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Using Operating Characteristic curves to quantify the effect of quality control on geometric properties in structural members: Use case for structural reliability assessments

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Abstract

Quality control has a favourable effect on material properties since quality requirements, as conformity control criteria, encourage producers to deliver high quality products. Previous studies indicate that this effect influences the stochastic models of building material properties — and, ultimately, the safety level of structures. Typically, for every sampling plan an operation characteristics curve (OC-curve) can be derived showing how the plan performs as lots of different quality levels are submitted to it. An OC-curve plots the discriminating capacity of conformity control criteria by establishing a relationship between the probability of accepting a lot and the percent defectives. In structural engineering, the utilisation of OC-curves is well established for the control of certain variables as concrete compressive strength. However, their utilisation for the control of attributes as geometric properties is less common and often neglected. This paper investigates how OC-curves can be derived for the conformity control of geometric properties and further utilisation in structural reliability assessments. This investigation is critical to unlock the potential offered by quality control in the identification and utilisation of existing safety margins during structural design.

Keywords

Quality control, Operating characteristic, Geometric properties, Structural Reliability

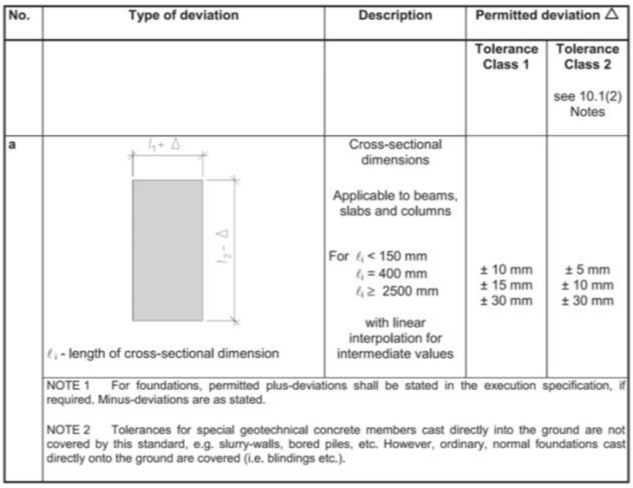
1 Introduction

To ensure the safety, cost-effectiveness, functionality, and durability in design, knowledge of component properties (e.g., concrete compressive strength and steel yield strength) and geometric properties (e.g., effective depth and other geometric dimensions such as column dimensions or slab thickness) utilised in a structure is a key requirement [1]. In this context, to conduct accurate structural reliability assessments, realistic probabilistic models for the characterisation of all relevant uncertain variables (usually known as basic variables) are needed (e.g., [1]-[4]). In principle, probabilistic models of basic variables, as those offered in reference bibliography – such as in JCSS Probabilistic Model Code [5] or, more recently, the *fib* Model Code 2020 [6] – can be utilised. Yet, the uncertainty in such models can be reduced through more up-to-date information.

In recent years, new means to reduce the uncertainty in the properties of structural components have been thoroughly investigated (e.g., [7]-[10]). Previous studies have demonstrated that the utilisation of quality control procedures enable the identification and quantification of errors in production. After quality control assessment, such procedures might play a relevant role on the outgoing properties (i.e., after being submitted to conformity control assessment) of structural components as material properties and, therefore, influencing the respective stochastic models of such properties [7]-[10]. When input from quality control is considered, Bayesian statistics offer a solid basis for the update of distributional parameters needed for the stochastic description of material and geometric properties [7]-[10]. Updating prior knowledge into a posterior belief is possible by means of direct information (e.g., using experimental test results as in [11]) or indirect information (e.g., using expert input) [9].

Then, the favourable filtering effect of quality control assessments on structural safety can be quantified by means of advanced reliability-based techniques, such as First- or Second Order Reliability Methods (Level II methods) or numerical simulations such as Crude Monte Carlo methods or Monte Carlo methods with variance reduction (Level III methods) (e.g.,[1]-[4],[7],[9],[12],[13]).

When a structural component is subjected to conformity control – as part of quality control assessment – compliance with certain criteria must be verified. Normally, such conformity control criteria are specified in codes of practice, guidelines or standards. Criteria for the control of concrete compressive strength is detailed in the European standard DIN EN 206 [14] depending on sample size provisions and on the specific production regime (i.e., initial production or continuous production). These criteria can be nationally adjusted to address country-specific provisions (e.g., DIN 1045-2 in Germany [15]). The use of conformity criteria for the quality control of reinforcing steel is mentioned in the German standard DIN 488-6 [16]. Likewise, criteria for the control of geometric properties are specified in DIN EN 13670 [17] on the grounds of distinct tolerance classes (classes 1 and 2) (Figure 1). Also, the European standard EN 13369 [18] provides criteria for the quality control of geometric properties of precast concrete components. Notwithstanding, conformity control criteria can be also found in specific harmonised European Standards (known as hEN), which are instruments for demonstrating the conformity of products with the essential requirements of the European harmonisation directives and regulations established in Construction Products Regulation (known as CPR).



No.	Type of deviation	Description	Permitted deviation Δ	
			Tolerance Class 1	Tolerance Class 2 see 10.1(2) Notes
a	$\ell_1 + \Delta$ $\ell_2 - \Delta$ ℓ_i - length of cross-sectional dimension	Cross-sectional dimensions Applicable to beams, slabs and columns For ℓ_i < 150 mm ℓ_i = 400 mm $\ell_i \geq$ 2500 mm with linear interpolation for intermediate values	± 10 mm ± 15 mm ± 30 mm	± 5 mm ± 10 mm ± 30 mm
	NOTE 1 For foundations, permitted plus-deviations shall be stated in the execution specification, if required. Minus-deviations are as stated. NOTE 2 Tolerances for special geotechnical concrete members cast directly into the ground are not covered by this standard, e.g. slurry-walls, bored piles, etc. However, ordinary, normal foundations cast directly onto the ground are covered (i.e. blindings etc.).			

Figure 1 Extract of conformity control criteria for cross-sectional properties on the basis of EN 13670 [17]

When a product is subjected to a quality control assessment, certain lots are considered *good* and others *defective* and, therefore, they are accepted or rejected, respectively [8]. To this, the discriminatory power (i.e., effectiveness) of different acceptance sampling plans shall be investigated by comparing how they perform at different levels of quality established in acceptance sampling plans. Acceptance sampling plans can be divided into two types [19]:

- *Control based on variable assessment* [20],[21]: A batch of products subjected to inspection should be considered conforming if the number of quality statistics does not exceed a qualifying number k (e.g., characteristic value for the concrete compressive strength as the 5% quantile value).
- *Control based on attribute assessment* [22],[23]: A batch of products subjected to inspection should be considered compliant if the number of non-compliant items in the tested sample does not exceed a qualifying number k (e.g., number of defective items).

When conformity control assessments are conducted, the quantification of the filtering effect is rather straightforward with the support of operating characteristics, also known as OC curves. These curves describe the discriminating capacity of the conformity control criteria considered in a specific quality control inspection [7]-[9]. An OC curve is a function $P_a(\theta)$ that illustrates the probability P_a that an inspection lot that is characterised by a certain fraction defective θ is accepted [7]-[9] (Figure 2; see also Section 2). In short, this curve is a quality control tool to ensure that (i) the quality of incoming parts satisfies certain requirements before they are released (or assembled, if it is the case), (ii) the quality of semi-finished products is acceptable before they are passed to the next production (or construction) stage or (iii) that the quality of finished products satisfies the customer's specifications before they are released to the customer [24].

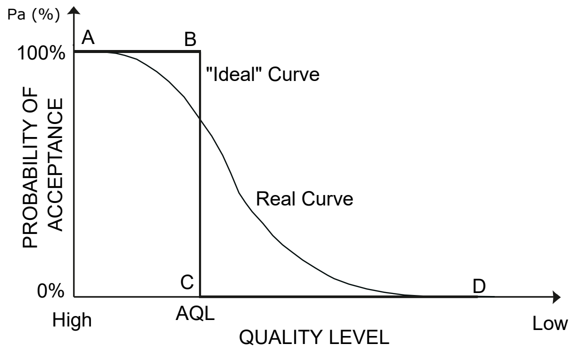


Figure 2 Illustration of OC curves for a specific sampling plan and comparison to an ideal situation (AQL: Acceptance Quality Level) (adapted from [25])

In the context of structural reliability, the work of Caspeele & Taerwe [7]-[9] demonstrates how OC curves can be utilised for the control of concrete compressive strength and to which extent this probability of acceptance affects the structural safety level. However, the utilisation of OC curves for the control of attributes, as geometric properties, is not widely covered in literature (apart from specific studies, e.g., [19],[26]) and, therefore, their potential for structural safety assessments remains to be investigated. To bridge such knowledge gap, this paper explores the potential offered through the utilisation of OC curves for a control based on attribute inspection applied to geometric properties and open avenues for further utilisation in structural safety assessments.

2 Operating characteristics for the quality control of attributes

The function $P_a(\theta)$ — described in Section 1 — is a required input for the Bayesian framework that enables the update of the incoming distribution function (into a posterior distribution function) due to quality control. Considering the Bayesian updating principle, the posterior (filtered) joint density function of the parameters of the distribution (of the material or geometric property) after conformity control is given by [7]-[9]:

$$f''_{M,\Sigma}(\mu,\sigma) = \frac{f'_{M,\Sigma}(\mu,\sigma)\cdot P_a(\mu,\sigma|x)}{\iint f'_{M,\Sigma}(\mu,\sigma)\cdot P_a(\mu,\sigma|x)\,d\mu\,d\sigma} \quad (1)$$

with $P_a(\mu,\sigma|x)$ being the probability of acceptance of a population with mean and standard deviation associated to the conformity criterion under consideration, $f'_{M,\Sigma}(\mu,\sigma)$ being the prior joint density function of the mean and standard deviation of the population and $f''_{M,\Sigma}(\mu,\sigma)$ the density function of the mean and standard deviation of the population. Numerical integration is typically adopted to solve Equation (1) [7]-[9].

According to [19], the simplest attribute assessment plan is a single plan classifying each item as *good* or *defective*. In such a plan, a batch defect is seen as $\theta = k/n$ with n being the number of samples in a lot and k being the allowable number of defective items in that lot. A single assessment plan — normally denoted as $k \parallel n$ — is a formal record specifying the size of a random sample taken from a lot at one time. To create such a plan, the acceptable quality level (AQL) that meets the inequality $\theta \leq \text{AQL}$ (Figure 2). Note that exceeding the allowable number of defective items k leads to a lot being classified as *defective* and, thus, rejected. An "ideal" OC curve would be a plan to ensure that all the lots with defectiveness $\theta \leq \text{AQL}$ are accepted and rejected when $\theta \geq \text{AQL}$. This "ideal" OC curve is represented by the lines ABCD in Figure 2, which can be only achieved if each individual production unit is subjected to conformity control or if the (hypothetical) sample is infinitely large for continuous processes in which the product cannot be discretely separated.

The probability that in a n-sample taken from a lot will have at most k defective items can be estimated through the following distribution function (Bernoulli type), which is suitable for independent samples (e.g., [19],[25],[27]):

$$P_a(\theta) = \binom{n}{k}\theta^k(1-\theta)^{n-k} \quad (2)$$

with $k = 0, 1, \dots, n$.

It should be mentioned that it is assumed that the lot size is large as compared to the sample size so that removing the sample does not significantly change the remainder of the lot, independently of the number of defects in the sample.

As a final remark, it should be noted that an attribute assessment plan should never be adopted prior to being tested for the behaviour of its OC curves. This is necessary to assure that it has the wanted characteristics [25].

3 Influence of sampling plan on OC curves

3.1 General considerations

In this sub-section, it is investigated how the probability of lot acceptance differs with variation of two parameters: number of inspected items in a batch n and acceptance number k. To this, two trends were investigated (Table 1). In Trend 1, alternative OC curves were determined by keeping n constant, while varying k. In Trend 2, alternative OC curves were estimated by keeping the acceptance number k constant and varying n.

Table 1 Effect of sampling plan on OC curves: assumptions for the parameters n and k

Trends	Sampling plans $k \parallel n$	Inspected items in a batch n	Acceptance number k
Trend 1: Constant n and variable k	0 ǁ 5	5	0
	0 ǁ 10	10	0
	0 ǁ 20	20	0
Trend 2: Variable n and constant k	0 ǁ 20	20	0
	1 ǁ 20	20	1
	2 ǁ 20	20	2

3.2 Results and discussion

The OC curves displayed in Figures 3 and 4 were determined based on Equation (2). Figure 3 illustrates a situation where the acceptance number k is kept constant with n varying as $n_1 < n_2 < n_3$. The results confirm that by keeping the acceptance number constant, the more items n are inspected, the shape of the OC curve becomes steeper and, thus, closer to what is considered ideal (i.e., when optimal control would be applicable) (see Figure 2).

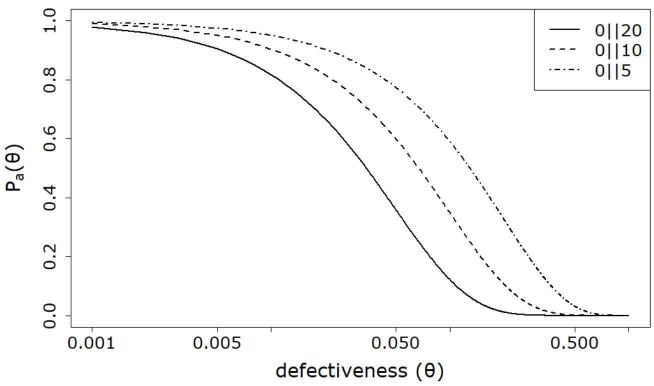


Figure 3 OC curves illustrating the sampling plans of Trend 1: Constant acceptance number, varying sample size

The reverse trend is demonstrated in Figure 4, in which the number of inspected items n is kept constant with k varying as $k_1 < k_2 < k_3$. Likewise, here the trend is also comprehensive: By keeping a constant n, the smaller the acceptance number k is, the closer the OC-curve is to an optimal control.

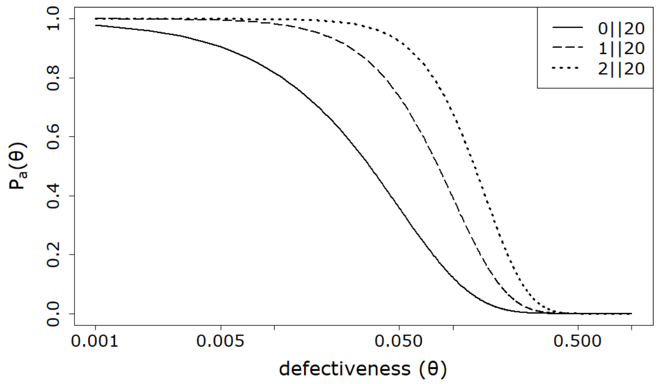


Figure 4 OC curves illustrating the sampling plans of Trend 2: Constant sample size, varying acceptance number

4 Influence of quality control on structural safety through the utilisation of OC curves

4.1 General considerations

In this sub-section, the influence of conformity control on the safety level of a structural component is assessed. In reliability studies, structural safety is typically measured through a failure probability P_f and a corresponding reliability index β (e.g., [2],[12]). For inferences regarding the level of structural safety, target reliability indices β_target are commonly established in reference structural codes such as in DIN EN 1990 [28] (e.g., reliability index $\beta_\mathrm{target} = 3.8$ for a reliability class RC 2 and a 50-year reference period).

In this investigation, the influence of conformity control on the safety level of a structural component is investigated. In this case, a flat slab without shear reinforcement subjected to punching shear was analysed, whose structural resistance R is expressed through the following limit state function $g(\vec{x})$:

$$g(\vec{x}) = \theta_\mathrm{R} \cdot v_\mathrm{Rm,c}\,(d, \rho_\mathrm{l}, f_\mathrm{c}) \cdot u_1\,(d, a) \cdot d - V_\mathrm{Rd,c} = 0 \tag{3}$$

with θ_R being a random variable to describe the model uncertainty of the design model for resistance. The term $v_\mathrm{Rm,c}\,(.)$ refers to the punching shear resistance whose key variables are affected by multiple uncertainties expressed by: d is the random variable expressing the geometric uncertainty on effective depth, a is the random variable expressing the geometric uncertainty on column dimensions, ρ_l as the random variable expressing the geometric uncertainty on the flexural reinforcement steel ratio, and f_c as the random variable expressing the uncertainty on concrete compressive strength. The term $u_1\,(.)$ refers to the critical perimeter at the distance of 2.0 d from the periphery of the loaded area.

The parameter $V_\mathrm{Rd,c}$ is the design value for punching shear resistance according to DIN EN 1992-1-1 [29]. By adopting the equation for the punching shear capacity given in DIN EN 1992-1-1 for $v_\mathrm{Rm,c}\,(.)$, and considering interior squared columns, Equation (3) can be redefined as [30]:

$$g(\vec{x}) = \theta_\mathrm{R} \cdot \left[0.18 \cdot \min\left\{2.0; 1 + \left(\frac{200}{d}\right)^{\frac{1}{2}}\right\} \cdot (100 \cdot \rho_1 \cdot f_\mathrm{c})^{\frac{1}{3}}\right] \cdot [4a + 2\pi \cdot 2d] \cdot d - V_\mathrm{Rd,c} = 0 \tag{4}$$

Note that $\vec{x} = \left(\theta_\mathrm{R}, d, a, \rho_\mathrm{l}, f_\mathrm{c}\right)^\mathrm{T}$ is the vector of realisations of the basic variables, whose stochastic models are described in Table 2.

Table 2 Stochastic models for the basic variables (based on [5] and [30])

X	Variable	Dist.	X_k	μ_X	σ_X
f_c	Concrete compressive strength (30/37); $\gamma_\mathrm{C} = 1.5$) [N/mm²]	LSt[(a)]	30	42.9	5.84
d	Effective depth [mm]	St[(b)]	d_nom	d_nom+10	10
a	Column dimensions [mm]	St[(b)]	a_nom: 350	a_nom + 0.003 a_nom	4+ 0.006 a_nom
ρ_l	Flexural reinf. steel [mm²]	N	$\rho_\mathrm{l,nom}$: 0.01	Nom.	0.02 $\rho_\mathrm{l,nom}$
θ_R	Model uncertainty for the resistance [-] based on [30]	LN	-	1.1343	0.2005

LSt: Log-Student-*t* distribution; St: Student-*t* distribution; N: Normal distribution; LN: Lognormal distribution; X_k: Nominal value; μ_X: Expected value; σ_X: Standard deviation

(a) Hyperparameters for the Log-student-*t* distribution interpolated from the values offered in **[5]** for the concrete class C30/37: $m' = 3.750$; $s' = 0.105$; $n' = 3$; $v' = 10$

(b) Hyperparameters for Student-*t* distribution determined according to the Maximum Likelihood Estimators described in [10**]** for a synthetic database of N = 10 000 normal distributed samples with sample size n = 15 (e.g., for geometric dimension a = 350 mm: $m' = 351.1$; $s' = 5.64$; $n' = 12.75$; $v' = 12.43$)

Note that for the characterisation of some basic variables, Student-*t* or Log-Student-*t* distributions were considered. These distributions are commonly utilised to estimate the population parameters of small sample sizes. For sufficiently large sample sizes, Student-*t* or Log-Student-*t* distributions tend to converge towards the ''base distribution'' (i.e., Normal or Lognormal distribution, respectively). Since the Student-*t* distribution has a higher chance for extreme values than Normal distributions, they typically have heavier (or fatter) tails (e.g., [31],[32]), which is particularly interesting for reliability-based analysis [10]. For the conformity control criteria, the provisions in DIN EN 13670 [17] were adopted namely Tolerance Class 1 from DIN EN 13670 [17] as displayed in Figure 1. Based on the discussion offered in Section 3.1., the sampling plan 0 ‖ 20 was assumed for this investigation. For the reliability-based calculation, the

First-**O**rder **R**eliability **M**ethod (FORM) was considered (Hasofer-Lind-Rackwitz-Fiessler algorithm [33]). The computational analysis was conducted in the *TesiproV* [34] — a structural reliability software package built in *R* [35]. The target value β_{target} of 3.8 was utilised for reference in this investigation.

4.2 Results and discussion

Figure 5 illustrates the influence of the conformity control criteria on the distribution of the effective depth d_{nom} equal to 200 mm by assuming two tolerances levels using the values proposed in DIN EN 13670 [17] as reference. In this case, it was considered a Tolerance Class 1 (TC 1) with a lower tolerance value of $d_{\text{nom}} - 10$ mm and a Tolerance Class 2 (TC 2) with a lower tolerance value of $d_{\text{nom}} - 5$ mm. When TC 1 is applied, the mean value of the effective depth *d* increases from an initial value of 210 mm (i.e., incoming mean value) to 210.4 mm (i.e., outgoing mean value). Likewise, due to this conformity control, the standard deviation of the effective depth *d* decreases from 10.50 mm (i.e., incoming standard deviation value) to 9.60 mm (i.e., outgoing standard deviation value).

When TC 2 is considered, the filtering effect is more prominent in comparison to the conformity control established through TC 1. With TC 2, the outgoing mean value increases to 210.9 mm and the outgoing standard deviation decreases to 9.22 mm.

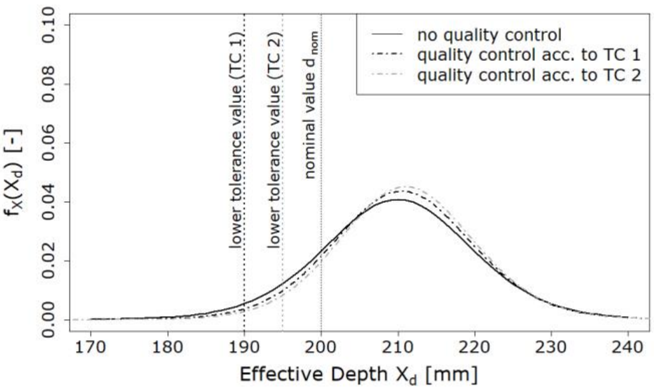


Figure 5 Influence of the investigated conformity criteria on the posterior distributions of the effective depth (d_{nom} = *200 mm*)

Figure 6 displays the variation of the reliability index β with and without considering the filtering effect of conformity control with respect to the effective depth *d*. When both tolerances are considered, the reliability indices β tend to increase over the range of values considered for the effective depth *d* with resulting β –values being always equal or above the recommended β_{target} of 3.8. When conformity control is applied to the effective depth *d*, the increase is more pronounced for small values of effective depth *d*. The positive influence of conformity assessment – regardless of the tolerance classes considered – becomes gradually inexpressive for high values of the effective depth ($d_{\text{nom}} \geq 400$ mm), with resulting β –values around 4.55. Considering the β – values for d_{nom} = *100* mm, the reliability index β increases from an initial value (i.e., without conformity control assessment) of 3.93 to 4.19 when TC 1 is applied and to 4.30 when TC 2 is considered. These increments in reliability values represent gains up to 9.4 % simply due to conformity control applied to the above-described geometric property.

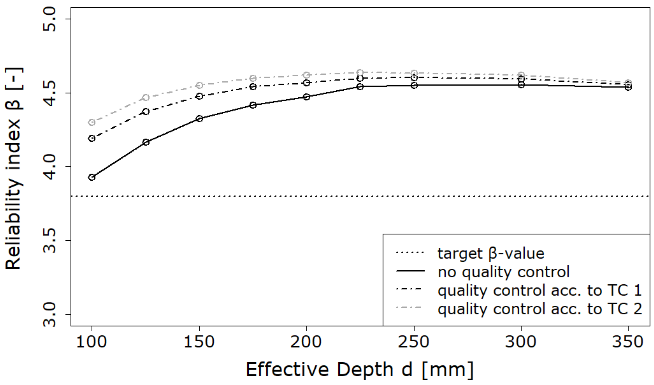


Figure 6 Reliability index β for a flat slab subject to punching shear (without shear reinforcement) with and without considering the filtering effect of conformity criteria with respect to the effective depth

It is important to highlight here that the effect of conformity control assessment on structural reliability may be different when distinct sampling plans and tolerances criteria are considered. The effect of different assumptions in terms of sampling plans and tolerance criteria can already be investigated from early stages of the design process in order to establish more efficient quality control processes and, ultimately, make use of possible safety reserves through adjusted partial actors.

5 Conclusions

The statistical inference needed for the Bayesian framework utilised to update the distributional characteristics of geometric properties due to effect of quality control assessment is addressed in this investigation, which yielded the following conclusions:

- For every accepting sampling plan (part of quality control assessment) there is a resulting OC curve that illustrates how the plan performs as lots of different quality levels are submitted to it. The characterisation of a sampling plan is, thus, critical to define key parameters as the number of items to be inspected in a lot as well as the number of defective items – and ultimately, derive suitable OC curves.
- Structural safety assessments can benefit from considering the positive influence of the filtering effect of quality control, which is quantified with support of advanced reliability-based methods such as FORM or Crude Monte Carlo simulation methods.
- The example addressed in this investigation demonstrates that after conformity control assessment both statistical parameters of the effective depth distribution (i.e., mean and standard deviation values) vary. After the consideration of tolerances on the geometric properties, the outgoing mean value increases and the outgoing standard deviation decreases.
- The outgoing distribution of the effective depth *d* after conformity control assessment introduces a positive effect on the reliability index β of the structural component under investigation. The

highest gains were attained for a slab height of 150 mm with a β –value increasing around 9.4 % after passing the conformity control assessment.

- These results suggest that additional gains might be attained when the combined effects of quality control (either on other geometric properties and/ or on other properties) are included in the structural safety assessment.
- Additionally, the proposed approach can be utilised to compare different sampling plans and conformity control criteria with respect to their effect on structural reliability.
- The results of this investigation highlight the potential of quality control for structural reliability optimisations in early design stages, where reduced partial safety factors can enable the utilisation of safety reserves without compromising the target reliability levels established in modern structural codes. With such optimisation mindset, material and resources efficiencies can be potentially attained alongside environmental and economic benefits.

Future work shall include a wider range of sampling plans and conformity control criteria applied to distinct geometric characteristics. Note that, for example, the precast sector is known for having stringent quality control requirements specifically tailored to each component type. Other failure mechanisms can also be considered. Additionally, application of the OC curves to other basic variables could be evaluated alongside their influence on structural safety and on aspects of structural durability. Finally, the influence of autocorrelation between consecutive measurements can be included in the analysis and investigated as recommended in [9].

Acknowledgments

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References

[1] Costa, L. G., & Beck, A. T. (2024) *A critical review of probabilistic live load models for buildings: Models, surveys, Eurocode statistics and reliability-based calibration.* Structural Safety 106, 102411.

[2] Melchers, R. E.; Beck, A. T. (2018) *Structural reliability analysis and prediction*. Hobeken, NL: John Wiley & Sons.

[3] Holický, M.; Retief, J. V.; Sýkora, M. (2016) *Assessment of model uncertainties for structural resistance*. Probabilistic Engineering Mechanics 45, pp. 188-197.

[4] Ricker, M.; Feiri, T.; Nille-Hauf, K.; Adam, V.; Hegger, J. (2021) *Enhanced reliability assessment of punching shear resistance models for flat slabs without shear reinforcement*. Engineering Structures 226, 111319.

[5] Joint Commitee on Structural Safety (JCSS) (2001) *JCSS Probabilistic Model Code.* Available on: https://www.jcss-lc.org.

[6] Federation Internationale du Beton (2024) *fib Model Code for Concrete Structures 2020*. Hoboken, NJ: John Wiley & Sons.

[7] Caspeele, R.; Sykora, M.; Taerwe, L. (2014) *Influence of quality control of concrete on structural reliability: assessment using a Bayesian approach*. Materials and Structures 47(1), pp. 105-116.

[8] Caspeele, R.; Taerwe, L. (2013) *Numerical Bayesian updating of prior distributions for concrete strength properties considering conformity control.* Advances in concrete construction 1(1), pp. 85-102.

[9] Caspeele, R. (2010) *Probabilistic evaluation of conformity control and the use of Bayesian updating techniques in the framework of safety analyses of concrete structures.* Doctoral Dissertation. Belgium: University of Ghent.

[10] Rackwitz, R, (1983) *Predictive distribution of strength under control.* Matériaux et Construction 16, pp. 259-267.

[11] Feiri, T.; Kuhn, S.; Wiens, U.; Ricker, M. (2023) *Updating the prior hyperparameters of concrete compressive strength through Bayesian statistics for structural reliability assessment.* Structures 58, 105636.

[12] Ellingwood, B.; Maes, M.; Bartlett, F. M.; Beck, A.T. et al. (2024) *Development of methods of structural reliability.* Structural Safety, 102474.

[13] Ricker, M.; Feiri, T.; Nille-Hauf, K. (2022) *Contribution to efficient structural safety assessments: A comparative analysis of computational schemes.* Probabilistic Engineering Mechanics 69, 103285.

[14] DIN-Deutsches Institut für Normung e.V. (2021) *DIN EN 206:2021, Concrete - Part 1: Specification, performance, production and conformity.* German version EN 206:2013+A2:2021. Berlin: Beuth Verlag.

[15] DIN-Deutsches Institut für Normung e.V. (2023) *DIN 1045-2: 2009, Tragwerke aus Beton, Stahlbeton und Spannbeton - Teil 2: Beton.* Berlin: Beuth Verlag.

[16] DIN-Deutsches Institut für Normung e.V. (2009) *DIN 488-6: 2009, Reinforcing steel – Part 6: Assessment of conformity.* Berlin: Beuth Verlag.

[17] DIN-Deutsches Institut für Normung e.V. (2011) *DIN 13670:2011, Execution of concrete structures.* German version EN 13369:2009. Berlin: Beuth Verlag.

[18] DIN-Deutsches Institut für Normung e.V. (2018) *DIN EN 13369: 2018, Common rules for precast concrete products.* German version EN 13369:2018. Berlin: Beuth Verlag.

[19] Skrzypczak, I. (2023) *Statistical quality inspection methodology in production of precast concrete*

elements. Materials 16(1), 431.

[20] ISO 3951-1 (2013) *Sampling Procedures for Inspection by Variables - Part 1: Specification for Single Sampling Plans Indexed by Acceptance Quality Limit (AQL) for Lot-by-Lot Inspection for a Single Quality Characteristic and a Single AQL*. Geneva, Switzerland: International Organization for Standardization/International Electrotechnical Commission.

[21] ISO 3951 (1997) *Sampling Procedures and Charts for Inspection by Variables for Percent Nonconforming.* Geneva, Switzerland: International Organization for Standardization/International Electrotechnical Commission.

[22] ISO 2859-0 (2002*) Sampling Procedures for Inspection by Attributes - Part 0: Introduction to the ISO 2859 Attribute Sampling System*. Geneva, Switzerland: International Organization for Standardization/International Electrotechnical Commission.

[23] ISO 2859-1 (2003) *Sampling Procedures for Inspection by Attributes - Part 1: Sampling Schemes Indexed by Acceptance Quality Limit (AQL) for Lot-by-Lot Inspection.* Geneva, Switzerland: International Organization for Standardization/International Electrotechnical Commission.

[24] Ng, S. H. (2003) *Studying the effect of the parameters of an attribute acceptance sampling plan on its operating characteristic curve: a visual approach.* Journal of Statistics Education 11(3).

[25] Chorafas, D. N. (2013) *Operating Characteristics Curves.* Quality Control Applications. Springer Series in Reliability Engineering. London: Springer-Verlag.

[26] Monteiro, A.; Gonçalves, A. (2011) *Assessment of concrete cover in structures. Part 1 – Statistical tolerance analysis approach.* In: *fib* Workshop performance-based specifications for concrete, Leipzig, Germany, pp. 220-229.

[27] Ang, A. H.; Tang, W. H. (2007) *Probability concepts in engineering planning: Emphasis on applications to civil and environmental engineering*. Hoboken, NJ: John Wiley and Sons.

[28] DIN-Deutsches Institut für Normung e.V. (2011) *DIN EN 1990:2011, Eurocode 0: Basis of structural design.* German version of EN 1990:2002 + A1:2005 + A1:2005/ AC:2010. Berlin: Beuth Verlag.

[29] DIN-Deutsches Institut für Normung e.V. (2011) DIN EN 1992-1-1:2011 + AC:2010, *Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings.* German version of EN 1992-1-1:2004 + AC:2010. Berlin: Beuth Verlag.

[30] Hegger, J.; Ricker, M.; Adam, V.; Feiri, T.; Nille-Hauf, K. (2022) *Überprüfung der Zuverlässigkeit der für die nächste Generation von EN 1992-1-1 vorgesehenen neuen Bemessungsansätze gegen Durchstanzen ohne Durchstanzbewehrung und Querkraft ohne Querkraftbewehrung.* Final Report DIBt research project (P 52-5-7.322.1-2032/18, Projektlaufzeit: 10/2018 - 01/2021, Projekt-Nr. IMB: F-2017-034, Projekt-Nr. IKI: P-18001). Available on: https://www.irbnet.de/daten/rswb/22029037458.pdd.

[31] Raiffa, H.; Schlaifer, R. et al. (1961) Applied statistical decision theory. New York: Wiley.

[32] Gelman, A.; Carlin, J.B. (2013) Bayesian data analysis. CRC Press.

[33] Rackwitz, R., Fliessler, B (1978) *Structural reliability under combined random load sequences*. Computers & Structures 9(5), pp. 489–94.

[34] Nille-Hauf, K.; Feiri, T.; Ricker, M. (2022) *TesiproV: Calculation of reliability and failure probability in civil engineering*. R Package Version 0.9.2.

[35] R Core Team, R (2013) *A Language and Environment for Statistical Computing.* R Foundation for Statistical Computing. Austria.