

Artificial Materials for Electromagnetic Wave-Front Manipulation Applied to Antenna Systems

Von der Fakultät für Elektrotechnik und Informationstechnik der
Rheinisch-Westfälischen Technischen Hochschule Aachen zur Erlangung des
akademischen Grades eines Doktors der Ingenieurwissenschaften genehmigte Dissertation

vorgelegt von
Taher Badawy, M.Sc.
aus Monofiya, Ägypten

Berichter:
Univ.-Professor Dr.-Ing. Peter Knott
Professor Miguel Beruete, Ph.D., Universidad de Navarra, Pamplona, Spanien

Tag der mündlichen Prüfung: 06.10.2025

Diese Dissertation ist auf den Internetseiten der Universitätsbibliothek online verfügbar.

Acknowledgements

I would like to express my sincere gratitude to my university supervisor, Prof. Dr.-Ing. Peter Knott, for giving me the opportunity to pursue my doctoral studies and for his guidance and support. I am also grateful to Prof. Miguel Beruete for his willingness to serve as the second examiner. Moreover, I would like to express my deep thanks to Dr.-Ing. Thomas Bertuch for his scientific and practical advice and for the discussions we had regarding this work.

I would also like to thank my colleagues at Fraunhofer FHR for their assistance: Claudius Löcker, Lukas Schuck, Carlos Galvis-Salzburg, Wasim Alshrafi, Diego Betancourt, Michael Jäger, Maik Hillmer and Alexander Kremring.

Furthermore, I am grateful to Fraunhofer FHR and especially Dr.-Ing. Frank Weinmann for providing an excellent working environment to perform my research tasks in the form of IT, simulation, and measurement facilities.

Finally, I would like to thank my wife, Aya, and my mother for their constant encouragement throughout this journey. Many thanks also go to my sister Walla and my brother Khaled, who continually motivated and supported me.

Summary

Artificial surfaces are engineered structures consisting of two-dimensional arrangements of subwavelength scattering elements that enable electromagnetic wave manipulation. They can be designed for many applications, including impedance matching surfaces, antenna beam scanning, and beam focusing lenses. With respect to the type of their interaction with incident electromagnetic waves, they can be categorized into three groups: wave guiding, transmission, and reflection.

Artificial surfaces are challenging to model and optimize using full-wave simulations as they represent a multi-scale problem. They consist of subwavelength unit cells (microscopic scale) that require dense geometrical meshes to capture all physical details. At the same time, the entire surface is electrically large (macroscopic scale). This highlights the need to develop a simple and efficient modeling method to analyze three-dimensional arbitrarily shaped artificial surfaces with any kind of polarization. In this work, we focus only on artificial surfaces that are periodic or continuous and can be considered as infinitely extended in one spatial dimension perpendicular to the plane of wave propagation.

The three-dimensional physical problem is simplified to a two-dimensional equivalent problem considering only the plane where the fields are not uniform. The subwavelength metallic patterns of the artificial surfaces are homogenized and represented by zero-thickness impedance sheets with equivalent electric currents. Thus, the two-dimensional equivalent model consists of the cross-sections of multilayer arbitrarily curved zero-thickness impedance sheets separated by dielectric layers. The fields within the dielectric regions are described by equivalent polarization currents according to the volume equivalent principle. A volume-surface integral equation is proposed to analyze the electromagnetic scattering by the two-dimensional equivalent model due to the equivalent currents. The continuous volume-surface integral equation is discretized and solved numerically using the method of moments (MoM). The MoM is formulated for both TE and TM polarized excitations. Since the problem is two-dimensional, the volume integrals of the original three-dimensional problem reduce to surface integrals over the cross-sections of the structure, and the surface integrals reduce to line integrals over the boundaries of the two-dimensional cross-sections.

To validate and evaluate the accuracy and efficiency of the proposed MoM modeling method, three illustrative examples are studied. These examples correspond to the different types of artificial surfaces as mentioned earlier and are outlined as below:

1. Guiding of waves along artificial surfaces
2. Manipulation of electromagnetic waves passing through transmissive metasurfaces
3. Manipulation of electromagnetic waves by reflective metasurfaces

Guided-wave-type artificial surfaces support the propagation of surface or leaky waves such as textured surfaces. These types of artificial surfaces exhibit many interesting and unconventional features that do not occur in nature. For example, electromagnetic bandgap (EBG) structures provide surface wave band gaps, which can be used to reduce the mutual coupling between array elements and act as ground planes to achieve low profile configurations. A printed Yagi-Uda director array is presented as an illustrative example of such artificial surfaces. It is designed to provide beam focusing and consists of 18 subwavelength director elements printed on a single dielectric substrate layer. The lengths and spacing between the director elements are tapered in the direction of wave propagation to reduce the sidelobe level meaning that the surface is non-uniform. An array of inclined slots in a rectangular hollow waveguide excites a TE_0 surface wave mode, which propagates along the surface and radiates at the end of the structure.

Transmissive-type artificial surfaces are used to control the amplitude and phase of transmitted waves enabling various applications such as transmissive reconfigurable intelligent surfaces for dynamic beam scanning and beam forming. As an illustrative example, a dual-linear polarized transmissive reconfigurable metasurface with an embedded biasing network is designed. The reconfigurable metasurface illuminated by a horizontally-polarized horn antenna designed to realize phase compensation and one-dimensional dynamic beam scanning in a single plane. The metasurface unit cell structure, with a total thickness of 1 mm, consists of three patterned metallic layers placed on dielectric substrates. The top and bottom layers consist of square patches surrounded by wire grids and behave as capacitive layers, while the middle layer is an inductive wire grid. The electromagnetic characteristics of the tunable metasurface can be controlled by applying direct current (DC) bias voltages to the varactor diodes integrated within the unit cell structure. The transmission phase of the tunable metasurface can be simultaneously controlled for both polarization, providing a phase range of up to 170° , and low insertion losses ranging from 1 dB to 4 dB at 10 GHz.

Reflective-type artificial surfaces are designed to control the amplitude and phase of reflected waves such as reflectarray antennas or reflective intelligent surfaces (RIS) for beam focusing and dynamic beam scanning. A dual-polarized reflective passive metasurface is selected from the literature as an illustrative example to validate the proposed MoM modeling method for TE polarization. Based on the unit cell structure, a reflective metasurface illuminated by a vertically-polarized horn antenna is designed to achieve beam focusing in a single plane at scanning angles 0° , 30° and 50° (i.e., -90° , -120° and -140°).

The proposed modeling method accurately predicts the radiation patterns of the illustrative examples and provides a better understanding of how these artificial surfaces function. It also simplifies the study and analysis of a wide range of artificial surfaces with complex shapes. Furthermore, the optimization process can be simplified by focusing on surface impedance parameters rather than the physical parameters, thereby reducing the computational time required for optimization.

Kurzfassung

Künstliche Oberflächen sind technische Strukturen, die aus zweidimensionalen Anordnungen von Streuteilchen im Subwellenlängenbereich bestehen und die Manipulation elektromagnetischer Wellen ermöglichen. Sie können für viele verschiedene Funktionen ausgelegt werden, darunter anomale Reflexion, Strahlschwenkung und Fokussierung. Hinsichtlich der Art ihrer Wechselwirkung mit einfallenden elektromagnetischen Wellen können sie in drei Gruppen eingeteilt werden: Wellenleitung, Transmission und Reflexion.

Wichtige Werkzeuge zur Untersuchung derartiger Materialien stellen rechnerbasierte numerische Verfahren dar, bei denen die Wellenausbreitung je nach Größe des Problems (numerisch) anhand des Lösen der Maxwell'schen Gleichungen (numerisch exakt, Vollwellensimulation) oder unter Ausnutzung asymptotischer Verfahren (Näherungsverfahren) berechnet wird. Da sich die geometrischen Abmessungen der Materialstrukturen zur Beeinflussung einer Welle im mikroskopischen Subwellenlängenbereich befinden, die Oberfläche der Gesamtstruktur für sinnvolle Anwendungen jedoch als makroskopisch anzusehen ist, stellen die simulationstechnische Aufbereitung (Modellierung) des Problems und dessen Lösung besondere Herausforderungen dar. Dies unterstreicht die Notwendigkeit, eine einfache und effektive Modellierungsmethode zu entwickeln, um beliebige dreidimensionale Objekte oder beliebig geformte Oberflächen zur Manipulation von elektromagnetischen Ausbreitungsphänomenen zu analysieren und maßgeschneiderte Oberflächen für spezielle Anwendungsfälle zu kreieren bzw. zu optimieren.

Das im allgemeinen dreidimensionale Problem wird auf eine zweidimensionale Problemstellung reduziert. Dabei wird nur die Ebene berücksichtigt, in der die elektromagnetischen Felder nicht homogen sind. Die metallischen Strukturen, deren geometrische Ausdehnungen sich weit unterhalb der sich bei der Betriebsfrequenz einstellenden Materialwellenlänge erstrecken, werden als effektiv lokal homogen betrachtet und durch einen beliebig gekrümmten Lagenaufbau von sich abwechselnden dielektrischen Schichten und Impedanzschichten repräsentiert. Jede Impedanzschicht wird durch äquivalente Ströme modelliert und hat eine senkrecht zur Oberflächennormalen gegen Null tendierende Ausdehnung. Die Felder innerhalb der dielektrischen Bereiche werden durch äquivalente Polarisationsströme nach dem Volumenäquivalenzprinzip beschrieben. Um die elektromagnetische Streuung der äquivalenten Ströme an dem Schichtenmodell zu analysieren, wird

zunächst eine Volumen-Oberflächen-Integralgleichung angesetzt. Die kontinuierliche Integralgleichung wird diskretisiert und numerisch mit der Momentenmethode (Method of Moments, MoM) gelöst. Die MoM wird sowohl für TE als auch TM polarisierte, auf das Schichtmedium einfallende Wellen formuliert. Da das betrachtete Problem zweidimensional ist, werden die Volumenintegrale des ursprünglich dreidimensionalen Problems zu Oberflächenintegralen über die Querschnittsflächen und die Oberflächenintegrale zu Linienintegralen über die Berandungen der Querschnittsflächen reduziert.

Um die Genauigkeit und Effizienz der vorgeschlagenen Modellierungsmethode zu validieren und zu bewerten, werden drei verschiedene Beispiele untersucht. Diese Beispiele entsprechen den drei bereits erwähnten Arten von künstlichen Oberflächen, wie bereits erwähnt, und werden im Folgenden beschrieben:

Guided-wave-type Künstliche Oberflächen zur Wellenleitung unterstützen die Ausbreitung von Oberflächen- oder Leckwellen. Diese Arten von Oberflächen weisen je nach Entwurf unkonventionelle Eigenschaften auf, die in der Natur nicht zu beobachten sind. Zur gezielten Reduzierung der Verkopplung von Antennen Elementen in einer Gruppenantenne können beispielsweise Oberflächen mit elektromagnetischen Bandlücken (electromagnetic band gap, EBG) eingesetzt werden, die in ihrem Frequenzspektrum Bereiche ausbilden, in denen keine Wellenausbreitung stattfindet. Ein gedrucktes Yagi-Uda-Direktor-Array wird als anschauliches Beispiel für derartige künstliche Oberflächen vorgestellt. Es ist für die Strahlfokussierung konzipiert und besteht aus 18 Subwellenlängen-Direktorelementen, die auf eine gemeinsame dielektrische Substratschicht gedruckt sind. Die Längen und Abstände zwischen den Direktionselementen sind in Richtung der Wellenausbreitung verjüngt, um die Nebenkeulen zu verringern, was bedeutet, dass die Oberfläche ungleichmäßig ist. Ein Array aus geneigten Schlitzten in einem speisenden Rechteckhohlleiter regt eine TE_0 Oberflächenwellenmode an, die sich entlang der Oberfläche ausbreitet und am Ende der Struktur abstrahlt.

Transmissive-type Zur Demonstration einer gezielten Beeinflussung der Transmissions eigenschaften einer künstlichen Oberfläche wird ein Transmitarray zur Strahlfokussierung und Strahlschwenkung vorgestellt. Die rekonfigurierbare Schicht besteht aus drei metallischen Lagen, welche durch dielektrische Substrate getrennt sind. Die außenliegenden Metallschichten bestehen aus gleichen, periodisch über die Oberfläche angeordneten Zellen. Jede Zelle ist mit einer Varactor-Diode ausgestattet, deren Kapazität über ein Gleichspannungsnetz gesteuert werden kann. Die beiden kapazitiven Schichten werden durch die mittlere Schicht, welche von induktiver Natur ist, getrennt. Je nach Wahl der Zellenspannung, welche eine Änderung der kapazitiven Last der Zelle bewirkt, ist die rekonfigurierbare Schicht in der Lage, eine linear (senkrecht oder horizontal) polarisierte Welle nach dem Durchgang durch die Schicht zu fokussieren (Linseneffekt durch Phasenkompensation) oder eine räumliche Richtwirkung zu erzeugen, die zur dynamische Strahlsteuerung (in der Azimut Ebene) genutzt werden kann.

Reflective-type künstliche Oberflächen werden entwickelt, um die Amplitude und Phase von reflektierten Wellen zu kontrollieren, wie z.B. Reflectarray-Antennen. Eine dualpolarisierte passive Metasurface wird aus der Literatur als illustratives Beispiel ausgewählt, um die vorgeschlagene MoM-Modellierungsmethode für TE-Polarisation zu validieren.

Basierend auf der gewählten Einheitszellenstruktur wird eine reflektierende Metasurface entworfen, die von einer vertikal polarisierten Hornantenne beleuchtet wird, um eine Strahlfokussierung in der Azimutalebene bei den Schwenkwinkeln 0° , 30° und 50° zu erreichen. Zur Vorhersage der Strahlungseigenschaften des betrachteten Beispiels werden die gedruckten metallischen Strukturen mit ihren im Vergleich zur Wellenlänge kleinen Abmessungen homogenisiert und durch Impedanzschichten mit infinitesimalen Dicken dargestellt. Jeder Einheitszellentyp der gesamten Oberfläche wird unter lokaler periodischer Annäherung simuliert, um die Impedanz jeder Einheitszelle auf der Grundlage eines äquivalenten Leitungsmodells zu extrahieren.

Contents

1	Introduction	1
1.1	Definition of Artificial Surfaces	1
1.2	Modeling Methods of Electromagnetic Artificial Surfaces	2
1.3	Methodology	3
1.4	Thesis Contributions and Outline	4
2	MoM Analysis of Arbitrarily Shaped Artificial Surfaces	7
2.1	Problem Description	8
2.2	Boundary Conditions	10
2.2.1	Impedance Boundary Conditions	10
2.2.2	Volume Equivalence Principle	10
2.3	Volume Surface Integral Equations	12
2.3.1	Discretization of the Two-Dimensional Equivalent Model	15
2.3.2	MoM Formulation	17
2.4	VSIEs: TE Polarization	24
2.4.1	Basis Functions	27
2.4.2	Solution Using Point Matching Method	29
2.4.3	Calculation of Far-field	32
2.5	VSIEs: TM Polarization	34
2.5.1	Basis Functions	36
2.5.2	Solution Using Galerkin's Method	43
2.5.3	Evaluation of Impedance Matrix Elements	44
2.5.4	Calculation of Far-field	47
2.6	Summary	48
3	Analysis and Modeling of Guided Waves on Artificial Surfaces	49
3.1	Design of Printed Yagi-Uda-type Director Array	51
3.2	Numerical Analysis	53
3.2.1	Homogenization Model Based on Equivalent Impedance	55
3.2.2	MoM Solution	60
3.2.3	Numerical Results	62
3.3	Summary	65

4	Analysis and Modeling of Reconfigurable Transmissive Metasurface	67
4.1	Design of Reconfigurable Transmissive Metasurface	68
4.1.1	Unit Cell Design	68
4.1.2	Lens Array Design	72
4.2	Reconfigurable Metasurface for Shaping Transmitted Fields	73
4.2.1	Phase Compensation	74
4.2.2	Beam Scanning	76
4.3	Numerical Analysis	79
4.3.1	Homogenization Model Based on Equivalent Impedance	81
4.3.2	MoM Solution	85
4.3.3	Numerical Results	86
4.4	Lens Array Measurements and Analysis	88
4.5	Summary	91
5	Analysis and Modeling of Reflective Metasurface	93
5.1	Design of Reflective Metasurface	94
5.1.1	Unit Cell Design	94
5.2	Reflective Metasurface for Shaping Reflected Fields	96
5.3	Numerical Analysis	101
5.3.1	Homogenization Model Based on Equivalent Impedance	102
5.3.2	MoM Solution	104
5.3.3	Numerical Results	104
5.4	Summary	108
6	Conclusion	109
	Bibliography	113
	List of Publications	121
	Curriculum Vitae	123

List of Figures

2.1	Illustrative example of an arbitrarily shaped artificial surface composed of multilayer subwavelength patch elements printed on dielectric layers.	8
2.2	Arbitrarily shaped two-dimensional equivalent model consists of N_{imp} impedance sheets separated by N_{die} dielectric layers. The equivalent model is illuminated by an incident electromagnetic field $\mathbf{E}^{\text{inc}}$ and generates scattered electromagnetic field $\mathbf{E}^{\text{scat}}$ due to the equivalent electric currents.	9
2.3	When the equivalent impedance sheet is illuminated by an incident field $\mathbf{E}^{\text{inc}}$, equivalent electric currents $\mathbf{J}_s$ are induced on the impedance sheet defined by the boundary C_s	11
2.4	Volume equivalence principle. (a) Cross-sectional surface of the dielectric layer. (b) The dielectric surface is removed and replaced by equivalent polarization currents $\mathbf{J}_p$	11
2.5	Discretization of the impedance sheet boundary C_s into line segments.	16
2.6	Discretization of the dielectric cross-sectional surface S_p : (a) Geometry. (b) Quadrilateral mesh. (c) Triangular mesh.	16
2.7	Two-dimensional arbitrarily shaped multilayer impedance sheets separated by dielectric layers and illuminated by either a TE plane wave or a TE cylindrical wave propagating in the xy -plane. The incident electric field has only z -component.	24
2.8	Expansion of the equivalent currents on the impedance sheet $\mathbf{J}_s$ using one-dimensional pulse basis functions.	28
2.9	Expansion of the equivalent polarization currents $\mathbf{J}_p$ on the dielectric cross-sectional surface using two-dimensional pulse basis functions.	29
2.10	Two-dimensional arbitrarily shaped multilayer impedance sheets separated by dielectric layers and illuminated by either a TM plane wave or a TM cylindrical wave propagating in the xy -plane. The incident electric field has x - and y -components.	34
2.11	Piecewise triangular basis functions. (a) Definition of the triangular expansion function [1, p. 92]. (b) Representation of the equivalent currents $\mathbf{J}_s$ using piecewise triangular basis functions.	38

List of Figures

2.12	(a) RWG basis function defined on two adjacent triangular elements $S_{n_p}^-$ and $S_{n_p}^+$ with an internal edge e_{n_p} . (b) Discretization of the cross-sectional surface of an arbitrarily shaped dielectric layer into triangular elements. . .	40
2.13	(a) Rooftop basis function defined on two adjacent quadrilateral elements $S_{n_p}^-$ and $S_{n_p}^+$. (b) Discretization of the cross-sectional surface of a flat dielectric layer into quadrilateral elements.	42
3.1	Vertical cross-sectional view of the proposed dual-band antenna for maritime harmonic radar, consisting of S-band and C-band antennas.	50
3.2	S-band antenna configuration. (a) Vertical cross-sectional view. (b) Single layer printed Yagi-Uda-type director array with its geometrical dimensions.	53
3.3	S-band antenna configuration. (a) Vertical cross-sectional view (physical model). (b) Two-dimensional equivalent impedance model with equivalent electric currents.	54
3.4	Surface impedance homogenization of the 18 director elements. (a) Single layer printed Yagi-Uda-type director array. (b) Equivalent impedance sheets represented by different colors.	55
3.5	Electric and magnetic field distribution of the dominant TE_0 surface wave mode.	56
3.6	Eigen mode solver setup with periodic boundary conditions (PBCs) in $\pm x$ -direction and perfect electric conductor (PEC) boundaries in $\pm z$ -direction. In the yz -plane, the top and bottom planes are perfect magnetic boundaries (PMC) with a height of 50 mm relative to the director element surface. . .	56
3.7	Illustrative example of the dispersion diagram of the dominant TE_0 surface wave mode for a uniform director array.	57
3.8	Normalized propagation constant (β_x) of the proposed non-uniform tapered Yagi-Uda-type director array at 2.925 GHz.	58
3.9	Transverse equivalent network for determining a relationship between the propagation constant (β_x) of the TE_0 surface wave and the equivalent sheet impedance Z_{sheet}	58
3.10	Extracted impedance values Z_{sheet} of the 18 director elements and their normalized propagation constant at 2.925 GHz.	59
3.11	Discretization of the two-dimensional equivalent model into line segments for MoM solution. The equivalent currents are expanded using pulse basis functions.	61
3.12	Normalized equivalent electric currents distribution $J_{z,s}$ on the non-uniform impedance sheets at 2.925 GHz.	63
3.13	Real part of the total near-field obtained from the MoM modeling method. (a) without printed Yagi-Uda-type director array. (b) with printed Yagi-Uda-type director array. The red circle in both figures represent the electric line source.	63
3.14	3D simulation model of two identical inclined slots with electric walls in $\pm z$ -direction. The included slots excite TE_0 surface wave mode propagates along the surface in the x -direction.	64

3.15	Comparison between the radiation patterns obtained from full-wave simulation and the proposed MoM method at 2.925 GHz.	65
4.1	Structure of the considered metasurface unit cell. (a) 3-D perspective view of four unit cells including varactor diodes. (b) Geometrical dimensions of the top and middle metallic layers of single unit cell. (c) Cross-sectional and front view of four unit cells indicating plated through vias. The four vias placed at the centers of the square patches connect the top and bottom metallic layers to the middle metalization layer, while the via in the center connects the top and bottom metallic layers. The red strips show the cathode side of the varactor diodes.	69
4.2	Schematic representation of the varactor diodes located on the top layer. .	70
4.3	Simulation model of the reconfigurable transmissive metasurface unit cell with Floquet boundary conditions.	71
4.4	Simulated transmission performance of the metasurface unit cell at normal incidence. (a) Transmission amplitude. (b) Transmission phase.	71
4.5	Proposed reconfigurable transmissive metasurface. (a) Top layer including 16 vertical modules. (b) Middle layer including the DC biasing lines. . . .	72
4.6	3D simulation model of the reconfigurable transmissive metasurface with magnetic boundary conditions (PMC) and illuminated by a horizontally-polarized horn antenna. A waveguide port highlighted by red is used to excite the horn antenna.	73
4.7	Illustration of phase compensation performed by the reconfigurable transmissive metasurface. The metasurface consists of N unit cells and the different colored line segments represent the metasurface unit cells. Each unit cell introduces a certain phase shift to manipulate the phase of transmitted wave.	74
4.8	Extracted capacitance values C_{var} required to achieve phase compensation at 10 GHz.	75
4.9	Simulated radiation patterns in the azimuth plane: reconfigurable transmissive metasurface with phase compensation compared to a horizontally-polarized horn antenna at 10 GHz.	76
4.10	Illustration of beam scanning at (θ_0) performed by the reconfigurable transmissive metasurface. The metasurface consists of N unit cells and the colored line segments represent different multilayer metasurface unit cells. Each metasurface introduces a certain phase shift to the manipulate the phase of transmitted wave.	77
4.11	Capacitance values of the metasurface unit cells required to achieve beam scanning at -20°	78
4.12	Simulated radiation patterns in the azimuth plane for beam steering at -20° : reconfigurable transmissive metasurface compared to a horizontally-polarized horn antenna at 10 GHz.	78
4.13	Two-dimensional equivalent impedance model of the reconfigurable multilayer transmissive metasurface.	80

List of Figures

4.14	Surface impedance homogenization. (a) Unit cell structure. (b) Equivalent impedance unit cell.	81
4.15	Two-port network representation of the multilayer metasurface unit cell structure.	81
4.16	Simulated S-parameters of the reconfigurable metasurface unit cell under TM excitation at 10 GHz for different incident angles.	84
4.17	Extracted sheet impedance values ($\Omega/\square$) under TM excitation at 10 GHz for different incident angles.	84
4.18	Discretization of the two-dimensional equivalent impedance model including the non-uniform impedance sheets, the dielectric substrate layers and the horn antenna.	85
4.19	Extracted reactive sheet impedance values for phase compensation at 10 GHz: (a) $Z_1 = Z_3$, (b) Z_2	86
4.20	Comparison of radiation patterns obtained from full-wave simulation and MoM for phase compensation at 10 GHz.	87
4.21	Extracted reactive sheet impedance values for beam scanning at -20° and 10 GHz: (a) $Z_1 = Z_3$, (b) Z_2	87
4.22	Comparison of radiation patterns obtained from full-wave simulation and MoM for beam scanning case at -20°	88
4.23	A scenario of using the proposed tunable metasurface as a transmissive reconfigurable intelligent surface (T-RIS).	89
4.24	Measurement setup of the fabricated metasurface together with the horn antenna. (a) Tunable metasurface illuminated by the horn. (b) Horizontally polarized horn antenna without the tunable metasurface.	90
4.25	Measured gain patterns in dB of the reconfigurable metasurface illuminated by the horn antenna for different scanning angles compared to the gain pattern of the horn antenna at 9.8 GHz in the horizontal plane.	90
4.26	Comparison between the measured and predicted radiation patterns for phase compensation at 10 GHz.	91
5.1	Geometry of the reflective metasurface unit cell designed in [2]. (a) Top view. (b) Cross-sectional view.	94
5.2	Simulation model of the reflective metasurface unit cell with Floquet PBCs.	95
5.3	Simulated reflection phase at 12 GHz for different incident angles (θ_{inc}).	95
5.4	Reflective metasurface illuminated by a horn antenna positioned at a Distance D from the metasurface.	96
5.5	3D simulation model of the reflective metasurface illuminated by a vertically-polarized horn antenna, with electric boundary conditions applied in the $\pm z$ -direction.	97
5.6	Beam focusing in the azimuth plane at 12 GHz for a scanning angle 0° (-90°). (a) Required phase distribution. (b) Geometrical dimensions L_d unit cells. (c) Simulated radiation pattern.	98
5.7	Beam focusing in the azimuth plane at 12 GHz for a scanning angle 30° (-120°). (a) Required phase distribution. (b) Geometrical dimensions L_d unit cells. (c) Simulated radiation pattern.	99

5.8	Beam focusing in the azimuth plane at 12 GHz for a scanning angle 50° (-140°). (a) Required phase distribution. (b) Geometrical dimensions L_d unit cells. (c) Simulated radiation pattern.	100
5.9	Two-dimensional equivalent impedance model of the considered reflective metasurface with equivalent electric currents.	101
5.10	Surface impedance homogenization. (a) Reflective metasurface. (b) Equivalent transmission line model.	103
5.11	Extracted equivalent sheet impedance Z_{sheet} for TE excitation at 12 GHz. .	104
5.12	Discretization of the two-dimensional equivalent impedance model including the non-uniform impedance sheets, the dielectric substrate layer, the ground plane and the horn antenna.	105
5.13	Extracted sheet impedance values for beam focusing at scanning angles: (a) 0° , (b) 30° , (c) 50°	106
5.14	Comparison between commercial simulation software (CST) and MoM for beam focusing at 12 GHz at scanning angles of (a) 0° , (b) 30° , and (c) 50° . .	107

List of Tables

3.1	Center-to-center spacing between directors [mm].	52
3.2	Lengths of director elements [mm].	52
3.3	Radiation characteristics of the printed Yagi-Uda-type director array at 2.925 GHz.	64
4.1	Geometrical dimensions of the metasurface unit cell structure [mm].	68
4.2	Radiation characteristics for phase compensation case at 10 GHz.	87
4.3	Radiation characteristics for beam scanning at -20°.	88
5.1	Geometrical dimensions of the unit cell structure [mm].	94
5.2	Radiation characteristics of the reflective metasurface [0°].	108
5.3	Radiation characteristics of the reflective metasurface [30°].	108
5.4	Radiation characteristics of the reflective metasurface [50°].	108
6.1	Comparison of the proposed MoM modeling method and commercial EM solver (CST Studio Suite) in CPU time and number of unknowns.	110

Chapter 1

Introduction

With the increasing use of wireless technologies, many engineering applications such as satellite communications, 5G wireless communications, and radar systems pose certain electrical and mechanical specifications on their antennas such as high gain, multi-beam or wide-angle beam scanning with compact and low profile design [3–6].

Traditionally, high gain antennas have been realized using conventional approaches such as reflectors or dielectric lenses [4, 7]. However, these designs are bulky and heavy. On the other hand, phased array antennas are widely employed to generate highly directional radiation patterns by electronically controlling discrete antenna elements. Nevertheless, they require complex feeding networks and expensive T/R modules [4, 7–9]. As electronic beam scanning using conventional approaches becomes increasingly complex, current research focuses on engineered artificial surfaces that can provide electromagnetic wave manipulation [6]. Artificial surfaces have a wide range of potential applications in electromagnetics at frequencies from microwave to optical, including anomalous reflection [10–13], absorbers, focusing lenses [14, 15], beam steering antennas [16–18], radar cross section reduction [18], wide-angle impedance matching [19, 20], and phase shifters [7, 21].

1.1 Definition of Artificial Surfaces

Metamaterials are engineered structures consisting of three-dimensional arrangements of unit cells designed to manipulate electromagnetic waves by changing their amplitude, phase, and polarization [7, 22]. They possess unique properties such as negative refraction [7]. However, they became less attractive due to their fabrication complexity and losses [22]. Now, the focus lies more on two-dimensional artificial surfaces such as metasurfaces and high impedance surfaces. Artificial surfaces are two-dimensional arrays of engineered subwavelength elements designed to control the amplitude, phase, and polarization of reflected, transmitted, and guided waves [15, 23]. They have several advantages, including low profile, low losses, and ease of fabrication [7].

The electromagnetic properties of passive artificial surfaces are fixed once they are fabricated. Consequently, they cannot meet the increasing demand for multifunctional devices [24]. Reconfigurable artificial surfaces have attracted great attention due to their capability to control electromagnetic waves dynamically. This is achieved by integrating reconfigurable elements into the unit cells such as varactor diodes [16, 17, 25], micromechanical systems (MEMS) [8, 25, 26], and liquid crystals (LCs) [25, 27, 28]. They have several applications such as dynamic beam scanning [16, 17, 29]. For example, wide-angle beam steering based on a reconfigurable conformal metasurface lens is demonstrated in [16]. The proposed active conformal metasurface lens transforms the incident spherical wavefront radiated by patch antenna elements into a planar wavefront with a predefined scanning direction providing a one-dimensional beam scanning of $\pm 60^\circ$.

Artificial surfaces can be categorized into three types based on their functionality: guided-wave-type [30–32], transmissive-type [21, 33], and reflective-type [12–14].

1.2 Modeling Methods of Electromagnetic Artificial Surfaces

The analysis of artificial surfaces is essential in the design and optimization process, as it enables to study and predict the performance of artificial surfaces [7, p. 133]. While analytical methods provide basic insight into the electromagnetic behavior of artificial surfaces, full-wave simulations are required to model complex structures and achieve high accuracy. Moreover, to capture the small physical details of artificial surface unit cells, numerical methods require very dense geometrical meshes of the subwavelength elements, which increase the computational complexity. On the other hand, optimization is necessary to improve the performance by tuning the design parameters of the unit cells. However, the optimization process of such complex structures presents challenges due to the complexity and the large number of design variables in the optimization process [24, 34]. This highlights the need to develop a simple and efficient numerical method to analyze and optimize artificial surfaces.

Periodic boundary conditions are widely used to simplify the analysis and characterization of artificial surfaces [7, p. 165], [13, 35]. In this method, the local scattering parameters of each unit cell of the artificial surface are determined by assuming the unit cell in a periodic environment of identical unit cells [7, p. 151], [13, 36–40]. While this method determines the radiation characteristics of artificial surfaces, it does not accurately model the mutual coupling between elements, particularly of non-uniform artificial surfaces, where the adjacent elements are different [7, p. 165], [13, 41].

Due to the subwavelength arrangements of artificial surfaces, they can be effectively modeled and homogenized as zero-thickness impedance sheets that create a discontinuity in the electromagnetic fields [7, pp. 37–42], [42–44]. By replacing the complex physical artificial surface with a much simpler sheet structure, the complexity of the mesh generation is simplified and the number of unknowns is reduced. Moreover, the optimization process is

simplified by focusing on the effective surface impedance parameters rather than geometrical parameters. This is essential in the optimization process, particularly in reconfigurable artificial surfaces where multiple simulations are required [23, 34, 45].

Much efforts have been recently dedicated to the analysis of artificial surfaces [13, 34, 41, 46–50]. A volume-surface integral equation solved by method of moments (MoM) is an effective approach to predict the radiation characteristics of artificial surfaces [13, 41]. The surface integral method offers advantages over other three-dimensional numerical methods like the finite element method (FEM) or finite-differences time-domain (FDTD) because it does not require meshing the entire problem space [34, 45]. Also, it can accurately model the mutual coupling between adjacent elements. For example, this approach is used in [13, 51] to design and model a non-uniform reflectarray for beam focusing. Similarly, [46, 52] demonstrates an array of printed metallic wires periodically loaded with subwavelength reactive elements, such as printed capacitors for wide-angle beamforming up to 60° and beam splitting. In [47], this method is applied to design and optimize conformal metasurfaces for beamforming. In [41, 53, 54], a dual-band stacked reflective-type metasurface is designed based on surface impedance homogenization to achieve beam focusing.

The computational studies found in the literature have been limited to the analysis of artificial surfaces under TE_z excitation only (i.e., the electric field of the incident wave is oriented along the z -direction perpendicular to the modeling plane of incidence). However, practical applications require a diversity of polarization. For example, dual-polarized reconfigurable transmissive or reflective intelligent surfaces are widely used in wireless communications to improve channel capacity [55]. This highlights the need to extend the current computational techniques for analyzing three-dimensional, arbitrarily shaped artificial surfaces to handle both TE_z and TM_z (i.e., the magnetic field of the incident wave is oriented along the z -direction perpendicular to the modeling plane of incidence) excitations.

1.3 Methodology

An efficient analysis method of three-dimensional electromagnetic scattering problems is presented where the considered geometry and excitation are invariant or effectively periodic with respect to one spatial dimension, and wave propagation occurs in the plane perpendicular to that direction only. In that plane of propagation, the scattering problem is reduced to a two-dimensional one. In contrast to a full three-dimensional problem where, at a given single frequency, the electromagnetic field can be expressed as a superposition of spherical wave functions, in the considered two-dimensional case, the field is expressed with the aid of cylindrical wave functions. Even though the scattered far-field magnitudes for three-dimensional finite problems exhibit a radial dependency with the factor $\frac{1}{kr}$ which in the two-dimensional case becomes $\frac{1}{\sqrt{kr}}$, in both cases, the field phases are expressed with the factor e^{-jkr} . Since in both cases, for an efficient scattered far-field computation, the radial behavior of the magnitude is neglected, the far-field determined by the proposed

two-dimensional analysis are meaningful although the real world geometry might not be really infinite along the direction perpendicular to the propagation plane.

To further simplify the problem and reduce the computational complexity, the printed metallic patterns of the artificial surfaces are homogenized due to the subwavelength periodicity of the unit cells and represented by zero-thickness uniform impedance sheets. A volume-surface integral equation is used to determine the electromagnetic scattering by the two-dimensional arbitrarily shaped impedance sheets. The continuous volume-surface integral equation is discretized and solved numerically using the MoM for both TE and TM polarized excitations.

1.4 Thesis Contributions and Outline

The main contributions of the PhD thesis are summarized as follows:

1. MoM modeling method:

- Development of a simple and efficient MoM modeling method to analyze artificial surfaces for both TE and TM polarized incident fields
- Capable of handling three-dimensional uniform and non-uniform artificial surfaces that are periodic or invariant in one spatial dimension
- Homogenization impedance models developed to determine the effective surface impedance for:
 - Wave guiding artificial surfaces
 - Multilayer transmissive reconfigurable artificial surfaces
 - Single layer reflective artificial surfaces

2. Design of novel artificial surfaces for electromagnetic wavefront manipulation:

- Design of a novel printed Yagi-Uda-type director array to shape the radiation pattern of a slotted waveguide antenna in the plane perpendicular to the waveguide axis at S-band
- Design, fabrication, and measurement of a low-profile dual-linear-polarized reconfigurable transmissive metasurface with an embedded biasing network at X-band, capable of:
 - Beam focusing (i.e., phase compensation)
 - One-dimensional beam scanning in a single plane

The thesis organization is as follows. **Chapter two** begins with the problem definition of the three-dimensional arbitrarily shaped artificial surface. A simple two-dimensional arbitrarily curved impedance model equivalent to the three-dimensional physical problem is developed. The mathematical formulation of the volume-surface integral equation is derived to determine the scattering by the two-dimensional equivalent impedance model. The chapter is then divided into two parts, where each part focuses on the solution of the volume-surface integral equation using the MoM for TE_z and TM_z polarized excitations. For TE_z incident wave, the equivalent electric surface currents on the impedance sheets are expanded into one-dimensional pulse basis functions, while the equivalent polarization currents within the dielectric layers are expanded into two-dimensional pulse basis functions. Point matching is used to solve the discretized volume-surface integral equation to form a system of linear equations.

For TM_z incident wave, the equivalent electric surface currents on the impedance sheets are expanded into one-dimensional piecewise triangular basis functions. The equivalent polarization currents within the dielectric layers are expanded using rooftop basis functions for planar configurations and RWG basis functions for curved dielectric layers. Galerkin's method is used to solve the discretized volume-surface integral equation to form a system of linear equations.

Chapter three focuses on the analysis and modeling of a printed Yagi-Uda-type director array designed for wavefront shaping using the proposed modeling method. The printed Yagi-Uda-type director array consists of 18 subwavelength director elements printed on a single dielectric substrate layer and is periodic in one spatial dimension. The lengths and spacing between the director elements are tapered in the direction of wave propagation to reduce the sidelobe level (SLL). An inclined-slot array excites a TE_0 surface wave mode, which propagates along the surface and radiates at the end of the structure. To analyze and predict the radiation characteristics of the printed Yagi-Uda-type director array using the proposed MoM modeling method, a two-dimensional equivalent model is constructed, where the subwavelength director elements are homogenized and replaced with zero-thickness impedance sheets. The original source is replaced by an infinite electric line source. To extract the equivalent impedances representing the director elements, each unit cell forming the entire surface is simulated under local periodic approximation and eigenmode analysis is performed to determine the phase constant of the dominant TE_0 surface wave mode. A dispersion relation, based on a transverse resonance method, is used to construct a relationship between the phase constant of the TE_0 surface wave mode and the equivalent sheet impedance. Once the director elements are homogenized and their equivalent surface impedances are determined, the MoM matrix is constructed. The unknown equivalent currents on the impedance sheets are then determined, and the far-field radiation pattern is calculated and compared with full-wave simulations.

Chapter four focuses on the analysis and modeling of a transmissive reconfigurable metasurface. For this purpose, a multilayer reconfigurable metasurface unit cell structure is designed and consists of three metallic patterns separated by two dielectric substrates. The transmission phase response of the proposed metasurface is controlled by surface-mounted varactor diodes placed on the top and bottom metallic layers, providing a transmission

1 Introduction

phase range of 170° at 10 GHz. To test the concept, a planar metasurface configuration is constructed using the designed reconfigurable metasurface unit cells and illuminated by a horizontally-polarized flared horn antenna positioned at a certain distance from the metasurface. The planar metasurface configuration is divided into 16 vertical modules, each consisting of an array of 2×10 metasurface unit cells. The transmission phase of each module can be individually controlled by applying a specific DC bias voltage. The main functionality of the proposed reconfigurable metasurface is to transform the incident spherical wavefront from the horn antenna into a planar wavefront. The subwavelength metallic patterns are homogenized and represented by zero-thickness impedance sheets, while the dielectric layers are replaced by equivalent polarization electric currents according to the volume equivalence principle. A magnetic line source oriented along the z -axis is used to illuminate the metasurface, radiating TM_z cylindrical waves ($\hat{\mathbf{z}}$ -directed H-field) in the xy -plane. The metal sheets forming the horn are replaced by equivalent electric surface currents. To accurately predict the radiation characteristics of the reconfigurable metasurface, the equivalent impedance of each metallic layer is extracted based on a two-port network representation of the metasurface unit cell structure. The unknown equivalent currents on the impedance sheets are then determined and the far-field radiation pattern is calculated and compared with full-wave simulations for beam focusing and beam scanning applications.

Chapter five focuses on the analysis and modeling of a reflective-type metasurface. The metasurface unit cell structure is selected from literature [2] as an illustrative example and consists of a subwavelength metallic pattern (Jerusalem cross-shaped unit cell) printed on a grounded dielectric substrate. The reflection phase response is controlled by changing the geometrical dimensions of the unit cell with a 360° degree phase range at 12 GHz. A non-uniform reflective metasurface is designed based on the unit cell structure to convert the incident spherical wavefront radiated by a vertically-polarized horn antenna to a focused beam at scanning angles 0° , 30° , and 50° . A two-dimensional impedance model is constructed to predict the radiation characteristics of the non-uniform reflective metasurface. An infinite electric line source oriented along the z -axis is used to replace the original source. It radiates TE_z cylindrical waves ($\hat{\mathbf{z}}$ -directed E-field) in the xy -plane. An equivalent transmission line model is developed to extract equivalent sheet impedance. Once the non-uniform sheet impedances are determined, the MoM matrix is constructed. The unknown equivalent currents are then determined and the far-field radiation pattern is calculated and compared with full-wave simulations for beam focusing at 0° , 30° and 50° , respectively. Finally, **chapter six** presents the conclusion.

Chapter 2

MoM Analysis of Arbitrarily Shaped Artificial Surfaces

This chapter focuses on the mathematical formulation of the MoM modeling method developed to model three-dimensional arbitrarily shaped artificial surfaces. The considered artificial surfaces consist of discrete subwavelength metallic patterns printed on dielectric substrate layers and illuminated by a specified excitation source. These artificial surfaces show periodicity in one spatial dimension and can be either uniform or non-uniform in the other dimension within the modeling plane where the wave propagates. To simplify the analysis, the subwavelength metallic patterns are homogenized and represented by zero-thickness uniform sheet impedances and equivalent electric surface currents. The dielectric layers are removed and replaced by equivalent polarization currents using the volume equivalence principle. Within the plane of wave propagation, the scattering problem is reduced to a two-dimensional one. The focus is now on a simple two-dimensional model that are equivalent to the three-dimensional physical problem. The scattering by the two-dimensional equivalent impedance model is described by volume surface integral equations (VSIEs), where the continuous VSIE is transformed into a discrete problem by line and surface discretization and the definition of a set of deterministic basis functions with unknown amplitudes for the equivalent surface and polarization currents.

The chapter is divided into three parts. Section 2.1 begins with the problem definition of three-dimensional arbitrarily shaped artificial surfaces and the construction of the two-dimensional equivalent impedance model and the mathematical formulation of VSIEs. Section 2.4 describes the solution of the VSIEs using the MoM for TE. Point matching is used to solve the discretized VSIEs to form a system of linear equations. Section 2.5 explains the solution of the VSIEs using the MoM for TM. Galerkin's method is used to solve the discretized VSIEs.

2.1 Problem Description

Consider an arbitrarily shaped artificial surface composed of N layers of arbitrarily shaped metallic patterns such as simple patch elements [7, p. 156]. These metallic patterns are separated by dielectric layers, where each dielectric layer is characterized by its thickness and relative permittivity, as illustrated in Fig. 2.1. The considered surface is periodic along the z -direction and shows only an electric response, with equivalent magnetic currents assumed to be zero. This assumption means that the tangential electric field remains continuous across the artificial surface interfaces and only electric currents are considered.

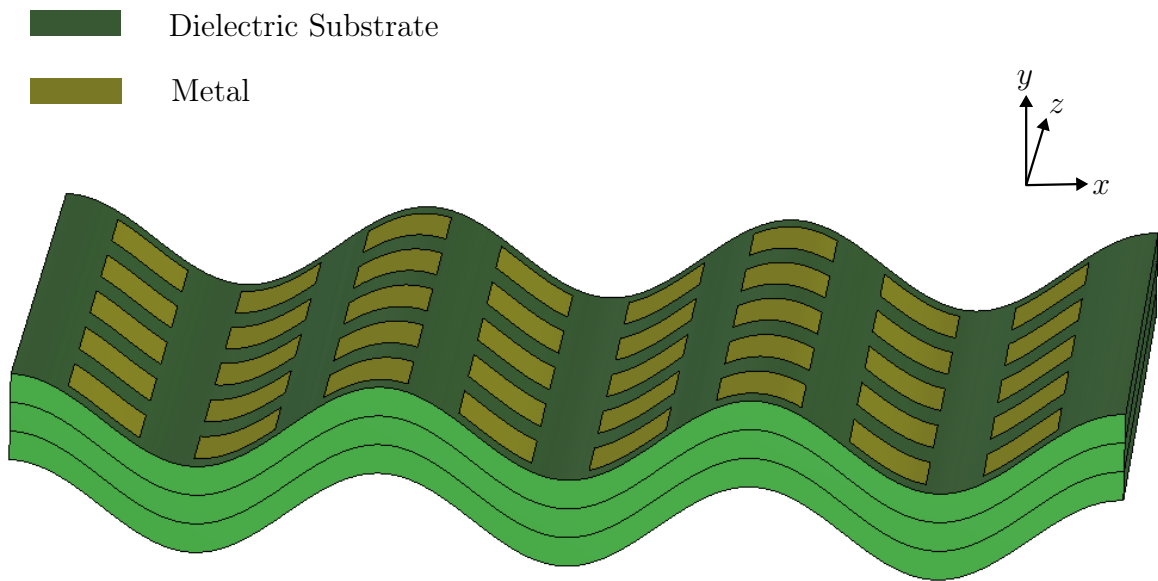


Fig. 2.1: Illustrative example of an arbitrarily shaped artificial surface composed of multilayer subwavelength patch elements printed on dielectric layers.

Due to the subwavelength nature of the printed metallic patterns forming the artificial surface, they can be effectively modeled by replacing the subwavelength metallic patterns with zero-thickness uniform impedance sheets through surface impedance homogenization [7, pp. 43–46] and equivalent surface electric currents, thereby simplifying the modeling and analysis of such surfaces. On the other hand, since the geometry of the artificial surface and the electromagnetic field are periodic in the z -direction, it can be considered as a two-dimensional scattering problem with the xy -plane being the propagation plane. Fig. 2.2 shows a two-dimensional model equivalent to the physical artificial surface, consisting of N_{imp} arbitrarily shaped impedance sheets separated by N_{die} dielectric layers.

An incident electromagnetic wave characterized by electric and magnetic fields $\{\mathbf{E}^{\text{inc}}, \mathbf{H}^{\text{inc}}\}$ illuminates the two-dimensional arbitrarily shaped multilayer impedance model. The associated parameters shown in Fig. 2.2 are defined as follows:

- Subscript s denotes zero-thickness impedance sheets and subscript p denotes cross-sectional dielectric layers
- N_{die} : Total number of dielectric layers
- $\epsilon_{r,q}$: Permittivity of the q -th dielectric layer
- d_q : Thickness of the q -th dielectric layer
- $S_{p,q}$: Cross-sectional surface of the q -th dielectric layer in the xy -plane
- N_{imp} : Total number of zero-thickness impedance sheets
- $Z_{s,i}$: Equivalent impedance assigned to the i -th zero-thickness sheet, defined by the boundary curve $C_{s,i}$

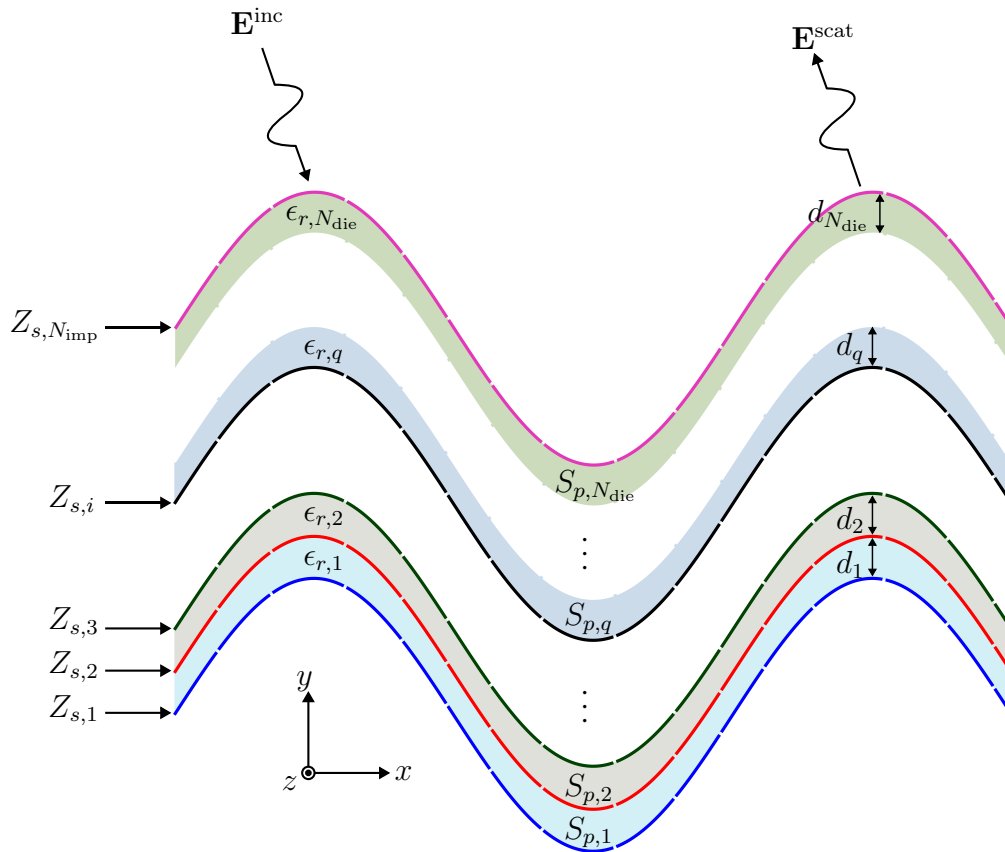


Fig. 2.2: Arbitrarily shaped two-dimensional equivalent model consists of N_{imp} impedance sheets separated by N_{die} dielectric layers. The equivalent model is illuminated by an incident electromagnetic field $\mathbf{E}^{\text{inc}}$ and generates scattered electromagnetic field $\mathbf{E}^{\text{scat}}$ due to the equivalent electric currents.

2.2 Boundary Conditions

In this section, we discuss the boundary conditions enforced on the equivalent impedance model. VSIEs are then formulated by applying the surface and volume equivalence principles, defining the equivalent electric currents on the impedance sheets and the equivalent polarization electric currents on the cross-sectional dielectric substrate layers.

2.2.1 Impedance Boundary Conditions

The equivalent electric currents $\mathbf{J}_s$ on the impedance layer satisfy the discontinuity of the tangential magnetic field on the both sides of the impedance layer, according to the following boundary condition [56], [57, p. 23]:

$$\mathbf{J}_s = \hat{\mathbf{n}} \times (\mathbf{H}_2(\mathbf{r}) - \mathbf{H}_1(\mathbf{r})), \quad (2.2.1)$$

where $\mathbf{H}_2(\mathbf{r})$ and $\mathbf{H}_1(\mathbf{r})$ represent the magnetic fields above and below the impedance layer, respectively. $\hat{\mathbf{n}}$ is the outward unit vector normal to the equivalent impedance sheet pointing from region 1 to region 2, as seen in Fig. 2.3, and $\times$ denotes the vector cross product. The equivalent electric currents on the impedance layer are related to the total electric field through electric surface impedance [56], [57, p. 23]:

$$\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \mathbf{E}^{\text{tot}}(\mathbf{r})) = -Z_s(\mathbf{r})\mathbf{J}_s(\mathbf{r}) \quad \mathbf{r} \in C_s, \quad (2.2.2)$$

where C_s is the contour of the arbitrarily shaped impedance sheet Z_s . The term $\mathbf{E}^{\text{tot}}(\mathbf{r})$ represents the total electric field.

2.2.2 Volume Equivalence Principle

The boundary condition enforced on the dielectric layer is expressed according to the volume equivalence principle [57, pp. 15–17], in which the dielectric layer is removed and replaced by equivalent electric polarization currents $\mathbf{J}_p$, as illustrated in Fig. 2.4. This boundary condition is expressed as [57, pp. 15–17], [58, 59]:

$$\mathbf{J}_p(\mathbf{r}) = j\omega\epsilon_0(\epsilon_r - 1)\mathbf{E}^{\text{tot}}(\mathbf{r}) \quad \mathbf{r} \in S_p, \quad (2.2.3)$$

where ϵ_0 and μ_0 represent the permittivity and permeability of free-space, respectively. ϵ_r is the relative permittivity of the substrate layer and S_p denotes the cross-sectional surface of the dielectric layer:

$$\mathbf{E}^{\text{tot}}(\mathbf{r}) = Z_p(\mathbf{r})\mathbf{J}_p(\mathbf{r}). \quad (2.2.4)$$

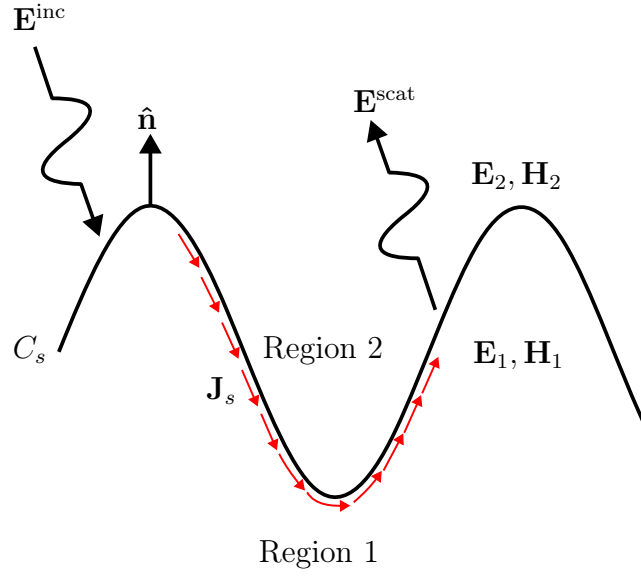


Fig. 2.3: When the equivalent impedance sheet is illuminated by an incident field $\mathbf{E}^{\text{inc}}$, equivalent electric currents $\mathbf{J}_s$ are induced on the impedance sheet defined by the boundary C_s .

The equivalent impedance Z_p due to the equivalent polarization currents is given by:

$$Z_p = \frac{1}{j\omega\epsilon_0(\epsilon_r - 1)}. \quad (2.2.5)$$

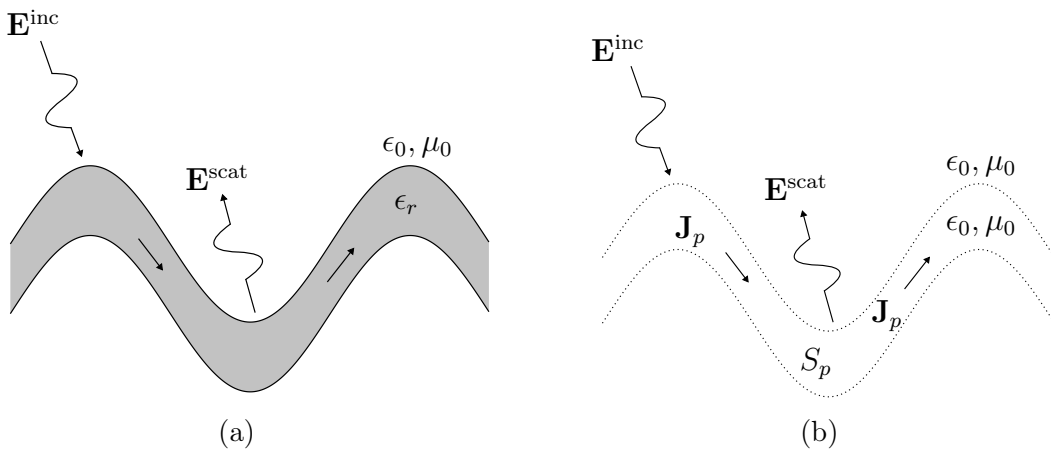


Fig. 2.4: Volume equivalence principle. (a) Cross-sectional surface of the dielectric layer. (b) The dielectric surface is removed and replaced by equivalent polarization currents $\mathbf{J}_p$.

2.3 Volume Surface Integral Equations

When the two-dimensional equivalent model representing the multilayer artificial surface depicted in Fig. 2.2 is illuminated by an incident electromagnetic field $\{\mathbf{E}^{\text{inc}}, \mathbf{H}^{\text{inc}}\}$, defined in the absence of the equivalent impedance model, equivalent electric surface currents $\mathbf{J}_{s,i}$ are induced on the i -th impedance sheet defined by the contour $C_{s,i}$ and equivalent polarization currents $\mathbf{J}_{p,q}$ are induced on the q -th dielectric layer bounded by the surface $S_{p,q}$. These equivalent electric currents generate the scattered fields $\{\mathbf{E}_{s,i}^{\text{scat}}, \mathbf{H}_{s,i}^{\text{scat}}\}$ due to $\mathbf{J}_{s,i}$ and $\{\mathbf{E}_{p,q}^{\text{scat}}, \mathbf{H}_{p,q}^{\text{scat}}\}$ due to $\mathbf{J}_{p,q}$. The total electric field $\mathbf{E}^{\text{tot}}$ is given by:

$$\mathbf{E}^{\text{tot}}(\mathbf{r}) = \mathbf{E}^{\text{inc}}(\mathbf{r}) + \sum_{i=1}^{N_{\text{imp}}} \mathbf{E}_{s,i}^{\text{scat}}(\mathbf{r}, \mathbf{J}_{s,i}) + \sum_{q=1}^{N_{\text{die}}} \mathbf{E}_{p,q}^{\text{scat}}(\mathbf{r}, \mathbf{J}_{p,q}). \quad (2.3.1)$$

Here, N_{imp} and N_{die} are the number of zero-thickness impedance sheets and the dielectric substrate layers, respectively. By using the boundary conditions defined in Eqs. 2.2.2 and 2.2.3 and substituting into Eq. 2.3.1, equations are obtained for $\mathbf{J}_{s,i}$ and $\mathbf{J}_{p,q}$ at the boundary of the impedance sheet and the dielectric layer, respectively. As a result, the VSIE is formulated for the i -th impedance sheet and expressed as:

$$\begin{aligned} -\mathbf{E}_{\text{tan}}^{\text{inc}}(\mathbf{r}) &= \sum_{i=1}^{N_{\text{imp}}} [\mathbf{E}_{s,i}^{\text{scat}}(\mathbf{r}, \mathbf{J}_{s,i})]_{\text{tan}} + \sum_{q=1}^{N_{\text{die}}} [\mathbf{E}_{p,q}^{\text{scat}}(\mathbf{r}, \mathbf{J}_{p,q})]_{\text{tan}} \\ &\quad - Z_{s,i}(\mathbf{r})\mathbf{J}_{s,i}(\mathbf{r}), \quad \mathbf{r} \in C_{s,i}, \quad i = 1, 2, \dots, N_{\text{imp}}, \end{aligned} \quad (2.3.2)$$

where the subscript **tan** indicates the tangential components. The VSIE for the q -th dielectric layer is given by:

$$\begin{aligned} -\mathbf{E}^{\text{inc}}(\mathbf{r}) &= \sum_{i=1}^{N_{\text{imp}}} \mathbf{E}_{s,i}^{\text{scat}}(\mathbf{r}, \mathbf{J}_{s,i}) + \sum_{q=1}^{N_{\text{die}}} \mathbf{E}_{p,q}^{\text{scat}}(\mathbf{r}, \mathbf{J}_{p,q}) \\ &\quad - Z_{p,q}(\mathbf{r})\mathbf{J}_{p,q}(\mathbf{r}), \quad \mathbf{r} \in S_{p,q}, \quad q = 1, 2, \dots, N_{\text{die}}. \end{aligned} \quad (2.3.3)$$

The scattered electric field $\mathbf{E}_{s,i}^{\text{scat}}(\mathbf{r})$ radiated by the equivalent electric currents $\mathbf{J}_{s,i}$ on the i -th impedance sheet can be represented in terms of the magnetic vector potential $\mathbf{A}_{s,i}$ and the electric scalar potential $\Phi_{s,i}$ as [57, pp. 147–150]:

$$\mathbf{E}_{s,i}^{\text{scat}}(\mathbf{r}) = -j\omega\mathbf{A}_{s,i}(\mathbf{r}) - \nabla\Phi_{s,i}(\mathbf{r}), \quad (2.3.4)$$

where ω is the angular frequency of operation and $\nabla\Phi_{s,i}$ is the gradient of the electric scalar potential. The magnetic vector potential $\mathbf{A}_{s,i}(\mathbf{r})$ and the electric scalar potential

2.3 Volume Surface Integral Equations

$\Phi_{s,i}$ can be expressed in terms of the equivalent electric currents $\mathbf{J}_{s,i}$ and the free-space Green's function $G_0(\mathbf{r}, \mathbf{r}')$ as:

$$\mathbf{A}_{s,i}(\mathbf{r}) = \mu_0 \int_{C_{s,i}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{s,i}(\mathbf{r}') d\mathbf{r}', \quad (2.3.5)$$

$$\Phi_{s,i}(\mathbf{r}) = \frac{j}{\omega\epsilon_0} \int_{C_{s,i}} G_0(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}_{s,i}(\mathbf{r}') d\mathbf{r}', \quad (2.3.6)$$

where $C_{s,i}$ is the contour defining the i -th impedance sheet and $\nabla' \cdot$ denotes the surface divergence operator with respect to the primed coordinates. μ_0 is the magnetic permeability of free-space and ϵ_0 is the free-space permittivity. Similarly, the scattered electric field $\mathbf{E}_{p,q}^{\text{scat}}(\mathbf{r})$ radiated by the equivalent polarization currents $\mathbf{J}_{p,q}$ is written as:

$$\mathbf{E}_{p,q}^{\text{scat}}(\mathbf{r}) = -j\omega \mathbf{A}_{p,q}(\mathbf{r}) - \nabla \Phi_{p,q}(\mathbf{r}). \quad (2.3.7)$$

The magnetic vector potential $\mathbf{A}_{p,q}(\mathbf{r})$ and the scalar potential $\Phi_{p,q}(\mathbf{r})$ for q -th dielectric layer are expressed in terms of the equivalent polarization currents $\mathbf{J}_{p,q}$ and the free-space Green's function $G_0(\mathbf{r}, \mathbf{r}')$:

$$\mathbf{A}_{p,q}(\mathbf{r}) = \mu_0 \iint_{S_{p,q}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{p,q}(\mathbf{r}') d\mathbf{r}', \quad (2.3.8)$$

$$\Phi_{p,q}(\mathbf{r}) = \frac{j}{\omega\epsilon_0} \iint_{S_{p,q}} G_0(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}_{p,q}(\mathbf{r}') d\mathbf{r}', \quad (2.3.9)$$

where $S_{p,q}$ is the cross-sectional surface of the q -th dielectric substrate layer, where the equivalent polarization currents are defined. Substituting the above expressions of the scattered fields radiated by $\mathbf{J}_{s,i}$ and $\mathbf{J}_{p,q}$ into Eqs. 2.3.2 and 2.3.3, we obtain the following VSIE for the i -th impedance sheet:

$$\begin{aligned}
 \mathbf{E}_{\text{tan}}^{\text{inc}}(\mathbf{r}) = & \sum_{i=1}^{N_{\text{imp}}} \left[j\omega\mu_0 \int_{C_{s,i}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{s,i}(\mathbf{r}') d\mathbf{r}' \right. \\
 & \left. + \frac{j}{\omega\epsilon_0} \nabla \nabla \cdot \int_{C_{s,i}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{s,i}(\mathbf{r}') d\mathbf{r}' \right]_{\text{tan}} \\
 & + \sum_{q=1}^{N_{\text{die}}} \left[j\omega\mu_0 \iint_{S_{p,q}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{p,q}(\mathbf{r}') d\mathbf{r}' \right. \\
 & \left. + \frac{j}{\omega\epsilon_0} \nabla \nabla \cdot \iint_{S_{p,q}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{p,q}(\mathbf{r}') d\mathbf{r}' \right]_{\text{tan}} \\
 & + Z_{s,i}(\mathbf{r}) \mathbf{J}_{s,i}(\mathbf{r}), \quad \mathbf{r} \in C_{s,i}.
 \end{aligned} \tag{2.3.10}$$

The VSIE for the q -th dielectric layer:

$$\begin{aligned}
 \mathbf{E}^{\text{inc}}(\mathbf{r}) = & \sum_{i=1}^{N_{\text{imp}}} j\omega\mu_0 \int_{C_{s,i}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{s,i}(\mathbf{r}') d\mathbf{r}' \\
 & + \frac{j}{\omega\epsilon_0} \nabla \nabla \cdot \int_{C_{s,i}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{s,i}(\mathbf{r}') d\mathbf{r}' \\
 & + \sum_{q=1}^{N_{\text{die}}} j\omega\mu_0 \iint_{S_{p,q}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{p,q}(\mathbf{r}') d\mathbf{r}' \\
 & + \frac{j}{\omega\epsilon_0} \nabla \nabla \cdot \iint_{S_{p,q}} G_0(\mathbf{r}, \mathbf{r}') \mathbf{J}_{p,q}(\mathbf{r}') d\mathbf{r}' \\
 & + Z_{p,q}(\mathbf{r}) \mathbf{J}_{p,q}(\mathbf{r}), \quad \mathbf{r} \in S_{p,q}.
 \end{aligned} \tag{2.3.11}$$

Eqs. 2.3.10 and 2.3.11 are recognized as the coupled VSIE having the unknowns $\mathbf{J}_s$, the equivalent electric currents on the impedance sheets, and the unknowns $\mathbf{J}_p$, the equivalent polarization currents on the cross-sectional dielectric layers [57, pp. 250–251]. Since the considered problem is two-dimensional, the two-dimensional free-space Green's function is used [1, pp. 29–30]:

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{1}{4j} H_0^{(2)}(k_0 |\mathbf{r} - \mathbf{r}'|), \tag{2.3.12}$$

where $H_0^{(2)}()$ is the Hankel function of the second kind of zero order and $\mathbf{r}$ is the observation point and $\mathbf{r}'$ is the source point.

In the next sections, we focus on the discretization and numerical solution of the continuous volume surface integral equations in 2.3.10 and 2.3.11. This is achieved by representing the equivalent electric currents $\mathbf{J}_s$ and $\mathbf{J}_p$ in terms of basis functions and unknown coefficients. The RWG basis function is written [57, p. 145]. This will generate a system of linear equations that can be solved by a method of moments procedure [57, pp. 168–170].

2.3.1 Discretization of the Two-Dimensional Equivalent Model

The first step in the numerical solution of the VSIEs in 2.3.10 and 2.3.11 is the discretization of the arbitrarily shaped impedance sheets and the cross-section surfaces of the dielectric layers. This involves dividing the zero-thickness impedance sheets into line segments and the dielectric layers into triangles or quadrilaterals elements depending on the complexity of the geometry.

Arbitrarily Curved Impedance Sheet

The contour of the shaped impedance sheet defined by the boundary C_s is discretized into N_s line segments, as shown in Fig. 2.5. The points on the n_s -th line segment can be parametrically represented as [57, p. 223]:

$$x(l) = x_{n_s-1} + l \cos \theta_{n_s}, \quad y(l) = y_{n_s-1} + l \sin \theta_{n_s}, \quad (2.3.13)$$

where x and y are the coordinates of any point along the line segment. The segment starts at the first vertex point (x_{n_s-1}, y_{n_s-1}) and extends in the direction defined by the angle θ_{n_s} to the second vertex point (x_{n_s}, y_{n_s}) . The variable l represents the distance from (x_{n_s-1}, y_{n_s-1}) to any point on the n_s line segment. The length of the n_s -th line segment is represented by Δl_{n_s} . The angle θ_{n_s} , defines the direction of the line segment with respect to the x -axis, is expressed as:

$$\theta_{n_s} = \tan^{-1} \left(\frac{y_{n_s} - y_{n_s-1}}{x_{n_s} - x_{n_s-1}} \right). \quad (2.3.14)$$

Arbitrarily Shaped Dielectric layer

The cross-section of the arbitrarily shaped dielectric layer, defined by the surface S_p , is discretized into N_p elements. The discretization can be performed using quadrilateral or triangular elements, depending on the complexity of the surface. For quadrilateral discretization, the surface S_p is divided into quadrilateral elements with dimensions $\Delta x_{n_p} \times \Delta y_{n_p}$, where Δx_{n_p} and Δy_{n_p} represent the lengths of the elements along the x - and y -axis,

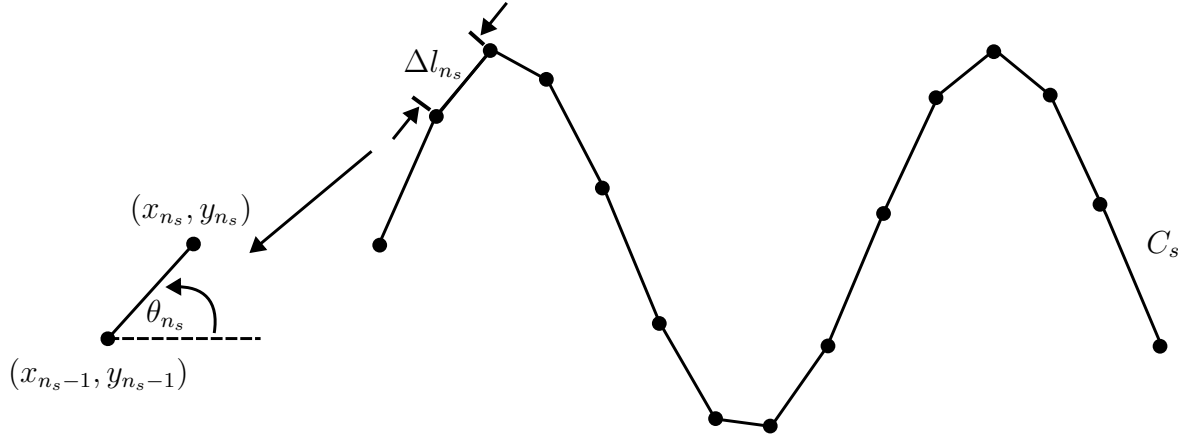


Fig. 2.5: Discretization of the impedance sheet boundary C_s into line segments.

as illustrated in Fig. 2.6b. On the other hand, triangular mesh is employed to better approximate surfaces with highly curved boundaries, as shown in Fig. 2.6c.

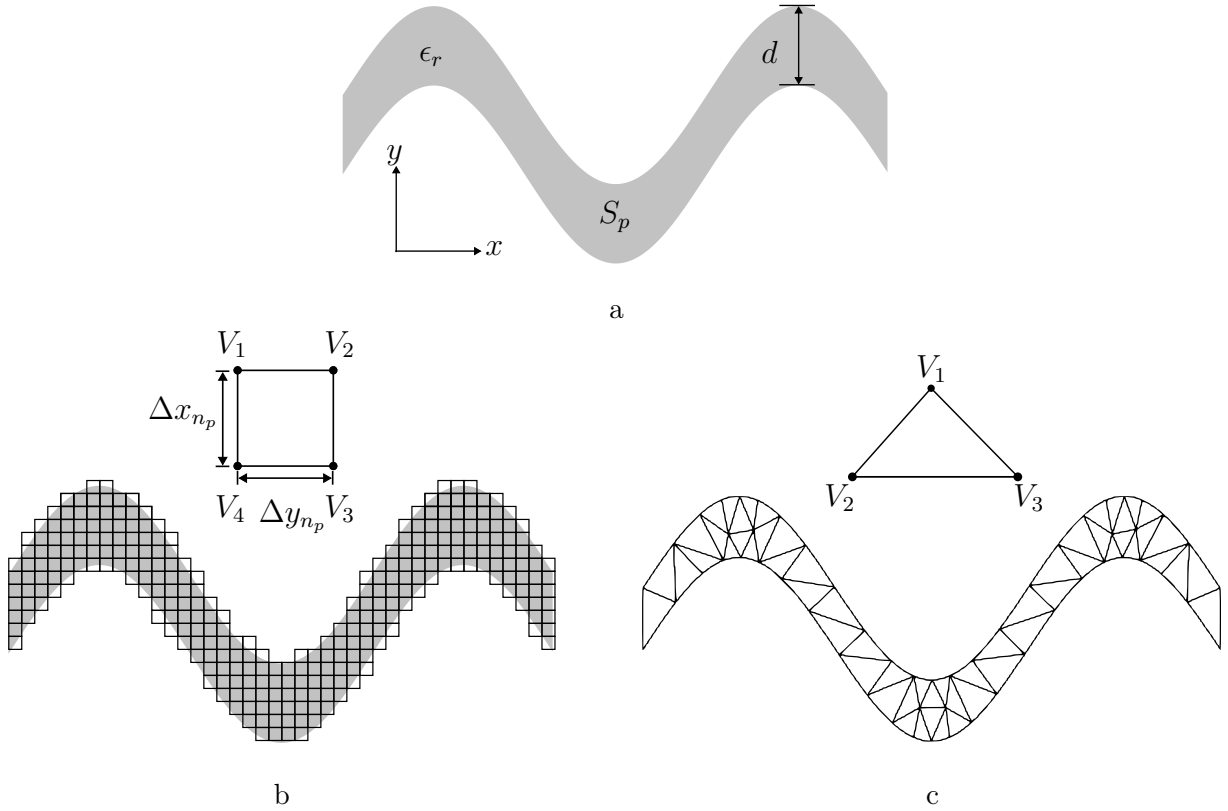


Fig. 2.6: Discretization of the dielectric cross-sectional surface S_p : (a) Geometry. (b) Quadrilateral mesh. (c) Triangular mesh.

2.3.2 MoM Formulation

Once the arbitrarily curved impedance sheets are discretized into line segments and the cross-sectional surfaces of the dielectric layers into quadrilateral or triangular elements, the next step is to expand the unknown equivalent currents over these discrete elements using a set of basis functions. The equivalent electric currents $\mathbf{J}_{s,i}(\mathbf{r})$ on the i -th impedance sheet are represented as a sum of $N_{s,i}$ basis functions with unknown coefficients:

$$\mathbf{J}_{s,i}(\mathbf{r}) = \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}), \quad (2.3.15)$$

where $\mathbf{T}_{n_s}^{(i)}(\mathbf{r})$ are the vector basis functions of the expansion and $I_{n_s}^{(i)}$ are unknown expansion coefficients. Similarly, the equivalent polarization currents $\mathbf{J}_{p,q}(\mathbf{r})$ on the cross-sectional surface of the q -th dielectric layer are also expanded using a sum of $N_{p,q}$ basis functions:

$$\mathbf{J}_{p,q}(\mathbf{r}) = \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}), \quad (2.3.16)$$

where $\mathbf{f}_{n_p}^{(q)}(\mathbf{r})$ are the vector basis functions and $I_{n_p}^{(q)}$ are unknown expansion coefficients. Substituting the equivalent current expansions in Eqs. 2.3.15 and 2.3.16 into the continuous VSIEs 2.3.10 and 2.3.11, we obtain the following discretized VSIEs:

$$\begin{aligned} \mathbf{E}_{\text{tan}}^{\text{inc}}(\mathbf{r}) = & \sum_{i=1}^{N_{\text{imp}}} \left[j\omega\mu_0 \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right. \\ & \left. + \frac{j}{\omega\epsilon_0} \nabla \nabla \cdot \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right]_{\text{tan}} \\ & + \sum_{q=1}^{N_{\text{die}}} \left[j\omega\mu_0 \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right. \\ & \left. + \frac{j}{\omega\epsilon_0} \nabla \nabla \cdot \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right]_{\text{tan}} \\ & + Z_{s,i}(\mathbf{r}) \mathbf{J}_{s,i}(\mathbf{r}), \quad \mathbf{r} \in C_{n_s}^{(i)}, \end{aligned} \quad (2.3.17)$$

2 MoM Analysis of Arbitrarily Shaped Artificial Surfaces

$$\begin{aligned}
\mathbf{E}^{\text{inc}}(\mathbf{r}) = & \sum_{i=1}^{N_{\text{imp}}} \left[j\omega\mu_0 \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right. \\
& \left. + \frac{j}{\omega\epsilon_0} \nabla\nabla \cdot \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] \\
& + \sum_{q=1}^{N_{\text{die}}} \left[j\omega\mu_0 \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right. \\
& \left. + \frac{j}{\omega\epsilon_0} \nabla\nabla \cdot \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] \\
& + Z_{p,q}(\mathbf{r}) \mathbf{J}_{p,q}(\mathbf{r}), \quad \mathbf{r} \in S_{n_p}^{(q)}. \tag{2.3.18}
\end{aligned}$$

Eq. 2.3.17 is tested with the weighting basis functions $\mathbf{W}_{m_s}^{(j)}(\mathbf{r})$ defined on the j -th observation impedance layer. The VSIE in Eq. 2.3.17 after applying the weighting basis functions $\mathbf{W}_{m_s}^{(j)}(\mathbf{r})$ on both sides becomes:

$$\begin{aligned}
\int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \mathbf{E}_{\text{tan}}^{\text{inc}}(\mathbf{r}) d\mathbf{r} = & \sum_{i=1}^{N_{\text{imp}}} \left[j\omega\mu_0 \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' d\mathbf{r} \right. \\
& \left. + \frac{j}{\omega\epsilon_0} \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \left[\nabla\nabla \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] d\mathbf{r} \right]_{\text{tan}} \\
& + \sum_{q=1}^{N_{\text{die}}} \left[j\omega\mu_0 \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' d\mathbf{r} \right. \\
& \left. + \frac{j}{\omega\epsilon_0} \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \left[\nabla\nabla \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] d\mathbf{r} \right]_{\text{tan}} \\
& + \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(j)} \int_{C_{m_s}^{(j)}} Z_{m_s}^{(j)}(\mathbf{r}) \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \mathbf{T}_{n_s}^{(i)}(\mathbf{r}) d\mathbf{r}, \quad \mathbf{r} \in C_{m_s}^{(j)}. \tag{2.3.19}
\end{aligned}$$

Here, $C_{m_s}^{(j)}$ represents the m_s -th line segment, where the weighting basis functions $\mathbf{W}_{m_s}^{(j)}(\mathbf{r})$ are defined and $Z_{m_s}^{(j)}$ is the equivalent zero-thickness impedance sheet that represent the subwavelength metallic patterns over the m_s -th line segment. The superscript j represents the observation arbitrarily curved impedance layer. By setting $m_s = 1, 2, 3, \dots, N_{s,j}$, a system of linear equations is constructed. It can be arranged into a matrix form as follows:

$$\mathbf{V}_s^j = \mathbf{Z}_{\text{self}}^j \mathbf{I}_s^j + \sum_{i=1}^{N_{\text{imp}}} \mathbf{Z}_{ss}^{ji} \mathbf{I}_s^i + \sum_{q=1}^{N_{\text{die}}} \mathbf{Z}_{sp}^{jq} \mathbf{I}_p^q, \quad j = 1, 2, \dots, N_{\text{imp}}. \quad (2.3.20)$$

Here, $\mathbf{Z}_{ss}^{ji}$ represents the interactions between the equivalent electric currents on the j -th observation impedance layer and on the i -th source impedance layer. $\mathbf{Z}_{sp}^{jq}$ represents the interactions between the equivalent electric currents on the j -th impedance layer and on the q -th source dielectric layer. The elements of the impedance matrices $\mathbf{Z}_{ss}^{ji}$ and $\mathbf{Z}_{sp}^{jq}$ are organized as:

$$\begin{aligned} Z_{m_s, n_s}^{ji} = & j\omega\mu_0 \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' d\mathbf{r} \\ & + \frac{j}{\omega\epsilon_0} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] d\mathbf{r}, \end{aligned} \quad (2.3.21)$$

$$\begin{aligned} Z_{m_s, n_p}^{jq} = & j\omega\mu_0 \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' d\mathbf{r} \\ & + \frac{j}{\omega\epsilon_0} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] d\mathbf{r}. \end{aligned} \quad (2.3.22)$$

The elements of the diagonal impedance matrix $\mathbf{Z}_{\text{self}}^j$, when $m_s = n_s$ and $i = j$, are expressed as:

$$Z_{m_s, m_s}^j = \int_{C_{m_s}^{(j)}} Z_{m_s}^{(j)}(\mathbf{r}) \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \mathbf{T}_{m_s}^{(j)}(\mathbf{r}) d\mathbf{r}. \quad (2.3.23)$$

2 MoM Analysis of Arbitrarily Shaped Artificial Surfaces

The elements of the excitation vector $\mathbf{V}_s^j$ are given by:

$$V_{m_s}^j = \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \mathbf{E}_{\text{tan}}^{\text{inc}}(\mathbf{r}) d\mathbf{r}. \quad (2.3.24)$$

Similarly, Eq. 2.3.18 is tested on both sides with the weighting basis functions $\mathbf{W}_{m_p}^{(k)}(\mathbf{r})$ defined on the k -th dielectric layer. This yields the following equation:

$$\begin{aligned} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \mathbf{E}^{\text{inc}}(\mathbf{r}) d\mathbf{r} = & \sum_{i=1}^{N_{\text{imp}}} \left[j\omega\mu_0 \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' d\mathbf{r} \right. \\ & \left. + \frac{j}{\omega\epsilon_0} \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] d\mathbf{r} \right] \\ & + \sum_{q=1}^{N_{\text{die}}} \left[j\omega\mu_0 \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' d\mathbf{r} \right. \\ & \left. + \frac{j}{\omega\epsilon_0} \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] d\mathbf{r} \right] \\ & + \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(k)} \iint_{S_{m_p}^{(k)}} Z_{m_p}^{(k)}(\mathbf{r}) \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \mathbf{f}_{n_p}^{(q)}(\mathbf{r}) d\mathbf{r}, \quad \mathbf{r} \in S_{m_p}^{(k)}. \end{aligned} \quad (2.3.25)$$

By setting $m_p = 1, 2, 3, \dots, N_{p,k}$, we can generate $N_{p,k}$ linear equations, which can be formulated as a linear matrix equation of the form:

$$\mathbf{V}_p^k = \mathbf{Z}_{\text{self}}^k \mathbf{I}_p^k + \sum_{i=1}^{N_{\text{imp}}} \mathbf{Z}_{ps}^{ki} \mathbf{I}_s^i + \sum_{q=1}^{N_{\text{die}}} \mathbf{Z}_{pp}^{kq} \mathbf{I}_p^q, \quad k = 1, 2, \dots, N_{\text{die}}. \quad (2.3.26)$$

Here, $\mathbf{V}_p^k$ represents the excitation vector of the k -th dielectric layer. $\mathbf{Z}_{ps}^{ki}$ represents the interaction between the testing functions on the k -th dielectric layer and the expansion functions on the i -th impedance sheet. $\mathbf{Z}_{pp}^{kq}$ represents the mutual impedance matrix for interactions between the testing functions on the k -th dielectric layer and the expansion functions of the q -th dielectric layer.

Each element of the impedance matrices $\mathbf{Z}_{ps}^{ki}$ and $\mathbf{Z}_{pp}^{kq}$ can be expressed as:

$$Z_{m_p, n_s}^{ki} = j\omega\mu_0 \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' d\mathbf{r} \\ + \frac{j}{\omega\epsilon_0} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] d\mathbf{r}, \quad (2.3.27)$$

$$Z_{m_p, n_p}^{kq} = j\omega\mu_0 \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' d\mathbf{r} \\ + \frac{j}{\omega\epsilon_0} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \right] d\mathbf{r}. \quad (2.3.28)$$

The elements of the diagonal self impedance matrix $\mathbf{Z}_{\text{self}}^k$, when $m_p = n_p$ and $k = q$, are expressed as:

$$Z_{m_p, m_p}^{(k)} = \iint_{S_{m_p}^{(k)}} Z_{m_p}^{(k)}(\mathbf{r}) \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \mathbf{f}_{m_p}^{(k)}(\mathbf{r}) d\mathbf{r}. \quad (2.3.29)$$

The corresponding excitation vector elements is given by:

$$V_{m_p}^{(k)} = \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \mathbf{E}^{\text{inc}}(\mathbf{r}) d\mathbf{r}. \quad (2.3.30)$$

Eqs. 2.3.20 and 2.3.26 are combined into a single matrix form:

$$\begin{bmatrix} \mathbf{V}_s \\ \mathbf{V}_p \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{ss} & \mathbf{Z}_{sp} \\ \mathbf{Z}_{ps} & \mathbf{Z}_{pp} \end{bmatrix} \begin{bmatrix} \mathbf{I}_s \\ \mathbf{I}_p \end{bmatrix}, \quad (2.3.31)$$

where the submatrices are $\mathbf{Z}_{ss}$, $\mathbf{Z}_{sp}$, $\mathbf{Z}_{ps}$ and $\mathbf{Z}_{pp}$ are defined as follows:

$$\mathbf{Z}_{ss} = \begin{bmatrix} [Z_{ss}^{1,1}] & [Z_{ss}^{1,2}] & \dots & [Z_{ss}^{1, N_{\text{imp}}}] \\ [Z_{ss}^{2,1}] & [Z_{ss}^{2,2}] & \dots & [Z_{ss}^{2, N_{\text{imp}}}] \\ \vdots & \vdots & \ddots & \vdots \\ [Z_{ss}^{N_{\text{imp}},1}] & [Z_{ss}^{N_{\text{imp}},2}] & \dots & [Z_{ss}^{N_{\text{imp}}, N_{\text{imp}}}] \end{bmatrix}, \quad (2.3.32)$$

$$\mathbf{Z}_{sp} = \begin{bmatrix} [Z_{sp}^{1,1}] & [Z_{sp}^{1,2}] & \dots & [Z_{sp}^{1,N_{\text{die}}}] \\ [Z_{sp}^{2,1}] & [Z_{sp}^{2,2}] & \dots & [Z_{sp}^{2,N_{\text{die}}}] \\ \vdots & \vdots & \ddots & \vdots \\ [Z_{sp}^{N_{\text{imp}},1}] & [Z_{sp}^{N_{\text{imp}},2}] & \dots & [Z_{sp}^{N_{\text{imp}},N_{\text{die}}}] \end{bmatrix}, \quad (2.3.33)$$

$$\mathbf{Z}_{ps} = \begin{bmatrix} [Z_{ps}^{1,1}] & [Z_{ps}^{1,2}] & \dots & [Z_{ps}^{1,N_{\text{imp}}}] \\ [Z_{ps}^{2,1}] & [Z_{ps}^{2,2}] & \dots & [Z_{ps}^{2,N_{\text{imp}}}] \\ \vdots & \vdots & \ddots & \vdots \\ [Z_{ps}^{N_{\text{die}},1}] & [Z_{ps}^{N_{\text{die}},2}] & \dots & [Z_{ps}^{N_{\text{die}},N_{\text{imp}}}] \end{bmatrix}, \quad (2.3.34)$$

$$\mathbf{Z}_{pp} = \begin{bmatrix} [Z_{pp}^{1,1}] & [Z_{pp}^{1,2}] & \dots & [Z_{pp}^{1,N_{\text{die}}}] \\ [Z_{pp}^{2,1}] & [Z_{pp}^{2,2}] & \dots & [Z_{pp}^{2,N_{\text{die}}}] \\ \vdots & \vdots & \ddots & \vdots \\ [Z_{pp}^{N_{\text{die}},1}] & [Z_{pp}^{N_{\text{die}},2}] & \dots & [Z_{pp}^{N_{\text{die}},N_{\text{die}}}] \end{bmatrix}. \quad (2.3.35)$$

The unknown current expansion coefficients $\mathbf{I}_s$ and $\mathbf{I}_p$ are formulated in a vector form:

$$\mathbf{I}_s = \begin{bmatrix} \mathbf{I}_s^{(1)} \\ \mathbf{I}_s^{(2)} \\ \vdots \\ \mathbf{I}_s^{(i)} \\ \vdots \\ \mathbf{I}_s^{(N_{\text{imp}})} \end{bmatrix}, \quad (2.3.36)$$

and

$$\mathbf{I}_p = \begin{bmatrix} \mathbf{I}_p^{(1)} \\ \mathbf{I}_p^{(2)} \\ \vdots \\ \mathbf{I}_p^{(q)} \\ \vdots \\ \mathbf{I}_p^{(N_{\text{die}})} \end{bmatrix}. \quad (2.3.37)$$

The excitation vectors $\mathbf{V}_s$ and $\mathbf{V}_p$ are given by:

$$\mathbf{V}_s = \begin{bmatrix} \mathbf{V}_s^{(1)} \\ \mathbf{V}_s^{(2)} \\ \vdots \\ \mathbf{V}_s^{(j)} \\ \vdots \\ \mathbf{V}_s^{(N_{\text{imp}})} \end{bmatrix}, \quad (2.3.38)$$

and

$$\mathbf{V}_p = \begin{bmatrix} \mathbf{V}_p^{(1)} \\ \mathbf{V}_p^{(2)} \\ \vdots \\ \mathbf{V}_p^{(q)} \\ \vdots \\ \mathbf{V}_p^{(N_{\text{die}})} \end{bmatrix}. \quad (2.3.39)$$

2.4 VSIEs: TE Polarization

Consider the problem of scattering by the two-dimensional arbitrarily shaped multilayer impedance sheets illustrated in Fig. 2.7 for TE excitation. The incident electric field is oriented along the z -direction perpendicular to the modeling xy -plane. The equivalent model lies in the xy -plane and extends infinitely along the z -axis.

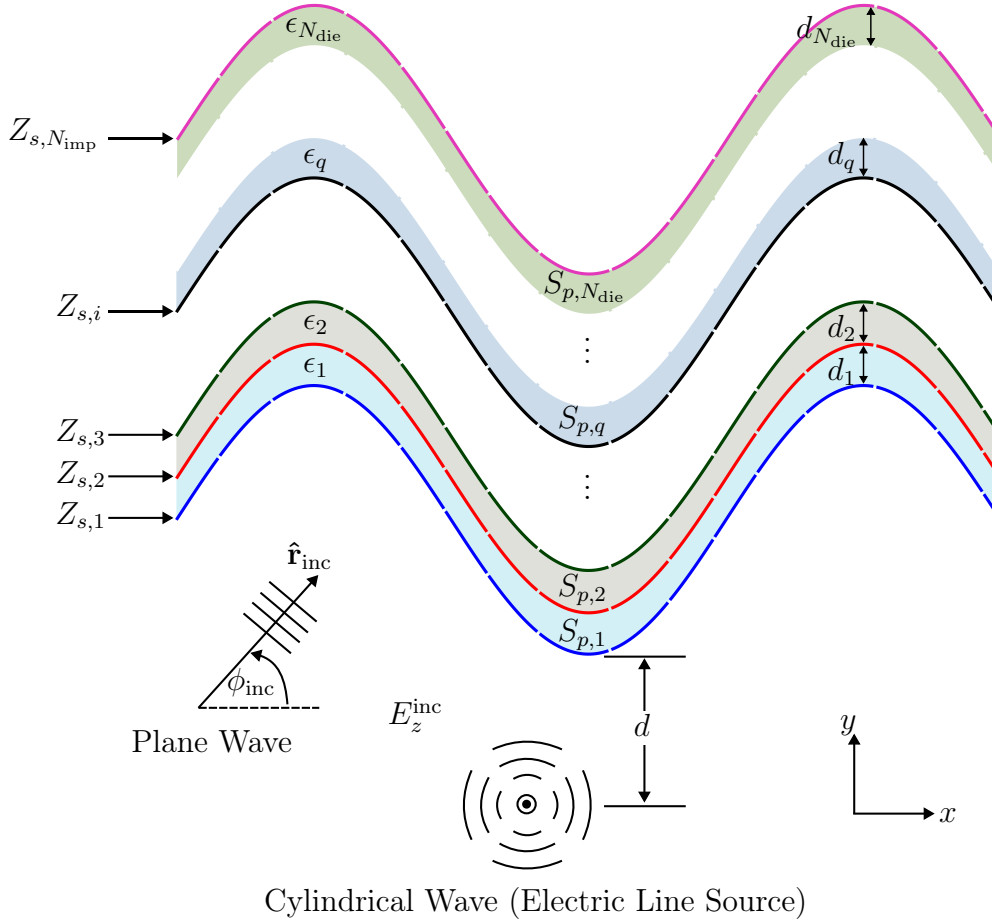


Fig. 2.7: Two-dimensional arbitrarily shaped multilayer impedance sheets separated by dielectric layers and illuminated by either a TE plane wave or a TE cylindrical wave propagating in the xy -plane. The incident electric field has only z -component.

Plane Wave

A plane wave propagating in the xy -plane illuminates the two-dimensional equivalent model at an incident angle ϕ_{inc} with respect to the x -axis, as shown in Fig. 2.7. The electric field of the incident plane wave is given by [1, p. 124], [57, p. 198]

$$\mathbf{E}^{\text{inc}}(\mathbf{r}) = \hat{\mathbf{z}} E_0 e^{j k_0 \hat{\mathbf{r}}_{\text{inc}} \cdot \mathbf{r}} = \hat{\mathbf{z}} E_0 e^{j k_0 (x \cos \phi_{\text{inc}} + y \sin \phi_{\text{inc}})}, \quad (2.4.1)$$

where E_0 is the amplitude of the plane wave, $\hat{\mathbf{r}}_{\text{inc}} = \hat{\mathbf{x}} \cos \phi_{\text{inc}} + \hat{\mathbf{y}} \sin \phi_{\text{inc}}$, k_0 is the wavenumber in free-space and $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ is the observation point.

Cylindrical Wave

An infinite electric line source is used as an excitation source and is positioned at $\mathbf{r}_{\text{src}} = x_{\text{src}}\hat{\mathbf{x}} + y_{\text{src}}\hat{\mathbf{y}}$. The electric line source radiates a TE polarized cylindrical wave propagating in the xy -plane with incident electric field parallel to the z -axis. The TE polarized incident electric field, $\mathbf{E}^{\text{inc}}(\mathbf{r}) = \hat{\mathbf{z}}E_z^{\text{inc}}(\mathbf{r})$ is known at the observation point $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ [57, p. 59]

$$E_z^{\text{inc}}(\mathbf{r}) = \frac{-I_0\eta_0k_0}{4}H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}_{\text{src}}|), \quad (2.4.2)$$

where I_0 is the strength of the line source in Ampere, η_0 is the free-space wave impedance in ohms, k_0 is the free-space wavenumber in rad/m, and $H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}_{\text{src}}|)$ is the Hankel function of the second kind and order zero. The total field $\mathbf{E}^{\text{tot}}$ and scattered field $\mathbf{E}^{\text{scat}}$ are also z -polarized. The equivalent electric currents $\mathbf{J}_s$ on the impedance layers and the equivalent polarization currents on the cross-sectional surface of the dielectric layers, both of which contribute to the scattered field, have only z -component

$$\mathbf{J}_s = J_{z,s}\hat{\mathbf{z}}, \quad \mathbf{J}_p = J_{z,p}\hat{\mathbf{z}}. \quad (2.4.3)$$

Since the equivalent currents have only z -component and are invariant along the z -direction, their divergences satisfy [1, p. 126]

$$\nabla' \cdot \mathbf{J}_s(\mathbf{r}') = 0, \quad \nabla' \cdot \mathbf{J}_p(\mathbf{r}') = 0. \quad (2.4.4)$$

Applying (2.4.4) and substituting the two-dimensional free-space Green's function (2.3.12) into Eqs. 2.3.19 and 2.3.25, the discretized VSIEs for the $(m_s \times j)$ -th equation with $m_s = 1, 2, \dots, N_{s,j}$ and $j = 1, 2, \dots, N_{\text{imp}}$ are reduced to

$$\begin{aligned} & \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \mathbf{E}_{\text{tan}}^{\text{inc}}(\mathbf{r}) d\mathbf{r} \\ &= \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(j)} \int_{C_{m_s}^{(j)}} Z_{m_s}^{(j)}(\mathbf{r}) \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \mathbf{T}_{n_s}^{(i)}(\mathbf{r}) d\mathbf{r} \\ &+ \sum_{i=1}^{N_{\text{imp}}} \left[\frac{\omega\mu_0}{4} \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \right]_{\text{tan}} \\ &+ \sum_{q=1}^{N_{\text{die}}} \left[\frac{\omega\mu_0}{4} \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \right]_{\text{tan}}. \end{aligned} \quad (2.4.5)$$

2 MoM Analysis of Arbitrarily Shaped Artificial Surfaces

and for the $(m_p \times k)$ -th equation with $m_p = 1, 2, \dots, N_{p,k}$ and $k = 1, 2, \dots, N_{\text{die}}$

$$\begin{aligned}
& \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \mathbf{E}^{\text{inc}}(\mathbf{r}) d\mathbf{r} \\
&= \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(k)} \iint_{S_{m_p}^{(k)}} Z_{m_p}^{(k)}(\mathbf{r}) \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \mathbf{f}_{n_p}^{(q)}(\mathbf{r}) d\mathbf{r} \\
&+ \sum_{i=1}^{N_{\text{imp}}} \left[\frac{\omega\mu_0}{4} \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \right] \\
&+ \sum_{q=1}^{N_{\text{die}}} \left[\frac{\omega\mu_0}{4} \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \right].
\end{aligned} \tag{2.4.6}$$

The impedance matrix elements that represent the interactions between the equivalent currents are arranged as:

$$Z_{m_s, n_s}^{ji} = \frac{\omega\mu_0}{4} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \tag{2.4.7}$$

$$Z_{m_s, n_p}^{jq} = \frac{\omega\mu_0}{4} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \tag{2.4.8}$$

$$Z_{m_p, n_s}^{ki} = \frac{\omega\mu_0}{4} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \tag{2.4.9}$$

$$Z_{m_p, n_p}^{kq} = \frac{\omega\mu_0}{4} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}. \tag{2.4.10}$$

Having established the simplified forms of the impedance matrix elements for TE excitation, we will define the expansion functions for the equivalent currents in the following sections.

2.4.1 Basis Functions

This section focuses on the definition of the expansion functions $T_{n_s}(\mathbf{r})$ in (2.3.15) and $f_{n_p}(\mathbf{r})$ in (2.3.16). These functions are employed to expand the equivalent currents on the impedance layers and the equivalent polarization currents on the cross-sectional surfaces of the dielectric layers.

One dimensional Pulse Basis Functions

The equivalent electric currents $\mathbf{J}_s(\mathbf{r}) = \hat{\mathbf{z}}J_{z,s}(\mathbf{r})$ on the impedance layers are expanded using a set of basis functions :

$$J_{z,s}(\mathbf{r}) = \sum_{n_s=1}^{N_s} I_{n_s} T_{n_s}(\mathbf{r}), \quad (2.4.11)$$

where $T_{n_s}(\mathbf{r})$ are the basis functions of the expansion and I_{n_s} are the unknown expansion coefficients. One-dimensional pulse basis functions are used to represent the equivalent currents on the impedance layers. The pulse basis function associated with the n_s -th line segment is defined as [1, p. 126] [57, p. 222], [60]

$$T_{n_s}(\mathbf{r}) = P_{n_s}(l - \frac{\Delta l_{n_s}}{2}) = \begin{cases} 1 & \text{if } 0 < l < \Delta l_{n_s}, \\ 0 & \text{elsewhere.} \end{cases} \quad (2.4.12)$$

Here, Δl_{n_s} is the length of the n_s -th line segment. The parametric representations for the points (x, y) on the n_s -th line segment are given by [57, p. 222]

$$x(l) = x_{n_s-1} + l \cos \theta_{n_s}, \quad y(l) = y_{n_s-1} + l \sin \theta_{n_s}, \quad (2.4.13)$$

where (x_{n_s-1}, y_{n_s-1}) and (x_{n_s}, y_{n_s}) are the starting and ending points of the n_s -th line segment, respectively. The angle θ_{n_s} is given by

$$\theta_{n_s} = \tan^{-1} \left(\frac{y_{n_s} - y_{n_s-1}}{x_{n_s} - x_{n_s-1}} \right). \quad (2.4.14)$$

Here, l is the distance measured along the n_s -th line segment starting from the point (x_{n_s-1}, y_{n_s-1}) , as illustrated in Fig. 2.8.

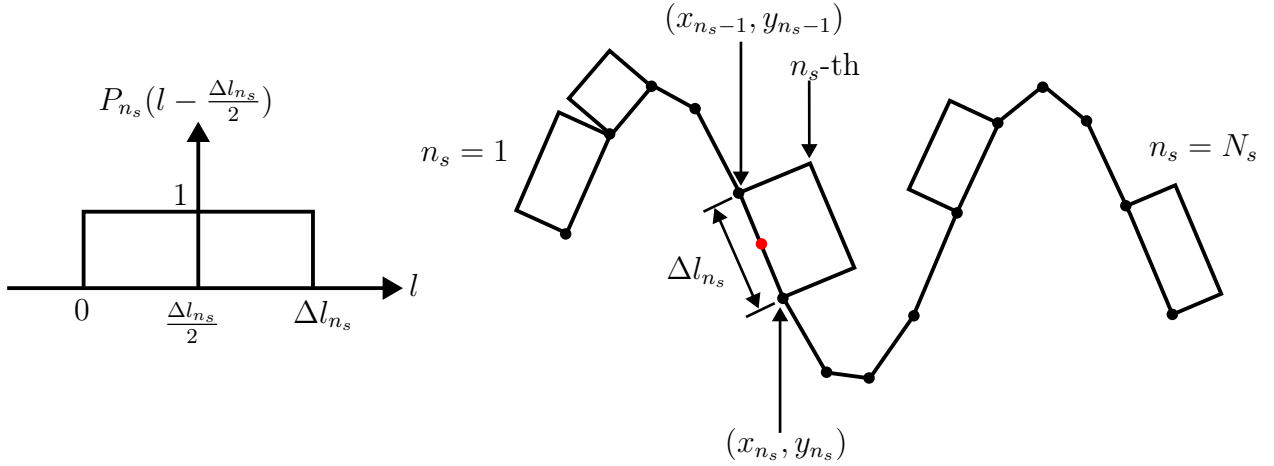


Fig. 2.8: Expansion of the equivalent currents on the impedance sheet $\mathbf{J}_s$ using one-dimensional pulse basis functions.

Two-dimensional Pulse Basis Functions

The equivalent polarization currents $\mathbf{J}_p(\mathbf{r}) = \hat{\mathbf{z}}J_{z,p}(\mathbf{r})$ on the cross-sectional surface of the dielectric layer are expanded into N_p two-dimensional pulse basis functions

$$J_{z,p}(\mathbf{r}) = \sum_{n_p=1}^{N_p} I_{n_p} f_{n_p}(\mathbf{r}), \quad (2.4.15)$$

where $f_{n_p}(\mathbf{r})$ are the basis functions of the expansion and I_{n_p} represents the unknown expansion coefficients. The two-dimensional pulse basis function $f_{n_p}(\mathbf{r})$ centered at (x_{n_p}, y_{n_p}) can be expressed as [57, pp. 244–249], [60]

$$f_{n_p}(x - x_{n_p}, y - y_{n_p}) = P_{\Delta x_{n_p}}(x - x_{n_p}) P_{\Delta y_{n_p}}(y - y_{n_p}). \quad (2.4.16)$$

The pulse functions $P_{\Delta x_{n_p}}$ and $P_{\Delta y_{n_p}}$, along the x - and y -directions, are given by

$$P_{\Delta x_{n_p}}(x - x_{n_p}) = \begin{cases} 1 & x_{n_p} - \frac{\Delta x_{n_p}}{2} \leq x \leq x_{n_p} + \frac{\Delta x_{n_p}}{2} \\ 0 & \text{elsewhere,} \end{cases} \quad (2.4.17)$$

and

$$P_{\Delta y_{n_p}}(y - y_{n_p}) = \begin{cases} 1 & y_{n_p} - \frac{\Delta y_{n_p}}{2} \leq y \leq y_{n_p} + \frac{\Delta y_{n_p}}{2} \\ 0 & \text{elsewhere.} \end{cases} \quad (2.4.18)$$

Here, $P_{\Delta x_{n_p}}$ being the pulse function centered at x_{n_p} of width Δx_{n_p} and $P_{\Delta y_{n_p}}$ being the pulse function centered at y_{n_p} of width Δy_{n_p} , as illustrated in Fig. 2.9.

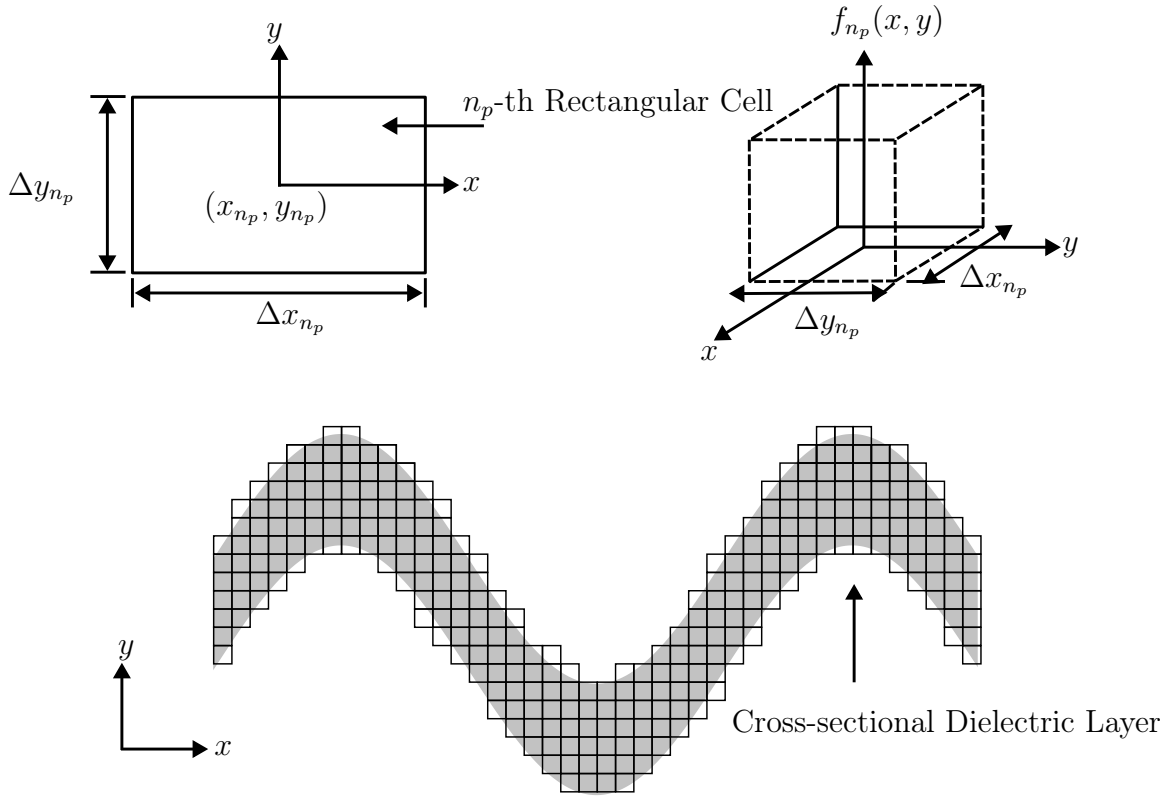


Fig. 2.9: Expansion of the equivalent polarization currents $\mathbf{J}_p$ on the dielectric cross-sectional surface using two-dimensional pulse basis functions.

2.4.2 Solution Using Point Matching Method

Point matching method is used to solve the discretized VSIEs in (2.4.5) and (2.4.6) to determine the unknown coefficients [1, p. 18] [57, pp. 198–200]. Because the equivalent currents have only z -component, we work with the scalar component of the weighting basis functions, which we denote as $W_{m_s}^{(j)}(\mathbf{r})$. The weighting basis functions $W_{m_s}^{(j)}(\mathbf{r})$ defined on the j -th impedance layer are selected as a Dirac delta function [1, p. 18] [57, pp. 198–200]

$$W_{m_s}^{(j)}(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}_{m_s+\frac{1}{2}}), \quad (2.4.19)$$

where $\mathbf{r}_{m_s+\frac{1}{2}} = x_{m_s+\frac{1}{2}}\hat{\mathbf{x}} + y_{m_s+\frac{1}{2}}\hat{\mathbf{y}}$ is the observation point located at the center of the m_s -th line segment on the j -th impedance layer with $(m_s = 1, 2, \dots, N_{s,j})$ and $(j = 1, 2, \dots, N_{\text{imp}})$. The midpoint of the m_s -th line segment $(x_{m_s+\frac{1}{2}}, y_{m_s+\frac{1}{2}})$ is given by [57, p. 223]

$$x_{m_s+\frac{1}{2}} = x_{m_s} + \frac{\Delta l_{m_s}}{2} \cos \theta_{m_s} \quad (2.4.20)$$

$$y_{m_s+\frac{1}{2}} = y_{m_s} + \frac{\Delta l_{m_s}}{2} \sin \theta_{m_s}, \quad (2.4.21)$$

2 MoM Analysis of Arbitrarily Shaped Artificial Surfaces

where (x_{m_s}, y_{m_s}) represents the starting point of the m_s -th line segment and Δl_{m_s} is the length of the m_s -th line segment. By substituting the expansion functions (2.4.12) and (2.4.16) into (2.4.5) and applying the point matching method. The impedance matrix elements Z_{m_s, n_s}^{ji} in (2.4.7) become

$$Z_{m_s, n_s}^{ji} = \frac{\omega\mu_0}{4} \int_0^{\Delta l_{m_s}^{(j)}} \delta(\mathbf{r} - \mathbf{r}_{m_s + \frac{1}{2}}) \int_0^{\Delta l_{n_s}^{(i)}} P_{n_s}^{(i)}(\mathbf{r}) H_0^{(2)}(k_0 |\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \quad (2.4.22)$$

and simplified to

$$Z_{m_s, n_s}^{ji} = \frac{\omega\mu_0}{4} \int_0^{\Delta l_{n_s}^{(i)}} H_0^{(2)}(k_0 |\mathbf{r}_{m_s + \frac{1}{2}} - \mathbf{r}'|) d\mathbf{r}' = \frac{\omega\mu_0}{4} \int_0^{\Delta l_{n_s}^{(i)}} H_0^{(2)}(k_0 R_{m_s, n_s}^{ji}) dl'. \quad (2.4.23)$$

The distance R_{m_s, n_s}^{ji} between the source and observation points is given by

$$R_{m_s, n_s}^{ji} = \left[\left((x_{m_s} + \frac{\Delta l_{m_s}}{2} \cos \theta_{m_s})^{(j)} - (x_{n_s} + l' \cos \theta_{n_s})^{(i)} \right)^2 + \left((y_{m_s} + \frac{\Delta l_{m_s}}{2} \sin \theta_{m_s})^{(j)} - (y_{n_s} + l' \sin \theta_{n_s})^{(i)} \right)^2 \right]^{\frac{1}{2}}. \quad (2.4.24)$$

Similarly, the impedance matrix elements Z_{m_s, n_p}^{jq} become

$$\begin{aligned} Z_{m_s, n_p}^{jq} &= \frac{\omega\mu_0}{4} \int_{C_{m_s}^{(j)}} \delta(\mathbf{r} - \mathbf{r}_{m_s + \frac{1}{2}}) \iint_{S_{n_p}^{(q)}} P_{n_p}^{(q)}(\mathbf{r}) H_0^{(2)}(k_0 |\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\ &= \frac{\omega\mu_0}{4} \iint_{S_{n_p}^{(q)}} H_0^{(2)}(k_0 |\mathbf{r}_{m_s + \frac{1}{2}} - \mathbf{r}'|) d\mathbf{r}'. \end{aligned} \quad (2.4.25)$$

Now, the above expression of the impedance matrix Z_{m_s, n_p}^{jq} is reduced to

$$Z_{m_s, n_p}^{jq} = \frac{\omega\mu_0}{4} \int_{y_{n_p}^{(q),-}}^{y_{n_p}^{(q),+}} \int_{x_{n_p}^{(q),-}}^{x_{n_p}^{(q),+}} H_0^{(2)}(k_0 R_{m_s, n_p}^{jq}) dx' dy', \quad (2.4.26)$$

where $(x_{n_p}^{(q),-}, y_{n_p}^{(q),-})$ and $(x_{n_p}^{(q),+}, y_{n_p}^{(q),+})$ represent, respectively, the lower and upper limits of the n_p -th rectangular element. The distance R_{m_s, n_p}^{jq} is calculated as

$$R_{m_s, n_p}^{jq} = \left[\left((x_{m_s} + \frac{\Delta l_{m_s}}{2} \cos \theta_{m_s})^{(j)} - (x')^{(q)} \right)^2 + \left((y_{m_s} + \frac{\Delta l_{m_s}}{2} \sin \theta_{m_s})^{(j)} - (y')^{(q)} \right)^2 \right]^{\frac{1}{2}}. \quad (2.4.27)$$

The weighting basis functions $W_{m_p}^{(k)}(\mathbf{r})$ defined on the k -th dielectric layer are chosen as a Dirac delta function

$$W_{m_p}^{(k)}(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}_{m_p}), \quad (2.4.28)$$

where $\mathbf{r}_{m_p} = x_{m_p} \hat{\mathbf{x}} + y_{m_p} \hat{\mathbf{y}}$ is the coordinates of the observation point at the center of the m_p -th rectangular element located on the k -th dielectric layer, with $m_p = 1, 2, \dots, N_{p,k}$ and $k = 1, 2, \dots, N_{\text{die}}$. By substituting the expansion functions (2.4.12) and (2.4.16) into the VSIE (2.4.6) and applying the point matching method. The impedance matrix elements Z_{m_p, n_s}^{ki} are given by

$$Z_{m_p, n_s}^{ki} = \frac{\omega \mu_0}{4} \int_0^{\Delta l_{n_s}^{(i)}} H_0^{(2)}(k_0 R_{m_p, n_s}^{ki}) dl', \quad (2.4.29)$$

where the distance R_{m_p, n_s}^{ki} is given by:

$$R_{m_p, n_s}^{ki} = \left[\left(x_{m_p}^{(k)} - (x_{n_s} + l' \cos \theta_{n_s})^{(i)} \right)^2 + \left(y_{m_p}^{(k)} - (y_{n_s} + l' \sin \theta_{n_s})^{(i)} \right)^2 \right]^{\frac{1}{2}}. \quad (2.4.30)$$

Similarly, the impedance matrix elements Z_{m_p, n_p}^{kq} are expressed as

$$Z_{m_p, n_p}^{kq} = \frac{\omega \mu_0}{4} \int_{y_{n_p}^{(q),-}}^{y_{n_p}^{(q),+}} \int_{x_{n_p}^{(q),-}}^{x_{n_p}^{(q),+}} H_0^{(2)}(k_0 R_{m_p, n_p}^{kq}) dx' dy', \quad (2.4.31)$$

where the distance R_{m_p, n_p}^{kq} is defined as

$$R_{m_p, n_p}^{kq} = \left[\left(x_{m_p}^{(k)} - (x')^{(q)} \right)^2 + \left(y_{m_p}^{(k)} - (y')^{(q)} \right)^2 \right]^{\frac{1}{2}}. \quad (2.4.32)$$

M-point Gaussian quadrature is employed to evaluate numerically the integrals involved in the impedance matrix elements for nonsingular case (i.e., $m \neq n$) [1, p. 403]. When the observation and source basis functions are the same (i.e., $m = n$), Hankel function has a logarithmic singularity, which requires special techniques to handle the singularity. The methods described in [1, pp. 265–266], [57] and [61, 62] are used to handle this singularity based on the small argument of the Hankel function and analytical integration.

2.4.3 Calculation of Far-field

Once the equivalent electric currents on the N_{imp} layers and the equivalent polarization currents on the cross-sectional surfaces of the dielectric layers are determined, the corresponding scattered electric field radiated by these equivalent currents in the far-field region is computed. For this purpose, the expression of the scattered electric field radiated by the equivalent electric currents on the i -th impedance layer

$$\mathbf{E}_{s,i}^{\text{scat}}(\mathbf{r}) = -\frac{\omega\mu_0}{4} \int_{C_{s,i}} \mathbf{J}_{s,i}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}', \quad (2.4.33)$$

where $C_{s,i}$ is the integration contour of the i -th arbitrarily shaped impedance sheet. The scattered electric field radiated by the equivalent polarization currents on the cross-sectional surface of the q -th dielectric layer

$$\mathbf{E}_{p,q}^{\text{scat}}(\mathbf{r}) = -\frac{\omega\mu_0}{4} \iint_{S_{p,q}} \mathbf{J}_{p,q}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}'. \quad (2.4.34)$$

Here, $S_{p,q}$ is the cross-sectional surface of the q -th dielectric layer. The term $|\mathbf{r} - \mathbf{r}'|$ is expanded in cylindrical coordinates [1, pp. 41–43]

$$\mathbf{R} = |\mathbf{r} - \mathbf{r}'| = \left[r^2 + (r')^2 - 2rr' \cos(\phi - \phi') \right]^{\frac{1}{2}}, \quad (2.4.35)$$

where $r' \cos(\phi - \phi') = \mathbf{r}' \cdot \hat{\mathbf{r}}$. In the far-field $r \gg r'$, the binomial expansion for $\mathbf{R}$ can be written as [1, pp. 41–43] [57, p. 49]:

$$\mathbf{R} \approx r - r' \cos(\phi - \phi') = r - \mathbf{r}' \cdot \hat{\mathbf{r}}. \quad (2.4.36)$$

In the above expression, the first two terms have been considered and the remaining terms are ignored due to $r \gg r'$ in the far-field. A large argument approximation of the Hankel function is used in case of $r \gg r'$ [1, p. 43]

$$H_0^{(2)}(k_0 \mathbf{R}) \approx \sqrt{\frac{2j}{\pi k_0}} \frac{e^{-jk_0 r}}{\sqrt{r}} e^{-jk_0 \mathbf{r}' \cdot \hat{\mathbf{r}}} \quad k_0 r \rightarrow \infty. \quad (2.4.37)$$

By substituting (2.4.37) into (2.4.33) and (2.4.34), we obtain expressions of the scattered fields in the far-field. For the i -th impedance layer, the scattered field due to the equivalent electric currents $\mathbf{J}_{s,i}(\mathbf{r}')$

$$\mathbf{E}_{s,i}^{\text{scat-FF}}(r, \phi) = -\frac{\omega \mu_0}{4} \sqrt{\frac{2j}{\pi k_0}} \frac{e^{-jk_0 r}}{\sqrt{r}} \int_{C_{s,i}} \mathbf{J}_{s,i}(\mathbf{r}') e^{-jk_0 \mathbf{r}' \cdot \hat{\mathbf{r}}} d\mathbf{r}'. \quad (2.4.38)$$

The scattered field due to the equivalent polarization currents $\mathbf{J}_{p,q}(\mathbf{r}')$ is expressed as

$$\mathbf{E}_{p,q}^{\text{scat-FF}}(r, \phi) = -\frac{\omega \mu_0}{4} \sqrt{\frac{2j}{\pi k_0}} \frac{e^{-jk_0 r}}{\sqrt{r}} \iint_{S_{p,q}} \mathbf{J}_{p,q}(\mathbf{r}') e^{-jk_0 \mathbf{r}' \cdot \hat{\mathbf{r}}} d\mathbf{r}'. \quad (2.4.39)$$

The expansion functions with their known expansion coefficients are substituted. The scattered field due to the equivalent currents on the impedance sheet for the i -th impedance layer

$$\mathbf{E}_{s,i}^{\text{scat-FF}}(r, \phi) = -\frac{\omega \mu_0}{4} \sqrt{\frac{2j}{\pi k_0}} \frac{e^{-jk_0 r}}{\sqrt{r}} \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{n_s}^{(i)}} e^{-jk_0 \mathbf{r}_{n_s} \cdot \hat{\mathbf{r}}} d\mathbf{r}', \quad (2.4.40)$$

where $\mathbf{r}_{n_s} = (x_{n_s} + l' \cos \theta_{n_s})^{(i)} \hat{\mathbf{x}} + (y_{n_s} + l' \sin \theta_{n_s})^{(i)} \hat{\mathbf{y}}$. The scattered field due to the equivalent polarization currents on the cross-sectional surface of q -th dielectric layer

$$\mathbf{E}_{p,q}^{\text{scat-FF}}(r, \phi) = -\frac{\omega \mu_0}{4} \sqrt{\frac{2j}{\pi k_0}} \frac{e^{-jk_0 r}}{\sqrt{r}} \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{n_p}^{(q)}} e^{-jk_0 \mathbf{r}_{n_p} \cdot \hat{\mathbf{r}}} d\mathbf{r}', \quad (2.4.41)$$

where $\mathbf{r}_{n_p} = x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}$ and $\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos(\phi) + \hat{\mathbf{y}} \sin(\phi)$. Once the scattered fields are determined, the total field is given by

$$\mathbf{E}^{\text{tot-FF}} = \mathbf{E}^{\text{inc}} + \sum_{i=1}^{N_{\text{imp}}} \mathbf{E}_{s,i}^{\text{scat-FF}} + \sum_{q=1}^{N_{\text{die}}} \mathbf{E}_{p,q}^{\text{scat-FF}}. \quad (2.4.42)$$

2.5 VSIEs: TM Polarization

Consider the problem of scattering by the two-dimensional arbitrarily shaped multilayer impedance sheets illuminated by a TM polarized incident wave as illustrated in Fig. 2.10. The incident magnetic field is oriented along the z -direction perpendicular to the modeling xy -plane, while the incident electric field possesses both x - and y -components.

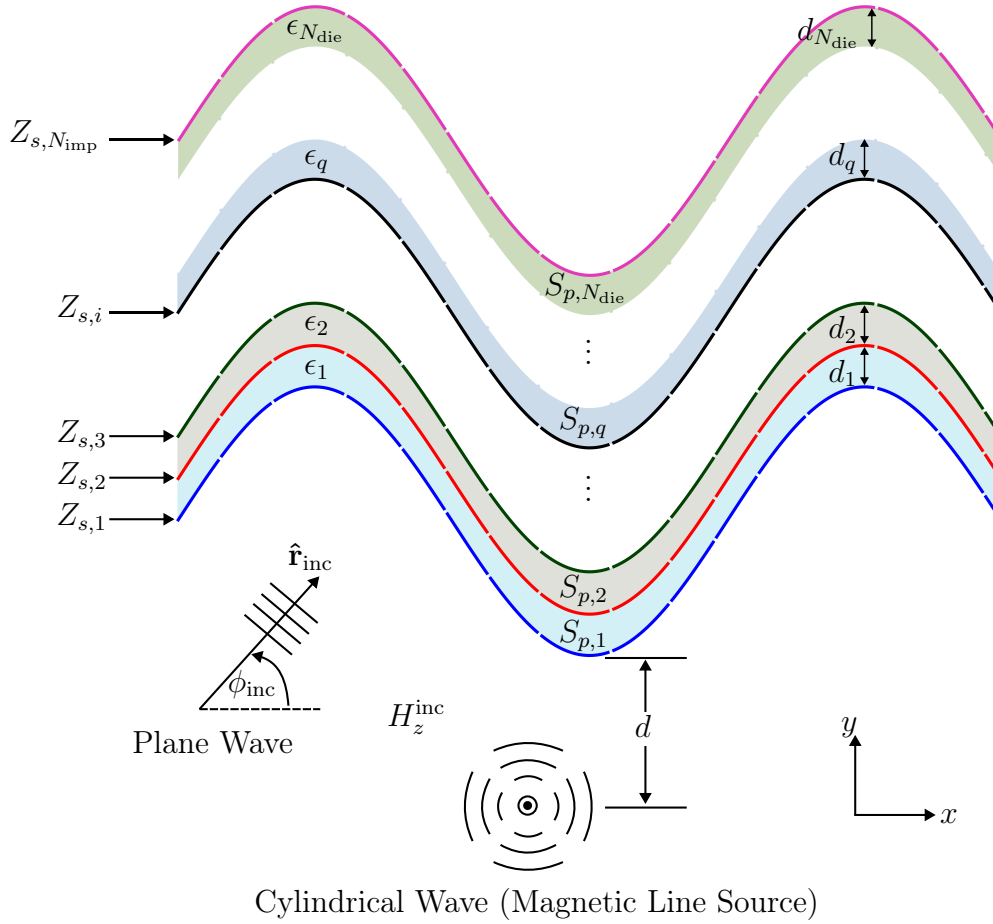


Fig. 2.10: Two-dimensional arbitrarily shaped multilayer impedance sheets separated by dielectric layers and illuminated by either a TM plane wave or a TM cylindrical wave propagating in the xy -plane. The incident electric field has x - and y -components.

Plane Wave Incidence

A plane wave propagating in the xy -plane illuminates the equivalent model at an incident angle ϕ_{inc} with respect to the x -axis, as shown in Fig. 2.10. The electric field of the incident plane wave is given by [1, p. 133]

$$\mathbf{E}^{\text{inc}}(\mathbf{r}) = E_0 e^{-jk_0 \hat{\mathbf{r}}_{\text{inc}} \cdot \mathbf{r}} = E_0 (\sin \phi_{\text{inc}} \hat{\mathbf{x}} - \cos \phi_{\text{inc}} \hat{\mathbf{y}}) e^{-jk_0 (x \cos \phi_{\text{inc}} + y \sin \phi_{\text{inc}})}, \quad (2.5.1)$$

where E_0 is the amplitude of the plane wave, $\hat{\mathbf{r}}_{\text{inc}} = \hat{\mathbf{x}} \cos \phi_{\text{inc}} + \hat{\mathbf{y}} \sin \phi_{\text{inc}}$, k_0 is the wavenumber in free-space and $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ is the observation point.

Cylindrical Wave

An infinite magnetic line source aligned to the z -axis and positioned at a distance d from the equivalent impedance model is used as an excitation source. The magnetic line source radiates a TM polarized cylindrical wave propagating in the xy -plane with magnetic field parallel to the z -axis. The incident electric field radiated by the magnetic line source has E_x and E_y field components. These field components are known at an observation point $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ [57, p. 60]

$$\begin{aligned} E_x^{\text{inc}}(\mathbf{r}) &= \frac{jk_0 I_m}{4} H_1^{(2)} \left(k_0 \sqrt{(x - x_{\text{src}})^2 + (y - y_{\text{src}})^2} \right) \frac{(x - x_{\text{src}})}{\sqrt{(x - x_{\text{src}})^2 + (y - y_{\text{src}})^2}} \\ E_y^{\text{inc}}(\mathbf{r}) &= \frac{-jk_0 I_m}{4} H_1^{(2)} \left(k_0 \sqrt{(x - x_{\text{src}})^2 + (y - y_{\text{src}})^2} \right) \frac{(y - y_{\text{src}})}{\sqrt{(x - x_{\text{src}})^2 + (y - y_{\text{src}})^2}}, \end{aligned} \quad (2.5.2)$$

where I_m is the strength of the magnetic source in Volts, k_0 is the free-space wavenumber in rad/m, $H_1^{(2)}(k_0 R)$ is the Hankel function of the second kind and first order, and $\mathbf{r}_{\text{src}} = x_{\text{src}}\hat{\mathbf{x}} + y_{\text{src}}\hat{\mathbf{y}}$ is the position of the magnetic line source. By substituting the free-space Green's function (2.3.12) into the discretized VSIEs (2.3.19) and (2.3.25), the impedance matrix elements become

$$\begin{aligned} Z_{m_s, n_s}^{ji} &= \frac{\omega \mu_0}{4} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0 |\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\ &\quad + \frac{1}{4\omega \epsilon_0} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^2(k_0 |\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' \right] d\mathbf{r}, \end{aligned} \quad (2.5.3)$$

$$\begin{aligned}
 Z_{m_s, n_p}^{jq} = & \frac{\omega\mu_0}{4} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\
 & + \frac{1}{4\omega\epsilon_0} \int_{C_{m_s}^{(j)}} \mathbf{W}_{m_s}^{(j)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' \right] d\mathbf{r}, \quad (2.5.4)
 \end{aligned}$$

$$\begin{aligned}
 Z_{m_p, n_s}^{ki} = & \frac{\omega\mu_0}{4} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\
 & + \frac{1}{4\omega\epsilon_0} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' \right] d\mathbf{r}, \quad (2.5.5)
 \end{aligned}$$

$$\begin{aligned}
 Z_{m_p, n_p}^{kq} = & \frac{\omega\mu_0}{4} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\
 & + \frac{1}{4\omega\epsilon_0} \iint_{S_{m_p}^{(k)}} \mathbf{W}_{m_p}^{(k)}(\mathbf{r}) \cdot \left[\nabla \nabla \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' \right] d\mathbf{r}. \quad (2.5.6)
 \end{aligned}$$

2.5.1 Basis Functions

This section focuses on the definition of expansion functions. Specifically, piecewise linear basis functions are used to represent the equivalent currents on the arbitrarily shaped impedance layers, while rooftop basis functions are employed to represent the equivalent polarization currents on the cross-sectional surfaces of the dielectric layers.

Piecewise Triangular Basis Functions

Each arbitrarily curved impedance layer is discretized into N_s line segments. The equivalent electric currents on each impedance layer are expanded as a sum of N_s-1 piecewise triangular basis functions as follows:

$$\mathbf{J}_s(\mathbf{r}) = \sum_{n_s=1}^{N_s-1} I_{n_s} \mathbf{T}_{n_s}(\mathbf{r}), \quad (2.5.7)$$

where $\mathbf{T}_{n_s}(\mathbf{r})$ are the piecewise triangular vector basis functions and I_{n_s} are the unknown coefficients. The piecewise triangular vector basis functions $\mathbf{T}_{n_s}(l)$ are defined on two adjacent line segments [57, p. 231]

$$\mathbf{T}_{n_s}(\mathbf{r}) = \hat{\mathbf{I}}_{n_s}^- T_{n_s}^-(l) - \hat{\mathbf{I}}_{n_s}^+ T_{n_s}^+(l), \quad (2.5.8)$$

where

$$T_{n_s}(l) = \begin{cases} T_{n_s}^-(l) = \frac{l}{\Delta l_{n_s}^-} & \text{for } 0 < l < \Delta l_{n_s}^- \text{ left segment,} \\ T_{n_s}^+(l) = \frac{\Delta l_{n_s}^+ - l}{\Delta l_{n_s}^+} & \text{for } 0 < l < \Delta l_{n_s}^+ \text{ right segment,} \\ 0 & \text{elsewhere.} \end{cases} \quad (2.5.9)$$

Here, $\Delta l_{n_s}^-$ and $\Delta l_{n_s}^+$ represent the lengths of the left and right adjacent segments. Fig. 2.11 illustrates the discretization of the arbitrarily curved impedance layer into N_s line segments and the definition of a piecewise triangular basis function over two adjacent line segments. The parametric equations corresponding to the left segment are given by [57, pp. 232–233]

$$x = x_{n_s-1} + l \cos \theta_{n_s}^-, \quad y = y_{n_s-1} + l \sin \theta_{n_s}^-. \quad (2.5.10)$$

For the right segment

$$x = x_{n_s} + l \cos \theta_{n_s}^+, \quad y = y_{n_s} + l \sin \theta_{n_s}^+. \quad (2.5.11)$$

The angles $\theta_{n_s}^-$ and $\theta_{n_s}^+$ are defined as [57, p. 233]

$$\theta_{n_s}^- = \tan^{-1} \left(\frac{y_{n_s} - y_{n-1}}{x_{n_s} - x_{n_s-1}} \right), \quad \theta_{n_s}^+ = \tan^{-1} \left(\frac{y_{n_s+1} - y_{n_s}}{x_{n_s+1} - x_{n_s}} \right). \quad (2.5.12)$$

The unit tangent vectors for the left and right segments are expressed as [57, p. 233]

$$\hat{\mathbf{I}}_{n_s}^- = \hat{\mathbf{x}} \cos \theta_{n_s}^- + \hat{\mathbf{y}} \sin \theta_{n_s}^-, \quad \hat{\mathbf{I}}_{n_s}^+ = \hat{\mathbf{x}} \cos \theta_{n_s}^+ + \hat{\mathbf{y}} \sin \theta_{n_s}^+. \quad (2.5.13)$$

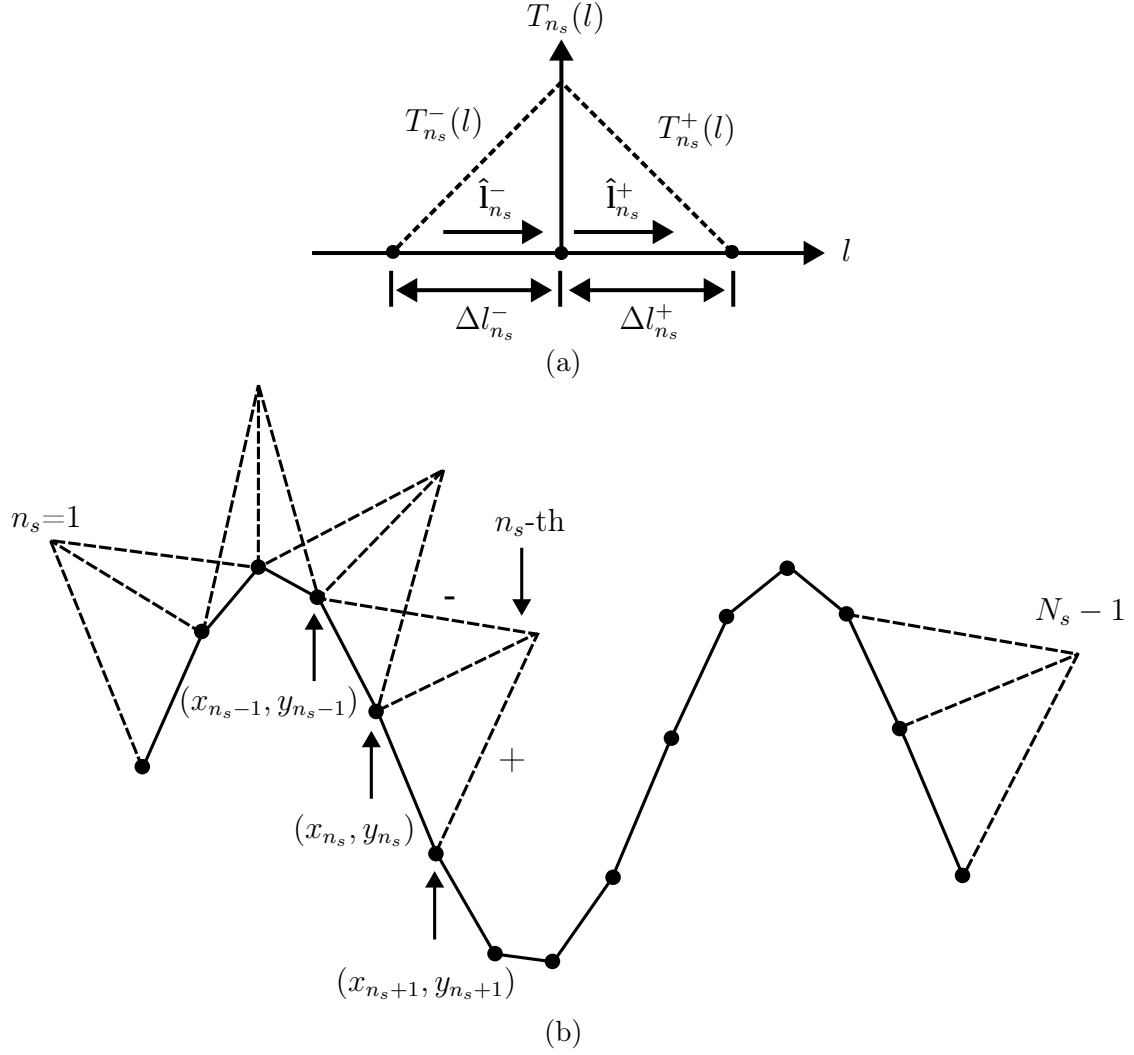


Fig. 2.11: Piecewise triangular basis functions. (a) Definition of the triangular expansion function [1, p. 92]. (b) Representation of the equivalent currents $\mathbf{J}_s$ using piecewise triangular basis functions.

RWG and Rectangular Rooftop Basis Functions

Based on the geometry of the dielectric layers, two types of expansion functions are used to represent the equivalent polarization currents $\mathbf{J}_p(\mathbf{r})$. RWG basis functions are used for curved dielectric layers, where the cross-sectional surfaces of the dielectric layers are discretized into triangular elements. In contrast, rooftop basis functions are employed for simple geometries, where the cross-sectional surfaces are discretized into quadrilateral elements.

RWG Basis Functions

The cross-sectional surface of an arbitrarily shaped dielectric layer is discretized into triangular elements as shown in Fig. 2.12b. The equivalent polarization currents on this surface are expanded as a sum of N_p basis functions

$$\mathbf{J}_p(\mathbf{r}) = \sum_{n_p=1}^{N_p} I_{n_p} \mathbf{f}_{n_p}(\mathbf{r}), \quad (2.5.14)$$

where $\mathbf{f}_{n_p}(\mathbf{r})$ are the RWG vector basis functions and I_{n_p} are unknown expansion coefficients. The RWG basis function is defined on a pair of triangular elements that share an internal edge e_{n_p} with a length L_{n_p} , as illustrated in Fig. 2.12a. The mathematical definition of the RWG basis function is written as [57, p. 268], [63, 64] :

$$\mathbf{f}_{n_p}(\mathbf{r}) = \begin{cases} \frac{L_{n_p}}{2A_{n_p}^-} \rho_{n_p}^-(\mathbf{r}) & \mathbf{r} \in S_{n_p}^-, \\ \frac{L_{n_p}}{2A_{n_p}^+} \rho_{n_p}^+(\mathbf{r}) & \mathbf{r} \in S_{n_p}^+, \\ 0 & \text{otherwise.} \end{cases} \quad (2.5.15)$$

Here, $S_{n_p}^-$ and $S_{n_p}^+$ represent the surfaces of the two adjacent triangular elements with areas $A_{n_p}^-$ and $A_{n_p}^+$, respectively. On $S_{n_p}^-$, the vector $\rho_{n_p}^-(\mathbf{r})$ starts from the free vertex $\mathbf{v}^-$ points toward the observation point $\mathbf{r}$:

$$\rho_{n_p}^-(\mathbf{r}) = \mathbf{r} - \mathbf{v}^- \quad \mathbf{r} \in S_{n_p}^-. \quad (2.5.16)$$

On $S_{n_p}^+$, the vector $\rho_{n_p}^+(\mathbf{r})$ extends from the observation point $\mathbf{r}$ toward the free vertex $\mathbf{v}^+$:

$$\rho_{n_p}^+(\mathbf{r}) = \mathbf{v}^+ - \mathbf{r} \quad \mathbf{r} \in S_{n_p}^+. \quad (2.5.17)$$

The divergence of the RWG basis function is given by [1, p. 256], [64] :

$$\nabla \cdot \mathbf{f}_{n_p}(\mathbf{r}) = \begin{cases} \frac{L_{n_p}}{A_{n_p}^-} & \mathbf{r} \in S_{n_p}^-, \\ \frac{-L_{n_p}}{A_{n_p}^+} & \mathbf{r} \in S_{n_p}^+, \\ 0 & \text{otherwise.} \end{cases} \quad (2.5.18)$$

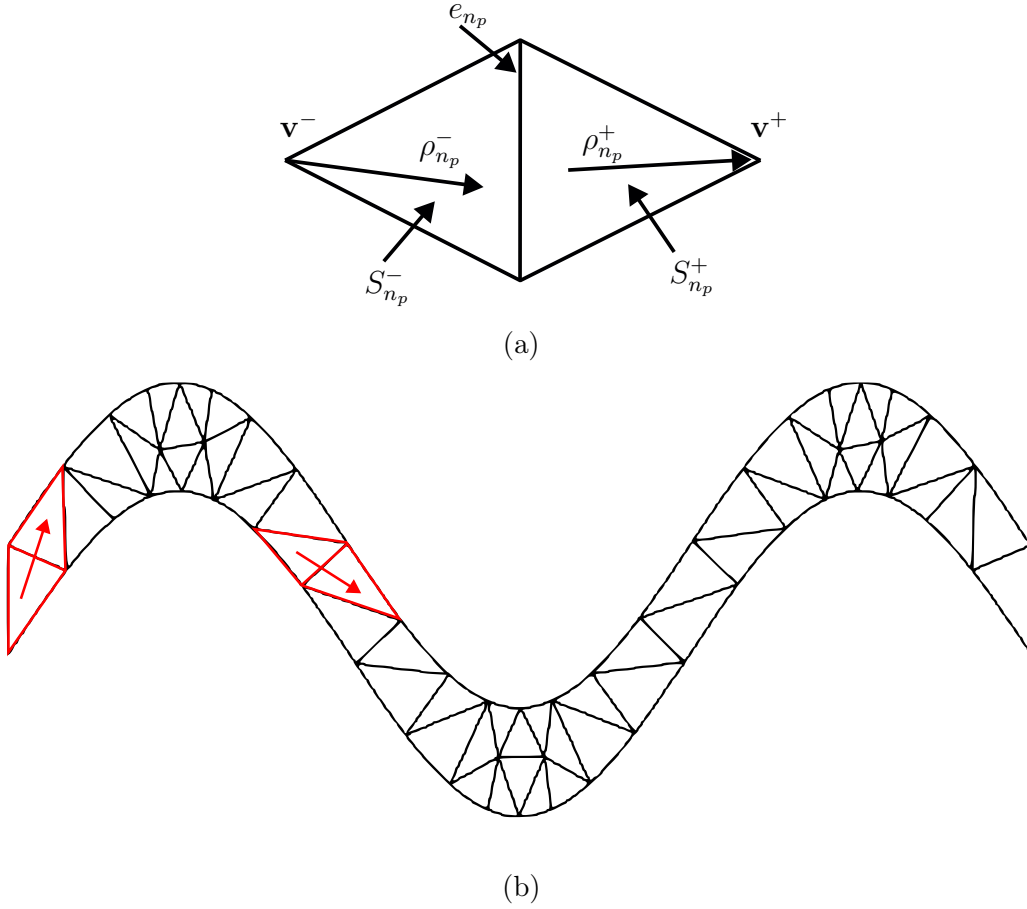


Fig. 2.12: (a) RWG basis function defined on two adjacent triangular elements $S_{n_p}^-$ and $S_{n_p}^+$ with an internal edge e_{n_p} . (b) Discretization of the cross-sectional surface of an arbitrarily shaped dielectric layer into triangular elements.

Rooftop Basis Functions

The cross-sectional surface of the dielectric layer is discretized into quadrilateral elements as shown in Fig. 2.13b. The length and width of each quadrilateral element are denoted by Δx_{n_p} and Δy_{n_p} , respectively.

The rooftop basis function is defined on a pair of adjacent quadrilateral elements, represented by $S_{n_p}^+$ and $S_{n_p}^-$, which share a common internal edge denoted as e_{n_p} [57, pp. 285–286],[65, 66]. The rooftop basis function is mathematically expressed as [57, pp. 285–286],[65, 66]

$$\mathbf{f}_{n_p}(\mathbf{r}) = \begin{cases} \frac{1}{L_{n_p}^-} \rho_{n_p}^-(\mathbf{r}) \cdot \hat{\mathbf{u}}^- & \mathbf{r} \in S_{n_p}^-, \\ \frac{1}{L_{n_p}^+} \rho_{n_p}^+(\mathbf{r}) \cdot \hat{\mathbf{u}}^+ & \mathbf{r} \in S_{n_p}^+, \\ 0 & \text{otherwise.} \end{cases} \quad (2.5.19)$$

Here, $L_{n_p}^-$ and $L_{n_p}^+$ are the lengths of the two quadrilateral elements, while $\rho_{n_p}^-(\mathbf{r})$ and $\rho_{n_p}^+(\mathbf{r})$ are piecewise linear vector basis functions that change linearly and vanish outside the surfaces $S_{n_p}^-$ and $S_{n_p}^+$. For the x-component, the unit vectors $\hat{\mathbf{u}}^\pm$ are aligned in the x -direction and the lengths $L_{n_p}^\pm$ are equal to Δx_{n_p} . The rooftop basis function for the x-component is written as:

$$\mathbf{f}_{n_p}(\mathbf{r}) = \begin{cases} \frac{1}{\Delta x_{n_p}} \rho_{n_p}^-(\mathbf{r}) \cdot \hat{\mathbf{x}} & \mathbf{r} \in S_{n_p}^-, \\ \frac{1}{\Delta x_{n_p}} \rho_{n_p}^+(\mathbf{r}) \cdot \hat{\mathbf{x}} & \mathbf{r} \in S_{n_p}^+, \\ 0 & \text{otherwise.} \end{cases} \quad (2.5.20)$$

The divergence of the rooftop basis function in the x -direction is given by

$$\nabla \cdot \mathbf{f}_{n_p}(\mathbf{r}) = \begin{cases} \frac{1}{\Delta x_{n_p}} & \mathbf{r} \in S_{n_p}^-, \\ \frac{-1}{\Delta x_{n_p}} & \mathbf{r} \in S_{n_p}^+. \end{cases} \quad (2.5.21)$$

For the y-component, the unit vectors $\hat{\mathbf{u}}^\pm$ are aligned in the y -direction and the lengths $L_{n_p}^\pm$ are equal to Δy_{n_p} . The rooftop basis function for the y-component becomes:

$$\mathbf{f}_{n_p}(\mathbf{r}) = \begin{cases} \frac{1}{\Delta y_{n_p}} \rho_{n_p}^-(\mathbf{r}) \cdot \hat{\mathbf{y}} & \mathbf{r} \in S_{n_p}^-, \\ \frac{1}{\Delta y_{n_p}} \rho_{n_p}^+(\mathbf{r}) \cdot \hat{\mathbf{y}} & \mathbf{r} \in S_{n_p}^+. \\ 0 & \text{otherwise.} \end{cases} \quad (2.5.22)$$

The divergence of the rooftop basis function for the y -directed equivalent polarization currents is given by

$$\nabla \cdot \mathbf{f}_{n_p}(\mathbf{r}) = \begin{cases} \frac{1}{\Delta y_{n_p}} & \mathbf{r} \in S_{n_p}^-, \\ \frac{-1}{\Delta y_{n_p}} & \mathbf{r} \in S_{n_p}^+. \end{cases} \quad (2.5.23)$$

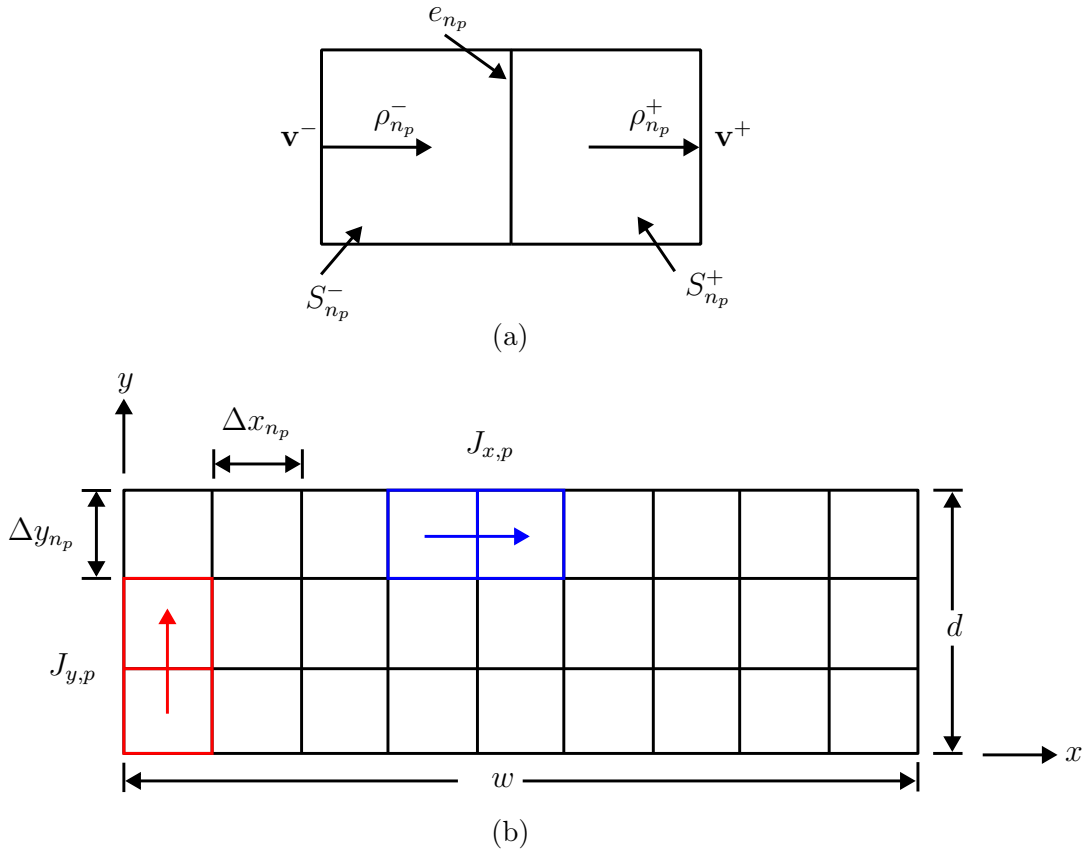


Fig. 2.13: (a) Rooftop basis function defined on two adjacent quadrilateral elements $S_{n_p}^-$ and $S_{n_p}^+$. (b) Discretization of the cross-sectional surface of a flat dielectric layer into quadrilateral elements.

2.5.2 Solution Using Galerkin's Method

Galerkin's method is used to solve the discretized VSIEs in case of TM excitation, where the weighting basis functions are chosen to be same as the expansion functions [1, 57]. The weighting basis function $\mathbf{W}_{m_s}^{(j)}(\mathbf{r})$ defined on the j -th impedance layer is chosen to be the same as the piecewise triangular basis function $\mathbf{T}_{n_s}^{(j)}(\mathbf{r})$:

$$\mathbf{W}_{m_s}^{(j)}(\mathbf{r}) = \mathbf{T}_{m_s}^{(j)}(\mathbf{r}). \quad (2.5.24)$$

By applying the vector identity and divergence theorem [1, p. 34], we transfer the gradient operator onto the testing function $\mathbf{T}_{m_s}^{(j)}(\mathbf{r})$. Now, the impedance matrix elements in Eqs. 2.3.21 and 2.3.22, representing the self and mutual interactions between the impedance and dielectric layers, are reformulated and become:

$$\begin{aligned} Z_{m_s, n_s}^{ji} = & \frac{\omega\mu_0}{4} \int_{C_{m_s}^{(j)}} \mathbf{T}_{m_s}^{(j)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\ & - \frac{1}{4\omega\epsilon_0} \int_{C_{m_s}^{(j)}} \nabla \cdot \mathbf{T}_{m_s}^{(j)}(\mathbf{r}) \int_{C_{n_s}^{(i)}} \nabla' \cdot \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \end{aligned} \quad (2.5.25)$$

$$\begin{aligned} Z_{m_s, n_p}^{jq} = & \frac{\omega\mu_0}{4} \int_{C_{m_s}^{(j)}} \mathbf{T}_{m_s}^{(j)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\ & - \frac{1}{4\omega\epsilon_0} \int_{C_{m_s}^{(j)}} \nabla \cdot \mathbf{T}_{m_s}^{(j)}(\mathbf{r}) \iint_{S_{n_p}^{(q)}} \nabla' \cdot \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}. \end{aligned} \quad (2.5.26)$$

The weighting function $\mathbf{W}_{m_p}^{(k)}$ defined on the k -th dielectric layer is selected to be the same as the RWG or rectangular rooftop basis functions $\mathbf{f}_{m_p}^{(k)}(\mathbf{r})$:

$$\mathbf{W}_{m_p}^{(k)}(\mathbf{r}) = \mathbf{f}_{m_p}^{(k)}(\mathbf{r}). \quad (2.5.27)$$

The impedance matrix elements in Eqs. 2.3.27 and 2.3.28, which represent the mutual interactions Z_{m_p, n_s}^{ki} and self interactions Z_{m_p, n_p}^{kq} between the impedance layers and the dielectric layers, are given by:

$$\begin{aligned}
 Z_{m_p, n_s}^{ki} = & \frac{\omega\mu_0}{4} \iint_{S_{m_p}^{(k)}} \mathbf{f}_{m_p}^{(k)}(\mathbf{r}) \cdot \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\
 & - \frac{1}{4\omega\epsilon_0} \iint_{S_{m_p}^{(k)}} \nabla \cdot \mathbf{f}_{m_p}^{(k)}(\mathbf{r}) \int_{C_{n_s}^{(i)}} \nabla' \cdot \mathbf{T}_{n_s}^{(i)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \quad (2.5.28)
 \end{aligned}$$

$$\begin{aligned}
 Z_{m_p, n_p}^{kq} = & \frac{\omega\mu_0}{4} \iint_{S_{m_p}^{(k)}} \mathbf{f}_{m_p}^{(k)}(\mathbf{r}) \cdot \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\
 & - \frac{1}{4\omega\epsilon_0} \iint_{S_{m_p}^{(k)}} \nabla \cdot \mathbf{f}_{m_p}^{(k)}(\mathbf{r}) \iint_{S_{n_p}^{(q)}} \nabla' \cdot \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}. \quad (2.5.29)
 \end{aligned}$$

2.5.3 Evaluation of Impedance Matrix Elements

Arbitrarily Curved Impedance Layers

To evaluate the impedance matrix elements Z_{m_s, n_s} given in Eq. 2.5.25, we substitute the expansion function $\mathbf{T}_{n_s}(\mathbf{r})$ given in Eq. 2.5.9 and the weighting function $\mathbf{T}_{m_s}(\mathbf{r})$. The impedance matrix elements integrals can be evaluated as sum of four integrals [57, p. 236]:

$$Z_{m_s, n_s} = I_{T_{m_s}^- T_{n_s}^-} + I_{T_{m_s}^- T_{n_s}^+} + I_{T_{m_s}^+ T_{n_s}^-} + I_{T_{m_s}^+ T_{n_s}^+}, \quad (2.5.30)$$

where

$$\begin{aligned}
 I_{T_{m_s}^- T_{n_s}^-} = & \int_0^{\Delta l_{m_s}^-} \int_0^{\Delta l_{n_s}^-} \left[(\hat{\mathbf{l}}_{m_s}^- \cdot \hat{\mathbf{l}}_{n_s}^-) \frac{l}{\Delta l_{m_s}^-} \frac{l'}{\Delta l_{n_s}^-} \right. \\
 & \left. + \frac{1}{\Delta l_{m_s}^- \Delta l_{n_s}^-} \right] H_0^{(2)}(k_0 R_{T_{m_s}^- T_{n_s}^-}) dl' dl, \quad (2.5.31)
 \end{aligned}$$

$$\begin{aligned}
 I_{T_{m_s}^- T_{n_s}^+} = & \int_0^{\Delta l_{m_s}^-} \int_0^{\Delta l_{n_s}^+} \left[(\hat{\mathbf{l}}_{m_s}^- \cdot \hat{\mathbf{l}}_{n_s}^+) \frac{l}{\Delta l_{m_s}^-} \left\{ \frac{\Delta l_{n_s}^+ - l'}{\Delta l_{n_s}^+} \right\} \right. \\
 & \left. - \frac{1}{\Delta l_{m_s}^- \Delta l_{n_s}^+} \right] H_0^{(2)}(k_0 R_{T_{m_s}^- T_{n_s}^+}) dl' dl, \quad (2.5.32)
 \end{aligned}$$

$$\begin{aligned}
 I_{T_{m_s}^+ T_{n_s}^-} = & \int_0^{\Delta l_{m_s}^+} \int_0^{\Delta l_{n_s}^-} \left[(\hat{\mathbf{l}}_{m_s}^+ \cdot \hat{\mathbf{l}}_{n_s}^-) \left\{ \frac{\Delta l_{m_s}^+ - l}{\Delta l_{m_s}^+} \right\} \frac{l'}{\Delta l_{n_s}^-} \right. \\
 & \left. - \frac{1}{\Delta l_{m_s}^+ \Delta l_{n_s}^-} \right] H_0^{(2)}(k_0 R_{T_{m_s}^+ T_{n_s}^-}) dl' dl, \quad (2.5.33)
 \end{aligned}$$

$$\begin{aligned}
 I_{T_{m_s}^+ T_{n_s}^+} = & \int_0^{\Delta l_{m_s}^+} \int_0^{\Delta l_{n_s}^+} \left[(\hat{\mathbf{l}}_{m_s}^+ \cdot \hat{\mathbf{l}}_{n_s}^+) \left\{ \frac{\Delta l_{m_s}^+ - l}{\Delta l_{m_s}^+} \right\} \left\{ \frac{\Delta l_{n_s}^+ - l'}{\Delta l_{n_s}^+} \right\} \right. \\
 & \left. + \frac{1}{\Delta l_{m_s}^+ \Delta l_{n_s}^+} \right] H_0^{(2)}(k_0 R_{T_{m_s}^+ T_{n_s}^+}) dl' dl. \quad (2.5.34)
 \end{aligned}$$

The distance between the source and observation points on the basis functions are given by [57, p. 236]:

$$\begin{aligned}
 R_{T_{m_s}^- T_{n_s}^-} = & \left[(x_{m_s-1}^- + l \cos \theta_{m_s}^- - x_{n_s-1}^- - l' \cos \theta_{n_s}^-)^2 \right. \\
 & \left. + (y_{m_s-1}^- + l \sin \theta_{m_s}^- - y_{n_s-1}^- - l' \sin \theta_{n_s}^-)^2 \right]^{1/2}, \quad (2.5.35)
 \end{aligned}$$

$$\begin{aligned}
 R_{T_{m_s}^- T_{n_s}^+} = & \left[(x_{m_s-1}^- + l \cos \theta_{m_s}^- - x_{n_s}^+ - l' \cos \theta_{n_s}^+)^2 \right. \\
 & \left. + (y_{m_s-1}^- + l \sin \theta_{m_s}^- - y_{n_s}^+ - l' \sin \theta_{n_s}^+)^2 \right]^{1/2}, \quad (2.5.36)
 \end{aligned}$$

$$\begin{aligned}
 R_{T_{m_s}^+ T_{n_s}^-} = & \left[(x_{m_s}^+ + l \cos \theta_{m_s}^+ - x_{n_s-1}^- - l' \cos \theta_{n_s}^-)^2 \right. \\
 & \left. + (y_{m_s}^+ + l \sin \theta_{m_s}^+ - y_{n_s-1}^- - l' \sin \theta_{n_s}^-)^2 \right]^{1/2}, \quad (2.5.37)
 \end{aligned}$$

$$\begin{aligned}
 R_{T_{m_s}^+ T_{n_s}^+} = & \left[(x_{m_s}^+ + l \cos \theta_{m_s}^+ - x_{n_s}^+ - l' \cos \theta_{n_s}^+)^2 \right. \\
 & \left. + (y_{m_s}^+ + l \sin \theta_{m_s}^+ - y_{n_s}^+ - l' \sin \theta_{n_s}^+)^2 \right]^{1/2}. \quad (2.5.38)
 \end{aligned}$$

Arbitrarily Curved Dielectric Layers

The evaluation of the impedance matrix elements Z_{m_p, n_p} given in Eq. 2.5.29 involves the evaluation of four double surface integrals [57, p. 269] as follows:

$$Z_{m_p, n_p} = I_{S_{m_p}^- S_{n_p}^-} + I_{S_{m_p}^- S_{n_p}^+} + I_{S_{m_p}^+ S_{n_p}^-} + I_{S_{m_p}^+ S_{n_p}^+}, \quad (2.5.39)$$

where

$$\begin{aligned} I_{S_{m_p}^- S_{n_p}^-} &= \frac{\omega\mu_0}{4} \iint_{S_{m_p}^-} \mathbf{f}_{m_p}^-(\mathbf{r}) \cdot \iint_{S_{n_p}^-} \mathbf{f}_{n_p}^-(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\ &\quad - \frac{1}{4\omega\epsilon_0} \iint_{S_{m_p}^-} \nabla \cdot \mathbf{f}_{m_p}^-(\mathbf{r}) \iint_{S_{n_p}^-} \nabla' \cdot \mathbf{f}_{n_p}^-(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \end{aligned} \quad (2.5.40)$$

$$\begin{aligned} I_{S_{m_p}^- S_{n_p}^+} &= \frac{\omega\mu_0}{4} \iint_{S_{m_p}^-} \mathbf{f}_{m_p}^-(\mathbf{r}) \cdot \iint_{S_{n_p}^+} \mathbf{f}_{n_p}^+(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\ &\quad - \frac{1}{4\omega\epsilon_0} \iint_{S_{m_p}^-} \nabla \cdot \mathbf{f}_{m_p}^-(\mathbf{r}) \iint_{S_{n_p}^+} \nabla' \cdot \mathbf{f}_{n_p}^+(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \end{aligned} \quad (2.5.41)$$

$$\begin{aligned} I_{S_{m_p}^+ S_{n_p}^-} &= \frac{\omega\mu_0}{4} \iint_{S_{m_p}^+} \mathbf{f}_{m_p}^+(\mathbf{r}) \cdot \iint_{S_{n_p}^-} \mathbf{f}_{n_p}^-(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\ &\quad - \frac{1}{4\omega\epsilon_0} \iint_{S_{m_p}^+} \nabla \cdot \mathbf{f}_{m_p}^+(\mathbf{r}) \iint_{S_{n_p}^-} \nabla' \cdot \mathbf{f}_{n_p}^-(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}, \end{aligned} \quad (2.5.42)$$

$$\begin{aligned} I_{S_{m_p}^+ S_{n_p}^+} &= \frac{\omega\mu_0}{4} \iint_{S_{m_p}^+} \mathbf{f}_{m_p}^+(\mathbf{r}) \cdot \iint_{S_{n_p}^+} \mathbf{f}_{n_p}^+(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r} \\ &\quad - \frac{1}{4\omega\epsilon_0} \iint_{S_{m_p}^+} \nabla \cdot \mathbf{f}_{m_p}^+(\mathbf{r}) \iint_{S_{n_p}^+} \nabla' \cdot \mathbf{f}_{n_p}^+(\mathbf{r}') H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}' d\mathbf{r}. \end{aligned} \quad (2.5.43)$$

The expansion functions $\mathbf{f}_{n_p}^-(\mathbf{r})$ and $\mathbf{f}_{n_p}^+(\mathbf{r})$ defined on $S_{n_p}^-$ and $S_{n_p}^+$ can be either RWG basis functions explained in 2.5.1 or rectangular rooftop basis functions explained in 2.5.1.

The nonsingular integrals are evaluated numerically using M-point Gaussian quadrature [1, p. 403], [57, p. 337]. For the evaluation of self and near terms, where the Hankel function exhibits a logarithmic singularity, the integrals are evaluated analytically based on the method described in [1, pp. 265–266], [61, 62].

2.5.4 Calculation of Far-field

Once the equivalent electric currents on the arbitrarily curved impedance sheets and the equivalent polarization currents within the dielectric layers are determined, the scattered fields in the far-field region can be determined. This is achieved by substituting the large argument approximation of the Hankel function expressed in Eq. 2.4.37 into the expression of the scattered field due to the equivalent electric currents on the i -th impedance layer expressed in Eq. 2.4.33, this yields the following expression [57, pp. 64–65]:

$$\mathbf{E}_{s,i}^{\text{scat-FF}}(r, \phi) = \frac{-\omega\mu_0}{4} \sqrt{\frac{2j}{\pi k}} \frac{e^{-jkr}}{\sqrt{r}} \int_{C_{s,i}} \mathbf{J}_{s,i}(\mathbf{r}') e^{jk(\mathbf{r}' \cdot \hat{\mathbf{r}})} d\mathbf{r}'. \quad (2.5.44)$$

Using the expression of the equivalent electric currents $\mathbf{J}_{s,i}$ defined in Eq. 2.5.7, the scattered field due to the equivalent electric currents $\mathbf{J}_{s,i}$ in the far-field region becomes:

$$\mathbf{E}_{s,i}^{\text{scat-FF}}(r, \phi) = \frac{-\omega\mu_0}{4} \sqrt{\frac{2j}{\pi k}} \frac{e^{-jkr}}{\sqrt{r}} \sum_{n_s=1}^{N_{s,i}} I_{n_s}^{(i)} \int_{C_{n_s}^{(i)}} \mathbf{T}_{n_s}^{(i)}(l') e^{jk(\mathbf{r}' \cdot \hat{\mathbf{r}})} dl', \quad (2.5.45)$$

where $\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi$ and $\mathbf{r}' = x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}$. The resulting integral can be evaluated using M-point Gaussian quadrature [1, p. 403]. Similarly, the scattered far-field due to the equivalent polarization currents within the q -th dielectric layer can be calculated according to the following equation [57, pp. 64–65]:

$$\mathbf{E}_{p,q}^{\text{scat-FF}}(r, \phi) = \frac{-\omega\mu_0}{4} \sqrt{\frac{2j}{\pi k}} \frac{e^{-jkr}}{\sqrt{r}} \iint_{S_{p,q}} \mathbf{J}_{p,q}(\mathbf{r}') e^{jk(\mathbf{r}' \cdot \hat{\mathbf{r}})} d\mathbf{r}'. \quad (2.5.46)$$

Using triangular or rectangular rooftop expansion functions defined in Eq. 2.5.14 into Eq. 2.5.46, we obtain:

$$\mathbf{E}_{p,q}^{\text{scat-FF}}(r, \phi) = \frac{-\omega\mu_0}{4} \sqrt{\frac{2j}{\pi k}} \frac{e^{-jkr}}{\sqrt{r}} \sum_{n_p=1}^{N_{p,q}} I_{n_p}^{(q)} \iint_{S_{n_p}^{(q)}} \mathbf{f}_{n_p}^{(q)}(\mathbf{r}') e^{jk(\mathbf{r}' \cdot \hat{\mathbf{r}})} d\mathbf{r}'. \quad (2.5.47)$$

The resulting integral is evaluated using M-point Gaussian quadrature [1, pp. 404–405]. Now, the total far-field is equal to the sum of the incident and scattered fields

$$\mathbf{E}^{\text{tot-FF}}(\mathbf{r}) = \mathbf{E}^{\text{inc}}(\mathbf{r}) + \sum_{i=1}^{N_{\text{imp}}} \mathbf{E}_{s,i}^{\text{scat-FF}}(\mathbf{r}) + \sum_{q=1}^{N_{\text{die}}} \mathbf{E}_{p,q}^{\text{scat-FF}}(\mathbf{r}). \quad (2.5.48)$$

2.6 Summary

A volume-surface integral equation is formulated to analyze the electromagnetic scattering by two-dimensional arbitrarily curved impedance sheets separated by dielectric layers equivalent to the three-dimensional problem. The continuous volume surface integral equations are discretized and solved numerically using MoM for TE_z and TM_z excitations. For TE_z excitation, the equivalent electric currents on the arbitrarily curved impedance sheets are expanded into one-dimensional pulse basis functions. The equivalent polarization currents within the dielectric layers are expanded into two-dimensional pulse basis functions. Point matching method is used to solve the discretized VSIEs to form a system of linear equations, where the unknown current expansion coefficients are determined.

For TM_z excitation, the equivalent electric currents on the arbitrarily curved impedance sheets are expanded into one-dimensional piecewise triangular basis functions. The equivalent polarization currents within the dielectric layers are expanded using rooftop basis functions for planar surfaces and RWG basis functions for curved surfaces. Galerkin's method is used to solve the discretized VSIEs to form a system of linear equations, where the unknown current expansion coefficients are determined.

Analysis and Modeling of Guided Waves on Single Artificial Surfaces

In this chapter, we will validate the proposed two-dimensional MoM modeling method against full-wave simulations by analyzing artificial surfaces that support the propagation of surface wave modes along their surfaces. We consider a printed Yagi-Uda-type director array as an illustrative example for validation. Before going into the analysis of these structures using our two-dimensional MoM modeling method, we will first present an overview of their functions and practical configurations.

Maritime radar antennas require fan-beam radiation patterns characterized by a very narrow half power beam-width (HPBW) in the azimuth plane and a relatively narrow HPBW in the elevation plane with high gain and low sidelobe level (SLL) [67, 68]. The narrow HPBW in the elevation plane is necessary to cope with the movements of the ship platform [32, 68]. Slotted waveguide antenna arrays (SWAs) are widely used in maritime radar applications because they provide excellent features such as high gain, low losses, low SLL and ease of fabrication [32]. As needed, they produce a very narrow beam in the azimuth plane (along the array) but in the elevation plane, the isolated slotted waveguide antenna, produces very broad beam. For this reason, printed Yagi-Uda-type director arrays and dielectric slab waveguide structures are used to shape the radiation patterns of maritime radar antennas in the elevation plane [68, 69].

Fig. 3.1 shows the configuration of a dual-band antenna for maritime harmonic radar. The proposed antenna is used for an innovative maritime search and rescue system, which uses harmonic radar technology to locate passive and active transponders integrated into life jackets [68–70]. The transponders receive horizontally polarized waves in the S-band and re-radiate a part of the received power at the second harmonic frequency in the C-band with vertical polarization [68, 69]. The maritime harmonic radar antenna structure consists of two main parts, an S-band antenna array with horizontal polarization and a C-band antenna array with vertical polarization. The S-band antenna array operates at the center frequency of 2.925 GHz with a $1.9^\circ \times 26^\circ$ HPBW.

It is implemented as a one-dimensional array of 44 inclined slot radiators cut in the narrow wall of a standard WR-284 rectangular waveguide. A polarization grid is used as a polarization filter to reduce the level of the cross-polarization components. A planar printed array, consisting of 44 identical one-dimensional tapered printed Yagi-Uda-type director arrays arranged side by side, is positioned in front of the 44 inclined slot radiators to narrow the HPBW in the elevation plane. The S-band antenna is fed by an N-type connector through a coaxial-to-waveguide transition on one side, and terminated by a $50\ \Omega$ matched load connected to the other side of the waveguide. The C-band antenna operates at the center frequency of 5.85 GHz with a $1.9^\circ \times 25^\circ$ HPBW. It is based on a number of longitudinal slots cut in the broad wall of a standard WR-137 rectangular waveguide. A tapered dielectric slab waveguide is efficiently fed with the help of a short flared horn structure to reduce the HPBW of the C-band slot array antenna in the elevation plane. Similar to the S-band antenna, the C-band slot array antenna is fed by an N-type connector through a coaxial-to-waveguide transition on one side and terminated by a $50\ \Omega$ matched load connected to the other side of the waveguide. Both S- and C-band waveguide slot arrays share the same housing, where the C-band antenna is mounted on top of the S-band antenna [68].

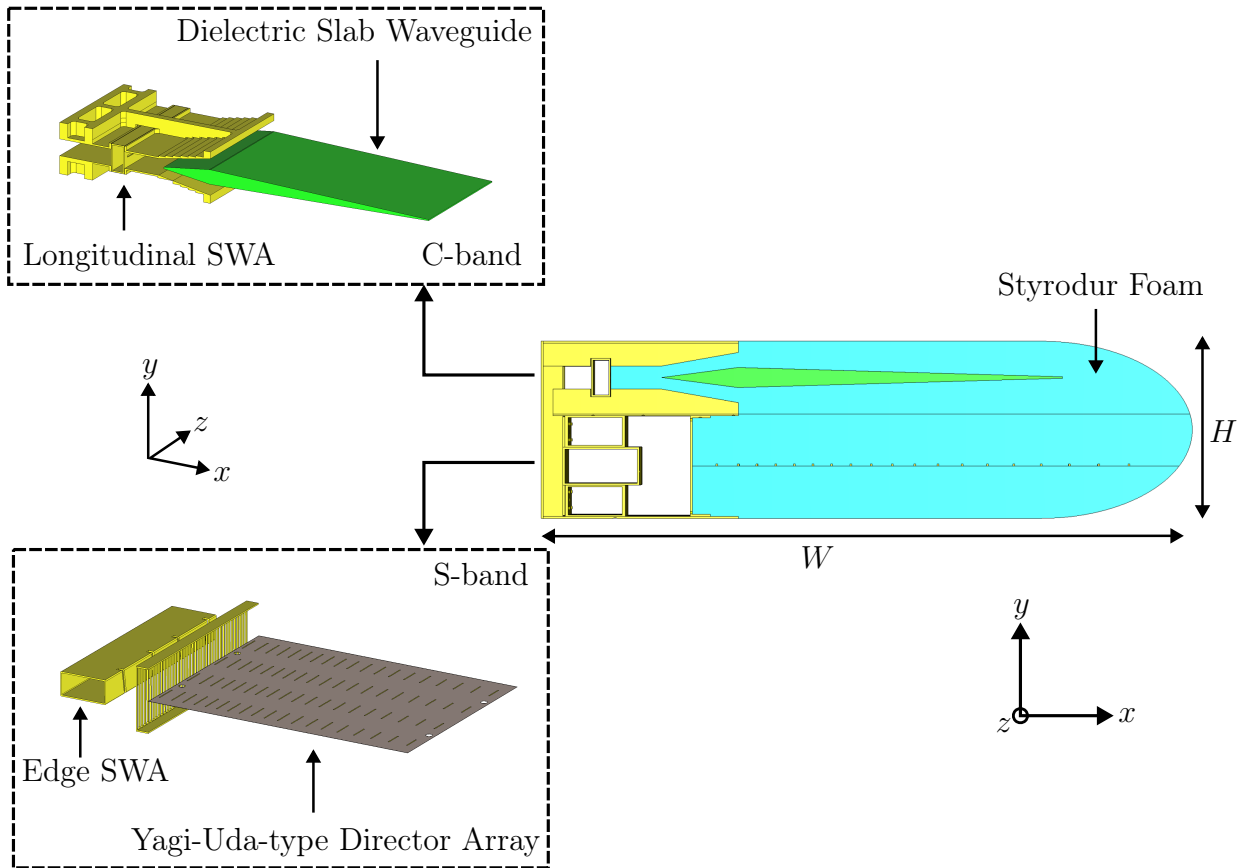


Fig. 3.1: Vertical cross-sectional view of the proposed dual-band antenna for maritime harmonic radar, consisting of S-band and C-band antennas.

The goal is to validate the two-dimensional MoM modeling method by determining the far-field radiation properties of the S-band antenna including the printed Yagi-Uda-type director array in the elevation plane. Commercial simulation software CST Studio Suite is used to validate the far-field radiation obtained by the MoM. By comparing the results obtained from the MoM solution with those from the full-wave simulations, the efficiency and accuracy of the modeling method can be evaluated.

3.1 Design of Printed Yagi-Uda-type Director Array

A conventional Yagi-Uda antenna is widely used in TV and amateur radio applications because of its simplicity and high directivity. It typically consists of an array of linear dipoles, including a driven element and several parasitic elements (reflector and director elements). It radiates end-fire radiation pattern [71, pp. 577-580].

In this work, a printed Yagi-Uda-type director array is designed to shape the radiation pattern of an S-band SWA [32, 68]. Fig. 3.2a shows a vertical cross-sectional view of the S-band antenna configuration including SWA array cut into the narrow wall of an S-band rectangular waveguide, metallic rectangular housing and printed Yagi-Uda-type director array. The printed Yagi-Uda-type director consists of 18 metallic strips, referred to as parasitic directors, parallel to each other along a straight line in the x -direction, as shown in Fig. 3.2b. The directors are printed on a Rogers substrate material with a permittivity of 3.48 and a thickness of 0.254 mm. The design begins by determining the length of the printed Yagi-Uda-type director array. The length of the array is chosen based on the Hansen-Woodyard (H-W) condition to achieve the maximum directivity in the endfire direction. The theoretical HPBW of an H-W endfire antenna can be determined using the following equation [71, pp. 305-309][68]:

$$\theta_{3dB} = 2 \cos^{-1} \left(1 - 0.1398 \frac{\lambda_0}{L_s} \right), \quad (3.1.1)$$

where L_s represents the length of the Yagi-Uda-type director array and λ_0 is the free-space wavelength. Eq. 3.1.1 is used as an initial design step to determine the length of the array to achieve a 27° HPBW in the elevation plane. Based on this equation and the desired HPBW in the elevation plane, the total length of the printed array is chosen to be 430 mm, which corresponds to $4\lambda_0$, where λ_0 is the free-space wavelength at 2.925 GHz. The propagation constant β_{H-W} that meets the H-W condition of the TE_0 surface wave mode which propagates along the structure needs to be determined. This propagation constant can be calculated using the following equation [68, 72]:

$$\beta_{H-W} = k_0 + \frac{2.94}{L_s}, \quad (3.1.2)$$

where k_0 is the wave number in free-space. Based on Eq 3.1.2 and the length of the printed Yagi-Uda-type director array ($L_s = 430$ mm), the propagation constant β_{H-W} is

3 Analysis and Modeling of Guided Waves on Artificial Surfaces

equal to 68 rad/m. In order to reduce the SLL in the elevation plane, the length of each director and the distances between directors are linearly tapered along the direction of propagation [32, 68]. This tapering changes the propagation constant β_x slowly towards the end of the array. In other words, tapering the propagation constant β_x along the length of the array produces a non-uniform amplitude distribution along the length of the Yagi-Uda-type director array [32, 68]. It is necessary to determine the propagation constant β_x to optimize the geometrical dimensions of the tapered director array. This is achieved through eigenmode analysis, described in detail in Section 3.2.1. Tables 3.1 and 3.2 list the geometrical dimensions of the proposed printed Yagi-Uda-type director array.

Table 3.1: Center-to-center spacing between directors [mm].

S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
18.6	18.6	18.6	18.6	18.6	19.2	20.2	21.2	22.2
S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	
23.2	24.2	25.2	26.2	27.2	28.2	29.2	30.2	

Table 3.2: Lengths of director elements [mm].

L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9
39	38.5	38	37.5	37	36.5	36	35.5	35
L_{10}	L_{11}	L_{12}	L_{13}	L_{14}	L_{15}	L_{16}	L_{17}	L_{18}
34.5	34	33.5	33	32.5	30	29	28	27

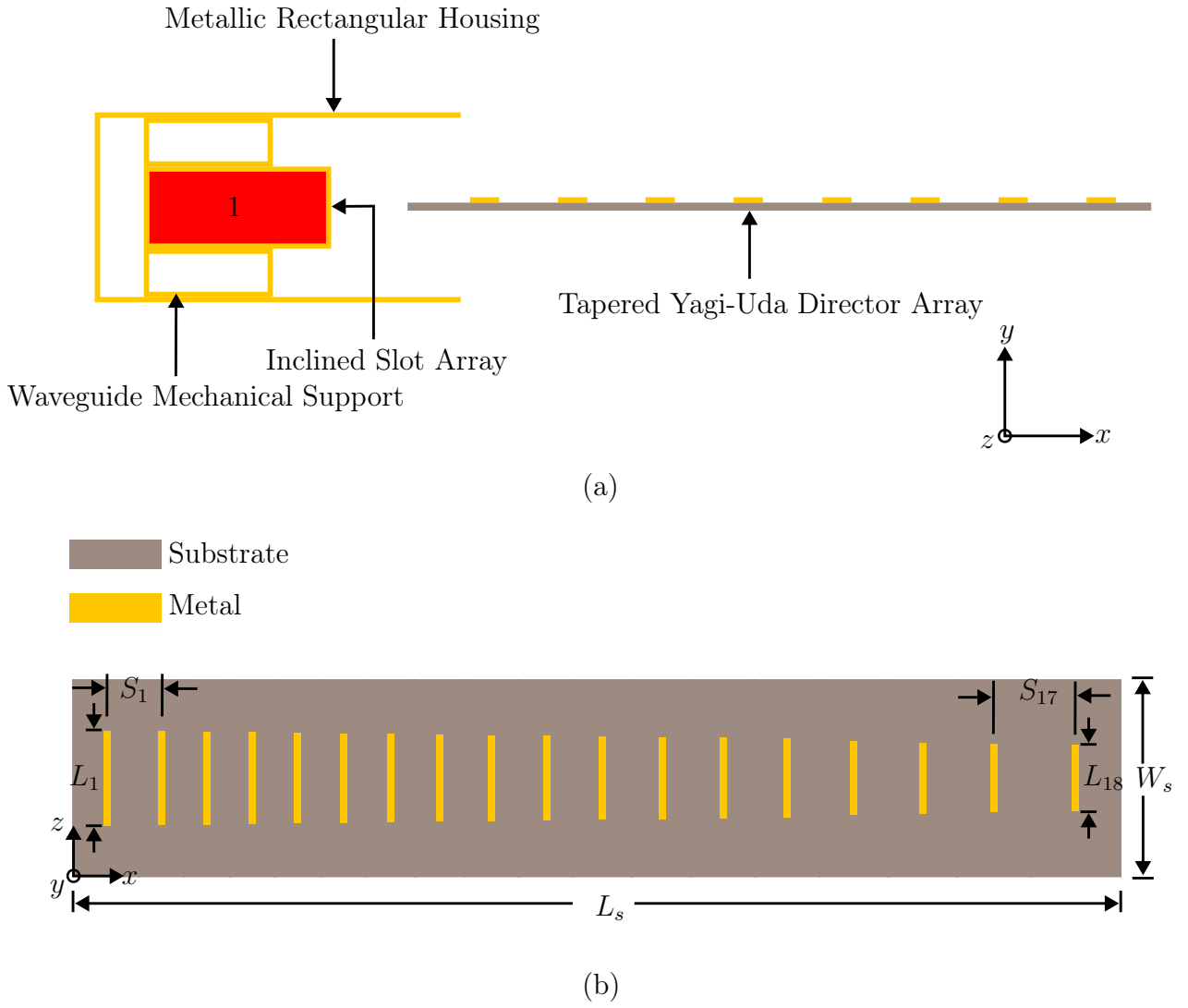


Fig. 3.2: S-band antenna configuration. (a) Vertical cross-sectional view. (b) Single layer printed Yagi-Uda-type director array with its geometrical dimensions.

3.2 Numerical Analysis

The analysis and modeling of the printed Yagi-Uda-type director array using the proposed MoM modeling method is divided into:

1. Construction of a two-dimensional equivalent model for the cross-sectional geometry shown in Fig. 3.2a including the metallic rectangular housing and the printed Yagi-Uda-type director array
2. Homogenization and modeling of the subwavelength director elements as zero-thickness non-uniform impedance sheets
3. Solve for the unknown induced equivalent electric currents on the impedance sheets and the metallic rectangular housing

4. Calculation of the total and scattered far-fields

An equivalent two-dimensional model is constructed based on the surface equivalence principle. The considered geometry shown in Fig. 3.2a and the excitation (i.e., inclined slot array) are periodic in the z -direction perpendicular to the two-dimensional modeling plane (i.e., xy -plane) and the wave propagation occurs in the modeling plane only. By applying the surface equivalence principle, the metallic rectangular housing is replaced with equivalent surface electric currents denoted as $J_{z,h}(\mathbf{r})$. The subwavelength director elements are homogenized and represented by zero-thickness impedance sheets with equivalent electric surface currents denoted as $J_{z,s}(\mathbf{r})$. The impedance sheets are placed along the xz -plane and have a total length L_s corresponding to the overall aperture length of the printed Yagi-Uda-type director array. The original excitation (i.e., inclined slot array) is replaced by an infinite electric line source oriented along the z -axis. It radiates TE cylindrical waves with electric field parallel to the z -axis and the wave propagates in the xy -plane. Fig. 3.3 illustrates a cross-sectional view of the original problem and its two-dimensional equivalent model.

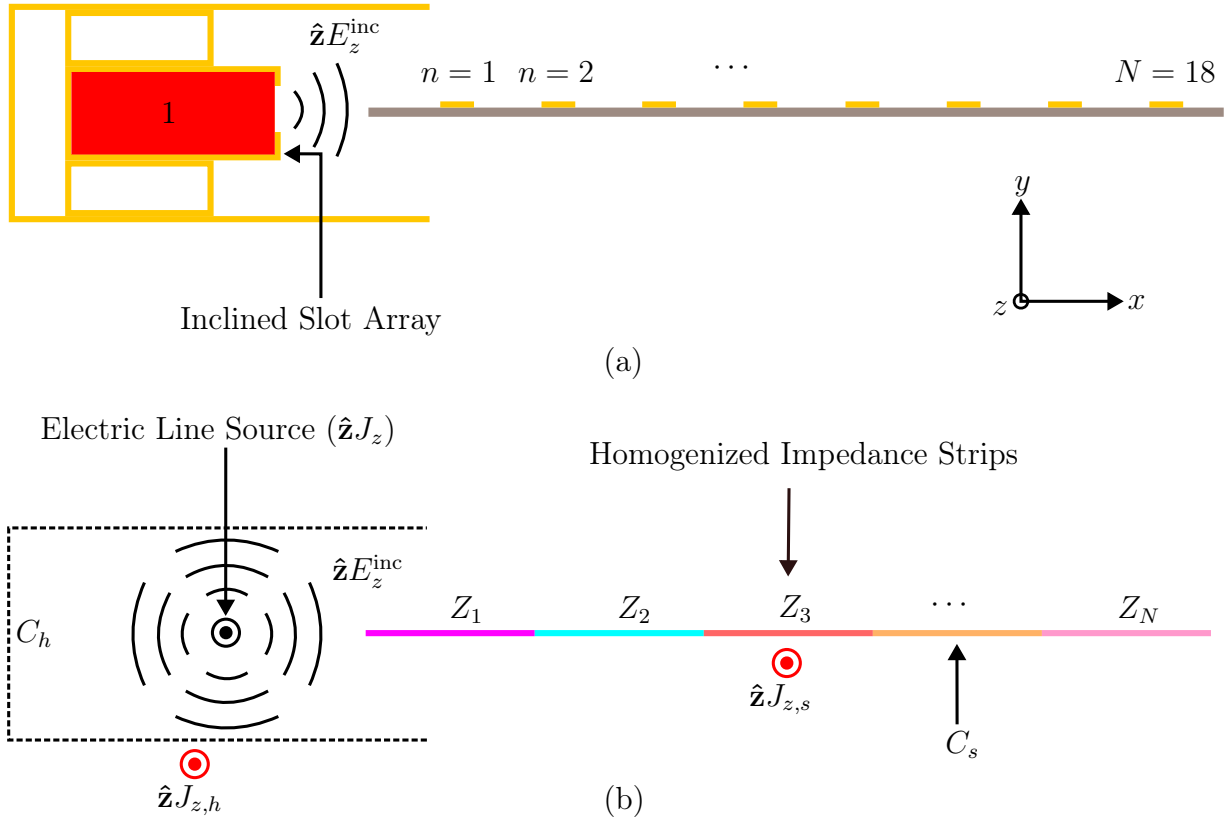


Fig. 3.3: S-band antenna configuration. (a) Vertical cross-sectional view (physical model). (b) Two-dimensional equivalent impedance model with equivalent electric currents.

3.2.1 Homogenization Model Based on Equivalent Impedance

In this section, the N director elements are modeled with zero-thickness impedance sheets denoted as $Z_n [\Omega/\square]$ ($n \in [1, N]$), as shown in Fig. 3.4. In order to determine the equivalent surface electric impedances that represent the subwavelength director elements, eigenmode analysis is performed to determine the phase constant of the dominant surface wave mode for each director element. Subsequently, a dispersion equation is derived, which relates the phase constant with the equivalent surface impedance of the director element using a transverse resonance condition [73].

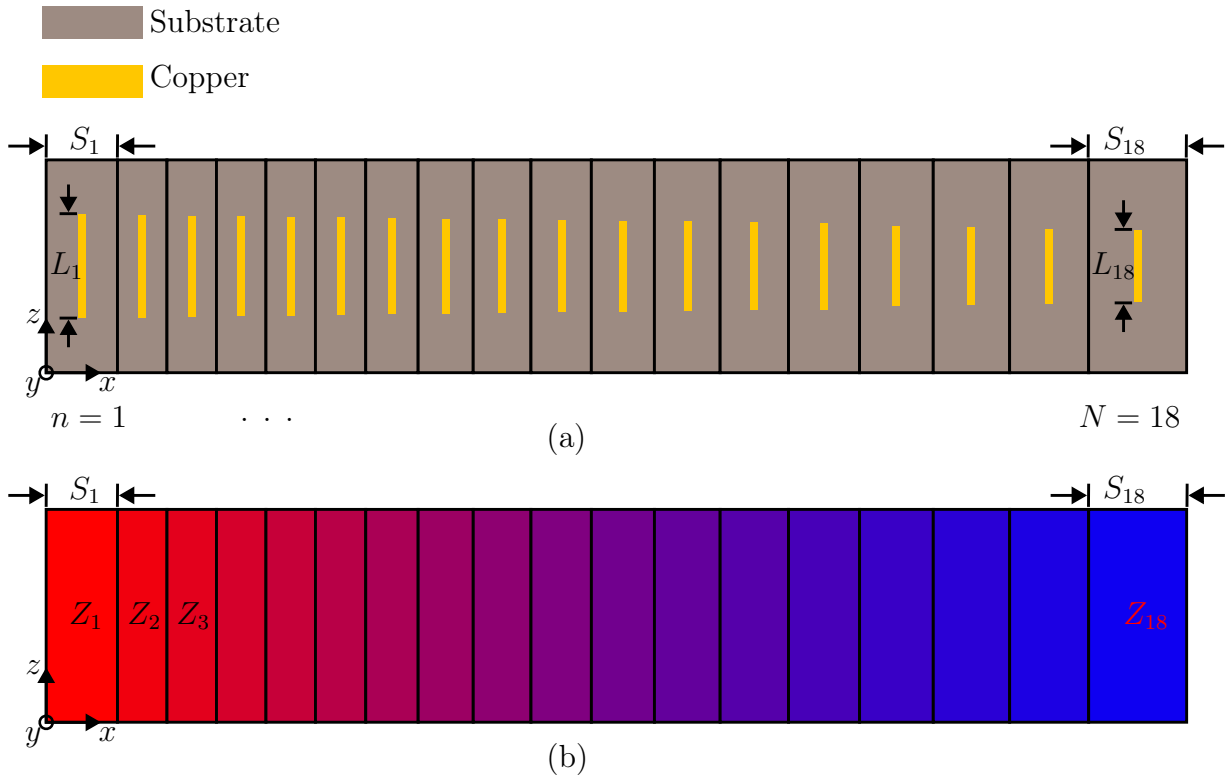


Fig. 3.4: Surface impedance homogenization of the 18 director elements. (a) Single layer printed Yagi-Uda-type director array. (b) Equivalent impedance sheets represented by different colors.

Eigenmode Analysis

The inclined radiating slots excite a TE_0 surface wave mode that propagates along the surface of the Yagi-Uda-type director array in the x -direction. The TE_0 surface wave mode is characterized by an exponential decay k_{y0} away from the surface in the y -direction and having a phase constant β_x in the x -direction. The phase constant β_x and the attenuation constant k_{y0} in the y -direction satisfy [74, pp. 54–56]

$$k_{y0}^2 = \beta_x^2 - k_0^2, \quad (3.2.1)$$

where k_0 is the free-space wavenumber. Fig. 3.5 shows the E-field distribution of the TE_0 surface wave mode in xy -plane supported by the printed Yagi-Uda-type director array.

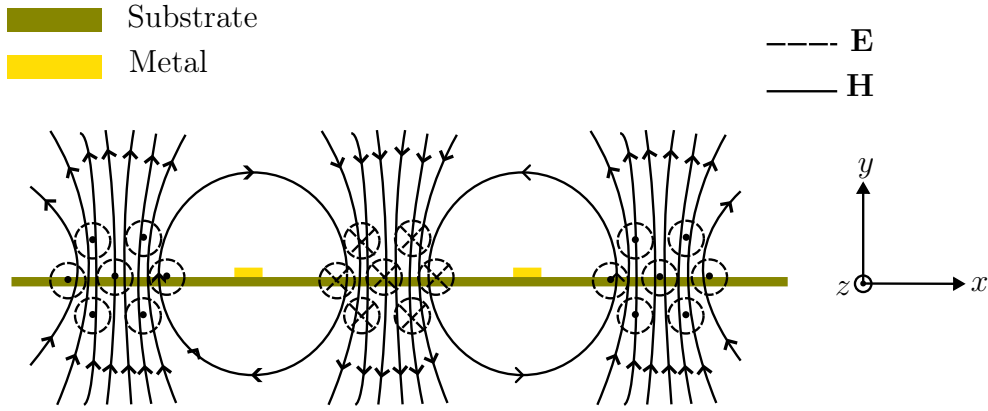


Fig. 3.5: Electric and magnetic field distribution of the dominant TE_0 surface wave mode.

To determine the phase constant β_x of the TE_0 surface wave mode traveling along the tapered Yagi-Uda-type director array, we assume that each unit cell of the tapered director array is locally periodic. In other words, the phase constant β_x corresponding to each unit cell of the tapered director array is calculated by simulating each unit cell (i.e. unit cell of interest) in a periodic array environment. For this purpose, an eigenmode solver in CST Studio Suite is used to determine the dispersion diagram of each unit cell forming the tapered director array, as illustrated in Fig. 3.6. The phase shift ϕ across the unit cell of width W_{dir} in the x -direction is related to the phase constant β_x through $\phi = \beta_x W_{dir}$.

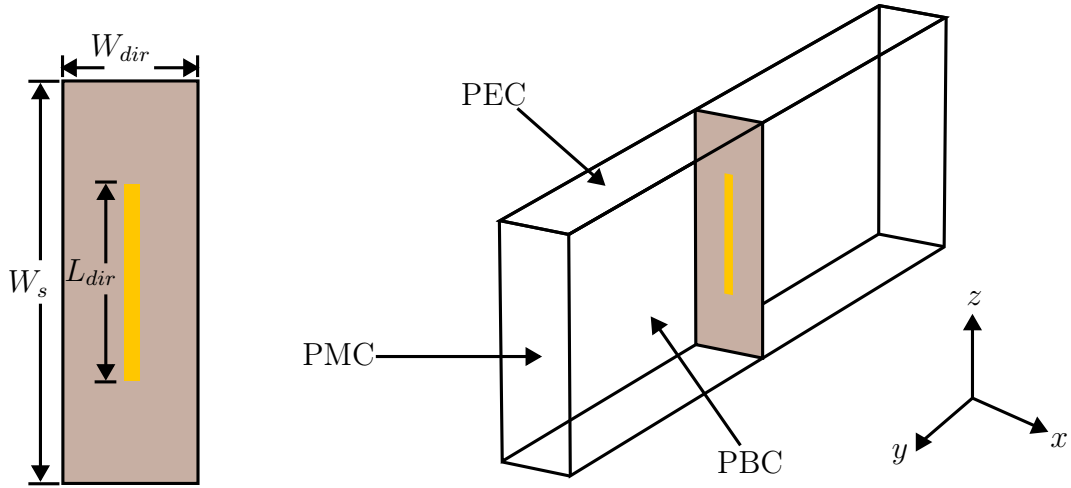


Fig. 3.6: Eigen mode solver setup with periodic boundary conditions (PBCs) in $\pm x$ -direction and perfect electric conductor (PEC) boundaries in $\pm z$ -direction. In the yz -plane, the top and bottom planes are perfect magnetic boundaries (PMC) with a height of 50 mm relative to the director element surface.

For demonstration, Fig. 3.7 shows the dispersion diagram of the TE_0 surface wave mode for a unit cell with a width W_{dir} of 18.6 mm and a director length L_{dir} of 39 mm. It can be

observed that the dispersion curve starting from 2 GHz lies below the light line, indicating bound surface waves (i.e., slow waves).

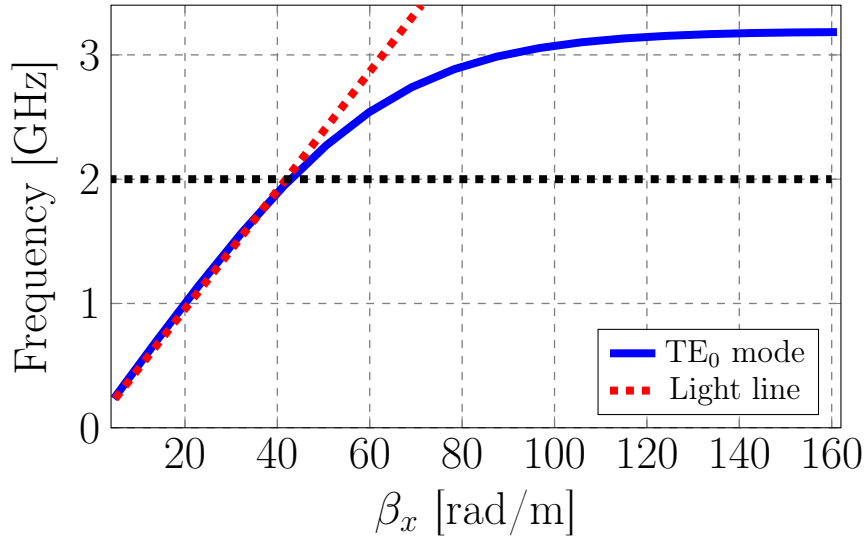


Fig. 3.7: Illustrative example of the dispersion diagram of the dominant TE_0 surface wave mode for a uniform director array.

Based on the above eigenmode analysis, the phase constant of each unit cell forming the tapered director array is determined at 2.925 GHz. Fig. 3.8 shows the normalized phase constant of the tapered Yagi-Uda-type director array. The values of the propagation constant β_x of the tapered design is changed from $k_1 = 79$ rad/m at the start of the director array to a value of $k_2 = 62$ rad/m at the open end of the array. The operating frequency and free-space wavenumber k_0 are equal to 2.925 GHz and 62 rad/m, respectively. The tapering technique reduces the phase constant of the surface wave mode traveling along the length of structure to match the free-space wavenumber k_0 at the end of the array.

Transverse Resonance Method

Once the phase constant β_x of each director element forming the tapered Yagi-Uda-type array is determined, a dispersion relation is derived between the propagation wavenumber k_x of the dominant TE_0 surface wave mode and the equivalent surface impedance Z_{sheet} . The dispersion equation is obtained by constructing a transverse equivalent network including the equivalent surface impedance Z_{sheet} and then applying a transverse resonance condition [73]. A cross-sectional view of a uniform periodic printed director array with its transverse equivalent network is shown in Fig. 3.9. The equivalent surface impedance representing the periodic director array is in parallel with the TE free-space wave impedance Z_0^{TE} . The dispersion relation is obtained by enforcing the transverse resonance condition, at $z = 0$, on the equivalent network [73]:

$$1/Z_0^{\text{TE}} + 1/Z_{\text{sheet}} + 1/Z_0^{\text{TE}} = 0, \quad (3.2.2)$$

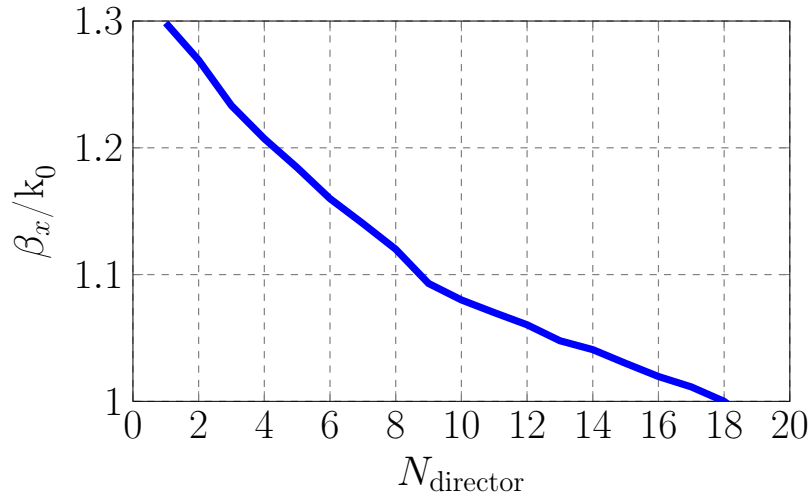


Fig. 3.8: Normalized propagation constant (β_x) of the proposed non-uniform tapered Yagi-Uda-type director array at 2.925 GHz.

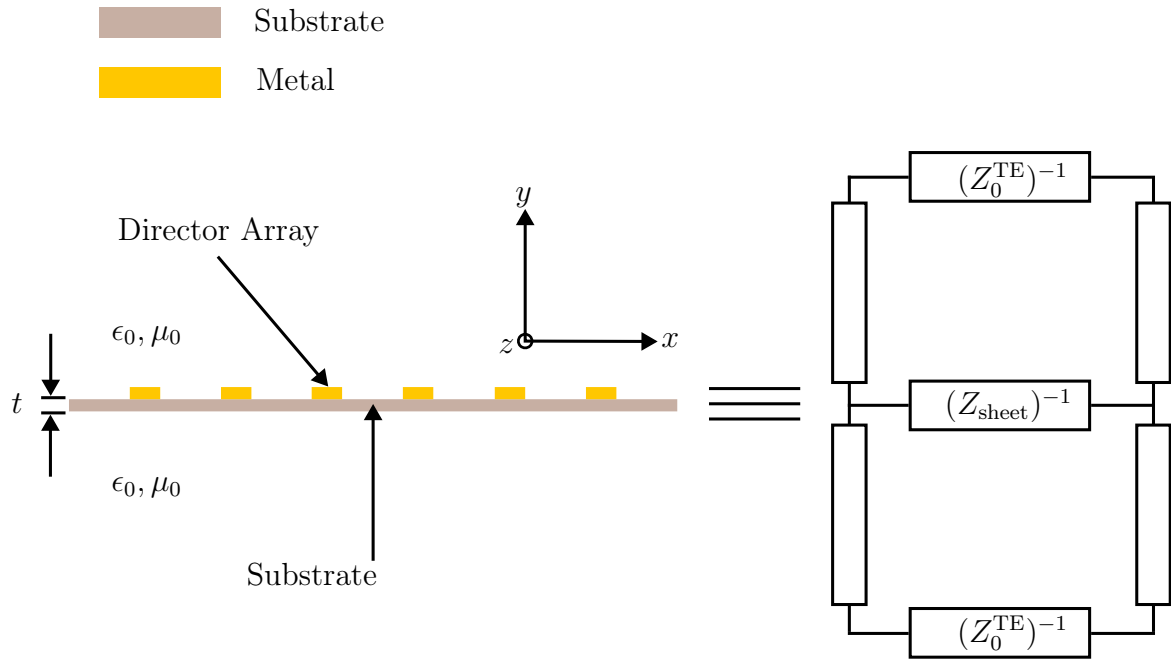


Fig. 3.9: Transverse equivalent network for determining a relationship between the propagation constant (β_x) of the TE_0 surface wave and the equivalent sheet impedance Z_{sheet} .

where Z_0^{TE} represents the TE wave impedance in free-space and Z_{sheet} is the sheet impedance used to represent the periodic Yagi-Uda-type structure. Now, the sheet impedance is related to the TE free-space wave impedance (Z_0^{TE}) by

$$Z_{\text{sheet}} = -\frac{Z_0^{\text{TE}}}{2}. \quad (3.2.3)$$

The TE free-space wave impedance (Z_0^{TE}) is expressed by [73]

$$Z_0^{\text{TE}} = j\eta_0 \frac{k_0}{k_{y0}} = \frac{j\eta_0}{\sqrt{\frac{k_x^2}{k_0^2} - 1}}, \quad (3.2.4)$$

where η_0 is the characteristics impedance of free-space. By substituting (3.2.4) into (3.2.3), the relation between the propagation wavenumber k_x and the sheet impedance (Z_{sheet}) is expressed as

$$Z_{\text{sheet}} = -\frac{j\eta_0}{2\sqrt{\frac{k_x^2}{k_0^2} - 1}}. \quad (3.2.5)$$

Eq. 3.2.5 indicates that we can determine the equivalent impedance of the printed Yagi-Uda-type director array from the propagation wavenumber k_x . Using this equation, the equivalent surface impedances representing the 18 director elements are determined and plotted in Fig. 3.10. As expected, the equivalent impedances show capacitive behavior required to support the propagation of TE₀ surface wave mode. Since the geometrical dimensions of the printed Yagi-Uda director array vary gradually along the surface, the impedance profile similarly exhibits a gradual variation.

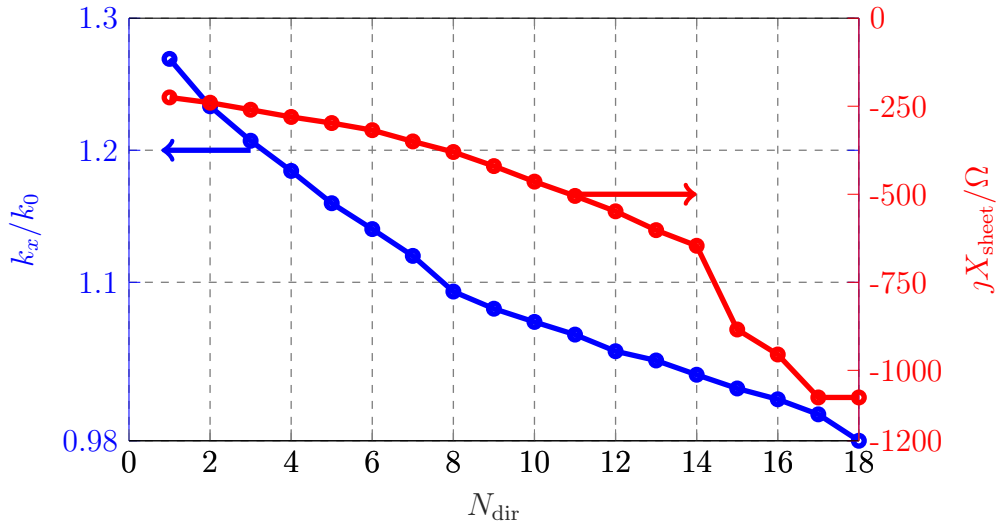


Fig. 3.10: Extracted impedance values Z_{sheet} of the 18 director elements and their normalized propagation constant at 2.925 GHz.

3.2.2 MoM Solution

Once the N director elements are homogenized and their equivalent surface impedances are determined, the MoM matrix is constructed. The MoM solution begins by first defining the boundary conditions of the two-dimensional equivalent scattering problem shown in Fig. 3.3b, followed by discretizing the equivalent model. The boundary condition is satisfied at the equivalent impedance sheet as follows:

$$E_z^{\text{tot}}(\mathbf{r}) = Z_{\text{sheet}}(\mathbf{r})J_{z,s}(\mathbf{r}), \quad \mathbf{r} \in C_s, \quad (3.2.6)$$

where C_s is a line along which the surface impedance is defined. The tangential component of the electric field vanishes on the metallic wall of the rectangular housing as follows:

$$E_z^{\text{tot}}(\mathbf{r}) = 0, \quad \mathbf{r} \in C_h, \quad (3.2.7)$$

where C_h is the curve along which the equivalent electric currents $J_{z,h}$ are distributed. The equivalent impedance strips, C_s , are discretized into N_s line segments of widths Δs_{n_s} ($n_s = 1 : N_s$). The PEC rectangular housing C_h is discretized into N_h line segments of widths Δs_{n_h} ($n_h = 1 : N_h$). Using pulse basis functions, the equivalent electric surface currents $J_{z,s}$ and $J_{z,h}$ are expanded as follows:

$$J_{z,s}(x) = \sum_{n_s=1}^{N_s} I_{n_s} f_{n_s}(x), \quad (3.2.8)$$

$$J_{z,h}(\mathbf{r}) = \sum_{n_h=1}^{N_h} I_{n_h} f_{n_h}(\mathbf{r}). \quad (3.2.9)$$

Here, I_{n_s} and I_{n_h} are, respectively, the unknown coefficients of the pulse basis functions associated with $J_{z,s}$ and $J_{z,h}$ in the n_s -th and n_h -th line segments. N_s and N_h are the number of segments used to discretize the impedance sheets and the PEC rectangular housing, respectively. The MoM matrix equation is constructed as follows:

$$\begin{bmatrix} V_{m_s} \\ V_{m_h} \end{bmatrix} = \begin{bmatrix} Z_{\text{sheet}} + Z_{m_s n_s} & Z_{m_s n_h} \\ Z_{m_h n_s} & Z_{m_h n_h} \end{bmatrix} \begin{bmatrix} I_{n_s} \\ I_{n_h} \end{bmatrix}. \quad (3.2.10)$$

Here, $Z_{m_s n_s}$ is the self-impedance matrix, where it represents the interaction between the observation and source pulse basis functions on the impedance sheets. Similarly, $Z_{m_h n_h}$ denotes the self-impedance matrix of the equivalent surface currents $J_{z,h}$ on the PEC rectangular housing. This represents the interaction between the m_h -th observation and the n_h -th source pulse basis functions located on the PEC rectangular housing C_h . $Z_{m_s n_h}$ and $Z_{m_h n_s}$ correspond to the mutual coupling between the equivalent electric currents on

the impedance sheets $J_{z,s}$ and the PEC rectangular housing $J_{z,h}$. Z_{sheet} is a diagonal matrix containing the equivalent surface impedances of the subwavelength director elements. Fig. 3.11 illustrates the discretization of the equivalent two-dimensional model using pulse basis functions.

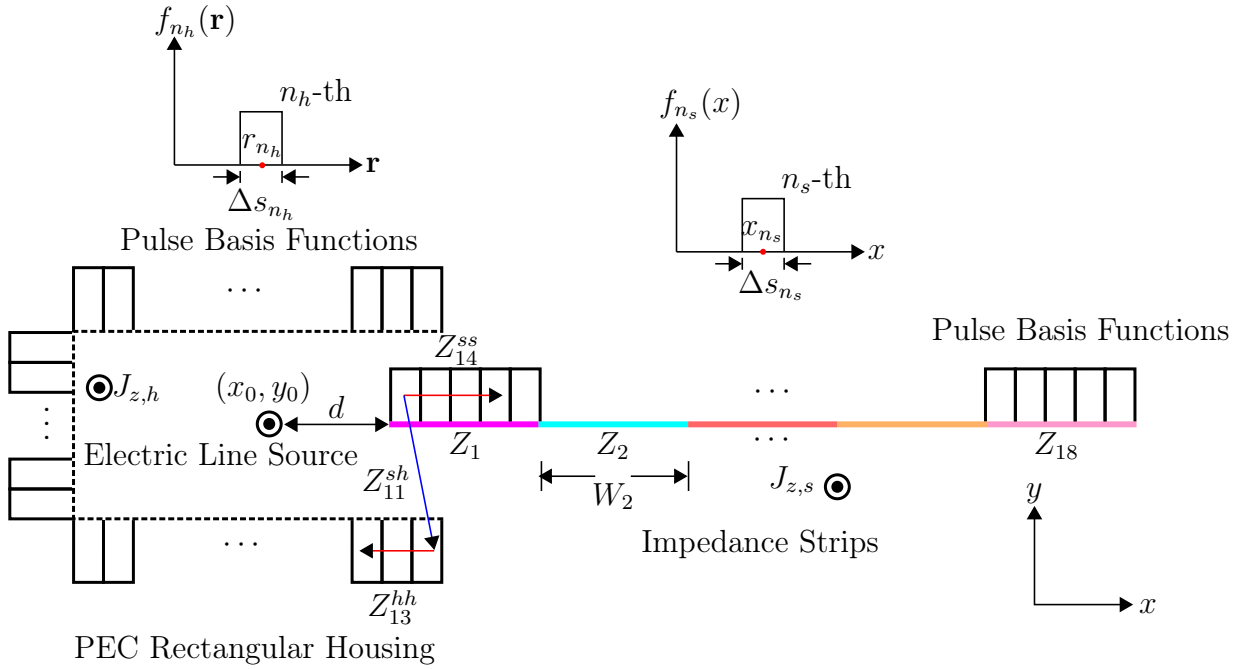


Fig. 3.11: Discretization of the two-dimensional equivalent model into line segments for MoM solution. The equivalent currents are expanded using pulse basis functions.

V_{m_s} is a 1-D vector of dimension $N_s \times 1$, representing the incident electric field produced by the electric line source at matching point $\mathbf{r} = \mathbf{r}_{m_s}$ on the impedance strips. It is expressed as

$$[V_{m_s}] = \frac{\eta_0 k_0}{4} H_0^{(2)} \left(k_0 \sqrt{(x_{m_s} - x_0)^2 + (y_{m_s} - y_0)^2} \right). \quad (3.2.11)$$

Similarly, V_{m_h} is a 1-D vector of dimension $N_h \times 1$, denoting the incident field produced by the electric line source at point matching $\mathbf{r} = \mathbf{r}_{m_h}$ on the PEC rectangular housing. It is expressed as

$$[V_{m_h}] = \frac{\eta_0 k_0}{4} H_0^{(2)} \left(k_0 \sqrt{(x_{m_h} - x_0)^2 + (y_{m_h} - y_0)^2} \right). \quad (3.2.12)$$

3 Analysis and Modeling of Guided Waves on Artificial Surfaces

The nonsingular impedance elements are expressed as [13, 41]

$$[Z_{m_s n_s}] = \frac{\eta_0 k_0 \Delta s_{n_s}}{4} H_0^{(2)} \left(k_0 \sqrt{(x_{m_s} - x_{n_s})^2 + (y_{m_s} - y_{n_s})^2} \right), \quad (3.2.13)$$

$$[Z_{m_s n_h}] = \frac{\eta_0 k_0 \Delta s_{n_h}}{4} H_0^{(2)} \left(k_0 \sqrt{(x_{m_s} - x_{n_h})^2 + (y_{m_s} - y_{n_h})^2} \right), \quad (3.2.14)$$

$$[Z_{m_h n_s}] = \frac{\eta_0 k_0 \Delta s_{n_s}}{4} H_0^{(2)} \left(k_0 \sqrt{(x_{m_h} - x_{n_s})^2 + (y_{m_h} - y_{n_s})^2} \right), \quad (3.2.15)$$

$$[Z_{m_h n_h}] = \frac{\eta_0 k_0 \Delta s_{n_h}}{4} H_0^{(2)} \left(k_0 \sqrt{(x_{m_h} - x_{n_h})^2 + (y_{m_h} - y_{n_h})^2} \right). \quad (3.2.16)$$

When the observation and source points are equal (i.e., $m_s = n_s$ or $m_h = n_h$) [13, 41]

$$[Z_{m_s n_s}] = \frac{\eta_0 k_0 \Delta s_{n_s}}{4} - j \frac{\eta_0 k_0 \Delta s_{n_s}}{2\pi} \left(\ln \left(\frac{k_0 \Delta s_{n_s} \gamma}{4} \right) - 1 \right), \quad (3.2.17)$$

$$[Z_{m_h n_h}] = \frac{\eta_0 k_0 \Delta s_{n_h}}{4} - j \frac{\eta_0 k_0 \Delta s_{n_h}}{2\pi} \left(\ln \left(\frac{k_0 \Delta s_{n_h} \gamma}{4} \right) - 1 \right). \quad (3.2.18)$$

Once the impedance elements are determined, the resulting system of linear equation can be solved directly to obtain the unknown currents coefficients (I_{n_s} and I_{n_h}). For the application of MoM, 195 line segments with size of $\Delta s_{n_h} = 0.01\lambda_0$ at 2.925 GHz describe the details of PEC rectangular housing and 434 segments with size of $\Delta s_{n_s} = 0.01\lambda_0$ describe the equivalent impedance strips. Fig. 3.12 shows the equivalent current distribution $J_{z,s}$ obtained from the proposed MoM solution along the impedance strips in the x -direction. It can be clearly seen that the equivalent current distribution is tapered along the length of the Yagi-Uda-type director array resulting in reduced SLL of the radiated field.

3.2.3 Numerical Results

Near-field Distributions

The real part of the scattered near-field in the two-dimensional modeling plane, produced by the induced equivalent surface electric currents $J_{z,s}$ and $J_{z,h}$, is determined. Fig. 3.13a shows the scattered field without the printed Yagi-Uda-type director array, while Fig. 3.13b illustrates the scattered field with the printed Yagi-Uda-type director array produced from reactive sheet impedances. As can be seen in Fig. 3.13b, the printed Yagi-Uda-type director guides TE₀ surface wave along the surface.

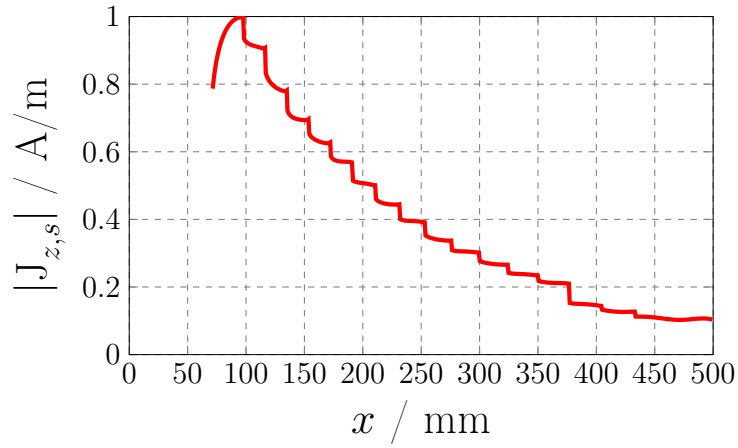
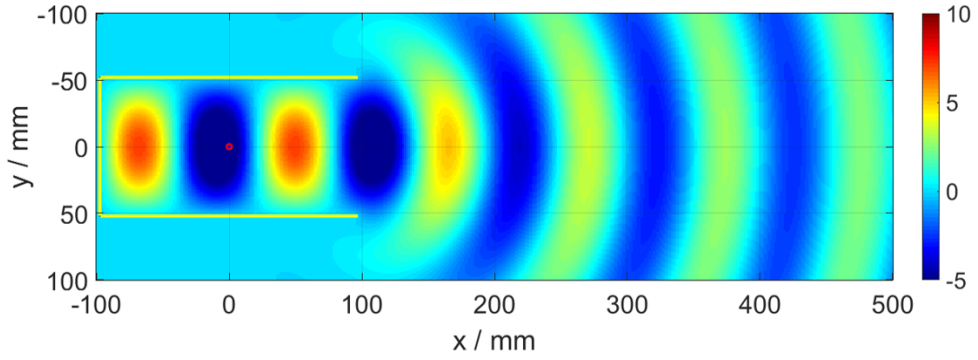
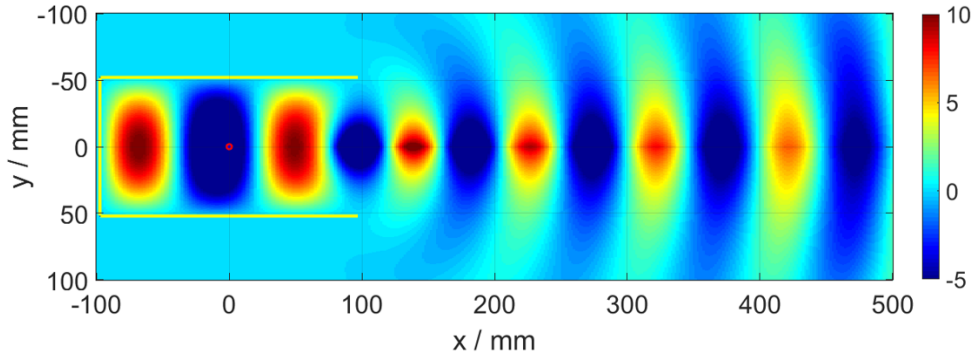


Fig. 3.12: Normalized equivalent electric currents distribution $J_{z,s}$ on the non-uniform impedance sheets at 2.925 GHz.



(a)



(b)

Fig. 3.13: Real part of the total near-field obtained from the MoM modeling method. (a) without printed Yagi-Uda-type director array. (b) with printed Yagi-Uda-type director array. The red circle in both figures represent the electric line source.

Radiation Characteristics

To validate the far-field radiation pattern obtained from the MoM solution, the general EM solver using CST Studio Suite is employed. Two inclined slots integrated with two printed Yagi-Uda-type director arrays have been simulated with PEC in $\pm z$ -direction, as shown in Fig. 3.14.

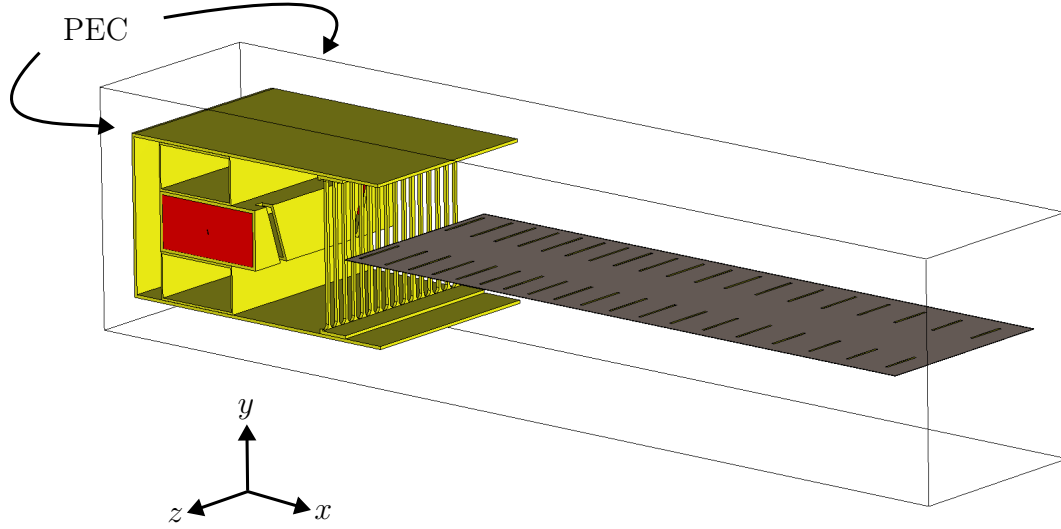


Fig. 3.14: 3D simulation model of two identical inclined slots with electric walls in $\pm z$ -direction. The included slots excite TE_0 surface wave mode propagates along the surface in the x -direction.

Fig. 3.15 presents a comparison between the far-field radiation patterns obtained from the MoM solution and the commercial EM solver (CST Studio Suite). A good agreement between the obtained results can be observed, demonstrating the effectiveness of the proposed MoM. Table 3.3 presents the radiation characteristics: half-power beamwidth (HPBW), main beam direction and the maximum side lobe level (SLL). As can be seen, the far-field results obtained by the MoM solution close to the simulated values by CST. The discrepancy can be attributed to the non-uniform sheet impedances extracted based on the local periodic assumption. In other words, the surface impedances used by the MoM method are determined by assuming each unit cell in a periodic environment. This depends on the local periodic assumption which is not completely valid.

Table 3.3: Radiation characteristics of the printed Yagi-Uda-type director array at 2.925 GHz.

Parameter	MoM	CST
HPBW	30°	31°
Main beam direction	0°	0°
Maximum SLL	−18 dB	−19 dB

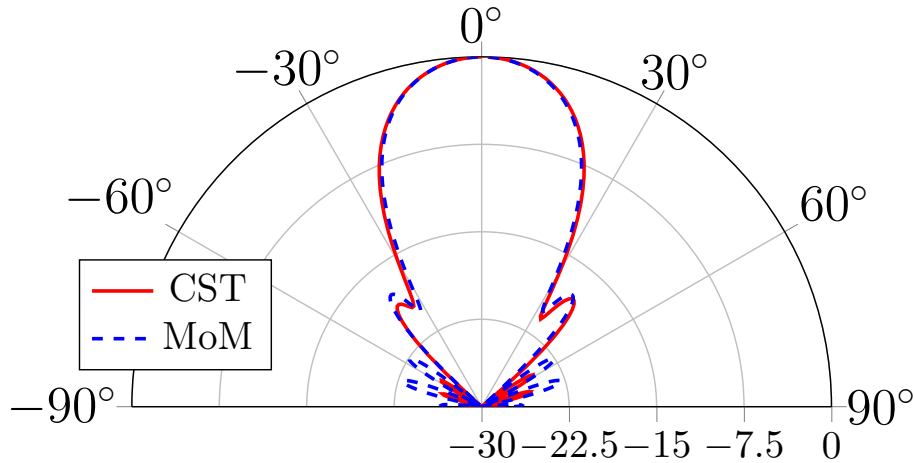


Fig. 3.15: Comparison between the radiation patterns obtained from full-wave simulation and the proposed MoM method at 2.925 GHz.

3.3 Summary

A printed Yagi–Uda-type director array is selected as an illustrative example to validate the proposed MoM modeling method against full-wave simulations. This type of artificial surface supports the propagation of guided TE_0 surface wave mode. It is designed to shape the radiation pattern of a slotted waveguide antenna in the plane perpendicular to the waveguide axis at S-band. The Yagi–Uda-type director array consists of 18 subwavelength elements called parasitic directors and is printed on a single dielectric substrate layer.

To predict the radiation characteristics using the proposed MoM modeling method, the subwavelength director elements are homogenized and represented by zero-thickness impedance sheets. The slot waveguide antenna array is replaced by an infinite electric line source to generate a TE_z cylindrical wave parallel to the z -axis. Each unit cell is simulated under local periodic approximation and an eigenmode analysis is performed to determine the phase constant of the dominant TE_0 surface wave mode. A transverse resonance method is used to construct a dispersion relation between the phase constant of the dominant surface wave mode and the equivalent sheet impedance. Based on the dispersion relation, the sheet impedances are extracted, where they show capacitive behavior to support the propagation of TE_0 surface wave mode.

Once the director elements are homogenized and their equivalent sheet impedances are extracted. The unknown equivalent currents are then calculated and the far-field radiation pattern is determined. The far-field result predicted by the proposed MoM modeling method is in a good agreement with the result obtained from CST Studio Suite.

Analysis and Modeling of Reconfigurable Transmissive Metasurface

Metasurfaces are two-dimensional counterparts of metamaterials, consisting of discrete subwavelength elements whose electromagnetic properties can be controlled either by changing the geometrical dimensions of the metasurface unit cell [13, 41] or by introducing reconfigurable elements such as varactor diodes [3, 16] or PIN diodes [21, 24, 75, 76]. Reconfigurable metasurfaces provide dynamic control of the amplitude and phase of a reflected or transmitted wave to achieve different functionalities such as dynamic beam steering [18, 21, 77, 78]. This chapter will focus on the analysis and modeling of a reconfigurable transmissive-type artificial surface using the proposed MoM modeling method and validation of the obtained results with full-wave simulations.

The chapter is divided into two parts: the first part presents the design and simulations of a dual-polarized reconfigurable transmissive metasurface used as an illustrative example. The second part focuses on the analysis of the MoM modeling method to model multilayer reconfigurable transmissive metasurface for two applications: beam focusing and beam scanning at 10 GHz.

The considered reconfigurable transmissive metasurface is designed to convert the incident spherical wavefront from a horizontally-polarized horn antenna to a focused beam directed at a certain scanning angle [7, p. 272]. This is achieved by controlling the transmission phase response of each metasurface unit cell to introduce a spatially varying phase delay across the incident spherical wavefront. This controlled phase delay can be used to compensate for phase variations or to introduce a specific phase profile to achieve desired functionalities, such as phase equalization for beam focusing or generation of phase gradients for beam scanning.

This chapter is structured as follows: Sections 4.1 and 4.2 detail the design and simulations of a dual-polarized reconfigurable transmissive metasurface, as well as its application for

beam focusing and one-dimensional beam scanning. Section 4.3 focuses on the analysis and modeling of the reconfigurable transmissive metasurface using the MoM modeling method. Section 4.3 is divided into two parts: the first part focuses on the surface impedance homogenization of the proposed metasurface unit cell structure using an equivalent impedance transmission line model. The second part discusses the expansion of the equivalent electric currents induced on the two-dimensional equivalent impedance model and the solution of the MoM using Galerkin's method. The far-field radiation patterns obtained by the MoM method are compared with the general EM solver (CST Studio Suite) and discussed in Subsection 4.3.3. In Section 4.4, the proposed reconfigurable transmissive metasurface is fabricated and measured to demonstrate the one-dimensional beam scanning capability.

4.1 Design of Reconfigurable Transmissive Metasurface

4.1.1 Unit Cell Design

The considered unit cell structure consists of three metallic patterns separated by two dielectric substrates [21, 79]. The top and bottom layers are made of square loop apertures, while the middle layer consists of a cross dipole. Fig. 4.1a illustrates the proposed metasurface unit cell structure, highlighting the different metallic layers. The dielectric substrates are Rogers R4003C with a permittivity of 3.55 and total thickness equal to 1.1 mm. Rogers RO4450B is used as a bonding layer to bond the two dielectric substrate layers. Fig. 4.1b shows the geometrical parameters of the reconfigurable metasurface unit cell structure.

The reconfigurable metasurface has been designed and optimized to operate in the X-band with a center frequency equal to 10 GHz and bandwidth equal to 0.3 GHz. The period of the metasurface unit cell is equal to $\lambda_0/4$, in which λ_0 is the wavelength at 10 GHz. Table 4.1 lists the geometrical dimensions of the unit cell structure.

Table 4.1: Geometrical dimensions of the metasurface unit cell structure [mm].

Parameters	Value
P_x	7.5
P_y	7.5
w_1	6
g	0.15
w_2	1.7

The transmission phase response of the metasurface unit cell structure is controlled by surface-mounted varactor diodes (MACOM MA46H120) placed on the top and bottom metallic layers as illustrated in Fig. 4.1c. The varactor diode provides a capacitance range between 0.4 pF and 1.2 pF for a reverse bias voltage between 16 V and 0 V, respectively,

4.1 Design of Reconfigurable Transmissive Metasurface

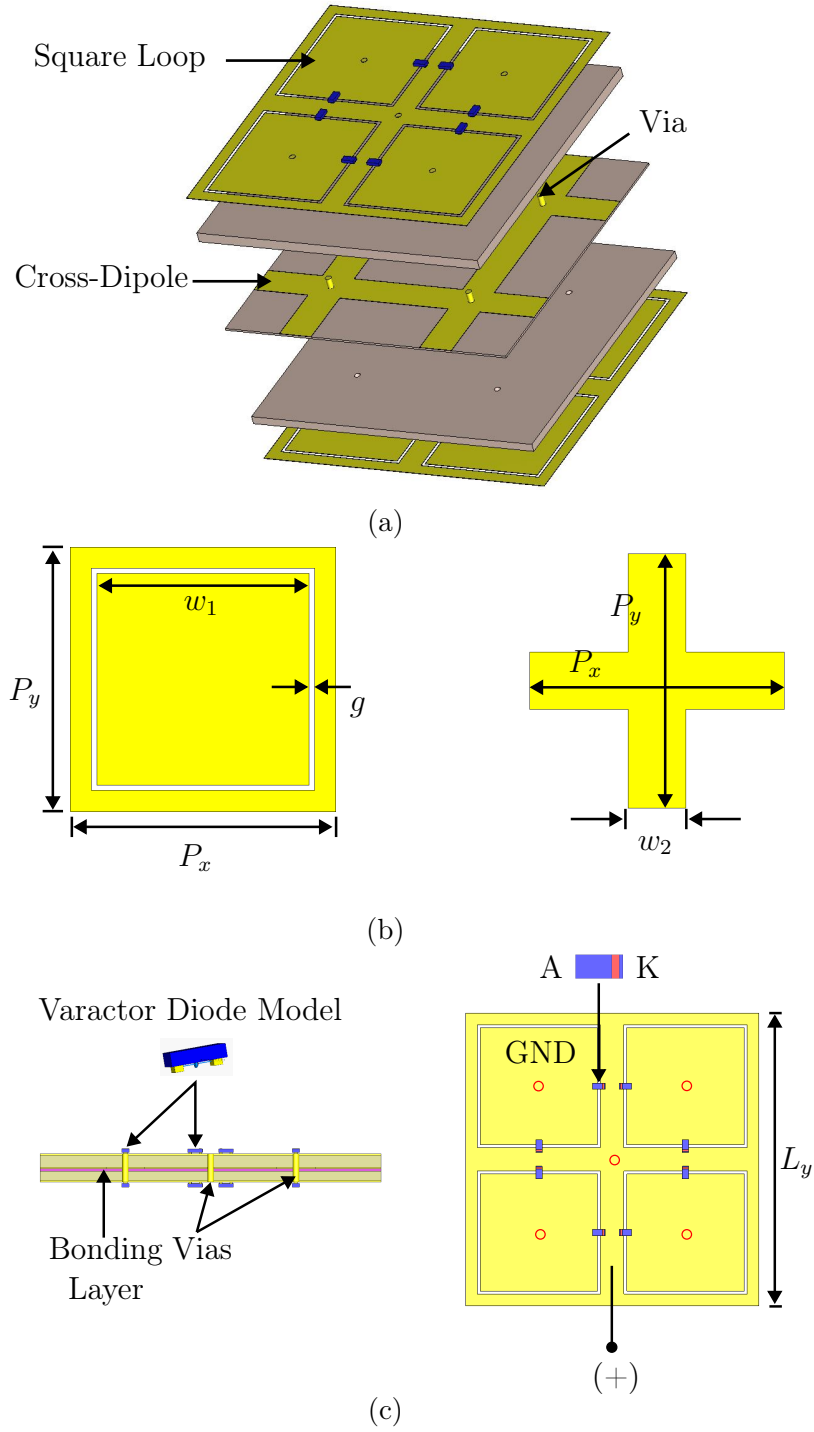


Fig. 4.1: Structure of the considered metasurface unit cell. (a) 3-D perspective view of four unit cells including varactor diodes. (b) Geometrical dimensions of the top and middle metallic layers of single unit cell. (c) Cross-sectional and front view of four unit cells indicating plated through vias. The four vias placed at the centers of the square patches connect the top and bottom metallic layers to the middle metalization layer, while the via in the center connects the top and bottom metallic layers. The red strips show the cathode side of the varactor diodes.

with a parasitic resistance equal to 2Ω . Each metasurface unit cell is equipped with four varactor diodes enabling simultaneous phase control for both TE and TM polarizations. The varactor diodes are positioned across the gap between the square patches and the wire grids. Four metallic vias located at the centers of the square patches are used to provide electrical connections between the three metallic layers as illustrated in Fig. 4.1c. Additionally, a central via connects the top and bottom wire grid layers. To reverse bias the varactor diodes, the square patches with the middle layer are connected to the DC ground, while the positive terminal of the DC voltage source is connected to the top and bottom wire grids as shown in Fig. 4.2.

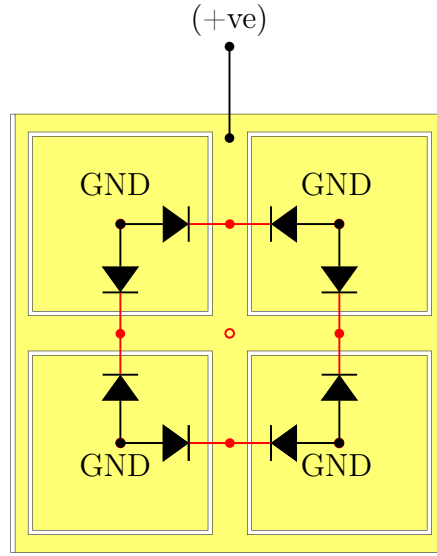


Fig. 4.2: Schematic representation of the varactor diodes located on the top layer.

In order to determine the transmission coefficient, the metasurface unit cell is simulated in CST Studio Suite in the frequency range from 8 GHz to 12 GHz with normal incidence plane wave and by employing periodic boundary conditions and so-called Floquet ports, as shown in Fig. 4.3. The varactor diode is modeled as an RLC serial lumped element with a series resistance equal to 2Ω and an inductance equal to 0.1 nH. The simulated transmission amplitude and phase response at the normal incidence are shown in Fig. 4.4. It can be seen that by changing the capacitance C_{var} values of the series RLC network from 0.19 pF to 0.4 pF, a 170° change in the transmission phase is achieved with an insertion loss from 1 dB to 4 dB at 10 GHz.

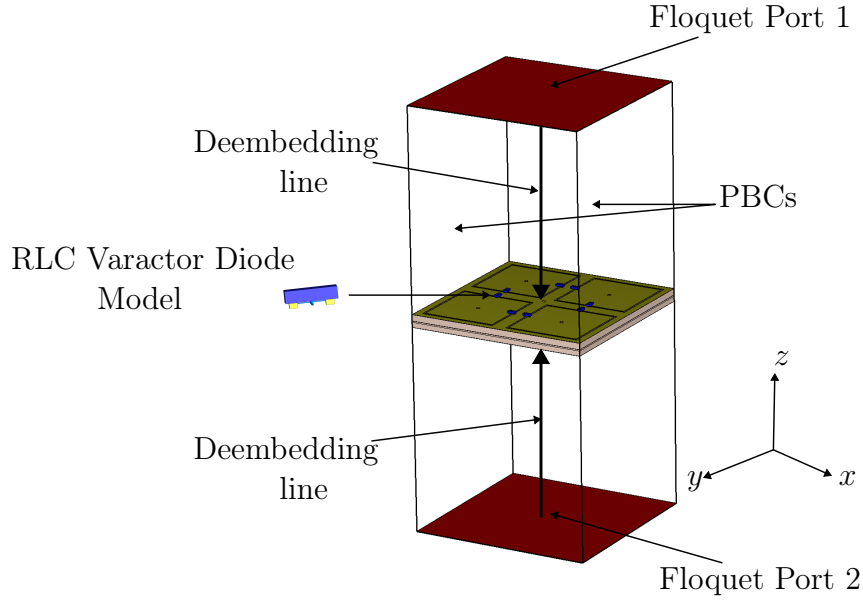


Fig. 4.3: Simulation model of the reconfigurable transmissive metasurface unit cell with Floquet boundary conditions.

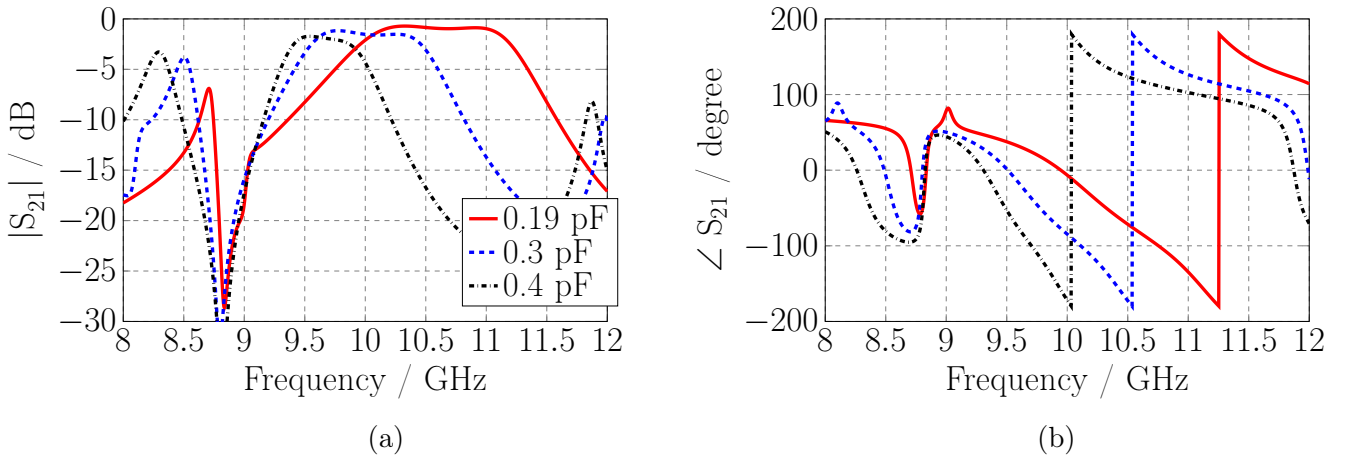
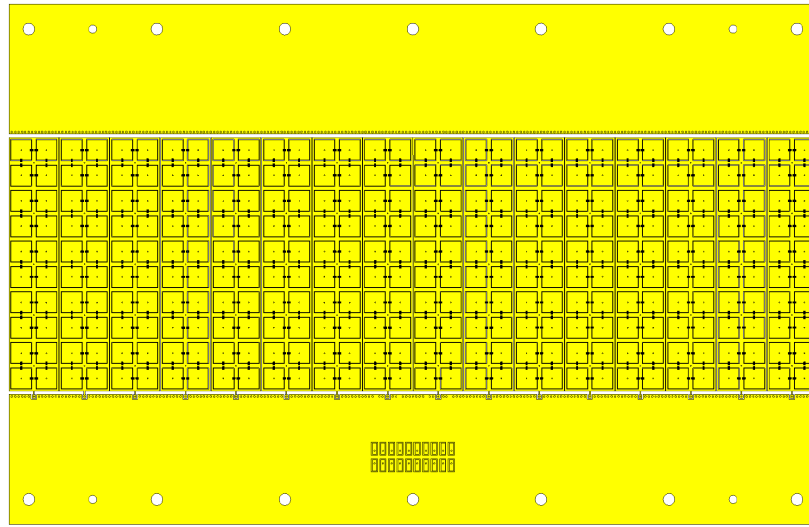


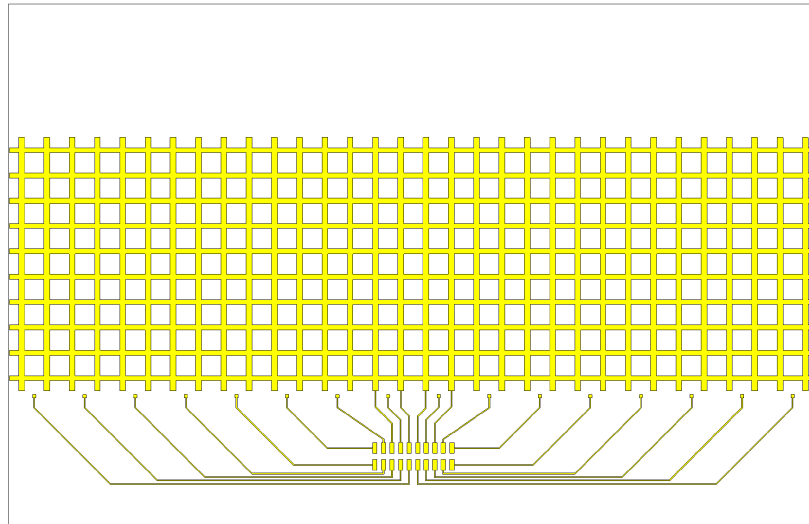
Fig. 4.4: Simulated transmission performance of the metasurface unit cell at normal incidence. (a) Transmission amplitude. (b) Transmission phase.

4.1.2 Lens Array Design

A planar reconfigurable metasurface based on the presented unit cell structure is designed and shown in Fig. 4.5. The planar reconfigurable metasurface PCB is divided into 16 vertical modules and each module consists of an array of 2×10 metasurface unit cells [21]. The 16 vertical modules are electrically separated in the horizontal direction by introducing vertical gaps of 0.3 mm between the adjacent modules to independently control each module. The DC biasing lines used to control the modules are placed in the middle layer and run to a 20-pin connector located on the top layer through metallized vias as illustrated in Fig. 4.5.



(a)



(b)

Fig. 4.5: Proposed reconfigurable transmissive metasurface. (a) Top layer including 16 vertical modules. (b) Middle layer including the DC biasing lines.

4.2 Reconfigurable Metasurface for Shaping Transmitted Fields

The proposed reconfigurable transmissive metasurface is used to perform phase compensation and one-dimensional beam scanning in the azimuth plane. A horizontally-polarized horn antenna positioned at a distance D from the metasurface is used to illuminate the transmissive metasurface. The width of the reconfigurable metasurface in the x -direction is $8\lambda_0$, where λ_0 represents the free-space wavelength at 10 GHz. The distance between the horn antenna and the metasurface is 150 mm. Since the wave propagates in the xy -plane and the transmissive metasurface is periodic along the z -direction, magnetic boundary conditions are applied in the $\pm z$ -directions to enforce horizontal polarization. Fig. 4.6 shows the 3D simulation model of a single row of the reconfigurable metasurface with magnetic walls illuminated by the horizontally-polarized horn antenna.

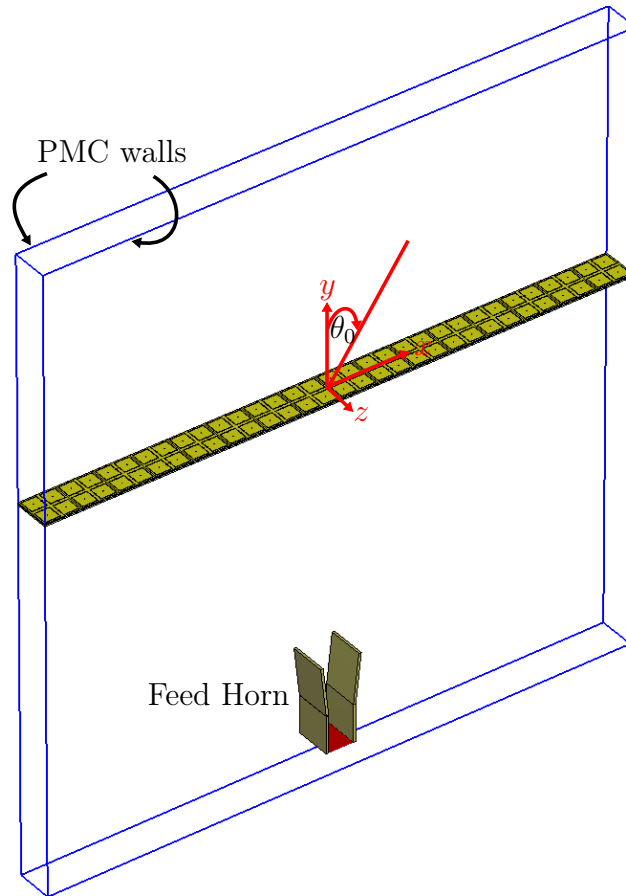


Fig. 4.6: 3D simulation model of the reconfigurable transmissive metasurface with magnetic boundary conditions (PMC) and illuminated by a horizontally-polarized horn antenna. A waveguide port highlighted by red is used to excite the horn antenna.

4.2.1 Phase Compensation

The goal is to transform the transmitted spherical wavefront generated by the horn antenna into a nearly planar wavefront using the reconfigurable transmissive metasurface, as illustrated in Fig. 4.7. Phase compensation is achieved by introducing a spatially varying phase shift denoted by ϕ_n , where $n = 1, 2, \dots, N$, to the incident spherical wavefront.

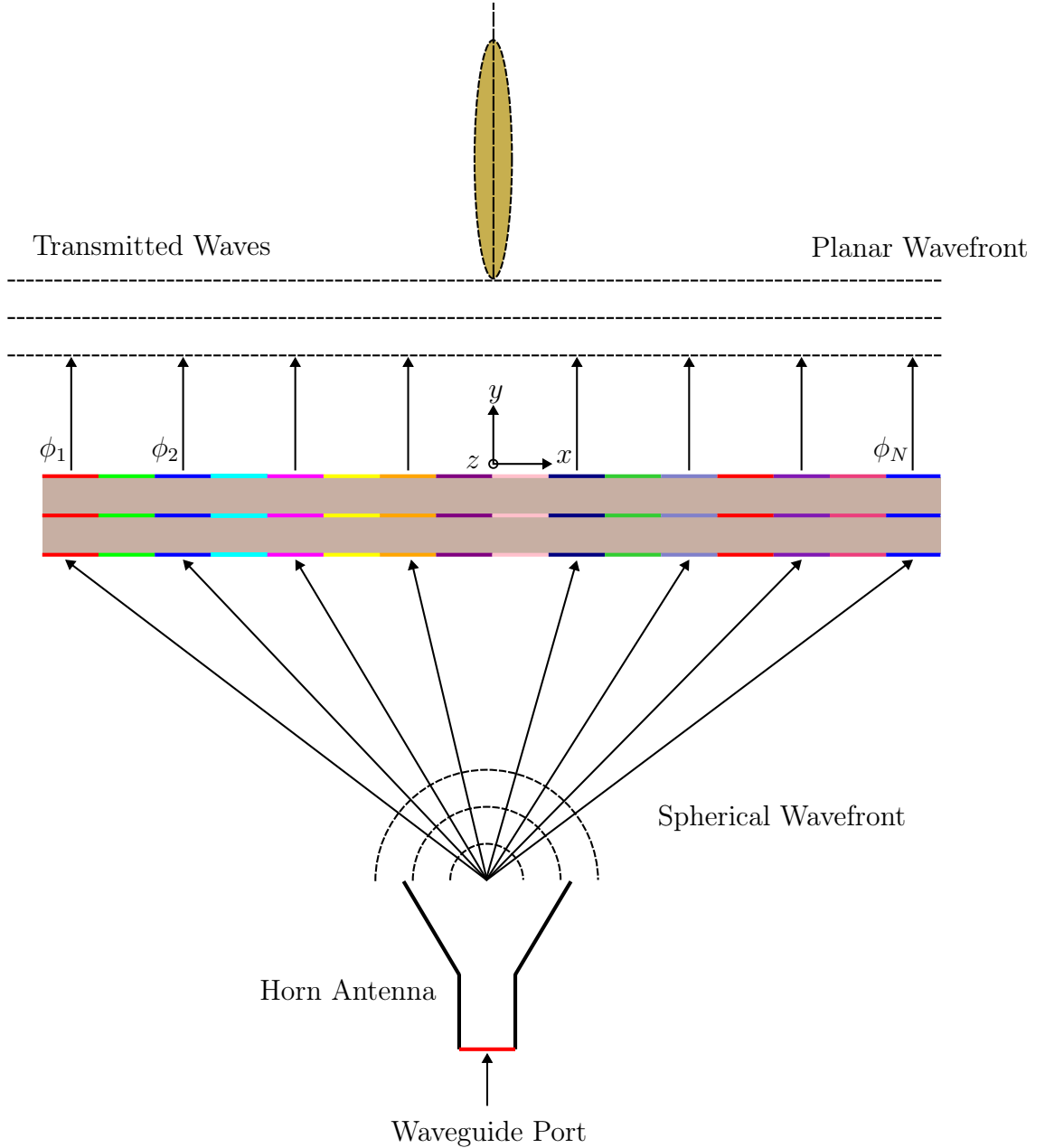


Fig. 4.7: Illustration of phase compensation performed by the reconfigurable transmissive metasurface. The metasurface consists of N unit cells and the different colored line segments represent the metasurface unit cells. Each unit cell introduces a certain phase shift to manipulate the phase of transmitted wave.

4.2 Reconfigurable Metasurface for Shaping Transmitted Fields

The phase shift ϕ_n introduced by the n -th metasurface unit cell located at (x_n, y_n) is given by the difference between the desired output phase profile for a plane wave $\phi_{\text{desired}}(x_n, y_n)$ and the phase of the incident spherical wavefront $\phi_{\text{incident}}(x_n, y_n)$ along the metasurface:

$$\phi_n(x_n, y_n) = \phi_{\text{desired}}(x_n, y_n) - \phi_{\text{incident}}(x_n, y_n). \quad (4.2.1)$$

Based on the above equation, the reconfigurable transmissive metasurface is used to perform phase compensation by determining the transmission phase profile along the metasurface. The transmission phase profile is then used to determine the corresponding capacitance values C_{var} of the RLC varactor diodes integrated into the metasurface unit cells. The capacitance values are extracted from a lookup database. This database maps the transmission phase response of the proposed metasurface unit cell to the capacitance values C_{var} (see Fig. 4.16). Fig. 4.8 shows the extracted capacitance values of the 16 modules required to achieve phase compensation in the azimuth plane for horizontal polarization at 10 GHz. It is important to note that the variation of the determined capacitances across the metasurface is smooth. This smooth variation is critical for the impedance extraction process, which relies on the local periodic approximation.

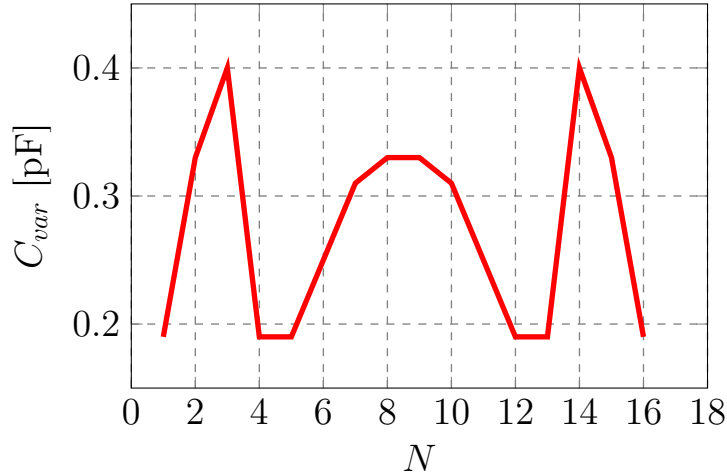


Fig. 4.8: Extracted capacitance values C_{var} required to achieve phase compensation at 10 GHz.

The 3D simulation model shown in Fig. 4.6 including the capacitance values is simulated using CST Studio Suite. Fig. 4.9 presents a comparison of the simulated radiation patterns in the azimuth plane for the horn antenna with and without the reconfigurable transmissive metasurface. As can be seen, the HPBW is reduced from 44° (horn antenna without metasurface) to 10° using the reconfigurable transmissive metasurface demonstrating the effectiveness of the proposed reconfigurable transmissive metasurface to provide phase compensation. Furthermore, the sidelobe level of the simulated radiation pattern using the metasurface is -11 dB. This is mainly attributed to the uniform amplitude distribution.

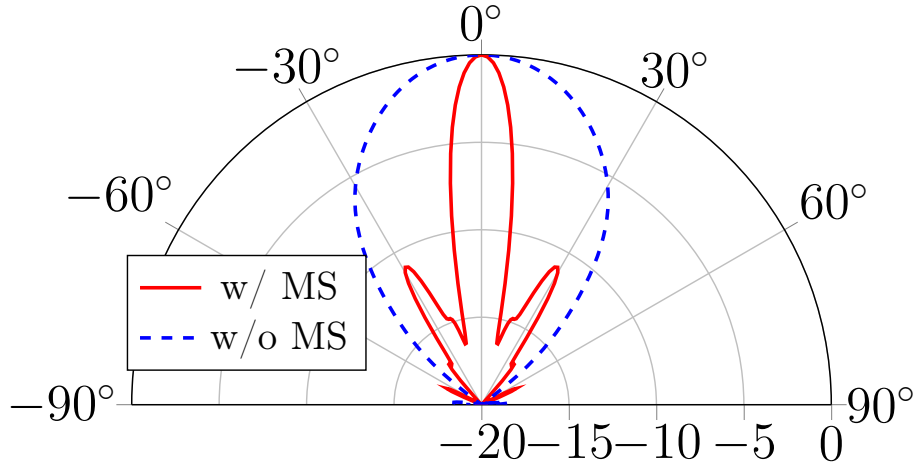


Fig. 4.9: Simulated radiation patterns in the azimuth plane: reconfigurable transmissive metasurface with phase compensation compared to a horizontally-polarized horn antenna at 10 GHz.

4.2.2 Beam Scanning

Fig. 4.10 illustrates the principle of beam scanning by a transmissive metasurface. As can be seen, each metasurface unit cell introduces a phase shift to convert the spherical wavefront from a horizontally-polarized horn antenna into a focused beam directed at scanning angle θ_0 . The phase shift ϕ_n introduced by the n -th metasurface unit cell to steer the beam to scanning angle θ_0 is determined by:

$$\phi_n(x_n, y_n) = \frac{2\pi}{\lambda_0} x_n \sin(\theta_0) - \phi_{\text{incident}}(x_n, y_n), \quad (4.2.2)$$

where the first term $\frac{2\pi}{\lambda_0} x_n \sin(\theta_0)$ determines the output progressive phase shift along the metasurface. x_n is the position of the n -th metasurface unit cell along the x -direction (i.e., one-dimensional array). $\phi_{\text{incident}}(x_n, y_n)$ represents the incident spherical phase profile of the horizontally-polarized horn antenna (i.e., phase compensation).

The reconfigurable transmissive metasurface is used to achieve beam scanning at -20° in the azimuth plane at 10 GHz. For this purpose, Eq. 4.2.2 is used to determine the transmission phase profile $\phi_n(x_n, y_n)$. Based on the transmission phase response, the corresponding capacitance values are then retrieved from the lookup database and shown in Fig. 4.11. We can observe that the capacitance profile shows abrupt variations at the edge elements (i.e., $N = 10$ to $N = 18$). Furthermore, as mentioned previously, the transmission phase range provided by the proposed reconfigurable metasurface unit cell is limited to 170° . To overcome this limitation, a partial phase compensation is applied, particularly at the edge elements [21].

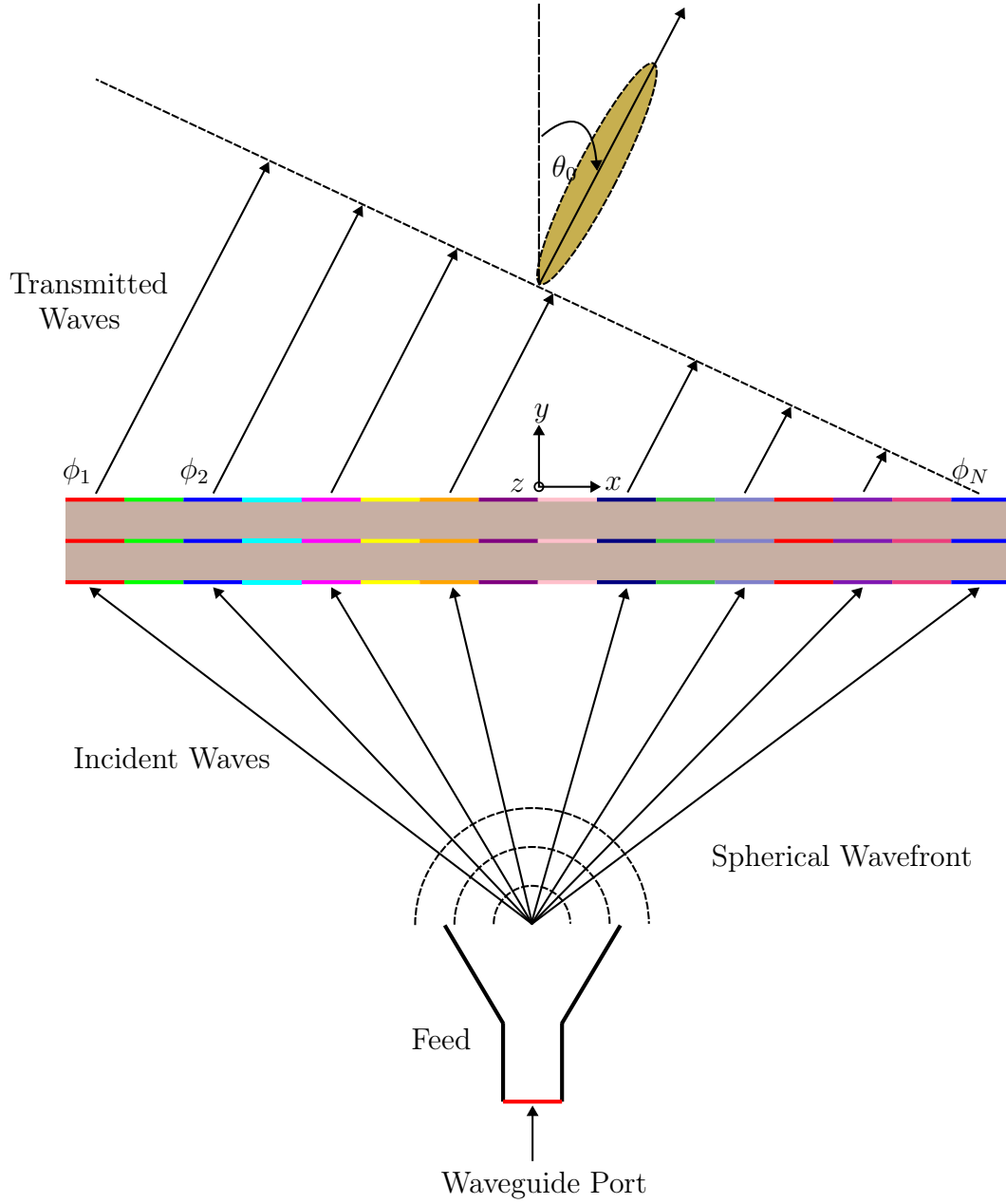


Fig. 4.10: Illustration of beam scanning at (θ_0) performed by the reconfigurable transmissive metasurface. The metasurface consists of N unit cells and the colored line segments represent different multilayer metasurface unit cells. Each metasurface introduces a certain phase shift to the manipulate the phase of transmitted wave.

The 3D simulation model shown in Fig. 4.6 including the capacitance values is simulated in CST Studio Suite. Fig. 4.12 shows the simulated radiation pattern of the reconfigurable transmissive metasurface compared with the radiation pattern of the horizontally-polarized horn antenna.

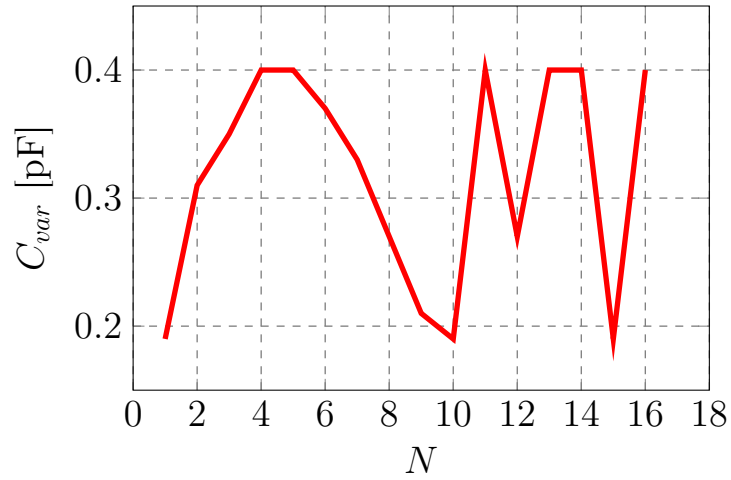


Fig. 4.11: Capacitance values of the metasurface unit cells required to achieve beam scanning at -20° .

The simulated main beam direction is at -20° with a half-power beamwidth equal to 9° . It can be clearly observed that the maximum SLL is -8 dB at 27° . The relatively high SLL is primarily due to the limited phase range of 170° provided by the reconfigurable metasurface.

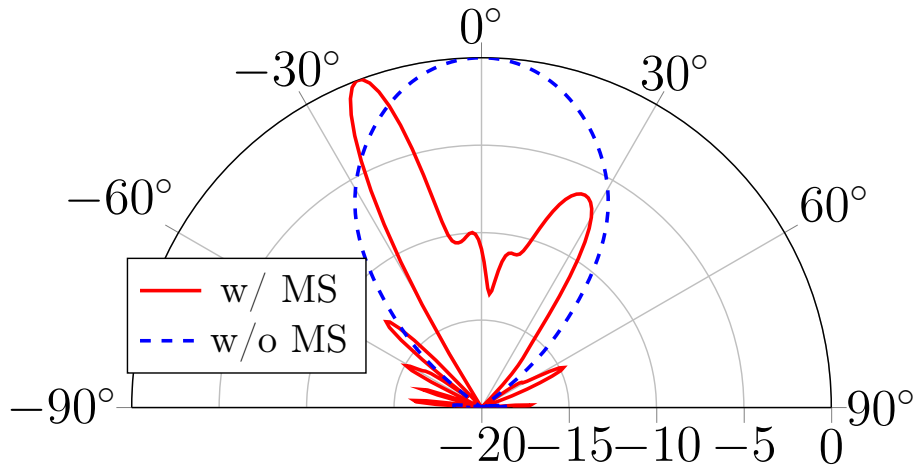


Fig. 4.12: Simulated radiation patterns in the azimuth plane for beam steering at -20° : reconfigurable transmissive metasurface compared to a horizontally-polarized horn antenna at 10 GHz.

4.3 Numerical Analysis

This section will focus on the analysis of the reconfigurable transmissive metasurface using the MoM modeling method for two applications phase compensation explained in Section 4.2.1 and one-dimensional beam scanning explained in Section 4.2.2. The results obtained from the MoM modeling method will be validated with those obtained from CST Studio Suite to demonstrate the accuracy and efficiency of the proposed method. The analysis of the transmissive metasurface using the MoM modeling method is divided into:

1. Construction of a two-dimensional multilayer impedance model equivalent to the physical problem shown in Fig. 4.6
2. Surface impedance homogenization of the subwavelength metallic patterns
3. Solve for the unknown equivalent electric currents
4. Calculation of the total and scattered far-fields
5. Validation against full-wave simulations

Since the considered reconfigurable transmissive metasurface shown in Fig. 4.6 is periodic along the z -direction, an equivalent two-dimensional impedance model is constructed. This is achieved by applying the surface equivalence principle, the metal sheets forming the horn are replaced by equivalent electric surface currents denoted by $\mathbf{J}_h$. The subwavelength metallic patterns of the multilayer metasurface are homogenized and represented by zero-thickness impedance sheets, denoted as $Z_{L_1,n}$, $Z_{L_2,n}$ and $Z_{L_3,n}$, where n refers to the n -th unit cell. The corresponding equivalent surface currents on the impedance sheets are represented as $\mathbf{J}_{s,L_1}$, $\mathbf{J}_{s,L_2}$ and $\mathbf{J}_{s,L_3}$. The dielectric substrate layers removed and replaced by equivalent polarization electric currents according to the volume equivalence principle. The corresponding equivalent polarization currents on the dielectric layers are represented as $\mathbf{J}_p$. A magnetic line source oriented along the z -axis and radiates TM_z cylindrical waves (z -directed H-field) with respect to the xy -plane is used to illuminate the reconfigurable metasurface. The incident electric field radiated by the magnetic line source has E_x and E_y field components. The incident magnetic field has only H_z component. Fig. 4.13 illustrates the two-dimensional equivalent model with the corresponding equivalent currents.

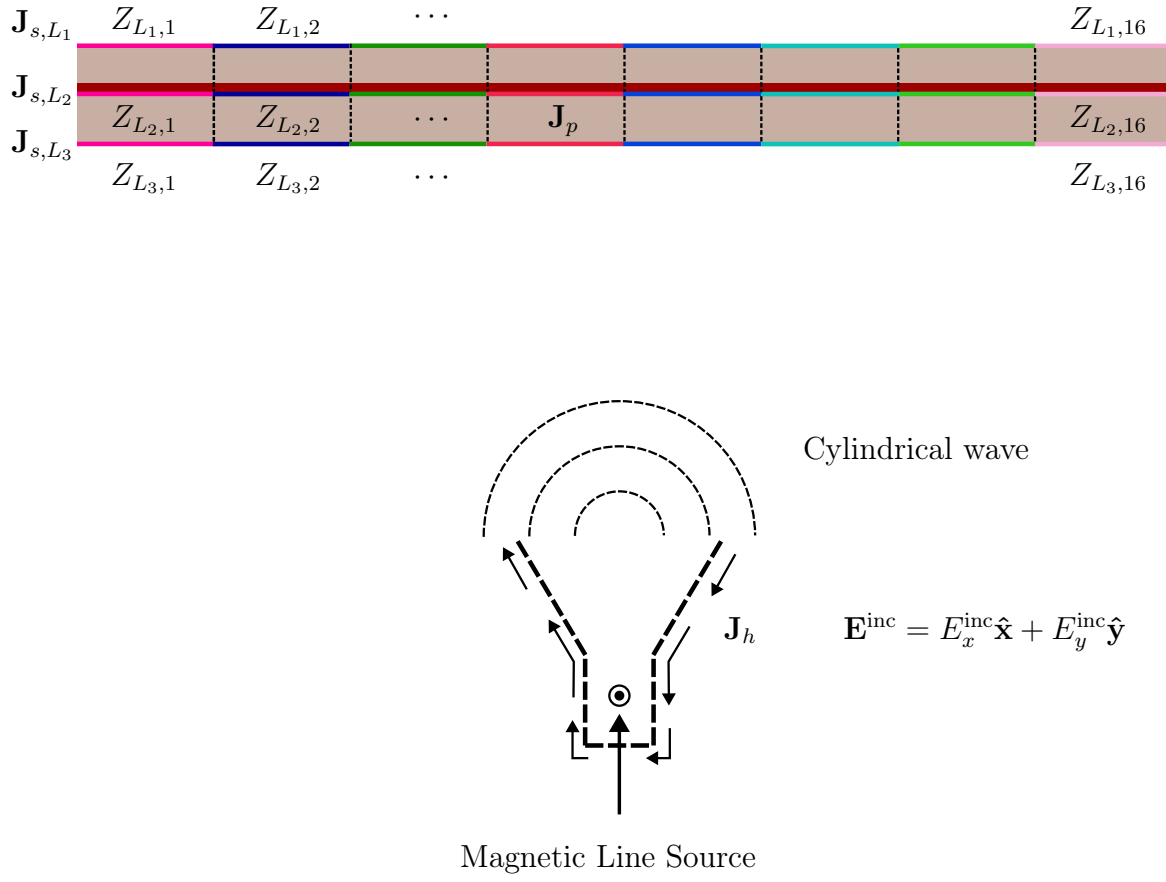


Fig. 4.13: Two-dimensional equivalent impedance model of the reconfigurable multilayer transmissive metasurface.

4.3.1 Homogenization Model Based on Equivalent Impedance

For a given polarization, the three metallic patterns of the reconfigurable transmissive metasurface unit cell are homogenized and modeled as zero-thickness electric impedance sheets, denoted as Z_1 , Z_2 , and Z_3 . In this model, Z_1 and Z_3 correspond to the outer metallic layers, while Z_2 represents the impedance of the middle layer. Due to the symmetric design of the unit cell, Z_1 and Z_3 are equal [80]. Fig. 4.14 shows the transmissive multilayer unit cell structure and its equivalent impedance model.

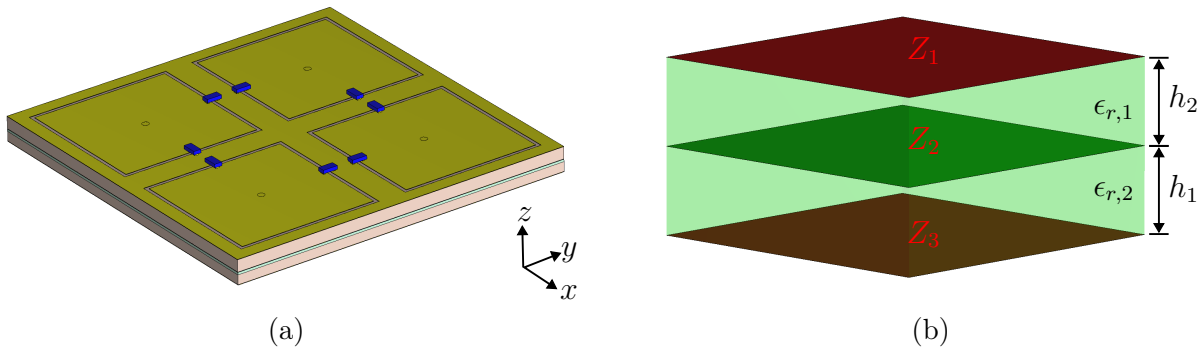


Fig. 4.14: Surface impedance homogenization. (a) Unit cell structure. (b) Equivalent impedance unit cell.

A two-port network representation of the metasurface unit cell structure is used to extract the impedance of each metallic layer. It consists of three shunt admittances representing the three metallic layers, Y_1 , Y_2 , and Y_3 , along with two sections of lossless transmission lines that model the two dielectric substrate layers [81], as illustrated in Fig. 4.15. The lengths of the transmission lines correspond to the thicknesses of the dielectric substrate layers, denoted as h_1 and h_2 .

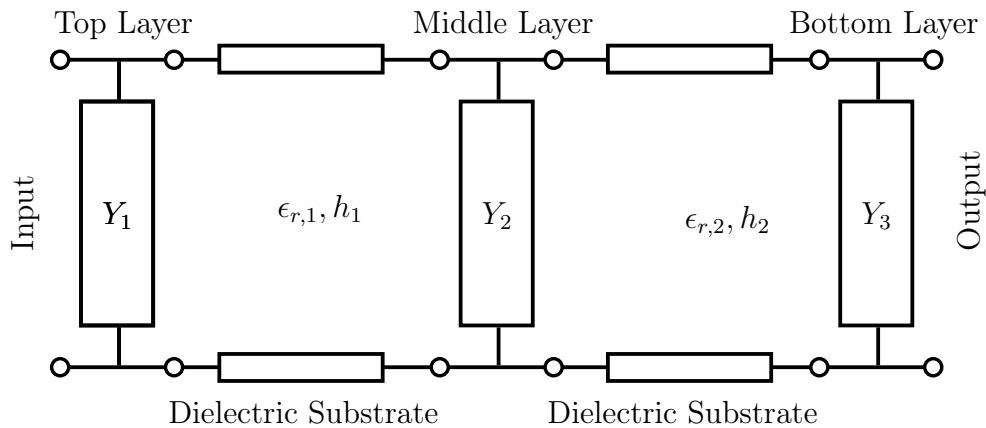


Fig. 4.15: Two-port network representation of the multilayer metasurface unit cell structure.

The metasurface unit cell structure can be analyzed using an ABCD matrix representation as follows [7, p. 154][81, 82]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\beta_1 h_1) & jZ_d \sin(\beta_1 h_1) \\ \frac{j \sin(\beta_1 h_1)}{Z_d} & \cos(\beta_1 h_1) \end{bmatrix} \quad (4.3.1)$$

$$\cdot \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\beta_2 h_2) & jZ_d \sin(\beta_2 h_2) \\ \frac{j \sin(\beta_2 h_2)}{Z_d} & \cos(\beta_2 h_2) \end{bmatrix} \quad (4.3.2)$$

$$\cdot \begin{bmatrix} 1 & 0 \\ Y_3 & 1 \end{bmatrix}. \quad (4.3.3)$$

Here, Y_1 and Y_3 represent the admittances of the outer layers, while Y_2 corresponds to the admittance of the middle layer. β_1 and β_2 are the propagation constants in the z -direction of the dielectric substrates. h_1 and h_2 represent the thickness of the dielectric substrates. The characteristic impedance $Z_d^{\text{TE, TM}}$ of the transmission line model for the TM and TE excitations under oblique incidence is given by [83]:

$$Z_d^{\text{TM}} = \frac{Z_0}{\sqrt{\epsilon_r}} \cos(\theta_{\text{ref}}), \quad (4.3.4)$$

$$Z_d^{\text{TE}} = \frac{Z_0}{\sqrt{\epsilon_r} \cos(\theta_{\text{ref}})}, \quad (4.3.5)$$

$$\beta = \frac{2\pi\sqrt{\epsilon_r} \cos(\theta_{\text{ref}})}{\lambda_0}. \quad (4.3.6)$$

The refraction angle θ_{ref} is determined using Snell's law:

$$\theta_{\text{ref}} = \sin^{-1}\left(\frac{\sin(\theta_{\text{inc}})}{\sqrt{\epsilon_r}}\right). \quad (4.3.7)$$

Here θ_{inc} is the incident angle on the metasurface unit cell structure with respect to the z -axis. The complete ABCD matrix characterizing the metasurface unit cell can be derived from its simulated S-parameters [81]:

$$A = \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}, \quad (4.3.8a)$$

$$B = Z_0 \frac{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}{2S_{21}}, \quad (4.3.8b)$$

$$C = \frac{1}{Z_0} \frac{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}{2S_{21}}, \quad (4.3.8c)$$

$$D = \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{2S_{21}}. \quad (4.3.8d)$$

Based on the ABCD matrix representation, the admittances of the three metallic layers can be written as [7, 82]:

$$Y_1 = \frac{D - jZ_0 \sin(\beta h_1) \cos(\beta h_1) Y_2 - \cos^2(\beta h_1) + \sin^2(\beta h_2)}{2jZ_0 \sin(\beta h_2) \cos(\beta h_1) - Z_0^2 \sin^2(\beta h_1) Y_2}, \quad (4.3.9)$$

$$Y_2 = \frac{B - 2jZ_0 \sin(\beta h_1) \cos(\beta h_2)}{-Z_0^2 \sin^2(\beta h_2)}, \quad (4.3.10)$$

$$Y_3 = \frac{A - jZ_0 \sin(\beta h_1) \cos(\beta h_1) Y_2 - \cos^2(\beta h_1) + \sin^2(\beta h_2)}{2jZ_0 \sin(\beta h_1) \cos(\beta h_1) - Z_0^2 \sin^2(\beta h_1) Y_2}. \quad (4.3.11)$$

A database is constructed to map the capacitance values C_{var} and incident angles θ_{inc} to the corresponding equivalent impedances representing the metasurface unit cell. This database is based on the simulated S-parameters of the metasurface unit cell under different incident angles, as well as Eqs. 4.3.9, 4.3.10, and 4.3.11. Fig. 4.16 displays the simulated S-parameters of the unit cell as functions of capacitance values C_{var} and the incident angle θ_{inc} at 10 GHz under TM excitation. The capacitance values range from 0.19 pF to 0.4 pF, while the incident angle varies from 0° to 30° . The ABCD matrix is determined from the simulated S-parameters, and the equivalent impedances (Z_1 , Z_2 and Z_3) are then retrieved as functions of C_{var} and θ_{inc} , as shown in Fig. 4.17. It can be observed that the top and bottom impedances, Z_1 and Z_3 , exhibit capacitive behavior, while the middle layer impedance Z_2 is inductive.

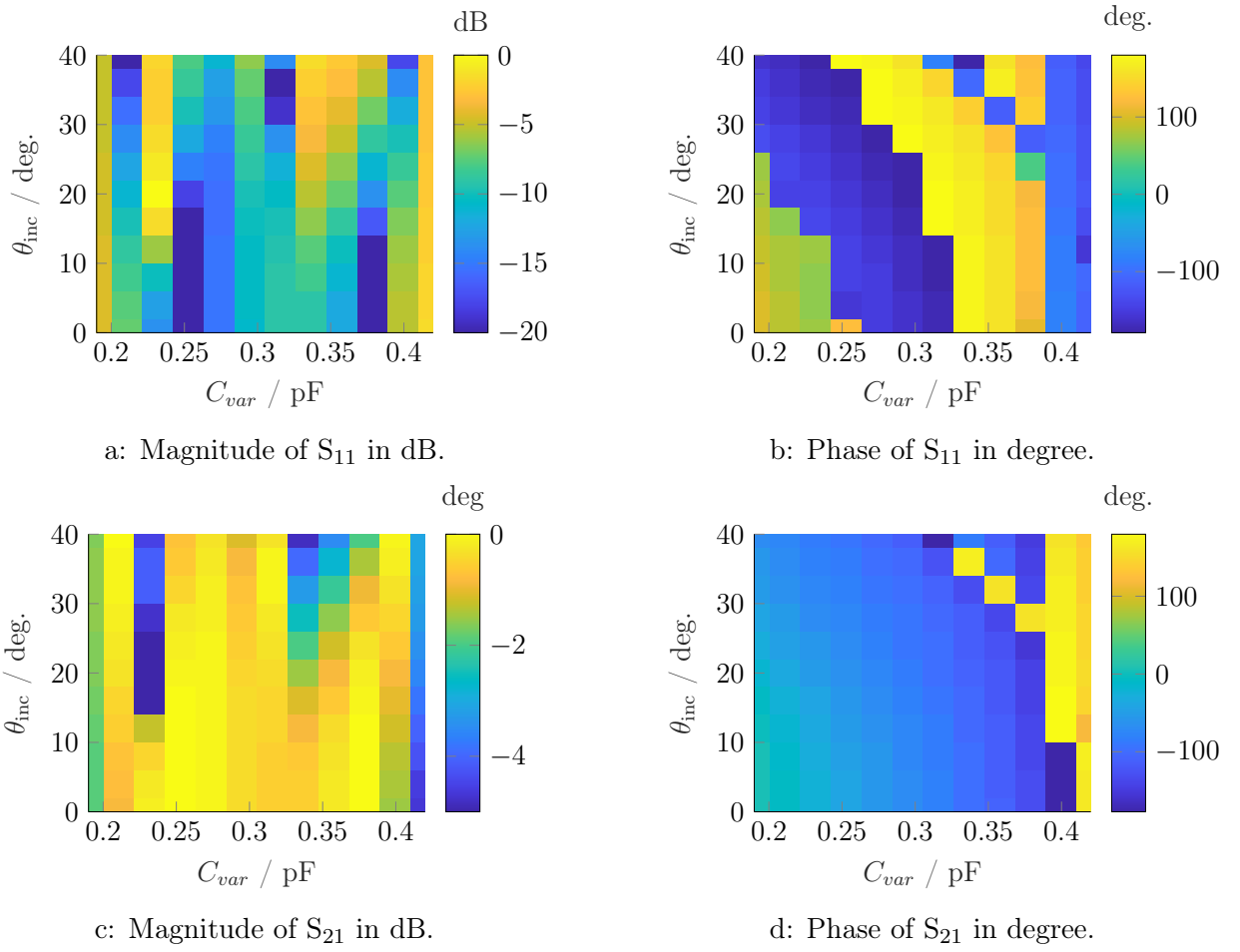


Fig. 4.16: Simulated S-parameters of the reconfigurable metasurface unit cell under TM excitation at 10 GHz for different incident angles.

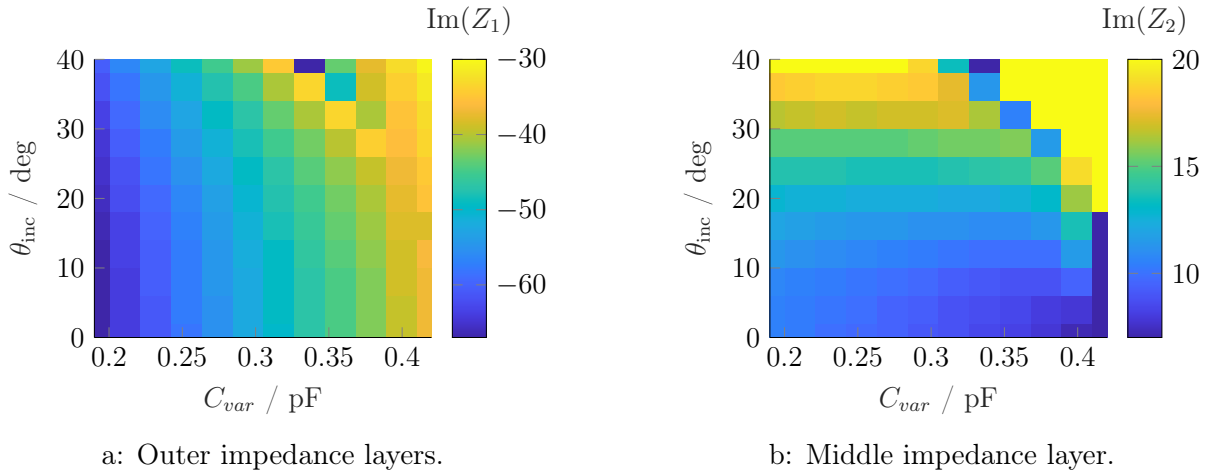


Fig. 4.17: Extracted sheet impedance values ($\Omega/\square$) under TM excitation at 10 GHz for different incident angles.

4.3.2 MoM Solution

To determine the unknown equivalent currents on the impedance sheets and the equivalent polarization currents within the dielectric substrate layers, the two-dimensional equivalent model is discretized. Fig. 4.18 illustrates the discretization of the two-dimensional equivalent impedance model. Each impedance layer is divided into line segments, where piecewise linear basis functions are used to expand the equivalent electric currents on the impedance layers. The dielectric substrate layers are discretized into small rectangular elements of size $\Delta x_p \times \Delta y_p$ with rooftop basis functions employed to expand the equivalent polarization currents $J_{x,p}$ and $J_{y,p}$. The geometry of the feeding horn is similarly discretized into line segments and the equivalent electric surface currents $\mathbf{J}_h$ are expanded using triangular basis functions [84].

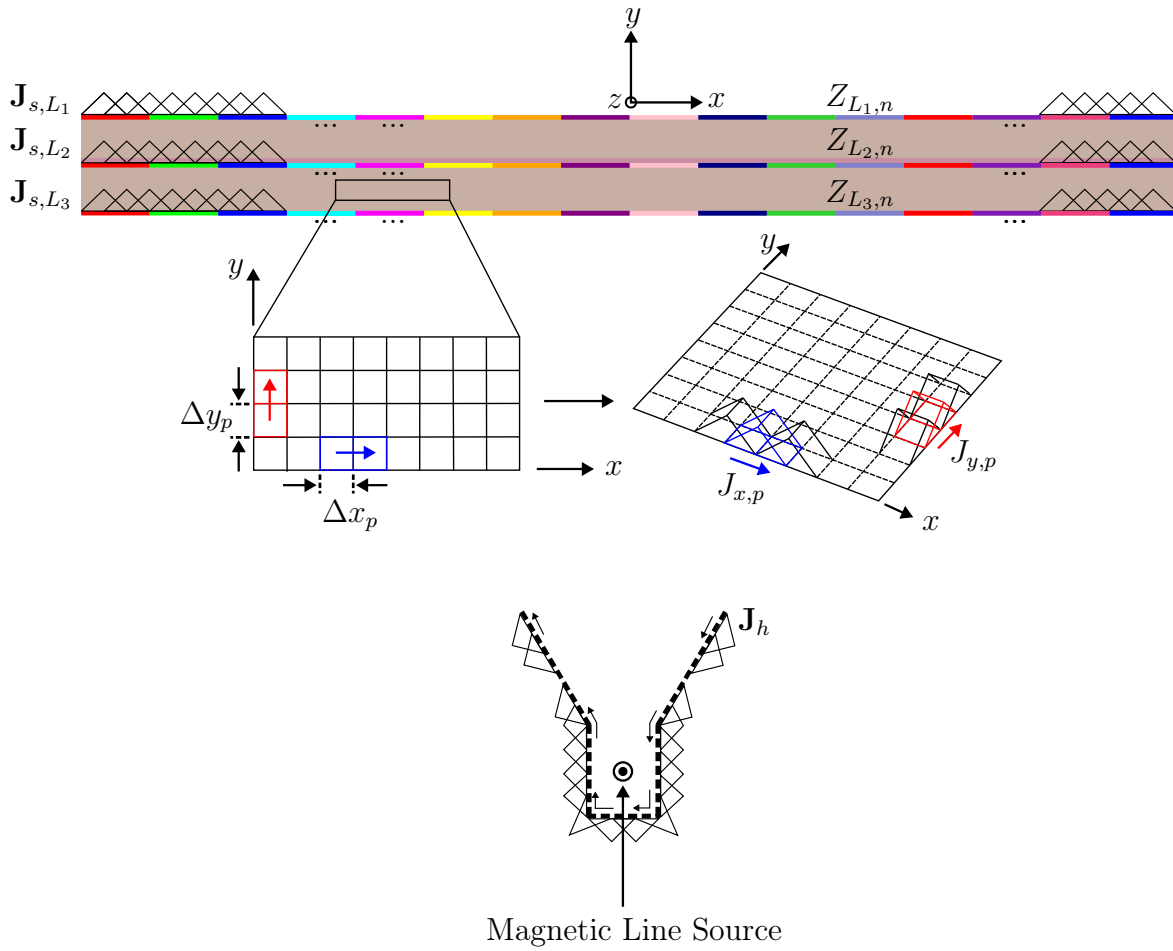


Fig. 4.18: Discretization of the two-dimensional equivalent impedance model including the non-uniform impedance sheets, the dielectric substrate layers and the horn antenna.

4.3.3 Numerical Results

Phase Compensation

To predict the far-field radiation pattern for the phase compensation example explained in 4.2.1 using the MoM modeling method, the impedances Z_1 , Z_2 and Z_3 corresponding to the capacitance values of the reconfigurable transmissive metasurface unit cell structure (Fig. 4.8) are extracted based on the lookup database shown in Fig 4.17. The extraction is performed based on the local periodic approximation, where each unit cell with the corresponding capacitance value is simulated under periodic boundary condition.

Fig. 4.19 shows the impedance values of the top, bottom and middle layers at 10 GHz for phase compensation configuration. Since the unit cell is symmetric, the top impedance Z_1 and bottom impedance Z_3 are equal. As expected, the top and bottom impedances Z_1 and Z_3 exhibit capacitive behavior, while the the middle layer Z_2 is inductive. Moreover, the extracted impedance profiles show smooth variation across the metasurface.

The obtained impedance values are then used in the proposed MoM modeling method to predict the radiation pattern for phase compensation configuration. Fig. 4.20 compares the radiation pattern results from the proposed MoM modeling method and the simulated radiation pattern obtained from simulating the physical model using CST Studio Suite (Fig. 4.9). As can be seen, the radiation pattern predicted by the proposed MoM modeling method is in a good agreement with the result obtained from the full-wave simulation. Table 4.2 summarizes the radiation characteristics for phase compensation configuration at 10 GHz. The good agreement between the MoM solution and the full-wave simulation can be primarily attributed to the smooth spatial variation of the artificial surface's properties. The capacitance profile shown in Fig. 4.8 changes slowly, which ensures that the local periodic assumption used to extract the surface impedance remains valid.

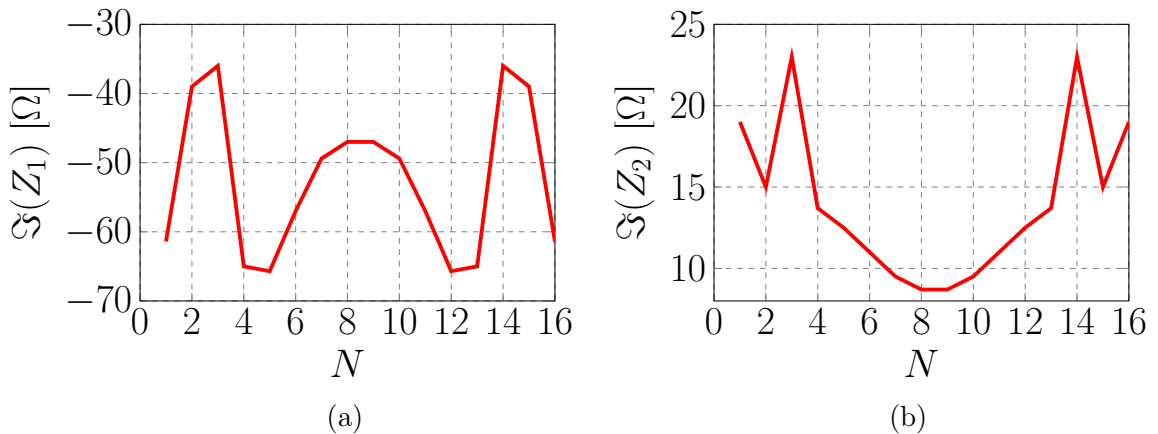


Fig. 4.19: Extracted reactive sheet impedance values for phase compensation at 10 GHz: (a) $Z_1 = Z_3$, (b) Z_2 .

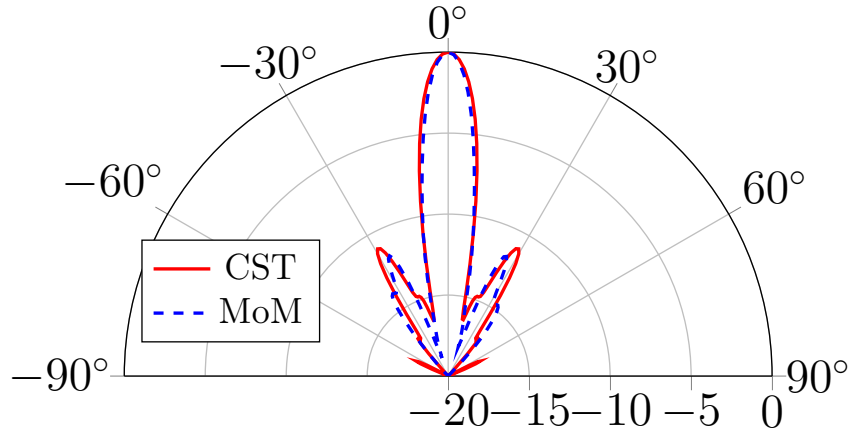


Fig. 4.20: Comparison of radiation patterns obtained from full-wave simulation and MoM for phase compensation at 10 GHz.

Table 4.2: Radiation characteristics for phase compensation case at 10 GHz.

Parameter	MoM	CST
HPBW	9°	10°
Maximum SLL	-12 dB	-11 dB

Beam Scanning

The impedances Z_1 , Z_2 and Z_3 corresponding to the capacitance values (Fig. 4.11) used in the beam scanning example are extracted from the lookup database and presented in Fig. 4.21. Similar to the phase compensation example, the top and bottom impedances Z_1 and Z_3 exhibit capacitive behavior and the middle layer Z_2 shows inductive behavior. In addition, the extracted impedance profiles show smooth spatial variation from $n = 1$ to $n = 10$ and abrupt changes at the edge elements from $n = 11$ to $n = 16$.

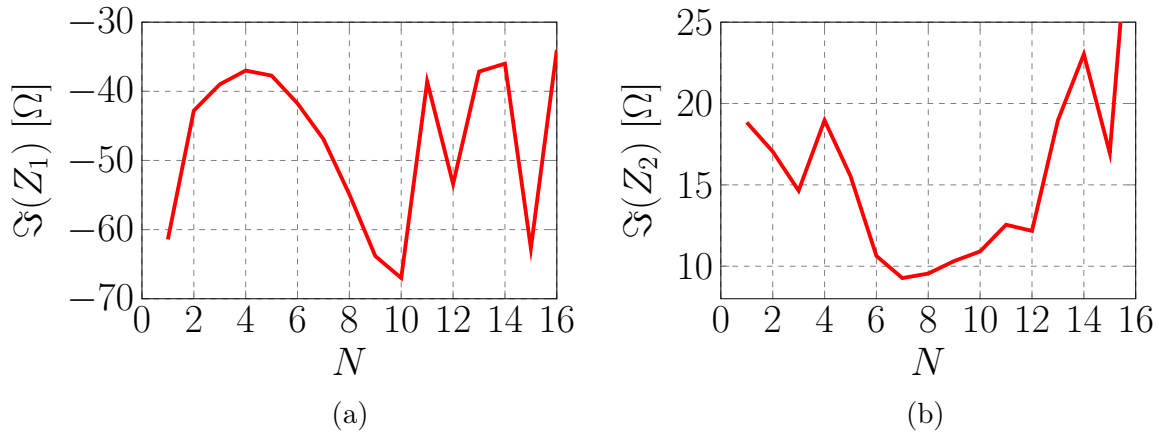


Fig. 4.21: Extracted reactive sheet impedance values for beam scanning at -20° and 10 GHz: (a) $Z_1 = Z_3$, (b) Z_2 .

The extracted non-uniform impedance sheets are used by the MoM to predict the radiation pattern. Fig. 4.22 presents a comparison between the far-field radiation patterns obtained by the MoM and CST Studio Suite. As can be seen, the radiation pattern predicted by the MoM is in a good agreement with CST. Table 4.22 presents the radiation characteristics: main beam direction, half-power beamwidth and the maximum SLL.

Differences in the radiation patterns at approximately $\pm 60^\circ$ can be observed between the two methods. These discrepancies can be attributed to abrupt variations in the capacitance values particularly at the edge elements, which breaks the fundamental assumption of the local periodic approximation and lead to inaccuracies in impedance extraction.

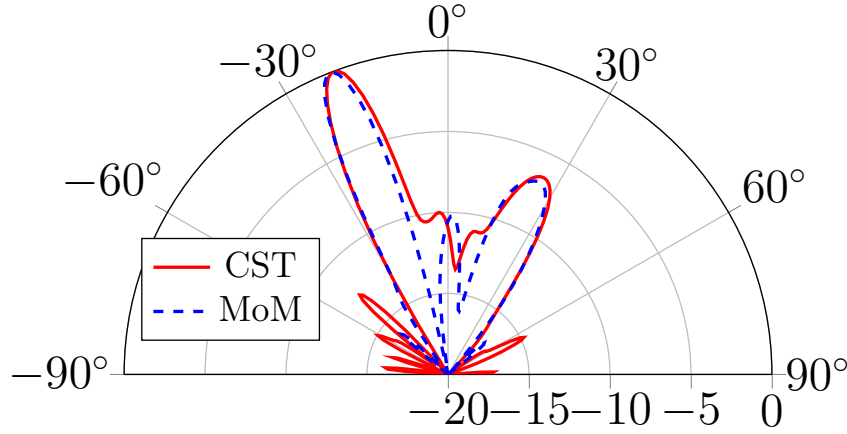


Fig. 4.22: Comparison of radiation patterns obtained from full-wave simulation and MoM for beam scanning case at -20° .

Table 4.3: Radiation characteristics for beam scanning at -20° .

Parameter	MoM	CST
Beam direction	-21°	-21°
HPBW	8°	9°
Maximum SLL	-8.2 dB	-8 dB

4.4 Lens Array Measurements and Analysis

The proposed reconfigurable metasurface can function as a transmissive reconfigurable intelligent surface (T-RIS) to improve the link budget of a wireless communication channel and/or enhance the accuracy of indoor sensing and localization functionalities. This is achieved by focusing and scanning electromagnetic (EM) waves toward the receiver or target. In various scenarios, such as communication between adjacent rooms (as depicted in Fig. 4.23) or outdoor-to-indoor communication, the T-RIS effectively minimizes the blockage losses that EM waves typically encounter when penetrating walls. Therefore, the proposed reconfigurable metasurface demonstrator is tested as a T-RIS by measuring

its ability to focus and steer electromagnetic waves emitted by a horizontally-polarized horn antenna positioned at a distance. Fig. 4.24 illustrates the measurement setup for the fabricated tunable metasurface, which is illuminated by the horizontally-polarized horn antenna within the measurement chamber. The setup comprises a dielectric frame that holds the metasurface in front of the horn antenna, while the metasurface's controller is positioned behind it. To ensure that only the EM waves passing through the T-RIS are characterized, the holder was covered with EM absorbers. This step mimics the scenario of an EM wave passing through an opening between two rooms, where the absorber blocks the waves that would typically be obstructed by the wall as in Fig. 4.23, allowing us to focus solely on the waves passing through the opening. The distance between the horn antenna and the center of the metasurface is maintained at 30 cm ($10 \lambda_0$ at 10 GHz), which maximize the distance while preserving the mechanical stability of the holder during far-field measurements in the anechoic chamber. The horn antenna aperture is designed to produce a nearly planar wavefront in the vertical plane, effectively illuminating the tunable metasurface.

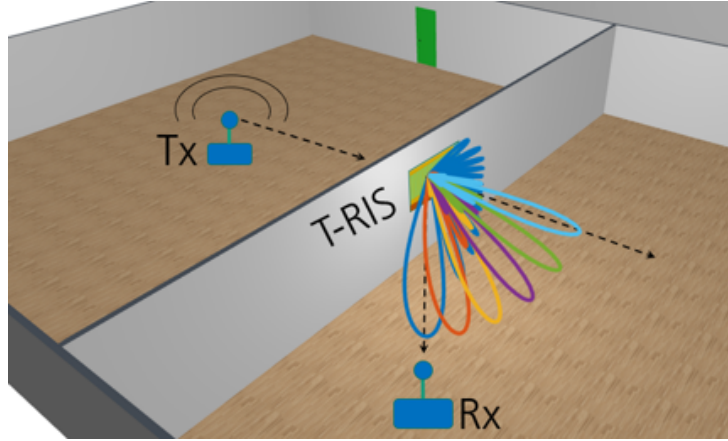


Fig. 4.23: A scenario of using the proposed tunable metasurface as a transmissive reconfigurable intelligent surface (T-RIS).

The measurement results, shown in Fig. 4.25, demonstrate that the reconfigurable metasurface has beam steering capabilities ranging from -60° to 60° . It is clear that the proposed metasurface focuses the EM wave passing through it in the desired direction. At 0° , beam focusing narrowed the half-power beamwidth from 25° to 8° and increased the gain of the EM wave passing through the T-RIS by 5 dB. Moreover, at a 60° angle, the gain increased by 16 dB compared to the horn antenna without T-RIS, resulting in a significant improvement in the received power at the receiver.

Fig. 4.26 presents a comparison between the measured radiation pattern of the fabricated reconfigurable transmissive metasurface and the radiation pattern obtained using the MoM modeling method for phase compensation example. As can be observed, the HPBW obtained from the MoM simulation is slightly broader than that of the measured radiation pattern. Moreover, the measured SLL is higher than the one predicted by the MoM. These discrepancies could be attributed to these reasons. The MoM modeling does not consider



Fig. 4.24: Measurement setup of the fabricated metasurface together with the horn antenna. (a) Tunable metasurface illuminated by the horn. (b) Horizontally polarized horn antenna without the tunable metasurface.

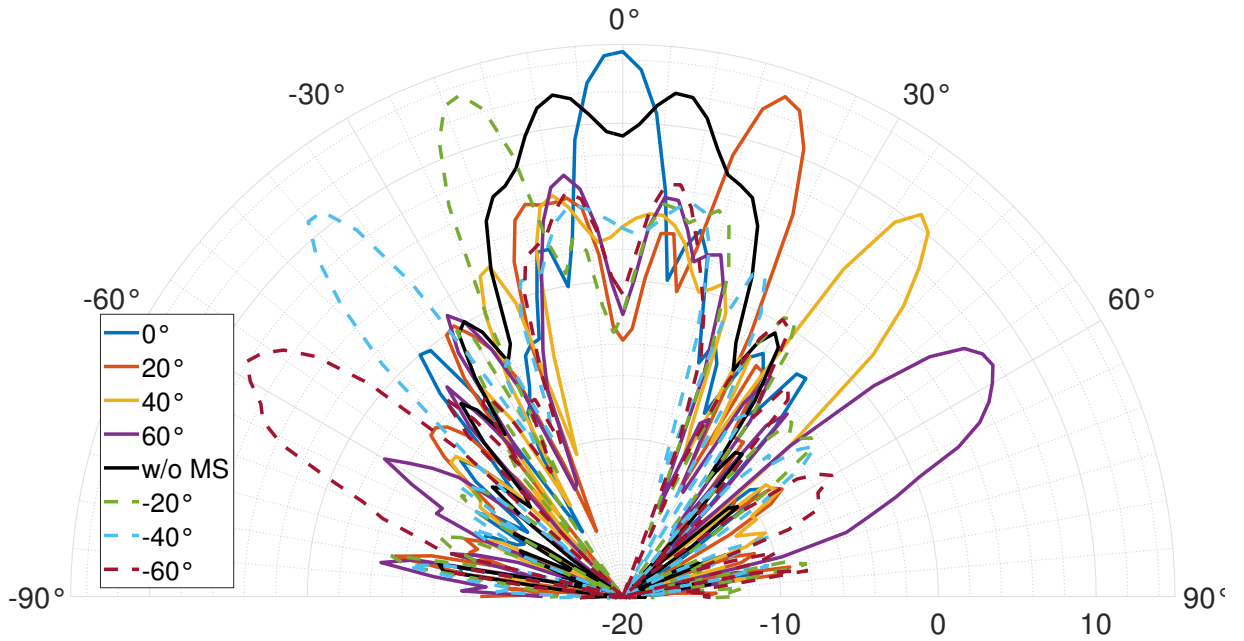


Fig. 4.25: Measured gain patterns in dB of the reconfigurable metasurface illuminated by the horn antenna for different scanning angles compared to the gain pattern of the horn antenna at 9.8 GHz in the horizontal plane.

the edge effects resulting from the finite size of the fabricated metasurface. Further, the usage of absorbers around the fabricated metasurface in the measurement setup can also influence the measured radiation pattern.

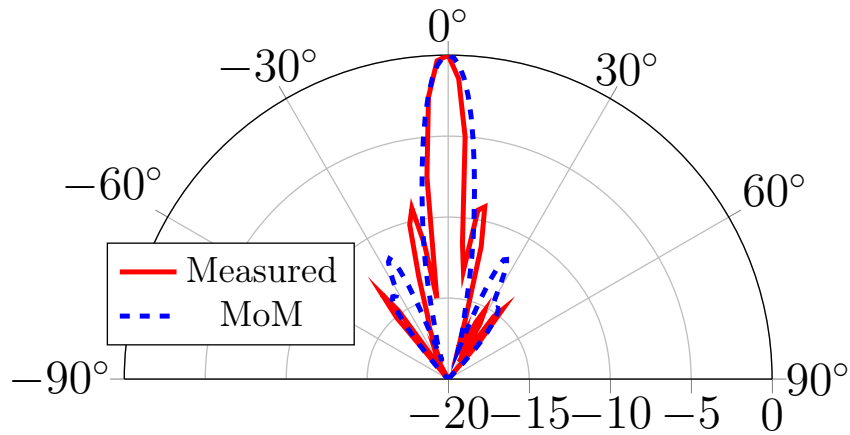


Fig. 4.26: Comparison between the measured and predicted radiation patterns for phase compensation at 10 GHz.

4.5 Summary

Two illustrative examples, phase compensation, and one-dimensional beam scanning, have been demonstrated to show the accuracy and efficiency of the proposed MoM modeling method. For this purpose, a dual-linear-polarized reconfigurable transmissive metasurface is designed to achieve phase compensation and one-dimensional beam scanning in the azimuth plane at the X-band. The metasurface unit cell structure consists of three metallic layers separated by two dielectric substrates. The transmission phase of the metasurface can be simultaneously controlled in both polarizations, with a phase range of up to 170° in the X-band. The tunability is achieved by using surface-mounted varactor diodes placed on the top and bottom metallic layers.

A homogenization impedance model has been developed based on a two-port network representation to extract the equivalent impedances that represent the multilayer transmissive metasurfaces. It has been demonstrated that the proposed modeling method accurately predicts the radiation characteristics with good agreement with full-wave simulations using CST Studio Suite.

Finally, a prototype of the reconfigurable transmissive metasurface is fabricated and measured to achieve phase compensation and dynamic beam scanning in a one-dimensional plane at the X-band. The measured results demonstrate that the fabricated reconfigurable transmissive metasurface enables dynamic beam scanning up to $\pm 60^\circ$.

Analysis and Modeling of Reflective Metasurface

In this chapter, we focus on the analysis and modeling of reflective metasurfaces using the proposed modeling method. Reflective metasurfaces are used to control the amplitude and phase of the reflected waves, enabling different applications such as beam focusing [2, 12–14]. A single-layer dual-polarized reflective metasurface is used as an illustrative example to perform phase compensation of an incident spherical wave at 12 GHz. The considered unit cell structure is based on the design in [2].

The chapter is divided into two parts: the first part presents the simulations of the dual-polarized reflective metasurface unit cell to determine the complex reflection coefficients. The reflective metasurface is then used to convert the incident spherical wavefront from a vertically-polarized horn antenna to a planar wavefront, resulting in a focused beam. The phase compensation is achieved by controlling the reflection phase response of each metasurface unit cell forming the surface to introduce a spatially varying phase delay along the incident spherical wavefront. The second part of the chapter focuses on the analysis and modeling of the reflective metasurface by the MoM modeling method and a comparison with full-wave simulations.

This chapter is structured as follows: Sections 5.1 and 5.2 include the simulations and characterization of a dual-polarized reflective metasurface unit cell, as well as its application for beam focusing. Section 5.3 focuses on the analysis and modeling of the reflective metasurface using the proposed MoM modeling method. It is divided into two main parts: the first part explains the homogenization model developed to determine the equivalent zero-thickness impedance sheets used to represent the subwavelength physical unit cell structures. The second part discusses the expansion of the equivalent electric currents using pulse basis functions and the solution of the MoM. The far-field radiation pattern predicted by the MoM modeling method is compared with full-wave simulations using CST Studio Suite and discussed in 5.3.3. Finally, summary is stated in 5.4.

5.1 Design of Reflective Metasurface

5.1.1 Unit Cell Design

The metasurface unit cell structure consists of a Jerusalem cross with an outer square loop printed on a grounded dielectric substrate with a dielectric constant equal to 2.2 and $\tan \delta = 0.002$ [2]. Fig. 5.1 shows the considered reflective metasurface unit cell structure. The periodicity of the unit cell in x - and y -directions is equal to 8.3 mm, corresponding to $\lambda_0/3$ (λ_0 is the free-space wavelength at 12 GHz). Table 5.1 lists the geometrical dimensions of the metasurface unit cell structure at 12 GHz. The ratios between the lengths of the Jerusalem cross I_{d1} , I_{d2} and the length of the outer square loop L_d are given by:

$$M = \frac{I_{d1}}{L_d}, \quad N = \frac{I_{d2}}{L_d}, \quad M = 0.7, \quad N = 0.6. \quad (5.1.1)$$

The reflection phase response is controlled by changing the geometrical dimensions of the unit cell L_d , I_{d1} , and I_{d2} [2]. It is worth mentioning that the considered unit cell and the geometrical dimensions are taken from [2] to validate the proposed MoM modeling approach against full-wave simulations.

Table 5.1: Geometrical dimensions of the unit cell structure [mm].

Parameters	Value
P	8.3
I_{w1}	0.4
I_{w2}	0.6
h	3

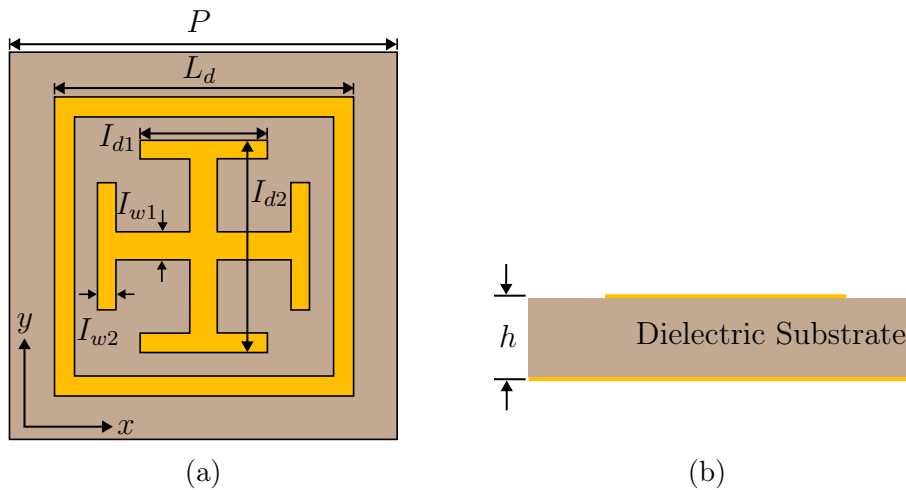


Fig. 5.1: Geometry of the reflective metasurface unit cell designed in [2]. (a) Top view. (b) Cross-sectional view.

Periodic boundary conditions (PBCs) with Floquet port are employed to determine the complex reflection coefficients using CST Studio Suite in the frequency range from 10 GHz to 13 GHz, as illustrated in Fig. 5.2. The incident angle θ_{inc} defined in the xz -plane with respect to the z -axis is changed from 0° to 50° . The reflection phase of the metasurface unit cell is controlled by changing the length of the outer square loop L_d from 4.2 mm to 7.2 mm. The lengths of the Jerusalem cross I_{d1} and I_{d2} are changed according to Eq. 5.1.1. Fig. 5.3 displays the simulated reflection phase of the unit cell under different incident angles at 12 GHz. The unit cell provides a reflection phase range equal to 360° in the frequency range from 10 GHz to 13 GHz [2].

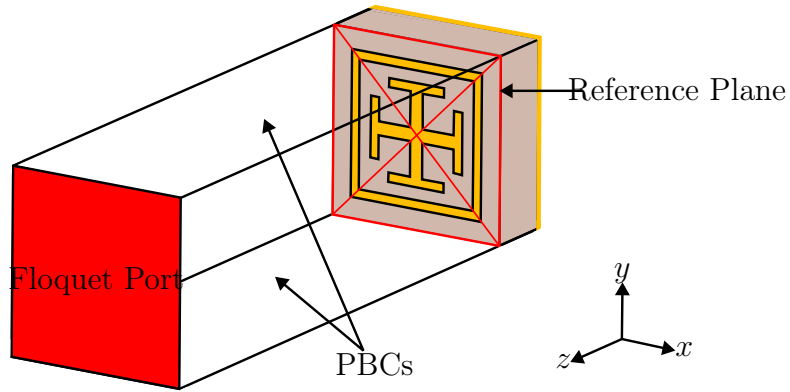


Fig. 5.2: Simulation model of the reflective metasurface unit cell with Floquet PBCs.

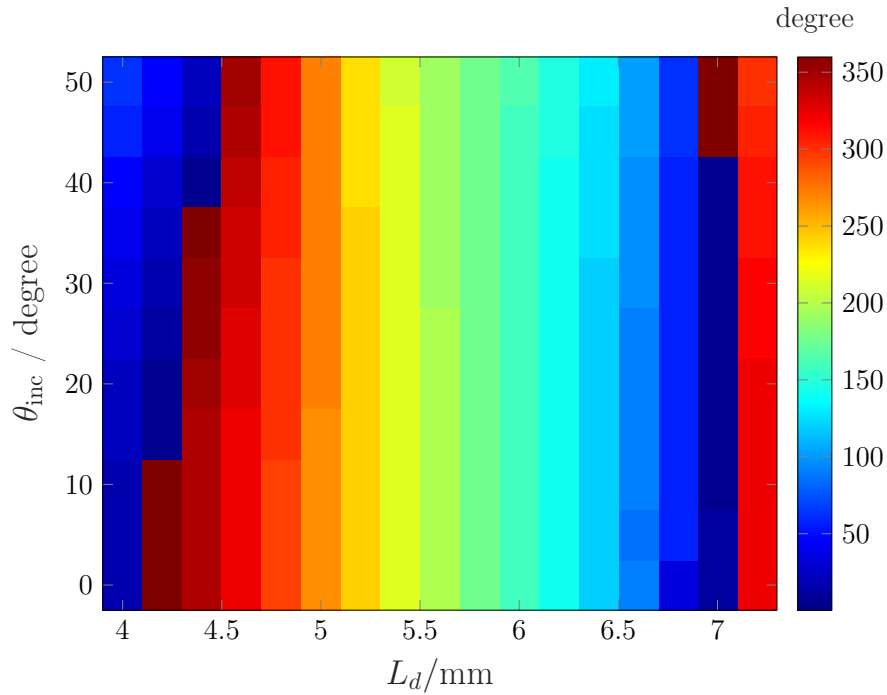


Fig. 5.3: Simulated reflection phase at 12 GHz for different incident angles (θ_{inc}).

5.2 Reflective Metasurface for Shaping Reflected Fields

Fig. 5.4 shows a reflective metasurface illuminated by a horn antenna. The horn antenna radiates a spherical wavefront, where the phase of the reflected waves is controlled by the reflective metasurface to achieve a nearly planar wavefront directed to a certain angle φ_0 . The reflective metasurface consists of N unit cells placed on a grounded dielectric substrate. The desired reflection phase shift of the m -th element for a given scan angle can be calculated by [2]:

$$\phi_m = k_0 (R_m + \mathbf{r}_m \cdot \hat{\mathbf{u}}_0) + \Delta\phi_0 \quad (5.2.1)$$

, where $R_m = |\mathbf{r}_m - \mathbf{r}_{\text{source}}|$ is the spatial distance between the source antenna and the m -th element. $\mathbf{r}_m$ is the position vector of the m -th element, as illustrated in Fig. 5.4. $\Delta\phi_0$ is a constant phase. $\hat{\mathbf{u}}_0$ is the unit vector in the desired main beam pointing direction and is expressed in the xy -plane by:

$$\hat{\mathbf{u}}_0 = \hat{\mathbf{x}} \cos \varphi_0 + \hat{\mathbf{y}} \sin \varphi_0. \quad (5.2.2)$$

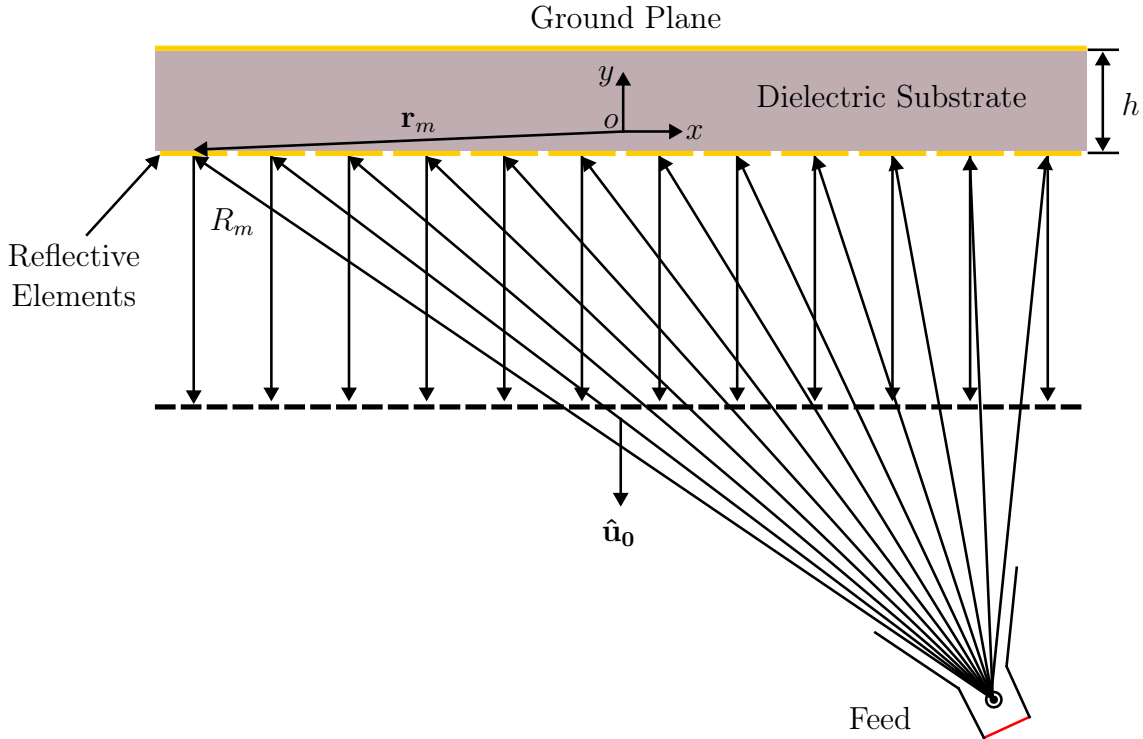


Fig. 5.4: Reflective metasurface illuminated by a horn antenna positioned at a Distance D from the metasurface.

5.2 Reflective Metasurface for Shaping Reflected Fields

A planar reflective metasurface, consisting of 21 unit cells, is designed based on the presented unit cell structure to achieve beam focusing in the azimuth plane at scanning angles 0° , 30° and 50° . The metasurface is periodic in the z -direction and the width in the x -direction is $8\lambda_0$, where λ_0 represents the free-space wavelength at 12 GHz. A vertically-polarized horn antenna is used to illuminate the reflective metasurface, with a distance $D = 150$ mm between the source antenna and the metasurface. According to Eq. 5.2.1, the required phase shift ϕ_m of each element is designed to compensate the spatial phase delay from the source antenna so that a certain phase distribution can be realized to point the beam to the desired direction. The geometrical dimensions of each element is then determined to achieve the required phase distribution. Figs. 5.6, 5.7, and 5.8 illustrate the phase distributions required to achieve beam focusing at scanning angles 0° , 30° and 50° , respectively, along with the corresponding geometrical dimensions of each reflective metasurface unit cell.

Fig. 5.5 illustrates the 3D simulation model of the reflective metasurface used for full-wave simulations. Since the considered metasurface is periodic in the $\pm z$ -direction and the polarization is vertical, electric wall boundary conditions are applied. A waveguide port is used to excite the horn antenna, which transmits a cylindrical wave that illuminates the reflective metasurface. Figs. 5.6c, 5.7c, and 5.8c show the simulated radiation patterns of the proposed reflective metasurface at 12 GHz, focused at scanning angles of 0° , 30° and 50° , respectively. As can be seen, a directive beam with a narrow beamwidth is achieved using the reflective metasurface compared to the broad beam of the horn antenna without the reflective metasurface.

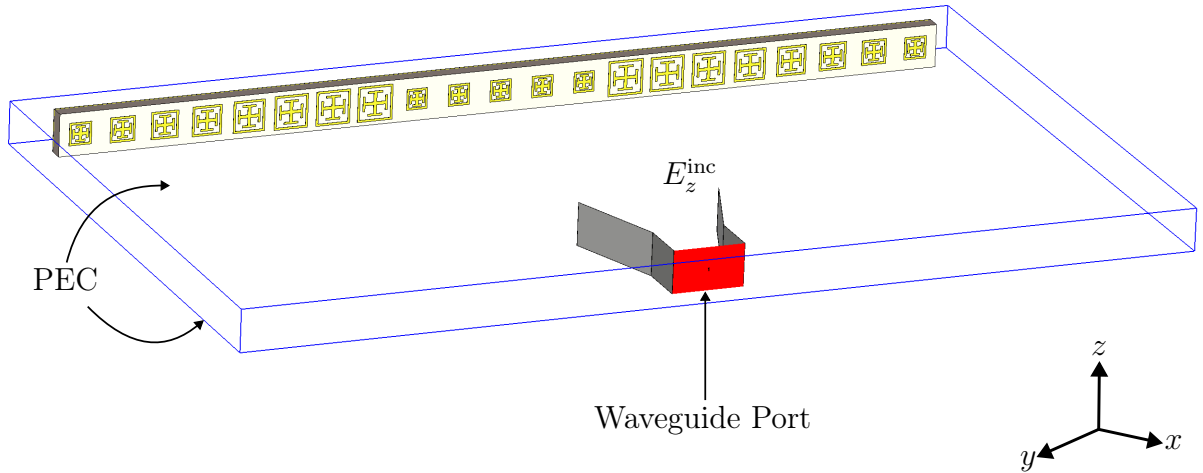


Fig. 5.5: 3D simulation model of the reflective metasurface illuminated by a vertically-polarized horn antenna, with electric boundary conditions applied in the $\pm z$ -direction.

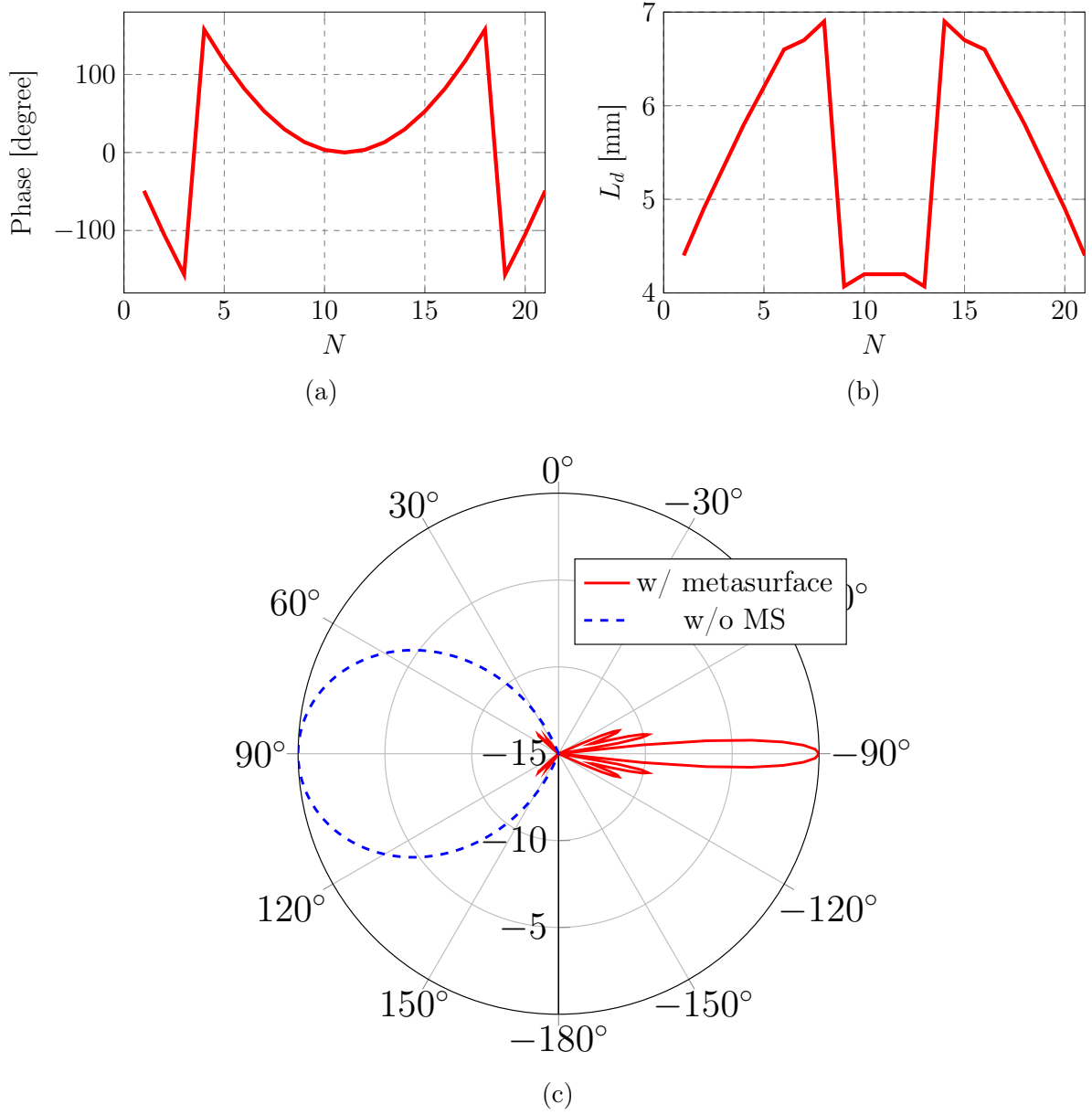


Fig. 5.6: Beam focusing in the azimuth plane at 12 GHz for a scanning angle 0° (-90°). (a) Required phase distribution. (b) Geometrical dimensions L_d unit cells. (c) Simulated radiation pattern.

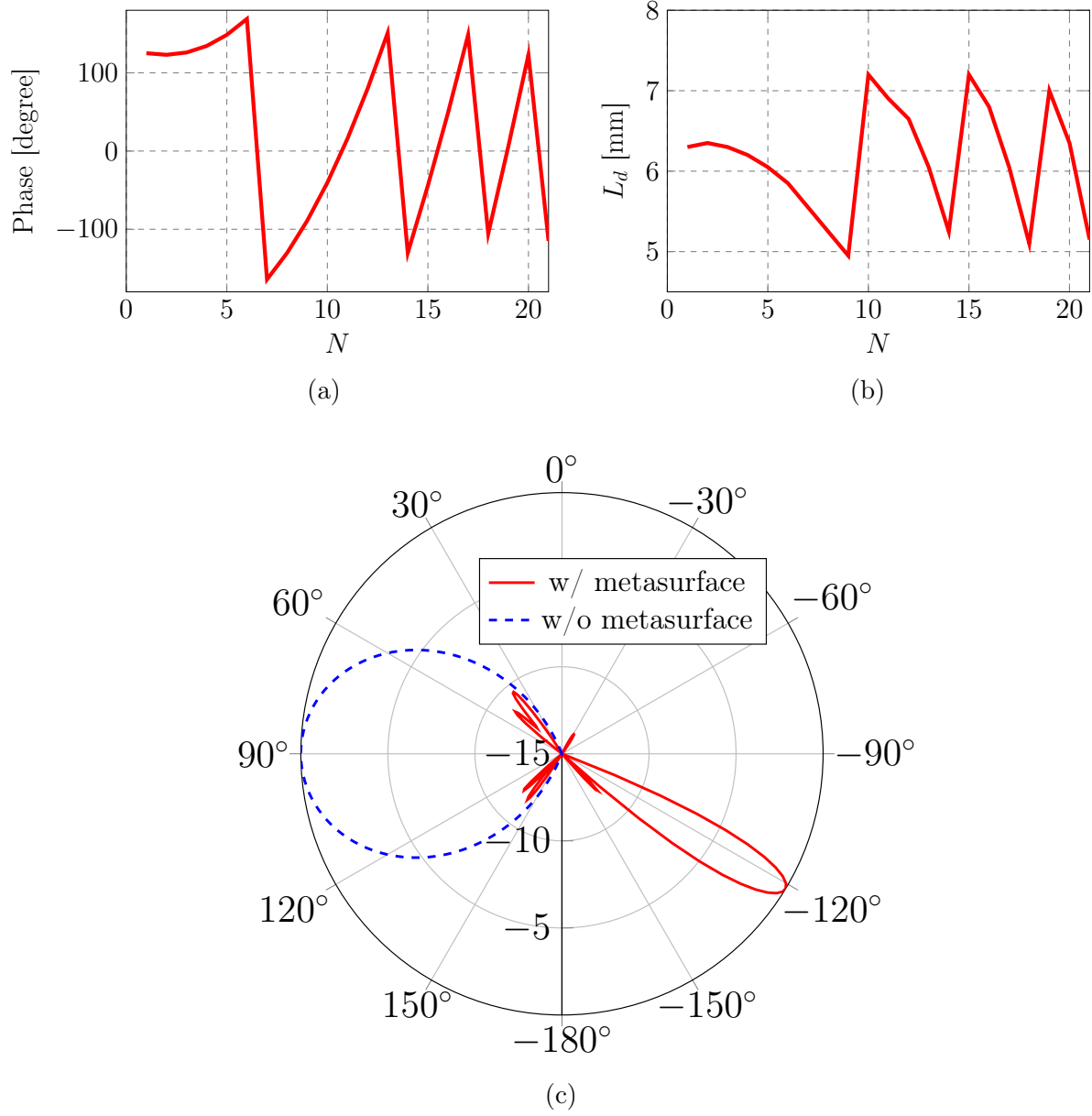


Fig. 5.7: Beam focusing in the azimuth plane at 12 GHz for a scanning angle 30° (-120°). (a) Required phase distribution. (b) Geometrical dimensions L_d unit cells. (c) Simulated radiation pattern.

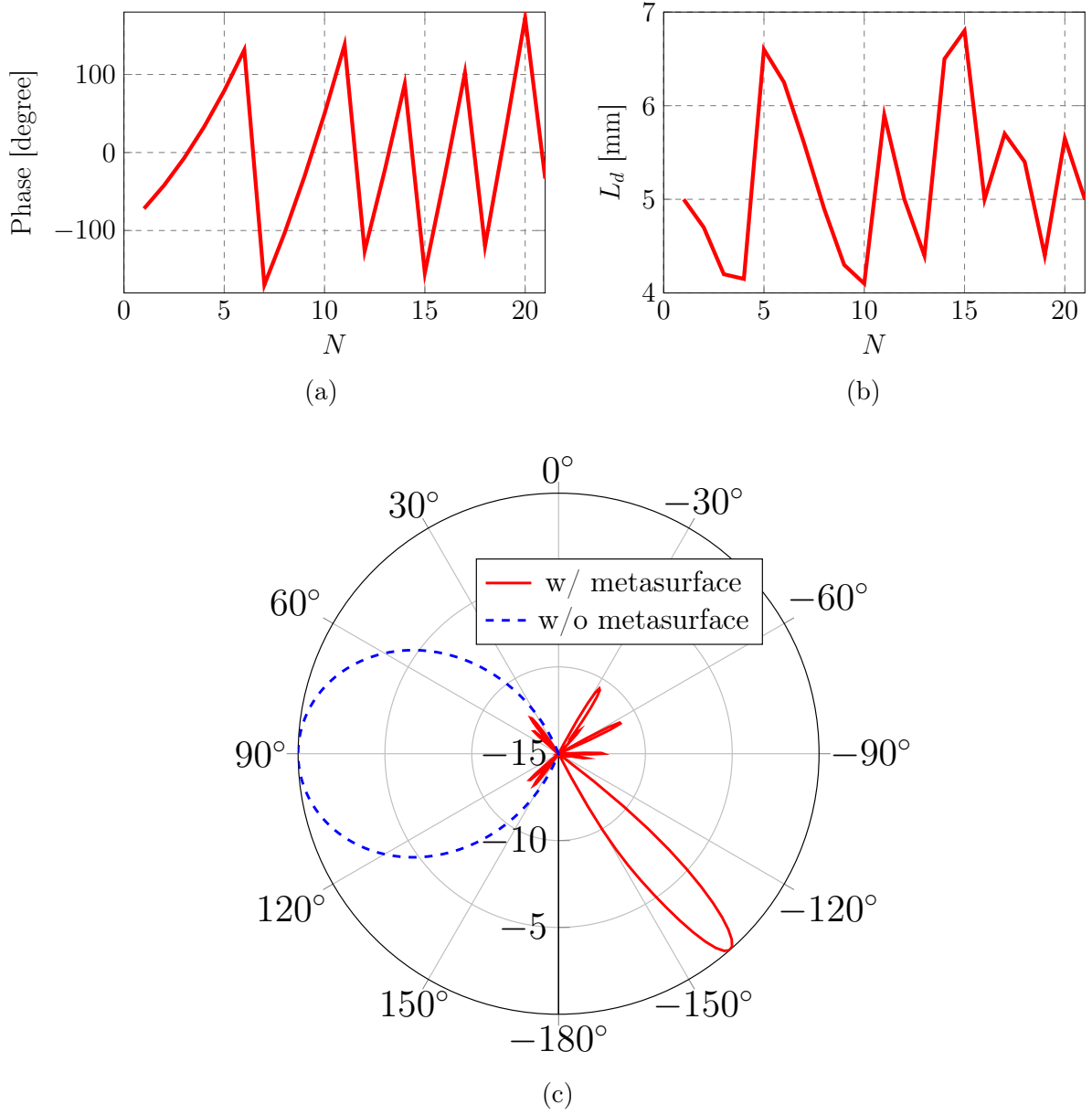


Fig. 5.8: Beam focusing in the azimuth plane at 12 GHz for a scanning angle 50° (-140°). (a) Required phase distribution. (b) Geometrical dimensions L_d unit cells. (c) Simulated radiation pattern.

5.3 Numerical Analysis

This section will focus on the analysis and modeling of the considered reflective metasurface using the proposed MoM modeling method for phase compensation. Since the reflective metasurface is periodic along the z -direction, an equivalent two-dimensional impedance model is constructed. This is achieved by applying the surface equivalence principle, the metal sheets forming the horn are replaced by equivalent electric surface currents denoted by $J_{z,h}$. The subwavelength metallic patterns of the reflective metasurface are homogenized and represented by zero-thickness impedance sheets with equivalent electric surface currents $J_{z,s}$. The dielectric substrate layers removed and replaced by equivalent polarization electric currents according to the volume equivalence principle $J_{z,p}$. An electric line source oriented along the z -axis and radiates TE_z cylindrical waves ($\hat{z}$ -directed E-field) is used to illuminate the metasurface. Fig. 5.9 illustrates the two-dimensional equivalent model with equivalent currents. As can be seen, the metasurface is modeled as N sub-wavelength surface impedance sheets, with the n -th sheet having an impedance value of Z_n ($\Omega/\square$), placed on top of the grounded dielectric substrate.

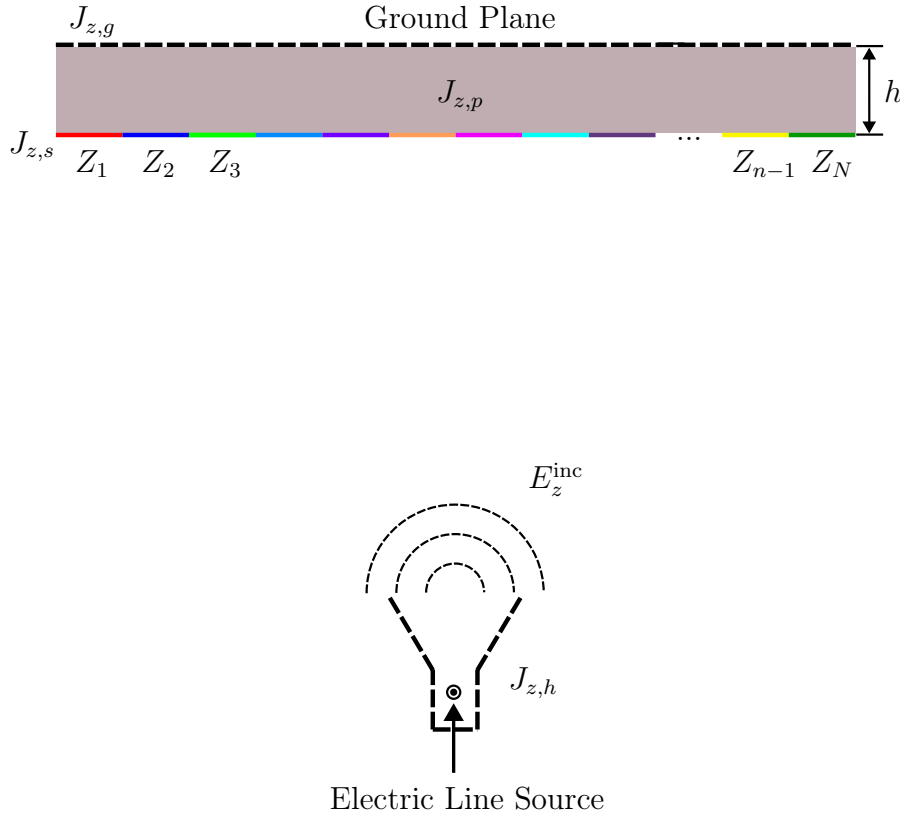


Fig. 5.9: Two-dimensional equivalent impedance model of the considered reflective metasurface with equivalent electric currents.

5.3.1 Homogenization Model Based on Equivalent Impedance

Fig. 5.10 shows a reflective metasurface with its equivalent circuit model. The dielectric substrate has thickness h and ϵ_r . The subwavelength top metallic layer is homogenized and represented as a zero-thickness sheet impedance Z_{sheet} , while the grounded dielectric slab is modeled as a short-circuited transmission line [85, 86]. The impedance of the grounded dielectric slab $Z_d^{\text{TE/TM}}$ for TE and TM polarization is given by [87, 88]:

$$Z_d^{\text{TE/TM}} = jZ^{\text{TE/TM}} \tan(k_z h), \quad (5.3.1)$$

where $Z^{\text{TE/TM}}$ represents the characteristics impedance of the dielectric substrate along the transmission line towards z -direction and h represents the length of the transmission line. The characteristic impedance for TE and TM polarization is given by [88]:

$$Z^{\text{TE}} = \frac{\omega \mu_0 \mu_r}{k_z} \quad ; \quad Z^{\text{TM}} = \frac{k_z}{\omega \epsilon_0 \epsilon_r}. \quad (5.3.2)$$

The normal propagation constant within the dielectric substrate is expressed as [87, 88]:

$$k_z = k_0 \sqrt{\epsilon_r - \sin^2 \theta_{\text{inc}}}, \quad (5.3.3)$$

where k_0 is the propagation constant of the incident plane wave, and θ_{inc} is the incidence angle as shown in Fig. 5.10a. As illustrated in Fig. 5.10a, the input impedance of the entire surface $Z_{\text{input}}^{\text{TE/TM}}$ is equal to the parallel connection between the sheet impedance $Z_{\text{sheet}}^{\text{TE/TM}}$ and the impedance of the grounded dielectric slab $Z_d^{\text{TE/TM}}$ [87, 88]:

$$Z_{\text{input}}^{\text{TE/TM}} = \frac{Z_{\text{sheet}}^{\text{TE/TM}} Z_d^{\text{TE/TM}}}{Z_{\text{sheet}}^{\text{TE/TM}} + Z_d^{\text{TE/TM}}}. \quad (5.3.4)$$

The input impedance of the entire surface is computed according to the complex reflection coefficient as follows [87, 88]:

$$\Gamma^{\text{TE/TM}} = \frac{Z_{\text{input}}^{\text{TE/TM}} - Z_0^{\text{TE/TM}}}{Z_{\text{input}}^{\text{TE/TM}} + Z_0^{\text{TE/TM}}}, \quad (5.3.5)$$

where $Z_0^{\text{TE/TM}}$ is the free-space impedance for TE and TM polarization, respectively. Eq. 5.3.5 demonstrates that the equivalent sheet impedance Z_{sheet} can be determined from the complex reflection coefficient of the reflective metasurface. The complex reflection coefficient can be determined by simulating the reflective metasurface unit cell structure in a periodic boundary condition. Once the input impedance of the entire surface is determined, the equivalent sheet impedance can be extracted using Eq. 5.3.4. Based on the simulated reflection coefficient of the Jerusalem reflective metasurface shown in Fig. 5.3, the equivalent sheet impedance Z_{sheet} is extracted and shown in Fig. 5.11.

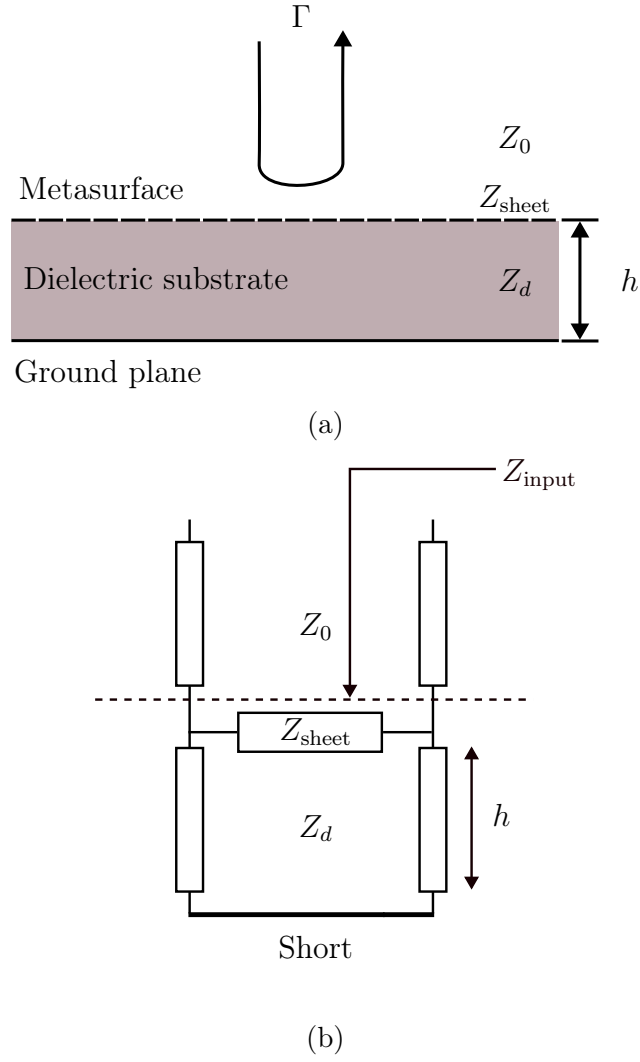


Fig. 5.10: Surface impedance homogenization. (a) Reflective metasurface. (b) Equivalent transmission line model.

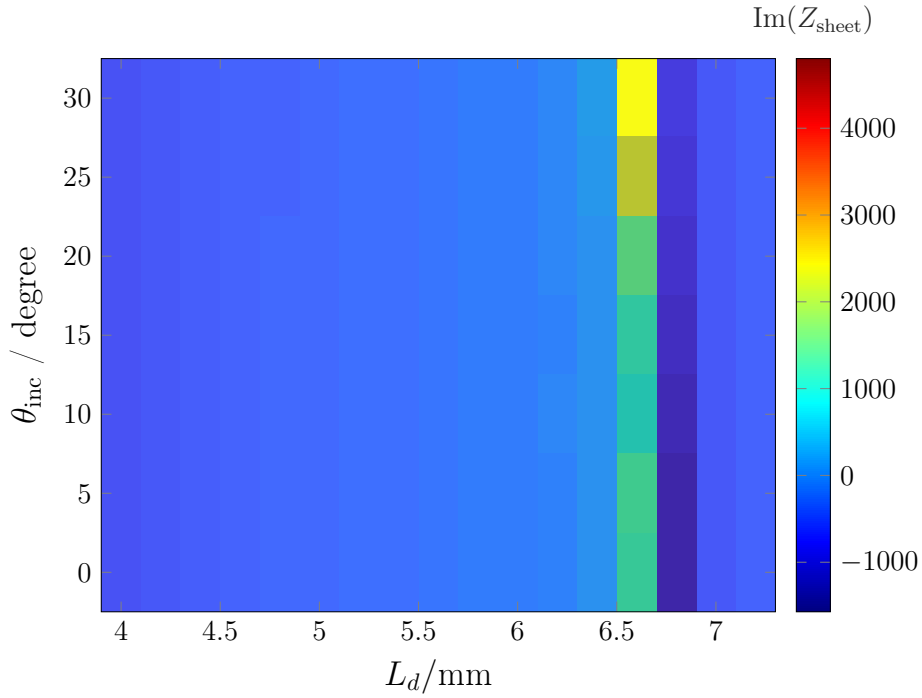


Fig. 5.11: Extracted equivalent sheet impedance Z_{sheet} for TE excitation at 12 GHz.

5.3.2 MoM Solution

Fig. 5.12 illustrates the discretization of the two-dimensional equivalent impedance model shown in Fig. 5.9. The impedance layer is divided into line segments along the x -direction, each with a width of 1 mm. Pulse basis functions are used to expand the equivalent electric currents $J_{z,s}$ on the impedance layer. The cross-sectional surface of the dielectric substrate layer is discretized into a total of 147 rectangular elements, with 58 elements along the x -direction and three elements along the y -direction. The size of each element is 3 mm $\times$ 1 mm. Two-dimensional pulse basis functions are employed to expand the equivalent polarization currents $J_{z,p}$. The geometry of the horn antenna is discretized into a total of 50 line segments, each with a width of 1 mm. The equivalent electric surface currents $J_{z,h}$ are expanded using one-dimensional basis functions. The PEC ground layer is divided into 27 line segments along the x -direction, each with a width of 3 mm and the equivalent electric currents $J_{z,g}$ on the ground layer are expanded using 1D pulse basis functions.

5.3.3 Numerical Results

To determine the radiation characteristics of the considered reflective-type metasurface, the non-uniform sheet impedances representing the top metallic patterns of the metasurface are extracted. To accurately determine these sheet impedances, the local incident angle on each unit cell is taken into account due to the spatial dispersion characteristics of the metasurface unit cell structure. The lookup database shown in Fig. 5.11 is then

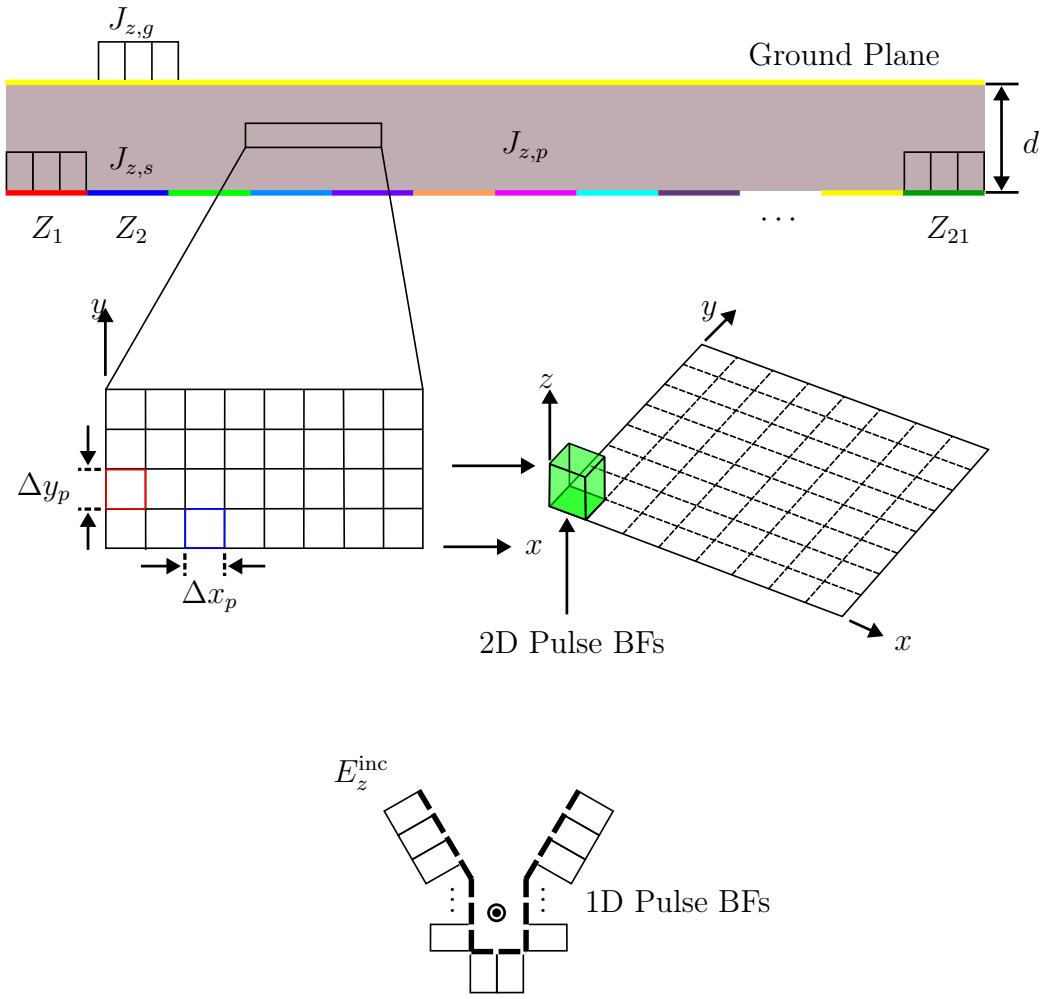
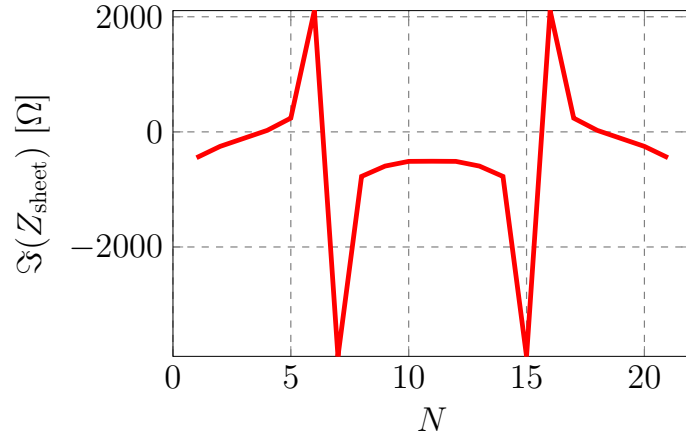


Fig. 5.12: Discretization of the two-dimensional equivalent impedance model including the non-uniform impedance sheets, the dielectric substrate layer, the ground plane and the horn antenna.

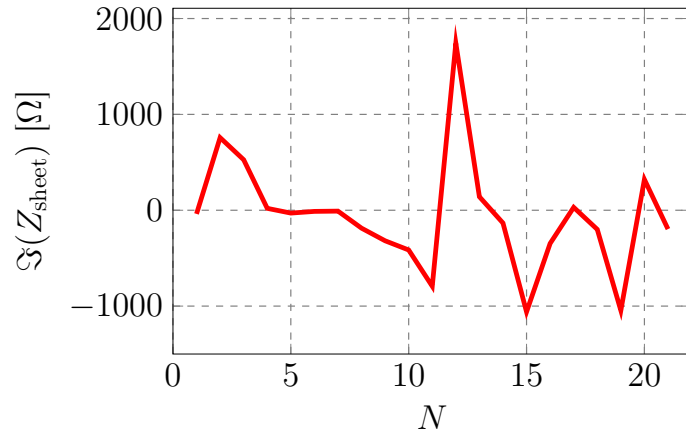
employed to map the geometrical parameters to their corresponding sheet impedance values. Figs. 5.13 present the extracted impedances for scanning angles 0° , 30° , and 50° . As can be seen, the impedances are purely reactive, where the losses were neglected. The impedance profile for 0° scanning angle shows smooth variations while for scanning angles 30° and 50° , the impedance profiles exhibit abrupt changes. This is mainly due to the spatial properties of the reflective metasurface at larger scanning angles.

The far-field radiation patterns are then obtained by the proposed modeling method for beam scanning angles 0° , 30° , and 50° and compared with the simulated results. Figs. 5.14 present the radiation patterns predicted by the MoM and compared with those obtained from full-wave simulations of the physical metasurface model.

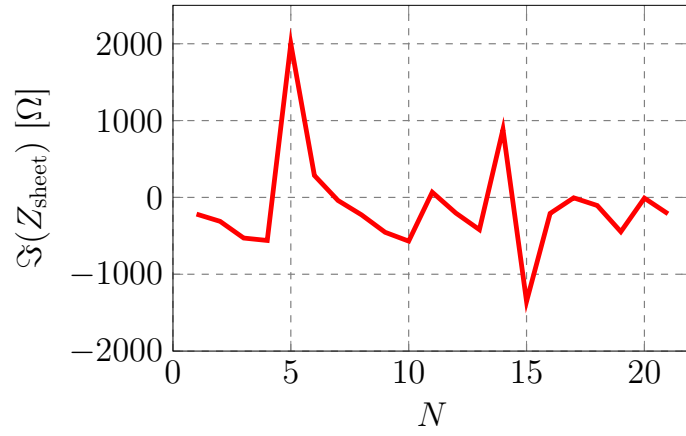
For the 0° scanning angle, the predicted and simulated radiation patterns show very agree-



(a)



(b)



(c)

Fig. 5.13: Extracted sheet impedance values for beam focusing at scanning angles: (a) 0° , (b) 30° , (c) 50° .

ment in terms of half-power beamwidth and side lobe level. At scanning angles 30° and 50° , the radiation patterns predicted by the modeling method are in good agreement with the simulated results in terms of half-power beamwidth and main beam direction. However, differences in the side lobe levels can be observed mainly due to abrupt variations in the geometrical dimensions of the unit cells required to achieve larger scan angles (i.e. 30° and 50°). Tables 5.2, 5.3, and 5.4 presents the radiation characteristics: main beam direction and half-power beamwidth.

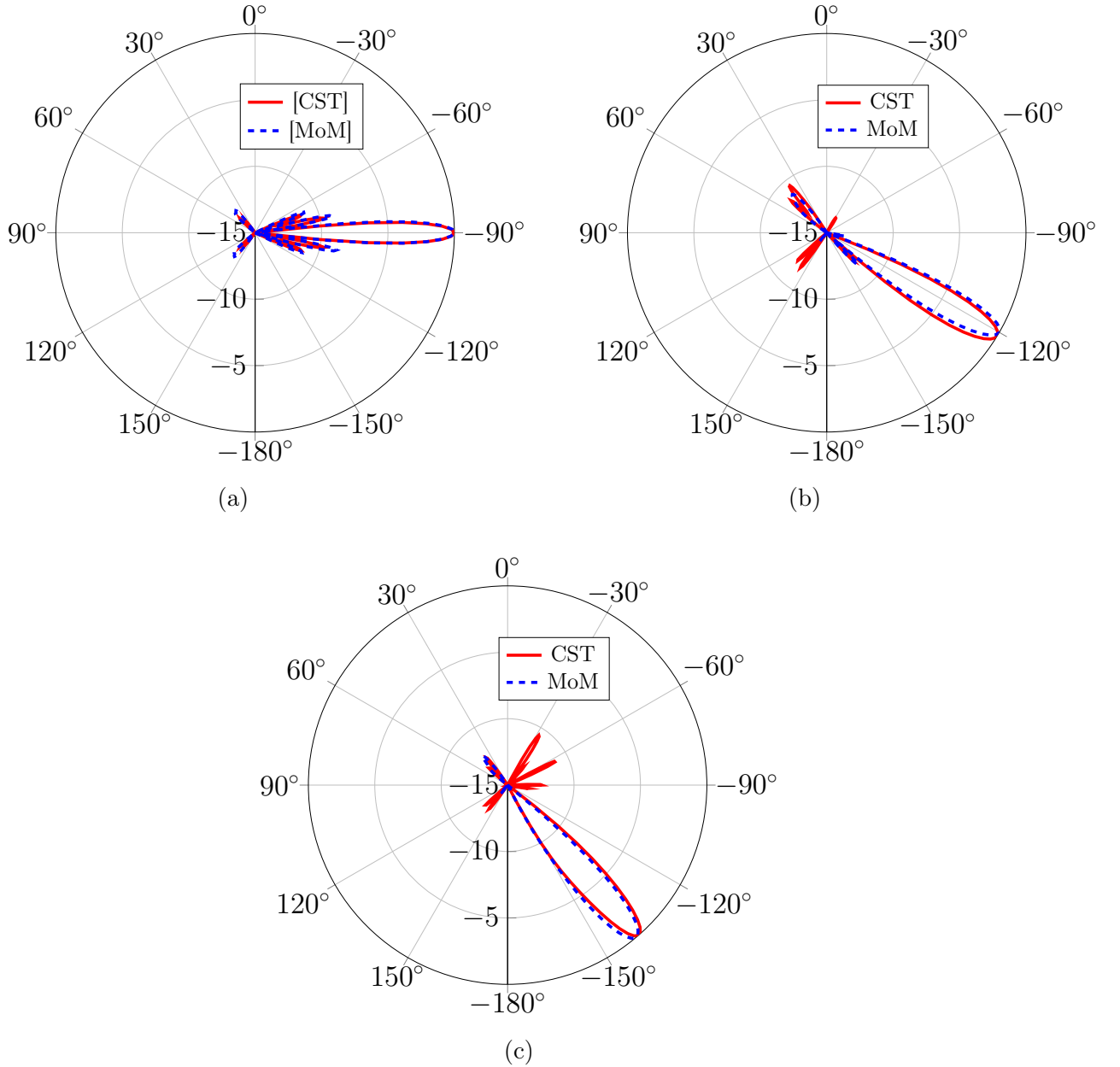


Fig. 5.14: Comparison between commercial simulation software (CST) and MoM for beam focusing at 12 GHz at scanning angles of (a) 0° , (b) 30° , and (c) 50° .

Table 5.2: Radiation characteristics of the reflective metasurface $[0^\circ]$.

Parameter	MoM	CST
Beam direction	-90°	-90°
HPBW	7°	7°

Table 5.3: Radiation characteristics of the reflective metasurface $[30^\circ]$.

Parameter	MoM	CST
Beam direction	-120°	-121°
HPBW	10°	11.2°

Table 5.4: Radiation characteristics of the reflective metasurface $[50^\circ]$.

Parameter	MoM	CST
Beam direction	-140°	-139°
HPBW	10°	11°

5.4 Summary

A reflective-type metasurface consists of non-uniform unit cells illuminated by a spherical wavefront radiated from a vertically-polarized horn antenna is designed to achieve beam focusing at scanning angles 0° , 30° and 50° , respectively. Each unit cell consists of a Jerusalem cross surrounded by a square ring printed on a grounded dielectric substrate. The reflection phase response is controlled by changing the geometrical dimensions of the unit cells, providing a 360° degree phase range at 12 GHz. This configuration is used as an illustrative example to show the accuracy and efficiency of the MoM modeling method to model reflective-type artificial surfaces.

To predict the radiation characteristics of the considered example, the subwavelength metallic patterns are homogenized and represented by zero-thickness impedance sheets. For the homogenization, each type of unit cell of the artificial surface is simulated separately assuming a periodic environment to extract an equivalent surface impedance based on an equivalent transmission line model. Once the subwavelength elements are homogenized and their equivalent surface impedances are determined. The unknown equivalent currents are then calculated, and the far-field radiation patterns are determined. The obtained results show good agreement with full-wave simulations, demonstrating the efficiency of the proposed MoM modeling method.

Conclusion

A simple and efficient modeling method has been developed to analyze electromagnetic scattering by three-dimensional arbitrarily shaped artificial surfaces that are periodic or invariant in one spatial dimension. The three-dimensional physical problem is simplified and reduced to a two-dimensional equivalent model. The subwavelength metallic patterns on the interfaces of the metasurface's dielectric layers are homogenized and represented by zero-thickness impedance sheets. A volume-surface integral equation formulation is presented to determine the electromagnetic scattering by the two-dimensional equivalent arbitrarily curved impedance model. While existing works in the literature mainly focus on formulating volume-surface integral equation for a single polarization, specifically TE polarization, the proposed method is formulated for both TE and TM polarized excitations.

Homogenization impedance models have been developed based on equivalent transmission line network representations and local periodic approximation to extract the zero-thickness sheet impedance of uniform and non-uniform wave-guiding, transmissive and reflective artificial surfaces.

The accuracy and efficiency of the proposed modeling method have been evaluated and demonstrated through three illustrative examples: a wave-guiding artificial surface through the design of a single layer Yagi-Uda-type printed director array at 2.925 GHz, a multilayer reconfigurable transmissive artificial surface at 10 GHz and a reflective passive artificial surface at 12 GHz. These examples cover the three types of artificial surfaces and demonstrate their capabilities for electromagnetic wavefront manipulation.

It has been demonstrated that the proposed modeling method accurately predicts the radiation characteristics with results showing good agreement with those obtained from three-dimensional full-wave simulations and by measurements on manufactured technology demonstrators.

It is found that to accurately predict the radiation patterns of non-uniform transmissive or reflective artificial surfaces illuminated by a spherical incident wave, it is necessary

6 Conclusion

to consider the local incident angle on each unit cell forming the artificial surface in the impedance homogenization process. This is due to the spatial dispersive behavior of artificial surfaces.

Compared to commercial simulation software such as CST Studio Suite, the proposed MoM modeling method demonstrates improved numerical efficiency. The proposed MoM modeling method uses surface impedance homogenization to represent the subwavelength artificial surface unit cells. As a result, the effort of mesh generation for the two-dimensional equivalent problem is reduced compared to the full-wave simulations, which require very dense three-dimensional geometrical meshes of the subwavelength elements. This simplification in the mesh generation directly leads to fewer unknowns in the MoM formulation. Consequently, CPU time and memory requirements are reduced. As an example, the CPU times and the number of unknowns of the MoM and commercial simulation software (CST Studio Suite) for one row of unit cells (with periodic boundary conditions in case of the commercial solver) of the reflective-type metasurface discussed in chapter five are presented in Table 6.1. From these results, we can observe that the computation time using the proposed MoM is almost two orders of magnitude faster than the commercial simulation software. Moreover, the number of unknowns is significantly lower than that of CST. It is important to note that the MoM code has not yet been optimized for numerical efficiency and further optimization of the numerical efficiency could be considered a future research topic.

Table 6.1: Comparison of the proposed MoM modeling method and commercial EM solver (CST Studio Suite) in CPU time and number of unknowns.

Method	MoM	CST - Frequency Domain
Number of unit cells	21	21
CPU time (sec)	34	2202
Unknown number	514	7104707

Although the proposed MoM modeling method is simple and efficient, it has certain limitations. It is developed to model only three-dimensional artificial surfaces that are either periodic or invariant in one spatial dimension. The homogenization approach is accurate for subwavelength artificial surfaces, but its accuracy decreases when the periodicity of the structure becomes comparable to the operating wavelength. Moreover, the extraction of the equivalent impedances that represent the subwavelength elements depends on the local periodic approximation, which works well for uniform and non-uniform artificial surfaces with smooth variations of the spatial properties. However, its accuracy is reduced in case of abrupt variations.

Throughout this thesis, we consider the analysis of linearly polarized artificial surfaces that exhibit only an electric response. However, artificial surfaces that support both electric and magnetic responses such as Huygens' metasurfaces need to be considered as they provide additional degrees of freedom for advanced wavefront manipulation including polarization control, perfect anomalous refraction and beam focusing. Another aspect that has not been addressed in the analysis of artificial surfaces is the evaluation of losses, as

the subwavelength artificial surfaces have been modeled as purely reactive (i.e., lossless) surface impedances to simplify the analysis. In practical applications such as the design of high-gain antennas, it is important to consider lossy homogenized impedance models.

As previously discussed, three different types of artificial surfaces have been designed and modeled using MoM combined with surface impedance homogenization to effectively manipulate electromagnetic waves enabling beam focusing and dynamic beam steering using reconfigurable elements. These artificial surfaces have a wide range of applications in wireless communications and radar systems.

In wireless communications, the printed Yagi-Uda-type director array (chapter three) can be used in point-to-point communication systems due to its simple, planar structure, directive end-fire beam and ease of fabrication using PCB technology. The proposed reconfigurable transmissive (chapter four) and the reflective (chapter five) metasurfaces can be used to enhance the communication between different parts of wireless networks including base stations and mobile users. Real-time beam steering improves the coverage for both fixed and mobile elements of communication systems, particularly in indoor environments, by dynamically focusing and scanning electromagnetic waves toward the receiver or target. Furthermore, it is particularly useful in tracking applications where stable communication between moving platforms is required such as maintaining data connections between moving vehicles [6, 7].

The reconfigurable transmissive metasurface designed to dynamically control the surface impedance patterns, can function as a transmissive reconfigurable intelligent surface capable of redirecting electromagnetic waves toward targets. This is useful for improving the link budget and enhancing the received power when the line of sight is blocked by objects [6]. Furthermore, it can be employed to generate multiple beams when multiple feeds illuminate the metasurface. This has great potential in 5G communications and MIMO systems, where multiple communication links can be established with the use of only one single metasurface aperture [6].

In radar systems, reconfigurable metasurfaces can be employed in unmanned vehicles such as small drones to enable real-time tracking and target detection, where payload weight and available space are limited.

Potential Future Work

Based on the outcomes of this dissertation, the following future research topics can be suggested:

- The proposed modeling method can be extended to analyze three-dimensional artificial surfaces using three-dimensional Green's functions to enable a more general analysis of artificial surfaces
- The development of fast optimization algorithms is essential for designing non-uniform artificial surfaces particularly with tunable elements due to the large number of degrees of freedom and the inherently non-linear nature of the electromagnetic scattering process

6 Conclusion

- We focused on the forward scattering problem by determining the unknown equivalent currents based on the known extracted sheet impedances. However, studying the inverse scattering problem will facilitate the optimization process of artificial surfaces, where the desired far-field radiation pattern or near-field distribution along the metasurface is specified and the required sheet impedances are subsequently determined. Furthermore, the homogenization process might also be inverted to determine the unit cell layout from the required sheet impedance
- Further optimization of the numerical efficiency of the developed code for example by using a compiler-based environment such as C/C++
- Huygens' metasurfaces offer several advantages compared to anisotropic artificial surfaces with only electric responses, so the analysis of Huygens' metasurfaces could be an interesting future research topic to achieve new functionalities
- Varactor diodes have been used to achieve dynamic control of electromagnetic properties, the usage of other tunable components like MEMS switches or tunable capacitors could be useful as they provide lower losses and high power handling capability
- A one-dimensional beam scanning capability has been considered; however, the design of reconfigurable metasurfaces with two-dimensional beam scanning can provide new functionalities and better performance

Bibliography

- [1] W. C. Gibson, *The Method of Moments in Electromagnetics*, 2nd ed. Chapman and Hall/CRC, 2014.
- [2] G.-B. Wu, S.-W. Qu, S. Yang, and C. H. Chan, “Low-cost 1-D beam-steering reflectarray with $\pm 70^\circ$ scan coverage,” *IEEE Transactions on Antennas and Propagation*, vol. 68, no. 6, pp. 5009–5014, 2020.
- [3] S. V. Hum and J. Perruisseau-Carrier, “Reconfigurable reflectarrays and array lenses for dynamic antenna beam control: A review,” *IEEE Transactions on Antennas and Propagation*, vol. 62, no. 1, pp. 183–198, 2014.
- [4] O. Quevedo-Teruel, H. Chen, A. Díaz-Rubio, G. Gok, A. Grbic, G. Minatti, E. Martini, S. Maci, G. V. Eleftheriades, M. Chen, N. I. Zheludev, N. Papasimakis, S. Choudhury, Z. A. Kudyshev, S. Saha, H. Reddy, A. Boltasseva, V. M. Shalaev, A. V. Kildishev, D. Sievenpiper, C. Caloz, A. Alù, Q. He, L. Zhou, G. Valerio, E. Rajo-Iglesias, Z. Sipus, F. Mesa, R. Rodríguez-Berral, F. Medina, V. Asadchy, S. Tretyakov, and C. Craeye, “Roadmap on metasurfaces,” *Journal of Optics*, vol. 21, no. 7, p. 073002, jul 2019.
- [5] M. Di Renzo, F. H. Danufane, and S. Tretyakov, “Communication models for reconfigurable intelligent surfaces: From surface electromagnetics to wireless networks optimization,” *Proceedings of the IEEE*, vol. 110, no. 9, pp. 1164–1209, 2022.
- [6] V. G. Ataloglou, S. Taravati, and G. V. Eleftheriades, “Metasurfaces: physics and applications in wireless communications,” *National Science Review*, vol. 10, no. 8, p. nwad164, 06 2023. [Online]. Available: <https://doi.org/10.1093/nsr/nwad164>
- [7] F. Yang and Y. Rahmat-Samii, *Surface Electromagnetics: With Applications in Antenna, Microwave, and Optical Engineering*. Cambridge: Cambridge University Press, 2019.
- [8] B. Rana, S.-S. Cho, and I.-P. Hong, “Review paper on hardware of reconfigurable intelligent surfaces,” *IEEE Access*, vol. 11, pp. 29 614–29 634, 2023.
- [9] C. Ma, H. Li, B. Zhang, D. Ye, J. Huangfu, Y. Sun, W. Zhu, C. Li, and L. Ran, “Implementation of a 2-D reconfigurable fresnel-zone-plate antenna,” *IEEE Transactions on Antennas and Propagation*, vol. 69, no. 1, pp. 520–525, 2021.

- [10] A. M. H. Wong and G. V. Eleftheriades, “Perfect anomalous reflection with a bipartite Huygens’ metasurface,” *Phys. Rev. X*, vol. 8, p. 011036, Feb 2018. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevX.8.011036>
- [11] O. Rabinovich, Y. Komarovskiy, D. Dikarov, and A. Epstein, “Printed circuit board (PCB) metagratings for perfect anomalous refraction: Theory and experiment,” in *2019 International Conference on Electromagnetics in Advanced Applications (ICEAA)*, 2019, pp. 1070–1070.
- [12] J. Budhu and A. Grbic, “A reflective metasurface for perfect cylindrical to planar wavefront transformation,” in *2020 Fourteenth International Congress on Artificial Materials for Novel Wave Phenomena (Metamaterials)*, 2020, pp. 234–236.
- [13] —, “Perfectly reflecting metasurface reflectarrays: Mutual coupling modeling between unique elements through homogenization,” *IEEE Transactions on Antennas and Propagation*, vol. 69, no. 1, pp. 122–134, 2021.
- [14] —, “A rigorous approach to designing reflectarrays,” in *2019 23rd International Conference on Applied Electromagnetics and Communications (ICECOM)*, 2019, pp. 1–3.
- [15] V. G. Ataloglou and G. V. Eleftheriades, “Arbitrary wave transformations with Huygens’ metasurfaces through surface-wave optimization,” *IEEE Antennas and Wireless Propagation Letters*, vol. 20, no. 9, pp. 1750–1754, 2021.
- [16] H. Li, C. Ma, F. Shen, K. Xu, D. Ye, J. Huangfu, C. Li, L. Ran, and T. A. Denidni, “Wide-angle beam steering based on an active conformal metasurface lens,” *IEEE Access*, vol. 7, pp. 185 264–185 272, 2019.
- [17] H. Li, C. Ma, T. Zhou, J. Wang, D. Ye, Y. Sun, W. Zhu, T. A. Denidni, and L. Ran, “Reconfigurable fresnel lens based on an active second-order bandpass frequency-selective surface,” *IEEE Transactions on Antennas and Propagation*, vol. 68, no. 5, pp. 4054–4059, 2020.
- [18] J. Yu, W. Jiang, and S. Gong, “Low-RCS beam-steering antenna based on reconfigurable phase gradient metasurface,” *IEEE Antennas and Wireless Propagation Letters*, vol. 18, no. 10, pp. 2016–2020, 2019.
- [19] M. Ebrahimpouri, L. F. Herran, and O. Quevedo-Teruel, “Wide-angle impedance matching using glide-symmetric metasurfaces,” *IEEE Microwave and Wireless Components Letters*, vol. 30, no. 1, pp. 8–11, 2020.
- [20] A. O. Bah, R. W. Ziolkowski, P.-Y. Qin, and Y. J. Guo, “Design and analysis of a wide angle impedance matching metasurface for wideband antenna arrays,” in *12th European Conference on Antennas and Propagation (EuCAP 2018)*, 2018, pp. 1–4.
- [21] T. Badawy, T. Bertuch, F. Weinmann, and P. Knott, “Investigations on improving wide angular beam scanning of conformal X-band array antennas by tunable metasurfaces,” in *2024 IEEE International Symposium on Phased Array Systems and Technology (ARRAY)*, 2024, pp. 1–6.

- [22] C. L. Holloway, E. F. Kuester, J. A. Gordon, J. O'Hara, J. Booth, and D. R. Smith, "An overview of the theory and applications of metasurfaces: The two-dimensional equivalents of metamaterials," *IEEE Antennas and Propagation Magazine*, vol. 54, no. 2, pp. 10–35, 2012.
- [23] M. Dehmollaian, N. Chamanara, and C. Caloz, "Wave scattering by a cylindrical metasurface cavity of arbitrary cross section: Theory and applications," *IEEE Transactions on Antennas and Propagation*, vol. 67, no. 6, pp. 4059–4072, 2019.
- [24] I. Jung, Z. Peng, and Y. Rahmat-Samii, "Recent advances in reconfigurable electromagnetic surfaces: Engineering design, full-wave analysis, and large-scale optimization," *Electromagnetic Science*, vol. 2, no. 3, pp. 1–25, 2024.
- [25] J. Wang, V. Manohar, and Y. Rahmat-Samii, "Enabling the internet of things with cubesats: A review of representative beamsteerable antenna concepts," *IEEE Antennas and Propagation Magazine*, vol. 63, no. 6, pp. 14–28, 2021.
- [26] T. Debogovic and J. Perruisseau-Carrier, "Low loss mems-reconfigurable 1-bit reflectarray cell with dual-linear polarization," *IEEE Transactions on Antennas and Propagation*, vol. 62, no. 10, pp. 5055–5060, 2014.
- [27] P.-Y. Wang, B. Sievert, J. T. Svejda, N. Benson, F.-Y. Meng, A. Rennings, and D. Erni, "A liquid crystal tunable metamaterial unit cell for dynamic metasurface antennas," *IEEE Transactions on Antennas and Propagation*, vol. 71, no. 1, pp. 1135–1140, 2023.
- [28] L. Xu, K. Li, G. Zhang, J. Yang, Z. Yin, M. Zhong, H. Lu, G. Deng, and Y. Li, "Fully electrically driven liquid crystal reconfigurable intelligent surface for terahertz beam steering," *IEEE Transactions on Terahertz Science and Technology*, vol. 14, no. 5, pp. 708–717, 2024.
- [29] H. Li, Y. Fan, Y. Zhuang, T. Zhou, and T. A. Denidni, "Experimental validation of an active parallel-plate-waveguide-loaded metasurfaces lens for wide-angle beam steering," *IEEE Antennas and Wireless Propagation Letters*, vol. 21, no. 11, pp. 2156–2160, 2022.
- [30] W. Fuscaldo, P. Burghignoli, and A. Galli, "The transition between reactive and radiative regimes for leaky modes in planar waveguides based on homogenized partially reflecting surfaces," *IEEE Transactions on Microwave Theory and Techniques*, vol. 68, no. 12, pp. 5259–5269, 2020.
- [31] W. Fuscaldo, A. Galli, and D. R. Jackson, "Optimization of 1-D unidirectional leaky-wave antennas based on partially reflecting surfaces," *IEEE Transactions on Antennas and Propagation*, vol. 70, no. 9, pp. 7853–7868, 2022.
- [32] T. Badawy and T. Bertuch, "Slotted waveguide antenna integrated with printed Yagi-Uda director array," in *2019 13th European Conference on Antennas and Propagation (EuCAP)*, 2019, pp. 1–5.

Bibliography

- [33] J. Y. Lau and S. V. Hum, "Reconfigurable transmitarray design approaches for beamforming applications," *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 12, pp. 5679–5689, 2012.
- [34] S. A. Stewart, S. Moslemi-Tabrizi, T. J. Smy, and S. Gupta, "Scattering field solutions of metasurfaces based on the boundary element method for interconnected regions in 2-d," *IEEE Transactions on Antennas and Propagation*, vol. 67, no. 12, pp. 7487–7495, 2019.
- [35] P. Nayeri, A. Z. Elsherbeni, and F. Yang, "Radiation analysis approaches for reflectarray antennas [antenna designer's notebook]," *IEEE Antennas and Propagation Magazine*, vol. 55, no. 1, pp. 127–134, 2013.
- [36] S. Targonski and D. Pozar, "Analysis and design of a microstrip reflectarray using patches of variable size," in *Proceedings of IEEE Antennas and Propagation Society International Symposium and URSI National Radio Science Meeting*, vol. 3, 1994, pp. 1820–1823 vol.3.
- [37] J. Encinar, "Design of two-layer printed reflectarrays using patches of variable size," *IEEE Transactions on Antennas and Propagation*, vol. 49, no. 10, pp. 1403–1410, 2001.
- [38] P. Harms, R. Mittra, and W. Ko, "Implementation of the periodic boundary condition in the finite-difference time-domain algorithm for FSS structures," *IEEE Transactions on Antennas and Propagation*, vol. 42, no. 9, pp. 1317–1324, 1994.
- [39] D. Pozar, S. Targonski, and H. Syrigos, "Design of millimeter wave microstrip reflectarrays," *IEEE Transactions on Antennas and Propagation*, vol. 45, no. 2, pp. 287–296, 1997.
- [40] S. V. Hum, M. Okoniewski, and R. J. Davies, "Modeling and design of electronically tunable reflectarrays," *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 8, pp. 2200–2210, 2007.
- [41] J. Budhu, E. Michielssen, and A. Grbic, "The design of dual band stacked metasurfaces using integral equations," *IEEE Transactions on Antennas and Propagation*, vol. 70, no. 6, pp. 4576–4588, 2022.
- [42] N. Chamanara, K. Achouri, and C. Caloz, "Efficient analysis of metasurfaces in terms of spectral-domain GSTC integral equations," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 10, pp. 5340–5347, 2017.
- [43] X. Liu, F. Yang, S. Xu, and M. Li, "Design method for modulated metasurface antennas composed of anisotropic elements based on generalized boundary conditions," *IEEE Antennas and Wireless Propagation Letters*, vol. 18, no. 9, pp. 1848–1852, 2019.
- [44] J. Budhu and A. Grbic, "Recent advances in bianisotropic boundary conditions: theory, capabilities, realizations, and applications," *Nanophotonics*, vol. 10, no. 16, pp. 4075–4112, 2021.

- [45] S. Sandeep and S. Y. Huang, “Simulation of circular cylindrical metasurfaces using GSTC-MoM,” *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, vol. 3, pp. 185–192, 2018.
- [46] G. Xu, V. G. Ataloglou, S. V. Hum, and G. V. Eleftheriades, “Extreme beam-forming with impedance metasurfaces featuring embedded sources and auxiliary surface wave optimization,” *IEEE Access*, vol. 10, pp. 28 670–28 684, 2022.
- [47] J. Budhu, L. Szymanski, and A. Grbic, “Design of planar and conformal, passive, lossless metasurfaces that beamform,” *IEEE Journal of Microwaves*, vol. 2, no. 3, pp. 401–418, 2022.
- [48] T. J. Smy, J. Connor, S. A. Stewart, and S. Gupta, “General formulation of the boundary element method (bem) for curvilinear metasurfaces in the presence of multiple scattering objects,” in *2020 14th European Conference on Antennas and Propagation (EuCAP)*, 2020, pp. 1–4.
- [49] S. C. Sierra, R. Zhao, R. Chen, and H. Bagci, “An SIE-GSTC solver for simulation of monoanisotropic metasurfaces,” in *2023 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (USNC-URSI)*, 2023, pp. 553–554.
- [50] M. A. Francavilla, E. Martini, S. Maci, and G. Vecchi, “On the numerical simulation of metasurfaces with impedance boundary condition integral equations,” *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 5, pp. 2153–2161, 2015.
- [51] J. Budhu and A. Grbic, “Passive reflective metasurfaces for far-field beamforming,” in *2021 15th European Conference on Antennas and Propagation (EuCAP)*, 2021, pp. 1–4.
- [52] G. Xu, S. V. Hum, and G. V. Eleftheriades, “Extreme beam-forming with metagrating-assisted planar antennas,” in *2021 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (APS/URSI)*, 2021, pp. 95–96.
- [53] J. Budhu, A. Grbic, and E. Michielssen, “Design of multilayer, dualband metasurface reflectarrays,” in *2020 14th European Conference on Antennas and Propagation (EuCAP)*, 2020, pp. 1–4.
- [54] —, “Dualband stacked metasurface reflectarray,” in *2020 IEEE International Symposium on Antennas and Propagation and North American Radio Science Meeting*, 2020, pp. 821–822.
- [55] R. Xie, Y. Wang, X. Ma, J. Huang, K. Cheng, L. Zhang, Z. Zhang, and T. Zhang, “Dual-polarization reconfigurable metasurface for broadband vortex beam generation,” *Opt. Express*, vol. 32, no. 22, pp. 38 087–38 096, Oct 2024.

- [56] X. Liu, F. Yang, M. Li, and S. Xu, “Generalized boundary conditions in surface electromagnetics: Fundamental theorems and surface characterizations,” *Applied Sciences*, vol. 9, no. 9, 2019. [Online]. Available: <https://www.mdpi.com/2076-3417/9/9/1891>
- [57] K. S. John L. Volakis, *Integral Equation Methods for Electromagnetics*, ser. Electromagnetic Waves. Institution of Engineering and Technology, 2012. [Online]. Available: <https://digital-library.theiet.org/content/books/ew/sbew045e>
- [58] C. Pelletti, G. Bianconi, R. Mittra, and A. Monorchio, “Volume integral equation analysis of thin dielectric sheet using sinusoidal macro-basis functions,” *IEEE Antennas and Wireless Propagation Letters*, vol. 12, pp. 441–444, 2013.
- [59] A. Bostani and H. Oraizi, “Analysis and design of tapered uniaxial-anisotropic dielectric rod antennas,” *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 11, pp. 5787–5795, 2017.
- [60] A. Glisson and D. Wilton, “Simple and efficient numerical methods for problems of electromagnetic radiation and scattering from surfaces,” *IEEE Transactions on Antennas and Propagation*, vol. 28, no. 5, pp. 593–603, 1980.
- [61] S. Bao, D. Wang, Y. Mo, S. Hu, J. Gu, and W. Tang, “Fully analytical evaluation of singular integrals with RWG and rooftop basis functions,” *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, vol. 5, pp. 217–226, 2020.
- [62] D. Wilton, S. Rao, A. Glisson, D. Schaubert, O. Al-Bundak, and C. Butler, “Potential integrals for uniform and linear source distributions on polygonal and polyhedral domains,” *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 3, pp. 276–281, 1984.
- [63] A. I. Mackenzie, S. M. Rao, and M. E. Baginski, “Method of moments solution of electromagnetic scattering problems involving arbitrarily-shaped conducting/dielectric bodies using triangular patches and pulse basis functions,” *IEEE Transactions on Antennas and Propagation*, vol. 58, no. 2, pp. 488–493, 2010.
- [64] S. Rao, D. Wilton, and A. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape,” *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 3, pp. 409–418, 1982.
- [65] J. T. du Plessis and M. M. Botha, “A macro basis function formulation for thick wires in the method of moments,” in *2021 15th European Conference on Antennas and Propagation (EuCAP)*, 2021, pp. 1–4.
- [66] M. Kuipers, “Mutual coupling computation and effects in phased array microstrip antennas,” in *VTC '98. 48th IEEE Vehicular Technology Conference. Pathway to Global Wireless Revolution (Cat. No.98CH36151)*, vol. 2, 1998, pp. 1181–1185 vol.2.
- [67] I. E. Commission, “Maritime navigation and radio-communication equipment and systems - shipborne radar - performance requirements, methods of testing and required test results,” *International Electrotechnical Commission*, 2007.

- [68] T. Badawy, T. Bertuch, and P. Knott, “A compact maritime harmonic radar antenna for navigation and search-and-rescue,” *IEEE Access*, vol. 13, pp. 190 136–190 151, 2025.
- [69] T. Badawy, A. Kremring, D. Nulwalla, and T. Bertuch, “Mechanical and environmental aspects of antennas for a novel maritime search and rescue system,” in *2020 14th European Conference on Antennas and Propagation (EuCAP)*, 2020, pp. 1–4.
- [70] H. Heuermann, T. Harzheim, and T. Cronenbroeck, “First SIMO harmonic radar based on the SFCW concept and the HR transfer function,” *Remote Sensing*, vol. 13, no. 24, 2021. [Online]. Available: <https://www.mdpi.com/2072-4292/13/24/5088>
- [71] C. A. Balanis, *Antenna Theory: Analysis and Design*, 3rd ed. John Wiley and Sons, Inc., 2005.
- [72] Q. Fu, H. Ni, G. Q. Luo, L. Zhu, and L. Liu, “A high aperture efficiency endfire antenna based on spoof surface plasmon polaritons,” *IEEE Transactions on Antennas and Propagation*, vol. 71, no. 1, pp. 50–57, 2023.
- [73] M. Li, S. Xiao, J. Long, and D. F. Sievenpiper, “Surface waveguides supporting both TM mode and TE mode with the same phase velocity,” *IEEE Transactions on Antennas and Propagation*, vol. 64, no. 9, pp. 3811–3819, 2016.
- [74] A. Ishimaru, *Electromagnetic Wave Propagation, Radiation, and Scattering: From Fundamentals to Applications*, ser. IEEE Press Series on Electromagnetic Wave Theory. Wiley, 2017.
- [75] A. Clemente, “Wideband 400-element electronically reconfigurable transmitarray in X-band,” *IEEE Transactions on Antennas and Propagation*, vol. 61, no. 10, pp. 5017–5027, 2013.
- [76] C. Huang, “1-bit reconfigurable circularly polarized transmitarray in X-band,” *IEEE Transactions on Antennas and Propagation*, vol. 15, no. 10, pp. 448–451, 2016.
- [77] P. Sangmahamad, P. Janpugdee, and Y. Zhao, “A metasurface-based electronically reconfigurable and dual-polarized reflectarray antenna for beam-steering applications,” *IEEE Access*, vol. 11, pp. 137 414–137 425, 2023.
- [78] S. Cho, A. Salim, and H.-J. Song, “Two-dimensional beam steering active lens with simple grid bias lines at 19 GHz,” *IEEE Transactions on Vehicular Technology*, vol. 70, no. 10, pp. 9716–9724, 2021.
- [79] K. Zhang, W. Jiang, S. Gong, and T. Hong, “X-band tunable frequency selective surface with embedded bias network,” in *2016 International Symposium on Antennas and Propagation (ISAP)*, 2016, pp. 880–881.
- [80] A. Olk and D. Powell, “Accurate metasurface synthesis incorporating near-field coupling effects,” *Phys. Rev. Appl.*, vol. 11, p. 064007, Jun 2019.
- [81] D. M. Pozar, *Microwave Engineering*, 4th ed. John Wiley & Sons, 2011.

Bibliography

- [82] T. Brown and P. Mojabi, “Cascaded metasurface design using electromagnetic inversion with gradient-based optimization,” *IEEE Transactions on Antennas and Propagation*, vol. 70, no. 3, pp. 2033–2045, 2022.
- [83] Q. Wu, C. P. Scarborough, D. H. Werner, E. Lier, and X. Wang, “Design synthesis of metasurfaces for broadband hybrid-mode horn antennas with enhanced radiation pattern and polarization characteristics,” *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 8, pp. 3594–3604, 2012.
- [84] S. R. Zang and J. R. Bergmann, “Analysis of omnidirectional dual-reflector antenna and feeding horn using method of moments,” *IEEE Transactions on Antennas and Propagation*, vol. 62, no. 3, pp. 1534–1538, 2014.
- [85] X. Liu, F. Yang, M. Li, and S. Xu, “Analysis of reflectarray antenna elements under arbitrary incident angles and polarizations using generalized boundary conditions,” *IEEE Antennas and Wireless Propagation Letters*, vol. 17, no. 12, pp. 2208–2212, 2018.
- [86] —, “Generalized boundary conditions in surface electromagnetics: Fundamental theorems and surface characterizations,” *Applied Sciences*, vol. 9, no. 9, 2019. [Online]. Available: <https://www.mdpi.com/2076-3417/9/9/1891>
- [87] F. Costa and A. Monorchio, “Closed-form analysis of reflection losses in microstrip reflectarray antennas,” *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 10, pp. 4650–4660, 2012.
- [88] F. Costa and M. Borgese, “Circuit modelling of reflecting intelligent surfaces,” in *2021 IEEE 22nd International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2021, pp. 546–550.

List of Publications

- T. Badawy, T. Bertuch, and P. Knott, “A compact maritime harmonic radar antenna for navigation and search-and-rescue,” *IEEE Access*, vol. 13, pp. 190 136–190 151, 2025.
- T. Badawy, T. Bertuch, F. Weinmann, and P. Knott, “Investigations on improving wide angular beam scanning of conformal X-band array antennas by tunable metasurfaces,” in *2024 IEEE International Symposium on Phased Array Systems and Technology (AR-RAY)*, 2024, pp. 1–6.
- T. Badawy, T. Bertuch, C. Loecker, and P. Knott, “High gain frequency scanning antenna using dumbbell-shaped inclined slots and parallel plate waveguide polariser,” *IET Microwaves, Antennas & Propagation*, vol. 16, no. 9, pp. 553–566, 2022. [Online]. Available: <https://ietresearch.onlinelibrary.wiley.com/doi/abs/10.1049/mia2.12248>
- T. Badawy, A. Kremring, D. Nulwalla, and T. Bertuch, “Mechanical and environmental aspects of antennas for a novel maritime search and rescue system,” in *2020 14th European Conference on Antennas and Propagation (EuCAP)*, 2020, pp. 1–4.
- T. Badawy and T. Bertuch, “Slotted waveguide antenna integrated with printed Yagi-Uda director array,” in *2019 13th European Conference on Antennas and Propagation (EuCAP)*, 2019, pp. 1–5.
- C. Wasserzier, T. Badawy, J. Klimek, M. Caris, H. Kuschel, T. Bertuch, C. Löcker, F. Klöppel, J. Wilcke, and A. Saalman, “Radar demonstrator for bird monitoring in wind farms,” in *2018 22nd International Microwave and Radar Conference (MIKON)*, 2018, pp. 119–122.

Curriculum Vitae

06.10.2025	Doctoral Examination for Dr.-Ing.
2020–2025	Doctoral Candidate Chair of Radar Systems Engineering RWTH Aachen University, Germany
2016–Present	Research Assistant Fraunhofer Institute for High Frequency Physics and Radar Techniques, Germany
<i>Dec. 2015</i>	Master of Science Ulm University, Germany
<i>May 2009</i>	Bachelor of Science Monofiya University, Egypt
2001–2004	General Higher Education Entrance Qualification (Abitur) El Sadat Secondary School, <i>Monofiya, Egypt</i>
1987	born in <i>Monofiya, Egypt</i>