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Where to market flexibility? Integrating continuous intraday trading into multi-market participation of industrial multi-energy systems

Niklas Nolzen [a,b], Alissa Ganter [a,b], Nils Baumgärtner [c], Florian Joseph Baader [a], Ludger Leenders [a], André Bardow [a,d,*]

[a] Energy & Process Systems Engineering, Department of Mechanical and Process Engineering, ETH Zürich, Tannenstrasse 3, 8092 Zürich, Switzerland
[b] Institute of Technical Thermodynamics, RWTH Aachen University, Schinkelstraße 8, 52062 Aachen, Germany
[c] Energy Trading and Dispatching, Currenta GmbH & Co. OHG, Kaiser-Wilhelm-Allee 80, 51373 Leverkusen, Germany
[d] Institute of Energy and Climate Research - Energy Systems Engineering (IEK-10), Forschungszentrum Jülich GmbH, Wilhelm-Johnen-Straße, 52425 Jülich, Germany

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ABSTRACT

The rising share of volatile renewable electricity generation increases the demand for flexibility. Flexibility can be offered by industrial multi-energy systems and marketed either on the continuous intraday, day-ahead, or balancing-power markets. Thus, industrial multi-energy systems face the question where to market their flexibility. We propose a two-step method to integrate trading on the continuous intraday market into a multi-market optimization for flexible industrial multi-energy systems. First, we estimate revenues from continuous trading in the intraday market, employing option-price theory. Second, a multi-stage stochastic optimization allocates the flexibility to the three markets. The case study of an industrial multi-energy system demonstrates that coordinated bidding in all three markets reduces costs the most. A sensitivity analysis reveals that the optimal split between the different markets strongly depends on the intraday market volatility. Overall, the proposed method provides a practical decision-support tool for multi-energy systems participating in short-term electricity and balancing-power markets.

1. Introduction

The rise of variable renewable generation increases supply uncertainty in the electricity grid. To ensure stability, more short-term system balancing is needed. This growing need for grid balancing is addressed by balancing-power (Ocker and Ehrhart, 2017) and continuous intraday markets (Koch and Hirth, 2019).

Conventionally, flexibility for system balancing is provided by fossil power plants (Rancilio et al., 2022), which are, however, being phased out during the energy transition. As a substitute, industrial multi-energy systems are promising candidates to provide flexibility (Nolzen et al., 2023; Srinivasan et al., 2023; Feng et al., 2023). As multi-energy systems commonly handle large amounts of energy in a highly integrated fashion, these systems typically achieve the provision of flexibility by fuel switching (Bauer et al., 2022). For instance, heating demands can be met by gas-driven combined-heat-and-power units or electrode boilers, while cooling demands can be fulfilled by either electric or heat-powered absorption chillers. Depending on the units utilized, these systems vary their electricity production and consumption and market the resulting flexibility.

Both the balancing-power and the continuous intraday market require participants to have sufficient flexible capacity, but differ in their market characteristics: On the continuous intraday market, flexible capacity can exploit price volatilities by asset-backed trading (Löhndorf and Wozabal, 2021). Asset-backed trading continuously trades the flexible capacity based on the real-time electricity price and the marginal electricity costs, with the sum of all trades determining the revenue and electricity supplied. On the balancing-power market, the flexible capacity commits positive and/or negative balancing power for a certain period in the future, e.g., the next day or the following week. The remuneration comprises the capacity price for the provision of balancing-power capacity and/or the energy price for the request of balancing power (Barbero et al., 2020). In addition, the day-ahead market facilitates the purchase and sale of electricity one day before delivery to fix the price and volume in advance.

While marketing flexibility can reduce costs, successful implementation is often challenging for two reasons: First, industrial multi-energy

* Corresponding author at: Energy & Process Systems Engineering, Department of Mechanical and Process Engineering, ETH Zürich, Tannenstrasse 3, 8092 Zürich, Switzerland.
E-mail address: abardow@ethz.ch (A. Bardow).

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Table 1
Overview on relevant literature and models on multi-market participation, categorized by participation in day-ahead and balancing-power markets (top), day-ahead and intraday markets (middle), and all three markets (bottom).

Features	Day-ahead market	Balancing-power market	*Capacity price bidding*	*Energy price bidding*	Intraday market	*Continuous trading*	Multi-energy system
Our paper	✓	✓	✓	✓	✓	✓	✓
Swider (2007)	✓	✓	✓				
Boomsma et al. (2014)	✓	✓		✓			
Muche et al. (2016)	✓	✓					✓
Kumbartzky et al. (2017)	✓	✓	✓				✓
Schäfer et al. (2019)	✓	✓	✓				
Leenders et al. (2020)	✓	✓		✓			✓
Bohlayer et al. (2020)	✓	✓	✓	✓			
Fusco et al. (2023)	✓	✓		✓			✓
Finhold et al. (2023)	✓	✓	✓	✓			
Dalle Ave et al. (2019)	✓				✓		
Finnah et al. (2022)	✓				✓		
Corinaldesi et al. (2020)	✓				✓	✓	
Germscheid et al. (2022)	✓				✓		
Germscheid et al. (2023)	✓				✓		
Dowling et al. (2017)	✓	✓			✓		✓
Kraft et al. (2022)	✓	✓	✓		✓		

systems usually generate several energy forms in a highly integrated fashion, requiring adjustments to their entire operation to provide flexibility. Second, the sequential market clearing, typical for European electricity markets, reveals market uncertainties only gradually. However, bidding on different markets couples the decisions and leads to a sequential decision-making process in which the available flexibility to be traded on one market depends on the commitments to the other markets.

Optimal participation in sequential electricity markets thus requires so-called coordinated bidding (Löhndorf and Wozabal, 2022). Coordinated bidding requires the value of the opportunity to trade on the continuous intraday market when deciding on participation in the balancing-power and day-ahead market (Klaboe and Fosso, 2013).

1.1. Relevant literature on coordinated bidding for multi-market participation

This section reviews approaches for coordinated bidding, including the day-ahead, balancing-power and intraday markets (Table 1). Subsequently, we discuss methods for continuous trading on the intraday market.

Coordinated bidding considers participation in multiple markets in one single optimization problem (Aasgård et al., 2019). Typically, multi-stage stochastic optimization problems are employed to account for price uncertainties in the balancing-power market (Swider, 2007; Boomsma et al., 2014) and the intraday market (Germscheid et al., 2022, 2023). The balancing-power market is often simplified by either considering fixed average capacity and energy prices (Muche et al., 2016), only optimizing capacity-price bids (Kumbartzky et al., 2017; Schäfer et al., 2019), or energy-price bids (Leenders et al., 2020; Fusco et al., 2023). While average prices neglect the effect of optimized bidding decisions, neglecting capacity or energy prices potentially underestimates the revenue opportunities. Conversely, optimizing both capacity and energy-price bids results in a computationally challenging multi-stage stochastic optimization problem (Bohlayer et al., 2020) that may require decomposition approaches (Fusco et al., 2023; Finhold et al., 2023). Apart from the computational complexity, all reviewed publications (Swider, 2007; Boomsma et al., 2014; Muche et al., 2016; Kumbartzky et al., 2017; Schäfer et al., 2019; Leenders et al., 2020; Bohlayer et al., 2020; Fusco et al., 2023; Finhold et al., 2023) show clear cost benefits from utilizing flexibility in the balancing-power market. However, the intraday market is not considered as a potential alternative.

Coordinated bidding between the day-ahead and the intraday market often approximates the intraday market as a single stage, accounting for price (Germscheid et al., 2022, 2023), or demand uncertainties (Dalle Ave et al., 2019), but neglecting continuous trading. Dowling et al. (2017) and Kraft et al. (2022) model the participation in all three markets: intraday, day-ahead, and balancing-power. However, both simplify the intraday market to one stage with a single recourse decision, neglecting potential revenue streams from continuous trading (Dowling et al., 2017; Kraft et al., 2022).

Continuous trading means that orders can be placed over several hours from the start of the trading session until the actual delivery. During this time, the market price varies continuously due to updates in renewables and demand forecasts (Kiesel and Paraschiv, 2017; Kremer et al., 2021). Market participants need to decide on the optimal sequence of order placements in a trading session.

This continuous trading can be optimized with the order-placement problem (Guo et al., 2017), modeled for example with machine learning algorithms (Boukas et al., 2021), dynamic programming (Aïd et al., 2016; Glas et al., 2020; Finnah et al., 2022) or rolling-horizon optimization (Corinaldesi et al., 2020). As these approaches are computationally challenging or require information that is typically unavailable at the time coordinated bidding decisions have to be made, they are less promising for coordinated bidding problems that also account for other markets. However, coordinated bidding does not need a complete continuous trading strategy but rather potential revenues from reserving flexible capacity for continuous trading in the intraday market. To estimate these revenues, Weber et al. (2022) and Muthirayan et al. (2021) propose using option-price theory. The option value estimates the revenues per mega-watt traded at the continuous intraday market as a function of parameters, such as the expected price volatility and the electricity production cost. Consequently, the option value is promising to estimate intraday market revenues and integrate continuous intraday trading into the already computationally challenging multi-stage stochastic problem for balancing-power and day-ahead markets.

1.2. Contribution of this work

Despite the growing importance of the continuous intraday market, the reviewed contributions on multi-market participation either neglect the continuous intraday market or the intraday market stage is represented without considering its key characteristic of continuous trading (Table 1).

Here, we present a coordinated bidding strategy derived from a multi-stage stochastic optimization that explicitly considers revenues from continuous trading in the intraday market. At the same time,

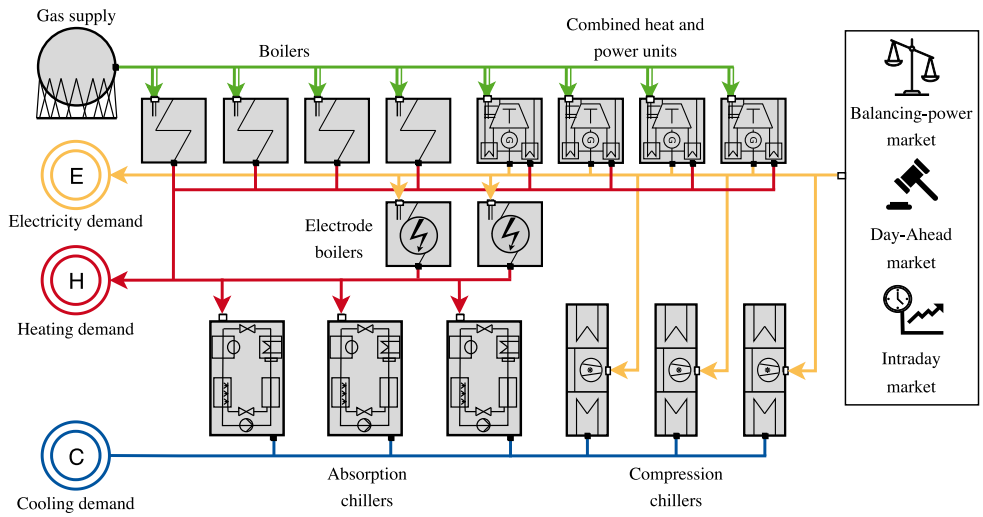


Fig. 1. Overview of a typical multi-energy system supplying an industrial site: absorption chillers, compression chillers, boilers, electrode boilers, and combined-heat-and-power units cover hourly varying electricity, heating, and cooling demands. The multi-energy system participates in the balancing-power market, day-ahead market, and continuous intraday market.

the presented strategy optimizes the balancing-power market revenues from both the capacity and the energy price explicitly, while the day-ahead market facilitates the purchase and sale of electricity. For the continuous intraday market, we propose using option-price theory to estimate the trading revenues one day in advance, allowing us to integrate continuous trading into a multi-stage stochastic optimization without losing computational tractability. The method considers the optimal operation of industrial multi-energy systems and optimally allocates the flexibility between the balancing-power market, day-ahead market, and the continuous intraday market.

The main contribution of this work, thus, lies in the integration of the continuous intraday market into a multi-market optimization to optimize the allocation of flexible capacity across markets. By combining option-price theory and multi-stage stochastic optimization, our method considers interdependencies between the balancing-power market, day-ahead market, and trading on the continuous intraday market to answer the question: Where to market flexibility?

2. The multi-energy system as a provider of flexibility in sequential electricity markets

2.1. Flexibility provision by multi-energy systems

Multi-energy systems are integrated energy systems that combine multiple energy sources to supply multiple energy demands, such as electricity, heat, and cooling. These energy systems, also called utility systems in the chemical industry, are usually located on-site and comprise multiple, often redundant, energy conversion units (Frangopoulos, 2018; Mancarella, 2014). Industrial multi-energy systems can offer flexibility by participating in electricity and balancing-power markets as they demand but also produce electricity. The multi-energy system in Fig. 1 represents a typical industrial market participant comprising electrode and gas-fired boilers, combined-heat-and-power units, and absorption and compression chillers.

The flexibility for power-market participation is provided by adjusting the operation of the energy conversion units. Here, we define flexibility as the difference between the highest and lowest possible electricity consumption, resulting in a varying flexibility for each time step. Electricity production is limited by all specified energy demands from the industrial site due to the interconnected co-generation: The heating and cooling demands limit the positive flexibility since the heat surplus from co-generation needs to be utilized, while the electricity demand constrains the negative flexibility. Fig. 2 illustrates the flexibility range available in a multi-energy system for trading on the markets depending on the time-varying electricity, heat, and cooling demands.

2.2. Allocation of flexibility in sequential markets

The flexibility can be marketed on the sequential balancing-power, day-ahead, and continuous intraday market, presented in the introduction. Thereby, we study a market setting that is common in Europe and oriented towards the German market design (Fig. 3).

For the balancing-power market, we consider both a capacity and energy auction taking place one day in advance (9 a.m., d − 1), covering various market settings. Providers submit tenders for four-hour time slices for the next day, specifying the amount of positive and/or negative balancing power, along with capacity and energy-price bids. The procurement of balancing power is organized as a daily two-stage pay-as-bid auction: The first stage considers the capacity-price bid that remunerates the provision of flexible capacity for balancing power. The bids with the lowest capacity prices are accepted, while the cut-off is made at the total demand of balancing power. Among all accepted participants, the second stage considers the energy-price bids. The energy-price bids remunerate the request of flexible capacity. If balancing power is needed, tenders with the lowest energy prices are requested first. In both stages, the pay-as-bid principle causes a trade-off between prices and probabilities to maximize market revenues: high capacity (energy) price bids have low acceptance (request) probabilities, and vice versa.

After the results of the first balancing-power market stage are published, electricity can be traded as hourly contracts on the day-ahead market until the market closes at noon (12 p.m., d−1). These trades are based on the market-clearing price, which we assume is deterministic.

Finally, trading on the continuous intraday market starts at 3 p.m., d − 1, trading quarter-hourly, half-hourly, and hourly contracts. The contracts are continuously traded for the next day until shortly before delivery (gate-closure at t_0 - 5 min, d). Thus, the trading sessions for each of the contracts last from 9 h to 32 h (Fig. 4). Trading on the continuous intraday market follows the double-sided pay-as-bid principle, i.e., the trade is executed at the bidding price if a buy and sell order matches. Although contracts with finer intervals (e.g., quarter-hourly or half-hourly) could capture more price volatility, we use hourly contracts to maintain manageable complexity in this market setting. Under these assumptions, the industrial multi-energy system engages in continuous intraday trading of hourly contracts (Löhndorf and Wozabal, 2021), in response to fluctuating electricity prices. As a result of continuous trading, the system either ends up selling its remaining positive flexibility (upper end of the flexibility range) or buying its negative flexibility (lower end of the flexibility range),

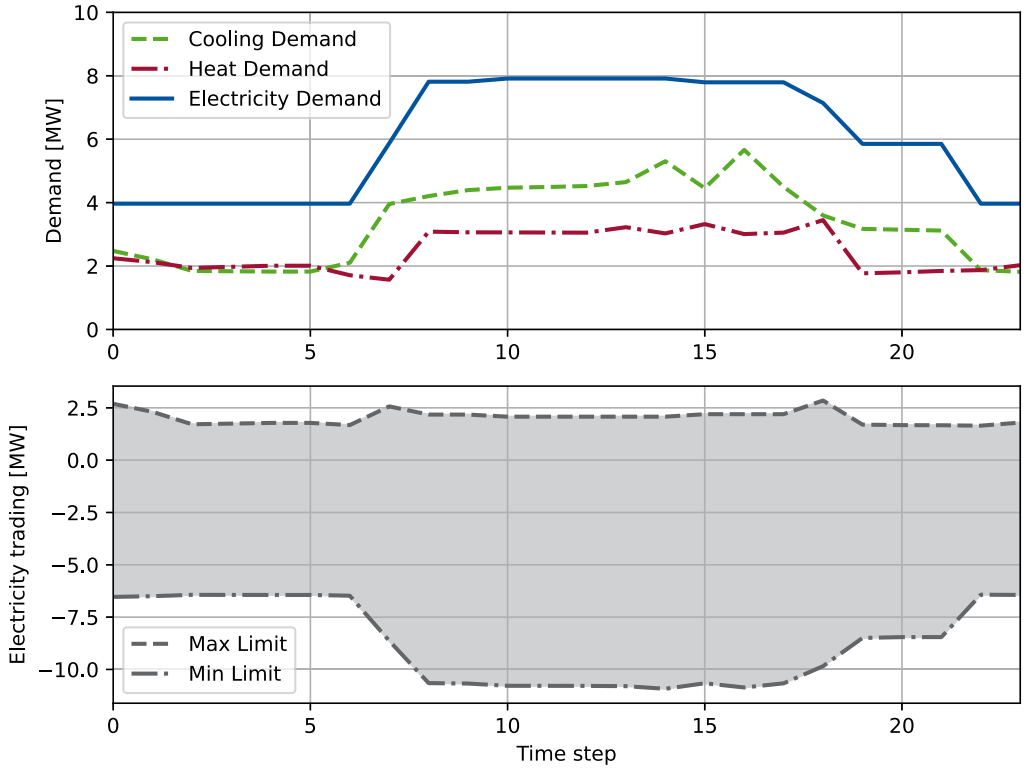


Fig. 2. Hourly varying demands of the multi-energy system (top) and flexibility available for trading on the markets (bottom) for one example day. After covering the demands, electricity production can either be increased (positive value) or decreased (negative value) within the limits of the flexibility range.

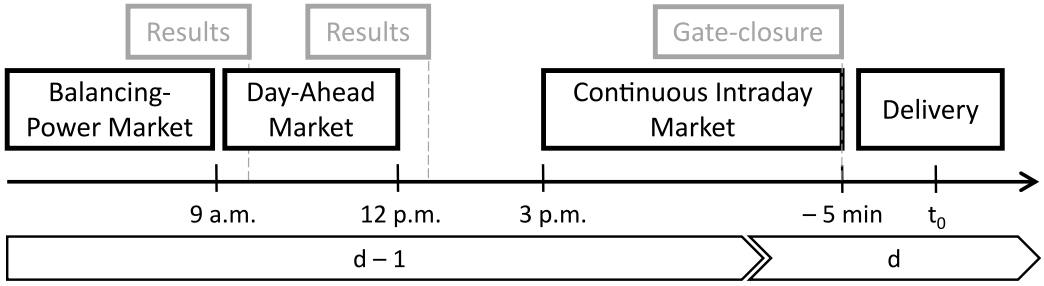


Fig. 3. Trading deadlines for sequential decision-making in the balancing-power, day-ahead, and continuous intraday markets, oriented towards the German market design, up to the delivery time t_0 on day d.

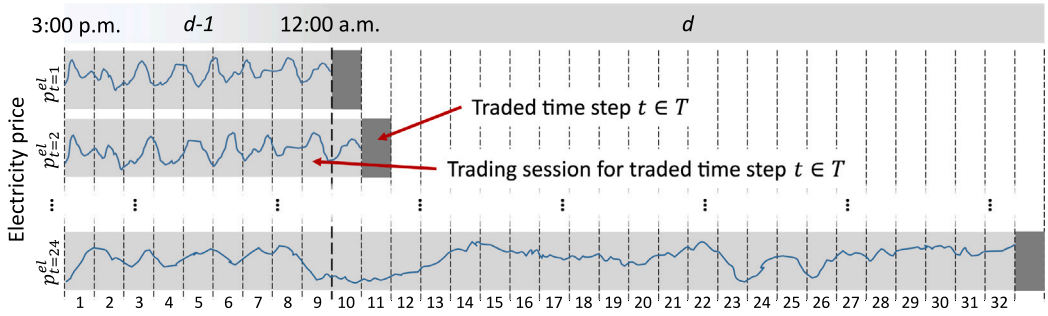


Fig. 4. Trading sessions on the continuous intraday market for hourly contracts for the next day. The trading session for each of the contracts lasts from 9 h to 32 h.

thereby participating in the intraday market by either selling or buying electricity.

In conclusion, we study a sequential market setting, where participants first submit their bids for positive and negative balancing power. These bids can result in four outcomes: accepted for both positive and negative balancing power (±), only accepted for positive balancing power (+), only accepted for negative balancing power (−), or not accepted for any balancing power (0). To place the balancing-power market bid, participants have to simultaneously decide on the day-ahead market bid, the flexible capacity reserved for continuous trading on the intraday market, and the operation of the multi-energy system for all four outcomes of balancing-power acceptance (±, +, −, 0), while accounting for any potential requests of balancing power.

3. Method to allocate flexible capacity in sequential electricity markets

3.1. Problem statement

Given the multi-energy system and introduced market setting, we present a method that optimally allocates flexibility of a multi-energy system for the next day on the balancing-power market, day-ahead market, and continuous intraday market, resulting in the following problem statement.

Given (Fig. 5 , bottom):

- A flexible multi-energy system that:
  - is qualified for participation in the balancing-power, day-ahead, and continuous intraday market.
  - has to supply specified deterministic industrial energy demands (e.g., heat, cooling, electricity) for the next day.
  - consists exclusively of dispatchable energy conversion units and no storage or intermittent renewable energy sources.
  - is modeled on a single-period basis, thus neglecting time-coupling constraints (e.g., minimum up/down times, ramp rates, or storage operation).
- Market forecasts:
  - day-ahead market price for selling ($p_t^{\mathrm{DA,sell}}$) and buying ($p_t^{\mathrm{DA,buy}}$),
  - price drift (μ_t) of the continuous intraday market, representing the average expected price deviation throughout a trading session,
  - the price volatility (σ_t) of the continuous intraday market, modeling the distribution of prices at the end of a trading session,
  - deterministic capacity ($cp_t^{+/-}$) and energy prices ($ep_t^{+/-}$) with corresponding acceptance $\pi_{t,\omega}^{\mathrm{BP,cp}}$ and request probabilities $\pi_{t,\omega}^{\mathrm{BP,ep}}$ for the balancing-power market.

Determine for the next day:

- Tenders for the balancing-power market, including quantities, capacity prices, and energy prices.
- For all four cases of balancing-power acceptance (±, +, −, 0):
  - The amount of electricity sold or purchased on the day-ahead market.
  - Amount of flexibility to be used for continuous trading on the intraday market.
  - For both selling and buying at the intraday market and all cases of balancing-power requests:
    - Optimal operation of the multi-energy system.

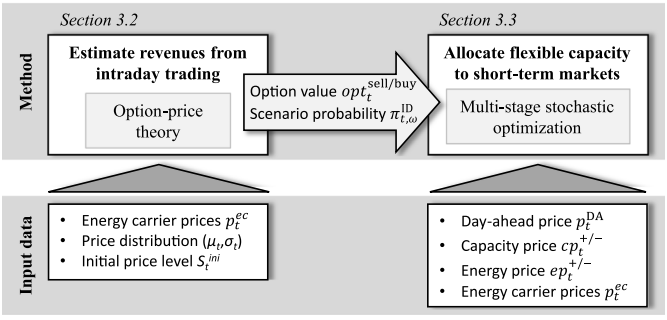


Fig. 5. Flow-chart of the proposed method to optimally allocate flexibility in short-term markets: Option-price theory approximates the value of intraday trading (Section 3.2) which serves as input for the multi-stage stochastic optimization (Section 3.3).

Our focus is on the method to integrate trading on the continuous intraday market into the multi-market bidding problem. We assume that accurate price forecasts are available as electricity-price forecasting is a distinct and separate research field (Hong et al., 2020). Day-ahead prices exhibit relatively low forecast errors, with symmetric mean absolute percentage errors of 14–18% for the German market (Lago et al., 2021). In contrast, intraday and balancing-power market forecasts are newer fields, with higher errors due to shorter lead times and evolving market rules (Narajewski and Ziel, 2020; Kraft et al., 2020; Hirsch and Ziel, 2024). Therefore, we refer to the literature for day-ahead market (Lago et al., 2021), intraday (Narajewski and Ziel, 2020) and balancing-power market forecasts (Jahns and Weber, 2019; Kraft et al., 2020). To demonstrate our method's capabilities, we derive price forecasts and other necessary input parameters from historical data spanning a one-year period from 08-2019 to 07-2020. The input data preparation is detailed in the Supporting Information.

In the following, we present our two-step method (Fig. 5, top): First, we present the approximation of revenues from continuous trading with the option value (Section 3.2). Second, we present its integration into the multi-stage stochastic optimization to optimally allocate the available flexible capacity (Section 3.3).

3.2. Approximating the revenues from the continuous intraday market

Flexible capacity can be monetized on the intraday market, via continuous trading, through the virtual trading of electricity depending on the price development (Löhndorf and Wozabal, 2021). Finally, electricity is physically sold or purchased based on the trading result. Both trading with positive and negative flexibility is considered: Positive flexibility describes the ability of the multi-energy system to increase its electricity production and, as a result of trading, sell it on the intraday market when intraday prices exceed marginal electricity costs. Conversely, negative flexibility describes the ability of the multi-energy system to reduce its electricity production and purchase additional electricity from the intraday market when prices are lower than marginal electricity costs. Continuous trading, thus, utilizes the flexible capacity reserved for the intraday market, allowing the system to respond to fluctuating prices until delivery by continuously buying and selling electricity based on current prices.

We approximate the revenues from trading in the continuous intraday market with the option value (Weber et al., 2022; Muthirayan et al., 2021), which estimates the revenues per mega-watt of traded positive/negative flexible capacity. Option-price theory assumes a risk-neutral trader who aims to replicate the option value calculated beforehand, independent of the actual price development.

The option value is estimated using the so-called multi-period binomial model, representing each trading session in discrete time (Cox et al., 1979) (Section 3.2.1). This approximation has the advantage that it avoids explicit computation of the trading strategy by relying on only a few key parameters (cf. Fig. 5):

- the stochastic price process, characterized by the initial electricity price level at the start of trading S_t^{ini}, the price drift μ_t and the price volatility σ_t (Section 3.2.2),
- the marginal electricity costs to provide flexible capacity using the energy carrier prices p_t^{ec}, such as the gas price (Section 3.2.3).

Forecasts for these parameters are typically available one day before delivery and are assumed to be given for our approach.

3.2.1. Option value as revenues from intraday market trading

The option value for positive flexible capacity opt_t^{sell} and negative flexible capacity opt_t^{buy} for each traded time step t is derived from the multi-period binomial model following Björk (2009) with:

$$opt_t^{\text{sell/buy}} = \sum_{k=0}^{N_t} \underbrace{\binom{N_t}{k}}_{(a)} \cdot \underbrace{\left(\frac{u_t}{u_t - d_t}\right)^{k}}_{(b)} \cdot \underbrace{\left(\frac{-d_t}{u_t - d_t}\right)^{N_t-k}}_{(c)} \cdot \underbrace{\Phi^{\text{sell/buy}}(S_{t,k})}_{(d)} \forall t \in T. \quad (1)$$

Overall, the option value sums up and weights the revenues from all k last prices, at the end of a trading session. The option value $opt_t^{\text{sell/buy}}$ represents the revenues from continuous trading for positive (sell) and negative (buy) flexible capacity. The computation of the required input parameters is presented in the next subsections Sections 3.2.2 and 3.2.3.

Compared to Björk (2009), Eq. (1) is adapted to the continuous intraday market, by setting the interest rate to zero to account for the short-term nature of the market. In Eq. (1), the binomial coefficient (a) denotes the absolute frequency that the last price $S_{t,k}$ is reached. The factors (b) and (c) are the risk-neutral probabilities for an up-movement and down-movement, determining the probability of one specific price path. Thus, the factors (a), (b), and (c) model the discrete distribution of the last prices. The factor (d) evaluates the option value $\Phi^{sell/buy}(S_{t,k})$ at the end of a trading session for the sell option and the purchase option for each traded time step t and last price k:

$$\Phi^{\text{sell}}(S_{t,k}) = \begin{cases} S_{t,k} - mc_t & \text{if} \quad S_{t,k} > mc_t \\ 0 & \text{if} \quad S_{t,k} \leq mc_t \end{cases} \quad (2)$$

$$\Phi^{\text{buy}}(S_{t,k}) = \begin{cases} 0 & \text{if} \quad S_{t,k} \geq mc_t \\ mc_t - S_{t,k} & \text{if} \quad S_{t,k} < mc_t. \end{cases} \quad (3)$$

As a result of continuous trading, either positive or negative flexible capacity is utilized. Whether the respective flexible capacity is utilized, depends on the last price on the continuous intraday market $S_{t,k}$ compared to the marginal electricity costs mc_t (cf. Eqs. (2) and (3)).

To determine the scenario probabilities $\pi_{t,\omega}^{\text{ID}}$ for utilizing positive or negative flexible capacity, we evaluate the distribution of the last prices against the marginal electricity costs. Specifically, we use the cumulative distribution function $P(\cdot)$ of the last prices, characterized by the price drift μ_t adjusted by the initial price level S_t^{ini} and the price volatility σ_t. The scenario probabilities $\pi_{t,\omega}^{\text{ID}}$ are then obtained from the cumulative distribution function, indicating the probability that the last price is higher than the marginal electricity costs:

$$\pi_{t,\omega}^{\text{ID}} = P(S_{t,k} \geq mc_t). \quad (4)$$

The option values $opt_t^{\text{sell/buy}}$ and the scenario probabilities $\pi_{t,\omega}^{\text{ID}}$ are the input parameters for the multi-stage stochastic optimization presented in Section 3.3.

3.2.2. Stochastic price process

The option value requires the stochastic price process $S_{t,k}$ for each trading session t as input parameter. Each trading session (cf. Fig. 4) starts with the initial price level S_t^{ini} which can be assumed to equal the corresponding day-ahead market price (Han et al., 2022).

Subsequently, the electricity price follows a stochastic price process which can be modeled as an arithmetic Brownian motion (Alexander et al., 2012). The arithmetic Brownian motion models the absolute price change by the price drift μ_t and the price volatility σ_t. Thus, the arithmetic Brownian motion assumes that the absolute price deviation from the initial price level S_t^{ini} is normally distributed with $N(\mu_t, \sigma_t^2)$ at gate-closure. Contrary to the common geometric Brownian motion, the arithmetic Brownian motion can capture negative electricity prices, which have become more frequent in recent years, particularly during windy or sunny periods with low residual load (Halbrügge et al., 2021).

The multi-period binomial model discretizes the stochastic price process into N_t trading opportunities. Throughout a trading session, the price moves up u_t or down d_t at each trading opportunity, determined as follows for the arithmetic Brownian motion:

$$u_t = \mu_t \cdot \frac{1}{N_t} + \sigma_t \cdot \sqrt{\frac{1}{N_t}} \quad \forall t \in T,$$
$$d_t = \mu_t \cdot \frac{1}{N_t} - \sigma_t \cdot \sqrt{\frac{1}{N_t}} \quad \forall t \in T. \quad (5)$$

In Eq. (5), the price drift μ_t and the price volatility σ_t are multiplied with the trading frequency $\frac{1}{N_t}$ to distribute the price drift μ_t and the price volatility σ_t over the trading session.

The underlying arithmetic Brownian motion determines the last electricity price $S_{t,k}$ used in part (d) of Eq. (1) as the summation of up-movements and down-movements:

$$S_{t,k} = S_t^{\text{ini}} + k \cdot u_t + (N_t - k) \cdot d_t \quad \forall t \in T, k \in \{0, 1, \ldots, N_t\}. \quad (6)$$

Eq. (6) determines the $N_t + 1$ possible last prices $S_{k,t}$ for each traded time step. As discussed above, the normal distribution of the last prices is reflected in the option value calculation in Eq. (1), using the binomial coefficient (a) and the risk-neutral probabilities for an up-movement (b) and down-movement (c).

3.2.3. Marginal electricity cost estimation

The option value requires the marginal electricity costs mc_t of the market participant as an input parameter (see Eq. (1)). The marginal electricity costs depend on the time step due to changing price parameters (e.g. gas price) and exogenous demands. The marginal electricity cost in a multi-energy system depends also on the operational state, i.e., which units are active and how they adjust their operations, making these costs, in principle, a nonlinear function of the load.

To determine the marginal electricity costs, we calculate the influence of changing electricity demand on the operational cost. As the flexibility needs to be provided by the energy system itself, the opportunity to participate in electricity and balancing-power markets is excluded here. For each time step $t \in T$, we vary the electricity demand between zero and the maximum possible electricity output for the energy system. For each electricity demand, we determine the cost-minimal operational schedule for the energy system by an operational optimization.

A least-square linear regression is performed to approximate the operational cost as an affine function of the electricity demand. The slope of the linear regression of the operational cost approximates the marginal electricity costs. The linear approximation provides a good fit of the marginal electricity costs for the multi-energy system considered in Section 4 ($R^2 > 0.97$), justifying the assumption of constant marginal electricity cost. In the Supporting Information, we exemplarily visualize the fit and discuss the consequences of imperfect prediction of the marginal costs.

3.3. Multi-stage stochastic optimization model

A multi-stage stochastic optimization allocates the flexibility to the three short-term markets (Fig. 5) based on the option value and the intraday scenario probabilities from the previous section, and parameters from the day-ahead and balancing-power markets. This section presents the multi-stage stochastic optimization, modeling the stochastic process presented in Section 3.3.1. The objective function and the

Table 2
Decision variables and revealed information at each stage of the multi-stage stochastic optimization model.

Stage	Decision Variables	Revealed Uncertain Information
1st stage	BP_t^+, BP_t^-, $\lambda_{t,c}^{\mathrm{BP}}$	
2nd stage	DA_t^{sell}, DA_t^{buy}, ID_t^{buy}, ID_t^{sell}	Acceptance/rejection in the balancing-power market
3rd stage		Results of intraday trading, i.e., purchase/sale of electricity depending on price development
4th stage	$P_{u,t,\omega,p}$	Operation incl. request of balancing power

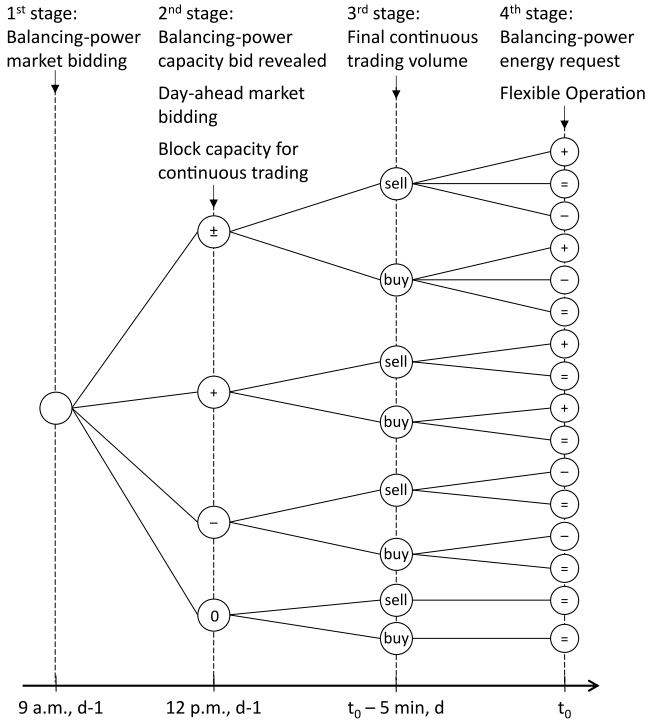


Fig. 6. Stochastic process of the multi-stage stochastic optimization model to obtain the optimal bidding strategy for a flexible market participant participating in the balancing-power market, day-ahead market, and continuous intraday market for each time step. Each stage reveals new information or makes a decision regarding market participation and operation.

multi-market bidding problem are presented in Section 3.3.2. Subsequently, Section 3.3.3 presents the model of the balancing-power market and Section 3.3.4 the model of the intraday market. The remaining constraints of the day-ahead market and the underlying flexible multi-energy system are presented in the Supporting Information.

3.3.1. Stochastic process of multi-market participation

We formalize the sequential decision-making process, which is modeled as a stochastic process that considers all possible outcomes during market participation and operation. Therein, we divide the stochastic process into four stages (Table 2).

The resulting scenario tree comprises four stages and 16 scenarios $\omega \in \Omega$ for each time step $t \in T$ (Fig. 6). At the first stage (9 a.m., d − 1), a tender comprising the capacity price $cp_t^{+/-}$ and energy price $ep_t^{+/-}$ and the amount of positive and negative balancing power $BP_t^{+/-}$ is submitted to the balancing-power market. The capacity and energy prices are selected with the binary variable $\lambda_{t,c}^{\mathrm{BP}}$ that is 1, if the price combination c is selected for time step t and zero otherwise. After the results have been revealed, the second stage (12 p.m., d − 1) decides on how much electricity to sell $DA_{t,\omega}^{\mathrm{sell}}$, or buy $DA_{t,\omega}^{\mathrm{buy}}$ at the day-ahead market and how much flexible positive capacity $ID_{t,\omega}^{\mathrm{sell}}$ (sell option) and negative capacity $ID_{t,\omega}^{\mathrm{buy}}$ (purchase option) is reserved for trading in the continuous intraday market.

In the third stage (t_0 − 5 min, d), the information is revealed whether the last price of the continuous intraday trading session is above or below the marginal costs. Here, two alternative scenarios $\omega^{\mathrm{ID}} \in \{\mathrm{sell}, \mathrm{buy}\}$ arise for the final continuous trading volume. Suppose the last price on the continuous intraday market is greater than the marginal electricity costs ($S_{t,k} > mc_t$). In this scenario, positive flexible capacity is entirely utilized, and electricity is sold on the continuous intraday market. If the last price on the continuous intraday market is less than the marginal electricity costs ($S_{t,k} < mc_t$), negative flexible capacity is entirely utilized, and electricity is purchased on the continuous intraday market.

At the fourth stage (t_0, d), the information is revealed whether positive balancing power is requested (+), negative balancing power is requested (-), or no balancing power is requested (=). Finally, the energy system units $u \in U$ convert products $p \in P$ by adjusting their operation $P_{u,t,\omega,p}$ according to the realized scenario.

Based on this stochastic price process, we formulate the multi-market bidding problem in the following section as a multi-stage single-period stochastic optimization problem. While this formulation avoids the exponential growth of the scenario tree (Fig. 6), it inherently neglects time-coupling constraints in the multi-energy system.

3.3.2. Multi-market bidding problem

The objective of the multi-market bidding problem is to minimize the expected operational cost $OPEX_t^{\mathrm{exp}}$ summed up over all time steps $t \in T$:

$$\min \underbrace{\sum_{t \in T} \sum_{\omega \in \Omega} \pi_{t,\omega} \left(C_{t,\omega}^{\mathrm{energy}} - R_{t,\omega}^{\mathrm{BP}} - R_{t,\omega}^{\mathrm{DA}} - R_{t,\omega}^{\mathrm{ID}} \right)}_{OPEX_t^{\mathrm{exp}}} \tag{P1}$$

$$\text{s.t.} \quad \pi_{t,\omega} = \pi_{t,\omega}^{\mathrm{BP,cp}}(cp_t^{+/-}) \cdot \pi_{t,\omega}^{\mathrm{ID}} \cdot \pi_{t,\omega}^{\mathrm{BP,ep}}(ep_t^{+/-}) \quad \forall t \in T,\ \omega \in \Omega \tag{P2}$$

$$R_{t,\omega}^{\mathrm{BP}} = f_{\text{Balancing-power market model}}(\cdot) \quad \forall t \in T,\ \omega \in \Omega \tag{P3}$$

$$R_{t,\omega}^{\mathrm{ID}} = f_{\text{Intraday market model}}(\cdot) \quad \forall t \in T,\ \omega \in \Omega \tag{P4}$$

$$R_{t,\omega}^{\mathrm{DA}} = f_{\text{Day-ahead market model}}(\cdot) \quad \forall t \in T,\ \omega \in \Omega \tag{P5}$$

$$C_{t,\omega}^{\mathrm{energy}} = f_{\text{Energy system model}}(\cdot) \quad \forall t \in T,\ \omega \in \Omega \tag{P6}$$

In the objective function (P1), the operational cost of a specific scenario ω is calculated from the costs of the energy system operation $C_{t,\omega}^{\mathrm{energy}}$ minus the revenues from the balancing-power market ($R_{t,\omega}^{\mathrm{BP}}$), day-ahead market ($R_{t,\omega}^{\mathrm{DA}}$), and continuous intraday market ($R_{t,\omega}^{\mathrm{ID}}$). The probability of a scenario $\pi_{t,\omega}$ is the product of the acceptance probability in the balancing-power market $\pi_{t,\omega}^{\mathrm{BP,cp}}$, the probability of selling or buying in the intraday market $\pi_{t,\omega}^{\mathrm{ID}}$, and the request probability in the balancing-power market $\pi_{t,\omega}^{\mathrm{BP,ep}}$ (P2). As the probabilities in the balancing-power market $\pi_{t,\omega}^{\mathrm{BP,cp}}$, $\pi_{t,\omega}^{\mathrm{BP,ep}}$ are functions on the offered capacity price $cp_t^{+/-}$ and energy price $ep_t^{+/-}$ (cf. (P2)), the optimization problem includes endogenous uncertainty, i.e., the probabilities depend on the optimization choices (Hellemo et al., 2018). Thus, the optimization problem (P) is in the general formulation nonlinear, which usually requires customized decomposition techniques, see, e.g., Schäfer et al. (2019). Note that due to this decision-dependent uncertainty, a deterministic reference scenario does not exist for comparison. As a result, calculating the value of the stochastic solution that measures the advantage of a stochastic optimization model over its deterministic

counterpart is generally difficult (Rathi and Zhang, 2022).

While the probability of selling and buying in the intraday market $\pi^{\mathrm{ID}}_{t,\omega}$ is an output of the option price calculation (cf. Eq. (4)), we follow the approach by Leenders et al. (2020) to derive the acceptance probability and by Loesch et al. (2018) to derive the request probability. The acceptance probability $\pi^{\mathrm{BP,cp}}_{t,\omega}$ is derived as a continuous function of the ratio of four-hour time slices with marginal capacity prices exceeding the offered capacity price $cp^{+/-}_{t}$ to the total number of slices. Similarly, the request probability $\pi^{\mathrm{BP,ep}}_{t,\omega}$ is calculated as a continuous function based on the marginal energy prices exceeding the offered energy price $ep^{+/-}_{t}$. As bidding in the balancing-power market is discretized (see Section 3.3.3), we select discrete capacity and energy prices from these continuous curves, each with their associated probabilities, to capture the trade-off between prices and probabilities. In our case study in Section 4, we use 3 positive and negative capacity prices and 4 positive and negative energy prices to ensure the optimization problem can be solved in under an hour. A higher resolution improves accuracy but substantially increases computation time (details in Supporting Information).

In this work, we discretize the nonlinearities to an MILP formulation. The discretization, together with the balancing-power market model (P3), is explained in Section 3.3.3. The intraday market model (P4) is explained in Section 3.3.4. The day-ahead market model (P5) and the energy system model (P6) are explained in the Supporting Information as these are mostly standard energy system constraints not specific to market participation.

3.3.3. Discretization and balancing-power market modeling

To avoid the nonlinearities caused by the endogenous uncertainty (cf. (P1) and (P2)), we introduce discrete capacity and energy prices via the set C, extending the idea from Leenders et al. (2020). The set C contains discrete combinations of capacity price $cp^{+/-}_{t,c}$ and energy price $ep^{+/-}_{t,c}$ for positive and negative balancing power. For every price combination $c \in C$, we introduce the revenues from the balancing-power market $R^{\mathrm{BP}}_{t,c,\omega}$ (cf. (P1)), and the probabilities $\pi^{\mathrm{BP,cp}}_{t,c,\omega}$, $\pi^{\mathrm{BP,ep}}_{t,c,\omega}$ (cf. (P2)), which are now constant for a given c. The scenario-dependent operational cost $OPEX^{\mathrm{scenario}}_{t,c,\omega}$ is calculated as:

$$OPEX^{\mathrm{scenario}}_{t,c,\omega} = C^{\mathrm{energy}}_{t,\omega} - R^{\mathrm{BP}}_{t,c,\omega} - R^{\mathrm{DA}}_{t,\omega} - R^{\mathrm{ID}}_{t,\omega}, \quad \forall t \in T,\ c \in C,\ \omega \in \Omega. \tag{7}$$

These scenario-dependent operational costs $OPEX^{\mathrm{scenario}}_{t,c,\omega}$ are weighted with the scenario probability $\pi_{t,c,\omega}$. The scenario probability is now, in contrast to (P1), a predefined parameter as $\pi^{\mathrm{BP,cp}}_{t,c,\omega}$, $\pi^{\mathrm{BP,ep}}_{t,c,\omega}$ are constant for every price combination c. Thereby, we obtain the operational costs $OPEX^{\mathrm{price\ comb}}_{t,c}$ linearly for each time step $t \in T$, and price combination $c \in C$ using:

$$OPEX^{\mathrm{price\ comb}}_{t,c} = \sum_{\omega \in \Omega} OPEX^{\mathrm{scenario}}_{t,c,\omega} \cdot \pi_{t,c,\omega}, \quad \forall t \in T,\ c \in C. \tag{8}$$

To ensure that only one price combination c is selected, we introduce the binary decision variable $\lambda^{\mathrm{BP}}_{t,c}$, which indicates if a price combination is selected ($\lambda^{\mathrm{BP}}_{t,c} = 1$) or not ($\lambda^{\mathrm{BP}}_{t,c} = 0$). For each time step, exactly one price combination is selected:

$$\sum_{c \in C} \lambda^{\mathrm{BP}}_{t,c} = 1, \quad \forall t \in T. \tag{9}$$

The selected price combination determines the expected operational costs $OPEX^{\mathrm{exp}}_{t}$, which are identified using a Big-M formulation with M^{BP} being a sufficiently large number:

$$OPEX^{\mathrm{exp}}_{t} \geq OPEX^{\mathrm{price\ comb}}_{t,c} - \mathrm{M}^{\mathrm{BP}} \cdot (1 - \lambda^{\mathrm{BP}}_{t,c}), \quad \forall t \in T,\ c \in C. \tag{10}$$

Since we consider a cost-minimization problem, inequality constraint Eq. (10) sets the expected operational costs $OPEX^{\mathrm{exp}}_{t}$ to the operational costs $OPEX^{\mathrm{price\ comb}}_{t,c}$ of the selected price combination as all other price combinations yield a large negative number on the right-hand side of Eq. (10).

The revenues in the balancing-power market $R^{\mathrm{BP}}_{t,c,\omega}$ are modeled for each time step $t \in T$, price combination $c \in C$, and scenario $\omega \in \Omega$ with:

$$\begin{aligned} R^{\mathrm{BP}}_{t,c,\omega} =& BP^{+}_{t} \cdot (s^{\mathrm{cp+}}_{\omega} \cdot cp^{+}_{t,c} + s^{\mathrm{ep+}}_{\omega} \cdot ep^{+}_{t,c}) \\ &+ BP^{-}_{t} \cdot (s^{\mathrm{cp-}}_{\omega} \cdot cp^{-}_{t,c} + s^{\mathrm{ep-}}_{\omega} \cdot ep^{-}_{t,c}), \quad \forall t \in T,\ \omega \in \Omega. \end{aligned} \tag{11}$$

The offered amount of positive balancing power BP^{+}_{t} and negative balancing power BP^{-}_{t} is compensated with the capacity price $cp^{+/-}_{t,c}$ and energy price $ep^{+/-}_{t,c}$ depending on the scenario $\omega \in \Omega$. The binary parameters $s^{\mathrm{cp+}}_{\omega}$, $s^{\mathrm{cp-}}_{\omega}$, $s^{\mathrm{ep+}}_{\omega}$, $s^{\mathrm{ep-}}_{\omega}$ are equal to 1 if compensation is awarded in the respective scenario $\omega \in \Omega$. The binary parameters $s^{\mathrm{cp+}}_{\omega}$ and $s^{\mathrm{cp-}}_{\omega}$ indicate the acceptance in the balancing-power market for positive and negative balancing power. The binary parameters $s^{\mathrm{ep+}}_{\omega}$ and $s^{\mathrm{ep-}}_{\omega}$ indicate the request of positive and negative balancing power. Hence, we assume a full request of balancing power (cf. Kumbartzky et al., 2017; Bohlayer et al., 2020).

Usually, balancing power is offered for several time steps, e.g., for time slices of four hours in the German balancing-power market. Thus, the price combination c of capacity and energy price, as well as the offered amount of positive and negative balancing power, have to be equal for each four-hour time slice. Eq. (12)–(14) ensure that the tender is the same for each four-hour time slice:

$$BP^{+}_{t} = BP^{+}_{t+a}, \quad \forall t \in \{T \mid (t-1) \mod 4 = 0\},\ a \in \{1,2,3\}, \tag{12}$$

$$BP^{-}_{t} = BP^{-}_{t+a}, \quad \forall t \in \{T \mid (t-1) \mod 4 = 0\},\ a \in \{1,2,3\}, \tag{13}$$

$$\lambda^{\mathrm{BP}}_{t,c} = \lambda^{\mathrm{BP}}_{t+a,c}, \quad \forall t \in \{T \mid (t-1) \mod 4 = 0\},\ a \in \{1,2,3\}. \tag{14}$$

3.3.4. The revenues from intraday market participation

The revenues from trading flexibility on the continuous intraday market $R^{\mathrm{ID}}_{t,\omega}$ are determined with the option value. We assume that positive flexible capacity $ID^{\mathrm{sell}}_{t,\omega}$, and negative flexible capacity $ID^{\mathrm{buy}}_{t,\omega}$ can be monetized on the continuous intraday market. Hence, the revenues in the intraday market $R^{\mathrm{ID}}_{t,\omega}$ are determined based on the amount of positive and negative flexible capacity $ID^{\mathrm{sell}}_{t,\omega}$ and $ID^{\mathrm{buy}}_{t,\omega}$ blocked for continuous trading, the respective option value opt^{sell}_{t} and opt^{buy}_{t}, and the marginal electricity costs mc_t.

The option value is realized independently of the last price in the intraday market. However, the utilization of positive or negative flexible capacity, and thus electricity exchange with the grid, depends on whether the last price is higher or lower than the marginal electricity costs, leading to two alternative scenarios $\omega^{\mathrm{ID}} \in \{\mathrm{sell}, \mathrm{buy}\}$.

Following, the revenues in the continuous intraday market are determined as:

$$\begin{aligned} R^{\mathrm{ID}}_{t,\omega} =& ID^{\mathrm{sell}}_{t,\omega} \cdot (opt^{\mathrm{sell}}_{t} + s^{\mathrm{sell}}_{\omega} \cdot mc_t) \\ &+ ID^{\mathrm{buy}}_{t,\omega} \cdot (opt^{\mathrm{buy}}_{t} - s^{\mathrm{buy}}_{\omega} \cdot mc_t), \quad \forall t \in T,\ \omega \in \Omega. \end{aligned} \tag{15}$$

We introduce the binary parameters $s^{\mathrm{sell}}_{\omega}$ and $s^{\mathrm{buy}}_{\omega}$ in Eq. (15) to model the scenario-dependent remuneration on the continuous intraday market and the physical electricity exchange with the continuous intraday market.

If the sell scenario materializes, the positive flexible capacity $ID^{\mathrm{sell}}_{t,\omega}$ is utilized. The multi-energy system delivers electricity to the intraday market and receives the option value opt^{sell}_{t}. Since additional operational costs arise endogenously, the binary parameter $s^{\mathrm{sell}}_{\omega}$ is set to one. We compensate the additional operational costs by the marginal electricity costs mc_t, as the operational costs from participating in the continuous intraday market are considered in the option value and in the stochastic optimization problem. In the stochastic optimization problem, we thus avoid the operational costs being taken into account twice.

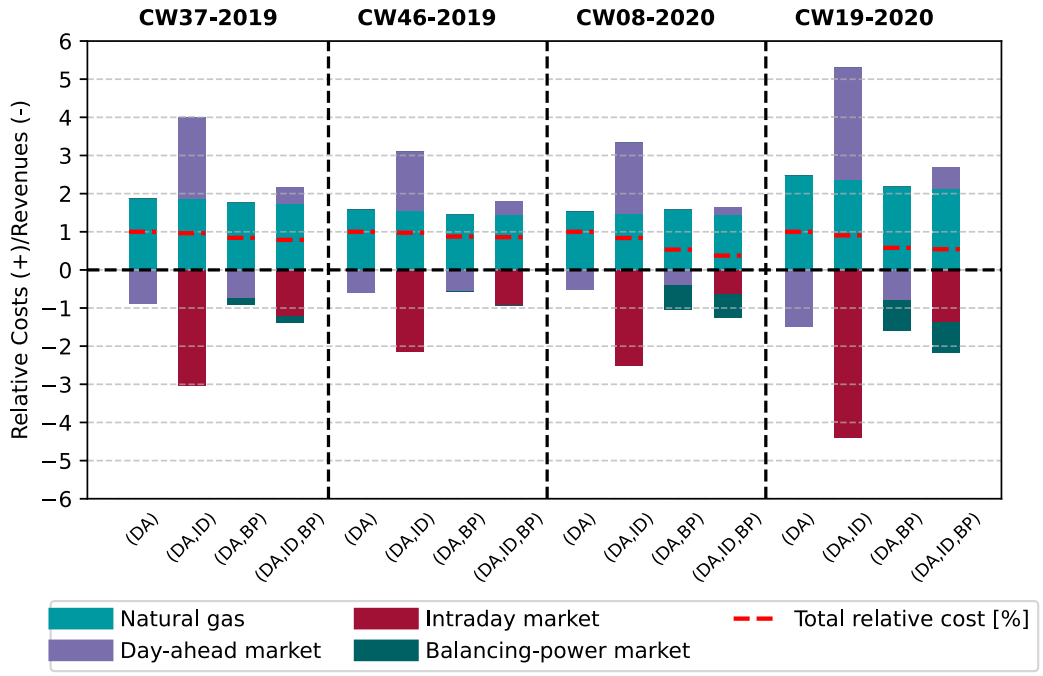


Fig. 7. Relative costs (+) and revenues (-) of the multi-energy system relative to the costs in case (DA) for the four typical calendar weeks (CWs) in 2019/2020 when participating in the day-ahead market (DA), and in addition the continuous intraday market (DA,ID), or the balancing-power market (DA,BP), and in all three markets (DA,ID,BP).

However, in the sell scenario, no electricity is purchased on the intraday market ($s_{\omega}^{\text{buy}} = 0$), while the negative flexible capacity $ID_{t,\omega}^{\text{buy}}$ is remunerated with the option value opt_{t}^{buy}. In the buy scenario, the same concept applies vice versa.

Finally, we need to introduce non-anticipativity constraints to ensure that the intraday market decision does not depend on the request of balancing power, which is only revealed later:

$$ID_{t,\omega}^{\text{sell}} = ID_{t,\omega'}^{\text{sell}}, \quad \forall t \in T, \omega, \omega' \in \Omega : (s_{\omega}^{\text{cp+}}, s_{\omega}^{\text{cp-}}) = (s_{\omega'}^{\text{cp+}}, s_{\omega'}^{\text{cp-}}), \tag{16}$$

$$ID_{t,\omega}^{\text{buy}} = ID_{t,\omega'}^{\text{buy}} \quad \forall t \in T, \omega, \omega' \in \Omega : (s_{\omega}^{\text{cp+}}, s_{\omega}^{\text{cp-}}) = (s_{\omega'}^{\text{cp+}}, s_{\omega'}^{\text{cp-}}). \tag{17}$$

Overall, Section 3 presents a two-step method, which in the first step approximates revenues from continuous trading with the option value. Using the option value as input for the second step, a multi-stage stochastic optimization allocates the flexibility range of a multi-energy system to the balancing-power, day-ahead, and continuous intraday market.

4. Case study for multi-market participation of a multi-energy system

The proposed method is applied to a flexible multi-energy system (Fig. 1) based on Baumgärtner et al. (2020), which allocates its flexibility to the sequential German balancing-power, day-ahead, and continuous intraday markets. This system is capable of providing secondary balancing power and trades hourly contracts in the day-ahead and intraday markets. The multi-energy system includes absorption chillers, compression chillers, gas-driven boilers, electrode boilers, and combined-heat-and-power units (Supporting Information, Tab. B.1), supplying an industrial site with real-world, hourly varying demands for electricity, heat, and cooling (Baumgärtner et al., 2020).

Our analysis uses historical data from 08-2019 to 07-2020 (see Supporting Information). To reflect seasonal variability, we select four typical weeks using k-means clustering on the hourly demand time series: calendar week 37 in 2019 (CW37-2019), calendar week 46 in 2019 (CW46-2019), calendar week 08 in 2020 (CW08-2020), and calendar week 18 in 2020 (CW18-2020). For the four typical weeks, we compare the cases of participation in the day-ahead market (DA), day-ahead market and continuous intraday market (DA,ID), day-ahead market and balancing-power market (DA,BP), and day-ahead market, continuous intraday market and balancing-power market (DA,ID,BP).

Table 3
Characteristics of the MILP model.

Model Characteristic	Value
# Continuous Variables	8501
# Integer Variables	8858
# Binary Variables	8846
Average Runtime (s)	1834

We derive daily optimizations with 24 hourly time steps each, using the SecMOD MILP framework (Reinert et al., 2022, 2023). All optimizations are solved using the solver Gurobi 9.5.0. The optimizations are successfully solved within a time limit of 3600 s and a gap of 1% on an AMD EPYC 7F52 (16 cores, 3.5 GHz) with 192 GB RAM. Table 3 summarizes the average model size and runtime for each daily optimization.

4.1. Results for the participation in sequential electricity markets

The multi-energy system saves the most cost by simultaneous and coordinated participation in all three markets in case (DA,ID,BP) (Fig. 7). In this case, the proposed method indicates savings of 14% to 63% for the four typical weeks, in comparison to sole day-ahead market participation (case (DA)). With expanding participation in the day-ahead market to the continuous intraday market, costs decrease between 3% and 16% (case (DA,ID)). Additional participation in the balancing-power market leads to savings between 12% and 47% (case (DA,BP)). Due to the decision-dependent uncertainties, a formal calculation of the value of the stochastic solution is not directly possible (Rathi and Zhang, 2022). However, the cost savings reported in Fig. 7 provide an indirect measure of the value of the stochastic solution when assuming sole day-ahead market participation as the deterministic counterpart.

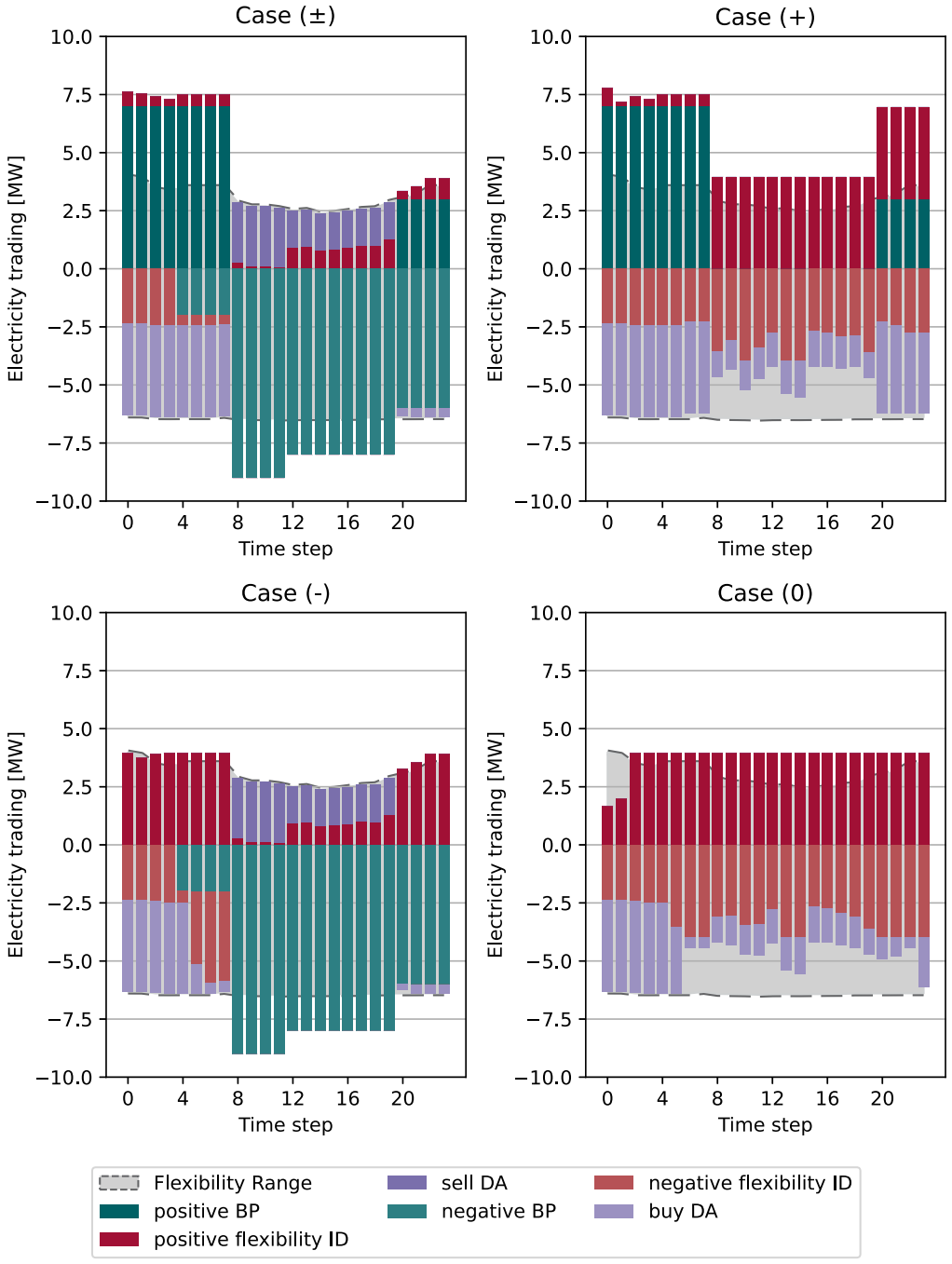


Fig. 8. Hourly allocation of the available flexibility range to the balancing-power (BP), day-ahead (DA), and continuous intraday (ID) markets, depending on the tenders accepted in the balancing-power market for an example day: accepted for both positive and negative balancing power (case ±), only accepted for positive balancing power (case +), only accepted for negative balancing power (case −), or not accepted for any balancing power (case 0).

The continuous intraday market achieves the largest additional revenues in CW08-2020 (compare case (DA) with (DA,ID) in Fig. 7). During this week, the price volatility in the continuous intraday market is the highest due to many hours of high feed-in from wind generation. The volatility is estimated rather conservatively based on the ID_1 price for hourly contracts (cf. Supporting Information). Quarter-hourly contracts and utilization of shorter lead times result in higher volatilities with higher option values (Han et al., 2022). Thus, the multi-energy system may achieve even higher revenues in the continuous intraday market.

In case (DA,ID,BP), the multi-energy system exploits the possibility to market flexibility in both the balancing-power and the continuous intraday market. The hourly bidding behavior (Fig. 8) for allocating flexibility depends on the acceptance of tenders in the balancing-power

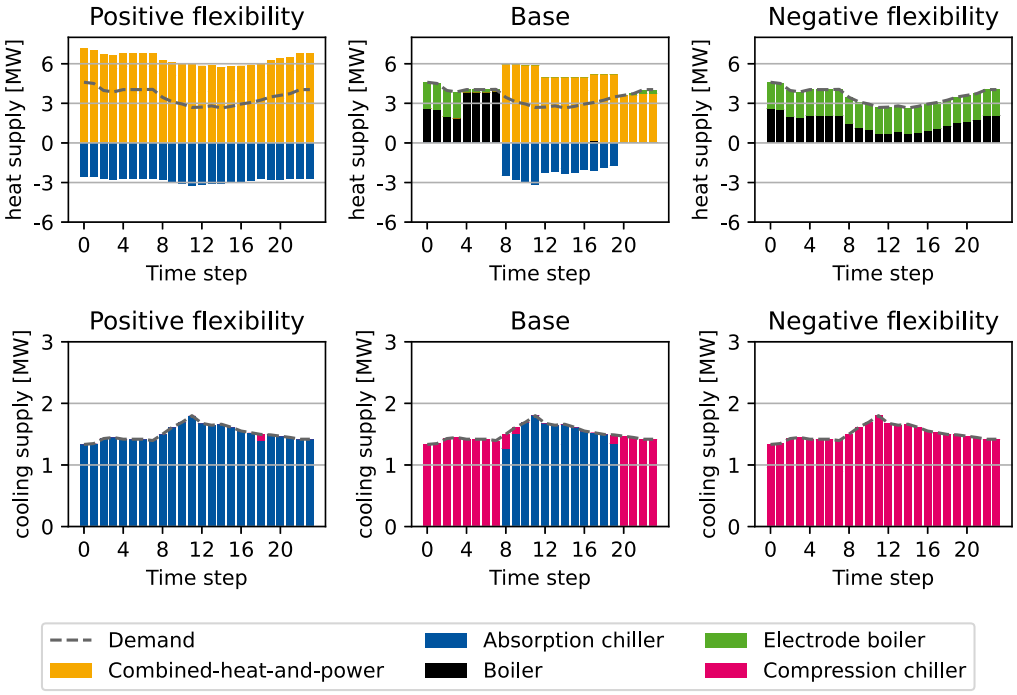


Fig. 9. Hourly heating (top) and cooling supply (bottom) by the multi-energy system for the utilization of the flexible capacity, while covering the exogenous demands (dashed gray) for an example day. Positive flexibility (left) is provided for the request of positive balancing power and continuous trading of positive flexibility. Negative flexibility (right) is provided for the request of negative balancing power and continuous trading of negative flexibility. Base operation refers to a scenario with no request of balancing power (middle).

market (cases ±, +, −, 0), ensuring the flexibility range is fully allocated across the markets.

The flexibility range is derived by maximizing and minimizing the multi-energy system's electricity production at each time step while respecting the time-varying energy demands (cf. Section 2.1). Then, the difference between these maximum and minimum values defines the flexibility range. Note that in this case study, the energy demands are assumed deterministic. Hence, no uncertainties are associated with the range.

The multi-energy system prioritizes deploying flexible capacity in the balancing-power market (case ±). If a tender is unsuccessful in the capacity-price auction of the balancing-power market (cases + and −), the continuous intraday market is used instead. In these cases, the flexible capacity is used for continuous trading in the intraday market.

To optimize the use of positive or negative flexibility, the day-ahead market is employed. Using this market, the flexibility range can be shifted in one direction at the expense of the other: Electricity is purchased to enhance positive flexibility during time steps 0–8 (cases ±, +) and sold to enhance negative flexibility during time steps 8–24 (cases ±, −). Overall, the method effectively exploits the potential of sequential bidding in the multi-stage stochastic optimization.

The multi-energy system utilizes its flexibility by a complex and interconnected adjustment of its operation: The electricity production is adjusted based on the market signals, followed by adjusting the operation of the multi-energy system for the heat and cooling supply (Fig. 9). In the event that positive flexible capacity is utilized (Fig. 9, left), the multi-energy system commonly increases electricity production by combined-heat-and-power units. Due to their co-generation, electricity production increases heat production, which is used to cover heating demands or cooling demands via absorption chillers. In the event that negative flexible capacity is utilized (Fig. 9, right), the multi-energy system either increases internal electricity use or decreases electricity production, as electricity is drawn from the grid. Subsequently, electrode boilers mostly provide heat, and compression chillers provide cooling.

In conclusion, the results indicate substantial savings when participating in all markets since the flexible capacity is utilized both in the balancing-power market and the continuous intraday market. The employed bidding strategies and operations are complex, justifying the use of the proposed method to allocate the flexibility range to the markets optimally.

4.2. Sensitivity analysis of intraday market volatility

The previous analysis reveals that the utilization of flexible capacity is highly beneficial. However, the balancing-power market tends to be preferred for the typical weeks considered in this study. To better understand market preferences, we analyze the sensitivities of the model with respect to the volatilities on the intraday market. Higher volatilities on the intraday market add to the profitability of continuous trading and increase the option value of the intraday market in our model.

We perform the sensitivity analysis for CW37-2019 and consider the case with participation in all markets (DA,ID,BP). Fig. 10 compares the average amount of flexible capacity offered in the balancing-power market and continuous intraday market, assuming a successful capacity price auction for positive and negative balancing power (case ±). Thereby, we analyze both the balancing-power market bids of the first stage and the subsequent strategy on the continuous intraday market at the same time.

The sensitivity analysis highlights the importance of the intraday opportunity within a multi-market optimization (Fig. 10). We vary the

volatility between a factor of 0.5 to 6 to reflect the opportunity for flexible capacity to utilize higher volatilities by trading quarter-hourly contracts and exploiting shorter lead times. With the base volatility (1σ), flexible capacity is mainly utilized in the balancing-power market (88%), offering low capacity prices. In this case, the average acceptance probability is 66% for positive and 48% for negative balancing power. Higher volatilities increase both the flexible capacity traded on the intraday market and the capacity prices offered in the balancing-power market, reducing the acceptance probabilities. With increasing volatility ($2\sigma - 4\sigma$), the capacity prices are increased first. For even higher volatilities ($4\sigma - 6\sigma$), the flexible capacity is shifted towards the continuous intraday market. Consequently, the average acceptance probability reduces to 48% for positive and 10% for negative balancing power. Thus, the provision of balancing power is less likely. In case 6σ, almost the entire flexible capacity (85%) is allocated to the continuous intraday market.

In conclusion, the sensitivity analysis shows that higher intraday volatilities shift both the capacity prices and the flexible capacity to favor the continuous intraday market. Thereby, the bidding strategy reflects the multi-energy system's capabilities to exploit the volatility of the continuous intraday market. By considering all three markets, the method is, thus, able to optimally allocate the available flexibility depending on the current market situation to obtain further savings.

4.3. Limitations and method discussion

This study models multi-market participation as a multi-stage stochastic optimization problem. While the model captures sequential bidding across different markets by considering the different market outcomes and operational scenarios for each time step, our model does not include a multi-period formulation. Consequently, the model excludes storage and neglects time-linking constraints, such as minimum up- and downtimes, and maximum ramp rates. This simplification may overestimate the revenues generated from marketing flexibility. Future work could extend the model to a multi-period formulation, as demonstrated in recent literature by Bohlayer et al. (2020), Fusco et al. (2023) and Kraft et al. (2022).

Beyond the temporal structure of the model, another critical aspect relates to how uncertainties are represented: While operational uncertainties are captured by operational scenarios, probabilistic market forecasts, including their confidence intervals, are not explicitly incorporated. The multi-period formulation could account for full-day or multi-day scenarios, enabling the inclusion of uncertainty ranges for forecasted market parameters. Uncertainty could also be included for the energy demands, for industries with less reliable forecasts. In addition, supply variability would become more important for volatile renewable energy sources such as wind and solar, making their accurate representation essential for future utility systems. Thus, probabilistic forecasts would also facilitate the integration of renewables into the model. Addressing these gaps in future work with a probabilistic multi-period approach would improve the robustness and accuracy of the model, offer the possibility to consider risk-averse decision-making optimizing the conditional value at risk (Germscheid et al., 2022, 2023), and also enable the integration of variable renewable energy sources (Fusco et al., 2023).

In our approach, revenues from the continuous intraday market are estimated using the option price, providing a fast and efficient method to approximate trading revenues. Therein, potential price paths are modeled as arithmetic Brownian motion. This assumption effectively captures the occurrence of negative prices but implicitly results in normally distributed prices at the end of a trading session. The validity of this assumption is assessed by comparing the fitted price distribution with observed intraday market data, showing adequate alignment (see Supporting Information). However, recent studies suggest that alternative distributions using normalizing flows may better represent observed price behavior (Cramer et al., 2023). Future work could refine the option price model by integrating recent forecast models, i.e., to explore the suggested distributions (Cramer et al., 2023) and time-dependent volatilities (Hirsch and Ziel, 2024) to improve the accuracy of the pricing model. Furthermore, a multi-period structure could capture potential dependencies between trading sessions (Hirsch and Ziel, 2024).

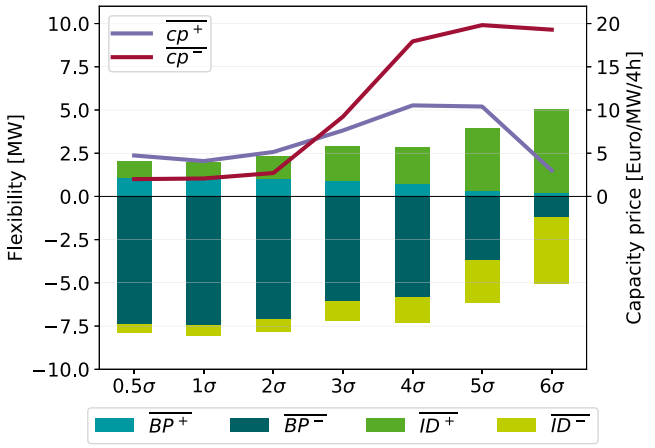


Fig. 10. Flexible capacity offered on the balancing-power $\overline{BP^{+/-}}$ and continuous intraday market $\overline{ID^{+/-}}$ for CW37-2019 after the successful capacity price auction (case ±) as a function of the intraday volatility σ. Additionally, the volume-weighted average capacity price $\overline{cp^{+/-}}$ is shown at which price positive and negative balancing power is offered.

In addition to these modeling considerations, we compare our expected savings from intraday trading with the literature. In our case study, the intraday market saves roughly 5300 Euro per MW and year. Given the associated uncertainty, this value seems to be well in line with revenues reported by Kern et al. (2019) that are between 6200 and 6400 Euro per MW and year for a dispatchable generator, which indicates the option price represents a rather conservative estimate of market returns. Our results of this study highlight the importance of considering the intraday market opportunity within a sequential decision-making framework. However, our method does not optimize actual intraday market trading. Full validation should include an intraday market bidding model that would, in theory, be continuously solved during the trading session to update trading decisions in the intraday market, or employ empirical evidence from actual trading.

5. Conclusions

A method is presented to integrate trading on the continuous intraday market in a multi-market optimization for the day-ahead, balancing-power, and continuous intraday market. To approximate the revenues from continuous trading, we employ option-price theory. The option value is integrated into a multi-stage stochastic optimization that models multiple markets, including the energy system's operations. As a result, the method is able to determine an optimal coordinated bidding strategy for all markets.

We apply our method to a multi-energy system participating in the German market as of July 2020, representing a typical European sequential electricity market. The largest cost reductions can be achieved when participating in all three considered markets, utilizing the entire flexibility range of the multi-energy system. A sensitivity analysis shows that larger volatilities in the intraday market support a shift towards increased participation in the continuous intraday market: Tenders on the balancing-power market are offered with increasing capacity prices

and thus reduced acceptance probabilities. The analysis highlights the importance of considering the intraday opportunity.

In conclusion, our method derives a coordinated and optimized bidding strategy in short-term electricity and balancing-power markets by exploiting the flexibility of the market participant's energy system to achieve the largest cost reductions.

The performance of our method depends on accurate price forecasts. Since price forecasting is not in the scope of this paper, future research could focus on developing price forecasting methods that are consistent across the markets. In particular, the volatility on the continuous intraday market greatly influences market preferences. Hence, forecasting methods for the continuous intraday market are desirable. Additionally, we consider risk-neutral decision-making. Since energy supply companies often act risk-averse, our approach could thus be extended by considering risk in the optimization model (Kraft et al., 2022; Germscheid et al., 2022). Finally, our approach could consider storage to increase the flexibility of an energy system.

Our method and the results motivate market participants to implement joint optimization in all markets. Joint market participation increases the flexibility of the power system and ultimately supports the energy transition.

Nomenclature

Sets	
Symbol	Explanation
T	Time steps
Ω	Scenarios
Ω^{ID}	Intraday market scenarios
C	Capacity and energy price combinations
U	Units
P	Products

Variables	
Symbol	Explanation
$OPEX_{t}^{\mathrm{exp}}$	Expected operational cost
$OPEX_{t,c}^{\mathrm{price\ comb}}$	Operational cost per price combination
$OPEX_{t,c,\omega}^{\mathrm{scenario}}$	Operational cost per price combination and scenario
$C_{t,\omega}^{\mathrm{energy}}$	Energy cost
$R_{t,c,\omega}^{\mathrm{BP}}$	Balancing-power market revenues
$R_{t,\omega}^{\mathrm{DA}}$	Day-ahead market revenues
$R_{t,\omega}^{\mathrm{ID}}$	Intraday market revenues
$\lambda_{t,c}^{\mathrm{BP}}$	Binary decision for price combination in the balancing-power market
$\lambda_{t,\omega}^{\mathrm{DA}}$	Binary decision to buy/sell electricity in the day-ahead market
$BUY_{t,\omega,p}$	Amount of purchased energy carrier (e.g. natural gas)
$DA_{t,\omega}^{\mathrm{sell}}$	Amount of sold electricity in the day-ahead market
$DA_{t,\omega}^{\mathrm{buy}}$	Amount of purchased electricity in day-ahead market
$ID_{t,\omega}^{\mathrm{sell}}$	Continuous intraday market trading volumes for positive flexible capacity
$ID_{t,\omega}^{\mathrm{buy}}$	Continuous intraday market trading volumes for negative flexible capacity
BP_{t}^{+}	Amount of positive balancing power
BP_{t}^{-}	Amount of negative balancing power
$P_{u,t,\omega,p}$	Product supply/demand of unit

Parameters	
Symbol	Explanation
$\pi_{t,c,\omega}$	Scenario probability
$\pi_{t,c,\omega}^{\mathrm{BP,cp}}$	Acceptance probability in the balancing-power market
$\pi_{t,\omega}^{\mathrm{ID}}$	Probability of positive or negative flexible capacity being utilized in the intraday market
$\pi_{t,c,\omega}^{\mathrm{BP,ep}}$	Request probability in the balancing-power market
M^{BP}	Big M (large number)
M^{DA}	Big M (large number)
p_{t}^{ec}	Energy carrier price
$p_{t}^{\mathrm{DA,sell}}$	Day-ahead market selling price
$p_{t}^{\mathrm{DA,buy}}$	Day-ahead market purchasing price
$cp_{t,c}^{+}$	Capacity price for acceptance of positive balancing power
$cp_{t,c}^{-}$	Capacity price for acceptance of negative balancing power
$ep_{t,c}^{+}$	Energy price for request of positive balancing power
$ep_{t,c}^{-}$	Energy price for request of negative balancing power
$s_{\omega}^{\mathrm{cp+}}$	Binary parameter for acceptance of positive balancing power
$s_{\omega}^{\mathrm{cp-}}$	Binary parameter for acceptance of negative balancing power
$s_{\omega}^{\mathrm{ep+}}$	Binary parameter for request of positive balancing power
$s_{\omega}^{\mathrm{ep-}}$	Binary parameter for request of negative balancing power
$s_{\omega}^{\mathrm{sell}}$	Binary parameter to sell in the continuous intraday market
$s_{\omega}^{\mathrm{buy}}$	Binary parameter to buy in the continuous intraday market
$d_{p,t}$	Product demand
opt_{t}^{sell}	Option price for positive flexible capacity
opt_{t}^{buy}	Option price for negative flexible capacity
mc_{t}	Marginal electricity cost
u_{t}	Up-movement
d_{t}	Down-movement
μ_{t}	Price drift
σ_{t}	Price volatility
N_{t}	Number of trading opportunities
S_{t}^{ini}	Initial price-level
$S_{t,k}$	Last prices
Φ^{sell}	Option value for positive flexible capacity at the end of the trading session
Φ^{buy}	Option value for negative flexible capacity at the end of the trading session

CRediT authorship contribution statement

Niklas Nolzen: Writing – original draft, Visualization, Validation, Software, Project administration, Methodology, Investigation, Data curation, Conceptualization. **Alissa Ganter:** Writing – review & editing, Visualization, Software, Methodology, Investigation, Data curation, Conceptualization. **Nils Baumgärtner:** Writing – review & editing, Conceptualization. **Florian Joseph Baader:** Writing – review & editing, Visualization, Supervision, Conceptualization. **Ludger Leenders:** Writing – review & editing, Visualization, Supervision, Funding acquisition, Conceptualization. **André Bardow:** Writing – review & editing, Supervision, Resources, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: AB has ownership interests in firms that render services to industry, some of which may participate in energy markets. AB has served on review committees for research and development at ExxonMobil and TotalEnergies, oil and gas companies that are also active in energy markets. All other authors have no competing interests.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.compchemeng.2025.109026.

Data availability

Data will be made available on request.

References

Aasgård, E.K., Fleten, S.-E., Kaut, M., Midthun, K., Perez-Valdes, G.A., 2019. Hydropower bidding in a multi-market setting. Energy Syst. 10 (3), 543–565. http://dx.doi.org/10.1007/s12667-018-0291-y.

Aïd, R., Gruet, P., Pham, H., 2016. An optimal trading problem in intraday electricity markets. Math. Financ. Econ. 10 (1), 49–85. http://dx.doi.org/10.1007/s11579-015-0150-8.

Alexander, D.R., Mo, M., Stent, A.F., 2012. Arithmetic Brownian motion and real options. Eur. J. Oper. Res. 219 (1), 114–122. http://dx.doi.org/10.1016/j.ejor.2011.12.023.

Barbero, M., Corchero, C., Canals Casals, L., Igualada, L., Heredia, F.-J., 2020. Critical evaluation of European balancing markets to enable the participation of demand aggregators. Appl. Energy 264, 114707. http://dx.doi.org/10.1016/j.apenergy.2020.114707.

Bauer, T., Prenzel, M., Klasing, F., Franck, R., Lützow, J., Perrey, K., Faatz, R., Trautmann, J., Reimer, A., Kirschbaum, S., 2022. Ideal–typical utility infrastructure at chemical sites – definition, operation and defossilization. Chem. Ing. Tech. 94 (6), 840–851. http://dx.doi.org/10.1002/cite.202100164.

Baumgärtner, N., Shu, D., Bahl, B., Hennen, M., Hollermann, D.E., Bardow, A., 2020. DeLoop: Decomposition-based long-term operational optimization of energy systems with time-coupling constraints. Energy 198 (4), 117272. http://dx.doi.org/10.1016/j.energy.2020.117272.

Björk, T., 2009. Arbitrage theory in continuous time, third ed. Oxford University Press, Oxford.

Bohlayer, M., Fleschutz, M., Braun, M., Zöttl, G., 2020. Energy-intense production-inventory planning with participation in sequential energy markets. Appl. Energy 258, 113954. http://dx.doi.org/10.1016/j.apenergy.2019.113954.

Boomsma, T.K., Juul, N., Fleten, S.-E., 2014. Bidding in sequential electricity markets: The nordic case. Eur. J. Oper. Res. 238 (3), 797–809. http://dx.doi.org/10.1016/j.ejor.2014.04.027.

Boukas, I., Ernst, D., Théate, T., Bolland, A., Huynen, A., Buchwald, M., Wynants, C., Cornélusse, B., 2021. A deep reinforcement learning framework for continuous intraday market bidding. Mach. Learn. 110 (9), 2335–2387. http://dx.doi.org/10.1007/s10994-021-06020-8.

Corinaldesi, C., Schwabeneder, D., Lettner, G., Auer, H., 2020. A rolling horizon approach for real-time trading and portfolio optimization of end-user flexibilities. Sustain. Energy, Grids Netw. 24, 100392. http://dx.doi.org/10.1016/j.segan.2020.100392.

Cox, J.C., Ross, S.A., Rubinstein, M., 1979. Option pricing: A simplified approach. J. Financ. Econ. 7 (3), 229–263. http://dx.doi.org/10.1016/0304-405X(79)90015-1.

Cramer, E., Witthaut, D., Mitsos, A., Dahmen, M., 2023. Multivariate probabilistic forecasting of intraday electricity prices using normalizing flows. Appl. Energy 346, 121370. http://dx.doi.org/10.1016/j.apenergy.2023.121370.

Dalle Ave, G., Harjunkoski, I., Engell, S., 2019. A non-uniform grid approach for scheduling considering electricity load tracking and future load prediction. Comput. Chem. Eng. 129, 106506. http://dx.doi.org/10.1016/j.compchemeng.2019.06.031.

Dowling, A.W., Kumar, R., Zavala, V.M., 2017. A multi-scale optimization framework for electricity market participation. Appl. Energy 190, 147–164. http://dx.doi.org/10.1016/j.apenergy.2016.12.081.

Feng, Y., Jia, H., Wang, X., Ning, B., Liu, Z., Liu, D., 2023. Review of operations for multi-energy coupled virtual power plants participating in electricity market. Energy Rep. 9, 992–999. http://dx.doi.org/10.1016/j.egyr.2023.04.149.

Finhold, E., Gärtner, C., Grindel, R., Heller, T., Leithäuser, N., Röger, E., Schirra, F., 2023. Optimizing the marketing of flexibility for a virtual battery in day-ahead and balancing markets: A rolling horizon case study. Appl. Energy 352, 121667. http://dx.doi.org/10.1016/j.apenergy.2023.121667.

Finnah, B., Gönsch, J., Ziel, F., 2022. Integrated day-ahead and intraday self-schedule bidding for energy storage systems using approximate dynamic programming. Eur. J. Oper. Res. 301 (2), 726–746. http://dx.doi.org/10.1016/j.ejor.2021.11.010.

Frangopoulos, C.A., 2018. Recent developments and trends in optimization of energy systems. Energy 164 (4), 1011–1020. http://dx.doi.org/10.1016/j.energy.2018.08.218.

Fusco, A., Gioffrè, D., Francesco Castelli, A., Bovo, C., Martelli, E., 2023. A multi-stage stochastic programming model for the unit commitment of conventional and virtual power plants bidding in the day-ahead and ancillary services markets. Appl. Energy 336, 120739. http://dx.doi.org/10.1016/j.apenergy.2023.120739.

Germscheid, S.H.M., Mitsos, A., Dahmen, M., 2022. Demand response potential of industrial processes considering uncertain short–term electricity prices. AIChE J. 68 (11), http://dx.doi.org/10.1002/aic.17828.

Germscheid, S.H., Röben, F.T., Sun, H., Bardow, A., Mitsos, A., Dahmen, M., 2023. Demand response scheduling of copper production under short-term electricity price uncertainty. Comput. Chem. Eng. 178, 108394. http://dx.doi.org/10.1016/j.compchemeng.2023.108394.

Glas, S., Kiesel, R., Kolkmann, S., Kremer, M., Graf von Luckner, N., Ostmeier, L., Urban, K., Weber, C., 2020. Intraday renewable electricity trading: advanced modeling and numerical optimal control. J. Math. Ind. 10 (3), http://dx.doi.org/10.1186/s13362-020-0071-x.

Guo, X., de Larrard, A., Ruan, Z., 2017. Optimal placement in a limit order book: an analytical approach. Math. Financ. Econ. 11 (2), 189–213. http://dx.doi.org/10.1007/s11579-016-0177-5.

Halbrügge, S., Schott, P., Weibelzahl, M., Buhl, H.U., Fridgen, G., Schöpf, M., 2021. How did the german and other European electricity systems react to the COVID-19 pandemic? Appl. Energy 285, 116370. http://dx.doi.org/10.1016/j.apenergy.2020.116370.

Han, C., Hilger, H., Mix, E., Böttcher, P.C., Reyers, M., Beck, C., Witthaut, D., Gorjão, L.R., 2022. Complexity and persistence of price time series of the European electricity spot market. PRX Energy 1 (1), 013002. http://dx.doi.org/10.1103/PRXEnergy.1.013002.

Hellemo, L., Barton, P.I., Tomasgard, A., 2018. Decision-dependent probabilities in stochastic programs with recourse. Comput. Manag. Sci. 15, 369–395. http://dx.doi.org/10.1007/s10287-018-0330-0.

Hirsch, S., Ziel, F., 2024. Multivariate simulation-based forecasting for intraday power markets: Modeling cross-product price effects. Appl. Stoch. Models Bus. Ind. 40 (6), 1571–1595. http://dx.doi.org/10.1002/asmb.2837.

Hong, T., Pinson, P., Wang, Y., Weron, R., Yang, D., Zareipour, H., 2020. Energy forecasting: A review and outlook. IEEE Open Access J. Power Energy 7, 376–388. http://dx.doi.org/10.1109/OAJPE.2020.3029979.

Jahns, C., Weber, C., 2019. Probabilistic forecasting of reserve power prices in Germany using quantile regression. In: Proceedings of 16th International Conference on the European Energy Market. EEM, IEEE, Ljubljana, pp. 1–5. http://dx.doi.org/10.1109/EEM.2019.8916228.

Kern, T., Hinterstocker, M., von Roon, S., 2019. The value of intraday electricity trading: Evaluating situation-dependent opportunity costs of flexible assets.

Kiesel, R., Paraschiv, F., 2017. Econometric analysis of 15-minute intraday electricity prices. Energy Econ. 64, 77–90. http://dx.doi.org/10.1016/j.eneco.2017.03.002.

Klaboe, G., Fosso, O.B., 2013. Optimal bidding in sequential physical markets — A literature review and framework discussion. In: Proceedings of 2013 IEEE Grenoble Power Tech Conference. IEEE, Grenoble, pp. 1–6. http://dx.doi.org/10.1109/PTC.2013.6652371.

Koch, C., Hirth, L., 2019. Short-term electricity trading for system balancing: An empirical analysis of the role of intraday trading in balancing Germany's electricity system. Renew. Sustain. Energy Rev. 113, 109275. http://dx.doi.org/10.1016/j.rser.2019.109275.

Kraft, E., Keles, D., Fichtner, W., 2020. Modeling of frequency containment reserve prices with econometrics and artificial intelligence. J. Forecast. 39 (8), 1179–1197. http://dx.doi.org/10.1002/for.2693.

Kraft, E., Russo, M., Keles, D., Bertsch, V., 2022. Stochastic optimization of trading strategies in sequential electricity markets. Eur. J. Oper. Res. 238 (3), 400–421. http://dx.doi.org/10.1016/j.ejor.2022.10.040.

Kremer, M., Kiesel, R., Paraschiv, F., 2021. An econometric model for intraday electricity trading. Philos. Trans. R. Soc. A 379 (2202), 20190624. http://dx.doi.org/10.1098/rsta.2019.0624.

Kumbartzky, N., Schacht, M., Schulz, K., Werners, B., 2017. Optimal operation of a CHP plant participating in the german electricity balancing and day-ahead spot market. Eur. J. Oper. Res. 261 (1), 390–404. http://dx.doi.org/10.1016/j.ejor.2017.02.006.

Lago, J., Marcjasz, G., de Schutter, B., Weron, R., 2021. Forecasting day-ahead electricity prices: A review of state-of-the-art algorithms, best practices and an open-access benchmark. Appl. Energy 293 (4), 116983. http://dx.doi.org/10.1016/j.apenergy.2021.116983.

Leenders, L., Starosta, A., Baumgärtner, N., Bardow, A., 2020. Integrated scheduling of batch production and utility systems for provision of control reserve. In: Proceedings of ECOS 2020 - 33rd International Conference on Efficiency, Cost, Optimization, Simulation and Environmental Impact of Energy Systems. Osaka, pp. 712–723. http://dx.doi.org/10.3929/ethz-b-000423722.

Loesch, M., Rominger, J., Nainappagari, S., Schmeck, H., 2018. Optimizing bidding strategies for the german secondary control reserve market: The impact of energy prices. In: Proceedings of 15th International Conference on the European Energy Market. EEM, IEEE, Lodz, pp. 1–5. http://dx.doi.org/10.1109/EEM.2018.8469993.

Löhndorf, N., Wozabal, D., 2021. Gas storage valuation in incomplete markets. Eur. J. Oper. Res. 288 (1), 318–330. http://dx.doi.org/10.1016/j.ejor.2020.05.044.

Löhndorf, N., Wozabal, D., 2022. The value of coordination in multimarket bidding of grid energy storage. Oper. Res. 71 (1), http://dx.doi.org/10.1287/opre.2021.2247.

Mancarella, P., 2014. MES (multi-energy systems): An overview of concepts and evaluation models. Energy 65 (3), 1–17. http://dx.doi.org/10.1016/j.energy.2013.10.041.

Muche, T., Höge, C., Renner, O., Pohl, R., 2016. Profitability of participation in control reserve market for biomass-fueled combined heat and power plants. Renew. Energy 90, 62–76. http://dx.doi.org/10.1016/j.renene.2015.12.051.

Muthirayan, D., Kalathil, D., Li, S., Poolla, K., Varaiya, P., 2021. Selling demand response using options. IEEE Trans. Smart Grid 12 (1), 279–288. http://dx.doi.org/10.1109/TSG.2020.3011382.

Narajewski, M., Ziel, F., 2020. Ensemble forecasting for intraday electricity prices: Simulating trajectories. Appl. Energy 279 (2), 115801. http://dx.doi.org/10.1016/j.apenergy.2020.115801.

Nolzen, N., Leenders, L., Bardow, A., 2023. Flexibility-expansion planning of multi-energy systems by energy storage for participating in balancing-power markets. Front. Energy Res. 11, 116923. http://dx.doi.org/10.3389/fenrg.2023.1225564.

Ocker, F., Ehrhart, K.-M., 2017. The "german paradox" in the balancing power markets. Renew. Sustain. Energy Rev. 67, 892–898. http://dx.doi.org/10.1016/j.rser.2016.09.040.

Rancilio, G., Rossi, A., Falabretti, D., Galliani, A., Merlo, M., 2022. Ancillary services markets in europe: Evolution and regulatory trade-offs. Renew. Sustain. Energy Rev. 154, 111850. http://dx.doi.org/10.1016/j.rser.2021.111850.

Rathi, T., Zhang, Q., 2022. Capacity planning with uncertain endogenous technology learning. Comput. Chem. Eng. 164, 107868. http://dx.doi.org/10.1016/j.compchemeng.2022.107868.

Reinert, C., Nolzen, N., Frohmann, J., Tillmanns, D., Bardow, A., 2023. Design of low-carbon multi-energy systems in the secmod framework by combining milp optimization and life-cycle assessment. Comput. Chem. Eng. 172, 108176. http://dx.doi.org/10.1016/j.compchemeng.2023.108176.

Reinert, C., Schellhas, L., Frohmann, J., Nolzen, N., Tillmanns, D., Baumgärtner, N., Deutz, S., Bardow, A., 2022. Combining optimization and life cycle assessment: Design of low-carbon multi-energy systems in the secmod framework. In: 32nd European Symposium on Computer Aided Process Engineering. In: Computer Aided Chemical Engineering, 51, Elsevier, pp. 1201–1206. http://dx.doi.org/10.1016/B978-0-323-95879-0.50201-0.

Schäfer, P., Westerholt, H.G., Schweidtmann, A.M., Ilieva, S., Mitsos, A., 2019. Model-based bidding strategies on the primary balancing market for energy-intense processes. Comput. Chem. Eng. 120, 4–14. http://dx.doi.org/10.1016/j.compchemeng.2018.09.026.

Srinivasan, A., Wu, R., Heer, P., Sansavini, G., 2023. Impact of forecast uncertainty and electricity markets on the flexibility provision and economic performance of highly-decarbonized multi-energy systems. Appl. Energy 338, 120825. http://dx.doi.org/10.1016/j.apenergy.2023.120825.

Swider, D.J., 2007. Simultaneous bidding in day-ahead auctions for spot energy and power systems reserve. Int. J. Electr. Power & Energy Syst. 29 (6), 470–479. http://dx.doi.org/10.1016/j.ijepes.2006.11.005.

Weber, C., Möst, D., Fichtner, W., 2022. Economics of power systems: Fundamentals for sustainable energy. Springer eBook Collection, Cham, http://dx.doi.org/10.1007/978-3-030-97770-2.