

Stochastic Modelling of Galactic Cosmic Rays: Spectral Features and Diffuse Emission

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DOCTORAL THESIS

Stochastic Modelling of Galactic Cosmic Rays: Spectral Features and Diffuse Emission

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Institute for Theoretical Particle Physics and Cosmology

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March 23, 2026

Eidesstattliche Erklärung

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I, Anton Stall

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7. Parts of this work have been published before as stated in the List of Publications on page VI.

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List of Publications

This thesis is based on the research I carried out during my doctoral studies at the Institute for Theoretical Particle Physics and Cosmology at RWTH Aachen University. This research led to several scientific publications, listed below.

Journal articles:

1. **Anton Stall**, Chun Khai Loo, and Philipp Mertsch (2025). *Investigating the CREDIT History of Supernova Remnants as Cosmic-Ray Sources*, The Astrophysical Journal Letters **980**, L21. DOI: [10.3847/2041-8213/adaea8](https://doi.org/10.3847/2041-8213/adaea8). In arXiv: [2409.11012](https://arxiv.org/abs/2409.11012) [[astro-ph.HE](#)].
2. **Anton Stall** and Philipp Mertsch (2026). *Stochastic modelling of cosmic-ray sources for Galactic diffuse emissions*, Journal of Cosmology and Astroparticle Physics **03**, 026. DOI: [10.1088/1475-7516/2026/03/026](https://doi.org/10.1088/1475-7516/2026/03/026). In arXiv: [2509.06857](https://arxiv.org/abs/2509.06857) [[astro-ph.HE](#)].

Conference proceedings:

1. **Anton Stall**, Leonard Kaiser, and Philipp Mertsch (2023). *Stochastic modelling of cosmic ray sources for diffuse high-energy gamma-rays and neutrinos*, In: Proceedings of 38th International Cosmic Ray Conference — PoS(ICRC2023). Vol. 444, p. 687. DOI: [10.22323/1.444.0687](https://doi.org/10.22323/1.444.0687). arXiv: [2309.02860](https://arxiv.org/abs/2309.02860) [[astro-ph.HE](#)].
2. **Anton Stall** (2024). *Stochastic modelling of cosmic ray sources for diffuse high-energy gamma-rays and neutrinos*, In: Proceedings of the 58th Rencontres de Moriond on VHEPU 2024. Ed. by J. Dumarchez, E. Augé and T. V. Tran. Rencontres de Moriond. La Thuile, Italy: ARISF, pp. 175–182. ISBN: 979-10-96879-29-8. DOI: [10.58027/dt1e-e224](https://doi.org/10.58027/dt1e-e224).
3. **Anton Stall**, Chun Khai Loo, and Philipp Mertsch (2025). *Investigating the CREDIT history of supernova remnants as cosmic-ray sources*, In: Proceedings of 39th International Cosmic Ray Conference — PoS(ICRC2025). Vol. 501, p. 135. DOI: [10.22323/1.501.0135](https://doi.org/10.22323/1.501.0135). arXiv: [2507.07088](https://arxiv.org/abs/2507.07088) [[astro-ph.HE](#)].
4. **Anton Stall** and Philipp Mertsch (2025). *Diffuse emission from stochastic sources*, In: Proceedings of 39th International Cosmic Ray Conference — PoS(ICRC2025). Vol. 501, p. 848. DOI: [10.22323/1.501.0848](https://doi.org/10.22323/1.501.0848). arXiv: [2509.14776](https://arxiv.org/abs/2509.14776) [[astro-ph.HE](#)].

Abstract

Doctoral Thesis

Stochastic Modelling of Galactic Cosmic Rays: Spectral Features and Diffuse Emission

by Anton Stall

Cosmic rays are ubiquitous in our Galaxy and bombard the Earth continuously from all directions. They also interact with the interstellar medium throughout the Galaxy, producing so-called diffuse emission in γ -rays and neutrinos. Direct observations by space-borne high-precision experiments such as AMS-02, DAMPE, and CALET provide measurements of the local cosmic-ray spectrum, while ground-based γ -ray and neutrino observatories such as LHAASO and IceCube are beginning to probe diffuse emission at energies beyond 100 TeV, extending the energy range of previous experiments such as *Fermi*-LAT.

Cosmic rays are commonly assumed to be accelerated at the shock fronts of supernova remnants. However, observations indicate that Galactic cosmic-ray sources must accelerate particles efficiently up to the so-called cosmic-ray knee at energies around 3 PeV. Achieving such energies is theoretically challenging, and realistic source models must address this issue. Magnetic field amplification through streaming instabilities provides a possible solution, but it also influences the time at which cosmic rays escape into the interstellar medium. In this thesis, we introduce a model that incorporates energy-dependent cosmic-ray injection times.

Models of Galactic cosmic rays are inherently limited by our incomplete knowledge of the spatial distribution and ages of their sources. To address this, we adopt a stochastic approach to model the population of Galactic cosmic-ray sources, treating their positions and ages as random variables. Using extensive Monte Carlo simulations, we characterise the impact of individual sources, quantify the uncertainties arising from the unknown source population, and investigate whether observations of the local cosmic-ray spectrum and diffuse emission can constrain source properties. Our results demonstrate a close connection between the statistical properties of cosmic-ray intensities and those of the resulting diffuse emission.

Finally, we show that the proposed energy-dependent injection model leaves distinct features in the local cosmic-ray spectrum. Using machine-learning techniques, we explore the potential to distinguish between different injection scenarios based on direct cosmic-ray measurements. We highlight the prospects of this approach for current and future experimental analyses.

Zusammenfassung

Doktorarbeit

Stochastische Modellierung galaktischer kosmischer Strahlung: Spektrale Merkmale und diffuse Emission

von Anton Stall

Kosmische Strahlung ist in unserer Galaxie allgegenwärtig und bombardiert die Erde kontinuierlich aus allen Richtungen. Sie interagiert auch mit dem interstellaren Medium in der gesamten Galaxie und erzeugt dabei sogenannte diffuse Emissionen in Form von γ -Strahlen und Neutrinos. Direkte Beobachtungen durch hochpräzise Experimente im Weltraum wie AMS-02, DAMPE und CALET liefern Messungen des lokalen Spektrums kosmischer Strahlung, während bodengestützte γ -Strahlen- und Neutrino-Observatorien wie LHAASO und IceCube beginnen, diffuse Emissionen bei Energien über 100 TeV zu untersuchen und damit den Energiebereich früherer Experimente wie *Fermi*-LAT zu erweitern.

Es wird allgemein angenommen, dass kosmische Strahlung an den Schockfronten von Supernovaüberresten beschleunigt wird. Beobachtungen deuten jedoch darauf hin, dass galaktische Quellen kosmischer Strahlung Teilchen effizient bis zum sogenannten *Knie* bei Energien um 3 PeV beschleunigen müssen. Das Erreichen solcher Energien ist eine theoretische Herausforderung und realistische Quellmodelle müssen sich mit diesem Problem befassen. Die Verstärkung des Magnetfelds durch Strömungsinstabilitäten bietet eine mögliche Lösung, beeinflusst jedoch auch den Zeitpunkt, zu dem kosmische Strahlung in das interstellare Medium entweicht. In dieser Arbeit stellen wir ein Modell vor, das energieabhängige Injektionszeiten kosmischer Strahlung berücksichtigt.

Modelle galaktischer kosmischer Strahlung sind naturgemäß durch unser unvollständiges Wissen über die räumliche Verteilung und das Alter ihrer Quellen begrenzt. Um diesem Problem zu begegnen, verwenden wir einen stochastischen Ansatz zur Modellierung der Population galaktischer Quellen kosmischer Strahlung, wobei wir ihre Positionen und Alter als Zufallsvariablen behandeln. Mithilfe umfangreicher Monte-Carlo-Simulationen charakterisieren wir den Einfluss einzelner Quellen, quantifizieren die Unsicherheiten, die sich aus der unbekannten Quellpopulation ergeben, und untersuchen, ob Beobachtungen des lokalen Spektrums kosmischer Strahlung und der diffusen Emission die Eigenschaften der Quellen einschränken können. Unsere Ergebnisse zeigen einen engen Zusammenhang zwischen den statistischen Eigenschaften der Intensität kosmischer Strahlung und denen der resultierenden diffusen Emission.

Schließlich zeigen wir, dass das vorgeschlagene energieabhängige Injektionsmodell deutliche Merkmale im lokalen Strahlungsspektrum hinterlässt. Mithilfe maschinellen Lernens untersuchen wir die Möglichkeit, anhand direkter Messungen der kosmischen Strahlung verschiedene Injektionsszenarien zu unterscheiden. Wir heben die Aussichten dieses Ansatzes für aktuelle und zukünftige experimentelle Analysen hervor.

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List of Abbreviations

CMB	Cosmic Microwave Background
CRDB	Cosmic Ray Database
TOA	Top Of Atmosphere
LIS	Local Interstellar Spectrum
DM	Dark Matter
IC	Inverse Compton
ISM	Interstellar Medium
SNR	Supernova Remnant
AMS-02	Alpha Magnetic Spectrometer
DAMPE	Dark Matter Particle Explorer
LHAASO	Large High Altitude Air Shower Observatory
<i>Fermi</i>-LAT	<i>Fermi</i> Large Area Telescope
GZK	Greisen-Zatsepin-Kuzmin
ISS	International Space Station
UHE	Ultra-High Energy
EAS	Extensive Air Showers
LHC	Large Hadron Collider
QCD	Quantum Chromodynamics
IACT	Imaging Atmospheric Cherenkov Telescope
CTA	Cherenkov Telescope Array
PWN	Pulsar Wind Nebula
AGN	Active Galactic Nucleus
GRB	Gamma-Ray Burst
SWGO	Southern Wide-field Gamma-ray Observatory
CGRO	Compton Gamma Ray Observatory
PMT	Photomultiplier Tube
DOM	Digital Optical Module
DSA	Diffusive Shock Acceleration
QLT	Quasilinear Theory
MHD	Magnetohydrodynamics
ISRF	Interstellar Radiation Field
NRH	Non-Resonant Hybrid
CREDIT	Cosmic-Ray Energy-Dependent Injection Time

ST	Sedov-Taylor
GDE	Galactic Diffuse Emission
SN	Supernova
IAU	International Astronomical Union
PDF	Probability Density Function
CDF	Cumulative Distribution Function
CLT	Central Limit Theorem
LOS	Line Of Sight
CRINGE	Cosmic Ray-fitted Intensities of Galactic Emission
GPU	Graphics Processing Unit
ML	Machine Learning
DT	Decision Tree

Nomenclature

Constants and conversion factors

$\text{erg} \rightarrow \text{GeV}$	Conversion factor	$1 \text{ erg} = 10^{-7} \text{ J} \approx 624 \text{ GeV}$
$\text{pc} \rightarrow \text{cm}$	Conversion factor	$1 \text{ pc} \approx 3.086 \cdot 10^{18} \text{ cm}$
$\text{V} \rightarrow \text{G cm}$	Conversion factor (SI to Gaussian)	$1 \text{ V} \hat{=} \frac{1}{300} \text{ G cm}$
$\text{yr} \rightarrow \text{s}$	Conversion factor	$1 \text{ yr} \approx 3.15 \cdot 10^7 \text{ s}$
m_p	Proton mass	$938.272 \text{ MeV}/c^2 = 1.672 \, 62 \cdot 10^{-24} \text{ g}$
c	Speed of light in vacuum	$299 \, 792 \, 458 \text{ m s}^{-1}$

General quantities

β	Velocity in units of c	$\beta = \frac{v}{c}$
$\mathcal{R}$	Rigidity	$\mathcal{R} = \frac{pc}{Ze}; [\text{V}]$
$\mathcal{R}_{\text{knee}}$	Cosmic-ray knee rigidity	$\approx 3 \cdot 10^{15} \text{ V} = 3 \text{ PV}$
κ	Diffusion coefficient	$[\text{cm}^2 \text{ s}^{-1}]$
δ	Diffusion spectral index	
$f(t, \mathbf{x}, \mathbf{p})$	Cosmic-ray phase space density	
ψ_X	Isotropic differential cosmic-ray density	e.g., $[\text{cm}^{-3} \text{ V}^{-1}]$
Φ_X	Isotropic differential cosmic-ray intensity	e.g., $[\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ V}^{-1}]$
J	Differential diffuse emission intensity	$[\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ eV}^{-1}]$

Galaxy parameters

R	Galactocentric radius	$[\text{pc}]$
$R_{\odot}$	Solar Galactocentric radius	$\approx 8 \text{ kpc}$
H	Half-height of the diffusion halo	$\approx 5 \text{ kpc}$
R_{Gal}	Radius of the Galaxy	$\approx 15 \text{ kpc}$

Source parameters

f_{SN}	Galactic supernova rate	$\approx 0.03 \text{ yr}^{-1}$
$\rho(t, \mathbf{x})$	Cosmic-ray source density	$[\text{kpc}^{-3} \text{ yr}^{-1}]$
$Q_{\mathcal{R}}(\mathcal{R})$	Universal cosmic-ray source spectrum	$[\text{V}^{-1}]$
α	Source spectral index	≈ 2

CREDIT parameters

t_{ST}	Sedov time (onset of Sedov-Taylor phase)	$\approx 1 \text{ kyr}$
t_{life}	Life-time of the cosmic-ray source	$\approx 100 \text{ kyr}$
$\mathcal{R}_{\text{b}}$	Break rigidity	$\approx 10 \text{ TV}$

Other symbols

$\mathcal{O}$	Notation denoting the order of magnitude	e.g. $\mathcal{O}(10)$ sources
Θ	Heaviside step function	$\Theta(x) = 1$ if $x \geq 0$, else 0
$\delta(x)$	Dirac delta distribution	$\int_{-\infty}^{\infty} dx \delta(x - x_0) f(x) = f(x_0)$

1

Introduction

Cosmic rays are nonthermal, charged, high-energy particles that are ubiquitous in our Galaxy and throughout the Universe. They are observed with energies spanning more than twelve orders of magnitude, from a few MeV up to more than 10^{20} eV. Cosmic rays up to the so-called cosmic-ray knee at about 3 PeV are believed to be mainly of Galactic origin, while cosmic rays with energies above 10^{18} eV are thought to originate from extragalactic sources, although the transition between these two populations is still not well understood (Navas et al., 2024). While cosmic rays with energies below 1 GeV can influence star formation processes by ionising and heating dense molecular clouds (Padovani et al., 2020), cosmic rays with higher energies trace extreme astrophysical acceleration processes in our Galaxy and beyond (Gaisser et al., 2016).

Since their discovery by Victor Hess, 1912, cosmic rays have been actively studied both experimentally and theoretically. The manifold questions raised by their study stimulated research in various fields like observational astrophysics, particle physics, and plasma physics. In the early days, those questions concerned the nature of cosmic rays, their main composition, and their origin. With the advancement of observational techniques and theoretical models, the questions became more detailed and complex. However, despite more than a century of research, some longstanding questions still lack definitive answers. Two of the most fundamental ones are:

- What is the origin of cosmic rays?
- How are cosmic rays accelerated to PeV energies in our Galaxy?

Supernova Remnants (SNRs) are the most prominent candidates for being the main sources of Galactic cosmic rays based on their energetics and the presence of strong shocks for particle acceleration in their expanding shells. Compared to the diffusive cosmic-ray transport scales, the acceleration and injection of cosmic rays by these sources is approximately point-like in space and instantaneous in time. The influence of individual SNRs on the local cosmic-ray spectrum is hard to determine observationally, since the arrival directions of cosmic rays are strongly isotropised upon interactions with the turbulent Galactic magnetic field, and contributions from multiple sources overlap. However, it is conceivable that nearby and recent injections of cosmic rays might leave observable imprints in the data.

Modelling the source population of Galactic cosmic rays is inherently limited by our incomplete knowledge of the exact locations and times of the injection of cosmic rays in our Galaxy. In this thesis, we take a stochastic approach, treating the positions and ages of sources as random variables allowing us to study the influence of individual sources on the cosmic-ray density throughout the Galaxy. This can inform models of the local cosmic-ray spectrum as well as of so-called Galactic Diffuse Emissions (GDEs) in γ -rays and neutrinos, which are produced by cosmic rays interacting with the Interstellar Medium (ISM) throughout the entire Galaxy. On the one hand, this allows us to quantify the influence of the discrete nature of cosmic-ray sources on observables, thereby estimating the uncertainties arising from our incomplete knowledge of the source population. On the other hand, it allows us to investigate whether observations of the local cosmic-ray spectrum and of GDEs can be used to constrain properties of the Galactic cosmic-ray source population.

Structure

We structure this thesis into three main parts. In Part I, we review the theoretical background of cosmic-ray physics that is relevant for this work. Then, in Part II, we study the influence of stochastic cosmic-ray sources on the local cosmic-ray spectrum. Parts of this chapter have been published in Stall et al., [2025](#). Finally, in Part III, we investigate the impact of stochastic cosmic-ray sources on GDEs. Some of the results presented in this chapter have been submitted for publication (Stall and Mertsch, [2026](#)).

Part I We start with a historical overview of the discovery and early research on cosmic rays in Chap. 2. Over the decades, cosmic-ray astrophysics was established as a research field of its own. Its development was driven by advances in observational techniques, which we summarise in Chap. 3. This includes high-precision measurements of the local cosmic-ray spectrum over a broad energy range (Secs. 3.3 and 3.4) as well as observations of secondary emissions produced by cosmic rays interacting with the ISM. We single out the importance of GDE observations in the high-energy γ -ray regime (Sec. 3.5) and in neutrinos (Sec. 3.6). Both messengers provide crucial information about the cosmic-ray population beyond our local Galactic neighbourhood.

Observations and theoretical advancements led to the establishment of a fundamental set of *paradigms* of cosmic-ray theory, which we review in Chap. 4. These paradigms include the assumption that SNRs are the main sources of Galactic cosmic rays and that the main transport process for Galactic cosmic rays is diffusion in the Galactic halo.

In Chap. 5, we introduce this plasma-physical description in detail. This includes the derivation of the diffusive transport of cosmic rays in the Galaxy in the framework of Quasilinear Theory (QLT) (Sec. 5.3) and the derivation of the transport equation for cosmic rays (Sec. 5.4), which forms the basis for most of the modelling work in this thesis.

Then, we review the theory of Diffusive Shock Acceleration (DSA) (Sec. 5.5), discuss how cosmic rays can be accelerated to PeV energies in SNRs (Sec. 5.6), and how this is linked to the escape from their sources into the ISM (Sec. 5.8).

In Chap. 6, we introduce the theoretical background for the production of hadronic GDEs by cosmic rays interacting with interstellar gas. For this, we discuss the main model ingredients, which are the distribution of interstellar gas in our Galaxy (Sec. 6.2), interaction cross-sections (Sec. 6.3), and absorption effects for high-energy γ -rays (Sec. 6.4). We also discuss the relation between γ -ray and neutrino emissions from those hadronic interactions (Sec. 6.5).

Finally, we introduce the relevant theoretical background for the stochastic modelling of cosmic-ray sources in Chap. 7. We start with a motivation for this approach (Sec. 7.1), discuss how to model the spatial and temporal distribution of the source population (Sec. 7.2), present an analytical solution for the cosmic-ray transport equation for protons injected by point sources (Sec. 7.3), and review previous work in this field (Sec. 7.4).

Part II In this part, we use the stochastic modelling of Galactic cosmic-ray sources to study their influence on the local cosmic-ray spectrum. We first discuss our Monte Carlo simulation framework to generate realisations of the source population and to compute corresponding local cosmic-ray spectra efficiently using Graphics Processing Unit (GPU) programming techniques (Chap. 8). For this, we consider different models for the injection of cosmic rays by their sources: a burst-like injection and an energy-dependent injection over an extended period of time.

In Chap. 9, we investigate how well different source models can be determined based on observations of the local cosmic-ray spectrum. We use Machine Learning (ML) for this classification problem and discuss the classification performance in relation to various model parameters. Finally, we discuss our results in the light of their potential use in current and future experimental analyses.

Part III In this final part, we study the influence of stochastic cosmic-ray sources on GDEs using extensive Monte Carlo simulations. Diffuse emissions produced by cosmic-ray interactions with interstellar gas in other parts of our Galaxy probe the cosmic-ray population beyond the local environment. In Chap. 10, we discuss the setup of our model and the Monte Carlo simulations.

In Chap. 11, we present our results and investigate the influence of stochastic cosmic-ray sources on hadronic GDEs in γ -rays and neutrinos. Our results highlight the close connection between the statistical properties of the cosmic-ray intensities and those of the resulting diffuse emissions. Furthermore, we quantify the impact of our limited knowledge of the source positions and ages on GDE model uncertainties and determine the influence different source models have on the diffuse sky.

Finally, we summarise our findings and conclude in Chap. 12.

Part I

Cosmic-ray physics

2

History of cosmic-ray physics

The significance of cosmic-ray physics for astrophysics has been growing since its early days at the start of the 20th century. It has evolved in tandem with advances in particle physics and observational astrophysics. Today it thrives through synergies with radio, X-ray, γ -ray, and neutrino astronomy. Together, these developments further our understanding of the Galaxy and the Universe. In this historical introduction, we will give an overview of the historical context of some of the most important discoveries in cosmic-ray physics, that have shaped our modern understanding of the field. This overview broadly follows the presentation in Ginzburg, [1996](#), De Angelis, [2014](#), [2023](#), and Walter and Wolfendale, [2012](#).

2.1 Early developments

The discovery of cosmic rays is attributed to Victor Hess, who conducted a series of balloon flights between 1911 and 1913. But this discovery was not an isolated event; it was the consequence of a series of experiments to understand the nature of ionising radiation. As early as the late 18th century, it was known that electroscopes discharge over time, even in well-isolated environments. This has been reported by Coulomb, [1785](#) and confirmed with greater accuracy by Faraday, [1844](#). Crookes, [1879](#) investigated the pressure dependence of the discharge rate and found that at lower ambient pressures the discharge slowed, leading him to conclude that ionised air was responsible for the observed effect (De Angelis, [2014](#)).

With the discovery of radioactivity by Becquerel, [1896](#) and further studies by Curie et al., [1898](#), a natural hypothesis emerged. It was soon understood that bringing radioactive materials near electroscopes increased the discharge rate, because such materials emit charged particles. This supported the idea that ionising radiation from radioactive materials — perhaps in the ground, the Sun, or the atmosphere — might explain the persistent ionisation.

Elster and Geitel, [1900](#) further improved the isolation of electroscopes and concluded that air must possess some electrical conductivity. What they named a *dark current* was present even in well-isolated setups. A series of experiments followed to determine the

origin of the ionising radiation. Wilson, 1901, who independently came to the same conclusion as Elster and Geitel, actually hypothesised that the radiation came from space, but soon abandoned this idea after he could not detect a reduction of ionisation when taking measurements in a tunnel, likely due to experimental uncertainties (De Angelis, 2014). Elster and Geitel actually reported a reduction when going underground in a salt mine in 1908, but still concluded that most radiation came from the ground. They assumed that the surrounding materials must be efficient absorbers of radioactivity from the Earth (De Angelis, 2023).

In 1909 and 1910, the German scientist and Jesuit priest Theodor Wulf travelled from Valkenburg to Paris to test the hypothesis of the terrestrial origin of the ionising radiation with multiple measurements at the bottom and on top of the Eiffel Tower. The decrease in ionisation with height was smaller than expected if the ground were the only source. For the Eiffel Tower's height of 300 m, a decrease of the ionisation rate to "just a few percent of the ground radiation" was expected under the terrestrial origin hypothesis, while a decrease to "not even half of its ground value" was measured (Wulf, 1909, 1910). However, he still concluded that radiation from the soil might be the most likely explanation (De Angelis, 2014). Still, those measurements were very valuable and considered reliable references for the study of the altitude dependence of ionisation. This was mainly due to their reproducibility under different conditions (De Angelis, 2014).

Early balloon experiments by Karl Bergwitz, 1910 and Albert Gockel, 1910, 1911 did not find a significant decrease in ionisation with altitude, as would be expected from the ground source hypothesis. Erwin Schrödinger also studied this problem during his thesis work and demonstrated that the expected decrease in ionisation with altitude could be offset by a second component of radiation from above. However, this included contributions from both potential radioactive elements accumulated at the top of the atmosphere and from *extraterrestrial* sources (De Angelis, 2023). While there was no conclusive result yet, a fitting term for the hypothesis of an extraterrestrial origin was already coined by Gockel and Wulf, 1908: *Kosmische Strahlung* (cosmic radiation).

The next decisive experiment did not involve balloon flights, but rather measurements on and underwater. Pacini, 1912 executed a crucial underwater test. He observed a reduction in ionisation beneath water compared to the surface. The absorption coefficient for water was well-known. Thus, both measurements were shielded from radiation from the ground. He concluded that a significant fraction of the radiation must be coming from above, but could not yet determine its exact origin - atmospheric or extraterrestrial. Pacini, 1912 also referenced the early balloon experiments by Albert Gockel, 1911 and Victor Hess, 1911, which had not yet reached firm conclusions.

2.2 Victor Hess' discovery

The definite proof for the existence of radiation from above came from a series of balloon flights by Victor Hess, 1912. After multiple flights that did not reach sufficient altitudes, his seventh flight on August 7, 1912, reached about 5200 m. This flight from Aussig a. d. Elbe (today Usti nad Labem) via Pirna and Cottbus to Pieskow (about 50 km east of Berlin) lasted roughly six hours. In Hess, 1912, the conclusions from all flights are summarised (see also Fig. 2.1). While the radiation decreases slightly above the ground, at altitudes above 1000 m it increases again, reaching values about 50 % higher than at the ground between 3000 m and 4000 m. At the highest altitude of 5200 m, the ionisation was found to be twice as high as on the ground. Further, Hess provides arguments against an atmospheric origin of the radiation. Neither meteorological conditions nor measurements after flights with a potentially contaminated balloon caused any changes in the measurements, while he observed fluctuations in the ionisation that could not be explained by any of these factors. For a more detailed discussion of why an atmospheric origin is unlikely, see Hess, 1912. Hess also ruled out the Sun as a source by the lack of a day-to-night variation and by measurements during a partial solar eclipse on his first flight.

Hess's results were quickly confirmed by Kolhörster, 1913, 1914, who reached altitudes up to 9300 m on his flights close to Halle a. d. Saale. He also found that the ionisation increased with altitude, supporting Hess's conclusions. His highest flight was on June 28, 1914, the day of the assassination of Archduke Franz Ferdinand, which marked the beginning of World War I and a shift of the scientific community's focus away from Europe to the United States (De Angelis, 2023). The results of these flights are also displayed in Fig. 2.1.

On the other side of the Atlantic, Millikan and his collaborators were leading the research on cosmic rays. Initially opposed to the idea of an extraterrestrial origin of the radiation, they eventually came to the same conclusion as Hess and Kolhörster and published their results in a series of papers in 1926, coining the term *cosmic rays* (Millikan and Bowen, 1926; Millikan and Otis, 1926; Millikan and Cameron, 1926). There was, however, some controversy with regard to the priority of the discovery. Millikan's conclusions were first published in *Science and Nature* at the end of 1925 (Millikan, 1925a,b), without giving due credit to Hess and Kolhörster. One of the reasons might have been that Millikan thought to have found a mistake in the measurements of Hess and Kolhörster. Millikan measured at even higher altitudes in Texas, while not finding an increase of the ionisation as strong as his European colleagues. This discrepancy would prove to be a result of the latitude effect due to the charged nature of cosmic rays, which was not yet known at the time. Instead, Millikan assumed that cosmic rays were γ -rays. At that time, only α , β , and γ radiation were known and among those, only γ -rays could explain the large penetration depths of cosmic rays (De Angelis, 2023).

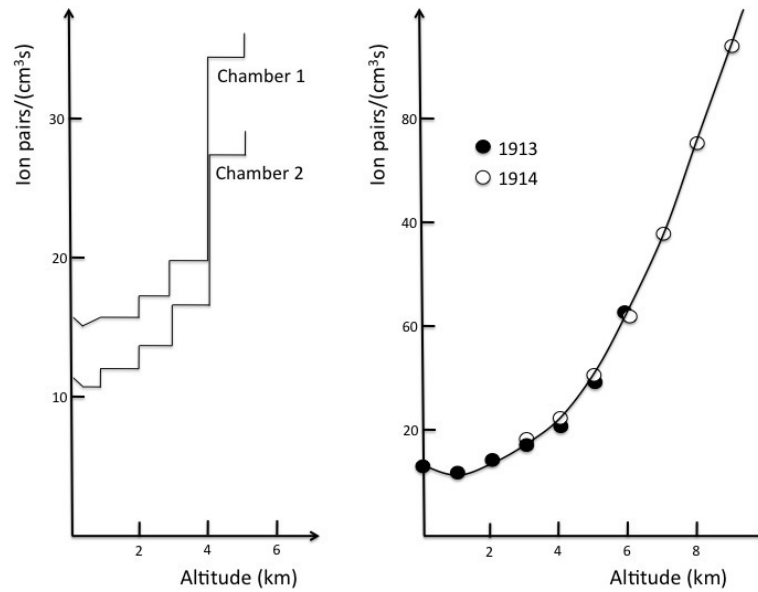


Figure 2.1: Results from balloon flights by Hess and Kolhörster. The figures show the increase of ionisation with altitude. The left panel shows the measurements by Hess, 1912, while the right panel shows the results of multiple measurements by Kolhörster, 1913, 1914. The figure is taken from (De Angelis, 2011). (CC BY-SA 3.0 licence)

The influential publications by Millikan prompted quotations to name the radiation after him: *Millikan rays* (New York Times, 1925), to which Hess, 1926b objected. Hess, 1926a,b and Kolhörster, 1926a,b defended their works against criticism. E.g., Kolhörster's determination of the absorption coefficient of cosmic rays in air was heavily criticised by Millikan and Bowen, 1926. While Kolhörster and v. Salis, 1926 inferred them from measurements at the glacier at Jungfraujoch, Millikan and Bowen, 1926 relied on unmanned balloons for their high-altitude measurements.

While Millikan and Cameron, 1926 acknowledged these objections, they did not reference Hess' and Kolhörster's original publications in their concluding paper. Nevertheless, the results of Millikan and his collaborators had a huge impact on the development of cosmic-ray physics. They established the extraterrestrial origin of cosmic rays in the scientific community and also added new insights regarding the isotropy of cosmic rays by comparing measurements where the Milky Way was visible and where it was not (Millikan and Cameron, 1926).

2.3 The nature of cosmic rays

Although the hypothesis of the cosmic origin of the radiation was now widely accepted, it was not at all clear what the nature of cosmic rays was. A controversy followed over

whether cosmic rays were γ -rays or charged particles. Clay, 1927, rather unexpectedly, discovered a latitude dependence. As pointed out by Bothe and Kolhörster, 1929, this suggested that cosmic rays were charged particles. This latitude effect was followed up on in a worldwide survey led by Compton, 1932, 1933, where Compton collaborated with more than 60 scientists to resolve this controversy (De Angelis, 2023). The conclusion was that at least some of the cosmic rays are charged particles and geomagnetic effects changed the cosmic-ray intensity depending on the observer's latitude.

Naturally, this prompted the question of the sign of the charge predominant in cosmic rays. This could also be disentangled using the Earth's magnetic field. The measurement showed a greater flux from the West than from the East indicating predominantly positively charged particles (Alvarez and Compton, 1933; Johnson, 1933a,b; Rossi, 1934).

Were it not for the discovery of the positron by Anderson, 1932, 1933¹, cosmic rays would probably have been expected to be protons right away (Ginzburg, 1996). It took some time and more careful studies to conclude that protons are the main component of cosmic rays. Johnson, 1938 came to this conclusion by studying the geomagnetic effects in more detail. Schein et al., 1941 showed — using differences in shower patterns between protons and electrons/positrons — that the primary cosmic rays are predominantly protons. More details on the modern understanding of the composition of cosmic rays will be discussed later in Sec. 3.1.2.

2.4 Towards modern cosmic-ray astrophysics

In 1936, the Nobel Prize in Physics was awarded to Hess for the discovery of cosmic rays and to Anderson for the discovery of the positron. This shows the close connection between cosmic-ray and particle physics in the early days of cosmic-ray physics, before the advent of powerful particle accelerators. After the positron, e.g., also the muon (Neddermeyer and Anderson, 1937; Street and Stevenson, 1937) and pion (Lattes et al., 1947) were first discovered in cosmic-ray showers (Ginzburg, 1996). After modern accelerators were built, however, cosmic rays lost their significance for particle physics. It is worth noting that, although cosmic rays can reach energies that clearly exceed those of human-made accelerators, they only hit fixed targets in the atmosphere. This limits the centre-of-mass energy that can be reached in cosmic-ray interactions. Modern accelerators can collide particles head-on. The Large Hadron Collider (LHC) at CERN

¹The often-shown positron track in Anderson's cloud chamber actually was very curious as it indicated that the positron moved upwards. Carlson, 2023 discusses this in detail and argues that the positron most likely came from above and was bent by the magnetic field applied around the cloud chamber. He also points out that Anderson first did not believe his own results and thought that his student must have played a trick on him by reversing the magnet's polarity.

can accelerate protons to a beam energy of about $E_{\text{LHC}} \approx 7 \text{ TeV}$. This corresponds to a cosmic-ray proton hitting a fixed target with an energy of

$$E_{\text{CR}} = \frac{2E_{\text{LHC}}^2}{m_p c^2} - m_p c^2 \approx 10^{17} \text{ eV}, \quad (2.1)$$

where m_p is the proton mass. The flux of cosmic rays of this energy is very low as we will discuss later in Sec. 3.1.1.

Before we conclude this historical introduction, we want to highlight two more important contributions to cosmic-ray physics before 1950: First, the question of the origin of cosmic rays was already addressed by Baade and Zwicky, [1934a,b](#). They proposed that the newly discovered supernovae could be their sources. However, they thought this mainly because there was no reported dependence of the cosmic-ray intensity on the Galactic coordinates. Thus, they considered extragalactic supernovae as source candidates mainly based on energetic arguments. We will address the supernova paradigm in more detail later in Sec. 4.4. Second, the question of how cosmic rays could be accelerated to such high energies was addressed by Fermi, [1949](#), who proposed an acceleration mechanism in astrophysical plasmas. Although his first proposition proved to be not efficient enough, this was the first step towards our modern understanding of cosmic-ray acceleration. We will also discuss this in more detail in Chap. 5.

3

Observations in cosmic-ray astrophysics

The previous section displays the early development of cosmic-ray physics and some of the big early-days enigmas that the scientific community had to solve. Today, we have learned much more about cosmic rays, and the field has been transformed into its own branch of astrophysics. The major developments in the field were driven by theoretical advancements and experimental feats alike. In the 1940s, the bulk of cosmic rays was identified to be protons of extraterrestrial origin, but many questions remained open. Now, we have reached a rich understanding of the origin and transport of cosmic rays, which we will explore in the following chapters.

In this chapter, we give an overview on the state-of-the-art measurements in modern cosmic-ray physics, we discuss how cosmic-ray astrophysics was established as its own branch of astrophysics, and we introduce the experiments that drive the modern development of the field. We put a special emphasis on the high-precision measurements by space-borne cosmic-ray detectors like the Alpha Magnetic Spectrometer (AMS-02) and the Dark Matter Particle Explorer (DAMPE). For γ -ray observations, we introduce the *Fermi* Large Area Telescope (*Fermi*-LAT) and the Large High Altitude Air Shower Observatory (LHAASO), and we also discuss the IceCube neutrino observatory.

We start this overview by defining what we actually understand as cosmic rays (Navas et al., 2024):

Cosmic rays are a nonthermal population of charged, high-energy particles that pervade the Universe.

This includes, e.g., protons, helium, and other nuclei, but also electrons, positrons, and other anti-particles like anti-protons. Other important messengers in the context of astroparticle physics like γ -rays and neutrinos are closely linked to cosmic rays, as we will discuss later, but they do not fall under this definition of cosmic rays. Sometimes, the term charged cosmic rays is used to distinguish them from those other messengers. In this thesis, we will use the term *cosmic ray* exclusively for the population of charged, relativistic, nonthermal particles, that are ubiquitous in the Universe (Mertsch, 2020). We also exclude Solar energetic particles (Reames, 2021) from our definition of cosmic rays in the proper sense.

3.1 Cosmic-ray data

The sensitivity of cosmic-ray measurements has been increasing over the decades. This made it possible to develop and test theories of cosmic-ray origin and transport. We will discuss those later in Chaps. 4 and 5. Here, we want to give an overview of the most relevant data from state-of-the-art experiments. Most importantly, this includes the spectrum of cosmic rays, the composition of cosmic rays, and their anisotropy (Navas et al., 2024).

3.1.1 Spectrum

Modern detection methods made it possible to measure cosmic rays reaching the Earth very precisely and determine their intensity at different energies. These measurements span remarkable twelve orders of magnitude in energy and reveal that the spectrum follows approximately a power law in energy. This can be seen in Fig. 3.1, where the differential intensity, dJ/dE , denoting the number of particles per unit time, unit energy, unit area, and unit solid angle, is plotted over the respective kinetic cosmic-ray energy, E . The approximate power law $dJ/dE \propto E^{-\gamma}$ is illustrated well as a straight line in this double-logarithmic plot. For the spectrum of all particles, the spectral index, γ , is close to 2.7 for energies up to the so-called cosmic-ray *knee* at about 3 PeV. For higher energies, it softens to about $\gamma \approx 3$. This indicates a change in the origin or transport of cosmic rays, which is an actively studied topic in cosmic-ray astrophysics. The spectrum at higher energies is characterised by further spectral features that are associated with changes in the population of Galactic cosmic-ray sources and the onset of as well as transition to an extragalactic source component. Specifically, this includes the so-called *second knee* at around $5 \cdot 10^{17}$ eV, where the spectral index softens even further to $\gamma \approx 3.3$ and the *ankle* at around $3 \cdot 10^{18}$ eV, where it hardens again to $\gamma \approx 2.5$ (Navas et al., 2024).

The spectrum in Fig. 3.1 also includes intensity measurements for individual cosmic-ray species. We can see that protons are clearly dominating the all-particle spectrum, followed by helium and heavier hadronic components. There is also a leptonic contribution by electrons and positrons. They are far less abundant than the proton component by two and three orders of magnitude, respectively. They also follow slightly different spectral indices. Fig. 3.1 also displays the intensities of γ -ray and neutrino signals that are related to cosmic-ray physics like measurements of *Fermi*-LAT (Ackermann et al., 2015), LHAASO (Cao et al., 2023a), and IceCube (Icecube Collaboration et al., 2023).

The steep power-law leads to vastly different rates of cosmic rays at different energies, which are indicated by the different grey-shaded regions in Fig. 3.1. Cosmic rays of energies below 1 TeV are incident with a rate of more than $1 \text{ m}^{-2} \text{ s}^{-1}$, which makes direct detections in compactly built detectors like AMS-02 or DAMPE (Aguilar et al., 2021; An et al., 2019) feasible (see Sec. 3.3). However, the rate quickly declines to $1 \text{ m}^{-2} \text{ d}^{-1}$ around

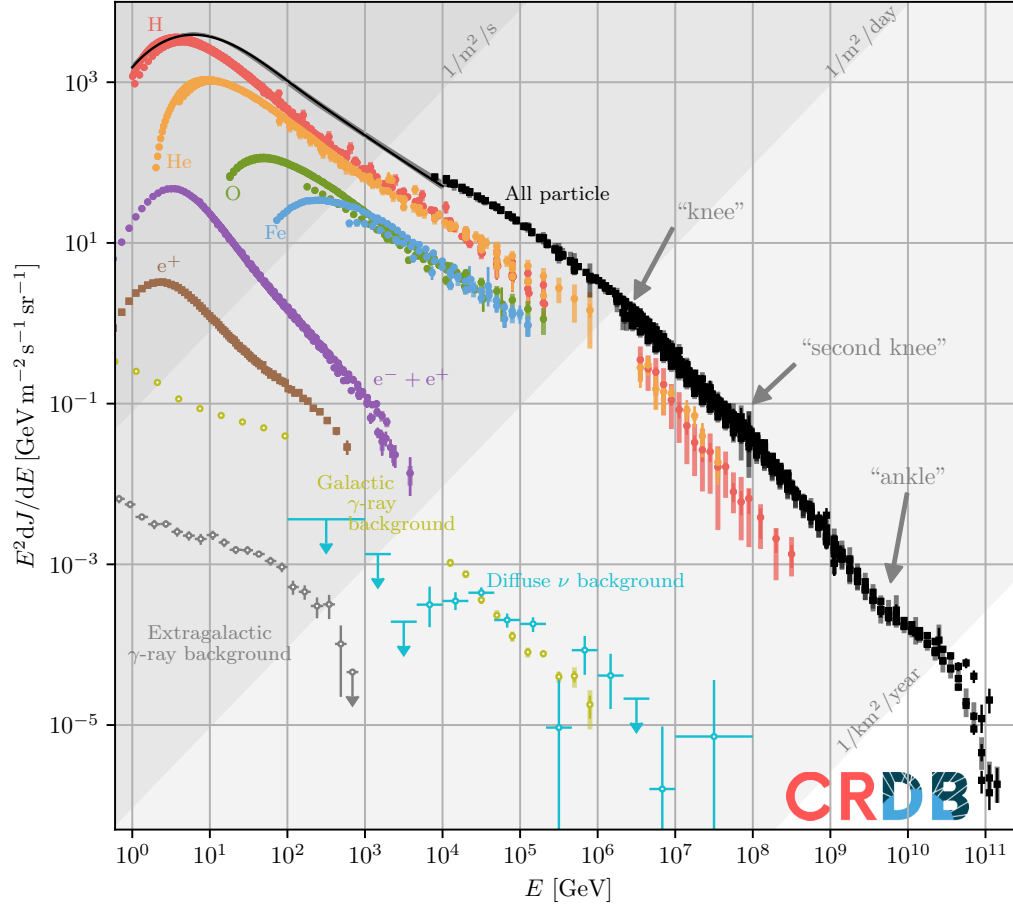


Figure 3.1: The cosmic-ray spectrum. The differential intensity, dJ/dE , is plotted over the kinetic energy of cosmic rays (here E). It is rescaled by E^2 to better illustrate the spectral features. The cosmic-ray data is taken from the Cosmic Ray Database (CRDB) (Maurin et al., 2023), while the γ -ray and neutrino data is taken from (Ackermann et al., 2015; Cao et al., 2023a; Icecube Collaboration et al., 2023). The grey-shaded regions indicate the rates of incident cosmic rays on Earth. The figure is taken from Mertsch, 2023, where also the references for the cosmic-ray data can be found. (CC BY 4.0 licence)

the *knee*-energy and further to $1 \text{ km}^{-2} \text{ yr}^{-1}$ above 10^{19} eV . With such low rates, only indirect detection methods (see Sec. 3.4) make it possible to observe those high-energy cosmic-ray particles (Gaisser et al., 2016).

The low- and high-energy ends of the all-particle spectrum can be explained by two effects: The cut-off of the intensity at low energies is due to solar modulation effects (Moraal, 2013). This means that charged cosmic-ray particles entering the solar system have to overcome the electric potential created by the solar wind. This stops cosmic-ray particles of low energies to enter the Solar system. A simple model for this effect is the so-called force-field approximation described by Gleeson and Axford, 1968. For more details on solar modulation, we refer to App. B. The high-energy cut-off is due to the so-called Greisen-Zatsepin-Kuzmin (GZK) effect (Greisen, 1966; Zatsepin and Kuz'min, 1966), which describes the strong energy losses of Ultra-High Energy (UHE) cosmic rays through photo-pion interactions with the Cosmic Microwave Background (CMB) (Penzias and Wilson, 1965).

3.1.2 Composition

The possibility to measure cosmic-ray intensities in such a great detail offers a huge potential for testing our understanding of the origin and transport of cosmic rays. As already shown in Fig. 3.1, there are measurements of individual species of cosmic rays. In Fig. 3.2, we can see a more extensive overview on such measurements of intensities of different nuclei. The spectra are shown with respect to rigidity, $\mathcal{R}$, instead of kinetic energy (for conversions, see App. A) and are rescaled with $\mathcal{R}^{2.7}$ to make spectral features, i.e., deviations from a perfect power law, visible more easily. The differential intensities are rescaled by the indicated factors, such that the spectra can be compared in a single figure. It can be seen that the spectral features occur for all species in the left and right panel, respectively, at about the same rigidities. This hints to a dependence of the cosmic rays' acceleration and transport on the rigidity, rather than (kinetic) energy or individual properties of the particle species (see Sec. 4.3).

The cosmic-ray species in Fig. 3.2 are divided in two groups: primary and secondary cosmic rays. This distinction concerns the origin of the respective cosmic rays. *Primaries* are accelerated in the cosmic-ray sources, while *secondaries* are produced during the transport of cosmic rays through spallation of primaries with particles of the Interstellar Medium (ISM). This effect can be seen in the abundances of the different cosmic-ray species shown in Fig. 3.3. The orange line shows the measured abundance of the respective cosmic-ray species at a kinetic energy per nucleon of 20 GeV. It is contrasted to the abundance of the different elements in the Solar system (Lodders, 2003). It can be assumed that the environments in which cosmic rays are accelerated have roughly the same elemental abundances as the Solar system. Thus, differences between the relative abundances must be caused by the production of secondaries (Ginzburg and Syrovatskii, 1964).

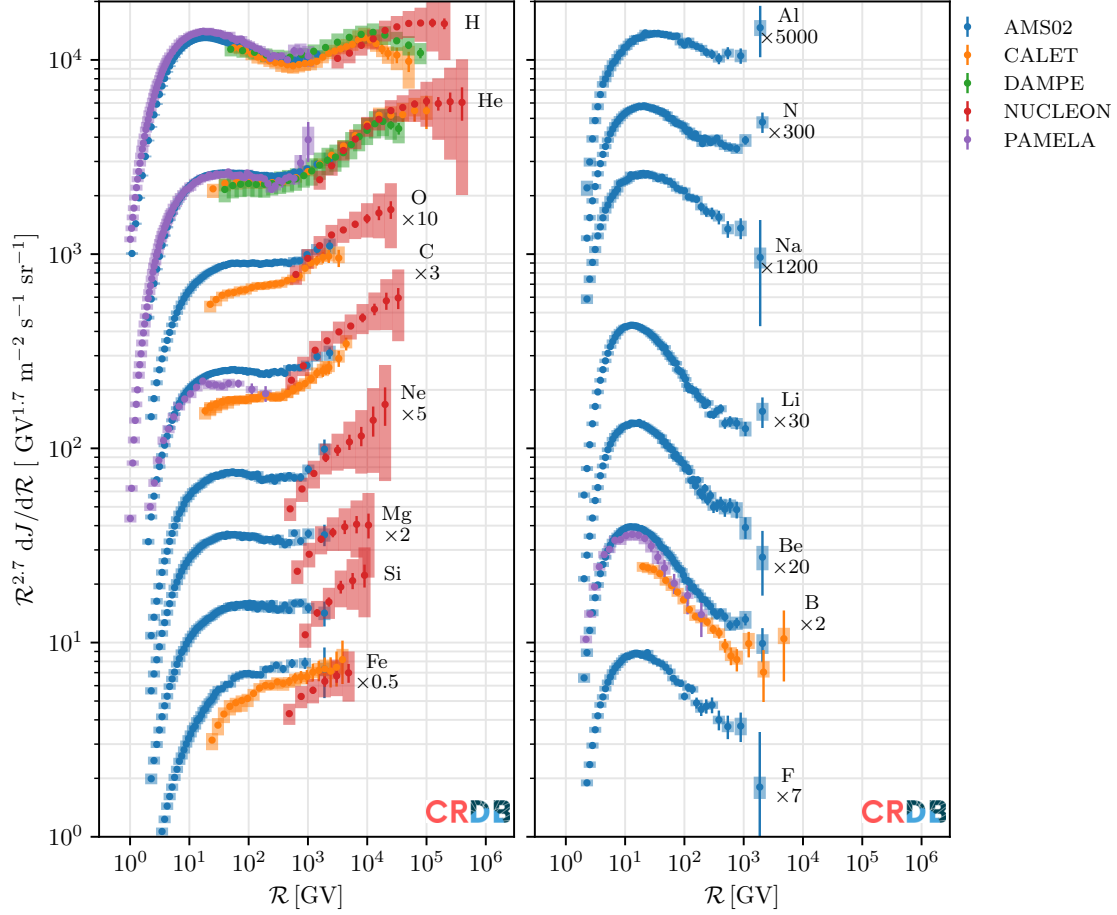


Figure 3.2: Primary and secondary cosmic-ray measurements. The differential intensity spectra for multiple primary (left) and secondary (right) cosmic rays are shown as functions of rigidity, $\mathcal{R}$. The spectra are rescaled by $\mathcal{R}^{2.7}$ to bring out spectral features and are shifted by the indicated factors to make it possible to compare spectra of different cosmic-ray species in a single figure. The statistical uncertainties are indicated by error bars, while systematic uncertainties are shown as shaded bars. The figure is taken from Navas et al., 2024, Fig. 30.3, where the references for the experimental data are also given. (CC BY 4.0 licence)

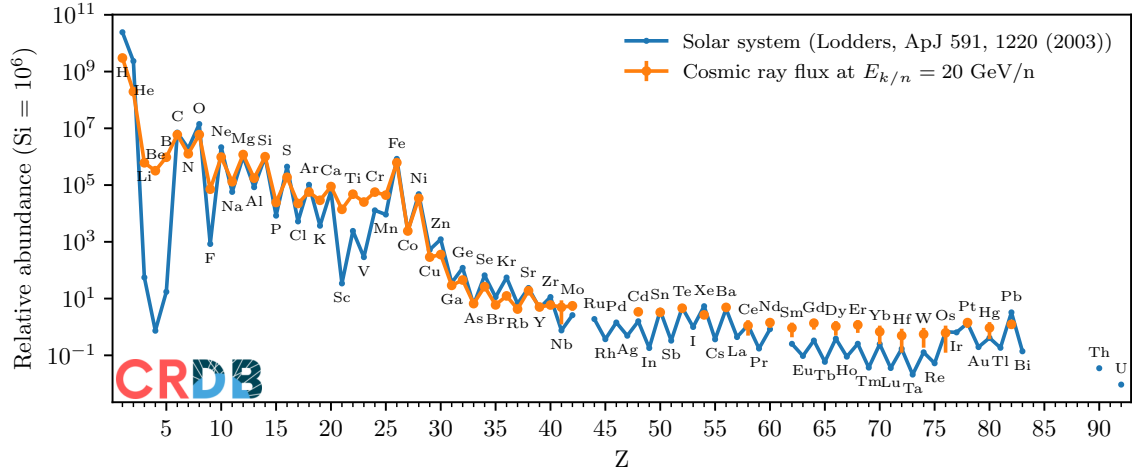


Figure 3.3: Abundances of cosmic rays compared to Solar system ones. The abundances are normalised such that $\text{Si} = 10^6$. The values for the Solar system abundances are taken from Lodders, 2003, while the cosmic-ray abundances correspond to the cosmic-ray flux at a kinetic energy per nucleon, i.e., E_{kin}/A , where A is the mass number, of 20 GeV. The figure is taken from Navas et al., 2024, Fig. 30.2. It has used data from the CRDB and has been adapted from the corresponding publication Maurin et al., 2023, where the references for the measurements are given. (CC BY 4.0 licence)

The origin of elements is studied in the field of nucleosynthesis. There, the many processes influencing the production of elements are studied. This includes the production of the lightest elements H, He, and Li in the big bang nucleosynthesis, and the synthesis of heavier elements in multiple different environments, e.g., low and intermediate mass stars, asymptotic giant branch stars, core-collapse supernovae, Type Ia supernovae, or neutron star mergers. The prediction of abundances in Galaxy simulations is involved and requires a good understanding of nuclear physics and the interaction rates for all processes involved. For reviews on these topics, we refer to Kobayashi et al., 2006, 2020; Kobayashi and Taylor, 2023; Kobayashi, 2026.

The Solar abundances in Fig. 3.3 show patterns that are universal for the elemental abundances in our Galaxy. The elements H and He were mostly produced in big bang nucleosynthesis and are most abundant. We also observe a clear even-odd pattern: The elements with an even atomic number are more abundant than the ones with an odd atomic number (Bergemann et al., 2026). This is a well-known pattern in the relative abundance of elements first observed by Oddo, 1914 and Harkins, 1917 and thus also known as the Oddo-Harkins rule related to nuclear stability.

For the secondaries already mentioned in the context of Fig. 3.2, we can make the following observations: Be and B are strongly under-abundant in the Solar system, because they are mainly produced as secondary cosmic rays (Prantzos et al., 1993). Li receives primordial as well as other contributions (Grisoni et al., 2019), but as indicated by Fig. 3.3, it is also produced as a secondary cosmic-ray component. For other secondaries like F, Na, and Al,

we can also see that the relative abundances in the cosmic-ray population vary from the Solar system abundances. We will discuss the importance of the so-called *secondary-to-primary ratio* for the understanding of cosmic-ray transport later in Chap. 4.

The interpretation of the composition of Galactic cosmic rays is also linked to the volatility of different elements. Volatile elements have low condensation temperatures, whereas refractory elements have high condensation temperatures. As a general trend, it has been found that refractory elements are enhanced in Galactic cosmic rays compared to the solar abundance as material clumped in grains can be accelerated to cosmic-ray energies more efficiently (Ellison et al., 1997; Meyer et al., 1997). For a recent review on the composition of Galactic cosmic rays and its interpretation, we refer to Tatischeff et al., 2021.

A closer investigation of different isotopes of individual elements can also give further insights. This includes radioactive isotopes, whose abundance holds information about the transport timescales of the respective cosmic rays (Gaisser et al., 2016). We will also discuss this later in Chap. 4. Further, specific isotopic ratios like $^{22}\text{Ne}/^{20}\text{Ne}$ inform us about the specific environments where cosmic rays are accelerated (Tatischeff et al., 2021). We will come back to this specific aspect briefly in Sec. 5.6.

3.1.3 Anisotropy

Besides the spectral information and composition of cosmic rays, their arrival directions are studied and reveal crucial information about their transport. We have already mentioned in Chap. 2, that the isotropic arrival of cosmic rays can be used to draw conclusions about their sources. It can also be used to constrain transport models. For this, it is important to quantify the anisotropy.

The anisotropy in cosmic-ray arrival directions is defined as the deviation of the directional flux from an isotropic flux. We can define the relative intensity in a given direction $\hat{\mathbf{n}}$, $I(\hat{\mathbf{n}})$, as

$$I(\hat{\mathbf{n}}) = \frac{\phi(\hat{\mathbf{n}})}{\phi^{\text{iso}}} = 1 + \delta I(\hat{\mathbf{n}}), \quad (3.1)$$

where $\hat{\mathbf{n}}$ is a unit vector pointing in the arrival direction, $\phi(\hat{\mathbf{n}})$ is the directional flux, and ϕ^{iso} is the angular average of the flux. Thus, $\delta I(\hat{\mathbf{n}})$ is the anisotropy. The relative intensity can be expanded in spherical harmonics, which allows the study of the anisotropy on different angular scales (Ahlers and Mertsch, 2017).

The dipole anisotropy is expected to be the dominant contribution. The reported amplitudes of the dipole anisotropy are $\mathcal{O}(10^{-4})$ at cosmic-ray energies of hundreds of GeV and increase with energy. At tens of TeV, they are $\mathcal{O}(10^{-3})$ (Navas et al., 2024). This low level of anisotropy constrains models of the origin and sources of cosmic rays. It indicates, e.g., that either the transport of cosmic rays is not ballistic or the number of

sources is very high. The dipole anisotropy in the scenario of ballistic transport would be $\approx 1/\sqrt{N_{\text{src}}}$ (Kulsrud, 2005).

Under the assumption that cosmic-ray transport can be explained by diffusion, a dipole anisotropy of the following amplitude, a , is expected (Ahlers and Mertsch, 2017):

$$a = \frac{\phi_{\text{max}} - \phi_{\text{min}}}{\phi_{\text{max}} + \phi_{\text{min}}} = \frac{3}{c} \frac{|\boldsymbol{\kappa} \cdot \nabla n|}{n}, \quad (3.2)$$

where ϕ_{max} and ϕ_{min} are the maximum and minimum directional cosmic-ray fluxes, respectively, $\boldsymbol{\kappa}$ is the diffusion tensor, and n is the isotropic cosmic-ray number density (Ahlers and Mertsch, 2017; Ginzburg and Syrovatskii, 1964). It turns out that the predicted anisotropy from a simple isotropic diffusion model overpredicts the measured anisotropies (Blasi and Amato, 2012a), and more sophisticated transport models have to be invoked (Mertsch and Funk, 2015) (see also a brief discussion in Sec. 5.3.8).

The observed anisotropies on smaller scales, i.e., higher multipoles, are in turn higher than expected in standard diffusion theories (Gabici et al., 2019; Navas et al., 2024). For a review on the explanations for the origin of those small-scale anisotropies in Galactic cosmic rays, we refer to Ahlers and Mertsch, 2017.

Besides the amplitude of the dipole anisotropy, it is also possible to measure its phase, i.e., the direction of the maximum flux. Below 100 TeV, the phase is roughly constant at a direction of about 45° in the equatorial coordinate system. At about 100 TeV, however, the phase flips to a value of about 250° , which is closer to the Galactic centre direction (Navas et al., 2024). Measurements on the amplitude and phase of the anisotropy in the TeV to PeV energy range are important for testing Galactic cosmic ray transport and source models. For a detailed theoretical account of the phase flip and amplitude changes in this energy range, we refer to Ahlers, 2016, where the influence of local cosmic-ray sources, ordered local magnetic fields, and experimental reconstruction biases are discussed.

For most energies, the anisotropy in arrival directions of cosmic rays is measured by indirect observations (see Sec. 3.4), e.g. at the Pierre Auger Observatory. It has been found that a dipole anisotropy with an amplitude of $a = 0.074^{+0.010}_{-0.008}$ in a direction of about 115° away from the Galactic centre is present at cosmic-ray energies above $8 \cdot 10^{18}$ eV (Abdul Halim et al., 2024). Cosmic rays at these energies have a limited range due to the GZK effect (see Sec. 3.1.1), and they are only weakly deflected by magnetic fields. Thus, the anisotropy at these energies is expected to be linked to the distribution of sources (Navas et al., 2024). The detection of a dipole anisotropy pointing away from the Galactic centre at these energies is a strong indication that the sources of UHE cosmic rays are extragalactic (Pierre Auger Collaboration et al., 2017).

3.2 Cosmic-ray astrophysics

In the early years of cosmic-ray research, it was not clear, to which extent the pursuit of cosmic-ray astronomy would be possible. Contrary to the electromagnetic radiation from astrophysical objects, cosmic-rays seemed to arrive almost perfectly isotropically and no individual sources could be observed by measuring the charged particles bombarding Earth. Ginzburg, 1996 compares this to a scenario of living on a planet with a very dense atmosphere which diffuses all the light from the Sun, stars, and other celestial objects such that only their integrated flux is measurable. It would hardly be possible to develop the field of astrophysics, as we know it, in such a scenario.

In the early phases of cosmic-ray physics, when high-precision measurements of the cosmic rays like the ones introduced in the previous Sec. 3.1 were not available yet, the progress in the field was driven by developments in other parts of astrophysics. Advancements of observational methods for electromagnetic radiation in the 1950s (Ginzburg, 1996) and progress in the understanding of atomic physics (Schlickeiser, 2002) made it possible to study cosmic rays away from Earth using spectral information from electromagnetic observations. In the beginning, those were observations of radio waves, which were linked to the synchrotron emission of cosmic-ray electrons (Alfvén and Herlofson, 1950; Ginzburg, 1951, 1953; Kiepenheuer, 1950).

Charged particles that are accelerated emit electromagnetic radiation. Three main processes are relevant in astrophysical environments: synchrotron radiation, bremsstrahlung, and inverse Compton scattering. Although electrons only make up a small fraction of the cosmic-ray flux, their energy losses through these processes are much stronger than those of the much heavier protons and other nuclei, because of the large mass ratio $m_p/m_e \approx 1836$ (Berezinskii et al., 1990, Chap. V). The synchrotron radiation is emitted by charged particles that are accelerated in magnetic fields. Synchrotron radiation mainly consists of non-thermal radio emissions, but can also produce X-rays in extreme environments like pulsars (Ginzburg, 1996). Assuming that the differential electron density follows a power law, $\phi_E \propto E^{-\alpha}$, the synchrotron emissivity is expected to follow a power law as well (Rybicki and Lightman, 1979):

$$I_\nu \propto \nu^{-\frac{\alpha-1}{2}}, \quad (3.3)$$

where I_ν is the differential energy intensity and ν is the frequency of the synchrotron radiation. The undisturbed propagation of the radio waves from the production sites of those synchrotron emissions allows it to study the distribution of cosmic-ray electrons in different objects in our Galaxy and beyond.

X-ray radiation is another especially valuable messenger because of its close link to the emission of bremsstrahlung by cosmic-ray electrons. X-ray astronomy started after the discovery of X-ray sources outside the Solar system by Giacconi et al., 1962. X-ray observatories have to be space-borne, as the Earth's atmosphere is opaque to X-rays, which meant that progress in this field was closely linked to the development of space

technology (Berezinskii et al., 1990). For a review on the early development of this field of astronomy, we refer to Holt and McCray, 1982. One experiment that is currently operating is Chandra (Weisskopf et al., 2003). We will reference the observations of radio and X-ray emissions relevant for cosmic-ray theory in Chap. 4.

Besides these, observations in other parts of the electromagnetic spectrum complement the study cosmic rays and the environments they are transported in. E.g., microwave observations of synchrotron emissions, as detected by satellites like COBE (Boggess et al., 1992) and WMAP (Bennett et al., 2013), or observations in the UV, visual, or near-infrared spectrum, e.g., by the Hubble Space Telescope (Dalcanton, 2009) or the James Webb Space Telescope (Gardner et al., 2006) provide valuable insights on the influence of cosmic rays in other parts of the Galaxy. The results of this thesis discussed in Parts II and III, however, focus on two other messengers: γ -rays and neutrinos.

γ -ray astronomy had its beginnings in the 1970s (Ginzburg, 1996), when the first significant detections of γ -ray emissions from the Galaxy were reported. Such emissions are produced by the decay of neutral pions and kaons, which in turn are produced in hadronic interactions of cosmic-ray protons and nuclei with the ISM. Many experiments were developed to observe γ -rays. There have been satellite experiments like EGRET (Fichtel, 1996), AGILE (Tavani et al., 2009), INTEGRAL (Winkler et al., 2003), and *Fermi*-LAT (Atwood et al., 2009), but also ground-based experiments like, e.g., ARGO-YBJ (Aielli et al., 2012), HAWC (Tibolla, 2023), H.E.S.S. (Ohm et al., 2023), MAGIC (Tescaro, 2014), VERITAS (Weekes et al., 2002), Tibet AS γ (Amenomori et al., 2021), and LHAASO (Cao et al., 2019).

The interactions producing neutral pions also produce charged pions, which decay into neutrinos. Thus, neutrino emissions can be expected from the same sources. Prominent neutrino observatories include IceCube (Halzen and Klein, 2010) in the Antarctic ice, as well as ANTARES (Albert et al., 2025) and its successor KM3NeT (Adrián-Martínez et al., 2016) in the Mediterranean Sea. We will discuss the production mechanisms of γ -rays and neutrinos in more detail in Chap. 6 and introduce the modern γ -ray and neutrino observatories that drive the field of cosmic-ray astrophysics today in Secs. 3.5 and 3.6. We will focus especially on *Fermi*-LAT, LHAASO, and IceCube.

With measurements of cosmic rays at our position in the Galaxy and the use of additional messengers like radio waves, X-rays, γ -rays, and neutrinos, the field of cosmic-ray astrophysics could be established. We will discuss the main paradigms of cosmic-ray theory in Chap. 4. In the following sections, we will briefly introduce the modern observatories and their basic working principles.

3.3 Direct observation methods

Cosmic rays of energies up to the TeV range arrive at the Earth with sufficiently high rates such that they can be detected in compact detectors (see Fig. 3.1 and its discussion in Sec. 3.1.1). However, the charged particles making up cosmic rays interact with particles in the Earth's atmosphere and thus get absorbed. To measure the cosmic-ray intensity directly, detectors have to be placed at the top of the atmosphere or in space.

In the past, this has mainly been done with instrumented balloon flights like, e.g., ATIC (Guzik et al., 2004), BESS (Ajima et al., 2000), CAPRICE/WiZard (Circella et al., 2000), or CREAM (Seo et al., 2004). With different focuses, they have provided valuable insights in the spectrum and composition of cosmic rays and built upon the early balloon measurements by Hess, Kolhörster, and others (see Chap. 2). Further, there were satellite experiments like, e.g., CRIS (Stone et al., 1998), HEAO (Bouffard et al., 1982), and PAMELA (Picozza et al., 2007). We also want to mention the Voyager missions (Stone et al., 2013), that made it possible to measure the incident cosmic rays outside the heliosphere, i.e., without the influence of solar modulation (see App. B). Launched in 1977, Voyager 1 passed the heliopause in late 2012 (Cummings et al., 2016) and contributed greatly to our understanding of the effect of the heliosphere in the context of solar modulation (see also App. B). Further, it provided direct measurements of low-energy cosmic rays, which are assumed to play an important role in the ionisation of dense molecular clouds (Phan et al., 2018; Phan et al., 2021).

More recently, high-precision calorimetric experiments in space have been developed. Notably, those are CALET (Akaike et al., 2024), ISS-CREAM (Yoon et al., 2024), and DAMPE (Chang et al., 2017). Here, we want to focus on AMS-02 (Aguilar et al., 2021), which, in contrast to the aforementioned, is equipped with a magnet. Because of it, AMS-02 can measure the sign and value of the charge, the momentum, the rigidity, and the flux of elementary particles like protons, electrons, antiprotons, and positrons, as well as nuclei, and anti-nuclei (Aguilar et al., 2021).

AMS-02 is installed on the International Space Station (ISS) since May 19, 2011 and is taking data since then. This makes it the first long duration mission of a large acceptance magnetic spectrometer in space (Aguilar et al., 2021). In Fig. 3.4, we can see the different components of the AMS-02 detector. It consists of a transition radiation detector (TRD), time-of-flight (TOF) systems, a silicon tracker with multiple layers, a ring imaging Cherenkov detector (RICH), a permanent magnet, an electromagnetic calorimeter (ECAL), and an anti-coincidence counter (ACC). The TRD is used to distinguish electrons and positrons from protons by their different transition radiation signatures. It also provides information about the charge value of nuclei by measuring their energy loss rate. The TOF counters are used to determine the direction, velocity, and charge of the incoming particles. The silicon tracker with its nine layers measures the trajectories of incident particles in the magnetic field of the permanent magnet. This way, the rigidity can be determined. As ionisation energy losses deposited in the silicon layers depend on the

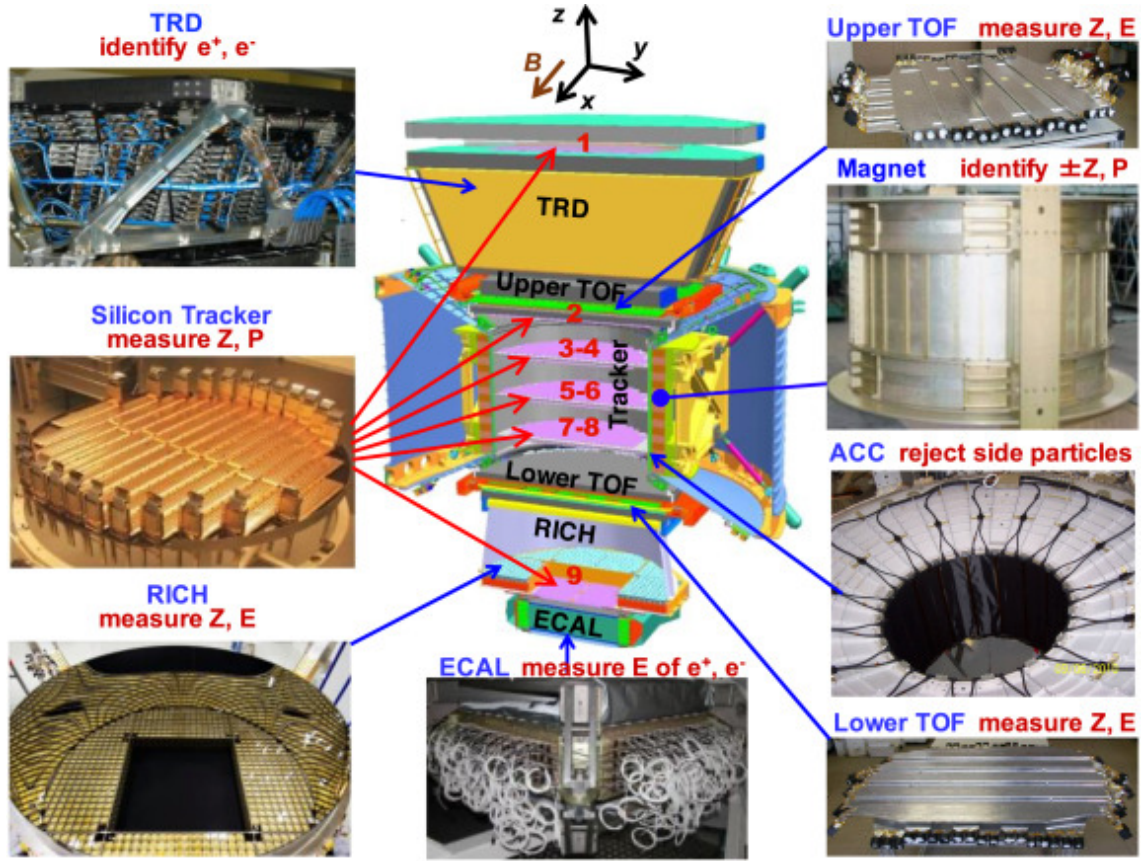


Figure 3.4: AMS-02 detector setup. This figure shows the different components of the AMS-02 detector with their functions. It consists of a transition radiation detector (TRD), time-of-flight (TOF) systems, a silicon tracker with multiple layers, a ring imaging Cherenkov detector (RICH), a permanent magnet, an electromagnetic calorimeter (ECAL), and an anti-coincidence counter (ACC). The figure is taken from Aguilar et al., [2021](#), Fig. 2 (Creative Commons CC-BY-NC-ND licence).

charge value, the tracker also measures the total charge. The RICH-counter measures the velocity and charge of particles passing through it by the geometry of the Cherenkov light they emit. The ECAL is the main component for measuring the energy of electrons and positrons and is also used to separate them from protons. Finally, the ACC is used to veto particles that do not pass through the full detector or enter from the side (Aguilar et al., 2021).

The AMS-02 detector provides high-precision measurements in a broad energy range from hundreds of MeV to several TeV. Before its launch, it has been tested extensively at CERN. In test beams of different particles, more than 2000 combinations of particle types, energies, and angles were tested that helped to calibrate the detector. The statistics of AMS-02 are very good due to the long exposure time and the large acceptance of the detector. The systematic error gets contributions due to uncertainties from the trigger and acceptance, the rigidity resolution function and unfolding, as well as the absolute rigidity scale (Aguilar et al., 2015, 2021). For analyses of the data, it can be crucial to understand the correlations between those uncertainties in different energy bins. While AMS-02 has not reported those correlations so far, some assumptions can be made: statistical uncertainties are uncorrelated, the rigidity scale uncertainty is fully correlated, and the other systematic uncertainties are partially correlated over some energy range (Cavazonza et al., 2017; Derome et al., 2019; Heisig et al., 2020). We will discuss this in more detail in the context of our analysis in Part II.

3.4 Indirect observation methods

The rate of cosmic-ray particles with energies above the PeV range is too low for direct measurements in detectors that can be launched on balloons or in space. Instead, interactions of cosmic rays with the Earth's atmosphere are used to detect those UHE cosmic rays indirectly. They induce cascades of secondary particles, so-called Extensive Air Showers (EAS), which can be observed to reconstruct the properties of the primary cosmic-ray particle (Navas et al., 2024). There are ground-based experiments like Pierre Auger (Abdul Halim et al., 2023), H.E.S.S. (Ohm et al., 2023), HAWC (Tibolla, 2023), TUNKA (Lubsandorzhiev and Tunka Collaboration, 2008), IceTop (Abbasi et al., 2013), LHAASO (Cao et al., 2019), KASCADE (Antoni et al., 2003), KASCADE-Grande (Apel et al., 2010), and the Telescope Array (TA) experiment (Kim et al., 2023). However, there are programs like JEM-EUSO (Caruso, 2025) or POEMMA (Probe Of Extreme Multi-Messenger Astrophysics) (Olinto, 2023) to observe UHE cosmic rays from space, thereby having the potential to increasing the statistics by several orders of magnitude due to much larger observed volumes (Caruso, 2025).

Conventional ground-based experiments observe the EAS through large arrays of particle detectors. This includes scintillators, water-Cherenkov stations, and underground muon detectors (Navas et al., 2024). The most notable detector of UHE cosmic rays of this kind

is the Pierre Auger Observatory in Malargüe, Argentina. Its name honours the seminal work on EAS by Auger et al., 1939.

The detection principle is hybrid, combining measurements by a surface array with those of fluorescence telescopes. Surface detectors measure a slice of the EAS by detecting the electromagnetic and muonic component of the shower in sensitive water-Cherenkov detectors. Additionally, fluorescence telescopes can be used to measure the fluorescence light emitted by nitrogen molecules that are excited by the passage of the EAS through the atmosphere (see, e.g., Pierre Auger Collaboration, 2015). While the surface detectors can take data around the clock, the fluorescence telescopes can only operate during clear, moonless nights.

There are several challenges in the reconstruction of the properties of primary UHE cosmic-ray particles from the measurements of the EAS. First, the interactions of cosmic rays with the air nuclei are governed by non-perturbative Quantum Chromodynamics (QCD), for which approximate models have to be used. Further, the shower development is mainly in the forward region, which is not well covered by the main collider experiments at the Large Hadron Collider (LHC) at CERN. However, there are dedicated forward experiments like LHCf (Sako, 2025) and the Forward Physics Facility (FPF) (Adhikary et al., 2025), which provide valuable data to tune the hadronic interaction models used in EAS simulations. Finally, primary cosmic rays with energies above 10^{17} eV = 0.1 EeV exceed the energies accessible to with particle accelerators on Earth at the LHC (see Eq. (2.1)).

All of this makes it necessary to extrapolate the tested models in energy and phase space, which introduces systematic uncertainties in the interpretation of UHE cosmic-ray data (Parsons et al., 2011). There are multiple available codes for the simulation of the EAS, like CORSIKA (Heck et al., 1998) or AIRES (Sciutto, 1999). For the hadronic interactions involved in the shower development, several models are available and permanently updated. Some of the most widely adopted models are EPOS-LHC (Pierog et al., 2015), QGSJET (Ostapchenko, 2024), and SIBYLL (Riehn et al., 2020). Considering the challenges mentioned above, it is crucial to be aware of the differences between those models and take those into account when interpreting the data with respect to systematic uncertainties.

3.5 γ -ray observatories

γ -rays are photons with energies above 100 keV. Since the Earth's atmosphere is opaque to γ -rays, observations of them face similar challenges as those of cosmic rays (see Secs. 3.3 and 3.4). As direct observations from the ground are not possible, balloon- and satellite-borne experiments operating above the atmosphere or large ground-based detectors that infer γ -ray signals indirectly are needed (Funk, 2015). At low energies, γ -ray detection is limited by background contributions such as cosmic rays, intrinsic detector radioactivity, and the diffuse γ -ray background (Dean et al., 1991). At the highest energies, the main limitation arises from the extremely low photon flux, which requires

large detection volumes, motivating the use of ground-based instruments (Gabici et al., 2019). Due to size-limitations, space-based detectors are thus limited to energies below some hundreds of GeV (Funk, 2015).

3.5.1 General considerations

γ -rays are produced by a variety of physical processes, including electron-positron annihilation, inverse Compton scattering, bremsstrahlung, and neutral pion and kaon decay (Funk, 2015). Several space-based missions have been dedicated to observing these γ -rays, including EGRET (Fichtel, 1996), AGILE (Tavani et al., 2009), INTEGRAL (Winkler et al., 2003), and in particular the *Fermi*-LAT (Atwood et al., 2009) experiment. Ground-based facilities such as H.E.S.S. (Ohm et al., 2023), MAGIC (Tescaro, 2014), VERITAS (Mukherjee and VERITAS Collaboration, 2018), HAWC (Tibolla, 2023), Tibet AS γ (Amenomori et al., 2021), and LHAASO have extended observations to the highest energies, with the future Cherenkov Telescope Array (CTA) (Acharya et al., 2013) and Southern Wide-field Gamma-ray Observatory (SWGGO) (SWGGO Collaboration et al., 2025) representing the next-generation experiment currently under development. This work focuses primarily on results from *Fermi*-LAT and LHAASO as the main instruments covering the γ -ray energy range from GeV to TeV energies.

γ -ray observations provide information about both Galactic and extragalactic sources, including blazars, pulsars, SNRs, Pulsar Wind Nebulae (PWNe), Active Galactic Nuclei (AGNs), Gamma-Ray Bursts (GRBs), starburst galaxies, and potentially even dark matter annihilation or decay. In addition to discrete sources, Galactic diffuse γ -ray emissions arising from hadronic and leptonic interactions of cosmic rays with the ISM are key observables (Atwood et al., 2009; Funk, 2015). This diffuse component has been measured by *Fermi*-LAT at GeV to TeV energies (Ackermann et al., 2012), and more recently by LHAASO at TeV to PeV energies (Cao et al., 2023a, 2025). These measurements are of particular importance for the study presented in Part III.

Since high-energy γ -rays cannot be reflected or refracted, measuring them is challenging. Space-based telescopes are mainly Compton or pair-conversion telescopes (Siegert et al., 2022). In Compton telescopes, the direction and energy of the incident γ -ray is reconstructed from the Compton scattering of the photon in the detector material. The scattered photon either scatters again in the detector or gets absorbed. The ejected electron might be tracked as well. This makes it possible to reconstruct the γ -ray's energy and constrain its direction to a circle segment. Compton telescopes work well up to a few MeV, where Compton scattering is the main interaction mechanism (Kierans et al., 2022). Pair-conversion telescopes, which reconstruct γ -ray properties from measuring the tracks of electron-positron pairs produced in the detector, are only sensitive above 100 MeV. Below this energy, the angular resolution with this method is too bad for an effective background rejection (Gros and Bernard, 2017).

Ground-based detectors can detect γ -rays of energies above some tens of GeV and generally employ the Imaging Atmospheric Cherenkov Telescope (IACT) technique or water-Cherenkov detectors to capture EAS produced by high-energy γ -rays (Navas et al., 2024). While IACTs offer excellent sensitivity and angular resolution, they are limited by a small field of view and a low duty cycle. In contrast, water-Cherenkov detectors provide continuous operation with a large field of view, albeit at somewhat lower angular resolution. However, they also have a higher energy threshold of about 100 GeV as the γ -rays have to reach the ground (Funk, 2015). The first detection of TeV γ -rays was made by Weekes et al., 1989 from the Crab Nebula using the IACT technique at the Whipple Observatory in Amado, Arizona in 1989.

3.5.2 MeV gap

The energy range between GeV and PeV is of particular interest for cosmic-ray studies as it provides crucial insights into the most energetic astrophysical processes in the Universe, and we will mainly focus on this range later. The MeV range, however, does also offer plentiful scientific opportunities (Kierans et al., 2022). γ -rays in this range are also produced through bremsstrahlung, inverse Compton, and synchrotron and thus carry valuable information about cosmic-ray accelerators. As most of the γ -ray energy budget lies in the low-energy range, MeV-observations can also constrain the thermal and non-thermal energy budget for high-energy Galactic and extragalactic sources. Further, decays of many radioactive processes, including secondary γ -rays from β -decays, produce γ -rays in this energy range, which can constrain nuclear processes (Kierans et al., 2022).

Despite this, there is a lack of high-sensitivity γ -ray detectors in the range of 0.1 MeV to 100 MeV, sometimes referred to as the *MeV-gap* (de Angelis et al., 2018; Lucchetta et al., 2022). Reasons for this include the low interaction cross-section with matter in this energy band and large backgrounds, but also the technological challenge of building large-enough detectors to stop Compton scattered photons within the detector. The main instrument in the energy range 0.8 MeV to 30 MeV was the COMPTEL instrument (Schoenfelder et al., 1993) on the Compton Gamma Ray Observatory (CGRO). One successor will be the COSI (Tomsick et al., 2021) mission scheduled to launch in 2027.

3.5.3 Fermi-LAT

The *Fermi*-LAT instrument is part of the *Fermi* Gamma-ray Space Telescope (formerly known as the Gamma-ray Large Area Space Telescope (GLAST)). It was launched in 2008 and is still operational. The mission also includes the Gamma-ray Burst Monitor (GBM). *Fermi*-LAT serves as the successor to the EGRET (Fichtel, 1996) instrument on the CGRO, which operated from 1991 to 2000 (Atwood et al., 2009).

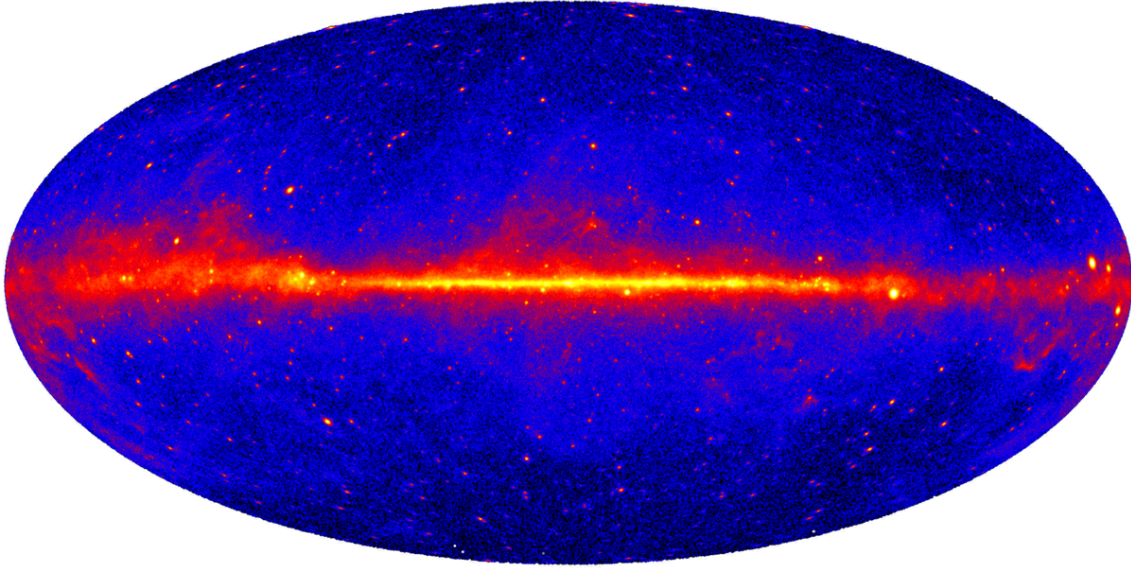


Figure 3.5: *Fermi*-LAT 5-year sky map. This figure shows the all-sky map of γ -ray emissions greater than 1 GeV as observed by *Fermi*-LAT over a period of 5 years. The map is presented in Galactic coordinates, with the Galactic plane running horizontally and the Galactic centre in the centre of the map. The map is colour-coded to represent the intensity of γ -ray emissions, with brighter regions indicating higher intensities. The figure is taken from NASA/DOE/*Fermi* LAT Collaboration, 2013 (public domain).

Fermi-LAT is a pair-conversion telescope, consisting of a precision tracker, a calorimeter, and an anti-coincidence detector. *Fermi*-LAT can detect γ -rays by measuring the tracks and energies of electron-positron pairs created upon interactions of the incident γ -rays with dense material. The energies and trajectories of these charged particles are then measured to reconstruct the incident photon's properties. The telescope's geometry is optimised so that most conversion events in the tracker are followed through the calorimeter, allowing for a precise energy reconstruction. The instrument covers an energy range from about 20 MeV to beyond 300 GeV, with a large field of view of approximately 2.4 sr (Atwood et al., 2009).

Fermi-LAT had a transformative impact on γ -ray astronomy and cosmic-ray physics (Funk, 2015). It has enabled the identification of thousands of Galactic and extragalactic sources, the characterization of the Galactic diffuse emission (Ackermann et al., 2012), and measurements of the isotropic diffuse γ -ray background (Ackermann et al., 2015). Furthermore, its observations have been a huge driver in the field of indirect Dark Matter (DM) detection. The so-called Galactic centre excess (Goodenough and Hooper, 2009; Hooper and Goodenough, 2011) has been discussed as a potential signal of DM annihilation, among other possible astrophysical explanations like a population of unresolved millisecond pulsars (Bartels et al., 2016; Lee et al., 2016). Measurements by *Fermi*-LAT placed constraints on dark matter annihilation and decay scenarios (e.g., Ackermann et al., 2014, for more examples, see Funk, 2015). The observed sources are listed in the

fourth *Fermi*-LAT source catalogue (4FGL) (Abdollahi et al., 2020). In Fig. 3.5, we can see the all-sky map of γ -ray emissions greater than 1 GeV as observed by *Fermi*-LAT over a period of 5 years.

3.5.4 Large High Altitude Air Shower Observatory

LHAASO aims to survey γ -ray sources at energies above 30 TeV and to measure the cosmic-ray spectrum and composition with high precision. γ -ray astronomy in this energy domain is closely connected to the study of cosmic-ray acceleration mechanisms and therefore plays a key role in addressing the long-standing question of the origin of cosmic rays (Cao et al., 2019).

Located at an altitude of 4410 m above sea level in the Sichuan Province in China, LHAASO began operations in 2019 and still continues to expand. The facility comprises several sub-detector systems: the KM2A array with 5195 electromagnetic particle detectors (EDs) and 1171 underground muon detectors (MDs); the water-Cherenkov detector array (WCDA), consisting of large water pools instrumented with Photomultiplier Tubes (PMTs); and the wide field-of-view Cherenkov telescope array (WFCTA) (Cao et al., 2019). An additional extension, the Large Array of imaging atmospheric Cherenkov Telescopes (LACT), is currently under development (Zhang et al., 2025b).

LHAASO has already published a range of notable results (Cao et al., 2023b), including the detection of numerous Galactic γ -ray sources (see 1LHAASO source catalogue (Cao et al., 2024b)), the identification of the first PeVatron candidates (Cao et al., 2021), and precise measurements of the diffuse γ -ray emission from the Galactic plane across TeV to PeV energies (Cao et al., 2023a, 2025). Further, it provided measurements of the GRB 221009A beyond 10 TeV (Cao et al., 2023c) and helps advance our understanding of microquasars (Cao et al., 2023b). Moreover, it has provided data on the all-particle (Cao et al., 2024a) and proton spectra of cosmic rays (The LHAASO Collaboration et al., 2025) valuable for linking direct and indirect measurements of cosmic rays.

3.6 Neutrino observatories

Neutrinos are important messengers for astrophysical phenomena. Because of their very weak interaction cross-sections, they traverse the Universe unimpededly from their sources to us. This way, they carry directional information not accessible by cosmic rays and information from opaque environments not accessible by observations via electromagnetic channels. However, their elusive nature also makes it hard to detect them. Large volumes have to be instrumented to detect products of the rare neutrino interactions, and they have to be shielded from the influence of other messengers.

Huge (at least) cubic kilometre sized detectors like the IceCube observatory (Halzen and Klein, 2010) in the Antarctic ice are used to measure the flux of neutrinos of astrophysical origin. In this section, we will introduce the working principle of such detectors, focusing specifically on the IceCube observatory, which has provided the most sensitive measurements to date. Other constructed or proposed future detectors include KM3Net (Adrián-Martínez et al., 2016) in the Mediterranean Sea, Baikal-GVD (Avrornin et al., 2018) in Lake Baikal, and P-ONE (Agostini et al., 2020) in the Pacific Ocean close to Vancouver. There are also multiple proposed future experiments in the South Chinese Sea: HUNT (Huang et al., 2023), NEON (Zhang et al., 2025a), and TRIDENT (Ye et al., 2023).

Neutrino observatories like IceCube detect Cherenkov light from charged particles produced in the interactions of neutrinos with nuclei in or close to the detector. Cherenkov radiation (Čerenkov, 1937) is produced when charged particles move through a medium faster than the respective speed of light in this medium. The superluminal motion produces an asymmetry in the radiation field caused by the deexcitation of polarised molecules in the medium. Its formation is analogous to the formation of a sonic boom caused by supersonic motion in a medium (Sigl, 2017).

3.6.1 Neutrino interactions and event topologies

Two kinds of interactions of neutrinos with nuclei in the detector can be distinguished. In neutral current (NC) interactions, an incident neutrino scatters on a nucleon. This interaction is mediated by a neutral Z boson. The interaction produces a hadronic shower and a neutrino of the same flavour as the incident one:

$$\nu_\ell (\bar{\nu}_\ell) + N \rightarrow \nu_\ell (\bar{\nu}_\ell) + X \quad \text{with } \ell \in \{e, \mu, \tau\}, \quad (3.4)$$

where N is a nucleon and X is the hadronic shower. In contrast, a charged current (CC) interaction is mediated by a charged W boson and produces a charged lepton of the same flavour as the incident neutrino in addition to a hadronic shower:

$$\nu_\ell (\bar{\nu}_\ell) + N \rightarrow \ell^- (\ell^+) + X \quad \text{with } \ell \in \{e, \mu, \tau\}. \quad (3.5)$$

Depending on the energy and flavour of the neutrinos involved, there are several event topologies that can be distinguished. We present a schematic overview of those in Fig. 3.6. If a neutrino interaction takes place in the instrumented detector volume, a hadronic cascade can be observed. In ν_e -CC interactions, the electron produces an additional electromagnetic cascade (Heitler, 1954). Compared to the much heavier muons, electrons lose their energy very fast in the detector material and do not produce pronounced tracks.

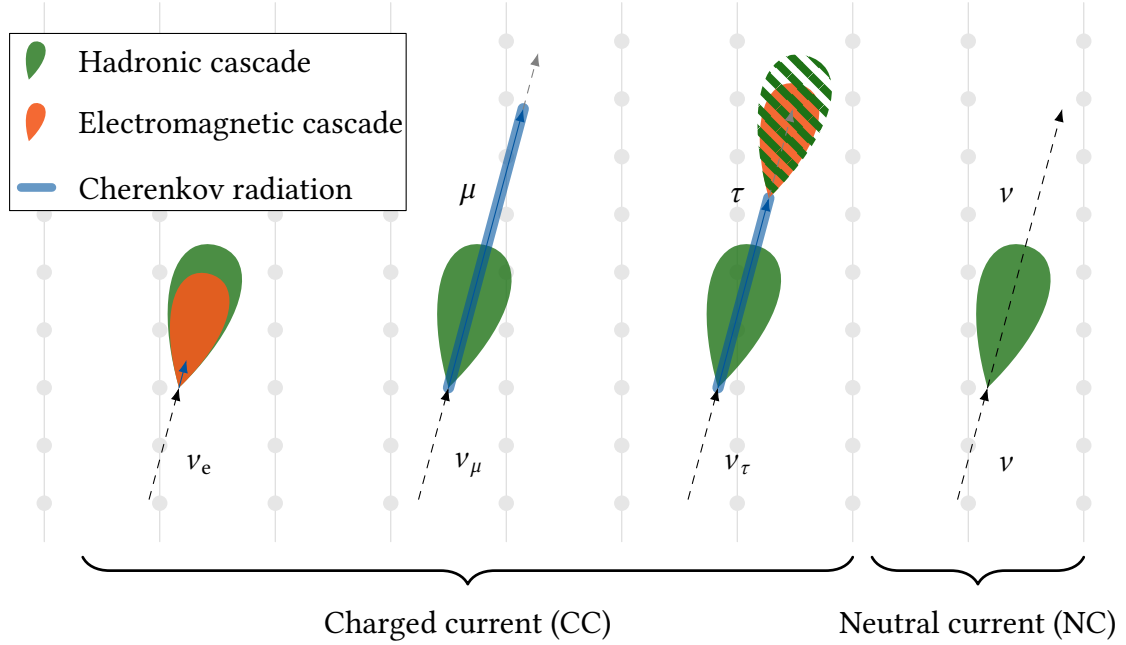


Figure 3.6: Schematic illustration of neutrino interaction topologies in detectors. The light produced by Cherenkov radiation (blue), hadronic cascades (green), and electromagnetic cascades (red) is shown. In charged current interactions, different signatures are observed for electron, muon, and tau neutrinos. Electron neutrinos produce hadronic and electromagnetic cascades. The electron produced in this interaction quickly loses its energy and does not produce a track. Muon neutrinos, however, produce a muon that can travel a longer distance in the detector before losing all its energy or decaying. If its energy is high enough, it can produce a track of Cherenkov radiation in addition to a hadronic cascade. Tau neutrinos can produce a double-bang signature if the tau produced in the interaction has enough energy to travel a measurable distance before decaying into hadronic or electromagnetic products, muons, or further neutrinos. In neutral current interactions, a hadronic cascade is observed. In the background, a grid of Digital Optical Modules (DOMs) is shown to illustrate the instrumented detector volume. The figure is modified from Reimann, 2019, Fig. 4.6 which itself is based on Hallen, 2013; Wallraf, 2010.

In a ν_μ -CC interaction, most of the energy of the incident muon-neutrino is transferred to the produced muon (Cooper-Sarkar et al., 2011). If the energy transferred to the muon is high enough, one can observe the Cherenkov radiation of a μ -track besides the hadronic cascade. This feature is especially useful for the detection of muon neutrinos. Low-energetic ν_μ -CC interactions, where the muon does not have enough energy to produce a long track, are harder to separate from ν_e -CC interactions.

Finally, ν_τ -CC interactions produce a hadronic cascade where the τ is produced. For ν_τ -energies beyond 1 PeV, a so-called *double-bang* signature can be observed. Besides the Cherenkov radiation from a τ -track, a hadronic or electromagnetic cascade initiated by the decay products of the τ might be observed in the detector (Abbasi et al., 2024). The high energy of the ν_τ is required because of the short decay length of a τ in the detector material due to their shorter lifetime.

Besides these charged and neutral current interactions, also the so-called *Glashow resonance* (Glashow, 1960) can be observed in neutrino detectors. This process occurs when an electron antineutrino interacts with an electron in the detector material to produce a W^- boson:

$$\bar{\nu}_e + e^- \rightarrow W^- . \quad (3.6)$$

This interaction has a resonance at an energy of about 6.3 PeV leading to an enhanced detection probability of electron antineutrinos at this energy. This can be used to test astrophysical neutrino production models (Navas et al., 2024). A candidate event at the Glashow resonance has already been detected by IceCube (IceCube Collaboration et al., 2021).

Several detection channels can be used by neutrino detectors. As described above, there are tracks and cascades. Depending on the actual position of the interaction point, the observed topologies can be used for different analyses. Cascades are localised and mostly spherical, with only a slight elongation along the ν -direction (Aartsen et al., 2020). They provide good information about the total energy of the incident neutrino, but have limited information about its direction. The main background for those are muons that have a single *catastrophic* interaction in the detector (Aartsen et al., 2016). Such muon-induced events are tried to be rejected.

The track topology naturally provides a much better angular resolution than cascades, but the energy resolution depends on the interaction point. For *starting tracks*, this is inside the detector volume. As also seen in Fig. 3.6, this means that the information from the μ -track as well as from the initial hadronic cascade is available. This gives a very good angular and energy resolution (Aartsen et al., 2015, 2019; Abbasi et al., 2021). If the interaction point lies outside the detector volume, tracks from high-energy muons can still pass the detector volume. These *through-going tracks* allows for a good angular resolution, but it only provides a lower bound for the neutrino energy. μ -tracks without initial interaction points are indistinguishable from atmospheric muons. However, if the tracks come from directions below the horizon, they cannot have been caused by atmospheric muons as they would have been absorbed by the Earth. Thus, tracks coming

from below the horizon must have been caused by neutrinos passing through the Earth and interacting before entering the detector. For the IceCube detector, a zenith angle cut of 85° is used as the detector is 2 km below the surface, with a lot of ice surrounding it (Abbasi et al., 2022, see also Fig. 3.7).

3.6.2 IceCube observatory

The IceCube observatory at the South Pole instruments a volume of roughly 1 km^3 of deep, ultra-transparent ice for the detection of high-energy neutrinos (Halzen and Klein, 2010). For the observation of the Cherenkov light produced by the products of the neutrino interactions (see Fig. 3.6), a total of 5160 Digital Optical Modules (DOMs) are mounted. Each DOM houses a PMT, electronics, and calibration devices within a glass pressure sphere (Aartsen et al., 2017). The DOMs are mounted on 86 vertical, kilometre-long strings, that are deployed between 1450 m and 2450 m below the surface.

In Fig. 3.7, the configuration of the IceCube observatory is shown. 78 strings are placed in a triangular grid with a horizontal spacing of 125 m, while the remaining strings are placed in a denser sub-array called DeepCore (Aartsen et al., 2017). In addition to the in-ice array, there is also the cosmic-ray air shower array IceTop on the surface (Abbasi et al., 2013). Its 162 ice-filled tanks are equipped with PMTs to detect Cherenkov radiation. Besides its function as a cosmic-ray observatory, it is also used as a partial veto of events that can be confused with downward-going neutrinos (Aartsen et al., 2017). Finally, there is the IceCube laboratory. It houses the necessary infrastructure for taking and sending the data.

To properly reconstruct the observed events, the propagation of light in the detector volume has to be calibrated very precisely. The Antarctic ice is transparent, but the exact spatially resolved properties, specifically the diffusion and absorption coefficients have to be determined. This is done by calibrating ice models with LED flashers built into the DOMs (Aartsen et al., 2013). The ice models are under constant development (Abbasi et al., 2025).

The IceCube detector registers events with a high trigger rate of about 3 kHz, but these events are strongly dominated by atmospheric muons created in cosmic-ray air showers. The second-largest component are atmospheric neutrinos created in such air showers. To extract astrophysical neutrinos from this background, different strategies are employed, and additional information from, e.g., IceTop is used (Gaisser et al., 2014; Halzen and Kheirandish, 2023; Schönert et al., 2009). For every observed astrophysical neutrino, about 100 million muons are observed (Icecube Collaboration et al., 2023).

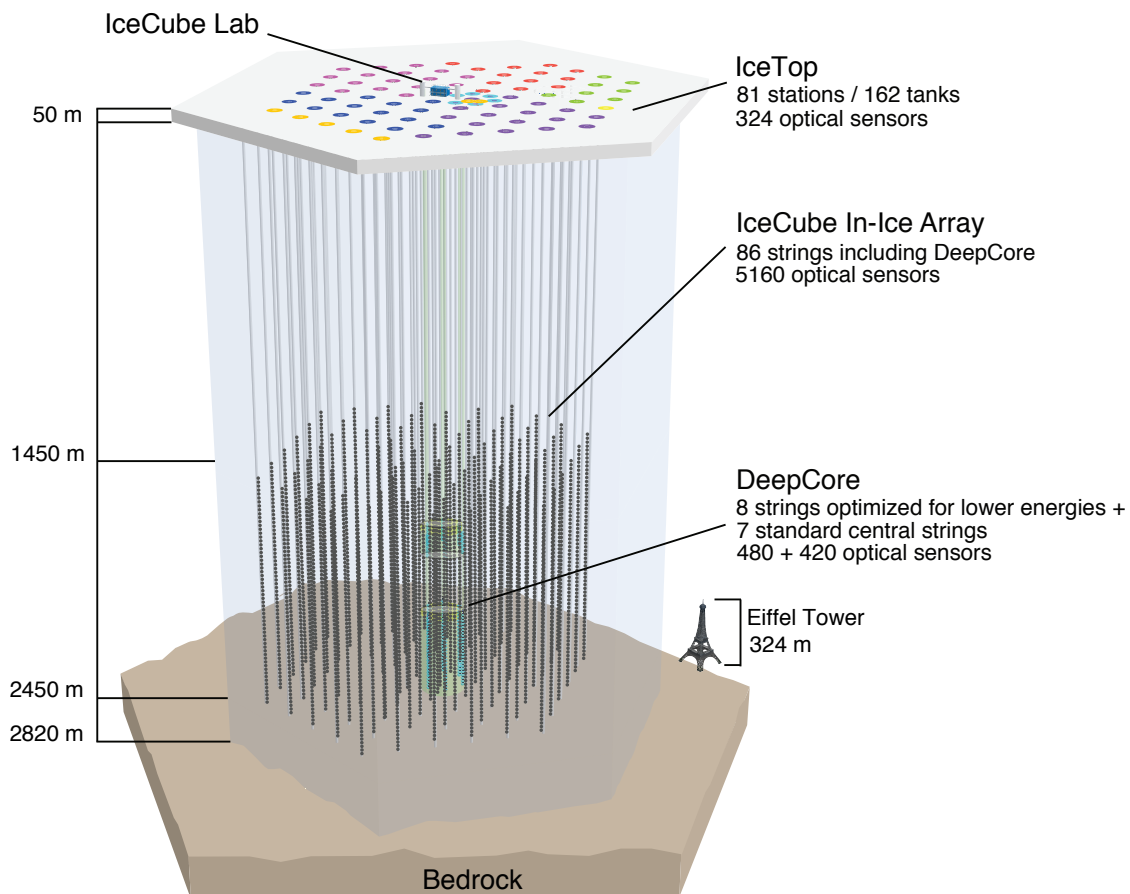


Figure 3.7: Graphic visualisation of the IceCube observatory. The different components of the observatory are shown. This includes the main strings and the strings of the DeepCore sub-array in the ice and the cosmic-ray air shower array IceTop. The Eiffel Tower is shown for scale. Used with permission of IOP Publishing Limited, from Aartsen et al., 2017, Fig. 1. (permission conveyed through Copyright Clearance Center, Inc.)

3.6.3 High-energy neutrinos measured at Earth

Neutrinos are very interesting messengers for astrophysical processes as they do not interact electromagnetically, thus pointing straight to their sources (Navas et al., 2024). In the context of this thesis, we are especially interested in astrophysical neutrinos. However, astrophysical neutrinos are a subdominant component of the flux of neutrinos arriving at Earth up to about 100 TeV. Up to this energy, atmospheric neutrinos are dominant.

Neutrinos are mainly produced by the decay of charged pions and, to a lesser extent, kaons, which can be created in hadronic interactions of cosmic rays with gas or photon fields. The decay chains of positively and negatively charged pions are given by:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad \text{followed by} \quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu, \quad (3.7)$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad \text{followed by} \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu. \quad (3.8)$$

Although an electronic decay of the pion would be energetically allowed, it is suppressed by roughly 10^{-4} . Angular momentum conservation forces the lepton in the pion-decay in the wrong helicity state as neutrinos and antineutrinos always have to be left-chiral. Roughly speaking, this is easier to accommodate for higher-mass particles. Thus, the decay is suppressed for small lepton masses (Sigl, 2017).

Atmospheric neutrinos originate from products of cosmic-ray interactions in the Earth's atmosphere. So-called *conventional* atmospheric neutrinos are produced by the decay of pions, and, to a lesser extent, kaons, that are produced by interactions of cosmic rays with nuclei in our atmosphere. They can reach the neutrino detector from all directions. The *conventional* atmospheric neutrino component dominates the neutrino flux measured at Earth in the GeV to TeV region. Its flux follows a power law: $dJ_\nu/dE \propto E_\nu^{-3.7}$. The spectral index is related to the spectral index of the cosmic-ray spectrum, but it is softer by ≈ 1 , because of interactions of the pions (and kaons) with the atmosphere (Gaisser et al., 2016; Honda et al., 2007; Volkova and Zatsepin, 2001). At higher energies, a more exotic *prompt* atmospheric neutrino component becomes relevant. This originates from the decay of charmed mesons. As this decay happens much faster than the decay of pions, the charmed mesons practically do not interact in the atmosphere. Thus, the *prompt* neutrino spectrum almost directly follows the power law of the cosmic ray component with a harder spectral index of $\gamma \approx -2.7$. This makes it the dominant atmospheric muon neutrino component at a few PeV (Böttcher, 2025).

In 2013, a high-energy astrophysical neutrino component was detected for the first time (IceCube Collaboration, 2013). Neutrinos are expected to be produced in various astrophysical environments, mainly through the decay of pions produced in interactions of highly energetic particles. In the analyses of neutrino data, a diffuse extragalactic neutrino component sourced outside the Milky Way with a power law index $\gamma \in [-2.5, -2.2]$ has been detected. The flavour composition of those neutrinos can help to identify the processes causing these neutrino emissions. The origin through pion decay as described in Eqs. (3.7) and (3.8) is favoured (Navas et al., 2024).

Additionally, individual neutrino point sources were tried to be identified. Most source candidates are associated with AGNs. E.g., a flare of the blazar TXS0506+056 was found to be coincident with a high-energy neutrino event at the 3σ level (IceCube Collaboration et al., 2018a), and further evidence at 3.5σ for neutrino emission from this source was found in archival IceCube data (IceCube Collaboration et al., 2018b). A coincidence search with γ -ray sources has also resulted in a 4.2σ evidence for neutrino emissions associated with the Seyfert-2 galaxy NGC 1068 (IceCube Collaboration et al., 2022). This observation is valuable for testing AGN models, as the inferred neutrino flux is much higher than the related observed γ -ray flux, which suggests that the electromagnetic emissions are obscured (Navas et al., 2024).

Most important for this work, there have also been searches for neutrino emissions from the Galactic plane. Just as interactions of cosmic rays with the gas component in the Galaxy predict the emissions of γ -rays through the decay of neutral pions produced in those collisions, neutrinos are produced in the decay of charged pions as described in Eqs. (3.7) and (3.8). In 2023, IceCube has measured a Galactic neutrino flux with a significance of 4.5σ (Icecube Collaboration et al., 2023). As the Galactic centre lies in the southern sky, which is more exposed to atmospheric muon backgrounds in the IceCube detector, cascade events were used in the analysis. Those only have atmospheric neutrinos as their main background. With the help of machine learning techniques applied to 10 yr of Ice Cube data, an excess of neutrino emissions from the Galactic plane was found. For this analysis, different diffuse emission templates were used (Icecube Collaboration et al., 2023). So far, the IceCube observatory is the only neutrino detector that was able to measure the Galactic diffuse neutrino emission with this significance. We will come back to this measurement in Part III.

Neutrinos at the highest energies are also of great interest. The highest energetic neutrinos that are expected to be observed are so-called *cosmogenic* neutrinos. At EeV energies, the neutrino flux is expected to be dominated by neutrinos produced in the interactions of UHE cosmic rays with the CMB, the GZK effect (Beresinsky and Zatsepin, 1969). To date, the highest energy neutrino candidate event has been KM3-230213A (KM3NeT Collaboration et al., 2025) registered by the ARCA detector of KM3NeT at an energy of 120^{+110}_{-60} PeV. For this event, many possible origins are discussed, and its origin remains an open question.

4 Paradigms of cosmic-ray theory

In the previous chapter, we have given a brief overview of the wealth of observations in the field of cosmic-ray astrophysics. These observations have shaped our understanding of the sources, acceleration mechanisms, and transport processes of cosmic rays, and they are constantly used to build and challenge theoretical models. In this section, we want to describe the modern theoretical framework that is used in cosmic-ray astrophysics and establish the so-called *paradigms* of cosmic-ray physics (Gabici et al., [2019](#)).

Those theoretical paradigms are a group of fundamental insights that have been gained in the last century and still guide our understanding of some of the most pressing questions of the field:

- What is the origin of cosmic rays?
- Which classes of astrophysical objects source the measured flux of cosmic rays?
- How is the power-law spectrum of cosmic rays, extending to such high energies, produced?
- How does the transport of cosmic rays work?
- How do we explain the difference in composition between cosmic rays and the Solar environment?

A basic understanding of these questions has been gained, but the wealth of direct and indirect observations that has become available in the past few decades has posed many new questions and challenged some of the *orthodoxy* developed in the field of cosmic-ray theory (Gabici et al., [2019](#)). We will lay out the most fundamental insights from the field and mention some of the challenges posed by modern observations. For more some general reviews on cosmic-ray physics, we refer to, e.g, Gabici et al., [2019](#); Grenier et al., [2015](#); Strong et al., [2007](#) or the textbooks by Berezhinskii et al., [1990](#); Gaisser et al., [2016](#); Ginzburg and Syrovatskii, [1964](#).

4.1 Galactic origin

In Sec. 3.1, we have given an overview of the measured cosmic-ray fluxes. This naturally poses the question, where these cosmic rays originate from. Possible origins can be summed up in three different models that were discussed in the past (Ginzburg, 1996): (1) Solar origin, (2) Galactic origin, and (3) meta-Galactic origin.

The Solar origin hypothesis suggests that cosmic rays originate in the Sun and fill the heliosphere. However, this idea has only been considered before the advent of cosmic-ray astrophysics in the 1950s (as discussed in Sec. 3.2). With the observation of radio emissions linked to the presence of cosmic-ray electrons in other parts of the Galaxy, there was evidence that cosmic rays pervade the entire Milky Way roughly uniformly. Further contradictions of a purely Solar origin arise from the composition of cosmic rays that is very different from the Solar composition (see Sec. 3.1.2) (Ginzburg and Syrovatskii, 1964). Still, the Sun is an important source for so-called Solar energetic particles (Reames, 2021) at energies below 1 GeV.

While the Galactic origin model assumes that cosmic rays observed at Earth are produced by sources in our own Galaxy, the meta-Galactic origin model suggests that cosmic rays are produced outside of it and pervade the entire Universe. It turns out that a closer investigation of the measured cosmic rays at the Earth's position can help to answer this question. In Sec. 3.1.1, we have seen the measured spectrum of cosmic rays. This gives us the differential intensity, $\Phi_{E_{\text{kin}}}$, from which we can determine the total integrated intensity of cosmic rays, I_{CR} :

$$I_{\text{CR}} = \int_{E_{\text{kin}, \text{min}}}^{\infty} dE_{\text{kin}} \Phi_{E_{\text{kin}}}(E_{\text{kin}}) \approx 0.2 - 0.3 \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}, \quad (4.1)$$

depending on the choice of the lower integration limit, $E_{\text{kin}, \text{min}} \approx 1 \text{ GeV}$. Similarly, the energy density of cosmic rays, w_{CR} , can be determined from the differential intensity (Ginzburg and Syrovatskii, 1964):

$$w_{\text{CR}} = \frac{4\pi}{c} \int_{E_{\text{kin}, \text{min}}}^{\infty} dE_{\text{kin}} E_{\text{kin}} \Phi_{E_{\text{kin}}}(E_{\text{kin}}) \approx 1 \text{ eV cm}^{-3}. \quad (4.2)$$

Due to the falling power law of the cosmic-ray spectrum, the energy density is dominated by cosmic rays at GeV energies.

Under the assumption of the meta-Galactic origin model, this energy density measured at Earth should be representative for the energy density of cosmic rays in and in-between galaxies. If we take the local energy density of cosmic rays as a proxy for the Galactic energy density of cosmic rays, we have:

$$w_{\text{CR, meta-Gal}} \approx w_{\text{CR, Galactic}} \equiv w_{\text{CR}} \approx 1 \text{ eV cm}^{-3}, \quad (4.3)$$

which is rather high¹ and hard to obtain for the whole meta-Galactic volume, even with the most powerful sources (Ginzburg and Syrovatskii, 1964). This indicates that the cosmic-ray energy can likely not be uniformly distributed on meta-Galactic scales.

However, there are more direct arguments against a meta-Galactic origin of the bulk of cosmic rays. The Universe is permeated by many radiation fields, most prominently by the CMB, discovered by Penzias and Wilson, 1965. The presence of these radiation fields puts strong bounds on the distance relativistic electrons of energies above 1 GeV can traverse before losing their energy due to Inverse Compton (IC) scattering. Even cosmic-ray electrons from the nearest neighbouring radio galaxy Centaurus A, at a distance of about 4 Mpc, could not reach us before losing their energy (Berezinskii et al., 1990). The presence of cosmic-ray electrons in our own Galaxy thus strongly suggests that those electrons must be produced in our own Galaxy. Further assuming that hadronic cosmic rays are produced in the same class of sources as electrons, this leads to the conclusion that at least some of the locally measured cosmic rays must originate from sources in our own Galaxy (Berezinskii et al., 1990; Ginzburg and Syrovatskii, 1964).

A more direct observational proof against a meta-Galactic origin of the hadronic cosmic-ray component is provided by observations of the small and the large Magellanic clouds in γ -rays. The argument goes as follows (Ginzburg, 1972): If Eq. (4.3) was true, the energy density of cosmic rays in the Magellanic clouds would cause a measurable flux of diffuse γ -rays through the decay of neutral pions produced in hadronic interactions of cosmic rays with the gas in these galaxies. These diffuse emissions should only scale with the gaseous mass of the Magellanic clouds and their distance from us. Observations by EGRET (Sreekumar et al., 1993) and more recently by *Fermi*-LAT (Abdo et al., 2010) have confirmed that the observed γ -ray fluxes fall short of the fluxes expected under the meta-Galactic origin hypothesis. Thus, they suggest that the cosmic-ray density differs significantly on meta-Galactic scales, and that the bulk of the local cosmic-ray energy density is produced by Galactic sources.

If we exclude the possibility that the flux of cosmic rays on meta-Galactic scales is highly anisotropic, the only remaining option is that cosmic rays observed at Earth are predominantly produced by sources in our own Galaxy. For more details on this argument, we refer to Ginzburg and Syrovatskii, 1964. Note, however, that these arguments for the Galactic origin of cosmic rays concern the bulk of cosmic rays. UHE cosmic rays of energies in excess of PeV are in fact believed to originate from extragalactic sources (Gabici et al., 2019).

¹This value puts the cosmic-ray energy density in the same range as many other energy densities in the ISM like the magnetic field energy density, the CMB, starlight, etc. (Draine, 2011). We will come back to this later in Sec. 5.2.

4.2 Grammage and lifetime

In the preceding section, we have given arguments for the Galactic origin of the bulk of cosmic rays observed at Earth. In this section, we want to understand, how long cosmic rays stay in the Galaxy on average. This is crucial for understanding the power that cosmic-ray sources have to provide to maintain the measured cosmic-ray energy density and to constrain Galactic cosmic-ray transport models. We want to sketch how this information can be extracted from cosmic-ray data.

In Sec. 3.1.2, we have introduced the measurements of so-called cosmic-ray primaries and secondaries. Primaries are defined as the cosmic-ray component that is directly accelerated in the cosmic-ray sources, while secondaries are produced in spallation reactions during the transport of cosmic rays. One such spallation reaction would be the production of boron from the spallation of a ^{12}C nucleon on a nucleon X of the ISM to produce ^{11}B and some other spallation products Y :



The kinetic energy per nucleon is approximately conserved in this reaction according to the so-called *straight-ahead-approximation*. This approximation is good enough for this derivation, but note that recoils of the fragments can lead to a broadening of the energy distribution (Tsao et al., 1995).

The production of secondaries is governed by the ISM material that the respective primaries traverse during their residence time in the Galaxy. To quantify this, we define the quantity *grammage*, χ , as:

$$\chi = \int ds \rho_{\text{ISM}}(s), \quad (4.5)$$

where s is the path length traversed by the cosmic ray and ρ_{ISM} is the mass density of the ISM. Thus, grammage has the dimension of mass per area and measures the amount of ISM material traversed by the cosmic-ray particle. In an approximate way, Eq. (4.5) can be rewritten as:

$$\chi \approx \rho_{\text{ISM}} L = m n_{\text{ISM}} \tau_{\text{res}} v, \quad (4.6)$$

where L is the total path length traversed by the cosmic ray, $m \approx 1.4 m_p$ is the mean target mass in the ISM for a composition of 90 % hydrogen and 10 % helium, and n_{ISM} is the number density of the ISM. Further, $L = \tau_{\text{res}} v$, where τ_{res} is the residence time of cosmic rays in the Galaxy and v is the velocity of the cosmic ray. In the energy range we are considering here, we usually have $v \simeq c$.

Some elements like boron are almost exclusively produced as secondaries (see Sec. 3.1.2). The differential rate of production for such secondaries, q_s , can be approximated as:

$$q_s(E) \approx \psi_{E_{\text{kin}},p}(E') n_{\text{ISM}} v \sigma_{p \rightarrow s}, \quad (4.7)$$

where $\psi_{E_{\text{kin}},p}(E')$ is the differential density of the primary species at an energy E' that is related to the energy E according to the *straight-ahead-approximation*, and $\sigma_{p \rightarrow s}$ is the cross-section for the spallation process.

Assuming a steady state situation, the differential density of secondaries $\psi_{E_{\text{kin}},s}$ can be approximated as:

$$\psi_{E_{\text{kin}},s}(E) \approx q_s(E) \tau_{\text{res}}. \quad (4.8)$$

Combining Eqs. (4.7) and (4.8) and using Eq. (4.6), we can express the secondary-to-primary ratio as:

$$\frac{\psi_{E_{\text{kin}},s}}{\psi_{E_{\text{kin}},p}} \approx \sigma_{p \rightarrow s} n_{\text{ISM}} \tau_{\text{res}} v = \frac{\sigma_{p \rightarrow s} \chi}{m}. \quad (4.9)$$

The number density of the ISM is known to be highest in the Galactic disk, where it has an approximate value of $n_{\text{ISM}} \approx 1 \text{ cm}^{-3}$ (Draine, 2011). The gaseous Galactic disk has a half-thickness of $h \approx 100 \text{ pc}$, or more precisely, the average surface density of the ISM material in the disk is $\mu_{\text{disk}} \approx 2.3 \cdot 10^{-3} \text{ g cm}^{-2}$ (Ferrière, 2001).

The measurements of secondary-to-primary ratios such as B/C suggest that the grammage for cosmic rays of a rigidity of $\mathcal{O}(10 \text{ GV})$ is about $\mathcal{O}(10 \text{ g cm}^{-2})$ (Aguilar et al., 2018). Via Eq. (4.6), it is possible to estimate the residence time of cosmic rays in the Galaxy to be $\mathcal{O}(\text{Myr})$ (Gabici et al., 2019). For more detailed calculations and reviews on this topic, we refer to Berezhinskii et al., 1990; Evoli et al., 2019; Gaisser et al., 2016.

It should be noted that the residence time τ_{res} estimates the time, a cosmic ray spends in a region with a density n_{ISM} . Under our assumptions, this is the time, that cosmic rays spend in the Galactic disk. This estimate is based on the assumption that the cosmic ray solely accumulates grammage by traversing the Galactic disk with a roughly homogeneous density. The approximate relation (4.6) does not take into account the possibility that a large fraction of the grammage might have been already accumulated in the vicinity of the sources (Gabici et al., 2019). However, cosmic rays might also spend a significant fraction of their lifetime in the Galaxy in low-density regions where they do not accumulate a lot of grammage. We will explore this further in the next section.

4.3 Diffusive transport in the Galactic halo

A comparison of the accumulated grammage and the surface density of the Galactic disk indicates that the average cosmic ray crosses the Galactic disk multiple times. If the grammage was just accumulated by crossing the Galactic disk vertically by travelling in straight lines, the average cosmic ray would cross the disk about $N \approx \chi/\mu_{\text{disk}} = \mathcal{O}(10^3)$ times. This raises the question of how cosmic rays are transported in the Galaxy to achieve this. If they are not travelling on straight lines, they must be scattered or deflected in some way.

Further, we just pointed out in Sec. 4.2 that τ_{res} just gives us an estimate for the time an average cosmic-ray particle spends in the Galactic disk. In addition, we are also interested how this timescale compares to the total time cosmic rays spend in the Galaxy before escaping, τ_{esc} . For this, radioactive cosmic-ray isotopes can be used as so-called *cosmic-ray clocks*. The prime example for this is the ratio of the radioactive isotope ^{10}Be to the stable isotope ^9Be . Both are produced as secondaries in spallation reactions. ^{10}Be decays to ^9B with a half-life of about $\tau_{^{10}\text{Be}} \approx 1.5 \text{ Myr}$, which is comparable to our estimate for τ_{res} .

The measurement of the ratio of $\phi_{E_{\text{kin}}, ^{10}\text{Be}}$ to $\phi_{E_{\text{kin}}, ^9\text{Be}}$ gives us a way to test the relation between the cosmic-ray escape time, τ_{esc} , and $\tau_{^{10}\text{Be}}$: If $\tau_{\text{esc}} < \tau_{^{10}\text{Be}}$, the ratio between the two isotopes should be close to the production ratio. However, we find that the measured density of ^{10}Be is clearly suppressed compared to the ratio expected from production. Thus, we can conclude that $\tau_{\text{esc}} > \tau_{^{10}\text{Be}} \approx \tau_{\text{res}}$ (Gaisser et al., 2016). In a careful quantitative analysis, Garcia-Munoz et al., 1977 found that $\tau_{\text{esc}} \approx 17 \text{ Myr}$.

This suggests that cosmic rays spend most of their lifetime in the Galaxy outside the Galactic disk in under-dense regions. This is usually referred to as the so-called *cosmic-ray halo*. A further proof for the existence of such cosmic-ray halos was given by radio astronomical observations of other Galaxies. E.g., Ekers and Sancisi, 1977 reported the observation of a radio continuum halo in the galaxy NGC 4631. Also, γ -ray observations by SAS-2 could be used to strengthen the hypothesis of a cosmic-ray halo in our own Galaxy (Stecker and Jones, 1977).

Still, the question about the transport mechanism of cosmic rays in the Galaxy remains open. For this, the isotropy in the arrival directions of cosmic rays as presented in Sec. 3.1.3 can be used. We argued that the bulk of cosmic rays up to PeV energies originates from sources in our own Galaxy. The observed isotropy suggests that the transport of cosmic rays in the Galaxy is randomising their directions efficiently. This points cosmic rays undergoing a random walk through the Galaxy before reaching the Earth.

However, this scattering cannot be caused by interactions with nuclei in the ISM, as the mean free path for Coulomb interactions is much too large to cause this effect:

$$\lambda_{\text{Coulomb}} = \frac{1}{n_{\text{ISM}} \sigma_{\text{T}}} \approx 500 \text{ kpc} \left(\frac{n_{\text{ISM}}}{\text{cm}^{-3}} \right)^{-1}, \quad (4.10)$$

where $\sigma_{\text{T}} \approx 6.65 \cdot 10^{-25} \text{ cm}^2$ is the Thomson cross-section. Thus, another scattering mechanism must be responsible for the transport in the Galaxy.

If we compare the spectra of different cosmic-ray species as presented in Fig. 3.2, we see that all primaries and all secondaries, respectively, have a very similar spectral shape when presented as a function of rigidity. This can be taken as a hint that the transport of cosmic rays in the Galaxy depends mainly on their rigidity, rather than on other variables such as their total energy, kinetic energy, or kinetic energy per nucleon (Gabici et al., 2019). As rigidity is defined as momentum per charge, this points towards a scattering mechanism depending on electromagnetic interactions.

Astrophysical plasmas are ubiquitous, and the ISM itself is a magnetised plasma (Draine, 2011). Thus, electromagnetic interactions of cosmic rays with this plasma are a natural possibility for explaining the transport of cosmic rays in the Galaxy. A theory of plasma astrophysics was developed in the past decades (see e.g., Fitzpatrick, 2022; Schlickeiser, 2002), and it has been shown that cosmic-ray observations can be explained well under the paradigm of diffusive transport of cosmic rays in a magnetised Galactic halo (Gabici et al., 2019). The height of this halo, H , is usually estimated to be a few kpc (Evoli et al., 2020). Today, the theoretical framework of plasma physics is one of the pillars of cosmic-ray theory. We will give more details on the theory of cosmic-ray transport, acceleration, and escape in Chap. 5.

Following this paradigm, we can already see that UHE cosmic rays of rigidities exceeding $10^{18} \text{ V} = 1 \text{ EV}$ cannot be confined in the Galaxy. Similarly to Eq. (A.16) in App. A.2, we can estimate the Larmor radius of a cosmic ray in the Galactic magnetic field as:

$$r_{\text{L,kpc}} \approx \mathcal{R}_{18} B_{\mu\text{G}}^{-1}, \quad (4.11)$$

where $r_{\text{L,kpc}}$ is the Larmor radius in units of kpc, $\mathcal{R}_{18}$ is the rigidity in units of 10^{18} V , and $B_{\mu\text{G}}$ is the magnetic field strength in units of μG . For typical values of the Galactic magnetic field of a few μG , cosmic rays with rigidities above a few EV have Larmor radii comparable to the size of the Galactic halo and thus, cannot be confined in the Galaxy. Hence, it is usually assumed that cosmic rays of higher energies must have an extra-Galactic origin (Ginzburg and Syrovatskii, 1964). On the other hand, this also means that cosmic rays of energies below PV rigidities, which have Larmor radii below $\mathcal{O}(\text{pc})$, can be efficiently confined in the Galaxy. This is usually reflected in naming the cosmic rays of rigidities below the so-called cosmic-ray knee at about 3 PV *Galactic cosmic rays*, while naming cosmic rays with energies above the so-called ankle at about $3 \cdot 10^{18} \text{ V}$ *extragalactic cosmic rays*. The rigidity region in-between is usually understood to be a transition between the two (Gabici et al., 2019).

4.4 Supernova remnant paradigm

In the previous sections, we have discussed the paradigms of the Galactic origin of cosmic rays and their diffusive transport in the Galactic halo. This still leaves the question of the actual sources of cosmic rays open. The so-called *supernova remnant paradigm*, which states that Galactic cosmic rays are mainly accelerated in SNRs, is based on an energy-budget argument, the presence of shocks for Diffusive Shock Acceleration (DSA), and direct observations of signatures of particle acceleration at those objects. In this section, we will summarise those arguments and briefly discuss for further source candidates.

Now that we have an estimate for the total time, cosmic rays spend in the Galaxy, $\tau_{\text{esc}} \approx 17 \text{ Myr}$, and a value for the local energy density of cosmic rays, $w_{\text{CR}} \approx 1 \text{ eV cm}^{-3}$, we can estimate the total power Galactic cosmic-ray sources have to provide to sustain

this. We also know that cosmic rays are confined in a Galactic halo. The exact height of this halo is under debate, but current estimates point towards a value of $H \lesssim 5$ kpc (Evoli et al., 2020). Thus, the total cosmic-ray power, P_{CR} , can be estimated as:

$$P_{\text{CR}} = \frac{w_{\text{CR}} V_{\text{halo}}}{\tau_{\text{esc}}} \approx \frac{1 \text{ eV cm}^{-3} 3500 \text{ kpc}^3}{20 \text{ Myr}} \approx 2 \cdot 10^{41} \text{ erg s}^{-1}, \quad (4.12)$$

where $V_{\text{halo}} \approx \pi (15 \text{ kpc})^2 \cdot 5 \text{ kpc} \approx 9.5 \cdot 10^{67} \text{ cm}^3$ is the volume of the cosmic-ray halo, assuming a Galactic radius of 15 kpc. The conversion factors can be found in the overview on page XXIII.

To satisfy this large power requirement, Supernovae (SNe) were proposed as cosmic ray sources already by Baade and Zwicky, 1934a,b. However, their estimates were based on the assumption that cosmic rays are of extragalactic origin as ‘none of the objects of our Milky Way seem to produce any cosmic rays.’ (Baade and Zwicky, 1934a) In the context of the Galactic origin of cosmic rays, Shklovskii, 1953; Ter Haar, 1950 argued that the kinetic energy released in SN explosions would provide plenty of energy to power a Galactic cosmic-ray population. A typical SN releases about $E_{\text{SN}} \approx 10^{51}$ erg of kinetic energy. With a Galactic SN rate of about $f_{\text{SN}} \approx 0.03 \text{ yr}^{-1}$, the total power provided by SNe in the Galaxy is:

$$P_{\text{SN}} = f_{\text{SN}} \cdot E_{\text{SN}} \approx 0.03 \text{ yr}^{-1} \cdot 10^{51} \text{ erg} \approx 10^{42} \text{ erg s}^{-1}. \quad (4.13)$$

If only $\mathcal{O}(10\%)$ of this power went into the acceleration of cosmic rays², SNRs alone could power the entire Galactic cosmic-ray population. It turns out that the strong shock fronts present in SNRs can efficiently accelerate particles through DSA. The theoretical framework is based on so-called *first-order Fermi acceleration*, which is a modification of acceleration mechanism originally proposed by Fermi, 1949, which is now commonly known as *second-order Fermi acceleration* (Malkov and Drury, 2001). DSA produces a power-law spectrum of accelerated particles, which is very useful for explaining the measured power-law spectrum of cosmic rays (Gabici et al., 2019). We will discuss DSA in more detail in Sec. 5.5.

Finally, there is observational evidence for cosmic-ray acceleration in SNRs. Observations of non-thermal emissions in the X-ray and γ -ray bands reveal the presence of nonthermal very high-energy particles (Berezinskii et al., 1990). This can be seen as clear evidence for particle acceleration in SNRs (Drury et al., 1994). Observations of non-thermal X-ray emissions dominated by synchrotron radiation from SN 1006 with the ASCA satellite (Koyama et al., 1995) provided first evidence for the acceleration of electrons to energies of about 100 TeV. Later, TeV γ -rays from the same object were observed with the CANGAROO 3.8 m telescope (Tanimori et al., 1998), further confirming the presence of very high-energy particles. Further γ -ray observations of the SNR RX J1713.7–3946 with H.E.S.S. (Aharonian et al., 2004) revealed a shell-like morphology of TeV γ -ray

²This fraction is the acceleration efficiency and is usually denoted by η_{CR} .

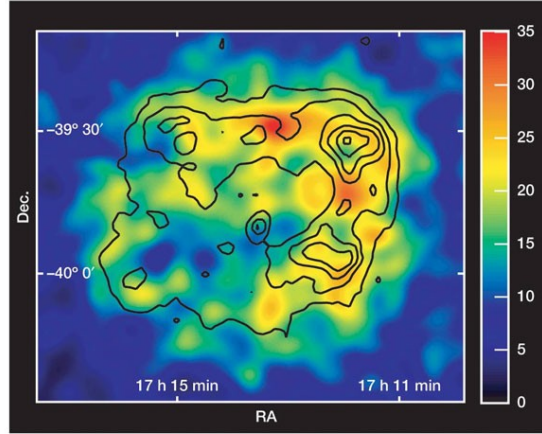


Figure 4.1: γ -ray image of the SNR RX J1713.7–3946 obtained with the HESS telescopes. This image shows γ -ray emissions from this SNR in the energy range above 800 GeV. The shell-like morphology is clearly visible and indicates particle acceleration at the shock front of the SNR. The linear colour scale is in units of counts, the angular coordinates are right ascension (RA) and declination (Dec). The contour lines indicate the X-ray intensity as measured with the ASCA satellite in the energy range between 1 keV and 3 keV (Uchiyama et al., 2002). Used with permission of Springer Nature, from Aharonian et al., 2004, Fig. 2; permission conveyed through Copyright Clearance Center, Inc.

emissions, which is in line with the predictions of DSA (Aharonian et al., 2006). This observation is shown in Fig. 4.1. Other observations that support the SNR paradigm include the detection of the characteristic pion-decay signature in the γ -ray spectra of SNRs (Ackermann et al., 2013; Jogler and Funk, 2016).

Other source candidates like explosive phenomena in the Galactic centre (Ptuskin and Khazan, 1981), PWNe, microquasars, or stellar winds (Casse and Paul, 1980) are discussed as well. In the last years, star clusters gained more attention as possible cosmic-ray sources, as DSA at the wind termination shock might efficiently accelerate cosmic rays to PeV energies (Morlino et al., 2021).

The SNR paradigm faces observational as well as theoretical challenges, as it is not clear whether SNRs can accelerate cosmic rays to energies up to the cosmic-ray knee at about 3 PV. On the one hand, γ -ray observatories like LHAASO have not firmly identified SNRs as so-called *PeVatrons*, i.e., sources that can accelerate cosmic rays to PeV energies (Cao et al., 2021). On the other hand, it is challenging to explain theoretically how cosmic rays can be accelerated to energies up to the cosmic-ray knee at about 3 PV in SNRs at all. We will discuss this further in Sec. 5.6. While SNRs might not suffice as the only sources of Galactic cosmic rays, there is little doubt that they are the prototypical sources and accelerate the bulk of Galactic cosmic rays (Gabici et al., 2019).

4.5 Dimensional analysis arguments

Before we will introduce the theoretical framework of cosmic-ray transport and acceleration in more detail in Chap. 5, we want to end this chapter with a few useful dimensional analysis arguments that help us with the understanding of the interplay between cosmic-ray acceleration and transport. This will provide us with a useful intuition for interpreting features in the cosmic-ray spectrum. We will also discuss the significance of measurements of secondary-to-primary and unstable-to-stable ratios in this context.

The following discussion is based on Evoli and Dupletsa, 2023. The propagated differential density can be approximated in the steady-state case as:

$$\text{Density} \sim \text{Injection Rate} \cdot \frac{\text{Relevant lifetime}}{\text{Relevant volume}} \propto Q(\mathcal{R}) \cdot \frac{\tau(\mathcal{R})}{V(\mathcal{R})}, \quad (4.14)$$

where Q is the source spectrum, τ is the relevant lifetime, and V is the relevant volume. All these quantities can depend on the rigidity, $\mathcal{R}$, of the cosmic rays.

The injection rate, $Q(\mathcal{R})$, is dependent on whether the cosmic rays are primaries or secondaries. For primaries, it is a source spectrum that follows a power law in rigidity as predicted by DSA (see Sec. 5.5). For secondaries, on the other hand, the production mechanism is spallation during transport (see Eq. (4.7)).

The relevant lifetime, $\tau(\mathcal{R})$, and volume, $V(\mathcal{R})$, are determined by the dominant loss mechanism for the cosmic rays. For stable cosmic rays, this is usually the escape from the Galaxy. As the transport is diffusive, the escape timescales as $\tau_{\text{esc}} \propto H^2/\kappa(\mathcal{R})$, where H is the halo height and $\kappa(\mathcal{R})$ is the rigidity-dependent diffusion coefficient. The relevant volume is the volume of the cosmic-ray halo, $V_{\text{halo}} \propto H$. For unstable cosmic rays at sufficiently low energies, the relevant lifetime is determined by the half-life of the respective isotope in the observer frame, $\tau_d(\mathcal{R})$, which leads to a relevant volume, scaling with the halo height, of $V(\mathcal{R}) \propto \sqrt{\kappa(\mathcal{R}) \tau_d(\mathcal{R})}$.

With this, we can give approximate expressions for the differential densities of primary, secondary, and unstable cosmic rays:

- Primaries³:

$$\psi_{\text{prim}}(\mathcal{R}) \propto Q_{\text{prim}}(\mathcal{R}) \cdot \frac{H}{\kappa(\mathcal{R})}, \quad (4.15)$$

- secondaries:

$$\psi_{\text{sec}}(\mathcal{R}) \propto \psi_{\text{prim}}(\mathcal{R}) \sigma_{p \rightarrow s} v n_{\text{disk}} V_{\text{disk}} \cdot \frac{H}{\kappa(\mathcal{R})}, \text{ and} \quad (4.16)$$

³This relation is very useful for understanding the dependence of the measured cosmic-ray spectrum on the source spectrum and the diffusion coefficient. In App. D, we will show that this relation can indeed be derived as a steady-state solution from the full transport equation given in Sec. 5.4.

- unstable secondaries:

$$\psi_{\text{unst}}(\mathcal{R}) \propto \psi_{\text{prim}}(\mathcal{R}) \sigma_{p \rightarrow s} v n_{\text{disk}} V_{\text{disk}} \cdot \sqrt{\frac{\tau_d(\mathcal{R})}{\kappa(\mathcal{R})}}. \quad (4.17)$$

With these relations, we can directly see, how measurements of cosmic-ray ratios can constrain the various parameters of cosmic-ray transport:

- Stable secondary-to-primary ratio (e.g. B/C):

$$\frac{\psi_{\text{sec}}(\mathcal{R})}{\psi_{\text{prim}}(\mathcal{R})} \propto \chi(\mathcal{R}) \propto \frac{H}{\kappa(\mathcal{R})} \quad \text{and} \quad (4.18)$$

- unstable-to-stable ratio (e.g. $^{10}\text{Be}/^{9}\text{Be}$):

$$\frac{\psi_{\text{unst}}(\mathcal{R})}{\psi_{\text{prim}}(\mathcal{R})} \propto \frac{\sqrt{\kappa(\mathcal{R})}}{H}. \quad (4.19)$$

Measurements of both ratios break the degeneracy between the halo height, H , and the normalisation of the diffusion coefficient $\kappa(\mathcal{R})$ that is present when only looking at stable secondary-to-primary ratios. Mind, however, that the scaling relations given here are approximations and depend on the dominant relevant lifetime. The decay timescale of unstable isotopes is strongly dependent on the particle's energy due to time dilation effects: $\tau_d(\mathcal{R}) = \gamma \tau_0$, with γ being the Lorentz factor. For more details, see [Evoli et al., 2020](#).

5

Transport, acceleration, and escape

The theory of cosmic-ray transport and acceleration is based on the interaction of charged particles with turbulent magnetic fields in astrophysical plasmas. In this chapter, we will give an introduction to the relevant concepts of plasma physics necessary for the understanding of the diffusive nature of cosmic-ray transport and the mechanism of Diffusive Shock Acceleration (DSA). Later in this chapter, we will also discuss the limitations of rigidities that cosmic rays can be accelerated to in Supernova Remnants (SNRs). We will explore the mechanism of magnetic field amplification that is sometimes invoked to overcome these limitations. At the end, we will also review the escape of cosmic rays from their acceleration sites.

We use cgs-units in this thesis. For a conversion to SI-units, we refer to the Appendix on Units and Dimensions in Jackson, [1998](#).

5.1 Basics of plasma physics

A plasma is a state of matter that is characterised by high electric conductivity in (partially) ionised substances. This allows for long-range electromagnetic interactions between charged particles and leads to a vast variety of collective dynamical modes that can be excited (Fitzpatrick, [2022](#)).

Plasmas are of fundamental importance for understanding astrophysical phenomena (Schlickeiser, [2002](#)). Many astrophysical environments are in a plasma state, e.g., the solar wind, the ISM, or the intracluster medium. In total, between 95 % and 99 % of the baryonic matter in the Universe is in a plasma state (Fitzpatrick, [2022](#)).

Plasmas are generally quasi-neutral. Oppositely charged fluid components interact electromagnetically, which leads to the so-called electric-field screening. As a consequence, plasmas appear neutral on length scales larger than the so-called Debye length, λ_D . It is related to the plasma frequency, ω_p , which is defined as

$$\omega_p = \sqrt{\frac{4\pi n q^2}{m}}, \quad (5.1)$$

where n is the number density of the charged particles, q is the elementary charge, and m is the mass of the charged particles. The plasma frequency is the characteristic frequency of oscillations caused by small charge separations.

The Debye length is defined as the distance that a thermal particle travels during ω_p^{-1} :

$$\lambda_D = \frac{v_{\text{th}}}{\omega_p} = \sqrt{\frac{k_B T}{4\pi n q^2}}, \quad (5.2)$$

where k_B is the Boltzmann constant and T is the temperature of the plasma, and we used $v_{\text{th}} = \sqrt{k_B T/m}$.

We will usually operate on time and length scales, t and ℓ , for which the following assumptions are true:

$$\omega_p^{-1} \ll t \ll t_{\text{dyn}} \quad \text{and} \quad \lambda_D \ll \ell \ll L_{\text{dyn}}, \quad (5.3)$$

where t_{dyn} and L_{dyn} are the characteristic dynamical time and length scales in which the system under consideration changes.

Another important quantity is the plasma parameter Λ , which is defined as three times the number of particles in a Debye sphere, a sphere of radius λ_D :

$$\Lambda = 4\pi n \lambda_D^3. \quad (5.4)$$

While $\Lambda \ll 1$ indicates a strongly coupled plasma with a sparsely populated Debye sphere, we will be more interested in weakly coupled plasmas with $\Lambda \gg 1$. Those can be usually described well by a Fokker-Planck equation (Fitzpatrick, 2022).

5.2 Interstellar medium

There are various different phases of the ISM, including the hot ionised medium (HIM), the warm ionised medium (WIM), the warm neutral medium (WNM), the cold neutral medium (CNM), and molecular clouds. They are characterised by different typical plasma parameters. We will not go into detail here but refer to Draine, 2011 for a detailed overview. According to Fitzpatrick, 2022, typical values in the CNM with a number density of about $n \approx 1 \text{ cm}^{-3}$ and a temperature of about $T \approx 100 \text{ K}$ lead to a Debye length of about $\lambda_D \approx 70 \text{ cm}$, a plasma frequency of about $\omega_p \approx 6 \cdot 10^4 \text{ s}^{-1}$, and a plasma parameter of about $\Lambda \approx 4 \cdot 10^6$. So, the ISM is clearly a weakly coupled plasma.

The diffusive transport of cosmic rays in the ISM can be derived in the so-called *test particle approach*, where the back-reaction of the cosmic rays on the plasma is neglected. This assumption is frequently used as an approximation and gives valuable insights in

Component	w [eV cm ⁻³]
Cosmic microwave background ($T_{\text{CMB}} = 2.725$ K)	0.265
Far-infrared radiation from dust	0.31
Starlight ($h\nu < 13.6$ eV)	0.54
Thermal kinetic energy $(3/2) nk_{\text{B}} T$	0.49
Turbulent kinetic energy $(1/2) \rho v^2$	0.22
Magnetic energy $B^2/8\pi$	0.89
Cosmic rays	~ 1

Table 5.1: Energy densities of interstellar medium components. The value for the local cosmic-ray energy density is taken from Eq. (4.2). This table is adapted from Draine, 2011, Tab. 1.5.

common transport scenarios, but is not true in most environments of cosmic-ray acceleration (Malkov and Drury, 2001) or in the context of *self-generated turbulence* (Aloisio and Blasi, 2013) (see Sec. 5.3.8). However, the usefulness of the test particle approach does not mean that cosmic rays are an unimportant component of the ISM. In fact, their energy density is comparable to other components of the ISM, as summarised in Tab. 5.1. Although this does not imply equipartition between the different components (Gabici et al., 2019), it shows that cosmic rays are an important ingredient of the ISM dynamics (Draine, 2011). Low-energy cosmic rays play an important role in heating and ionising the interstellar gas. Thus, they influence the evolution of galaxies and star formation (Orlando et al., 2019).

5.3 Quasilinear theory and diffusion

In plasmas like the magnetised ISM, plasma waves can be excited. Those are collective modes in a plasma described by coupled fluctuations of particle densities, electromagnetic fields, and particle velocities, which arise because charged particles interact through electromagnetic forces. We want to describe interactions of plasma waves in the magnetised ISM plasma with the phase space density of cosmic rays. Those plasma waves will mainly be so-called shear-Alfvén waves (Alfvén, 1942), which are transverse waves that propagate along magnetic field lines with the Alfvén speed, v_{A} :

$$v_{\text{A}} = \frac{B_0}{\sqrt{4\pi\rho}}, \quad (5.5)$$

where B_0 is the background magnetic field strength and ρ is the mass density of the plasma. We can get an estimate for the Alfvén speed in the ISM by inserting typical values, and we find:

$$v_{\text{A,km s}^{-1}} \approx 1.84 B_{0,\mu\text{G}} n_{\text{ISM,cm}^{-3}}^{-1/2}, \quad (5.6)$$

where, we have assumed again that the typical mass of an ISM particle is μm_p , where $\mu \approx 1.4$ is the molecular weight (Draine, 2011).

Our goal is to derive an equation that describes the evolution of the ensemble-averaged and pitch-angle averaged differential density of cosmic rays, $\psi(t, \mathbf{x}, \mathcal{R})$, under the influence of interactions with those plasma waves. In the case of small-amplitude magnetic field fluctuations, we can use Quasilinear Theory (QLT) to derive a Fokker-Planck equation for the evolution of the phase space density (Hall and Sturrock, 1967; Hasselmann and Wibberenz, 1970; Jokipii, 1966; Kennel and Engelmann, 1966).

5.3.1 Vlasov equation

Our starting point is the Lorentz-invariant phase space density, $f(t, \mathbf{x}, \mathbf{p})$ as a fundamental quantity to describe the distribution of cosmic rays in phase space. Its evolution is described by the Boltzmann equation:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{d\mathbf{x}}{dt} \cdot \nabla_{\mathbf{x}} f + \frac{d\mathbf{p}}{dt} \cdot \nabla_{\mathbf{p}} f = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}} \equiv 0, \quad (5.7)$$

where the right-hand side describes changes of f due to collisions, which can be assumed to be negligible for cosmic rays (Mertsch, 2020).

The equations of motion for a charged particle in an electromagnetic field are then given by

$$\frac{d\mathbf{p}}{dt} = q \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \quad \text{and} \quad \frac{d\mathbf{x}}{dt} = \mathbf{v}, \quad (5.8)$$

where q is the charge of the particle, $\mathbf{E}$ is the electric field, and $\mathbf{B}$ is the magnetic field. Inserting the equations of motion Eq. (5.8) into the collisionless Boltzmann equation Eq. (5.7) leads to the Vlasov equation:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + q \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \cdot \nabla_{\mathbf{p}} f = 0. \quad (5.9)$$

The evolution of the phase space density depends on the exact configuration of the electromagnetic fields. However, our knowledge of those is usually very limited. Thus, we will deal with the electromagnetic fields in a statistical sense. We will derive an equation for the mean of the phase space density f over the ensemble of realisations.

5.3.2 Ensemble average

First, we introduce the ensemble average, $\langle \cdot \rangle$:

$$\langle X \rangle = \int d\mathcal{P} X, \quad (5.10)$$

where $\mathcal{P}$ is the probability measure of the random variable X .

We assume that the magnetic field can be split into a large-scale background field, $\langle \mathbf{B} \rangle = \mathbf{B}_0$, where we assume $\mathbf{B}_0 = B_z \hat{z}$ without loss of generality, and small-scale turbulent fluctuations, $\delta \mathbf{B}$, where $\langle \delta \mathbf{B} \rangle = \mathbf{0}$:

$$\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B} \quad \text{with} \quad \langle \delta \mathbf{B} \rangle = \mathbf{0} \text{ and } \mathbf{B}_0 = B_z \hat{z}. \quad (5.11)$$

We consider time and length scales that are large compared to the characteristic plasma scales. Thus, as we have argued in Sec. 5.1, large-scale electric fields can be neglected (electric-field screening), and we assume $\langle \mathbf{E} \rangle = \mathbf{0}$. Small-scale electric field fluctuations, $\delta \mathbf{E}$, however, are a necessary consequence of magnetic field fluctuations due to Faraday's law of induction. Their magnitude can be estimated as $|\delta E| \sim (v_A/c) |\delta B|$ (Mertsch, 2020). As we have seen in Eq. (5.6), the Alfvén speed in the ISM is usually much smaller than the speed of light, $v_A \ll c$. Thus, we can neglect the contribution of electric field fluctuations: $\delta \mathbf{E} \approx \mathbf{0}$.

The phase space density can also be split in an expectation value $\langle f \rangle$ and fluctuations δf such that $\langle \delta f \rangle = 0$:

$$f = \langle f \rangle + \delta f. \quad (5.12)$$

By applying the ensemble average, $\langle \cdot \rangle$, to the Vlasov equation Eq. (5.9), we find (Jokipii, 1966):

$$\frac{d\langle f \rangle}{dt} = \frac{\partial \langle f \rangle}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \langle f \rangle + q \frac{\mathbf{v} \times \mathbf{B}_0}{c} \cdot \nabla_{\mathbf{p}} \langle f \rangle = -q \left\langle \frac{\mathbf{v} \times \delta \mathbf{B}}{c} \cdot \nabla_{\mathbf{p}} \delta f \right\rangle, \quad (5.13)$$

as $\langle \delta \mathbf{B} \rangle = \mathbf{0}$ and $\langle \delta f \rangle = 0$. We can see that the right-hand side does not vanish, contrary to the full Vlasov equation Eq. (5.9). This means that the ensemble-averaged phase space density is not conserved.

The central idea of QLT is to find an approximate solution for the fluctuations δf along the unperturbed trajectories of particles in the background magnetic field $\mathbf{B}_0$ (Mertsch, 2020). We obtain an equation for the evolution of the fluctuation δf by subtracting Eq. (5.13) from Eq. (5.9):

$$\frac{d\delta f}{dt} = \frac{\partial \delta f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \delta f + q \frac{\mathbf{v} \times \mathbf{B}_0}{c} \cdot \nabla_{\mathbf{p}} \delta f \simeq -q \frac{\mathbf{v} \times \delta \mathbf{B}}{c} \cdot \nabla_{\mathbf{p}} \langle f \rangle, \quad (5.14)$$

where we have neglected terms that are of second order in the fluctuations δf and $\delta \mathbf{B}$. This holds true, as long as the fluctuations are small, i.e., $|\delta \mathbf{B}| \ll |\mathbf{B}_0|$, which implies $\delta f \ll \langle f \rangle$ (Mertsch, 2020).

The solution of Eq. (5.14) can be determined with the method of characteristics: The equations of motion Eq. (5.8) with $\mathbf{B}$ replaced by $\mathbf{B}_0$ determine the characteristics $P(t) = (\mathbf{x}(t), \mathbf{p}(t))$. Those are the unperturbed orbits of particles in the background magnetic field $\mathbf{B}_0$. By integrating Eq. (5.14) along those, we find:

$$\delta f = \delta f_0 - q \int_{t_0}^t dt' \left[\frac{\mathbf{v} \times \delta \mathbf{B}}{c} \cdot \nabla_{\mathbf{p}} \langle f \rangle \right]_{P(t')}, \quad (5.15)$$

where δf_0 is the initial phase space density at time t_0 .

Now, we can insert this solution, Eq. (5.15), into the right-hand side of the ensemble-averaged Vlasov equation Eq. (5.13). We drop the initial phase space density fluctuation δf_0 and find:

$$\frac{d\langle f \rangle}{dt} \simeq q^2 \int_{t_0}^t dt' \left\langle \frac{\mathbf{v} \times \delta \mathbf{B}}{c} \cdot \nabla_{\mathbf{p}} \left[\frac{\mathbf{v} \times \delta \mathbf{B}}{c} \cdot \nabla_{\mathbf{p}} \langle f \rangle \right]_{P(t')} \right\rangle. \quad (5.16)$$

5.3.3 Assumptions and change of variables

It is already possible to perceive the structure of a diffusion interaction in the right-hand side of Eq. (5.16) due to the two momentum derivatives (Mertsch, 2020). To get from this equation to a Fokker-Planck equation with a diffusion term, further assumptions have to be made (Risken, 1989). The general idea is to derive an equation for the ensemble averaged phase space density $\langle f \rangle$, while making assumptions on how this quantity is influenced by fluctuations of the stochastic quantity $\delta \mathbf{B}$. We will summarise them here (see also Mertsch, 2020):

1. *Small perturbations:* As mentioned already, we only consider small perturbations of the magnetic field, i.e., $|\delta \mathbf{B}| \ll |\mathbf{B}_0|$.
2. *Gyrotropy:* We assume symmetry in the azimuthal angle around the background magnetic field direction: $\frac{\partial \langle f \rangle}{\partial \phi} = 0$.
3. *Adiabatic approximation:* We assume that the phase space density varies slowly compared to the correlation timescale of the magnetic field fluctuations (see Eq. (5.19) below):

$$\tau_{\text{corr}} \ll \left(\frac{\partial \ln \langle f \rangle}{\partial t} \right)^{-1}. \quad (5.17)$$

4. *Correlation timescale:* We assume that the correlation timescale of the magnetic field fluctuations is large compared to the particle's gyroperiod: $\tau_{\text{corr}} \gg \tau_{\text{gyro}} = \Omega^{-1}$, where $\Omega = qB_0/(mc)$ is the relativistic gyrofrequency of a particle with mass m .
5. *Homogeneous and stationary turbulence:* We assume homogeneous and stationary turbulence, i.e., the statistical properties of the magnetic field fluctuations do not depend on time or position.
6. *Alfvén speed:* Finally, we assume $v_A \ll v$, i.e., the Alfvén speed is much smaller than the particle speed.

The correlation length and timescales of the turbulent magnetic fields are defined by the 2-point correlation function of the magnetic field fluctuations in the following way (Mertsch, 2020):

$$\lambda_{\text{corr}} \equiv \frac{1}{\langle \delta B^2 \rangle} \int_0^\infty dx \langle \delta \mathbf{B}(\mathbf{x}_0) \delta \mathbf{B}(\mathbf{x}_0 + \mathbf{x}) \rangle, \quad (5.18)$$

$$\tau_{\text{corr}} \equiv \frac{1}{\langle \delta B^2 \rangle} \int_0^\infty dt \langle \delta \mathbf{B}(t_0) \delta \mathbf{B}(t_0 + t) \rangle. \quad (5.19)$$

Getting from the Vlasov equation Eq. (5.9) to the Fokker-Planck equation is an effective coarse-graining in time, only considering the influence of the turbulent magnetic fields on the ensemble averaged phase space density $\langle f \rangle$. Mind, that this limits the validity of this equation to times larger than the typical *scattering timescale*.

Now, we introduce a new set of variables to describe the momentum of the phase space density. Instead of using the Cartesian momentum coordinates $\mathbf{p} = (p_x, p_y, p_z)$, we use the magnitude of the momentum, $p = |\mathbf{p}|$, the pitch-angle cosine, $\mu = \cos(\theta) = p_z/p$, and the azimuthal angle, $\phi = \arctan(p_y/p_x)$. Thus, we get

$$\mathbf{p} = p \left(\sqrt{1 - \mu^2} \cos(\phi), \sqrt{1 - \mu^2} \sin(\phi), \mu \right)^T. \quad (5.20)$$

5.3.4 Fokker-Planck equation and pitch-angle diffusion coefficient

With these variables and the list of assumptions described above, the following Fokker-Planck equation can be derived (Mertsch, 2020):

$$\frac{\partial \langle f \rangle}{\partial t} + v\mu \frac{\partial \langle f \rangle}{\partial z} = \frac{\partial}{\partial \mu} \left(\kappa_{\mu\mu} \frac{\partial \langle f \rangle}{\partial \mu} \right), \quad (5.21)$$

where $\kappa_{\mu\mu}$ is the pitch-angle diffusion coefficient.

The pitch-angle diffusion coefficient $\kappa_{\mu\mu}$ can be derived under the assumptions made above by making choices on the magnetic field 2-point correlation function

$$C_{ij}(t) \equiv \langle \delta B_i(t_0) \delta B_j(t_0 + t) \rangle, \quad \text{with } \delta B_i(t) = [\delta B(\mathbf{x})]_{p(t)}, \quad (5.22)$$

where $i, j \in \{x, y, z\}$, and the fluctuations are evaluated at the specified time along the unperturbed orbits.

Instead of following the derivation of the pitch-angle diffusion coefficient from the microphysical relations under assumptions on the magnetic field 2-point correlation function Eq. (5.22), we will use the Fokker-Planck equation (Eq. (5.21)) as a starting point. For equations of this form, the Taylor-Green-Kubo relations (Green, 1951; Kubo, 1957; Taylor, 1922) can be used. They state that

$$\kappa_{\mu\mu} = \int_0^\infty dt \langle \dot{\mu}(t_0) \dot{\mu}(t_0 + t) \rangle. \quad (5.23)$$

The dot denotes the time derivative, i.e., $\dot{\mu} = \frac{d\mu}{dt}$.

From the equations of motion Eq. (5.8) and the change of variables for the momentum Eq. (5.20), we know that $p_z = p\mu$. As the sole influence of the magnetic field does not change the magnitude of the momentum, p , we find $\dot{p}_z = p\dot{\mu}$. With the equation of motion for the momentum, Eq. (5.8), this yields

$$\dot{\mu} = \frac{q}{pc} (\mathbf{v} \times \mathbf{B})_z = \frac{1}{r_L B_0} (v_x \delta B_y(\mathbf{x}) - v_y \delta B_x(\mathbf{x})), \quad (5.24)$$

where we have used that the background magnetic field is aligned with the z -axis and inserted the Larmor radius, $r_L = \frac{pc}{qB_0}$. Thus, we find the pitch-angle diffusion coefficient with Eq. (5.23):

$$\kappa_{\mu\mu} = \frac{1}{(r_L B_0)^2} \int_0^\infty dt \left\langle [v_x \delta B_y - v_y \delta B_x]_{t_0} [v_x \delta B_y - v_y \delta B_x]_{t_0+t} \right\rangle. \quad (5.25)$$

The specific form of this diffusion coefficient depends on the magnetic turbulence model. Before we mention the most common model, we will first discuss how pitch-angle diffusion leads to spatial diffusion in the so-called *diffusion approximation*.

5.3.5 Diffusion approximation

In the diffusion approximation, it is assumed that the pitch-angle diffusion is efficient enough to isotropise the cosmic-ray distribution quickly. This motivates an expansion of the mean phase space density $\langle f \rangle(t, \mathbf{x}, p, \mu)$ in an isotropic and a small anisotropic part

$$\langle f \rangle(t, \mathbf{x}, p, \mu) = g(t, \mathbf{x}, p) + h(t, \mathbf{x}, p, \mu), \quad (5.26)$$

where g is the pitch-angle averaged mean phase space density

$$g(t, \mathbf{x}, p) = \langle \langle f \rangle \rangle_\mu = \frac{1}{2} \int_{-1}^1 d\mu \langle f \rangle(t, \mathbf{x}, p, \mu), \quad (5.27)$$

and h is the small anisotropic part, i.e., $\langle h \rangle_\mu = 0$ and $h \ll g$. This last assumption is valid if g only varies slowly compared to the pitch-angle scattering scales (Mertsch, 2020). We define a temporal and spatial scale, τ_g and λ_g , on which the isotropic part g varies in analogy to Hasselmann and Wibberenz, 1970:

$$\frac{\partial g}{\partial t} = \mathcal{O}\left(\frac{g}{\tau_g}\right) \quad \text{and} \quad \frac{\partial g}{\partial z} = \mathcal{O}\left(\frac{g}{\lambda_g}\right). \quad (5.28)$$

Those scales are large compared to the pitch-angle scattering scales:

$$\tau_{sc} \sim \kappa_{\mu\mu}^{-1} \ll \tau_g \quad \text{and} \quad \lambda_{sc} \sim v \tau_{sc} \ll \lambda_g. \quad (5.29)$$

Taking the pitch-angle average, $\langle \cdot \rangle_\mu$, of the Fokker-Planck equation Eq. (5.21) leads to

$$\frac{\partial g}{\partial t} + v \frac{\partial}{\partial z} \langle \mu h \rangle_\mu = 0, \quad (5.30)$$

where we used that $\langle h \rangle_\mu = 0$, $\langle \mu \rangle_\mu = 0$, and

$$\left\langle \frac{\partial}{\partial \mu} \left(\kappa_{\mu\mu} \frac{\partial \langle f \rangle}{\partial \mu} \right) \right\rangle_\mu = \frac{1}{2} \int_{-1}^1 d\mu \frac{\partial}{\partial \mu} \left(\kappa_{\mu\mu} \frac{\partial h}{\partial \mu} \right) = \frac{1}{2} \left[\kappa_{\mu\mu} \frac{\partial \langle f \rangle}{\partial \mu} \right]_{\mu=-1}^{\mu=1} = 0. \quad (5.31)$$

The last identity follows as $\kappa_{\mu\mu}$ has to vanish for $\mu = \pm 1$, because particles with those pitch angles follow the unperturbed orbits perfectly. This means that there is no flux across the boundaries of the pitch-angle space, $\mu = \pm 1$.

Now, we can subtract the pitch-angle averaged equation Eq. (5.30) from the full Fokker-Planck equation Eq. (5.21) to find an equation for the anisotropic part h :

$$\underbrace{\frac{\partial h}{\partial t}}_{\mathcal{O}(\tau_g^{-1})} + \underbrace{v\mu \frac{\partial h}{\partial z}}_{\mathcal{O}(v\lambda_g^{-1})} - \underbrace{v \left\langle \mu \frac{\partial h}{\partial z} \right\rangle_\mu}_{\mathcal{O}(v\lambda_g^{-1})} - \underbrace{\frac{\partial}{\partial \mu} \kappa_{\mu\mu} \frac{\partial h}{\partial \mu}}_{\mathcal{O}(\tau_{sc}^{-1})} = -v\mu \frac{\partial g}{\partial z}. \quad (5.32)$$

Due to our scale assumptions Eq. (5.29), we can neglect the first three terms on the left-hand side in comparison to the remaining ones. Thus, we get

$$v\mu \frac{\partial g}{\partial z} = \frac{\partial}{\partial \mu} \kappa_{\mu\mu} \frac{\partial h}{\partial \mu}. \quad (5.33)$$

Eq. (5.33) can be integrated between $\mu' = -1$ (where $\kappa_{\mu\mu} = 0$) and μ . This leads to

$$\frac{\partial h}{\partial \mu} = \frac{v}{\kappa_{\mu\mu}} \frac{\partial g}{\partial z} \int_{-1}^{\mu} d\mu' \mu' = \frac{v}{2} \frac{\mu^2 - 1}{\kappa_{\mu\mu}} \frac{\partial g}{\partial z}. \quad (5.34)$$

Now, we manipulate Eq. (5.30) and insert the solution obtained in Eq. (5.34) to find

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial z} \left(\frac{v}{2} \int_{-1}^1 d\mu \mu h \right) \quad (5.35)$$

$$= -\frac{\partial}{\partial z} \left(\frac{v}{2} \int_{-1}^1 d\mu \frac{\partial}{\partial \mu} \left(\frac{\mu^2 - 1}{2} \right) h \right) \quad (5.36)$$

$$\stackrel{\text{PI}}{=} \frac{\partial}{\partial z} \left(\frac{v}{2} \int_{-1}^1 d\mu \frac{\mu^2 - 1}{2} \frac{\partial h}{\partial \mu} \right) \quad (5.37)$$

$$= \frac{\partial}{\partial z} \left(\frac{v^2}{2} \int_{-1}^1 d\mu \frac{(1 - \mu^2)^2}{4\kappa_{\mu\mu}} \right) \frac{\partial g}{\partial z}. \quad (5.38)$$

Thus, we have derived the diffusion equation for the isotropic part of the mean phase space density g :

$$\frac{\partial g}{\partial t} - \frac{\partial}{\partial z} \left(\kappa_{zz} \frac{\partial g}{\partial z} \right) = 0 \quad \text{with} \quad \kappa_{zz} = \frac{v^2}{2} \int_{-1}^1 d\mu \frac{(1 - \mu^2)^2}{4\kappa_{\mu\mu}}. \quad (5.39)$$

κ_{zz} is the parallel diffusion coefficient along the background magnetic field direction. Our derivation shows how spatial diffusion arises from pitch-angle diffusion in the diffusion approximation. The pitch-angle diffusion itself is caused by interactions of cosmic rays with turbulent magnetic fields. In the following section, we want to describe how the power spectrum of those turbulent magnetic fields is linked to the pitch-angle diffusion coefficient and thus to the spatial diffusion coefficient.

5.3.6 Slab turbulence and diffusion

We will consider shear-Alfvén waves as the main form of turbulence. Then, the diffusion process can be understood as a scattering of cosmic rays off those plasma waves. In the framework of QLT, these shear-Alfvén waves characterise the turbulent magnetic field fluctuations $\delta \mathbf{B}$. They are described by a power spectrum, $C_{ij}(\mathbf{k})$, where $\mathbf{k}$ is the wave vector:

$$C_{ij}(\mathbf{k}) \delta(\mathbf{k} - \mathbf{k}') = \langle \delta B_i(\mathbf{k}) \delta B_j^*(\mathbf{k}') \rangle. \quad (5.40)$$

This tensor is the Fourier transform of the magnetic field 2-point correlation function, we described earlier in Eq. (5.22).

Since shear-Alfvén waves are transverse waves, the geometry of the turbulence is described by the so-called *slab turbulence* model. There, it is assumed that the turbulent field has no component along the background magnetic field direction, i.e., $\delta B_z = 0$. The power spectrum for such a turbulence model can be written as (Mertsch, 2020):

$$C_{ij}^{\text{slab}}(\mathbf{k}) = \xi^{\text{slab}}(k_{\parallel}) \frac{\delta(k_{\perp})}{k_{\perp}} \delta_{ij}, \quad \text{and } 0 \text{ for } i, j = z, \quad (5.41)$$

where $k_{\parallel} = k_z$ and $k_{\perp} = \sqrt{k_x^2 + k_y^2}$ are the components of the wave vector $\mathbf{k}$ parallel and perpendicular to the background magnetic field direction, respectively. The function $\xi^{\text{slab}}(k_{\parallel})$ describes how turbulent power is distributed on different scales.

To calculate the pitch-angle diffusion coefficient according to Eq. (5.25), we need to specify the magnetic field 2-point correlation function Eq. (5.22). For slab turbulence, this can be determined from the slab turbulence power spectrum, Eq. (5.41), via an inverse Fourier transform:

$$C_{ii}^{\text{slab}}(\mathbf{x}(t)) = \int d^3k C_{ii}^{\text{slab}}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}(t)} = 2\pi \int_{-\infty}^{\infty} dk_{\parallel} \xi^{\text{slab}}(k_{\parallel}) e^{ik_{\parallel} z(t)}. \quad (5.42)$$

The factor 2π comes from the evaluation of the azimuthal integration (Kuhlen, 2022).

To determine the pitch-angle diffusion coefficient for slab turbulence, we go back to Eq. (5.25) and find:

$$\kappa_{\mu\mu} = \frac{1}{(r_L B_0)^2} \int_0^{\infty} dt \left[v_x(t_0) v_x(t_0+t) C_{yy}(t_0+t) + v_y(t_0) v_y(t_0+t) C_{xx}(t_0+t) \right]. \quad (5.43)$$

Again, the velocities are to be evaluated along the unperturbed orbits. From the equations of motion, Eq. (5.8), in a constant background magnetic field $\mathbf{B}_0 = B_z \hat{z}$, we find the following velocity components:

$$v_x(t) = v_{\perp} \cos(\phi_0 - \Omega t), \quad v_y(t) = v_{\perp} \sin(\phi_0 - \Omega t), \quad v_z(t) = v_{\parallel}, \quad (5.44)$$

where $v_{\perp} = v \sqrt{1 - \mu^2}$ is the velocity component perpendicular to the background magnetic field, $v_{\parallel} = v\mu$ is the velocity component parallel to the background magnetic field, $\Omega = \frac{qB_0}{mc}$ is the gyrofrequency, and ϕ_0 is the initial azimuthal angle at time t_0 . Inserting those into Eq. (5.43) gives

$$\kappa_{\mu\mu} = \frac{v^2 (1 - \mu^2)}{(r_L B_0)^2} \int_0^{\infty} dt \cos(\Omega t) C_{yy}(t_0 + t). \quad (5.45)$$

Inserting Eq. (5.42) into Eq. (5.45) using $z(t_0 + t) = v_{\parallel}t = v\mu t$ leads to:

$$\kappa_{\mu\mu}^{\text{slab}} = \frac{2\pi v^2 (1 - \mu^2)}{(r_L B_0)^2} \int_0^{\infty} dt \int_{-\infty}^{\infty} dk_{\parallel} \xi^{\text{slab}}(k_{\parallel}) \cos(\Omega t) e^{ik_{\parallel} v \mu t} \quad (5.46)$$

$$= \frac{\pi v^2 (1 - \mu^2)}{(r_L B_0)^2} \int_0^{\infty} dt \int_{-\infty}^{\infty} dk_{\parallel} \xi^{\text{slab}}(k_{\parallel}) (e^{i\Omega(t+t_0)} + e^{-i\Omega(t+t_0)}) e^{ik_{\parallel} v \mu (t+t_0)} \quad (5.47)$$

$$= \frac{\pi^2 v^2 (1 - \mu^2)}{(r_L B_0)^2} \int_{-\infty}^{\infty} dk_{\parallel} \xi^{\text{slab}}(k_{\parallel}) (\delta(k_{\parallel} v \mu - \Omega) + \delta(k_{\parallel} v \mu + \Omega)) \quad (5.48)$$

$$= \frac{\pi^2 v^2 (1 - \mu^2)}{(r_L B_0)^2 v |\mu|} \left(\xi^{\text{slab}}\left(k_{\parallel} = \frac{\Omega}{v \mu}\right) + \xi^{\text{slab}}\left(k_{\parallel} = -\frac{\Omega}{v \mu}\right) \right), \quad (5.49)$$

where δ denotes the Dirac delta distribution.

Under the assumption that the slab turbulence function $\xi^{\text{slab}}(k_{\parallel})$ is an even function, we finally find:

$$\kappa_{\mu\mu}^{\text{slab}} = \frac{2\pi^2 \Omega^2 (1 - \mu^2)}{B_0^2 v |\mu|} \xi^{\text{slab}}\left(k_{\parallel} = \frac{\Omega}{v |\mu|}\right). \quad (5.50)$$

This shows that the pitch-angle diffusion coefficient is directly proportional to the turbulence power spectrum evaluated at the resonant wave number, i.e., where $k_{\parallel} |\mu| r_L = 1$. We can interpret this as the cosmic-ray particle scattering on shear-Alfvén waves with a wavelength that matches their Larmor radius.

One main caveat to this result is the so-called *90°-problem*. According to Eq. (5.50), the pitch-angle diffusion coefficient vanishes for $\mu = 0$. This is related to the sharpness of the resonance condition, that collapses to a delta-peak in our derivation (Voelk, 1975). Both come from the assumption that unperturbed orbits are unchanged for $t \rightarrow \infty$ (Kuhlen, 2022). A relaxation of this assumption leads to a broadening of the resonance function and thus a non-vanishing pitch-angle diffusion coefficient at $\mu = 0$ (Mertsch, 2020). Many modifications of QLT have been proposed. For a review on those, see Shalchi, 2020. One promising suggestion is, e.g., to include interactions with magnetic mirrors (Cesarsky and Kulsrud, 1973; Felice and Kulsrud, 2001).

5.3.7 Turbulence spectra

We are interested in the final form of the spatial diffusion coefficient, κ_{zz} . It is related to the pitch-angle diffusion coefficient, $\kappa_{\mu\mu}$, via Eq. (5.39). To understand how it depends on the properties of the turbulent magnetic fields, we have to specify the turbulence power spectrum, $\xi^{\text{slab}}(k_{\parallel})$. First, we observe that the Larmor radius is directly proportional to the particle rigidity: $r_L \propto \mathcal{R}$. At the wave number specified by the resonance condition, we can thus write $k_{\parallel} \propto r_L^{-1} \propto \mathcal{R}^{-1}$.

The functional dependence of the turbulence spectral function, $\xi^{\text{slab}}(k_{\parallel})$, on the wave number, $k_{\parallel}$, is studied in the theory of turbulence. A general overview is given in Frisch, 1995. There are some prominent cascade models (Iroshnikov, 1963; Kolmogorov, 1941; Kraichnan, 1965). They assume that energy is injected on the largest scales (e.g., through supernova explosions in the Galaxy). Then, non-linear interactions cascade the energy down to smaller scales over a so-called *inertial range*. At the smallest scales, the energy dissipates. These cascade models lead to power-law turbulence spectra in the inertial range. An overview on turbulence spectra in the ISM can be found in Armstrong et al., 1995.

The power-law behaviour can be expressed as (Mertsch, 2020):

$$\xi^{\text{PL}}(k) = \xi_0 \left(\frac{k}{k_0} \right)^{-s} \quad \text{for } k \geq k_0 \quad \text{and 0 otherwise,} \quad (5.51)$$

where k_0 is the minimum wave number, corresponding to the maximum turbulence scale. For a Kolmogorov turbulence spectrum, we have $s = 5/3$, and for a Kraichnan spectrum, we have $s = 3/2$. The normalisation constant ξ_0 is determined by the condition (compare with Eqs. (5.22) and (5.42)):

$$\delta B^2 = \langle \delta B^2(x) \rangle = C_{xx}^{\text{slab}}(0) + C_{yy}^{\text{slab}}(0) = 4\pi \int_{-\infty}^{\infty} dk_{\parallel} \xi^{\text{PL}}(k_{\parallel}). \quad (5.52)$$

Depending on the spectral index s , we can now determine the rigidity dependence of the pitch-angle diffusion coefficient, $\kappa_{\mu\mu}$, (see Eq. (5.49)):

$$\kappa_{\mu\mu}(\mathcal{R}) \propto \beta k_{\parallel}^{-s} r_L^{-2} \propto \beta r_L^{s-2} \propto \beta \mathcal{R}^{s-2}. \quad (5.53)$$

With this, we can finally determine the rigidity-dependence of the spatial diffusion coefficient, κ_{zz} , via Eq. (5.39):

$$\kappa_{zz} \propto \beta^2 \kappa_{\mu\mu}^{-1} \propto \beta \mathcal{R}^{2-s} \propto \begin{cases} \beta \mathcal{R}^{1/3} & \text{for Kolmogorov turbulence } (s = 5/3), \\ \beta \mathcal{R}^{1/2} & \text{for Kraichnan turbulence } (s = 3/2). \end{cases} \quad (5.54)$$

Before we go on to discuss the actual diffusion coefficient, we want to mention so-called *Bohm diffusion*. It is often used as a limiting case for strong turbulence ($\delta B \sim B$), well outside the range of applicability for QLT (see assumption 1 in Sec. 5.3.3) (Bell, 2013). It is applicable if the Larmor radius, r_L , is of the order of the mean free path, λ_{mfp} . Then, the spatial diffusion coefficient can be expressed as

$$\kappa_{\text{Bohm}} = \frac{\lambda_{\text{mfp}} v}{3} \approx \frac{1}{3} v r_L \propto \beta \mathcal{R}. \quad (5.55)$$

It is often invoked in the context of maximum rigidities achievable in DSA (Bell, 2013). We will discuss this in more detail in Sec. 5.6.

To see that Bohm diffusion in fact describes an extreme diffusion scenario, we look at the pitch angle diffusion coefficient in Eq. (5.50) again. It can be rewritten as

$$\kappa_{\mu\mu} = (1 - \mu^2) \nu_s \quad \text{with} \quad \nu_s = \frac{\pi}{4} \Omega \left[\frac{k \xi(k)}{B_0^2/8\pi} \right]_{k=\frac{\Omega}{v|\mu|}}, \quad (5.56)$$

where ν_s is the scattering frequency. The term in the brackets gives the ratio between the turbulent magnetic field energy density at the resonant wave number and the background magnetic field energy density. We define this ratio as:

$$\mathcal{F}(k) \equiv \frac{k \xi(k)}{B_0^2/8\pi}. \quad (5.57)$$

From this, we find that the spatial diffusion coefficient can be expressed as (Amato, 2014)

$$\kappa_{zz}(p) \sim \frac{v^2}{3\nu_s} \sim \frac{\kappa_{\text{Bohm}}}{\mathcal{F}(1/r_L(p))}, \quad (5.58)$$

where we used the resonance condition $k \approx 1/r_L$ and the definition of the Bohm diffusion coefficient in Eq. (5.55). The derivation of QLT assumes that this ratio is small, i.e., $\mathcal{F}(k) \ll 1$. However, if the ratio in Eq. (5.57) is of order unity, i.e., $\mathcal{F}(k) \approx 1$, this reaches the Bohm diffusion limit.

5.3.8 Cosmic-ray diffusion coefficient in the Galaxy

In this section, we want to discuss properties of the cosmic-ray diffusion in the Galaxy. First, we want to discuss how the results from the previous sections can be used to get a description of the 3-dimensional diffusive transport of cosmic rays in the Galaxy. In the 3D case, the diffusion is described by a diffusion tensor κ , and the diffusion term in the transport equation changes compared to the one we have derived in Eq. (5.39):

$$\frac{\partial}{\partial z} \left(\kappa_{zz} \frac{\partial f}{\partial z} \right) \rightarrow \nabla \cdot (\kappa \cdot \nabla f). \quad (5.59)$$

The diffusion tensor can be decomposed into parallel and perpendicular components by choosing a coordinate system in which the z -axis is aligned with the background magnetic field:

$$\kappa = \begin{pmatrix} \kappa_{xx} & \kappa_{xy} & \kappa_{xz} \\ \kappa_{yx} & \kappa_{yy} & \kappa_{yz} \\ \kappa_{zx} & \kappa_{zy} & \kappa_{zz} \end{pmatrix} \sim \begin{pmatrix} \kappa_{\perp} & 0 & 0 \\ 0 & \kappa_{\perp} & 0 \\ 0 & 0 & \kappa_{\parallel} \end{pmatrix}, \quad (5.60)$$

as the off-diagonal components can usually be neglected (Berezinskii et al., 1990).

QLT predicts that the parallel diffusion coefficient, $\kappa_{\parallel}$, is much larger than the perpendicular diffusion, $\kappa_{\perp}$. Perpendicular diffusion can be caused by scattering with plasma waves that cause the gyrocentre to jump field lines or by the transport of field lines themselves. The latter is known as field line random walk (FLRW) (Mertsch, 2020).

While this makes the diffusive transport anisotropic on small scales, it is usually assumed that the diffusion tensor becomes isotropic on scales larger than the background magnetic field correlation length scale:

$$\boldsymbol{\kappa}_{\text{iso}} = \kappa \mathbf{I}_3 \quad \text{and} \quad \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla f) \rightarrow \kappa \nabla^2 f. \quad (5.61)$$

A more detailed discussion on this argument can be found in Berezhinskii et al., 1990, Chap. IX, §4. Observations suggest that the correlation length of the magnetic field is in the range of 1 pc to 100 pc (Haverkorn et al., 2008; Iacobelli et al., 2013). We will assume homogeneous and isotropic diffusion on Galactic scales in this work.

The actual value and spectral shape of the isotropic diffusion coefficient, κ , can be constrained by observations. From Eq. (5.54), we expect a power-law behaviour of the diffusion coefficient with rigidity:

$$\kappa(\mathcal{R}) = \kappa_0 \beta \left(\frac{\mathcal{R}}{\mathcal{R}_0} \right)^{\delta}, \quad (5.62)$$

where κ_0 is the normalisation of the diffusion coefficient at a reference rigidity $\mathcal{R}_0$, and δ is the spectral index. The most prominent constraints on those parameters come from measurements of secondary-to-primary ratios, e.g., the boron-to-carbon ratio. We discussed how this works qualitatively in Sec. 4.5. To break the degeneracy between the diffusion coefficient normalisation, κ_0 , and the size of the diffusion halo, H , additional measurements of unstable-to-stable secondary ratios are necessary. Such measurements point towards a value of the diffusion coefficient normalisation of $\kappa_0 \sim 1.1 \cdot 10^{28} \text{ cm}^2 \text{ s}^{-1}$ at a reference rigidity of $\mathcal{R}_0 = 1 \text{ GV}$ ($\kappa_0 \simeq \kappa(1 \text{ GV})$) (Evoli et al., 2019). This corresponds to a rather low turbulence level $\delta B/B = \mathcal{O}(10^{-4})$, i.e., $\mathcal{F} \ll 1$ (Blasi, 2013).

The spectral index of the diffusion coefficient can also be constrained by secondary-to-primary ratios. Measurements point towards a value in the range of $\delta \sim 0.3 - 0.6$ (Gabici et al., 2019). Observations indicate a break in the diffusion coefficient at a rigidity of about $\mathcal{R}_{\text{br}} \sim 300 \text{ GV}$ (Evoli et al., 2019). For rigidities below the break, the spectral index is roughly $\delta \sim 0.5 - 0.6$, while above the break, it is $\delta \sim 0.3$.

This break is associated with a break in the underlying power spectrum of magnetic turbulence. While the turbulence at large scales (related to the diffusion coefficient at high rigidities) is compatible with a Kolmogorov spectrum, whose energy might be injected by supernova explosions (Blasi et al., 2012; Evoli et al., 2018), the turbulence at small scales (related to the diffusion coefficient at low rigidities) is believed to be produced by the cosmic-ray streaming instability caused by cosmic rays escaping the disk (Evoli et al., 2018). This is usually called *self-generated turbulence* (Aloisio and Blasi, 2013).

Further constraints on the normalisation of the diffusion coefficient come from measurements of the dipole anisotropy via the relation that we pointed out in Eq. (3.2) in Sec. 3.1.3 (Ahlers and Mertsch, 2017). As has been pointed out in Gabici et al., 2019, the dipole anisotropy should roughly be

$$a_{-4} \sim \kappa_{0,28} L_{3\text{kpc}}^{-1}, \quad (5.63)$$

where L is a characteristic length scale of the local cosmic-ray gradient. However, this result might be influenced by the presence of a nearby source (Ptuskin et al., 2006).

5.4 Cosmic-ray transport equation

To get a complete picture of the transport of cosmic rays in the Galaxy, we need to add more physical processes. However, before we will state and discuss the complete cosmic-ray transport equation, it should be noted that the spatial diffusion that we derived in the last sections is the dominant transport process for hadronic cosmic rays with rigidities above a few GV on Galactic scales. As we will model Galactic cosmic-ray protons in this thesis, this is the main transport process we will consider in our model.

We will state the transport equation in terms of the isotropic differential density, $\psi_{\mathcal{R}} \propto p^2 f$ (see App. A.3 for more details). For conciseness, we will write ψ instead of $\psi_{\mathcal{R}}$ in the following. The relation of different isotropic differential densities can also be found in App. A.3.

The complete transport equation for a cosmic ray of species i is (Berezinskii et al., 1990)

$$\begin{aligned} \frac{\partial \psi_i}{\partial t} - \nabla \cdot (\kappa \cdot \nabla \psi_i - \mathbf{V} \psi_i) - \frac{\partial}{\partial p} \left(p^2 \kappa_{pp} \frac{\partial \psi_i}{\partial p} \frac{1}{p^2} \right) + \frac{\partial}{\partial p} \left(\frac{dp}{dt} \psi_i - \frac{p}{3} (\nabla \cdot \mathbf{V}) \psi_i \right) \\ = Q_i(\mathcal{R}, t, \mathbf{x}) - \left(v n_{\text{ISM}} \sigma_{\text{inel},i} + (\gamma \tau_{\text{decay},i})^{-1} \right) \psi_i + \sum_{i < j} \left(v n_{\text{ISM}} \sigma_{j \rightarrow i} + (\gamma \tau_{j \rightarrow i})^{-1} \right) \psi_j. \end{aligned} \quad (5.64)$$

It is usually solved under the assumption of free escape boundary conditions at the borders of the Galactic halo. This halo is modelled as a cylinder with half-height H and radius R_{Gal} . This means that one assumes $\psi_i(z = \pm H) = 0$ and $\psi_i(R = R_*) = 0^1$. The different terms in the transport equation describe the following physical processes:

- **Spatial diffusion and advection:** We already discussed how spatial diffusion arises from pitch-angle diffusion in turbulent magnetic fields in the previous sections. Above some GV, spatial diffusion is the dominant transport process for hadronic cosmic rays. The normalisation and spectral shape of the diffusion coefficient can be fitted to

¹The radial boundary, R_* , is often neglected as most sources and points of interest are closer to the vertical boundary of the halo than to the radial one.

observational data. One example for such a fitted diffusion coefficient, which we will also use later in this work, is given in Schwefer et al., 2023.

The additional process of advection arises in the presence of a large-scale background flow with velocity $\mathbf{V}$. It is usually assumed that those are cosmic-ray driven winds (Rechcia et al., 2016). If the medium in which cosmic rays diffuse is not at rest, they also get advected with the flow. One common assumption for the velocity profile is that it is symmetric around the Galactic disk and increases linearly with the distance from the disk: $\mathbf{V}(z) \propto z \mathbf{e}_z$. This process can be neglected above some GV, but it is important to describe transport effects at sub-GV rigidities (Gabici et al., 2019).

- **Momentum diffusion (reacceleration):** In our derivation of the Fokker-Planck equation (see Eq. (5.21)), we did not consider the influence of electric fields. Those give rise to additional terms in Eq. (5.21) (Mertsch, 2020). One of them is related to momentum diffusion: $\frac{\partial}{\partial p} \left(\kappa_{pp} \frac{\partial f}{\partial p} \right)$. By changing to the isotropic differential density ψ , this term transforms to the one shown in Eq. (5.64).

The physical picture behind momentum diffusion is that cosmic rays can also change their momentum by scattering on magnetic inhomogeneities. Again, we assume that those are shear-Alfvén waves that move with the Alfvén speed, v_A . This leads to a second-order Fermi acceleration process, as has been proposed by Fermi, 1949. It is possible to derive a direct correspondence between the spatial diffusion coefficient κ_{zz} and the momentum diffusion coefficient, κ_{pp} (Thornbury and Drury, 2014), considering that both are related to the same scattering processes.

According to kinetic theory, the diffusion coefficient is related to the mean free path λ_{mfp} and the scattering time τ_{sc} via $\kappa_{zz} \approx \lambda_{\text{mfp}}^2 / (3\tau_{\text{sc}}) = v^2 \tau_{\text{sc}} / 3$. The momentum gained in each interaction is approximately $\Delta p \approx p v_A / v$, which leads to a momentum diffusion coefficient of $\kappa_{pp} \approx (\Delta p)^2 / (3\tau_{\text{sc}}) = p^2 v_A^2 / (3v^2 \tau_{\text{sc}})$. Combining both expressions, we find the relation between spatial and momentum diffusion coefficient (Thornbury and Drury, 2014):

$$\kappa_{zz} \kappa_{pp} = \frac{p^2 v_A^2}{9}. \quad (5.65)$$

With this relation, it can be argued that reacceleration of cosmic rays can become relevant at low rigidities, where the spatial diffusion coefficient is small. As most of the energy density of Galactic cosmic rays is contained at rigidities around 1 GV, reacceleration actually significantly contributes to the overall energy budget of Galactic cosmic rays (Drury and Strong, 2017). At rigidities above a few GV, however, reacceleration can usually be neglected.

- **Continuous momentum losses (incl. adiabatic losses/gains):** All cosmic-ray species can lose momentum through processes like ionisation or Coulomb interactions. For hadronic cosmic rays, these effects can usually be neglected above a few GV (Gabici et al., 2019). For leptonic cosmic rays, however, interactions with the ISM through bremsstrahlung, synchrotron radiation, and inverse Compton scattering on

Interstellar Radiation Fields (ISRFs) are relevant for all rigidities. The momentum loss rate due to synchrotron and inverse Compton losses can be expressed in the Thomson approximation. It turns out that it is proportional to the energy squared (Crusius and Schlickeiser, 1988)

$$\frac{dp}{dt} = -\frac{4\sigma_T c}{3} (\rho_B + \rho_{\text{ISRF}}) \left(\frac{E}{mc^2} \right)^2, \quad (5.66)$$

where σ_T is the Thomson cross-section, ρ_B is the energy density of the magnetic field, and ρ_{ISRF} is the energy density of the ISRF. At higher energies, Klein-Nishina corrections have to be taken into account (Blumenthal and Gould, 1970). They lead to a suppression of the loss rate compared to the Thomson approximation, which influences γ -ray emissions of leptonic origin (Stawarz et al., 2010).

Additionally, adiabatic momentum losses or gains can be caused by diverging or converging flows in the background plasma. They are important if the Galactic wind velocity profile is non-uniform. According to thermodynamics, the internal energy U changes by $dU = nV dE = -P dV$ if the volume changes. For a relativistic gas, we have $P = nE/3$. The change of volume in a flow with velocity $\mathbf{V}$ is $\frac{dV}{dt} = (\nabla \cdot \mathbf{V}) V$. Thus, the energy and momentum change rate can be expressed as

$$\frac{dE}{dt} = -\frac{P}{nV} \frac{dV}{dt} = -\frac{E}{3} (\nabla \cdot \mathbf{V}) \quad \Rightarrow \quad \frac{dp}{dt} = -\frac{p}{3} (\nabla \cdot \mathbf{V}). \quad (5.67)$$

- **Source term:** The source term $Q_i(\mathcal{R}, t, \mathbf{x})$ describes the injection of cosmic rays of species i into the ISM. This term encodes all the information about the injection of cosmic rays. This includes their injection spectrum, which is usually derived from DSA. We will discuss this further in the following Sec. 5.5. Additionally, this term describes the spatial and temporal distribution of cosmic-ray sources in the Galaxy. It can also contain information about how sources inject cosmic rays (e.g., instantaneous burst-like injection or continuous injection over some time). Common assumptions are that the injection spectrum is universal and that the source term can be factorised:

$$Q_i(\mathcal{R}, t, \mathbf{x}) = Q_{\mathcal{R}}(\mathcal{R}) \rho(t, \mathbf{x}), \quad (5.68)$$

where $Q_{\mathcal{R}}(\mathcal{R})$ is the universal injection spectrum and $\rho(t, \mathbf{x})$ describes the spatial and temporal distribution of sources in the Galaxy.

- **Spallation and decay losses and sources:** Cosmic rays can interact with the ISM and produce lighter secondary cosmic rays through spallation processes. For secondaries, the spallation of heavier species acts as a source term (e.g., for boron from carbon and oxygen spallation). Radioactive decay can also function as a source or sink in the transport equation. However, these processes are not relevant for the transport of cosmic-ray protons.

This concludes the introduction to the general cosmic-ray transport equation. There are several numerical codes that solve the transport equation for different species under

varying assumptions, e.g., GALPROP² (Moskalenko and Strong, 1998; Porter et al., 2022; Strong and Moskalenko, 1998), DRAGON³ (Evoli et al., 2008, 2017), PICARD (Kissmann, 2014), USINE⁴ (Maurin, 2020), or CRPROPA⁵ (Alves Batista et al., 2022; Armengaud et al., 2007).

As we will focus on cosmic-ray protons above rigidities of about 10 GV in this thesis, we can simplify the transport equation significantly. We will also assume an isotropic diffusion coefficient according to Eq. (5.61). Thus, the transport equation for cosmic-ray protons for $\mathcal{R} > 10$ GV reduces to:

$$\frac{\partial \psi(\mathcal{R}, t, \mathbf{x})}{\partial t} - \kappa(\mathcal{R}) \nabla^2 \psi(\mathcal{R}, t, \mathbf{x}) = Q(\mathcal{R}, t, \mathbf{x}) . \quad (5.69)$$

5.5 Diffusive shock acceleration

The standard theory of cosmic-ray acceleration is based on Diffusive Shock Acceleration (DSA), sometimes also called first-order Fermi acceleration. Fermi's original idea was that particles could gain energy via collisions with moving magnetic clouds (Fermi, 1949). While particles gain energy in head-on collisions proportional to $\Delta E/E \propto u_{\text{cloud}}/c$, they lose energy in overtaking collisions. Only a slight asymmetry in the collision rates leads to a net energy gain proportional to $\Delta E/E \propto (u_{\text{cloud}}/c)^2$. This is why this process is called second-order Fermi acceleration. It is similar to the reacceleration process that we discussed in Sec. 5.4 and is not efficient enough to explain the observed cosmic-ray spectrum.

In shock environments, however, the situation is different and DSA, a first-order Fermi acceleration process with a net energy gain proportional to $\Delta E/E \propto u_{\text{sh}}/c$, where u_{sh} is the shock speed, can take place. In this section, we want to give a brief introduction to the theory of DSA. First, we want to motivate the emergence of strong shocks in SNRs. Then, we will discuss how the spectrum of cosmic rays accelerated in DSA emerges. There is a macroscopic derivation based on the transport equation for the phase space density and a microscopic derivation that is based on the understanding of the interactions on a particle-level. Both derivations give us an intuition for the spectral index that is characteristic for DSA in SNR shocks.

²<https://galprop.stanford.edu/>

³<https://github.com/cosmicrays/DRAGON>

⁴<https://lpsc.in2p3.fr/usine>

⁵<https://github.com/CRPropa>

5.5.1 Shocks in supernova remnants

First, we want to introduce shocks and discuss how they arise. A flow, e.g., of a plasma, is described by the fluid dynamical variables density (ρ), pressure (P), and velocity ($\mathbf{V}$). Those variables have a spatial dependence. If they change abruptly over a small spatial region, we call this discontinuity in the flow a *shock*. Shocks necessarily arise if there is a transition from supersonic to subsonic flows, i.e., if the flow velocity exceeds the medium's speed of sound, c_s , in one region, but is below it in another region.

This is the case in the first phase of SNR evolution when the ejected material from the supernova explosion expands supersonically into the surrounding ISM. This is called the *free-expansion phase*. The root mean squared speed of the ejecta can be estimated from the supernova's kinetic energy and the mass of the ejecta (Draine, 2011):

$$\sqrt{\langle v_{\text{ejecta}}^2 \rangle} = \sqrt{\frac{2E_{\text{SN}}}{M_{\text{ejecta}}}} = 10^4 \text{ km s}^{-1} E_{\text{SN},51}^{1/2} M_{\text{ejecta},\odot}^{-1/2}, \quad (5.70)$$

where $M_{\text{ejecta},\odot}$ is the ejecta mass in units of solar masses and $E_{\text{SN},51}$ is the kinetic energy in units of 10^{51} erg. The speed of sound in the ISM is typically of the order of (Draine, 2011):

$$c_s = \sqrt{\Gamma \frac{P}{\rho}} \stackrel{\text{ISM}}{\approx} 10 \text{ km s}^{-1}, \quad (5.71)$$

where Γ is the adiabatic index of the medium and P is the pressure.

Thus, $u_{\text{sh}} = \sqrt{\langle v_{\text{ejecta}}^2 \rangle} \gg c_s$. Far away from the supernova explosion, however, the flow is subsonic, and a shock has to form. The two regions separated by the shock front are called *upstream* (before the shock) and *downstream* (after the shock). The naming scheme is based on the flow direction in the shock rest frame. The unshocked medium upstream has yet to pass the shock front, while the shocked medium downstream has already passed it.

A shock can be characterised by its *compression ratio*, r , which is defined as the ratio of the downstream to upstream plasma density: $r = \rho_d / \rho_u$. The relation between the flow parameters on both sides of the shock can be derived from the conservation of mass, momentum, and energy flux across the shock front. Those conservation laws are given by (Amato, 2014; Draine, 2011; Fitzpatrick, 2022):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \quad (5.72)$$

$$\frac{\partial}{\partial t} (\rho \mathbf{U}) + \nabla \cdot \mathcal{T} = 0, \quad (5.73)$$

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \mathbf{w} = 0, \quad (5.74)$$

where $\mathcal{T} = \rho \mathbf{U} \otimes \mathbf{U} + P \mathbf{I}$ is the stress tensor, $\mathcal{E} = \frac{1}{2} \rho U^2 + \rho \varepsilon$ is the energy density, ε is the internal energy, and $\mathbf{w} = (\frac{1}{2} \rho U^2 + w) \mathbf{U}$ is the energy flux with enthalpy density $w = \rho \varepsilon + P$.

We assume the shock radius to be large. This allows us to treat the shock as planar and renders the problem one-dimensional along the normal direction of the shock front (Draine, 2011). Further, we assume the shock to be stationary in the shock rest frame. Integrating the conservation relations in Eqs. (5.72) to (5.74) over the shock leads to the shock jump conditions (also known as *Rankine-Hugoniot conditions* (Hugoniot, 1887; Rankine, 1870)). They are given by (de Hoffmann and Teller, 1950; Draine, 2011; Fitzpatrick, 2022):

$$\rho_u u_u = \rho_d u_d, \quad (5.75)$$

$$\rho_u u_u^2 + P_u = \rho_d u_d^2 + P_d, \quad (5.76)$$

$$\left(\frac{1}{2} u_u^2 + \varepsilon_u + \frac{P_u}{\rho_u} \right) \rho_u u_u = \left(\frac{1}{2} u_d^2 + \varepsilon_d + \frac{P_d}{\rho_d} \right) \rho_d u_d, \quad (5.77)$$

where the subscripts refer to up- and downstream quantities, respectively.

The last equation can be further simplified under the assumption of an ideal polytropic fluid with adiabatic index Γ . By using $\varepsilon = \frac{1}{\Gamma-1} \frac{P}{\rho}$ for the internal energy of an ideal gas and the mass conservation in Eq. (5.75), we find:

$$\frac{1}{2} u_u^2 + \frac{\Gamma}{\Gamma-1} \frac{P_u}{\rho_u} = \frac{1}{2} u_d^2 + \frac{\Gamma}{\Gamma-1} \frac{P_d}{\rho_d}. \quad (5.78)$$

From those shock jump conditions, we can derive an expression for the compression ratio, r , in terms of the upstream and downstream pressures and the upstream Mach number $\mathcal{M}_u \equiv u_u/c_{s,u}$ (see App. C.1 and Fitzpatrick, 2022):

$$r = \frac{\rho_d}{\rho_u} = \frac{u_u}{u_d} = \frac{\frac{\Gamma+1}{\Gamma-1} P_d + P_u}{P_d + \frac{\Gamma+1}{\Gamma-1} P_u} = \frac{(\Gamma+1) \mathcal{M}_u^2}{(\Gamma-1) \mathcal{M}_u^2 + 2}. \quad (5.79)$$

We have already noted that $u_{sh} \gg c_s$ in the case of SNRs. This means that the upstream Mach number is large: $\mathcal{M}_u \gg 1$. This is usually referred to as a *strong shock*. Especially, this means that $P_d \gg P_u$, which can be formally derived from the shock jump conditions by expressing the pressure ratio in terms of the Mach number. In this case, the compression ratio reduces to

$$r = \frac{\rho_d}{\rho_u} = \frac{u_u}{u_d} \simeq \frac{\Gamma+1}{\Gamma-1} = 4 \quad \text{for mono-atomic ideal gas with } \Gamma = \frac{5}{3}. \quad (5.80)$$

With these shock jump conditions, we have a basic understanding of shocks. For the derivation of DSA, it is instructive to introduce the different rest frames around a shock front. The basic picture of the different rest frames around a shock front is illustrated in Fig. 5.1.

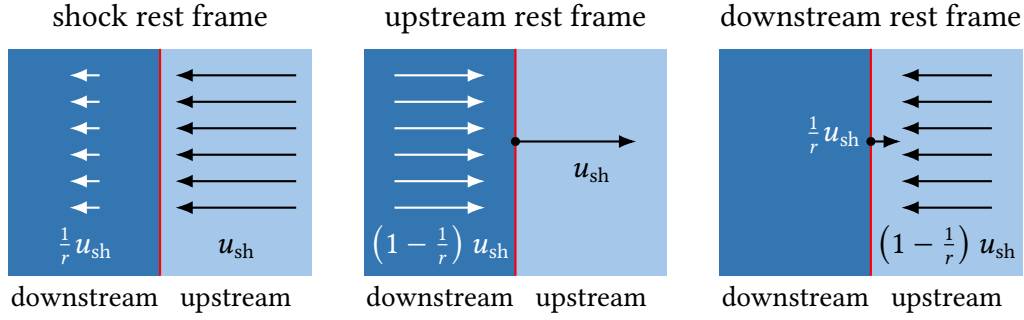


Figure 5.1: Rest frames around shock fronts. This figure illustrates the different rest frames that are commonly used when working with shocks. The dark blue region is the (denser) shocked downstream region, while the light blue region is the (less dense) unshocked upstream region. The compression ratio is the ratio of the upstream to downstream velocities in the shock rest frame: $r = u_u/u_d$. The figure is inspired by Mertsch, 2010, Fig. 1.2, which was inspired by Longair, 2011.

The shock jump conditions have been derived in the shock rest frame. Transforming into the upstream and downstream rest frames, respectively, shows that the flow in the other region is always moving towards the shock front with velocity $(1 - 1/r) u_{sh}$.

5.5.2 Macroscopic derivation

DSA can be derived from a one-dimensional transport equation under simplified assumptions. For this derivation, we use the one-dimensional version of the general transport Eq. (5.64) for the isotropic phase space density $f(t, x, p)$, while just considering spatial diffusion, advection, adiabatic losses, and a source term. This derivation has first been presented in Axford et al., 1977; Blandford and Ostriker, 1978; Krymskii, 1977. We will follow the explanation given in Drury, 1983.

We assume a planar shock, which renders the problem effectively one-dimensional. The one spatial coordinate is x , with the shock front at $x = 0$, the upstream region at $x < 0$, and the downstream region at $x > 0$. The transport equation for the isotropic pitch angle-averaged phase space density $f(t, x, p)$ in the shock rest frame is given by (Drury, 1983):

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} - \frac{\partial}{\partial x} \left(\kappa \frac{\partial f}{\partial x} \right) - \frac{1}{3} \frac{\partial u}{\partial x} p \frac{\partial f}{\partial p} = Q(x, p), \quad (5.81)$$

where $u(x)$ is the flow velocity in the shock rest frame, $\kappa(x, p)$ is the spatial diffusion coefficient, and $Q(x, p) = Q_0 \delta(p - p_{inj}) \delta(x)$ is a source term, i.e., we assume that particles are injected with momentum p_{inj} at the shock (Gieseler et al., 2000; Malkov, 1998; Morlino et al., 2021). We will discuss the source term further below. The other terms that can be added in analogy to the full transport equation, Eq. (5.64), are subdominant

here. E.g., momentum diffusion is significantly less efficient in accelerating cosmic rays than DSA, as we argued in the beginning of this section.

We solve Eq. (5.81) in a steady state, i.e., $\frac{\partial f}{\partial t} = 0$. For $x \neq 0$, Eq. (5.81) simplifies to $u \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial f}{\partial x} \right)$, and a solution is readily found:

$$f(x, p) = g_1(p) \exp \left(\int_0^x dx' \frac{u(x')}{\kappa(x', p)} \right) + g_2(p), \quad (5.82)$$

where $g_1(p)$ and $g_2(p)$ are functions to be determined by boundary conditions. We assume that far upstream ($x \rightarrow -\infty$), the phase space density is some given function $f_u(p)$, while far downstream ($x \rightarrow +\infty$), the phase space density remains finite. This leads to:

$$f(x, p) = \begin{cases} f_u(p) + g_u(p) \exp \left(\int_0^x dx' \frac{u(x')}{\kappa(x', p)} \right) & x < 0, \\ f_d(p) & x \geq 0. \end{cases} \quad (5.83)$$

The scale height of the exponential in the upstream region determines the distance that particles diffuse away from the shock before being lost to the acceleration process (see also Sec. 5.6). As the diffusion coefficient is larger for larger momenta, lower energy particles are confined closer to the shock. This also describes a balance between diffusion and advection which can only be achieved upstream (Drury, 1983).

The functions $g_u(p)$ and $f_d(p)$ can be determined by integrating the phase space density f and the whole transport equation Eq. (5.81) over an infinitesimal region around the shock front at $x = 0$ after multiplying it with the test function $\int_0^x dx' \kappa^{-1}$ (Drury, 1983). This yields the following two conditions:

$$[f]_{-\epsilon}^{+\epsilon} = \mathcal{O}(\epsilon) \quad \text{and} \quad \left[\kappa \frac{\partial f}{\partial x} \right]_{-\epsilon}^{+\epsilon} + \frac{1}{3} \left[u p \frac{\partial f}{\partial p} \right]_{-\epsilon}^{+\epsilon} - Q_0 \delta(p - p_{\text{inj}}) = \mathcal{O}(\epsilon), \quad (5.84)$$

which can be written as the following two identities at $x = 0$, noting that $\left[\kappa \frac{\partial f}{\partial x} \right]_{-\epsilon}^{+\epsilon} = -u_u g_u(p)$ according to Eq. (5.83):

$$f_u(p) + g_u(p) = f_d(p), \quad (5.85)$$

$$u_u g_u(p) + u_u \frac{p}{3} \frac{\partial (f_u + g_u)}{\partial p} + Q_0 \delta(p - p_{\text{inj}}) = u_d \frac{p}{3} \frac{\partial f_d}{\partial p}. \quad (5.86)$$

The last equation can be rearranged by eliminating $g_u(p)$ with the help of Eq. (5.85):

$$(u_u - u_d) \frac{p}{3} \frac{\partial f_d}{\partial p} = u_u (f_u - f_d) + Q_0 \delta(p - p_{\text{inj}}). \quad (5.87)$$

Dividing this by u_d and introducing the compression ratio $r = u_u/u_d$, we find:

$$p \frac{\partial f_d}{\partial p} = -\frac{3r}{r-1} (f_d - f_u) + \frac{3r}{u_u(r-1)} Q_0 \delta(p - p_{\text{inj}}). \quad (5.88)$$

We define the quantity $s \equiv 3r/(r-1)$. For strong shocks with $r = 4$ (see Eq. (5.80)), we have $s = 4$. We can rewrite Eq. (5.88):

$$\frac{\partial (p^s f_d)}{\partial p} = s f_u p^{s-1} + \frac{s}{u_u} Q_0 \delta(p - p_{\text{inj}}) p^{s-1}. \quad (5.89)$$

Integrating this equation yields our final result:

$$f_d(p) = p^{-s} s \int_{p_{\text{min}}}^p dp' (f_u(p') p'^{s-1}) + \frac{s Q_0}{u_u p_{\text{inj}}} \left(\frac{p}{p_{\text{inj}}} \right)^{-s}. \quad (5.90)$$

The first term shows that any existing phase-space density in the upstream region that has been convected to the shock gets accelerated to a power law in momentum with index s . If the upstream phase-space density has a spectrum that is softer than p^{-s} , i.e., it decreases faster than p^{-s} with increasing momentum, the resulting accelerated spectrum at the shock will have the power-law index s . However, if the upstream spectrum is harder than p^{-s} , the accelerated spectrum will reflect the upstream spectrum.

The second term shows that particles coming from the thermal population of particles present in the plasma can also be injected at the shock itself (Gieseler et al., 2000; Malkov, 1998). The source term $Q(x, p) = Q_0 \delta(p - p_{\text{inj}}) \delta(x)$ reflects that only particles with momenta higher than some injection momentum p_{inj} can take part in the acceleration process. Physically, p_{inj} is determined by the minimum momentum a particle needs to cross the shock front and participate in the acceleration process. This momentum lies at the high-energy tail of the thermal particle distribution, which is steeply falling at high momenta. Thus, the approximation of injecting particles just at the momentum p_{inj} via a delta-injection is justified (Gieseler et al., 2000). The normalisation Q_0 can be determined by assuming that a fraction η_{CR} of the incoming flux of thermal particles enters the acceleration process (Blasi, 2013):

$$Q_0 = \frac{\eta_{\text{CR}} n_u u_u}{4\pi p_{\text{inj}}^2}, \quad (5.91)$$

where n_u is the upstream number density of thermal particles.

For a strong shock, we have $s = 4$. Mind, that we usually work with the isotropic differential density, ψ_p , instead of the phase space density f . As we have shown in App. A.3, they are related via $\psi_p = 4\pi p^2 f$. Thus, the standard result from DSA tells us that the source term in the cosmic-ray transport equation Eq. (5.64) is proportional to $\mathcal{R}^{-2}$ for strong shocks:

$$Q(\mathcal{R}, t, \mathbf{x}) \propto \mathcal{R}^{-2}. \quad (5.92)$$

The solution in Eq. (5.90) is derived under a number of simplifying assumptions. The standard result for a power-law spectrum $\propto p^{-4}$ can be altered by many effects, e.g., non-linear feedback of accelerated particles on the shock structure, magnetic field amplification, or time-dependent effects (Cristofari et al., 2022; Malkov and Drury, 2001).

Further, the relative speed of the scattering centres (Bell, 1978a; Caprioli et al., 2020) or shock obliquity (Bell et al., 2011) can explain deviations from the canonical result.

A closer investigation of the result in Eq. (5.90) shows that it leads to a diverging cosmic-ray pressure at the shock (Morlino, 2017):

$$P_{\text{CR}} = \frac{4\pi}{3} \int_{p_{\text{inj}}}^{p_{\text{max}}} dp p^2 (pv(p)) f_d(p) \propto \int_{p_{\text{inj}}}^{p_{\text{max}}} dp p^{3-s} \propto \ln(p_{\text{max}}) \xrightarrow[s=4]{p_{\text{max}} \rightarrow \infty} \infty. \quad (5.93)$$

One way to remedy this is to impose a maximum momentum that particles can be accelerated to, p_{max} . Eq. (5.90) has been derived under the assumption that particles can diffuse arbitrarily far upstream without being completely lost to the acceleration process. However, in reality there will be limitations, because the transport is time-dependent or the size of the system is finite. Those are usually used to derive a maximum momentum, and we will discuss this in more detail in Sec. 5.6. Here, we just want to note that one way to impose a finite size of the system is to set a free-escape boundary some distance x^* upstream from the shock. Particles, whose diffusion length exceeds this distance, are then considered to be lost to the acceleration process.

5.5.3 Microscopic derivation

In this section, we want to understand the physical mechanism behind DSA better by re-deriving the result in a microscopic picture. The fundamental idea behind first- and second-order Fermi acceleration is that particles can gain energy by scattering off magnetic inhomogeneities that are moving with respect to the particle. We will introduce this picture and DSA following Morlino, 2017 and Drury, 1983.

For simplicity, we want to assume that the particles participating in the acceleration mechanism are relativistic, i.e., $E \sim pc$. The clouds are assumed to move with some velocity $v = \beta c$. A particle of energy E_1 whose velocity has a pitch angle θ_1 with respect to the cloud velocity will have an energy E'_1 in the cloud rest frame:

$$E'_1 = \gamma E_1 (1 - \beta \mu_1), \quad (5.94)$$

where $\mu_1 = \cos \theta_1$ is the pitch angle cosine and $\gamma = (1 - \beta^2)^{-1/2}$ is the Lorentz factor.

We assume that the energy is conserved in the scattering event, $E'_2 = E'_1$, while the momentum changes direction, $p'_2 = -p'_1$. Transforming back into the lab frame, we find for the energy after the scattering:

$$E_2 = \gamma E'_2 (1 + \beta \mu'_2), \quad (5.95)$$

where $\mu'_2 = \cos \theta'_2$ is the pitch angle cosine after the scattering in the cloud rest frame.

The energy gain (or, equivalently, the momentum gain) is then given by

$$\frac{\Delta p}{p_1} \simeq \frac{\Delta E}{E_1} = \frac{E_2 - E_1}{E_1} = \frac{1 - \beta\mu_1 + \beta\mu'_2 - \beta^2\mu_1\mu'_2}{1 - \beta^2} - 1. \quad (5.96)$$

We can determine the mean gain by averaging over the incoming and outgoing angles. First, we assume that the scattering is isotropic in the cloud frame, i.e., $\langle \mu'_2 \rangle = 0$. An asymmetry occurs in the flux of particles entering the cloud, as the cloud is moving with respect to the lab frame. The flux of particles entering the cloud from the lab frame is proportional to $\beta_r = 1 - \beta\mu_1$. For an isotropic flux of particles in the lab frame, we find:

$$\langle \mu_1 \rangle = \frac{\int_{-1}^{+1} d\mu_1 \mu_1 \beta_r}{\int_{-1}^{+1} d\mu_1 \beta_r} = \frac{\int_{-1}^{+1} d\mu_1 \mu_1 (1 - \beta\mu_1)}{\int_{-1}^{+1} d\mu_1 (1 - \beta\mu_1)} = -\frac{1}{3}\beta. \quad (5.97)$$

Plugging this into Eq. (5.96) gives the mean energy (momentum) gain per collision:

$$\left\langle \frac{\Delta p}{p} \right\rangle \simeq \left\langle \frac{\Delta E}{E} \right\rangle = \frac{1 + \frac{1}{3}\beta^2}{1 - \beta^2} - 1 \simeq \frac{4}{3}\beta^2, \quad (5.98)$$

where we assume that the clouds are non-relativistic, i.e., $\beta \ll 1$. This is the original result from Fermi, 1949. The fact that this energy gain is proportional to β^2 coined the term *second-order Fermi acceleration*. Typical cloud speeds are relatively small ($\beta = \mathcal{O}(10^{-4})$) and particles have relatively long mean free paths between collisions with different clouds. A typical mean free path of 0.1 pc would lead to only a few collisions per year. This makes second-order Fermi acceleration inefficient in explaining the observed cosmic-ray spectrum (Morlino, 2017).

A shock provides the needed asymmetry in the probability of head-on and overtaking collisions that makes DSA more efficient than this. We have just seen in Fig. 5.1, that in both the upstream and downstream rest frames, the flow in the other region is moving towards the shock front. This means, that particles crossing the shock will always experience head-on collisions. This idea was first developed by Bell, 1978a,b.

We can see in Fig. 5.2, that the situation is similar to the scattering off clouds moving with relative velocity $u_r = \beta_r = u_u - u_d = (1 - 1/r) u_{sh}$. The difference to the previous case of second-order Fermi acceleration lies in the averaging of the pitch angle cosine μ_1 . With the assumption of an isotropic particle density n , the flux of particles crossing the shock from the downstream to the upstream region is:

$$J_{d \rightarrow u} = \int_{-1}^0 d\mu_1 \frac{nc\mu_1}{2} = -\frac{nc}{4}. \quad (5.99)$$

Mind, that the integration is only over negative μ_1 because only particles with positive pitch angle cosines can cross the shock in this direction. The factor 1/2 in the integrand

is a normalisation as the integration of the whole pitch angle cosine space has to yield the total density, i.e., $\int_{-1}^{+1} d\mu \mu n/2 = n$.

Thus, the average pitch angle cosine of particles crossing the shock from downstream to upstream is:

$$\langle \mu_1 \rangle = \frac{1}{J_{d \rightarrow u}} \int_{-1}^0 d\mu_1 \mu_1 \frac{nc\mu_1}{2} = 2 \int_0^{-1} d\mu_1 \mu_1^2 = -\frac{2}{3}. \quad (5.100)$$

Similarly, the average pitch angle cosine of particles crossing the shock from upstream to downstream is: $\langle \mu_2' \rangle = \frac{2}{3}$. Plugging those averages into Eq. (5.96), we find for the mean energy (momentum) gain per shock crossing cycle:

$$\left\langle \frac{\Delta p}{p} \right\rangle \simeq \left\langle \frac{\Delta E}{E} \right\rangle = \frac{1 + \frac{4}{3}\beta_r + \frac{4}{9}\beta_r^2}{1 - \beta_r^2} - 1 = \frac{4}{3}\beta_r + \mathcal{O}(\beta_r^2), \quad (5.101)$$

Because this scales linearly with β_r , DSA is a *first-order Fermi acceleration* process. This makes DSA much more efficient than second-order Fermi acceleration.

In each cycle of a particle crossing the shock from downstream to upstream and back, there is also a probability that the particle escapes the acceleration process to downstream infinity. This is given by the ratio between the particle flux escaping to downstream infinity and the total particle flux crossing the shock from upstream to downstream. Under a steady-state assumption, the particle flux to downstream infinity is given by $J_\infty = nu_d$, while flux conservation gives $J_{u \rightarrow d} = J_\infty + J_{d \rightarrow u}$. This leads to an escape probability of (Morlino, 2017):

$$P_{\text{esc}} = \left| \frac{J_\infty}{J_{u \rightarrow d}} \right| = \left| \frac{nu_d}{-nc/4 + nu_d} \right| \simeq \frac{4u_d}{c}, \quad (5.102)$$

where we used that $u_d \ll c$. This does also imply that the escape probability is small, i.e., most particles will stay in the acceleration process for many crossings of the shock.

At each cycle, an average momentum gain of $\zeta = \langle \Delta p/p \rangle$ is achieved. This means that the momentum after k cycles is given by $p = p_0 (1 + \zeta)^k$. Inverting this, we find that the number of cycles needed to reach a given momentum p is given by

$$k = \frac{\ln(p/p_0)}{\ln(1 + \zeta)}, \quad (5.103)$$

Noting that after each cycle, only a fraction of $1 - P_{\text{esc}}$ of the particles remain in the acceleration process, the total number of particles with an energy exceeding a set momentum p is proportional to

$$N(> p) \propto \sum_{i=k}^{\infty} (1 - P_{\text{esc}})^i = \frac{(1 - P_{\text{esc}})^k}{P_{\text{esc}}} = \frac{1}{P_{\text{esc}}} \left(\frac{p}{p_0} \right)^{-q}, \quad (5.104)$$

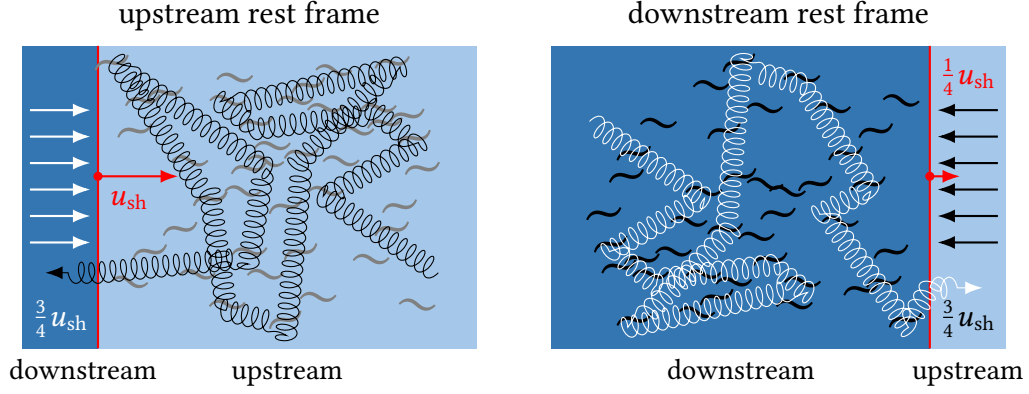


Figure 5.2: Sketch of Diffusive Shock Acceleration. This figure illustrates the physical picture behind DSA at strong shocks (compression ratio $r = 4$). On either side of the shock, a particle gets transported along magnetic field lines and scatters on magnetic turbulence (depicted by waves). The scattering isotropises the particle distribution in the local rest frame of either side of the shock. Because the flow on the other side of the shock is always moving towards the shock front, the particle experiences head-on collisions in each crossing of the shock.

where we defined $q \equiv -\ln(1 - P_{\text{esc}}) / \ln(1 + \zeta)$. As $P_{\text{esc}} \ll 1$ and $\zeta \ll 1$, we can approximate $q \simeq P_{\text{esc}} / \zeta$.

This leads to a differential spectrum of accelerated particles at the shock front following a power law: $Q(p) \propto p^{-q-1}$. Using the expressions for P_{esc} in Eq. (5.102) and ζ in Eq. (5.101), we find:

$$q + 1 = 1 + \frac{P_{\text{esc}}}{\zeta} = 1 + \frac{3u_d}{u_u - u_d} = \frac{r + 2}{r - 1}, \quad (5.105)$$

where we used the compression ratio $r = u_u / u_d$. For strong shocks with $r = 4$, we again find the standard result of a source spectrum: $Q(\mathcal{R}, t, \mathbf{x}) \propto \mathcal{R}^{-2}$.

5.6 Maximum rigidities

We already noted that the solution in Eq. (5.90) leads to a diverging cosmic-ray pressure at the shock if no maximum momentum, p_{max} , is imposed. A closer investigation of the cosmic-ray spectrum and the knee feature arising at rigidities of a few PV (see Fig. 3.1 in Sec. 3.1.1) suggest that Galactic accelerators need to accelerate cosmic rays up to a maximum rigidity in that range (Blasi, 2013; Gabici et al., 2019). One criterion that can be used for the derivation of a maximum rigidity is to consider the finite age of the accelerator. While the macroscopic derivation of DSA in Sec. 5.5.2 assumed a steady state, in reality the acceleration process takes some finite time. For this, we can use the

physical picture of DSA developed in Sec. 5.5.3. The maximum momentum achievable in DSA in accelerators with a finite lifetime can be estimated by demanding that:

$$\tau_{\text{acc}}(p_{\text{max}}) \lesssim \min(\tau_{\text{age}}, \tau_{\text{loss}}(p_{\text{max}})) , \quad (5.106)$$

where τ_{acc} is the available acceleration time, τ_{age} is the age of the accelerator, and τ_{loss} is the energy loss timescale of the particles. For hadronic cosmic rays, energy losses are usually negligible compared to the age of the accelerator, i.e., $\tau_{\text{loss}} \gg \tau_{\text{age}}$. Thus, we will focus on the age of the accelerator in the following. Further, it turns out that considerations of the finite age of the accelerator are most restrictive in deriving maximum rigidities for SNRs. For a discussion and comparison to other criteria, we refer the reader to Blasi, 2013 and Bell, 2013.

To determine the acceleration timescale, τ_{acc} , we note that a particle needs to spend some time on either side of the shock before being able to cross the shock again. In Fig. 5.2, this is illustrated by the particle scattering on the turbulent magnetic fields on either side of the shock and isotropising before crossing the shock again.

As we have shown before, the flux of particles crossing the shock from downstream to upstream is given by $|J_{\text{d} \rightarrow \text{u}}| = nc/4$ (see Eq. (5.99)). This means that a total number of $A(nc/4) \tau_{\text{diff,u}}$ particles cross the shock during the diffusion time $\tau_{\text{diff,u}}$, passing through an area A . From particle number conservation, this has to equal the total number of particles within a volume AL_{u} , where $L_{\text{u}} = \kappa_{\text{u}}/u_{\text{u}}$ is the diffusion length upstream of the shock. This leads to the condition:

$$\frac{nc}{4} \tau_{\text{diff,u}} A = n \frac{\kappa_{\text{u}}}{u_{\text{u}}} A \quad \Rightarrow \quad \tau_{\text{diff,u}} = \frac{4\kappa_{\text{u}}}{cu_{\text{u}}} . \quad (5.107)$$

Similarly, we find for the downstream region, that $\tau_{\text{diff,d}} = \frac{4\kappa_{\text{d}}}{cu_{\text{d}}}$ (Blasi, 2013). The timescale for a full shock crossing cycle is simply the sum of those two: $\tau_{\text{cycle}} = \tau_{\text{diff,u}} + \tau_{\text{diff,d}}$.

The total acceleration timescale can then be estimated by dividing the time per cycle by the average fractional momentum gain per cycle ζ (see Eq. (5.101)):

$$\tau_{\text{acc}} = \frac{\tau_{\text{cycle}}}{\zeta} = \frac{\frac{4}{c} \left(\frac{\kappa_{\text{u}}}{u_{\text{u}}} + \frac{\kappa_{\text{d}}}{u_{\text{d}}} \right)}{\frac{4}{3} \frac{u_{\text{u}} - u_{\text{d}}}{c}} = \frac{3}{u_{\text{u}} - u_{\text{d}}} \left(\frac{\kappa_{\text{u}}}{u_{\text{u}}} + \frac{\kappa_{\text{d}}}{u_{\text{d}}} \right) , \quad (5.108)$$

This expression obtained by this simplified derivation is in agreement with a more rigorous derivation that is used, e.g., in Lagage and Cesarsky, 1983a and later in Lagage and Cesarsky, 1983b:

$$\tau_{\text{acc}} = \frac{3}{u_{\text{u}} - u_{\text{d}}} \int_0^p dp' \frac{1}{p'} \left(\frac{\kappa_{\text{u}}(p')}{u_{\text{u}}} + \frac{\kappa_{\text{d}}(p')}{u_{\text{d}}} \right) . \quad (5.109)$$

By assuming a strong shock and that the diffusion coefficient downstream is approximately suppressed by the compression ratio $r = 4$, i.e., $\kappa_d \approx \kappa_u/4$ (Bell, 2013; Morlino, 2017), we find:

$$\tau_{\text{acc}} = \frac{4}{u_{\text{sh}}} \left(\frac{\kappa_u}{u_{\text{sh}}} + \frac{4\kappa_d}{u_{\text{sh}}} \right) \approx 8 \frac{\kappa_u}{u_{\text{sh}}^2}. \quad (5.110)$$

For our estimate of a maximum rigidity, we will assume a spatially constant diffusion coefficient on either side of the shock. Following our considerations in Sec. 5.3.7 that lead to Eq. (5.58), we can write the diffusion coefficient as:

$$\kappa_u(\mathcal{R}_{\text{max}}) = \frac{1}{3} \frac{c r_L(\mathcal{R}_{\text{max}})}{\mathcal{F}(1/r_L(\mathcal{R}_{\text{max}}))}, \quad (5.111)$$

where $\mathcal{F}(k)$ is the power per logarithmic bandwidth at wavenumber k as introduced in Eq. (5.57) (Amato, 2014).

We get an estimate for the maximum rigidity by plugging this into Eq. (5.110) and equating the acceleration timescale to the age of the accelerator, τ_{SNR} . Further, we will assume that the age of the SNR is related to the shock radius, R_{sh} , via $\tau_{\text{SNR}} \approx R_{\text{sh}}/u_{\text{sh}}$. This is true for the *free-expansion phase* of the SNR, where the shock velocity is approximately constant (Draine, 2011). We will argue below, why we consider this phase of SNR evolution for the maximum rigidities. This leads to:

$$\frac{R_{\text{sh}}}{u_{\text{sh}}} = \frac{8}{3} \frac{c r_L(\mathcal{R}_{\text{max}})}{u_{\text{sh}}^2 \mathcal{F}(1/r_L(\mathcal{R}_{\text{max}}))} \quad (5.112)$$

$$\Rightarrow \mathcal{R}_{\text{max}} \approx \frac{3}{8c} B u_{\text{sh}} R_{\text{sh}} \mathcal{F}(1/r_L(\mathcal{R}_{\text{max}})) \quad (5.113)$$

$$\Rightarrow \mathcal{R}_{\text{max}} \approx 10 \text{ TV} \left(\frac{B}{1 \mu\text{G}} \right) \left(\frac{u_{\text{sh}}}{c/30} \right) \left(\frac{R_{\text{sh}}}{1 \text{ pc}} \right) \mathcal{F}(\mathcal{R}_{\text{max}}), \quad (5.114)$$

where we used that the Larmor radius is given by $r_L(\mathcal{R}) = \mathcal{R}/B$ (see App. A.2). In the last line, we used that the shock speed is usually of the order of $10^4 \text{ km s}^{-1} \approx c/30$ in young SNRs (see Eq. (5.70)). This is related to the famous *Hillas criterion*, where Bohm diffusion is assumed, i.e., $\mathcal{F} = 1$ (Hillas, 1984).

The result in Eq. (5.114) shows how the maximum rigidity achievable in DSA depends on the magnetic field strength, B , the shock speed, u_{sh} , the shock radius, R_{sh} , and the level of turbulence at the scale of the Larmor radius, $\mathcal{F}(\mathcal{R}_{\text{max}})$. If the magnetic field strength was of the order of μG and the level of turbulence was as inferred from local observations, which suggest $\mathcal{F} \ll 1$, this would clearly fall short of the knee rigidities at a PV level, hardly exceeding the GV range (Blasi, 2013). Thus, this result suggests that acceleration up to the knee requires a significant amplification of the magnetic field and a high level of turbulence, $\mathcal{F}$, on all scales up to the Larmor radius of PV particles (Blasi, 2013).

The estimate of the maximum rigidity in Eq. (5.114) is similar to the one derived by Lagage and Cesarsky, 1983a,b. They showed that the consideration of a spatially dependent

diffusion coefficient leads to a further suppression of the estimate by a factor of order 10. Even with a high level of turbulence, $\mathcal{F} \approx 1$, this means that there is a serious problem in achieving rigidities in excess of $\mathcal{O}(10 \text{ TV})$ in SNR shocks (Bell, 2013). Before we come to a discussion of how a turbulence level of $\mathcal{F} \approx 1$ can be achieved, we discuss this result a bit further and argue, why the highest rigidities are expected to be achieved at the end of the free-expansion phase of the SNR evolution.

We have derived this result under the assumption that the age of the SNR is given by $\tau_{\text{SNR}} \approx R_{\text{sh}}/u_{\text{sh}}$, i.e., that the acceleration happens in the free-expansion phase of the SNR evolution. To justify this, we note that in later phases of the SNR evolution, the shock speed decreases significantly with time, while the shock radius increases only slowly. The free-expansion phase, where the ejecta travel ballistically continues until the mass swept up by the SNR shock equals the mass of the ejecta (Draine, 2011). This marks the beginning of the so-called *Sedov-Taylor (ST) phase*. From this condition, we can estimate the shock radius at the beginning of the ST phase as (Draine, 2011):

$$R_{\text{ST}} \approx \left(\frac{3M_{\text{ejecta}}}{4\pi\rho_0} \right)^{1/3} \approx 2 \text{ pc} \left(\frac{M_{\text{ejecta}}}{M_{\odot}} \right)^{1/3} \left(\frac{n_0}{1 \text{ cm}^{-3}} \right)^{-1/3} \left(\frac{\mu}{1.4} \right)^{-1/3}, \quad (5.115)$$

where μ is the mean molecular weight, i.e., $\rho_0 = n_0 \mu m_p$ with m_p being the proton mass.

Using the estimate of the shock speed in Eq. (5.70), we can determine that the Sedov time, t_{ST} , is given by:

$$t_{\text{ST}} \approx \frac{R_{\text{ST}}}{u_{\text{sh}}} \approx 200 \text{ yr} \left(\frac{M_{\text{ejecta}}}{M_{\odot}} \right)^{5/6} \left(\frac{E_{\text{SN}}}{10^{51} \text{ erg}} \right)^{-1/2} \left(\frac{n_0}{1 \text{ cm}^{-3}} \right)^{-1/3} \left(\frac{\mu}{1.4} \right)^{-1/3}. \quad (5.116)$$

In the ST phase, the evolution of the shock can be approximated by a self-similar solution that can be obtained by idealising the SNR as a point explosion in a uniform medium. Such solutions have been first derived by Sedov, 1946; Taylor, 1950; Von Neumann, 1941. The time evolution of the shock radius and shock speed in this phase can be derived by dimensional analysis arguments (Draine, 2011). The only dimensional parameters of the problem are the explosion energy, E_{SN} , the ambient density, ρ_0 , and the time since the explosion, t . From these, we can construct a length scale that has to correspond to the shock radius R_{sh} (Truelove and McKee, 1999):

$$R_{\text{sh}}(t) = A E_{\text{SN}}^{1/5} \rho_0^{-1/5} t^{2/5} \propto t^{2/5} \Rightarrow u_{\text{sh}}(t) \propto t^{-3/5}, \quad (5.117)$$

where A is a dimensionless constant that can be determined in the exact solution.

We can see that while the shock radius, R_{sh} , still increases after the beginning of the ST phase, the shock speed, u_{sh} , decreases significantly with time. Thus, we expect that the maximum rigidity in DSA in a SNR environment can be achieved at the end of the

free-expansion phase, i.e., around the Sedov time, t_{ST} (Blasi, 2013). Equating Eq. (5.116) with the acceleration time in Eq. (5.110) gives:

$$\mathcal{R}_{\text{max}} \approx 20 \text{ TV} \left(\frac{B}{1 \mu\text{G}} \right) \left(\frac{M_{\text{ejecta}}}{M_{\odot}} \right)^{-1/6} \left(\frac{E_{\text{SN}}}{10^{51} \text{ erg}} \right)^{1/2} \left(\frac{\mu n_0}{1.4 \text{ cm}^{-3}} \right)^{-1/3} \mathcal{F}(\mathcal{R}_{\text{max}}). \quad (5.118)$$

The result in Eq. (5.118) shows that achieving rigidities in excess of some tens of TV requires a high level of turbulence $\mathcal{F} \approx 1$ or a significant amplification of the magnetic field strength B compared to typical ISM values.

Before we discuss possible mechanisms to achieve, both, high turbulence levels and stronger magnetic fields, we briefly want to mention other proposed solutions for accelerating cosmic rays to PV rigidities in Galactic accelerators. One might be that oblique shocks can increase the maximum rigidity achievable in DSA. Because of the shock geometry, particles can cross the shock front multiple times during gyrations around the background magnetic field lines, which can lead to a reduced acceleration time (Jokipii, 1982, 1987). However, advection away from the shock front can limit this effect (Morlino, 2017).

Another solution might be to consider sources other than SNRs. Recently, the winds of star clusters have gained new attention. For an overview on this topic, we refer to Morlino et al., 2021. High energies can be achieved in those environments because of the special shock geometry of the wind termination shock (see Fig. 5.3) and the long lifetime of those sources. The supersonic wind from the massive stars in the cluster creates a shock upon interaction with the ISM, the forward shock. However, the continuous wind from the star cluster also creates a termination shock when interacting with the shocked stellar wind material (R_s in Fig. 5.3).

Because of the continuous energy input from the stellar winds, the termination shock can last for several million years and is approximately stationary (Morlino et al., 2021). Most importantly, the shock geometry is such that the upstream of the termination shock is on the inside of the wind bubble, such that an escape upstream is not possible. Sizeable particle acceleration and maximum rigidities in excess of 100 TV are indicated by γ -ray observations of a few compact star clusters like, e.g., Westerlund 1 (Abramowski et al., 2012), Westerlund 2 (Yang et al., 2018), or the Cygnus cocoon (Ackermann et al., 2011; Lhaaso Collaboration, 2024). Enhancements of the $^{22}\text{Ne}/^{20}\text{Ne}$ -ratio, as already mentioned in Sec. 3.1.2, also indicate that cosmic rays might be accelerated at least partially from material in winds of massive stars (Gupta et al., 2020; Prantzos, 2012). While this class of sources provides an interesting alternative to the standard SNR paradigm, it is still unclear how large their contribution to the total Galactic cosmic-ray flux can be. In the following, we will focus on SNRs as the main prototypical sources of Galactic cosmic rays. We discuss possible mechanisms to amplify magnetic fields and achieve high turbulence levels in those sources.

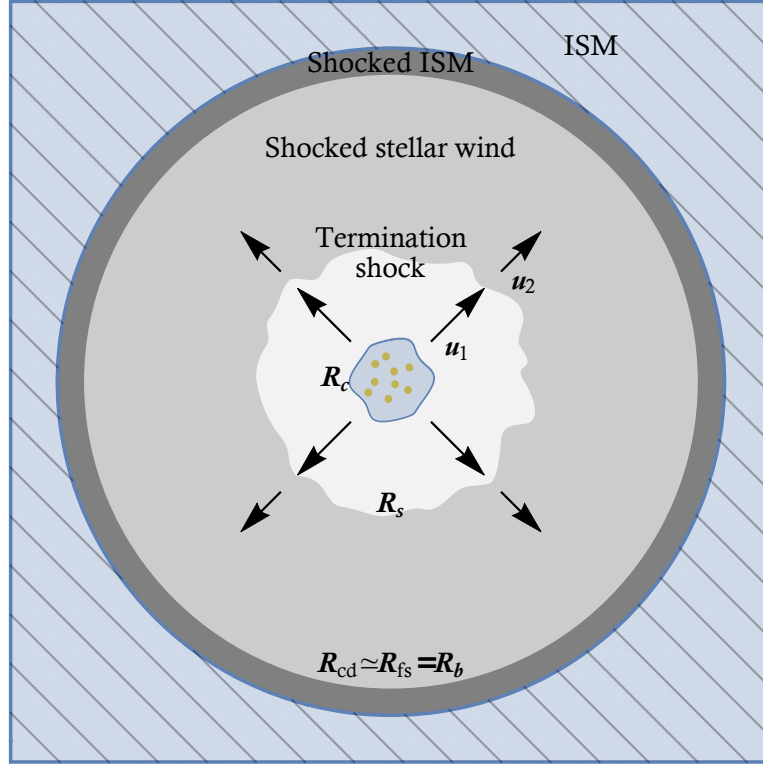


Figure 5.3: Schematic view of a star cluster wind bubble. The collective wind of massive stars in a star cluster creates shock fronts upon interaction with the ISM. R_s denotes the position of the termination shock, R_{cd} the contact discontinuity, and R_{fs} the forward shock propagating into the ISM. The figure is taken from Morlino et al., 2021, Fig. 1 (Creative Commons CC BY-NC-SA 4.0 licence).

5.7 Magnetic field amplification

One main difficulty in achieving high rigidities in SNR shocks lies in the need for a high level of turbulence, $\mathcal{F}$, upstream of the shock. This is necessary to reduce the diffusion coefficient and thus the acceleration timescale. It has been noted by several authors that the streaming of cosmic rays upstream of the shock can lead to an amplification of magnetic turbulence (Bell, 1978a,b; Lagage and Cesarsky, 1983a,b; Skilling, 1975a,b). This phenomenon is usually referred to as the *resonant streaming instability* or *Alfvén instability*. It is a non-linear effect in DSA, as the cosmic rays themselves induce turbulence at scales in resonance with their Larmor radius, which in turn are needed for their confinement close to the shock front. Calculations of this effect include a coupling of the transport equations for the cosmic rays and the equations that describe the evolution of magnetic turbulence. For a review on this topic, we refer the reader to (Amato, 2014; Bell, 2013; Blasi, 2013; Malkov and Drury, 2001).

By invoking the resonant streaming instability, it is possible to achieve turbulence levels of $\mathcal{F} \lesssim 1$ (see, e.g., Lagage and Cesarsky, 1983b). However, the instability saturates once the turbulent component of the magnetic field becomes comparable to the background magnetic field: $\delta B \sim B_0$. As we have seen in Eqs. (5.114) and (5.118), an effective transition to Bohm diffusion alone ($\mathcal{F} \sim 1$) cannot explain cosmic-ray acceleration beyond some tens of TV, unless the magnetic field strength is significantly amplified compared to typical ISM values.

There is observational evidence from SNRs that magnetic fields can be amplified to strengths of the order of hundreds of μG and that Bohm diffusion is present (Ballet, 2006; Vink, 2012). Those conclusions can be made by observations of synchrotron emissions from non-thermal cosmic-ray electrons close to SNR shocks in the X-ray band. For electrons, the relevant loss timescale for the estimation of maximum rigidities (see Eq. (5.106)) is the energy loss timescale due to synchrotron radiation. The mean free path of electrons can be obtained from a turnover in the measured synchrotron spectrum. It can be shown that this turnover is independent of the magnetic field and directly relates to the mean free path of the electrons (see, e.g., Bell, 2013). There is indeed evidence for $\lambda_{\text{mfp}} \sim r_L$, i.e., Bohm diffusion. Further, the scale length of synchrotron emissions in the rims can be related to the magnetic field strength, implying $B = \mathcal{O}(100 \mu\text{G})$ (Blasi, 2013).

While it is well-established that the resonant streaming instability saturates at $\delta B \sim B_0$, Magnetohydrodynamics (MHD) simulations by Lucek and Bell, 2000 showed further growth of the magnetic field beyond this limit. Later, Bell, 2004, 2005 showed that this is not due to the Alfvén instability, but a new-found Non-Resonant Hybrid (NRH) instability (often referred to as *non-resonant streaming instability*). This instability is non-resonant as it does not rely on the resonance between the Larmor radius of the cosmic rays and the wavelength of the excited turbulence, and it is hybrid as it couples the kinetic non-thermal cosmic-ray current to the thermal plasma (Bell, 2013). Now, we want to derive the growth rate of the magnetic fields in the NRH instability based on the simplified derivation in Bell, 2005, 2013. This allows us to estimate the maximum magnetic fields that can be reached by invoking this mechanism.

On scales larger than the Larmor radius, the cosmic rays are transported along the background magnetic field $\mathbf{B}$, and cause an electric current which can be described by a current density $\mathbf{j}_{\text{CR}} \parallel \mathbf{B}$. In the case of a cold background plasma, the pressure gradient is small compared to the $\mathbf{j}_{\text{CR}} \times \mathbf{B}$ -force. The cosmic-ray current must be balanced by a return current in the background plasma, $\mathbf{j}_r$. We assume that $\nabla \times \mathbf{B} = 4\pi/c (\mathbf{j}_{\text{CR}} + \mathbf{j}_r) \approx 0$, such that $\mathbf{j}_r = -\mathbf{j}_{\text{CR}}$.

Let $\mathbf{v}$ be the velocity of the background plasma and ρ its mass density. The equation of motion of the background plasma is then given by:

$$\rho \frac{d\mathbf{v}}{dt} = \frac{\mathbf{j}_r \times \mathbf{B}}{c} = -\frac{\mathbf{j}_{\text{CR}} \times \mathbf{B}}{c}. \quad (5.119)$$

Further, Faraday's law and the Lorentz force can be combined, and the time derivative of the mass density, ρ , can be given:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad \text{and} \quad \frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v}. \quad (5.120)$$

The last two equations given in Eq. (5.120) can be combined to give:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\mathbf{B}}{\rho} \right) &= \frac{1}{\rho} (-\mathbf{B} (\nabla \cdot \mathbf{v}) + (\mathbf{B} \cdot \nabla) \mathbf{v} - (\mathbf{v} \cdot \nabla) \mathbf{B}) + \frac{(\mathbf{v} \cdot \nabla) \mathbf{B}}{\rho} + \frac{\mathbf{B}}{\rho^2} (\rho \nabla \cdot \mathbf{v}) \\ &= \left(\frac{\mathbf{B}}{\rho} \right) \cdot \nabla \mathbf{v}. \end{aligned} \quad (5.121)$$

Assuming that cosmic rays only stream along the unperturbed field lines of the initial magnetic field, $\mathbf{B}_0$, which we assume to be along the z -direction, and that all quantities only vary in the direction parallel to $\mathbf{j}_{\text{CR}}$, we get simplified versions of Eqs. (5.119) and (5.121) (Bell, 2013):

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{j}_{\text{CR}} \times \frac{\mathbf{B}}{c\rho} \quad \text{and} \quad \frac{\partial}{\partial t} \left(\frac{\mathbf{B}}{c\rho} \right) = \left(\frac{B_0}{c\rho_0} \right) \frac{\partial \mathbf{v}}{\partial z}. \quad (5.122)$$

There is an exponentially growing circularly polarised solution to those equations. In particular, the magnetic field lines take the form of spirals with constant wavelength $\lambda = 2\pi/k$ around the initial magnetic field direction, but with an exponentially growing amplitude. We can determine the growth rate, Γ , of this instability by assuming such a solution and using Eq. (5.122). We define $\Phi \equiv \mathbf{B}/(c\rho)$ and find:

$$\frac{\partial^2 \Phi}{\partial t^2} \sim \frac{B_0}{c\rho_0} k j_{\text{CR}} \Phi \quad \Rightarrow \quad \Gamma = \sqrt{\frac{k B_0 j_{\text{CR}}}{c\rho_0}}. \quad (5.123)$$

The growth of these spirals is slowed when they run into each other, but simulations show that the magnetic field strength still grows very fast (Bell, 2004). The NRH instability creates low density cavities as the plasma is frozen to the field lines. Those are surrounded by regions of high density and strong magnetic fields (Bell, 2013). Because of the scaling of the growth rate with the wavenumber k in Eq. (5.123) ($\Gamma \propto \sqrt{k}$), the instability grows most rapidly on the smallest scales, i.e., for large k .

We can determine the maximum growth rate by considering the smallest scale on which the instability can grow. In the derivation above, we have assumed that magnetic tension forces are negligible. Specifically, we neglected the magnetic tension term $\propto \mathbf{B} \times (\nabla \times \mathbf{B})$ on the right-hand side of Eq. (5.119). This is a valid assumption as long as $4\pi j_{\text{CR}} > ckB_0$ holds⁶. Thus, the maximum wavenumber for which the instability can grow is given by

⁶The full force equation is $\rho d\mathbf{v}/dt = -\mathbf{j}_{\text{CR}} \times \mathbf{B}/c - \mathbf{B} \times (\nabla \times \mathbf{B})/(4\pi)$. A comparison between $\mathbf{j}_{\text{CR}}/c$ and $\nabla \times \mathbf{B}/(4\pi)$ gives the desired inequality.

$k_{\max} \sim 4\pi j_{\text{CR}} / (cB_0)$. Plugging this into Eq. (5.123), we find the maximum growth rate of the NRH instability (Bell, 2004, 2005; Blasi and Amato, 2019):

$$\Gamma_{\max} = \frac{j_{\text{CR}}}{c} \sqrt{\frac{4\pi}{\rho_0}} = k_{\max} v_A, \quad (5.124)$$

where we inserted the Alfvén speed $v_A = B_0 / \sqrt{4\pi\rho_0}$ (see Eq. (5.6)).

The natural lower limit in wavenumber for the instability to operate is $k_{\min} = r_L^{-1}$. Once the upper and lower limit coincide, the NRH instability can no longer operate. From this, we can estimate a maximum magnetic field strength that the instability can amplify. Following Bell, 2013, the cosmic-ray current close to the shock front can be estimated by assuming that a fraction η_{CR} of the kinetic energy flux through the shock is converted into cosmic rays. Thus, the cosmic-ray energy density close to the shock is given by $\sim \eta_{\text{CR}} \rho_0 u_{\text{sh}}^2$, which yields an energy flux of $\sim \eta_{\text{CR}} \rho_0 u_{\text{sh}}^3 \equiv pc j_{\text{CR}} / q$, i.e., a cosmic-ray current of:

$$j_{\text{CR}} \approx \frac{\eta_{\text{CR}} \rho_0 u_{\text{sh}}^3}{\mathcal{R}}. \quad (5.125)$$

Then, the condition $k_{\min} = k_{\max}$ relates the maximum magnetic field strength to the cosmic-ray current as:

$$B_{\max} \approx \sqrt{\frac{4\pi j_{\text{CR}} \mathcal{R}_{\max}}{c}} = \sqrt{\frac{4\pi \eta_{\text{CR}} \rho_0 u_{\text{sh}}^3}{c}}. \quad (5.126)$$

The efficiency η_{CR} is uncertain, but values of $\mathcal{O}(10\%)$ are typically assumed (see our discussion in Sec. 4.4).

This leads to an estimate for the maximum magnetic field strength of:

$$B_{\max} \approx 300 \mu\text{G} \left(\frac{\eta_{\text{CR}}}{0.1} \right)^{1/2} \left(\frac{n_0}{1 \text{ cm}^{-3}} \right)^{1/2} \left(\frac{u_{\text{sh}}}{c/30} \right)^{3/2}, \quad (5.127)$$

where we used that $\rho_0 = n_0 \mu m_p$ with m_p being the proton mass and $\mu \approx 1.4$ the mean molecular weight for a typical ISM composition. As pointed out by Bell, 2013, this rough estimate is in good agreement with the magnetic field strengths inferred from SNR observations by Vink, 2006, 2008, who found that the data is fitted best by:

$$B \sim 700 \mu\text{G} \left(\frac{u_{\text{sh}}}{c/30} \right)^{3/2} \left(\frac{n_0}{\text{cm}^{-3}} \right)^{1/2}. \quad (5.128)$$

This shows that magnetic field amplification to the levels inferred from SNR observations is indeed possible via the NRH instability. The strong dependence on the shock speed u_{sh} also shows that the most efficient magnetic field amplification is expected in the free-expansion phase early in the SNR evolution. For a further discussion of the maximum energy achievable in DSA when considering the NRH instability, we refer the reader to

Bell et al., 2013; Schure and Bell, 2013. Together with the assumption of Bohm diffusion, this can explain acceleration of cosmic rays up to the knee energies in SNR shocks (Bell, 2013; Blasi, 2013).

5.8 Escape from the sources

In the previous sections, we have discussed how particles can be accelerated in SNR shocks up to high rigidities. In deriving those limits, we have often argued that cosmic rays need to stay close to the shock front to be efficiently accelerated. This raises the questions of how and when particles actually escape their acceleration sites, which is especially relevant for particles with rigidities above some TV that require strong magnetic field amplification and high turbulence levels to be confined close to the shock front. We are mainly interested in the escape of these high-rigidity cosmic rays. Specifically, we want to discuss how they can escape upstream before losing their energy adiabatically through the expansion of the SNR. At the end of this section, we will briefly discuss the escape of cosmic rays with lower rigidities.

The escape of cosmic rays from SNRs is closely linked to the SNR evolution. As discussed in the preceding sections, the magnetic field strength, the turbulence level, and hence the maximum rigidities grow in the early free-expansion phase. Additionally, the shock speeds are highest and roughly constant during this phase. The maximum rigidity that can be achieved during this phase increases with the growth of the turbulent magnetic field in the upstream region. Thus, particles accelerated to the maximum rigidity cannot diffuse far enough upstream to efficiently escape the shock front, as the turbulence level is increasing and the shock speed is high (Caprioli et al., 2009b). After the transition to the ST phase, however, the shock speed decreases significantly (see Eq. (5.117)). This makes the NRH streaming instability less efficient (see Eq. (5.127)). Further, as the shock slows down, magnetic field damping becomes more efficient (Ptuskin and Zirakashvili, 2005). Non-linear wave interactions reduce the level of turbulence (Voelk et al., 1988), and ion-neutral collisions can damp magnetic turbulence efficiently (Bell, 1978b; Drury et al., 1996). Hence, particle escape becomes possible during the ST phase. The mechanism of cosmic-ray escape has been discussed in many works (see, e.g., Bell et al., 2013; Caprioli et al., 2009b, 2010; Schure and Bell, 2013, 2014).

As a consequence of the magnetic field damping and decreasing shock speeds, the maximum rigidity achievable in DSA, $\mathcal{R}_{\max}(t)$, decreases after the transition from the free-expansion to the ST phase (Caprioli et al., 2009b). This decrease in the maximum rigidity that particles can be accelerated to is directly linked to a decrease of the maximum rigidity of particles that can be confined close to the shock front. Thus, particles with rigidities above $\mathcal{R}_{\max}(t)$ can escape to upstream infinity.

This leads to a continuous escape of the highest-rigidity particles during the ST phase of the SNR evolution (Caprioli et al., 2009a,b). The particles of the highest rigidities

escape first, followed by particles of lower rigidities, as the SNR shock slows down. At a time t^* , particles with rigidity $\mathcal{R}_{\max}(t^*)$ can escape upstream without suffering adiabatic energy losses through the expansion of the SNR. Lower rigidity particles are still efficiently confined, and higher rigidity particles have already escaped or suffered energy losses (Caprioli et al., 2009b, 2010).

In Caprioli et al., 2009b, the decrease of $\mathcal{R}_{\max}$ with time has been tested based on a semi-analytic kinetic calculation introduced in Caprioli et al., 2009a. The time-dependence of $\mathcal{R}_{\max}(t)$ can be roughly estimated from the evolution of the SNR in the ST phase under the simplifying assumption that the acceleration efficiency stays constant. It is $\mathcal{R}_{\max}(t) \propto t^{-11/10}$ when the NRH instability operates efficiently. This can be seen by noting that $\mathcal{R}_{\max} \propto Bu_{\text{sh}}R_{\text{sh}}$ under the assumption of Bohm diffusion (see Eq. (5.113)). Combining the time dependencies of the shock radius ($\propto t^{2/5}$) and shock speed ($\propto t^{-3/5}$) during the ST phase, Eq. (5.117), and the dependence of the magnetic field strength through the evolution of the NRH instability on the shock speed ($\propto u_{\text{sh}}^{3/2}$), Eq. (5.127), we find:

$$\mathcal{R}_{\max}(t) \propto B u_{\text{sh}} R_{\text{sh}} \propto u_{\text{sh}}^{5/2} R_{\text{sh}} \propto t^{-15/10} t^{2/5} \propto t^{-11/10}. \quad (5.129)$$

In cases where the resonant streaming instability dominates, this time dependence changes to $\mathcal{R}_{\max}(t) \propto t^{-1/2}$. However, the scaling can be expected to be more complicated once non-linear effects through the interaction with accelerated particles are included, which can alter the acceleration efficiencies (Caprioli et al., 2009b).

It is important to note that the spectrum of escaping particles integrated over the whole SNR evolution is hard to infer observationally. The particles sourcing secondary radiation like radio, X-ray, or γ -ray emissions are usually confined downstream of the shock. They do not need to have the same spectrum as the escaping particles (Caprioli et al., 2009b).

The energy-dependent escape of cosmic rays from their sources has been used for the modelling of cosmic-ray sources before (Blasi and Amato, 2012b; Celli et al., 2019; Gabici et al., 2009; Thoudam and Hörandel, 2012). It is usually assumed that $\mathcal{R}_{\max}(t)$ follows a single power law from the maximum rigidity achieved at the Sedov time, t_{ST} , down to some minimum rigidity at the end of the ST phase, i.e., when the *radiative phase* begins. In Draine, 2011, the transition to the radiative phase is estimated to happen at:

$$t_{\text{rad}} \approx 49.3 \text{ kyr} \left(\frac{E_{\text{SN}}}{10^{51} \text{ erg}} \right)^{0.22} \left(\frac{n_0}{1 \text{ cm}^{-3}} \right)^{-0.55}. \quad (5.130)$$

When that happens, the shock speed has decreased significantly to $u_{\text{sh}} \approx 200 \text{ km s}^{-1}$. The radius of the shock at that time is given by inserting Eq. (5.130) into Eq. (5.117):

$$R_{\text{sh}}(t_{\text{rad}}) \approx 24.4 \text{ pc} \left(\frac{E_{\text{SN}}}{10^{51} \text{ erg}} \right)^{0.29} \left(\frac{n_0}{1 \text{ cm}^{-3}} \right)^{-0.42}. \quad (5.131)$$

In the model we just referenced, the minimum rigidity is usually chosen to be in the GV range, but this is quite uncertain. The escape from the sources and residence in the

vicinity of the sources under the consideration of the resonant streaming instability have been studied for lower-rigidity cosmic rays, e.g. in Jacobs et al., 2022; Nava et al., 2016, 2019; Recchia et al., 2022. However, the inclusion of the NRH instability might change the results presented there as pointed out in Recchia et al., 2022. A comparison with the time-evolution of $\mathcal{R}_{\text{max}}$ in Caprioli et al., 2009b, where the NRH instability is considered, shows that the power-law dependence of $\mathcal{R}_{\text{max}}(t)$ might only extend down to rigidities in the TV range when the SNR enters the radiative phase (see Eq. (5.130)). This indicates that the time-dependence of $\mathcal{R}_{\text{max}}$ inferred from Caprioli et al., 2009b might only be modified slightly by the considerations presented in, e.g., Recchia et al., 2022.

It is pointed out in Caprioli et al., 2010, that the escape of lower-rigidity cosmic rays becomes more complicated than the continuous time-dependent escape of cosmic rays at higher rigidities. The SNR shock environment might still be able to confine all particles below rigidities of some TV until the radiative phase is reached, the shock dies out, and all particles are released. Later in this thesis in Part III, we will discuss how we implemented this in a source model for cosmic rays called the Cosmic-Ray Energy-Dependent Injection Time (CREDIT) model to capture the effects of an energy-dependent escape on the cosmic-ray spectrum (Stall et al., 2025).

6

Hadronic diffuse emissions

Upon interactions with the Interstellar Medium (ISM), cosmic rays can produce observable signals. In Chap. 3, we have discussed that this can lead to emissions in various energy bands of the electromagnetic spectrum, as well as to neutrino emissions. In the context of this thesis, we are interested in the diffuse emissions produced by interactions of Galactic cosmic-ray protons in the rigidity range from a few GV up to the cosmic-ray knee at about 3 PV. These lead to diffuse emissions in γ -rays and neutrinos through the decay of pions and kaons produced in hadronic interactions of cosmic rays and interstellar gas. The ubiquity of cosmic rays in the Milky Way and the presence of interstellar gas throughout the Galaxy leads to a guaranteed emission of γ -rays and neutrinos that bears information about distribution and intensity of cosmic rays throughout the Galaxy. These emissions are produced by cosmic rays of a factor of $\mathcal{O}(10)$ larger energies due to inelasticity (e.g., Kelner et al., 2006; Koldobskiy et al., 2021).

The basic scheme of hadronic interactions in the ISM leading to diffuse emissions in γ -rays and neutrinos can be understood as follows (Gaisser et al., 2016). Nuclei in cosmic rays interact with the interstellar gas. If a sufficient amount of energy is available in the centre of mass system, pions can be produced in these interactions. In their main decay channels, neutral pions decay into γ -rays, while charged pions decay into neutrinos and charged leptons as follows:

$$\text{CR} + \text{ISM} \rightarrow \begin{cases} \pi^0 + X \rightarrow 2\gamma + X \\ \pi^+ + X \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu + X \\ \pi^- + X \rightarrow e^- + \bar{\nu}_e + \nu_\mu + \bar{\nu}_\mu + X \end{cases}, \quad (6.1)$$

where CR denotes a cosmic ray (typically a proton or a heavier nucleus), ISM denotes the interstellar medium (mainly hydrogen and helium gas), X represents other secondary particles produced in the interaction, π^0 are neutral pions decaying into γ -rays, and $\pi^\pm$ are charged pions decaying into neutrinos and charged leptons. For details on the latter two decay chains, see Chap. 3, Eqs. (3.7) and (3.8).

As those hadronic interactions are mainly interactions of cosmic-ray protons with hydrogen gas in the ISM, the interactions are often referred to as p-p-interactions. A pion can only be created if a production threshold is overcome. For p-p-interactions of the form $p + p \rightarrow p + p + \pi^0$, the cosmic-ray proton needs a total energy of at least 1.22 GeV

in the laboratory frame to produce a neutral pion (Kelner et al., 2006). The pions decay quickly after their production with a mean lifetime of about $2.6 \cdot 10^{-8}$ s for charged pions and $8.5 \cdot 10^{-17}$ s for neutral pions (Particle Data Group et al., 2020).

The signal produced by these interactions of Galactic cosmic rays with the interstellar gas is referred to as *hadronic Galactic Diffuse Emissions (GDEs)*. There is also a leptonic component of the GDEs in γ -rays, which arises from processes such as inverse Compton scattering, synchrotron radiation, and bremsstrahlung of cosmic-ray electrons. We will only be concerned with the hadronic component in this chapter. Thus, we will refer to hadronic GDEs simply as GDEs in the following.

Modelling GDEs is strongly motivated by the search for the origin of cosmic rays. It was noted early on, that a correspondence between GDEs and the intensity of cosmic rays throughout the Galaxy might be used to infer a spatial source distribution (e.g., Stecker, 1975). In the light of modern observatories, such as *Fermi*-LAT, LHAASO, and IceCube, the precise modelling of GDEs has gained relevance. We will discuss our study, Stall and Mertsch, 2026, of GDEs based on stochastic source modelling (see Chap. 7) in Part III.

In this chapter, we will describe how GDEs can be calculated from the underlying cosmic-ray intensities, the relevant interaction cross-sections, and the distribution of interstellar gas in the Galaxy. We will also discuss how γ -ray absorption due to pair production can influence the observed GDEs in γ -rays at energies in excess of some tens of TeV. Finally, we will discuss how diffuse γ -ray and neutrino emissions are connected due to their common origin in the hadronic interactions described in Eq. (6.1).

6.1 Emissivity

GDEs are a cumulative effect of hadronic interactions occurring all the way along specific Lines Of Sight (LOS). A LOS can be parametrised by the Galactic longitude, l , and Galactic latitude, b . In the following, we will focus on GDEs of γ -rays. The GDEs of neutrinos are closely related to the γ -ray emissions as both arise from the same initial hadronic interactions producing pions. In Sec. 6.5, we will discuss this connection.

The calculation of the intensity of GDEs along a LOS (l, b) at a certain energy E , i.e., $J(l, b, E)$, requires an integration of the hadronic emissivity, ϵ , along the LOS. γ -rays produced at some location in the Galaxy can also be absorbed on their way to the observer. This needs to be accounted for by including an attenuation factor. Thus, the GDE intensity, J , can be expressed as (Bloemen, 1989; Dundovic et al., 2021):

$$J(l, b, E) = \frac{1}{4\pi} \int_0^\infty ds e^{-\tau(E, \mathbf{x}(l, b, s))} \epsilon(\mathbf{x}(l, b, s), E), \quad (6.2)$$

where s is the distance along the LOS, $\mathbf{x}(l, b, s)$ describes the 3D position in the Galaxy corresponding to the direction (l, b) and distance s , and $\tau(E, \mathbf{x})$ is the optical depth accounting for absorption effects, which will be discussed in Sec. 6.4.

The hadronic emissivity, ϵ , describes the production rate of γ -rays per unit volume, energy, and time at a certain position $\mathbf{x}$ in the Galaxy.

$$\epsilon(\mathbf{x}(l, b, s), E) = 4\pi \sum_{m,n} n_{\text{gas},n}(\mathbf{x}) \int_E^\infty dE' \frac{d\sigma_{m,n}}{dE}(E', E) \Phi_m(\mathbf{x}, E'), \quad (6.3)$$

where Φ_m is the cosmic-ray intensity of species m (for conversions to, e.g, the differential cosmic-ray density, see App. A), $d\sigma_{m,n}/dE(E', E)$ is the differential cross-section for the production of γ -rays of energy E by inelastic collisions of cosmic rays of *kinetic energy* E' with gas of species n , and $n_{\text{gas},n}(\mathbf{x})$ describes the 3D distribution of Galactic gas of species n (e.g., Bloemen, 1989; Dundovic et al., 2021; Kaiser, 2023; Schwefer et al., 2023). We will discuss the different components of Eq. (6.3) in the following sections.

6.2 Galactic gas maps

A main ingredient for the calculation of GDEs is the distribution of interstellar gas in the Galaxy, $n_{\text{gas},n}(\mathbf{x})$, where n denotes the gas species. About 5 % of the baryonic mass of the Milky Way is present in the form of interstellar gas (Longair, 2011). The most abundant component of the interstellar gas is hydrogen with about 90 %. Most of it can be found in electrically neutral form as atomic (H I) or molecular hydrogen (H₂)¹. With about 10 %, helium is the second most abundant component, followed by heavier elements, which only contribute roughly 1 % to the total mass of the interstellar gas (Longair, 2011). It is typically assumed that the spatial distribution of these heavier components roughly follows that of hydrogen, allowing to include them by applying scaling factors to the hydrogen distribution (e.g., Mori, 2009).

The distribution of the hydrogen gas in the Galaxy can be reconstructed from observations. H I can be observed through its characteristic 21 cm line emission, which arises from the hyperfine transition of the hydrogen atom. H₂ is typically traced by observing line emissions of the CO molecule, because it is assumed that those two components are spatially correlated (e.g., Söding et al., 2025). The distribution of H₂ can then be inferred from the CO distribution by assuming a linear relation that gets calibrated by comparison of the H₂ column density and the velocity-integrated CO brightness temperature (Bolatto et al., 2013). The HI4PI survey (HI4PI Collaboration et al., 2016) provides a velocity-resolved full-sky map of the H I distribution in the Milky Way and dedicated radio

¹Ionised hydrogen (H II) is a subdominant component of the ISM, only comprising about 15 % of the hydrogen mass fraction of the Galaxy with low number densities and a scale height of a few 100 pc (Ferrière, 2001). Thus, we neglect this component here.

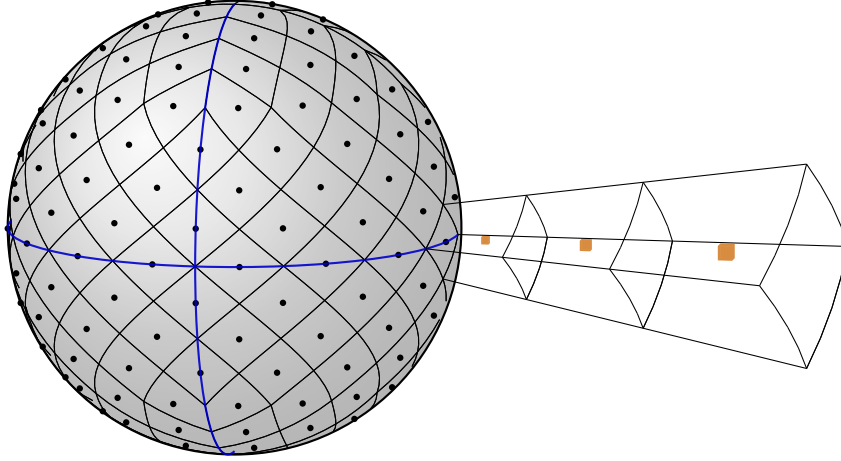


Figure 6.1: Schematic representation of the grid used in Söding et al., 2025. This sketch shows an orthographic projection illustrating the HEALPix $\times \log(R)$ -grid with a reduced $N_{\text{side}} = 4$ for illustrative purposes. Three radial layers are shown. The HEALPix scheme ensures that the areas of the individual spherical pixels are equal. The equator and one meridian are highlighted by blue lines. The orange cubes indicate individual voxels at different radial levels, where the size of the cubes represents the volume belonging to each voxel. (This figure has been kindly provided by Laurin Söding.)

surveys also provide maps of the CO distribution (e.g., Dame et al., 2001). However, these observations only provide direct information about the column densities of the gas along specific LOS. Reconstructing the full 3D distribution of the gas from these observations is a non-trivial task, which requires additional information or assumptions.

Under the assumption of a simple rotation model of the Galaxy with a set rotation curve, the observed Doppler shifts of the line emissions can be used to infer distances to the emitting gas clouds. Ackermann et al., 2012 used this method to reconstruct the gas density in $\mathcal{O}(10)$ annular rings around the Galactic centre. Besides this simple ring model, analytical models of the Galactic gas distribution like, e.g., Lipari and Vernetto, 2018, have been developed.

More sophisticated methods to reconstruct the 3D distribution of the Galactic gas employ Bayesian inference techniques to combine different data sets and prior assumptions about the gas distribution. Reconstructions of this kind have been carried out by Mertsch and Vittino, 2021; Mertsch and Phan, 2023. More recently, Söding et al., 2025 presented a new reconstruction of the 3D distribution of both H I and CO in the Galaxy. They used Gaussian processes to model the correlations in the ISM and also inferred the velocity field of the gas self-consistently from data. They provide estimates for the uncertainties of their reconstruction by giving multiple samples of their posterior gas distribution. In this work (see Part III), we will use the gas maps provided by Söding et al., 2025.

For the calculation of the hadronic emissivity, ϵ , in Eq. (6.3), the total hydrogen gas number density is needed. This is given by

$$n_{\text{H}} = n_{\text{H I}} + 2 \cdot n_{\text{H}_2}. \quad (6.4)$$

This hydrogen comprises the main target material for the hadronic interactions of cosmic rays producing GDEs.

The gas maps provided by Söding et al., 2025 give the number densities of H I and the derived H₂ densities on a 3D grid, reflecting the spatial resolution achievable in their reconstruction. The grid is centred on the Sun's position in the Galaxy. At 388 logarithmically spaced radial distances from the Sun, grid points are distributed on a sphere according to a HEALPix pixellation scheme (Górski et al., 2005) with $N_{\text{side}} = 64$. This corresponds to 49 152 grid points for each of the in total 388 radial distances. A graphical illustration of this grid is provided in Fig. 6.1. Due to its construction, this grid is especially well-suited for the calculation of LOS integrals as required for the calculation of the GDE intensity, J , in Eq. (6.2).

6.3 Cross-sections and scaling factors

The hadronic interactions described in Eq. (6.1) are not solely interactions of cosmic-ray protons with hydrogen gas. Instead, both heavier cosmic-ray nuclei and heavier gas components in the ISM contribute to the total hadronic interaction rate. In the previous Sec. 6.2, we have already discussed that the interstellar gas is mainly composed of hydrogen and helium. More precisely, it is assumed that the heavier components of the interstellar gas have the same spatial distribution as the hydrogen gas.

A similar argument can be made for the heavier components of the cosmic-ray intensity. Following the procedure in Casandjian, 2015 and Honda et al., 2004, interactions of heavier cosmic-ray nuclei with the complete ISM and interactions of cosmic-ray protons with heavier components of the ISM can be approximately included by applying scaling factors to the proton-proton interaction cross-section, $\sigma_{\text{p-p}}$. With these considerations, the hadronic emissivity in Eq. (6.3) can be rewritten in a simplified form, only including cosmic-ray protons and the hydrogen distribution:

$$\epsilon(\mathbf{x}(l, b, s), E) = 4\pi n_{\text{H}}(\mathbf{x}) \int_E^\infty dE' \frac{d\sigma}{dE}(E', E) \Phi(\mathbf{x}, E'), \quad (6.5)$$

where Φ is the cosmic-ray proton intensity and $d\sigma/dE(E', E)$ is the effective cross-section including the scaling factors for heavier cosmic rays and gas components. This effective cross-section depends on the p-p and the p-He cross-sections as follows:

$$\frac{d\sigma}{dE}(E', E) = (1 + \xi) \frac{d\sigma_{\text{p-p}}}{dE}(E', E) + \frac{n_{\text{He}}}{n_{\text{H}}} \frac{d\sigma_{\text{p-He}}}{dE}(E', E), \quad (6.6)$$

where $n_{\text{He}}/n_{\text{H}} \approx 0.096$ (Casandjian, 2015) is the ratio of the number densities of helium to hydrogen in the ISM and ξ is a scaling factor accounting for interactions of cosmic-ray protons with components of the ISM heavier than helium, and for interactions of heavier cosmic-ray species with the complete ISM:

$$\xi = \xi_{\text{p,heavy}} + \xi_{\text{He+heavier,ISM}} = 0.021 + 0.397 = 0.418, \quad (6.7)$$

where the numbers were taken from Casandjian, 2015, p. 10, assuming that the cosmic-ray helium intensity follows the cosmic-ray proton one with a ratio of 0.055 (Casandjian, 2015).

Under the simplifying assumption that the p-He cross-section can be approximated by scaling the p-p cross-section roughly by a factor of four due to the larger number of nucleons, the effective cross-section in Eq. (6.6) can be rewritten as (Mori, 2009)

$$\frac{d\sigma}{dE}(E', E) \approx (1 + \xi) \frac{d\sigma_{\text{p-p}}}{dE}(E', E) \quad \text{with } \xi \approx 0.8, \quad (6.8)$$

using the same values as in Eq. (6.7).

For the p-p and p-He γ -ray production cross-sections, multiple parametrisations with varying energy ranges are available in the literature (e.g., Kafexhiu et al., 2014; Kamae et al., 2006; Kelner et al., 2006, see overview in Koldobskiy et al., 2021). They are usually based on different hadronic interaction models (see our brief discussion in Sec. 3.4) and assumptions concerning the intermediate particle production and decay (see, e.g., Aharonian and Atoyan, 2000; Dermer, 1986 for more details). The whole energy range of interest for this work is covered by the AAFRAG cross-sections (Kachelrieß et al., 2019), based on the hadronic interaction model QGSJET-II-04m. They are available via the Python package AAFRAGPY (Koldobskiy et al., 2021).

6.4 Absorption

Next, we discuss how absorption affects the GDEs in γ -rays at very high energies. Up to about some tens of TeV, γ -rays can travel through the Galaxy almost unimpeded. However, at higher energies, interactions of γ -rays with ISRFs become relevant and lead to absorption of the γ -rays. The absorption happens via the production of electron-positron pairs in photon-photon interactions, $\gamma\gamma \rightarrow e^+e^-$ (Dundovic et al., 2021). The interaction cross-section of this process is given by (Vernetto and Lipari, 2016)

$$\sigma_{\gamma\gamma} = \sigma_{\text{T}} \frac{3}{16} (1 - \beta^2) \left[2\beta(\beta^2 - 2) + (3 - \beta^4) \ln \left(\frac{1 + \beta}{1 - \beta} \right) \right], \quad (6.9)$$

where σ_{T} is the Thomson cross-section and $\beta = \sqrt{1 - 1/x}$.

β depends on the variable x , which is related to the centre of mass energy of the interaction, $s = 2E_\gamma E_{\text{ph}} (1 - \cos \theta)$ via $x = s/(4m_e^2)$, where m_e is the electron mass, E_γ is the γ -ray energy, E_{ph} is the energy of the target photon, and θ is the angle between the momenta of the interacting photons. The cross-section, $\sigma_{\gamma\gamma}$, vanishes below the threshold $\sqrt{s} = 2m_e$, and falls off quickly around its maximum which is reached for a γ -ray energy of roughly $E_\gamma \approx m_e^2/E_{\text{ph}}$ (Vernetto and Lipari, 2016).

To include the absorption effects in the calculation of the GDEs according to Eq. (6.2), we need to know how different target ISRFs contribute to the absorption of γ -rays. This is captured by the optical depth, $\tau(E, \mathbf{x})$, which describes the probability of a γ -ray of energy E being absorbed on its way from position $\mathbf{x}$ to the observer. It can be calculated by integrating the absorption coefficient K , which we will define in Eq. (6.11), along the LOS from the source position to the observer:

$$\tau(E, \mathbf{x}(l, b, s)) = \int_0^s dt K(E_\gamma, \hat{\mathbf{u}}, t \cdot \hat{\mathbf{u}}) . \quad (6.10)$$

Here, $\hat{\mathbf{u}}$ is the unit vector pointing from the source to the observer's position. From this optical depth, the attenuation factor in Eq. (6.2) can be calculated as $e^{-\tau}$. It describes the transmittance, i.e., the probability of a γ -ray to survive the trip from the source position to the observer without being absorbed (Moskalenko et al., 2006).

The absorption coefficient can depend on the γ -ray's energy, position, and direction of travel. Given the differential number density of the target photon field, $dn_{\text{ph}}/dE_{\text{ph}}$, and the interaction cross-section, $\sigma_{\gamma\gamma}$, the absorption coefficient can be calculated as follows (Moskalenko et al., 2006):

$$K(E_\gamma, \hat{\mathbf{u}}, \mathbf{x}) = \int_{E_{\text{ph}}^{\text{min}}}^\infty dE_{\text{ph}} \int_{4\pi} d\Omega \frac{1 - \cos \theta}{2} \frac{dn_{\text{ph}}}{dE_{\text{ph}}}(E_{\text{ph}}, \hat{\mathbf{u}}, \mathbf{x}) \sigma_{\gamma\gamma}(E_\gamma, E_{\text{ph}}, \theta) . \quad (6.11)$$

The factor $(1 - \cos \theta)/2$ accounts for the dependence on the relative velocity on the interaction angle θ and is normalised such that $\int_{-1}^1 d \cos(\theta) (1 - \cos \theta)/2 = 1$.

One especially important target ISRF is the CMB. Its differential number density is isotropic and homogeneous throughout the Galaxy and is given by a black-body spectrum:

$$\frac{dn_{\text{ph}}}{dE_{\text{ph}}}(E_{\text{ph}}) = \frac{8\pi}{(hc)^3} \frac{E_{\text{ph}}^2}{\exp\left(\frac{E_{\text{ph}}}{k_B T_{\text{CMB}}}\right) - 1} \quad (6.12)$$

where we use a CMB temperature of $T_{\text{CMB}} = 2.7255$ K (Fixsen, 2009). Its isotropic and homogeneous nature simplifies the calculation of the absorption coefficient according to Eq. (6.11) and shows that it does not depend on the position or direction. Thus, the corresponding optical depth only depends on the γ -ray energy and the distance along the LOS.

While the absorption due to the CMB only becomes relevant at γ -ray energies above some hundreds of TeV, other target ISRFs lead to noticeable absorption effects already at lower energies. Infrared radiation emitted by Galactic dust heated by starlight provides an important target ISRF for γ -ray absorption in the Milky Way (Porter et al., 2018; Vernetto and Lipari, 2016). However, this radiation field has a non-trivial distribution in energy, space, and angle which complicates the calculation of the corresponding optical depths.

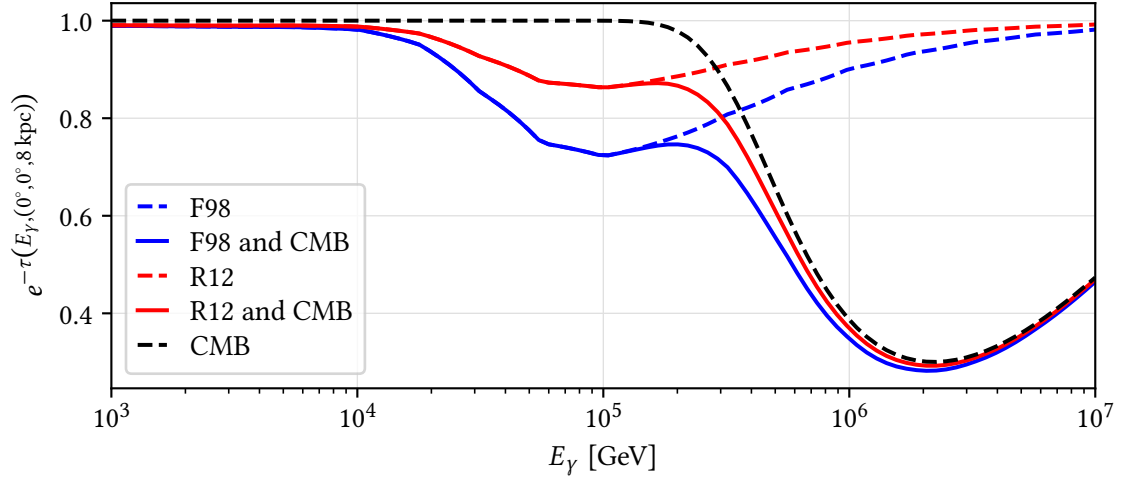


Figure 6.2: Transmittance for LOS towards Galactic centre. The transmittance (or survival probability), $e^{-\tau}$, for γ -rays travelling from the Galactic centre $(l, b) = (0^\circ, 0^\circ)$ to the Sun (here 8.0 kpc) is shown as a function of the γ -ray energy, E_γ . The absorption due to the CMB is shown as a black dashed line. The absorption due to the dust-emitted infrared radiation fields are shown as dashed lines in red (R12 model) and blue (F98 model). The total absorption including both the CMB and the infrared radiation fields is shown as solid lines. We use the same setup for the calculation as in Part III.

In GALPROP v.57 (Porter et al., 2022), the absorption coefficients due to the dust-emitted infrared radiation field have been precomputed on a spatial grid covering the Galaxy. This builds on calculations presented in Porter et al., 2018, where two different models for the dust distribution were considered: one based on Robitaille et al., 2012 (R12) including a spiral structure and one based on Freudenreich, 1998 (F98) with an asymmetric stellar bar. In Fig. 6.2, we show the transmittance, $e^{-\tau}$, for γ -rays travelling from the Galactic centre to the Sun, calculated using these two different dust models and the CMB as described in Eq. (6.12). The F98 model generally leads to higher absorption in the inner Galaxy due to a higher dust density in this region in this model.

Further components of the ISRF are subdominant and not relevant for γ -ray absorption as considered in this work. Those other components include the extragalactic background light and radiation emitted by stars directly (Vernetto and Lipari, 2016). At sub-TeV γ -ray energies, absorption due to pair-production can be neglected altogether.

6.5 γ -ray–neutrino relation

The diffuse γ -ray emissions discussed in this chapter are closely related to diffuse neutrino emissions due to their common origin in hadronic interactions of cosmic rays with the interstellar gas as described in Eq. (6.1). The ratio of charged to neutral pions produced in p-p-interactions allows us to relate the intensities of diffuse γ -ray and neutrino emissions. It is approximately given by $N_{\pi^\pm}/N_{\pi^0} \simeq 2$ (Gaisser et al., 2016).

In the decay of neutral pions, $\pi^0 \rightarrow \gamma\gamma$, each γ -ray takes about half of the pion energy. For the decay of charged pions, $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ followed by $\mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$, the three neutrinos produced take only about a quarter of the pion energy (Ahlers and Murase, 2014; Gaisser et al., 2016). Thus, each neutrino is about half as energetic as each γ -ray produced in the decay of an equally energetic neutral pion, $E_\gamma \approx 2E_\nu$, and for each two γ -rays produced, a total of three neutrinos is produced.

Considering these two factors, the GDEs intensities for γ -rays and neutrinos can be related. However, it is important that absorption only effects γ -rays and not neutrinos. Thus, the relation between the GDEs intensities in γ -rays without absorption, $J_\gamma^{\text{no abs}}$, and the GDEs intensities for neutrinos, J_ν , is given by (Ahlers and Murase, 2014):

$$E_\gamma J_\gamma^{\text{no abs}}(E_\gamma) \approx \frac{1}{3} \sum_{\ell \in \{e, \mu, \tau\}} E_\nu J_{\nu_\ell}(E_\nu) \quad \text{with } E_\gamma \approx 2E_\nu, \quad (6.13)$$

where the sum runs over the neutrino flavours. This accounts for the effect of neutrino oscillations.

6.6 Further contributions and models

In addition to the hadronic GDEs discussed in this chapter, there can be further Galactic contributions to the γ -ray and neutrino emissions in our energy range of interest (GeV to PeV energies). Both γ -rays and neutrinos can also be produced by individual sources in the Galaxy. However, some of these sources can be too faint to be resolved individually or the limitations of the angular resolution of current experiments can lead to them being indistinguishable from the diffuse emissions. These are called *unresolved sources* and can easily be misinterpreted as GDEs. There are some models in the literature that specifically address the contribution of unresolved sources to the GDEs (e.g., Cataldo et al., 2020; Vecchiotti et al., 2022a). The development of those was partially driven by excesses in the measured diffuse γ -ray data compared to standard GDE models, e.g., in the Tibet AS γ data (Vecchiotti et al., 2022b).

The observation of GDEs, first in γ -rays and more recently also in neutrinos (see Chap. 3), has led to the development of various theoretical and phenomenological models. Just to name a few, the so-called π^0 -model (Ackermann et al., 2012) was developed in conjunction

with the analysis of *Fermi*-LAT data. The KRA_γ model (Gaggero et al., 2015a,b) tries to explain an apparent spectral hardening of the *Fermi*-LAT diffuse γ -ray flux towards the inner Galaxy by assuming that the isotropic diffusion coefficient has a spectral index depending on the Galactocentric radius. If the diffusion is more efficient in the outer Galaxy, this leads to a hardening of the cosmic-ray spectrum and thus also of the GDE spectrum towards the inner Galaxy. It has been proposed that this effect could arise from anisotropic diffusion along a large-scale magnetic field pointing out of the Galactic plane in the inner Galaxy and being more aligned with the Galactic plane in the outer Galaxy (Cerri et al., 2017). However, a full explanation of the apparent spectral hardening is still lacking.

Further, the Cosmic Ray-fitted Intensities of Galactic Emission (CRINGE) model (Schwefer et al., 2023) has been developed as a comprehensive data-driven model for both diffuse γ -ray and neutrino emissions in the Galaxy. It is based on a fit of Galactic transport parameters to local cosmic-ray data and takes uncertainties due to the various model ingredients discussed in this chapter into account. More recently, Vecchiotti et al., 2025 presented a GDE model in the context of LHAASO observations of diffuse γ -ray emissions. They also consider various model uncertainties and discuss their results in the context of a possible spectral hardening and unresolved source contributions, finding that their influence should be suppressed in the masking applied to the LHAASO data. However, He et al., 2025 claim that the fraction of unresolved sources is constrained to a maximum of 5-27 % in the LHAASO diffuse γ -ray data.

Further contributions to GDEs from leptonic processes only influence γ -ray emissions (see, however, Hooper and Plant, 2023). A mismatch between γ -ray and neutrino GDEs could thus point towards a fraction of leptonic GDEs, but it can also arise from a different fraction of unresolved sources in neutrino and γ -ray data. Various studies have tried to constrain the leptonic contribution to the diffuse γ -ray emissions by comparing measured γ -ray and neutrino data. For example, Vecchiotti et al., 2023a,b used data from ANTARES and IceCube, respectively, to determine whether Galactic sources are hadronic or leptonic in nature. Fang and Halzen, 2024 determined that leptonic source contribution of 20-50 % is needed to explain that LHAASO sources were not detected with IceCube.

The leptonic contribution to the diffuse γ -ray emissions is uncertain, but studies suggest that it is subdominant, at least at low Galactic latitudes ($|b| < 10^\circ$). Lipari and Vernetto, 2018 estimate that leptonic contributions make up less than 10 % of the total diffuse γ -ray emissions in this region. Further, local measurements of the cosmic-ray electron spectrum suggest that it has a cut-off at some tens of TeV (Aharonian et al., 2024; Mertsch, 2018), which would even limit the leptonic contributions at energies above 1 TeV to a maximum of 5 % (Lipari and Vernetto, 2018). This can be estimated by approximating the inverse Compton scattering in the Thomson limit. Due to the fast cooling of high-energy electrons, it has also been suggested that leptonic contributions at energies above some tens of TeV cannot contribute to truly diffuse emissions far away from their sources, but are rather part of a (unresolved) leptonic source component (Kaci and Giacinti, 2025).

7

Stochastic source modelling

In this chapter, we want to motivate and introduce the concept of *stochastic source modelling* of Galactic cosmic-ray sources. With this, we want to lay the foundation for our treatment of the source term in the cosmic-ray transport equation (Eq. (5.64) or Eq. (5.69), respectively) in the work presented in Parts II and III. We start by discussing the dimensions of cosmic-ray sources and what this implies for modelling them in Sec. 7.1. The source positions and ages are generally not known. In Sec. 7.2, we introduce the underlying distributions for those. In Sec. 7.3, we discuss the solution of the transport equation for protons, Eq. (5.69), for discrete sources. Finally, in Sec. 7.4, we introduce the existing literature for the stochastic treatment of the cosmic-ray intensities and prospects of the stochastic source modelling in the light of the unprecedented high-precision measurements of cosmic-ray intensities and diffuse emissions.

7.1 Source dimensions

In our discussion of the SNR paradigm in Sec. 4.4, we explored the observational and theoretical reasons for considering SNRs as the main sources of Galactic cosmic rays up to the cosmic-ray knee at a rigidity of about 3 PV. In Chap. 5, we outlined how the acceleration and escape of cosmic rays works in SNRs. We discussed that cosmic rays are accelerated in the shock fronts of SNRs and escape into the Interstellar Medium (ISM) during the Sedov-Taylor (ST) phase of the SNR evolution. This gives us a characteristic size and timescale for the sources of Galactic cosmic rays. From Eq. (5.116) and Eq. (5.130), we learned that the ST phase, during which the injection of cosmic rays into the ISM takes place, lasts for $\mathcal{O}(100 \text{ kyr})$. The corresponding size of the SNR lies between the estimates given in Eqs. (5.117) and (5.131), which corresponds to $\mathcal{O}(10 \text{ pc})$ for typical SNRs.

The timescale for the transport of cosmic rays is set by their main loss processes. These can be the diffusive or advective escape from the Galaxy through the halo boundaries, energy losses via cooling processes, radioactive decay, ionisation losses, interactions with

the ISM, etc. The most efficient of these processes sets the *loss timescale*, $\tau_{\text{loss}}(\mathcal{R})$, which generally depends on the cosmic-ray rigidity, $\mathcal{R}$:

$$\begin{aligned} \tau_{\text{loss}}(\mathcal{R}) &\simeq \frac{1}{\frac{1}{\tau_{\text{diff}}(\mathcal{R})} + \frac{1}{\tau_{\text{adv}}(\mathcal{R})} + \frac{1}{\tau_{\text{dec}}(\mathcal{R})} + \frac{1}{\tau_{\text{cool}}(\mathcal{R})} + \frac{1}{\tau_{\text{ion}}(\mathcal{R})} + \dots} \\ &\approx \min\{\tau_{\text{diff}}(\mathcal{R}), \tau_{\text{adv}}(\mathcal{R}), \tau_{\text{dec}}(\mathcal{R}), \tau_{\text{cool}}(\mathcal{R}), \tau_{\text{ion}}(\mathcal{R}) \dots\}. \end{aligned} \quad (7.1)$$

The loss timescale poses a limit on the number of sources that contribute to the local cosmic-ray intensity at a given rigidity. For a general loss timescale, $\tau_{\text{loss}}(\mathcal{R})$, only sources within a distance given by the diffusion length can contribute (e.g., Evoli and Dupletsa, 2023):

$$\lambda_{\text{loss}}(\tau_{\text{loss}}) \equiv \sqrt{2 \kappa(\mathcal{R}) \tau_{\text{loss}}(\mathcal{R})}. \quad (7.2)$$

The total number of contributing sources, N_{src} , is approximately given by all sources within a distance λ_{loss} that have injected cosmic rays within the loss timescale τ_{loss} . If we assume for a moment that SNRs are homogeneously distributed in the Galactic disk with radius R_{disk} and that they occur with a Galactic SN rate, f_{SN} , we can estimate the number of contributing sources as (e.g., Evoli and Dupletsa, 2023):

$$N_{\text{src}}(\mathcal{R}) \approx f_{\text{SN}} \tau_{\text{loss}}(\mathcal{R}) \frac{\pi \lambda_{\text{loss}}^2(\mathcal{R})}{\pi R_{\text{disk}}^2}. \quad (7.3)$$

There are some scenarios where the number of contributing sources can be especially small. E.g., for MV cosmic rays, ionisation losses dominate and severely limit the lifetime and range of such cosmic rays in the ISM. This becomes more constraining with lower rigidities. It can be expected that below some threshold, only a single source determines the cosmic-ray intensity (see, e.g., Phan et al., 2018; Phan et al., 2021; Phan, 2020).

Another example is given by leptonic cosmic rays, i.e. cosmic-ray electrons and positrons. They lose energy through cooling processes like synchrotron radiation and inverse Compton scattering. Due to the strong dependence of these processes on energy, the number of contributing sources decreases with increasing rigidity and reduces to $\mathcal{O}(1)$ at some tens of TV (see, e.g., Evoli et al., 2021a,b; Mertsch, 2011; Mertsch, 2018).

In both of these cases, it is clearly necessary to model individual sources to correctly predict the local cosmic-ray intensity. Modelling individual sources, however, requires detailed knowledge about the positions and ages of all the sources that contribute to the local intensity. This includes knowledge about sources that injected their cosmic rays some time ago and are no longer visible in electromagnetic emissions. It can also include sources that are obscured (see, e.g., discussion in Mertsch, 2018). Our limited knowledge about the source coordinates imprints on uncertainties in the predicted cosmic-ray intensities. These uncertainties can be studied in a stochastic framework, but an analytic treatment is not possible. Hence, a Monte Carlo approach is especially well-suited for this task. Source positions and ages can be sampled from underlying distributions. Then,

the resulting variations in the local cosmic-ray intensity can be studied. We will discuss this in more detail in Sec. 7.4.

In this work, we are specifically interested in cosmic-ray protons at rigidities above 10 GV. For those, ionisation losses are negligible. A simple estimate shows that their escape is dominated by the diffusive escape from the Galaxy. Typically, advection speeds, v_{adv} , between 10 and 100 km s⁻¹ are considered (Blasi and Amato, 2019). We assume a typical wind speed of $v_{\text{adv}} \approx 40 \text{ km s}^{-1} \approx 40 \text{ pc Myr}^{-1}$ (Recchia et al., 2016) to determine the advection loss timescale:

$$\tau_{\text{adv}} \sim \frac{H}{v_{\text{adv}}} \approx \frac{5 \text{ kpc}}{40 \text{ pc Myr}^{-1}} \approx 125 \text{ Myr}. \quad (7.4)$$

This can be compared to the diffusive escape timescale, which can be determined from setting the diffusion length (see Eq. (7.2)) to the halo height, H . Assuming $H \approx 5 \text{ kpc}$ and $\kappa(10 \text{ GV}) \approx 5 \cdot 10^{28} \text{ cm}^2 \text{ s}^{-1}$ (Evoli et al., 2019), we find:

$$\tau_{\text{diff}} \simeq \frac{(5 \text{ kpc})^2}{2 \cdot 0.165 \text{ kpc}^2 \text{ Myr}^{-1}} \approx 75 \text{ Myr} \quad (7.5)$$

where we used that $1 \text{ kpc}^2 \text{ Myr}^{-1} \approx 3 \cdot 10^{29} \text{ cm}^2 \text{ s}^{-1}$. This is clearly smaller than the advection timescale in Eq. (7.4) and keeps decreasing for $\mathcal{R} > 10 \text{ GV}$.

If the loss timescale is given by the diffusive escape from the Galaxy, τ_{esc} , the loss length, λ_{loss} , corresponds to the halo height, H , and the number of contributing sources (according to Eq. (7.3)) is given by:

$$N_{\text{src}}(\mathcal{R}) \approx f_{\text{SN}} \tau_{\text{esc}}(\mathcal{R}) \frac{H^2}{R_{\text{disk}}^2} \propto \kappa^{-1}(\mathcal{R}). \quad (7.6)$$

In Fig. 7.1, we show an estimate for the escape timescale for cosmic-ray protons as a function of rigidity. With increasing rigidity, the diffusion coefficient increases and the escape timescale decreases. Accordingly, the number of contributing sources decreases. While at some tens of GV, $\mathcal{O}(10^5)$ sources contribute to the local cosmic-ray intensity, this number decreases to $\mathcal{O}(100)$ at some PV.

Contrary to the extreme cases of low-rigidity protons in the MV range dominated by ionisation losses or high-rigidity electrons dominated by cooling losses, where only one source might contribute to the local intensity, the number of contributing sources for cosmic-ray protons is generally clearly larger than one. Fig. 7.1 shows that $\mathcal{O}(10^4 - 10^5)$ sources contribute at GV rigidities. With such a high number of sources, it is reasonable to approximate the contributions of the many discrete cosmic-ray sources by a continuous source term. Due to their high number, the individual source positions and ages do not influence the local cosmic-ray intensity significantly, and the continuous source approximation is well justified (e.g., Genolini et al., 2017). Often, it is further assumed

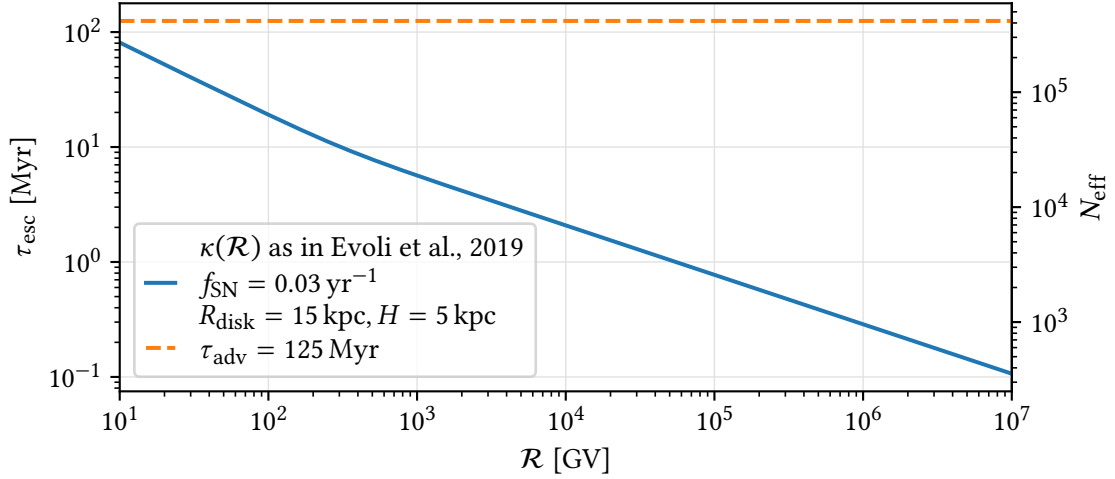


Figure 7.1: Diffusive escape timescale. Estimate of the diffusive escape timescale from the Galaxy as a function of rigidity, assuming a halo height of $H = 5$ kpc and a diffusion coefficient following a power law with one break as determined by Evoli et al., 2019. The second axis shows the corresponding number of contributing sources as estimated by Eq. (7.6), assuming a Galactic SN rate of $f_{\text{SN}} = (30 \text{ yr})^{-1}$ and a disk radius of $R_{\text{disk}} = 15$ kpc. The dashed orange line indicates the approximate advection timescale as estimated in Eq. (7.4).

that the source distribution is also steady in time, which motivates steady-state solutions of the transport equation (e.g., Berezhinskii et al., 1990; Ginzburg and Syrovatskii, 1964), but we will not discuss this further here.

However, as the rigidity increases, the number of contributing sources decreases and this approximation becomes less accurate. Our estimate in Fig. 7.1 shows that the number of contributing sources is still in the range of $\mathcal{O}(100 - 1000)$ at some PV, but not all of those sources contribute equally. In fact, it is possible that a few nearby and young sources dominate the local cosmic-ray intensity at those rigidities. However, it is important to quantify how likely this is. Some previous studies have discussed these stochastic effects. We will give an overview of them in Sec. 7.4. Our own study of the influence of stochastic sources on the local cosmic-ray proton spectrum will be discussed in Part II.

Besides the possibility that individual sources significantly influence the cosmic-ray proton intensities at our position in the Galaxy, they might also impact the cosmic-ray intensities in other regions of the Galaxy. Variations of the cosmic-ray intensity throughout the Galaxy can imprint on the Galactic Diffuse Emissions (GDEs) in γ -rays and neutrinos (for an introduction, see Chap. 6). We discuss the results of our study on the impact of stochastic sources on the GDEs in Part III.

7.2 Source rates and distributions

To model the cosmic-ray density throughout the Galaxy, in principle, we need to know the positions and ages of all cosmic-ray sources. However, this information is not available, as we discussed above. Two possibilities emerge: Either, the source coordinates can be treated as random quantities sampled from some distributions, or the discrete sources can be approximated by a continuous source term. For both methods, we need to know the underlying source rate and spatial distribution of cosmic-ray sources in the Galaxy. In this section, we discuss, how those can be determined and give an overview of existing models.

7.2.1 Galactic supernova rate

We have argued that SNRs are the most likely sources for the bulk of cosmic rays up to the cosmic-ray knee in Sec. 4.4. Thus, the Galactic SN rate, f_{SN} , sets the rate of Galactic cosmic-ray sources. Estimates of this rate for our Galaxy are hard to obtain as direct measurements have not been recorded for a long enough time. There are some historical records of SNe in our Galaxy, dating back some hundreds of years. An extensive compilation and discussion of historical SNe can be found, e.g., in Alsabti and Murdin, 2017; Stephenson and Green, 2002. Five Galactic SNe have been recorded firmly in the last millennium: SN 1006 (Lupus), SN 1054 (Crab Nebula), SN 1181, SN 1572 (Tycho's SN), and SN 1604 (Kepler's SN). In addition, there are observations from the year 1680 that are likely associated with the SN that created the SNR Cassiopeia A.

To obtain an estimate of the average Galactic SN rate, observations of SNe in other galaxies are used as well. Such an average rate can be defined as (van den Bergh and Tammann, 1991):

$$\bar{f}_{\text{SN}} = \frac{N_{\text{SN}}}{N_{\text{gal}} \Delta t}, \quad (7.7)$$

where N_{SN} is the number of observed SNe in a sample of N_{gal} galaxies over an observation time Δt . Observational constraints limit the accuracy of such estimates. It is not clear that all SNe in the observed galaxies can be detected, because they might be obscured or are only visible for a short time (van den Bergh and Tammann, 1991).

Estimates of the Galactic SN rate, f_{SN} , can be obtained by combining historical records of Galactic SNe with observations of SNe in other galaxies. A broad overview of such estimates is given in van den Bergh and Tammann, 1991. A commonly adopted value for the Galactic SN rate is $f_{\text{SN}} \approx 0.03 \text{ yr}^{-1} = 3 \text{ century}^{-1}$, which lies within the range of estimates obtained by Tammann et al., 1994, who find $f_{\text{SN}} = 2.5^{+0.8}_{-0.5} \text{ century}^{-1}$. However, this value is still uncertain and other estimates determined by different methods, e.g., the measurements of γ -rays from radioactive ^{26}Al decay (Diehl et al., 2006), suggest slightly lower values of $f_{\text{SN}} \approx 0.02 \text{ yr}^{-1}$.

7.2.2 Radial and vertical distribution

Besides the overall rate of Galactic cosmic-ray sources, we also need to know their spatial distribution throughout the Galaxy. Studies (see below) suggest that their distribution is not expected to be homogeneous throughout the Galactic disk, but to follow some more complicated spatial distribution. Population studies based on SNRs, massive stars as SNR progenitors, or pulsars as relics tracing SNe inform analytical parametrisations of source distributions. Those usually assume axisymmetry around the Galactic centre and a functional dependence on the Galactocentric radius, R , and the height above the Galactic plane, z . Some models also assume a spiral arm structure for the source distribution. We give a brief overview of existing models in the following (see also Figs. 7.2 and 7.3).

One of the first studies on the distribution of SNRs in our Galaxy was performed by Ilovaisky and Lequeux, 1972a,b. By using radio-observations of SNRs and assuming a simple relation between the surface brightness and diameter of SNRs ($\Sigma - D$ relation¹), they derived a radial distribution of SNRs and rate of SNe. Multiple similar studies with reanalyses of varying SNR catalogues followed (see, e.g., Case and Bhattacharya, 1996; Case and Bhattacharya, 1998; Green, 2015; Kodaira, 1974; Stecker and Jones, 1977). A commonly used functional form for the radial distribution of SNRs is:

$$\rho(R) = \left(\frac{R + R_1}{R_\odot + R_1} \right)^A \exp \left[-B \left(\frac{R - R_\odot}{R_\odot + R_1} \right) \right], \quad (7.8)$$

where the parameters A , B , and R_1 are determined by fitting to observational data, and $R_\odot$ is the distance of the Sun from the Galactic centre.

The parameter R_1 was introduced to allow for a non-vanishing surface density of SNRs in the Galactic centre (Yusifov and Küçük, 2004). Before the introduction of this parameter, other functional forms were explored to allow for this, e.g., Case and Bhattacharya, 1998, Eq. (15):

$$\rho(R) = A \sin \left(\frac{\pi R}{R_0} + \theta_0 \right) \exp(-\beta R), \quad (7.9)$$

where R_0 , θ_0 , and β are fit parameters.

Later, studies based on pulsar observations were performed. An extensive catalogue was obtained in the Parkes Multibeam Pulsar Survey (Manchester et al., 2001). Fitted distributions based on this survey include those by Yusifov and Küçük, 2004, Lorimer et al., 2006, and Xie et al., 2024.

Ferrière, 2001 gives different parametrisations for type-II and type-I SNe. Type-II SNe can be associated to H II regions (McKee and Williams, 1997), which are produced by massive OB stars, as well as pulsars which are likely remnants of core-collapse SNe. Type-I SNe,

¹Typically, the relation $\Sigma = AD^\delta$ with $\delta < 0$ is used, i.e., SNRs with higher surface brightness correspond to smaller diameters (see, e.g., Case and Bhattacharya, 1998). As the surface brightness is distance-independent, this can be used to estimate distances to SNRs (Green, 2015).

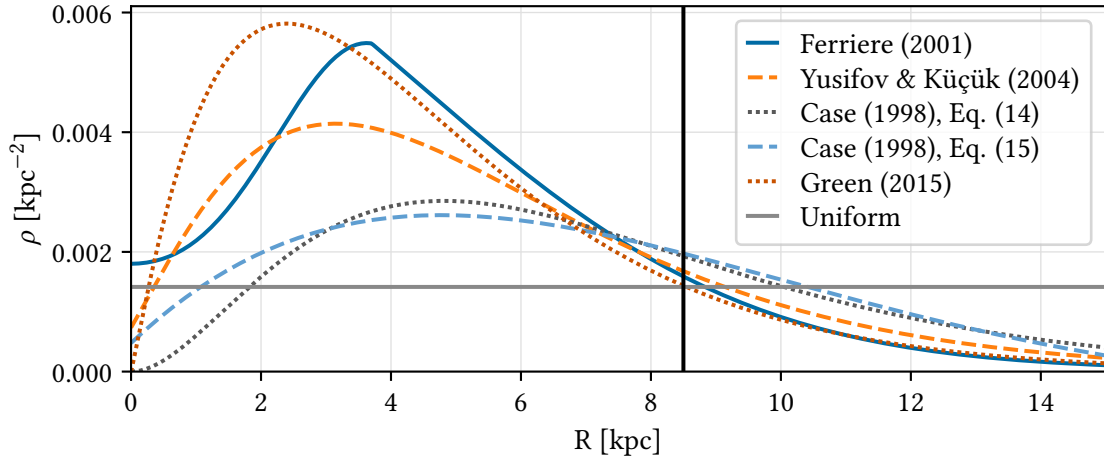


Figure 7.2: Radial source density profile normalised as a density. The radial profiles of different SNR distributions according to Case and Bhattacharya, 1998; Ferrière, 2001; Green, 2015; Yusifov and Küçük, 2004 are shown. For comparison, we also show a uniform homogeneous density profile. They are normalised such that $\int_0^{R_{\text{Gal}}} dR 2\pi R \rho(R) = 1$. The Solar Galactocentric radius is indicated by the vertical line. We use the standard value $R_{\odot} = 8.5$ kpc as recommended by the International Astronomical Union (IAU) (Kerr and Lynden-Bell, 1986). Here, we assume a Galactic radius of $R_{\text{Gal}} = 15$ kpc, up to which sources can be found.

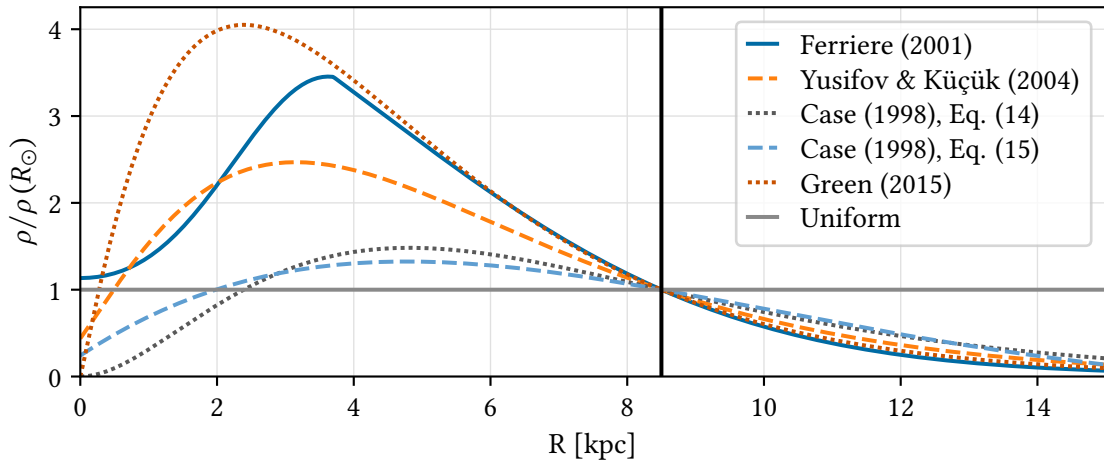


Figure 7.3: Radial source density profile normalised to local value. The same radial profiles as in Fig. 7.2 are shown, but normalised to their value at the position of the Sun, $R_{\odot}$, indicated by the vertical line. Again, we use the standard value $R_{\odot} = 8.5$ kpc as recommended by the IAU (Kerr and Lynden-Bell, 1986).

on the other hand, are expected to follow the distribution of old disk stars (Freeman, 1987). According to Ferrière, 2001, the functional form for the distribution of type-I SNe is given by

$$\rho_I(r) \propto \exp\left(-\frac{r - R_\odot}{4.5 \text{ kpc}}\right), \quad (7.10)$$

and that for type-II SNe by

$$\rho_{II}(r) \propto \begin{cases} \exp\left(-\left(\frac{r-3.7 \text{ kpc}}{2.1 \text{ kpc}}\right)^2\right) & r < 3.7 \text{ kpc}, \\ \exp\left(-\frac{r^2 - R_\odot^2}{(6.8 \text{ kpc})^2}\right) & r \geq 3.7 \text{ kpc}, \end{cases} \quad (7.11)$$

Often, the two distributions are combined. For the relative contribution of the respective SN types, the normalisations given in Ferrière, 2001 under the assumption of a Galactic radius, R_{Gal} , can be used.

We show a comparison of a few of the discussed radial source distributions in Figs. 7.2 and 7.3. The two figures differ in their normalisation. In Fig. 7.2, the distributions are normalised such that their integral over the Galactic volume is unity, while in Fig. 7.3, they are normalised to their value at the position of the Sun, $R_\odot = 8.5 \text{ kpc}$. For a fixed Galactic SN rate, f_{SN} , the first normalisation is more sensible for comparisons of the overall source density throughout the Galaxy². It can be seen that the different distributions agree reasonably well in the region around the position of the Sun, but differ significantly in the inner Galaxy.

For the dependence of the source distribution on the height above the Galactic plane, z , either an exponential or a Gaussian dependence is commonly assumed:

$$\rho(z) \propto \exp\left(-\frac{|z|}{z_0}\right) \quad \text{or} \quad \rho(z) \propto \exp\left(-\left(\frac{z}{2z_0}\right)^2\right), \quad (7.12)$$

where z_0 is the scale height of the source distribution. Typically, values for z_0 of $\mathcal{O}(100 \text{ pc})$ are adopted (e.g., Ferrière, 2001; Lorimer et al., 2006; Yusifov and Küçük, 2004).

7.2.3 Spiral models

In the previous section, we discussed axisymmetric source distributions. However, it can be expected that the source distribution has a more complicated structure. There is reason to believe that the distributions of SNRs follow the distribution of their progenitor stars, e.g., massive OB stars for type-II SNe. There is some evidence that regions with high star formation rates are arranged in a spiral structure in our Galaxy (see, e.g., Churchwell

²The second normalisation is still shown often, but it suggests that we were somehow more certain about our local source density, which is not necessarily the case.

et al., 2009; Reid et al., 2019; Steiman-Cameron et al., 2010). Thus, it has been suggested to impose some spiral structure on the distribution of cosmic-ray sources (e.g., Blasi and Amato, 2012b; Evoli et al., 2021a,b; Mertsch, 2011).

Observations from our location within the Galactic disk make it hard to determine the structure of the Milky Way. Many models for the different components of the Milky Way exist. Some of them suggest some kind of spiral structure. For a review of the various ISM components, see, e.g., Draine, 2011. We just want to point out a meta study undertaken by Vallée, 2002, who compiled different measurements of the spiral structure of the Milky Way. This provides an instructive overview of the broad range of existing models in terms of the number of spiral arms, their pitch angles, the existence and orientation of a central bar, etc.

Now, we want to mention some commonly used spiral models. This includes the one compiled by Vallée, 2005, which is based on an extension of the meta study mentioned above. Further, Steiman-Cameron et al., 2010 fitted a spiral model based on maps of C II and N II line intensities as measured by the *FIRAS* instrument on board of the COBE satellite. More recently, Reid et al., 2019 presented a spiral model based on trigonometric parallaxes and proper motions of molecular masers associated with very young high-mass stars. They used data from the BeSSeL Survey using the VLBA and the Japanese VERA project. Models that include a central bar structure usually assume a bar angle of 20° as suggested by Churchwell et al., 2009.

The centres of the spiral arms can be modelled parametrically as logarithmic spirals (Steiman-Cameron et al., 2010):

$$\theta_i(R) = \frac{1}{\alpha} \ln \left(\frac{R}{a_i} \right), \quad (7.13)$$

where θ_i is the azimuthal angle of the i -th spiral arm at Galactocentric radius R , a_i is a scaling parameter setting the orientation of the spiral arm, and α is the tangent of the pitch angle of the spiral arm. The relative emissivity, ϵ_i , of each spiral arm can also be included to account for different star formation rates in the various arms. In Steiman-Cameron et al., 2010, those were inferred from observations of C II and N II line intensities. The parameters for a model with four spiral arms as determined by Steiman-Cameron et al., 2010 are summarised in Tab. 7.1. There, we used the relative emissivities inferred from C II observations.

7.2.4 Inverse transform sampling

After we introduced a range of source distribution models, we want to conclude this section with a description of how to sample source coordinates from those distributions. A commonly used method for this task is *inverse transform sampling* (e.g., Devroye, 1986).

Arm Number	Pitch Angle [°]	α	a	Rel. ϵ [%]	Common Name
1	13.6 ± 1.0	0.242	0.246	17.8	Sagittarius–Carina
2	15.6 ± 1.2	0.279	0.608	28.0	Scutum–Crux
3	13.5 ± 2.0	0.249	0.449	35.7	Perseus
4	13.5 ± 3.0	0.240	0.378	18.5	Norma–Cygnus

Table 7.1: Spiral arm parameters from Steiman-Cameron et al., 2010. The values are taken from Steiman-Cameron et al., 2010, Tabs. 3 and 4.

Suppose that the Probability Density Function (PDF) of a random variable X is given by f_X , i.e.: $P(X \in A) = \int_A dx f_X(x)$, for some measurable set A . Then, the corresponding Cumulative Distribution Function (CDF), $F_X(x)$, is given by:

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x dx' f_X(x'). \quad (7.14)$$

The range of the CDF is $[0, 1]$. Thus, the inverse³ CDF, F_X^{-1} , maps the unit interval $[0, 1]$ to the domain of X . Inverse transform sampling now uses the fact that if a random variable U is uniformly distributed on the unit interval, then the random variable $X = F_X^{-1}(U)$ is distributed according to the PDF f_X (Devroye, 1986).

Thus, to sample from a given PDF, f_X , we can follow these steps:

1. Calculate the CDF, $F_X(x)$, from the PDF, $f_X(x)$, using Eq. (7.14).
2. Calculate the inverse CDF, $F_X^{-1}(u)$.
3. Sample a random number, u , from a uniform distribution on the unit interval.
4. Calculate $x = F_X^{-1}(u)$, a sample from the PDF, $f_X(x)$.

7.3 Discrete source modelling

Our goal is to model the density of cosmic-ray protons with rigidities above about 10 GV throughout the Galaxy, taking into account the discrete nature of their sources that we discussed in Sec. 7.1. To this end, we need to solve the cosmic-ray transport equation Eq. (5.69). For reference, we repeat it here:

$$\frac{\partial \psi(\mathcal{R}, t, \mathbf{x})}{\partial t} - \kappa(\mathcal{R}, t) \nabla^2 \psi(\mathcal{R}, t, \mathbf{x}) = Q(\mathcal{R}, t, \mathbf{x}). \quad (7.15)$$

³Note, that a well-defined inverse function only exists if the CDF is strictly monotonically increasing. If this is not the case, one can still define a generalised inverse CDF as $F_X^{-1}(u) \equiv \inf \{x \in \mathbb{R} : F_X(x) = u \wedge u \in (0, 1)\}$.

Note, that we have introduced the possibility of a time-dependent diffusion coefficient, $\kappa(\mathcal{R}, t)$, to account for possible temporal variations of the diffusion properties (for an example of this, see Part III).

The source term on the right-hand side receives contributions from all cosmic-ray sources throughout the Galaxy, i.e.,

$$Q(\mathcal{R}, t, \mathbf{x}) = \sum_{j=1}^{N_{\text{src}}} Q_j(\mathcal{R}, t, \mathbf{x}) , \quad (7.16)$$

where N_{src} is the total number of sources considered and $Q_j(\mathcal{R}, t, \mathbf{x})$ is the contribution from the j -th source.

We argued in Sec. 7.1, that the spatial extent of individual sources can be neglected, i.e., they can be modelled as point sources in space. Further, it is commonly assumed that all sources inject cosmic rays according to the same universal injection spectrum, $Q_{\mathcal{R}}(\mathcal{R})$, which is expected to follow a power law in rigidity, as we have discussed in the context of Diffusive Shock Acceleration (DSA) in Sec. 5.5. Finally, our discussion of the escape of cosmic rays from their sources in Sec. 5.8 suggests that the injection time for cosmic rays may depend on rigidity. Due to the mechanisms confining cosmic rays close to their sources, higher-rigidity cosmic rays are expected to escape earlier than lower-rigidity ones, but cosmic rays of one specific rigidity are expected to be injected in a short time interval (for more details, see Sec. 5.8 and, e.g., Stall et al., 2025). Thus, it is a common assumption that all cosmic rays of a specific rigidity, $\mathcal{R}$, are injected instantaneously at an escape time, $t_{\text{esc}}(\mathcal{R})$, by the cosmic-ray source.

In summary, the contribution from an individual source, $Q_j(\mathcal{R}, t, \mathbf{x})$, as introduced in Eq. (7.16) is modelled as:

$$Q_j(\mathcal{R}, t, \mathbf{x}) = Q_{\mathcal{R}}(\mathcal{R}) \delta(t - t_{\text{esc},j}(\mathcal{R})) \delta(\mathbf{x} - \mathbf{x}_j) , \quad (7.17)$$

where $\mathbf{x}_j$ is the position of the j -th source and $t_{\text{esc},j}(\mathcal{R})$ is the escape time of cosmic rays with rigidity $\mathcal{R}$ from that source, i.e.,

$$t_{\text{esc},j}(\mathcal{R}) = t_j + \Delta t_{\text{esc}}(\mathcal{R}) , \quad (7.18)$$

where t_j is the time of the SN explosion of the j -th source and $\Delta t_{\text{esc}}(\mathcal{R})$ describes the time dependence of the escape of cosmic rays of rigidity $\mathcal{R}$.

In the following, we discuss how the transport equation Eq. (7.15) can be solved with the source term as defined in Eqs. (7.16) to (7.18). The central tool is the Green's function, which we introduce in Sec. 7.3.1. It is defined as the solution of the transport equation, Eq. (7.15), for a delta-injection in space and time. After that, we discuss the influence of the free escape boundary condition at the halo height in Sec. 7.3.2. Further, we discuss how this solution can be modified at early times after the injection of cosmic rays, when the diffusion approximation is not valid yet, in Sec. 7.3.4.

7.3.1 Green's function

A Green's function is defined as the solution of a differential equation, where the source term is given by a delta-function. First, we want to determine the Green's function for the cosmic-ray transport equation Eq. (7.15) without considering the boundary conditions. We call this the *free* case. We will ensure the boundary conditions later in Sec. 7.3.2 by the *method of image charges*.

The Green's function, $G_{\text{free}}(\mathcal{R}, t, \mathbf{x} | t_s, \mathbf{x}_s)$, for the free case is the solution of:

$$\frac{\partial}{\partial t} G_{\text{free}} - \kappa(\mathcal{R}, t) \nabla^2 G_{\text{free}} = \delta(t - t_{\text{esc}}(\mathcal{R} | t_s)) \delta(\mathbf{x} - \mathbf{x}_s), \quad (7.19)$$

where the source is located at position $\mathbf{x}_s$ and injects cosmic rays at time t_{esc} , which depends on the rigidity $\mathcal{R}$ and the time of the SN explosion, t_s (see Eq. (7.18))⁴.

For conciseness, we omit the functional arguments in the following derivation. We reinstate them in the final expression for the Green's function. The Green's function can be determined by performing a Fourier transform in the spatial coordinates⁵:

$$\frac{\partial \mathcal{G}_{\text{free}}}{\partial t} + \kappa k^2 \mathcal{G}_{\text{free}} = \delta(t - t_{\text{esc}}) e^{-i\mathbf{k} \cdot \mathbf{x}_s}. \quad (7.20)$$

This can be brought into a different form by multiplying with an integrating factor, $e^{\int_{t_0}^t dt' \kappa k^2}$, where $t_0 < t_s$ is some initial time *before* the injection of cosmic rays. This gives:

$$\frac{\partial}{\partial t} \left[\mathcal{G}_{\text{free}} e^{\int_{t_0}^t dt' \kappa k^2} \right] = \delta(t - t_{\text{esc}}) e^{-i\mathbf{k} \cdot \mathbf{x}_s} e^{\int_{t_0}^t dt' \kappa k^2}. \quad (7.21)$$

Integrating both sides gives:

$$\mathcal{G}_{\text{free}} = \Theta(t - t_{\text{esc}}) e^{-i\mathbf{k} \cdot \mathbf{x}_s} e^{-\int_{t_{\text{esc}}}^t dt' \kappa k^2}, \quad (7.22)$$

where Θ is the Heaviside step function, which vanishes for negative arguments and is equal to one otherwise. We define:

$$\sigma^2 \equiv 2 \int_{t_{\text{esc}}}^t dt' \kappa. \quad (7.23)$$

With this, we can rewrite Eq. (7.22) as:

$$\mathcal{G}_{\text{free}} = \Theta(t - t_{\text{esc}}) e^{-i\mathbf{k} \cdot \mathbf{x}_s} e^{-\frac{1}{2} k^2 \sigma^2}. \quad (7.24)$$

⁴The $|$ in the function argument separates the source coordinates from the other ones.

⁵We define the Fourier transform such that all prefactors of 2π appear in the inverse transform.

Now, we apply the inverse Fourier transform and get:

$$G_{\text{free}} = \frac{\Theta(t - t_{\text{esc}})}{(2\pi)^3} \int d^3k e^{ik \cdot (\mathbf{x} - \mathbf{x}_s)} e^{-\frac{1}{2}k^2 \sigma^2}. \quad (7.25)$$

This integral results in a Gaussian (see a detailed derivation in App. C.2), giving:

$$G_{\text{free}} = \frac{\Theta(t - t_{\text{esc}})}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{|\mathbf{x} - \mathbf{x}_s|^2}{2\sigma^2}\right). \quad (7.26)$$

A Green's function of this form has been derived by Syrovatskii, 1959 for the case of cosmic-ray electrons, where also continuous energy losses have to be taken into account.

Now, we can reinstate all functional dependencies that are not effected by the derivation. By using $d \equiv |\mathbf{x} - \mathbf{x}_s|$ to denote the distance between the source and the observation point, we can write the final expression for the free Green's function as:

$$G_{\text{free}}(\mathcal{R}, t, \mathbf{x} | t_s, \mathbf{x}_s) = \frac{\Theta(t - t_{\text{esc}}(\mathcal{R} | t_s))}{(2\pi\sigma^2(\mathcal{R}, t | t_s))^{3/2}} \exp\left(-\frac{d^2}{2\sigma^2(\mathcal{R}, t | t_s)}\right), \quad (7.27)$$

where the quantity σ^2 has the following functional dependencies:

$$\sigma^2(\mathcal{R}, t | t_s) = 2 \int_{t_{\text{esc}}(\mathcal{R} | t_s)}^t dt' \kappa(\mathcal{R}, t'). \quad (7.28)$$

For time-independent diffusion coefficients, this simplifies to:

$$\sigma^2(\mathcal{R}, t | t_s) = 2\kappa(\mathcal{R})(t - t_{\text{esc}}(\mathcal{R} | t_s)). \quad (7.29)$$

7.3.2 Boundary condition

From this free Green's function, we can construct a solution to Eq. (7.19) that also satisfies the boundary conditions at the halo height, i.e., $G(z = \pm H) \equiv 0$. We neglect an additional radial boundary condition, as discussed in Sec. 5.4, because the radial boundary is usually farther away from the sources and observation points of interest than the halo height. For this, the *method of image charges*, commonly used in electrostatics (e.g., Jackson, 1998), can be employed. It can be used to construct solutions of *linear* differential equations satisfying certain boundary conditions within a volume of interest. In our case, we want to find a solution of Eq. (7.19) that is valid within the volume bounded by the planes at $z = \pm H$ and satisfies the free escape boundary conditions there. The central idea is to place additional image sources outside the volume of interest, such that the superposition of the Green's functions corresponding to the real and image sources satisfies the boundary conditions. The image sources do not correspond to physical

sources. They are merely a mathematical tool to ensure the boundary conditions are satisfied and are only placed outside the volume of interest.

To satisfy a free escape boundary condition on one plane, e.g., at $z = H$, an image source with opposite sign can be placed at the mirrored position with respect to that plane, i.e., at $z' = 2H - z_s$ for the real source located at z_s . This ensures that the Green's function vanishes at that plane. Including a second plane at $z = -H$ requires an infinite series of image sources to satisfy the boundary conditions at both planes simultaneously. E.g., we need to place an image source of opposite sign for the real source to satisfy the boundary condition at $z = -H$, but we also have to place an image source for the image source we placed at $z' = 2H - z_s$, etc.

The final Green's function solves Eq. (7.19) and satisfies the free escape boundary conditions at $z = \pm H$. It is given by (e.g., Mertsch, 2011):

$$G(\mathcal{R}, t, \mathbf{x} | t_s, \mathbf{x}_s) = \sum_{n=-\infty}^{\infty} (-1)^n G_{\text{free}}(\mathcal{R}, t, \mathbf{x} | t_s, \mathbf{x}_{s,n}) \text{ with } \mathbf{x}_{s,n} = \begin{pmatrix} x_s \\ y_s \\ 2Hn + (-1)^n z_s \end{pmatrix}. \quad (7.30)$$

7.3.3 Differential cosmic-ray density

With the Green's function at hand, we can determine the differential cosmic-ray density, $\psi(\mathcal{R}, t, \mathbf{x})$, that solves the transport equation Eq. (7.15). The linearity of this transport equation allows us to write down the solution for an arbitrary source term, $Q(\mathcal{R}, t, \mathbf{x})$ as (e.g., Jackson, 1998):

$$\psi(\mathcal{R}, t, \mathbf{x}) = \iint dt' d^3x' G(\mathcal{R}, t, \mathbf{x} | t', \mathbf{x}') Q(\mathcal{R}, t', \mathbf{x}'). \quad (7.31)$$

In the preceding sections, we have developed the source term for discrete sources, as given in Eqs. (7.16) to (7.18). The source term for a smooth source distribution is given as in Eq. (5.68). In summary, we get:

$$Q(\mathcal{R}, t, \mathbf{x}) = Q_{\mathcal{R}}(\mathcal{R}) \begin{cases} \sum_{i=1}^{N_{\text{src}}} \delta(t - t_{\text{esc},i}(\mathcal{R})) \delta(\mathbf{x} - \mathbf{x}_i) & \text{(discrete)} \\ \rho(t, \mathbf{x}) & \text{(smooth)} \end{cases}. \quad (7.32)$$

Thus, the solution for the differential cosmic-ray density in the case of discrete sources is given by a sum of Green's functions:

$$\psi(\mathcal{R}, t, \mathbf{x}) = Q_{\mathcal{R}}(\mathcal{R}) \sum_{i=1}^{N_{\text{src}}} G(\mathcal{R}, t, \mathbf{x} | t_{\text{esc},i}(\mathcal{R}), \mathbf{x}_i). \quad (7.33)$$

The density function, $\rho(t, \mathbf{x})$, describes the distribution of sources in space and time. It is normalised such that:

$$\int_{t_{\text{start}}}^{t_{\text{end}}} dt \int_V d^3x \rho(t, \mathbf{x}) = N_{\text{src}}(t \in [t_{\text{start}}, t_{\text{end}}], \mathbf{x} \in V), \quad (7.34)$$

i.e., the integral over some time interval, $[t_{\text{start}}, t_{\text{end}}]$, and volume, V , gives the expected number of sources within that interval and volume.

If the source positions and times of the SN explosions in the discrete case are sampled *independently* of the distribution described by the density function $\rho(t, \mathbf{x})$, the ensemble average (for reference, see definition in Eq. (5.10) in Sec. 5.3.2) over the possible source realisations of the discrete source term yields the corresponding smooth source term in the limit of $N_{\text{src}} \rightarrow \infty$:

$$\langle Q^{\text{discrete}}(\mathcal{R}, t, \mathbf{x}) \rangle = Q_{\mathcal{R}}(\mathcal{R}) \langle \rho(t, \mathbf{x}) \rangle = Q^{\text{smooth}}(\mathcal{R}, t, \mathbf{x}). \quad (7.35)$$

Consequently, the ensemble average of the differential cosmic-ray density over the realisations of the source population coincides with the solution for a smooth source distribution (e.g., Genolini et al., 2017):

$$\psi^{\text{smooth}}(\mathcal{R}, t, \mathbf{x}) = Q_{\mathcal{R}}(\mathcal{R}) \iint dt' d^3x' G(\mathcal{R}, t, \mathbf{x} | t', \mathbf{x}') \rho(t', \mathbf{x}') = \langle \psi(\mathcal{R}, t, \mathbf{x}) \rangle. \quad (7.36)$$

Via this correspondence, it is always possible to obtain an approximate solution for the differential cosmic-ray density that solves the transport equation for a smooth source term. It can be obtained by averaging over many realisations of the discrete source term:

$$\psi^{\text{smooth}}(\mathcal{R}, t, \mathbf{x}) \approx \frac{1}{N_{\text{real}}} \sum_{k=1}^{N_{\text{real}}} \psi_k^{\text{discrete}}(\mathcal{R}, t, \mathbf{x}), \quad (7.37)$$

where N_{real} is the number of realisations of the discrete source term considered.

This is useful to compare the results of discrete source modelling to the commonly used smooth source distribution approximation. Using the sample mean of discrete source realisations to estimate the differential cosmic-ray density for a smooth source distribution is effectively a Monte Carlo integration of the convolution integral in Eq. (7.36).

However, there is an important caveat to the accuracy of this estimation. The convergence of the sample mean to the ensemble mean is slow in this case as the variance of the differential cosmic-ray density diverges (e.g., Bernard et al., 2012 and Sec. 7.4.2). This means, that extreme outliers can strongly influence the sample mean, even for many realisations, N_{real} . We refer to (Nolan, 2020) for a detailed discussion of the robustness of the sample mean for distributions with infinite variance and other statistical estimators that can be used in such cases. Still, the sample mean is a reasonably good estimator for the ensemble mean in cases where outliers are not too extreme and do not dominate the sample. We will discuss and make use of this in the following Parts II and III.

7.3.4 Diffusion approximation and causality

In Chap. 5, we have derived the cosmic-ray transport equation, Eq. (7.15). Part of this derivation was the diffusion approximation, which we discussed in Sec. 5.3.5. This assumes that we are interested in the evolution of the isotropic part of the cosmic-ray phase-space density on timescales much longer than the scattering time and on spatial scales much larger than the mean free path (see Eq. (5.29)). This assumption breaks down at very early times after the injection of cosmic rays by a source. Here, we want to discuss how this affects the applicability of the Green's function derived in Sec. 7.3.1 as a solution of the transport of cosmic-ray protons on the scales we are interested in.

On a similar note, it should be pointed out that the diffusion transport equation, Eq. (7.15), which we used to derive the Green's function in Sec. 7.3.1, is not Lorentz invariant, i.e., it is not causal. The solution is non-zero for all distances d at any time $t > t_{\text{esc}}$, which would imply superluminal propagation of cosmic rays. This is especially problematic at early times when the diffusion radius (see Eq. (7.2)) can be larger than the causal horizon, i.e., the distance that cosmic rays could have propagated ballistically since their injection. This is a well-known issue of the diffusion equation (e.g., Aloisio et al., 2009).

The diffusion approximation is not valid at very early times when there was not enough time for the cosmic-ray phase space density to isotropise via scattering. Thus, the pitch-angle averaging procedure we used in Sec. 5.3.5 to derive the diffusion equation is not valid at early times (e.g., Axford, 1965). On a microphysical level, this means that the transport of cosmic rays is rather described by ballistic motion than by a diffusion process then, with a smooth transition to diffusion at some point. Malkov and Sagdeev, 2015 discuss this problem and a possible solution provided by the so-called *telegraph model*, which has been introduced for the problem of cosmic-ray transport by Litvinenko and Schlickeiser, 2013. They showed that a so-called *telegraph equation* arises when further terms related to the evolution of the anisotropic part of the phase-space density are retained in the derivation of the transport equation. This takes into account finite propagation speeds of cosmic rays and provides a better description of the transport at early times.

A comparative assessment of the diffusion approximation and the solution of the telegraph equation using simulations is given in Effenberger and Litvinenko, 2014. It is shown that the diffusion approximation describes the particle density similarly well as the telegraph model if the initial injection of particles is isotropic, as is the case for the point-like source injections we are considering in our isotropic treatment of the cosmic-ray intensity. However, the diffusion approximation wrongfully predicts superluminal propagation leading to a non-zero particle density at distances $d > c(t - t_{\text{esc}})$, where c is the speed of light ⁶.

⁶As we are considering cosmic-ray protons with rigidities above 10 GV, their speed is very close to the speed of light, i.e., $v \approx c$. Thus, the causality argument coincides with the physical limit of how far particles can have propagated ballistically since their injection.

We can expect that the error made by the diffusion approximation is big if a large fraction of the particle density is located beyond that causal horizon. We estimate this by equating it with the diffusion radius (Effenberger and Litvinenko, 2014). Our solution for the Green's function in Eq. (7.27) together with the definition of σ^2 in the case of a time-independent diffusion coefficient in Eq. (7.29) shows that the diffusion radius we defined in Eq. (7.2) coincides with the standard deviation of the Gaussian spatial distribution of the particle density, i.e., $\lambda_{\text{diff}} = \sigma$. Thus, we know that a fraction of about 68 % of the particle density is enclosed within a diffusion radius. We estimate that the diffusion approximation is reasonable if the diffusion radius is smaller than the causal horizon, i.e., if:

$$\lambda_{\text{diff}}(\tau) = \sqrt{2\kappa\tau} < c\tau \quad \Rightarrow \quad \tau > \frac{2\kappa}{c^2} \equiv \tau_{\text{min}}, \quad (7.38)$$

where we defined $\tau \equiv t - t_{\text{esc}}$ as the time since the injection of cosmic rays.

We can estimate the minimum time since injection, τ_{min} , for which the diffusion approximation is reasonable by inserting typical values for the diffusion coefficient. As in the estimates presented in Fig. 7.1, we use the diffusion coefficient determined by Evoli et al., 2019. We find that $\tau_{\text{min}}(10 \text{ GV}) \approx 3 \text{ yr}$, which is very small compared to the transport and escape timescales. Even at much higher rigidities, we find moderate values, e.g., $\tau_{\text{min}}(1 \text{ PV}) \approx 1 \text{ kyr}$. Thus, we conclude that the diffusion approximation is reasonable for our purposes. To avoid unphysical superluminal propagation in our modelling, we will impose a causality condition by setting the cosmic-ray density to zero for distances $d > c(t - t_{\text{esc}})$ in our models (see Parts II and III and, e.g., Genolini et al., 2017).

7.4 Stochastic analysis

In this section, we give an overview of previous studies related to the stochastic modelling of discrete cosmic-ray sources. This includes studies of the impact of discrete sources on the local cosmic-ray spectrum as well as on the GDEs. In Sec. 7.4.2, we highlight results on the distribution functions of the differential cosmic-ray densities as presented in Genolini et al., 2017. We conclude by summarising the prospects of stochastic source modelling for constraining the origins of cosmic rays in the Galaxy in Sec. 7.4.3.

7.4.1 Previous studies

The arguments we presented in Sec. 7.1 were already recognised in the early days of cosmic-ray research (e.g., Ginzburg and Syrovatskii, 1964). One of the first works to systematically argue for the stochastic nature of SN explosions and their impact on the cosmic-ray density was provided by Jones, 1970. This work was guided by whether the discrete nature of cosmic-ray sources has an impact on the observed cosmic-ray spectrum

and anisotropy. A formalisation of the stochastic treatment of the cosmic-ray density was provided by Lee, 1979. Related to this work, Cowsik and Lee, 1979 also argued that the discreteness of sources is especially important for high-energy electrons due to their rapid synchrotron energy losses (see Sec. 7.1).

The impact of the *stochasticity*⁷ of cosmic-ray sources always has to be seen relative to the achievable experimental precisions. Thus, further developments in this field can usually be related to corresponding experimental advancements. As observations became more precise, the interest in stochastic source modelling rekindled around the turn of the millennium. With X-ray and γ -ray observations available, the SNR paradigm (see Sec. 4.4) was strengthened, which also affirmed the discrete nature of cosmic-ray sources. Studies by Pohl and Esposito, 1998 and Strong and Moskalenko, 2001 discussed the impact of discrete sources on GDEs in γ -rays. Jones and Streitmatter, 2005; Ptuskin et al., 2006 also revisited the stochastic treatment of cosmic-ray sources to determine whether fluctuations in the cosmic-ray density due to source stochasticity are significant.

A special interest in the stochastic modelling of cosmic-ray electrons arose from measurements in γ -rays by *Fermi*-LAT (Abdo et al., 2009) and direct measurements of the electron spectrum, e.g., by PAMELA (Adriani et al., 2011), AMS-02 (Aguilar et al., 2014), CALET (Adriani et al., 2017), DAMPE (DAMPE Collaboration et al., 2017), and H.E.S.S. (Aharonian et al., 2008 and recently Aharonian et al., 2024). Based on the Green's function for cosmic-ray electrons (Syrovatskii, 1959), multiple studies focused on stochastic source effects on the local cosmic-ray electron spectrum by employing Monte Carlo simulations. This includes work by, e.g., Mertsch, 2011, followed up by Mertsch, 2018, Evoli et al., 2021a,b, and more recently under the use of machine learning methods by Frediani et al., 2025.

Direct measurements of cosmic-ray nuclei in the MV rigidity range by Voyager 1 (Stone et al., 2013) posed questions about the spectral shape of the cosmic-ray spectrum at low rigidities. As demonstrated by Phan et al., 2021, this can be explained by the stochasticity of cosmic-ray sources in conjunction of the low number of sources contributing due to the small loss lengths for MV cosmic rays.

Interest in the stochastic treatment of cosmic-ray nuclei at high rigidities and their influence on GDEs has flared up in the past decade due to the availability of high-precision measurements by experiments like AMS-02 (Aguilar et al., 2021) and DAMPE (An et al., 2019), as well as measurements of diffuse γ -ray emissions at very high energies, most recently by LHAASO (Cao et al., 2023a, 2025). Detailed analyses of stochastic models for high-energy nuclei were presented by, e.g., Blasi and Amato, 2012a,b and Genolini et al., 2017. We will briefly summarise the main results of the latter one in Sec. 7.4.2. Our results, presented in Stall et al., 2025, are concerned with the detectability of signatures

⁷By this, we mean that treating the source positions and ages as random variables renders the cosmic-ray observables to random variables as well. If this results in a significant variance in the predictions, we say that the source *stochasticity* has an impact or is high.

of discrete sources in the local cosmic-ray proton spectrum. We discuss them in more detail in Part II.

Other recent studies also discussed the influence of source stochasticity on GDEs. This includes work by Thaler et al., 2023, Marinos et al., 2023, 2025, and an explorative study by Giacinti and Semikoz, 2023. More recently, Kaci and Giacinti, 2025 discussed stochastic models for GDEs in light of the LHAASO observations. Our own study, Stall and Mertsch, 2026, which will be presented in Part III, provides a broad analysis of the relevance of discrete source modelling for GDEs in γ -rays and neutrinos.

7.4.2 Stable laws

Stable probability distributions (also known as *stable laws*) naturally arise in the description of the distribution of cosmic-ray intensities. In this section, we briefly summarise the main results presented in Genolini et al., 2017 regarding stable laws in the context of stochastic cosmic-ray source modelling. We start by introducing stable laws and their most relevant properties. For a detailed introduction to stable laws and their main applications, we refer to the monograph by Nolan, 2020.

Stable laws were first introduced by Lévy, 1925. Over time, it was found that they have useful applications in various fields of physics and beyond, e.g., in finance (Uchaikin and Zolotarev, 1999). Stable laws are defined as the family of probability distributions that are stable under convolution. If a random variable X is distributed according to a univariate stable distribution, we can write:

$$P(X) \sim S(\alpha, \beta, \gamma, \delta) , \quad (7.39)$$

where $\sim$ denotes *is distributed as* and S denotes the stable law, which is specified by four parameters: the index of stability $\alpha \in (0, 2]$, the skewness $\beta \in [-1, 1]$, the scale $\gamma > 0$, and the location $\delta \in \mathbb{R}$ (Nolan, 2020). The index of stability, α , determines the asymptotic behaviour of the distribution's tails. By that, we mean that if X 's probability distribution follows a stable law with a certain α , the cumulative distribution function F_X fulfils:

$$\lim_{X \rightarrow \infty} X^\alpha C(X) = c > 0 , \quad (7.40)$$

where c is a constant. The skewness parameter, β , determines the asymmetry of the distribution, where $\beta = 1$ corresponds to a maximally right-skewed distribution. For $\alpha < 2$, power-law tails appear and the variance diverges. The mean exists only for $\alpha > 1$ and coincides with the location parameter δ in that case. The standard deviation σ is only defined for $\alpha = 2$, where its relation to the scale parameter is given by $\sigma = \sqrt{2}\gamma$. A comparison of stable laws with different indices of stability to a Gaussian distribution is shown in Fig. 7.4. The corresponding PDFs are shown on linear axes on the left and on log-log axes on the right to highlight the power-law tails.

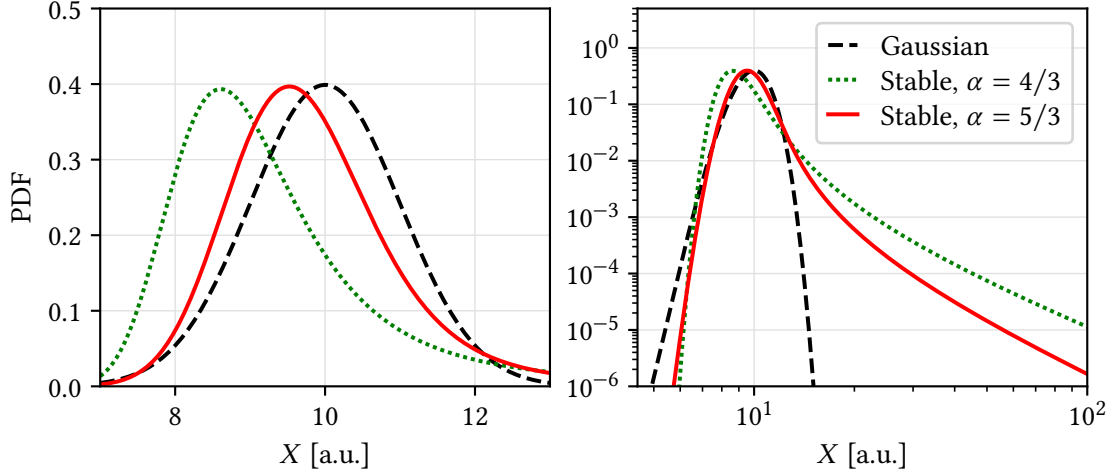


Figure 7.4: Demonstration of stable laws. The PDFs of a Gaussian distribution (black dashed) and two stable laws with $\alpha = 4/3$ (green dotted) and $\alpha = 5/3$ (red) are shown. The mean of the Gaussian and location parameters of the stable laws are set to $\mu = \delta = 10$, while the standard deviation of the Gaussian is set to $\sigma = 1$ and the scale parameter of the stable laws is set to $\gamma = 1/\sqrt{2}$ to match the scale approximately. The skewness parameter is set to $\beta = 1$ for the stable laws, in agreement with the theoretical expectations for cosmic-ray intensities. On the left, the axes are linear, while on the right, a log-log scale is used to highlight the power-law tails of the stable laws.

In a stochastic source model with discrete point sources, the single-source contribution to the local intensity can become arbitrarily large if sources are very close and young (see Eqs. (7.27) and (7.28)). The finite probability of very high contributions from such sources results in heavy tails of the intensity distribution for a single source. Bernard et al., 2012 showed that these single-source contributions follow distributions with power-law tails leading to a diverging variance. To obtain the total cosmic-ray intensity, contributions from many sources are summed (see Eq. (7.33)). Due to the diverging variance of the single-source contributions, the standard Central Limit Theorem (CLT) cannot be used to determine what distribution the total cosmic-ray intensity converges to in the limit of many sources. Instead, a generalised CLT for stable laws applies (Nolan, 2020, Theorem 1.4).

Genolini et al., 2017 determined the PDFs for the total cosmic-ray intensity resulting from stochastic source populations in two idealised geometric setups. They considered a distribution of sources in an infinitesimally thin disk (2D) and in a disk with finite height (3D). They determined that the total cosmic-ray intensity, Φ , follows a stable law:

$$P(\Phi) \sim S(\alpha, \beta = 1, \gamma = \sigma_N, \delta = \langle \Phi \rangle), \quad (7.41)$$

where the index of stability, α , depends on the geometry ($\alpha = 4/3$ in the 2D case and $\alpha = 5/3$ in the 3D case), the skewness is strongly positive ($\beta = 1$) as negative intensities

are not possible, the scale parameter γ depends on the source rate, spatial density and transport parameters, and the location parameter δ coincides with the ensemble mean.

A Monte Carlo study of the cosmic-ray intensity distributions confirmed the stable law predictions in Genolini et al., 2017. Yet, there are also discussions, to which extent the high-intensity tails emerging from diverging single-source contributions are physical in the context of our local environment (e.g., Blasi and Amato, 2012b). However, a precise observational determination of the maximum possible single-source contribution is highly uncertain (see discussion in Mertsch, 2018). The possibility of strong contributions from nearby and young sources cannot be excluded and can be expected to occur with a finite probability.

This strengthens the argument that the intensity distribution needs to be determined using the generalised CLT instead of the standard CLT due to the diverging variance of the single-source contributions. However, the causality condition discussed in Sec. 7.3.4 effectively limits source contributions. This has been implemented in the Monte Carlo simulations of Genolini et al., 2017. As the causality criterion actually renders the variance of the single-source contributions finite, the standard CLT technically becomes applicable. Still, the resulting intensity distributions are often better described by stable laws than by normal distributions predicted from the standard CLT. A normal distribution would only be approached in the limit of an infinite source rate. Yet, the causality criterion softens the high-intensity tail of the distributions.

The choice of a distribution most suitable to describe the CR intensity at a given location in the Galaxy depends on the surrounding source population and the relevant transport parameters. In Genolini et al., 2017, the authors considered a position in the centre of the Galactic disk and found that a 2D stable distribution with $\alpha = 4/3$ generally provides a better fit for intensities similar to the mean intensity, whereas a 3D distribution with $\alpha = 5/3$ is more appropriate for the extreme, high-intensity tail of the distribution. The reason is that realisations with high intensities are typically dominated by a single, *nearby* source, whose spatial distribution is effectively three-dimensional in a Galactic disk with a finite thickness. By contrast, less extreme intensity realisations are mainly shaped by *more distant* sources, for which the flat geometry of the thin Galactic disk dominates. A more detailed analysis of the intensity regimes in which the respective distributions perform better can be found in Genolini et al., 2017.

In Part III, we want to employ these results in the context of GDEs. In particular, this means that we are not only interested in cosmic-ray intensities at locations whose surroundings are densely populated with sources, such as the Galactic disk, but also at locations situated above or below the Galactic disk, further along the LOS (see Eq. (6.2) in Chap. 6). At such positions, the probability of receiving a strong contribution from a nearby source is significantly reduced. Consequently, the total CR intensity, arising as the sum of many small contributions, is expected to be described by a normal distribution following the standard CLT. As one moves away from the Galactic disk into the surrounding halo, a corresponding shift is therefore anticipated: intensity distributions that are well

described by stable laws in the disk should gradually give way to distributions that are better approximated by a Gaussian.

7.4.3 Prospects of stochastic source modelling

The stochastic modelling of discrete cosmic-ray sources provides a promising avenue for constraining the origins of cosmic rays in the Galaxy. The observation of subtle spectral features in the local cosmic-ray spectrum that deviate from smooth power laws could be indicative of contributions from nearby sources. Further, observations of GDEs over a broad energy range, from GeV to PeV energies, can provide insights into the spatial and temporal distribution of cosmic-ray sources in the Galaxy.

Our discussion in Sec. 7.1 shows that the stochasticity of sources is expected to have a different impact on cosmic-ray observables at different energies. This can lead to a more structured cosmic-ray intensity throughout the Galaxy at higher energies in the TV to PV range, compared to the smoother intensity distribution at lower energies in the GV range. Different transport scales in the different energy regimes can also lead to energy-dependent morphologies of GDEs.

With increasing accuracies of cosmic-ray measurements, such as those provided by AMS-02, DAMPE, and future missions like AMS-100 (Schael et al., 2019), the detection of spectral features arising from discrete sources becomes more feasible. Similarly, advancements in γ -ray and neutrino observatories, such as LHAASO and IceCube enhance the sensitivity to GDEs and their potential signatures of discrete sources. We refer to Parts II and III for detailed discussions of the results of the work presented in this thesis.

Part II

Local Galactic cosmic-ray spectrum

8

Monte Carlo simulations

In this chapter, we want to give an introduction to the Monte Carlo simulations we have used to investigate the influence of the discreteness of cosmic ray sources on the cosmic-ray proton spectrum observed at Earth. As we have discussed in Chap. 7, the discrete nature of cosmic-ray sources can potentially lead to deviations of the observed cosmic-ray spectrum from the mean spectrum predicted by continuous source models.

Studying these effects requires a stochastic framework that can account for the random spatial and temporal distribution of sources. The statistics of cosmic-ray intensities are characterised by heavy-tailed distributions, i.e., extreme results can have a non-negligible probability to occur. To capture these effects accurately, large sample sizes are necessary. Under idealised assumptions, approximate analytical solutions for the resulting cosmic-ray intensity distributions can be derived (see, e.g., Sec. 7.4.2), but with more complicated cosmic-ray source or transport models, numerical methods are required.

As we discussed in Chap. 7, Monte Carlo methods can be employed to capture the stochastic nature of cosmic-ray source positions and ages. We use Monte Carlo simulations to generate millions of samples of discrete source realisations with the resulting cosmic-ray spectra at Earth. Because millions of sources need to be simulated for each sample and large sample sizes of $\mathcal{O}(10^6)$ are required to accurately capture the heavy-tailed distributions (see Sec. 7.4.2), running such simulations can be computationally expensive. However, the calculations of the contributions of individual sources are independent of each other, making the problem straightforward to parallelise on dedicated modern high-performance computing architectures like Graphics Processing Units (GPUs).

In Sec. 8.1.5, we describe our efficient implementation of the Monte Carlo simulations, which allows us to explore different cosmic-ray source models, source distributions, and parameter choices in comparison to each other. This includes a test of our previously introduced Cosmic-Ray Energy-Dependent Injection Time (CREDIT) model (Stall et al., 2025), which is based on a physically motivated modelling of the injection of cosmic rays from individual sources¹. In the light of the unprecedented precision of current and future cosmic-ray experiments (see Sec. 3.3), it is imperative to evaluate whether the discreteness

¹For a discussion of the theoretical motivation for a time-dependent escape of cosmic rays from their sources and previously developed models of similar scenarios, see Sec. 5.8.

of cosmic-ray sources can lead to observable signatures in the cosmic-ray spectrum at Earth. We aim to identify such signatures and assess their potential to discriminate between different source models. For this, we also explore the application of machine learning methods to classify the resulting cosmic-ray spectra (see discussion in Sec. 8.5). The results of these simulations and classifications are presented in the following Chap. 9. Parts of this study have been discussed in our previous publication Stall et al., 2025.

We will start by describing the model setup used for the Monte Carlo simulations in Sec. 8.1. This includes a discussion of the Galactic propagation model parameters, the source distributions considered, and the setup of the CREDIT model. This is followed by an in-depth analysis of the emergence of characteristic peaks in the cosmic-ray spectra resulting from the CREDIT model in Sec. 8.2. The effects of the collective contribution from many discrete sources are discussed in Sec. 8.3. Considering the experimental sensitivities of current and future cosmic-ray detectors (Sec. 8.4), different source models might be discriminated based on the presence or absence of these spectral features. We explore this possibility in Sec. 8.5.

8.1 Model setup

For the modelling of discrete cosmic-ray sources, we implement the theoretical framework introduced in Chap. 7. This includes the use of the Green's function method to solve the cosmic-ray transport equation for protons (Eq. (5.69)). We consider standard diffusion and source injection models (see Chap. 5) and introduce the CREDIT model to account for a time-dependent escape of cosmic rays from their sources. Effects of solar modulation (see App. B) do not influence our results significantly, as we focus on cosmic-ray protons at rigidities above 10 GV, and solar modulation effects are only relevant at lower rigidities.

8.1.1 Source injection models

We consider two different source injection models for the Monte Carlo simulations. The first is the standard burst-like injection model, where cosmic rays of all rigidities are injected by a source instantaneously at the same time. This corresponds to the assumption that the acceleration and release of cosmic rays from the source happens on timescales much shorter than the propagation time to Earth. This injection model has been widely used in previous studies of discrete cosmic-ray sources (e.g., Genolini et al., 2017; Mertsch, 2011; Mertsch, 2018; Phan et al., 2021).

However, it is not clear that this assumption is justified. Source catalogues do not allow us to exclude the presence of nearby and young sources in our Galactic neighbourhood reliably (see discussion in Mertsch, 2018). Thus, the burst-like injection model might not

be a good approximation for all relevant sources. As we have discussed in Sec. 5.8, the escape of cosmic rays from their sources likely happens over an extended period of time that corresponds to the full duration of the Sedov-Taylor (ST) phase of the SNR evolution. As estimated in Eqs. (5.116) and (5.130), this phase lasts for $\mathcal{O}(100 \text{ kyr})$. It can be assumed that a time-dependent escape of cosmic rays from their sources over such a timescale can lead to observable signatures in the cosmic-ray spectrum at Earth if the age of the source is comparable to or smaller than this timescale.

Time-dependent injection models have been considered in previous studies (e.g., Blasi and Amato, 2012b; Celli et al., 2019; Gabici et al., 2009; Thoudam and Hörandel, 2012), but they usually assume that the escape time follows a power law in rigidity down to the lowest rigidities in the GV range. However, as we have discussed in Sec. 5.8 and argued in our previous work Stall et al., 2025, a time-dependence only from the highest rigidities down to a few TV is more physically motivated in concordance with the amplification of magnetic fields in the source environment due to the Non-Resonant Hybrid (NRH) instability (see Secs. 5.6 to 5.8).

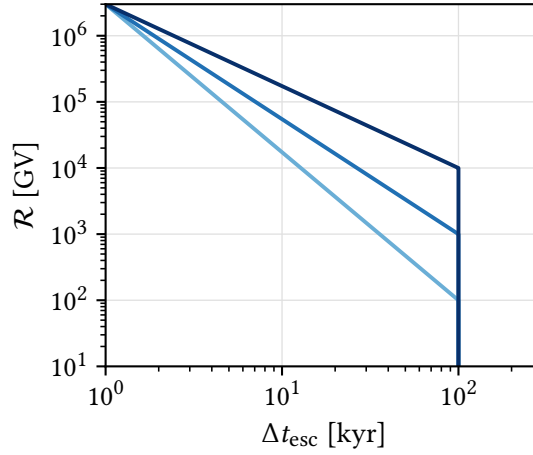


Figure 8.1: CREDIT escape rigidity as a function of time. This plot shows after which escape time delay, Δt_{esc} , cosmic rays of rigidity $\mathcal{R}$ are injected according to the CREDIT model (see Eq. (8.1) and Stall et al., 2025). In different shades of blue, three choices for the parameter $\mathcal{R}_b$ are illustrated (0.1 TV, 1 TV, and 10 TV, from light to dark blue).

In our fiducial CREDIT model, we assume that Galactic cosmic rays are injected by SNRs following a time-dependent escape model where cosmic rays of the highest rigidities escape first. We assume that the injection starts at the beginning of the ST phase, where a typical Sedov time is given by $t_{\text{ST}} = 1 \text{ kyr}$ (compare with Eq. (5.116)). Further, we assume that the escape of cosmic rays continues until the end of the ST phase at a typical time of $t_{\text{life}} = 100 \text{ kyr}$ (compare with Eq. (5.130)). The escape time delay of a cosmic ray with rigidity $\mathcal{R}$ is described by Δt_{esc} , which we assume to follow the power law that connects

the injection of cosmic rays at the maximum rigidity $\mathcal{R}_{\max} = \mathcal{R}_{\text{knee}} = 3 \text{ PV}$ at t_{ST} to the injection of cosmic rays at a rigidity $\mathcal{R}_b = \mathcal{O}(10 \text{ TV})$ at t_{life} :

$$\Delta t_{\text{esc}}(\mathcal{R}) = t_{\text{ST}} \left(\frac{\mathcal{R}}{\mathcal{R}_{\text{knee}}} \right)^{-a}, \quad a = \frac{\ln(t_{\text{life}}/t_{\text{ST}})}{\ln(\mathcal{R}_{\text{knee}}/\mathcal{R}_b)}, \quad (8.1)$$

for $\mathcal{R} > \mathcal{R}_b$ and $\Delta t_{\text{esc}}(\mathcal{R}) = t_{\text{life}}$ otherwise (Stall et al., 2025). The functional form of the escape time delay is illustrated in Fig. 8.1. The motivation for choosing $\mathcal{R}_b$ in the TV range is discussed in Sec. 5.8 and can be motivated by simulations in Caprioli et al., 2009b.

8.1.2 Diffusion models

For the propagation of cosmic rays from their sources to Earth, we consider a standard diffusion model as described in Sec. 5.3.8. We assume an isotropic, spatially constant diffusion coefficient that follows a (broken) power law in rigidity. We base our Monte Carlo simulations on a diffusion coefficient that has been fitted to locally measured cosmic-ray data in Schwefer et al., 2023. It is given by:

$$\kappa(\mathcal{R}) = \kappa_0 \left(\frac{H}{6 \text{ kpc}} \right) \beta \left(\frac{\mathcal{R}}{\mathcal{R}_{12}} \right)^{\delta_1} \prod_{i=1}^4 \left(1 + \left(\frac{\mathcal{R}}{\mathcal{R}_{i(i+1)}} \right)^{1/s_{i(i+1)}} \right)^{s_{i(i+1)}(\delta_{i+1} - \delta_i)}, \quad (8.2)$$

where β is the ratio of the particle's speed to the speed of light. The spectral indices δ_i , break rigidities, $\mathcal{R}_{i(i+1)}$, and break softness parameters, $s_{i(i+1)}$, are given in Schwefer et al., 2023, Tab. 1. As the model developed in Schwefer et al., 2023 is called CRINGE, we will refer to this diffusion coefficient as the CRINGE diffusion coefficient in the following. Its rigidity dependence in comparison to another, simpler *Default* diffusion coefficient is shown as the red line in Fig. 8.2.

As we have discussed in Sec. 4.5, there is a degeneracy between the diffusion coefficient normalisation, κ_0 , and the halo height, H , when we want to determine the diffusion coefficient from the measurement of secondary-to-primary ratios like B/C. The fit in Schwefer et al., 2023 assumes a halo height of $H = 6 \text{ kpc}$. To account for the possibility of different halo heights, we explicitly include this dependence in Eq. (8.2).

The fit in Schwefer et al., 2023 makes use of a degeneracy between the diffusion coefficient and the source spectrum for primary cosmic rays like protons to accommodate for both transport and injection effects in the parametrisation of the diffusion coefficient. We discussed this degeneracy in Sec. 4.5 Eq. (4.15).

The fit in Schwefer et al., 2023 does not assume a specific origin of the spectral breaks, but a physical interpretation of the breaks is possible: The first break at some GV does not influence our simulations of cosmic-ray protons above 10 GV. It has been suggested

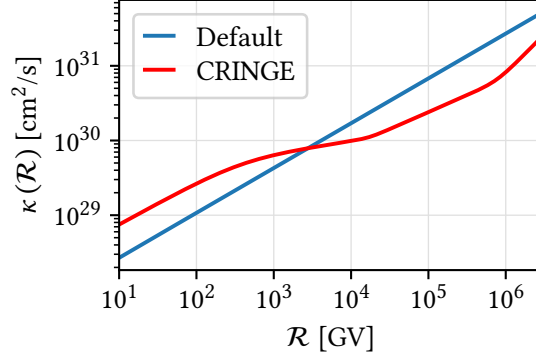


Figure 8.2: Diffusion coefficients. The diffusion coefficient from the fit in Schwefer et al., 2023 (CRINGE, red) and a simplified single power-law diffusion coefficient (Default, blue) are shown as a function of rigidity. The reference halo height of $H = 6$ kpc, as in Schwefer et al., 2023, is assumed here.

that it originates from stochastic source effects of low-energy cosmic rays (Phan et al., 2021) or from self-confinement of cosmic rays close to their sources (Jacobs et al., 2022). It is likely rather a source than a transport effect. The second break at $\mathcal{R} \simeq 300$ GV is likely related to a transition in the turbulence power spectrum and thus to transport effects (see discussion in Sec. 5.3.8). The third break at $\mathcal{R} \simeq 10$ TV remains unexplained², while the final break at $\mathcal{R} = \mathcal{O}(1$ PV) is usually attributed to the maximum rigidity of Galactic cosmic-ray sources, i.e., the cosmic-ray knee feature, or the transition to small-angle scattering (see, e.g., Plotnikov et al., 2011).

For comparison, we also consider a simplified diffusion coefficient that follows a single power law in rigidity without any breaks:

$$\kappa(\mathcal{R}) = \kappa_0 \left(\frac{H}{6 \text{ kpc}} \right) \beta \left(\frac{\mathcal{R}}{\mathcal{R}_0} \right)^\delta, \quad (8.3)$$

with $\kappa_0 = 6.8 \cdot 10^{27} \text{ cm}^2 \text{ s}^{-1}$, $\mathcal{R}_0 = 1$ GV, and $\delta = 0.6$. The values are chosen such that the diffusion coefficient matches the one from the fit in Schwefer et al., 2023 roughly throughout the rigidity range relevant for our simulations. We illustrate both diffusion coefficients in Fig. 8.2.

²Although this seems statistically unlikely (Genolini et al., 2017), if, e.g., the third break at around 10 TV was caused by a strong influence from a local source (Fornieri et al., 2021; Malkov and Moskalenko, 2021), the feature would need to be removed from the parametrisation of the diffusion coefficient as single-source effects are not degenerate with transport effects.

8.1.3 Green's function implementation

To calculate the total cosmic-ray proton intensity, solutions of the cosmic-ray transport equation for discrete sources, i.e., Green's functions, need to be added. We derived the Green's function for the cosmic-ray transport equation for protons in Sec. 7.3. In our Monte Carlo simulations of the local cosmic-ray proton spectrum, we want to calculate the contribution of millions of discrete sources at our position in the Galaxy, which we assume to be located at a Galactocentric radius of $R_\odot = 8.3$ kpc (as in Schwefer et al., 2023). Further, we assume that we and all cosmic-ray sources are located in a thin disk³, such that the vertical position of the sources and us is $z \approx z_s \approx 0$ pc. In this study, we only consider a time-independent diffusion coefficient, but we discuss models of this in Part III in the context of predictions for GDEs.

We introduce the notations $d \equiv |\mathbf{x} - \mathbf{x}_s|$ for the distance between the source and the observer and $\tau \equiv t - t_{\text{esc}}$ for the time since the cosmic rays have been injected at the source, where $t_{\text{esc}} = t_s + \Delta t_{\text{esc}}(\mathcal{R})$ (see Eq. (7.18)). For burst-like injection, $\Delta t_{\text{esc}}(\mathcal{R}) = 0$, while for the CREDIT model, it is given by Eq. (8.1). The free Green's function from Eqs. (7.27) and (7.29) can then be written as:

$$G_{\text{free}}(\mathcal{R}, d, \tau) = \frac{1}{(2\pi\sigma^2(\mathcal{R}, \tau))^{3/2}} \exp\left(-\frac{d^2}{2\sigma^2(\mathcal{R}, \tau)}\right), \quad \sigma^2 = 2\kappa\tau. \quad (8.4)$$

To account for the vertical free escape boundary conditions at $z = \pm H$, we use the final Green's function from Eq. (7.30), which we can rewrite as:

$$G(\mathcal{R}, d, \tau) = \sum_{n=-\infty}^{\infty} (-1)^n G_{\text{free}}(\mathcal{R}, d_n, \tau), \quad (8.5)$$

for positive τ , where the distances of the image sources are given by:

$$d_n^2 = (x_s - x)^2 + (y_s - y)^2 + (2Hn)^2 = d_0^2 + (2Hn)^2, \quad (8.6)$$

for sources located in the Galactic mid-plane at $z_s = 0$ and the observer at $z = 0$. This simplifies Green's function given in Eq. (8.5), and using Eq. (8.4), we obtain:

$$G(\mathcal{R}, d, \tau) = G_{\text{free}}(\mathcal{R}, d, \tau) \underbrace{\sum_{n=-\infty}^{\infty} (-1)^n \exp\left(-\frac{(Hn)^2}{\kappa\tau}\right)}_{\equiv \vartheta\left(\ell \equiv \frac{\kappa\tau}{H^2}\right)}. \quad (8.7)$$

³As we will discuss below and in Sec. 8.2, only the absolute distance between the source and the observer matters for young and nearby sources, which have the largest contributions to the local cosmic-ray intensity. While this assumption does not alter the distances of sources, which are commonly assumed to lie in the Galactic mid-plane close to $z = 0$ (see Sec. 7.2.2), it simplifies the computations of the boundary conditions (see below and Sec. 10.1.1 in comparison).

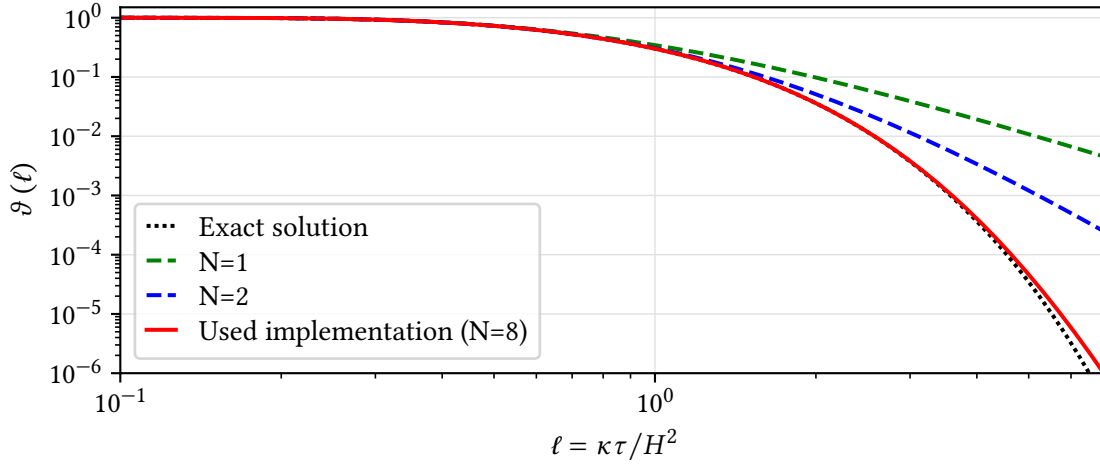


Figure 8.3: Implementation of the suppression factor for the boundary conditions. The suppression factor, $\vartheta(\ell)$, from Eq. (8.7) is shown as a function of the parameter $\ell = \kappa\tau/H^2$. The dotted black line shows the exact result, while the coloured lines show the product representation from Eq. (8.8) for different numbers of terms, N , taken into account in the product.

The infinite sum term, ϑ , in Eq. (8.7) does only depend on the parameters in the combination $\ell \equiv \kappa\tau/H^2$, which determines the ratio between the diffusion length (see Eq. (7.2)) and the halo height, H . The function ϑ can be identified as a Jacobi theta function of the fourth kind, which can also be expressed as a product (Whittaker and Watson, 1996):

$$\vartheta(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n e^{-\frac{n^2}{\ell}} = K(\ell) \cdot \prod_{n=1}^{\infty} \left(1 - e^{-\frac{(2n-1)}{\ell}}\right)^2, \quad (8.8)$$

where $K(\ell) = \prod_{k=1}^{\infty} (1 - e^{-2\frac{k}{\ell}})$.

This product representation converges very fast. The range of ϑ is $[0, 1]$. As it determines the influence of the vertical escape boundaries on the Green's function, it becomes very small if the diffusion length exceeds the halo height, i.e., for $\ell > 1$. Computationally, the suppression factor, ϑ , which can span many orders of magnitude, can be evaluated more accurately and reliably using the product representation in Eq. (8.8) rather than the sum representation in Eq. (8.5). There, rounding errors can occur, potentially leading to negative values of ϑ for large ℓ . We illustrate the functional behaviour of $\vartheta(\ell)$ and the quick convergence of the product representation in Fig. 8.3. The variable N denotes the number of terms taken into account in the product representation in Eq. (8.8), for both products alike. In our Monte Carlo simulations, we use $N = 8$, which guarantees accurate results up to suppressions of less than 10^{-3} on a percent level. Sources that are suppressed by such a low factor have a vanishingly small influence on the total cosmic-ray intensity. Thus, small numerical errors in the calculation of the suppression factor, ϑ , beyond this suppression level do not affect the overall results.

In agreement with our discussion in Sec. 7.3.4, we implement a causality condition in our Green's function evaluation. This means that we set the Green's function to zero if the injection of cosmic rays at the source lie outside the observer's past light cone, i.e., if $c\tau < d$.

8.1.4 Source distribution, rate, and spectrum

For the SN rate, we assume a fiducial value of $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$ (see Sec. 7.2 for a discussion of possible values). For the spatial distribution of cosmic-ray sources in the Galaxy, we consider two different models. For a general overview of possible source distributions, see Sec. 7.2. Our fiducial model is an axisymmetric distribution based Ferrière, 2001 as described in Eqs. (7.10) and (7.11). The radial distribution is illustrated in Fig. 7.2 (blue line). We rescale it to our assumed Galactocentric radius of $R_{\odot} = 8.3 \text{ kpc}$ (Schwefer et al., 2023).

As a second source distribution, we consider a spiral model that is inspired by the model used by Evoli et al., 2021a,b, but with the same radial source density profile as our fiducial axisymmetric model. It is based on the four-arm spiral model from Steiman-Cameron et al., 2010 with the relative emissivities of the spiral arms as inferred from the C II observations as described in Sec. 7.2.3. We also add a Galactic bar extending to about 2.9 kpc with an angle of 20° with respect to the line connecting the Galactic centre and the Sun's position (Churchwell et al., 2009), and a scale width of 1 kpc (Evoli et al., 2021a,b). The spiral arms start at the end of the bar.

We assume two different scalings for the spiral model which we illustrate in Fig. 8.4. First, we assume that the centre of the closest spiral arm towards the Galactic centre (Sagittarius-Carina) is located at a distance of 1.5 kpc from the Sun's position, as in Evoli et al., 2021b. This places the Sun in an inter-arm region. Second, we assume that the closest spiral arm towards the Galactic centre is located as given in Steiman-Cameron et al., 2010, rescaled to our assumed Galactocentric radius of $R_{\odot} = 8.3 \text{ kpc}$. This places the Sun closer to the centre of the Sagittarius-Carina arm at a distance of about 0.95 kpc.

For the source spectrum, $Q_{\mathcal{R}}(\mathcal{R})$ (see, e.g., Eq. (7.17)), we assume a power law in rigidity with a spectral index, α :

$$Q_{\mathcal{R}}(\mathcal{R}) = Q_0 \left(\frac{\mathcal{R}}{\mathcal{R}_0} \right)^{-\alpha}, \quad (8.9)$$

where Q_0 is the normalisation at a reference rigidity $\mathcal{R}_0 = 1 \text{ GV}$.

The spectral index, α , is chosen such that the resulting mean cosmic-ray proton spectrum at Earth matches the locally observed spectrum after propagation. For our default diffusion model, we choose $\alpha = 2.2$, while for the diffusion coefficient in Eq. (8.2), we choose $\alpha = 2.383$, as determined in Schwefer et al., 2023. The deviation from the canonical $\mathcal{R}^{-2}$ spectrum expected from DSA can have multiple reasons like the relative speed of the

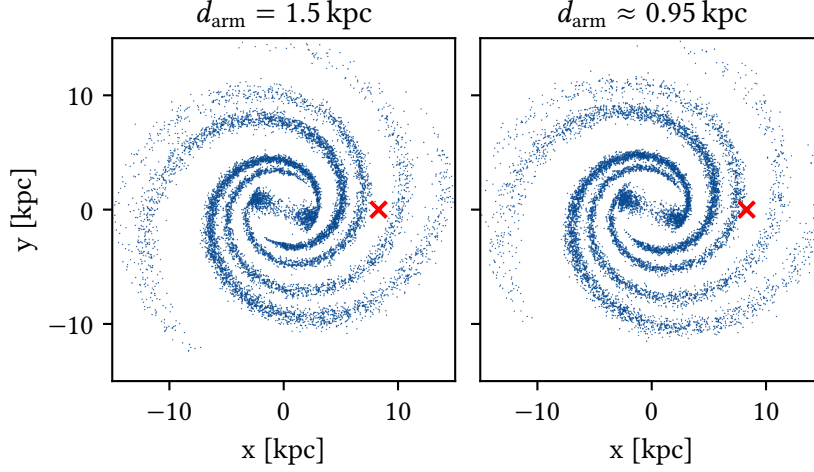


Figure 8.4: Spiral models. We show spiral arm models based on Churchwell et al., 2009; Evoli et al., 2021a,b; Steiman-Cameron et al., 2010 by plotting 10^4 randomly sampled source positions in the Galactic plane. The Sun’s position is indicated by the red cross. On the left, we rescale the spiral arm positions such that the distance to the closest spiral arm towards the Galactic centre is set to 1.5 kpc, while on the right, it is set according to Steiman-Cameron et al., 2010 to about 0.95 kpc.

scattering centres (Bell, 1978a; Caprioli et al., 2020) or shock obliquity (Bell et al., 2011), as discussed in Sec. 5.5.

A reasonable estimate for Q_0 can be determined by demanding that the total energy injected in cosmic-ray protons per SN is a certain fraction, η_{CR} , of the total kinetic energy released by the SN explosion, E_{SN} :

$$\int_{E_{\text{kin}, \text{min}}}^{\infty} dE_{\text{kin}} E_{\text{kin}} Q_{\mathcal{R}}(E_{\text{kin}}) = \eta_{\text{CR}} E_{\text{SN}}, \quad (8.10)$$

where we perform the integration in kinetic energy, E_{kin} (for the conversions, see App. A). In accordance with Phan et al., 2021, we choose a minimum kinetic energy of $E_{\text{kin}, \text{min}} = 100$ MeV, a total SN energy of $E_{\text{SN}} = 10^{51}$ erg, and an acceleration efficiency of $\eta_{\text{CR}} = 8.7\%$.

For our default case with $\alpha = 2.2$, we find $Q_0 = 6.85 \cdot 10^{51} \text{ GV}^{-1}$. For the case based on the CRINGE model, we find $Q_0 = 2.828 \cdot 10^{52} \text{ GV}^{-1}$, where we attributed the first break of the cosmic-ray spectrum to the source spectrum (see also Stall and Mertsch, 2026). Otherwise, the source spectrum normalisation can also be fitted such that the resulting spectra match the locally observed cosmic-ray proton spectrum after propagation, but we do not follow this approach here.

8.1.5 Implementation

We implement the Green’s function formalism described above in a computational framework that allows us to efficiently calculate the cosmic-ray proton spectrum at Earth resulting from millions of discrete sources. For each Monte Carlo realisation, we randomly sample source positions and times according to the chosen source distribution and rate using inverse transform sampling (see Sec. 7.2.4). We limit the maximum source age to 500 Myr. At a rigidity of 10 GV, this is already a multiple of the typical cosmic-ray proton escape time from the Galaxy (see, e.g., Fig. 7.1). For higher rigidities, the escape time is even shorter, such that this maximum source age ensures that all relevant sources contributing to the local cosmic-ray intensity are taken into account.

Then, we calculate the contribution of an array of sources to the cosmic-ray intensity at Earth for a range of rigidities using the Green’s function from Eq. (8.7). Our rigidity grid is inspired by the binning of the AMS-02 (Aguilar et al., 2021) and DAMPE (An et al., 2019) experiments to facilitate a comparison of our results to the measured cosmic-ray proton spectrum at Earth. To be precise, we use the following rigidity grid:

$$\mathcal{R}_{\text{AMS}} = \{1.10655^n \cdot 11.06 \text{ GV} \mid n = 0, 1, \dots, 49\} \quad (8.11)$$

$$\mathcal{R}_{\text{DAMPE}} = \{1.583^n \cdot 1983 \text{ GV} \mid n = 0, 1, \dots, 8\} . \quad (8.12)$$

This results in a total of 58 rigidity bins from 10 GV to 78 TV, with a transition between the two grids at about 2 TV.

After calculating the contributions from all sources to the cosmic-ray intensity at Earth for the given rigidity grid, we sum them up to obtain the total cosmic-ray proton spectrum. Computation efficiency can be significantly improved by using array programming with the NUMPY library (Harris et al., 2020). The computations can be further sped up by parallelising the calculations using GPUs with the JAX library (Bradbury et al., 2018). Compared to an implementation of the Monte Carlo simulations on a single CPU using array programming with NUMPY, we achieve speed-ups of about a factor of 500 when using a single GPU, bringing down the runtime for a single realisation to $\mathcal{O}(1 \text{ s})$. Even for parallelisation on 24 CPUs, we still achieve speed-ups of about an order of magnitude compared to a single CPU implementation. We used the resources of the RWTH HPC cluster CLAIX, specifically a H100 NVIDIA GPU and Intel Xeon 8468 Sapphire CPUs.

8.2 CREDIT peaks

In this section, we discuss the emergence of characteristic peaks in the cosmic-ray proton spectrum at Earth resulting from the CREDIT model for time-dependent escape from discrete sources. For this, we consider the contributions from individual sources separately. The contribution from a single source depends on its distance and the time since the

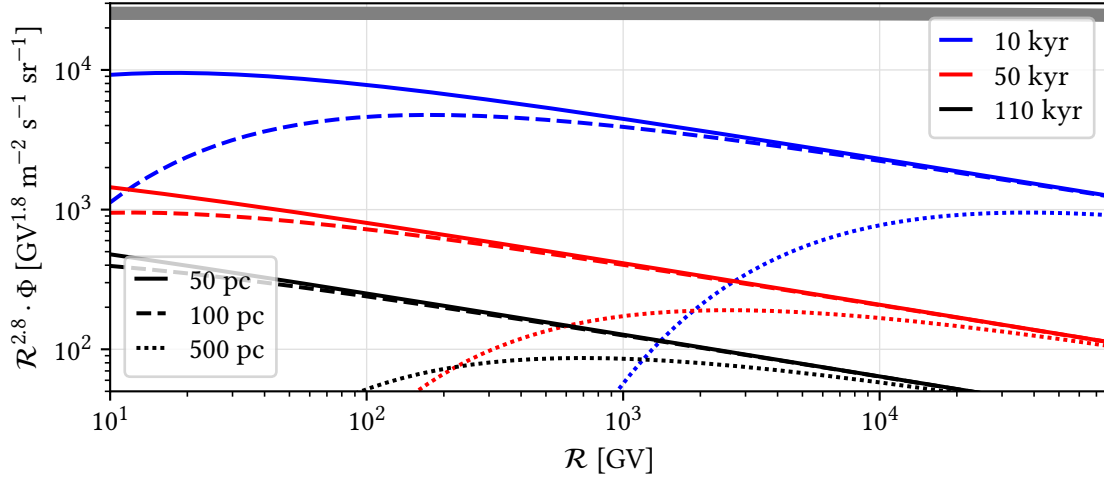


Figure 8.5: Single source spectra in the burst-like injection model. We show the contribution from single sources at different distances and ages to the local cosmic-ray proton intensity at Earth for burst-like injection. The mean total cosmic-ray proton spectrum is shown as a thick grey line for comparison. The spectra are multiplied by $\mathcal{R}^{2.8}$ to let the total spectrum appear as a flat line and enhance visibility.

cosmic rays have been injected. Young and nearby sources have a stronger influence on the local cosmic-ray intensity than old and distant ones. As we are interested in the effects of the most contributing sources, we can neglect the influence of the boundary conditions for the vertical escape of cosmic rays from the Galaxy for now.

Thus, the free Green's function, Eq. (8.4), is multiplied by the source spectrum, Eq. (8.9), and the conversion factor to an intensity (see App. A), describes the contribution from a single source to the local cosmic-ray intensity at Earth, which we denote by Φ_s :

$$\Phi_s(\mathcal{R}, d, \tau) = \frac{c}{4\pi e} \frac{Q_{\mathcal{R}}}{(4\pi\kappa\tau)^{3/2}} \exp\left(-\frac{d^2}{4\kappa\tau}\right), \quad (8.13)$$

where the additional causality condition is implied. c is the speed of light and e is the elementary charge (see App. A).

In Fig. 8.5 and Fig. 8.6, we illustrate the contribution from single sources at different distances and ages for burst-like injection and for the CREDIT model, respectively. In both cases, we also show the total cosmic-ray proton spectrum, that we determined as a sample average from our full Monte Carlo simulation including all sources (see discussion in relation to Eq. (7.36)). It follows roughly a power law in rigidity with a spectral index of about $2.8 = \alpha + \delta$ (because of Eq. (4.15)).

There is a stark difference between the contributions in the two injection models. While the burst-like injection model leads to smooth moderate single source contributions, the CREDIT model leads to pronounced peaks that can clearly exceed the total cosmic-ray

intensity. We refer to those characteristic features as *CREDIT peaks* and investigate them further in the following.

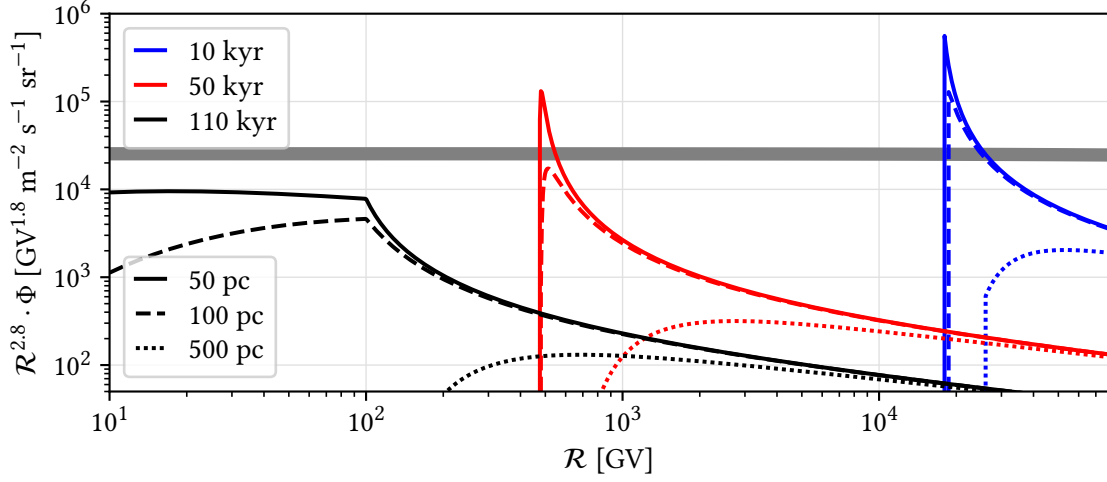


Figure 8.6: Single source spectra in the CREDIT injection model. We show the same configuration as in Fig. 8.5, but for the CREDIT injection model. The parameter $\mathcal{R}_b$ is set to 100 GV (for illustrative purposes) here.

8.2.1 Description of peaks

To understand the spectral shape of the single source contributions, we can determine the parameter dependence of the spectral index for those. The spectral index is the slope of a spectrum in a double-logarithmic plot. From our functional form in Eq. (8.13) and noting that $\kappa(\mathcal{R}) \propto \mathcal{R}^\delta$ and $Q_{\mathcal{R}}(\mathcal{R}) \propto \mathcal{R}^{-\alpha}$, we find that:

$$\frac{d \ln \Phi_s}{d \ln \mathcal{R}} = \frac{\partial \ln \Phi_s}{\partial \ln \mathcal{R}} + \frac{\partial \ln \Phi_s}{\partial \ln(\kappa \tau)} \frac{d \ln(\kappa \tau)}{d \ln \mathcal{R}} \quad (8.14)$$

$$= -\alpha + \left(-\frac{3}{2} + \frac{d^2}{4\kappa\tau} \right) \frac{d \ln(\kappa\tau)}{d \ln \mathcal{R}}. \quad (8.15)$$

In the case of burst-like injection, the term $\kappa\tau$ only depends on rigidity via the diffusion coefficient, and we find:

$$\frac{d \ln \Phi_s}{d \ln \mathcal{R}} = -\alpha + \left(-\frac{3}{2} + \frac{d^2}{4\kappa\tau} \right) \delta \xrightarrow{\tau \rightarrow \infty} -\alpha - \frac{3}{2}\delta. \quad (8.16)$$

It can be seen that the resulting propagated single-source spectrum has approximately a power-law dependence in rigidity, $\Phi_s \propto \mathcal{R}^{-\gamma}$ with $\gamma \equiv \alpha + \frac{3}{2}\delta$. Note, that this approximate

power law deviates from the power law of the total spectrum from all sources with index $\alpha + \delta$, which comes from the combined effect of the contribution of all sources of different ages and the boundary condition, i.e., the escape from the Galaxy.

Deviations from a power law with index γ are determined by the term

$$\mathcal{A} \equiv \frac{d^2}{4\kappa\tau}, \quad (8.17)$$

which is the squared ratio of the distance to the source and the diffusion length (see Eq. (7.2)). It can be attributed to the suppression of the intensity for $\mathcal{A} \gg 1$, i.e., when the diffusion length is still smaller than the distance to the source. This can be seen in Fig. 8.5, for low rigidities and young ages. However, this is the only change of the spectral index compared to the asymptotic value of γ .

Burst-like sources might be observable in the spectrum as they naturally have a softer spectrum than the total CR spectrum and might stick out as a *bump* if their individual contribution to the spectrum is high enough. However, as can be seen in Fig. 8.5, the contribution from single sources is only sizeable for rather extreme parameter choices. We will discuss their emergence and detectability later in Chap. 9.

In the CREDIT model, there is an additional rigidity-dependence in the injection time for $\mathcal{R} > \mathcal{R}_b$ (see Eq. (8.1)), which leads to a different behaviour of the spectral index. We find:

$$\frac{d \ln(\kappa\tau)}{d \ln \mathcal{R}} = \delta + \frac{a}{\tau(\mathcal{R})} t_{\text{ST}} \left(\frac{\mathcal{R}}{\mathcal{R}_{\text{knee}}} \right)^{-a} = \delta + a \frac{\Delta t_{\text{esc}}(\mathcal{R})}{\tau(\mathcal{R})} > \delta \quad \text{as } a > 0. \quad (8.18)$$

We denote the additional term by

$$\mathcal{B} \equiv a \frac{\Delta t_{\text{esc}}(\mathcal{R})}{\tau(\mathcal{R})}. \quad (8.19)$$

Together with a strict low-rigidity cut-off for cosmic rays that have not been injected yet, this additional term, $\mathcal{B}$, explains the characteristic CREDIT spectra that emerge in Fig. 8.6. Especially the CREDIT peaks potentially lead to pronounced detectable features in the local cosmic-ray proton spectrum as they can clearly exceed the total cosmic-ray intensity.

For $\mathcal{B} \ll 1$, i.e., if the escape time delay is negligible compared to the age of the source, the behaviour is similar to the burst-like injection case. However, for $\mathcal{B} \gtrsim 1$, the spectral index can increase significantly. This corresponds to the steep fall-off of the spectrum after the peaks that can be seen in Fig. 8.6.

The interplay of the two terms, $\mathcal{A}$ and $\mathcal{B}$, determines the exact spectral shape around the peak. If $\mathcal{A}$ is also large, the peak is suppressed, as can be seen for the more distant sources in Fig. 8.6. For small $\mathcal{A}$ and large $\mathcal{B}$, however, a pronounced peak emerges. This is the

case for close and young sources. An increase of the diffusion coefficient normalisation would lead to a smaller $\mathcal{A}$ at a given rigidity and age, which would reduce the suppression and thus enhance the peak.

The parameter $\mathcal{B}$ can be influenced by the choice of the parameters in the CREDIT model. Longer source lifetimes, t_{life} , can lead to larger escape time delays, Δt_{esc} , at a given rigidity, which increases $\mathcal{B}$. A longer time difference between the injection of adjacent rigidities leads to larger spectral features. A reasonable change of the parameter t_{ST} would slightly change the parameters a and Δt_{esc} accordingly, but would not lead to significant changes of the spectra.

8.2.2 Parameter influence on the total mean spectrum

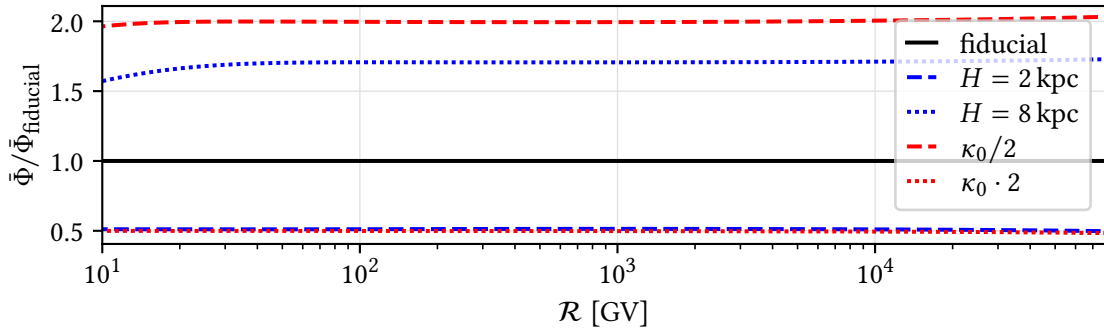


Figure 8.7: Comparison of total cosmic-ray proton spectra. We show the mean total cosmic-ray proton spectra, $\bar{\Phi}$, resulting from our Monte Carlo simulations for different values of either the halo height, H , or the diffusion coefficient normalisation, κ_0 , relative to our fiducial case with $H = 4$ kpc and the diffusion coefficient as in Eq. (8.3).

In this section, we want to understand the influence of different parameters on the emergence of the CREDIT peaks in more detail. Ultimately, we are interested in the detectability of those peaks in the local cosmic-ray proton spectrum, i.e., how large a spectral feature is *relative to* the total cosmic-ray proton spectrum. For this, it is crucial to understand how different parameters influence both the single-source spectra and the total cosmic-ray proton spectrum.

Before we will discuss the influence of different parameters on the single-source spectra, we summarise the influence of source and transport parameters on the total spectrum. In Fig. 8.7, we vary solely either the halo height, H , or the diffusion coefficient normalisation, κ_0 , and show the resulting mean cosmic-ray proton spectra relative to our fiducial case with $H = 4$ kpc and the diffusion coefficient as in Eq. (8.3). It can be seen that the mean spectrum is strongly influenced by those parameters.

Doubling κ_0 has the same effect as halving H , which is to reduce the mean spectrum by a factor of two. Conversely, halving κ_0 leads to a doubling of the mean spectrum. The deviation of $H = 8$ kpc from a factor two can be attributed to the fact that a direct scaling is only expected for $R_{\text{Gal}} > H$. Sources that lie in a circle of radius $\sim H$ around the observer can contribute to the mean spectrum (see Sec. 7.1). As H gets larger, larger parts of this region lie in regions of low source density or even outside the Galaxy. Thus, the enhancement of the mean spectrum is suppressed.

Further, also other parameters like the source rate, f_{SN} , or the radius of the source distribution, R_{Gal} , influence the total spectrum in an intuitive way. An increase of the source rate or decrease of the source distribution radius leads to a higher local source density and thus a higher mean spectrum, and vice versa. The influence of other parameters like the diffusion spectral index, δ , is more complex, but can in principle also be understood from our discussion of the diffusion coefficient normalisation above. Variations of the source spectrum parameters, α and Q_0 , influence the total and single-source spectra in the same way and thus do not change the relative peak heights.

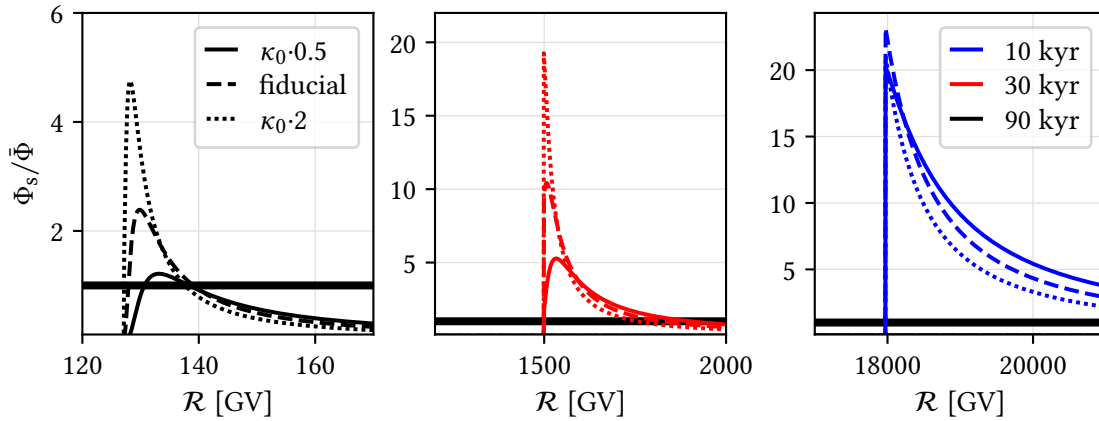


Figure 8.8: Single-source spectra with varying diffusion coefficient normalisation (with causality condition). We show single-source spectra, Φ_s , for a fixed source distance of 50 pc and a range of source ages (10 kyr, 30 kyr, and 90 kyr) for different values of the diffusion coefficient normalisation, κ_0 compared to the fiducial value given below Eq. (8.3). All spectra are normalised to their respective mean total cosmic-ray proton spectrum, $\bar{\Phi}$.

8.2.3 Parameter influence on the CREDIT peaks

Now, we discuss the influence of different parameters on the single-source spectra, Φ_s , relative to the total cosmic-ray proton spectrum, $\bar{\Phi}$. We show the influence of different parameters on the single-source spectra in Figs. 8.8 to 8.11. There, we show single-source spectra, Φ_s , for a fixed source distance of 50 pc and a range of source ages (10 kyr, 30 kyr,

and 90 kyr). All of those spectra are normalised to their respective mean cosmic-ray proton spectrum, such that the y-axis shows the relative contribution of the single source to the total intensity.

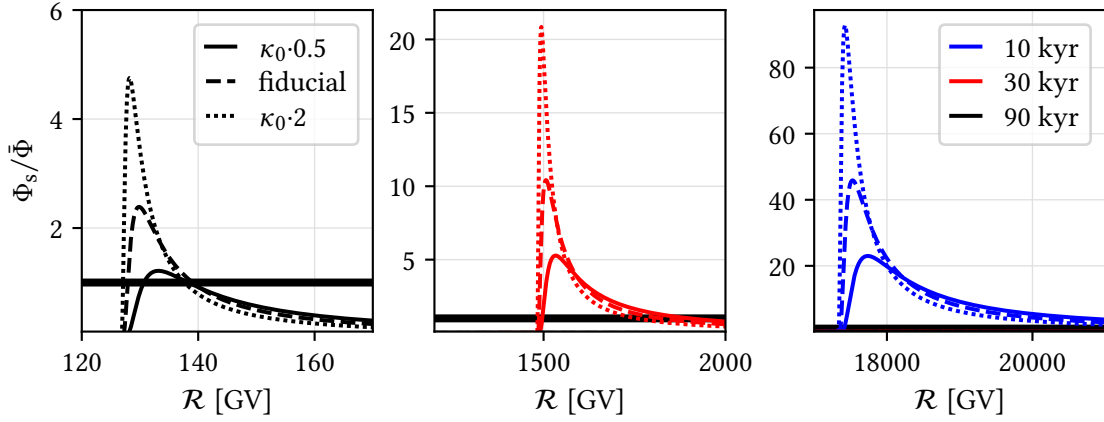


Figure 8.9: Single-source spectra with varying diffusion coefficient normalisation (without causality condition). Same as in Fig. 8.8, but without the causality condition implemented.

In Fig. 8.8, we vary solely the diffusion coefficient normalisation, κ_0 , while keeping all other parameters fixed to our fiducial values. It can be seen that increasing κ_0 leads to steeper and narrower peaks, while decreasing κ_0 suppresses the peaks. This is in line with our previous discussion of the parameter $\mathcal{A}$ in Eq. (8.17). Increasing κ_0 leads to a smaller $\mathcal{A}$ at a given rigidity and age, which reduces the suppression and thus enhances the peak.

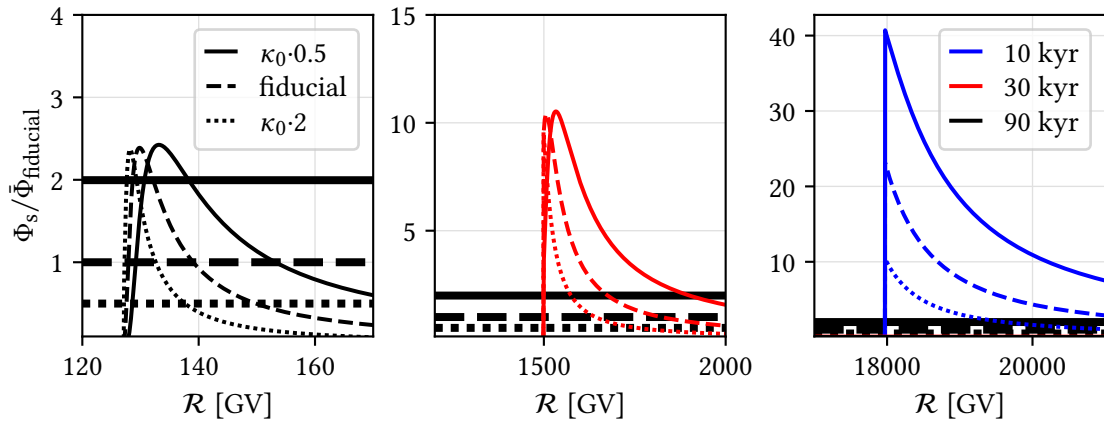


Figure 8.10: Single-source spectra with varying diffusion coefficient normalisation (normalised to fiducial mean). Same as in Fig. 8.8, but all spectra are normalised to the mean total cosmic-ray proton spectrum with the fiducial diffusion coefficient normalisation, κ_0 .

However, at high rigidities, the height of the peaks does not vary significantly with κ_0 (Fig. 8.8, right panel). This can be attributed to our implementation of the causality condition, which suppresses the arrival of recently injected cosmic-ray protons as their arrival would only be possible with superluminal speeds (see Sec. 7.3.4). To show that this is indeed the case, we show the same configuration as in Fig. 8.8, but without the causality condition in Fig. 8.9. There, we can see that the peak heights at high rigidities do depend on κ_0 as expected. This shows that the causality condition limits the peak heights at high rigidities.

The peak shapes in the previous Figs. 8.8 and 8.9 are influenced by the change of the diffusion coefficient normalisation, κ_0 . The peak heights, however, are mainly determined by their ratio to the total cosmic-ray proton spectrum. To illustrate this, we show the same single-source spectra as in Fig. 8.8, but all normalised to the mean with the fiducial κ_0 in Fig. 8.10. The peak heights are not influenced by the change of κ_0 , but their position and shape still are.

Finally, we show the influence of different halo heights, H , on the single-source spectra in Fig. 8.11. H has no influence on the single-source spectra, Φ_s , as it does not appear in the free Green's function from Eq. (8.4). Thus, the peak shapes are the same for different H , and only the height changes.

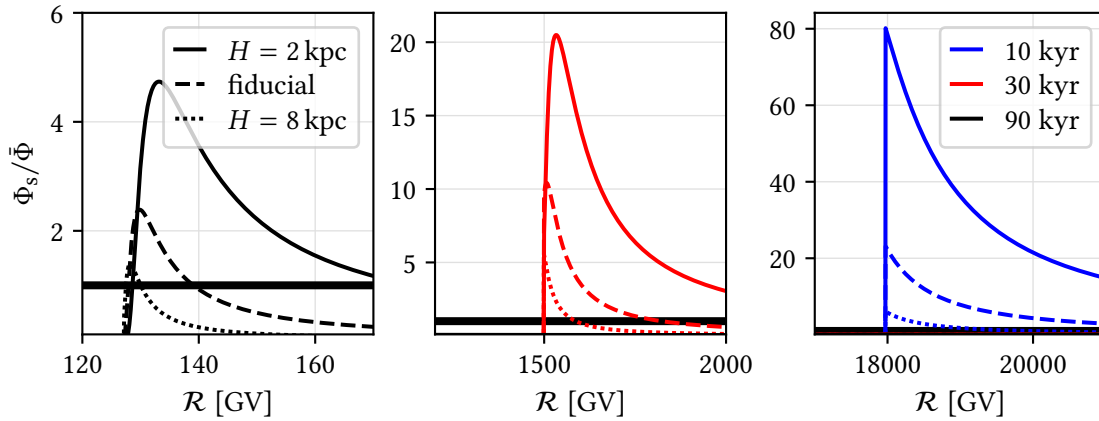


Figure 8.11: Single-source spectra with varying halo height. Same as in Fig. 8.8, but for different values of the halo height, H , compared to the fiducial value of 4 kpc.

8.3 Collective effects of discrete sources

In a realistic scenario, many sources contribute to the local cosmic-ray spectrum at Earth. Thus, a realisation of the local cosmic-ray proton spectrum reflects the collective effect of many sources with their respective spectral features. In Fig. 8.12, we show ten random

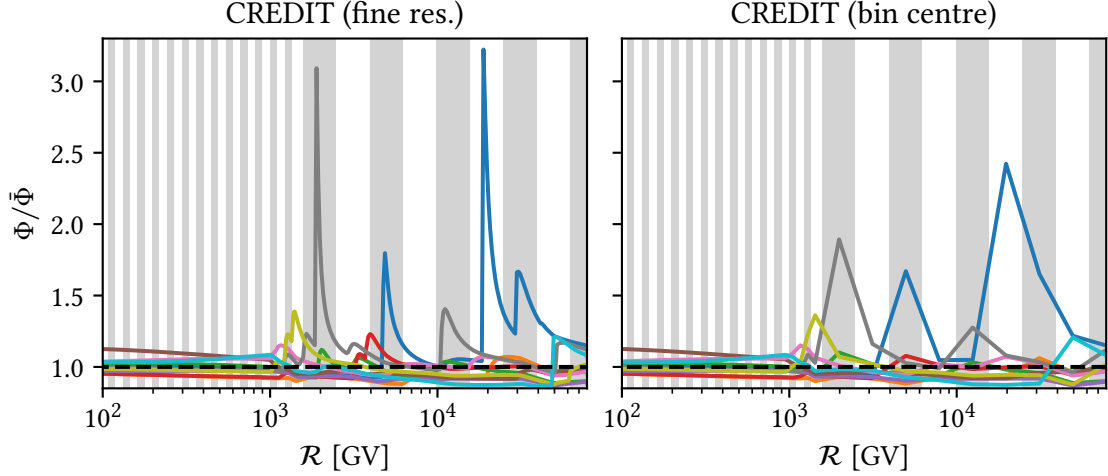


Figure 8.12: Example CREDIT spectra (fine resolution vs. bin centre). We show ten example realisations of local cosmic-ray proton spectra in the CREDIT model with $\mathcal{R}_b \in [1, 25]$ TV, normalised by the mean total spectrum, $\bar{\Phi}$. The alternating white and gray vertical lines indicate the rigidity bins with bin centres as described in Eqs. (8.11) and (8.12). The left panel shows the spectra with a very fine rigidity resolution, much higher than currently realised by experiments. The right panel shows the same realisations, but sampled at the centres of the rigidity bins.

realisations of local cosmic-ray proton spectra in the CREDIT model with parameters $\mathcal{R}_b$ in the range of 1 TV to 25 TV. The parameter $\mathcal{R}_b$ is chosen for each source individually from a distribution $p(\mathcal{R}_b) \propto 1/\mathcal{R}_b$ (Stall et al., 2025).

In the left panel of Fig. 8.12, we show the spectra with a very fine rigidity resolution, much higher than the spacing of the bins currently realised by experiments like AMS-02 and DAMPE. Variations of the spectra around the mean are present at all rigidities, but get more drastic above a rigidity of about 1 TV, when the escape time delays start to become relevant for some sources (see Eq. (8.1)). For $\mathcal{R} \lesssim 1$ TV, the spectral features are rather smooth and have a limited extent of only a few percent above or below the mean. However, for $\mathcal{R} \gtrsim 1$ TV, the spectra show the characteristic CREDIT peaks.

Experimental bins are typically wider than the spectral features shown in the left panel of Fig. 8.12. Hence, the contributions to a given experimental bin need to be determined by integrating the spectrum over the bin width. Running an extended Monte Carlo simulation with a very fine rigidity resolution is computationally expensive. Thus, we checked whether sampling the spectrum at the central rigidity of each experimental bin catches the same statistical properties as integrating over the bin width. We found that this is indeed the case, and such an approximation can be used. As fewer rigidity points need to be evaluated, this approximation helps speed up the computations a lot. We can use this to sample more realisations and increase the statistical power of our approach.

The right panel of Fig. 8.12 shows the same example spectra as in the left panel but sampled only at the central rigidities of bins employed by experimental collaborations. While the features are not as high and as nicely resolved as before, the most dramatic ones are still easy to make out by eye. Still, a finer rigidity resolution or potentially the provision of unbinned data by experimental collaborations would be beneficial to resolve the features in more detail (Stall et al., 2025).

While some of the realisations show noticeable spectral features, others are rather smooth. To determine whether features are detectable by current or future experiments, we have to quantify the sensitivity of experiments to those features, given our limited knowledge of the source coordinates realised in our own Galaxy. The CREDIT injection model leads to characteristic spectral features, but as our local cosmic-ray spectrum corresponds to only a single realisation of the underlying source distribution, it is not guaranteed that we can detect those features. To quantify the sensitivity of experiments to those features, we need to check detectability on large numbers of Monte Carlo realisations. A key ingredient to this are the experimental uncertainties, which we will discuss in the next section.

8.4 Sensitivity of experiments

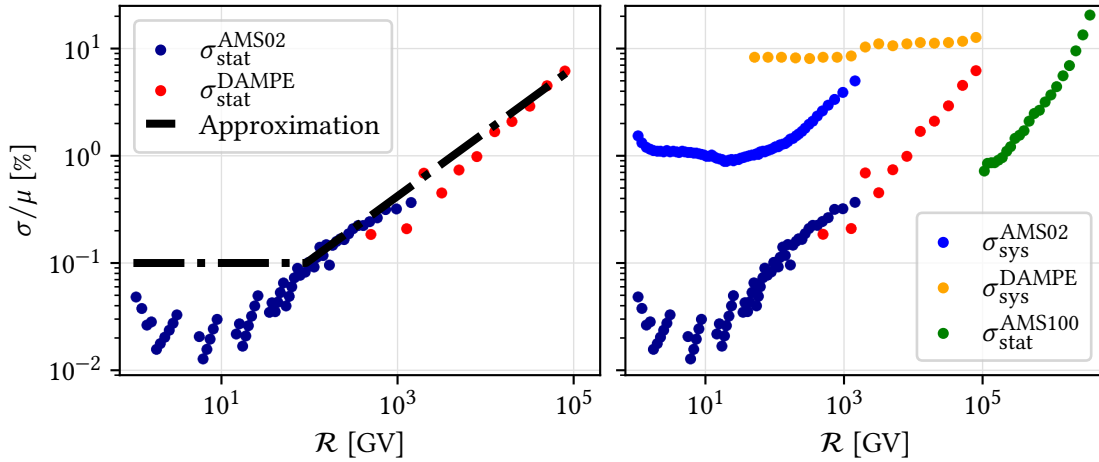


Figure 8.13: Experimental uncertainties for AMS-02, DAMPE, and the proposed AMS-100. We show the relative statistical and systematic uncertainties for the AMS-02 (Aguilar et al., 2021) and DAMPE (An et al., 2019) experiments as well as the projected uncertainties for the proposed AMS-100 experiment (Schael et al., 2019). In the left panel, we also show our approximation of the statistical uncertainties according to (8.20) as a dash-dotted line.

In Fig. 8.13, we show the relative statistical and systematic uncertainties for the AMS-02 (Aguilar et al., 2021) and DAMPE (An et al., 2019) experiments as well as the projected uncertainties for the proposed AMS-100 experiment (Schael et al., 2019). The systematic uncertainties of current experiments are at the level of a few percent and dominate the total uncertainty budget at most rigidities.

Statistical uncertainties are generally much smaller. They arise due to the finite number of registered events. If the number of events per bin is large enough, the distribution of the measured intensities will be Gaussian, and the measured intensities in different bins can be considered as independent. We find that the reported statistical errors of both current experiments AMS-02 and DAMPE can be approximated by the relation:

$$\sigma_{\text{stat}}[\%] = \begin{cases} 0.42 (\mathcal{R}[\text{TV}])^{0.6} & \text{for } \mathcal{R} > 0.1 \text{ TV} \\ 0.1 & \text{for } \mathcal{R} \leq 0.1 \text{ TV} \end{cases}, \quad (8.20)$$

which we also show in the left panel of Fig. 8.13 as a dash-dotted line.

Spectral features as the CREDIT peaks described in this chapter are characterised by changes in the deviation of the measured spectrum from a smooth reference spectrum from one rigidity bin to the next. Thus, parts of the total uncertainty that are correlated across rigidity bins, like, e.g., systematic energy scale uncertainties, will not influence the detectability of those features. Accordingly, we assume that only the uncorrelated part of the total uncertainty limits detectability. In the best case, this is the statistical uncertainty only. Then, only the statistical uncertainties, Eq. (8.20), are relevant to assess the detectability of features. However, systematic uncertainties can also have uncorrelated parts. Unfortunately, the detailed correlation structure of systematic uncertainties is not published by the experimental collaborations. According to Cavasonza et al., 2017, an additional uncorrelated systematic uncertainty of about 1 % should be added in quadrature to account for this for AMS-02 (see also Sec. 3.3). In the following section, we discuss how the statistical and uncorrelated systematic uncertainties compare to the expected features in the burst-like and CREDIT injection models.

8.5 Discriminating source models

We identify two main questions that can be addressed considering the discussions in this chapter:

1. Can we test the hypothesis of discrete sources with current and future experiments by detecting features in the local cosmic-ray proton spectrum?
2. Is it possible to discriminate different source models, e.g., the burst-like injection model and the CREDIT model, based on their characteristic features?

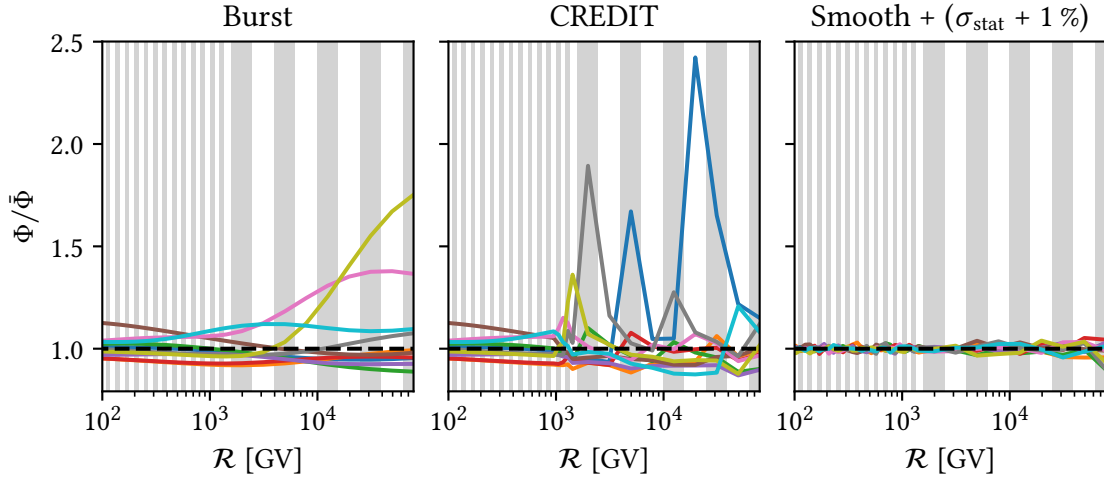


Figure 8.14: Example spectra for the burst-like, CREDIT, and smooth injection models. As in Fig. 8.12, we show ten example realisations of local cosmic-ray proton spectra normalised by the mean total spectrum, $\bar{\Phi}$. The left panel shows the burst-like injection model, the middle panel the same configuration as in Fig. 8.12 (sampled at the bin centres, as in the right panel there), and the right panel the smooth injection model with additional uncorrelated noise according to Eq. (8.20) and an additional component of 1 % added in quadrature.

Both questions are closely related, as they require us to determine what it is that actually *defines a feature* in the local cosmic-ray proton spectrum.

In Fig. 8.14, we show ten example realisations of local cosmic-ray proton spectra for the burst-like injection model (left panel), the CREDIT model (middle panel), and purely uncorrelated fluctuations around the smooth mean spectrum (right panel). All spectra are normalised by the mean total spectrum, $\bar{\Phi}$. By eye, it seems clear that the discrete source models produce clear deviations from the mean spectrum, much more pronounced than the statistical errors (Question 1?), and the features in the CREDIT model look clearly different, somehow more peaked, than in the burst-like injection model (Question 2?).

However, a closer look reveals that the situation is more complex. Some of the realisations only show very moderate features, comparable to the scale of the statistical fluctuations. Especially for those, the difference between the burst-like injection model and the CREDIT model is less obvious. A criterion that only works for the most extreme cases is not sufficient to answer the questions posed above. We need a robust test statistic, that can reliably discriminate between the different scenarios to really answer the questions above with a high degree of confidence. We can test such a criterion on our large Monte Carlo data sets of different source models. Only if the criterion yields the correct answer for a large fraction of those realisations, we can be confident that it will also work for the actual local cosmic-ray proton spectrum.

Finding a suitable test statistic from scratch is not straightforward. For an extended overview of the different approaches we have explored, we refer to the Master's thesis of Loo, 2024. Because we need to define a criterion that also works for moderate features, simple approaches like determining the height of jumps in the deviation between different bins fails. It proved most promising to employ techniques that use the full information of the spectrum. A classical approach included the use of Fourier analyses (see Loo, 2024 for details). However, the most successful approach we found was to make use of Machine Learning (ML) techniques, specifically Decision Trees (DTs). We will discuss this approach in detail in Chap. 9.

9

Classification of source models

In the preceding chapter, we have introduced our Monte Carlo model setup for simulating local cosmic-ray proton spectra from discrete sources. We found that different source models lead to local spectra that might be distinguishable from each other. Specifically, we consider a discrete source model with an instantaneous, burst-like injection of cosmic rays, discrete source models with a rigidity-dependent injection of cosmic rays (CREDIT), and a smooth source model as a limiting case without imprints from individual sources.

In Sec. 8.5, we identified two classification tasks (Question 1 and 2). In this chapter, we will address these and explore how well the available information from current cosmic-ray experiments is suitable to fulfil these tasks. If a distinction between different source models is possible, this would allow us to draw conclusions about the properties of the sources of Galactic cosmic rays, e.g.,

- Are the implications of the SNR paradigm concerning their rate and injection mechanism valid?
- Do cosmic rays escape from their sources in a rigidity-dependent manner?
- Are cosmic-ray sources discrete?
- What can we learn from features in the local cosmic-ray spectrum?

The information available to us consists of measured cosmic-ray intensities in different rigidity bins, each with a certain experimental uncertainty. As discussed in Eqs. (8.11) and (8.12), we consider 58 bins above 10 GV as set by AMS-02 and DAMPE. Information about the source model is thus encoded in a 58-dimensional space, whose dependence on the high-dimensional space of possible realisations of source coordinates, ages, and injection models is highly non-trivial. It is hard to distinguish if effects are from a specific model or a possible realisation of the latent space. Machine Learning (ML) can *learn* patterns without us having to explicitly program them through the process of training (Mohri et al., 2018).

Based on large samples of simulated local cosmic-ray spectra for the different source models: Burst, CREDIT, and Smooth, we want to train a ML model to classify those spectra according to their source model. This is a typical ML classification problem (Mohri et al., 2018). In the following, we give a brief introduction to the ML methods we will use for

this purpose in Sec. 9.1. Then, we present the results of the classification in Sec. 9.2 and discuss their implications in Sec. 9.3.

9.1 Machine learning for classification

We identified that our problem of classifying the source models¹ based on the local cosmic-ray spectra is a typical ML classification task, where we want to assign one of three discrete labels (the source models) to an input (the local spectrum, represented by intensity values in 58 rigidity bins). This classification requires drawing boundaries between the different classes in a 58-dimensional space, which is hard to do with non-ML methods.

One option to tackle this problem are so-called *linear classifiers* (Mohri et al., 2018). These split the input space with hyperplanes, which are defined by linear equations. Their big advantage lies in their simplicity, interpretability, and low computational cost. However, they are limited to problems where the classes can be separated by linear boundaries. As Loo, 2024 has shown, this is not the case for our problem of classifying the source models based on the local cosmic-ray spectra². Correlations between different rigidity bins in the burst-like and CREDIT scenarios necessitate non-linear decision boundaries. As determined by Loo, 2024, (boosted) *Decision Trees* (DTs) are a suitable choice for this task.

9.1.1 Decision trees

The basic idea of DTs is to approximate the decision boundaries between different classes by a series of splits of the input space using hypercuboids. With a sufficient number of splits, those hypercuboids can approximate any shape. Thus, DTs are in principle able to approximate highly non-linear boundaries (Bishop, 2006). The splits are chosen in a way that minimises quantities like the Gini index or the entropy, which encode the gain in information (Mohri et al., 2018). These splits can be visualised as a tree structure, where each internal node represents a split of the input space and each leaf node represents a class label. Given a set of training data with known labels, the DT algorithm iteratively finds the splits that lead to the highest information gain until a stopping criterion is

¹For now, further goals like the inference of source parameters (e.g. source positions, ages, R_b , etc.) are out of the scope considering current experimental uncertainties and the complexity of the problem.

²Loo, 2024 tested various linear classification models like `LogisticRegression`, `LinearSVC`, and `LinearDiscriminantAnalysis` from the `SKLEARN` Python library for the classification of discrete against smooth source models. It was found that all of those had an accuracy score close to 0.5, i.e., they did not work better than random guesses. This is an indication for an inherent non-linearity in our classification problem.

met. This can be, e.g., that all samples in a leaf node belong to the same class or that a maximum tree depth is reached.

As DTs can capture non-linear dependencies, they are a suitable choice for our classification problem. However, they are prone to overfitting the training data, i.e., learning patterns that do not generalise well to unseen data but are due to specific random realisations of the training data (Bishop, 2006). Checking for this is part of the validation process, which we will describe below.

To decrease the time required to train DTs, ensemble methods like *boosting* can be used (Mohri et al., 2018). Here, multiple DTs are trained sequentially and combined to form a stronger classifier. For a more detailed introduction to DTs, we refer to, e.g., Breiman et al., 1984; Mohri et al., 2018; Quinlan, 1986; Sarker, 2021.

In this work, we use the implementation of DTs provided by the SCIKIT-LEARN (SKLEARN) library in Python (Pedregosa et al., 2011) out of the box. Specifically, we use the model provided by the class `HistGradientBoostingClassifier`, which is an ensemble method based on boosting of DTs that is optimised for large datasets. In the following, we briefly outline how we train and validate our ML model.

9.1.2 Training

The training data is a set of local cosmic-ray spectra simulated for the three different source models (Burst, CREDIT, and Smooth). In this work, we usually generate $4 \cdot 10^6$ samples for each source model to ensure a good coverage of the input space. Those are represented by arrays of length 58, containing the intensities in the rigidity bins defined in Eqs. (8.11) and (8.12) and a corresponding label indicating the source model. In a preprocessing step, we normalise the intensities by the mean intensities in each rigidity bin to ensure that all features are on a similar scale, which helps the training process (Mohri et al., 2018).

The training data for the Burst and CREDIT models is generated by the Monte Carlo procedure described in Chap. 8. For the Smooth model, we sample uncorrelated errors around the average spectrum as described in Sec. 8.4. To account for those experimental uncertainties, we have four options:

- use the simulated Burst and CREDIT spectra as they are, i.e., without adding any additional uncertainties and only include σ_{stat} for the Smooth model according to Eq. (8.20)
- include σ_{stat} and an additional uncorrelated systematic uncertainty of 1 % in quadrature only in the Smooth model
- include σ_{stat} in all models
- include σ_{stat} and an additional uncorrelated systematic uncertainty of 1 % in quadrature in all models

We will explore the influence of these choices on the classification results in Sec. 9.2.

9.1.3 Validation

To assess the performance of our trained ML model, we use *k-fold cross-validation*. This means that we split our dataset into k equally sized subsets, so-called *folds*. This allows us to train the model k times, each time using $k - 1$ folds for training and the remaining fold for validating (Berrar, 2019). For the validation, we use the trained model to predict the labels of the samples in the validation fold and compare them to the true labels. The fraction of correctly classified samples is called the *accuracy score*.

The accuracy score is a simple metric to evaluate the performance of a classifier. By checking how consistent it is across the different folds, we can also get an estimate of the consistency of the model. If it has a small variance across the folds, it indicates that the model generalises well to unseen data. This also indicates that the model is not overfitting, but for more details on this, we refer to Vakili et al., 2020.

A more detailed representation of the classification performance can be obtained by using a *confusion matrix*. This is a table that summarises the number of correct and incorrect predictions for each class. Each row of the matrix corresponds to the true class, while each column corresponds to the predicted class. The diagonal elements represent the number of correct predictions, while the off-diagonal elements represent the number of misclassifications (e.g., Vakili et al., 2020). We will use confusion matrices as the main tool to present and discuss our classification results in Sec. 9.2.

For the implementation of the cross-validation and the confusion matrix, we again use the `SKLEARN` library in Python (Pedregosa et al., 2011). Specifically, we use the class `StratifiedKFold` for the cross-validation with $k = 10$ folds³ and the function `confusion_matrix` to compute the confusion matrices.

9.2 Classification results

In this section, we discuss the performance of our ML model for the classification of the source models. We start with the results for our fiducial model setup in Sec. 9.2.1. We discuss how well the model can classify the three source models and how this depends on the inclusion of experimental uncertainties (see Sec. 8.4) in the training data. Further, we check how the choice of the CREDIT model parameter $\mathcal{R}_s$ influences the classification results.

³Our training data sets are large enough to use many folds without compromising the size of the training sets in each fold. As we are not limited by training time either, we decided that $k = 10$ instead of the default value of $k = 5$ is a reasonable choice to get a more reliable estimate of the model performance.

Then, we explore how the classification performance changes when varying different model parameters. In Sec. 9.2.2, we study the influence of the source distribution. In Sec. 9.2.3, we explore the impact of the parameters H and κ_0 . Finally, in Sec. 9.2.4, we investigate the influence of other parameter variations like the source rate, f_{SN} , the source lifetime, t_{life} , and our sampling of the parameter $\mathcal{R}_b$.

9.2.1 Fiducial model

We define our fiducial model setup as follows: For both discrete source models (Burst and CREDIT), we use the default diffusion coefficient as defined in Eq. (8.3), the axisymmetric source distribution from Ferrière, 2001, a halo height of $H = 4$ kpc and a source rate of $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$. In the CREDIT model, we set the source lifetime to $t_{\text{life}} = 100$ kyr, the start of the Sedov phase to $t_{\text{ST}} = 1$ kyr, and choose the parameter $\mathcal{R}_b$ in the range $[1, 25]$ TV (see below).

For each of the three source models, we generate $4 \cdot 10^6$ samples of local cosmic-ray spectra as described in Chap. 8. For the realisations in the CREDIT model, we choose 20 logarithmically spaced values of $\mathcal{R}_b$ in the range $[1, 25]$ TV and simulate $2 \cdot 10^5$ samples for each of those. This allows us to cover a wide range of possible behaviours of the CREDIT model. As we have discussed in Sec. 8.2, the parameter $\mathcal{R}_b$ sets the rigidity above which peaks characteristic for the CREDIT model might appear in the local spectrum. As we have argued in Sec. 5.8, values of $\mathcal{O}(10 \text{ TV})$ are reasonable. Choosing a range of values for $\mathcal{R}_b$ allows us to account for the uncertainty in this parameter, and the maximum value of 25 TV ensures that there are at least three rigidity bins above $\mathcal{R}_b$ in our analysis (see Eqs. (8.11) and (8.12)).

In Fig. 9.1, we show the confusion matrices obtained from the validation of our DT. We show results for the four different cases regarding the inclusion of experimental uncertainties in the training data as described in Sec. 9.1.2. While the detailed values in the confusion matrices differ between the four cases, some general trends can be observed:

1. All confusion matrices have a strong diagonal component, indicating that the model is able to classify the source models well above random guessing.
2. All matrices are block-diagonal, i.e., misclassifications mainly occur between the Burst and CREDIT models, while the Smooth model is classified very reliably.
3. There are misclassifications between all models, i.e., there are realisations of each source model that resemble realisations of the other models.

The first point shows that our ML model is able to learn patterns in the local cosmic-ray spectra that allow it to distinguish between the different source models. This indicates that the information contained in the local spectra is useful to infer properties of the sources of Galactic cosmic rays, although this needs to be discussed in more detail. The

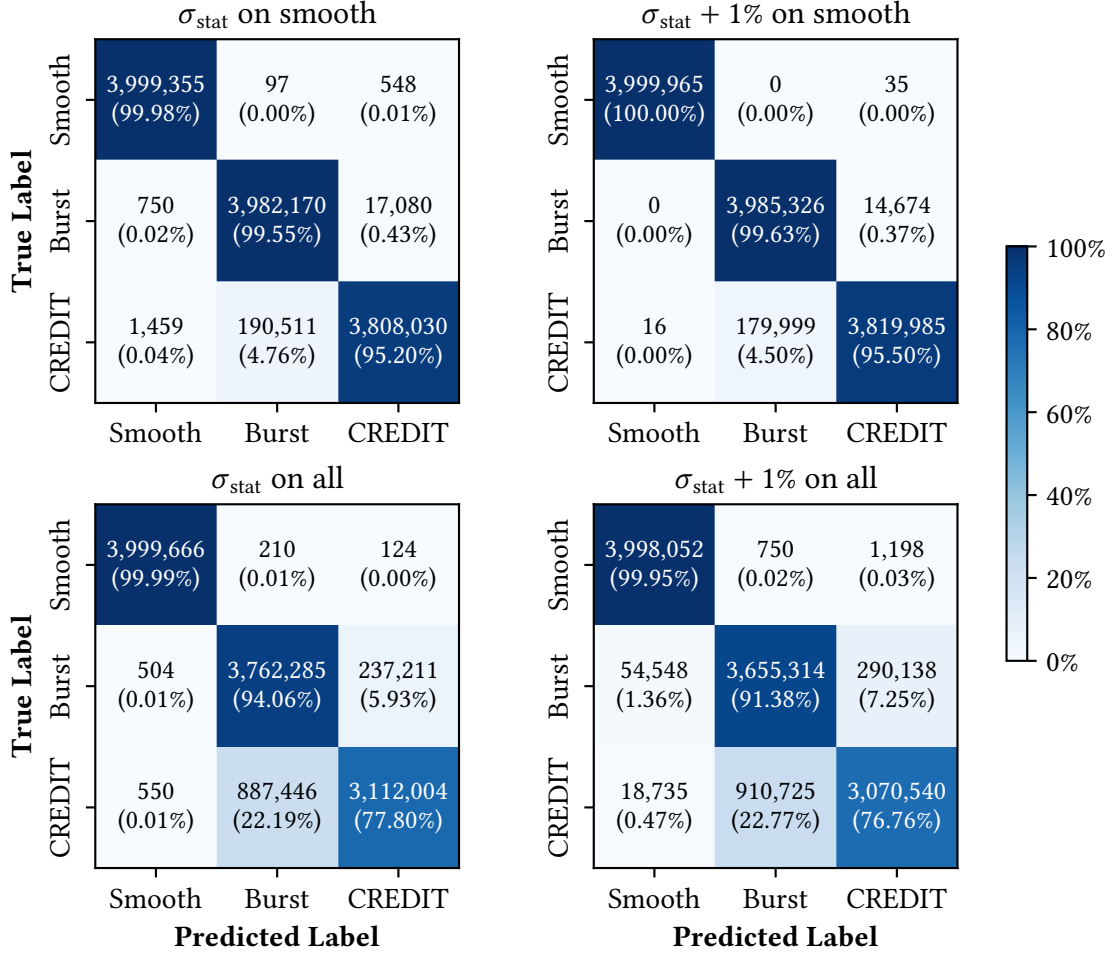


Figure 9.1: Confusion matrices for the fiducial model. (Default diffusion, Ferrière source distribution, $H = 4$ kpc, $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$, $t_{\text{life}} = 100$ kyr, $t_{\text{ST}} = 1$ kyr, fixed $\mathcal{R}_b \in [1, 25]$ TV) We show confusion matrices for four different cases regarding the inclusion of experimental uncertainties in the training data as described in Sec. 9.1.2.

second point shows that the Smooth model seems to be sufficiently different from the two discrete source models to be classified reliably. This is a good indication that we can answer Question 1 from Sec. 8.5 positively, i.e., we probably can distinguish discrete source models from a smooth source model based on the local cosmic-ray spectra. Finally, the third point shows that the classification task is not trivial, as there are misclassifications between all models. There are realistic scenarios in which the local spectrum from one source model can resemble that from another source model.

A closer look at the misclassifications between the Burst and CREDIT models in Fig. 9.1 shows that they mainly occur in the way that CREDIT samples are misclassified as Burst samples, while the opposite is much rarer. This is expected as realisations of the CREDIT model without pronounced peaks in the local spectrum because of a lack of nearby

sources young sources can resemble Burst model realisations, but the opposite is not true. This indicates that distinguishing between the Burst and CREDIT models (Question 2 from Sec. 8.5) is more challenging. However, the fact that many CREDIT samples are classified correctly shows that the model is able to learn patterns that are characteristic for the CREDIT model. With an extension of the rigidity range and improved experimental uncertainties, this task might be feasible (see Sec. 9.3).

A further observation from Fig. 9.1 is that the inclusion of experimental uncertainties by adding random noise to the training data decreases the classification performance. This is expected as the noise obscures patterns in the data that are characteristic for the different source models. The difference between including σ_{stat} only or also an additional 1 % of systematic error (left vs. right panels in Fig. 9.1) is small, while including the uncertainties just in the Smooth or in all models (top vs. bottom panels in Fig. 9.1) has a larger impact. This shows that including uncertainties in all models complicates the classification task, as the patterns that the model can learn are more obscured by the added noise, potentially making a clear distinction between the source models even impossible. Example spectra for the different entries of the confusion matrix in the case with inclusion of σ_{stat} and an additional uncorrelated systematic uncertainty of 1 % in quadrature in all models (lower right in Fig. 9.1) are shown in Fig. E.2 at the end of the Appendix.

In a real experimental situation, uncertainties will always be present in the measured data. A real measured spectrum will thus always resemble one of the cases with uncertainties included in the training data. For the remainder of this chapter, we focus on the case with inclusion of σ_{stat} and an additional uncorrelated systematic uncertainty of 1 % in quadrature in all models (like lower right in Fig. 9.1).

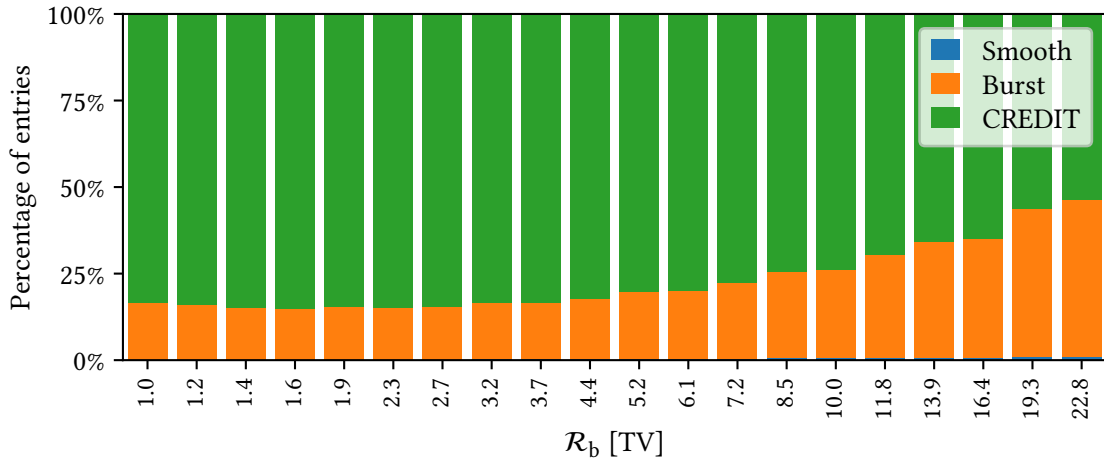


Figure 9.2: Dependence of classification on R_b . We show the distribution of predicted labels for CREDIT samples as a function of the parameter R_b used in their generation for the fiducial model setup with inclusion of σ_{stat} and an additional uncorrelated systematic uncertainty of 1 % in quadrature in all models (lower right in Fig. 9.1).

A further aspect we want to explore for our fiducial model setup is how the classification performance depends on the choice of the CREDIT model parameter $\mathcal{R}_b$. As discussed above, this parameter sets the rigidity above which peaks characteristic for the CREDIT model might appear in the local spectrum. Hence, we expect that the classification performance improves for lower values of $\mathcal{R}_b$, as more rigidity bins are potentially affected by respective features. In Fig. 9.2, we show the distribution of predicted labels for CREDIT samples as a function of the parameter $\mathcal{R}_b$ used in their generation, corresponding to the lower right panel in Fig. 9.1. As expected, we see that for low values of $\mathcal{R}_b$, more samples are classified correctly as CREDIT, while for higher values of $\mathcal{R}_b$, more samples are misclassified as Burst. Increasing $\mathcal{R}_b$ can be seen as a continuous transition from the CREDIT to the Burst model, as the rigidity-dependent escape becomes less relevant in the considered rigidity range.

Finally, we note that the standard deviation of the accuracy score across the different folds in all our tests is small ($\lesssim 0.1\%$), indicating that our DT generalises well. As discussed in Sec. 9.1.3, this is a good indication that the model is not overfitting the training data.

9.2.2 Dependence on source and diffusion models

In this section, we show the results obtained with the CRINGE diffusion coefficient (see Eq. (8.2)) for a range of different source distributions introduced in Sec. 8.1.4. In Fig. 9.3, we show the resulting confusion matrices. For reference, we also show the confusion matrix for the fiducial model setup with the Ferrière source distribution in the top left panel. Comparing the top left to the top right panel of Fig. 9.3 shows that a change of the diffusion coefficient only has a minor impact on the classification results. This can be attributed to our choice of the default diffusion coefficient, Eq. (8.3), to resemble the CRINGE diffusion coefficient in the relevant rigidity range (see Fig. 8.2).

As discussed in Sec. 8.1.4, we consider the axisymmetric source distribution from Ferrière, 2001 as our fiducial choice. In addition, we also show two different variants of a spiral arm source distribution based on Churchwell et al., 2009; Evoli et al., 2021a,b; Steiman-Cameron et al., 2010, that differ in the assumed distance of the Sun to the nearest spiral arm. One assumes a distance of about 0.95 kpc (in arm), while the other assumes about 1.5 kpc (out of arm).

Comparing the top to the bottom panels of Fig. 9.3 shows that the choice of the source distribution has a larger impact. The classification performance decreases strongly when using the spiral opposed to the axisymmetric source distribution. In both spiral models, the local source density is lower than in the axisymmetric Ferrière model. This leads to two competing effects: On the one hand, a lower local source density should lead to a stronger relative contribution of individual sources (compare with discussion in Secs. 8.2.2 and 8.2.3). However, on the other hand, it is less likely to have a young nearby source that can produce pronounced features in the local spectrum. Thus, the fraction of realisations

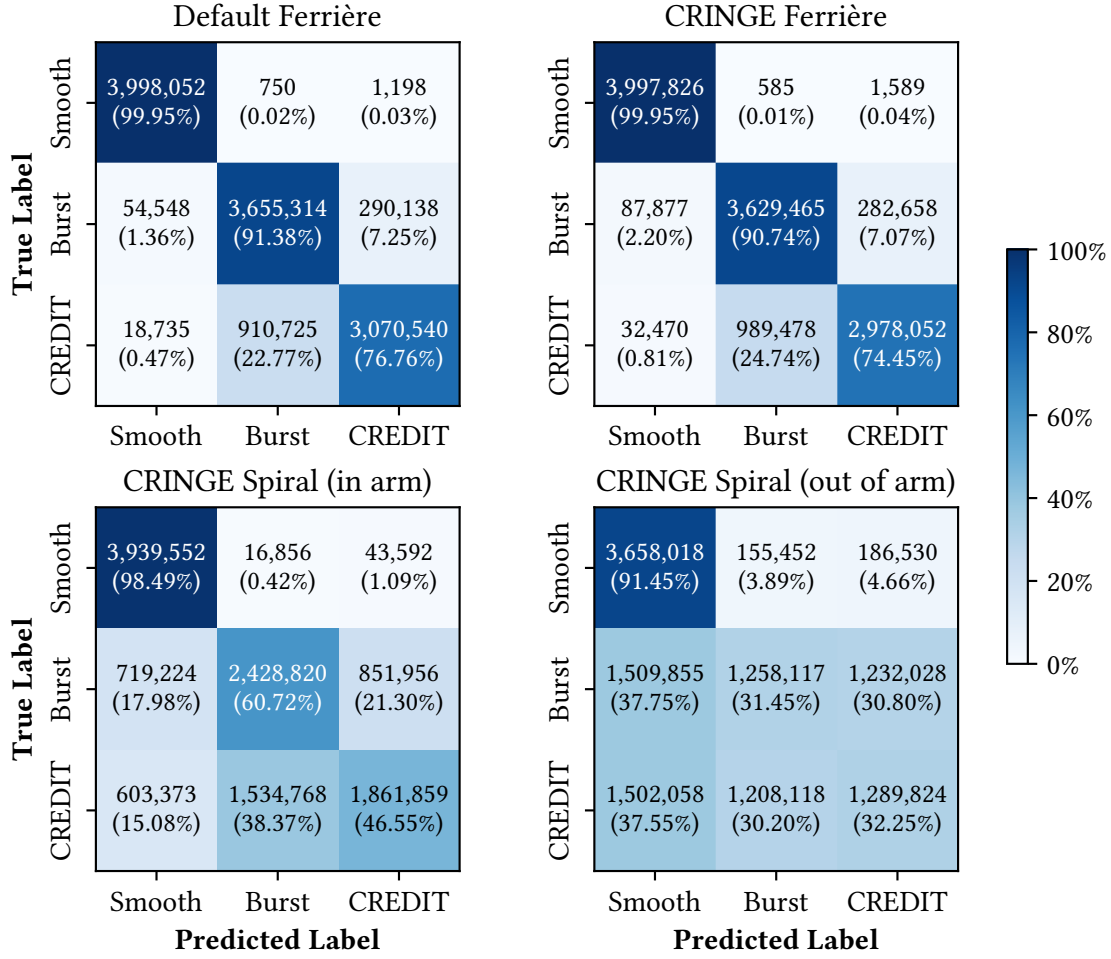


Figure 9.3: Confusion matrices for different source distributions and diffusion coefficients. (Default/CRINGE diffusion, $H = 4$ kpc, $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$, $t_{\text{life}} = 100$ kyr, $t_{\text{ST}} = 1$ kyr, fixed $\mathcal{R}_b \in [1, 25]$ TV, $\sigma_{\text{stat}} + 1\%$ in all models)

with pronounced features decreases with a lower local source density. In other words, our expectation to experience such an unlikely event also decreases.

The confusion matrices in the bottom panels of Fig. 9.3 show that the second effect dominates. The deterioration of the classification performance with an increased distance to the nearest spiral arm, i.e., a lower local source density, indicates that a sparsity of local sources makes the classification task more difficult. For a position out of a spiral arm, a large fraction of realisations lacks pronounced features in the local spectrum, which even hampers the distinction between the Smooth and the discrete source models.

9.2.3 Dependence on transport parameters

In this section, we explore how the classification performance depends on the transport parameters H and the normalisation of the diffusion coefficient. As we discussed in Sec. 8.1.2, those two parameters are in general degenerate as their ratio is constrained by secondary-to-primary ratios like B/C (see also Sec. 4.5). In Fig. 9.4, we show confusion matrices for three different combinations of H and the normalisation of the diffusion coefficient, which we denote by a parameter d that scales κ_0 . In Eq. (8.3), we have set this parameter to $H/(6 \text{ kpc})$. The top panel of Fig. 9.4 thus shows the fiducial model setup with $H = 4 \text{ kpc}$ and $d = 2/3$. For the bottom left panel, we set $H = 6 \text{ kpc}$ and $d = 1$, according to the degeneracy, while we effectively change the transport model by setting $H = 4 \text{ kpc}$ and $d = 1$ in the bottom right panel.

A comparison between the top and bottom left panel of Fig. 9.4 shows that changing H and the diffusion normalisation according to the degeneracy has only a minor impact on the classification results. The opposing effects of changing H and the diffusion normalisation seem to cancel out to a large extent. This is in line with our discussion in Secs. 8.2.2 and 8.2.3. Only increasing the diffusion normalisation while keeping H fixed (top vs. bottom right panel) or only decreasing H while keeping the diffusion normalisation fixed (bottom left vs. bottom right panel) enhances the classification performance. We have seen in Sec. 8.2.3, that the spectral features in the CREDIT model become more pronounced for those changes. As the local source density is not affected by those changes, the increased prominence of features in the local spectrum leads to an improved classification performance.

9.2.4 Other parameter variations

In this section, we explore how the classification performance changes when varying further model parameters. First, we study how the modelling of the parameter $\mathcal{R}_b$ in the CREDIT model influences the results. In the results shown so far, we have chosen an equal amount of samples for 20 logarithmically spaced values of $\mathcal{R}_b$ in the range $[1, 25] \text{ TV}$, which we fixed for all sources in a given realisation. In the context of Fig. 9.2, we have seen that the classification performance depends on the choice of $\mathcal{R}_b$, where lower values lead to a better performance.

Instead of allowing different choices of a fixed $\mathcal{R}_b$, it is also perceivable that $\mathcal{R}_b$ varies between different sources. To explore the impact of this modelling choice, we also sample the parameter $\mathcal{R}_b$ randomly for *each individual source*. To make both approaches comparable and also get a uniform distribution on a logarithmic scale, we sample $\mathcal{R}_b$ from a distribution $p(\mathcal{R}_b) \propto 1/\mathcal{R}_b$ for $\mathcal{R}_b \in [1, 25] \text{ TV}$.

In Fig. 9.5, we show the resulting confusion matrices for both approaches. The resulting confusion matrices are very similar, with an only slightly better classification performance

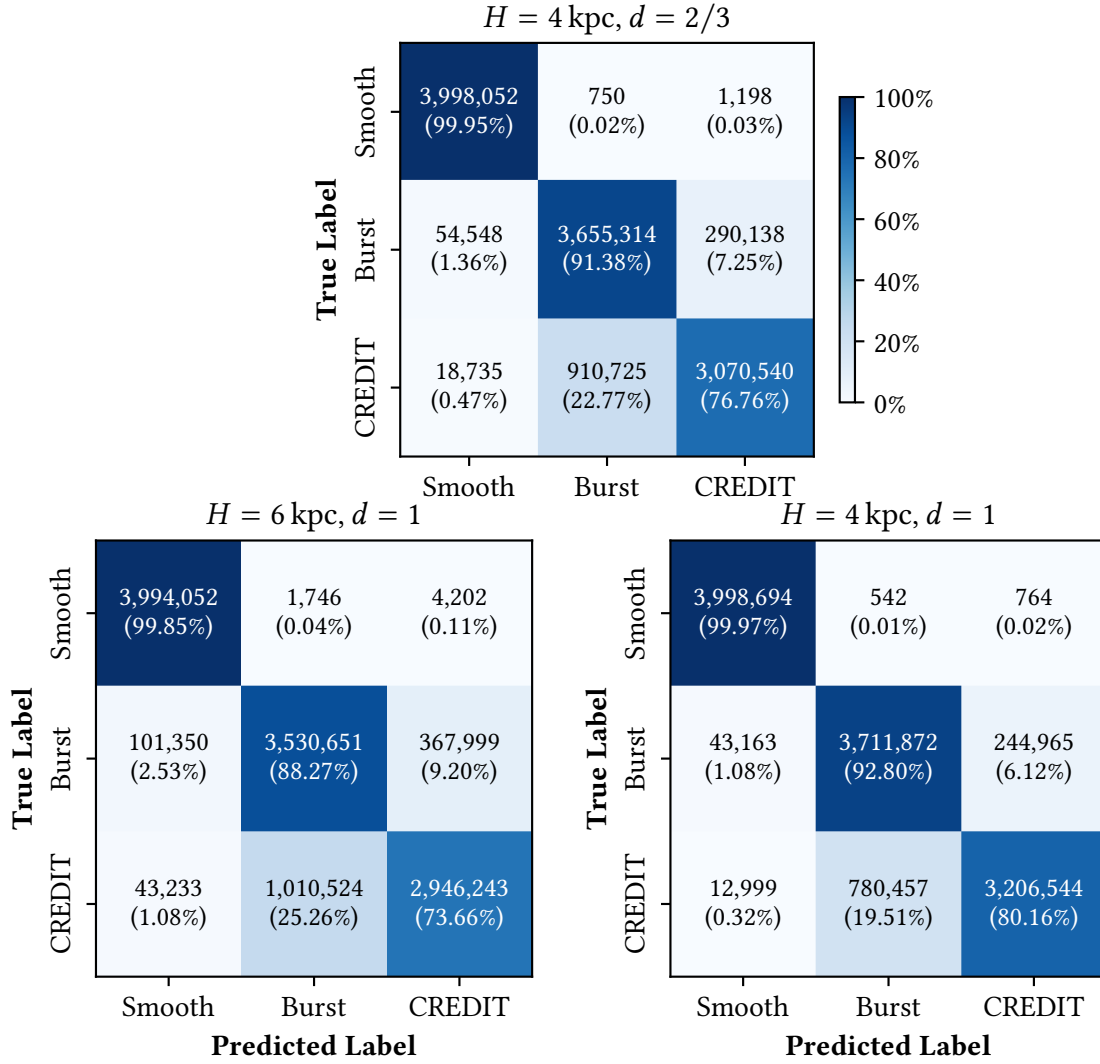


Figure 9.4: Confusion matrices for different transport parameters. (Default diffusion (with varying normalisation), Ferrière source distribution, $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$, $t_{\text{life}} = 100 \text{ kyr}$, $t_{\text{ST}} = 1 \text{ kyr}$, fixed $\mathcal{R}_b \in [1, 25]$ TV, $\sigma_{\text{stat}} + 1\%$ in all models) d denotes the scaling of the diffusion coefficient normalisation compared to Eq. (8.3) for $H = 6 \text{ kpc}$.

when using randomly sampled $\mathcal{R}_b$ for each source (right panel). The classification performance between the Burst and the CREDIT models is slightly improved, which can be attributed to a higher chance of finding a distinguishing CREDIT feature in the considered rigidity range when sampling $\mathcal{R}_b$ for each source.

Next, we explore how the source rate, f_{SN} , influences the classification results in Fig. 9.6. We compare the fiducial value of $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$ (left) to a halved value of $f_{\text{SN}} = 0.015 \text{ yr}^{-1}$ (right). At the first glance, the classification performance seems to be very similar in both cases. Due to the lower source rate, we expect the relative contribution of individual

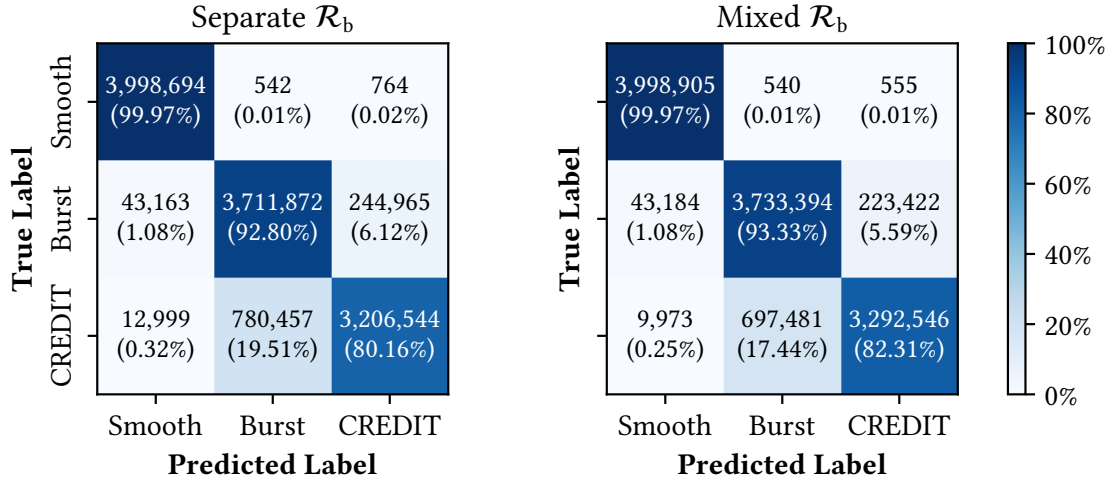


Figure 9.5: Confusion matrices for different modelling of $\mathcal{R}_b$. (Default diffusion (with $d = 1$), Ferrière source distribution, $H = 4$ kpc, $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$, $t_{\text{life}} = 100$ kyr, $t_{\text{ST}} = 1$ kyr, $\sigma_{\text{stat}} + 1\%$ in all models)

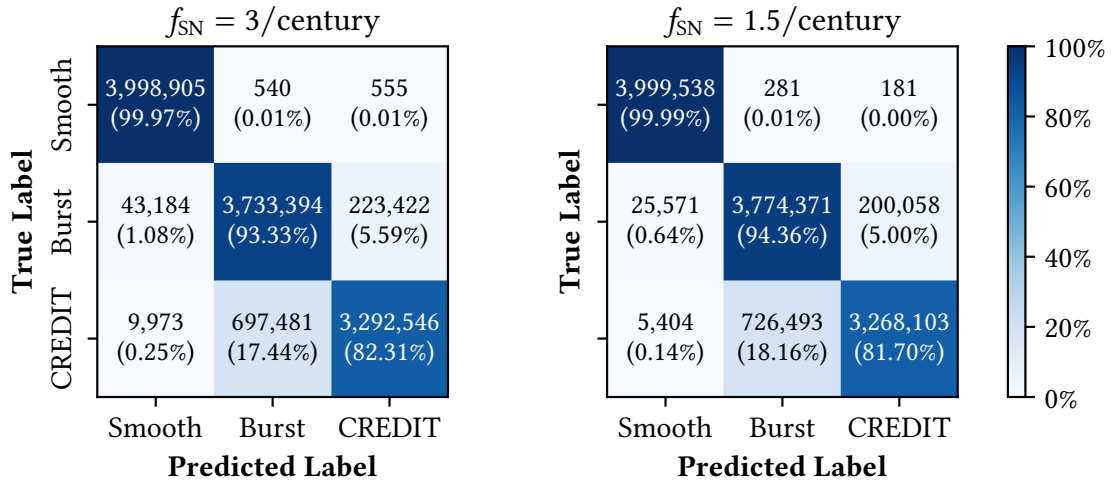


Figure 9.6: Confusion matrices for different source rates. (Default diffusion (with $d = 1$), Ferrière source distribution, $t_{\text{life}} = 100$ kyr, $t_{\text{ST}} = 1$ kyr, mixed $p(\mathcal{R}_b) \propto 1/\mathcal{R}_b$ for $\mathcal{R}_b \in [1, 25]$ TV, $\sigma_{\text{stat}} + 1\%$ in all models)

sources to increase, which should lead to more pronounced features in the local spectrum (see discussion in Secs. 8.2.2 and 8.2.3). However, the lower source rate also decreases the chance of having a young source that can produce pronounced features in the local spectrum. Contrary to the case of changing the source distribution in Sec. 9.2.2, those two effects seem to roughly cancel out here, leading to a similar classification performance. We conclude that the spatial source density seems to have a stronger impact on the classification performance than the source rate.

Halving the source rate only seems to slightly reduce the already rare misclassifications of Smooth samples as Burst or CREDIT and vice versa. We attribute this to the generally more pronounced impact of individual sources on the local spectrum for a lower source rate.

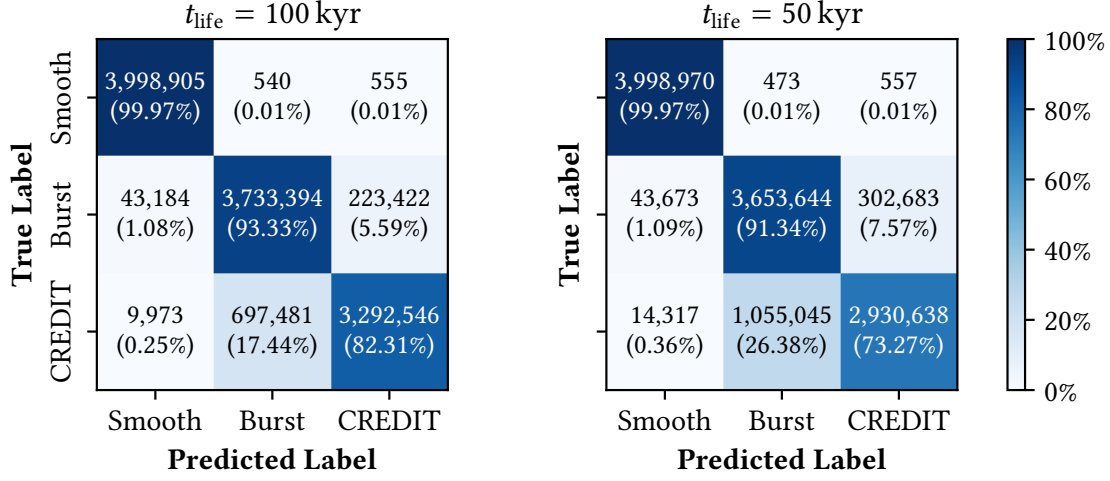


Figure 9.7: Confusion matrices for different source lifetimes. (Default diffusion (with $d = 1$), Ferrière source distribution, $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$, $t_{\text{ST}} = 1 \text{ kyr}$, mixed $p(\mathcal{R}_b) \propto 1/\mathcal{R}_b$ for $\mathcal{R}_b \in [1, 25]$ TV, $\sigma_{\text{stat}} + 1 \%$ in all models)

Finally, we explore how the source lifetime, t_{life} , influences the classification results in Fig. 9.7. As this is a parameter specific to the CREDIT model, we only expect changes in the classification performance between the Burst and CREDIT models, while the classification of the Smooth model should remain unaffected. Comparing the left panel with the fiducial value of $t_{\text{life}} = 100 \text{ kyr}$ to a lower value of $t_{\text{life}} = 50 \text{ kyr}$ (right), we see that the classification performance between the Burst and CREDIT models decreases slightly for the lower lifetime. This is expected following our discussion of the parameter $\mathcal{B}$ (see Eq. (8.19) in Sec. 8.2.1). A lower source lifetime leads to a lower value of $\mathcal{B}$, which decreases the prominence of peaks in the CREDIT model. Qualitatively, this makes sense: More transient sources (lower t_{life}) lead to a more burst-like behaviour, making it harder to distinguish between the two discrete source models.

9.3 Discussion

In the previous sections, we have demonstrated that DTs are a powerful tool to classify local cosmic-ray spectra according to their underlying source model. We have shown that DTs bear a high potential to distinguish discrete source models (Burst and CREDIT) from a smooth source model based on the local cosmic-ray proton spectrum. Coming back to Question 1 from Sec. 8.5, we are optimistic that this can be answered positively when

carefully applied to experimental data, but we also note that this approach is limited by current experimental uncertainties and our limited knowledge of model parameters. Most importantly, the classification performance strongly depends on the local source density, which is not well constrained.

The classification performance between the Burst and CREDIT models (Question 2 from Sec. 8.5) is more limited but still shows promising results. We have seen that the classification performance has strong dependencies on model parameters, which we discussed in detail in Sec. 9.2. The resulting changes in the classification performance could be understood in the context of our discussions in Chap. 8. This comprehensive understanding of the model dependencies is a good basis for future applications of ML models to classify experimental data in cosmic-ray astrophysics.

We introduced the CREDIT model as a physically motivated discrete source model in the context of Galaxy-wide stochastic cosmic-ray source models. Our discussion in Chap. 8 and this chapter has shown that the CREDIT model predicts pronounced features which potentially increase the observability of individual source contributions in the local cosmic-ray spectrum. This discussion shows that the current and future high-precision measurements of the cosmic-ray proton spectrum at high rigidities bear a high potential to constrain models of cosmic-ray sources and acceleration.

While most of the results shown in this chapter are based on the default diffusion coefficient defined in Eq. (8.3), we have also explored the impact of using the CRINGE diffusion coefficient (see Eq. (8.2)). The classification results are very similar in both cases. This shows that our results are robust against changes of the exact spectral shape of the diffusion coefficient. However, before applying a classifier trained on one diffusion model to experimental data, it is important to ensure that the chosen diffusion model is a good representation of reality. If spectral features in the data are caused by transport effects that are not captured by the chosen diffusion model, they could be misinterpreted as source features (Stall et al., 2025).

An extension of high-precision measurements to even higher rigidities beyond ≈ 100 TV would further enhance the classification performance. As we have discussed in Sec. 8.2, the parameter $\mathcal{R}_b$ sets the rigidity above which peaks characteristic for the CREDIT model might appear in the local spectrum. Extending the rigidity range would allow probing a wider rigidity range in which such features might appear, increasing the chance of observing them. In addition, the relative contribution of individual sources increases with rigidity, which also enhances the prominence of features in the local spectrum. The proposed space-based direct detection experiment AMS-100 (Schael et al., 2019) aims to extend high-precision proton measurements beyond the cosmic-ray knee, which would be highly beneficial in this context (see expected experimental uncertainties in Fig. 8.13).

Leading high-precision experiments like DAMPE are currently limited by statistics at the highest rigidities and suffer from increasingly limited power to discriminate proton and helium at rigidities beyond ≈ 100 TV (Alemanno et al., 2024). Thus, the inclusion of

heavier nuclei can also increase the predictive power of our model due to an extended availability of data.

The experimental uncertainties play an important role in the classification performance. We argued in Sec. 8.4 that the analysis of spectral features is mainly limited by uncorrelated uncertainties. A better understanding of the correlation structure of systematic uncertainties in cosmic-ray experiments could thus potentially improve the classification performance.

We also suspect that the use of unbinned data instead of the publicly available binned spectra could potentially further improve the sensitivity to features in the local cosmic-ray spectrum. Through the binning process, some spectral information is inevitably lost. We suggest considering this in future applications of ML models to classify experimental data in cosmic-ray astrophysics.

Finally, we want to stress that our results are just a first step towards applying ML models to classify experimental data in cosmic-ray astrophysics. We provided a proof of concept that DTs might be a powerful tool for this task. Future applications of ML models can extend the scope of our analysis in several ways. E.g., it might be possible to move beyond a pure classification task and instead infer the likelihood of different source models given a measured cosmic-ray spectrum. Along this line, it might also be possible to infer model parameters like the source rate or the local source density from measured data using ML models in the future.

Part III

Galactic diffuse emission

10

Preparation of the simulations

In this part, we want to extend our stochastic model of Galactic cosmic-ray sources to also include the calculation of Galactic Diffuse Emissions (GDEs). The observation of GDEs is an important tool to study cosmic rays in other parts of the Galaxy. Further, it is an important background in γ -ray and neutrino astronomy, which needs to be properly understood to identify and characterise potential point sources of γ -rays and neutrinos. In Chap. 6, we discussed in detail how hadronic GDEs are produced. This included an introduction to the relevant model ingredients, such as the gas distribution in the Galaxy, the cross-sections for the production of γ -rays and neutrinos, absorption processes, and the calculation of the LOS integrals to obtain the GDE intensities.

The stochastic nature of the cosmic-ray intensities in a discrete source model with uncertain source locations and ages was discussed in Chap. 7. Naturally, this *stochasticity* will also affect the GDEs, as they are produced by the interactions of cosmic rays with the interstellar gas. It is not inherently clear how strong the variations in the GDEs are when the stochasticity of the sources is taken into account. Our analysis in Part II revealed that a full-scale simulation of the Galactic cosmic-ray intensities is required to properly capture the effects of the stochastic source distribution. Multiple nearby and young sources can significantly influence the local cosmic-ray spectrum in non-trivial ways. For the modelling of the GDEs, this difficulty is further enhanced as we now need to consider many positions throughout the Galaxy, which can be influenced by different sets of sources, to properly capture the variations in the GDEs.

In this chapter, we want to describe the setup of our study, whose results will be presented in Chap. 11 and have been discussed in Stall and Mertsch, [2026](#). We start by describing the model setup in Sec. 10.1, which is closely connected to the one in Part II. In Sec. 10.2, we discuss how we efficiently calculate the GDEs for large sample sizes of source realisations. Finally, we show how known distributions of cosmic-ray intensities can inform our understanding of the distributions of GDEs and lead to testable predictions in Sec. 10.3.

10.1 Model setup

In this section, we summarise the main aspects of the model setup for the simulations of the GDEs. The setup for the simulations of the cosmic-ray intensities is similar to the one used in Part II, just extended to a full-scale Galactic simulation. We introduce the grid we use for the calculation of the GDEs, which is closely related to the gas maps from Söding et al., 2025. We also describe the used source distribution, and the cosmic-ray transport parameters. Then, we discuss different choices for the cosmic-ray injection and near-source transport models. And finally, we will briefly introduce the used cross-sections and absorption model.

10.1.1 Gas and cosmic-ray model

As we have discussed in Chap. 6, the gas distribution in the Galaxy is a crucial ingredient for the calculation of hadronic GDEs. It describes the target material for the interactions of cosmic rays that lead to the production of γ -rays and neutrinos. Thus, the gas distribution mainly determines the morphology of the hadronic GDEs. In Sec. 6.2, we introduced different models for the gas distribution in the Galaxy, mainly the one based on the work of Söding et al., 2025, which we will also use in this study.

Gas grid

The gas maps by Söding et al., 2025 are provided on a 3D grid. As illustrated in Fig. 6.1, the grid consists of a set of 388 logarithmically spaced concentric spheres around the Sun's position in the Galaxy. Each sphere is pixelated according to the HEALPix scheme (Górski et al., 2005) with $N_{\text{side}} = 64$, i.e., 49 152 grid points for each of 388 radial distances. The Galactocentric Solar distance was reconstructed to be $R_{\odot} = 8.0$ kpc by Söding et al., 2025, which we will adopt as our fiducial value in this study.

This grid is especially suitable for the calculation of GDEs, and we will also adopt it for the simulations of cosmic-ray intensities throughout the Galaxy. With this choice, the calculation of the GDEs becomes very efficient, as the LOS integrals (see Eq. (6.2)) can be performed by efficient matrix operations instead of more costly interpolation and numerical integration methods.

Source distribution and rate

As in Part II, we will assume a fiducial Galactic supernova rate of $f_{\text{SN}} = 0.03 \text{ yr}^{-1}$. For the cosmic-ray source distribution, we will adopt the spiral model we described in Sec. 8.1.4. To be consistent with our assumptions from the gas map, we rescale this model to a Galactocentric Solar distance of $R_{\odot} = 8.0 \text{ kpc}$ (e.g., the Galactic bar does only extend to $\approx 2.8 \text{ kpc}$ here), while still ensuring a distance of 1.5 kpc between the Sun and the nearest spiral arm. While we still assume that the Sun's position is at $z = 0 \text{ pc}$, we extend the source distribution in vertical direction by a Gaussian distribution with a scale height of $z_0 = 70 \text{ pc}$, as suggested by Evoli et al., 2021a,b. Specifically, we use (see also Eq. (7.12)):

$$\rho(z) \propto e^{-\left(\frac{z}{2z_0}\right)^2}. \quad (10.1)$$

Boundary conditions

Having sources out of the Galactic plane and needing to calculate cosmic-ray intensities throughout the Galaxy, i.e., also at positions with $z \neq 0$, we need to consider the treatment of the boundary conditions again (Stall and Mertsch, 2026). As in Sec. 8.1.3, Eq. (8.5), we can write the Green's function as:

$$G(\mathcal{R}, d, \tau) = \sum_{n=-\infty}^{\infty} (-1)^n G_{\text{free}}(\mathcal{R}, d_n, \tau). \quad (10.2)$$

Only this time, the distances d_n also depend on the vertical position of the source and observer, z_s and z :

$$\begin{aligned} d_n^2 &= (\mathbf{x}_{s,n} - \mathbf{x})^2 \\ &= (x_s - x)^2 + (y_s - y)^2 + (2Hn + (-1)^n z_s - z)^2 \\ &= d_0^2 + \begin{cases} 4H^2 n^2 - 4Hn(z - z_s) & n \text{ even} \\ 4H^2 n^2 - 4Hn(z + z_s) + 4zz_s & n \text{ odd} \end{cases}, \end{aligned} \quad (10.3)$$

where d_0 is the distance between source and observer.

This allows us to rewrite Eq. (10.2) as

$$G(t, \mathbf{x}; t_i, \mathbf{x}_i) = G_{\text{free}}(t, \mathbf{x}; t_i, \mathbf{x}_i) \cdot \underbrace{(A - B)}_{\equiv \vartheta(z, z_s, \sigma^2, H)}, \quad (10.4)$$

where

$$A = \sum_{n \text{ even}} e^{-\frac{2H^2 n^2 - 2Hn(z - z_s)}{\sigma^2}} \quad \text{and} \quad B = \sum_{n \text{ odd}} e^{-\frac{2H^2 n^2 - 2Hn(z + z_s) + 2zz_s}{\sigma^2}}. \quad (10.5)$$

A can be rewritten by setting $n = 2k$:

$$A = \sum_{k=-\infty}^{\infty} e^{-\frac{8H^2 k^2 - 4H(z-z_s)k}{\sigma^2}} \quad (10.6)$$

and B by setting $n = 2k + 1$, respectively,

$$B = e^{-\frac{2H^2 + 2zz_s - 2H(z+z_s)}{\sigma^2}} \sum_{k=-\infty}^{\infty} e^{-\frac{8H^2 k^2 - 4H(z+z_s)k + 8H^2 k}{\sigma^2}}. \quad (10.7)$$

Again, we can identify both sums in Eq. (10.6) and Eq. (10.7) as Jacobi theta functions. This time, they are of the third kind, which does also have a corresponding product representation (Whittaker and Watson, 1996):

$$\vartheta_3(\ell, \xi) = \sum_{k=-\infty}^{\infty} e^{-\ell k^2} e^{\xi k} = K(\ell) \cdot \prod_{k=1}^{\infty} [1 + 2e^{-\ell(2k-1)} \cosh(\xi) + e^{-\ell(4k-2)}], \quad (10.8)$$

where $K(\ell) = \prod_{k=1}^{\infty} (1 - e^{-2\ell k})$.

By comparing the functions A and B in Eqs. (10.6) and (10.7) with the functions in Eq. (10.8), we find that

$$\begin{aligned} \vartheta(z, z_s, \sigma^2, H) = K\left(\frac{8H^2}{\sigma^2}\right) \cdot \left[\vartheta_3\left(\frac{8H^2}{\sigma^2}, \frac{4H(z-z_s)}{\sigma^2}\right) \right. \\ \left. - e^{-\frac{2H^2 + 2zz_s - 2H(z+z_s)}{\sigma^2}} \vartheta_3\left(\frac{8H^2}{\sigma^2}, \frac{4H(z+z_s) - 8H^2}{\sigma^2}\right) \right]. \end{aligned} \quad (10.9)$$

The evaluation of the emerging third type theta functions via the product formula, Eq. (10.8), is very fast. As in Sec. 8.1.3, we find that the evaluation of 8 factors is sufficient to achieve a satisfactory accuracy for the suppression. The boundary condition leads to a strong suppression of the Green's function close to the boundaries at $z = \pm H$ and if the diffusion length is similar to the distance of the source from a boundary.

Cosmic-ray model parameters

For the diffusion coefficient, we will assume the CRINGE model diffusion coefficient (see Sec. 8.1.2, Eq. (8.2)) and a halo height of $H = 4$ kpc, as in Part II. The corresponding source injection spectrum is also the same as in Part II. This includes the source spectrum normalisation that we discussed in Sec. 8.1.4.

We also implement the causality condition that we motivated in Sec. 7.3.4. Further, we relax our assumption of a spatial point source injection. We assume that the sources have a finite extension of roughly 10 pc, which is similar to the size of a typical SNR at the time when it releases most of its accelerated cosmic rays during the ST phase (see Eq. (5.115)). We implement this by adding a constant $\sigma_0^2 = (10 \text{ pc})^2$ to the Green's functions' variance parameter (definition in Eq. (7.28))¹.

10.1.2 Source injection and near-source transport

We consider different scenarios for the injection of cosmic rays by their sources and their transport in the near-source region. In Part II, we already introduced the *burst-like* injection scenario, where all cosmic rays escape the source at the same time, as well as the *CREDIT* model of *energy-dependent escape* from sources (Stall et al., 2025). We will also consider a model of *time-dependent diffusion* in the near-source region (Kaci and Giacinti, 2025), which we introduce in this section.

In Chap. 5, especially in Secs. 5.7 and 5.8, we have already discussed how the near-source transport of cosmic rays can be significantly influenced by self-generated turbulence through streaming instabilities. It has been suggested that this increased level of turbulence can lead to a strong suppression of the diffusion coefficient in the vicinity of the sources (e.g., Fang et al., 2018; Jacobs et al., 2022; Jacobs, 2024; Mukhopadhyay and Linden, 2022). Following Kaci and Giacinti, 2025, it is possible to model this effect in a simplified way by assuming a time-dependent diffusion coefficient. The correspondence between the time after escape and the spatial distance from the source is given by the diffusion length (see Eq. (7.2)).

The diffusion is suppressed to a rigidity-independent constant diffusion coefficient $\kappa_{\text{start}} = 10^{28} \text{ cm}^2 \text{ s}^{-1}$ for a time $t_{\text{change}} = 10 \text{ kyr}$. This is in line with models of self-generated turbulence around some cosmic-ray sources, which find even smaller approximately constant values for the diffusion coefficient at rigidities below some tens of TV (Mukhopadhyay and Linden, 2022). The corresponding diffusion length after this time is given by $r_{\text{change}} \simeq 50 \text{ pc}$, which marks a typical size of low-diffusion zones according to Fang et al., 2018. After the time t_{change} , the standard rigidity-dependent diffusion coefficient, Eq. (8.2), sets in.

¹We checked that the implementation of the same finite source extent does not alter the results we presented in Part II. The reason for its inclusion in the calculation of GDEs is that, in contrast to the calculation of the cosmic-ray spectrum at just one position, the calculation of GDEs requires the evaluation of cosmic-ray intensities at a large number of grid points, which massively increases the potential influence of computational artefacts due to a false modelling of the source extent.

We implement this by assuming the following diffusion coefficient in this scenario (Stall and Mertsch, 2026):

$$\kappa_{\text{TDD}}(\mathcal{R}, t') = \begin{cases} \kappa_{\text{start}} & \text{if } t' < t_{\text{change}} \\ \kappa(\mathcal{R}) & \text{if } t' \geq t_{\text{change}} \end{cases}, \quad (10.10)$$

where $\kappa(\mathcal{R})$ is defined as in Eq. (8.2), and $t' = t - t_{\text{esc}}$ is the time since the escape of the respective particle of rigidity $\mathcal{R}$. Using Eq. (7.28), i.e., integrating this over the time since escape, the variance, $\sigma^2(\mathcal{R}, t | t_s)$, entering the Green's function, Eq. (7.27), can easily be calculated.

To summarise, we implement three different source injection and near-source transport scenarios:

1. *Burst-like* scenario: Protons of all rigidities escape the source at the same time and experience a time-independent diffusion coefficient.
2. *Energy-dependent escape* scenario: There is a time-dependent injection of protons by the source, where high-rigidity protons get injected earlier, but they experience a time-independent diffusion coefficient. We choose $\mathcal{R}_b = 10$ TV as a representative parameter.
3. *Time-dependent diffusion* scenario: Protons of all rigidities escape the source at the same time, but experience a time-dependent diffusion coefficient.

10.1.3 Cross-sections and absorption

We gave an overview of the relevant cross-section models in Sec. 6.3. For this study, we will use the AAFRAG cross-sections (Kachelrieß et al., 2019), which are based on the hadronic interaction model QGSJET-II-04m. They are available via the Python package AAFRAGPY (Koldobskiy et al., 2021). The scaling factors to account for heavier nuclei in both cosmic rays and the interstellar gas are as described in Sec. 6.3.

The cross-sections determine the range of cosmic-ray energies that contribute significantly to the production of γ -rays and neutrinos at a given energy (for conversions between energy and rigidity, see App. A). To understand this better, we repeat the formula for the hadronic γ -ray emissivity, Eq. (6.5):

$$\epsilon(\mathbf{x}(l, b, s), E) = 4\pi n_{\text{H}}(\mathbf{x}) \int_E^\infty dE' \frac{d\sigma}{dE}(E', E) \Phi(\mathbf{x}, E'). \quad (10.11)$$

We want to use this to identify the range of cosmic-ray proton energies that we need to consider for the calculation of the GDEs at different γ -ray energies. Assuming a power-law cosmic-ray proton intensity, $\Phi(E') \propto E'^{-\nu}$, we can determine the relative

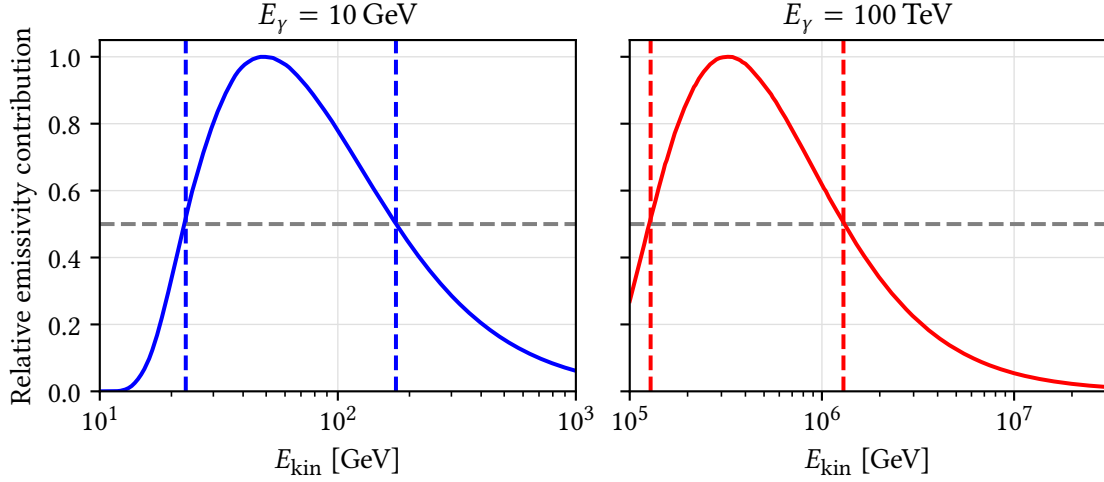


Figure 10.1: Contributions to the hadronic γ -ray emissivity. This plot shows the relative contribution of cosmic-ray proton intensities in different (logarithmic) energy bins to the hadronic γ -ray emissivity at 10 GeV (left) and 100 TeV (right), respectively. We assume a cosmic-ray proton power-law spectrum with a spectral index of $\gamma = 2.7$. The dashed lines indicate the full width at half maximum of the distributions. The y-axis is normalised to the maximum of each distribution.

contributions per logarithmic energy bin to the hadronic γ -ray emissivity at energy E . We show the resulting distributions for $\gamma = 2.7$ at 10 GeV and 100 TeV in Fig. 10.1. We see that contributions are significant in a range from roughly 2 to 200 times the γ -ray energy.

For the full-sky GDE simulations, we choose to calculate diffuse emissions at 12 energies around 10 GeV (in the range [5, 16] GeV) and at 12 energies around 100 TeV (in the range [40, 110] TeV). For this, we evaluated the cosmic-ray proton intensity at 48 kinetic energies for both ranges, accordingly ([10, 900] GeV and [80, 7000] TeV) (Stall and Mertsch, 2026).

We also account for the absorption of γ -rays due to pair production upon interactions with the Interstellar Radiation Field (ISRF), as discussed in Sec. 6.4. We include the CMB, Eq. (6.12), as well as the infrared dust emissions according to Porter et al., 2018. For the latter, we use the absorption coefficients provided in GALPROP v.57 (Porter et al., 2022). We use the R12 model based on Robitaille et al., 2012 which also assumes a spiral structure as we do for the source distribution. To make it consistent with our Galactocentric Solar distance of $R_\odot = 8.0$ kpc instead of the 8.5 kpc there, we rescale the model accordingly. Assuming the same total energy in infrared radiation in the Galaxy, the spectral number density of the radiation field produced by the dust emissions is scaling by a factor of $(8.5/8)^3$ while the path length element in the absorption coefficient, Eq. (6.11) scales as $(8/8.5)$. We interpolate the resulting optical depth to the spatial and energy grid described above and include it in our calculations of the GDEs.

10.2 Efficient calculation of diffuse emissions

The computation of the GDEs proceeds in two main stages: first, we determine the cosmic-ray proton intensity on the spatial grid described above, and second, we determine the hadronic emissivity and evaluate the LOS integrals to obtain the diffuse emissions. Depending on the energy range, the cosmic-ray intensity at a given position in the Galaxy can receive contributions from more than tens of millions of individual sources. The contributions from all of those need to be summed efficiently. In addition, the calculations of the GDEs require multiplications of large multidimensional arrays and summations along some of their axes to perform the LOS integrations.

Overall, the calculation of GDEs for many different source realisations is computationally rather expensive, but it is also an *embarrassingly* parallel problem. As in Sec. 8.1.5, we tackle this challenge by efficiently using the paradigms of array programming (Harris et al., 2020) together with the exploitation of extremely powerful hardware accelerators. To this end, we use the PYTHON package JAX (Bradbury et al., 2018), which allows us to perform the necessary calculations on GPUs. Further speed-ups are achieved by just-in-time (jit) compilation of functions which are called multiple times in the Monte Carlo simulations.

While this already leads to tremendous speed-ups compared to conventional CPU-based calculations, a full-scale Monte Carlo simulation of the GDEs remains computationally challenging if we require large sample sizes of source realisations. To further increase the statistical power of our analysis, we have a closer look at the contributions of cosmic-ray sources of different ages to the cosmic-ray intensity and, more importantly, to its variations across different source realisations. Younger sources contribute more strongly to the variations in the cosmic-ray intensity, as the cosmic rays injected by them are still more localised. Because of this, the cosmic-ray intensity produced by young sources also varies on much smaller spatial scales compared to that produced by older sources. We can exploit both of these aspects to reduce the computational cost of our Monte Carlo simulations.

First, we restrict the range of source ages that must be recomputed in the Monte Carlo simulation. Cosmic rays injected sufficiently long ago give rise to a nearly homogeneous background intensity that varies only minutely across different source realisations, since the precise source locations become less relevant once the cosmic rays diffused through the Galaxy for a long enough time. As a function of rigidity, we impose an upper limit on the ages that enter the foreground component:

$$t_{\text{bkg}}(\mathcal{R}) = \min [20, 5 \kappa(1 \text{ TV}) / \kappa(\mathcal{R})] \text{ Myr}. \quad (10.12)$$

This time is determined heuristically by demanding that the cosmic-ray intensity does not vary a lot across different source realisations for sources older than this age. In terms of GDEs, $\mathcal{O}(30 \%)$ of the total intensity is produced by this background component. Due to

the scaling with the rigidity-dependent diffusion coefficient, this stays roughly constant for different energies.

Sources older than t_{bkg} contribute to a background intensity that we can calculate as the mean of a low number of source realisations. The maximum age for the sources contributing to the background component, t_{max} , is chosen as roughly five times the time it takes for cosmic rays to diffuse to the halo boundaries, i.e.,

$$t_{\text{max}}(\mathcal{R}) = 5 \frac{H^2}{\kappa(\mathcal{R})}. \quad (10.13)$$

Second, we only need to evaluate the cosmic-ray intensity produced by young sources on a fine spatial grid to capture the small-scale variations properly. For older sources, we lower the resolution of the HEALPix $\times \log(R)$ -grid we introduced above. Instead of using $N_{\text{side}} = 64$ for the angular resolution, we use $N_{\text{side}} = 32$, which reduces the number of spatial grid points by a factor of four. We also change the number of radial shells from 388 to 199 by setting the minimum radial distance between shells to 50 pc. The spacing is linear until the logarithmic spacing sets in once it is larger than that. The split between young and old sources is defined at $t_{\text{split}} \approx 100$ kyr. We implement this by evaluating the cosmic-ray intensity from a total of $f_{\text{SN}} \cdot t_{\text{split}}$ contributing sources for each rigidity on the fine grid, while the contributions from older sources are evaluated on the coarse grid. We verified that these two approximations do not introduce artefacts in our simulations.

10.3 Galactic diffuse emission distributions

Discrete sources affect the GDEs by locally modifying the cosmic-ray intensity in their surroundings. If we treat source positions and ages as random variables with specified Probability Density Functions (PDFs), the resulting cosmic-ray intensities become stochastic quantities as well, as we discussed in Chap. 7. Our goal is to characterise their PDFs and to understand how these influence the distribution functions of the GDEs. The distribution of cosmic-ray intensities in a burst-like model has been analysed in detail by Genolini et al., 2017, which we discussed in Sec. 7.4.2. We will see that these results can be used to predict GDE intensity distributions and show that those are expected to either follow stable laws as well or, depending on the chosen LOS, can be represented as a combination of normal distributions and stable laws (Stall and Mertsch, 2026).

We employ the results from Sec. 7.4.2 to obtain informed expectations for the distributions of the GDEs based on the cosmic-ray intensity distributions. To this end, we approximate the diffuse intensity $J(l, b, E)$ defined in Sec. 6.1, Eqs. (6.2) and (6.3) (see also Eq. (10.11)). For simplicity, we restrict ourselves to the contribution from a single cosmic-ray energy

E' and approximate the LOS integral as a discrete sum over M voxels (i.e., volume elements),

$$J(l, b, E) \simeq \sum_{i=1}^M n_{\text{gas}}(\mathbf{x}_i) \frac{d\sigma}{dE}(E', E) \Phi(\mathbf{x}_i, E') = \sum_{i=1}^M n_{\text{gas},i} \sigma \Phi_i, \quad (10.14)$$

where, for brevity, we denote by σ the differential cross-section, by $n_{\text{gas},i}$ the gas density, and by Φ_i the cosmic-ray intensity in voxel i .

In the following, we exploit two basic properties of random variables that are distributed according to stable laws, S . First, if $X \sim S(\alpha, \beta, \gamma, \delta)$, then for any $a \neq 0$ one has $aX \sim S(\alpha, \text{sign}(a)\beta, |a|\gamma, a\delta)$. Second, if $\{X_i\}$ are independent random variables with $X_i \sim S(\alpha, \beta, \gamma_i, \delta_i)$, then their sum is again stable:

$$\sum_i X_i \sim S\left(\alpha, \beta, \left(\sum_i \gamma_i^\alpha\right)^{1/\alpha}, \sum_i \delta_i\right). \quad (10.15)$$

Note that this requires all X_i to share the same index of stability, α (Nolan, 2020).

These relations allow us to anticipate the form of the GDE distributions along different LOS. We first consider LOS that predominantly traverse the Galactic disk, where we expect the underlying cosmic-ray intensity distributions to follow stable laws. We assume that the voxelisation used in the approximation in Eq. (10.14) is coarse enough such that the cosmic-ray intensities in distinct voxels can be treated as statistically independent, and that in each voxel the intensity distribution is well described by a stable law with a common index $\alpha \in \{4/3, 5/3\}$ (see Sec. 7.4.2 and Genolini et al., 2017). Under these assumptions, the intensity along such LOS should follow

$$P(J(l, b, E)) \simeq S(\alpha, 1, \gamma_{\text{tot}}, \bar{J}) \quad \alpha \in \{4/3, 5/3\}, \quad (10.16)$$

where γ_{tot} denotes the overall scale parameter and $\bar{J}$ is the mean GDE intensity at the given energy, obtained as the sum of the mean contributions from all voxels.

The total scale parameter γ_{tot} receives contributions from all voxels along the LOS as well as from cosmic rays of different energies.² We do not attempt to predict γ_{tot} analytically. Instead, we determine it by fitting a stable-law distribution to the simulation results. For this fit, we employ a quantile-based estimation technique (McCulloch, 1986), as implemented in SciPy (Virtanen et al., 2020), while fixing the remaining parameters of the stable distribution according to Eq. (10.16).

Next, we turn to LOS that extend out of the Galactic plane. According to the approximation Eq. (10.14), the GDE intensity is built up from contributions of several voxels along

²Note that Eq. (10.15) cannot be applied directly to different cosmic-ray energies, because the intensities at different energies are not independent. Nonetheless, the qualitative statement that the effective scale parameter increases when adding contributions from multiple energies continues to hold.

such a LOS. Only a small subset of these voxels contains nearby cosmic-ray sources and therefore contributes to the GDE intensity through cosmic-ray intensity components following a stable law. In contrast, voxels located farther away from the disk host almost no sources. Cosmic-ray intensities in these regions, and hence their GDE contribution, are better approximated by normal distributions. The full GDE distribution is therefore anticipated to be a mixture of stable and normal distributions. To capture this behaviour, we assume that a fraction f of the contributions to the GDEs along the LOS is governed by cosmic-ray intensities following a stable law, while the remaining fraction $(1 - f)$ follows a normal distribution $\mathcal{N}(\mu, \sigma)$, as usual defined as

$$\mathcal{N}(\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \quad (10.17)$$

The total GDE intensity is then the sum of two random variables. Because these two components have different indices of stability, the simple stability property for sums does not apply, and one has to use the general convolution formula (see, e.g., Feller, 1971) to obtain the resulting distribution:

$$P(J(l, b, E)) \simeq S(\alpha, 1, f^{1/\alpha} \cdot \gamma_{\text{tot}}, f \cdot \bar{J}) * \mathcal{N}((1 - f) \cdot \bar{J}, (1 - f)^{1/2} \cdot \sigma_{\text{sim}}), \quad (10.18)$$

where γ_{tot} is the same scale parameter as for LOS in the Galactic centre region and σ_{sim} denotes the sample standard deviation obtained from the simulation. While the weighting of the mean values with f and $(1 - f)$ is straightforward, the appropriate scaling of the corresponding scale parameters is less obvious. We adopt the weighting scheme in Eq. (10.18) guided by the properties of sums of random variables that follow stable-law distributions, Eq. (10.15).

In Sec. 11.4, we confront these analytical expectations with the Monte Carlo results. Some deviations from a purely stable-law behaviour are anticipated because of the implementation of a causality condition and the simplifying assumptions underlying our model (Stall and Mertsch, 2026). Moreover, it is plausible that a description of the GDE distributions in terms of pure stable laws, or mixtures of stable and normal distributions, will not perform equally well in all scenarios of source injection and near-source transport.

10.4 Stochastic sky maps

Before concluding this chapter and going to the results in Chap. 11, we want to give an overview of how we produce the stochastic sky maps of the GDEs, and how these can be understood in the context of stochastic discrete sources. For this, we have produced a schematic overview that illustrates the *workflow* of our stochastic GDE modelling (see Fig. 10.2). In contrast to a smooth source model, where the sources are modelled by a smooth source term, we sample discrete source positions and ages from a source

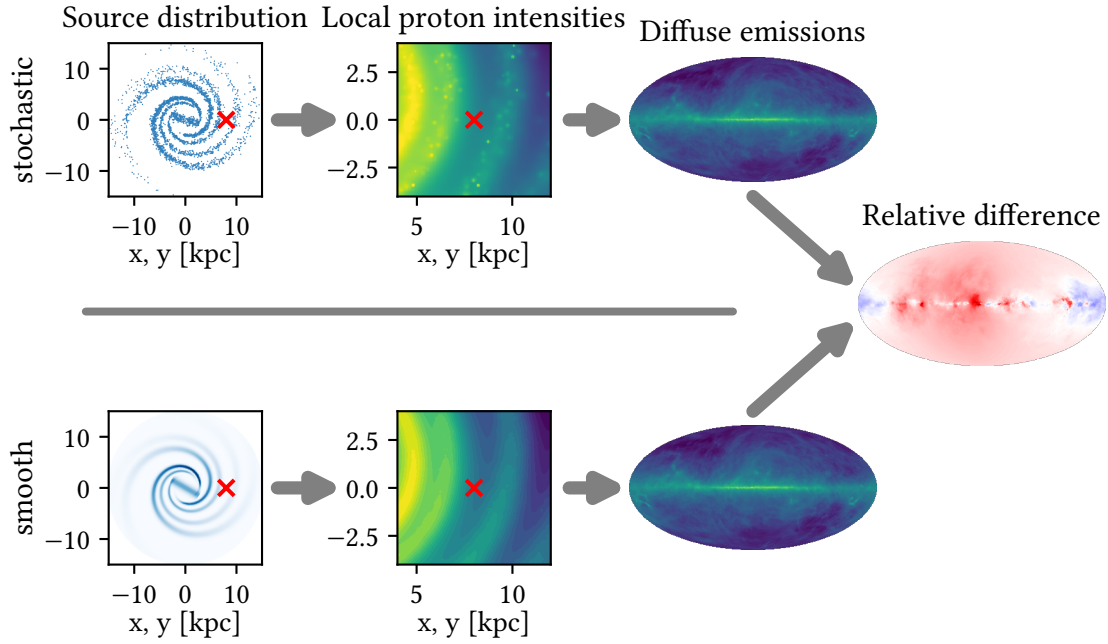


Figure 10.2: Schematic overview of stochastic diffuse modelling. This schematic illustrates the stochastic cosmic-ray source model (upper panels) and contrasts it with the smooth source model (lower panels). On the left, the source distributions are shown: discrete point sources in the stochastic case versus a smooth continuum in the smooth case, with the Sun marked by a red cross. To the right, the resulting cosmic-ray proton intensities within a few kpc of the Sun are displayed, where the stochastic scenario yields a patchier pattern with visible imprints of individual sources. From these cosmic-ray proton intensities, we calculate the hadronic GDEs for both models. Their relative difference highlights the signatures of discrete sources and quantifies their impact.

distribution. As we have discussed in Chap. 7, this leads to fluctuations of the predictions from a stochastic source model around the predictions of a corresponding smooth source model. Fig. 10.2 shows how the discrete sources lead to fluctuations in the cosmic-ray intensities around the smooth model predictions and how these fluctuations translate into fluctuations of the GDEs.

In Chap. 7, Eq. (7.36), we have discussed how the ensemble mean of the cosmic-ray intensity produced by a stochastic source model corresponds to the prediction of a smooth source model. In analogy to this, the ensemble mean of the GDEs produced by our stochastic source model also corresponds to the GDEs predicted by a smooth source model, as the calculation of the GDEs is a linear operation on the cosmic-ray intensities (see Eqs. (6.2) and (6.3)). As stated in Eq. (7.37), we can therefore obtain the smooth source model predictions by averaging over many stochastic source realisations:

$$J^{\text{smooth}} = \langle J \rangle \approx \bar{J}, \quad (10.19)$$

where $\bar{J}$ denotes the sample mean over many stochastic source realisations.

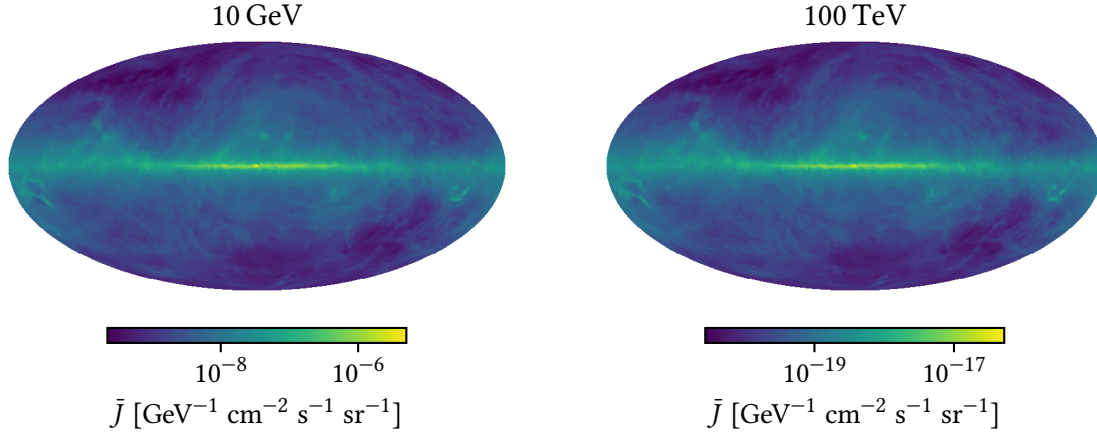


Figure 10.3: Sky maps of total diffuse emissions. We show the hadronic GDEs at 10 GeV (left) and 100 TeV (right) for a smooth source model, obtained as the sample mean over 1000 realisations in the burst-like scenario. The colour scale is logarithmic to better visualise the full-sky emission range.

To illustrate the GDE, we use sky maps. For our chosen HEALPix (Górski et al., 2005) pixelisation of the sky, there is a suitable way to visualise the full-sky maps in a so-called *Mollweide projection*. For this, we use the HEALPY Python package (Zonca et al., 2019). In Fig. 10.3, we show the hadronic GDEs at 10 GeV and 100 TeV for a smooth source model, obtained as the sample mean over 1000 realisations in the burst-like scenario. Many structures are visible, especially the Galactic plane with its enhanced gas density, leading to increased hadronic emissions in that region (compare to maps in Söding et al., 2025 or to *Fermi*-LAT data, Fig. 3.5). The morphology of the GDEs is similar at both energies, as the underlying gas distribution is the same. The main difference is the overall intensity, which is lower at 100 TeV due to the steeply falling cosmic-ray spectrum.

The impact of the discrete source models on the GDEs is most clearly seen by comparison with the GDE predictions of the smooth source model. From the production mechanism described in Chap. 6, it follows that variations in the GDEs are driven by changes in the cosmic-ray intensity and are modulated by the Galactic gas density in the corresponding regions of the Galactic volume. Moreover, contributions from multiple source regions accumulate along a given LOS. In Fig. 10.4, we illustrate how regions of enhanced GDE emission can be associated with directions in which the cosmic-ray proton intensity is increased. The left panel shows the sky map of the relative deviation from the smooth model predictions at 100 TeV for one realisation in the burst-like scenario, where several excess regions appear along the Galactic plane.

As a concrete example, we focus on the most extended excess region, indicated by a grey circle, and analyse its origin using the projection of the cosmic-ray proton intensities at 1 PeV shown in the right panel of Fig. 10.4. Because of the typical inelasticity of pion

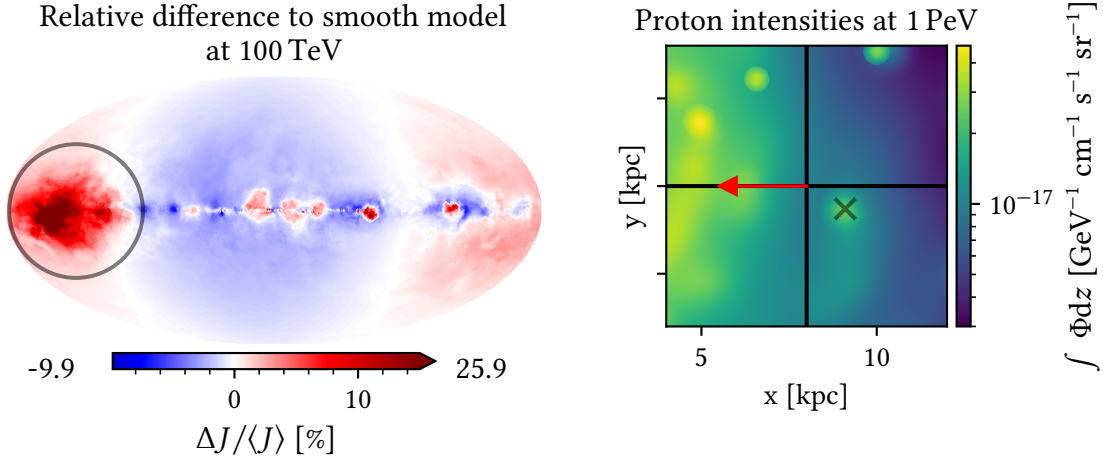


Figure 10.4: GDE and cosmic-ray proton intensity deviations. For a single realisation in the burst-like scenario, we show (left) the relative deviation of the GDEs from the smooth model and (right) the cosmic-ray proton intensities at 1 PeV around the Sun, integrated along the z -direction. The red arrow indicates the Galactic centre, located at the centre of the sky map on the left. The excess region marked by the grey circle in the sky map corresponds to the enhanced proton intensities marked by the crosses in the right panel.

production, the cosmic-ray proton intensity at this energy is a major contributor to the GDEs at 100 TeV (see Fig. 10.1). For the two-dimensional projection, we integrate the intensities along the vertical direction so that enhanced cosmic-ray proton intensities at $z \neq 0$ are also included. The increased GDEs in the marked region can then be traced back to stronger proton intensities around the position indicated by the grey cross. The morphology of this enhancement in the cosmic-ray proton intensity identifies it as the imprint of a relatively young source located in that region.

It should be noted that the excess GDEs exhibit considerably more structure than the corresponding excess in the cosmic-ray proton intensities, reflecting the inhomogeneous Galactic gas distribution. To illustrate this, we refer to Fig. E.1 in App. E.1, where we show the integrated cosmic-ray intensities along each LOS for one source realisation in the burst-like scenario. Comparing this to the corresponding GDE map in Fig. 10.3, it becomes clear that the gas is responsible for the more structured morphology of the GDEs. This already indicates that a simple association of excess regions with individual sources is, in general, challenging. Both the spatial extent and the maximum relative difference of an excess region depend in a degenerate way on the age and distance of the contributing SNRs and on the Galactic gas density in the near-source region. A robust inference of source parameters from individual excess regions would require more advanced morphological analyses and is beyond the scope of this work.

11

Stochastic diffuse emissions

In this chapter, we present our investigations of the effects of discrete cosmic-ray sources Galactic Diffuse Emissions (GDEs). Most of the results presented here have been published in Stall and Mertsch, 2026. We will discuss them in the context of this thesis, especially with respect to the discussions in Chaps. 6, 7 and 10.

We ran extensive Monte Carlo simulations of GDE produced by discrete cosmic-ray sources as described in Chap. 10. Here, we contrast the results of three different source injection and near-source transport scenarios (see Sec. 10.1.2) and an additional variation of the burst-like scenario with a halved source rate:

- The burst-like scenario,
- the energy-dependent escape scenario,
- the time-dependent diffusion scenario, and
- the burst-like scenario with a halved source rate.

There are multiple aspects of the GDEs produced by discrete sources that we want to investigate. For an analysis of the morphology of the GDEs and how discrete sources influence them, we present sky maps of the GDEs at different energies for our different model scenarios in Sec. 11.1. In Sec. 11.2, we study the most extreme relative deviations from the smooth model predictions that can be expected in the different models. For these analyses, we produced 1000 realisations of the whole diffuse sky for each scenario at various γ -ray energies. This allows us to understand the morphology of the GDEs and link fluctuations in the simulated GDEs to individual sources as pointed out in Sec. 10.4. Reasons for the differences between the burst-like and the energy-dependent escape scenario are discussed in Sec. 11.3.

In Sec. 11.4, we present tests of the predicted distributions of GDE intensities along selected LOS (see Sec. 10.3) with highly increased statistics. For this, we ran 10^6 realisations along each of the selected LOS to obtain well-resolved intensity distributions. In Sec. 11.5, we investigate GDE spectra in selected sky windows used by experimental collaborations to quantify the systematic modelling uncertainties coming from discrete sources. For those, we produced 1000 realisations for each sky window for each scenario in an extended energy range. We discuss the stochastic effects in longitude profiles of

the GDEs at a selected energy. Then, we investigate the correlation of GDE intensities at different energies in Sec. 11.6. Finally, we discuss our findings in Sec. 11.7.

11.1 First glimpse into the diffuse sky

With the procedures described in Chap. 10, we can calculate the GDEs at a range of energies for different source configurations in varying injection and near-source transport scenarios. In Sec. 10.4, we have explained how discrete sources can leave imprints on the GDEs, how these can be understood qualitatively, and how we can visualise them as sky maps. It is best to look at the relative deviations from the GDEs predicted by a smooth source model:

$$\frac{\Delta J}{\langle J \rangle} = \frac{J - \langle J \rangle}{\langle J \rangle}, \quad (11.1)$$

where J is the GDE intensity produced by discrete sources and $\langle J \rangle$ is the ensemble mean over many realisations, which is equivalent to the smooth model prediction. We approximate it by the sample mean over all 1000 realisations (see Eq. (10.19)). In this section, we present three specific realisations of the GDEs at two different energies, 10 GeV and 100 TeV, in each of the four scenarios (see Figs. 11.1 to 11.4).

Burst-like scenario

The sky maps presented in Fig. 11.1 illustrate the relative deviations of three out of the 1000 realisations in the burst-like scenario with respect to the smooth-model expectation, as defined in Eq. (11.1). In the vicinity of the Galactic plane, numerous compact areas with enhanced intensity are visible. These features originate from individual sources located close to the corresponding LOS, whose contributions to the GDEs are particularly strong (see Sec. 10.4). The colour scales are symmetric around zero and are saturated at $\pm 5\%$ for the lower-energy maps and at $\pm 15\%$ for the higher-energy maps. For each sky map, the largest absolute relative deviation is indicated at the ends of the corresponding colour bar. Even though the GDEs at both energies are generated by cosmic rays released by the same sources at identical times, the relative-difference maps corresponding to a given realisation exhibit distinct spatial patterns at different energies. This behaviour can be attributed to the substantially larger diffusion coefficient at 100 TeV compared to 10 GeV. As a result, although some correspondence between regions of enhanced emission can be identified in the maps at the two energies, these structures are generally more spatially extended in the higher-energy case (for example, the regions on the right-hand side of realisation 1).

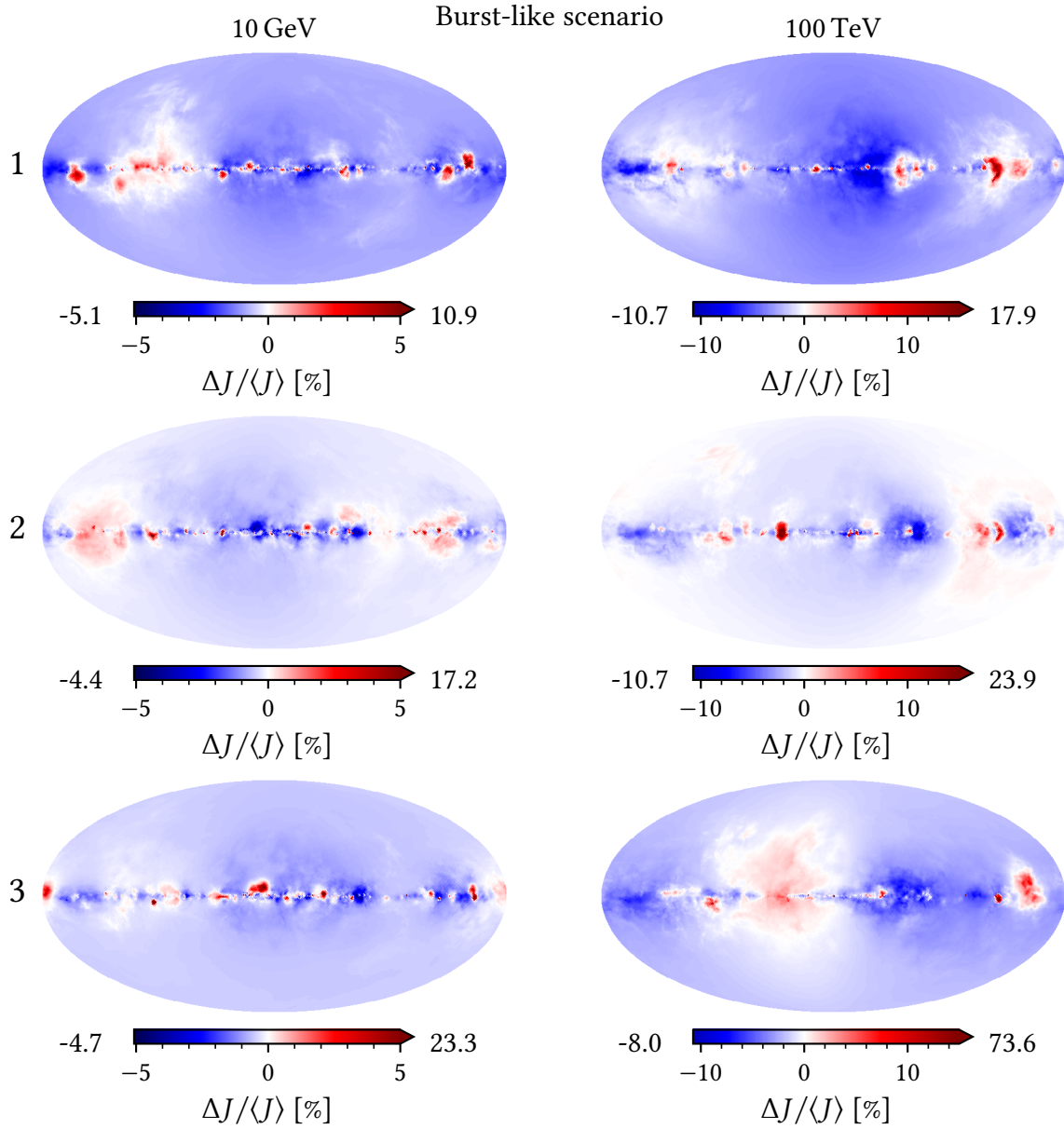


Figure 11.1: Three realisations, burst-like scenario. Sky maps at two energies are shown (10 GeV left, 100 TeV right). The panels show the relative deviations (see Eq. (11.1)) of three individual realisations. The colour bars are symmetric and saturate at ± 5 % (10 GeV) and ± 15 % (100 TeV). The numbers at their ends indicate the maximum relative deviations in each sky map.

Energy-dependent escape scenario

In the energy-dependent escape scenario illustrated in Fig. 11.2, any morphological similarity between sky maps at different energies is essentially lost. Because the injection of cosmic rays is time-dependent in this framework, the sources responsible for enhanced

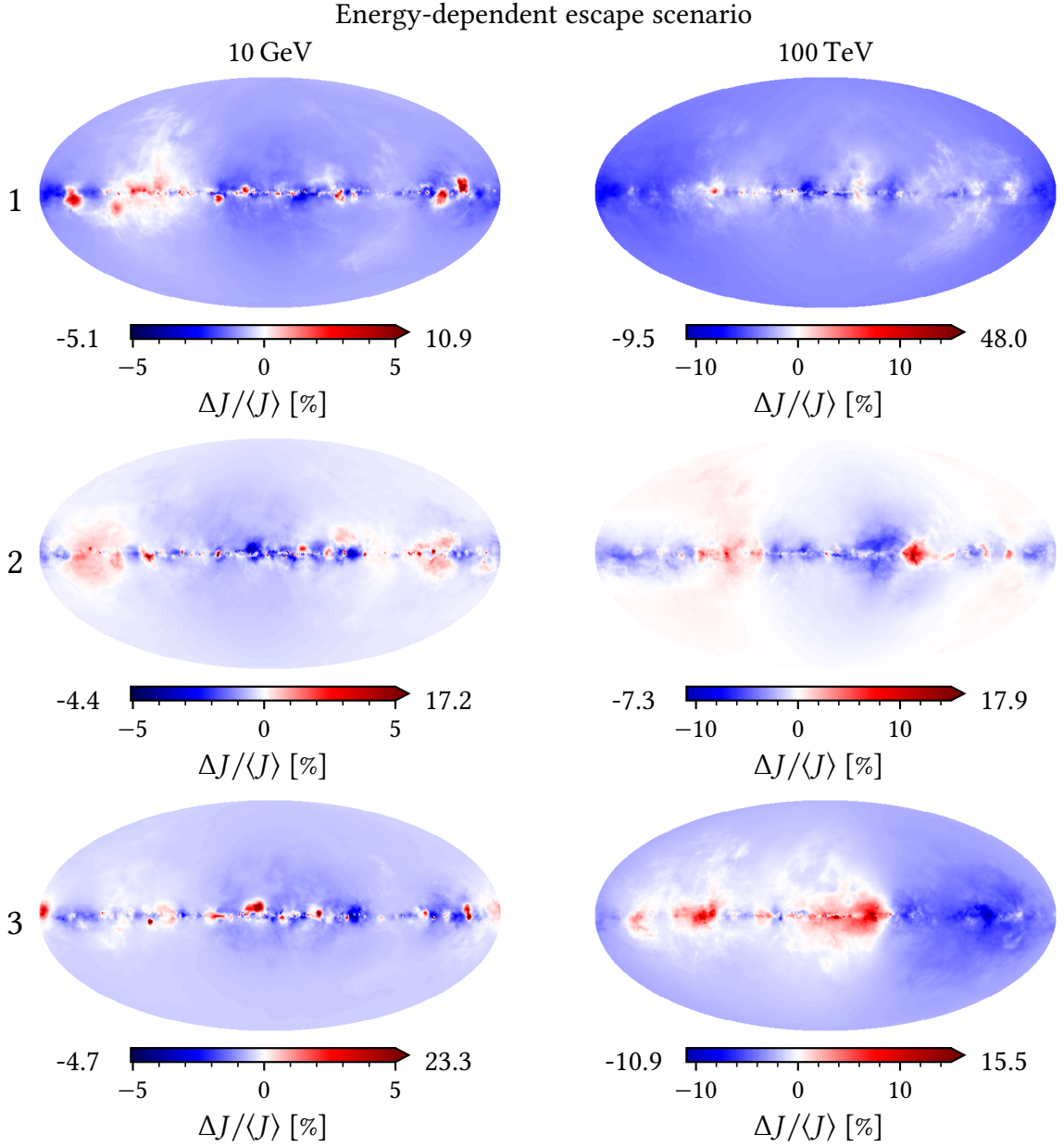


Figure 11.2: Three realisations, energy-dependent escape scenario, as in Fig. 11.1.

GDEs at 10 GeV are generally not the same as those dominating the emission at 100 TeV. The figure presents the relative-deviation maps for the same three source realisations already shown in Fig. 11.1. With the choice of $\mathcal{R}_b = 10$ TV, the resulting GDEs at 10 GeV coincide with those obtained in the burst-like scenario displayed in Fig. 11.1. In contrast, at 100 TeV the energy-dependent escape scenario yields GDEs patterns that differ markedly from their burst-like counterparts. Moreover, the high-energy relative deviations are somewhat less pronounced than those found in the burst-like case shown in Fig. 11.1. This reduction follows from the time-dependent injection, which confines the

impact of an individual source to a comparatively narrow energy interval (see discussion in Part II and Stall et al., 2025). While this mechanism can still produce strong cosmic-ray intensity enhancements, the restriction to a limited energy range results in a milder overall influence of individual sources on the GDEs in this scenario. A more detailed comparison between the burst-like and energy-dependent escape scenarios is given in Sec. 11.3.

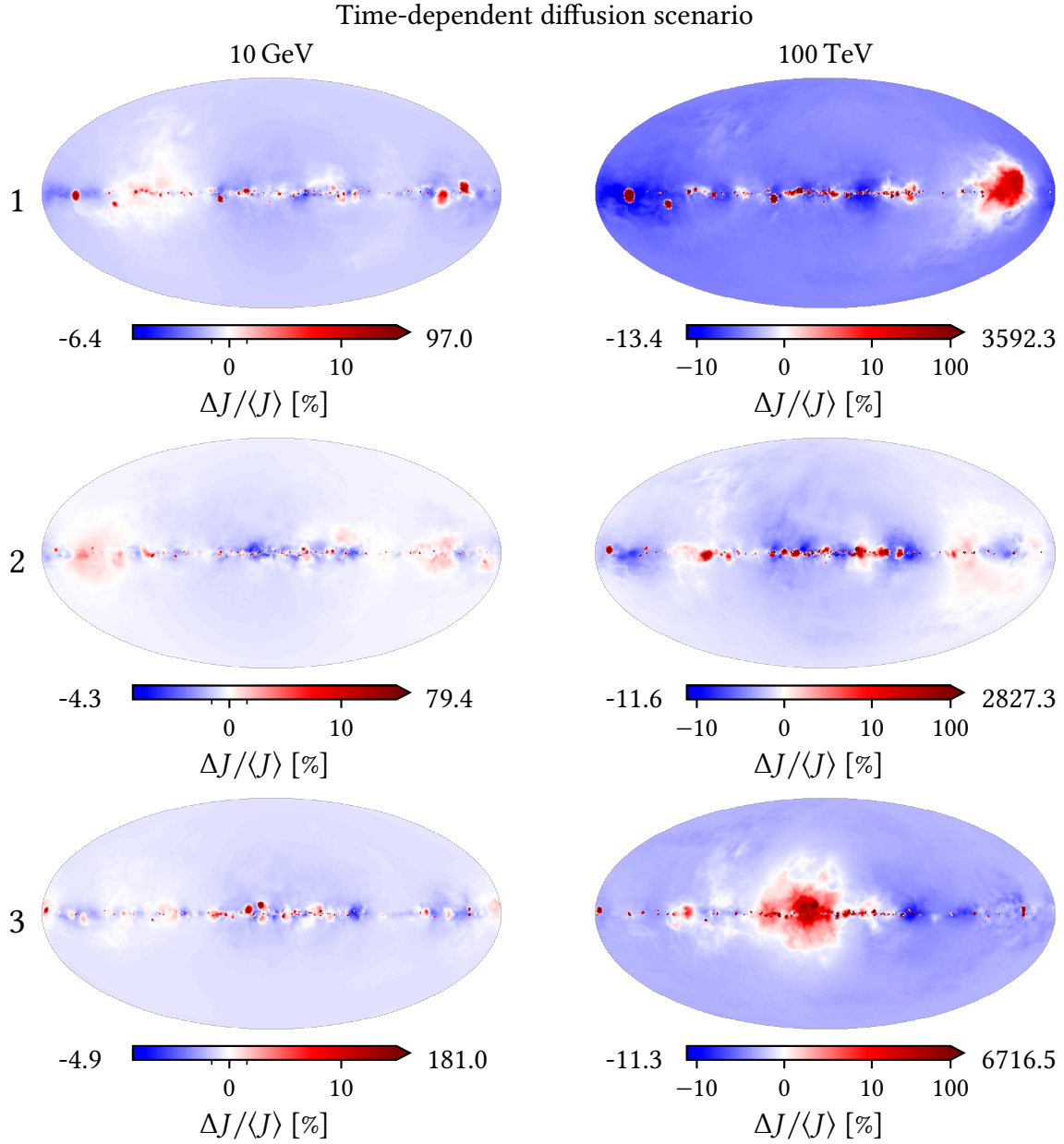


Figure 11.3: Three realisations, time-dependent diffusion scenario, as in Fig. 11.1. Due to more extreme relative deviations in this scenario, different colour bars are used. The colour bars are symmetric and saturate at ± 50 % (10 GeV) and ± 100 % (100 TeV). They are linear up to ± 5 % (10 GeV) and ± 10 % (100 TeV) and logarithmic beyond that.

Time-dependent diffusion scenario

Among the scenarios considered, the time-dependent diffusion case shown in Fig. 11.3 exhibits the largest relative departures from the smooth-model expectation. To accommodate these higher amplitudes, distinct colour scales are employed. They are symmetric and saturate at $\pm 50\%$ at 10 GeV and at $\pm 100\%$ at 100 TeV. In addition, the colour mapping is linear up to $\pm 5\%$ for 10 GeV and up to $\pm 10\%$ for 100 TeV, becoming logarithmic beyond these thresholds. At the lower energy, relative deviations exceeding 100% are present, while at the higher energy deviations can reach several thousand percent. Despite these large amplitudes, a clear morphological correspondence between the sky maps at the two energies can be identified. Furthermore, the spatial structures in this scenario resemble those seen in the burst-like scenario, reflecting the use of identical source positions and ages.

The strong confinement of cosmic rays in the vicinity of their sources results in a highly inhomogeneous distribution of the GDEs (Kaci and Giacinti, 2025). Localised enhancements of diffuse γ -ray emission on such small angular scales challenge the distinction between diffuse and discrete emission components. It has been suggested that the number of source-associated enhancement regions detected by LHAASO could place constraints on both source rates and models of near-source transport. Observing fewer than a certain number of such regions would either disfavour the time-dependent diffusion scenario or imply a lower rate of PeVatrons, i.e., sources capable of accelerating particles to energies above 1 PeV. In the implementation adopted here, a time-dependent diffusion scenario with a PeVatron rate of 100%, i.e., every Galactic source is a PeVatron, leaves clear imprints of the near-source transport model. Reducing the source rate would decrease the number of excess regions, but a systematic exploration of these cases is beyond the scope of this work. For a more comprehensive discussion, we refer the reader to Kaci and Giacinti, 2025.

Burst-like, halved source rate scenario

Finally, we present three realisations of the burst-like, halved source rate scenario in Fig. 11.4. This scenario is identical to the burst-like scenario, except that the source rate is reduced by a factor of two to $f_{\text{SN}} = 0.015 \text{ yr}^{-1}$. Thus, it is most instructive to compare these sky maps to the ones shown in Fig. 11.1. The general morphology and the correspondence between the two energies is similar to the burst-like scenario, as expected.

We can see that there are stronger variations in the burst-like, halved source rate scenario. To illustrate this, we use the same colour bars in this figure as in Fig. 11.1. They tend to be more saturated than in the burst-like scenario, indicating that the deviations are more extreme in this scenario. Fewer sources lead to a higher stochasticity as the relative

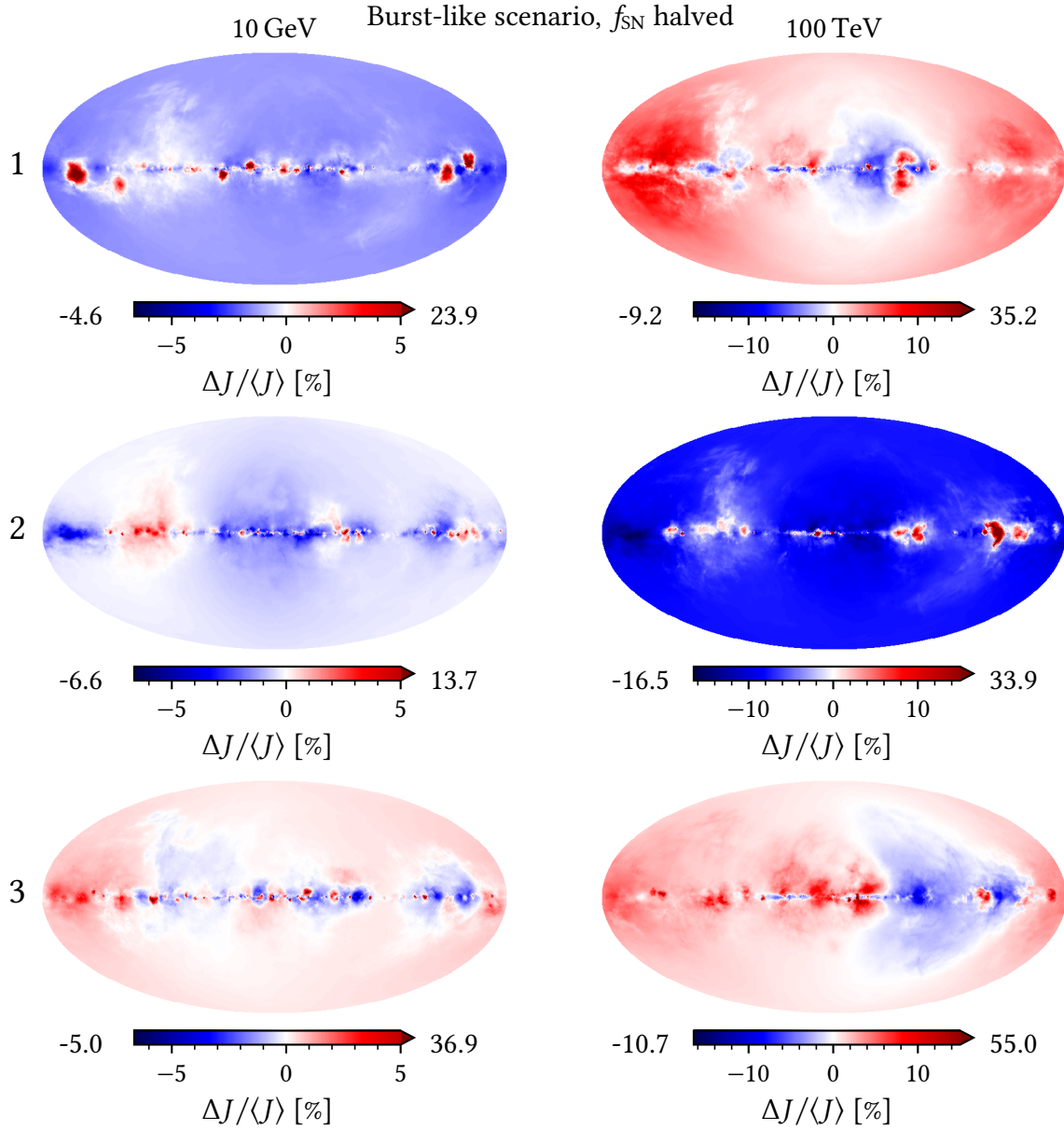


Figure 11.4: Three realisations, burst-like, halved source rate scenario, as in Fig. 11.1.

influence of every single source is increased and the presence or absence of a source close to a given LOS has a stronger effect. This is different from the results for reducing the source rate in the analysis in Part II (see Fig. 9.6 in Sec. 9.2.4). There, we concluded that an increase of the influence of individual sources by reducing the source rate is compensated by a smaller probability of having a young source close to the observer. This effect is apparently not dominating here, which is likely due to the large number of sources contributing strongly to GDEs along different LOS.

Summary

In this section, we compared sky maps of the GDEs produced by discrete sources in four different source scenarios. We have seen that the morphology of the GDEs can be understood as imprints of individual sources as described in Sec. 10.4. The comparison between GDEs at 10 GeV and 100 TeV revealed that the morphology can either be similar, as in the burst-like, the time-dependent diffusion, and the burst-like, halved source rate scenario, or very different, as in the energy-dependent escape scenario. This can be understood by considering the differences in the injection and near-source transport models. The scale of the relative deviations also varies strongly between the different scenarios, with the time-dependent diffusion scenario producing the most extreme deviations due to the strong confinement of cosmic rays close to their sources. Its extreme imprints can potentially be used to constrain this model (see Kaci and Giacinti, 2025). The stronger variations in the burst-like, halved source rate scenario compared to the burst-like scenario illustrate the effect of a reduced source rate on the stochasticity of the GDEs. We also mentioned the maximum relative deviation that can be expected in a sky map and how they can be understood in the different source models, but we discuss this in more detail in the following section.

11.2 Most extreme deviations

The sky maps presented in Sec. 11.1 already demonstrate that the amplitude of the relative deviations varies substantially with the assumed injection prescription, the near-source transport model, and the particular realisation of the source population. In the following, we quantify the dynamic range covered by our simulations by extracting, for each sky map and each realisation, the largest relative deviation among all pixels, i.e., LOS, and compiling their occurrence in histograms. This approach provides an estimate of the maximum GDE enhancements that can arise from discrete sources. As before, results are shown for GDE energies of 10 GeV and 100 TeV. To accommodate the wide spread of values obtained in the simulations, the histograms are displayed on a logarithmic x -axis. For improved readability, only values up to the 99th percentile are shown, such that the ten most extreme realisations are omitted. The mean ($\bar{X}$) and median ($\tilde{X}$) of the relative deviations, expressed in percent, are explicitly indicated in each histogram.

Burst-like scenario

The distributions corresponding to the burst-like scenario are shown in Fig. 11.5. For both energies, the histograms are clearly asymmetric and exhibit pronounced tails towards

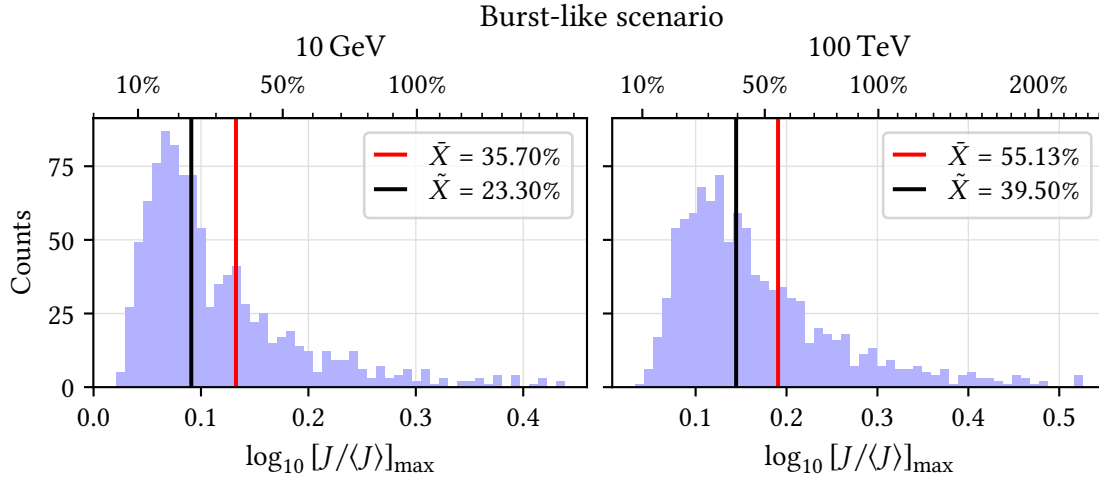


Figure 11.5: Extreme deviations, burst-like scenario. For each of the 1000 realisations, we select the LOS with the largest relative deviation from the ensemble mean, which represents the smooth model predictions (Sec. 10.4). The figure shows histograms of these extrema for 10 GeV (left) and 100 TeV (right) on a logarithmic x-axis up to the 99th percentile. We indicate the sample mean $\bar{X}$ and median $\tilde{X}$ of the relative deviations (in percent), and a secondary axis at the top shows the corresponding maximum relative difference $[\Delta J / \langle J \rangle]_{\max}$ in percent.

large relative deviations. This behaviour mirrors the properties of the underlying stable-law distributions of cosmic-ray intensities introduced in Secs. 7.4.2 and 10.3. Since the quantities shown here correspond to the most extreme GDE enhancements within each sky map, these distributions are dominated by the influence of those cosmic-ray sources that contribute most strongly at their Galactic locations. In particular, young sources play a key role in shaping the high-deviation tail. The histograms therefore provide insight into the maximum impact such sources can have on the GDEs.

In addition, the relative deviations are systematically larger at the higher GDE energy. This trend can be understood from the reduced number of cosmic-ray sources that effectively contribute to the proton intensity at higher energies, where the escape timescale from the Galaxy is shorter than at lower energies. Consequently, the relative contribution of an individual source is enhanced, which in turn leads to potentially larger relative deviations in the corresponding GDE sky maps.

Energy-dependent escape scenario

The histograms for the energy-dependent escape scenario are presented in Fig. 11.6. Because identical source realisations are used for the burst-like and energy-dependent escape scenarios, the distribution at 10 GeV is the same as that shown for the burst-like

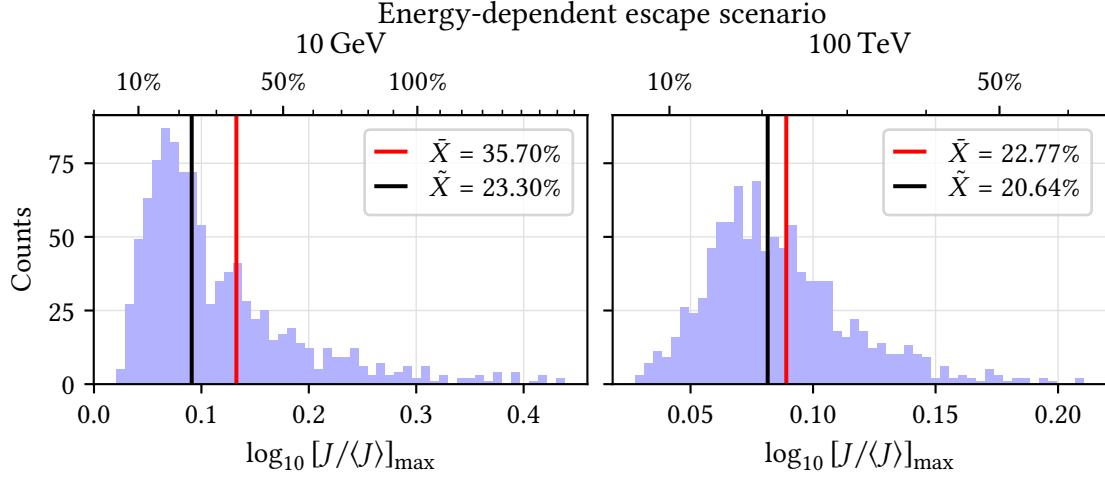


Figure 11.6: Extreme deviations, energy-dependent escape scenario, as in Fig. 11.5.

case in Fig. 11.5. At a GDE energy of 100 TeV, the distribution retains a similarly skewed shape with an extended high-value tail, but the overall magnitude of the deviations is reduced. As discussed in Sec. 11.3, this behaviour results from the restriction of the influence of individual sources to a narrower energy interval in the energy-dependent escape framework. Since the diffuse γ -ray emission is generated by cosmic-ray protons spanning a broad range of energies, this restricts the impact of any single source on the GDEs. Notably, the relative deviations at 100 TeV are typically smaller than those obtained at 10 GeV.

Time-dependent diffusion scenario

The histograms associated with the time-dependent diffusion scenario, shown in Fig. 11.7, also display strongly skewed distributions with extended tails, but with substantially more extreme deviations than in the other scenarios. In the high-energy case, relative differences exceeding 900 % occur in every realisation. The contrast between the maximum deviations at 10 GeV and 100 TeV can be interpreted analogously to the burst-like scenario: at higher energies, fewer sources contribute effectively to the cosmic-ray intensity, amplifying the relative importance of individual sources for the GDE. The particularly large deviations observed at 100 TeV, however, arise from the very efficient confinement of cosmic rays in the immediate vicinity of their sources at these energies. In this regime, the reduced diffusion coefficient, κ_{start} , is orders of magnitude smaller than in the burst-like and energy-dependent escape scenarios (see Sec. 10.1.2).

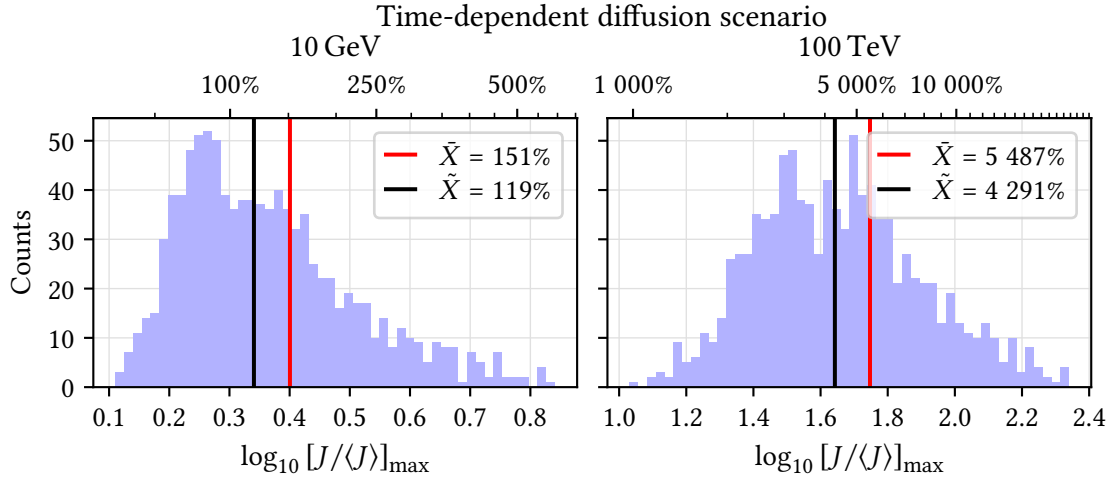


Figure 11.7: Extreme deviations, time-dependent diffusion scenario, as in Fig. 11.5.

Burst-like, halved source rate scenario

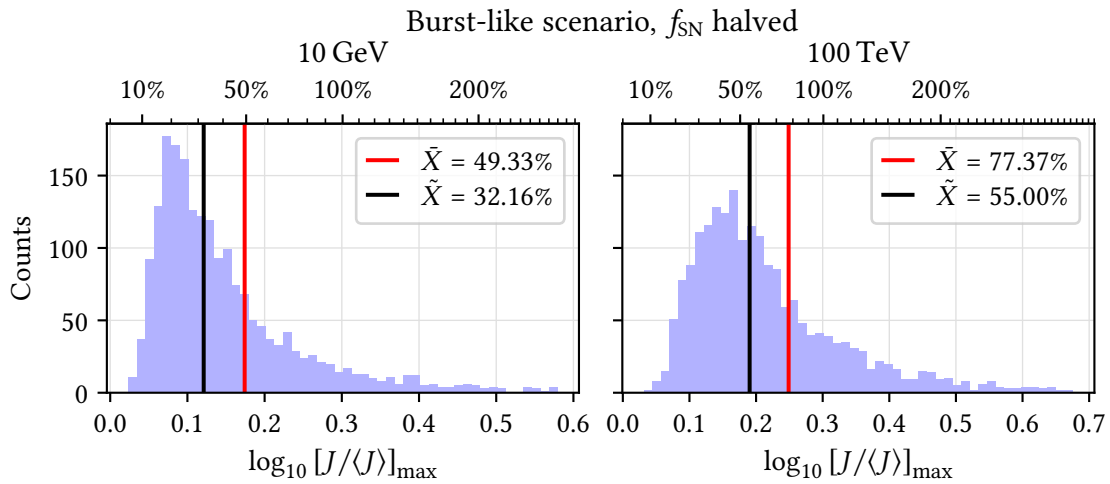


Figure 11.8: Extreme deviations, halved scenario, as in Fig. 11.5. The number of realisations is doubled to 2000 here.

The histograms for the burst-like, halved source rate scenario are displayed in Fig. 11.8. The reduced source rate leads to an increase in the influence of individual sources and thus, to more extreme relative deviations. A comparison with the burst-like scenario in Fig. 11.5 confirms this expectation. The shape of the distributions is very similar, but the range of relative deviations is extended. This is also reflected in the mean and median values, which are increased compared to the burst-like scenario. As in the burst-like scenario, the relative deviations are larger at 100 TeV than at 10 GeV. The number of

realisations is twice as high here (2000), because the lower source rate decreased the number of sources that needed to be included in a single realisation.

Summary

The histograms presented in this section quantify the most extreme relative deviations from the smooth-model predictions that can be expected in the different scenarios. This formalises our discussions based on a limited number of sky map realisations in Sec. 11.1. We found that the relative deviations are systematically larger at higher GDE energies in all except the energy-dependent escape scenario. This shows the limiting effect that the energy-dependent escape has on source imprints in that model. We discuss this further in the following Sec. 11.3.

11.3 Burst-like vs. energy-dependent escape

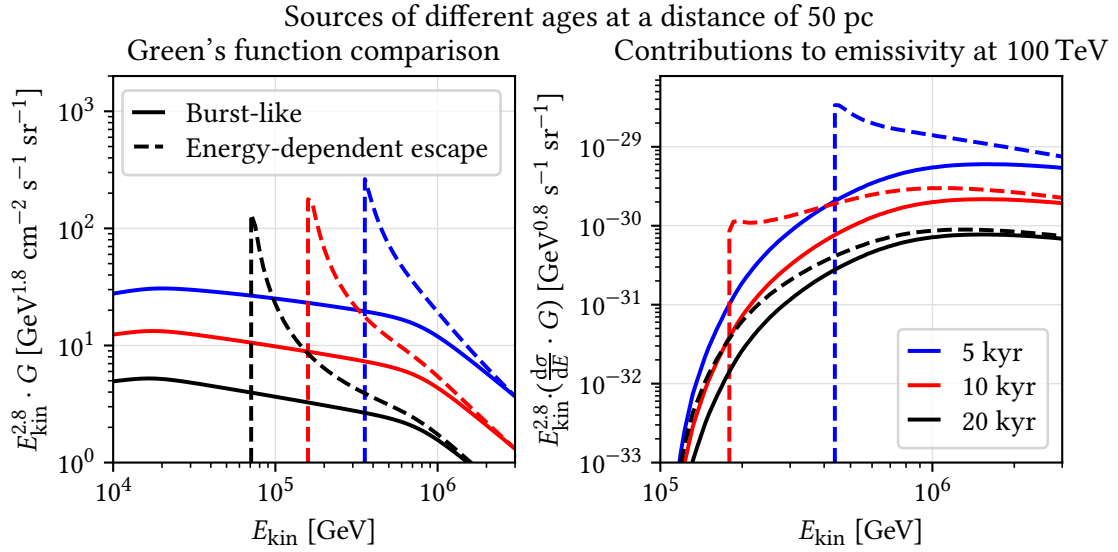


Figure 11.9: Spectra and emissivity contributions. *Left:* Cosmic-ray spectra from single sources (Green's functions) at a distance of 50 pc, multiplied by $E_{\text{kin}}^{2.8}$, are shown for the burst-like (solid) and energy-dependent escape (dashed) models; colours indicate different source ages. The energy-dependent escape scenario features a pronounced low-energy cut-off and subsequent peak (see Sec. 8.2). *Right:* Corresponding contributions to the γ -ray emissivity at 100 TeV as a function of cosmic-ray energy, obtained by multiplying the Green's functions with the differential cross-sections.

The statistical evaluation of our Monte Carlo realisations indicates that the burst-like scenario produces systematically more pronounced deviations than the energy-dependent escape scenario (see, e.g., Sec. 11.2). In this section, we investigate this behaviour. The left-hand panel of Fig. 11.9 displays free Green's functions (see Eq. (7.27)) for both the burst-like and energy-dependent escape scenarios for several source ages at a fixed distance of 50 pc. In the energy-dependent escape case, distinct peaks are visible, arising from the energy-dependent escape implemented in that model and previously discussed in Sec. 8.2. These peaks correspond to strong enhancements of the cosmic-ray intensity over limited energy intervals and could, in principle, generate particularly large contributions to the GDEs. At the same time, the low-energy suppression introduced by the energy-dependent escape restricts the range of energies over which an individual source contributes.

The right-hand panel of Fig. 11.9 shows the contributions to the emissivity (see Eq. (6.5) in Chap. 6¹) at a γ -ray energy of 100 TeV as a function of the cosmic-ray kinetic energy. Only cosmic-ray energies exceeding 100 TeV enter this calculation. By comparison with the left panel, the effect of the differential production cross-section becomes apparent. Depending on the source parameters, the emissivity contribution at a given cosmic-ray energy can be larger in either the energy-dependent escape or the burst-like scenario. The total emissivity is obtained by integrating these contributions over the cosmic-ray energy, as prescribed by Eq. (6.5).

The resulting total emissivity at 100 TeV is shown in the left-hand panel of Fig. 11.10. The curves represent the emissivity as a function of source age, such that each point corresponds to a cosmic-ray energy integral of a distribution similar to those shown in the right panel of Fig. 11.9. Different colours indicate different source distances. For all distances considered, the emissivity in the burst-like scenario reaches its maximum at earlier source ages and attains significantly larger values than in the energy-dependent escape case. This age dependence reflects the fact that, in the burst-like scenario, the cosmic-ray intensity is largest across the full energy range at early times, whereas particle escape is still ongoing in the energy-dependent escape scenario. As the source ages in the energy-dependent escape framework, the CREDIT peak (Sec. 8.2) of the cosmic-ray spectrum shifts towards lower energies. This initially leads to an increase in the total emissivity, followed by a decline with further ageing, and eventually converges towards the emissivity obtained in the burst-like scenario for sufficiently old sources.

To assess which scenario is expected to yield more extreme GDE intensities on average, we recall that, in our discrete source model, deviations are driven by individual sources. The largest deviations arise from particularly young sources located close to a given grid point. Consequently, the most extreme cases are associated with very young sources.

¹This is similar to our discussion of Fig. 10.1 in Sec. 10.1.3. Note however, that we plotted a different quantity there.

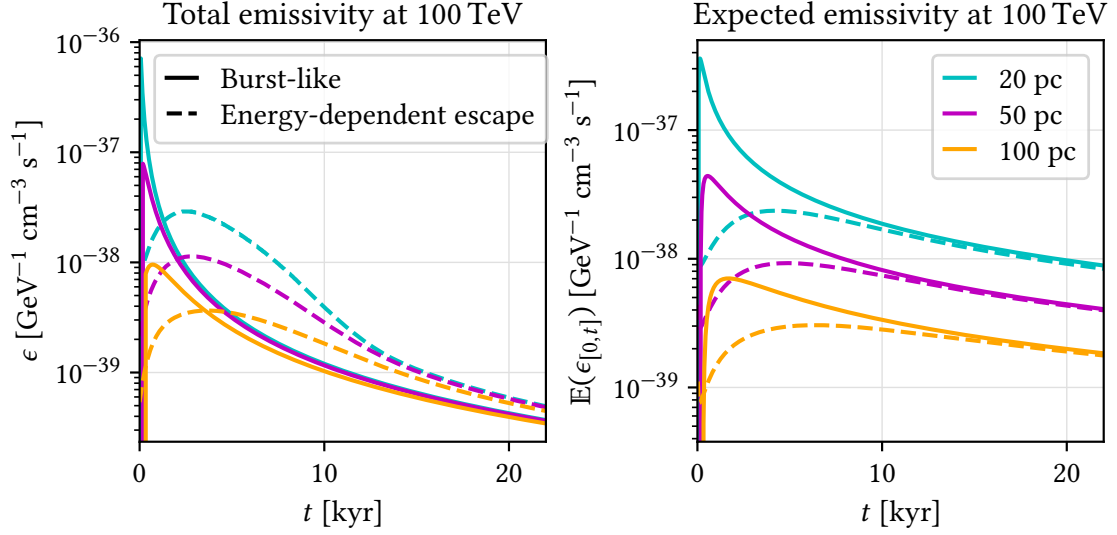


Figure 11.10: Total and time-averaged emissivity. Contributions of discrete sources to the emissivity at a γ -ray energy of 100 TeV. As in Fig. 11.9, line styles indicate the source model, and colours denote different source distances. The emissivity is plotted as a function of source age. *Left:* Single-source emissivity ϵ from Eq. (6.5) versus source age for different distances in the burst-like and energy-dependent escape models, assuming a gas density of 1 cm^{-3} . In the energy-dependent escape scenario, the emissivity peaks at later times because the peak of the cosmic-ray spectrum (see Fig. 11.9) moves into the energy range that contributes most. *Right:* Expected emissivity obtained from the time-averaged value $E(\epsilon_{[0,t]})$ for source ages between 0 and t (see Eq. (11.2)). This time average is consistently higher in the burst-like than in the energy-dependent escape model, explaining the larger GDE deviations in the burst-like scenario.

To compare the burst-like and energy-dependent escape scenarios in this regime, we compute the expectation value:

$$E(\epsilon_{[0,t]}) = \frac{1}{t} \int_0^t dt' \epsilon(t', d_0 | \mathbf{x}, 100 \text{ TeV}) , \quad (11.2)$$

where $\epsilon(t', d_0 | \mathbf{x}, 100 \text{ TeV})$ denotes the GDE emissivity at 100 TeV for a source of age t' located at a distance d_0 from a grid point $\mathbf{x}$. The prefactor $1/t$ accounts for a uniform distribution of source ages.

This expectation value is displayed in the right-hand panel of Fig. 11.10 and enables a direct comparison between the two scenarios. In all cases, the expected emissivity is larger for the burst-like scenario. Accordingly, realisations containing very young sources, which are responsible for the most extreme GDE intensities, are anticipated to exhibit stronger deviations in the burst-like model. Although the energy-dependent escape scenario features pronounced enhancements of the cosmic-ray intensity over narrow energy ranges around the spectral peaks (see Fig. 11.9), these do not compensate

for the broad and intense early-time contribution across the entire cosmic-ray energy range characteristic for the burst-like scenario.

11.4 Galactic diffuse emission distributions

Up to this point, our analysis has focused on the most pronounced GDE deviations, evaluated per sky map, between the discrete-source simulations and the smooth-model expectation. These extreme deviations originate from the local impact of individual, young cosmic-ray sources on the surrounding cosmic-ray intensity. In the following, we explore this relationship in greater depth by exploiting our understanding of cosmic-ray intensity distributions and the corresponding predictions developed in Sec. 10.3. A detailed study of the full GDE intensity distributions requires significantly larger statistics than the 1000 realisations employed so far. We therefore restrict the analysis to four selected LOS, for which the number of Monte Carlo realisations is increased to 10^6 each ($2 \cdot 10^6$ for the burst-like, halved source rate scenario). Specifically, we consider the directions $(l, b) = (0^\circ, 0^\circ)$ towards the Galactic centre, $(180^\circ, 0^\circ)$ towards the Galactic anticentre, $(0^\circ, 90^\circ)$ perpendicular to the Galactic plane, and $(-65^\circ, 0^\circ)$ pointing into the nearby Sagittarius–Carina spiral arm.

In what follows, we describe the simulation outcomes for the different injection and near-source transport scenarios. We further discuss how the theoretical expectations for cosmic-ray intensity distributions presented in Sec. 10.3 can be translated into an understanding of the resulting GDE distributions. As before, we present results at GDE energies of 10 GeV and 100 TeV as representative examples of low and high energies.

Burst-like scenario

The results for the burst-like scenario are shown in Fig. 11.11 for a γ -ray energy of 10 GeV and in Fig. 11.12 for 100 TeV. All histograms are displayed on double-logarithmic axes. The large number of realisations enables us to resolve the PDFs deep into their tails, spanning several orders of magnitude.

Alongside the simulated histograms, we also include the analytical predictions derived in Sec. 10.3. For the three LOS lying in the Galactic plane ($b = 0^\circ$), we overlay stable-law distributions with stability indices $\alpha = 4/3$ and $\alpha = 5/3$, respectively (see Eq. (10.16) in Sec. 10.3), with the scale parameters fitted to the data. In addition, a normal distribution with the sample mean and standard deviation is shown for comparison. For the LOS perpendicular to the plane, $(l, b) = (0^\circ, 90^\circ)$, we display a mixture model composed of a stable law with $\alpha = 5/3$ and a Gaussian component, using an appropriate mixing fraction f (see Eq. (10.18)).

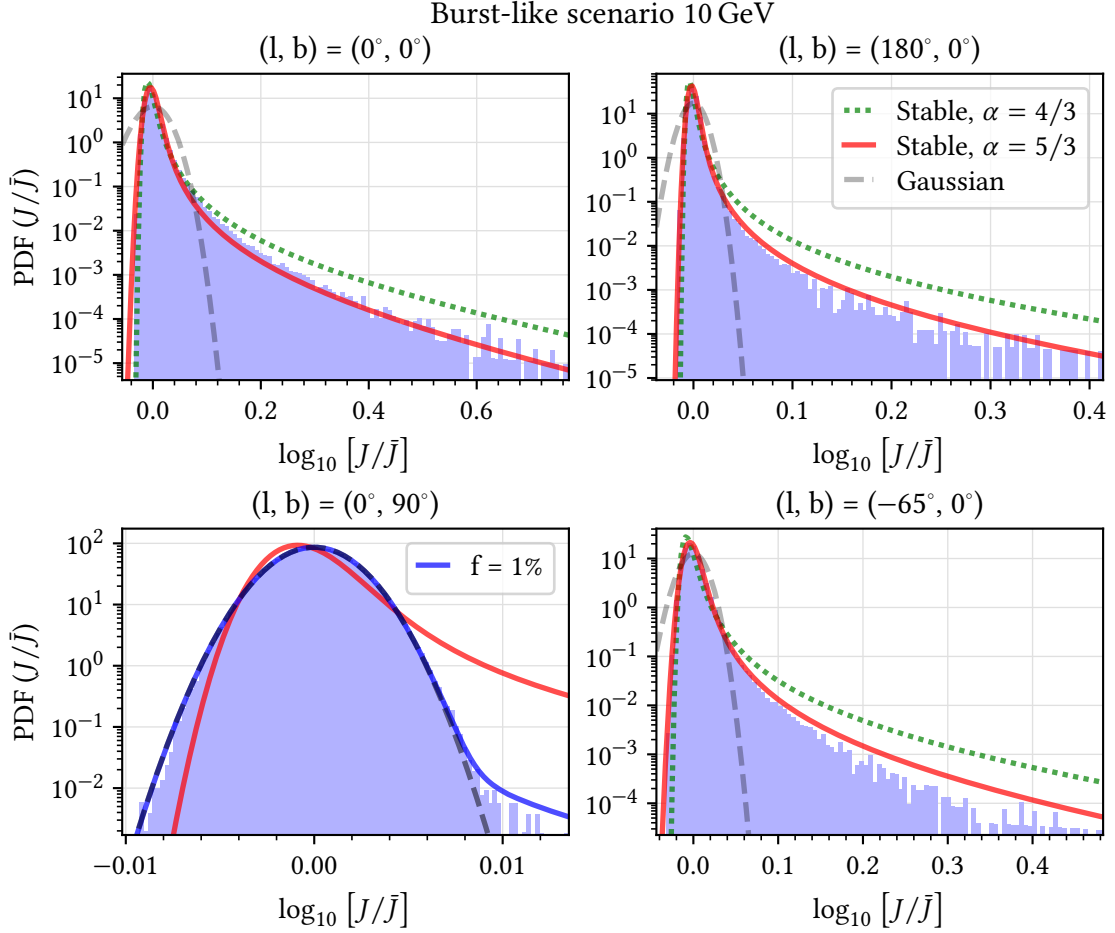


Figure 11.11: PDFs, burst-like scenario at 10 GeV. We simulate GDE intensities along several LOS for 10^6 realisations of source positions and ages. Each panel corresponds to one LOS, labelled by its Galactic longitude l and latitude b . The histograms show $J/\bar{J}$ on a double-logarithmic scale, normalised to the probability density function and excluding the ten largest values. The mean intensity $\bar{J}$ is the sample mean over all realisations. We overlay fitted stable-law distributions with $\alpha = 4/3$ (green dotted) and $\alpha = 5/3$ (red), as well as a normal distribution (grey dashed). For the $(0^\circ, 90^\circ)$ LOS, we additionally show the mixture model from Sec. 10.3 (blue), with the fraction f given in the legend.

The low-energy results are presented in Fig. 11.11. Among the four directions, the LOS towards the Galactic centre, $(0^\circ, 0^\circ)$, exhibits the largest GDE intensities. Overall, the shapes of the histograms are well reproduced by the stable-law models. At small values of J , the data is better described by a stable law with $\alpha = 4/3$, whereas at large J the index $\alpha = 5/3$ provides a better fit. This behaviour is consistent with the interpretation of the index of stability as encoding the effective dimensionality of the dominant source population: a two-dimensional, disk-like distribution corresponds to $\alpha = 4/3$, while a three-dimensional, volume-filling distribution yields $\alpha = 5/3$ (Genolini et al., 2017). Large

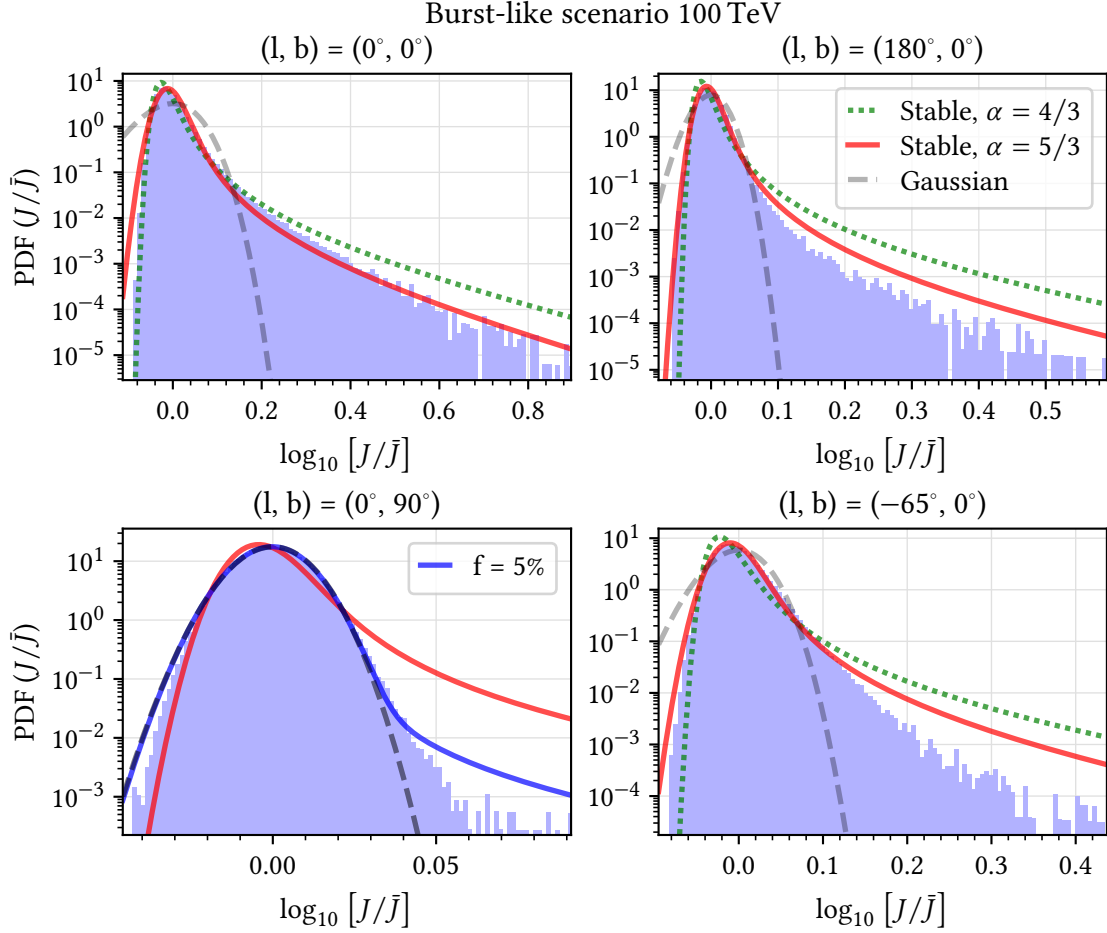


Figure 11.12: PDFs, burst-like scenario at 100 TeV, as in Fig. 11.11.

values of J require sources to be located close to the LOS. In regions near the Galactic plane, the source distribution is approximately uniform in three dimensions on spatial scales smaller than the vertical scale height, which we take to be 70 pc (see Sec. 10.1). At lower values of J , more distant sources dominate, and the flattened geometry of the Galactic disk becomes important (see also Sec. 10.3). Consequently, the transition from $\alpha = 4/3$ at low J to $\alpha = 5/3$ at high J mirrors the behaviour previously identified for cosmic-ray intensity distributions in Genolini et al., 2017 (see Sec. 7.4.2). The same qualitative trends are observed for the other two LOS in the Galactic plane. In all three cases, the Gaussian distribution fails to provide an adequate description.

For the LOS (0°, 90°), the intensity values are substantially less extreme, reflecting the reduced density of cosmic-ray sources along this direction. Here, the normal distribution reproduces the bulk of the data but does not capture the high-intensity tail. A mixture of a Gaussian and a stable law with $\alpha = 5/3$, using a fixed mixing fraction of $f = 1\%$, successfully describes both the central part and the tail of the distribution. This supports

our expectation that GDEs observed out of the Galactic plane receive contributions from regions where the cosmic-ray intensities follow either stable-law or Gaussian statistics.

The high-energy results for the burst-like scenario are shown in Fig. 11.12. As anticipated, the deviations are generally more extreme than at lower energies, owing to the increased relative influence of individual cosmic-ray sources. The same statistical models provide the best overall description of the data. However, at large values of J , the agreement deteriorates. This can be traced back to the causality condition implemented in the simulations (see Sec. 7.3.4), which predominantly affects the high-intensity tails at high energies, as discussed in Genolini et al., 2017. In addition, for the LOS $(0^\circ, 90^\circ)$, a larger mixing fraction f is required at this energy, indicating a stronger relative contribution from nearby sources.

Energy-dependent escape scenario

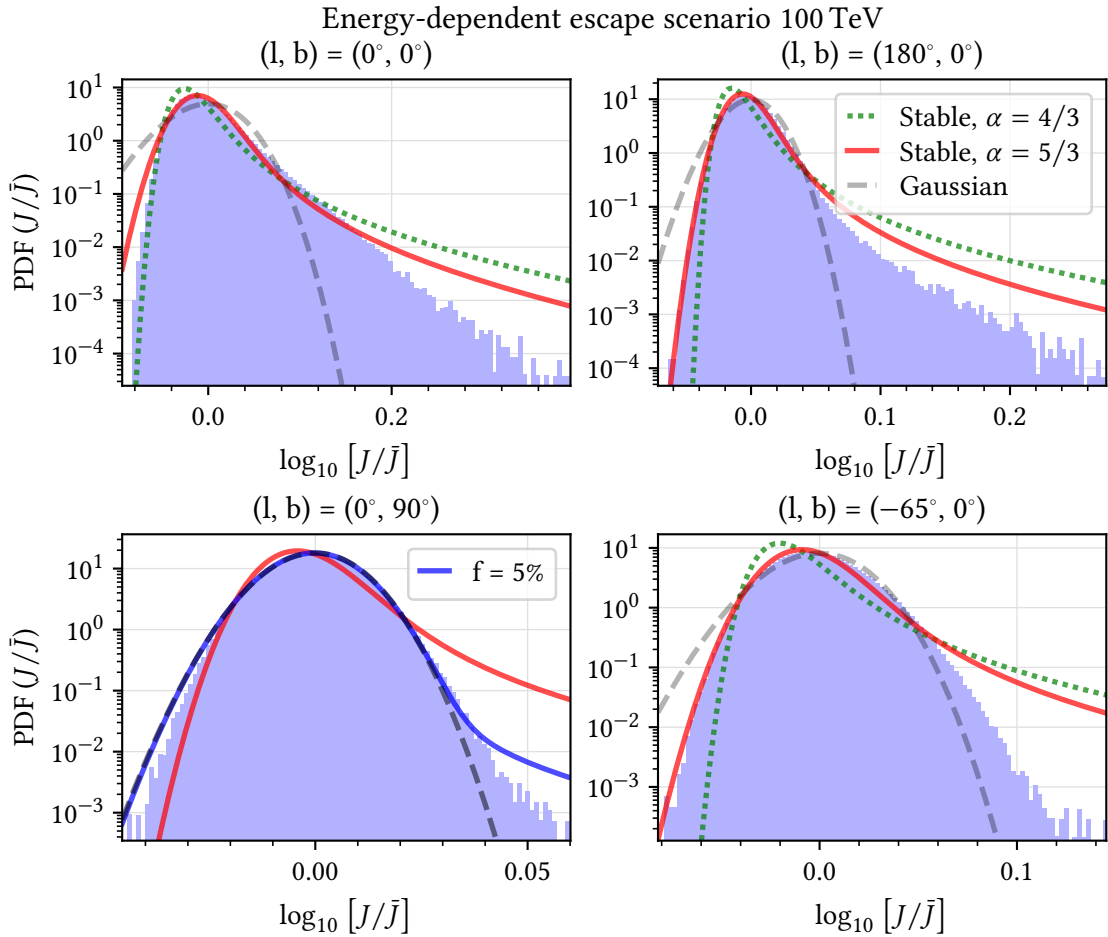


Figure 11.13: PDFs, energy-dependent escape scenario at 100 TeV, as in Fig. 11.11.

For the energy-dependent escape scenario, we restrict the presentation to 100 TeV, since the results at 10 GeV are identical to those obtained in the burst-like case. A comparison between Fig. 11.13 and the corresponding burst-like results in Fig. 11.12 reveals a reduced probability of very high intensities in the energy-dependent escape scenario. This effect can be understood in the context of our previous discussion in Sec. 11.3 and is likely related to the energy-dependent injection.

While the detailed shapes of the distributions differ slightly from those in the burst-like scenario, the stable-law models continue to provide an adequate overall description. As before, discrepancies at the highest values of J arise from the imposed causality condition. For the LOS ($0^\circ, 90^\circ$), the mixture model again captures the main features of the distribution.

Time-dependent diffusion scenario

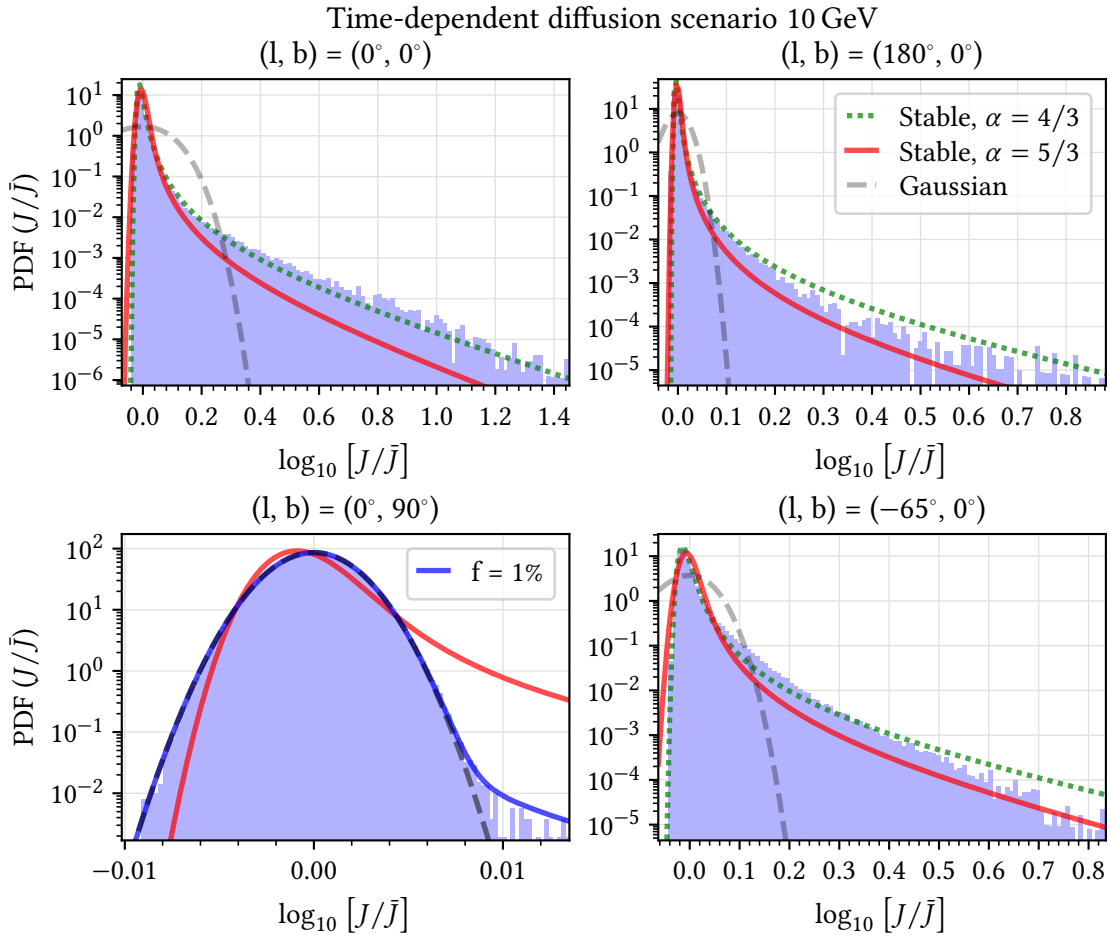


Figure 11.14: PDFs, time-dependent diffusion scenario at 10 GeV, as in Fig. 11.11.

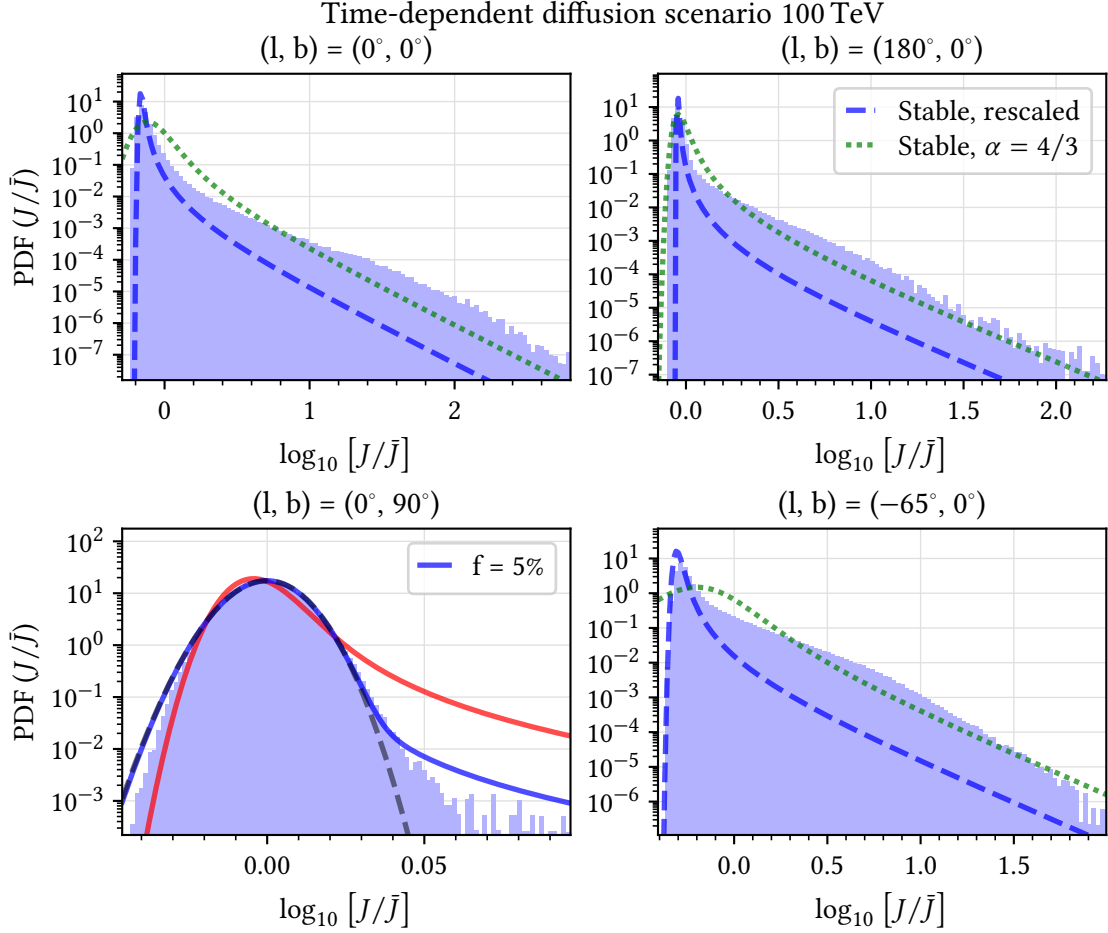


Figure 11.15: PDFs, time-dependent diffusion scenario at 100 TeV. As in Fig. 11.11, but for conciseness, the stable laws with $\alpha = 5/3$ are omitted. Instead, rescaled versions of the stable laws with $\alpha = 4/3$ in the time-dependent diffusion scenario presented in Fig. 11.14 are added for the LOS in the Galactic plane (blue dashed).

The intensity distributions obtained in the time-dependent diffusion scenario are shown in Figs. 11.14 and 11.15. As demonstrated previously, the strong confinement of cosmic rays in the vicinity of their sources leads to enhanced GDE intensities. This manifests itself in a markedly increased fraction of high-intensity deviations. We therefore check whether the analytical expectations developed in Sec. 10.3 remain applicable in this case.

We begin with the low-energy results displayed in Fig. 11.14. For the LOS within the Galactic plane, the histograms are clearly inconsistent with a normal distribution. Instead, stable laws with $\alpha = 4/3$ provide a good description, and generally outperform the $\alpha = 5/3$ models that were favoured in the burst-like scenario at the same energy. This may indicate a stronger contribution from sources located farther away from the LOS, such that the disk-like geometry of the source distribution dominates the high-intensity

tail. However, drawing firm conclusions is challenging, since the time-dependent diffusion scenario involves a more complex transport mechanism that is not incorporated in the cosmic-ray intensity distributions discussed in Genolini et al., 2017.

The high-energy results are shown in Fig. 11.15. Here, the histograms extend to extremely large values of J , and stable laws with fitted scale parameters fail to reproduce the data satisfactorily. For the LOS along the Galactic plane, we display the somewhat better-performing $\alpha = 4/3$ stable laws. While these capture parts of the high-intensity tails, the distributions exhibit additional structure induced by the time-dependent diffusion transport model. As an alternative, we rescale the scale parameters of the low-energy distributions shown in Fig. 11.14 by the corresponding mean values $\bar{J}$. The resulting PDFs are indicated by blue dashed curves in Fig. 11.15. Although this approach improves the description of the bulk of the data, significant residual structure remains that cannot be captured by this simplified model.

In contrast, the LOS ($0^\circ, 90^\circ$) is well described at both energies. We attribute this to the comparatively weak influence of nearby cosmic-ray sources along this direction, rendering the detailed injection and near-source transport properties less critical.

Burst-like, halved source rate scenario

The results for the burst-like, halved source rate scenario are presented in Figs. 11.16 and 11.17. Reducing the source rate enhances the influence of individual sources, leading to stronger deviations from the smooth-model predictions, as we discussed in Secs. 11.1 and 11.2. Accordingly, the histograms extend to larger values of $J/\bar{J}$ than in the burst-like scenario with the standard source rate. Nevertheless, the stable-law models continue to provide a satisfactory description of the data. The same trends regarding the preferred stability indices are observed as in the burst-like scenario. For the LOS ($0^\circ, 90^\circ$), the mixture model again captures the main features of the distribution.

Summary

To investigate the detailed properties of GDE intensity distributions, we simulated GDEs along selected LOS using large Monte Carlo samples comprising 10^6 realisations. This provides the statistical power to robustly resolve the tails of the distributions and thereby enable a meaningful comparison with predictions derived from the known distributions of cosmic-ray intensities (see Sec. 10.3). For LOS within the Galactic plane, we find that stable-law distributions successfully reproduce the main characteristics of the simulated GDE distributions, whereas normal distributions provide a poor description. This agreement is strongest in the burst-like scenario as well as in the burst-like, halved source rate scenario and remains acceptable for the energy-dependent escape scenario. At a GDE

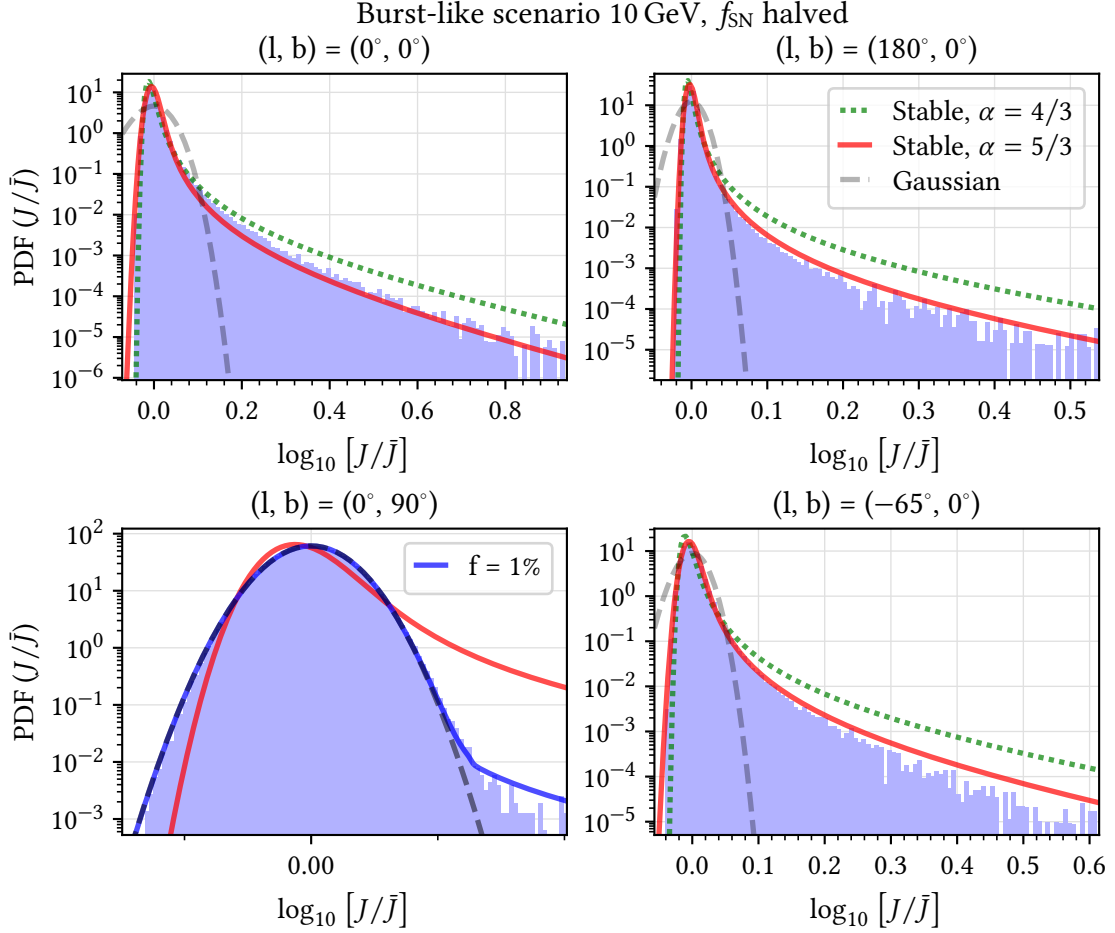


Figure 11.16: PDFs, burst-like, halved source rate scenario at 10 GeV, as in Fig. 11.11. Due to the larger sample size of $2 \cdot 10^6$ realisations, we exclude the 20 largest values here.

energy of 100 TeV, deviations from the theoretical expectations emerge at the highest intensities. These can be traced back to the causality condition implemented in the simulations. In contrast, for the time-dependent diffusion scenario, the simple analytical models are insufficient to capture the effects introduced by the more complex near-source transport.

For LOS outside the Galactic plane, the distributions are well described by a mixture model that combines components following a stable law with components that are approximately normally distributed, independent of the underlying injection and near-source transport scenario. By explicitly validating the predictions derived in Sec. 10.3, we have demonstrated the impact of discrete cosmic-ray sources on the GDEs. This provides a quantitative foundation for the connection between cosmic-ray intensity statistics and the resulting GDE intensity distributions.

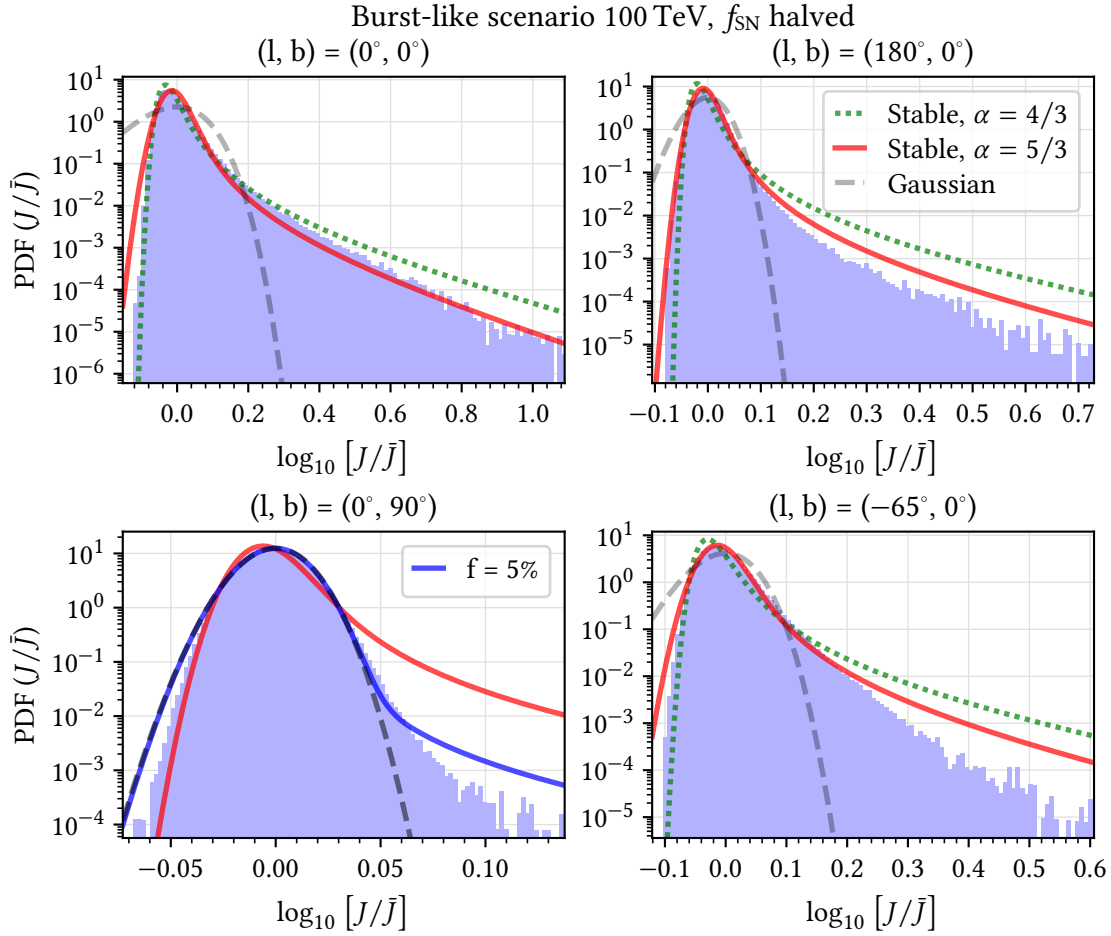


Figure 11.17: PDFs, burst-like, halved source rate scenario at 100 TeV, as in Fig. 11.11. Due to the larger sample size of $2 \cdot 10^6$ realisations, we exclude the 20 largest values here.

11.5 Sky windows and profiles

In the preceding sections, we examined how discrete sources shape both the morphology and the statistical properties of GDE intensities. This allowed us to qualitatively interpret the signatures of individual cosmic-ray sources and to quantify the largest deviations from smooth-model expectations along individual LOS. In observational practice, however, GDE measurements, particularly at energies above several TeV, are often not spatially resolved at the level of full sky maps. Instead, intensities are averaged over extended sky regions, and energy spectra integrated over these sky windows are reported (Amenomori et al., 2021; Cao et al., 2023a, 2025). This raises questions of whether and to what extent source discreteness affects GDE spectra averaged over such sky windows.

To address this issue, we study the impact of source stochasticity on predicted GDE spectra for different injection and near-source transport scenarios. We analyse 1000 realisations

across a broad energy range in two representative sky windows corresponding to the inner and outer Galaxy regions observed by LHAASO (Cao et al., 2023a, 2025). The uncertainty in GDE intensities arising from our incomplete knowledge of source positions and ages must be assessed in relation to other modelling uncertainties, such as those associated with interaction cross-sections, gas distributions, the assumed source population, and cosmic-ray model parameters. Estimates for these additional uncertainties can be found in the CRINGE model analysis (Schwefer et al., 2023).

With improving experimental sensitivity and increasingly sophisticated analysis techniques, it also becomes feasible to resolve diffuse emission as a function of Galactic longitude. In this case, the influence of discrete sources is expected to grow as the averaging region shrinks and finer spatial structure in the GDEs becomes accessible. We investigate this aspect by studying longitude profiles of GDE intensities at 100 TeV for the different injection and near-source transport models. As before, we use 1000 realisations per scenario to estimate the expected spread.

Burst-like scenario

Figure Fig. 11.18 shows the GDE intensity spectra in the inner and outer Galaxy sky windows defined by LHAASO (Cao et al., 2023a, 2025) for the burst-like scenario. The mean spectra and several uncertainty intervals are displayed to visualise the spread among realisations. As expected, the inner Galaxy exhibits higher intensities than the outer Galaxy, while the overall spectral shapes remain similar. In addition, we show ten individual realisations in grey to illustrate the typical spectral behaviour arising from discrete source configurations.

The lower panels present the same spectra normalised to the mean intensity $\bar{J}$, which facilitates a direct assessment of the relative spread. The light-green bands are used as a proxy for the uncertainty introduced by source stochasticity. This uncertainty remains below the percent level up to energies of order TeV and increases to less than 10 % at PeV energies. By comparison, uncertainties associated with cosmic-ray model parameters and interaction cross-sections are of order 10 % at GeV energies and rise to roughly 50 % around 100 TeV (Schwefer et al., 2023). Thus, in the burst-like scenario, source stochasticity contributes only a subdominant fraction to the overall modelling uncertainty for spectra averaged over these large sky windows. This behaviour is expected, since averaging over extended regions suppresses the localised impact of individual sources, which affect only limited areas in the sky (see Fig. 11.1 in Sec. 11.1).

We also overlay the LHAASO WCDA–KM2A measurements together with their statistical and systematic uncertainties (Cao et al., 2025). The small effect of source discreteness in the burst-like scenario implies that cosmic-ray source stochasticity alone cannot account for the discrepancy between the model predictions and the observed intensities. Addi-

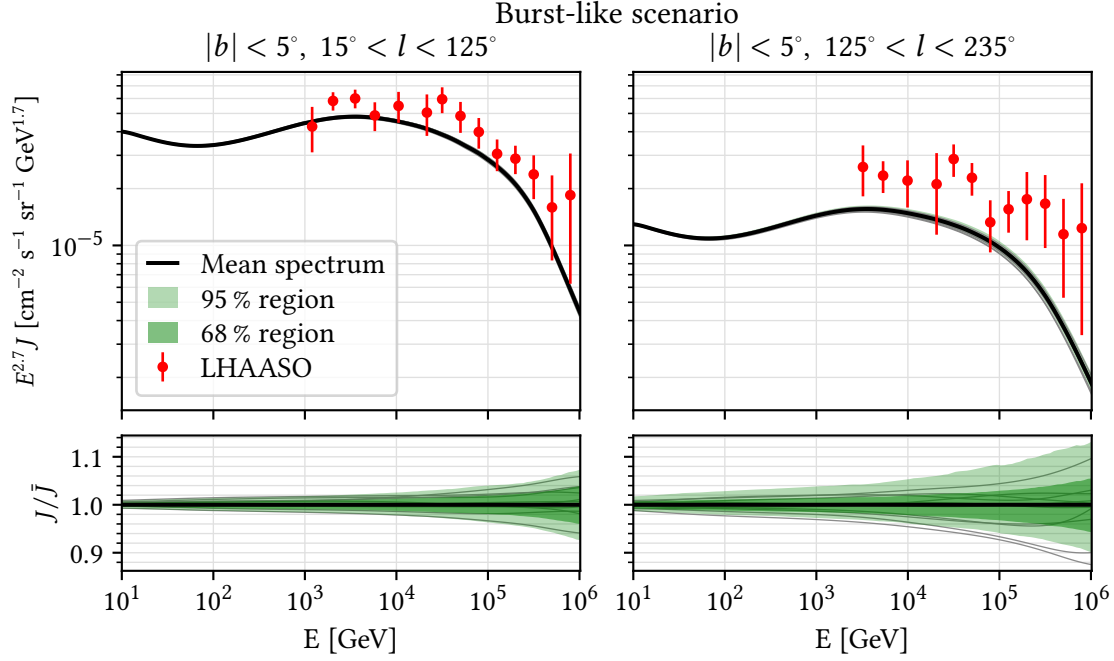


Figure 11.18: GDE spectra, burst-like scenario. This plot shows spectra of the GDE intensity averaged over the LHAASO inner and outer Galaxy sky windows as defined in Cao et al., 2023a, 2025. The black curve indicates the sample mean, with green shaded bands denoting uncertainty intervals. The lower panel shows the same spectra normalised to their respective means, with ten individual realisations plotted in grey. For comparison, we include the diffuse γ -ray emission intensities measured by LHAASO WCDA-KM2A (Cao et al., 2025), where the error bars represent 1σ statistical and systematic uncertainties added in quadrature.

tional modelling uncertainties or unresolved source populations are therefore required to reconcile the two.

Figure Fig. 11.19 shows the longitude profile of the GDE intensity at 100 TeV, averaged over $|b| < 5^\circ$, for the burst-like scenario. The inner Galaxy clearly dominates the emission. Normalising the profiles to their mean values in the lower panel reveals that source stochasticity introduces uncertainties at the level of approximately 10 %. We also display five individual realisations, which exhibit structured deviations confined to angular scales of a few degrees in longitude. As these fluctuations originate from individual sources, their limited spatial extent mirrors the behaviour observed in the full sky maps. It is therefore plausible that the actual source configuration of the Milky Way produces similar imprints, implying that source discreteness constitutes a genuine modelling uncertainty for longitude profiles.

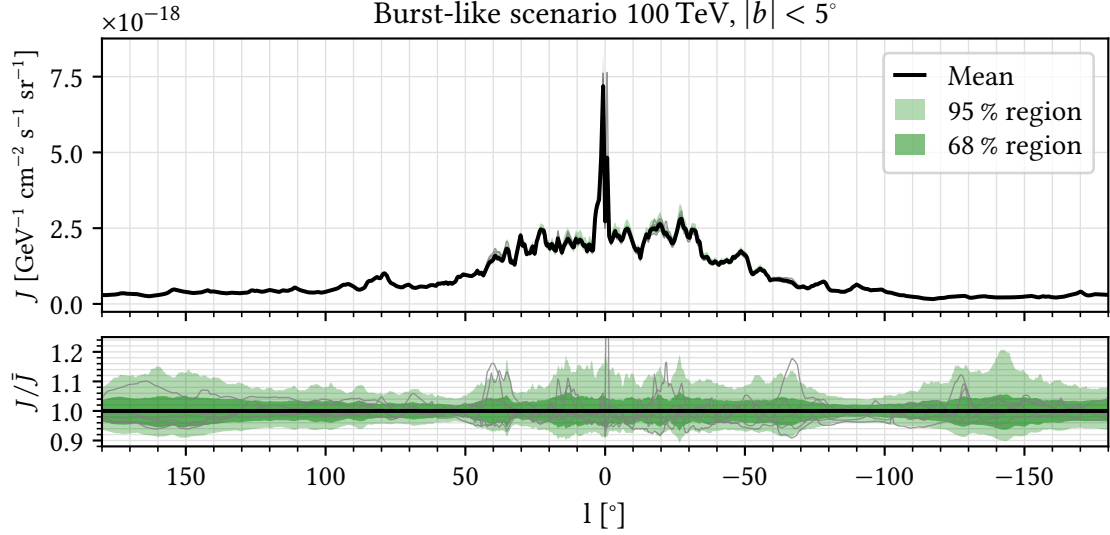


Figure 11.19: Longitude profile, burst-like scenario. Shown is the GDE intensity at 100 TeV for the burst-like scenario, averaged over $|b| < 5^\circ$ and plotted as a function of Galactic longitude. The black curve shows the sample mean, and green shaded bands indicate uncertainty intervals. The lower panel presents the same profiles normalised to their respective means, with five example spectra from individual source realisations plotted in grey.

Energy-dependent escape scenario

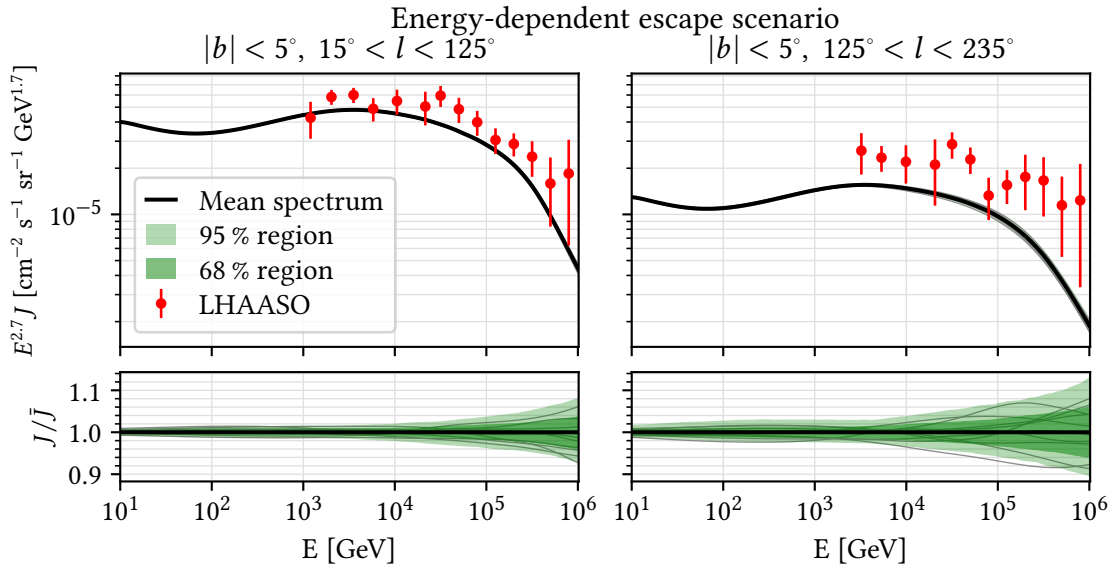


Figure 11.20: GDE spectra, energy-dependent escape scenario, as in Fig. 11.18.

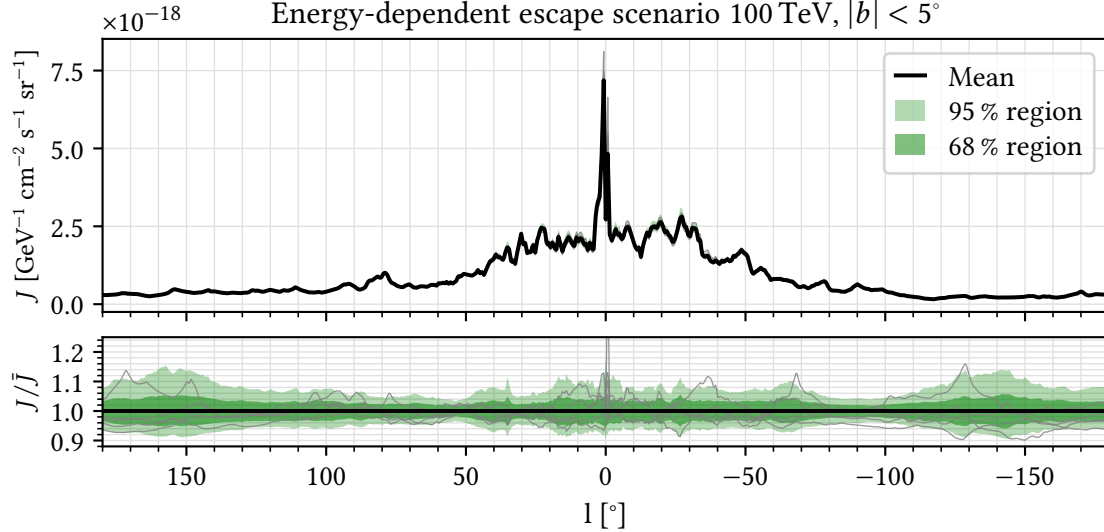


Figure 11.21: Longitude profile, energy-dependent escape scenario, as in Fig. 11.19.

The corresponding GDE spectra for the energy-dependent escape scenario are shown in Fig. 11.20, with longitude profiles presented in Fig. 11.21. As in previous analyses, the overall level of deviations is comparable to that in the burst-like scenario, though slightly reduced at GDE energies influenced by cosmic rays above $E_b \simeq e\mathcal{R}_b = 10$ TeV, where energy-dependent escape becomes important (see Sec. 8.2). Consequently, the conclusions regarding uncertainty estimates and consistency with LHAASO measurements closely mirror those of the burst-like scenario. The individual realisations again follow the mean spectral shape without exhibiting pronounced spectral features. The longitude profiles resemble those of the burst-like scenario but show narrower uncertainty bands, reflecting the reduced imprint of individual sources at high energies. This behaviour is related to our discussion in Sec. 11.3.

Time-dependent diffusion scenario

The time-dependent diffusion scenario exhibits a markedly stronger influence of discrete sources on the GDE spectra, as shown in Fig. 11.22. In this case, the uncertainty due to source stochasticity becomes a significant component of the total modelling uncertainty above several tens of TeV, comparable to the uncertainties reported in Schwefer et al., 2023. In addition, the mean spectrum itself is enhanced relative to the burst-like and energy-dependent escape scenarios. Together with the broadened uncertainty bands, this leads to an improved agreement with the LHAASO measurements in the inner Galaxy sky window, at least within uncertainties.

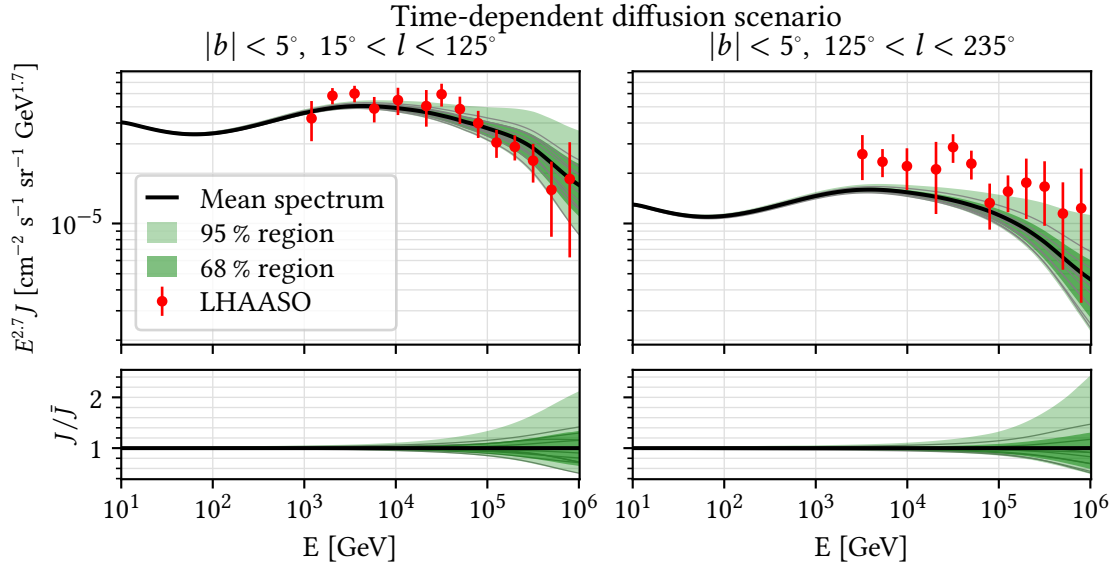


Figure 11.22: GDE spectra, time-dependent diffusion scenario, as in Fig. 11.18.

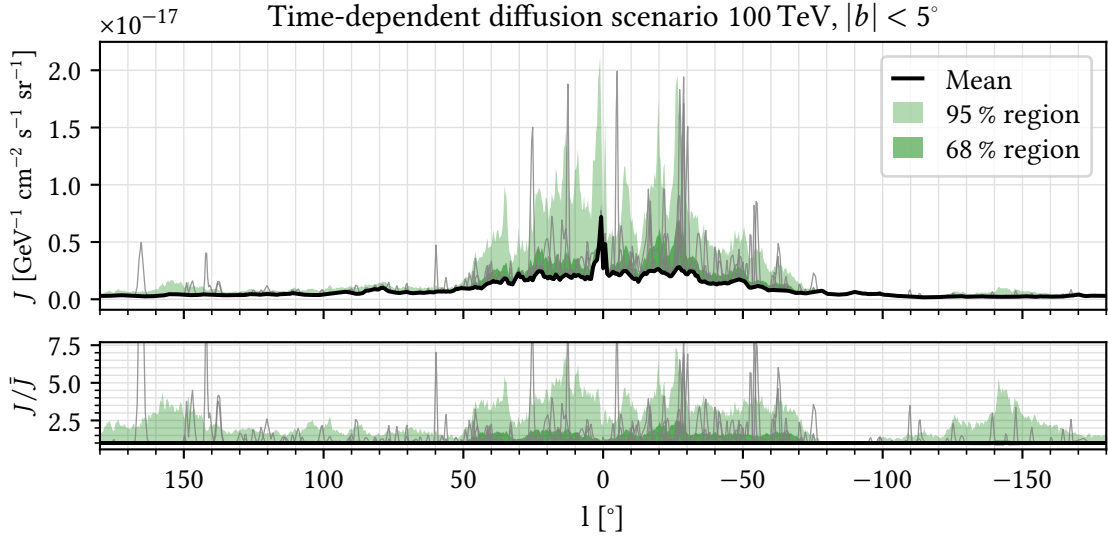


Figure 11.23: Longitude profile, time-dependent diffusion scenario, as in Fig. 11.19.

The longitude profiles for the time-dependent diffusion scenario are shown in Fig. 11.23. They display extremely strong source-induced features, with deviations reaching several hundred percent. Individual realisations reveal sharply peaked structures, consistent with the highly localised imprint expected from strong cosmic-ray confinement near sources in this scenario.

Such pronounced effects offer a potential avenue to constrain injection and near-source transport models. However, as already discussed in Sec. 11.1, the absence of features like

those shown in Fig. 11.23 could also indicate a lower rate of sources accelerating particles to PeV energies, rather than ruling out the time-dependent diffusion scenario entirely.

Burst-like, halved source rate scenario

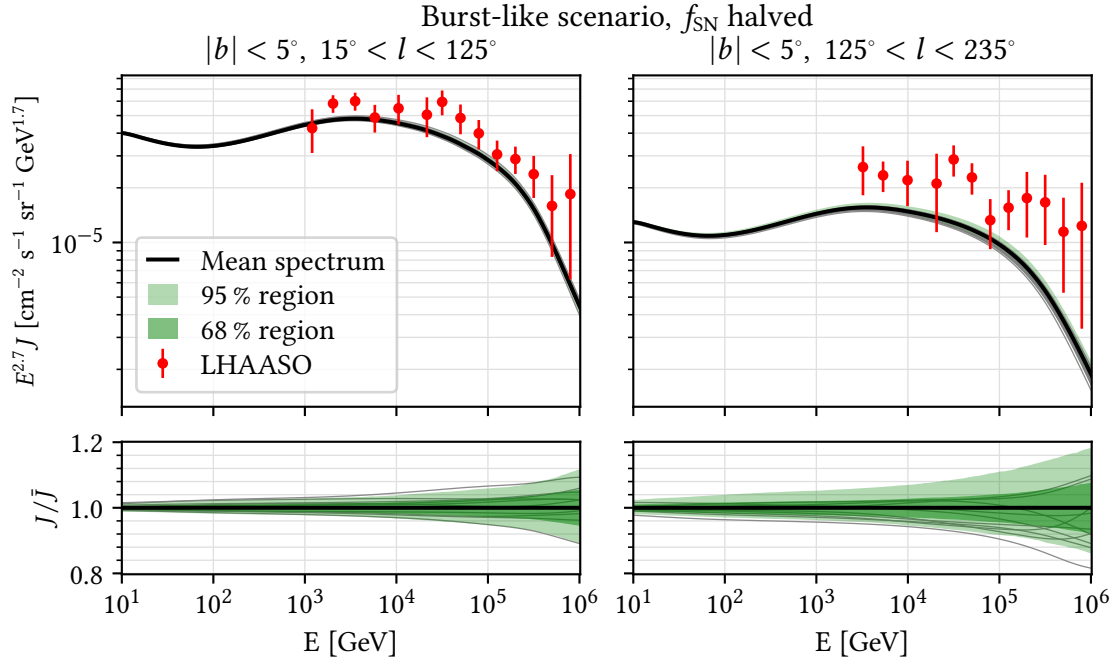


Figure 11.24: GDE spectra, burst-like, halved source rate scenario, as in Fig. 11.18.

The burst-like, halved source rate scenario results are presented in Figs. 11.24 and 11.25. As expected, the reduced source rate amplifies the influence of individual sources, leading to larger uncertainties in both the GDE spectra and longitude profiles compared to the burst-like scenario (note the larger uncertainty bands and the different y-axis scale in the lower panel in Fig. 11.24). Still, the total uncertainty due to source stochasticity remains subdominant compared to other modelling uncertainties in this scenario. To reconcile the model predictions with the LHAASO measurements, the source rate would need to be reduced even further, which would further enhance the impact of source discreteness. While this halved source rate is still plausible, further reductions may conflict with observational constraints on the Galactic supernova rate as discussed in Sec. 7.2.

Summary

In this section, we examined the stochastic impact of discrete cosmic-ray sources on GDE spectra in the LHAASO sky windows and on longitude profiles of GDE intensities. For

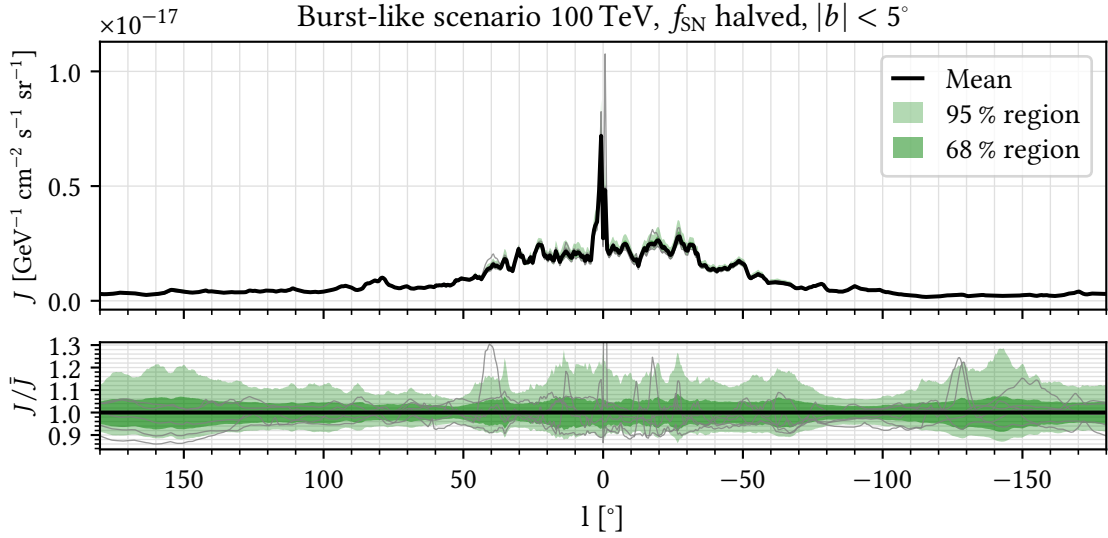


Figure 11.25: Longitude profile, burst-like, halved source rate scenario, as in Fig. 11.19.

spectra averaged over large sky regions, the effect of source discreteness is limited, and individual realisations largely follow the smooth-model spectral shape. The resulting uncertainty bands quantify the modelling uncertainty associated with source stochasticity and must be compared with uncertainties from other model ingredients such as cosmic-ray parameters and interaction cross-sections (Schwefer et al., 2023). These contributions are subdominant in the burst-like, the energy-dependent escape, and also the burst-like, halved source rate scenario, but become significant above several tens of TeV in the time-dependent diffusion scenario.

A comparison with LHAASO WCDA–KM2A data (Cao et al., 2025) shows that the burst-like (also with halved source rate) and energy-dependent escape scenarios undershoot the measured intensities for the chosen cosmic-ray model parameters, implying that additional uncertainties or unresolved sources must be invoked. In contrast, the time-dependent diffusion scenario is compatible with the data within uncertainties.

Longitude profiles exhibit a much stronger sensitivity to source discreteness, particularly in individual realisations. This is encouraging, as the actual Galactic source population may leave detectable imprints of this kind. While such uncertainties affect all scenarios, they are most pronounced in the time-dependent diffusion case. Overall, our results demonstrate that the influence of discrete sources on GDE predictions increases as the size of the probed region decreases. With continued experimental and methodological progress, measurements of longitude profiles may therefore provide a powerful tool to constrain extreme injection and near-source transport scenarios such as the time-dependent diffusion model (see also Kaci and Giacinti, 2025).

11.6 Correlations between different energies

The example sky maps presented in Sec. 11.1 already demonstrated that the imprint of discrete sources on the GDE can vary strongly with energy. In the following, we examine in more detail how GDE intensities at 10 GeV and 100 TeV are correlated, and how the deviations from smooth-model predictions behave at these two energies. We begin by showing that models assuming a continuous source distribution predict a strong correlation between low- and high-energy GDEs. We then investigate how this correlation is altered once discrete cosmic-ray sources are taken into account.

Widely used phenomenological models of GDEs, such as the π^0 model (Ackermann et al., 2012) and the KRA_γ models (Gaggero et al., 2015a,b), neglect source discreteness and instead assume a smooth distribution of cosmic-ray sources. Under this assumption, the predicted GDE sky maps at low and high energies are expected to be tightly correlated. In practice, this means that the GDE intensity at different energies can be approximated by a simple linear relation of the form

$$J_{100 \text{ TeV}}(l, b) \simeq X \cdot J_{10 \text{ GeV}}(l, b) \quad (11.3)$$

with a constant proportionality factor, X .

A convenient quantitative measure of such a linear relationship is the Pearson correlation coefficient ρ . For two random variables X and Y , it is defined as

$$\rho_{X,Y} = \sigma_{X,Y} / \sigma_X \sigma_Y, \quad (11.4)$$

where $\sigma_{X,Y}$ denotes the covariance of X and Y , and σ_X and σ_Y are their respective standard deviations. The coefficient ranges between -1 and 1 , with positive (negative) values indicating correlation (anti-correlation). Values of $|\rho_{X,Y}|$ close to unity correspond to a strong linear relationship.

As discussed in Sec. 10.4, the ensemble average of the burst-like scenario provides a good approximation to the predictions of a smooth source model. To test the validity of Eq. (11.3) for the ensemble-averaged intensities $\langle J_{10 \text{ GeV}} \rangle$ and $\langle J_{100 \text{ TeV}} \rangle$, we compute the Pearson correlation coefficient across all LOS. We obtain $\rho_{\langle J_{10 \text{ GeV}} \rangle, \langle J_{100 \text{ TeV}} \rangle} = 0.9988$, confirming an almost perfect correlation.

This result is readily understood in terms of the physical ingredients entering the calculation of diffuse γ -ray emission, as discussed in Chap. 6. The gas distribution is energy-independent, the interaction cross-sections primarily introduce an overall energy dependence, and the spatial variations of the cosmic-ray intensity are weak in smooth source models. The dominant energy-dependent spatial modulation arises from γ -ray absorption, which affects only the highest GDE energies and is confined to a limited region of the Galactic disk, in particular toward the Galactic centre.

Once discrete sources are included, however, additional structure appears in the sky maps, and this structure generally differs between low and high energies, as already illustrated in Sec. 11.1. Consequently, the simple relation expressed by Eq. (11.3) is no longer expected to hold for individual realisations. Instead, departures from the nearly perfect correlation of the smooth model should arise at 10 GeV and 100 TeV. In the following, we examine whether the deviations from the smooth predictions, denoted by $\Delta J_{10 \text{ GeV}}(l, b)$ and $\Delta J_{100 \text{ TeV}}(l, b)$, or the total intensities $J_{10 \text{ GeV}}(l, b)$ and $J_{100 \text{ TeV}}(l, b)$ remain correlated in the different injection and near-source transport scenarios.

Burst-like and energy-dependent escape scenario

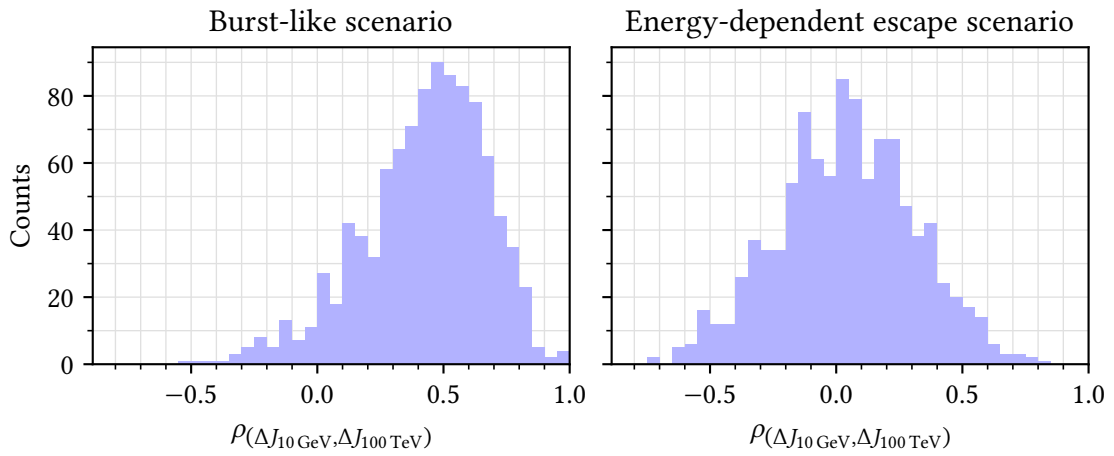


Figure 11.26: Correlation coefficients, burst-like and energy-dependent escape scenarios. Shown are histograms of the Pearson correlation coefficients ρ between $\Delta J_{10 \text{ GeV}}(l, b)$ and $\Delta J_{100 \text{ TeV}}(l, b)$, computed for each of 1000 source realisations. The distributions demonstrate that deviations of the GDE intensity from the mean at these two energies are only weakly correlated, as most coefficients lie well below 1. The correlation is generally stronger in the burst-like scenario, where the same sources contribute to the GDE intensity at both energies.

Fig. 11.26 shows histograms of the Pearson correlation coefficients between $\Delta J_{10 \text{ GeV}}(l, b)$ and $\Delta J_{100 \text{ TeV}}(l, b)$ for each of the 1000 realisations in the burst-like and energy-dependent escape scenarios. Each entry corresponds to the strength of an affine linear relation between the low- and high-energy deviations for a single realisation. In both cases, the correlation coefficients are significantly below unity, demonstrating that such a linear relation generally does not hold.

The deviations from the smooth-model predictions are primarily driven by a few dominant individual sources (see Sec. 10.4). Because cosmic-ray transport differs between low and high energies, the resulting imprints on the GDEs morphology are not the same. This

effect is particularly pronounced in the energy-dependent escape scenario, shown in the right panel of Fig. 11.26, where different sources located at different positions dominate the deviations at different energies. As a result, the correlation between ΔJ_{low} and ΔJ_{high} is almost entirely lost in this scenario.

Despite this, the deviations remain small compared to the smooth-model intensities themselves. Consequently, the correlation between the total intensities $J_{10 \text{ GeV}}(l, b)$ and $J_{100 \text{ TeV}}(l, b)$ remains very high, with most correlation coefficients deviating from unity by less than one percent. This indicates that, in these scenarios, the influence of discrete sources is spatially localised and does not substantially alter the global GDE intensity pattern. This behaviour is consistent with the findings of Sec. 11.5, where only a minor impact of source discreteness on spectra averaged over the LHAASO sky windows was observed for the burst-like and energy-dependent escape scenarios.

Time-dependent diffusion scenario

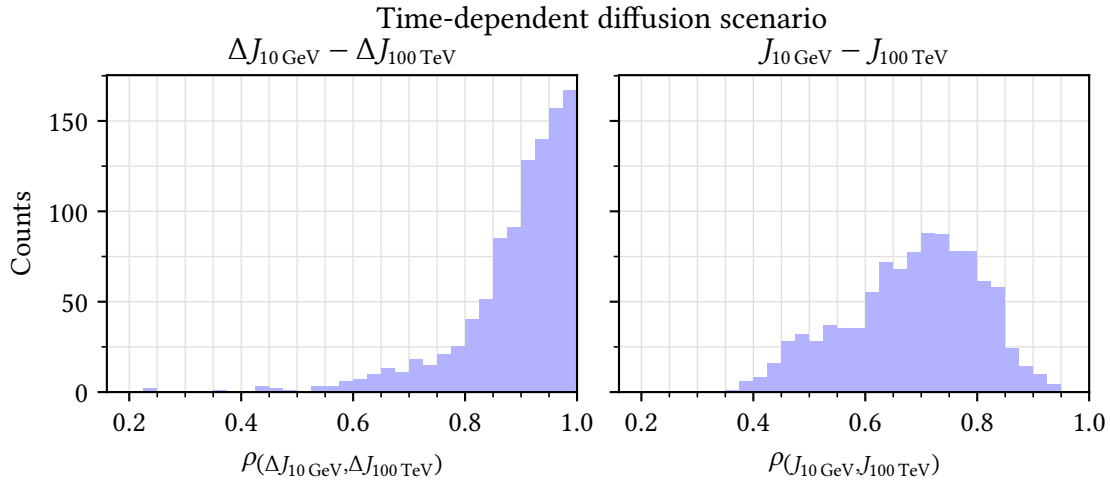


Figure 11.27: Correlation coefficients, time-dependent diffusion scenario. The left panel shows the same type of histogram as in Fig. 11.26, but for the time-dependent diffusion scenario. Here, the correlations are noticeably stronger than in the burst-like and energy-dependent escape scenarios. The right panel displays a histogram of the total GDE intensities $J_{10 \text{ GeV}}(l, b)$ and $J_{100 \text{ TeV}}(l, b)$, indicating that these absolute intensities are only weakly correlated.

The situation changes qualitatively in the time-dependent diffusion scenario, as illustrated in Fig. 11.27. Here, the deviations from the smooth-model predictions at 10 GeV and 100 TeV are often strongly correlated. The quantities ΔJ trace the imprint of some dominant sources. In the time-dependent diffusion scenario, the near-source transport is initially energy-independent, causing the propagation at different energies to follow

similar paths and thus producing correlated signatures in the GDEs. At the same time, contributions from older sources that have left the low-diffusion regime can reduce this correlation for some realisations. This interplay is reflected in the distribution of correlation coefficients shown in the left panel of Fig. 11.27.

Unlike in the burst-like and energy-dependent escape scenarios, discrete sources in the time-dependent diffusion model can also substantially enhance the total GDE intensity. As a consequence, the strong correlation between $J_{10\text{ GeV}}(l, b)$ and $J_{100\text{ TeV}}(l, b)$ seen in the previous scenarios is no longer preserved. This loss of correlation, visible in the right panel of Fig. 11.27, highlights the extreme impact that discrete sources can have on the GDEs in the time-dependent diffusion scenario, in agreement with the conclusions drawn in earlier sections.

Burst-like, halved source rate scenario

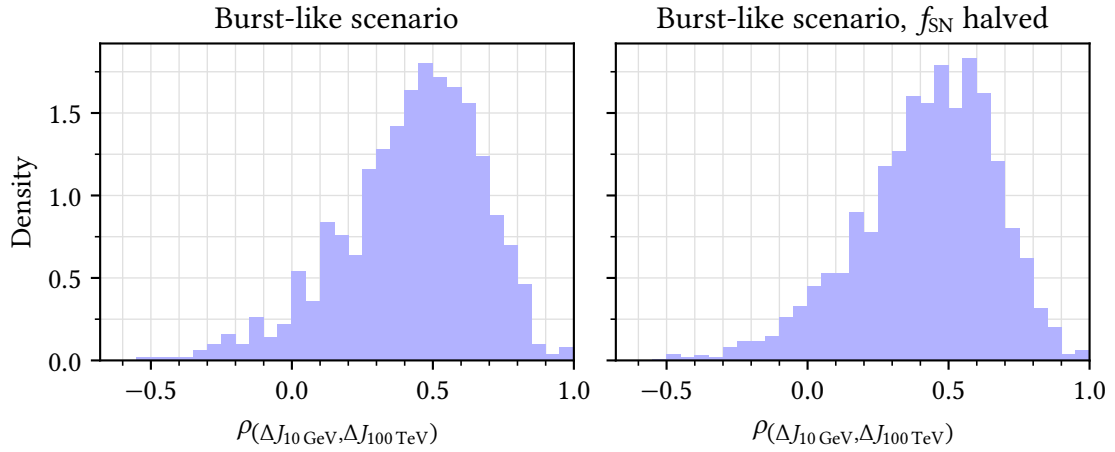


Figure 11.28: Correlation coefficients, burst-like and burst-like, halved source rate scenario, as in Fig. 11.26.

In Fig. 11.28, we contrast the correlation coefficients for the burst-like and burst-like, halved source rate scenarios. As expected, reducing the source rate does not alter the correlation between the deviations from the smooth-model predictions at 10 GeV and 100 TeV. The same physical considerations discussed for the burst-like scenario apply here. Increased stochasticity affects the low and the high energies similarly, leaving the correlation structure unchanged.

Summary

We have shown that, in models assuming a smooth source distribution, GDE sky maps at 10 GeV and 100 TeV are strongly correlated. Once discrete sources are included, this correlation is significantly reduced for the deviations from the smooth predictions, $\Delta J_{10 \text{ GeV}}(l, b)$ and $\Delta J_{100 \text{ TeV}}(l, b)$. The weakest correlation is found in the energy-dependent escape scenario, while the burst-like scenario (also for halved source rate) still exhibits poor correlation and the time-dependent diffusion scenario shows a comparatively strong one. These trends can be directly traced back to the specific characteristics of the respective injection and near-source transport models.

In contrast, the correlation between the total GDE intensities remains very high for the moderate burst-like and energy-dependent escape scenarios, but breaks down in the more extreme time-dependent diffusion case. This behaviour reflects the extent to which individual sources influence the overall GDE intensity. While their impact is confined to small regions around specific LOS in the burst-like and energy-dependent escape scenarios, discrete sources can significantly alter the global intensity distribution in the time-dependent diffusion scenario.

11.7 Discussion

In the preceding sections, we explored the generic signatures of discrete cosmic-ray source models in the GDEs, with particular emphasis on how different assumptions about source injection and near-source transport shape these signatures. In the following, we place these results into a broader context and discuss limitations of the present study.

One important simplifying assumption of our work is the adoption of a single population of SNRs, characterised by a universal source spectrum extending up to the cosmic-ray knee at a rigidity of 3 PV. This assumption greatly facilitates stochastic investigations of the kind presented here and has been widely employed in earlier studies (see, e.g., Blasi and Amato, 2012b; Evoli et al., 2021b; Genolini et al., 2017; Kaci and Giacinti, 2025; Mertsch, 2011; Phan et al., 2021). Arguments supporting the SNR paradigm were summarised in Sec. 4.4 and Sec. 7.4.1 of Chap. 7.

Restricting the analysis to a single source class enables a controlled comparison of the specific injection and near-source transport models introduced in Sec. 10.1.2. In reality, however, the Galactic source population is likely more heterogeneous, with source properties varying from object to object. While such complexity would provide a more realistic description, it remains largely unconstrained at present. A systematic investigation of multiple source populations should therefore build upon simpler single-population studies such as this one. Some of the effects identified here would likely be amplified, while others might be reduced. Analogous considerations apply to the

possibility of continuously injecting sources, such as star clusters, which were omitted here.

Another limitation of our approach is the use of a spatially homogeneous and isotropic diffusion model. This choice is common in stochastic source studies because it permits analytic solutions for the Green's functions and is supported by several arguments discussed in Sec. 5.3.8 of Chap. 5. Alternative frameworks, including anisotropic diffusion, have been proposed (see, e.g., Giacinti and Semikoz, 2023), but the Galactic magnetic-field structure and the spatial dependence of the diffusion coefficient are still poorly constrained. Anisotropic transport is expected to modify the morphology of source-induced features in the GDEs, but a detailed analysis of this is out of the scope of the present work.

Beyond the source injection and near-source transport models, a wide range of additional cosmic-ray model parameters could influence the results. A comprehensive exploration of this parameter space is, however, computationally prohibitive within a Monte Carlo framework of the size used here. This limitation is generic to studies of this kind. Recently, ML techniques have been proposed as a promising avenue to alleviate this problem (Frediani et al., 2025), potentially enabling more extensive parameter scans in the future.

As a specific example of parameter variation, we investigated the effect of reducing the source rate by a factor of two in the burst-like scenario. As expected, this enhances the stochastic nature of the GDEs. Nevertheless, the analysis presented in Part II already demonstrated that the response of stochastic observables to parameter changes is not always trivial. While a lower source rate increases the relative weight of individual sources, it simultaneously reduces the chance for a young source to be present. For the local cosmic-ray spectrum, these effects approximately compensated each other. In contrast, the results of the present study indicate that, for the GDEs, the enhanced stochasticity dominates when the source rate is halved. For a discussion of source-rate effects within the time-dependent diffusion scenario, we refer to Kaci and Giacinti, 2025.

The choice of the halo height, H , constitutes another relevant modelling uncertainty. Larger halo sizes, as suggested by measurements of stable and unstable isotopes at low energies (Evoli et al., 2020), require a higher normalisation of the diffusion coefficient in order to reproduce local secondary-to-primary ratios. This, in turn, reduces the level of stochasticity and is expected to suppress the magnitude of deviations in the GDEs. Different parametrisations of the diffusion coefficient are also expected to affect the results similarly: a smaller diffusion coefficient at a given energy would enhance stochastic effects and increase the expected deviations at that energy, whereas a larger diffusion coefficient would have the opposite effect. Moreover, changes in the spectral index of the diffusion coefficient directly translate into corresponding changes in the spectral slope of the GDE. A more detailed discussion of transport-parameter dependencies can be found in Sec. 9.2.3 of Part II.

Additional contributions to the total GDEs in γ -rays arise from leptonic diffuse emission processes. Both γ -ray and neutrino signals may also contain unresolved point-source contributions. These components were discussed in Sec. 6.6 of Chap. 6, but were not modelled explicitly in this study. There are strong indications that such contributions are subdominant at the energies of interest here. Nevertheless, stochastic studies including leptonic GDE components have been performed elsewhere (see, e.g., Marinos et al., 2025; Thaler et al., 2023).

Leptonic diffuse emission is expected to contribute less than 10 % to the total GDEs at low Galactic latitudes ($|b| < 10^\circ$) (Lipari and Vernetto, 2018). Furthermore, local measurements of the cosmic-ray electron spectrum indicate a cut-off at energies of a few tens of TV (Mertsch, 2018), which restricts the leptonic contribution above 1 TeV to at most 5 % (Lipari and Vernetto, 2018). Additional details can be found in Sec. 6.6 of Chap. 6.

Finally, our analysis exclusively considered hadronic interactions of cosmic-ray protons with the interstellar gas as the source of the GDEs. This deliberate restriction allowed us to isolate and quantify the impact of discrete proton sources on the *hadronic* diffuse emission. An important consequence is that our results can be straightforwardly translated to diffuse neutrino emission arising from the same hadronic interactions. Using the relation introduced in Eq. (6.13) of Sec. 6.5, the predicted γ -ray spectra can be converted to neutrino spectra with an appropriate energy rescaling. The spatial morphology and the magnitude of deviations from smooth-model expectations are therefore expected to carry over directly to diffuse neutrino emission.

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Summary and Conclusion

The origin of cosmic rays remains one of the most enduring mysteries in astrophysics. While significant theoretical and observational progress has been made over the past century (Chaps. 2 to 4), many aspects of the nature of Galactic cosmic-ray sources, acceleration mechanisms, and propagation are still not fully understood. Known as the *supernova remnant paradigm* (Sec. 4.4), it is commonly assumed that Supernova Remnants (SNRs) are the primary accelerators of Galactic cosmic rays up to the cosmic-ray knee at rigidities of a few PV. However, it is theoretically challenging to explain how SNRs can accelerate cosmic rays to such high rigidities (Sec. 5.6). Magnetic field amplification mechanisms (Sec. 5.7) could be the key to solving this problem. In addition, they also influence the escape of cosmic rays from their sources (Sec. 5.8), which can be captured in models of a rigidity-dependent escape of cosmic rays from their sources (e.g., the Cosmic-Ray Energy-Dependent Injection Time (CREDIT) model introduced in Part II).

A model of Galactic cosmic rays needs to describe the sources and how they inject cosmic rays into the Interstellar Medium (ISM). We argued that SNRs can be regarded as discrete in space and time (Sec. 7.1) and discussed that the often employed approximation of those discrete sources by a continuous source term must fail in some cases (Chap. 7, specifically Sec. 7.4). A model taking into account the discreteness and injection mechanisms of cosmic-ray sources is expected to make more accurate predictions of cosmic-ray intensities throughout the Galaxy.

Using the full power of astrophysical observations (Chap. 3), we can probe the properties of cosmic rays. This includes direct measurements of cosmic rays in the Solar system (Secs. 3.3 and 3.4), as well as observations of Galactic Diffuse Emissions (GDEs) of γ -rays (Sec. 3.5) and neutrinos (Sec. 3.6) produced by cosmic rays interacting with the ISM. These provide high-precision data of the cosmic-ray intensity in our local Galactic neighbourhood as well as information about cosmic-ray intensities throughout the entire Galaxy.

In this thesis, we investigated the impact of discrete cosmic-ray sources on the local cosmic-ray spectrum (Part II) and on GDEs (Part III). For this, we developed a stochastic framework to model discrete cosmic-ray sources and their injection mechanisms in extensive Monte Carlo simulations, accelerated by modern GPU programming paradigms.

We also explored the use of Machine Learning (ML) techniques to efficiently classify cosmic-ray spectra based on their underlying source models.

Local Galactic cosmic-ray spectrum

In Part II, we studied the impact of discrete cosmic-ray sources on the local cosmic-ray spectrum. For this, we revisited an often employed burst-like injection model, where all cosmic rays are injected instantaneously, and contrasted it with a rigidity-dependent escape model, that captures the physics of cosmic-ray escape from their sources more realistically (the CREDIT model). The latter is developed from previous models but adapted to be consistent with cosmic-ray acceleration up to the knee region (Sec. 5.8).

We found that the rigidity-dependent escape of cosmic rays from their sources can leave distinct imprints on the local cosmic-ray spectrum. We characterised them and determined how they depend on various model parameters (Sec. 8.2). The emergence of distinct spectral features in this model suggests that such features would be detectable in current and future cosmic-ray measurements. To address this, we employed ML techniques to train a Decision Tree (DT) classifier on Monte-Carlo-generated local cosmic-ray spectra from different injection models (Chap. 9).

We explored the potential of this approach and its dependence on various model parameters and experimental uncertainties (Sec. 9.2). This provides a proof of concept for the use of ML techniques to test cosmic-ray source models based on local cosmic-ray measurements. We conclude that such ML techniques are a promising avenue for future experimental analyses, especially with upcoming high-precision measurements of cosmic rays at higher rigidities. Increased event statistics and an improved understanding of the correlations of systematic uncertainties will be crucial to fully exploit the potential of this approach (Sec. 9.3).

Measurements of the local cosmic-ray spectrum provide important high-precision probes of cosmic-ray sources in our Galactic neighbourhood. However, they are inherently limited to a small volume of the Galaxy (Sec. 7.1). GDEs, however, probe cosmic-ray intensities throughout the Galaxy and thus provide complementary information about cosmic-ray sources on Galactic scales. A detailed understanding of diffuse γ -rays and neutrinos is therefore essential to use all the potential of astrophysical observations to answer the enduring questions about the origin of cosmic rays. We addressed this in Part III.

Galactic diffuse emission

In this part, we developed and analysed a stochastic framework for hadronic GDEs produced by cosmic-ray protons. We focused on three distinct cosmic-ray injection

and near-source transport scenarios (Sec. 10.1.2). The impact of discrete sources was quantified by comparing GDE predictions obtained from stochastic source realisations to those derived from the corresponding smooth source models, using relative deviations as a diagnostic (Sec. 10.4). Representative examples of these source-induced imprints on the diffuse sky were presented (Sec. 11.1). In addition, we characterised the most extreme deviations across thousands of realisations in order to determine the maximal excesses that can be induced by individual cosmic-ray sources in the different scenarios (Sec. 11.2).

A detailed investigation of GDEs along selected Lines Of Sight (LOS) was carried out with large sample sizes (10^6) for the different source models (Sec. 11.4). We demonstrated that the statistical properties of the underlying cosmic-ray intensity distributions (Sec. 7.4.2) are essential for understanding the characteristic features of the GDE intensity distributions along a single LOS. In the burst-like (also with halved source rate) and energy-dependent escape scenarios, the main features of the simulated GDE distributions can be described by stable laws and normal distributions (Sec. 10.3). This highlights the close link between spatial fluctuations of cosmic-ray intensities throughout the Galaxy and the observed properties of diffuse γ -ray emission. For more strongly altered source models, such as the time-dependent diffusion scenario, simple analytic descriptions become insufficient, and Monte Carlo simulations provide the most reliable means for studying the resulting GDE intensity distributions.

Further, we examined the impact of source discreteness on GDEs spectra averaged over extended sky windows, as well as on longitude profiles (Sec. 11.5). Uncertainties related to the unknown source positions and ages constitute an additional modelling uncertainty, complementing those associated with cosmic-ray transport parameters or interaction cross-sections. For the sky windows considered in experimental analyses of LHAASO, we found that the influence of discrete sources is small in the burst-like and energy-dependent escape scenarios and cannot account for the discrepancy between model predictions and LHAASO measurements. In contrast, the time-dependent diffusion scenario exhibits substantially larger uncertainties, making source stochasticity a relevant contribution to the overall model uncertainty. For more spatially resolved observables, such as longitude profiles, the effect of discrete sources becomes increasingly important, consistent with the highly localised morphology of individual source imprints on the GDEs. With improving angular resolution in high-energy GDE analyses, the discreteness of cosmic-ray sources therefore emerges as a key uncertainty.

We further investigated the correlation between GDEs at different energies, in particular at 10 GeV and 100 TeV (Sec. 11.6). We showed that this correlation depends sensitively on the extent to which the assumed injection and near-source transport models affect the total GDE intensity. In the moderate burst-like (also with halved source rate) and energy-dependent escape scenarios, the total GDE intensities at different energies remain strongly correlated, whereas this correlation is significantly reduced in the more extreme time-dependent diffusion scenario. By contrast, the correlation of the deviations from smooth-model predictions at different energies is governed by the details of the injection

and transport mechanisms. It is strong in the time-dependent diffusion scenario, where the early transport phase is energy-independent, but weak in the other scenarios. This provides an additional perspective on how cosmic-ray model assumptions shape the relationship between GDEs observed at different energies, and highlights a fundamental limitation of assuming a simple correspondence between them.

Throughout this work, our discussion of GDEs has focused on hadronic γ -ray emissions. These emissions are directly related to diffuse neutrino fluxes produced in hadronic pp-interactions. Consequently, our conclusions regarding GDEs can be extended to diffuse neutrino emission at appropriately rescaled energies (Sec. 6.5). In particular, this applies to our assessment of modelling uncertainties in sky window-averaged intensity spectra (Sec. 11.5), where source stochasticity was found to be subdominant in the burst-like and energy-dependent escape scenarios but significant in the time-dependent diffusion case.

Overall, the results presented here advance our understanding of how discrete cosmic-ray sources shape GDEs. They highlight the close connection between cosmic-ray intensity fluctuations and the resulting GDEs, and clarify how individual sources imprint characteristic structures on the diffuse emission. This, in turn, strengthens the role of GDE observations, such as those provided by LHAASO, as probes of cosmic-ray injection and near-source transport models. Our findings indicate that especially with future measurements achieving higher spatial resolution at energies beyond several tens of TeV, studies of GDEs will offer increasingly powerful constraints on the physics of Galactic cosmic-ray sources.

The study of cosmic rays throughout the last century has led to significant progress in shaping and refining our currently accepted paradigms of cosmic-ray physics. Theoretical and observational advancements alike have challenged and changed common assumptions. In recent years, cosmic-ray astrophysics has entered a new era of high-precision measurements, which are expected to provide deep insights in the years to come. We hope that the work presented in this thesis contributes to this endeavour and helps to shed light on one of the most enduring mysteries in cosmic-ray astrophysics: the origin of cosmic rays.

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A

Useful relations

In this section, we summarise useful relations for cosmic-ray physics. We start with the relations between the total energy, E , kinetic energy, E_{kin} , momentum, p , and rigidity, $\mathcal{R}$ of a particle with rest mass m_0 and charge Z . Then, we derive the relation between rigidity and Larmor radius. Finally, we introduce different ways to describe the distribution of cosmic rays and the relations between them.

A.1 Relations between E , E_{kin} , p , and $\mathcal{R}$

The total energy, E , of a particle is the sum of its kinetic energy, E_{kin} , and its rest mass energy, $E_0 = m_0 c^2$, where c is the speed of light in vacuum:

$$E = E_{\text{kin}} + E_0 = E_{\text{kin}} + m_0 c^2. \quad (\text{A.1})$$

A relation between the total energy, E , and the momentum, p , is provided by the relativistic energy-momentum relation:

$$E^2 = p^2 c^2 + m_0^2 c^4. \quad (\text{A.2})$$

A different way of expressing the total energy, E , is in terms of the Lorentz factor, γ

$$E = \gamma m_0 c^2, \quad (\text{A.3})$$

where $\gamma = 1/\sqrt{1 - \beta^2}$ and $\beta = v/c$ with v being the velocity of the particle. The momentum, p , can also be expressed in terms of γ :

$$p = \gamma m_0 v. \quad (\text{A.4})$$

This leads to the following relation between β , the total energy, E , and the momentum, p :

$$\beta = \frac{pc}{E}. \quad (\text{A.5})$$

$A(B) =$		B			
		E	E_{kin}	p	$\mathcal{R}$
A	E	—	$E_{\text{kin}} + E_0$	$\sqrt{(pc)^2 + E_0^2}$	$\sqrt{(Ze\mathcal{R})^2 + E_0^2}$
	E_{kin}	$E - E_0$	—	$\sqrt{(pc)^2 + E_0^2} - E_0$	$\sqrt{(Ze\mathcal{R})^2 + E_0^2} - E_0$
	p	$\frac{1}{c} \sqrt{E^2 - E_0^2}$	$\frac{1}{c} \sqrt{E_{\text{kin}}(E_{\text{kin}} + 2E_0)}$	—	$\frac{Ze\mathcal{R}}{c}$
	$\mathcal{R}$	$\frac{1}{Ze} \sqrt{E^2 - E_0^2}$	$\frac{1}{Ze} \sqrt{E_{\text{kin}}(E_{\text{kin}} + 2E_0)}$	$\frac{pc}{Ze}$	—

Table A.1: Relations between E , E_{kin} , p , and $\mathcal{R}$.

The rigidity, $\mathcal{R}$, of a particle is defined as

$$\mathcal{R} = \frac{pc}{Ze}, \quad (\text{A.6})$$

where Z is the charge of the particle and e is the elementary charge. Using the energy-momentum relation in Eq. (A.2), we can express this as a function of the total energy, E

$$\mathcal{R}(E) = \frac{1}{Ze} \sqrt{E^2 - (m_0c^2)^2} \quad (\text{A.7})$$

and similarly as a function of the kinetic energy, E_{kin} :

$$\mathcal{R}(E_{\text{kin}}) = \frac{1}{Ze} \sqrt{E_{\text{kin}}(E_{\text{kin}} + 2m_0c^2)}. \quad (\text{A.8})$$

A summary of the relations between E , E_{kin} , p , and $\mathcal{R}$ is given below in Tab. A.1. Later, we will need the derivatives of these quantities with respect to each other. We summarise them in Tab. A.2.

We can express β as a function of E , E_{kin} , p , and $\mathcal{R}$, respectively:

$$\beta(E) = \frac{\sqrt{E^2 - (m_0c^2)^2}}{E}, \quad (\text{A.9})$$

$$\beta(E_{\text{kin}}) = \frac{\sqrt{E_{\text{kin}}(E_{\text{kin}} + 2m_0c^2)}}{E_{\text{kin}} + m_0c^2}, \quad (\text{A.10})$$

$$\beta(p) = \frac{pc}{\sqrt{(pc)^2 + (m_0c^2)^2}}, \quad (\text{A.11})$$

$$\beta(\mathcal{R}) = \frac{\mathcal{R}}{\sqrt{\mathcal{R}^2 + \left(\frac{m_0c^2}{Ze}\right)^2}}. \quad (\text{A.12})$$

Those relations can be used conveniently to convert between different descriptions of the energy of a cosmic ray particle. We usually use the units of GeV for E and E_{kin} , GeV/c for

$\frac{da}{db}$		a			
		E	E_{kin}	p	$\mathcal{R}$
b	E	—	1	$(\beta c)^{-1}$	$(Ze\beta)^{-1}$
	E_{kin}	1	—	$(\beta c)^{-1}$	$(Ze\beta)^{-1}$
	p	βc	βc	—	$\frac{c}{Ze}$
	$\mathcal{R}$	$Ze\beta$	$Ze\beta$	$\frac{Ze}{c}$	—

Table A.2: Derivatives of E , E_{kin} , p , and $\mathcal{R}$ with respect to each other.

p , GeV/c^2 for m , and GV for $\mathcal{R}$. The relations given above can be used conveniently with these units. E.g., the relation between rigidity and energy in Eq. (A.7) becomes

$$\mathcal{R} (E [\text{GeV}]) [\text{GV}] = \sqrt{(E [\text{GeV}])^2 - (m_0 [\text{GeV}/c^2])^2}. \quad (\text{A.13})$$

In this thesis, we are working exclusively with protons, i.e., $Z = 1$ and $m_0 = m_p$, where $m_p \approx 1 \text{ GeV}/c^2$ is the proton rest mass, which simplifies the relations further.

A.2 Rigidity–Larmor radius relation

As we have already introduced in Eq. (A.7), the rigidity, $\mathcal{R}$, of a charged particle is defined as its momentum per unit charge. If a charged particle moves in a uniform, perpendicular magnetic field $\mathbf{B}$, it performs a circular motion with the Larmor radius r_L given by

$$r_L = \frac{pc}{ZeB} = \frac{\mathcal{R}}{B}, \quad (\text{A.14})$$

where Z is the charge of the particle, e is the elementary charge, c is the speed of light, and B is the magnetic field strength. Note, that particles with the same rigidity are influenced by magnetic fields in the same way. This makes rigidity a very useful quantity in the study of cosmic-ray acceleration and transport processes.

It is useful to express Eq. (A.14) in units that are suited for astrophysical applications:

$$r_L = \frac{10^{15} \cdot \frac{1}{300} \text{ G cm}}{10^{-6} \text{ G}} \left(\frac{\mathcal{R}}{\text{PV}} \right) \left(\frac{B}{\mu\text{G}} \right)^{-1} \approx 1 \text{ pc} \left(\frac{\mathcal{R}}{\text{PV}} \right) \left(\frac{B}{\mu\text{G}} \right)^{-1}, \quad (\text{A.15})$$

where we used $1 \text{ V} \approx \frac{1}{300} \text{ G cm}$ and $1 \text{ pc} \approx 3.086 \cdot 10^{18} \text{ cm}$. The conversion between the SI-unit V and the Gaussian unit G cm involves a numerical factor $c_{\text{CGS}} 10^{-8}$, where

c_{CGS} is the numerical value of the speed of light in the Gaussian system of units, i.e., $c_{\text{CGS}} \approx 3 \cdot 10^{10} \text{ cm s}^{-1}$.

In short, we can write this as

$$r_{\text{L, pc}} \approx \mathcal{R}_{\text{PV}} B_{\mu\text{G}}^{-1}, \quad (\text{A.16})$$

where the subscripts indicate the units used for the respective quantities. We use this shorthand notation at various places in this thesis.

A.3 Isotropic differential densities

There are different ways to describe the distribution of cosmic rays (Moraal, 2013; Navas et al., 2024). One fundamental quantity is the Lorentz invariant phase space density $f(\mathbf{x}, \mathbf{p}, t)$ (Forman, 1970), which describes the number density of particles in a given volume of phase space. It is defined as

$$f(\mathbf{x}, \mathbf{p}, t) = \frac{dN}{d^3\mathbf{x} d^3\mathbf{p}} = \frac{dn}{d^3\mathbf{p}}, \quad (\text{A.17})$$

where N is the total number of particles and $n = \frac{dN}{d^3\mathbf{x}}$ is the number density.

For an isotropic distribution, the phase space density only depends on the magnitude of the momentum $p = |\mathbf{p}|$ and not on its direction. The angular dependence can be integrated out:

$$f(\mathbf{x}, p, t) = \frac{1}{4\pi} \int d\Omega f(\mathbf{x}, \mathbf{p}, t), \quad (\text{A.18})$$

where $d\Omega = \sin \theta d\theta d\phi$ is the differential solid angle.

We are interested in the density of particles in a momentum interval $[p, p + dp]$. By replacing the volume element in momentum space $d^3\mathbf{p} = 4\pi p^2 dp$ and integrating over the solid angle, we find

$$\frac{dN}{d^3\mathbf{x} dp} = \frac{dn}{dp} = 4\pi p^2 f(\mathbf{x}, p, t). \quad (\text{A.19})$$

We can also consider intervals in kinetic energy, total energy, or rigidity instead of momentum. These quantities are called isotropic differential densities. We define them as follows:

$$\psi_p(\mathbf{x}, p, t) = \frac{dn}{dp} = 4\pi p^2 f(\mathbf{x}, p, t), \quad (\text{A.20})$$

$$\psi_{E_{\text{kin}}}(\mathbf{x}, E_{\text{kin}}, t) = \frac{dn}{dE_{\text{kin}}} = \frac{dp}{dE_{\text{kin}}} \psi_p(\mathbf{x}, p(E_{\text{kin}}), t), \quad (\text{A.21})$$

$$\psi_E(\mathbf{x}, E, t) = \frac{dp}{dE} = \frac{dn}{dE} \psi_p(\mathbf{x}, p(E), t), \quad (\text{A.22})$$

$$\psi_{\mathcal{R}}(\mathbf{x}, \mathcal{R}, t) = \frac{dn}{d\mathcal{R}} = \frac{dp}{d\mathcal{R}} \psi_p(\mathbf{x}, p(\mathcal{R}), t). \quad (\text{A.23})$$

Similar relations between the isotropic differential densities can be derived using the derivatives given in Tab. A.2:

$$\psi_X = \frac{dY}{dX} \psi_Y. \quad (\text{A.24})$$

A.4 Isotropic differential intensities

The measured quantity in cosmic-ray experiments, however, is usually a differential intensity. This is defined as the number of particles per unit area, unit time, unit solid angle, and unit energy (or momentum, rigidity, etc.). Assuming an isotropic distribution of particles, the isotropic differential intensity, Φ_X , can be expressed as

$$\Phi_X(\mathbf{x}, X, t) = \frac{dN}{dA dt d\Omega dX} = \frac{v(X)}{4\pi} \psi_X(\mathbf{x}, X, t), \quad (\text{A.25})$$

where X can be E , E_{kin} , p , or $\mathcal{R}$ and $v(X) = c \beta(X)$ is the velocity of the particle expressed as a function of X . Note, that the differential intensity is often also labelled as flux in the literature and is often denoted as J . Sometimes, the differential intensity w.r.t. a quantity X is also labelled dJ/dX (see, e.g, Fig. 3.1). We usually use the notation Φ_X or simply Φ , if the quantity X is clear from the context, for cosmic rays. For Galactic Diffuse Emissions (GDEs), we use the notation J_X or simply J .

We can use the relations summarised in Tab. A.2 to derive relations between the isotropic differential intensities. We just reference one result used in this thesis. The isotropic differential intensity w.r.t. kinetic energy is related to the isotropic differential density w.r.t. rigidity and the phase space density as

$$\Phi_{E_{\text{kin}}} = \frac{c}{4\pi Ze} \psi_{\mathcal{R}} = p^2 f. \quad (\text{A.26})$$

Eqs. (A.20) and (A.26) tell us how to translate power-law scaling of the phase space density, e.g., $f \propto p^\gamma$, into power-law scalings of isotropic differential densities and intensities. We find that they follow a power law with an index of $\gamma + 2$.

B

Solar modulation

In this section, we provide the details of solar modulation, which is used to include the effect of solar wind on the cosmic ray fluxes measured at the Earth. This mostly effects low energy cosmic rays (below a few GeV). We will give a short overview of the effect and introduce the force-field approximation by Gleeson and Axford, 1968. A more detailed discussion of solar modulation can be found in Moraal, 2013 and Potgieter, 2013.

The magnetised Solar wind influences the propagation of cosmic rays in multiple ways. It leads to advection with the wind, diffusion in the turbulence created by the wind on small scales, adiabatic energy losses due to expansion, and drifts due to changes in the magnetic field on large scales (Potgieter, 2013). The transport of cosmic rays in the heliosphere is described by the following transport equation (see Parker, 1965 and compare to Eq. (5.64)):

$$\frac{\partial f}{\partial t} + \nabla \cdot (C\mathbf{V}f - \boldsymbol{\kappa} \cdot \nabla f) + \frac{1}{3p^2} \frac{\partial}{\partial p} (p^3 \mathbf{V} \cdot \nabla f) = q, \quad (\text{B.1})$$

where f is the phase space density of the cosmic rays, p is the momentum in the observer frame, $\mathbf{V}$ is the solar wind velocity, $C = -\frac{1}{3}(\partial f / \partial p)$ accounts for the Compton-Getting effect (Compton and Getting, 1935), $\boldsymbol{\kappa}$ is the diffusion tensor, and q is the source term. From left to right, the terms describe temporal changes, convection, diffusion, adiabatic energy changes, and sources and sinks. The drift mentioned above is included as an antisymmetric component in the diffusion tensor $\boldsymbol{\kappa}$ (Moraal, 2013).

The most commonly used approximation for modelling solar modulation is the force-field approximation first described by Gleeson and Axford, 1968. It assumes a steady state, spherical symmetry, and no drifts. The simplified version of Eq. (B.1) is then solved using the method of characteristics, along which the phase space density is constant. Translated to the isotropic differential intensities, this leads to the following modulation equation:

$$\Phi_{E_{\text{kin}}}^{\text{TOA}}(E - \phi) = \frac{(E - \phi)^2 - E_0^2}{E^2 - E_0^2} \Phi_{E_{\text{kin}}}^{\text{LIS}}(E), \quad (\text{B.2})$$

where $\Phi_{E_{\text{kin}}}^{\text{TOA}}$ is the Top Of Atmosphere (TOA) differential intensity, $\Phi_{E_{\text{kin}}}^{\text{LIS}}$ is the Local Interstellar Spectrum (LIS), E is the total energy, E_0 is the rest mass energy, and ϕ describes

the modulation. In principle, ϕ depends on the cosmic-ray species, its energy and the time of observation (Gleeson and Axford, 1968). As the isotropic differential intensity and the phase space density are related via Eq. (A.26), this equation simply states that $f/p^2 = \text{const}$ along a characteristic, described by the function ϕ .

In the literature, the modulation equation is usually written differently where all E are replaced with $E + \phi$. This reflects that the TOA intensities are measured and the LIS intensities are in principle unknown (see, however, the Voyager measurements (Stone et al., 2013)). As we model the LIS intensities and then apply solar modulation to predict the measurements, we write the equation in the form given here.

Often, simplifying assumptions on the transport parameters are made (Caballero-Lopez and Moraal, 2004; Cholis et al., 2016; Gleeson and Axford, 1968), that relate ϕ to a solely time- and space-dependent modulation potential φ , as

$$\phi = |Z| e \varphi . \quad (\text{B.3})$$

Then, Eq. (B.2) becomes

$$\Phi_{E_{\text{kin}}}^{\text{TOA}}(E - |Z| e \varphi) = \frac{(E - |Z| e \varphi)^2 - E_0^2}{E^2 - E_0^2} \Phi_{E_{\text{kin}}}^{\text{LIS}}(E) . \quad (\text{B.4})$$

Written in terms of the kinetic energy $E_{\text{kin}} = E - E_0$, this becomes

$$\Phi_{E_{\text{kin}}}^{\text{TOA}}(E_{\text{kin}} - |Z| e \varphi) = \frac{(E_{\text{kin}} - |Z| e \varphi)(E_{\text{kin}} - |Z| e \varphi + 2E_0)}{E_{\text{kin}}(E_{\text{kin}} + 2E_0)} \Phi_{E_{\text{kin}}}^{\text{LIS}}(E_{\text{kin}}) . \quad (\text{B.5})$$

The modulation potential φ , also called Fisk potential, has the units of V. It is usually determined by fitting to data. However, modelling the whole complexity of solar modulation with only one parameter is a strong simplification. There are many approaches to solve Eq. (B.1). E.g., Kuhlen and Mertsch, 2019 made less limiting assumptions on the symmetry and included drifts in a semianalytical model. Often, the transport equation is solved numerically (Boschini et al., 2018; Fiandrini et al., 2021; Kappl, 2016; Vittino et al., 2017). Schlickeiser, 2002 pointed out that the description of the modulation of Galactic cosmic rays by the Sun is possibly the most complicated and detailed application of the cosmic-ray transport equation. For most practical purposes, however, the force-field approximation is sufficient.

C

Additional derivations

C.1 Compression ratio, Eq. (5.79)

First, we use the mass conservation in Eq. (5.75) and define:

$$A = \rho_u u_u = \rho_d u_d. \quad (\text{C.1})$$

Using the momentum conservation in Eq. (5.76), we can express A as:

$$A^2 = \frac{P_d - P_u}{\frac{1}{\rho_u} - \frac{1}{\rho_d}}. \quad (\text{C.2})$$

Inserting our definition for A in Eq. (C.1) into the energy conservation in Eq. (5.78), using the previous expression for A^2 , and rearranging for the compression ratio $r = \rho_d/\rho_u$, we find:

$$\begin{aligned} \frac{1}{2} \frac{A^2}{\rho_u^2} + \frac{\Gamma}{\Gamma-1} \frac{P_u}{\rho_u} &= \frac{1}{2} \frac{A^2}{\rho_d^2} + \frac{\Gamma}{\Gamma-1} \frac{P_d}{\rho_d} \\ \frac{A^2}{2} \left(\frac{1}{\rho_u^2} - \frac{1}{\rho_d^2} \right) &= \frac{\Gamma}{\Gamma-1} \left(\frac{P_d}{\rho_d} - \frac{P_u}{\rho_u} \right) \\ \frac{1}{2} (P_d - P_u) \left(\frac{1}{\rho_u} + \frac{1}{\rho_d} \right) &= \frac{\Gamma}{\Gamma-1} \left(\frac{P_d}{\rho_d} - \frac{P_u}{\rho_u} \right) \\ \frac{1}{\rho_u} \left(\frac{1}{2} (P_d - P_u) + \frac{\Gamma}{\Gamma-1} P_u \right) &= \frac{1}{\rho_d} \left(\frac{1}{2} (P_u - P_d) + \frac{\Gamma}{\Gamma-1} P_d \right) \\ \frac{1}{\rho_u} \left(\frac{\Gamma+1}{\Gamma-1} P_u + P_d \right) &= \frac{1}{\rho_d} \left(\frac{\Gamma+1}{\Gamma-1} P_d + P_u \right). \end{aligned} \quad (\text{C.3})$$

From this, the result in Eq. (5.79) follows immediately:

$$r = \frac{\rho_d}{\rho_u} = \frac{\frac{\Gamma+1}{\Gamma-1} P_d + P_u}{P_d + \frac{\Gamma+1}{\Gamma-1} P_u}. \quad (\text{C.4})$$

C.2 Inverse Fourier transform in Sec. 7.3.1

We want to evaluate an integral of the form:

$$f(\mathbf{x}) = \int d^3k \, e^{i\mathbf{k}\cdot\mathbf{x}} e^{-\frac{1}{2}a^2k^2}. \quad (\text{C.5})$$

This can be done by applying a gradient with respect to $\mathbf{x}$:

$$\nabla f(\mathbf{x}) = i\mathbf{k} \int d^3k \, e^{i\mathbf{k}\cdot\mathbf{x}} e^{-\frac{1}{2}a^2k^2} \quad (\text{C.6})$$

$$= -i \frac{2}{a^2} \int d^3k \, e^{i\mathbf{k}\cdot\mathbf{x}} \nabla_{\mathbf{k}} e^{-\frac{1}{2}a^2k^2} \quad (\text{C.7})$$

$$= i \frac{2}{a^2} \int d^3k \, \nabla_{\mathbf{k}} (e^{i\mathbf{k}\cdot\mathbf{x}}) e^{-\frac{1}{2}a^2k^2} \quad (\text{C.8})$$

$$= -\frac{2}{a^2} \int d^3k \, e^{i\mathbf{k}\cdot\mathbf{x}} e^{-\frac{1}{2}a^2k^2} \quad (\text{C.9})$$

$$= -\frac{2}{a^2} f(\mathbf{x}), \quad (\text{C.10})$$

where we integrated by parts and used that the boundary term vanishes. This gives a differential equation for $f(\mathbf{x})$, which can be solved to give:

$$f(\mathbf{x}) = C \exp\left(-\frac{|\mathbf{x}|^2}{2a^2}\right), \quad (\text{C.11})$$

where C is a constant that can be determined by evaluating the integral at $\mathbf{x} = 0$. This is a standard Gaussian integral and gives:

$$C = \int d^3k \, e^{-\frac{1}{2}a^2k^2} = \left(\frac{2\pi}{a^2}\right)^{3/2}. \quad (\text{C.12})$$

D

Steady state solution of the transport equation

Following the discussion in Blasi and Amato, 2019, we want to show that the approximate relation given in Eq. (4.15) in Sec. 4.5 can also be derived by solving the transport equation for cosmic-ray protons (Eq. (5.69)) under simplified steady state assumptions.

Compared to the age of our Galaxy, $\mathcal{O}(10 \text{ Gyr})$, the timescales of variability of the cosmic-ray density are rather short. In Sec. 4.3, we estimated that the residence time of nuclei in the Galaxy is at maximum a few tens of Myr. Thus, we can assume that the differential cosmic-ray proton density, ψ , is in a steady state, i.e., $\partial\psi/\partial t = 0$. Further, we assume that cosmic-ray sources are distributed homogeneously in an infinitesimally thin disk at $z = 0$, which is so extended that there is only negligible radial dependence of the differential cosmic-ray density. This renders our problem one-dimensional in the z -direction perpendicular to the disk and simplifies the transport equation for cosmic-ray protons, Eq. (5.69), to:

$$-\frac{\partial}{\partial z} \left(\kappa(\mathcal{R}) \frac{\partial \psi}{\partial z} \right) = Q_0(\mathcal{R}) \delta(z), \quad (\text{D.1})$$

with a free escape boundary condition at $z = \pm H$, i.e., $\psi(z = \pm H) = 0$.

For $z \neq 0$, the right-hand side of Eq. (D.1) vanishes, and by integrating twice, we find:

$$\kappa(\mathcal{R}) \frac{\partial \psi}{\partial z} = c_1(\mathcal{R}) \quad \Rightarrow \quad \psi(\mathcal{R}, z) = \frac{c_1(\mathcal{R})}{\kappa(\mathcal{R})} z + c_2(\mathcal{R}), \quad (\text{D.2})$$

for some integration constants $c_1(\mathcal{R})$ and $c_2(\mathcal{R})$. Using the boundary conditions at $z = \pm H$, we find:

$$\psi(\mathcal{R}, z) = \psi_0(\mathcal{R}) \left(1 - \frac{|z|}{H} \right), \quad (\text{D.3})$$

where we defined $\psi_0(\mathcal{R}) = \psi(\mathcal{R}, z = 0)$ as the differential cosmic-ray density in the disk. To determine $\psi_0(\mathcal{R})$, we integrate Eq. (D.1) over an infinitesimal interval around $z = 0$ and find:

$$-\kappa(\mathcal{R}) \frac{\partial \psi}{\partial z} \Big|_{z=0^-}^{z=0^+} = Q_0(\mathcal{R}). \quad (\text{D.4})$$

Using the solution in the halo given in Eq. (D.3), we find:

$$2 \frac{\psi_0(\mathcal{R}) \kappa(\mathcal{R})}{H} = Q_0(\mathcal{R}) \quad \Rightarrow \quad \psi_0(\mathcal{R}) = Q_0(\mathcal{R}) \frac{H}{2\kappa(\mathcal{R})}, \quad (\text{D.5})$$

which is in agreement with the result given in Eq. (4.15).

The rigidity-dependence of the source term $Q_0(\mathcal{R})$ is given by the acceleration process in the sources. For DSA, it can be modelled as a power law in rigidity, i.e., $Q_0(\mathcal{R}) \propto \mathcal{R}^{-\alpha}$, where $\alpha \approx 2$ as derived in Sec. 5.5. The diffusion coefficient, $\kappa(\mathcal{R})$, is also assumed to have a power-law dependence on rigidity. Thus, we find that the differential steady-state cosmic-ray proton density in the disk follows a power law in rigidity as well.

E.1 Additional figures

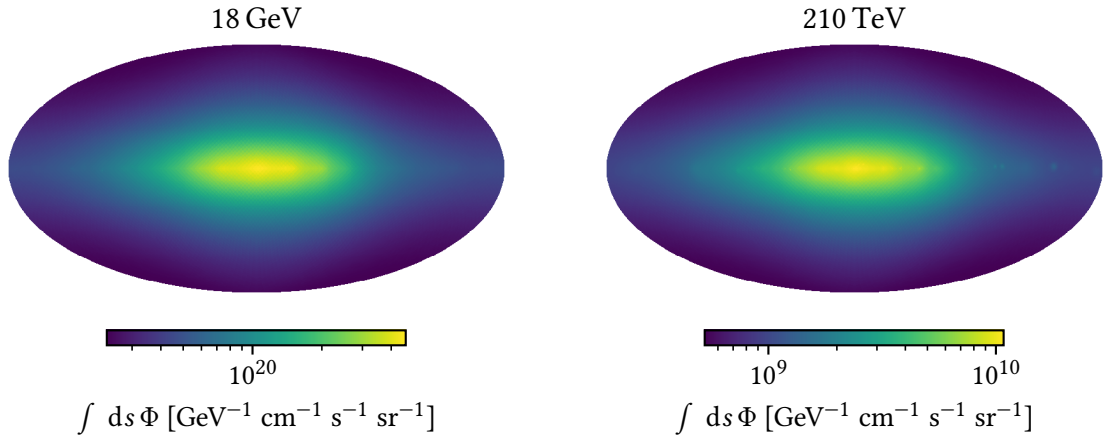


Figure E.1: Sky maps of integrated cosmic-ray intensity. We show the LOS integrated cosmic-ray intensities at 18 GeV (left) and 210 TeV (right) for a random source realisation in the burst-like scenario. The colour scale is logarithmic to better visualise the full-sky emission range.

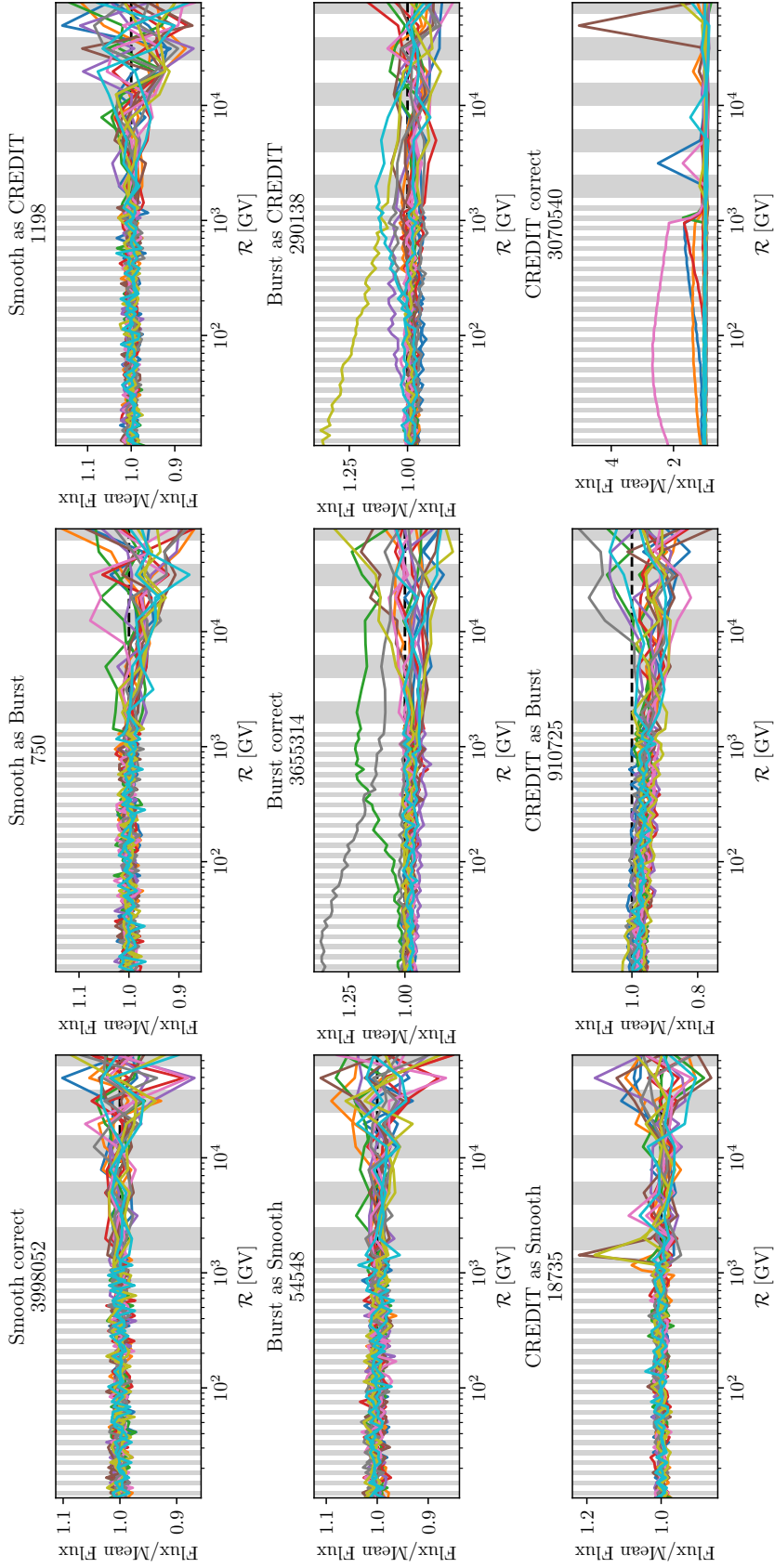


Figure E.2: Example spectra for the fiducial model classification. The different panels show 20 example spectra for each of the entries of the confusion matrix presented in Fig. 9.1 (lower right panel). This illustrates how the different model classes can be confused with each other.