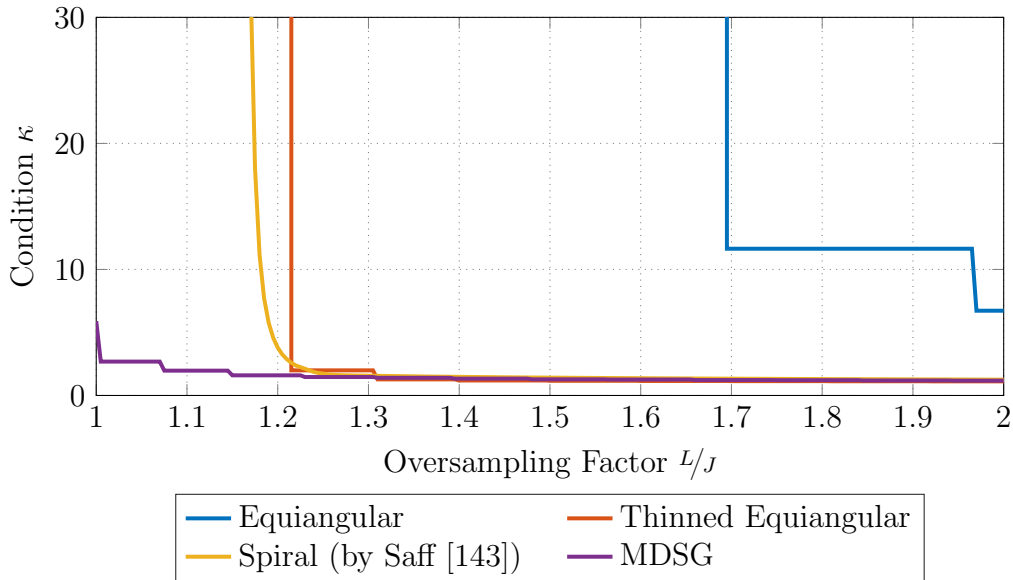


Figure 7.2: Comparison of the resulting condition number κ depending on the oversampling factor for different sampling strategies. The calculation of the condition is based on an AUT with $N_{m,n} = 27$ and a dipole probe antenna for a frequency of $f = 60$ GHz.

all tested sampling grids improves with increasing oversampling. Equiangular measurements achieve an acceptable stability for an oversampling factor of $L/J = 1.7$ or higher, although it is also evident that a condition number in the order of magnitude of the other sampling grids is only available for the typically used oversampling of $L/J = 2$. The thinned equiangular and spiral sampling grids require considerably less oversampling. Both variants exhibit approximately the same stability from an oversampling of $L/J \gtrsim 1.25$. However, while the thinned equiangular point distribution allows an acceptable error propagation abruptly from $L/J \gtrsim 1.22$, the spiral grid shows a more continuous trend. Thus, the exemplary chosen spiral implementation by Saff [143] could already be used from an oversampling of $1.18 \lesssim L/J \lesssim 1.20$, although the slope of the curve indicates a significantly increased error propagation towards even lower oversampling factors.

In contrast, the MDSG enables a numerically stable NFFFT, which exhibits only an exceptionally low dependence on a possible oversampling. Even in the case without any oversampling $L/J = 1$, the condition number is approximately 5.9, which is lower than for the equiangular sampling grid with twice as many sampling points ($\kappa = 6.7$ for $L/J = 2$). However, if a small oversampling is nevertheless applied, the condition number converges monotonically decreasing to the achievable stability of the thinned equiangular or spiral point distribution, resulting in $\kappa \approx 1.2$ for $L/J \gtrsim 1.31$.

In a second step, the stability of the NFFFT as a function of the number of AUT modes is investigated in Fig. 7.3 for a more comprehensive assessment of a possible oversampling requirement. The oversampling factors for equiangular and thinned equiangular point distributions are determined to $L/J \geq 2.0$ and $L/J = 1.25$, respectively, based on Fig. 7.2. Similarly, a slight oversampling of $L/J = 1.20$ as well as a higher oversampling of $L/J = 1.25$ are analyzed for the spiral sampling grids.

In contrast to conventional sampling grids, the MDSG can be scaled with mode-exact precision. Therefore, variants without oversampling ($N_{\text{MDSG}} = N_{m,n}$) and with a slight oversampling with regard to one or three further calculateable modes, i.e. $N_{\text{MDSG}} = N_{m,n} + 1$ and $N_{\text{MDSG}} = N_{m,n} + 3$, are examined.

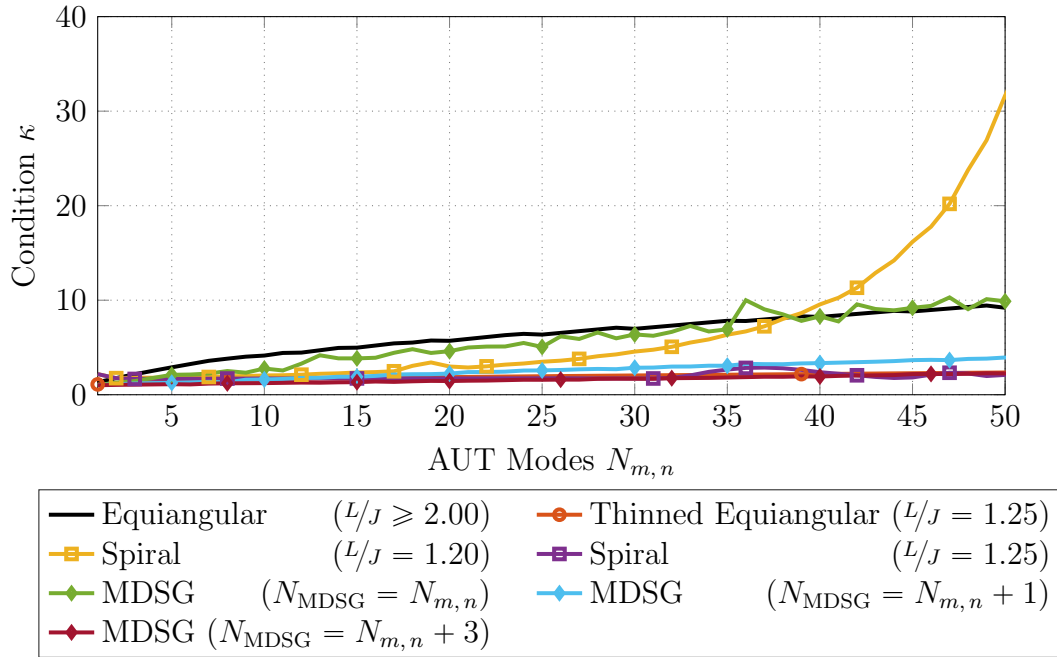


Figure 7.3: Comparison of the resulting condition number κ depending on the amount of AUT modes $N_{m,n}$ for different sampling grids and appropriate oversampling factors derived from Fig. 7.2. A dipole probe antenna and a frequency of $f = 60$ GHz is used to maintain comparability to the aforementioned analysis.

For the calculation of the condition number κ for an equiangular distribution of sampling points, the requirement of the oversampling factor has been reduced to $L/J \geq 2.00$. This is necessary to ensure that the angular step size of the equiangular sampling grid is an integer multiple of the angular range and, thus, a valid equiangular sampling grid is generated. Due to the scalability of this sampling grid, this can result in an oversampling factor $L/J \geq 2.00$, where the requirement for a valid sam-

pling grid instead of enforcing to $L/J = 2.00$ prevails which is also in accordance to the established measurement practices [43]. Using this method of calculation yields a slight increase in the condition number κ for an increasing complexity of the AUT. In particular for $N_{m,n} \geq 25$, the resulting condition number is comparable to the stability of the MDSG for $N_{\text{MDSG}} = N_{m,n}$ and consistently amounts to $\kappa < 10$ for $N_{m,n} \leq 50$.

The spiral sampling grid with $L/J = 1.2$ reveals a rapidly decreasing numerical stability for increasing AUT complexity ($N_{m,n} \gtrsim 45$), which is due to the significant gradient of the condition number demonstrated Fig. 7.2 for the given oversampling factor. However, the cross-comparison with the spiral point distribution with $L/J = 1.25$ indicates that the stability can be recovered if the oversampling is increased, which leads to the conclusion that it is advisable to rely on a higher oversampling for spiral grids to ensure reasonable results are provided independent of the AUT. In this case, a condition number in the order of $\kappa \lesssim 2$ is achieved, which is analogous to the thinned equiangular sampling grid.

The same order of magnitude is also achieved by the MDSG if its degree is increased, i.e. a slight oversampling is considered for this grid as well. This case is shown exemplarily for the MDSG with $N_{\text{MDSG}} = N_{m,n} + 1$ and $N_{\text{MDSG}} = N_{m,n} + 3$, where especially the latter variant provides a comparable stability to the spiral or thinned equiangular sampling grid with $L/J = 1.25$.

This proves that the point distribution of an MDSG is a non-redundant and numerically stable sampling grid to perform a NFFFT resulting in a condition number comparable to equiangular sampling grids, but requiring only approximately half of the sampling points. However, an additional improvement in the numerical stability can be achieved for the MDSG as well if a slight oversampling is performed, as exemplarily shown for one or three additional AUT modes. Hence, a condition number of comparable order of magnitude to the spiral or thinned equiangular sampling grids using $L/J = 1.25$ can be achieved in this case.

7.2 Path Generation Algorithms

The sampling points defined by an MDSG enable a numerically stable NFFFT with half the number of measurement data compared to an equiangular measurement. However, for the practical feasibility of an MDSG measurement, an efficient measurement path for the acquisition of the sampling points is required. For conventional roll-over-azimuth positioners, the irregular distribution of the sampling points poses a challenge which, depending on the system, may only be possible with stepwise measurements, thus turning a possible speed advantage into the opposite. Robot-based systems provide new possibilities in this respect, although the appropriate design of a measurement path impacts the resulting measurement duration.

Therefore, two methods for path generation are presented, namely a minimization of the total length of the path based on a traveling salesman problem (TSP) as well as an optimization with regard to wider curve radii (WCR). The WCR approach allows to achieve higher path speeds of the robot through larger curve radii and, thus, contributes to a reduction of the measurement duration despite an increased overall path length.

A comparison of the resulting measurement duration, the achievable average speed on the generated path and the total path length is conducted directly after the introduction of the WCR optimization in Section 7.2.2.

7.2.1 Traveling Salesman Optimization

The TSP refers to a combinatorial optimization problem in the areas of operations research and theoretical computer science. It denotes the problem of a sales representative to find the shortest, or more generally, most efficient route through a given set of cities, where none but the first city is visited more than once. The predecessor of the problem is either a guide for traveling salesmen published in 1831 [150] or the Icosian Game by the Irish mathematician William Rowan Hamilton from 1857 as explained in [151]. The first explicit identification of this challenge as a mathematical optimization problem was published in 1932 by Karl Menger [152] or, in a more formal way, in the mathematical treatment from 1935 by Hassler Whitney [153]. A theoretical justification for the difficult solvability of the problem was provided by the proof of $\mathcal{NP}$ -completeness in 1972 [154], resulting in an exponential increase in complexity for an enlarging number of cities. Since then, the TSP has been subject of scientific work, for example on obtaining possibly non-optimal solutions through the use of heuristics in a more efficient runtime. In this respect, a breakthrough has been achieved by an algorithm published in 2014 providing a solution in polynomial runtime that is at most 1.4 times longer than the classical, optimal solution [155].

The first robot-based implementation of an MDSG in antenna measurement technology has been presented in [129], where the path for determining the sequence of sampling points is calculated based on an optimal solution of a TSP. Since the TSP is well-established, the procedure for determining a solution is only briefly outlined hereinafter. First all unique combinations for the sequence of sampling points are determined and the efficiency for all pairs of sampling points based on a cost function, in this case based on the Euclidean distance is calculated. The global cost function to be optimized results from the sum of the distances between all sampling points in the path. Binary decision variables are used for this purpose, which determine whether a sampling point is already contained in a path. In addition, an integer linear condition is introduced which specifies that each sampling point occurs on exactly two sub-paths, ensuring that each sampling point is traversed exactly once (or in other words, there is exactly one arrival and one departure connection to this sampling point). In a subsequent step, it is checked that no closed-loop sub-paths exist. If these are identified, additional inequality constraints are added to the problem, which exclude these sub-paths as a solution. This is repeated iteratively until only one single closed-loop path exists, which defines the sequence of the sampling points of the MDSG.

Both, to reduce interference during a measurement and for safety reasons to avoid any risks of collisions, the robot arm should be prevented from reaching over the AUT. For this reason, the sampling point distribution is divided into quarters of the hemisphere so that the robot arm performs the measurement always in the same spatial quarter while the AUT is rotated using the additional axis of rotation. This is not only advantageous in terms of the measurement procedure, but also contributes to reduce the computational complexity of the algorithm for solving

the TSP. It is, therefore, possible to calculate the path even for larger MDSG in an acceptable runtime in the order of minutes on a normal work station. Furthermore, to exclude unpredictable transitional movements of the robot between the quarters, the sequence in the respective quarters is coordinated to ensure that the starting point of a subsequent quarter has the minimum Euclidean distance to the target point of the previous quarter. The result of this algorithm for calculating the measurement path of the same MDSG already presented in Fig. 7.1b is visualized in Fig. 7.4. In the interests of better visualization, only two exemplary quarters are plotted. Furthermore, the actual path of the robot is depicted including a qualitative assessment of the path velocity due to the color-coded trajectory. The illustrated

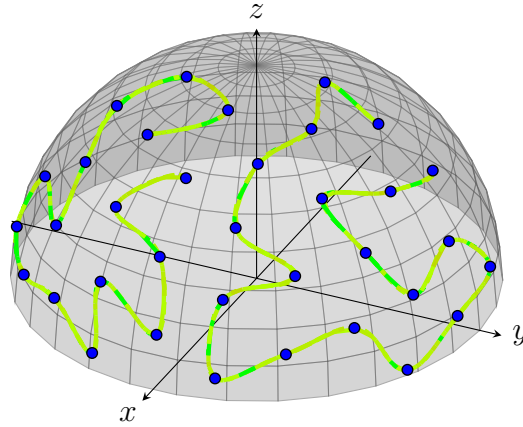


Figure 7.4: Two quarters of the determined measurement sequence based on the solution of the TSP for the MDSG_{11} already presented in Fig. 7.1b. The starting point of a subsequent quarter is selected based on the minimum Euclidean distance to the final point of the previous quarter. The color coding enables a qualitative representation of the actual path velocity.

path demonstrates that even if it is the variant with shortest Euclidean distance over a closed-loop path (on a quarter), it contains a relatively large amount of sharp curves. The color-coding thereby reveals that the RC reduces the path speed at many of these curves where the turning radius is narrow. In addition, it is apparent that the path speed is also adjusted immediately before reaching a sampling point so that the robot can pass accurately through this point.

Alternatively, the point distribution of the MDSG can also be halved instead of quartered, which increases the number of sampling points for the solution of the TSP. Since the TSP is $\mathcal{NP}$ -complete, this considerably increases the calculation complexity, but the larger data set facilitates the computation of a path with a shorter total length. However, negative effects such as cable flexing during the dynamic movement of the robot over half the hemisphere are more pronounced in this case and, therefore, this approach is not considered at this point.

The resulting durations of the MDSG measurements based on the solution of the TSP are provided in Fig. 7.5. The comparison is made against the corresponding equiangular measurements, which are performed using both, the canonical as well as the spline-based path implementation. To evaluate the scalability of MDSG-based measurements, four comparative cases are considered, specifically equiangu-

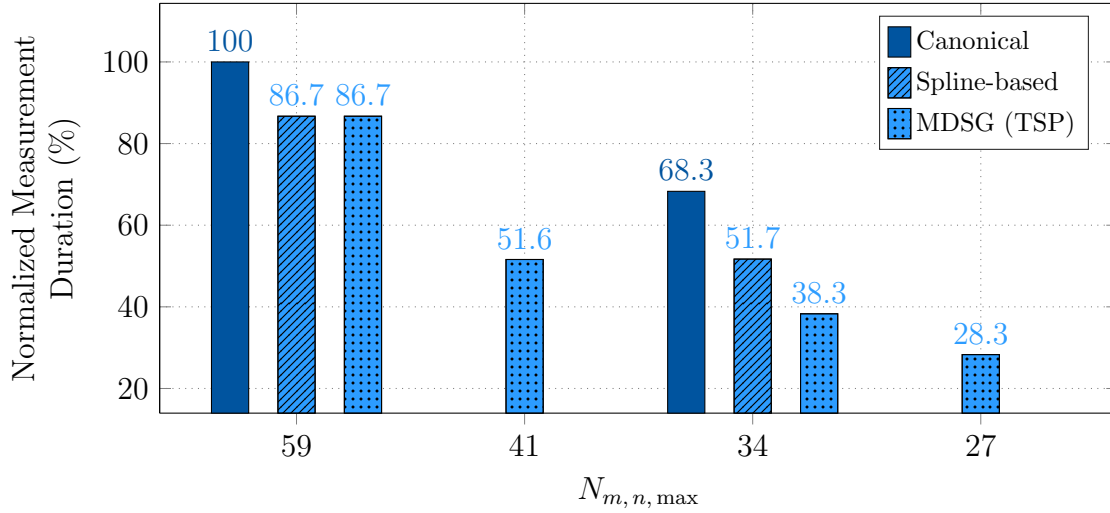


Figure 7.5: Normalized acquisition time for equiangular (canonical and spline-based implementation) and MDSG measurements where the measurement path is derived by solving the associated TSP for minimum Euclidean distance.

lar sampling grids for $N_{m,n,max} = 59$ and $N_{m,n,max} = 34$, which correspond to an angular step size of $\Delta = 3^\circ$ and $\Delta = 5^\circ$, respectively, as well as the associated MDSG implementations. In addition, the measurement durations of a larger dimensioned MDSG_{41} and the non-redundant minimum configuration of a MDSG_{27} are also included in the comparison, for both of which no corresponding equiangular measurement exists. For a relative comparison, all determined measurement durations are normalized to the canonical implementation of the equiangular sampling grid with $N_{m,n,max} = 59$.

As already examined in Section 6.3, conventional canonical measurements can be accelerated by a spline-based implementation regardless of the velocity profile used. While in the canonical implementation each measurement cut is communicated separately to the RC, spline paths may comprise several measurement cuts. This reduces standstill times at the beginning and end of a path due to a reduced communication overhead with the RC. For a complete equiangular measurement, this allows to reduce the measurement time by approximately 13% for $N_{m,n,max} = 59$ and 16% for $N_{m,n,max} = 34$ compared to the canonical implementation.

Due to the irregular point distribution of the MDSG, only spline-based implementations can be considered in this case. The MDSG_{59} results in the same measurement duration as the spline-based implementation of the equiangular sampling grid for $N_{m,n,max} = 59$. This is due to the fact that the relatively dense distribution of sampling points requires the robot to decelerate more frequently in order to precisely traverse the requested coordinates. Nevertheless, the MDSG scales mode-accurately in contrast to equiangular measurements. It is, therefore, possible to select the sampling grid according to the requirements of the AUT, so that for an AUT described by $35 < N_{m,n} < 59$, an MDSG of a lower order can be applied. This is exemplified by the MDSG_{41} , which achieves a significant reduction in measurement time to approximately 51.6% of the equiangular reference. Thus, the finely graduated

scalability, depending on the AUT, may contribute to a reduction in measurement time even for more complex measurement grids.

This tendency continues for the equiangular sampling grid with $N_{m,n,\max} = 34$, where the measurement duration is reduced by 13.4 % if the corresponding MDSG₃₄ is applied. Furthermore, the minimum configuration of a MDSG₂₇ is included, which corresponds to the AUT explained in Appendix C for a frequency of 60 GHz. This sampling grid facilitates a further reduction in measurement time of 10 % compared to the MDSG₃₄ and more than 23 % in relation to the corresponding equiangular measurement with $N_{m,n,\max} = 34$. Also in this case, the mode-accurate scaling of these sampling grids is emphasized, which enables a precise adjustment of the measurement in accordance to the AUT.

The holistic view of the comparison further demonstrates that the duration of the MDSG measurements based on the TSP path optimization does not scale for increasingly dense sampling grids as exemplified for the MDSG₅₉. This is due to the fact that the generated path based on the shortest total length contains many tight curves, which require the robot to operate at a reduced speed. However, for sampling grids of decreasing order, for example for $N_{\text{MDSG}} \leq 41$, the duration of the MDSG measurements using the TSP path calculation scales accordingly, enabling the acceleration of measurement projects.

7.2.2 Wider Curve Radii Optimization

Although the solution calculated using the TSP determines the closed-loop path with the shortest total length, the result is not guaranteed to be optimal for a robot-based system. Since the path may contain many sharp turns that require continuous accelerations and decelerations, the average path speed will be reduced. Instead, the optimization for WCR presented in [149] is discussed in the following, which can achieve a reduction in measurement time due to a higher average speed on the resulting path, even if the overall length is increased.

The approach is based on a similar principle to the TSP, but instead of the Euclidean distance as the cost function, the change in direction on a section of the path is considered. This requires the simultaneous consideration of at least three sampling points, which are used to construct a spline path. For this part of the resulting trajectory, the maximum change of the tangent for this constructed spline path is used as the cost function, described as

$$f_{\text{WCR}} = \max_t \left| \ddot{\xi}(t) \right| \quad (7.2)$$

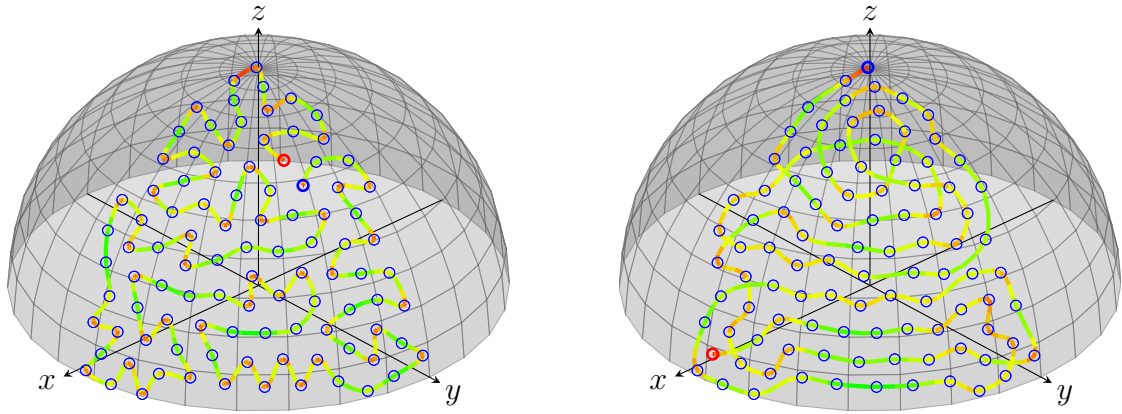
where ξ is the vectorial definition of a point on the parametric description of the constructed spline. Therefore, the tangent at the respective point is given by its derivative $\dot{\xi}(t)$. However, since the exact implementation of a spline-based movement is proprietary, the calculation of $\xi(t)$ is only an estimate and, therefore, the same applies to the tangent $\dot{\xi}(t)$. Nevertheless, the analyses in Section 6.1 demonstrate that the cubic spline calculation is for complex scenarios a reasonable approximation of the actual motion of the robot, as illustrated in Fig. 6.4. It is, therefore, assumed that the change in direction of the actual spline path is only slightly deviating from the change in direction of the calculated path.

Consequently, the solution is calculated based on the weights f_{WCR} analogously to the TSP, whereas the simultaneous consideration of three sampling points instead of the previously accounted two points for the Euclidean distance severely increases the calculation complexity. For this reason, not all possible combinations are examined, but a neighborhood of points located within a radius d_{min} is considered only. The filter radius d_{min} is determined empirically and chosen to be twice the minimum distance between all sampling points of the MDSG, so that the neighborhood to be evaluated provides sufficient variability. The larger the filter radius is selected, the better the results of the WCR optimization, but at the cost of an increasing calculation complexity. Moreover, to ensure a valid path, additional boundary conditions are introduced. The corresponding optimization problem, therefore, results in

$$\begin{aligned}
& \min_x && F_{\text{WSR}}^T x \\
& \text{subject to} && A_{\text{ieq}} \cdot x \leq 0^{2N_e \times 1} \\
& && A_{\text{eq}} \cdot x = 1^{3N_s \times 1} \\
& && x \in \mathbb{N}_0^{N_e \times 1} \text{ with } x_i \in \{0,1\}
\end{aligned} \tag{7.3}$$

where $F_{\text{WSR}} \in \mathbb{R}^{N_e \times 1}$ is the vector containing the weights defined in (7.2) and N_e as well as N_s are the amount of edges and sampling points, respectively. To ensure the continuity of the computed path, for each edge of three points (P_i, P_{i+1}, P_{i+2}) it is required that both edges (P_n, P_i, P_{i+1}) and (P_{i+1}, P_{i+2}, P_m) are included as part of the solution, where P_n and P_m are arbitrary sampling points within the neighborhood of P_i . This requirement is implemented by the inequality constraints $A_{\text{ieq}} \in \mathbb{Z}^{2N_e \times N_e}$. For the edge itself, this condition is set to 1 and the edges (P_n, P_i, P_{i+1}) and (P_{i+1}, P_{i+2}, P_m) are each assigned to -1 . Furthermore, the equality conditions $A_{\text{eq}} \in \mathbb{Z}^{3N_s \times N_e}$ ensure that each sampling point is start, middle and end point of an edge exactly once. In this case, however, it should be noted that additional equality conditions must be inserted for the start and end point of the entire path. Finally, the last condition specifies the solution vector x to be binary, resulting in a mixed-integer linear optimization problem. In order to obtain a continuous trajectory without isolated sub-paths, such cases are excluded from the solution of the optimization problem by introducing additional inequality constraints. This process is repeated iteratively, similar to the TSP, until a valid solution is identified. The results of the path generation for the two approaches based on the TSP and WCR optimizations are compared in Fig. 7.6. In contrast to Fig. 7.4, a denser point distribution of an MDSG_{27} is considered to allow a better assessment of the arising differences. The color coding of the measurement track allows a qualitative comparison of the actual path velocity of the robot, which is, on average, 53.1 % of the specified maximum for the path of the TSP optimization containing many curves with narrow radii. In contrast, the path generated using the WCR optimization contains significantly larger curve radii. In particular, a relatively large number of almost straight trajectories can be achieved in the middle section of the quarter shown. As a result, the average path speed can be increased to 71.9 % of the specified maximum velocity, which represents an increase of approximately 19 % compared to path generation based on the TSP.

However, the increase in the average path speed is not directly transferred to the actually resulting measurement duration, as the path of the WCR optimization is



(a) The path generation based on the TSP facilitates an average path speed of 53.1 % of the specified maximum speed.

(b) The WCR path optimization results in an average path speed of 71.9 % of the specified maximum speed.

Figure 7.6: Comparison of the two path optimization approaches based on TSP and WCR for one quarter of an MDSG_{27} . The color-coded path allows a qualitative comparison of the actual path speed of the robot, which is on average approximately 19 % higher for the solution based on the WCR optimization. The sampling points highlighted in bold blue and red indicate the start and end position of the measurement path.

longer. Furthermore, in order to increase the optimization potential for larger curve radii, the WCR-based approach does not require a closed-loop path, but only a continuous one. In this respect, although it depends on the actual implementation, a repositioning movement to the pole of the measurement sphere may be advisable in order to be able to start the measurement for the next quarter from a well-defined reference point. This minimizes possible risks of collisions with the AUT, which is, however, inherently ensured by the closed-loop measurement path of the TSP optimization.

The measurement durations and path lengths resulting from the TSP and WCR optimization are compared in Fig. 7.7 and neglect potential repositioning drives, as these are specific to the actual implementation. Since absolute numbers depend heavily on the measurement setup used, a relative assessment normalized to the corresponding spline-based implementation of the equiangular reference measurement for $N_{m,n,\max} = 34$ is provided. All comparative measurements have been performed with the same speed and acceleration parameters to ensure comparability. The results presented in Fig. 7.7a proves that the measurement duration can be effectively reduced by applying an MDSG. The larger MDSG_{41} based on the TSP optimization requires only 87 % of the measurement time of the reference case. A further enhancement of 15 % to 72 % of the measurement duration is possible through the application of the WCR path optimization. For the direct comparison of the MDSG_{34} to the equiangular reference, the measurement time is reduced to 66 % using the TSP approach, whereas the WCR optimization enables a further improvement of 10 %. The mode-accurate scalability yields further potential, as exemplified for the MDSG_{27} . If such a point distribution is sufficient for the AUT used, the measurement du-

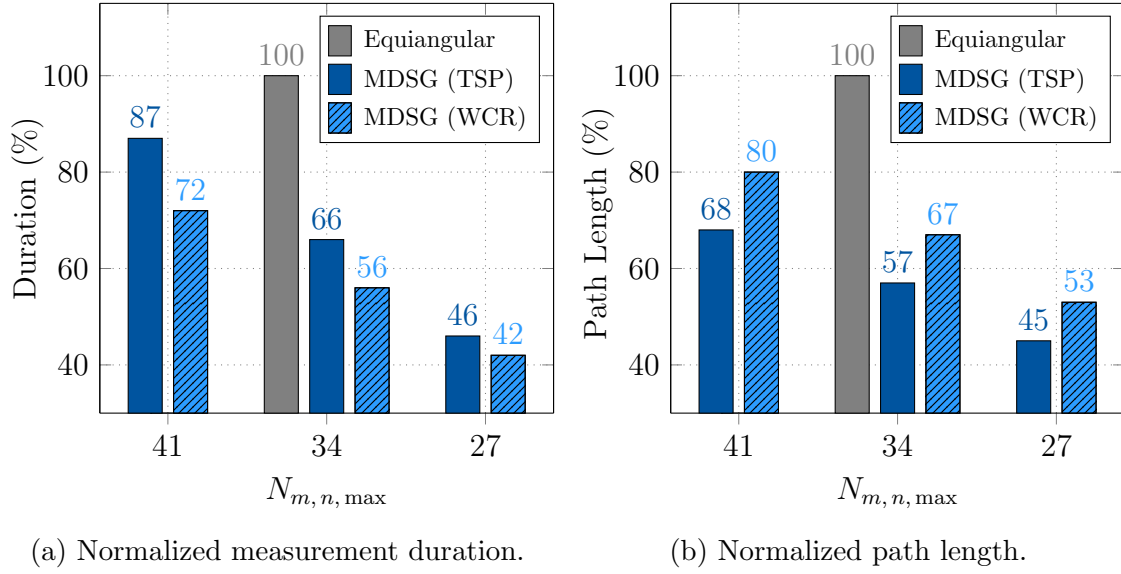


Figure 7.7: Relative comparison of the resulting measurement duration and path length for the MDSG_{41} , MDSG_{34} and MDSG_{27} in relation to the corresponding spline-based equiangular measurement. For the MDSG measurements, the paths are compared using the TSP as well as WCR optimization.

ration can be reduced to 46% compared to the equiangular measurement. Especially for MDSG of increasing order, the WCR optimization provides advantages in terms of the measurement duration. While an additional improvement of 10% to 15% is possible for the MDSG_{41} and MDSG_{34} , it decreases to only 4% in case of the MDSG_{27} . This indicates that the results of the WCR optimization scale effectively for denser distributions of sampling point, in contrast to the TSP approach as explained in Fig. 7.5 for $N_{m,n,max} = 59$. Thus, the WCR optimization is a viable alternative especially for complex AUTs. For AUTs of decreasing order, the TSP path generation can be applied instead, as the resulting time differences are rather negligible in this case.

The supplementary comparison in Fig. 7.7b illustrates the associated path lengths normalized to the equiangular reference measurement. The TSP approach for minimizing the total Euclidean distance reduces the length of the MDSG_{41} measurement path to only 68% of the reference case. Although the path of the WCR optimization is 12% longer, the unproportionally higher path speed enables to improve the measurement duration by 15%. For the MDSG_{34} , the path length can be reduced to 57% and 67% using the TSP and WCR optimizations. Similarly, the total path length for the MDSG_{27} is only 45% and 53%, respectively. Although the result of the WCR path optimization leads to approximately 8% to 12% longer trajectories, the increased path speed achieved by the larger curve radii still ensures a further reduction of the measurement duration in the range from 4% to 15%.

For additional analyses of the two path generation algorithms, reference is made to Appendix D. The illustrations provided in Fig. D.1 to Fig. D.6 present the realized paths over all quarters Q_i of the hemisphere, for the MDSG_{41} , MDSG_{34} as well as for the MDSG_{27} . In addition, the actual length ℓ_i of the spline trajectory

generated by the RC and the corresponding average path speed $\bar{v}_i$ are stated for each quarter individually. The presented data confirms that the optimization based on the TSP generate measurement paths that are comparably efficient, independent of the quarter under consideration. With regard to both, the length of the measurement path and the average path speed, the deviations for the MDSG₂₇ are less than 2.6 % for the respective quarters. For more complex TSP generated measurement paths, the variations decrease further and are less than 2.2 % for the MDSG₃₄ and, finally, less than 1.6 % for the MDSG₄₁.

The same decreasing trend in the variations is also evident for the measurement paths generated by the WCR optimization, although the underlying variability is in this case generally higher. The largest difference in the average path velocity of 8.6 % occurs with the MDSG₂₇ combined with a path length variation of only 2.8 %. This variability is reduced to 5.7 % and 4.2 % for the MDSG₃₄. For the more complex MDSG₄₁, the range of variation is further reduced to only 3.4 % in the average path velocity and 3.0 % in the path length, which means that both measures of variance not only decrease, but also converge to each other. Therefore, especially for the application of the WCR-based path optimization and lower-order MDSGs, different path velocities must be taken into account in the parameterization of the measurement in order to comply with the maximum possible triggering rate of the instrumentation. Alternatively, however, the individual quarters of a measurement can also be performed with different velocity profiles, thus avoiding this problem.

7.3 Measurement Results

Complementing the analyses of the resulting measurement durations, path lengths and the actual path velocities presented in Section 7.2.2, the functional principle of SNF measurements based on sampling point distributions defined by an MDSG is verified in the following. For this purpose, different MDSGs based on the electrical size of the AUT described in Appendix C are considered in Section 7.3.1, specifically an MDSG₃₄ with reference to an equiangular measurement with a step size of $\Delta = 5^\circ$ as well as the MDSG₂₇ which is the minimum requirement for the AUT and the MDSG₄₁ as a more oversampled variant. The numerical stability of these sampling grids is firstly related to the equiangular measurement and, afterwards, the effects occurring in a real measurement and the resulting radiation patterns transformed into the far field are discussed.

Subsequently, based on the PDT implementation explained in Section 6.2, a method for generating additional sampling points on the resulting path of the WCR optimization, i.e. without increasing in the measurement duration, is presented in Section 7.3.2.

7.3.1 Order of the Sampling Grid

For the verification of the functional principle of SNF measurements based on a point distribution defined by an MDSG, three different orders of the sampling grid are examined below in comparison with conventional equiangular measurements. The OEWG probe antenna and AUT described in Appendix C are used for this

purpose, with the latter being the basis for the design of the experimental verification. At a frequency of 60 GHz and based on (2.5), it is represented by $N_{m,n} = 27$, so that the next appropriate equiangular sampling grid is determined by a step size of $\Delta = 5^\circ$ or $N_{m,n,\max} = 35$, respectively. However, since the AUT is described by $N_{m,n} = 34$ at its highest operating frequency, the number of modes considered for the equiangular sampling grid and the MDSG is limited to the same amount. Therefore, the MDSG₃₄ is selected as the corresponding equivalent, as well as the minimum configuration corresponding to the AUT through the MDSG₂₇ as well as a more oversampled variant, specifically the MDSG₄₁.

All comparative measurements are performed in the near field of the AUT at a measurement radius of $r = 200$ mm using an OEWG probe antenna, which is represented by $N_{\mu,\nu} = 12$. Based on (2.42), both, the measurement distance r by the translation operator $C_{\sigma\mu\nu}^{sn(c)}(kr)$ within the probe response constants $P_{s\mu n}(kr)$ calculation of the actually used probe antenna as well as the respective measurement coordinates used for the rotation of $P_{s\mu n}(kr)$ affect the condition of the measurement matrix Ψ . Therefore, specifically for the applied measurement setup as well as the AUT and probe antenna used, the most important parameters of the test measurements, such as the respective mode configurations, the amount of sampling points L and the resulting condition number κ are summarized in Table 7.1. The results demonstrate that the numerical stability of the MDSG measurements is for $N_{m,n} = N_{m,n,\max}$ always of the same order of magnitude or slightly lower than for the equiangular reference measurement, even for the specific case of the described near-field measurements. As already explained in Section 7.1, the oversampled case $N_{m,n} < N_{m,n,\max}$ allows for a further improvement, so that the resulting condition number for the applicable MDSGs reduces to $\kappa < 1.4$. Moreover, it should be emphasized that even the significantly larger MDSG₄₁ requires only 66 % of the number of sampling points of the equiangular measurement. The MDSG₃₄ and MDSG₂₇ sampling grids, which correspond to the equiangular reference measurement and AUT, respectively, ensure a further reduction in the number of sampling points to approximately 46 % and 29 %.

Until now, all robot-based measurements have been performed stepwise in ϕ using the additional rotation axis and continuously in θ employing the robot arm. This is advantageous since the robot always performs the measurements along the same path. Thus, positional uncertainties of the robot can be reduced and difficult poses or the robot reaching over the AUT can be avoided. In addition, possible collision risks between the robot and the AUT in the event of a misconfiguration are also minimized. However, further advantages exist with regard to the actual antenna measurement, as interferences such as room or scanner scattering as well as cable flexing effects are identical for each of the performed measurement cuts.

Nevertheless, for the MDSG-based measurements, this established scheme is modified by extending the robot's range of motion to a quarter of the hemisphere, as already explained in Section 7.2 concerning the path generation. This approach enables to allocate a sufficiently large working area to the robot, allowing it to traverse through the irregularly distributed sampling points using spline-based motion sequences and, thereby, obtain the required measurement data in this process. Beyond this, however, an increase in the measurement uncertainties is anticipated,

Table 7.1: Condition number κ and amount of sampling points L for different configurations of equiangular sampling and MDSG specifically for the actually performed near-field measurements.

$N_{m,n}$	$N_{m,n,\max}$	Sampling Grid	L	κ
34	35 ($\Delta = 5^\circ$)	Equiangular	5328	7.60
27	35 ($\Delta = 5^\circ$)	Equiangular	5328	6.44
41	41	MDSG	3528	7.62
34	41	MDSG	3528	1.33
27	41	MDSG	3528	1.14
34	34	MDSG	2450	6.61
27	34	MDSG	2450	1.27
27	27	MDSG	1568	5.87

both, with regard to increasing positional uncertainties of the robot due to the extended range of motion as well as regarding the antenna measurement due to the emerging differences in contrast to the established measurement scheme. To evaluate the impact of these effects, as a first step the resulting mode spectrum in n of the conventional measurement scheme is compared in Fig. 7.8 to an equiangular measurement which is likewise performed on a quarter, i.e. analogous to the case of the MDSG implementation, as well as to the actual MDSG measurements.

The mode spectrum in n determined by the respective measurements denotes the summation of all modes along m and s , indicating the relative power of the respective modes according to their index. Since the measurements have been aligned towards the aperture of the AUT instead of the phase center, which is unknown at the time of measurement, this results in an offset of the origin of the mode expansion to the actual radiation center of approximately 3 mm. Therefore, the most power is concentrated in the area $n \approx 5$ instead of $1 \lesssim n \lesssim 3$, which is, however, not decisive for the following analysis. Instead, it can be observed that the respective measurements exhibit different power decay behaviors for higher-order modes, with the conventional full- ϕ scheme of a SNF measurement revealing the largest power decay. Nevertheless, even in this case a second maximum is evident for $n > 32$, which indicates scattering effects of the AUT mounting structure being filtered using the conventional mode truncation with $n_1 = 10$ according to (2.5). As illustrated in Fig. A.2, the AUT is mounted spatially close to the absorbers of the measurement setup and scattering caused from these or other components is, therefore, modeled in an increased power of higher-order modes.

The quarter-wise performed equiangular measurement, i.e. analogously to the MDSG implementations, reproduces the second peak in good agreement, but additionally shows a significantly reduced power drop in the range $13 \leq n \leq 31$. Since the parameterization of this measurement is identical with regard to the maximum amount of modes that can be calculated, this effect can be attributed to the quarter-wise implementation. This includes dynamic effects due to increased position uncertainties of the robot resulting from the extended range of motion or stronger cable bending effects, but also spatial conditions. In contrast to roll-over-azimuth positioners, the probe antenna is physically moved through the measurement chamber and, thus,

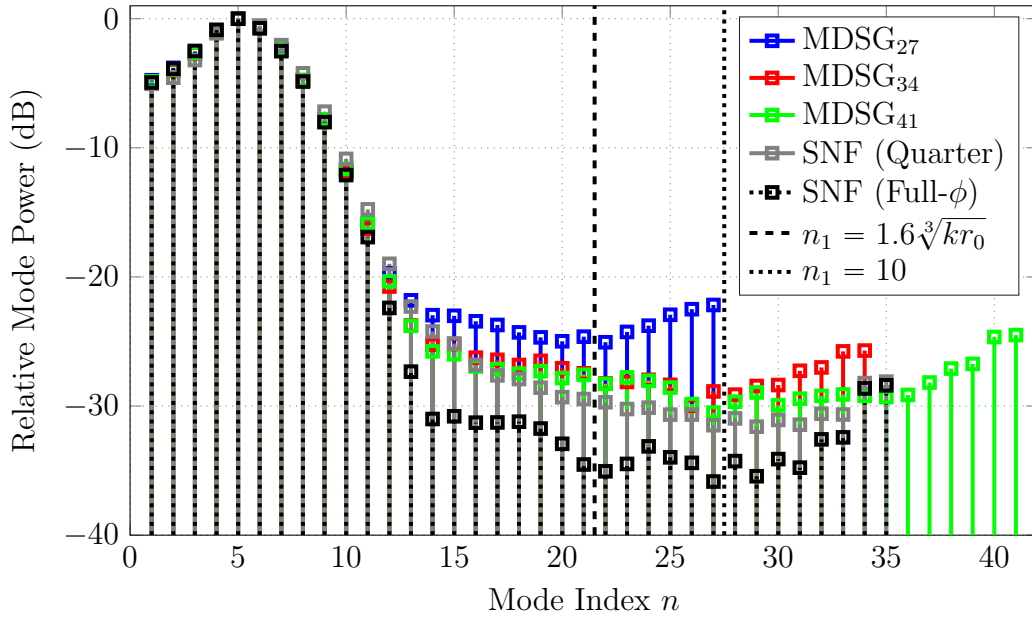


Figure 7.8: The measured SMC spectrum in n for the considered AUT compared for different measurement configurations at a frequency of 60 GHz.

different room and scanner scattering effects occur for each individual measurement position within the quarter. In this way, this measurement scheme is in the broadest sense comparable to the mathematical absorber reflection suppression (MARS) technique [122, 156–158], but based on the movement of the probe antenna instead of a purposely chosen offset of the AUT from the origin of the measurement coordinate system.

For the MDSG₄₁, the power decay in the range $13 \leq n \leq 31$ is minimally lower, which can be explained, for example, by the more dynamic spline-based movement as well as the irregular distribution of sampling points. With the equiangular measurement performed on the quarter, the spatial positions of the probe antenna are identical for each quarter, as the distribution of the sampling points is regular. However, this is no longer valid for MDSG-based measurements, as the sampling grid is irregular and, therefore, all sampling points are located at spatially different positions.

The tendency of the reduced power drop in the region $n > 13$ continues for the MDSG₃₄ and MDSG₂₇, where the reduced order of these sampling grids intensifies this effect, which is particularly evident for the MDSG₂₇. Since these sampling grids can only resolve $N_{m,n,\max} = 34$ or even only $N_{m,n,\max} = 27$ AUT modes, spatial aliasing of the higher-order modes occurs. As a result, effects that are identifiable in the higher-order modes, such as scattering due to the antenna mounting structure, cannot be filtered out. This effect is discussed in the field of acoustics in [159] and also on the basis of the SWE of electromagnetic fields in [146]. Within the SWE, all modes up to the band-limitation $N_{m,n,\max}$ form an orthogonal basis. However, if an electromagnetic field of higher order than $N_{m,n,\max}$ is sampled, it cannot be represented by this orthogonal basis and, thus, causes errors in the entire mode spectrum. The effects of spatial aliasing depend on the spatial distribution of the sampling points, as is simulated in [146] using various sampling grids, includ-

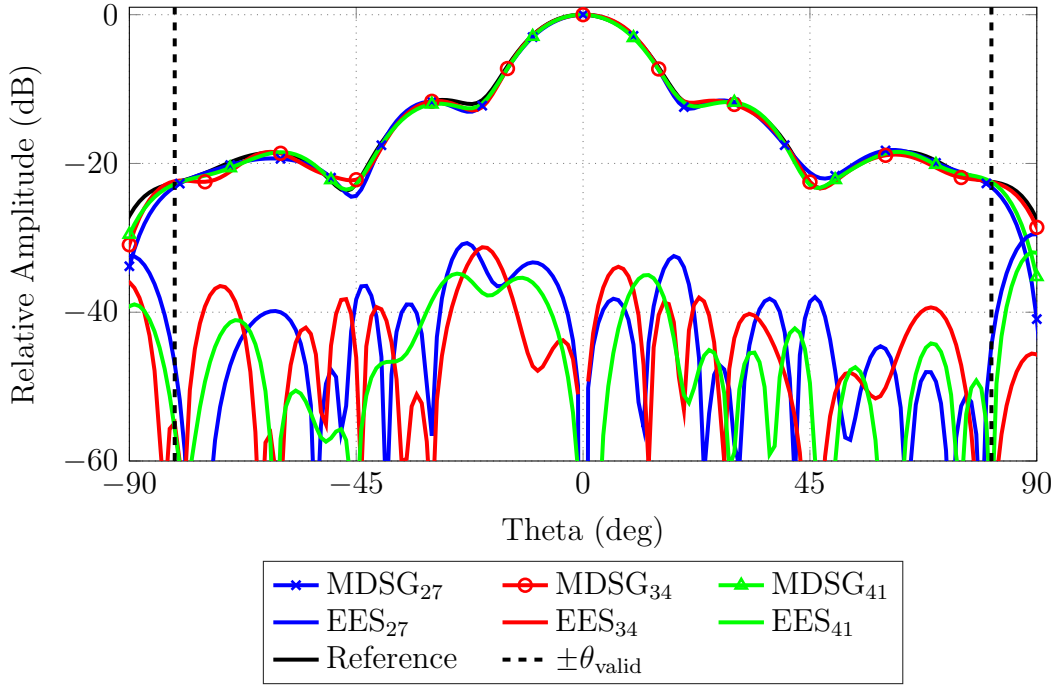


Figure 7.9: Transformed far-field radiation patterns ($\phi = 0^\circ$) of the MDSG-based measurements in comparison to an equiangular reference measurement performed using the conventional full- ϕ scheme for a frequency of 60 GHz. The respective EES for the MDSG results indicates the difference to the reference measurement.

ing an MDSG. In the case of the MDSG, the resulting errors are mostly uniformly distributed disturbances of the spectrum, which, however, intensify for modes in the region of the band-limitation. As further explained in [146], oversampling is a suitable method to overcome spatial aliasing errors, which, conversely, causes non-redundant sampling grids such as the MDSG to be more susceptible than inherently oversampled variants. In fact, this is also confirmed by the mode spectra of the performed MDSG measurements, which contain only 29 %, 46 % or 66 % of the sampling points of the corresponding equiangular measurement and, thus, demonstrate an increasing error in the mode spectrum for a decreasing degree of oversampling. Furthermore, an MDSG with the same amount of sampling points as an equiangular measurement is more robust to spatial aliasing, indicating this is enabled by the spatial distribution of the sampling points rather than their quantity [146].

To suppress the described interference effects, the power-based mode truncation [60] with $n_1 = 1.6 \sqrt[3]{kr_0}$ according to (2.7), which is compared in Fig. 7.8 with the truncation constant conventionally used according to [45] is, therefore, applied in the following. Although the effect of the mode truncation is most significant for the MDSG₂₇, the other measurements also take advantage of this due to the relatively low power decrease in the range $13 \leq n \leq N_{m,n,\max}$ compared to the standard scheme of a full- ϕ measurement. On the basis of this mode truncation, Fig. 7.9 compares the transformed far-field radiation patterns of the MDSG measurements with a conventionally performed full- ϕ reference measurement. The respective EES given for to the MDSG results indicates the difference to the aforementioned reference case.

The results demonstrate that all MDSG measurements reproduce the reference for wide parts of the angular range in an acceptable agreement, although, in particular, the MDSG₂₇ exhibits increased errors for both zeros at $\theta \approx \pm 45^\circ$. In contrast, the MDSG₃₄ accurately reproduces the zero at $\theta \approx 45^\circ$, but not the other one. Only the more oversampled MDSG₄₁ reproduces both zeros in accordance with the reference. Furthermore, since only a hemisphere has been measured, the truncation effect is stronger for all MDSG measurements than for the equiangular reference measurement. However, taking into account the commonly accepted validity range of a truncated SNF measurement given by $\theta_{\text{valid}} = \pm\theta_{\text{max}} \mp \arcsin(r_0/r) \approx 82^\circ$, these deviations only occur to a greater extent outside this range. In addition, the deviations outside the validity range are asymmetrical for all MDSG measurements, which may be attributable to the irregular distribution of sampling points.

Nevertheless, all MDSG measurements are within the validity range in acceptable agreement with the reference by achieving a maximum EES of less than -30 dB. The strongest deviations occur in the region $\theta \approx -23^\circ$ and amount to -30.7 dB and -31.3 dB for the MDSG₂₇ and MDSG₃₄, respectively. However, the MDSG₄₁ performs considerably better, with a maximum EES of -34.8 dB and is, according to Fig. 7.7a, still 28 % faster than the corresponding equiangular measurement despite the oversampling.

Therefore, the functional principle of SNF measurements based on sampling point distributions defined by MDSGs is confirmed, where it is emphasized that the MDSG₂₇ requires only 29 % of the sampling points of the comparable equiangular measurement. Nonetheless, it is also important to note that oversampling contributes to an improvement of the results which can be implemented on a fine basis due to the mode-accurate scalability of these sampling grids. Another alternative to implement oversampling without increasing the order of these sampling grids and, thus, without increasing the measurement duration is discussed hereinafter in Section 7.3.2.

7.3.2 Additional On-Path Sampling Points

As discussed in Section 7.3.1, oversampling improves the stability of the NFFFT and helps to suppress or filter out interference effects. While oversampling is a fundamental requirement for equiangular measurements to avoid an ill-conditioned system of linear equations, it is an optional step for MDSG-based sampling grids. Nevertheless, a certain degree of oversampling allows to suppress spatial aliasing issues or scattering effects due to the antenna mounting structure, which are in particular evident in the higher-order modes. Therefore, the following section presents a solution for acquiring additional measurement data while maintaining the order of the sampling grid and, thus, without affecting the measurement duration.

The approach is based on the coexistence of two trigger sources, specifically the SPT and PDT implementations described in Section 6.2. Therefore, the sampling points defined by the MDSG are realized using the SPT implementation, which is based on the native trigger functionality of the RC and, thus, features a high level of accuracy. Furthermore, the RC uses the predetermined SPT for the calculation of the spline path through the sampling grid and, hence, SPTs are also referred to as path-defining triggers.

This contrasts with the PDT implementation, in which the sampling points are not explicitly specified. Instead, a cyclic background task is used to generate further trigger signals depending on the passed distance on the measurement path. Consequently, the accuracy of the PDT implementation is lower compared to SPTs, which is also emphasized in Fig. 6.5. However, PDTs do not define a measurement path, but are generated based on the moved distance and are, therefore, also referred to as path-dependent triggers.

The combination of the both allows to ensure that the MDSG sampling points optimized for the electromagnetic problem are implemented using the more accurate SPTs. The PDTs are additionally generated without affecting the calculated trajectory based on the SPTs, so that the RC is granted further degrees of freedom for path optimization. Consequently, the measurement data obtained using the PDTs is optimized with regard to the mechanics and control of the robot and, in this context, not for performing antenna measurements. Since neither the position nor the orientation for these sampling points are specified, but instead are determined by the RC, it is required to apply a PPC as described in Section 5 based on the position and orientation data provided by the RC.

The resulting measurement with the concurrent application of both trigger sources is illustrated in Fig. 7.10 and represents for reasons of clarity the MDSG_{27} as it contains fewer sampling points. The sampling points of the MDSG_{27} implemented using SPTs determine the path and are marked with circles, where the bold blue and red markers indicate the start and stop position. The additional data acquired on the resulting path by the application of PDTs are highlighted by crosses. The trigger distance is configured to generate at least one PDT between every two SPTs.

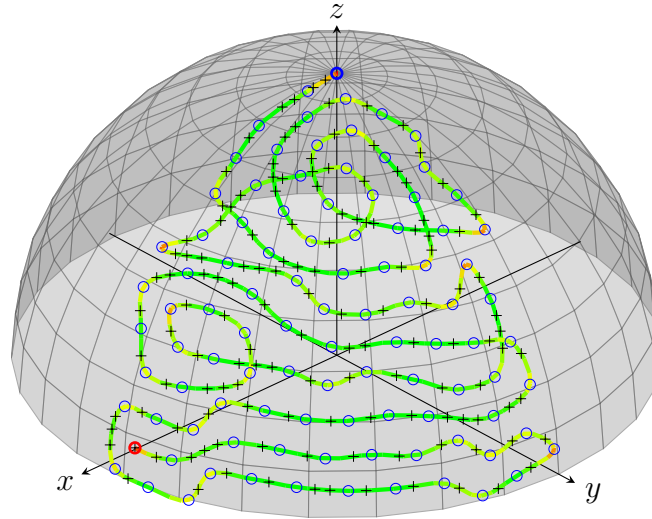


Figure 7.10: Measurement path of the WCR optimization for the first quarter of an MDSG_{27} as well as the qualitative comparison of the robot's actual path velocity via the color-coded measurement trajectory. The sampling points of the MDSG are measured using the SPT implementation and are represented as circles. The additional triggers acquired on the resulting measurement path are implemented by PDT and are depicted as crosses. The sampling points highlighted in bold blue and red indicate the start and end position of the measurement path.

Since the operating velocity of the mechanical axes is typically not limited by the maximum possible measurement rate of the instrumentation, this approach, thus, enables to reduce idle times of the measurement equipment. Consequently, the co-existence of SPTs and PDTs allows to obtain additional data without increasing the measurement duration. The amount of the additionally provided measurement data due the PDT implementation is compared in Table 7.2 for the aforementioned configuration of the triggering distance as well as for the different MDSGs. Here, all numbers stated refer to a hemispherical measurement for a single polarization.

Although the trigger distance for the PDT implementation has been chosen conservatively to ensure at least one PDT-generated data point between every two SPTs, the total amount of sampling points can be significantly increased. Accordingly, the largest number of additionally generated data points results for the spatially denser distribution of sampling points given by the MDSG_{41} . In this case, 1227 additional sampling points are acquired, resulting in total of approximately 54 % more data points than for the equiangular reference measurement, which is already over-sampled by a factor of two. Under consideration of the PDT, the total amount of sampling points of the MDSG_{34} is approximately 4 % higher than that of the reference measurement and is, therefore, relatively comparable. In contrast, the total number of measurement data for the spatially more widely spaced sampling point distribution of the MDSG_{27} amounts only 74 % of the reference case, despite the application of the PDTs. Since the sampling points of the lower-order MDSGs are spatially more separated, correspondingly less additional data is obtained, provided that at least one PDT is acquired between two SPT data points. However, as the measurement system still covers relatively comparable distances on the quarter to be measured, it is possible to reduce the trigger distance until the same measurement rate is achieved and, thus, if necessary, the required amount of additional data can be recorded.

For the analysis of the effects of the additional PDT-generated data points on the NFFFT, it should be noted that the selection of the MDSG already defines the mandatory sampling points including their optimized spatial coordinates. Consequently, the additional measurement data results in a redundancy or an overdetermination of the linear system of equations in order to determine the SMC, similar to an equiangular measurement. Analogously, this method of oversampling contributes

Table 7.2: Amount of sampling points generated by the SPT and PDT implementation for MDSG-based measurements compared to the equiangular sampling grid. The specified numbers refer to a hemispherical measurement of a single polarization.

Sampling Grid	Sampling Points				
	Absolute			Relative	
	SPT	PDT	Total	SPT	Total
Equiangular ($\Delta = 5^\circ$)	1368	0	1368	100 %	100 %
MDSG_{27}	388	629	1017	28.3 %	74.3 %
MDSG_{34}	612	807	1419	44.7 %	103.7 %
MDSG_{41}	879	1227	2106	64.3 %	153.9 %

to an improvement of the measurement results, also in case of an MDSG, for example by mitigating noise effects or spatial aliasing. To evaluate such aspects, the mode spectrum in n is, therefore, analyzed in Fig. 7.11 for both test cases, with and without the additional measurement data obtained by the PDT implementation for the three analyzed MDSG sampling grids.

As expected, the PDT-generated additional sampling points do not have a substantial effect on the dominant lower-order modes. For all three MDSGs considered, the power difference of the modes $n < 13$ is typically less than 0.1 dB. Similarly, the power differences in the area of the plateau $13 \leq n \lesssim N_{m,n,\max} - 10$, which demonstrates a significantly lower power decrease in comparison to the full- ϕ measurement scheme, are only minor. However, considerably power reductions are recognizable in the range of the higher-order modes, depending on the MDSG, approximately in the range $N_{m,n,\max} - 10 \lesssim n \leq N_{m,n,\max}$. In the case of the MDSG₂₇, the highest power degradation of 5.5 dB is observed for the highest-order mode. The same applies to the MDSG₃₄, although the power decrease in this case is lower and amounts only 3.4 dB. However, it should be noted that the mode power levels of the MDSG₃₄ without additional points are already significantly lower than those of the MDSG₂₇ and are instead of a comparable order of magnitude to the MDSG₄₁. In fact, the MDSG₄₁ reveals the largest attenuation of 6.4 dB for the second highest-order mode, where the highest-order mode is also suppressed by approximately 6 dB. Furthermore, the range of distinctly attenuated modes is larger than for the other sampling grids. Under consideration of the additional sampling points, all spherical modes for $n \geq 32$ indicate an attenuation of more than 1.5 dB. This increased range of power-reduced higher-order modes is, therefore, due to the total amount of additional sampling points, which is the largest for the MDSG₄₁. In a direct comparison, it contains 52 % more PDT-generated data points than the MDSG₃₄ and 95 % more than in the case of the MDSG₂₇.

Thus, it is evident that the additional sampling points enable a significant reduction of the power contained in the higher-order modes. However, this does not eliminate the cause of the power increase in this modes, which is attributable to the mounting of the AUT, for example. Instead, there is a power shift towards higher-order modes, which is exemplified for the MDSG₄₁ by the additionally calculated modes presented in gray color. In principle, the additional data points enable the calculation of further higher-order modes, although the stability of the resulting system of equations is not guaranteed in this case, as the data points generated by the PDT are not optimized with respect to the MDSG.

Nevertheless, oversampling based on the PDT-generated additional measurement points provides improved capabilities for modal filtering to separate direct from scattering sources, as for example caused by the AUT mount. However, dynamic effects such as cable bending or position-dependent room and scanner scattering are not affected. The comparison in Fig. 7.8 for an equiangular measurement, which has been performed both, quarterly, i.e. analogous to the MDSG, and according to the conventional full- ϕ scheme, identified these in the generally reduced power decrease in the middle part of the mode spectrum. Based on this spectral analysis, the additional PDT-generated measurement data do not result in any significant improvement with regard to these aspects. However, it should be noted that this

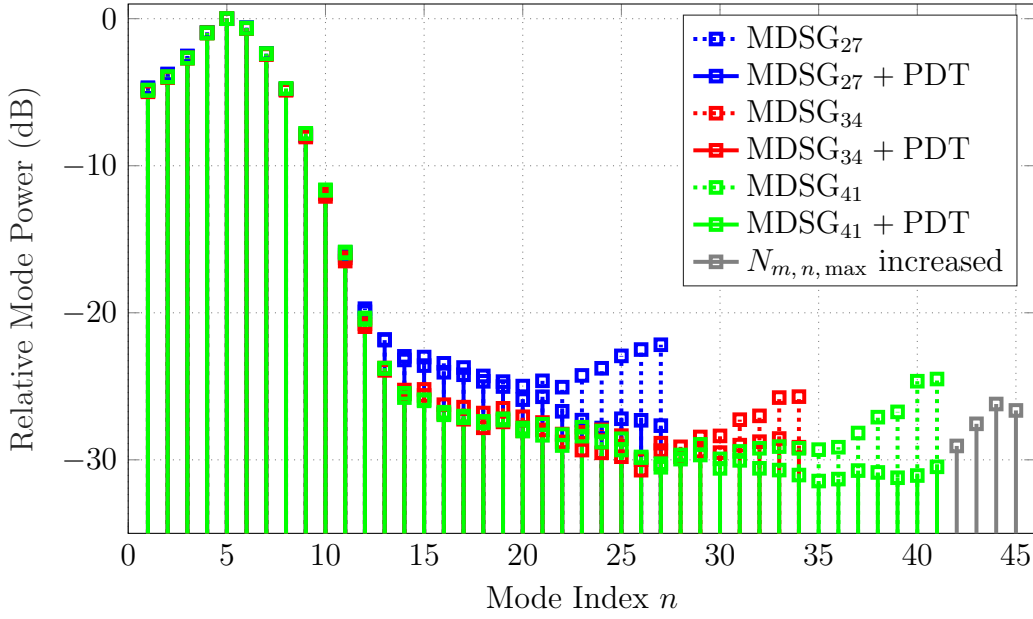


Figure 7.11: Impact of the additional, redundant sampling points of the PDT implementation on the resulting mode spectrum in n as a continuation of the analyses from Fig. 7.8.

effect depends on the combination of measurement system and AUT used and does not depend on the MDSG.

Furthermore, the effects of the PDT-generated additional data on the radiation pattern transformed into the far field are examined. Since the orientation of the probe antenna for the PDT sampling points is not specified but depends on the calculated trajectory of the RC, the application of the PPC introduced in Section 5 is recommended. In fact, the fourth sampling point highlighted in Fig. 5.2 and Fig. 5.3 is actually one of the PDT-generated data points, whereby it is evident that the orientation of the probe antenna deviates considerably from the ideal case. Therefore, the results of the PPC are compared with both, the MDSG measurement based exclusively on SPT, but also using SPT as well as PDT, but without the application of the PPC. The results are exemplified for the MDSG₃₄ in Fig. 7.12, where the same analysis is given for the MDSG₂₇ in Fig. D.7 and for the MDSG₄₁ in Fig. D.8. The results refer to a frequency of 60 GHz and the mode truncation proposed in Fig. 7.8 is applied according to (2.7). The probe antenna is described by $N_{\mu,\nu} = 12$, where a maximum of $\mu_{\max} = \nu_{\max} = 29$ modes are considered depending on the orientation error to be corrected. The indicated EES denotes the difference of the results considering SPT and PDT including the PPC compared to the results based on SPT only.

The inclusion of PDT-generated sampling points in the NFFFT without the application of the PPC demonstrates for all measurements a degradation of the cross-polarized results, which are in general highly sensitive against positioning or orientation errors. However, if the actual orientation of the probe antenna is included in the NFFFT by the application of the PPC, a significant improvement can be achieved. In this case, the power of the cross-polarization can be improved below

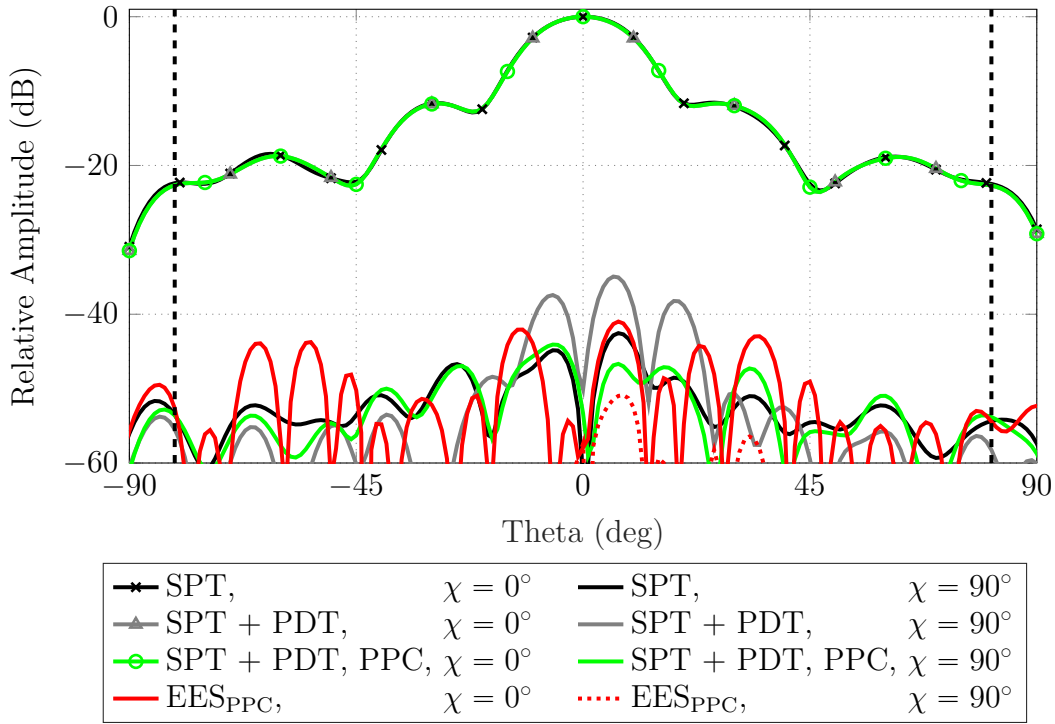


Figure 7.12: Comparison of the far-field transformed radiation pattern ($\phi = 0^\circ$) for an MDSG₃₄ considering SPT and SPT + PDT with and without the application of PPC. The EES denotes the difference from the original results based on SPT only to the application of PPC in case of SPT + PDT.

the level of the reference measurement based exclusively on SPT, which confirms the functional principle of the PPC for MDSG measurements. Consequently, the maximum EES for all MDSGs is in the order of approximately -40 dB for the co-polarization and -45 dB to -50 dB for the cross-polarized results.

In addition, a global comparison of the cross-polarized results without the application of the PPC indicates that the degradation increases in particular for MDSGs of lower order, whereas the effect on the cross-polarization for the MDSG₄₁ in Fig. D.8 is only marginal. This is due to the fact that the spatially denser distribution of sampling points reduces the degrees of freedom for the RC to optimize the orientation guidance of the probe antenna on the resulting path. For the MDSG₂₇, in contrast, the sampling points are spatially further apart, so that larger orientation deviations can occur on the path between two SPTs.

Furthermore, it is to be emphasized that the OEWG probe antenna used provides a very broad receive characteristic and is, therefore, potentially robust against misorientations. For more directive probe antennas as well as for spatially more distributed sampling points, however, the application of the PPC to compensate for possible orientation errors in the case of PDT-generated data points is, hence, of even higher importance.

7.4 Summary

The equiangular sampling grid is the most established approach in SNF measurement technology. Both, the mechanical implementation using roll-over-azimuth positioners as well as the transformation into the far field using the Wacker algorithm allow for an efficient implementation. However, equiangular sampling grids are oversampled twice to achieve a stable system of equations, which increases the measurement time. Therefore, the MDSG is introduced, which describes a mathematically optimized non-redundant sampling point distribution on a sphere and, thus, facilitates a stable system of equations for the calculation of the SMC entirely without oversampling.

The numerical stability of the MDSG is compared to other less redundant implementations of sampling grids, specifically spiral and thinned equiangular sampling grids, however, both still requiring approximately 25 % oversampling for a stable solution. Compared to this, the numerical stability of the MDSG is of a comparable order of magnitude, even without any form of oversampling, although implementing it can further improve the numerical stability.

Afterwards, two methods for optimizing a measurement path through the sampling points defined by the MDSG are introduced and compared. The TSP optimization minimizes the overall path length, which is particularly efficient for MDSGs of lower order. However, for more complex MDSGs, the paths become increasingly inefficient due to many sharp turns causing frequent deceleration and acceleration. In contrast to this stands the WCR optimization, in which the occurring curve radii are estimated and consequently maximized. Thus, it is particularly suitable for higher-order MDSGs and allows to reduce the measurement time, as a higher average path speed can be achieved, even if the total length is increased compared to the TSP optimization. In principle, a combination of both methods is also conceivable, i.e. the optimization based on distance (TSP) as well as the achievable speed (WCR). However, given the complexity of both methods, this combination is no longer efficiently solvable, especially for MDSGs of higher order.

Furthermore, actual MDSG measurements are performed and evaluated against equiangular reference measurements. The resulting mode spectrum is used to identify effects of a non-ideal measurement environment, such as scattering effects due to the mounting of the AUT, spatial aliasing of higher-order modes or room and scanner scattering, which have a higher impact than for the conventional full- ϕ measurement scheme due to the quarterwise implementation of the MDSG.

In common with the other measurement implementations, the MDSG also requires oversampling to suppress these effects. On the one hand, this can be achieved by using a higher-order MDSG, since the MDSG can be adjusted very finely due to its mode-accurate scalability, although this leads to an increase in measurement time. On the other hand, the order of the MDSG can also be maintained by using the PDT implementation to generate additional measurement points on the resulting measurement path. This allows to acquire significantly more sampling points without increasing the path length or measurement duration. However, as the orientation of the probe antenna cannot be guaranteed for PDT-generated sampling points, the application of the PPC is recommended in this case. Based on the position and orientation feedback of the RC, this allows to compensate for the non-ideal orienta-

tion of the probe antenna for each sampling point individually in the NFFFT. The effects of such non-ideal sampling points are examined using the radiation pattern transformed into the far field. In particular, the cross-polarized results are sensitive to a misorientation of the probe antenna. However, by using the PPC, the signal level of the cross-polarization can be improved to a comparable or even better level compared to the MDSG measurement without PDT-generated sampling points. Furthermore, it is emphasized that the application of the PPC is of crucial importance for spatially more widely spread sampling point distributions, i.e. MDSGs of lower order, or if more directive probe antennas are employed.

In contrast to conventional roll-over-azimuth positioners, robot-based systems provide an efficient implementation of the irregular sampling point distribution defined by the MDSG. For example, the measurement of an MDSG_{34} can be performed in only 56 % of the measurement time of the corresponding equiangular measurement with an angular step size of 5° . The EES with respect to the equiangular reference measurement is approximately -30 dB for the MDSG_{27} and MDSG_{34} and improves to -35 dB for the MDSG_{41} , which is considered an acceptable agreement, but still provides potential for further optimization. Therefore, although further research is required on the robust design of an MDSG measurement, taking into account non-ideal measurement effects, the previously rather theoretically interesting MDSG is gaining practical relevance due to the flexibility of robot-based measurement technology.

Conclusion and Outlook

Robot-based positioning systems have become indispensable in many branches of modern industry, but, however, only a few robot-based systems are currently used in SNF measurement technology. In addition, these extremely versatile and dynamic positioning systems are in most cases employed as a replacement for planar, cylindrical or spherical positioners and, therefore, apply only a fraction of the potentials provided by robots.

Thus, this dissertation focuses on the optimized operation of a robot for performing SNF measurements. This involves, on the one hand, transitioning from conventionally used canonical path definitions to more complex spline-based measurement sequences. On the other hand, methods are investigated to compensate for typical disadvantages of robot-based positioning systems, such as the absolute positioning accuracy, by applying and extending sophisticated SWE techniques to robot-based systems. With the computational capacities available today, the underlying theory of the SWE allows to compensate for irregular sampling grids as well as non-ideal orientations of the probe antenna for each sampling point separately. In this way, increased tolerances of robot-based positioning systems can be accounted for.

To investigate these research topics, a robot-based measurement system in the millimeter-wave range has been developed and installed as part of the dissertation. The most important components such as the mechanical structure, the RF design including the link budget, safety measures, but also the components and signal flows required for operation have been presented. In addition, two hardware units for digitizing the most important control signals as well as an LO/IF distribution unit have been developed. During the design of the measurement system and the in-house developed control software, the focus has been on maintaining the flexibility of the robot, even for deployment in a complex integrated measurement system.

Multi-purpose positioners such as robot arms are nowadays regarded as off-the-shelf components and are, therefore, significantly less expensive than positioning systems specifically developed for the respective application. In order to maintain this cost aspect, external measurement devices such as laser trackers, which are approximately twice as expensive as a robot, have been avoided. Instead, an error model has been presented in which the most significant uncertainties regarding displacements and tiltings are reflected. In addition, an electrical alignment procedure has been introduced to isolate and identify the individual error contributions, enabling the robot to adapt its measurement path to compensate for mechanical tolerances. Furthermore, the error model has been validated by comparative measurements in the CATR of the IHF, whereby a close agreement has been achieved with a maximum EES in the order of -40 dB, thus confirming the measurement capabilities of the developed system.

To compensate for position and orientation uncertainties, the actual measurement coordinates have been considered within the NFFFT. For this purpose, the actual position and orientation feedback of the RC is used to apply the direct solution method of the NFFFT, thus allowing for arbitrary measurement coordinates. Since robot-based systems provide six degrees of freedom, a non-ideal orientation of the probe antenna may occur at each of these sampling points. Therefore, the PPC has been introduced as the theoretical pendant to the mechanical flexibility of the robot and, hence, enables the representation of deviating orientations individually for each sampling point within the NFFFT. For verification, an intentional offset measurement has been performed in which the non-ideal orientation of the probe antenna resulting from the offset has been compensated by the PPC in the NFFFT. While the transformation to the far field fails without this correction, the application of the PPC achieves an almost error-free reconstruction with a maximum EES of -40 dB compared to an aligned reference measurement.

Furthermore, spline-based motion sequences have been utilized to implement irregular sampling grids and optimize the operation of the robot. In this type of movement, the sampling points are communicated to the RC for calculating a possible spline path. Furthermore, it allows to exploit the native trigger capabilities of the RC since the requested sampling points are known before the start of a measurement. Compared to the conventional trigger generation and for the same measurement speed, this method enables a tenfold reduction in the deviations of the generated trigger signals. Therefore, the application of spline-based path implementations is recommended for all measurements, regardless of the type of sampling grid as well as especially for measurements in the millimeter-wave range, where a high number of closely spaced sampling points have to be acquired.

The potentials of spline-based measurements have been demonstrated using the irregular distribution of sampling points defined by MDSGs. These describe a mathematically optimized spatial distribution of sampling points, which provide a numerically stable calculation of the SMC without any form of oversampling. Thus, only half of the sampling points are required for an MDSG compared to an equiangular measurement, which contributes to a reduction of the required measurement time. The implementation of such sampling grids is for most conventional roll-over-azimuth positioners not or only in stepwise operation possible, which prevents to achieve potential time advantages in practical measurements. In contrast, the flexible spline-based trajectories of a robot-based measurement system inherently provide a continuous and, therefore, efficient implementation. This has been verified for an MDSG_{34} , which corresponds to an equiangular measurement with an angular step size of 5° . For identical velocity parameters, an MDSG measurement can be completed in only 56 % of the time required for the equiangular counterpart. Furthermore, the conventional trigger implementation has been combined with the native trigger functionalities in spline-based measurements to allow the acquisition of additional measurement data on the resulting path. Thus, oversampling has been realized without increasing the measurement duration in order to suppress the interferences effects of a non-ideal measurement environment. Since the additional sampling points are neither predetermined with regard to their position nor orientation, the application of the PPC has been demonstrated in this case for an appropriate implementation of these sampling points within the NFFFT. Thus, the

enhanced measurement capabilities of robot-based systems allow to transform this previously rather theoretically interesting sampling grid into a relevant practical implementation.

Outlook

Numerous potentials of robot-based systems are currently not exploited and, thus, provide further research opportunities, which are briefly outlined in the following. Non-redundant sampling grids as the MDSG accelerate antenna measurements and, thus, increase the efficiency of measurement facilities, but are sensitive to noise effects compared to twice oversampled equiangular measurements. Although the MDSG₃₄ already achieves with a maximum EES to the reference measurement of -30 dB an acceptable agreement, it has also been demonstrated that oversampling using the MDSG₄₁ allows to reduce the resulting errors to -35 dB. Therefore, further research on the robust design of non-redundant sampling grids to account for scattering of non-ideal measurement environments is generally required. This includes spatial aliasing of higher-order modes, position-dependent effects of room and scanner scattering, but also a quantification of cable flexing due to the dynamic movements of robot-based systems. Although such interferences depend on the chamber and the measurement system and, therefore, also occur for analogically performed equiangular measurements, the expected noise effects are more severe in the case of non-redundant sampling grids. A possibility for suppressing such effects is to use more directive probe antennas, although depending on their design and aperture size other interference effects such as increased standing waves between AUT and probe may be induced. However, the flexibility of robot-based systems allows to perform measurements with a tilted probe antenna and, thus, to suppress standing waves. With the application of the PPC in the NFFFT such misalignments can be accounted to provide an error-free reconstruction of the far-field radiation pattern. Other novel approaches describe the application of probe signal derivatives to accelerate SNF measurements [160]. So far, the approach has been validated by acquiring two equiangular measurements with a slight offset along the θ -axis, for example 0.2° [161, 162]. Although the required measurement data can also be recorded within a single measurement run using a roll-over-azimuth positioner, it requires the complex and simultaneous control of the θ and ϕ axes in order to realize a sinusoidal movement. However, this sinusoidal movement of the probe antenna is either inherent to spline-based measurement paths or can at least be achieved efficiently by introducing additional support points to the spline path in order to enforce a wobbling path.

In addition, all measurements performed always referred to spherical surfaces, although this is neither required by the mechanics of the robot nor by the underlying theory of the SWE. In contrast, there are approaches in the field of phaseless measurement technology that provide an improved phase recovery if the sampling points have different radii [163]. Simulations confirm a reduced reconstruction error if the data is, for example, acquired on a surface described by an octahedron. Although the actual implementation using a roll-over-azimuth positioner is not possible in this case, robot-based systems offer an efficient solution.

Appendix

A

Robot-Based Measurement Setup

Table A.1: The main specifications of the robotic positioner. Further details are provided in [91].

Parameter	Value		
Model	KUKA KR8-R2010_2		
Reach	2010 mm		
Payload	nominal:	8 kg	
	point mass:	9.7 kg	
Weight	≈ 255 kg		
Repeatability	0.04 mm		
Angular range and speed	A1	$\pm 185^\circ$	$200^\circ/\text{s}$
	A2	$-185^\circ \dots 65^\circ$	$175^\circ/\text{s}$
	A3	$-138^\circ \dots 175^\circ$	$190^\circ/\text{s}$
	A4	$\pm 350^\circ$	$430^\circ/\text{s}$
	A5	$\pm 130^\circ$	$430^\circ/\text{s}$
	A6	$\pm 350^\circ$	$630^\circ/\text{s}$
Additional options	Absolute accuracy package		
	Safe operation package		
	EthernetKRL		

Table A.2: Extract of the relevant information on the additional linear axis [94].

Parameter	Value
Model	ROLLON TX100 SN2505
Drive	Ball screw
Guide system	Linear guide type D, double-guided
Stroke length	3500 mm
Maximum speed	24 m/min
Pitch accuracy	52 μm /300 mm
Spindle supports	3
Efficiency	0.93
Maximum static load	$F_y = F_z = 15\,900\text{ N}$
Dynamic load capacity	$F_y = F_z = 5850\text{ N}$
Static load torque	$M_x = 450\text{ Nm}$ $M_y = 1600\text{ Nm}$ $M_z = 1600\text{ Nm}$
Dynamic load torque	$M_x = 170\text{ Nm}$ $M_y = 600\text{ Nm}$ $M_z = 600\text{ Nm}$

Table A.3: Overview of the important information on the rotary axis mounted on the linear axis. Further details are given in [95]

Parameter	Value
Model	MDT360150-SG
Design	Hollow shaft ($\varnothing = 60\text{ mm}$)
Gear rotation	48:1
Backlash	$< 0.10^\circ$
Efficiency	0.37
Maximum static load	$F_x = 7500\text{ N}$ $F_y = 7500\text{ N}$ $F_z = 12\,000\text{ N}$
Dynamic load torque	$M_x = 200\text{ Nm}$ $M_y = 200\text{ Nm}$ $M_z = 30\text{ Nm}$

Table A.4: Relevant specifications of the two-channel RF rotary joint [98].

Parameter	Channel 1 (centric)	Channel 2 (excentric)
Frequency range	up to 18 GHz	
Nominal rotation speed	30 min^{-1}	
Peak power capability	1.0 kW	
Isolation	50 dB	
DC capability	0.5 A, 48 V	0.5 A, 24 V
Max. insertion loss variation	0.06 dB	0 – 4 GHz: 0.10 dB
		4 – 8 GHz: 0.35 dB
		8 – 12 GHz: 0.70 dB
		12 – 18 GHz: 1.50 dB
Max. phase variation	0 – 8 GHz: 0.5°	0 – 8 GHz: 4.0°
	8 – 18 GHz: 1.0°	8 – 12 GHz: 10.0°
		12 – 18 GHz: 25.0°

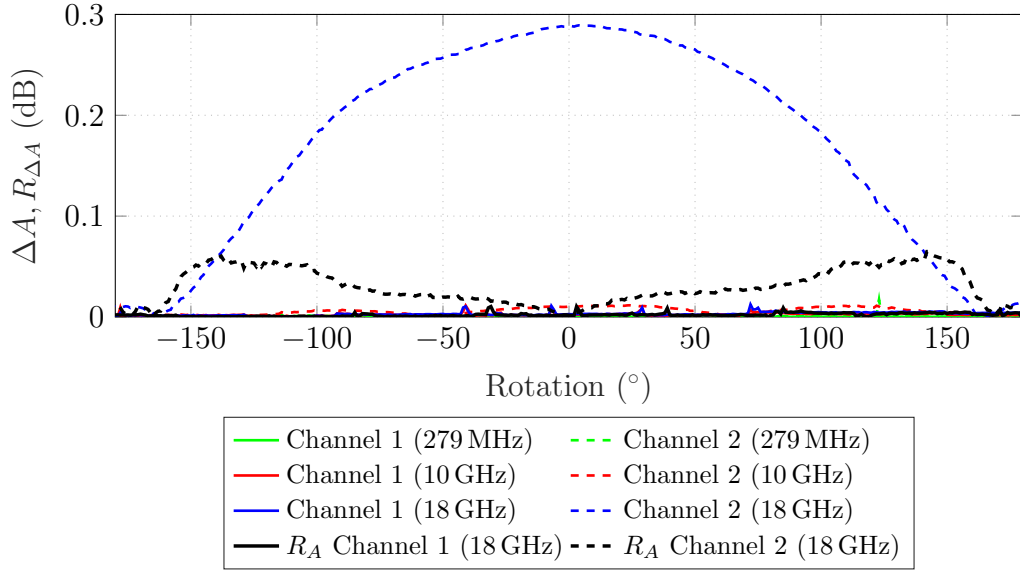


Figure A.1: Measured amplitude deviation ΔA and repeatability $R_{\Delta A}$ for a revolution of the rotary joint specified in Table A.4 and [98]. The measurements have been performed for $f = f_{\text{IF}} = 279$ MHz, as well as for $f = 10$ GHz and $f = 18$ GHz, which represents the lower and upper design frequency.

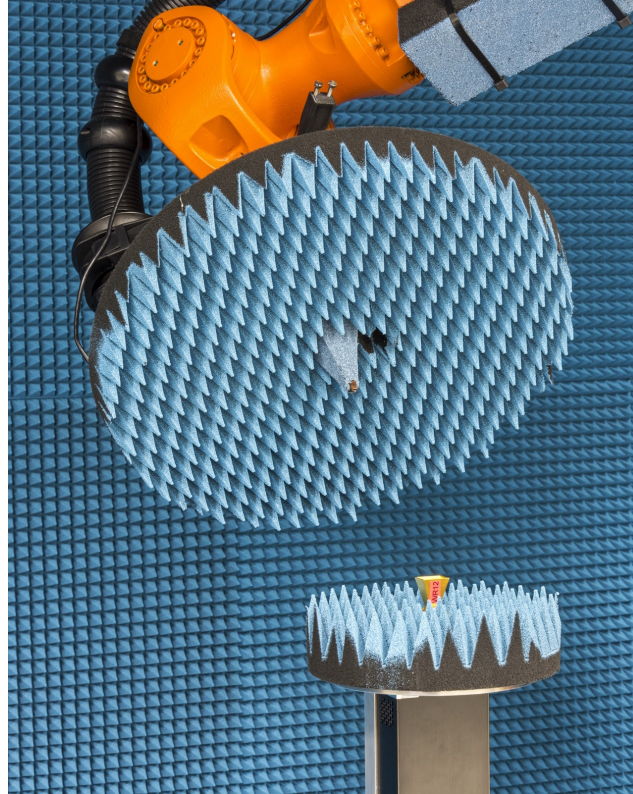


Figure A.2: Picture of the absorber ring mounted on the robot's TCP during a typical measurement run. The relatively large absorber ring is intended to reduce reflections on the robot's cable routing (black energy guiding chain in the picture) and on the metal mounting structure holding the Rx module and the probe antenna.

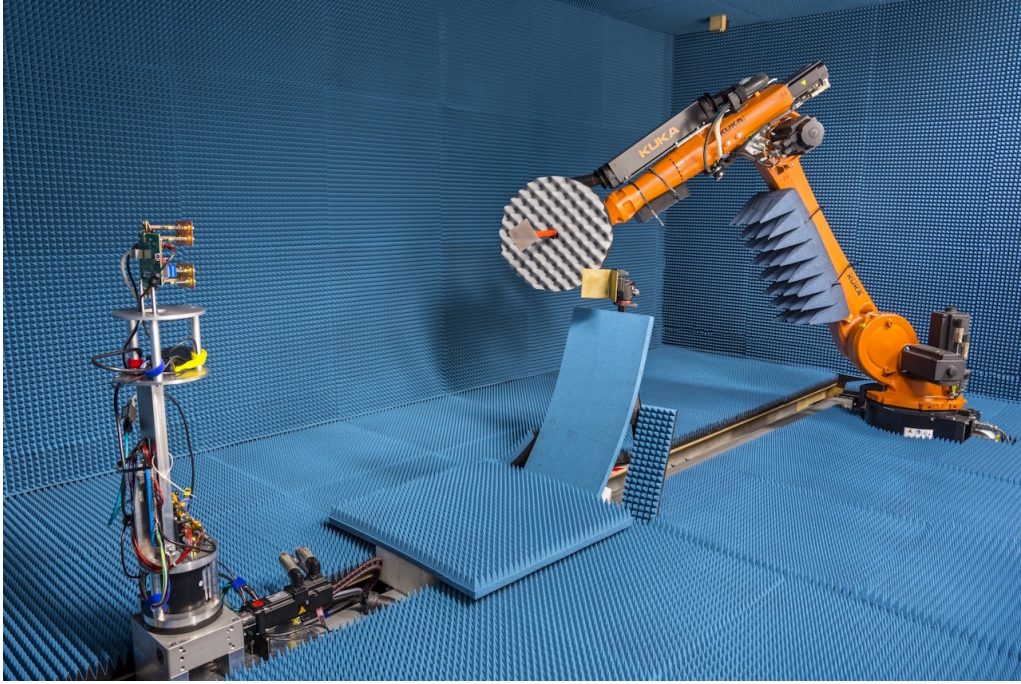


Figure A.3: Picture of the robot-based measurement system during the evaluation of the angular separation capability of a planar periodic leaky wave antenna for two targets in the same range to the radar [164]. This type of analysis is especially relevant for radar applications in the automotive sector and emphasizes the flexible application capabilities of the developed measurement system.

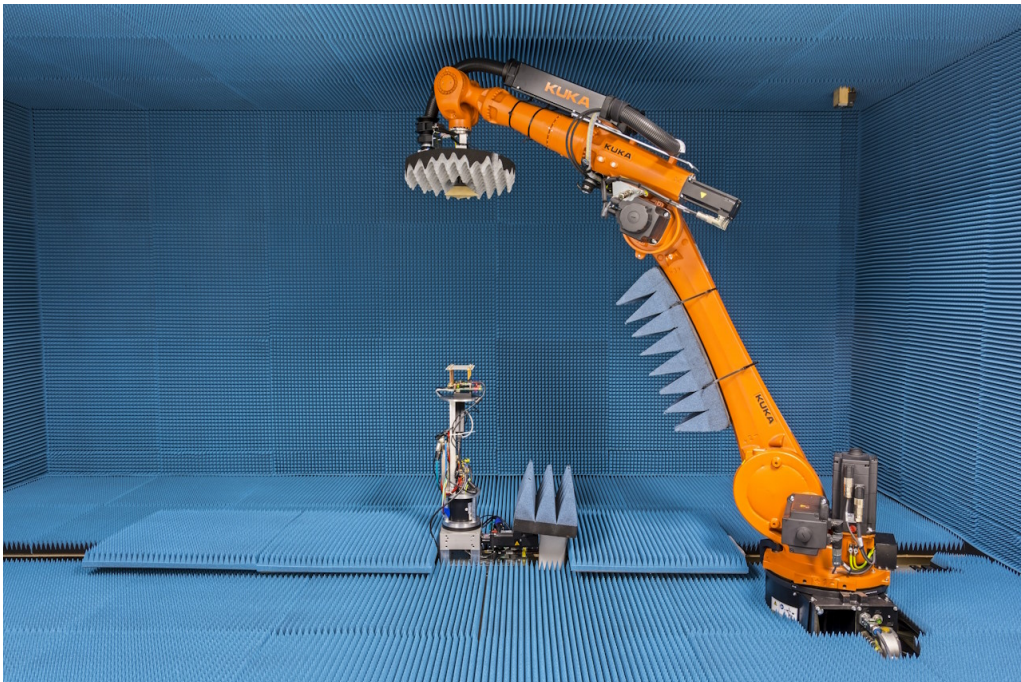


Figure A.4: Exemplary illustration of the measurement setup for determining the radiation pattern of an active radar module using a trihedral reflector [165,166]. The additional axis of rotation is used to perform the measurement over a hemisphere, thus demonstrating the wide range of applications for this system.

Appendix B

Optical Alignment Procedure

As an alternative or supplementing the electrical calibration methods for quantifying the individual terms of the error model presented in Section 4, an optical alignment method can also be applied. It requires an imaging sensor, whereby a digital microscope is used within the scope of this work. The sensor is either mounted directly at the TCP of the robot and, thus, enables an analysis of the setup for mounting the AUT or it can also be mounted in a fixed position in space to examine the probe antenna attached to the robot.

Therefore, a reference target is required whose image features can be recognized and extracted using appropriate image processing methods. A robust feature extraction technique is the Hough transform, which enables the detection of straight lines, circles or otherwise parameterizable objects [167]. However, a circular object is recommended, for example, a circular shaped calibration standard or a scalar feed horn antenna are suitable as a reference target and can, therefore, be introduced into the measurement setup in a minimally invasive manner. Numerous studies on the application of the Hough transformation to circular objects confirm this choice of reference target [168–171].

However, it should be noted that the projection into a two-dimensional image space does not allow a reliable determination of relatively small inclinations, e.g. tilts of less than 1° . Accordingly, this method refers to spatial offsets such as the AMO in relation to the AUT mounting structure or the PMO with respect to the mounting of the probe antenna and can, therefore, only be used to supplement a complete estimation of the error model.

An exemplary image for determining the AMO is presented in Fig. B.1 and depicts the the circular shaped calibration standard, which is mounted on the waveguide flange of the TxRx module, i.e. instead of the AUT. The red circle corresponds to the result of the Hough transformation for the detection of circular shapes, with the red cross indicating its center position. By repeating this procedure for different positions of the rotation axis on which the TxRx module is mounted, the AMO is determined by the course of the circle's center position. As the evaluations are based on a relative comparison, no specific alignment is required with regard to the position of the reference target in the camera image. To determine the actual AMO based on the results of the image processing techniques, either a microscope with calibrated optics can be used or the generated results can be corrected by the known mechanical dimensions of the reference target, for example its radius.

The procedure for determining the PMO is analogous, whereby the digital microscope is positioned on a tripod below the probe antenna in this case. Instead of the additional axis of rotation, the sixth axis of the robot arm is used to generate a rotation. To avoid unnecessary modifications and save time, depending on the probe

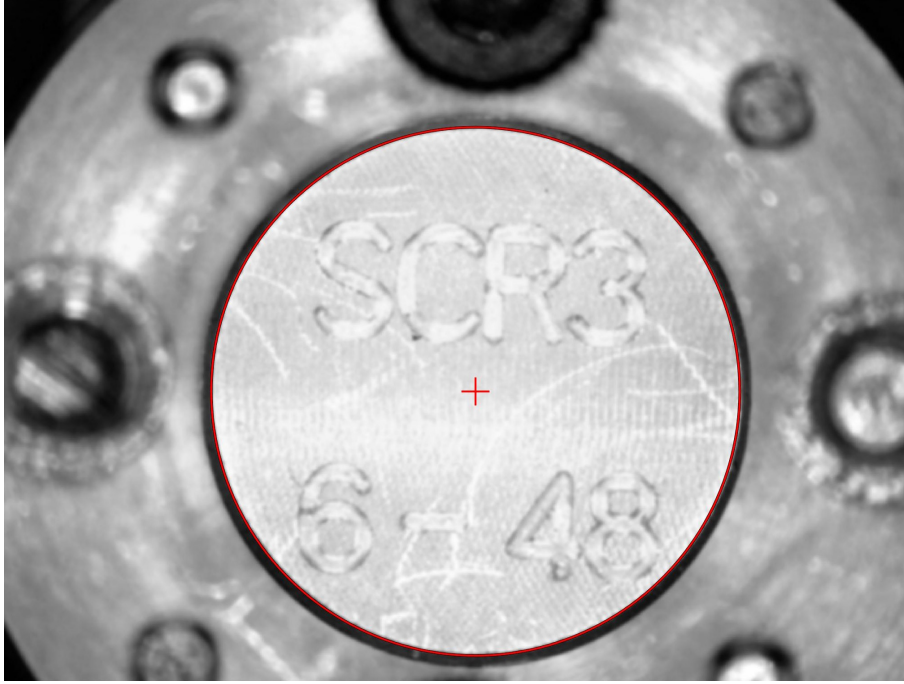


Figure B.1: Exemplary picture demonstrating the identification of a calibration standard used as a reference target mounted on the waveguide flange. Image processing algorithms allow the identification of the circular shape of the calibration standard, thereby determining the center and radius. Using multiple optical measurements for different rotational axis positions, it is possible to determine an error model for the measurement setup used, as further described in Section 4. In the background of the image, the screw connections and dowel pins of the calibration standard can be recognized.

used, the reference target can be omitted and, instead, the probe antenna can be detected by the image processing algorithms. This is exemplified in Fig. B.2, which depicts the aperture of the probe antenna as recorded by the microscope. Here, the inner structure of the scalar feed horn antenna is used to detect circular shapes by means of the Hough transformation. Since the probe antenna has a complex inner structure consisting of several corrugations, it is important to restrict the parameter space of the Hough transformation so that only the desired circular shapes are detected and false detections are avoided.

The results of the optical calibration method for determining the AMO are compared in Fig. 4.3 with the results of the electrical alignment methods and demonstrate a very good consistency with deviations of less than 0.4 %.

The results of the optical determination of the PMO are presented in Fig. 4.5. The complex structure of the probe antenna causes several outliers of the image processing results, although these do not significantly affect the overall evaluation due to the high number of data points averaged. Furthermore, it should be noted that the PMO offset is more than eight times smaller than the AMO.

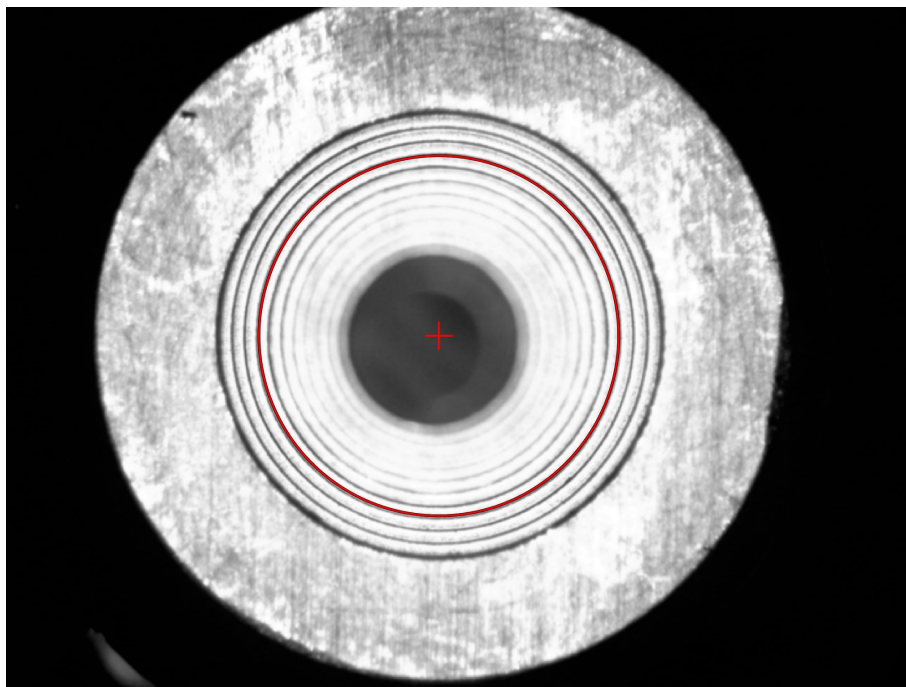


Figure B.2: Exemplary picture illustrating the probe antenna misalignment identification using image processing algorithms that identify the circular grooves of the scalar feed horn antenna. The red cross marks the center of the probe antenna. Multiple measurements for different rotational axis positions of the robot's TCP can be used to identify the offset of the probe antenna from the rotational axis, as described in more detail in Section 4.

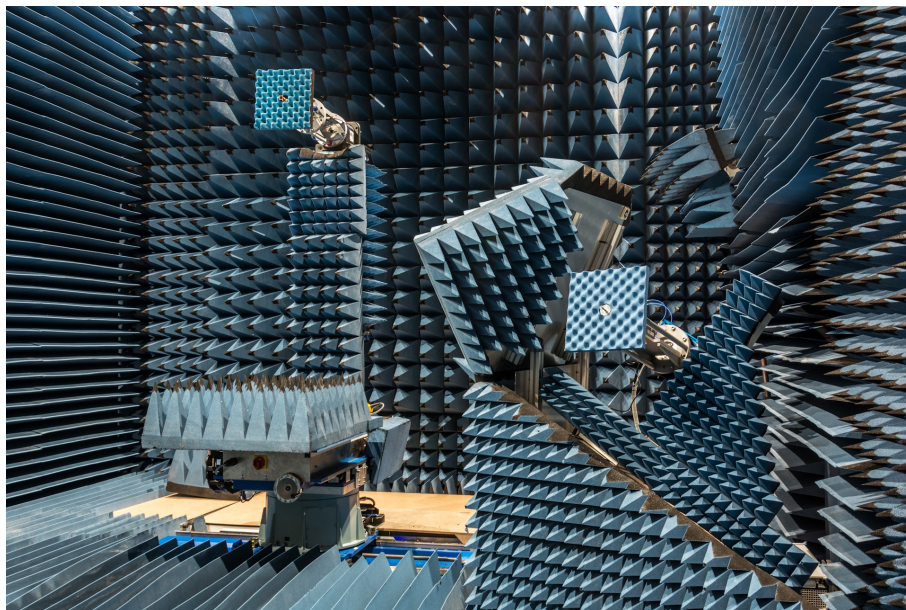


Figure B.3: Picture of the setup used for the comparative measurements in the side-fed single-reflector CATR. The qualitative comparison in Fig. 4.10 reveals the presence of spill-over effects for the angular range in which the AUT is oriented in the direction of the feed antenna.

AUT and Probe Antenna

The most important characteristics of the AUT and probe antenna used within this dissertation are briefly outlined in the following. All information is either based on simulation data or can be found in the respective data sheet.

C.1 AUT

The AUT used within this dissertation is the standard gain horn antenna shown in Fig. C.1. It is linearly polarized and specified for a frequency range of 60 GHz to 90 GHz. The half-power beam width (HPBW) is between 14° to 19° depending on the frequency. Further parameters are listed in Table C.1 and [172].



Figure C.1: Picture of the AUT used during the reference measurements performed in the CATR of the IHF.

C.2 OEWG Probe Antenna

The OEWG probe antenna used in the robot-based measurement setup has been manufactured in the IHF's mechanical workshop. The probe's length has been determined to allow for proper coverage with an absorber while maintaining the largest possible measurement radius. A picture of this setup is provided in Fig. C.2. Analogously to the AUT, the probe antenna is designed for a frequency range of 60 GHz to 90 GHz and has a HPBW of 52° to 68° , depending on the frequency. Further data is provided in Table C.2.

Table C.1: Overview of the most important properties of the AUT.

Parameter	Value	Parameter	Value
Manufacturer	RFSpin	Gain	19 dBi to 21 dBi
Model	H-A90-W20	HPBW	14° to 19°
Type	Pyramidal horn antenna	Polarization	single, linear
Connector	WR12 UG-387/U flange	Width	21.6 mm
Material	Aluminum, gold-plated	Depth	27.7 mm
		Height	16.6 mm

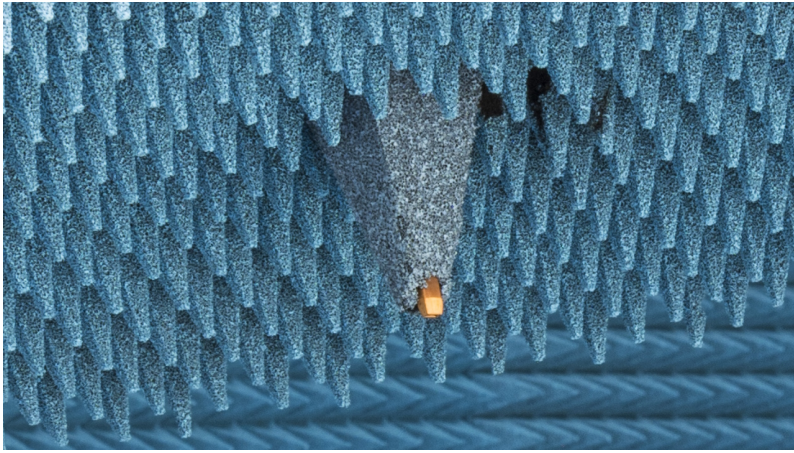


Figure C.2: Picture of the absorber-covered OEWG probe antenna mounted on the robot during an antenna measurement.

Table C.2: Overview of the most important properties of the OEWG probe antenna.

Parameter	Value
Manufacturer	In-house
Type	OWEG probe antenna with tapered aperture
Connector	WR12, UG-387/U flange
Gain	6.7 dBi to 7.0 dBi
3dB beamwidth	52° to 68°
Polarization	linear
Length	70 mm
Material	Copper, non-plated

C.3 Scalar Feed Horn Antenna

The scalar feed horn (SFH) antenna depicted in Fig. C.3 is only considered for the sake of completeness, as this antenna has been used to determine the PMO in Fig. 4.5 based on the image processing techniques illustrated in Fig. B.2. It has not been included in the measurements, as the antenna is susceptible to standing waves due to its relatively large aperture. Furthermore, test measurements have proven that

the cross-polarization suppression of this antenna is significantly lower compared to the OEWG probe antenna used. The SFH antenna is equipped with a transition from a circular waveguide with a diameter of 3.175 mm to a WR12 rectangular waveguide. An overview of the most important properties of this antenna is included in Table C.3 or refer to [173].



Figure C.3: Picture of the SFH antenna mounted in the CATR of the IHF.

Table C.3: Overview of the most important properties of the SFH antenna.

Parameter	Value
Manufacturer	Eravant
Model	SAF-6039031437-12-S1
Type	Scalar feed horn antenna
Connector	WR12, UG-387/U flange
Gain	14 dBi at 75 GHz
HPBW	37° at 75 GHz
Polarization	single, linear
Length	58.42 mm (including the waveguide transition)
Diameter (outer)	19.05 mm
Material	Brass, gold-plated

Appendix D

Maximum Determinant Sampling Grids

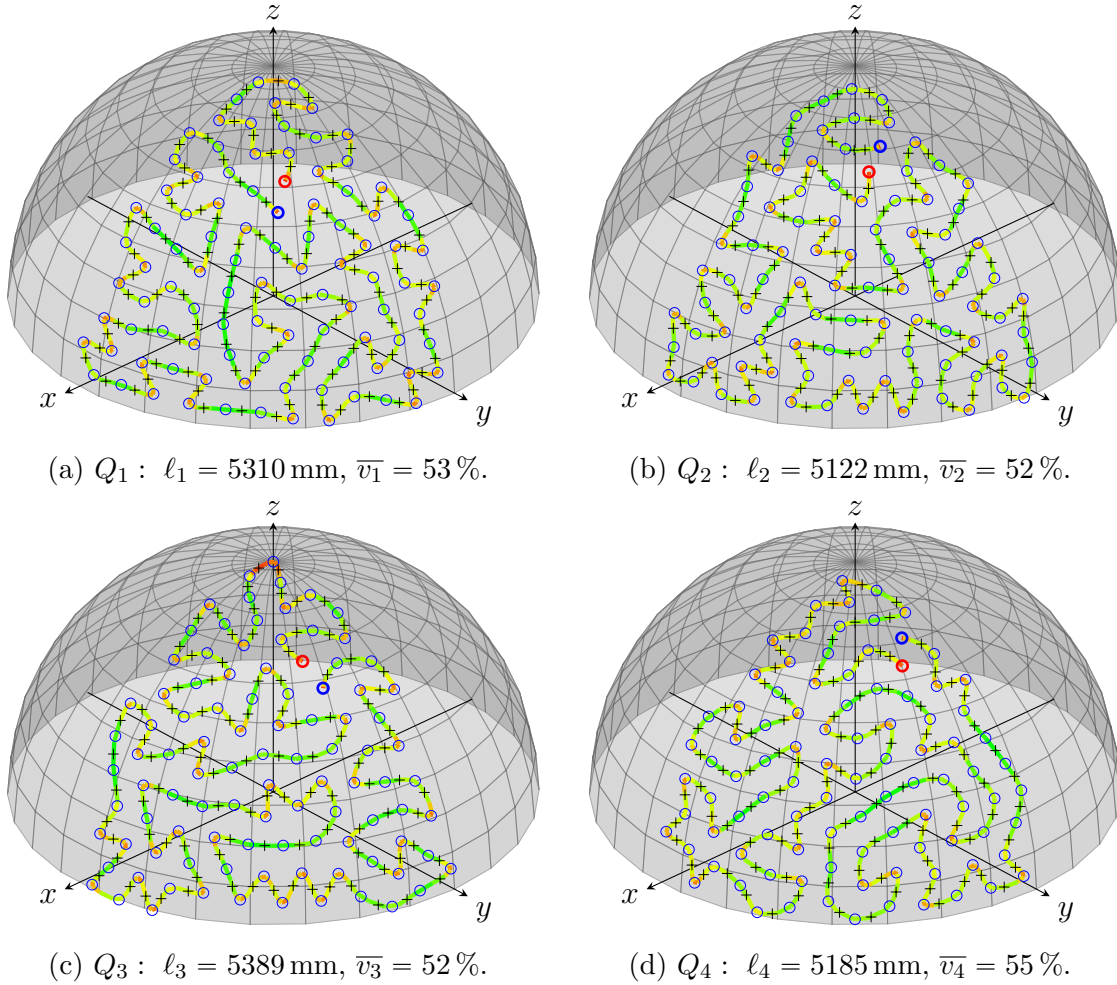


Figure D.1: Result of the TSP optimization for sampling point distributions defined by an MDSG_{27} including the actual robot path. The color-coded path velocity is provided to allow a qualitative comparison between the quarters or other variants of the MDSG. For each quarter Q_i , the path length ℓ_i and average velocity $\bar{v}_i$ normalized to the specified maximum velocity is stated. Circles and crosses represent SPT and PDT generated sampling points, with the markers highlighted in bold blue and red indicating the start and end points, respectively.

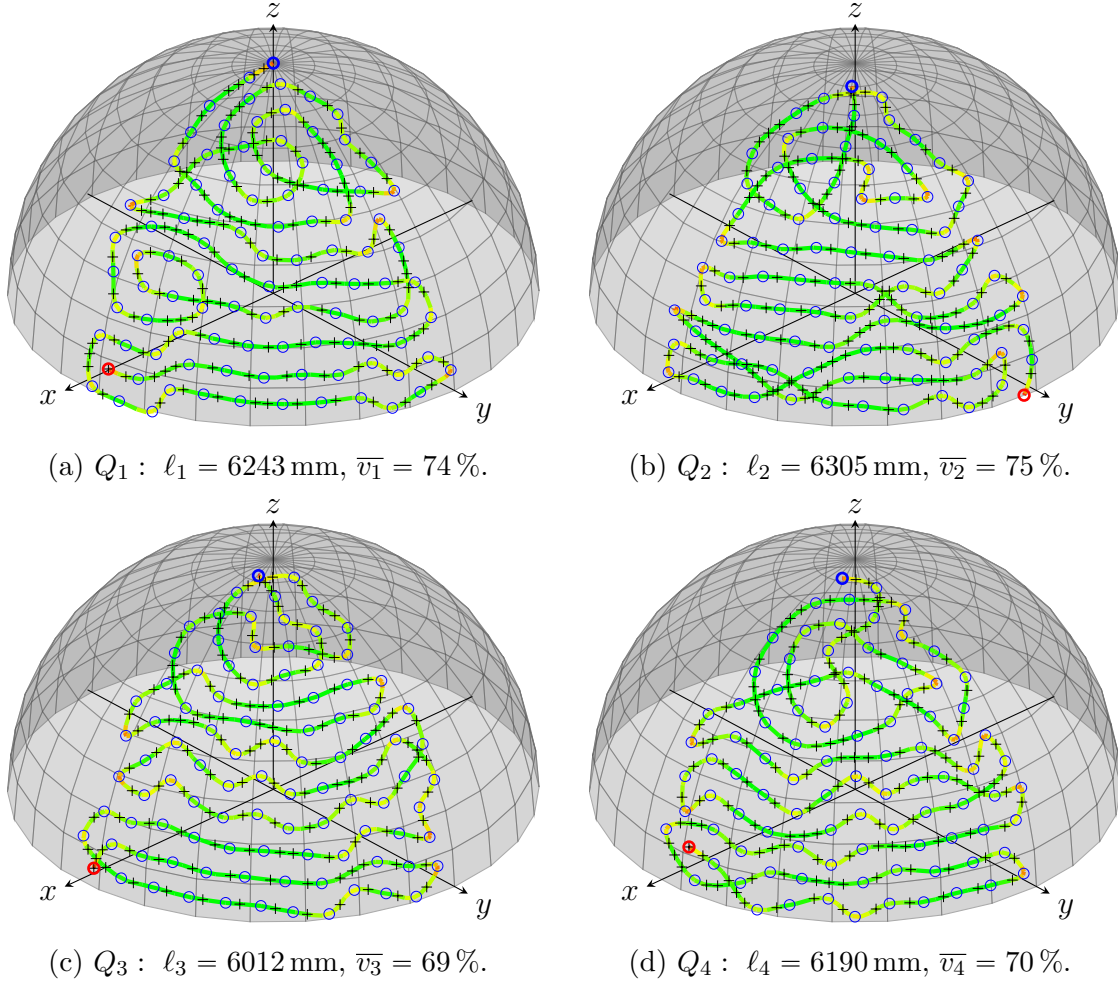


Figure D.2: Result of the WCR optimization for sampling point distributions defined by an MDSG_{27} including the actual robot path. The color-coded path velocity is provided to allow a qualitative comparison between the quarters or other variants of the MDSG. For each quarter Q_i , the path length ℓ_i and average velocity $\overline{v}_i$ normalized to the specified maximum velocity is stated. Circles and crosses represent SPT and PDT generated sampling points, with the markers highlighted in bold blue and red indicating the start and end points, respectively.

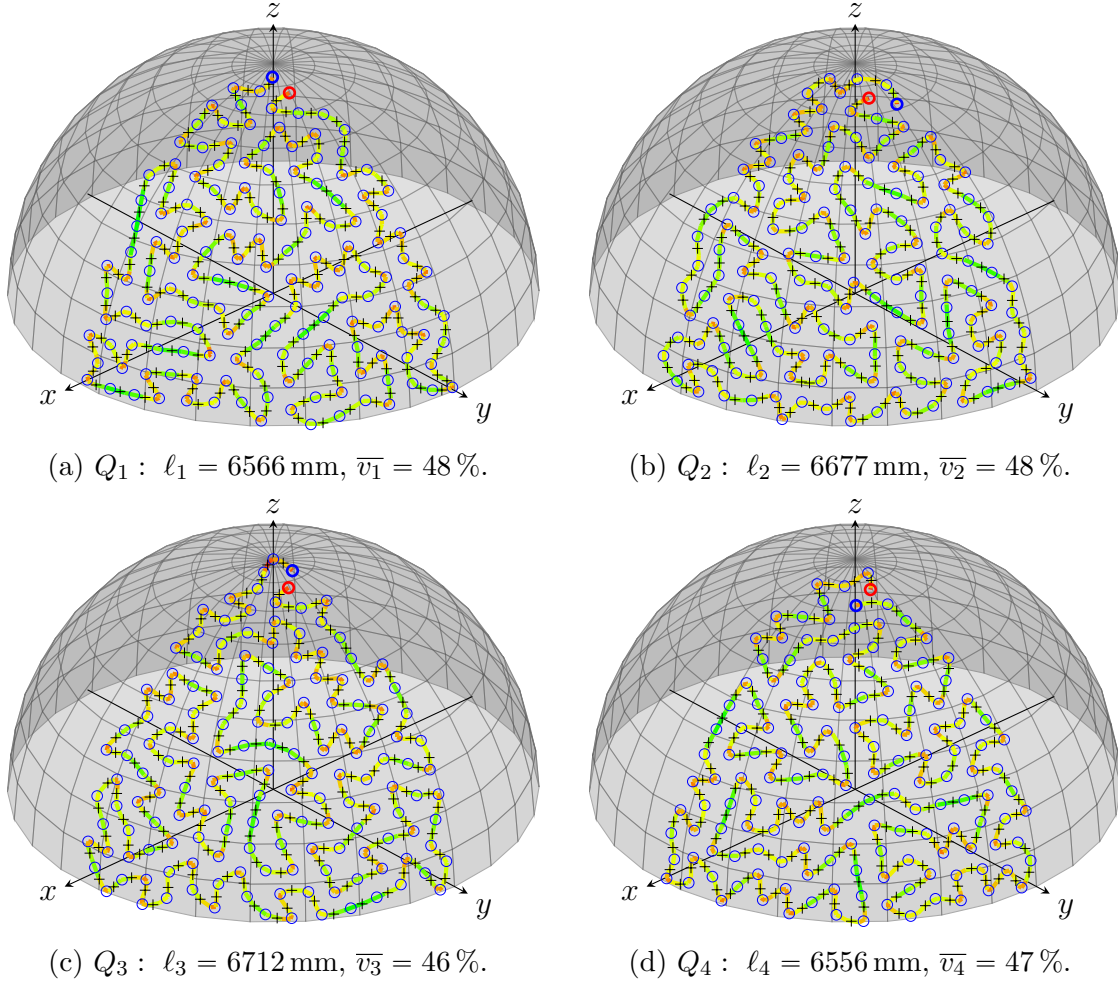


Figure D.3: Result of the TSP optimization for sampling point distributions defined by an MDSG_{34} including the actual robot path. The color-coded path velocity is provided to allow a qualitative comparison between the quarters or other variants of the MDSG. For each quarter Q_i , the path length ℓ_i and average velocity $\overline{v}_i$ normalized to the specified maximum velocity is stated. Circles and crosses represent SPT and PDT generated sampling points, with the markers highlighted in bold blue and red indicating the start and end points, respectively.

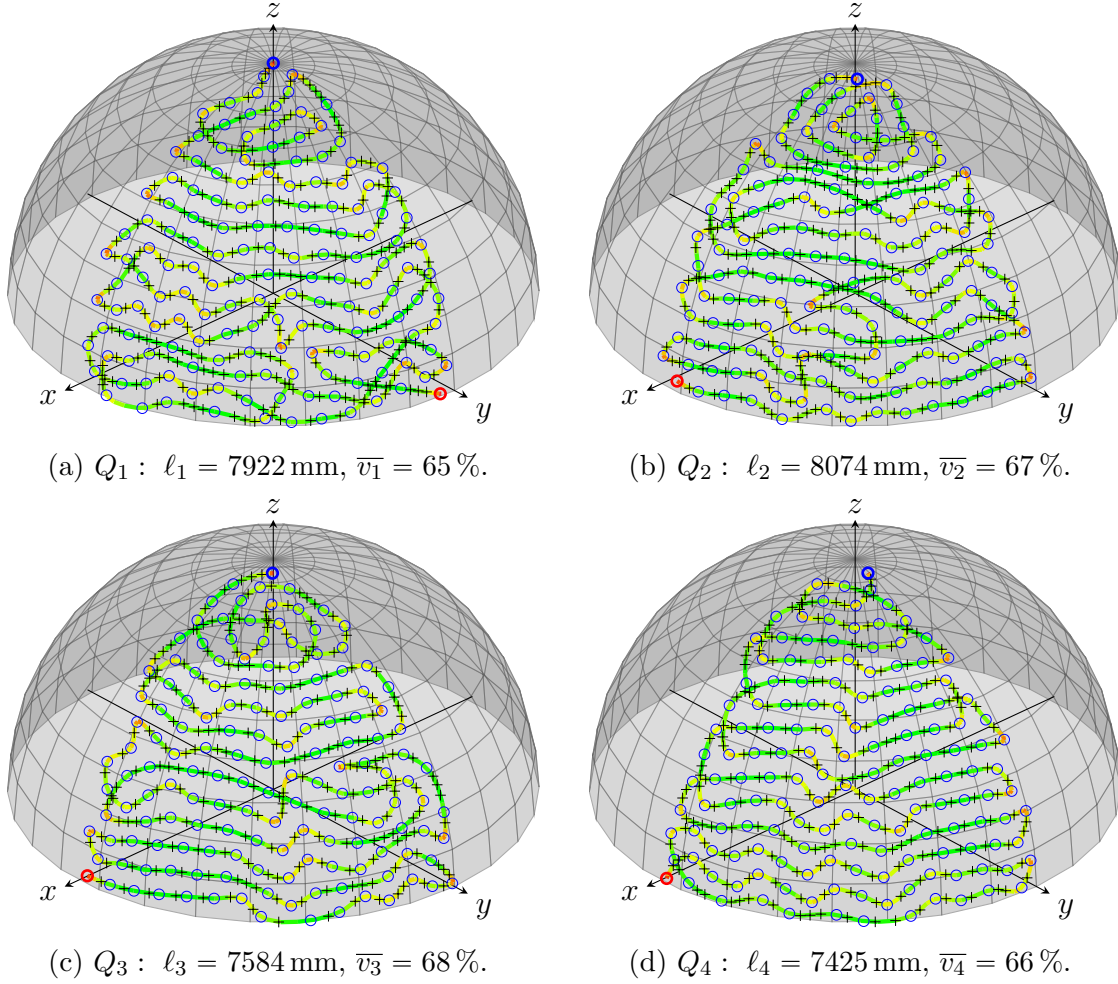


Figure D.4: Result of the WCR optimization for sampling point distributions defined by an MDSG_{34} including the actual robot path. The color-coded path velocity is provided to allow a qualitative comparison between the quarters or other variants of the MDSG. For each quarter Q_i , the path length ℓ_i and average velocity $\overline{v}_i$ normalized to the specified maximum velocity is stated. Circles and crosses represent SPT and PDT generated sampling points, with the markers highlighted in bold blue and red indicating the start and end points, respectively.

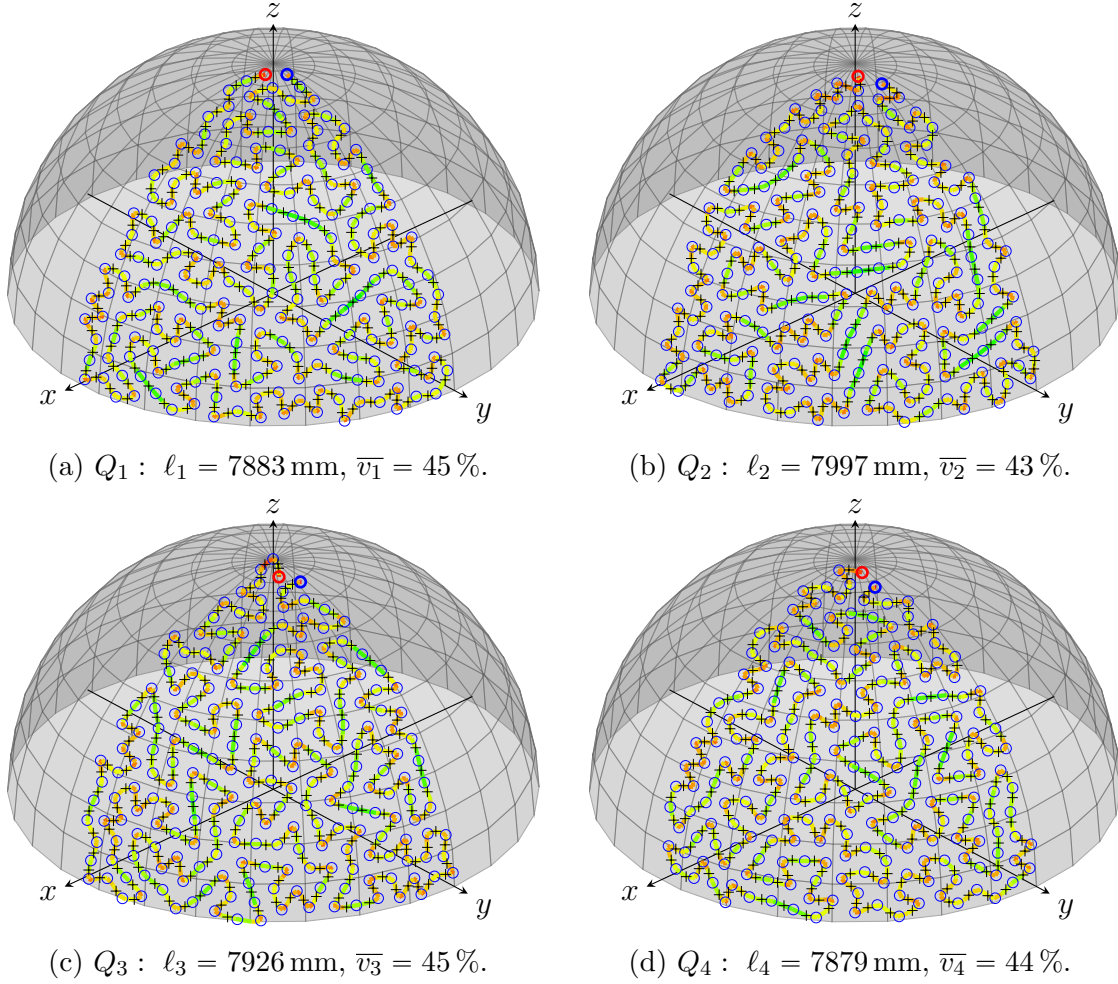


Figure D.5: Result of the TSP optimization for sampling point distributions defined by an MDSG_{41} including the actual robot path. The color-coded path velocity is provided to allow a qualitative comparison between the quarters or other variants of the MDSG . For each quarter Q_i , the path length ℓ_i and average velocity $\overline{v}_i$ normalized to the specified maximum velocity is stated. Circles and crosses represent SPT and PDT generated sampling points, with the markers highlighted in bold blue and red indicating the start and end points, respectively.

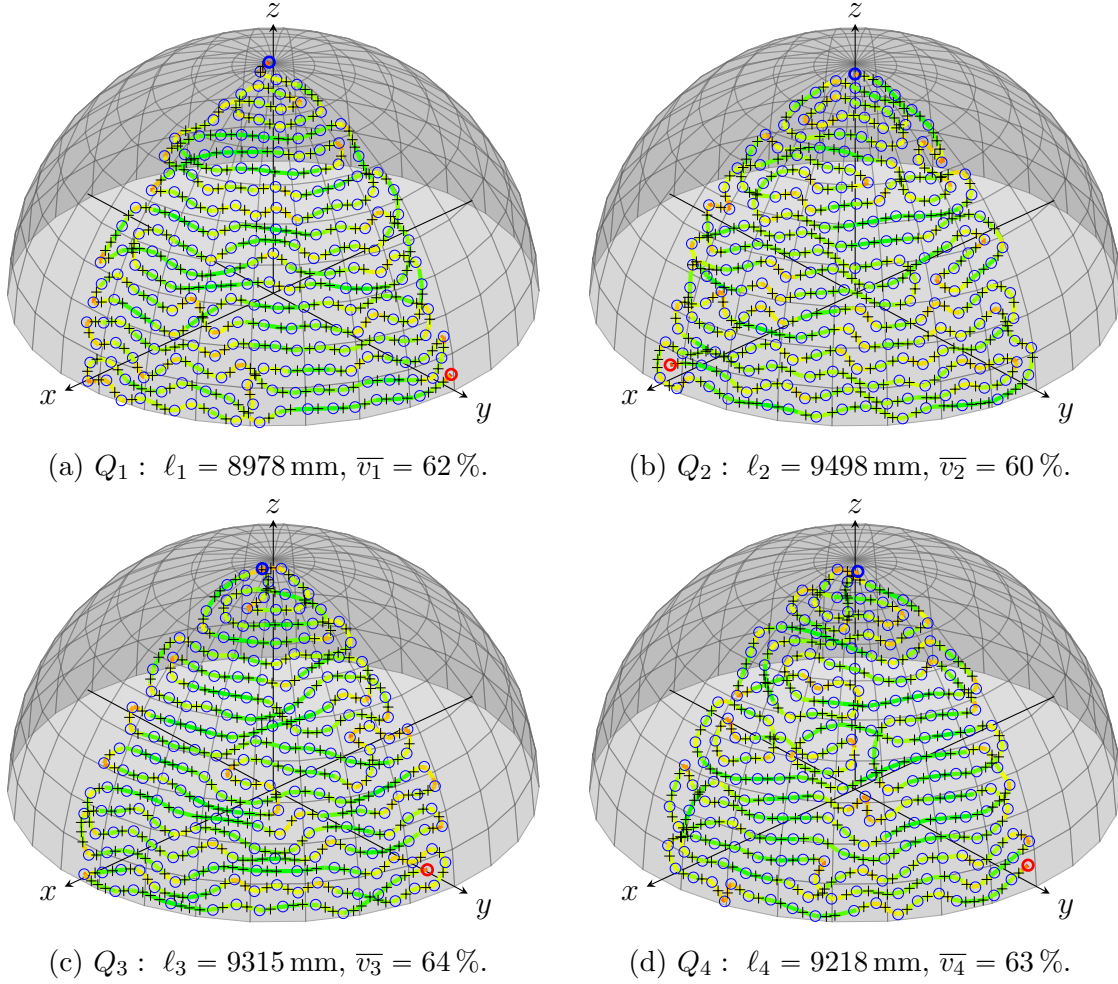


Figure D.6: Result of the WCR optimization for sampling point distributions defined by an MDSG_{41} including the actual robot path. The color-coded path velocity is provided to allow a qualitative comparison between the quarters or other variants of the MDSG. For each quarter Q_i , the path length ℓ_i and average velocity $\overline{v}_i$ normalized to the specified maximum velocity is stated. Circles and crosses represent SPT and PDT generated sampling points, with the markers highlighted in bold blue and red indicating the start and end points, respectively.

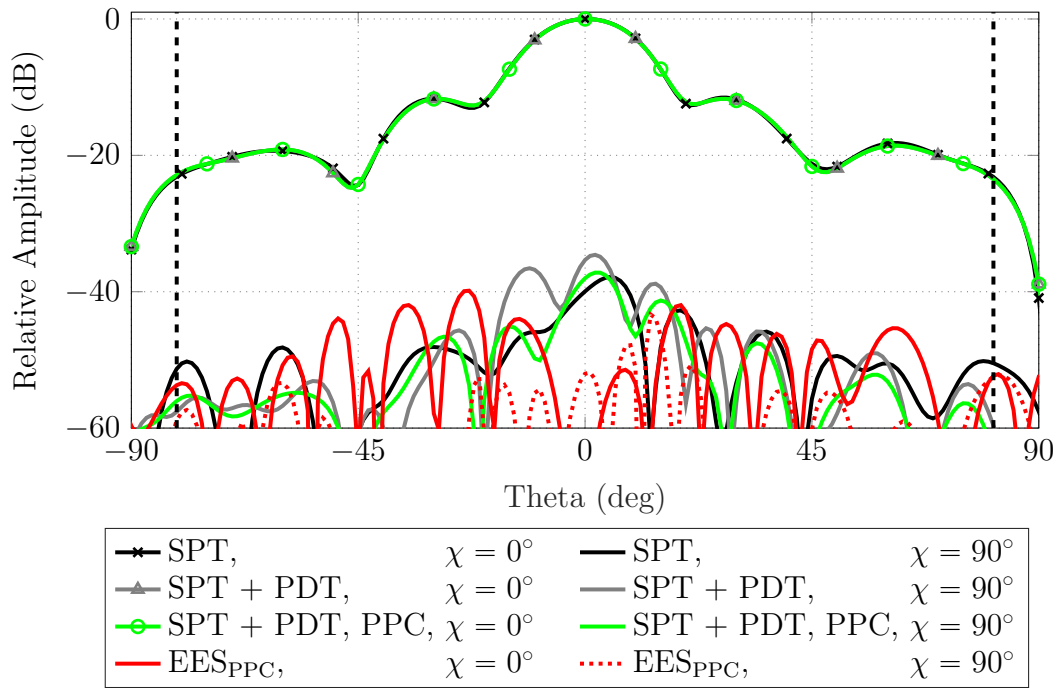


Figure D.7: Comparison of the far-field transformed radiation pattern ($\phi = 0^\circ$) for an MDSG₂₇ considering SPT and SPT + PDT with and without the application of PPC. The EES denotes the difference from the original results based on SPT only to the application of PPC in case of SPT + PDT.

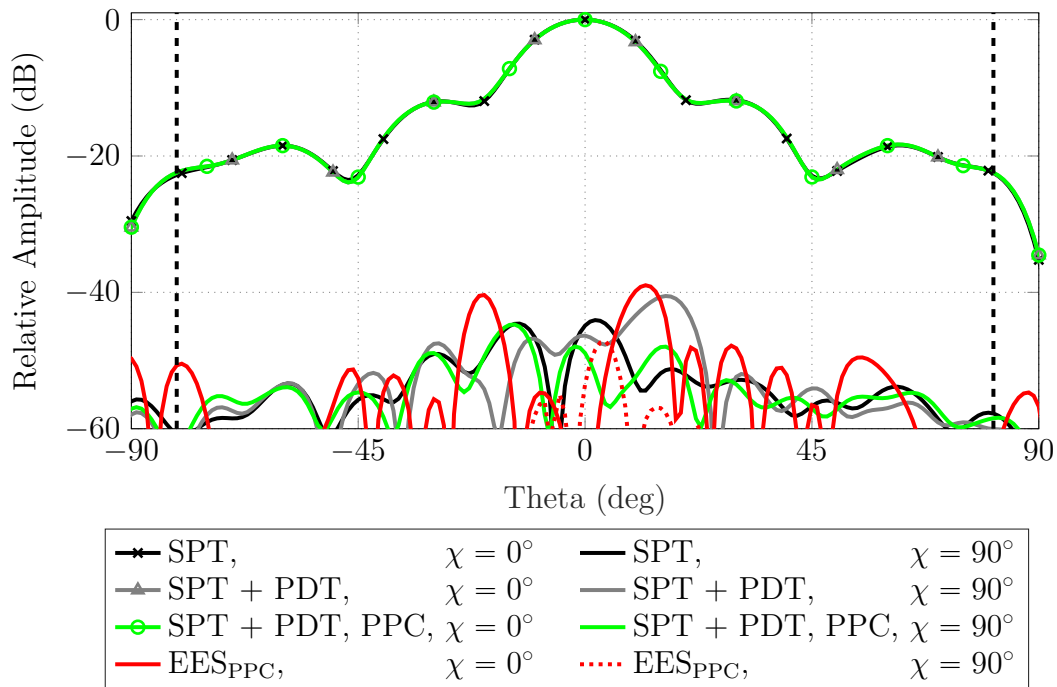


Figure D.8: Comparison of the far-field transformed radiation pattern ($\phi = 0^\circ$) for an MDSG₄₁ considering SPT and SPT + PDT with and without the application of PPC. The EES denotes the difference from the original results based on SPT only to the application of PPC in case of SPT + PDT.

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List of Abbreviations

AIO	axis intersection offset
AIT	axis intersection tilt
AMO	AUT mounting offset
AMT	AUT mounting tilt
AUT	antenna under test
BiCG	biconjugate gradient
BiCGSTAB	biconjugate gradient stabilized
CATR	compact antenna test range
CG	conjugate gradient
CNF	cylindrical near-field
DFT	discrete Fourier transform
DH	Denavit-Hartenberg
DIP	diplexer
DOF	degrees of freedom
DUT	device under test
EES	equivalent error signal
FEM	frequency extension modules
FOP	first-order probes
GMRES	generalized minimal residual
GU	gateway unit
HOP	higher-order probes
HPBW	half-power beam width
IF	intermediate frequency
IHF	Institute of High Frequency Technology
ISO	international organization for standardization
LO	local oscillator
MARS	mathematical absorber reflection suppression
MC	measurement completed
MCACK	measurement completed acknowledgement
MDSG	maximum determinant sampling grid
MLFM	multilevel fast multipole
NFFFT	near-field to far-field transformation
OEWG	open-ended waveguide
OSHA	occupational safety and health administration
OSSD	output signal switching device
PC	probe correction
PDF	probability density function
PDT	path-dependent trigger
PLC	programmable logic controller
PMO	probe mounting offset
PNF	planar near-field
PPC	pointwise probe correction

PPM probe polarization mismatch
PSU power supply unit
RC robot controller
RCS radar cross-section
RF radio frequency
RMS root mean square
Rx receive
SCU safety control unit
SFH scalar feed horn
SGH standard gain horn
SMC spherical mode coefficients
SNF spherical near-field
SPDT single-pole double-throw
SPT support point trigger
SWE spherical wave expansion
TACK trigger acknowledgement
TCP tool center point
TE transverse electric
TM transverse magnetic
TS trigger signal
TSP traveling salesman problem
TTL transistor-transistor logic
TxRx transmit and receive
VNA vector network analyzer
VPA variable programmable attenuator
WCR wider curve radii

List of Symbols

A_a	Amplitude pattern of an AUT
(α, β, γ)	Euler Rotations along the z -axis for an angle of α , along the rotated y' -axis for an angle of β and around the twice rotated z'' -axis for an angle of γ
A_m	Measured amplitude response of an AUT
$C_{\sigma\mu\nu}^{sn(3)}(kr)$	Translation operator for $\mathbf{F}_{smn}^{(3)}$ along the z -axis for a shift of r for a mode σ with order μ and degree ν
$C_{\sigma\mu\nu}^{sn(c)}(kr)$	Translation operator for $\mathbf{F}_{smn}^{(c)}$ with a radial dependency (c) along the z -axis for a shift of r for a mode σ with order μ and degree ν
C_{RT}	Reach-through (RT) safety buffer depending on the resolution of the light fence
δ	Linear offset corresponding to the axis of measured θ -cut, e.g. $\delta = \Delta y$ for $\phi = 90^\circ$
$d_{\mu m}^n(\theta)$	Rotation operator for $\mathbf{F}_{smn}^{(c)}$ along the y -axis for an angle of θ
ΔA	Difference of a measured amplitude
Δ	Uniform angular step size $\Delta = \Delta\theta = \Delta\phi$ applied for equiangular measurements
$d_{\mu'\mu}^\nu(\beta)$	Rotation operator for $\mathbf{F}_{smn}^{(c)}$ along the y -axis for an angle of β
$\hat{\delta}_x$	Estimated offset of the AUT relatively to the probe antenna in the x -direction
$\hat{\delta}_y$	Estimated offset of the AUT relatively to the probe antenna in the y -direction
d_{FF}	Far-field distance of the considered AUT
$\hat{\delta}$	The resulting estimation of the actual offset δ
$\Delta\varphi$	Difference of a measured phase
$\mathbf{E}(r, \theta, \phi)$	Electric field at a position in space specified by spherical coordinates (r, θ, ϕ)
$e^{i\mu\alpha}$	Rotation operator for $\mathbf{F}_{smn}^{(c)}$ along the z -axis for an angle of α
$e^{i\mu\chi}$	Rotation operator for $\mathbf{F}_{smn}^{(c)}$ along the z -axis for an angle of χ
$e^{i\mu'\gamma}$	Rotation operator for $\mathbf{F}_{smn}^{(c)}$ along the z -axis for an angle of γ
$e^{im\phi}$	Rotation operator for $\mathbf{F}_{smn}^{(c)}$ along the z -axis for an angle of ϕ_0

$\mathbf{E}_t(r, \theta, \phi)$	Electric field at a position in space specified by spherical coordinates (r, θ, ϕ) for an outward travelling wave
$\hat{\beta}_a$	Estimated tilt around the y_a -axis of the AUT coordinate system
$\hat{\beta}_{\text{axis}}$	Estimated tilt around the y_{axis} -axis of the axes coordinate system
$\hat{\gamma}_a$	Estimated tilt around the x_a -axis of the AUT coordinate system
$\hat{\gamma}_{\text{axis}}$	Estimated tilt around the x_{axis} -axis of the axes coordinate system
$\mathbf{F}_{smn}^{(c)}$	Spherical wave function with radial dependency c , describing a mode s with order m and degree n
$\mathbf{F}_{\sigma\mu\nu}^{(1)}$	Spherical wave function for radially standing waves (finite at the origin), describing a mode σ with order μ and degree ν
$\mathbf{F}_{smn}^{(3)}$	Spherical wave function for radially outward travelling waves, describing a mode s with order m and degree n
$\mathbf{F}_{s\mu n}^{(3)}$	Spherical wave function for radially outward travelling waves, describing a mode s with order μ and degree n
f_{IF}	Intermediate frequency of 279 MHz used by the FEM
F_x	Force applied in the direction of the x -axis
F_y	Force applied in the direction of the y -axis
F_z	Force applied in the direction of the z -axis
$h_n^{(l)}$	Spherical Hankel function of l -th kind and order n
IF_{M1}	The decoupled and downconverted reflected component in the RF_{Tx} path
IF_{M2}	The downconverted received signal of the Rx FEM
IF_{Ref}	The decoupled and downconverted reference component of the RF_{Tx} path
J	Total number of modes $2N_n(N_n + 2)$ in j -notation
$\mathcal{J}$	Jacobian matrix, e.g. used in order to solve the problem of inverse kinematics
j_n	Spherical Bessel function of order n
k	Wavenumber given by $k = 2\pi/\lambda$
κ	Condition number of a matrix
L	Total amount of acquired sampling points used to calculate the required amount of J modes
λ	Wavelength defined as $\lambda = c_0/f$, where c_0 is the speed of light and f the frequency
L/J	Oversampling factor to calculate J modes based on L measurements
η	Specific wave admittance given by $\mu = \sqrt{\epsilon/\mu}$

M_x	Torque applied around the x -axis
M_y	Torque applied around the y -axis
M_z	Torque applied around the z -axis
n_n	Spherical Neumann function of order n
N_f	The number of frequency points acquired for a certain measurement
N_{FEM}	Multiplication factor used by the FEM, in the current state $N_{\text{FEM}} = 6$
N_m	Modal truncation limit of the order m in relation to the band-limitation for a given AUT
N_{MDSG}	Degree of the point configuration of an MDSG
$N_{m,n}$	Modal truncation limit of the order m and degree n in relation to the band-limitation for a given AUT
$N_{m,n,\max}$	Maximum number of SMC that can be calculated depending on the applied sampling grid
N_ν	Modal truncation limit of the degree ν in relation to the band-limitation for a given probe antenna
N_n	Modal truncation limit of the degree n in relation to the band-limitation for a given AUT
N_μ	Modal truncation limit of the order μ in relation to the band-limitation for a given probe antenna
$N_{\mu,\nu}$	Modal truncation limit of the order μ and degree ν in relation to the band-limitation for a given probe antenna
N_s	The number of sampling points for given measurement
N_{TF}	Thinning factor applied to reduce the amount of spline support points
Δr_{offset}	Radial deviation based on the actual measurement coordinates for an offset measurement
$\Delta \theta_{\text{offset}}$	Deviation in theta based on the actual measurement coordinates for an offset measurement
$\Delta \phi_{\text{onset}}$	Deviation in phi based on the actual measurement coordinates for an aligned measurement
Δr_{onset}	Radial deviation based on the actual measurement coordinates for an aligned measurement
$\Delta \theta_{\text{onset}}$	Deviation in theta based on the actual measurement coordinates for an aligned measurement
$\overline{P}_n^{ m }$	Normalized Legendre polynomials with order m and degree n
Ψ	Measurement matrix in $\mathbb{C}^{L \times J}$ for L sampling points and J modes
$e_{p,x}$	Extent of the probe antenna in the x -direction
$e_{p,y}$	Extent of the probe antenna in the y -direction
$e_{p,z}$	Extent of the probe antenna in the z -direction

ϕ_{axis}	Position of the rotation axis on which the AUT is mounted
$\tilde{\chi}$	Ideal rotation used in the rotation operator $e^{i\mu\chi}$
χ	Ideal rotation used in the rotation operator $e^{i\mu\chi}$
$\tilde{\phi}$	Ideal rotation used in the rotation operator $e^{im\phi}$
ϕ	Ideal rotation used in the rotation operator $e^{im\phi}$
$\tilde{\theta}$	Ideal rotation used in the rotation operator $d_{\mu m}^n(\theta)$
θ	Ideal rotation used in the rotation operator $d_{\mu m}^n(\theta)$
ϕ_{m}	Phi coordinate in the measurement coordinate system $(x_{\text{m}}, y_{\text{m}}, z_{\text{m}})$
$\phi_{\text{p, ref}}$	Reference angle describing the radial direction to translate the axis coordinate system to the AUT coordinate system
$\phi_{\text{a, ref}}$	Reference angle describing the radial direction to translate the axis coordinate system to the AUT coordinate system
ψ_{a}	Intrinsic phase pattern of an AUT
$\Delta\psi$	Phase modulation caused by a non-ideal alignment of AUT and probe antenna
ψ_{m}	Measured phase response comprised of ψ_{a} and $\Delta\psi$
$\mathbf{d}$	Representation of the measured phase response ψ_{m} in vector form
$\mathbf{e}$	Representation of the sine terms corresponding to $\mathbf{d}$ in vector form
$P_{s\mu n}(kr)$	Probe response constants for a mode set $s\mu n$ in a radial distance r
$P_{s\mu n}^{(l)}(kr)$	Probe response constants for a mode set $s\mu n$ in a radial distance r for the l -th sampling point
$P_{s\mu n}^{(l)}(kr, \alpha, \beta, \gamma)$	Probe response constants for a mode set $s\mu n$ in a radial distance r for the l -th sampling point including additional Euler rotations (α, β, γ)
$\mathcal{Q}$	Conversion of a rotation matrix based on Euler rotations to quaternion representation
$Q_{smn}^{(c)}$	Spherical mode coefficients for complex weighting of the spherical wave functions $\mathbf{F}_{smn}^{(c)}$
$\vec{q}$	Vector in $\mathbb{C}^J$ containing the SMC q_j in general or T_j and R_j for the transmit and receive case, respectively
$\mathcal{Q}^{-1}$	Conversion of a quaternion representation to a rotation matrix based on Euler rotations
Q'_{r}	Representation of a non-ideal orientation using quaternions
Q_{r}	Representation of an ideal orientation using quaternions
$R_{\sigma\mu\nu}^{\text{p}}$	Spherical mode coefficients in receive mode

(r, θ, ϕ)	Spherical coordinates with radial distance r , polar angle θ and azimuthal angle ϕ
r_0	Minimum sphere of radius r_0 describing a volume which encloses sources
$R_{\Delta A}$	Repeatability of amplitude measurements
R_{actual}	Rotation matrix describing the actual orientation of a given sampling point
R_{actual}^{-1}	Inverted rotation matrix describing the actual orientation of a given sampling point in reversed order
RF_{Tx}	The signal transmitted by the TxRx module via the waveguide interface
R_{ideal}	Rotation matrix describing the ideal orientation of a given sampling point
$R_{\sigma\mu\nu}^{(l)\text{p}}$	Spherical mode coefficients in receive mode of the l -th probe or the l -th measurement position, respectively
$R_{\Delta\varphi}$	Repeatability of phase measurements
$(r_{\text{p}}, \theta_{\text{p}}, \phi_{\text{p}})$	Spherical coordinates of the probe coordinate system, analogous to (r, θ, ϕ)
(r, ϕ, θ, χ)	Spherical coordinates and orientation of the probe coordinate system, analogous to (r, θ, ϕ) , but including the polarization angle χ
R_{ref}	Reference orientation described as a rotation matrix
r_{TCP}	Vector describing the Cartesian position x, y, z and orientation a, b, c of the TCP, also referred to as the end-effector
r_{ϑ_i}	Vector describing the Cartesian position of the joint ϑ_i
S_{RT}	Reach-through (RT) safety distance to the dangerous area
T_{TD}	Total delay time of the overall system
T_{smn}	Spherical mode coefficients in transmit mode
α	Overall tilt around the y -axis
β	Overall tilt around the y -axis
$\hat{\beta}$	The resulting estimation of the actual tilt β
ϑ_6	Rotation axis of the robot's TCP
γ	Overall tilt around the x -axis
γ_{a}	Tilt around the x_{a} -axis of the AUT coordinate system
γ_{axis}	Tilt around the x_{axis} -axis of the axes coordinate system
$\hat{\gamma}$	The resulting estimation of the actual tilt γ
γ_{p}	Tilt around the x_{p} -axis of the probe coordinate system

θ_m	Theta coordinate in the measurement coordinate system (x_m, y_m, z_m)
T_{LF}	Reaction time of the light fence
T_R	Stopping time of the robot for a fully extended arm (slowest case)
T_{SCU}	Reaction time of the security control unit
T_{SR}	Reaction time of the security relay
v	Complex excitation amplitude
v_H	Standard approaching velocity of humans
$\vec{w}$	Measurement vector in $\mathbb{C}^L$ representing the acquired data for L sampling points
$w(r, \phi, \theta, \chi)$	Measurement signal of the probe depending on the relative position and orientation between probe and AUT
$w^{(l)}(r, \phi, \theta, \chi)$	Measurement signal of the probe depending on the relative position and orientation between probe and AUT for the l -th sampling point
$w_\mu(r, \phi, \theta)$	Coefficient in the Fourier series expansion in χ of $w(r, \phi, \theta, \chi)$
$w_{\mu m}(r, \theta)$	Coefficient in the Fourier series expansion in ϕ of $w_\mu(r, \phi, \theta)$
$w_{\mu m}^n(r)$	Coefficient in the Fourier series expansion in θ of $w_{\mu m}(r, \theta)$
(x'_p, y'_p, z'_p)	Cartesian coordinate system of the probe antenna in the non-ideal case
(x_f, y_f, z_f)	Cartesian coordinate system of the robot's TCP
ξ	Vector describing a point on a parametric description of a spline path
$(x_{axis}, y_{axis}, z_{axis})$	Coordinate system of the additional axes on which the AUT is mounted
(x_a, y_a, z_a)	Cartesian coordinate system of the AUT
(x_m, y_m, z_m)	Cartesian coordinate system of the measurement sphere
(x_p, y_p, z_p)	Cartesian coordinate system of the probe antenna
$z_n^{(c)}$	Radial dependency defined by superscript (c) and order n

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List of Publications

- **R. Moch**, Q. Ton, and P. Pelland, “Automated scan plane alignment for robot-based planar near-field antenna measurements,” in *47th Antenna Measurement Techniques Association Symposium (AMTA)*, Tucson, AZ, USA, Nov. 2025, to appear.
- H. Jansen, **R. Moch**, and D. Heberling, “Reduction of multiple reflections through intentional probe tilting enabled by robot-based measurement systems,” in *46th Antenna Measurement Techniques Association Symposium (AMTA)*, Cincinnati, OH, USA, Oct. 2024, pp. 1–6.
- H. Jansen, **R. Moch**, and D. Heberling, “Reduced-order maximum determinant sampling grids by acquisition of additional arbitrary sampling points on an optimized path,” in *18th European Conference on Antennas and Propagation (EuCAP)*, Glasgow, United Kingdom, Mar. 2024, pp. 1–5.
- H. Jansen, **R. Moch**, and D. Heberling, “Electrical alignment technique for offset-mounted and arbitrarily oriented AUTs in a robot-based mm-wave antenna test system,” in *45th Antenna Measurement Techniques Association Symposium (AMTA)*, Seattle, WA, USA, Oct. 2023, pp. 1–6.
- **R. Moch** and D. Heberling, “Maximum determinant sampling using spline-based trajectories in a robot-based mm-wave antenna test range,” in *44th Antenna Measurement Techniques Association Symposium (AMTA)*, Denver, CO, USA, Oct. 2022, pp. 1–6.
- **R. Moch** and D. Heberling, “Robot-based antenna measurements,” in *Short Course on the Application of Robotics for Antenna Measurements, 44th Antenna Measurement Techniques Association Symposium (AMTA)*, Denver, CO, USA, Oct. 2022.
- A. C. Granich, **R. Moch**, and D. Heberling, “Radiation pattern measurements using an active radar module,” in *44th Antenna Measurement Techniques Association Symposium (AMTA)*, Denver, CO, USA, Oct. 2022, pp. 1–5.
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- **R. Moch** and D. Heberling, “Pointwise probe correction applied to a robot-based mm-wave antenna test range,” in *43rd Antenna Measurement Techniques Association Symposium (AMTA)*, Daytona Beach, FL, USA, Oct. 2021, pp. 1–4.
- J. Jaeger, **R. Moch**, T. M. Gemmer, and D. Heberling, “Development of a motion-capable model for a robot-based antenna measurement system to simulate scattering-induced interference,” in *43rd Antenna Measurement Techniques Association Symposium (AMTA)*, Daytona Beach, FL, USA, Oct. 2021, pp. 1–6.
- A. C. Granich, **R. Moch**, and D. Heberling, “System parameter analysis of integrated radar sensors in a controlled test environment utilizing a robot-based measurement system,” in *43rd Antenna Measurement Techniques Association Symposium (AMTA)*, Daytona Beach, FL, USA, Oct. 2021, pp. 1–5.

- **R. Moch**, T. M. Gemmer, and D. Heberling, “Spherical antenna measurements performed in a robot-based mm-wave test range,” in *15th European Conference on Antennas and Propagation (EuCAP)*, Düsseldorf, Germany, Mar. 2021, pp. 1–5.
- **R. Moch** and D. Heberling, “Robot-based antenna and radar measurement system at the RWTH Aachen University,” in *42nd Antenna Measurement Techniques Association Symposium (AMTA)*, Newport, RI, USA, Nov. 2020, pp. 1–5.
- F. Weishaupt, A. Uzair, **R. Moch**, H.-L. Bloecher, J. F. Tilly, J. Dickmann, and D. Heberling, “Polarimetric scattering mechanism analysis of automotive landmarks,” in *21st International Radar Symposium (IRS)*, Oct. 2020, pp. 379–384.
- **R. Moch**, T. M. Gemmer, and D. Heberling, “Phase uncertainty analysis due to cable flexing in robot-based measurement systems,” in *42nd Antenna Measurement Techniques Association Symposium (AMTA)*, Newport, RI, USA, Nov. 2020, pp. 1–5.
- **R. Moch** and D. Heberling, “Height profiles of typical automotive landmarks using tomographic compact-range measurements,” in *14th European Conference on Antennas and Propagation (EuCAP)*, Copenhagen, Denmark, Mar. 2020, pp. 1–5.
- C. Culotta-López, **R. Moch**, R. Wilke, and D. Heberling, “On the influence of probe positioning errors due to mechanical uncertainties in spherical near-field measurements at terahertz frequencies in modern positioner systems,” in *Photonics & Electromagnetics Research Symposium (PIERS)*, Rome, Italy, Jun. 2019, pp. 1883–1888.
- **R. Moch** and D. Heberling, “Compensation of ambiguities in fast ISAR measurements,” in *13th European Conference on Antennas and Propagation (EuCAP)*, Krakow, Poland, Apr. 2019, pp. 1–5.
- X. Y. Wang, T. Dallmann, **R. Moch**, and D. Heberling, “ISAR tomography for full-polarimetric 3-D radar cross-section measurements,” *IEEE Transactions on Antennas and Propagation*, vol. 67, no. 4, pp. 2853–2858, Apr. 2019.
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Curriculum Vitae

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Education

08/2013 – 11/2015	Master of Science in Electrical Engineering, Information Technology and Computer Engineering, RWTH Aachen University, Germany Thesis: “Realization and Optimization of a High-Resolution Method for Source Separation of Radar Signals on a Multi-Core Processor System”
10/2009 – 08/2013	Bachelor of Science in Electrical Engineering, Information Technology and Computer Engineering, RWTH Aachen University, Germany Thesis: “Development of an Android-Client for Impedance Cystovolumetry”

Professional experience

04/2024 – present	NSI-MI Technologies AMETEK, Suwanee, Georgia, USA Robotic Systems Software Engineer
01/2016 – 03/2024	Institute of High Frequency Technology, RWTH Aachen University, Germany Research Assistant

Awards

2022	Best Technical Paper Award, 3 rd place 44 th Annual Meeting and Symposium of the Antenna Measurement Techniques Association (AMTA), Denver, CO, USA.
2022	Nominated for Best Measurement Paper Award 16 th European Conference on Antennas and Propagation, Madrid, Spain.
2021	Best Student Paper Award, 3 rd place 43 rd Annual Meeting and Symposium of the Antenna Measurement Techniques Association (AMTA), Daytona Beach, FL, USA (virtual).