

Structural joint inversion of seismic refraction tomography and surface wave dispersion curve data

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SUMMARY

We present here a structural joint inversion (SJI) scheme to couple *P*-wave first arrivals from seismic refraction tomography (SRT) and Rayleigh-wave dispersion curves from surface-wave analysis (SWA). Structural similarity is established through the cross-gradient constraint to build *P*-wave and *S*-wave velocity models with improved geometrical consistency. The SJI approach is applied to two synthetic models with blocky piecewise-constant structures and one field data experiment conducted in a shallow aquifer, where initial independent analysis revealed structural similarity between the seismic data sets. For both, synthetic and field data experiments, we observe that the SJI provides better results compared to the individual inversions regarding the reconstruction of consistent model boundaries, while maintaining a similar data fit. The differences between independent and joint inversion results for the field data experiment are less pronounced. The identification of relationships between *P*- and *S*-wave velocity is nevertheless improved with the resolved models corresponding well to the known local geology and a complementary electrical data set.

Key words: Joint inversion; Seismic tomography; Surface waves and free oscillations.

1 INTRODUCTION

Seismic refraction tomography (SRT) and seismic surface-wave analysis (SWA) are two frequently applied geophysical methods for near surface explorations. SRT is used to resolve the subsurface 2-D/3-D structure of the compressional (*P*)-wave velocity, whereas SWA is commonly applied to obtain the 1-D or pseudo-2-D shear (*S*) wave velocity distribution. The methods offer several synergies. For example, the data sets can be acquired simultaneously using optimized acquisition schemes (e.g. S. Foti *et al.* 2003) and the methods are sensitive to different elastic moduli, as for instance the propagation of *P* and *S* waves is affected differently in the presence of fluids (M.A. Biot 1956a, b).

Usually, when both SRT and SWA data are processed, the inversions are carried out independently and interpreted jointly (e.g. S.D. Sloan 2013; E. Cardarelli *et al.* 2014; S. Pasquet *et al.* 2016; E. Pegah & H. Liu 2016; M. Dangeard *et al.* 2021), which can be subject to inconsistent results and lead to interpretation ambiguities (e.g. S. Foti *et al.* 2003). The inconsistencies between *P*- and *S*-wave velocity models can arise from different intrinsic limitations, sensitivities, depth of investigations and resolutions of the two methods (e.g. S. Foti *et al.* 2003). SRT for instance often provides a higher investigation depth at the cost of low resolution close to the surface and the data may be affected by poor signal-to-noise ratio (S/N), especially in urban areas or difficult

terrain (e.g. M. Steiner *et al.* 2021). In most seismic surveys, surface waves are the dominating wave type, as most of the source energy is imparted to them (C.B. Park *et al.* 1999). SWA offers higher vertical resolution at shallow depth and is not limited by the hidden layer problem or velocity inversion in contrast to SRT (L.V. Socco & C. Strobbia 2004). The SWA method is further based on the assumption of a 1-D subsurface comprised of a stack of horizontal homogeneous elastic layers. Lateral variation is often assessed based on spatial windowing of the seismic traces (e.g. L.V. Socco *et al.* 2009; P. Bergamo *et al.* 2012), which can lead to smoothed lateral changes. As the SWA data is only evaluated in spatial windows fulfilling the 1-D assumption, lateral resolution can be low.

In a hydrogeological context, SRT is particularly suitable for imaging strong velocity contrasts, such as the transition between unsaturated zone and saturated zone at the water table, while surface-wave dispersion is more sensitive to small velocity variations within both zones (e.g. S. Pasquet *et al.* 2015; S. Foti *et al.* 2003). Surface waves are also influenced by the depth of the water table and can provide constraints on its position under favourable conditions (e.g. S.G. Solazzi *et al.* 2021; V. Sobolevskaya *et al.* 2024; J. Cunha Teixeira *et al.* 2025).

Another source for inconsistent velocity models is the solution non-uniqueness of the independent inversion (e.g. M.S. Zhdanov 2002), that is, that several or even an infinite number of

models can describe the data equally well. This renders the generation of a reliable subsurface model particularly challenging in case of complex environments, showing strong velocity contrasts and/or velocity inversion (F. Garofalo *et al.* 2015). J. Ivanov *et al.* (2006) address the issue of non-uniqueness of the SRT inverse problem. Through integration of a *S*-wave velocity reference model derived from SWA, the solution space is reduced and results obtained through the constrained inversion approach are found to be more geologically realistic (J. Ivanov *et al.* 2006).

Joint inversion (JI) of multiple geophysical data sets has been proposed to improve result consistency, and consequently reduce model uncertainty and aid interpretation. JI involves the integration of multiple geophysical methods into one parameter estimation (F.M. Wagner & S. Uhlemann 2021). The link between model parameters from various geophysical methods is typically established through either a structural or a (petro-)physical relationship. In the structural approach, the sensitivity of geophysical methods to common subsurface boundaries is leveraged by enforcing structural similarity between the models (e.g. L.A. Gallardo & M.A. Meju 2003). In the (petro-)physical approach, geophysical parameters are converted into one or more common (petro-)physical parameters using appropriate transformation relationships. These (petro-)physical parameters then form the inversion parameters to be estimated during the joint inversion (F.M. Wagner *et al.* 2019).

Several approaches for the simultaneous inversion of SRT and SWA data have been proposed, which use mutual constraints on the structure (i.e. shared layer thicknesses) and additionally couple the two methods based on a physical link. Poisson's ratio is directly related to the *P*- and *S*-wave velocity, which is a dimensionless elastic parameter that characterizes the deformation behaviour of isotropic materials under stress. It represents the ratio of lateral (transverse) strain to axial (longitudinal) strain when a material is subjected to uniaxial compression (R.E. Sheriff & L.P. Geldart 1995). G. Dal Moro (2008) uses a bi-objective evolutionary algorithm to perform a 1-D joint inversion of SWA and SRT data, where the coupling between the two model parameters is established through the Poisson's ratio. C. Piatti *et al.* (2013) present a 1-D least-squares joint inversion of SWA and SRT data based on geometrical and physical constraints, where layer thicknesses and *P*-wave velocities are shared and can vary according to the sensitivities on the SRT and SWA data. In their work, an optional link to the *S*-wave velocity is established through the Poisson's ratio. The Poisson's ratio is thereby constrained based on prior information, which improves the reliability and consistency of the results as unrealistic values are avoided. In C. Piatti *et al.* (2013), the initial joint thickness model is defined by interfaces obtained through a preliminary interpretation of the traveltimes with additional layers to account for possible hidden layers. D. Boiero & L.V. Socco (2014) extend the method from C. Piatti *et al.* (2013) to 2-D based on the laterally constrained inversion (LCI) scheme after E. Auker & A.V. Christiansen (2004). Z. Wang *et al.* (2023) present a joint iterative analysis of SRT and SWA, in which the SRT inversion is constrained by the *S*-wave velocity model from SWA, and the SWA inversion is constrained by the *P*-wave velocity model from SRT, with Poisson's ratio serving as the linking parameter between the two methods. F. Garofalo *et al.* (2022) use the porosity to couple *P*- and *S*-wave velocity and electrical resistivity in porous saturated media in a 1-D inversion framework. The model linking porosity and the seismic velocities is derived from Biot's theory (Biot, M.A. Biot 1956a, b),

while Archie's law (G.E. Archie 1942) is used to relate electrical resistivity to porosity. (Petro-)physical coupling based on parameters such as Poisson's ratio and porosity is constrained by the availability of reliable prior information. In the case of the porosity estimation, the applicability of the chosen petrophysical model to actual subsurface conditions must also be ensured (F. Garofalo *et al.* 2022), an aspect that can be challenging in 2-D investigations.

If common subsurface features are expected to be resolved by different geophysical methods, structural joint inversion (SJI) approaches can be used to enhance the structural similarity between the models. The most popular structural coupling mechanism is the cross-gradient constraint proposed by L.A. Gallardo & M.A. Meju (2003), which has been successfully applied to couple a variety of methods, for example, SRT and electrical resistivity data (e.g. L.A. Gallardo & M.A. Meju 2003; L.A. Gallardo 2004; C. Jordi *et al.* 2020), magnetotelluric and SW (L.A. Gallardo & M.A. Meju 2007), magnetotelluric and SRT (L.A. Gallardo & J. Zhang 2014). Most implementations are based on regular flat 2-D grids (e.g. L.A. Gallardo 2004; L.A. Gallardo & M.A. Meju 2007; P. Wu *et al.* 2020) and regular 3-D grids (e.g. N. Linde *et al.* 2006; M. Moorkamp *et al.* 2011), but extensions to structured 2-D grids with topography (G. Penta de Peppo *et al.* 2024) and irregular triangle or tetrahedral grids (C. Jordi *et al.* 2020) have been proposed. The coupling mechanism has also been successfully applied to hydrological targets (e.g. N. Linde *et al.* 2006; T. Lochbühler *et al.* 2013) and time-lapse inversions (J. Doetsch *et al.* 2010a; J.B. Rittgers *et al.* 2016). Compared to (petro-)physical coupling, the cross-gradient constraint serves as a relatively weak constraint particularly when the geophysical methods involved are not equally sensitive to the same subsurface features. As a result, the final models may still deviate from true subsurface petrophysical properties due to differing sensitivities. However, based on the cross-gradient constraint no artefacts can be introduced due to ill-suited (petro-)physical relations and lack of sufficient prior information (e.g. J.B. Rittgers *et al.* 2016).

In this study, we present a SJI framework to couple SRT and quasi-2-D SWA data based on cross-gradient constraints. The aim of the SJI is to (1) exploit the synergies of both methods, (2) enhance the structural coherency of the inverted *P*- and *S*-wave velocity models, (3) improve the delineation between (lithological) contacts based on the inverted models and (4) improve the reliability of *a posteriori* computed parameters (i.e. Poisson's ratio) in areas both methods are sensitive to without any *a priori* constraints. The framework is flexible and can be extended to include additional constraints, such as petrophysical relationships, when reliable information is available and when structural violations (e.g. hidden layers) are suspected.

We realize the cross-gradient constrained inversion of SWA and SRT data through regular 2-D grids with flat topography and solve the inverse problem with a least-squares approach. To solve for the 2-D *S*-wave velocity structure based on 1-D SWA data, we apply layer-based parametrization as suggested by D. Boiero & L.V. Socco (2014) in their joint inversion approach of SRT and SWA data and J.N. Ogunbo & J. Zhang (2014) in their cross-gradient joint inversion scheme of transient electromagnetic and SRT data. The joint inversion scheme is first tested on two block homogeneous synthetic examples and compared with independent inversion results using the same forward operators and initial model parametrization. To test the performance with

field-scale measurements, the SJI is then applied to field data collected in a shallow aquifer in Austria.

2 THEORY AND METHOD

2.1 Seismic refraction tomography

SRT is based on the propagation of body waves, particularly compressional (P) waves, through the Earth's interior to obtain a velocity model (typically P -wave velocity, v_p) of the subsurface. In the high-frequency limit, wave propagation can be approximated by ray theory, where energy is assumed to travel along paths determined by Snell's law in response to spatial variations in seismic velocity V . Červený (2001). At interfaces where velocity increases with depth, waves are critically refracted and generate head waves, which propagate along the interface and return energy to the surface at greater offsets.

The body waves are generated at known locations (i.e. shot stations) and the resulting ground motion is recorded by sensors (e.g. geophones) usually installed along a line. The SRT data processing consists of the picking of the first arrival traveltimes of the recorded seismic energy (e.g. M. Steiner & A. Flores Orozco 2023). The picking is usually performed manually. Uncertainty estimation in picking is often subjective and can be quantified by assigning a standard deviation to each arrival time based on repeated picking (e.g. S. Uhlemann *et al.* 2016).

In SRT, the first arrival traveltimes gathered from multiple shot gathers associated to different shot locations are inverted to reconstruct the variations in body-wave velocities in the subsurface in an imaging framework. The forward problem involves computing the theoretical arrival times by simulating the ray propagation for a given slowness distribution (s_p , inverse of P -wave velocity). The inverse problem is inherently nonlinear and ill-posed, requiring regularization such as smoothing to constrain the solution (D.J. White 1989).

Refraction tomography generally provides high lateral resolution in regions with dense ray coverage, especially across sharp velocity contrasts such as refracting interfaces. Its depth of investigation depends on the maximum source–receiver offset and on the ability of critically refracted waves to sample deeper layers.

2.2 Surface-wave analysis

Surface-wave methods generally exploit the dispersive nature of surface waves (typically Rayleigh waves) in a layered medium to obtain 1-D variations of S -wave velocity with depth. Rayleigh waves result from coupled P - and vertically polarized S -wave motion trapped along a surface boundary and are sensitive primarily to the shear-wave velocity structure (v_s) of the subsurface (K. Aki & P.G. Richards 2002). Rayleigh waves of different frequencies sample different depth ranges, with longer wavelengths (lower frequencies) probing deeper into the Earth. This frequency-dependent behaviour allows for the construction of dispersion curves of phase or group velocity as a function of frequency and retrieve the vertical variation of v_s through geophysical inversion.

Rayleigh waves propagate in multiple modes, each corresponding to a different solution of the dispersion relation for layered media. The fundamental mode is the simplest and often

most energetic form of Rayleigh-wave propagation. It is primarily sensitive to the shallowest layers, displaying lower phase velocities. Higher order modes exhibit higher sensitivity to deeper layers, with more complex displacement patterns, higher phase velocities, and typically lower amplitudes. The relative excitation of modes depends on the source characteristics and the velocity structure of the subsurface. In many near-surface applications, the fundamental mode dominates, but in certain cases, particularly in the presence of strong velocity inversions or complex layering, higher modes can carry essential information for constraining deeper velocity structures (K. Aki & P.G. Richards 2002; L.V. Socco & C. Strobbia 2004).

The acquisition is similar to SRT, but usually low-frequency geophones with a corner frequency of 4.5 Hz are deployed in near-surface studies. The data processing focuses on extracting the dispersion curves from multichannel seismic records. One of the most widely used methods is the multichannel analysis of surface waves (MASW; C.B. Park *et al.* 1999). In the MASW method, a multistation time-offset domain record is transformed into the frequency-phase velocity or frequency-wavenumber domain, where the energy maxima corresponding to the different modes are extracted. Confidence levels can be assigned to each frequency component of the extracted dispersion curves by combining all curves associated with the same surface location (e.g. E.A. Olafsdottir *et al.* 2018).

The forward problem involves calculating theoretical dispersion curves for layered elastic media, often using the propagator matrix method (F. Gilbert & G.E. Backus 1966). It involves finding the roots of the secular equation of Rayleigh waves:

$$F_r(v_r(z), f, \rho(z), v_p(z), v_s(z)) = 0, \quad (1)$$

where Rayleigh phase velocity (v_r), P -wave velocity (v_p), S -wave velocity (v_s) and density (ρ) are functions of the depth (z), and f is the frequency. Subsurface properties are thereby assumed to only vary with depth, that is, the model domain consists of a stack of horizontal layers, where each layer is defined by a thickness and constant material properties (v_p , v_s , ρ). The nonlinear and non-unique inverse problem is solved based on global optimization methods (e.g. genetic algorithms, Monte Carlo sampling; J. Xia *et al.* 1999; M. Wathelet *et al.* 2004) or linearized iterative least-squares inversion frameworks with model regularization (e.g. R.B. Herrmann 1994).

SWA offers higher vertical resolution at shallow depth and is not limited by the hidden layer problem or velocity inversion in contrast to SRT (L.V. Socco & C. Strobbia 2004). The vertical resolution at the surface is limited by one-third of the smallest seismic wavelength, while for the maximum depth of investigation commonly one-third to one-half of the maximum wavelength are assumed (e.g. B.R. Cox & D.P. Teague 2016). The MASW method assumes homogeneous lateral conditions. Lateral variation is often assessed based on spatial windowing of the seismic traces (e.g. L.V. Socco *et al.* 2009; P. Bergamo *et al.* 2012) and comparison of forward and reverse offset shots (e.g. E.A. Olafsdottir *et al.* 2020). The window size is thereby chosen as a trade-off between spectral and lateral resolution, while keeping the assumption of a horizontally layered subsurface valid (S. Pasquet & L. Bodet 2017).

Beyond their methodological differences, the distinct wave propagation physics of SRT and SWA lead to different subsurface illumination patterns, resolution and depth of investigation (e.g. S. Foti *et al.* 2003; L.V. Socco & C. Strobbia 2004; S.G. Solazzi *et al.* 2021). In SRT, body-wave propagation is governed by

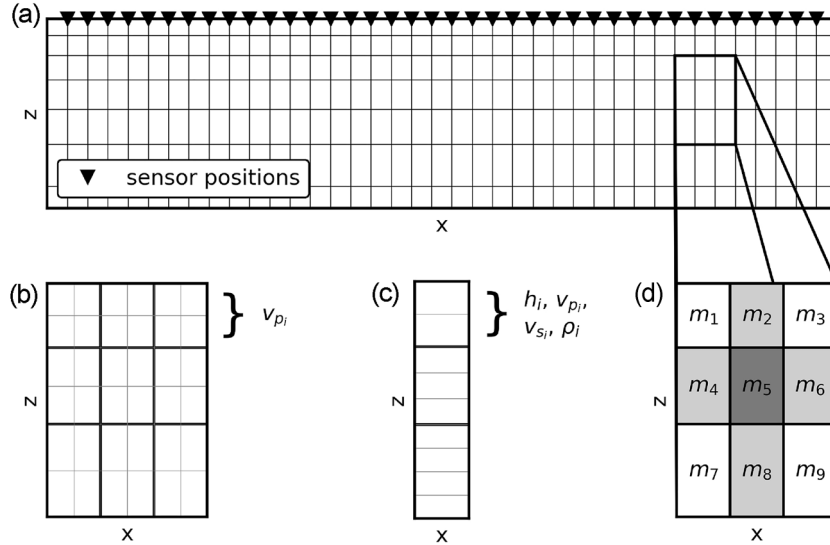


Figure 1. (a) Discretization of the joint inversion domain using quadrilateral grid cells. The black triangles represent both source and receiver positions. Refinement of the coarser parameter grid for the SRT (b) and SWA forward modelling (c). Thin lines represent the refined grid cells. Cell parameters required for each forward operation are denoted in the figure. (d) Illustration of the parameter grid cells involved in the gradient estimation for cell 5. Grid cells with grey shading are used to define four sets of finite difference operators to consider forward and backward finite differences.

ray paths controlled by velocity gradients. The ray paths preferentially sample sharp velocity contrasts, resulting in an illumination pattern concentrated along refracting interfaces. Consequently, SRT provides high sensitivity to strong velocity contrasts, but limited sensitivity elsewhere. In contrast, SWA exploits the dispersive behaviour of surface waves, with a depth-weighted sensitivity controlled by the wavelength. As a consequence, the vertical resolution at shallow depths is higher and the method is suitable to detect subtle velocity variations. Lateral resolution is constrained by the assumption of laterally homogeneous layers within analysis windows in classical SWA approaches. In terms of depth of investigation, SRT can provide sensitivity to greater depths depending on source–receiver geometry and refractor continuity, whereas SWA is typically limited to depths on the order of a fraction of the dominant surface-wave wavelengths. These fundamental differences highlight the complementary strengths and limitations of the two methods.

2.3 Data and forward modelling

The data for the joint inversion of SRT and SWA consist of P -wave first arrival traveltimes and (apparent) fundamental mode Rayleigh-wave dispersion curves, respectively.

The joint parameter domain is discretized into n quadrilateral model cells (see Fig. 1a), where each cell of the parameter grid has an assigned s_p, v_s, ρ and thickness (h) value.

The s_p model is required for the SRT forward solver. We use the algorithm by E. Dijkstra (1959) to solve the forward problem, which finds the shortest path from one cell node, where the source of the P wave is located, to all other nodes (K. Hellman et al. 2017). The accuracy of the forward solution is controlled by the size of the mesh cells, as ray paths are restricted to secondary nodes. To improve the accuracy of the predicted traveltimes, the parameter grid is refined for the forward modelling to include secondary nodes (B. Giroux & B. Larouche 2013), as shown in Fig. 1(b).

The s_p, v_s, ρ and h values of each model cell are required for the SWA forward modelling. We use Dunkin’s matrix (J.W. Dunkin 1965), which is based on the W.T. Thomson (1950) and N.A. Haskell (1953) method, to solve the forward problem in 1-D. For numerical precision, a fine homogeneous gridding is used for the forward simulations (J. Guillemeoteau et al. 2022), as shown in Fig. 1(c).

2.4 Laterally constrained inversion scheme

To solve the pseudo-2-D S-wave velocity structure from 1-D SWA forward models, we follow D. Boiero & L.V. Socco (2014) and adopt a laterally constrained inversion (LCI) scheme. The LCI, first presented by E. Auken & A.V. Christiansen (2004) for electrical resistivity soundings, is based on a pseudo-2-D layered parametrization and hence restricted to quasi-layered subsurfaces (L.V. Socco et al. 2009). In the LCI, all soundings (i.e. the fundamental mode dispersion curves) are simultaneously inverted by minimizing a common objective function containing the data misfit and lateral constraints between neighbouring models. This helps overcome instability and non-uniqueness in individual 1-D inversions by incorporating spatial continuity. The result of the LCI is a set of 1-D depth models, where each model corresponds to one dispersion curve and one 1-D forward solution. The 1-D models are linked to their neighbours based on the lateral constraints, which can be considered as *a priori* information on the subsurface variability: the higher the expected lateral variation between neighbouring models, the weaker the constraint (R. Wisén & A.V. Christiansen 2005). Since information from one model spreads to the neighbouring models through the lateral constraints, the resulting pseudo-2-D model will be laterally smooth.

2.5 Structural joint inversion scheme

In the SJI scheme, the joint data vector is

$$\mathbf{d} = [\mathbf{d}_{it}(rx, sx), \mathbf{d}_{dc}(x, f)]^T, \quad (2)$$

where the P -wave traveltimes $\mathbf{d}_t(rx, sx)$ are a function of receiver (rx) and source (sx) location and the dispersion curves $\mathbf{d}_{dc}(x, f)$ are a function of position (x) along a profile and frequency. The joint model vector $\mathbf{m}$ holds the P -wave slowness and S -wave velocity values for each model cell

$$\mathbf{m} = [\mathbf{m}_1, \mathbf{m}_2]^T. \quad (3)$$

Transformation of the model vector using logarithmic barriers is applied to constrain the values to plausible limits (H.J. Kim & Y. Kim 2011). The objective function to be minimized is

$$\Phi(\mathbf{m}) = \Phi_{d,1} + \Phi_{d,2} + \alpha_1^2 \Phi_{m,1} + \alpha_2^2 \Phi_{m,2} + \beta^2 \Phi_{cg}, \quad (4)$$

where $\Phi_{d,1}$ and $\Phi_{d,2}$ are the data misfit terms for the two methods, $\Phi_{m,1}$ and $\Phi_{m,2}$ are the corresponding smoothness regularization terms and Φ_{cg} represents the measure of structural similarity based on cross-gradients. The constants α_1 , α_2 and β are regularization weights. The data misfit term for data type k is defined as

$$\Phi_{d,k} = \|\mathbf{W}_{d,k}(\mathbf{d}_k - \mathcal{F}_k(\mathbf{m}_k))\|_2^2, \quad k = 1, 2 \quad (5)$$

where $\mathbf{W}_{d,k}$ is the diagonal data covariance matrix. Assuming uncorrelated data, $\mathbf{W}_{d,k}$ is a diagonal matrix and composed of the reciprocals of the data uncertainty. The $\mathbf{d}_k$ is the observed data and $\mathcal{F}_k(\mathbf{m}_k)$ is the forward response for model $\mathbf{m}_k$. The regularization term is defined as

$$\Phi_{m,k} = \|\mathbf{W}_{m,k}\mathbf{m}\|_2^2, \quad k = 1, 2 \quad (6)$$

where $\mathbf{W}_{m,k}$ is the smoothness regularization operator (S. Constable *et al.* 1987). The third term is the cross-gradient term for two data types

$$\Phi_{cg} = \|\mathbf{t}(\mathbf{m}_1, \mathbf{m}_2)\|_2^2, \quad (7)$$

where $\mathbf{t}(\mathbf{m}_1, \mathbf{m}_2)$ is the cross-gradient of the two models $\mathbf{m}_1$ and $\mathbf{m}_2$.

The objective function is nonlinear and the Gauss–Newton scheme is used for the iterative minimization. This leads to the following system of equations that is solved in a least-squares sense using the algorithm proposed by C.C. Paige & M.A. Saunders (1982)

$$\begin{bmatrix} \mathbf{W}_d \mathbf{J}^p \\ \mathbf{W}_m \\ \beta \mathbf{B}^p \end{bmatrix} [\Delta \mathbf{m}^{p+1}] = \begin{bmatrix} \mathbf{W}_d(\mathbf{d} - \mathcal{F}(\mathbf{m}^p)) \\ -\mathbf{W}_m \mathbf{m}^p \\ -\beta \mathbf{t}^p \end{bmatrix}, \quad (8)$$

$$\mathbf{W}_d = \begin{bmatrix} w_1 \mathbf{W}_{d,1} & 0 \\ 0 & w_2 \mathbf{W}_{d,2} \end{bmatrix},$$

$$\mathbf{W}_m = \begin{bmatrix} w_1 \alpha_1 \mathbf{W}_{m,1} & 0 \\ 0 & w_2 \alpha_2 \mathbf{W}_{m,2} \end{bmatrix},$$

$$\mathbf{J}^p = [\mathbf{J}_1^p \ \mathbf{J}_2^p]^T,$$

where $\Delta \mathbf{m}$ is the model update, $\mathbf{J}$ is the Jacobian matrix holding the changes in traveltimes and dispersion curves with respect to changes in the model values, and w_1 and w_2 are weighting factors to define individual weights for the two data sets. The matrix $\mathbf{B}$ contains the partial derivatives of $\mathbf{t}$ with respect to $\mathbf{m}_1$ and $\mathbf{m}_2$ to approximate the cross-gradient function by a first-order Taylor expansion

$$\mathbf{t}^{p+1} \cong \mathbf{t}^p + \mathbf{B}^p \Delta \mathbf{m}^{p+1}. \quad (9)$$

The SJI is realized through a joint 2-D grid for the inversion (see Fig. 1a). The parameter domain is characterized by coarse, quadrilateral grid cells with increasing cell size with depth to avoid an excessive number of parameters to be estimated. For

high numerical precision, the grids used for the forward modelling and inversion are decoupled as already mentioned in Section 2.3.

The elements of the Jacobian matrix $\mathbf{J}$ of the parameter domain are then calculated as

$$\mathbf{J}_i = \sum_{k=1}^K \mathbf{J}_{f,i,k}, \quad (10)$$

where $\mathbf{J}_f$ is the Jacobian matrix computed during the forward simulations based on the refined grid and K is the number of subcells of the refined grid contained within cell i of the parameter grid.

The independent SWA and SJI inversion are implemented within the open-source geophysical modelling and inversion library pyGIMLi (C. Rücker *et al.* 2017) adopting the LSQR algorithm implemented by F.M. Wagner *et al.* (2019).

2.6 Cross-gradient and cross-gradient sensitivity

Structural coupling is established through the cross-gradient function, first introduced by L.A. Gallardo & M.A. Meju (2003), which measures the geometrical similarity between two models

$$\mathbf{t}(\mathbf{m}_1, \mathbf{m}_2) = \nabla \mathbf{m}_1(x, y, z) \times \nabla \mathbf{m}_2(x, y, z). \quad (11)$$

When the directions of the model gradients $\nabla \mathbf{m}_1$ and $\nabla \mathbf{m}_2$ are collinear at a given location the cross-gradient becomes zero. Forcing the cross-gradient to vanish throughout the model domain implies solving for two models that are structurally identical. If a boundary is only sensed by one method (i.e. the gradient of one model is zero) the cross-gradient will also be zero. This implies that it is possible to admit the boundary in only one of the models (L.A. Gallardo & M.A. Meju 2003). In case of 2-D data, the x - and z -components of the cross-gradient $\mathbf{t}$ vanish and the y -component is

$$t_y = \frac{\partial \mathbf{m}_1}{\partial z} \frac{\partial \mathbf{m}_2}{\partial x} - \frac{\partial \mathbf{m}_1}{\partial x} \frac{\partial \mathbf{m}_2}{\partial z}. \quad (12)$$

The sensitivity matrix $\mathbf{B}_y$ is then given by

$$\mathbf{B}_y = [\mathbf{B}_{y,1} \ \mathbf{B}_{y,2}] = \left[\frac{\partial \mathbf{t}_y}{\partial \mathbf{m}_1} \ \frac{\partial \mathbf{t}_y}{\partial \mathbf{m}_2} \right]. \quad (13)$$

Since we use a regular inversion grid, the partial derivatives in eqs (12) and (13) can be estimated based on first-order finite differences as proposed by L.A. Gallardo & M.A. Meju (2003). The cross-gradient is then calculated as

$$t_y = D_z \mathbf{m}_1 D_x \mathbf{m}_2 - D_x \mathbf{m}_1 D_z \mathbf{m}_2, \quad (14)$$

where D_x and D_z represent the first-order difference operators. For the cross-gradient sensitivities we get

$$\mathbf{B}_{y,1} = D_x(D_z \mathbf{m}_2) - D_z(D_x \mathbf{m}_2), \quad (15)$$

$$\mathbf{B}_{y,2} = D_z(D_x \mathbf{m}_1) - D_x(D_z \mathbf{m}_1). \quad (16)$$

To avoid numerical errors at the right and bottom edges of the grid, instead of only using forward differences, we follow the recommendations of P.G. Lelièvre & D.W. Oldenburg (2009) and define four sets of finite difference operators $D_{x,i}$ and $D_{z,i}$ to consider all possible combinations of forward and backward finite differences (see Fig. 1b).

The grid for the model domain is not adapted during the inversion process, which reduces the computational load, as the difference operators must only be calculated once.

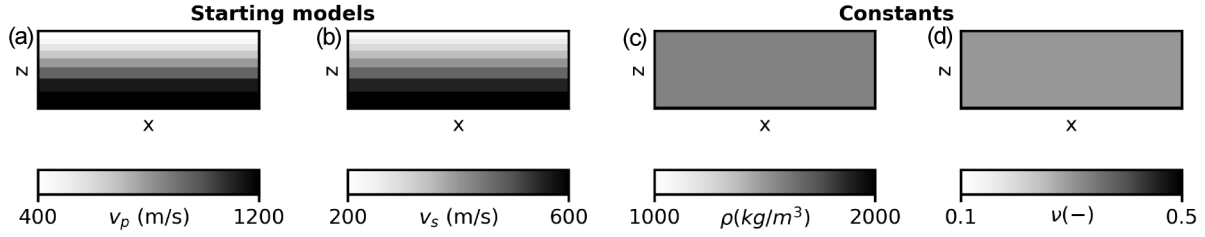


Figure 2. (a) and (b) Starting models for v_p and v_s , respectively. (c) Constant ρ model for the SWA forward operation used in both, independent and joint inversion schemes. (d) Constant ν model used in the independent SWA inversion to iteratively compute v_p based on the updated v_s model for forward modelling.

2.7 Data weighting

We adopt the data weighting approach proposed by C. Jordi *et al.* (2020) to balance the convergence speeds of the individual methods in the SJI. The weights w_1 and w_2 are dynamically adjusted using the following χ^2 -criterion

$$w_1 = \begin{cases} 1.0 & \text{if } \chi_1^2 > \chi_2^2 \\ \chi_1^2 / \chi_2^2 & \text{otherwise} \end{cases}$$

and

$$w_2 = \begin{cases} 1.0 & \text{if } \chi_1^2 < \chi_2^2 \\ \chi_2^2 / \chi_1^2 & \text{otherwise} \end{cases} \quad (17)$$

with $\chi^2 = \Phi_d/n$ and n is the number of data.

At each iteration, a lower weight is assigned to the method yielding a smaller χ^2 -value, helping to ensure that both methods converge toward an optimal data fit at a comparable pace.

For the regularization strengths α_1 and α_2 , we follow again C. Jordi *et al.* (2020) in the joint inversion and apply a cooling strategy to decrease the values by 20 per cent in each iteration until a target value is reached. As target values, the regularization strengths determined in the independent inversion are used. For the SRT method, α is fixed and selected empirically by testing values from 1 to 10^3 , choosing the one that achieves the best data fit ($\chi^2 = 1.0$). In the SWA method, a cooling strategy is employed: a high initial α ($= 150$) is reduced by 20 per cent per iteration until a target value is reached. For the synthetic data, the minimum value of α is set to 1, whereas for the field data, α_{min} is set to 15. The same anisotropic smoothing is applied to the independent SRT and SWA inversions, as well as the SJI.

For cross-gradient strength β , we test a wide range of values between 10^2 and 10^8 and choose a fixed value based on good convergence ($\chi^2 = 1.0$) and high structural similarity ($t_y = 0$).

2.8 Parametrization

The grid cells of the parameter grid are defined by constant P -wave velocity, S -wave velocity, thickness and density (see Figs 1b and c). The prediction of the traveltimes is based on the P -wave velocity model, while theoretical dispersion curves are simulated based on all four properties. S -wave velocity and thickness of the grid cells have the highest influence on the Rayleigh-wave propagation, while it is less sensitive to variations in the P -wave velocity and density (J. Xia *et al.* 1999). In order to reduce the degree of freedom in deterministic inversion approaches, usually only the S -wave velocity and thicknesses are considered variable, while density and P -wave velocity are fixed (e.g. G. Dal Moro & M. Pipan 2007). Since we provide a fine grid for the inversion, that is not iteratively changed, only the S -wave velocity is varied

in the SWA forward operator according to the sensitivity to the dispersion curves.

To obtain a rigorous parametrization of the initial s_p , v_s and ρ models, prior stochastic inversion of overparametrized models can be performed for deterministic inversion approaches (e.g. L.V. Socco *et al.* 2009). To investigate the performance of the cross-gradient constraint inversion, however, we define our initial models with minimum structure and use gradient starting models for the P -wave and S -wave velocity for both independent and joint inversion. Top and bottom P -wave velocities of the starting model (see Fig. 2a) are estimated directly from the observed traveltimes. The initial parametrization of the S -wave velocity model (see Fig. 2b) is obtained from the dispersion curve data, considering the ratio of S -wave velocity to Rayleigh-wave velocity ranging from 1.04 to 1.16 (F.E. Richart *et al.* 1970).

The additional physical parameters (ρ and s_p) for the SWA forward model are defined as follows. Since density has little influence on the dispersion curve, a homogeneous density distribution can be assumed (M. Wathelet *et al.* 2004). We follow here G. Dal Moro & M. Pipan (2007) and estimate a mean density value based on Gardner's law (G.H.F. Gardner *et al.* 1974), and the P -wave velocity range (see Fig. 2c). The propagation of surface waves is also less sensitive to variations in P -wave velocity, but sensitivity analyses suggest that, especially for materials with lower Poisson's ratios (ν), the P -wave velocity can have an effect on the dispersion curves (e.g. C. Piatti *et al.* 2013). For unsaturated, unconsolidated, or very stiff soil and rock layers, common ν range between 0.25 and 0.35 and are related to the soil skeleton (H. Gercek 2007). For saturated, soft soil the value is close to 0.5 and related to undrained conditions (H. Gercek 2007). In case of deep investigations (> 5 km) the assumption of a constant ν is usually valid (e.g. J. Guillemoteau *et al.* 2022) and can be used to estimate the P -wave velocity using

$$\frac{v_p}{v_s} = \sqrt{\frac{2(1-\nu)}{1-2\nu}}. \quad (18)$$

For shallower investigations, especially if the water table is not taken into consideration, an inappropriate definition of ν variation can lead to large errors in the resolved S -wave velocity models (S. Foti & C. Strobbia 2002).

For the independent SWA inversion, a constant and fixed ν model is defined (see Fig. 2d). Using this model, the s_p model required for the SWA forward simulation is iteratively recomputed based on the relationship given in eq. (18). In contrast, due to the aforementioned reason, during the joint inversion, the s_p model update is provided to the SWA forward operator in each iteration.

The extent and resolution of the parameter grid are determined based on the available experimental data. Horizontally, the grid is constrained by the surface locations of the dispersion curves. The vertical extent is set according to the estimated depth of investigation (DOI), taking the greater value derived from SRT and SWA. The DOI for SRT is estimated as one-fourth of the maximum receiver offset, while for SWA, it ranges from one-third to one-half of the longest wavelength observed in the Rayleigh-wave dispersion curves. The horizontal resolution of the parameter grid is defined by the receiver spacing. The minimum vertical resolution at the surface is estimated using the SWA data, which corresponds to one-third to one-half of the shortest wavelength, and increases slightly with depth following the layering strategy proposed by B.R. Cox & D.P. Teague (2016).

3 RESULTS

3.1 Synthetic data study

To test the SJI, we first build two synthetic models. The data sets are simulated using the same forward modelling algorithms used in the inversion. As acquisition set up, we choose a profile length of 150 m for both models with a geophone spacing of 2 m and shot positions at each geophone. For the SWA data, we define a frequency range of 2.5 to 50 Hz and simulate a dispersion curve between each geophone pair. Additional noise is superimposed on the data. SRT data is subjected to an absolute Gaussian noise of 1 ms, SWA data by a relative Gaussian noise of 2 per cent. The same values are used as errors in the inversions. The performance of both independent and joint inversion for the synthetic cases is evaluated based on the resulting data fit, the cross-gradient term, and the residuals of true ($\mathbf{m}_{\text{syn}}$) and inverted model ($\mathbf{m}_{\text{inv}}$) calculated as

$$\text{RES} = \sqrt{\frac{1}{K} \sum_{k=1}^K \left(\frac{\mathbf{m}_{\text{syn},k} - \mathbf{m}_{\text{inv},k}}{\mathbf{m}_{\text{syn},k}} \right)^2}. \quad (19)$$

3.1.1 Synthetic model A

Model A represents a simple two layer case [labels A and B in Figs 3(a and b)] with two anomalies [labels C and D in Figs 3(a and b)]. The features are constructed with reference to L.A. Gallardo (2004), the seismic velocities with reference to C. Piatti *et al.* (2013). The densities are estimated from the P -wave velocities using Gardner's law. The model parameters are listed in Table 1. Since the anomalies are placed close to the centre of the profile, we will focus on the results within the range of $x = 25$ to 125 m. The results for both independent (II) and joint (SJI) inversions are obtained with the same gradient starting models and isotropic smoothing. In the independent inversion, we use a constant smoothness regularization strength α of 100 for the SRT inversion and an initial high α -value of 150, that is iteratively decreased to a value of 1, for the inversion of the SWA data. The ν model is set to 0.3 over the whole model space. The II results in terms of the model values are shown in Figs 3(c) and (d). In case of the independent SRT inversion, convergence is reached after 20 iterations with $\chi^2 = 1.00$ and a root mean square error (RMS) of 1.00 ms. For the independent SWA inversion 15 iterations are required to reach a $\chi^2 = 0.98$ and a RMS of 3.06 m s^{-1} at an α of 5. Hence, the same final value of $\alpha_{\text{stop}} = 5$ is used for the SWA term in the SJI. Both methods are able to resolve some features

of the true models independently. In the P -wave velocity model (Fig. 3c) the top of both box anomalies is identified, but the lateral extents and vertical boundaries are not reconstructed. The true interface between top and bottom layer is poorly resolved. Fig. 3(d) shows, that the independent SWA inversion is generally able to recover the overall structure of the synthetic model, but the velocities of the anomalies are overestimated, while the thickness of the right box (defined by a different ν value of 0.35) is underestimated by 3 m.

For the SJI, we choose overall $\alpha_{1,2}$ -values of 150, which are iteratively decreased to 100 and 5, respectively. A fixed cross-gradient weight of $\beta = 5 \times 10^5$ is chosen after testing values between 10^2 and 10^8 . The SJI reaches an overall χ^2 of 1.00 after 26 iterations ($\chi_1^2 = 1.02$, $\chi_2^2 = 0.99$, $\text{RMS}_1 = 1.02 \text{ ms}$, $\text{RMS}_2 = 3.13 \text{ m s}^{-1}$). Beyond 26 iterations, the variations in the models and the data misfit obtained through the SJI are found to be insignificant. The resulting P - and S -wave velocity models (see Figs 3e–f), show higher structural similarity to each other, as imposed by the coupling strategy. The geometrical and physical aspects of the two anomalies are better reconstructed and the contact to the bottom layer is aligned in both models compared to the II results. The interface between the two layers is better resolved in the P -wave velocity, but appears to be smeared in the S -wave velocity image relative to the II results. The observed smearing is assumed to originate from the ν_p model, which is characterized by higher smoothness. Information on boundaries is propagated from the ν_p model into the ν_s model through the cross-gradient constraint and the link in the SWA forward operator and hence influences the evolution of the ν_s model. In Figs 3(g)–(i), the resulting ν_p models from the independent and joint inversions are compared along three 1-D depth profiles extracted from the 2-D model. Similarly, Figs 3(j)–(l) presents a comparison of the corresponding 1-D profiles for ν_s .

The evolution of the II and SJI χ^2 values and the objective function of the cross-gradient multiplied by the regularization strength β are shown Figs 4(a) and (b). The behaviour of the cross-gradient term, plotted in Fig. 4(b), shows a large increase in the cross-gradient term after the first iteration. This coincides with the large decrease in the data misfit. Beyond this point, the term converges towards zero and after 26 iterations the decrease in the cross-gradient term is found to be minimal.

In Figs 5(a)–(d), the final traveltime and dispersion curve data misfits for both II and SJI are compared. In Figs 5(e)–(h), we show the relative differences between the true model values and II and SJI results, respectively, which we calculate as

$$\frac{\Delta \mathbf{m}_k}{\mathbf{m}_k} = \frac{\mathbf{m}_{\text{syn},k} - \mathbf{m}_{\text{inv},k}}{\mathbf{m}_{\text{syn},k}}, \quad k = 1, 2. \quad (20)$$

The global residuals (RES) and residuals for each model feature, calculated using eq. (19), are provided in Table 2 to investigate the performance in each zone. Overall, the differences between the true models and models obtained with the SJI are lower relative to the differences to the II results. In case of the ν_p model, the RES of each true model feature decreased after the SJI. The highest change in comparison to the II occurs for the bottom layer B, while the lowest variation is obtained for anomaly D. For the ν_s model an increase of RES is obtained for unit B after the SJI, due to the assumed contribution from SRT. In all other zones the reconstruction of the true model structures/values is improved.

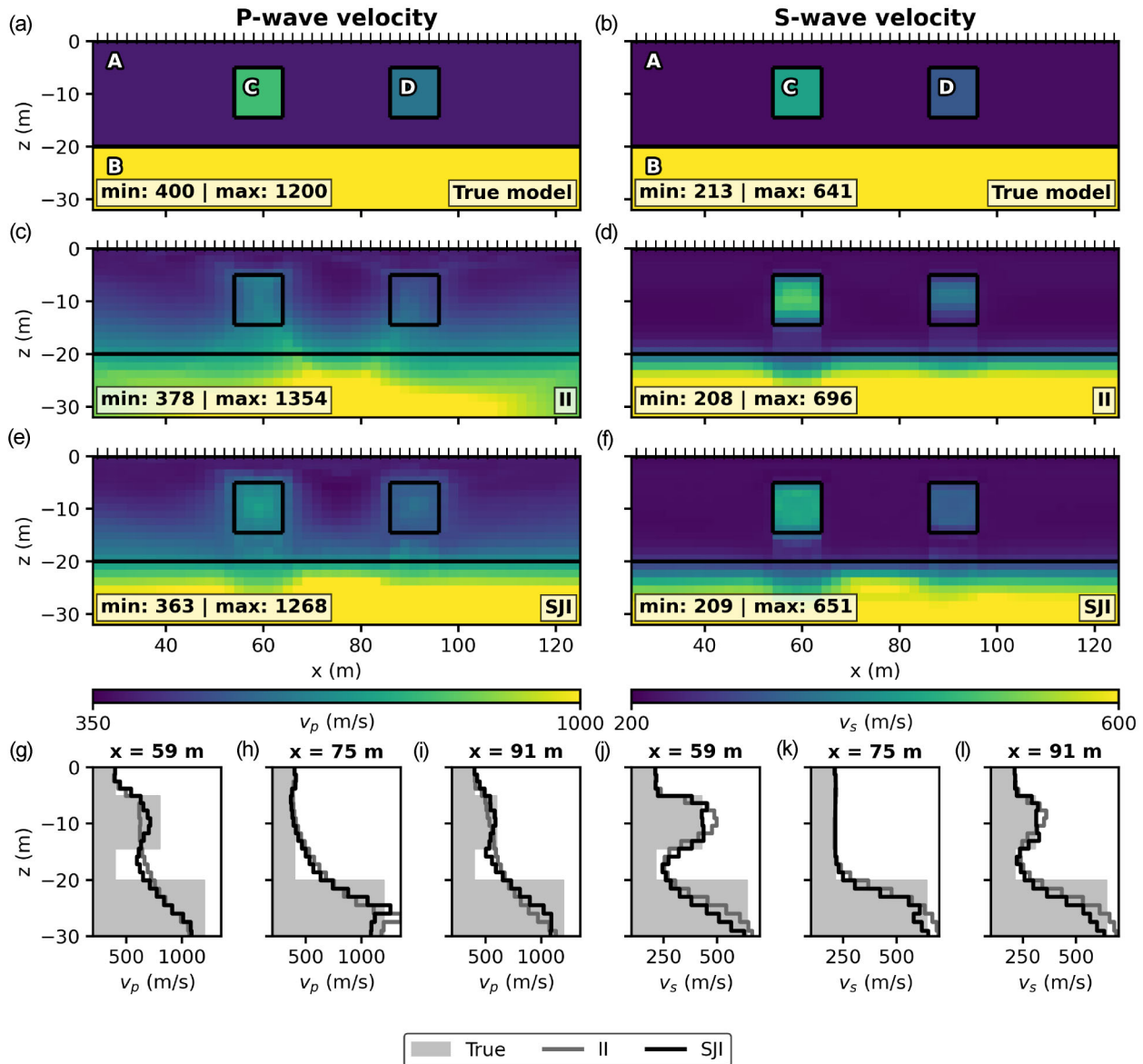


Figure 3. Results for synthetic model A. (a–b) True distribution of the *P*-wave and *S*-wave velocity. (c–d) Independent inversion (II) results for SRT and SWA. (e–f) Structural joint inversion (SJI) results based on the cross-gradient constraint. (g–j) 1-D depth models of v_p extracted at three distinct locations from the 2-D results. (k–l) 1-D depth models of v_s extracted at the same locations from the 2-D results.

Table 1. Model parameters of the synthetic models.

Model A	v_p (m s ⁻¹)	v_s (m s ⁻¹)	ν	ρ (kg m ⁻³)	Model B	v_p (m s ⁻¹)	v_s (m s ⁻¹)	ν	ρ (kg m ⁻³)
A	400	213	0.3	1386	A	294	180	0.2	1284
B	1200	641	0.3	1824	B	490	300	0.2	1459
C	800	428	0.3	1648	C	1530	300	0.48	1939
D	600	309	0.32	1534	D	2102	500	0.47	2099
	–	–	–	–	E	2942	700	0.47	2283
	–	–	–	–	F	1837	500	0.46	2029

Figs 5(i) and (j) show the cross-gradients obtained for the final II and SJI inversion results. Structural inconsistencies are observed around the edges of the model features, as denoted by the high values of the cross-gradient computed for the independent results (Fig. 5i). The cross-gradient values for the SJI results (Fig. 5j) are lower by one order of magnitude, suggesting that the

gradient fields are aligned through the cross-gradient constraint. The scatter plot of v_p versus v_s for every cell for the II, SJI, and true model values are shown in Fig. 5(k). No relationship between the two independently obtained models is easily identified. The model values obtained through the SJI group closer together and positive associations between v_p and v_s are observed.

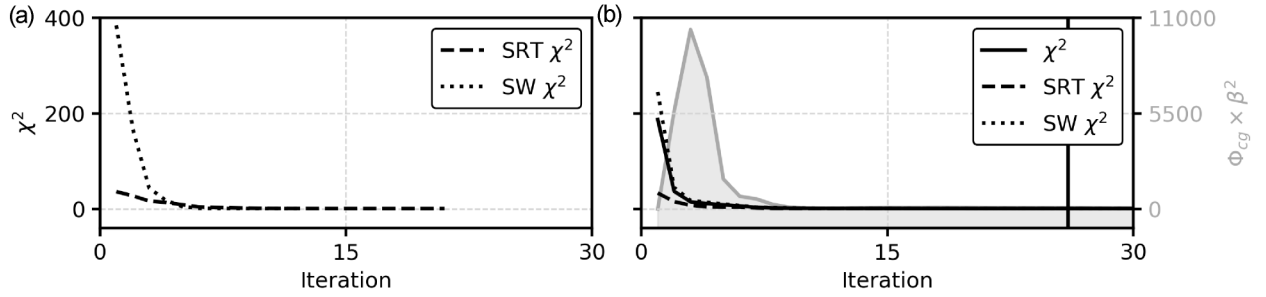


Figure 4. (a) Convergence curves of the independent inversion (II). (b) Convergence curves of the joint inversion (SJI).

Lastly, in Fig. 5(l) the *a posteriori* computed Poisson's ratio distribution of the inversion results is compared to the true model values. As shown in Fig. 5(j), after the SJI the computed ν values are closer to the true values and physically implausible values ($0 \leq \nu \leq 0.5$) are not obtained.

3.1.2 Synthetic model B

Model B is based on the synthetic model discussed in P. Wu *et al.* (2020) regarding the structure and C. Piatti *et al.* (2013) regarding the model values, which are listed in Table 1. Model B represents complex stratigraphic conditions and is characterized by the presence of a water table as v_p increases between layers B and C (see Figs 6a and b), while v_s remains constant. Sharp vertical boundaries (D) and a shallow high velocity zone (F) present a particularly challenging case for both SWA and SRT. To obtain the II and SJI results presented in Figs 6(c)–(f), the following smoothness regularization weights are used: $\alpha = 100$ for SRT, $\alpha_{\text{start}} = 150$ and $\alpha_{\text{stop}} = 1$ for SWA, $\alpha_{\text{start}} = 150$, $\alpha_{1,\text{stop}} = 100$ and $\alpha_{2,\text{stop}} = 10$ for the SJI. The cross-gradient weight is set to 5×10^5 . The independent SRT inversion is terminated after nine iterations resulting in a χ^2 of 1.02 and a RMS of 1.01 ms. The independent SWA inversion reaches a χ^2 of 0.99 and a RMS of 9.80 m s⁻¹ after 12 iterations at a α value of 10. Hence, the same final value of $\alpha_{\text{stop}} = 10$ is used for the SWA term in the SJI. The ν model is set to 0.4. The SJI converges after 18 iterations with an overall χ^2 of 1.05 ($\chi_1^2 = 1.11$, $\chi_2^2 = 1.02$, RMS₁ = 1.05 ms, RMS₂ = 9.49 m s⁻¹).

Between $x = 0$ to 100 m, the *P*-wave velocity model obtained through the II (shown in Fig. 6c) solves the true model structures fairly well, as the lateral boundaries to each layer can be identified and variation in velocity matches that of the true model. The area beneath the high velocity top layer ($x = 100$ to 150 m) is not resolved due to the lack of sensitivity of SRT. The *S*-wave velocity model (shown in Fig. 6d) reveals a sudden velocity jump in layer B from low to high values (180 to 500 m s⁻¹), while velocity values in layer C are underestimated and overestimated in layer D. The estimation of these structures is improved following the SJI (see Figs 6e and f).

In the v_p model, the sharpness of shared boundaries is improved (see Fig. 6g), while no significant changes are observed if the model features between v_p and v_s are different (see Fig. 6h). Variations are observed in the area to which SRT is not sensitive (i.e. the velocity inversion between units F and B at $x = 100$ –150 m) when comparing the II and SJI results (see Fig. 6i). However, the model features in this region—both in terms of structure and velocity values—are not well reconstructed in either case. No major improvements are expected in this area, as

the cross-gradient constraint becomes zero if only one method senses a boundary.

Following the SJI, the incorrectly modelled velocity jumps in the v_s model are reduced (see Figs 6j and k). The results show, that the change in *P*-wave velocity between units B and C is not imposed on the *S*-wave velocity model. The v_s values of the two units are similar (average values of 325 and 328 m s⁻¹ for B and C, respectively) following the SJI, which is closer to the true model values (see Fig. 6k). Since SRT is not sensitive to the area beneath unit F, the variation in the v_s model are similar to those resolved independently (see Fig. 6l).

The evolution of the misfit terms and the objective function of the cross-gradient are shown in Figs 7(a) and (b). In Figs 8(a)–(d), the final traveltime and dispersion curve data misfits for both II and SJI are compared, revealing the highest traveltime misfit between $x = 100$ –150 m. We compare the relative differences between inverted and true model values in Figs 8(e)–(h) and compute the RES for the II and SJI results for each model feature (see Table 3) considering only sensitive cells. Overall RES is reduced for both v_p and v_s models similar to the synthetic study A.

In case of the v_p model, major improvements concern layer D regarding the reconstruction of the step-like feature between $x = 0$ to 22 m. The *P*-wave velocity of the stiff top layer F is underestimated after the SJI (value range is 600 to 1930 m s⁻¹ for the SJI and 680 to 2080 m s⁻¹ for the II), resulting in an increase of RES. For the v_s model RES in all model features are reduced with the SJI. Differences are especially notable in units B, C and D due to the improved estimation of v_s without any velocity jumps as the ν estimate is improved. The cross-gradient of the SJI plotted in Fig. 8(j) shows some remaining spatial inconsistencies, which are highest at the vertical boundaries and related to the imposed smoothing. Nevertheless, the spatial consistency is overall increased when compared to the II result (Fig. 8i). The scatter plot of v_p versus v_s (Fig. 8k) shows that both methods are able to capture the true model variability, but the dispersion of the values is reduced when jointly inverting both methods. As for the histograms of the Poisson's ratio in Fig. 8(l), both II and SJI computed values show a similar distribution and capture the true range of the parameter.

3.2 Field data study

3.2.1 Overview of the study area and data

The joint inversion approach is applied to SRT and SWA field data collected in one soda lake of the National Park Lake Neusiedl-Seewinkel Basin, Austria (see Figs 9a and b). In this

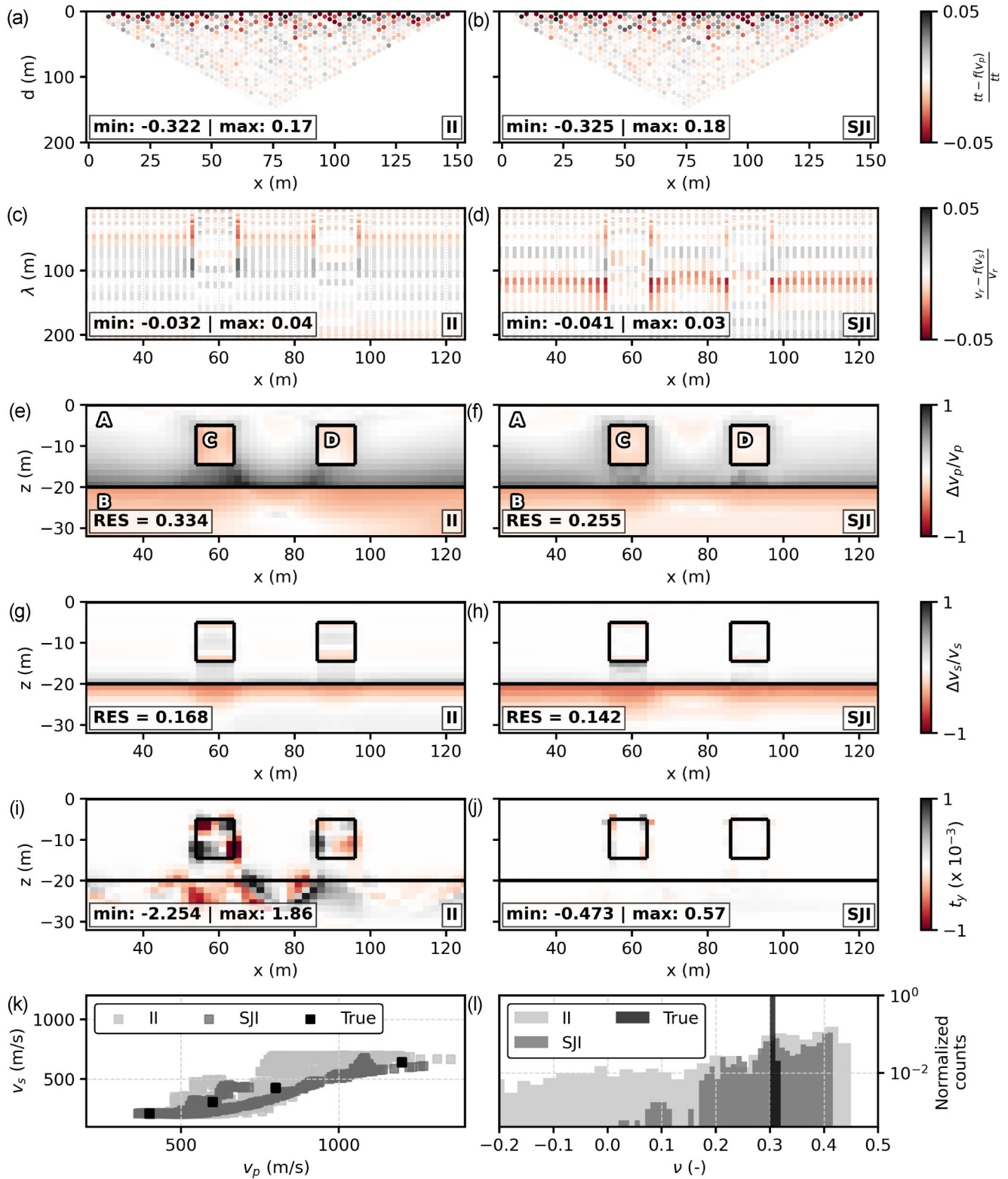


Figure 5. Assessment of results for synthetic model A. (a and b) Relative differences between synthetic traveltime data and final model response of the II and SJI. (c and d) Relative differences between synthetic dispersion curve data and final model response of the II and SJI. (e–h) Relative differences between true and inverse model for the II and SJI. (i and j) Cross-gradients for the II and SJI. In (a–f) the solid black lines indicate the true model structure. (k) Cross-plots of P -wave and S -wave velocity of the true model and II and SJI results. (l) Normalized histograms of the Poisson's ratio calculated for the true model and II and SJI results.

Table 2. Relative differences between true and inverted models.

Model A	RES $v_{p,II}$	RES $v_{p,SJI}$	RES $v_{s,II}$	RES $v_{s,SJI}$
A	0.355	0.273	0.1	0.068
B	0.289	0.217	0.198	0.269
C	0.253	0.186	0.137	0.064
D	0.138	0.09	0.129	0.055
Overall	0.334	0.255	0.168	0.142

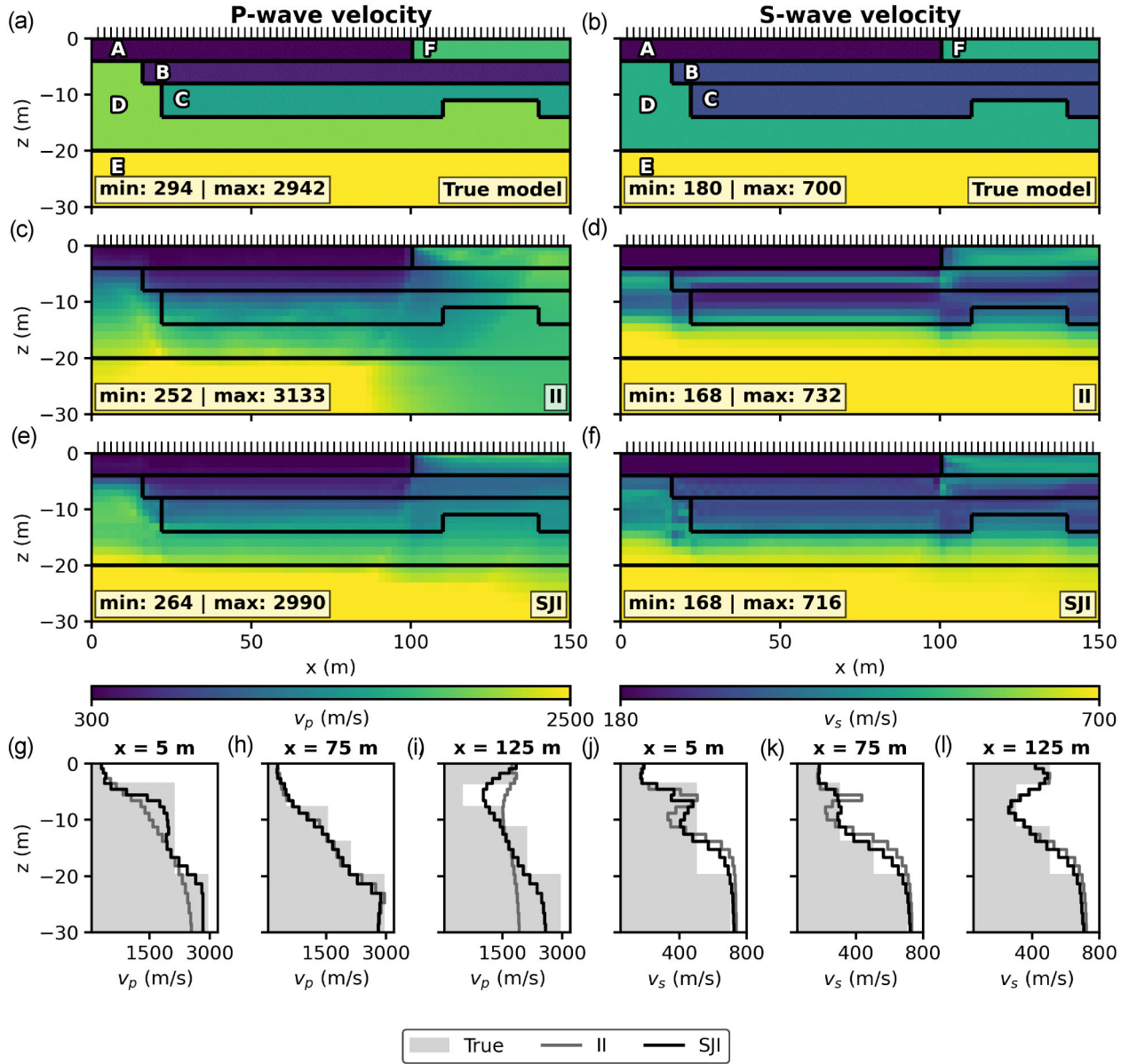


Figure 6. Results for synthetic model B. (a and b) True distribution of the P -wave and S -wave velocity. (c and d) Independent inversion (II) results for SRT and SWA. (e and f) Structural joint inversion (SJI) results based on the cross-gradient constraint. The solid black lines in the figures indicate the true model structures. (g–j) 1-D depth models of v_p extracted at three distinct locations from the 2-D results. (j–l) 1-D depth models of v_s extracted at the same locations from the 2-D results.

area, geophysical measurements are conducted to understand the water flow and storage within the subsurface. The soda lakes create a system of confined aquifers, where a shallow clay-rich impermeable layer hinders the infiltration of water, resulting

in the accumulation of water bodies at the surface. Water levels are shallow (up to 70 cm) and the lake and ground water salinity is relatively high (e.g. H. Häusler 2023). Prior measurements with electrical and electromagnetic methods point

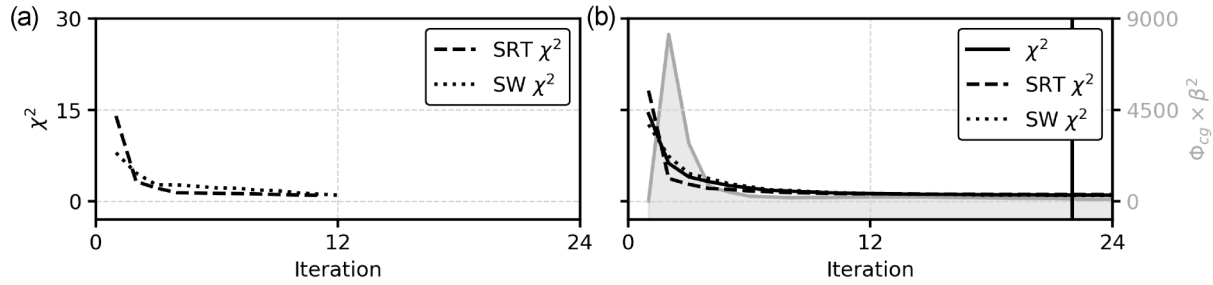


Figure 7. (a) Convergence curves of the independent inversion (II). (b) Convergence curves of the joint inversion (SJI).

at high heterogeneities in the lithology of the sediments especially in regards to a shallow impermeable clay layer. The analysis of the electrical conductivity, however, is not sufficient to discriminate between the contribution of clay (controlling water flow) and salt content (controlling the salinity of the lakes). Seismic methods are therefore applied at the site to provide complementary information on the variation in porosity and saturation.

The survey was conducted in February 2023. A thin (~ 5 – 10 cm) layer of partially frozen water covered parts of the lake-side during the campaign (see Figs 9c–d). The SWA data was collected with 96 geophones with a corner frequency of 4.5 Hz and nominally spaced at 1 m. Additionally, 48 30 Hz geophones with a separation of 2 m were deployed to collect data for SRT.

A ~ 4.5 kg sledge hammer was deployed as source with inline shot locations spaced at 2 m (at each 30 Hz geophone) and forward and reverse offset shots at 0.5, 5 and 10 m. Two shot locations in the center were skipped due to the thin water body, resulting in a total of 52 shot locations. The data set was processed to pick the first arrival traveltimes and extract the dispersion curves. In case of SRT, only the 46 inline shots were used. The manual picking of the first breaks was performed with the open-source library *formikoj* (M. Steiner & A. Flores Orozco 2023) and repeated once to obtain a rough estimate of the picking error of 0.7 ms, resulting in a total of 46 shot-receiver pairs. An estimated absolute error of 1.4 ms is used to construct the SRT data covariance matrix.

For the processing of the SWA data we adopt a windowing procedure after S. Pasquet & L. Bodet (2017), where a subset of the seismic data in time domain is selected based on a predefined window length and minimum/maximum offset requirements. The window size is thereby chosen as a trade-off between spectral and lateral resolution, while keeping the assumption of a horizontally layered subsurface valid. Minimum and maximum offsets (2 and 10 m, respectively) are chosen to avoid near- and far-field effects. The wave-field of the selected subset is transformed to the frequency-phase velocity spectrum (dispersion image) based on frequency domain beam forming (D.J. Zywicki 1999). The window is successively moved over all traces and the procedure is repeated for each shot location. Instead of stacking the dispersion images of each subset with a common receiver centre as performed by S. Pasquet & L. Bodet (2017), we extract each dispersion curve of each window. This allows us to combine dispersion curves associated with the same receiver centre to estimate the absolute error in the data, which ranges between 1 to 19 m s^{-1} , and is used to construct the SWA data covariance matrix. The combination is performed based on the binning procedure of E.A. Olafsdottir *et al.* (2018). Binned values are resampled to a common frequency range of 10 to 60 Hz.

In total, 429 dispersion curves were extracted, which resulted in 73 dispersion curves with error estimates at distinct surface locations.

The seismic inversion results are compared to a co-located electrical data set acquired on the same day to validate the results, due to the absence of direct information at the site. The electrical data was collected with a dipole-dipole normal-reciprocal configuration (D.J. LaBrecque *et al.* 1996) and the electrodes were distributed in a vertical electrical sounding layout with increasing electrode separation towards the end of the profile (see Fig. 9b).

3.2.2 Independent and joint inversion results

Fig. 10 shows the independent and joint inversion results obtained for the seismic data. Areas with no ray coverage are masked in the P -wave velocity images. Isolines of $v_p = 500$ and 1100 m s^{-1} and $v_s = 150$ and 220 m s^{-1} are superimposed on the velocity images. The independent SRT inversion was performed with an α of 75 and converged after 4 iterations to a χ^2 of 0.98 and RMS of 1.38 ms. The SWA inversion converged after 9 iterations to a χ^2 of 0.99 and RMS of 2.85 m s^{-1} . The regularization strength was thereby reduced from 150 to 15 by 20 per cent per iteration step, the ν -model was set to 0.4.

The independent v_p model shown in Fig. 10(a) reveals a horizontal layering, while more lateral variation is observed in the v_s model (see Fig. 10b). In the P -wave velocity model a 1.5 m thick top layer of low velocities is observed ($v_p < 500 \text{ m s}^{-1}$). Between $x = 48$ to 52 m, the values are slightly higher than 500 m s^{-1} in this depth range. Intermediate P -wave velocity values (1000 m s^{-1}) are found between 1.5 to 4 m depth. Below the 3 to 4 m depth, the P -wave velocity increases to values between 1500 and 1900 m s^{-1} . The S -wave velocity model reveals a surface layer in the northwestern part of the profile of low velocities ($v_s < 150 \text{ m s}^{-1}$) with a thickness of roughly 1.5 m. The southeastern part is characterized by top velocities v_s of 150 to 200 m s^{-1} to a depth of 3 m. This lateral change in S -wave velocity starts around $x = 45$ m. The increase in S -wave velocity with depth in the southeastern part of the profile is lower (from 200 to 250 m s^{-1}) in comparison with the northwestern part (from 110 to 280 m s^{-1}). The highest v_s ($> 280 \text{ m s}^{-1}$) are observed at $x = 50$ m towards the northwestern end of the profile. Low velocity zones with v_s values around 230 are identified in the S -wave velocity model beneath 4 m depth (at $x = 10$ to 20 m, $x = 30$ to 50 m and $x = 60$ to 70 m). These are not resolved with SRT and related to no ray coverage.

For the joint inversion α_{start} was set to 150 and decreased by 20 per cent. The α_{stop} values were set to 75 and 15 for SRT and SWA terms, respectively. The β value was set to 3×10^5 ,

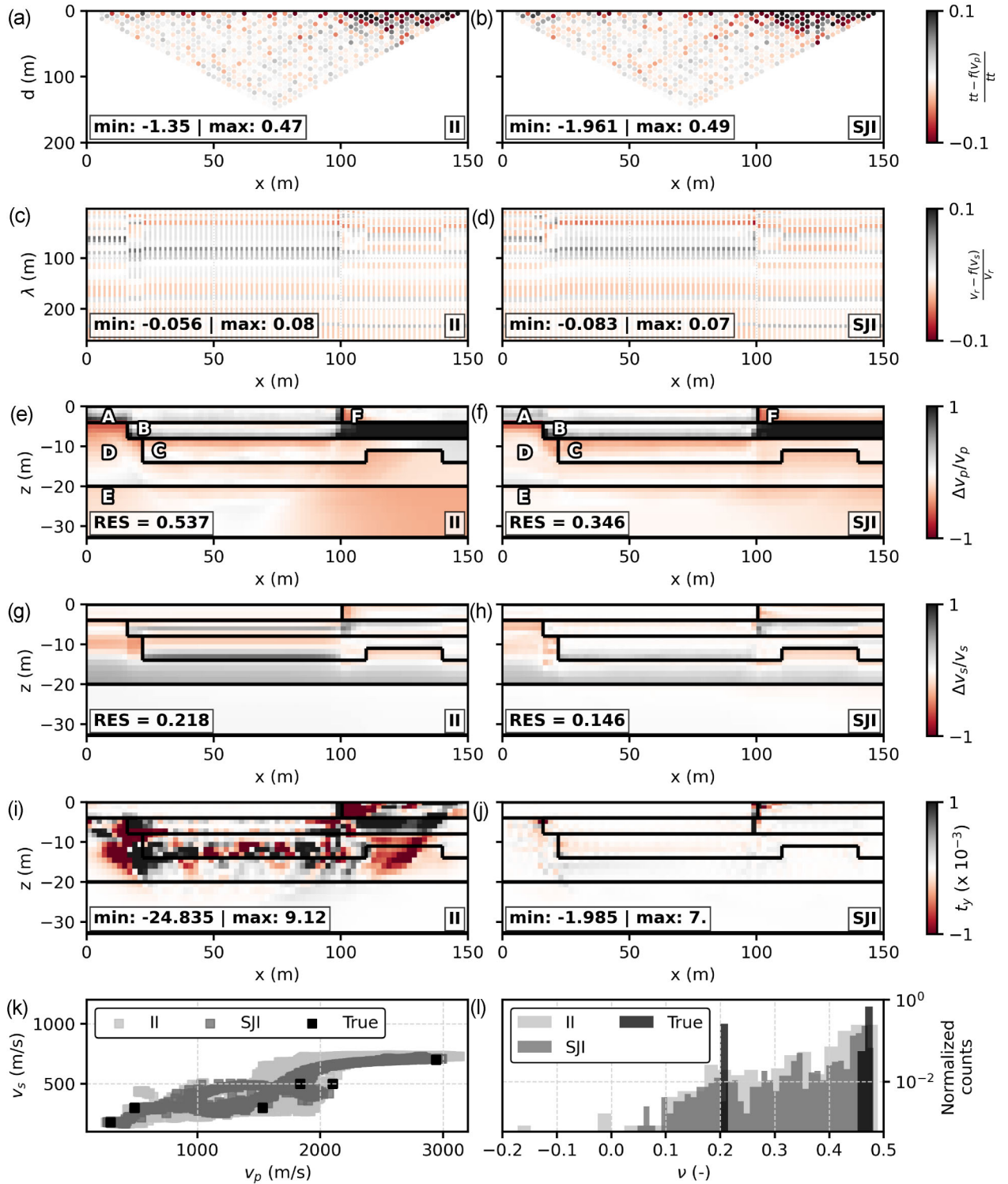


Figure 8. Assessment of results for synthetic model A. (a and b) Relative differences between synthetic traveltime data and final model response of the II and SJI. (c and d) Relative differences between synthetic dispersion curve data and final model response of the II and SJI. (e–h) Relative differences between true and inverse model for the II and SJI. (i–j) Cross-gradients for the II and SJI. In (a–f) the solid black lines indicate the true model structure. (k) Cross-plots of P -wave and S -wave velocity of the true model and II and SJI results. (l) Normalized histograms of the Poisson's ratio calculated for the true model and II and SJI results.

which resulted in an overall χ^2 of 1.12 ($\chi_1^2 = 1.07$, $\chi_2^2 = 1.15$, $\text{RMS}_1 = 1.45$ ms, $\text{RMS}_2 = 2.83$ m s $^{-1}$) after 20 iterations. The SJI results are shown in Figs 10(c) and (d) and reveal v_p and v_s

models consistent to those obtained through the II. Some lateral variations observed in the v_s model are propagated into the v_p model through the cross-gradient constraint, such as a more

Table 3. Relative differences between true and inverted models.

Model B	RES $v_{p,II}$	RES $v_{p,SJI}$	RES $v_{s,II}$	RES $v_{s,SJI}$
A	0.219	0.164	0.034	0.018
B	1.146	0.656	0.289	0.211
C	0.245	0.262	0.292	0.135
D	0.252	0.221	0.293	0.216
E	0.176	0.154	0.038	0.026
F	0.175	0.265	0.134	0.121
Overall	0.537	0.346	0.218	0.146

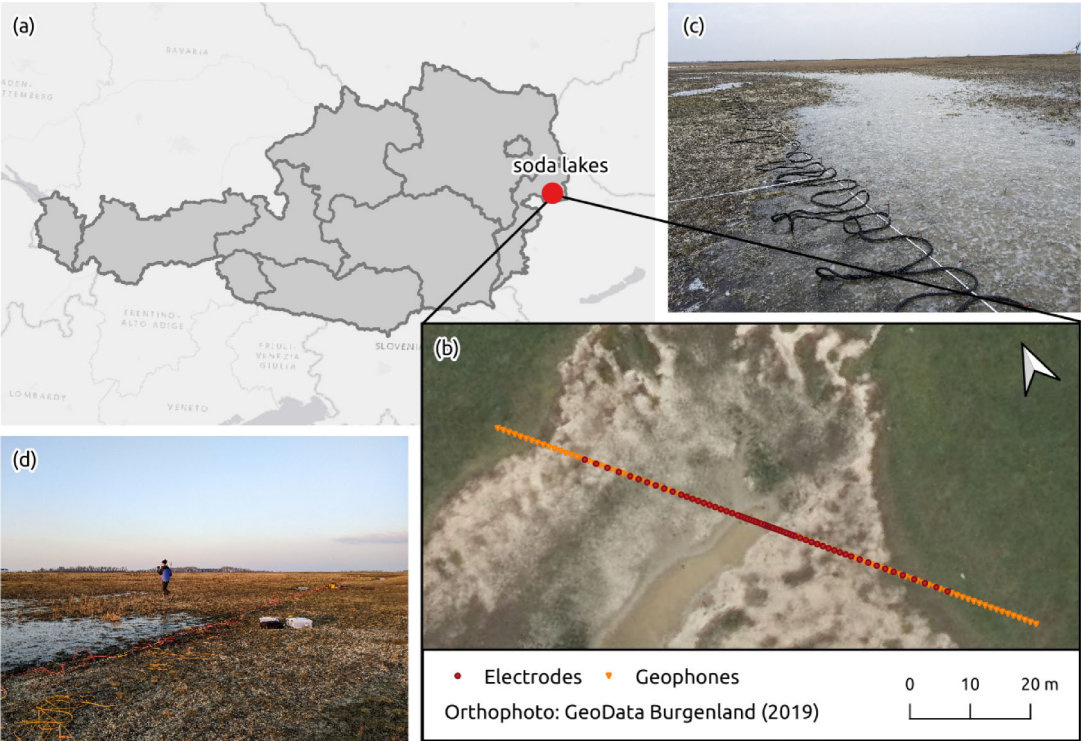


Figure 9. Overview of the study area. (a) Location of the soda lakes in Austria. (b) Position and extent of the seismic and electric measurement. (c) Picture of the field site and the electrical measurement layout. (d) Picture of the field site and the seismic measurement layout.

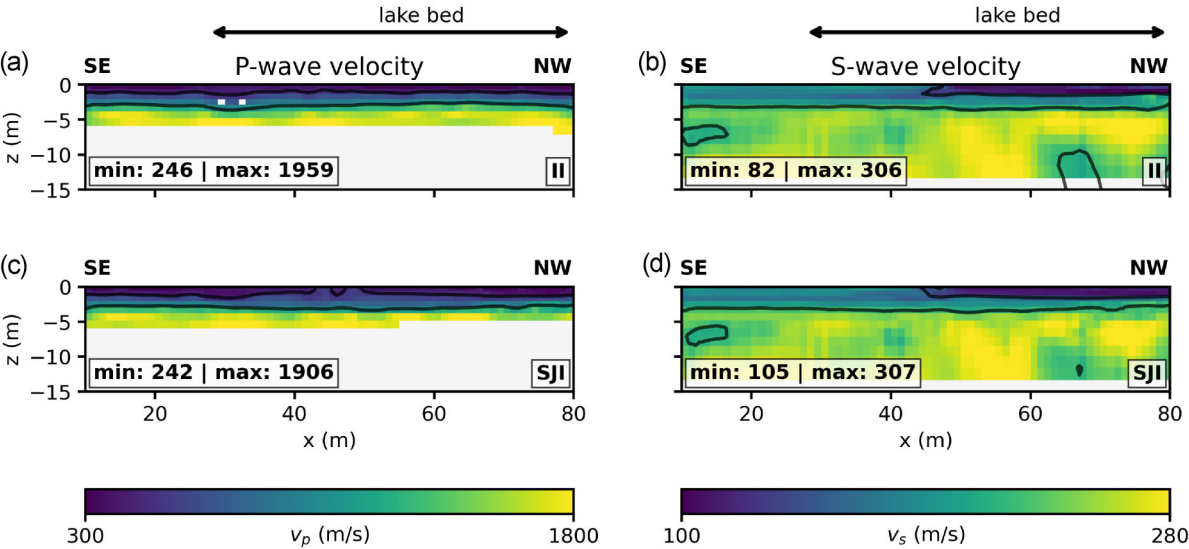


Figure 10. Results for profile P2. (a–b) Independent inversion results for SRT and SWA. (c–d) Joint inversion results. Isolines of $v_p = 500$ and 1100 m s^{-1} and $v_s = 150$ and 220 m s^{-1} are superimposed on the velocity images.

pronounced lateral discontinuity of the top P -wave velocity layer and the lateral velocity variation in the bottom layer.

To evaluate the results, we depict the relative differences between data and model response for the P -wave traveltimes and surface wave dispersion curve pseudosection in Figs 11(a)–(d). For both the II and SJI, the data are fitted within the respective error bounds, and the ranges of the data misfits are comparable. The relative differences between the final II and SJI results for v_p and v_s , shown in Figs 11(e)–(f), indicate the strongest discrepancies within the upper 5 m. Within this depth range, the cross-gradients (Figs 11g–h) exhibit a pronounced increase in structural similarity for the SJI, accompanied by a one-order-of-magnitude decrease in cross-gradient values. Below the 5 m depth, the models are primarily constrained by the SWA method alone, causing the cross-gradient to approach zero.

The scatter plot in Fig. 11(i) shows a reduction in scattering after applying the SJI, with emerging linear relationships between the v_p and v_s models obtained from the joint inversion. A negative association is observed for values corresponding to the shallow southeastern part of the model. The scatter increases at greater depths (characterized by $v_p > 1500 \text{ m s}^{-1}$, $v_s > 200 \text{ m s}^{-1}$), where the SRT method becomes insensitive to variations below approximately 5 m.

Finally, the histogram in Fig. 11(j) shows that the v values are predominantly distributed between 0.2 and 0.5, with a narrower distribution obtained with the SJI.

3.2.3 Comparison with complementary data and interpretation

Figs 12(a)–(c) compares the Poisson's ratio models obtained from the II and the SJI with the co-located electrical conductivity section to enable a joint interpretation. The v -model derived from the II exhibits more abrupt variations, whereas the SJI result displays smoother transitions due to the alignment of model boundaries achieved through structural coupling. The boundaries identified in the SJI model correspond more closely to the features observed in the electrical conductivity image. Consequently, the subsequent interpretation is based on the conductivity section and the SJI result. Overall consistent features can be observed in both models. The lake shore extends to roughly $x = 25 \text{ m}$ and is characterized by salt and clay-rich, unsaturated, slightly vegetated soil corresponding to the high conductivity values ($\sim 100 \text{ mS m}^{-1}$) and Poisson's ratio values in the range of $0.35 < \nu < 0.4$ to a depth of roughly 1.5 m. In the σ image this layer extends to $x = 40 \text{ m}$. In the ν image the values decrease below 0.35 towards $x = 40 \text{ m}$, characterizing low saturation, but fine grained material (i.e. the dry lake area). The centre of the profile (roughly $x = 40$ to 60 m) lies in the area of the lake that is predominantly flooded throughout most of the year. In the centre of the electrical profile ($x = 40$ to 50 m), a 0.5 m thick low conductivity zone is observed at the top due to the thin ice sheet. Beneath, the electrical conductivity increases to a depth of 1.5 to 2 m, as expected due to the high portion of fine grains (clays) of the confining layer. This lateral discontinuity at the shallow subsurface was already identified in the seismic images (isolines of $v_p = 500 \text{ m s}^{-1}$ are superimposed on the images in Figs 12(a) and (b) and is also reflected by the ν model due by the increase in values ($\nu > 0.45$). From $x = 45 \text{ m}$ onward to the north-west the shallow subsurface of the soda lake to a depth of roughly 2.5 m is characterized by ν values between 0.4 to 0.45 and high conductivity values ($> 100 \text{ mS m}^{-1}$) of the confining layer. Between

a depth of 1.5 to roughly 4 m ν is around 0.45 and the σ image shows low conductivity anomalies ($< 20 \text{ mS m}^{-1}$) due to the variation in textural properties (increase in coarse-grained material). Below 3 to 4 m ν is highest with values approaching 0.5, which is due to the increase in P -wave velocity, but minor change in S -wave velocity. The σ increases again below 3 to 4 m to values of around 30–50 mS m^{-1} , marking the depth to the aquifer at roughly 3 to 4 m.

4 DISCUSSION

We presented a framework to jointly invert SRT and SWA data based on structural coupling via cross-gradient constraints. The performance of the SJI approach was evaluated based on two synthetic and one field data experiment.

As demonstrated by both synthetic models, the performance of the SJI was superior to that of the independent inversions regarding the reconstruction of the true model structures and values, and identification of zonation patterns. Synthetic model A revealed a higher contribution of S -wave velocity model to the P -wave velocity model, as highlighted by the larger difference in residuals between II and SJI results (see Tables 2). The SWA method was able to reconstruct the sharp box-like features independently and enforced their development in the P -wave velocity model through the cross-gradient constraint where SRT was sensitive to these features. Improving the structural similarity between the model parameters has a significant impact on their distribution and thus the localization of subsurface units. Based on the scatter plot shown in Fig. 5(i) no clear relationship between v_p and v_s could be identified based on the independent results. Following the SJI, different slopes could be observed, where each trend suggests to belong to a different zone.

The impact of the coupling strategy was less pronounced in the field data, primarily due to the pronounced heterogeneity of the subsurface and no sensitivity of the SRT method below approximately 5 m depth. Nevertheless, the SJI results exhibited enhanced structural coherence and facilitated a more consistent interpretation, as a clearer relationship between the two model parameters could be established (see Fig. 11i). The *a posteriori* estimation of Poisson's ratio was also improved following the SJI, with no negative values observed above 5 m depth (see Fig. 12c). Moreover, the Poisson's ratio model derived from the SJI displayed increased structural similarity with an independently obtained electrical conductivity model, further supporting the consistency of the joint inversion results applied to a field data set (see Fig. 12).

The deterministic inversion approach is sensitive to the choice of the (initial) model parametrization. Iteratively updating the P -wave velocity model provided useful information for the forward computation of the SWA data, by correcting the initial homogeneous estimate of the Poisson's ratio. This leads to an improved estimate of the S -wave velocity as observed in both synthetic examples, without requiring *a priori* information to constrain the model parameters in our examples. Improving the estimation of the model values in conjunction with the enhanced structural similarity can further have significant impacts on the reliability of derived properties (e.g. the Poisson's ratio, as shown in Figs 5j and 8j) and hence on the quantitative interpretation of the results. Although no constraint on the Poisson's ratio was applied, especially in case of model A, the SJI clearly improved the overall distribution of the *a posteriori* computed Poisson's ratio.

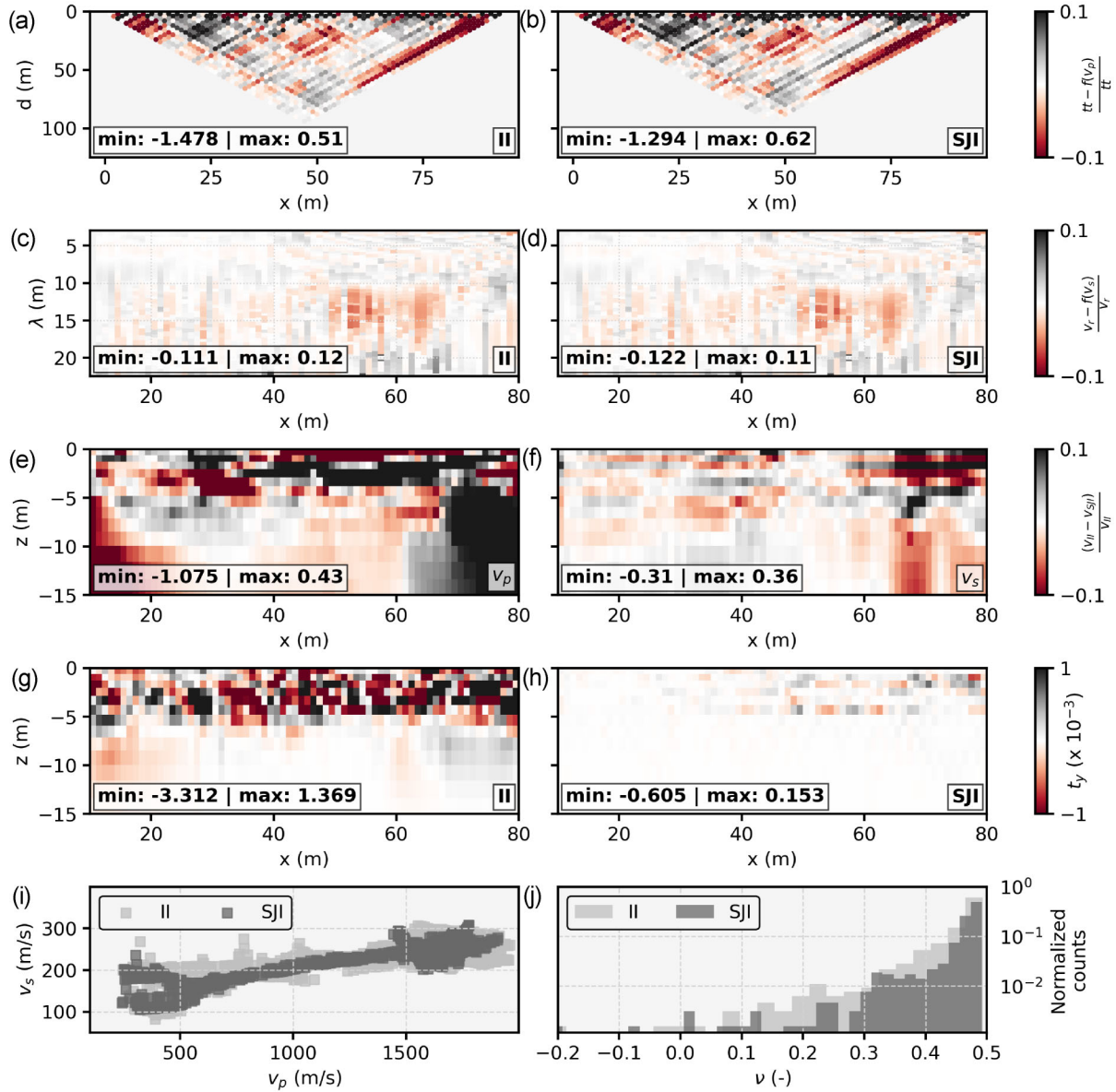


Figure 11. Continuation of results for the field data. (a–b) Relative differences between measured traveltime data and final model response of the II and SJI. (c–d) Relative differences between measured apparent fundamental mode dispersion curve data and final model response of the II and SJI. (e–f) Relative differences between independent and joint inversion results computed for the v_p and v_s models. (g–h) Cross-gradients of the independent and joint inversion. (i) Cross-plots of P -wave and S -wave velocity. (j) Normalized histograms of the Poisson's ratio calculated for independent and joint inversion results.

If the model features are not jointly constrained by both methods, the coupling will not enforce structural similarity as the cross-gradient becomes zero at the location of the feature. This gives the coupling strategy a certain flexibility, as the sensitive method can continue developing the model on its own. However, the final joint inversion outcome may not always justify the actual petrophysical or mechanical properties of the subsurface. For instance in case of hidden layers (velocity inversion or thin layers at depth), SWA is able to resolve the true model structure, but SRT not, as demonstrated with synthetic model B. The cross-gradient constraint then only poses a weak constraint on the model parameters and the obtained model values and derived parameters may still be affected by high uncertainties (see

Figs 8e and f). Alternative constraints based on prior information or region-specific coupling through appropriate petrophysical or mechanical relationships may be required to improve the reliability of the results in areas where only one method exhibits sensitivity, and to prevent the occurrence of unrealistic P - and S -wave velocity pairs. However, such approaches rely on the availability of reliable *a priori* information (e.g. Poisson's ratio estimated from expected or known soil types; e.g. C. Piatti *et al.* 2013; D. Boiero & L.V. Socco 2014; F. Garofalo *et al.* 2022). This information is not always available at the field scale, as is the case with the presented field data set, and may inadequately represent the true subsurface conditions in cases of pronounced lateral heterogeneity.

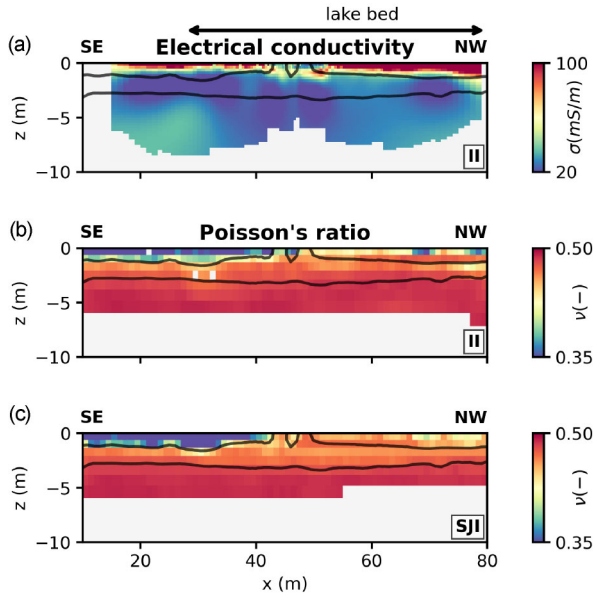


Figure 12. Complementary field data and derived properties. (a) Electrical conductivity obtained from an independent ERT inversion. (b) Poisson's ratio calculated from the independent inversion results. (c) Poisson's ratio calculated from the joint inversion results. Isolines of $v_p = 500$ and 1100 m s^{-1} are superimposed on the images. Areas with low sensitivity are blanked in (a). Areas with no ray coverage and negative Poisson's ratio are blanked in (b) and (c).

The goodness-of-fit of the final inversion result of the SJI is controlled by the inversion parameters. These are the weighting of the contributions of the individual methods and the weighting of the individual regularization terms. The regularization weights are systematically determined. The smoothness regularization strengths, α_1 and α_2 , are empirically determined during the independent inversions. For the SRT inversion, a fixed value between 50 and 150 resulted in a good data fit (see Fig. 13a). For the SWA inversion, a cooling strategy starting with $\alpha = 150$ provided better convergence, as initial tests with constant α -values produced a poor data fit (see Fig. 13b). This approach allows large-scale structures to be resolved first, followed by smaller-scale variations as α is progressively decreased. The same cooling strategy was applied during SJI, using the regularization strengths from II as target values to ensure comparable smoothness between the II and SJI results. In the SJI scheme proposed here, the cross-gradient constraint is applied with equal and constant weight to the whole joint model domain to ensure a strong and steady coupling. A good trade-off between data misfit and minimization of the cross-gradient is observed for values ranging between 10^5 and 10^6 . Within this interval, minimal variations of the results are observed. For decreasing β -values, the structural constraint becomes too weak to enforce coherent model gradients. Conversely, as β increases, the structural similarity between the models improves; however, beyond a certain threshold, the data fit begins to deteriorate (see Fig. 13c). The resulting increase in data misfit at higher β -values also leads to a stronger deviation from the true model (see Fig. 13d). For the data weighting, we applied the data fit based weighting method proposed by C. Jordi *et al.* (2020), due to the different convergence speeds observed in the independent inversion runs (see Figs 5a and 8a). Based on this method the evolution of the convergence parameters is balanced, as a higher weight is assigned

to the method with a poorer data fit (see Figs 5b and 8b). In Fig. 13(e), we compare the convergence curves obtained with and without data weighting. Since the initial SWA $\chi^2 (= 244)$ is much higher compared to the SRT $\chi^2 (= 33)$, a lower weight is assigned to the SRT term until both methods reach a similar misfit. In Fig. 13(e), the evolution of the cross-gradient is compared for the weighted and unweighted case, showing a slightly faster decrease in Φ_{cg} using additional data weighting.

In most applications, the cross-gradient constraint joint inversion is performed with homogeneous starting models to start from zero structure (e.g. L.A. Gallardo & M.A. Meju 2003; L.A. Gallardo 2004; L.A. Gallardo & M.A. Meju 2007). Due to the applied forward solution for SRT, which computes the shortest ray paths, a heterogeneous starting model was required for good convergence. It should also be mentioned that the proposed framework is currently limited to the inversion of the Rayleigh-wave fundamental mode, as the fundamental mode was the dominating mode in the field data experiment in the investigated range of 10 to 60 Hz.

5 CONCLUSION AND OUTLOOK

We presented the framework of a structural joint inversion algorithm to couple 2-D SRT and pseudo-2-D SWA data based on cross-gradient constraints. The cross-gradients are computed on regular meshes based on four difference operators in contrast to the typically applied forward difference computation. The pseudo-2-D inversion of the SWA data is achieved through lateral constraints. The joint inversion is tested based on two synthetic models and field measurements conducted in a shallow clay-rich aquifer and compared to independent inversion results using the same initial model parametrization. The synthetic experiments demonstrate the superior performance of the joint inversion regarding the discrimination of subsurface features and reconstruction of the true model values. The effects of the cross-gradient constraint on the field data are less pronounced, but the resolved models correspond well to the local geology and a complementary electrical data set. Overall, the synthetic and field data experiments show that the SJI based on cross-gradient constraints is applicable to 2-D SRT and pseudo-2-D SWA data, if the assumption of structural similarity is validated and the methods can justify the underlying petrophysics and mechanics of the subsurface. The examples show that the SJI improves the consistency of the resolved model parameters and solves for model values that are closer to the expectation, while maintaining a similar data fit.

The implemented SJI scheme is flexible and can be adapted to incorporate alternative constraints, for example, through the inclusion of prior information. In cases where the assumed subsurface petrophysical relationships are strongly violated, a petrophysically based approach may be more appropriate. Such an approach could establish a petrophysical link either via Poisson's ratio constrained by an *a priori* model (e.g. C. Piatti *et al.* 2013; D. Boiero & L.V. Socco 2014) or through porosity derived from rock physics modelling (e.g. F. Garofalo *et al.* 2022), provided that the underlying petrophysical relationships are valid for the investigated subsurface conditions.

Alternatively to the current strategy, the *P*-wave velocity could be treated as a variable in the SWA forward solution and adjusted according to its sensitivity to the dispersion curve, as demonstrated by C. Piatti *et al.* (2013). Moreover, local weighting of the

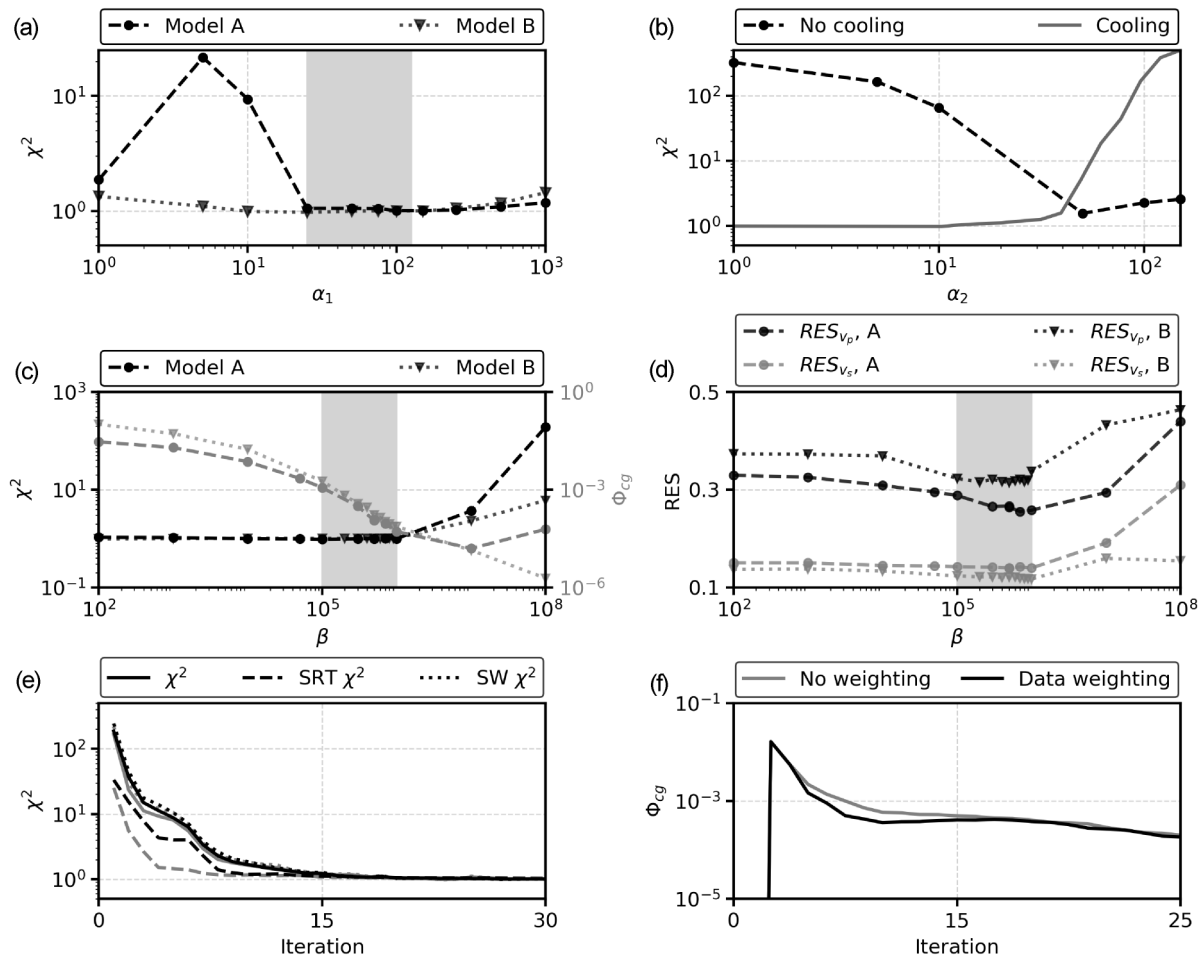


Figure 13. Empirical estimation of the regularization weights demonstrated for the synthetic models. (a) Data misfit for varying α values tested in the independent SRT inversion. (b) Data misfit for varying α values tested in the independent SWA inversion. The black curve represents the final data misfit obtained with fixed α values, while the grey curve illustrates the convergence achieved using the cooling strategy. (c) Data misfit for varying β values tested in the joint inversion. (d) Overall v_p and v_s residuals for varying β values in the joint inversion. The vertical black line marks the selected value, and the grey rectangle highlights the range of regularization strengths for which the final model variations and data misfit differences were minimal. (e) Convergence curve obtained with data weighting (black lines) and without (grey lines). (f) Evolution of the cross-gradient term with and without data weighting.

cross-gradient term based on the ratio of the model sensitivity distributions (J.B. Rittgers *et al.* 2016; L. Gross 2019) could further enhance the inversion results by explicitly accounting for the individual sensitivities during the model update.

Future research will focus on extending the framework to include other geophysical methods or regional constraints, for example, to incorporate prior information where available. Additionally, we will explore whether further improvements can be achieved by (1) sensitivity based weighting of the cross-gradient constraint, (2) improved strategies to set the inversion parameters, (3) the simultaneous estimation of the P -wave velocity within the SJI framework, that is, allowing it to vary based on the sensitivity to the dispersion curves, as well as the implementation of more accurate forward operators, (4) extension to multimodal inversion.

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DATA AVAILABILITY

The code and synthetic data supporting the findings of this study are openly available via the institutional repository of TU Wien at DOI: 10.48436/2zfz8-kb782. The code is distributed under the MIT License, and the synthetic data sets are available under a Creative Commons Attribution 4.0 International (CC BY 4.0) License. The synthetic data sets can be reproduced using the provided code. The field data is available on request from the corresponding author, N. Roser.

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APPENDIX A: SYNTHETIC DATA

The simulated data for synthetic models A and B in terms of traveltimes and dispersion curves is displayed in Fig. A1.

APPENDIX B: FIELD DATA

The raw seismic field data for two inline shots are shown in Figs B1(a) and (c). The dispersion images, computed from a subset of each field record using a window length of 24 channels, are shown in Figs B1(b) and (d). Fig. B2(a) shows the picked traveltimes for the whole field data set. Fig. B2(b) shows the picked apparent fundamental mode dispersion curves as pseudo-section.

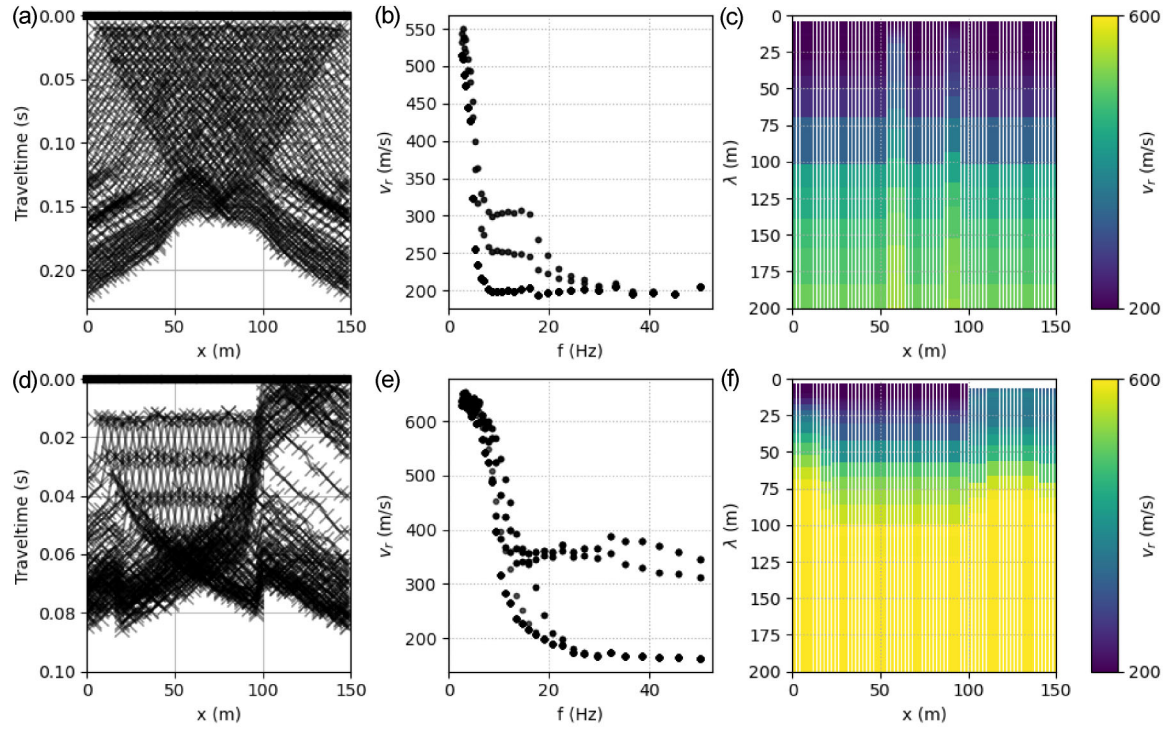


Figure A1. Synthetic data. (a) and (d) Simulated traveltimes for model A and B, respectively. (b) and (e) Simulated fundamental mode dispersion curve data for model A and B, respectively. (c) and (f) Simulated dispersion curves shown as pseudo-section for model A and B, respectively.

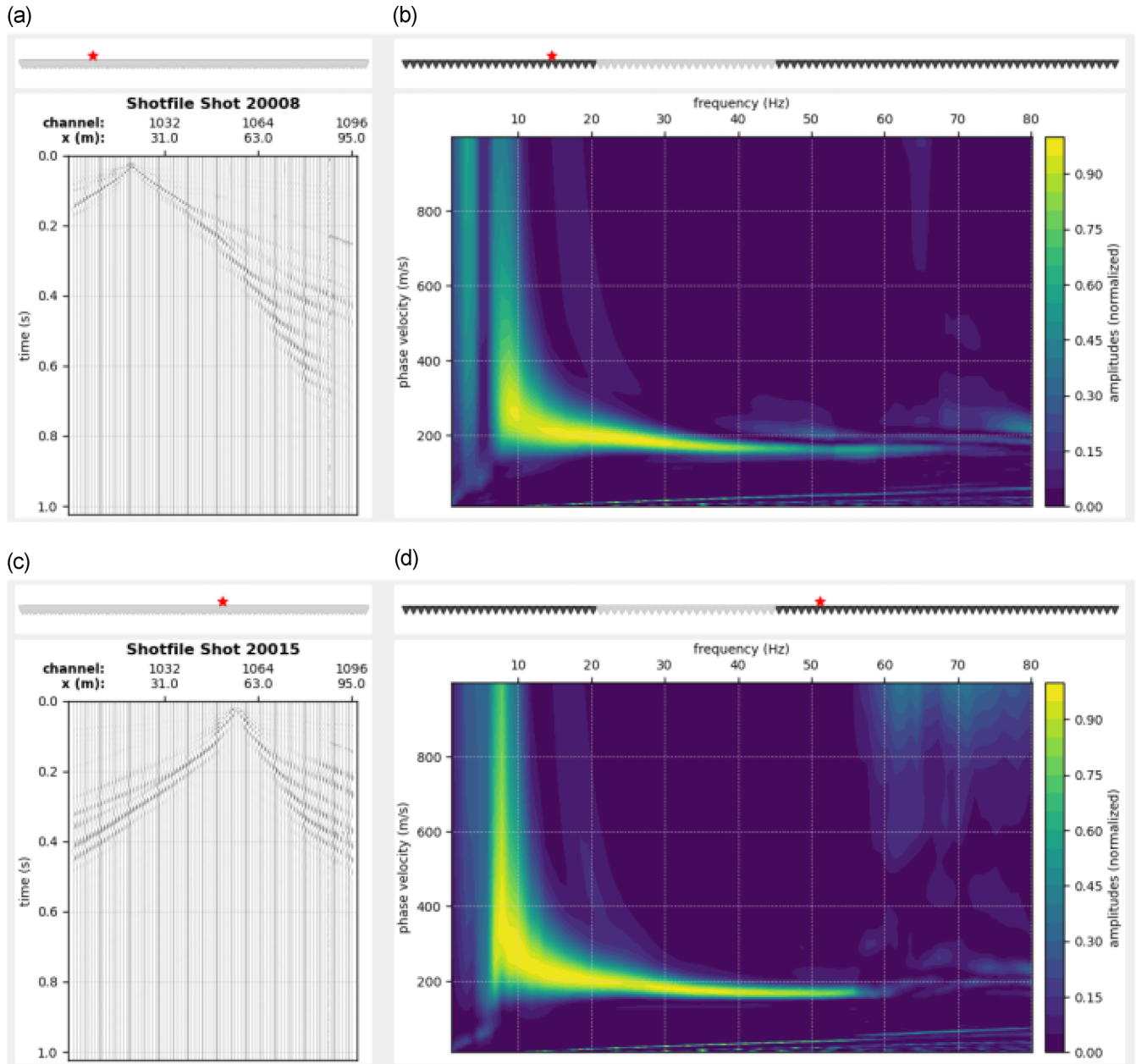


Figure B1. (a) and (c) Raw seismic field data. (b) and (d) Dispersion images computed from a subset of the records shown in (a) and (c), respectively. Above the seismograms and dispersion images, the corresponding survey design is displayed—that is, the geophone array and shot location associated with the displayed data. Triangles represent geophones, with grey triangles indicating the selected channels used for dispersion image computation. The star denotes the shot location.

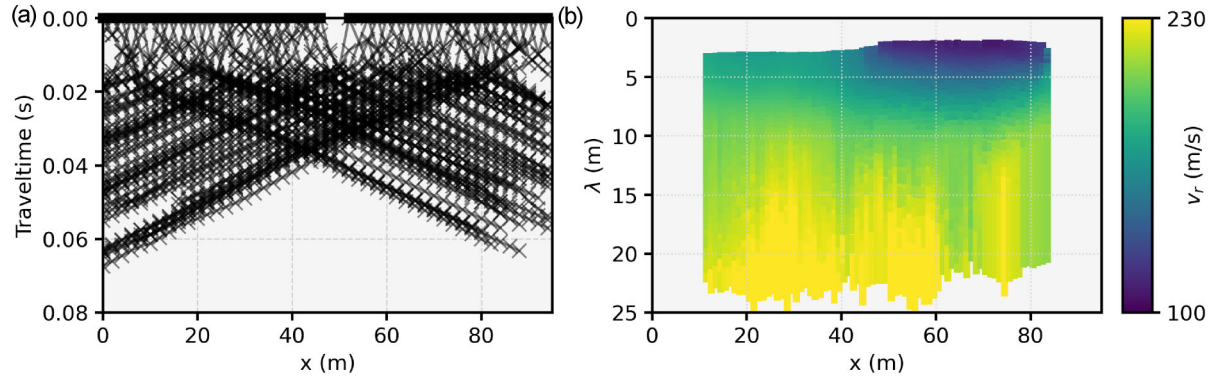


Figure B2. Field data. (a) Picked traveltimes. (b) Picked dispersion curves displayed as pseudo-section.