

Angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays with the full LHCb Run 1 and 2 data sample

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Abstract

The Standard Model of particle physics (SM) describes the fundamental interactions between the smallest constituents of matter with great precision. Over the past decades, it has been incredibly successful in predicting the outcome of a wide range of particle physics experiments. However, there are experimental observations that the SM in its current form can not explain. This implies the existence of yet undiscovered New Physics (NP) beyond the SM, which needs to be incorporated into the SM to complete the theory.

Precision measurements of observables from flavor-changing neutral currents are highly sensitive to NP effects due to their strong suppression in the SM. Previous analyses of these types of decays displayed intriguing tensions with the SM predictions. Among the most prominent results is the measurement of the angular observables of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ performed using data collected with the LHCb experiment during the years 2011, 2012, and 2016, where the optimized angular observable P'_5 displays local tensions with the SM prediction.

This thesis presents the legacy measurement of both the CP -symmetry and asymmetry angular observables as well as the branching fraction of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ using the Run 1 and 2016-2018 data sets collected with the LHCb experiment. These observables are determined in bins of the squared invariant dimuon mass (q^2) using a five-dimensional maximum likelihood fit. The signal decays are selected using a combination of requirements on kinematic and particle identification variables, as well as multivariate analysis techniques. The well-known angular observable P'_5 is measured to be

$$P'_5 = -0.481 \pm 0.073 \text{ (stat.)} \pm 0.006 \text{ (syst.)}$$

in the $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$ q^2 region and

$$P'_5 = -0.587 \pm 0.055 \text{ (stat.)} \pm 0.004 \text{ (syst.)}$$

in the $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$ q^2 region. This corresponds to local discrepancies at the level of 2.6σ and 3.1σ with respect to the SM predictions calculated using the Flavio theory framework. The deviation of the Wilson coefficient $\mathcal{R}e(C_9)$ from the SM value determined using constraints from the angular observables, and the branching fraction is found to be

$$\Delta \mathcal{R}e(C_9) = -0.92^{+0.18}_{-0.16},$$

which corresponds to a global significance of the deviation from the SM of 4.1σ . However, the significance of this deviation depends on assumptions made on the parameterization of the non-local form factors, which is currently a topic of debate in the theory community.

Zusammenfassung

Das Standardmodell der Teilchenphysik (SM) beschreibt die fundamentalen Interaktionen zwischen den kleinsten Bestandteilen der Materie in unserem Universum mit großer Präzision. Über die letzten Jahrzehnte konnten mit seiner Hilfe Messungen einer großen Bandbreite von physikalischen Experimenten erfolgreich vorhergesagt werden. Es gibt jedoch auch Beobachtungen, die das SM in seiner momentanen Form nicht erklären kann. Dies impliziert die Existenz einer noch nicht beobachteten Neuen Physik (NP), die über das Standardmodell hinausgeht und die Theorie komplettiert.

Präzisionsmessungen von Observablen in Flavor-verändernden neutralen Strömen besitzen eine große Sensitivität für eben solche NP-Effekte, da diese Prozesse im SM stark unterdrückt sind. Vorangegangene Analysen dieser Art von Zerfällen fanden Diskrepanzen zwischen den Messungen und den Vorhersagen des Standardmodells. Eines der prominentesten Resultate ist die Messung von CP -symmetrischen Winkelobservablen des $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ Zerfalls mit Daten des LHCb-Experiments von 2011, 2012 und 2016. Die optimierte Winkelobservable P'_5 zeigt lokale Diskrepanzen zur SM-Vorhersage.

Diese Dissertation präsentiert sowohl die Messungen der CP -symmetrischen und CP -asymmetrischen Winkelobservablen als auch das Verzweigungsverhältnis des Zerfalls $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ mit Daten des LHCb-Experiments, die während der Jahre 2011, 2012 und 2016-2018 aufgezeichnet wurden. Die Bestimmung der Observablen wird in Intervallen der quadrierten invarianten Dimuonenmasse mittels einer fünf-dimensionalen Maximum-Likelihood-Anpassung durchgeführt. Der Signalzerfall wird mit einer Kombination von Schnitten auf die Verteilungen sowohl von kinematischen als auch von Teilchenidentifikationsvariablen und multivariaten Analysemethoden selektiert. Der gemessene Wert der bekannten Winkelobservable P'_5 ist

$$P'_5 = -0.481 \pm 0.073 \text{ (stat.)} \pm 0.006 \text{ (syst.)}$$

im $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$ Intervall und

$$P'_5 = -0.587 \pm 0.055 \text{ (stat.)} \pm 0.004 \text{ (syst.)}$$

im $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$ Intervall. Dies entspricht lokalen Diskrepanzen zwischen der Messung und der SM-Vorhersage von 2.6σ und 3.1σ , welche mittels des Softwarepakets Flavio bestimmt wurden. Die auf Grundlage der Winkelobservablen und des Verzweigungsverhältnisses bestimmte Abweichung des gemessenen Wilson Koeffizienten $\mathcal{Re}(C_9)$ vom SM beträgt

$$\Delta \mathcal{Re}(C_9) = -0.92^{+0.18}_{-0.16}.$$

Dies entspricht einer globalen Signifikanz der Abweichung vom SM von 4.1σ . Die angegebenen Signifikanzwerte hängen jedoch von der Parametrisierung der nicht-lokalen Formfaktoren ab. Der Einfluss dieses Effekts auf die Theorievorhersage wird unter Theoretikern zu diesem Zeitpunkt noch diskutiert.

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Chapter 1

Introduction

The goal of particle physics is to describe the subatomic structure of the universe fully through a single comprehensive mathematical model. All the knowledge that science has accumulated in this field is condensed in the Standard Model of particle physics (SM), which is a quantum field theory (QFT) that has been incredibly successful in explaining the measurements of particle physics experiments around the globe with unparalleled precision [1–3]. As the SM describes the fundamental interactions between the smallest constituents of matter, it lays the groundwork for scientific descriptions of nature at all scales, from the structure of atoms to the life cycles of stars. However, there are also experimental observations that cannot be explained by the SM in its current form. Experiments that measure the relative movement of galaxies have found results that are inconsistent with the amount of visible matter observed in the universe [4, 5]. This implies the presence of a significant amount of heavy, so-called Dark Matter particles that interact feebly with other elementary particles. The current zoo of SM particles has no candidate for such a Dark Matter particle [6].

The SM also fails to explain the significant matter/antimatter asymmetry observed in the present universe. CP -violating processes present in the SM are not sufficient to explain an asymmetry of this scale [7]. Additional CP -violation can be introduced into the SM through the assumption of different non-zero neutrino masses for each lepton generation, leading to mixing between neutrino generations. This effect is described by the PMNS matrix [8], which can contain a CP -violating Dirac phase. Non-negligible neutrino masses have been confirmed through a multitude of neutrino experiments [9, 10]. However, measurements of the Dirac phase by T2K [11] and NOvA [12] do not indicate that the amount of CP -violation observed in the universe can be explained in this way.

These unexplained phenomena motivate a systematic search for physics beyond the Standard Model by the particle physics community. At the forefront of these efforts is the Large Hadron Collider (LHC) at CERN and its four major experiments CMS [13], ATLAS [14], ALICE [15], and LHCb [16]. These experiments study particle (mainly proton) collisions at high center-of-mass energies and search for signatures of particle interactions that are not predicted by the SM. Direct searches are conducted by reconstructing the neutrino invariant mass of the decay products of hypothetical New Physics (NP) particles. The largest success in this field has been the discovery of the Higgs boson by CMS and ATLAS in 2012 [17, 18]. Both experiments independently observed an excess at $125 \text{ GeV}/c^2$, which was clear proof of the existence of a previously

undiscovered particle. This discovery confirmed that the masses of bosons and fermions are generated by the Higgs mechanism [19].

Due to energy conservation, direct searches are limited to masses below the center-of-mass energy ($\sqrt{s}$) of the collider. New Physics scales above this $\sqrt{s}$ can only be probed through indirect searches. These are conducted through precision measurements of physics observables like angular observables or branching fractions of known SM decays. Heavy NP interactions can contribute to these processes as virtual particles and thereby alter the values of the aforementioned observables, as the mass of virtual particles is not constrained by energy conservation. A significant sensitivity to the NP effect is only achieved if the NP contributions to the final state of the decay are of similar size to the SM process. Therefore, the analysis of flavor-changing neutral current (FCNC) transitions is especially interesting as they are forbidden in the SM at tree-level and can only proceed at loop-level, which results in a strong suppression compared to potential NP tree-level processes. Such FCNC decays can occur in the decays of b -hadrons, where the b -quarks transition into a d or s quark. At the LHC, $b\bar{b}$ pairs are produced in large quantities [20]. The LHCb experiment has been specifically designed to reconstruct the decays of b -hadrons with high precision and efficiency. It has collected the largest sample of b -decays of all particle physics experiments to date, which makes it predestined to perform the indirect searches mentioned above with the highest precision.

In recent years, special attention has been paid to the $b \rightarrow s\ell^+\ell^-$ process as measurements of the branching fractions and angular observables have been found to be in tension with the SM predictions at the level of 2-3.5 standard deviations (σ) [21,22]. In order to clarify the situation, the continued analysis of these processes with larger data samples is of high importance. One of the key decays at LHCb is the $B^0 \rightarrow K^{*0}\mu^+\mu^-$ mode. As K^{*0} is a vector particle, which decays into $K^+\pi^-$, the decay has a rich angular structure that can be explored. The analysis has access to a set of optimized angular observables with reduced dependency on local hadronic form factors [23]. The LHCb collaboration has published measurements of the angular observables of $B^0 \rightarrow K^{*0}\mu^+\mu^-$ using the data sample collected in the years 2011, 2012 and 2016 corresponding to 1.1 fb^{-1} ($\sqrt{s} = 7\text{ TeV}$), 2.1 fb^{-1} ($\sqrt{s} = 8\text{ TeV}$) and 1.7 fb^{-1} ($\sqrt{s} = 13\text{ TeV}$). The optimized observable P'_5 displays local discrepancies with the SM expectation, and the global analysis of the full set of angular observables indicates a tension with the SM at the level of 3.3σ [24]. As the analysis is still statistically limited, an update of the analysis with a larger data sample is eagerly awaited. In the years 2017 and 2018, LHCb collected additional data corresponding to an integrated luminosity of 3.9 fb^{-1} at $\sqrt{s} = 13\text{ TeV}$. The combined analysis would, therefore, increase the precision of the measurement significantly.

This thesis presents the angular analysis of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$ with the LHCb data sets from 2011/12 and 2016-2018. This entails the implementation of a five-dimensional angular maximum likelihood fit to determine the full set of both CP -symmetric and CP -asymmetric angular observables, along with a simultaneous model-independent measurement of the branching fraction. The sets of angular observables are both fitted in the regular S_i and optimized P_i basis in bins of the invariant di-muon mass squared (q^2). The signal component is modeled using the theoretical parameterization of the differential decay rate, while the combinatorial background is modeled using Legendre polynomials. A simultaneous fit is performed

to samples of B^0 and $\bar{B}^0$ candidates to access the CP -asymmetry of the decay rate. The distortion of the angular and invariant mass distributions by reconstruction and selection effects is modeled from simulation using Legendre polynomials. Extensive validation of the fit procedure is performed by making use of simulated events, data samples, and pseudo-experiments. The fit is performed in five separate fit configurations to maximize the information extracted from the available data sample. Overall, over one thousand angular observable and branching fraction values are measured. Using the information from the angular fit, the deviations of the Wilson coefficients from their SM values are determined.

The thesis is structured as follows: Chapter 2 describes the theoretical framework upon which this analysis is based. An overview of the current experimental status of the flavor anomalies in $b \rightarrow s \ell^+ \ell^-$ decays can be found in Chapter 3. Chapter 4 gives an introduction to the LHCb experiment, which recorded the data sample used in this analysis. Chapter 5 is dedicated to the development of an HLT2 Software Trigger selection for rare b -decays during the current data taking in Run 3 at LHCb. Chapter 6 discusses the angular analysis of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ using the Run 1 and Run 2 LHCb data samples. Finally, conclusions and outlook are discussed in Chapter 7.

Chapter 2

Theory

2.1 Standard Model of particle physics

The Standard Model of particle physics (SM) is the most comprehensive mathematical description of the universe at the smallest scales to date. It describes the properties of the fundamental building blocks of our world, the elementary particles, and the forces that govern their interactions [1–3]. The particle content of the SM consists of seventeen indivisible particles and as many anti-particles, from which twelve are categorized as fermions with a spin of $1/2$ and five belong to the group of bosons of integer spin. The full list is displayed in Fig. 2.1. The fermions are the constituents of matter in the universe. They are again subdivided into two groups of six, the leptons and the quarks. The leptons occur in pairs of a massive particle with integer electric charge ($e^\pm, \mu^\pm, \tau^\pm$) and a corresponding neutral, massless neutrino (ν_e, ν_μ, ν_τ). The

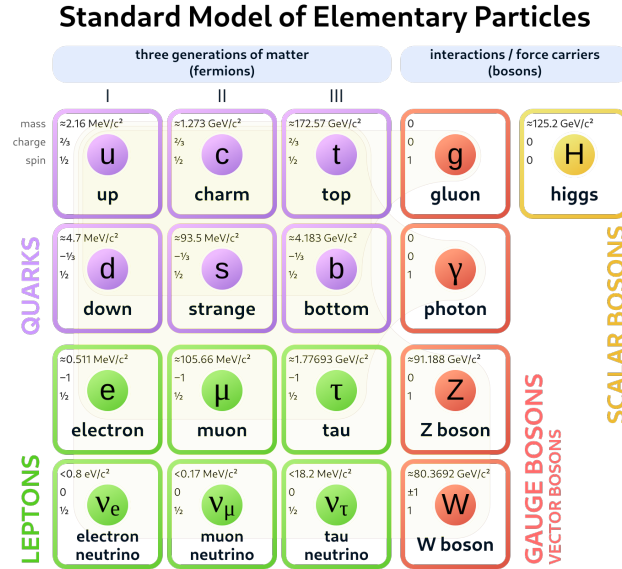


Figure 2.1: Overview of the fundamental particle content of the SM. The particles are grouped into quarks, leptons, gauge bosons, and one scalar boson. The figure is taken from Ref. [25].

only difference between these three generations of leptons is their mass, the electron (e) being the lightest generation, followed by the muon (μ) generation, and lastly the τ leptons, which are the heaviest.

The quarks display the same structure of three generations as the leptons. Here, a pair is formed by a charge $2/3$ up-type quark and a charge $-1/3$ down-type quark. Again, the particle mass is the only distinguishing property between generations. The pairs of quarks ordered in increasing particle mass are the down (d) and up quarks (u), the strange (s) and charm (c) quarks, and lastly the bottom (b) and top (t) quarks. For each of the particles in the SM, there is also a corresponding anti-particle with identical mass but inverted quantum numbers, like the electrical charge.

The SM knows three fundamental forces that govern the interactions between particles. Each of these forces is mediated by vector bosons, also called gauge bosons, of spin 1. The electromagnetic force governs the force between charged particles and is transmitted by the photon γ . Due to the vanishing mass of its vector boson, the effective range of the electromagnetic interaction is infinite. The orbital structure of atoms and the internal bonds of molecules originate from the electromagnetic force.

The radioactive beta decay of nuclei, on the other hand, is an example of the weak interaction between particles. Its corresponding vector bosons are the $W^\pm$ and Z^0 , which are, contrary to the photon, very heavy. With masses of 80 GeV and 91 GeV [26], respectively, those particles are as heavy as moderately sized atoms. This also explains the very small effective range of the weak interaction, which is at the level of 10^{-18} m [27]. A unique feature of the weak force is that the interaction with the $W^\pm$ boson is the only possibility to change the flavor of an interaction participant from an up-type to a down-type quark or a charged lepton to its corresponding neutrino and vice versa. It is noteworthy that in the SM, these flavor changes at tree-level can only occur via a charged current as transitions between quarks of the same charge are canceled by the GIM mechanism [28]. Similarly, the occurrence of CP -violation is a unique property of the weak interaction, which can be induced through the interaction of a strong phase with the weak phase of the CKM matrix [29, 30].

Lastly, the strong force is responsible for the binding of the large number of hadrons observed in nature. Those are either combinations of three quarks (baryons¹), like the proton, or a quark and an anti-quark (meson²), like the B -meson. The strong force facilitates the interactions between particles that can carry color charges, which are the quarks and the gauge boson of the strong force, the gluon. Defining properties of the strong force are the *asymptotic freedom* and the *quark confinement* [31]. These phenomena result from the dependence of the strong coupling on the transferred energy. Hard processes with large energy transfers lead to a strongly diminishing and asymptotically vanishing coupling strength. That means that at small distances within the confines of a hadron, the effect of the color charge becomes negligible. The quarks are asymptotically free. On the other hand, if one tries to separate quarks from within a hadron, with increasing distance between the color charges, the coupling strength increases to a point where the energy suffices to create quark-antiquark pairs from the vacuum, which in turn form new hadrons. This quark confinement means that color-charged quarks can never exist freely but must always be bound within a hadron.

¹Exotic baryons like penta-quarks can also contain additional $q\bar{q}$ pairs

²Exotic mesons like tetra-quarks can also contain additional $q\bar{q}$ pairs

In 2012, the CMS and ATLAS experiments discovered a scalar boson at a mass of $125 \text{ GeV}/c^2$ [17, 18]. This particle is the manifestation of the Higgs mechanism [19] in the SM that generates the masses of the heavy gauge bosons and the fermionic matter.

As mentioned before, the SM has some unique properties that distinguish it from extended models that include New Physics mechanisms. This includes the suppression of Flavor Changing Neutral Currents (FCNC) at the tree-level through the GIM mechanism [28]. This does not generally apply to New Physics interactions that might occur at the tree-level. Similarly, additional sources of CP -violation that do not originate from the CKM matrix may be introduced via a New Physics scenario.

These unique properties are opportunities for powerful tests of the SM through precision measurements, as any deviation from the SM prediction would be a clear sign of NP.

2.2 Quantum field theory

The SM is mathematically described through a relativistic quantum field theory approach. It combines the features of special relativity theory, which describes the behavior of physical objects at high energy, with quantum mechanics, including the quantization of force fields. The summary of this theory presented in this section is based on [27, 32, 33]. The dynamics and interactions of the system are encoded in the most general renormalizable Lagrangian that fulfills the following conditions:

1. The Lagrangian is invariant under the local gauge transformations defined by the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y. \quad (2.1)$$

$SU(3)_C$ is the gauge group of quantum chromodynamics, with the color charge C being the corresponding quantum number. The $SU(2)_L \times U(1)_Y$ group defines the electroweak sector of the theory. The corresponding quantum numbers are the third component of the weak Isospin T_3 and the weak hypercharge Y . The local transformations of each gauge group are defined by their specific generators. These are the 8 Gell-Mann matrices for the $SU(3)_C$ group, the three Pauli matrices for the $SU(2)_L$ group, and a single complex number for the $U(1)_Y$ group. Each of these generators corresponds to a gauge field that mediates the interactions of the theory.

2. The particle content of the theory consists of three generations of fermions grouped in left-handed doublets and right-handed singlets. Specifically, these are the left-handed doublet of quarks Q_{L_i} , where the index i describes the flavor of the quarks, the corresponding right-handed quark singlets U_{R_i} and D_{R_i} , the left handed lepton doublets L_{L_i} , and finally the right-handed leptons singlets E_{R_i} . The particle content is completed by the scalar field ϕ . The full list of fermionic matter is given in Tab. 2.1.
3. A mechanism that spontaneously breaks the gauge symmetry of the SM and lifts the degeneracy of the minimum energy states by assigning a specific non-zero vacuum expectation value of the Higgs field, and thereby creates renormalizable mass terms for the massive gauge bosons and fermionic matter.

Fermions			Quantum Numbers			
Gen. 1	Gen. 2	Gen. 3	Q	T	T_3	Y
u_L	c_L	t_L	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{3}$
d_L	s_L	b_L	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{3}$
u_R	c_R	t_R	$+\frac{2}{3}$	0	0	$+\frac{4}{3}$
d_R	s_R	b_R	$-\frac{1}{3}$	0	0	$-\frac{2}{3}$
e_L	μ_L	τ_L	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-1
$\nu_{e,L}$	$\nu_{\mu,L}$	$\nu_{\tau,L}$	0	$\frac{1}{2}$	$+\frac{1}{2}$	-1
e_R	μ_R	τ_R	-1	0	0	-2

Table 2.1: Fermionic particle content of the SM with their corresponding $SU(2)_L \times U(1)_Y$ quantum numbers; the electric charge Q , the weak isospin T , the third component of the weak isospin T_3 and the weak hypercharge Y . Quarks and leptons are organized in left-handed doublets and right-handed singlets.

The Lagrangian that fulfills the conditions detailed above can be written as

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kinetic}}^{\text{leptons}} + \mathcal{L}_{\text{kinetic}}^{\text{quarks}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}, \quad (2.2)$$

where the term $\mathcal{L}_{\text{kinetic}}^{\text{leptons}}$ describes the interaction between the gauge bosons and the leptonic fields. It is written as

$$\mathcal{L}_{\text{kinetic}}^{\text{leptons}} = \sum_{j=1}^3 i \bar{\ell}_{L_j} \not{D}_{\ell} \ell_{L_j} + i \bar{e}_{R_j} \not{D}_e e_{R_j}, \quad (2.3)$$

with the covariant derivatives being defined as

$$D_{\ell\mu} = \partial_\mu + ig \frac{1}{2} \tau^a W_\mu^a + ig' \frac{1}{2} Y B_\mu \quad (2.4)$$

$$D_{\ell\mu} = \partial_\mu + ig' \frac{1}{2} Y B_\mu, \quad (2.5)$$

where τ^a are the Pauli matrices and W_μ^a/B_μ are the gauge boson fields. The covariant derivatives are constructed such that the kinematic terms are invariant under the local gauge transformations. The constants g and g' describe the strength of the coupling between the leptons and gauge fields. The quark kinetic term $\mathcal{L}_{\text{kinetic}}^{\text{quarks}}$ has a similar structure, but the covariant derivatives also include the terms governing the interaction with the gluon fields.

$$\mathcal{L}_{\text{kinetic}}^{\text{quark}} = \sum_{j=1}^3 i \bar{q}_{L_j} \not{D}_q q_{L_j} + i \bar{u}_{R_j} \not{D}_u u_{R_j} + i \bar{d}_{R_j} \not{D}_d d_{R_j}, \quad (2.6)$$

$$(2.7)$$

with the covariant derivatives being defined as

$$D_{q\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig \frac{1}{2} \tau^a W_\mu^a + ig' \frac{1}{2} Y B_\mu \quad (2.8)$$

$$D_{u\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig' \frac{1}{2} Y B_\mu \quad (2.9)$$

$$D_{d\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig' \frac{1}{2} Y B_\mu. \quad (2.10)$$

The Higgs part of the Lagrangian $\mathcal{L}_{\text{Higgs}}$ contains the kinetic term of the scalar field and the Higgs potential.

$$\mathcal{L}_{\text{Higgs}} = D_\mu \phi^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{1}{2} \lambda (\phi^\dagger \phi)^2 \quad (2.11)$$

with the covariant derivative being defined as

$$D_\mu = \partial_\mu + ig \frac{1}{2} \tau^a W_\mu^a + ig' \frac{1}{2} Y B_\mu. \quad (2.12)$$

Lastly, the Yukawa part of the Lagrangian $\mathcal{L}_{\text{Yukawa}}$ describes the interaction between the fermion fields and the scalar field.

$$\mathcal{L}_{\text{Yukawa}} = \sum_{j=1}^3 -\bar{Q}_{L_k} \Gamma_{jk}^d D_{R_k} \phi - \bar{Q}_{L_j} \Gamma_{jk}^u U_{R_j} \tilde{\phi} - \bar{L}_{L_j} \Gamma_{jk}^e E_{R_k} \tilde{\phi}, \quad (2.13)$$

with $\tilde{\phi} = i\tau^2 \phi^\dagger$.

The Yukawa matrices Γ_{ij}^d , Γ_{ij}^u , and Γ_{ij}^e are general complex 3x3 matrices which are diagonalized in the mass basis and provide the mass terms for the fermions. A necessary prerequisite for these mass terms is the spontaneous symmetry breaking that lifts the degeneracy of the minimum energy states of the Higgs potential. For values of $\mu^2 < 0$, the ground state is defined by $\phi^\dagger \phi = -\frac{\mu^2}{2\lambda}$, which is degenerate as it is invariant under the gauge transformation. To break the symmetry, the minimum can be chosen to be real valued, which means that

$$\phi = \sqrt{\frac{\mu^2}{2\lambda}} + \frac{h}{\sqrt{2}} \quad (2.14)$$

This choice is justified as a different minimum value can be realized via a gauge transformation that leaves the rest of the Lagrangian invariant. Inserting this into Eq. 2.11 of $\mathcal{L}_{\text{Higgs}}$ one arrives at:

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \partial_\mu h \partial^\mu h + \mu^2 h^2 - \frac{\lambda}{4} h^4 - h^3 \sqrt{\lambda} |\mu| + \frac{\mu^4}{4\lambda} \quad (2.15)$$

This term includes a mass term for the physical Higgs field h and a constant term that defines the minimum energy state of the theory. Its value is explicitly not gauge symmetric. When inserting the new expression of the scalar field into the Yukawa part of the Lagrangian, one obtains terms of the following form:

$$\mathcal{L}_{\text{Yukawa}} = \sum_{i=1}^3 -\bar{D}_{L_i} \Gamma_{ij}^d D_{R_j} \sqrt{\frac{\mu^2}{2\lambda}} - \bar{U}_{L_i} \Gamma_{ij}^d U_{R_j} \sqrt{\frac{\mu^2}{2\lambda}} + \dots \quad (2.16)$$

As the Yukawa matrices are general 3x3 matrices, one has to perform a change of basis to diagonalize them. This is done using the unitary rotation matrices V_{uL} and V_{dL} that switch between the mass and interaction basis. It is particularly noteworthy that the weak eigenstates and the mass eigenstates of the quarks do not coincide, which allows flavor changes in the quark sector. This is illustrated by the change of basis to the mass basis in the charged current part of $\mathcal{L}_{\text{kinetic}}^{\text{quarks}}$.

$$\mathcal{L}_{\text{kinetic}}^{W^\pm} = \frac{g}{2} \sum_{j,k,l=1}^3 -\bar{u}_{L_j}^m V_{uL,jk}^\dagger V_{dL,kl} \gamma^\mu W_\mu^+ d_{L_l}^m - \bar{d}_{L_j}^m V_{dL,jk}^\dagger V_{uL,lk} \gamma^\mu W_\mu^- u_{L_l}^m \quad (2.17)$$

Non-zero off-diagonal elements of the matrix $V_{uL,jk}^\dagger V_{dL,kl}$ allow for interactions between quarks of different flavor in a joint vertex with a $W^\pm$ boson. In flavor physics, this matrix is called the Cabibbo-Kobayashi-Maskawa matrix V_{CKM} [29, 30]. The elements of the CKM matrix are free parameters of the SM that are determined through experiments. As it is obtained through the multiplication of two unitary matrices, the CKM matrix itself is also unitary. This leaves four independent parameters, which motivates the Wolfenstein parameterization [34] that expands the CKM matrix in terms of the parameter $\lambda \approx 0.22$. The remaining parameters are $A \approx 0.83$, $\rho \approx 0.16$ and $\eta \approx 0.35$.

$$V_{\text{CKM}} = \begin{bmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + \frac{1}{2}A\lambda^4(1 - 2(\rho + i\eta)) & 1 - \frac{1}{2}A^2\lambda^4 \end{bmatrix} + \mathcal{O}(\lambda^5) \quad (2.18)$$

The specific values of the CKM matrix elements directly impact the size of all measured observables in flavor physics. As this thesis discusses the analysis of a $b \rightarrow s\ell^+\ell^-$ flavor changing neutral current, the decay amplitudes include a factor of $V_{ts}^* V_{tb}$. This suppresses the abundance of the decay significantly as the leading order of λ in V_{ts} is λ^2 . Additionally, the CKM matrix element V_{ts} includes a complex phase. In conjunction with a strong phase difference between two interfering amplitudes contributing to a decay process, this weak CKM phase can lead to CP -violation in the decay, which is a part of the measurements in this thesis.

2.3 Production of B -mesons

B -mesons are strongly bound hadrons of a $\bar{b}$ -quark and another lighter valence quark. As b -quarks do not occur in a stable bound form in nature, they need to be produced in high-energy particle interactions at a particle collider like the LHC to study them in a controlled environment. Multiple production mechanisms of b -quarks arise from the QCD interactions of the partons [20] (*i.e.* quarks and gluons from the colliding protons). The Feynman diagrams of leading order and next-to-leading order contributions are displayed in Fig. 2.2.

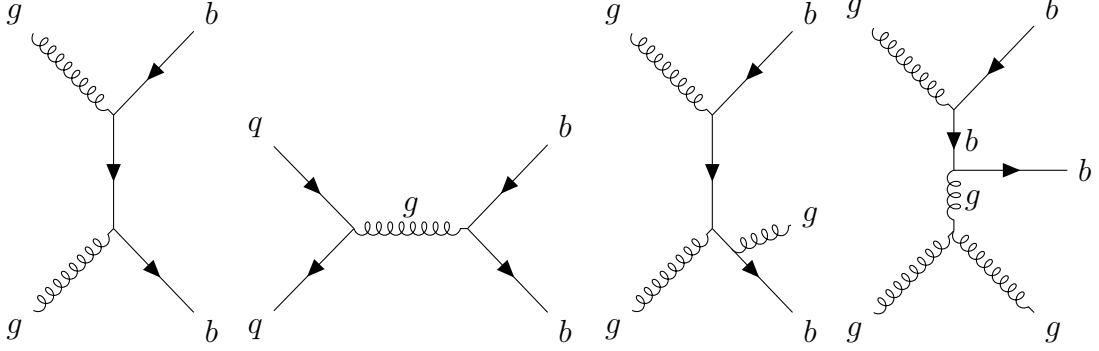


Figure 2.2: Feynman diagrams of relevant production mechanism of $b\bar{b}$ pairs at hadron colliders like the LHC. These diagrams are based on the discussion in Ref. [20].

As a result, b -quarks are always produced in $b\bar{b}$ pairs, which are heavily boosted into the forward beam direction. The quarks pairs are produced at the collision point of the LHCb detector with a cross-section of $\sigma_{pp \rightarrow b\bar{b}X} = 72.0 \pm 0.3 \pm 6.8 \mu\text{b}$ inside the LHCb acceptance at a center of mass energy of 7 TeV and $\sigma_{pp \rightarrow b\bar{b}X} = 144 \pm 1 \pm 21 \mu\text{b}$ at a center of mass energy of 13 TeV [35]. These cross-sections correspond to the collision conditions of the LHC Run 1 and Run 2, respectively. After the creation of the b -quark, the quark confinement enforces the hadronization into colorless B -hadrons. The most common of these are the B -mesons B^0 , B^+ , B_s^0 , B_c^+ , and the b -baryon Λ_b^0 . The relative abundance of these hadrons is given by the hadronization fractions. The joint hadronization fraction of the B^0 and B^+ is measured to be $f_{u/d} = 0.359 \pm 0.006 \pm 0.009$, while the fraction for the production of B_s^0 is found to be $f_s = 0.122 \pm 0.006$ and 0.259 ± 0.018 for the Λ_b^0 , respectively [36].

2.4 FCNC decay of B -mesons

This thesis in particular studies the decay of the B^0 meson into the final state $K^{*0}(\rightarrow K^+\pi^-)\mu^+\mu^-$ via a $b \rightarrow s\ell^+\ell^-$ FCNC transition, which is governed by the weak interaction. In order to choose an appropriate approach for the mathematical description of this decay, it first needs to be noted that this type of decay poses a multi-scale problem. The weak decay that governs the FCNC transition operates at the scale of the $W^\pm$ mass of about 80 GeV. The QCD processes governing the hadronic effects in the decay of the K^{*0} meson occur at the level of the Λ_{QCD} scale at about 0.2 GeV. The mass of the b -quark at 4.18 GeV/ c^2 forms another intermediate scale. From searches at the LHC, New Physics effects below the order of TeV have been largely excluded [37, 38]. This means that the New Physics scale is expected to be at least of $\mathcal{O}(1 \text{ TeV})$. In multiscale problems with processes that operate at very different energy scales, effective field theories (EFT) have proven to be a powerful tool [39]. The angular distributions of the final state particles are defined by the differential decay rate. In QFT, this rate is written as

$$\frac{d\Gamma}{\prod_f d^3p_f} = \frac{1}{(2\pi)^3} \frac{1}{2E_f} |\mathcal{M}(m_{B^0}, p_f)|^2 (2\pi)^4 \delta^{(4)}\left(p_{B^0} - \sum p_f\right), \quad (2.19)$$

with p_f and E_f being the momentum and energy of the final state. Furthermore, m_{B^0} and p_{B^0} denote the mass and momentum of the decaying initial state [33]. The matrix

element $\mathcal{M}$ is defined as

$$\mathcal{M} = \langle f | \mathcal{H}_{\text{eff}} | i \rangle, \quad (2.20)$$

with $|i\rangle$ being the initial state of the decay and $\langle f|$ the final state. The effective Hamiltonian operator $\mathcal{H}_{\text{eff}}$ encodes the physics of the model. In effective field theories, the effects from different scales are separated using an operator product expansion (OPE) [40]. Similar to the Fermi theory, the OPE integrates out the heavy degrees of freedom (*i.e.* the $W^\pm$ and the t) and treats the substructure of the interaction as point-like. The weak interaction vertices are replaced by an effective coupling constant. In this operator product expansion, the effective Hamiltonian is written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i f_{CKM}^i \mathcal{C}_i(\mu_b) \mathcal{O}_i(\mu_b), \quad (2.21)$$

where G_F is the Fermi constant, f_{CKM}^i is the CKM factor of the decay, the Wilson coefficients $\mathcal{C}_i$ are the effective couplings of the heavy degrees of freedom describing the physics above the chosen scale of the b -quark μ_b , and finally the operators $\mathcal{O}_i$ that encode the hadronic effects of the decay below the scale of μ_b . The sum runs over the different allowed $b \rightarrow s$ diagrams, which are each described with a separate effective coupling.

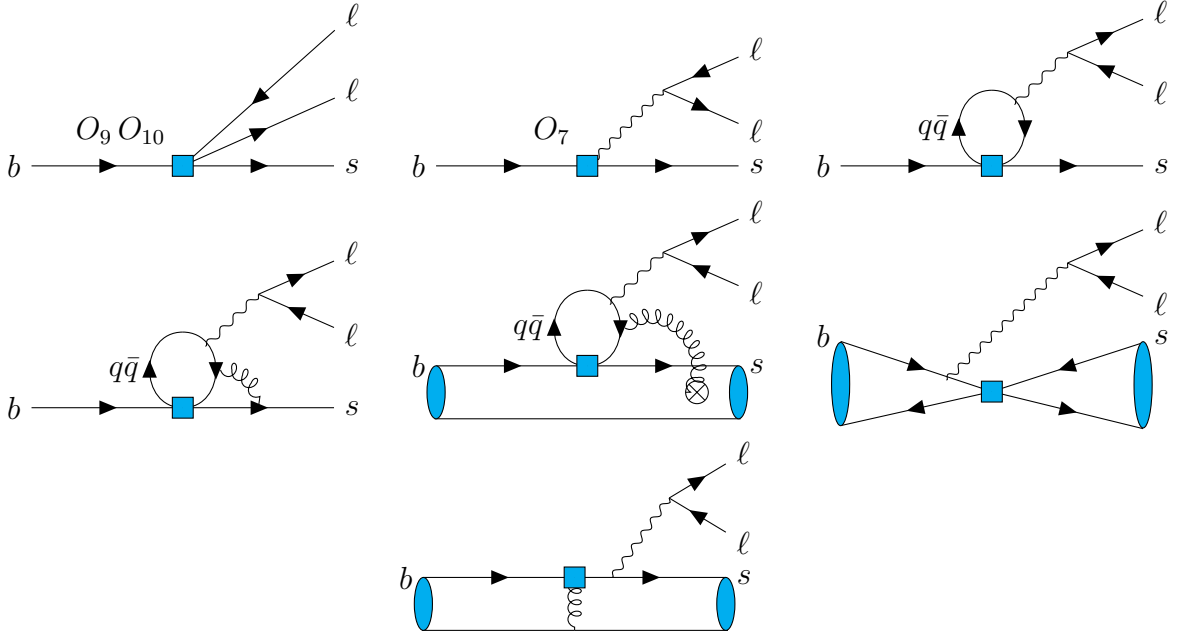


Figure 2.3: Diagrams of the local and non-local operators contributing to the $b \rightarrow s \ell^+ \ell^-$ decay amplitude. These diagrams are based on Ref. [21].

Figure 2.3 shows the Feynman diagrams' simplified representation of $b \rightarrow s \ell^+ \ell^-$ transitions in EFT. The EFT operators relevant for $b \rightarrow s \ell^+ \ell^-$ decays are displayed in Tab. 2.2 along with the corresponding Wilson coefficients. In precision measurements like the one presented in this thesis, the values of Wilson coefficients are of particular interest as New Physics effects are expected to contribute at a scale far exceeding μ_b , which means that such an effect would manifest itself as a modification of the effective coupling and cause a deviation from the value expected in the SM.

Table 2.2: EFT operators and Wilson coefficients relevant for $b \rightarrow s \ell^+ \ell^-$ decays [33].

Effective couplings in $b \rightarrow s \ell^+ \ell^-$ transitions		
Wilson coefficient	Operator	
γ -penguin	$C_7^{(\ell)}$	$\frac{e}{g^2} m_b (\bar{s} \sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}$
ew. penguin	$C_9^{(\ell)}$	$\frac{e^2}{g^2} (\bar{s} \gamma_\mu P_{L(R)} b) (\bar{\mu} \gamma^\mu \mu)$
	$C_{10}^{(\ell)}$	$\frac{e^2}{g^2} (\bar{s} \gamma_\mu P_{L(R)} b) (\bar{\mu} \gamma^\mu \gamma_5 \mu)$
scalar	$C_S^{(\ell)}$	$\frac{e^2}{16\pi^2} m_b (\bar{s} P_{R(L)} b) (\bar{\mu} \mu)$
pseudoscalar	$C_P^{(\ell)}$	$\frac{e^2}{16\pi^2} m_b (\bar{s} P_{R(L)} b) (\bar{\mu} \gamma_5 \mu)$

2.5 The rare decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

The decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ is a rare B -meson decay that is mediated via a $b \rightarrow s \ell \ell$ flavor-changing neutral current. This kind of transition is forbidden in the SM at tree-level due to the fact that flavor change can only happen via the emission of a charged $W^\pm$ boson. Therefore, this kind of decay is only allowed at the loop-level, where a virtual $W^\pm$ is emitted in the loop together with an up-type quark as depicted in Fig. 2.4. The decay rate of rare decays in the SM is strongly suppressed by the GIM mechanism [28]. Generally, there are three allowed loop-level Feynman diagrams for the $b \rightarrow s \ell \ell$ transition. One for every up-type quark that can be exchanged in the loop. The total decay rate is the sum of these diagrams. However, due to the unitarity of the CKM matrix, the different summed-up diagrams nearly cancel each other completely. The quark mass dependency of the total amplitude prevents a total cancellation, which would only happen if all up-type quarks had the same mass [33]. This also results in the $b \rightarrow s \ell \ell$ transition being dominated by the heavy top quark contribution. An additional suppression enters via the loop factor $g^2/16\pi^2$. The small expected branching fraction of FCNCs makes their study predestined for the search

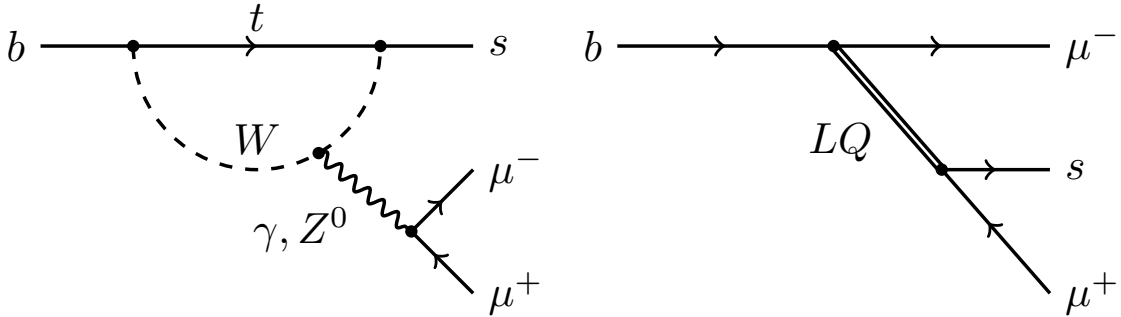


Figure 2.4: Feynman diagram of a $b \rightarrow s \ell^+ \ell^-$ flavor-changing neutral current transition in the SM involving two muons in the final state (left) and a hypothetical NP tree-level FCNC transition including a leptoquark (right).

for New Physics. Transitions via hypothetical particles like a leptoquark might be allowed at tree-level and therefore contribute significantly to the differential decay rate. An example of such a NP contribution can be found in Fig. 2.4 on the right side. Since NP contributions can appear as virtual particles, the probed NP scale is not limited by the initial energy of the b -quarks. In this way, the NP scale can be probed up to $\mathcal{O}(100 \text{ TeV})$ [41] depending on the nature of the NP coupling.

2.5.1 Angular basis

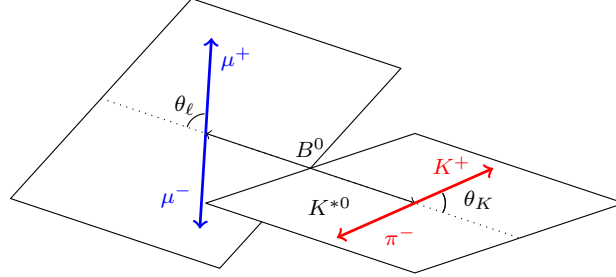
The B^0 meson decays into four final-state particles as the K^{*0} further decays into a K^+ and π^- . To describe the most general differential decay rate, a parameter basis must be found that fully parameterizes the kinematics and topology of the decay in the rest frame of the B^0 . The differential decay rate is constructed as a function of this complete parameter basis. The following section describes the angular basis employed by LHCb analysis to parameterize the relation between the angular distribution (*i.e.* the differential decay rate) and the Wilson coefficients introduced in Sec.2.4.

The kinematics of a four-body decay like $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ is fully described by the four-momenta of the final state particles [42]. This corresponds to 16 degrees of freedom. These are not independent and are constrained by several relations between the momenta. Firstly, the on-shell condition for each final state particle relates its energy to its momentum via the invariant mass. The four-momentum conservation introduces another 4 relations, which constrain the sum of final state four-momenta to the four-momentum of the mother particle. Lastly, the physics distributions and parameters are invariant under arbitrary rotations of the B^0 momentum direction. This allows the elimination of another three degrees of freedom, which correspond to three Euler angles. Finally, five independent degrees of freedom remain [42]. It is convenient to choose them as the di-muon invariant mass squared q^2 , the $K^+ \pi^-$ invariant mass $m_{K\pi}$, and the three decay angles $\cos \theta_\ell$, $\cos \theta_K$, and ϕ . These angles are defined as follows:

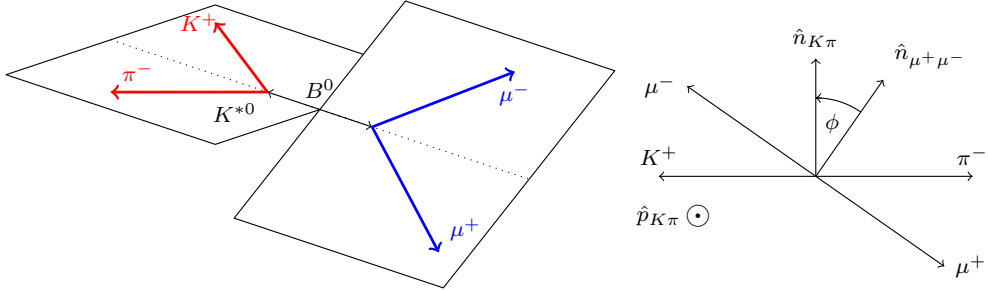
- ϕ is defined as the angle between the two normal vectors of the decay planes spanned by the $\mu^+ \mu^-$ and $K^+ \pi^-$ momentum vector pairs in the restframe of the B^0 .
- $\cos \theta_\ell$ is defined as the angle between the positively charged muon momentum vector in the di-muon rest frame and the flight direction of the di-muon system in the B^0 rest frame.
- $\cos \theta_K$ is defined as the angle between the K^+ momentum in the $K^+ \pi^-$ rest frame and the flight direction of the $K^+ \pi^-$ system in the B^0 rest frame.

Figure 2.5 shows a graphical representation of the decay angles' definition given above. The decay angles expressed in terms of the normalized momenta using the LHCb convention [24, 43, 44] are given in Eq. 2.22 to Eq. 2.25. They are defined as

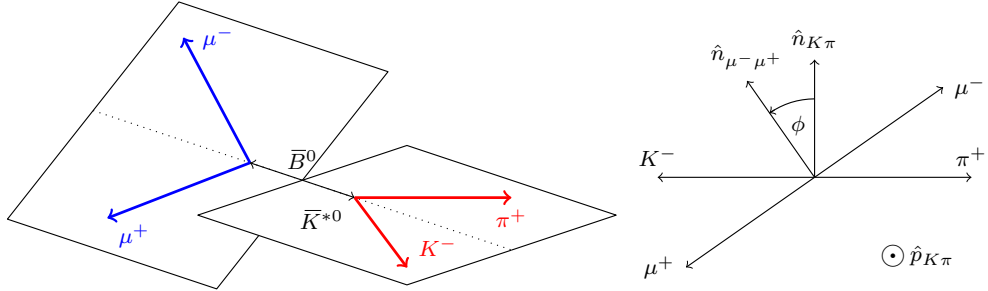
$$\begin{aligned} \cos \theta_\ell &= \left(\hat{p}_{\mu^+}^{(\mu^+ \mu^-)} \right) \cdot \left(\hat{p}_{\mu^+ \mu^-}^{(B^0)} \right) = \left(\hat{p}_{\mu^+}^{(\mu^+ \mu^-)} \right) \cdot \left(-\hat{p}_{B^0}^{(\mu^+ \mu^-)} \right) , \\ \cos \theta_K &= \left(\hat{p}_{K^+}^{(K^{*0})} \right) \cdot \left(\hat{p}_{K^+ \pi^-}^{(B^0)} \right) = \left(\hat{p}_{K^+}^{(K^{*0})} \right) \cdot \left(-\hat{p}_{B^0}^{(K^{*0})} \right) \end{aligned} \quad (2.22)$$



(a) θ_K and θ_ℓ definitions for the B^0 decay



(b) ϕ definition for the B^0 decay



(c) ϕ definition for the $\bar{B}^0$ decay

Figure 2.5: Graphical representation of the decay angles' definition for $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ and $\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$ decays [45]. $\hat{p}_x$ describes the normalized momentum direction of the particle system x, while $\hat{n}_x$ represents the normal vectors of the decay plane spanned by the two final state particles in system x.

for the B^0 and

$$\begin{aligned}\cos \theta_\ell &= \left(\hat{p}_{\mu^-}^{(\mu^+ \mu^-)} \right) \cdot \left(\hat{p}_{\mu^+ \mu^-}^{(\bar{B}^0)} \right) = \left(\hat{p}_{\mu^-}^{(\mu^+ \mu^-)} \right) \cdot \left(-\hat{p}_{\bar{B}^0}^{(\mu^+ \mu^-)} \right) , \\ \cos \theta_K &= \left(\hat{p}_{K^-}^{(\bar{K}^{*0})} \right) \cdot \left(\hat{p}_{K^{*0}}^{(\bar{B}^0)} \right) = \left(\hat{p}_{K^-}^{(\bar{K}^{*0})} \right) \cdot \left(-\hat{p}_{\bar{B}^0}^{(\bar{K}^{*0})} \right)\end{aligned}\quad (2.23)$$

for the $\bar{B}^0$ decay. The definition of the angle ϕ is given by

$$\begin{aligned}\cos \phi &= \left(\hat{p}_{\mu^+}^{(B^0)} \times \hat{p}_{\mu^-}^{(B^0)} \right) \cdot \left(\hat{p}_{K^+}^{(B^0)} \times \hat{p}_{\pi^-}^{(B^0)} \right) , \\ \sin \phi &= \left[\left(\hat{p}_{\mu^+}^{(B^0)} \times \hat{p}_{\mu^-}^{(B^0)} \right) \times \left(\hat{p}_{K^+}^{(B^0)} \times \hat{p}_{\pi^-}^{(B^0)} \right) \right] \cdot \hat{p}_{K^{*0}}^{(B^0)}\end{aligned}\quad (2.24)$$

for the B^0 and

$$\begin{aligned}\cos \phi &= \left(\hat{p}_{\mu^-}^{(\bar{B}^0)} \times \hat{p}_{\mu^+}^{(\bar{B}^0)} \right) \cdot \left(\hat{p}_{K^-}^{(\bar{B}^0)} \times \hat{p}_{\pi^+}^{(\bar{B}^0)} \right) , \\ \sin \phi &= - \left[\left(\hat{p}_{\mu^-}^{(\bar{B}^0)} \times \hat{p}_{\mu^+}^{(\bar{B}^0)} \right) \times \left(\hat{p}_{K^-}^{(\bar{B}^0)} \times \hat{p}_{\pi^+}^{(\bar{B}^0)} \right) \right] \cdot \hat{p}_{\bar{K}^{*0}}^{(\bar{B}^0)}\end{aligned}\quad (2.25)$$

for the $\bar{B}^0$ decay. The parameter $\hat{p}_x^y$ represents the normalized three-momenta of the particle x in the rest frame of the particle system y .

2.5.2 Matrix element

As described in Eq. 2.19, the differential decay rate of the process is directly proportional to the matrix element $\mathcal{M}$.

$$\mathcal{M}_{\lambda_1, \lambda_2} = \langle \bar{K}^{*0} (\rightarrow K^+ \pi^-) \mu_1(\lambda_1) \mu_2(\lambda_2) | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle, \quad (2.26)$$

where $\lambda_{1/2}$ is the helicity of the final state muons. This amplitude can be parameterized using a generalization of the helicity formalism used to determine the amplitude of a two-body decay. The following summary of this formalism is discussed in detail in Ref. [46].

At leading order, it can be assumed that the hadronic parts $\mathcal{H}_i$ and leptonic parts $\mathcal{L}_i$ of the process are factorizing up to small non-factorizing corrections $\delta_{\text{non-factorizing}}$. The matrix element can thus be written as

$$\mathcal{M}_{\lambda_1, \lambda_2} = \sum_i \mathcal{C}_i \cdot \underbrace{\langle \bar{K}^{*0} (\rightarrow K^+ \pi^-) | \mathcal{O}_i^{K^{*0} \rightarrow K\pi} | \bar{B}^0 \rangle}_{\mathcal{H}_i} \times \underbrace{\langle \mu_{\lambda_1} \mu_{\lambda_2} | \mathcal{O}_i^{\mu\mu} | 0 \rangle}_{\mathcal{L}_i} + \delta_{\text{non-factorizing}} \quad (2.27)$$

The full decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ can be described as a sequence of three two-body decays. The fact that B^0 is a spin 0 particle implies that the helicities of the dilepton γ and $K\pi$ system must be equal $\lambda \equiv \lambda_\gamma = \lambda_{K\pi}$ due to angular momentum conservation. As K^+ and π^- are both scalar particles, the helicity amplitude governing the decay $K^{*0} \rightarrow K^+ \pi^-$ is simply given by a scalar factor. The full decay amplitude is thus reduced to

$$\mathcal{M}(\Omega_{\ell\ell}, \Omega_{K\pi} | \lambda_K, \lambda_\pi, \lambda_1, \lambda_2) \propto \sum_{\lambda=-J_{K\pi}}^{J_{K\pi}} \bar{D}_{\lambda,0}(\Omega_{K\pi}) \bar{D}_{\lambda_\gamma, \lambda_{\ell_1}-\lambda_{\ell_2}}(\Omega_{\ell\ell}) \mathcal{H}_\lambda \mathcal{L}_{\lambda_{\ell_1}, \lambda_{\ell_1}}, \quad (2.28)$$

where $D_{\alpha,\beta}$ denotes the Wigner D-functions, $\Omega_{K\pi} = (0, \theta_K, 0)$ and $\Omega_{\ell\ell} = (\phi, \theta_\ell, -\phi)$. The helicity amplitudes are combined into the expression $\mathcal{A}_{\lambda,\lambda_{\ell_1},\lambda_{\ell_2}}^{J_\gamma} = \mathcal{H}_\lambda \mathcal{L}_{\lambda_{\ell_1},\lambda_{\ell_2}}$ in the following. $\mathcal{A}_{\lambda,\lambda_{\ell_1},\lambda_{\ell_2}}^{J_\gamma}$ contains the sum over the contributions from different pairs of local operators and Wilson coefficients. It is thus written as

$$\mathcal{A}_{0,\lambda_{\ell_1},\lambda_{\ell_2}}^0 = \mathcal{H}^S \mathcal{L}_{\lambda_1,\lambda_2}^S + \mathcal{H}^P \mathcal{L}_{\lambda_1,\lambda_2}^P \quad (2.29)$$

$$\mathcal{A}_{\lambda,\lambda_{\ell_1},\lambda_{\ell_2}}^1 = -\mathcal{H}_\lambda^V \mathcal{L}_{\lambda_1,\lambda_2}^V - \mathcal{H}_\lambda^A \mathcal{L}_{\lambda_1,\lambda_2}^A + \mathcal{H}_\lambda^T \mathcal{L}_{\lambda_1,\lambda_2}^T - 2\mathcal{H}_\lambda^{T_t} \mathcal{L}_{\lambda_1,\lambda_2}^{T_t}, \quad (2.30)$$

where the type of the contributing Feynman diagram is denoted in the superscripts. The amplitudes above are valid for arbitrary $K^+\pi^-$ resonances with spin $J_{K\pi}$. For the analysis of the rare decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$, only the S-wave ($J_{K\pi} = 0$) and P-wave ($J_{K\pi} = 1$) configurations are of significant importance. The total decay rate is then proportional to the absolute value squared of the decay amplitudes summed over the allowed spin configurations:

$$\begin{aligned} \frac{d^4\Gamma}{dq^2 d\cos\theta_K d\cos\theta_\ell d\phi dm_{K\pi}} &\propto \sum_{\lambda_{\ell_1}\lambda_{\ell_2}} \left| \sum_{J_{K\pi}=0}^1 \sum_{J_\gamma=0}^1 \mathcal{A}^{J_{K\pi}}(\Omega_{\ell\ell}, \Omega_{K\pi} | \lambda_K, \lambda_\pi, \lambda_{\ell_1}, \lambda_{\ell_2}) \right|^2 \\ &= \sum_{\lambda_{\ell_1},\lambda_{\ell_2}=-1/2}^{1/2} \sum_{J_{K\pi}=0}^1 \sum_{J_\gamma=0}^1 \sum_{\lambda=-\min(J_\gamma,J_{K\pi})}^{\min(J_\gamma,J_{K\pi})} \sum_{J'_{K\pi}=0}^1 \sum_{J'_\gamma=0}^1 \sum_{\lambda'=-\min(J'_\gamma,J'_{K\pi})}^{\min(J'_\gamma,J'_{K\pi})} \times \frac{\sqrt{2J_{K\pi}+1}}{4\pi} \\ &\quad \mathcal{A}_{\lambda,\lambda_{\ell_1},\lambda_{\ell_2}}^{J_\gamma,J_{K\pi}} \bar{\mathcal{A}}_{\lambda',\lambda_{\ell_1},\lambda_{\ell_2}}^{J'_\gamma,J'_{K\pi}} D_{\lambda,0}^{J_{K\pi}}(\Omega_{K\pi}) \bar{D}_{\lambda',0}^{J'_{K\pi}}(\Omega_{K\pi}) D_{\lambda,\lambda_{\ell_2}-\lambda_{\ell_1}}^{J_{K\pi}}(\Omega_\ell) \bar{D}_{\lambda',\lambda_{\ell_2}-\lambda_{\ell_1}}^{J'_{K\pi}}(\Omega_\ell) \end{aligned} \quad (2.31)$$

The complexity of the expression above can be reduced by applying the Clebsch-Gordan series, which reduces products of two Wigner-D functions into sums of single Wigner-D functions. These functions are explicitly given by

$$D_{m,0}^l(\phi, \theta, -\phi) = \sqrt{\frac{4\pi}{2l+1}} \bar{Y}_{lm}(\theta, \phi), \quad (2.32)$$

where $\bar{Y}_{lm}(\theta, \phi)$ is a spherical harmonic function. The explicit definitions of the hadronic $\mathcal{H}$ and leptonic $\mathcal{L}$ helicity amplitudes are given by

$$\mathcal{H}_0^V = \frac{4im_{B^0}m_{K^*0}((C_9 - C'_9)(m_B + m_{K^*0})A_{12} + m_b(C_7 - C'_7)T_{23})}{\sqrt{q^2}(m_B + m_{K^*0})} \quad (2.33)$$

$$\mathcal{H}_0^A = \frac{4im_{K^*0}m_{B^0}}{\sqrt{q^2}}(C_{10} - C'_{10})A_{12} \quad (2.34)$$

$$\begin{aligned} \mathcal{H}_\pm^V &= \frac{i}{2(m_{B^0} + m_{K^*0})} \left(\pm(C_9 + C'_9)\sqrt{\lambda_{B^0}}V - (m_{B^0} + m_{K^*0})^2(C_9 - C'_9)A_1 \right) \\ &\quad + \frac{im_b}{q^2} \left(\pm(C_7 + C'_7)\sqrt{\lambda_{B^0}}T_1 - (C_7 - C'_7)(m_{B^0}^2 - m_{K^*0}^2)T_2 \right) \end{aligned} \quad (2.35)$$

$$\mathcal{H}_\pm^A = \frac{i}{2(m_{B^0} + m_{K^*0})} \left(\pm(C_{10} - C'_{10})\sqrt{\lambda_{B^0}}V - (m_{B^0} + m_{K^*0})^2(C_{10} - C'_{10})A_1 \right) \quad (2.36)$$

$$\mathcal{H}^S = \frac{i\sqrt{\lambda_{B^0}}}{2} \frac{C_S - C'_S}{m_b + m_s} A_0 \quad (2.37)$$

$$\mathcal{H}^P = \frac{i\sqrt{\lambda_{B^0}}}{2} \left(\frac{C_P - C'_P}{m_b + m_s} + \frac{2m_\mu}{q^2} \frac{C_A - C'_A}{q^2} \right) A_0 \quad (2.38)$$

$$\mathcal{H}_0^T = \frac{2\sqrt{2}m_{B^0}m_{K^{*0}}}{m_{B^0} + m_{K^{*0}}} (\mathcal{C}_T + \mathcal{C}'_T) T_{23} \quad (2.39)$$

$$\mathcal{H}_0^{T_i} = \frac{2m_{B^0}m_{K^{*0}}}{m_{B^0} + m_{K^{*0}}} (\mathcal{C}_T + \mathcal{C}'_T) T_{23} \quad (2.40)$$

$$\mathcal{H}_\pm^T = \frac{1}{\sqrt{2q^2}} (\pm(\mathcal{C}_T + \mathcal{C}'_T)\sqrt{\lambda_B}T_1 - (\mathcal{C}_T - \mathcal{C}'_T)(m_{B^0}^2 - m_{K^{*0}}^2)T_2)T_{23} \quad (2.41)$$

$$\mathcal{H}_\pm^{T_i} = \frac{1}{2\sqrt{q^2}} (\pm(\mathcal{C}_T + \mathcal{C}'_T)\sqrt{\lambda_B}T_1 - (\mathcal{C}_T - \mathcal{C}'_T)(m_{B^0}^2 - m_{K^{*0}}^2)T_2)T_{23} \quad (2.42)$$

and

$$\mathcal{L}^V(\lambda_1, \lambda_2) = \begin{pmatrix} 2m_\ell & -\sqrt{2q^2} \\ -\sqrt{2q^2} & 2m_\ell \end{pmatrix} \quad (2.43)$$

$$\mathcal{L}^A(\lambda_1, \lambda_2) = \begin{pmatrix} 0 & \sqrt{2q^2}\beta_\ell \\ -\sqrt{2q^2}\beta_\ell & 0 \end{pmatrix} \quad (2.44)$$

$$\mathcal{L}^S(\lambda_1, \lambda_2) = \begin{pmatrix} \sqrt{2q^2}\beta_\ell & 0 \\ 0 & \sqrt{2q^2}\beta_\ell \end{pmatrix} \quad (2.45)$$

$$\mathcal{L}^P(\lambda_1, \lambda_2) = \begin{pmatrix} \sqrt{q^2} & 0 \\ 0 & -\sqrt{q^2} \end{pmatrix} \quad (2.46)$$

$$\mathcal{L}^T(\lambda_1, \lambda_2) = \begin{pmatrix} -i\sqrt{2q^2}\beta_\ell & 0 \\ 0 & i\sqrt{2q^2}\beta_\ell \end{pmatrix} \quad (2.47)$$

$$\mathcal{L}^{T_i}(\lambda_1, \lambda_2) = \begin{pmatrix} i\sqrt{q^2} & -2i\sqrt{2}m_\mu \\ -2i\sqrt{2}m_\mu & i\sqrt{q^2} \end{pmatrix} \quad (2.48)$$

Here λ_{B^0} is the Källen function of the B^0 decay. β_ℓ is defined as $\beta_\ell = \sqrt{1 - \frac{m_\ell^2}{q^2}}$. $A_0, A_1, A_2, T_1, T_2, T_3$ and V are hadronic form factors, which encode the QCD interactions of the $B^0 \rightarrow K^{*0}$ transition. The definitions of A_{12} and T_{23} are given by

$$A_{12} = \frac{(m_{B^0} + m_{K^{*0}})^2(m_{B^0}^2 - m_{K^{*0}}^2 - q^2)A_1 - \lambda_{B^0}A_2}{16m_{B^0}m_{K^{*0}}^2(m_{B^0} + m_{K^{*0}})} \quad (2.49)$$

$$T_{23} = \frac{(m_{B^0}^2 + m_{K^{*0}}^2)(m_{B^0}^2 + 3m_{K^{*0}}^2 - q^2)T_2 - \lambda_{B^0}T_3}{8m_{B^0}m_{K^{*0}}^2(m_{B^0} - m_{K^{*0}})} \quad (2.50)$$

In the literature, the differential decay rate is often expressed in terms of transversity amplitudes instead of helicity amplitudes. Specifically, this is the case for LHCb angular analysis publications, which is why the following section uses transversity amplitudes to be in line with LHCb notation. Helicity amplitudes can be transformed

into transversity amplitudes using the relations

$$A_0^{L/R} \equiv \mathcal{H}_0^V \mp \mathcal{H}_0^A \quad (2.51)$$

$$A_{\parallel}^{L/R} \equiv \frac{1}{\sqrt{2}} ((\mathcal{H}_+^V \mp \mathcal{H}_+^A) + (\mathcal{H}_-^V \mp \mathcal{H}_-^A)) \quad (2.52)$$

$$A_{\perp}^{L/R} \equiv \frac{1}{\sqrt{2}} ((\mathcal{H}_+^V \mp \mathcal{H}_+^A) - (\mathcal{H}_-^V \mp \mathcal{H}_-^A)) \quad (2.53)$$

2.5.3 Differential decay rate

In the following, the structure of the differential decay rate, its expression in terms of the angular coefficients $\vec{I}_i$, and the $m_{K\pi}$ dependence of the single terms are discussed. The full differential decay rate is given by

$$\frac{d^5 \vec{\Gamma}}{dq^2 dm_{K\pi} d\cos\theta_l d\cos\theta_K d\phi} = \vec{\Gamma}_P + \vec{\Gamma}_S. \quad (2.54)$$

The first term, $\vec{\Gamma}_P$, exclusively describes the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ contribution that decays via the spin-1 K^{*0} resonance, also denoted as P-wave in the following [42]. The spin-0 contribution from $K^{*0}(1430)$ and $\kappa(800)$, also called S-wave, and the interference terms between P- and S-wave are described by the second term $\vec{\Gamma}_S$ [47]. Both $\vec{\Gamma}_P$ and $\vec{\Gamma}_S$ are linear combinations of trigonometric functions and the angular coefficients $\vec{I}_i$. The P- and S-wave terms for the decay $\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$ are explicitly given by

$$\begin{aligned} \Gamma_P = \frac{9}{32\pi} [& I_1^s \sin^2 \theta_K + I_1^c \cos^2 \theta_K + (I_2^s \sin^2 \theta_K + I_2^c \cos^2 \theta_K) \cos 2\theta_\ell \\ & + I_3' \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + I_4' \sin 2\theta_K \sin 2\theta_\ell \cos \phi \\ & + I_5' \sin 2\theta_K \sin \theta_\ell \cos \phi + (I_6^s \sin^2 \theta_K + I_6^c \cos^2 \theta_K) \cos \theta_\ell \\ & + I_7' \sin 2\theta_K \sin \theta_\ell \sin \phi + I_8' \sin 2\theta_K \sin 2\theta_\ell \sin \phi \\ & + I_9' \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi] \end{aligned} \quad (2.55)$$

and

$$\begin{aligned} \Gamma_S = \frac{3}{16\pi} [& I_{10}' + I_{11}' \cos \theta_K + I_{12}' \cos 2\theta_\ell + I_{13}' \cos \theta_K \cos 2\theta_\ell \\ & + I_{14}' \sin \theta_K \sin 2\theta_\ell \cos \phi + I_{15}' \sin \theta_K \sin \theta_\ell \cos \phi \\ & + I_{16}' \sin \theta_K \sin \theta_\ell \sin \phi + I_{17}' \sin \theta_K \sin 2\theta_\ell \sin \phi], \end{aligned} \quad (2.56)$$

respectively. The trigonometric terms are functions of the decay angles $\cos \theta_K$, $\cos \theta_\ell$, and ϕ . The specific terms arise from the spin summation of the squared matrix element. The q^2 and $m_{K\pi}$ dependence is included in the angular coefficients $\vec{I}_i$. Analogously, the $\bar{\Gamma}_P$ and $\bar{\Gamma}_S$ describing the CP -conjugate decay mode $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ can be written as

$$\begin{aligned} \bar{\Gamma}_P = \frac{9}{32\pi} [& \bar{I}_1^s \sin^2 \theta_K + \bar{I}_1^c \cos^2 \theta_K + (\bar{I}_2^s \sin^2 \theta_K + \bar{I}_2^c \cos^2 \theta_K) \cos 2\theta_\ell \\ & + \bar{I}_3' \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + \bar{I}_4' \sin 2\theta_K \sin 2\theta_\ell \cos \phi \\ & + \bar{I}_5' \sin 2\theta_K \sin \theta_\ell \cos \phi + (\bar{I}_6^s \sin^2 \theta_K + \bar{I}_6^c \cos^2 \theta_K) \cos \theta_\ell \\ & + \bar{I}_7' \sin 2\theta_K \sin \theta_\ell \sin \phi + \bar{I}_8' \sin 2\theta_K \sin 2\theta_\ell \sin \phi \\ & + \bar{I}_9' \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi] \end{aligned} \quad (2.57)$$

and

$$\begin{aligned}\bar{\Gamma}_S = \frac{3}{16\pi} & \left[\bar{I}'_{10} + \bar{I}'_{11} \cos \theta_K + \bar{I}'_{12} \cos 2\theta_\ell + \bar{I}'_{13} \cos \theta_K \cos 2\theta_\ell \right. \\ & + \bar{I}'_{14} \sin \theta_K \sin 2\theta_\ell \cos \phi + \bar{I}'_{15} \sin \theta_K \sin \theta_\ell \cos \phi \\ & \left. + \bar{I}'_{16} \sin \theta_K \sin \theta_\ell \sin \phi + \bar{I}'_{17} \sin \theta_K \sin 2\theta_\ell \sin \phi \right].\end{aligned}\quad (2.58)$$

Table 2.3 displays the angular coefficients I' and $\bar{I}'$ in terms of the transversity amplitudes $A_{\parallel}^{L/R}$, $A_{\perp}^{L/R}$, $A_0^{L/R}$, $A_S^{L/R}$, A_{scalar} and functions $\mathcal{BW}_P$, $\mathcal{BW}_S$. These functions describe the $m_{K\pi}$ dependency of the decay rate and yield the biggest separation power between P- and S-wave contributions. The P-wave is described by a relativistic Breit-Wigner function [48], while the S-wave needs to be described by a more complex phenomenological model, the LASS function [49]. As the angular observables are exclusively bilinear combinations of the transversity amplitudes, the $m_{K\pi}$ dependency can be factored out and separated from the angular coefficients themselves. This allows them to be exclusively expressed as a function of q^2 .

When factoring out the Breit-Wigner function from pure P-wave amplitudes, the angular coefficients I'_i used in the equations above transform into the angular observables I_i for the P-wave as

$$I'_i(q^2, m_{K\pi}) = I_i(q^2) \times |\mathcal{BW}_P(m_{K\pi})|^2. \quad (2.59)$$

The terms for the S-wave observables are analogous. The resulting P-wave terms of the differential decay rate are

$$\begin{aligned}\Gamma_P = \frac{9}{32\pi} & \left[I_1^s \sin^2 \theta_K + I_1^c \cos^2 \theta_K + (I_2^s \sin^2 \theta_K + I_2^c \cos^2 \theta_K) \cos 2\theta_\ell \right. \\ & + I_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi \\ & + I_5 \sin 2\theta_K \sin \theta_\ell \cos \phi + (I_6^s \sin^2 \theta_K + I_6^c \cos^2 \theta_K) \cos \theta_\ell \\ & + I_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + I_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi \\ & \left. + I_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right] \times |\mathcal{BW}_P(m_{K\pi})|^2\end{aligned}\quad (2.60)$$

for $\bar{B}^0$ and

$$\begin{aligned}\bar{\Gamma}_P = \frac{9}{32\pi} & \left[\bar{I}_1^s \sin^2 \theta_K + \bar{I}_1^c \cos^2 \theta_K + (\bar{I}_2^s \sin^2 \theta_K + \bar{I}_2^c \cos^2 \theta_K) \cos 2\theta_\ell \right. \\ & + \bar{I}_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + \bar{I}_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi \\ & + \bar{I}_5 \sin 2\theta_K \sin \theta_\ell \cos \phi + (\bar{I}_6^s \sin^2 \theta_K + \bar{I}_6^c \cos^2 \theta_K) \cos \theta_\ell \\ & + \bar{I}_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + \bar{I}_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi \\ & \left. + \bar{I}_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right] \times |\mathcal{BW}_P(m_{K\pi})|^2,\end{aligned}\quad (2.61)$$

for B^0 . In the case of the interference terms, the expressions are more complicated as the bilinear combinations of P- and S-wave Breit-Wigner functions are not, by definition, real valued and thus do not simply factorize out of the imaginary and real part operations in the definitions of the angular observables. This causes the interference terms to split up as shown in Eq. 2.62.

i	I'_i	f_i
1s	$\left(\frac{(2+\beta_\mu^2)}{4}(A_\perp^L ^2 + A_\parallel^L ^2 + A_\perp^R ^2 + A_\parallel^R ^2) + \frac{4m_\mu^2}{q^2} \text{Re}[A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}]\right) \times \mathcal{BW}_P ^2$	$\sin^2 \theta_K$
1c	$\left((A_0^L ^2 + A_0^R ^2) + \frac{4m_\mu^2}{q^2}(A_t ^2 + 2 \text{Re}[A_0^L A_0^{R*}]) + \beta_\mu^2 A_{\text{scalar}} ^2\right) \times \mathcal{BW}_P ^2$	$\cos^2 \theta_K$
2s	$\frac{\beta_\mu^2}{4}(A_\perp^L ^2 + A_\parallel^L ^2 + A_\perp^R ^2 + A_\parallel^R ^2) \times \mathcal{BW}_P ^2$	$\sin^2 \theta_K \cos 2\theta_\ell$
2c	$-\beta_\mu^2(A_0^L ^2 + A_0^R ^2) \times \mathcal{BW}_P ^2$	$\cos^2 \theta_K \cos 2\theta_\ell$
3	$\frac{1}{2}\beta_\mu^2(A_\perp^L ^2 - A_\parallel^L ^2 + A_\perp^R ^2 - A_\parallel^R ^2) \times \mathcal{BW}_P ^2$	$\sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi$
4	$\frac{1}{\sqrt{2}}\beta_\mu^2 \text{Re}[A_0^L A_\parallel^{L*} + A_0^R A_\parallel^{R*}] \times \mathcal{BW}_P ^2$	$\sin 2\theta_K \sin 2\theta_\ell \cos \phi$
5	$\sqrt{2}\beta_\mu \left(\text{Re}[A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*}] - \frac{m_\mu}{\sqrt{q^2}} \text{Re}[A_\parallel^L A_{\text{scalar}}^{*} + A_\parallel^R A_{\text{scalar}}^{*}]\right) \times \mathcal{BW}_P ^2$	$\sin 2\theta_K \sin \theta_\ell \cos \phi$
6s	$2\beta_\mu \text{Re}[A_\parallel^L A_\perp^{L*} - A_\parallel^R A_\perp^{R*}] \times \mathcal{BW}_P ^2$	$\sin^2 \theta_K \cos \theta_\ell$
6c	$4\beta_\mu \frac{m_\mu}{\sqrt{q^2}} \text{Re}[A_0^L A_{\text{scalar}}^{*} + A_0^R A_{\text{scalar}}^{*}] \times \mathcal{BW}_P ^2$	$\cos^2 \theta_K \cos \theta_\ell$
7	$\sqrt{2}\beta_\mu \left(\text{Im}[A_0^L A_\parallel^{L*} - A_0^R A_\parallel^{R*}] + \frac{m_\mu}{\sqrt{q^2}} \text{Im}[A_\perp^L A_{\text{scalar}}^{*} + A_\perp^R A_{\text{scalar}}^{*}]\right) \times \mathcal{BW}_P ^2$	$\sin 2\theta_K \sin \theta_\ell \sin \phi$
8	$\frac{1}{\sqrt{2}}\beta_\mu^2 \text{Im}[A_0^L A_\perp^{L*} + A_0^R A_\perp^{R*}] \times \mathcal{BW}_P ^2$	$\sin 2\theta_K \sin 2\theta_\ell \sin \phi$
9	$\beta_\mu^2 \text{Im}[A_\parallel^L A_\perp^{L*} + A_\parallel^R A_\perp^{R*}] \times \mathcal{BW}_P ^2$	$\sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi$
10	$\frac{1}{2} \left(A_S^L ^2 + A_S^R ^2 + \frac{4m_\mu^2}{q^2}(A_t ^2 + 2\text{Re}[A_S^L A_S^{R*}])\right) \times \mathcal{BW}_S ^2$	1
11	$\sqrt{3} \left(\text{Re}[(A_S^L A_0^{L*} + A_S^R A_0^{R*} + \frac{4m_\mu^2}{q^2}(A_S^L A_0^{R*} + A_{\text{scalar},t} A_t^{*})) \times \mathcal{BW}_S \mathcal{BW}_P^{*}] + \text{Re}[\frac{4m_\mu^2}{q^2} A_0^L A_S^{R*} \times \mathcal{BW}_P \mathcal{BW}_S^{*}]\right)$	$\cos \theta_K$
12	$-\frac{1}{2}\beta_\mu^2 (A_S^L ^2 + A_S^R ^2) \times \mathcal{BW}_S ^2$	$\cos 2\theta_\ell$
13	$-\sqrt{3}\beta_\mu^2 \text{Re}[(A_S^L A_0^{L*} + A_S^R A_0^{R*}) \times \mathcal{BW}_S \mathcal{BW}_P^{*}]$	$\cos \theta_K \cos 2\theta_\ell$
14	$\sqrt{\frac{3}{2}}\beta_\mu^2 \text{Re}[(A_S^L A_\parallel^{L*} + A_S^R A_\parallel^{R*}) \times \mathcal{BW}_S \mathcal{BW}_P^{*}]$	$\sin \theta_K \sin 2\theta_\ell \cos \phi$
15	$2\sqrt{\frac{3}{2}}\beta_\mu \text{Re}[(A_S^L A_\perp^{L*} - A_S^R A_\perp^{R*}) \times \mathcal{BW}_S \mathcal{BW}_P^{*}]$	$\sin \theta_K \sin \theta_\ell \cos \phi$
16	$2\sqrt{\frac{3}{2}}\beta_\mu \text{Im}[(A_S^L A_\parallel^{L*} - A_S^R A_\parallel^{R*}) \times \mathcal{BW}_S \mathcal{BW}_P^{*}]$	$\sin \theta_K \sin \theta_\ell \sin \phi$
17	$\sqrt{\frac{3}{2}}\beta_\mu^2 \text{Im}[(A_S^L A_\perp^{L*} + A_S^R A_\perp^{R*}) \times \mathcal{BW}_S \mathcal{BW}_P^{*}]$	$\sin \theta_K \sin 2\theta_\ell \sin \phi$

Table 2.3: Definition of the angular coefficients I'_i as functions of the P-wave transversity amplitudes $A_{0,\perp,\parallel,t}^{L/R}$ and the S-wave amplitudes $A_S^{L/R}$ as well as the scalar amplitude A_{scalar} [42].

The parameter β_μ is defined $\beta_\mu = \sqrt{1 - 4m_\mu^2/q^2}$. The $\bar{I}'_i$ coefficients are obtained by substituting $A \rightarrow \bar{A}$, *i.e.* by complex conjugation of the weak phases in the amplitudes. For convenience the abbreviation $A_{\text{scalar},t} = A_{\text{scalar},t}^R - A_{\text{scalar},t}^L$ is used [45].

$$\begin{aligned}
\text{Re}[(A_S^L A_{0,\parallel,\perp}^{L*} \pm A_S^R A_{0,\parallel,\perp}^{R*}) \times \mathcal{BW}_S \mathcal{BW}_P^{*}] &= \underbrace{\text{Re}[A_S^L A_{0,\parallel,\perp}^{L*} \pm A_S^R A_{0,\parallel,\perp}^{R*}]}_{I_i^{\text{re}}} \text{Re}[\mathcal{BW}_S \mathcal{BW}_P^{*}] \\
&\quad - \underbrace{\text{Im}[A_S^L A_{0,\parallel,\perp}^{L*} \pm A_S^R A_{0,\parallel,\perp}^{R*}]}_{I_i^{\text{im}}} \text{Im}[\mathcal{BW}_S \mathcal{BW}_P^{*}] \\
\text{Im}[(A_S^L A_{0,\parallel,\perp}^{L*} \pm A_S^R A_{0,\parallel,\perp}^{R*}) \times \mathcal{BW}_S \mathcal{BW}_P^{*}] &= \underbrace{\text{Re}[A_S^L A_{0,\parallel,\perp}^{L*} \pm A_S^R A_{0,\parallel,\perp}^{R*}]}_{I_i^{\text{re}}} \text{Im}[\mathcal{BW}_S \mathcal{BW}_P^{*}] \\
&\quad + \underbrace{\text{Im}[A_S^L A_{0,\parallel,\perp}^{L*} \pm A_S^R A_{0,\parallel,\perp}^{R*}]}_{I_i^{\text{im}}} \text{Re}[\mathcal{BW}_S \mathcal{BW}_P^{*}] \quad (2.62)
\end{aligned}$$

Factoring out the $m_{K\pi}$ dependence from any interference term of this kind results in

two independent angular coefficients $I_i^{re/im}$, which are exclusively q^2 dependent. The full resulting S-wave and interference part of the decay rate is then given by

$$\begin{aligned} \Gamma_S = \frac{3}{16\pi} & \left[(I_{10} + I_{12} \cos 2\theta_\ell) \times |\mathcal{BW}_S(m_{K\pi})|^2 \right. \\ & + \cos \theta_K (I_{11}^{re} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] - I_{11}^{im} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & + \cos \theta_K \cos 2\theta_\ell (I_{13}^{re} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] - I_{13}^{im} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & + \sin \theta_K \sin 2\theta_\ell \cos \phi (I_{14}^{re} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] - I_{14}^{im} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & + \sin \theta_K \sin \theta_\ell \cos \phi (I_{15}^{re} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] - I_{15}^{im} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & + \sin \theta_K \sin \theta_\ell \sin \phi (I_{16}^{re} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] + I_{16}^{im} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & \left. + \sin \theta_k \sin 2\theta_\ell \sin \phi (I_{17}^{re} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] + I_{17}^{im} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \right] \quad (2.63) \end{aligned}$$

for $\bar{B}^0$ and

$$\begin{aligned} \bar{\Gamma}_S = \frac{3}{16\pi} & \left[(\bar{I}_{10} + \bar{I}_{12} \cos 2\theta_\ell) \times |\mathcal{BW}_S(m_{K\pi})|^2 \right. \\ & + \cos \theta_K (\bar{I}_{11}^{re} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] - \bar{I}_{11}^{im} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & + \cos \theta_K \cos 2\theta_\ell (\bar{I}_{13}^{re} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] - \bar{I}_{13}^{im} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & + \sin \theta_K \sin 2\theta_\ell \cos \phi (\bar{I}_{14}^{re} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] - \bar{I}_{14}^{im} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & + \sin \theta_K \sin \theta_\ell \cos \phi (\bar{I}_{15}^{re} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] - \bar{I}_{15}^{im} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & + \sin \theta_K \sin \theta_\ell \sin \phi (\bar{I}_{16}^{re} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] + \bar{I}_{16}^{im} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \\ & \left. + \sin \theta_k \sin 2\theta_\ell \sin \phi (\bar{I}_{17}^{re} \times \text{Im}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})] + \bar{I}_{17}^{im} \times \text{Re}[\mathcal{BW}_S(m_{K\pi})\mathcal{BW}_P^*(m_{K\pi})]) \right] \quad (2.64) \end{aligned}$$

for B^0 . As detailed in Tab. 2.3, the angular observables are dependent on the mass of the leptons in the final state. This dependency usually occurs as a ratio of m_μ^2 over the invariant di-muon mass squared q^2 . In cases where it is appropriate to assume that the muon mass is small compared to q^2 , the expressions for the angular observables simplify significantly. Assuming negligible lepton masses leads to the following relations between angular observables.

$$\begin{aligned} I_1' &= 3I_2' \\ I_1' &= -I_2' \\ I_{10}' &= -I_{12}' \\ I_{11}' &= -I_{13}' \end{aligned} \quad (2.65)$$

2.5.4 Hadronic form factors

A key ingredient to the theoretical prediction of the angular coefficients is the calculation of the hadronic form factors (FFs). They encode the QCD part of the interaction governing the transition of the B^0 meson into the K^{*0} meson. The short-distance matrix elements are parametrized in terms of seven q^2 dependent hadronic FFs $A_0, A_1, A_2, T_1, T_2, T_3$ and V [50]. They enter the matrix elements in Eq. 2.27 via the terms

$$\langle K^{*0} | \bar{s} \gamma^\mu (1 \mp \gamma_5) b | \bar{B} \rangle \quad (2.66)$$

$$\langle K^{*0} | \bar{s} i q_\nu \sigma^{\mu\nu} (1 \mp \gamma_5) b | \bar{B} \rangle. \quad (2.67)$$

As no perturbative calculation of the FFs from first principles is possible, non-perturbative methods need to be used. In principle, there are two methods available that allow the calculation of the FFs in different regimes of q^2 . The approach of Light Cone Sum Rules (LCSR) expands the correlator of the time-ordered product of two quark currents evaluated between the final state on-shell K^{*0} meson and the vacuum on the light cone [50]. This approach is typically valid in the low to mid q^2 regime up to approximately $10 \text{ GeV}^2/c^4$. The Lattice QCD (LQCD) approach calculates the FFs numerically on a discretized space-time lattice. This method is valid at high q^2 where the K^{*0} is nearly at rest in the rest-frame of the B^0 [51]. In global analyses that require knowledge of the FFs over the full q^2 range, usually a z -expansion is employed to extend the FFs parameterization into the q^2 region outside the validity of the aforementioned approaches [50, 52, 53]. A large effort has been invested into the improvement of the precision of the local hadronic FFs. At this point in time, the local form factors are no longer the main source of theoretical uncertainty in the prediction of the angular observables. The more relevant contribution comes from the non-local contributions displayed in Fig. 2.3 [54]. These non-local FFs are much more difficult to compute, as the assumption of factorization of weak and QCD interactions is not valid for these processes. The size of the theoretical uncertainty is also much harder to control. In the literature, the contributions from non-local processes like charm loops are calculated either using the QCD factorization framework [55, 56] or using light cone operator expansions (LCOPE) in the space like $q^2 < 0$ region and hadronic dispersion relations to interpolate the form factors between $q^2 < 0$ and $q^2 = m_{J/\psi}^2$ as well as $q^2 = m_{\psi(2S)}^2$ using experimental data from $B^0 \rightarrow K^{*0} J/\psi$ and $B^0 \rightarrow K^{*0} \psi(2S)$ decays [54]. While the QCD factorization approach breaks down in the vicinity of the charmonium resonances ($q^2 < 6.0 \text{ GeV}^2/c^4$), the dispersion relation approach can provide a reliable prediction up to the J/ψ resonance at about $q^2 < 8.0 \text{ GeV}^2/c^4$.

2.5.5 SM predictions and Wilson coefficient fits

Using the calculations of the local and non-local hadronic FFs, as well as the analytic expression of the angular coefficients in terms of the transversity amplitudes given in Tab. 2.3, it is possible to predict the values of the angular observables studied in this thesis. Flavor physics software packages, such as Flavio [57] or EOS [58], provide tools for these predictions, as well as for inferring the Wilson coefficients and determining their uncertainties from experimental constraints. SM predictions for all angular observables from Flavio and EOS are shown in Sec. 6.7 along with the results of the thesis. A more detailed explanation of the estimation of Wilson coefficients in this thesis can be found in Sec. 6.7.6. The estimation of the non-local form factors directly impacts the uncertainties on predictions and Wilson coefficients. Conservative theoretical uncertainties have been assigned to these contributions, but full agreement on how to best parameterize these contributions still has to be reached [54, 56, 59]. The determination of the Wilson coefficients from $b \rightarrow s \ell^+ \ell^-$ processes in particular has attracted attention in the past due to observed tensions with the SM predictions. These measurements are discussed in the following chapter.

Chapter 3

Flavor anomalies in $b \rightarrow s\ell^+\ell^-$ decays

In recent years, several experimental measurements of observables in the $b \rightarrow s\ell^+\ell^-$ sector were found to be in tension with the SM. These discrepancies occurred in measurements of the differential branching fractions of FCNC b -hadron decays like $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [60–64], $B^+ \rightarrow K^+\mu^+\mu^-$ [60–62, 65], $B_s^0 \rightarrow \phi\mu^+\mu^-$ [66], $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ [62, 67], $B^0 \rightarrow \mu^+\mu^-$ and $B_s^0 \rightarrow \mu^+\mu^-$ [68–70]. Similarly, the measurements of angular observables of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [24, 71–76] and $B^+ \rightarrow K^{*+}\mu^+\mu^-$ [76, 77] found a considerable deviation from the SM. These discrepancies are collectively referred to as the flavor anomalies and indicate a consistent negative shift of the Wilson coefficient $\mathcal{Re}(C_9)$. However, the tensions are not yet confirmed to originate from New Physics contributions, as on the experimental side significances of single measurements do not reach the 5σ level, and on the theory side, the parameterization of charm-loop effects is still under discussion [54, 56, 59]. The following chapter gives a brief overview of the mentioned measurements and their results.

3.1 Branching fraction measurements

The measurements of differential branching fractions of rare $b \rightarrow s\ell^+\ell^-$ decays are performed in bins of q^2 . Usually, branching fractions are determined relative to a normalization mode with similar kinematic properties and high statistics to cancel systematic uncertainties, for example, arising from the determination of the integrated luminosity. In $b \rightarrow s\ell^+\ell^-$ decays, this role is often assumed by the corresponding tree-level decays, like for example $B^0 \rightarrow K^{*0}J/\psi$, $B^+ \rightarrow K^+J/\psi$ or $B_s^0 \rightarrow \phi J/\psi$. Measurements of the differential branching fractions of the decays $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [60–64], $B^+ \rightarrow K^+\mu^+\mu^-$ [60–62, 65], $B_s^0 \rightarrow \phi\mu^+\mu^-$ [66] and $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ [62, 67] are displayed in Fig. 3.1. The most precise measurements of these branching fractions are performed by LHCb due to the experiment’s high b -production cross-section combined with an excellent reconstruction efficiency.

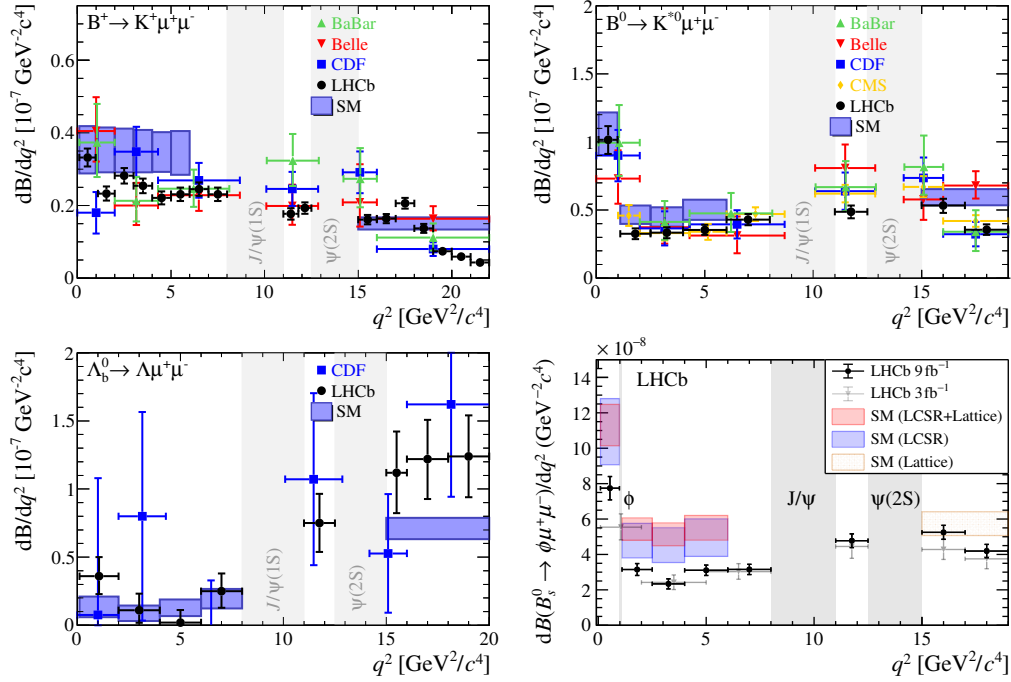


Figure 3.1: Differential branching fractions in bins of the invariant di-muon mass squared q^2 for the decays $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ [60–64], $B^+ \rightarrow K^+ \mu^+ \mu^-$ [60–62, 65], $B_s^0 \rightarrow \phi \mu^+ \mu^-$ [66] and $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ [62, 67]. The figures showing results from multiple experiments are taken from Ref. [22].

Generally, the measurements of the branching fractions seem to prefer smaller values than the SM prediction. One of the largest tensions is present in the branching fractions of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$ measured by LHCb with a local significance of 3.6σ in the q^2 region $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$ [66]. The branching fraction of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ is also found to be consistently smaller than the SM prediction [65]. However, due to the significant theoretical uncertainties, the tension is not as significant. Due to the suppression of the decays in the SM, the statistical uncertainties of all modes are still significant. The most relevant systematic uncertainties arise from the measured branching fraction of the normalization modes, which are usually determined by B -factories like Belle (II) or BaBar. After the completion of the LHCb experiment's Run 3, the uncertainties of these measurements are no longer expected to be statistically dominated. The SM predictions of the branching fraction carry significant theoretical uncertainties, which are dominated by the hadronic form factor uncertainties. Additionally, the uncertainty on the parameterization of non-factorizing charm-loop contributions to the $b \rightarrow s \ell^+ \ell^-$ transitions also contributes directly to the total theory uncertainty. The purely leptonic $b \rightarrow s \ell^+ \ell^-$ modes $B_s^0 \rightarrow \mu^+ \mu^-$ and $B^0 \rightarrow \mu^+ \mu^-$ are even more rare in the SM due to an additional helicity suppression and thus highly sensitive to NP effects. Due to the absence of hadrons in the final state, the decay amplitude is not dependent on local or non-local hadronic form factors, which makes the analysis very theoretically clean. The measurement of the branching fraction is sensitive to the Wilson coefficient $\mathcal{R}e(C_{10})$ as well as the scalar and pseudo-scalar Wilson coefficients $\mathcal{R}e(C_S)$ and $\mathcal{R}e(C_P)$. Combined measurements from the LHCb [68], CMS [69], and ATLAS [70] experiments have found the branching

fraction to be in tension with the SM prediction at the level of about 2σ [78].

3.2 Angular analyses

While the branching fraction measurements discussed above measure the integrated decay rate of the decays in bins of q^2 , angular analyses measure the observables that parameterize the angular differential decay rate of the process. These analyses are generally much more complicated to perform, but also give access to the full information about the rich internal structure of the decay. Similar to the branching fractions, the angular observables are determined in bins of q^2 . Additionally, as the angular observables are ratios of decay amplitudes, they are less affected by theoretical uncertainties from the form factors and the charm-loop effects. If the decay flavor of the b -hadron is known, the angular analysis has access to CP -symmetric as well as CP -asymmetric observables of the decay. Theory uncertainties arising from hadronic form factors can be canceled to leading order by a reparameterization of the angular observables [23]. Measurements of different angular observables of the decays $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [24, 71–76] and $B^+ \rightarrow K^{*+}\mu^+\mu^-$ [77] are displayed in Fig. 3.2.

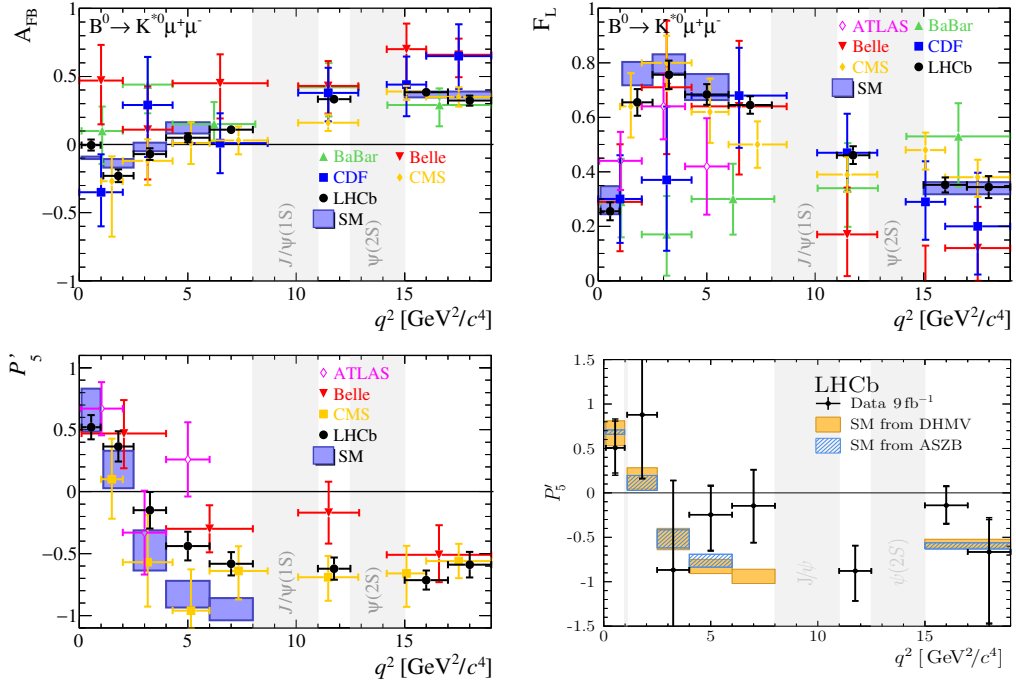


Figure 3.2: Angular observables in bins q^2 for the decays $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [24, 71–76] and $B^+ \rightarrow K^{*+}\mu^+\mu^-$ [77] (bottom right). The figures showing results from multiple experiments are taken from Ref. [22].

The angular analysis of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$ performed by LHCb using data collected during 2011, 2012, and 2016 finds a clear tension between the measurement and the SM prediction in P'_5 . The global significance of the tension with the SM, considering the full set of measured angular observables, is determined to be at the level of 3.3σ [24]. The corresponding angular analysis of the decay $B^0 \rightarrow K^{*+}\mu^+\mu^-$ also reports a consistent tension with the SM at the level of 3.1σ [77]. The angular analysis

of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$ by LHCb finds good agreement with the SM [79]. However, due to its flavor-unspecific final state, it is not sensitive to the observables P_5' and A_{FB} . The uncertainties on the angular observables are still statistically dominated. The precision of the analysis can be significantly improved by collecting a larger data sample. Also, for the CP -symmetric angular observables, the theory uncertainties are significant. The hadronic form factors and the charm-loop parameterization are again the leading contributions to the total theoretical uncertainty. For the CP -asymmetries, the theory uncertainties are completely negligible compared to the statistical ones. The LHCb collaboration conducted multiple q^2 -unbinned amplitude analyses of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay mode that parameterize and measure the non-local amplitudes explicitly [80–82]. These analyses extract the WC directly under the assumption of a set of hadronic form factors and are thus model-dependent and cannot be reinterpreted with improved theory calculations. The results show that the tensions with the SM cannot be trivially explained by the non-local contributions under the assumption of a specific theoretical model.

3.3 Test of lepton flavor universality

In the SM, the coupling between gauge bosons and leptons from different generations (e , μ , τ) is equivalent up to lepton mass effects. This is not strictly required for NP effects that could potentially prefer to couple to a specific generation. Lepton flavor universality (LFU) in $b \rightarrow s \ell^+ \ell^- \nu$ can be tested at collider experiments by comparing angular observables or branching fractions for a given decay mode. Specialized LFU observables like the R_X ratios, which are ratios of specific $b \rightarrow s e^+ e^-$ and the corresponding $b \rightarrow s \mu^+ \mu^-$ branching fractions, are theoretically very clean [83]. The same is true for the observables Q_i that are constructed as differences between angular observables extracted from the electron and muon modes [84]. The analysis of decays involving electrons in the final state poses significant challenges compared to muon modes. Most notably, the electrons are affected by Bremsstrahlung effects, which degrade invariant mass resolutions and lead to increased background levels. In general, the selection efficiencies for decays involving electrons are significantly lower, resulting in increased statistical uncertainties on observables compared to the muon modes. The LHCb collaboration published measurements of R_X ratios for the $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ [85], $B^+ \rightarrow K^+ \ell^+ \ell^-$ [85], $B_s^0 \rightarrow \phi \ell^+ \ell^-$ [86], $B^+ \rightarrow K^+ \pi^+ \pi^- \ell^+ \ell^-$ [87] and $\Lambda_b^0 \rightarrow \Lambda^0 \ell^+ \ell^-$ [88] decay modes. All of these measurements are found to be consistent with the LFU hypothesis. This means that the branching fractions of the electron modes are consistent with the tensions with respect to the SM displayed by the corresponding muon modes. The LHCb collaboration has also conducted LFU tests on angular observables for the $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ [89] and $B_s^0 \rightarrow \phi \ell^+ \ell^-$ [90] decay modes. The measurements similarly display no significant deviation from the LFU hypothesis. The precision on these angular LFU observables is expected to increase significantly with future data collected at the LHC, as the $b \rightarrow s e^+ e^-$ modes are even more statistically limited as the $b \rightarrow s \mu^+ \mu^-$ modes at this point in time. Similar analyses have been conducted by the CMS [91] and Belle [92] collaborations, which are also in agreement with the LFU hypothesis but with significantly larger uncertainties.

3.4 Global analysis

Multiple $b \rightarrow s\ell^+\ell^-$ decays are found to be in tension with the SM. However, none of these exceed the 5σ boundary on their own. Global analyses combine the results from different decays in order to achieve a higher sensitivity to the New Physics effects than each analysis can achieve on its own. The basis for this combination is the EFT theory approach, discussed in Sec. 2, which separates the $b \rightarrow s\ell^+\ell^-$ part of the decay from the hadronic interactions. Thus, all of these decays are parameterized by the same sets of Wilson coefficients, which can be determined from the combined set of experimental constraints using, for example, Bayesian inference [58] or the fast likelihood method [93]. A multitude of these global analyses have been performed by theorists, including constraints from a wide range of physical observables from radiative, semi-leptonic, and purely leptonic decays [94–98]. The experimental observables include branching fractions, angular observables, and theoretically clean tests of lepton flavor universality. These analyses consider different New Physics scenarios, floating different subsets of Wilson Coefficients in their fits to data. In general, the global fits prefer shifts of the Wilson coefficient $\mathcal{Re}(\mathcal{C}_9)$ to smaller values compared to the SM expectation with significance well above 5σ . However, it has also been established that underestimated effects from the previously mentioned charm loops could, in principle, mimic at least parts of the seen deviations. A thorough theory estimation from first principles is not yet available and would require a significant theoretical breakthrough [99]. Current estimations have to rely on data-driven methods. Until a comprehensive theoretical understanding of these contributions is achieved, it remains unclear if the flavor anomalies are due to genuine New Physics contributions or an insufficient understanding of SM effects. In any case, these global fits demonstrate that the SM calculations in their current form consistently fail to explain the experimental measurements of $b \rightarrow s\ell^+\ell^-$ decays. The sensitivity to the Wilson coefficients is limited by experimental as well as theoretical uncertainties. As the analysis of FCNC decays is still statistically dominated at LHCb, an enormous effort is ongoing to update the existing measurements with Run 3 data.

The analysis presented in this thesis provides an essential update to the angular observables of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$, which are one of the driving contributions of the flavor anomalies. This update additionally includes the LHCb data sets from 2017/18, which effectively doubles the statistics compared to the previous LHCb analysis.

Chapter 4

The LHCb detector

The data used in the angular analysis of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ presented in this thesis are collected with the LHCb experiment [16], which is one of the four large experiments located at the Large Hadron Collider (LHC) [100] in Geneva. With a circumference of 27 km and a $\sqrt{s}$ between 7 TeV and 13.6 TeV, the LHC is the largest and most powerful particle accelerator in the world. It accelerates two opposite beams of protons or ions to velocities close to the speed of light. The protons travel the accelerator ring through a vacuum tube in 2808 bunches of up to 10^{11} protons at a time. The charged particles are kept on their orbit with different types of superconducting magnets, which also maintain the focus of the beam. To accelerate the protons, 16 RF cavities are located around the accelerator ring. In each cavity, the particles pass a high-frequency electromagnetic field that is tuned for an optimal energy transfer to the particles. Repeated passes through the RF cavities accelerate the protons to their final kinematic energy, which has been 6.5 TeV in the years 2016-2018 (Run 2) and 3.5 TeV and 4.0 TeV in 2011 and 2012 (Run 1), respectively. The maximal particle energy is limited by the field strength of the dipole magnets and the radius of the accelerator ring. With a period time of 25 ns, the proton bunches are brought to collision at four interaction points that are housed inside the four major experiments CMS [13], ATLAS [14], ALICE [15], and LHCb [16]. Heavy quarks, like the b-quark, are produced through these collisions as described in Sec. 2.3. The LHCb detector is a single arm forward spectrometer with an angular coverage of about 10 to 300 mrad in the bending plane and 10 to 250 mrad in the non-bending plane. Figure 4.1 shows a cross-section of the detector and all its subcomponents. LHCb is dedicated to the study of heavy flavor decays, especially to the measurement of NP in CP -violation and rare decays of bottom and charm hadrons [16]. The geometry of the detector is tailored to the analysis of b -hadron decays since $b\bar{b}$ quark pairs are predominantly produced boosted in forward and backward direction as shown in Fig. 4.2.

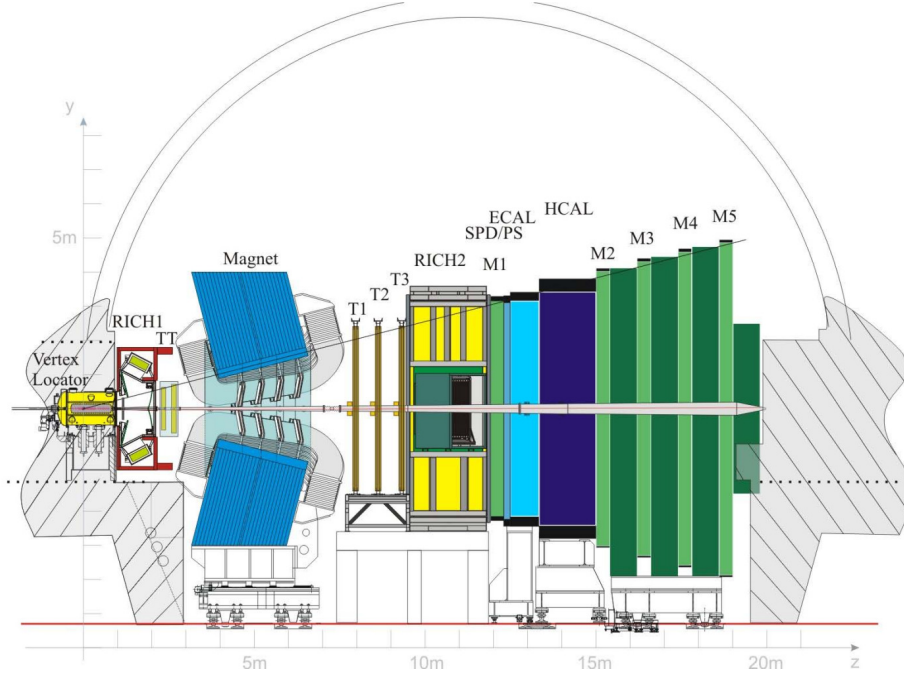


Figure 4.1: Cross section of the LHCb detector and its subsystems. [16]

Proton-proton collisions at the high center of mass energy of the LHC produce B^0 -, B^{+-} -, B_s^0 -, and B_c^+ -mesons with a total production cross section of $527.3 \mu\text{b}$ at 14 TeV [101], which is significantly larger compared to the B-factories. Through manipulation of the beam overlap at the interaction point, called luminosity leveling [102], the instantaneous luminosity $\mathcal{L}$ is artificially reduced to control the number of pp collisions per event in the LHCb detector. Through automatic adjustments of the beam overlaps, the number of interactions is kept at a constant level through the majority of the LHC fill. This allows a cleaner reconstruction of the event and reduces the radiation damage to the detector components.

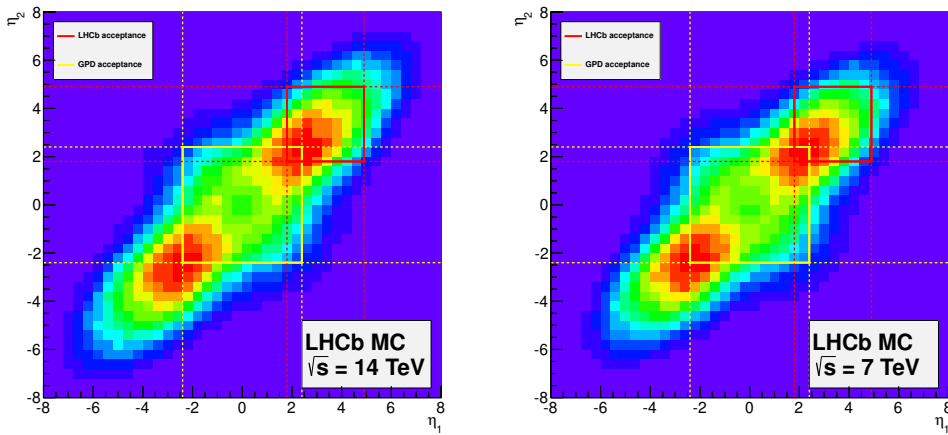


Figure 4.2: Production of $b\bar{b}$ quark pairs as a function of the pseudo-rapidity at the LHCb experiment [101].

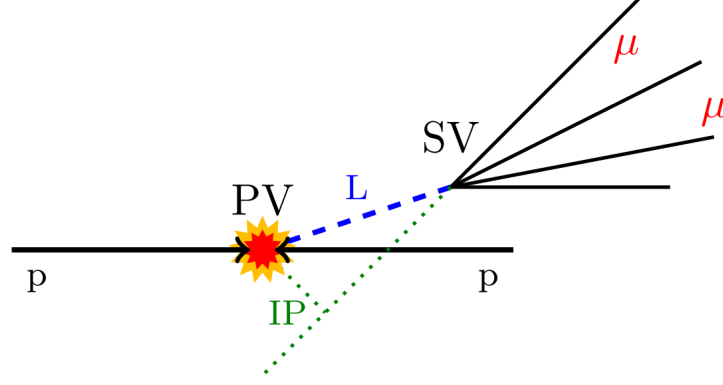


Figure 4.3: Topology of a typical B^0 decay.

4.1 Decay topology of heavy flavor decays

The main purpose of the LHCb detector is the precise reconstruction of heavy flavor decay chains, including the decay vertices and the momentum vectors of their final state particles. This allows the clean separation of the signal decay from background tracks that originate from the proton-proton collision point, the **primary vertex** (PV). Due to the significant lifetime of the B-meson, it can fly for several millimeters before it decays. By reconstructing the decay chain of intermediate particles, it is possible to determine the location of the initial B-meson decay, the **secondary vertex** (SV). If the SV is significantly detached from the PV, quantities like the distance between the PV and the SV, the **flight distance** (FD), allow for efficient separation of signal from background processes. In practice, one often uses the **flight-distance significance** (χ_{FD}^2), which is the FD divided by its uncertainty squared. Another quantity yielding high separation power between tracks from the PV and from a significantly detached SV is the **impact parameter** (IP), which is defined as the closest distance between the particle track and the PV. Its geometrical definition is displayed in Fig. 4.3. Similarly to the χ_{FD}^2 , analyses often use the **impact parameter significance** (χ_{IP}^2) as IP and FD are measured quantities with varying precision. If the decay of a B-meson is reconstructed, the summed-up momentum vectors of the final state particles are identical to the flight direction of the mother particle. The **direction angle** (DIRA) is defined as the angle between the particle momentum and the vector between PV and SV. For a fully reconstructed signal decay, the DIRA is expected to be close to zero, while partially reconstructed decays of combinations of random tracks can also show large values. Another key task of the detector is to identify the type of final state particles. This is crucial to analyze exclusive decays like $B^0 \rightarrow K^{*0} \mu^+ \mu^-$. This is achieved through the combination of information from different subdetectors, which is combined into quantities with high separation power between different particle types. The analysis presented in this thesis uses three different types of variables provided by the LHCb framework: the DLL_x variable, the ProbNN_x variable, and the boolean `isMuon` classifier. Using the detectors' Ring Imaging Cherenkov (RICH) detectors, muon chambers, and calorimeters, likelihoods for possible particle hypotheses are constructed. The **delta log likelihood** (DLL_x) is the difference between the likelihood of the particle hypothesis x and the pion hypothesis, which is

the most abundant particle in the detector [103]. The neural network-based ProbNN variables combine information from the RICH, muon system, tracking system, and calorimeter system to provide a probability-like variable between one and zero for a given particle hypothesis [103]. The ProbNN variables generally perform much better than the DLL variables. The isMuon criterion is a simple binary classifier determining whether a track is consistent with the muon hypothesis. It is based on a minimum requirement of muon chambers containing hits in an extrapolated field of interest around the track, depending on the track momentum [103]. The evaluation of this quantity is computationally very efficient, which allows it to be used in the trigger system, also discussed later in this chapter.

The LHCb detector is comprised of different subsystems designed to provide the specific information necessary to analyze the recorded data. In the following, the single subdetectors are discussed in more detail with a focus on the components relevant for the reconstruction of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$.

4.2 VELO

The interaction region inside the LHCb detector is surrounded by the Vertex Locator (VELO), which is a silicon strip detector that reconstructs charged tracks from the interaction point inside the VELO vacuum vessel. A cross-section of the detector can be found in Fig. 4.4. It consists of 42 pairs of sensors, where one (R-sensor) delivers information about the radial distance of the charged hits from the beam and the other one (ϕ -sensor) gives information about the azimuthal angle. Additionally, 4 R-sensors are located upstream of the remaining sensors and are used as a pile-up veto system in the trigger. The R-sensors are arranged in concentric semi-circles around the beam region to ensure a constant radius from the beam region for each strip. To avoid high detector occupancies in the inner region of the Velo and limit the spacing between the strips of the ϕ sensors in the outer region of Velo, it is divided into an inner ($r < 17.25$ mm) and an outer ($r > 17.25$ mm) region [16], with the pitch varying between 38 and 97 μm . The z-coordinate is inferred from the exact location of the sensor in the VELO. Depending on the pitch of the individual sensors, this allows for measuring the location of the charged hits in the silicon strips with a resolution of down to 4 μm [102]. To reconstruct the decay vertices with the highest possible precision, the distance between the interaction point and the sensors is reduced as much as possible ($r=8.2$ mm). The decay vertices inside the Velo can thus be determined with extremely high precision. Such a vertex reconstruction is indispensable since bottom and charm hadrons have a significant lifetime and therefore decay at a detached SV. This key signature distinguishes signal decays from the large number of tracks originating from the PV. The IP provides information about whether a specific track detected in the VELO originates from the PV or a potential SV. This parameter can be determined with a transverse momentum p_T dependent resolution of $\left(15 + \frac{29}{p_T[\text{GeV}/c]}\right) \mu\text{m}$.

4.3 Dipole magnet

The final state particle momenta are central to every analysis in LHCb. They can be inferred from the bending of the track of charged particles through a strong

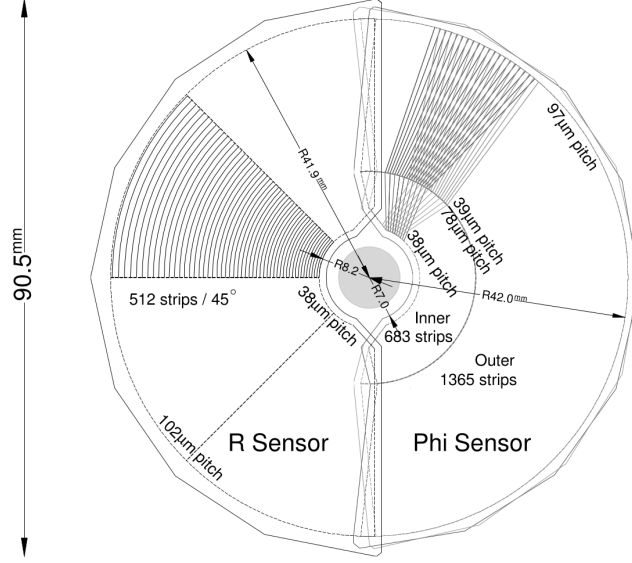


Figure 4.4: Cross-section of the Velo subdetector. Two types of silicon strip detector modules are arranged such that they provide information about the radial and azimuthal coordinates of hits in the detector. The figure is taken from Ref. [16].

magnetic field that deflects their trajectory. The magnetic field is generated by a large, saddle-shaped dipole magnet. A warm magnet design was chosen, which allows quick ramp-ups of the magnetic fields and regular inversion of the magnetic field direction as displayed in Fig. 4.5. The magnetic field strength integrated over a typical track length of 10 m amounts to 4 Tm [16]. While the magnetic field needs to be as strong as possible in the region between the VELO and the tracking stations, it needs to be small in other regions of the detector to allow, for example, particle identification. A high-precision map of the magnetic field is obtained by scanning the relevant volume inside the detector with a set of Hall probes. The mapping shows excellent agreement with the simulation. The knowledge of the exact layout of the magnetic field is vital for the precise calculation of the momenta.

4.4 Tracking stations

To perform a full reconstruction of the tracks of charged particles and measurement of the particle momenta, multiple tracking stations are located upstream and downstream of the magnet, in addition to the Velo. One silicon strip detector station located upstream of the magnet is denoted as Tracker Turicensis (TT). The tracking stations T1-T3, which are located downstream of the magnet, consist of silicon strips in the inner region (IT) and straw-tube detectors in the outer regions (OT). Each of the silicon tracker stations is comprised of four layers of silicon strip sensors with a pitch of 200 μm [16]. While the strips in the first and last layer are arranged vertically, the strips of the second and third layer are slightly rotated in opposite directions with respect to the other layers in order to allow a good spatial resolution in the x and y planes. This allows determination of the location of a charged hit in the TT and IT with a precision of about 50 μm [102]. An illustration of both TT and IT is

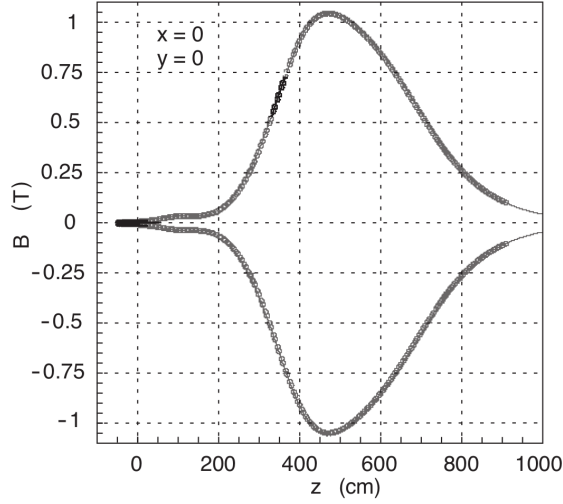


Figure 4.5: The magnetic field along the z -axis of the LHCb detector. The direction of the magnetic field can be reversed. The figure is taken from Ref. [16].

shown in Fig. 4.6. Below this threshold, the momentum resolution of the detector is mostly determined by the uncertainty introduced by multiple scattering of the charged particles on the tracking detector material. The OT is comprised of an array of small tube-shaped gas detectors that measure the track of charged final state particles via the drift time of ionized gas atoms between high voltage anodes and cathodes in the tube. Similar to IT, the straw-tube modules in each station are organized in four layers, where layers two and three are slightly tilted with respect to layers one and four by a stereo angle of -5° and $+5^\circ$, respectively. This stereo angle is required to determine both the x and y coordinates of the hits in the tracking detector. A single-cell hit resolution of about $200 \mu\text{m}$ [102] is achieved. The synergy of the different tracking stations enables the reconstruction of tracks bent by the magnetic field inside the detector and the extraction of the particle momentum components with a resolution of about $\Delta p/p = 0.5\%$ at low momenta up to a resolution of about 1% at $200 \text{ GeV}/c$.

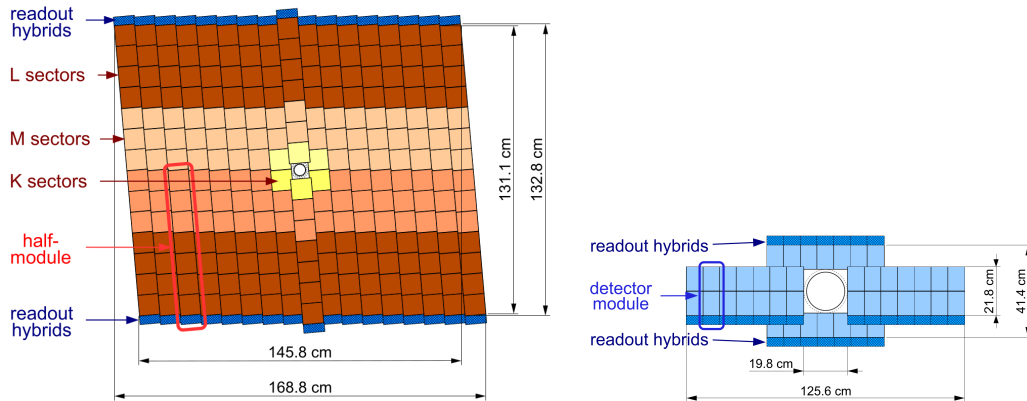


Figure 4.6: Illustration of the TT (left) and IT (right) silicon strip detectors. The figures are taken from Ref. [16].

4.5 RICH

Particle identification and especially the distinction of kaons and pions is an essential feature of the LHCb detector, which is crucial for the analysis of rare B-decays. LHCb uses two Ring Imaging Cherenkov Detectors (RICH1 and RICH2) to perform this identification, exploiting the β dependency of the Cherenkov angle. An illustration of both detectors is shown in Fig. 4.7. The RICH1 subdetector is located downstream of the dipole magnet. The radiators used to produce Cherenkov radiation are Aerogel and C_4F_{10} gas. The detector module is designed to cover the low momentum range from 1 to 60 GeV/c. At the end of Run 1, the Aerogel radiator was removed in order to comply with the changed requirements of the event reconstruction. Meanwhile, RICH2, which is located downstream of the magnet, covers the high momentum range from 15 to beyond 100 GeV/c, exclusively relying on CF_4 gas as radiator [16]. In both modules, the Cherenkov light is reflected onto photon detectors outside of the LHCb acceptance region using a construction of spherical and planar mirrors. The Cherenkov photons free a single photoelectron from the photocathode in the pixel hybrid photon detector. The electron is subsequently detected by an array of silicon pixels. Using the information about the track direction, it is possible to reconstruct a Cherenkov angle for each hit in the silicon sensor array of the hybrid photon detectors. Hadron identification using the RICH system is performed by constructing a global likelihood of the observed Cherenkov photon pattern given the particle identity hypothesis for each track in the events. This likelihood is maximized by varying the particle hypothesis until the best estimation is found. The excellent performance of this system is reflected in the kaon identification efficiency $\epsilon(K^+|K^+) = 95\%$ [102] and a pion to kaon misidentification probability of $\epsilon(K^+|\pi^+) = 5\%$ [102] in the momentum range of 2 to 100 GeV/c. The K^+ identification efficiency as a function of the momentum is shown in Fig. 4.7 on the right side.

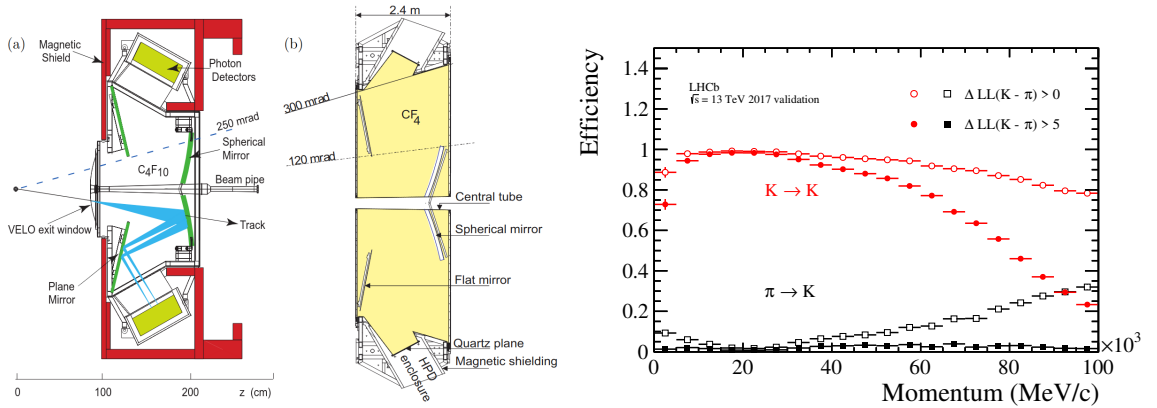


Figure 4.7: Illustration of the RICH1 and RICH2 detectors [104] (left) and plot of the kaon identification efficiency as a function of the particle momentum [105] (right).

4.6 Muon chambers

For the identification of muons, information from an additional subdetector system plays an important role. An illustration of the muon system is shown in Fig. 4.8. The muon chambers M1-M5 are mostly built from Multi-wire proportional chambers. Only the inner part of M1 is comprised of Gaseous Electron Multiplier chambers (GEM). The chambers are filled with a fast, non-flammable gas mix of Argon, CO₂, and CF₄. The spacing between the wire anodes is about 2 mm. The charged muons passing through the chambers ionize the gas inside, which causes an avalanche of secondary electrons proportional to the initially deposited energy. The small spacing between the anodes allows a precise determination of the muon hit location. The extremely high particle flux and the resulting radiation damage in the central region of M1 require the use of GEM chambers in this region. These gas detectors accelerate the ionization electrons, which are generated by charged muons, in the direction of the anode. GEM foils inside the gas chamber are used to concentrate the electric field in small gaps in the foil. Electron avalanches are created inside these choke points, which allows the preservation of spatial information. In each of these GEM chambers, 3 foils are installed. The anode is subdivided into several pads that provide a spatial resolution for the ionizing tracks. Muon identification is performed by extrapolating charged tracks reconstructed by the tracking system with a momentum of more than 3 GeV/c into the region of the muon chambers. A charged track is considered a muon candidate if at least two to four detector modules contain hits in the expected region (isMuon). This identifies muons with a sufficient momentum to reach the muon chambers with an efficiency of $\epsilon(\mu^+|\mu^+) = 94\%$ [102] and a pion to muon misidentification rate of only $\epsilon(\mu^+|\pi^+) = 3\%$ [102]. The muon identification efficiency as a function of the momentum is shown in Fig. 4.8 on the right side.

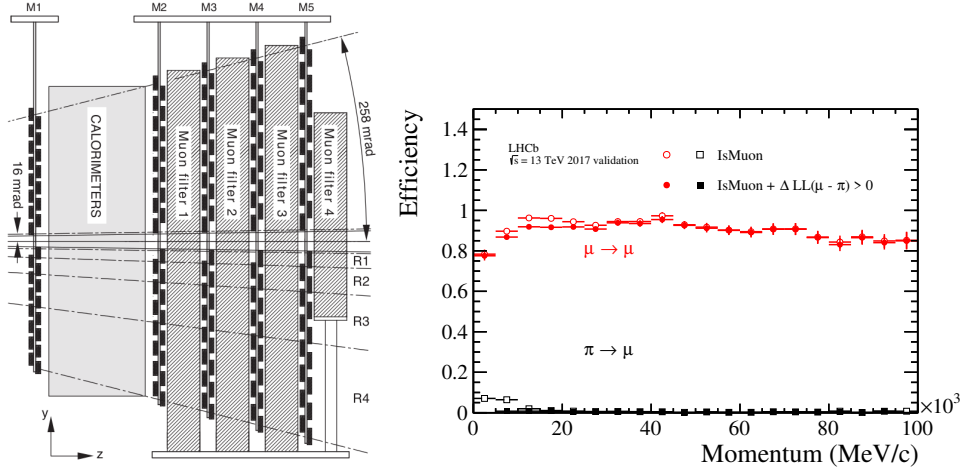


Figure 4.8: Illustration of the muon system [16] (left) and plot of the muon identification efficiency as a function of the particle momentum [105] (right).

4.7 ECAL and HCAL

The calorimeter subsystem, consisting of the electromagnetic calorimeter (ECAL) and hadronic calorimeter (HCAL), measures energy clusters deposited through hadronic or electromagnetic showers induced by charged or neutral particles. The calorimeters are used for the identification of electrons, photons, and hadrons. They play a vital role in the reconstruction of neutral particles and the reconstruction of Bremsstrahlung losses of electrons. Due to their fast readout speed, information from the calorimeters can also be used in the hardware trigger system.

The ECAL is designed to have a shashlik-like structure, which means that it is composed of alternating plates of lead and blocks of plastic scintillator. A picture of the ECAL and its modules is shown in Fig. 4.9. This composition is chosen due to its fast response time, radiation hardness, and reliability. A disadvantage of plastic scintillator tiles is the mediocre energy resolution compared to, for example, crystal scintillators used in CMS [106], but they also come at a fraction of the cost. The energy resolution of the ECAL is roughly $\sigma_E/E = 10\%/\sqrt{E[\text{GeV}/c]} \oplus 1\%$. As the ECAL is used for energy measurements of photons originating from signal decays, the subdetector must contain the complete particle avalanche caused by the high-energy photon. Therefore, a thickness of 25 radiation lengths is chosen. The energy deposited by electromagnetic showers induced by final state particles in the calorimeter volume is released in the form of scintillation photons, which are then detected using photomultiplier tubes. The number of emitted photons is directly related to the initially deposited energy.

The HCAL is a sampling calorimeter of alternating scintillator and steel tiles. As it is not used for precise measurement of hadron energies, a depth of 5.6 hadronic interaction lengths was deemed sufficient.

Two additional calorimeter modules, the scintillating pad detector (SPD) and the preshower detector (PS), are located upstream of the M1 chamber. Both detectors are constructed from scintillator pads with high granularity and separated by a lead absorber. The SPD is designed to separate electrons from photons at the hardware trigger level, as only charged particles induce a signal in the scintillator pads. The PS, on the other hand, is used to separate pions from electrons. While electrons induce a strong electromagnetic shower in the PS, pions deposit significantly less energy due to the small hadronic interaction length of the lead absorber.

4.8 Trigger system

The LHC operates at a bunch crossing rate of 40 MHz, while the reduced instantaneous luminosity of the LHCb experiment results in an average number of visible interactions per bunch crossing μ of about 1.6 in Run 1 [107] and 1.1 in Run 2 [108]. However, the rate of events that can be saved to permanent storage is limited to about 5 kHz in Run 1 [107] and 12.5 kHz in Run 2 [108]. An online selection of events is needed to exclude events that are not of interest for further physics analysis. This task is performed by the two-stage trigger system of the LHCb experiment.

The first stage (L0) is implemented in custom-designed hardware directly in the detector. Only information about high transverse energy E_T hadrons, electrons, and photons in the calorimeters and high p_T muons in the muon chambers can be read out



Figure 4.9: Picture of the ECAL in the detector cavern (left) and ECAL modules from different detector regions (right). The figures are taken from Ref. [16].

at a sufficient speed to keep pace with the initial bunch crossing rate. As B-meson decays are associated with high p_T and E_T due to their high invariant mass, minimum requirements on these measured p_T and E_T are imposed for the selection of the events. Additionally, information about the **Pile Up** and the number of tracks in the event are extracted by the **VELO** and calorimeters to reject events with exceedingly large track combinatorics. This selection reduces the event rate down to 1 MHz.

In a second step, the selected events are sent to a server farm where they are asynchronously processed by a software trigger system (**Hlt**). At the significantly reduced event rate of 1 MHz, it is possible to read out the complete detector and combine the information to perform a simplified reconstruction of the events that have been selected by the **L0** trigger. The selection process is divided into two stages, **Hlt1** and **Hlt2**, inside the software trigger. The task of **Hlt1** is to reconstruct the hits in the **VELO** and the tracking stations into a track and match it to the energy clusters in the calorimeters or hits in the muon chambers that triggered the positive **L0** decision. Multivariate selections are performed combining momentum and track topology information to reduce the event rate to about 80 kHz in 2012 [107] and about 110 kHz for Run 2 [108]. For Run 2, an online calibration and alignment system was introduced that uses data collected by the **Hlt1** stage to automatically rederive calibration and alignment parameters on the fly. Calibration and alignment are performed for the **Velo**, **Tracker**, **Muon**, and **RICH** subsystems, using an **Hlt1** data buffer system that holds the **Hlt1** calibration data until the automatic calibration has been performed. This significantly improves the resolution of invariant masses to offline quality at the **Hlt2** stage, allowing physics analysis directly on the output of the trigger [108]. At the **Hlt2** stage, a full track reconstruction of the event is performed, and the tracks are combined into composite particles. This way, full B-decay candidates are created, which are selected using requirements on the invariant masses, final state, and intermediate particle momenta, track and vertex quality, and also decay topology parameters like the impact parameter. This selection task is predestined for the usage of multivariate techniques like Boosted Decision Trees (BDT). A full diagram of the trigger data flow for Run 1 and Run 2 is given in Fig. 4.10.

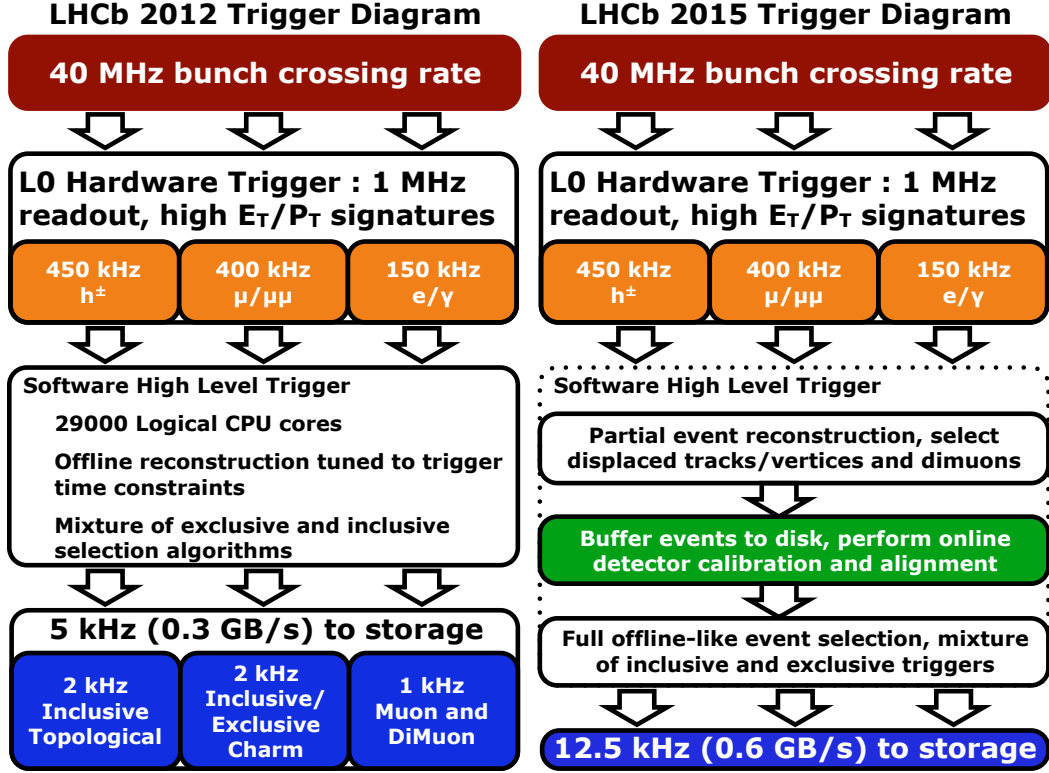


Figure 4.10: Schematic diagram of the LHCb trigger data-flow during Run 1 (left) and Run 2 (right) [109].

4.9 Track reconstruction

The analysis presented in this thesis uses only charged final state tracks that leave hits in the VELO as well as the tracking stations. These kinds of tracks are denoted as **long tracks** in the LHCb terminology. As a first step to construct a long track, the hit clusters in the VELO are combined to **VELO tracks** using pattern recognition algorithms [16]. As the VELO is mostly free of magnetic fields, these tracks follow a straight trajectory. To form a long track, the VELO tracks are combined with single hits in the tracking stations. Using only this information, the track is projected into the detector. Additional hit clusters in the tracker that coincide with the projection are added to the long track. This algorithm is called the forward tracking algorithm [110]. Another method reconstructs standalone tracks in the T tracking stations first and subsequently matches them with the list of VELO tracks using a χ^2 approach. This is called the track matching algorithm [110]. Hits from the TT are matched to the found track candidates to improve the momentum resolution. Given the detector hits of a track candidate, the Kalman filter technique [111] is used to iteratively determine the track parameters at the z location of each detector plane. The algorithm returns a track state vector containing the x and y coordinates of the hit, the slopes dx/dz and dy/dz of the track, and the ratio of the charge over momentum κ as a function of the z coordinate, which runs along the beam pipe. The Kalman filter technique is preferred over a global track fit as it is computationally less expensive.

4.10 Detector upgrade

During the LHC's first two run periods between 2010 and 2018, the LHCb experiment successfully collected a large amount of physics data equivalent to an integrated luminosity of 9 fb^{-1} in proton-proton collisions. However, to this point in time, the precision of a large number of measurements are still limited by the size of the available data set. Initially, the LHCb experiment has been designed to run at a reduced instantaneous luminosity to limit the pile-up. However, in practice, the detector was able to run at significantly higher luminosities than initially expected, due to the excellent performance of its subdetectors. For the LHC Run 3 the LHCb collaboration has decided to operate the detector at an instantaneous luminosity, which is 10 times higher than the initial design target from Run 1 and at the full LHC bunch crossing rate of 40 MHz [112]. The main limitation of the previous detector setup was the restriction on the L0 trigger output rate. To overcome this, a fully software-based online trigger system is required. Furthermore, at the target luminosity of $\mathcal{L} = 2 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$, the pile-up per bunch crossing is expected to increase to an average of 7.6 [112]. To facilitate these changes, a comprehensive upgrade of all subdetector systems, the LHCb software, and computing facilities was required. The silicon strip detectors from the VELO were replaced with silicon hybrid pixel detectors, and the distance between the first detector layer and the interaction point was reduced. Detector modules that were previously solely used in the hardware trigger are removed. The TT is replaced with the new UT silicon strip tracker. Hits detected in the UT are combined with the VELO tracks to significantly reduce the computational cost of the matching algorithms between the VELO and the T1-T3 trackers. The T1-T3 trackers in the pre-upgrade configuration were comprised of a combination of micro silicon strip detectors in the IT and straw-tube detectors in the OT. This detector design would be overwhelmed by the projected detector occupancy and was thus replaced by a Scintillating Fibre Tracker [113] that is read out using silicon photomultipliers. The general structure of the RICH system remains unchanged. However, the hybrid photomultipliers are exchanged as their readout electronics are designed to work at the old readout frequency of 1 MHz. As the L0 hardware trigger was retired and the trigger system was moved to a fully software-based online trigger that needs to process events at the bunch crossing rate of 40 MHz, the high-level trigger selections also needed to be revised. They are required to reduce the output bandwidth of the data saved to storage to approximately 10 GB/s [112]. Contrary to Run 2, all detector subsystems are read out at the full bunch crossing rate. The Hlt1 trigger stage performs a partial event reconstruction and an inclusive event selection based on detached, high p_T single tracks and track pairs, and lepton tracks that might also originate from the PV. Additionally, specific selections for the collection of calibration data samples are present. The Hlt1 stage is able to reduce the event rate after the first trigger stage in comparison to the visible bunch crossing rate by a factor of 30 to 60 [112]. To process the data read out from the subdetector systems at the speed dictated by the bunch crossing rate, the Hlt1 trigger software is run on a farm of GPU cards, exploiting the fact that the tracking algorithms are highly parallelizable. The data processed by the Hlt1 stage are stored in a buffering system that holds the data for processing in the Hlt2 stage and for real-time alignment and calibration. Similarly to Run 2, the alignment and calibration constants are determined in real-time to ensure offline level reconstruction

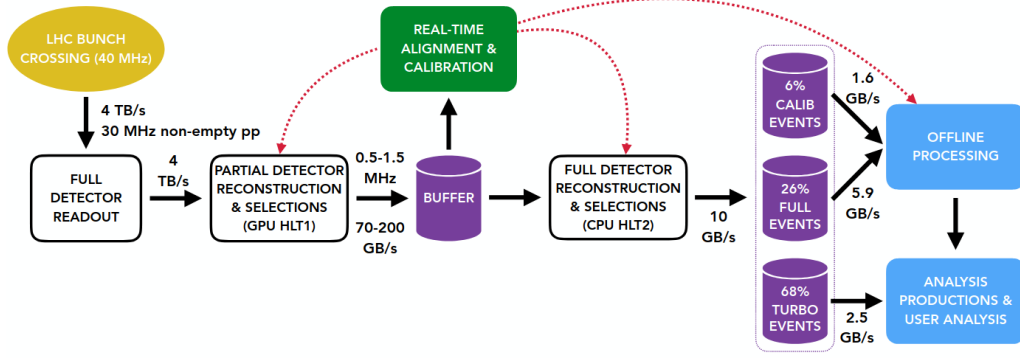


Figure 4.11: Diagram of the LHCb trigger data-flow during Run 3 [112].

quality at the Hlt2 stage. At the Hlt2 stage, a set of inclusive and exclusive offline quality selections is performed. A full event reconstruction is performed on a CPU farm. Depending on the type of the selection line, either the full event (Full stream) or only part of the event relevant for the specific selected signal candidate (Turbo stream) is saved to offline data storage. More details on the Hlt2 selection can be found in Chapter 5. A diagram of the trigger data flow during Run 3 is shown in Fig. 4.11. Within the framework of this thesis, a set of neural network-based, inclusive high-level trigger lines for rare decays with two charged leptons in the final state has been developed. These lines are designed to, amongst others, select the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ during Run 3, which is the main topic of this thesis. The development and performance of these trigger lines are described in the following chapter.

Chapter 5

Inclusive trigger for decays with detached dilepton pairs in Run 3

5.1 Real Time Analysis

Until the end of Run 2, the LHCb trigger system consisted of three trigger stages. The first stage was a hardware trigger system that selected events using p_T and E_T information from the muon chambers and calorimeters, because those were the only detector components that could be read out at sufficient speed. However, the Run 3 LHCb upgrade increases the instantaneous luminosity by such a large margin that this approach ceases to be efficient for the selection of hadronic decays [112]. To avoid this loss in efficiency, a fully software-based high-level trigger system is implemented that reads out all detector subsystems at the full bunch crossing rate. The average storage size of a complete event is expected to be approximately 100 kB. At the instantaneous target luminosity, the available bandwidth of the LHCb experiment is not sufficient to save all events of interest to the data storage. Thus, for the majority of the selections, a fully inclusive selection approach in Hlt2, where the full event is stored in the offline data storage after a positive trigger decision, is no longer feasible. To circumvent this problem, exclusive selection lines targeting specific decay modes are designed that exploit invariant mass windows and PID requirements to select the signal. These exclusive trigger lines only save the reconstructed signal candidate responsible for the positive trigger decision and some high-level information. This reduces the average size of a single event to about 10 kB. Inclusive Hlt2 selection lines aim to save the complete event to tape storage, which provides more space compared to the hard drives used for the exclusive trigger lines. As inclusive selections can not rely on decay-specific requirements like invariant mass windows of intermediate resonances, they mostly depend on multivariate classifiers with high separation power to suppress the background. At a later stage, exclusive selections, the sprucing lines, are used to select specific signal decay candidates from the events collected by the inclusive trigger lines and make them available to the analyst.

5.2 Inclusive Detached Dilepton HLT2 trigger

A major part of the LHCb physics program consists of the analysis of b -hadron decays via decay chains with two leptons in the final state. This also includes some of the most prominent analysis of LHCb in the past and present like the tests of lepton flavor universality $\mathcal{R}_K/\mathcal{R}_{K^*0}$ [85] and the angular analysis of $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [24], which is presented in this thesis. Analyses of rare B-decays like $B^0 \rightarrow K^{*0}\ell^+\ell^-$, $B_s^0 \rightarrow \phi\ell^+\ell^-$, $B^+ \rightarrow K^+\ell^+\ell^-$ and $B_{(s)}^0 \rightarrow \mu^+\mu^-$ are of key interest for the study of the flavor anomalies discussed in Sec. 3. Here, $\ell^+\ell^-$ represents any combination of electrons and muons ($\mu^+\mu^-$, e^+e^- , $e^+\mu^-$), including lepton flavor-violating decays. Key modes for the LHCb experiment are also, for example, the tree-level decays $B_s^0 \rightarrow J/\psi\phi$ and $B^0 \rightarrow J/\psi K_S$, where the J/ψ decays into two muons in the final state.

The selection of the aforementioned decays at the Hlt2 stage of the trigger is therefore a vital part of the real-time analysis strategy of the LHCb experiment. This chapter discusses the development of a set of Hlt2 selection lines that aim to inclusively select B -decays with detached pairs of charged leptons in the final state. These trigger lines are called the Inclusive Detached Dilepton (IDDL) lines in the following. In contrast to the exclusive Hlt2 selections that aim to select a specific decay channel, the IDDL selection aims to select all decays of b -hadrons into final states with two charged leptons. Due to the significantly increased instantaneous luminosity in Run 3 compared to previous runs, the selection has to exhibit a high background rejection power to meet the bandwidth restrictions of the experiment. The complete LHCb data infrastructure can store data with a bandwidth of about 10 GB/s [112]. The exclusive Hlt2 lines rely on invariant mass windows of the b -hadron and the intermediate resonances, and also PID variables. This is only partly possible when using an inclusive selection approach since the final state particles and intermediate resonances are not fully determined. Instead, the inclusive lines need to rely on a more efficient multivariate analysis (MVA) selection to suppress the combinatorial background. The approach is similar to the strategy of the topological trigger in previous data-taking periods [114]. In the topological trigger in Run 2, loose preselection cuts and a subsequent Boosted Decision Tree selection, which was trained on a cocktail of different signal decays, were applied. A key difference is the ability of the IDDL trigger lines to exploit cuts on the lepton PID, as each line of the IDDL trigger selects a specific type of dilepton pairs. The use of PID requirements reduces the amount of background significantly. Thus, this hybrid approach can be very efficient compared to both exclusive lines, which have the disadvantage that they do not usually use MVAs, and the topological trigger lines [115], which are not able to use PID cuts and therefore have to apply a more stringent MVA requirement. In contrast to the exclusive selections, the inclusive approach of the IDDL lines furthermore allows the selection of not fully reconstructed decays that might not have been envisioned at the time of development. This safeguards against the potential loss of analysis opportunities at later stages of the data taking.

5.2.1 Reconstruction and preselection

The IDDL trigger lines aim to reconstruct all relevant final state configurations with a pair of leptons detached from the PV. An example of such a signal decay can be seen in Fig. 4.3. The simplest topology are two body decays like $B_s^0 \rightarrow \mu^+\mu^-$ or

Table 5.1: This table summarizes the loose preselection cuts applied to the lepton tracks used to form the b -hadron candidate for all IDDL Hlt2 trigger lines.

Variable	Dimuon	Dielectron	MuE
p_T	$> 0.1 \text{ GeV}$	$> 0.25 \text{ GeV}$	$> 0.25 \text{ GeV}$
p	$> 2.0 \text{ GeV}$	$> 2.0 \text{ GeV}$	$> 2.0 \text{ GeV}$
χ_{IP}^2	> 3	> 3	> 3
PID_μ	> -3.0	—	> -3
PID_e	—	> 2.0	> 2.0
IsMuon	True	—	True

Table 5.2: This table summarizes the loose preselection cuts applied to the di-lepton candidate for all IDDL trigger lines. The di-lepton candidate coincides with the b -hadron candidate for the two-body Hlt2 lines.

Variable	Dimuons	Dielectrons	Electron/Muon
χ_{DOCA}^2	< 30	< 10	< 10
χ_{FD}^2	> 10	> 10	> 10
$\chi_{\text{vtx}}^2/\text{ndof}$	< 30	< 20	< 20
$m(\ell\ell)$	$< 6.5 \text{ GeV}$	$< 6.5 \text{ GeV}$	$< 6.5 \text{ GeV}$

$B^0 \rightarrow \mu^+ \mu^-$. These decays are fully reconstructed by two tracks, which are identified as leptons using PID requirements. Six separate Hlt2 selection lines (Two-body lines) are designed for $\mu^+ \mu^-$, $e^+ e^-$ and $\mu^+ e^-$ pairs, which in turn are also reconstructed with same-sign lepton charges. As the B-meson has a significant lifetime, the lepton candidates originate from a significantly detached SV. The combinatorial background mostly consists of random tracks from the PV. This means that the χ_{IP}^2 of the final state parameters and the χ_{FD}^2 of the B-meson candidate yield significant separation power. Additionally, combinatorial background tracks on average tend to have lower p_T compared to signal events. Requiring a small χ_{vtx}^2 ensures that the tracks of the two reconstructed leptons point back to the same vertex. This rejects a lot of background combinations that originate from different vertices. The dilepton invariant mass spectrum is restricted to be smaller than 6.5 GeV as there are no relevant signal candidates above this threshold. Loose preselection cuts are applied to reduce the number of track combinations. This has a significant impact on the number of possible signal candidates and, therefore, the run time of the trigger selection, which is also restricted by the hardware performance. The specific selection cuts on the final state leptons and the di-lepton candidate can be found in Tab. 5.1 and Tab. 5.2. The selection lines reconstructing lepton pairs with same-sign charges are used to collect data samples that are often used for background studies in analyses of corresponding decays with oppositely charged leptons. Additionally, the same-sign lepton charge selections are used in searches for heavy Majorana particles.

For signal decays with additional tracks on top of the dilepton pair in the final state, the two-body lines are expected to exhibit a reduced selection efficiency. This is due to the fact that the decay is not fully reconstructed, which leads to degraded resolution of

Table 5.3: This table summarizes the loose preselection cuts applied to the additional tracks which are added to the b -hadron candidate for the multi-body lines.

Variable	Dimuons	Dielectrons	Electron/Muon
p_T	$> 0.1 \text{ GeV}$	$> 0.1 \text{ GeV}$	$> 0.1 \text{ GeV}$
p	$> 2.0 \text{ GeV}$	$> 2.0 \text{ GeV}$	$> 2.0 \text{ GeV}$
χ_{IP}^2	> 6	> 7	> 7
PID_x	-	-	-

Table 5.4: The loose preselection requirements applied to the b -hadron candidate for the multi-body IDDL Hlt2 trigger lines.

Variable	Dimuons	Dielectrons	Electron/Muon
$B \chi_{\text{FD}}^2$	> 10	> 10	> 10
$B \chi_{\text{vtx}}^2$	< 16	< 16	< 16

relevant parameters like χ_{vtx}^2 used in the selection. The resolution for these parameters, and thus the selection efficiency, can be improved by considering additional tracks pointing to the same SV as the detached dilepton pair. No assumption on the type of these additional tracks is made. More sets of Hlt2 lines are added for three-body and four-body topologies. One or two additional tracks are added to the di-lepton candidate, respectively. The additional tracks are also required to originate from the same secondary vertex as the di-lepton candidate. To be inclusive with respect to all di-lepton decay channels, no specific PID requirements are applied to these tracks. The requirements imposed on the additional tracks are listed in Tab. 5.3. Additional requirements are imposed on the b -hadron candidate, which can be found in Tab. 5.4. These lines fully reconstruct all decays with four or fewer particles in the final state. However, since all tracks are required to originate directly from the secondary vertex, decays containing an intermediate resonance with significant lifetime like $\Lambda_b \rightarrow \Lambda^0 \mu^+ \mu^-$ are suppressed. Therefore, additional lines are added, allowing for a neutral intermediate resonance (V^0) that decays into two oppositely charged tracks. Thus, the additional tracks are constrained to a tertiary vertex, which is detached from the secondary vertex. To reduce overlap with the four-body line, a minimum on the χ_{FD}^2 of the intermediate resonance ($V^0 \chi_{\text{FD}}^2 > 4$) is required.

5.2.2 Multivariate selection

After the preselection has been applied, the main part of the background reduction is performed by a neural net. Overall, twelve separate multivariate analysis (MVA) classifiers are trained for two-body, three-body, four-body, and V^0 Hlt2 lines with three different di-lepton configurations each. Lines with same-sign and opposite-sign lepton charges use the same MVA to preserve the possibility of using the same-sign lepton data sample as a control sample for combinatorial background.

The MVA is trained on Monte Carlo simulation, which is generated using a specific detector configuration. As MVA model, monotone Lipschitz neural networks [116]

are chosen. This type of neural network has properties that are advantageous for the specific use case during the data taking at LHCb. The configuration of the experiment is expected to slightly change over time due to adjustments in the selection and reconstruction algorithms or through physical changes caused by the harsh radiation environment. Therefore, it is a necessity that the response of the MVA is robust against small changes in the input distributions. Secondly, some rare events are of exceptional interest to the physics analysis. These events include candidates that are highly detached from the primary vertex and have very high transverse momentum. Due to the rare nature of these outlier candidates, they might not be contained in the training sample in large quantities. Thus, the neural net might learn to reject events in this parameter space if no external constraints are enforced.

The robustness of the network output against small changes is achieved by enforcing that the difference between the network output of two different input vectors $\mathbf{x}$ and $\mathbf{y}$ is smaller than a boundary proportional to the Lipschitz-1 norm of $\mathbf{x} - \mathbf{y}$ [116].

$$|g(\mathbf{x}) - g(\mathbf{y})| \leq \lambda \|\mathbf{x} - \mathbf{y}\|_1 \quad (5.1)$$

Here, λ is the Lipschitz constant, which is chosen before the training of the network. $\|\mathbf{s}\|_1$ is the Lipschitz-1 norm of the vector $\mathbf{s}$. A densely connected neural network $g(x)$ with an activation function σ can be expressed in terms of its weight matrices W^i and bias vectors b_i as the recursive function

$$g(\mathbf{x}) = g^m(\mathbf{x}) = W^m \sigma(g^{m-1}(\mathbf{x})) + b^m, \quad (5.2)$$

where m is the maximum number of densely connected layers of the neural network. Equation 5.1 is enforced by rescaling the Weight matrices with the factor

$$W^i \rightarrow W^i \times \text{diag} \left(\frac{1}{\max(1, \lambda^{-1/m} \sum_j |W_{jk}^i|)} \right). \quad (5.3)$$

This automatically enforces

$$\prod_{i=0}^m \|W^i\|_1 < \lambda, \quad (5.4)$$

which is in turn satisfactory to fulfill Eq. 5.1. This generally ensures that the gradient of the neural network output can never be unexpectedly large, which prevents the network response from being highly dependent on small changes in the inputs [116].

To prevent the network from rejecting interesting but rare candidates, which might lie outside of the usual signal phase space of the training sample, it makes sense to force the network response to be monotone in some input variables. As mentioned before, highly detached and high p_T candidates are of particular interest. Therefore, the b -hadron χ_{FD}^2 and the transverse momenta p_T are chosen to be forced to be monotone in the MVA response. This can be done by adding the following adversarial term to the structure of the densely connected neural network [116].

$$f(\mathbf{x}) = g(\mathbf{x}) + \lambda \sum_{i \in I} x_i \quad (5.5)$$

Table 5.5: List of all signal Monte Carlo simulation decay modes used to train the Lipschitz Monotonic Neural Networks.

Dimuon	Dielectron	Electron/Muon
$B^0 \rightarrow \mu^+ \mu^-$	$B^0 \rightarrow (K^{*0} \rightarrow K^+ \pi^-) e^+ e^-$	$B^0 \rightarrow (K^{*0} \rightarrow K^+ \pi^-) \mu^+ e^-$
$B^0 \rightarrow K_S^0 \mu^+ \mu^-$	$B^0 \rightarrow (K_S^0 \rightarrow \pi^+ \pi^-) e^+ e^-$	$B^+ \rightarrow \mu^+ (N \rightarrow \pi^- e^+)$
$B^+ \rightarrow \Lambda^0 p^+ \mu^+ \mu^-$	$B^+ \rightarrow K^+ e^+ e^-$	$B^+ \rightarrow e^+ (N \rightarrow \pi^- \mu^+)$
$B_s^0 \rightarrow (\phi \rightarrow K^+ K^-) \mu^+ \mu^-$	$B^+ \rightarrow (K^{*+} \rightarrow (K_S^0 \rightarrow \pi^+ \pi^-) \pi^+) e^+ e^-$	$B^+ \rightarrow K^+ e^+ \mu^-$
$B^0 \rightarrow (K^{*0} \rightarrow K^+ \pi^-) \mu^+ \mu^-$	$B^+ \rightarrow (K_1^+ \rightarrow K^+ \pi^- \pi^+) e^- e^+$	$B^+ \rightarrow (K^{*+} \rightarrow (K_S^0 \rightarrow \pi^+ \pi^-) \pi^+) e^+ \mu^-$
$B^+ \rightarrow K^+ \mu^+ \mu^-$	$B^+ \rightarrow \Lambda^0 p^+ e^+ e^-$	$B_s^0 \rightarrow e^+ \mu^-$
$B^+ \rightarrow (K_1^+ \rightarrow K^+ \pi^- \pi^+) \mu^- \mu^+$	$B_s^0 \rightarrow e^+ e^-$	$B_s^0 \rightarrow (\phi \rightarrow K^+ K^-) e^+ \mu^-$
$B_s^0 \rightarrow \mu^+ \mu^-$	$B_s^0 \rightarrow (\phi \rightarrow K^+ K^-) e^+ e^-$	$\Lambda_b^0 \rightarrow (\Lambda^0 \rightarrow p^+ \pi^-) e^+ \mu^-$
$B^0 \rightarrow \mu^+ \mu^- \mu^+ \mu^-$	$\Lambda_b^0 \rightarrow (\Lambda^0 \rightarrow p^+ \pi^-) e^+ e^-$	
$B^+ \rightarrow (K^{*+} \rightarrow K_S^0 \pi^+) \mu^+ \mu^-$		
$B_s^0 \rightarrow \gamma \mu^+ \mu^-$		
$\Lambda_b^0 \rightarrow p^+ K^- \mu^+ \mu^-$		
$B_s^0 \rightarrow p^+ p^- \mu^+ \mu^-$		
$\Lambda_b^0 \rightarrow (\Lambda^0 \rightarrow p^+ \pi^-) \mu^+ \mu^-$		

Here, the symbol I denotes the group of all input variables that are forced to be monotone with respect to the network response. The derivative of this new expression for the network $f(\mathbf{x})$ with respect to an input variable contained in I is therefore always larger than zero due to the fact that the absolute value of the derivative of $g(\mathbf{x})$ is bounded by the Lipschitz constant λ . This prevents the MVA efficiency from decreasing in important phase space regions where no large quantity of signal simulation events is present.

All neural networks used in these trigger lines are fully trained on Monte Carlo simulation. The signal proxy consists of a cocktail of simulated rare b -hadron decays with two leptons in the final state. These simulation samples are generated with the expected Run 3 conditions and fully processed up to the Hlt2 level, which also includes an HLT1 filtering. A full list containing all decay modes used in the training can be found in Tab. 5.5. As background proxy for the neural network training, a special Monte Carlo sample is used that simulates particles produced by the proton-proton collisions without a specific signal decay that needs to be contained in any event. This so-called minimum bias simulation resembles the data seen by the LHCb experiment at the Hlt2 stage. The input variables used for the different types of neural nets are shown in Tab. 5.6. In addition to the variables already mentioned before, the neural nets use DIRA angles, χ_{vtx}^2 , and corrected masses M_{corr} for different composite particles. Especially M_{corr} is very efficient in rejecting the combinatorial background. This quantity aims to estimate the mass of the b -hadron, correcting for missing momentum [114]. While the signal peaks in the region of the b -hadron mass, the combinatorial background is flat, as it mostly does not originate from a physical particle decay whose mass could be reconstructed. The signal and background samples are split into a training and a testing sample. The testing sample consists of 25% of the background and signal proxy. The remaining simulation is used for the training. A potential pitfall that can occur when training complicated neural networks is overtraining. When an MVA classifier with a large number of degrees of freedom is optimized on a small sample of training events, the neural network might learn to distinguish signal from background based on statistical fluctuations in the training

Table 5.6: List of all training variables used for the different types of neural networks. Input variables marked with a M are forced to be monotone in the MVA response.

2-Body MVA	3-Body MVA	4-Body MVA	V0 MVA
$(\ell\ell) \theta_{\text{DIRA}}$	$(\ell\ell) \theta_{\text{DIRA}}$	$(\ell\ell) \theta_{\text{DIRA}}$	$(\ell\ell) \theta_{\text{DIRA}}$
$(\ell\ell) \chi_{\text{vtx}}^2$	$(\ell\ell) \chi_{\text{vtx}}^2$	$(\ell\ell) \chi_{\text{vtx}}^2$	$(\ell\ell) \chi_{\text{vtx}}^2$
$(\ell\ell) \chi_{\text{FD}}^2 \text{ (M)}$	$(\ell\ell) \chi_{\text{FD}}^2 \text{ (M)}$	$(\ell\ell) \chi_{\text{FD}}^2 \text{ (M)}$	$(\ell\ell) \chi_{\text{FD}}^2 \text{ (M)}$
$(\ell\ell) \min(\chi_{\text{IP}}^2)$	$(\ell\ell) \min(\chi_{\text{IP}}^2)$	$(\ell\ell) \min(\chi_{\text{IP}}^2)$	$(\ell\ell) \min(\chi_{\text{IP}}^2)$
$(\ell\ell) M_{\text{corr}}$	$(\ell\ell) M_{\text{corr}}$	$(\ell\ell) M_{\text{corr}}$	$(\ell\ell) M_{\text{corr}}$
$(\ell\ell) p_T \text{ (M)}$	$(\ell\ell) p_T \text{ (M)}$	$(\ell\ell) p_T \text{ (M)}$	$(\ell\ell) p_T \text{ (M)}$
$(\ell) \min(\chi_{\text{IP}}^2)$	$(\ell) \min(\chi_{\text{IP}}^2)$	$(\ell) \min(\chi_{\text{IP}}^2)$	$(\ell) \min(\chi_{\text{IP}}^2)$
$(\ell) p_T \text{ (M)}$	$(\ell) p_T \text{ (M)}$	$(\ell) p_T \text{ (M)}$	$(\ell) p_T \text{ (M)}$
	$(B) \theta_{\text{DIRA}}$	$(B) \theta_{\text{DIRA}}$	$(B) \theta_{\text{DIRA}}$
	$(B) \chi_{\text{vtx}}^2$	$(B) \chi_{\text{vtx}}^2$	$(B) \chi_{\text{vtx}}^2$
	$(B) \chi_{\text{FD}}^2 \text{ (M)}$	$(B) \chi_{\text{FD}}^2 \text{ (M)}$	$(B) \chi_{\text{FD}}^2 \text{ (M)}$
	$(B) \min(\chi_{\text{IP}}^2)$	$(B) \min(\chi_{\text{IP}}^2)$	$(B) \min(\chi_{\text{IP}}^2)$
	$(B) M_{\text{corr}}$	$(B) M_{\text{corr}}$	$(B) M_{\text{corr}}$
	$(B) p_T \text{ (M)}$	$(B) p_T \text{ (M)}$	$(B) p_T \text{ (M)}$
	$\pi \min(\chi_{\text{IP}}^2)$	$\pi \min(\chi_{\text{IP}}^2)$	$\pi \min(\chi_{\text{IP}}^2)$
	πp_T	πp_T	πp_T
			$(V^0) \chi_{\text{vtx}}^2$
			$(V^0) \chi_{\text{FD}}^2 \text{ (M)}$
			$(V^0) \min(\chi_{\text{IP}}^2)$
			$(V^0) p_T \text{ (M)}$

sample, which do not reflect the true distribution in data. This would reduce the performance of the MVA classifier depending on its severity. To rule out potential overtraining, the MVA classifier response distributions of the testing and training samples are displayed in Figs. 5.1 and 5.2. If overtraining were present, there would be large differences in the response distributions of training and testing samples, as the model is optimized on the fluctuations of the training samples, which do not occur in the testing samples. No significant deviations of the training and testing distributions can be observed. Thus, there is no overtraining present. The plots also show excellent separation power between signal and background.

The performance of the neural nets depends on the decay mode in question. This is evident from Fig. 5.3 and Fig. 5.4, which show the selection efficiencies as a function of the chosen MVA cut. The plots contain the efficiencies of different signal decays, and also simulated minimum bias events. Decay channels that are fully reconstructed perform significantly better in the corresponding MVA selection. The decay mode $B_s^0 \rightarrow \mu^+ \mu^-$, for example, is still nearly 100% efficient until very high MVA cut values

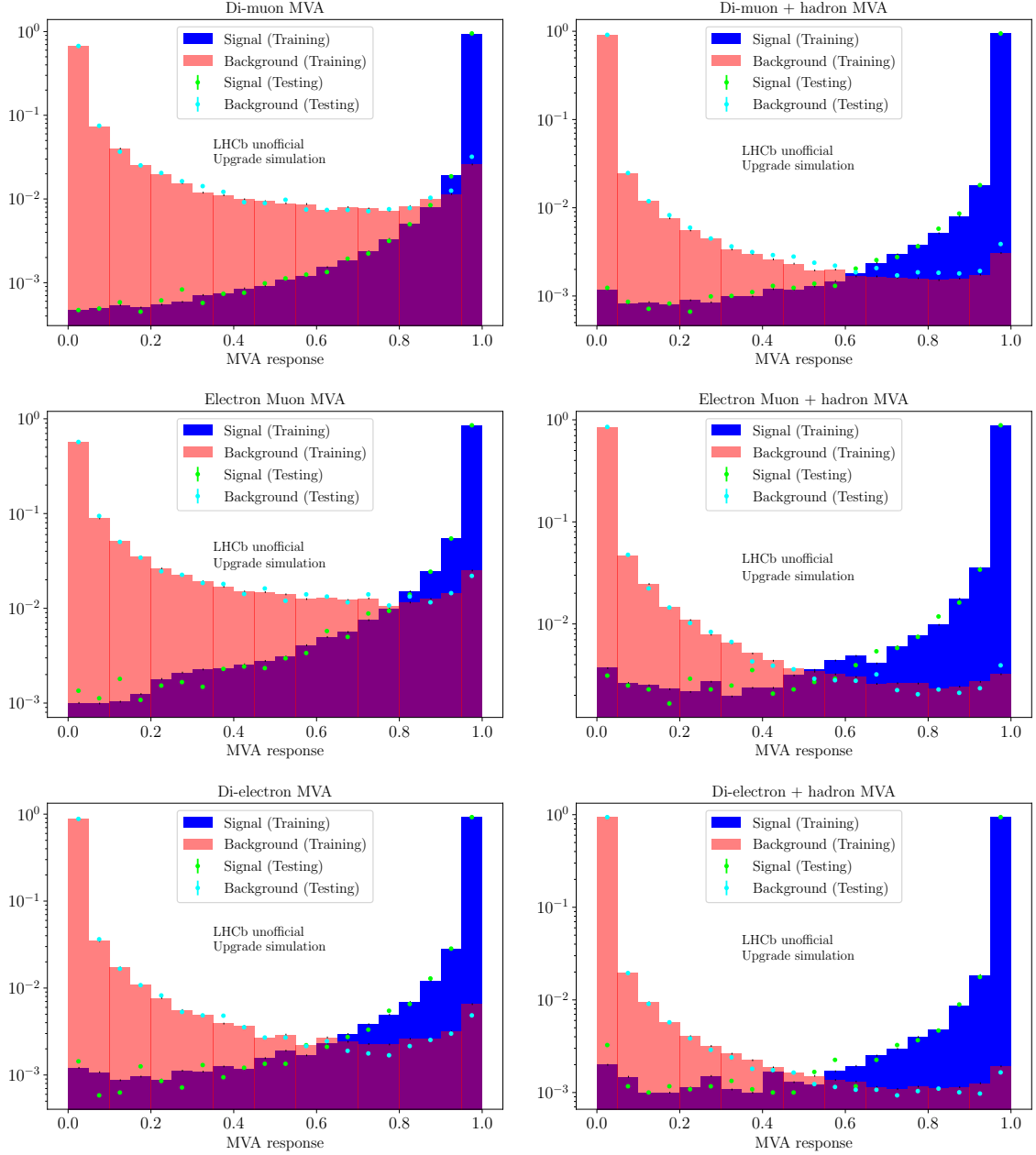


Figure 5.1: Plots comparing the training and testing distributions of the two-body and three-body MVA response for signal (blue) and combinatorial background (red). Significant differences between training and testing responses would be a sign of overtraining. However, no such effect is observed for either of the different MVAs.

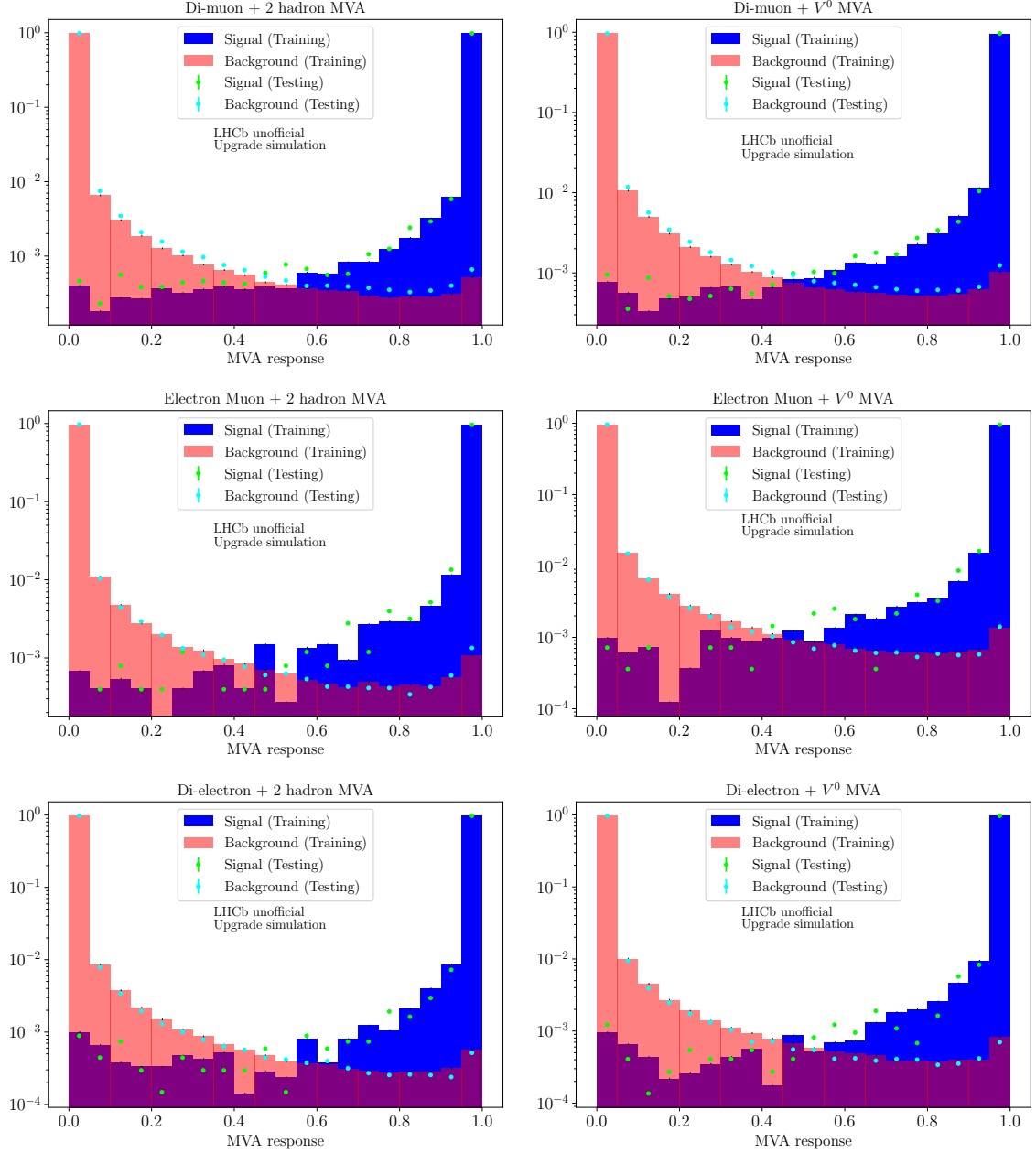


Figure 5.2: Plots comparing the training and testing distributions of the four-body and V^0 MVA response for signal (blue) and combinatorial background (red). Significant differences between training and testing responses would be a sign of overtraining. However, no such effect is observed for either of the different MVAs.

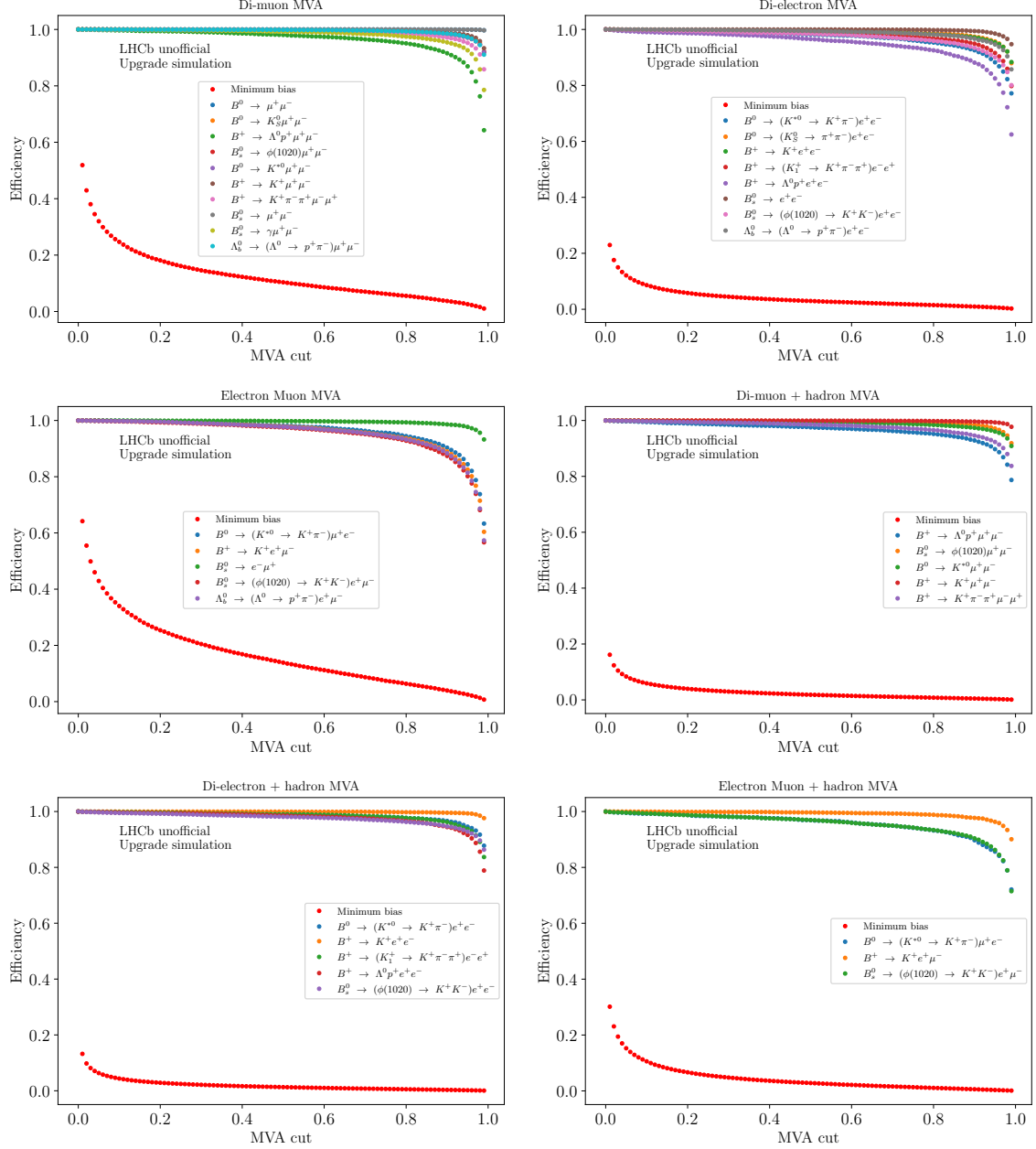


Figure 5.3: Plots comparing the selection efficiencies of two-body and three-body IDDL trigger MVA classifiers as a function of the MVA cut value for different decay modes.

of the two-body di-muon MVA, while the efficiency of three and four-body decays already drops off much earlier. This is expected as the full information of all final state particles maximizes the resolution of the B -hadron parameter distributions, like χ^2_{vtx} or M_{corr} . This improves the separation power of those variables significantly. The same is true for the three-body MVA where the $B^0 \rightarrow K^+ \mu^+ \mu^-$ mode is significantly more efficient compared to four-body decays. The V^0 MVA is especially efficient for $B^0 \rightarrow K_S^0 \mu^+ \mu^-$ and $\Lambda_b \rightarrow \Lambda^0 \mu^+ \mu^-$ as it can distinguish secondary and tertiary vertices.

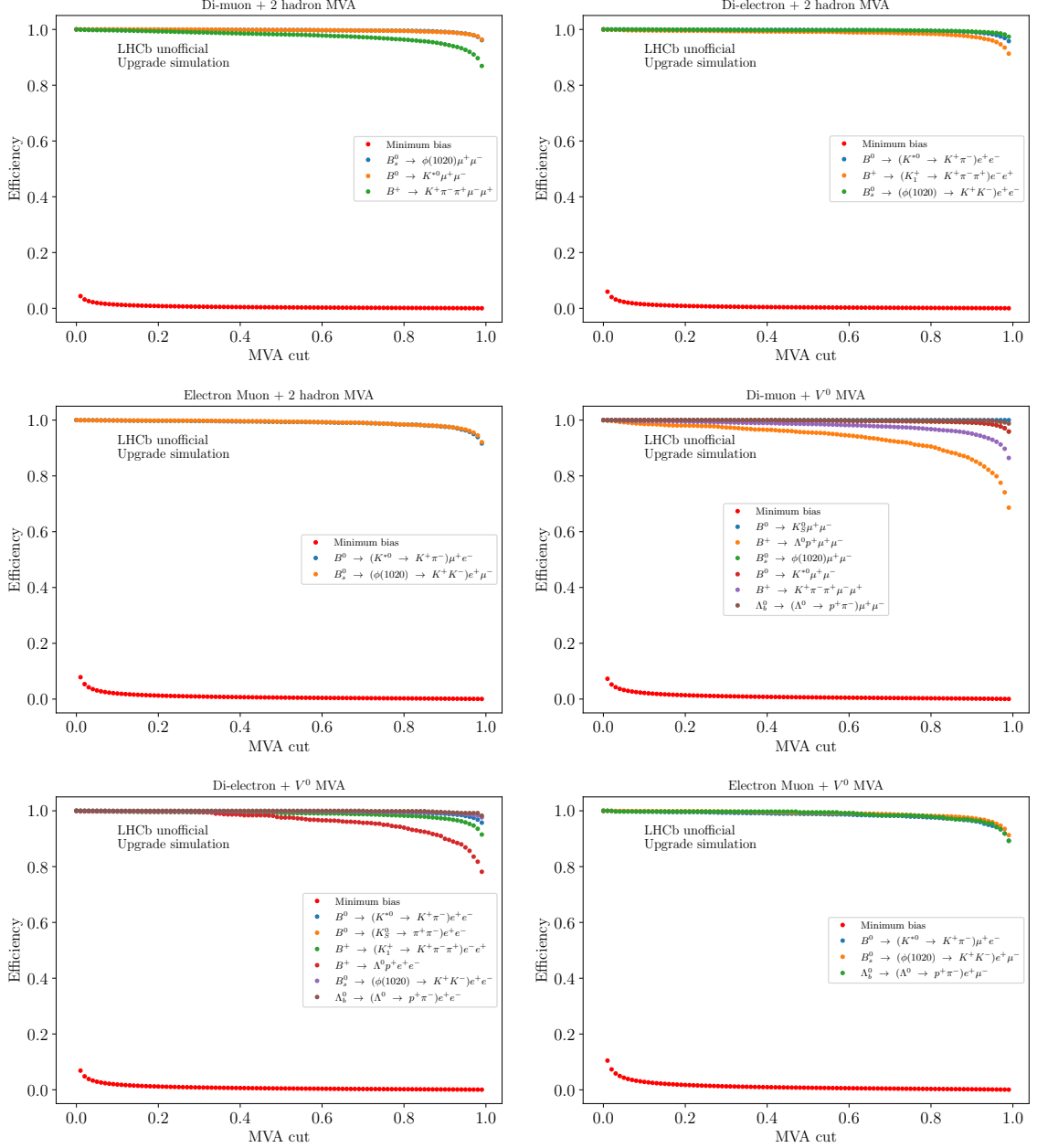


Figure 5.4: Plots comparing the selection efficiencies of the four-body and V^0 IDDL trigger MVA classifier as a function of the MVA cut value for different decay modes.

5.2.3 Estimation of minimum bias rate and bandwidth

The performance of the Hlt2 trigger lines is mainly limited by the allowed bandwidth of the LHCb experiment. As mentioned before, the amount of data per second that can be stored on disk or tape is limited to about 10 GB/s for the complete Hlt2 output. This bandwidth is split between different selections according to the expected rate and importance of the selected decay modes. The IDDL lines are a special case, as the

complete selected event with all tracks is saved, while for the exclusive Hlt2 lines, only the selected candidate is written to disk. This results in a much larger typical storage size of a single event selected by the IDDL lines compared to the exclusive lines. An average event storage size for this trigger is about 100 kB. This is a constant number and can not be influenced by the design of the Hlt2 line, but is an inherent property of the storage process and also the experiment. The only other variable impacting the total bandwidth of the inclusive lines is the rate of selected minimum bias events. It describes the number of events that are expected to be selected by the trigger lines per second. This number is directly dependent on the selection criteria of the Hlt2 lines. The rate is estimated using minimum bias simulation samples, which are filtered with an up-to-date Hlt1 selection and subsequently are selected by the studied Hlt2 lines. The efficiency on minimum bias events is directly proportional to the resulting output rate. The rate is also dependent on the run conditions, as pile-up and Hlt1 output rate can differ depending on the collider and detector setup. The expected conditions of the 2024 run period result in an expected total minimum bias rate of about 9 kHz for all IDDL lines combined. The rates of each line can be found in Tab. 5.7. The inclusive rate describes the rate of all events that are triggered by one specific trigger line. The exclusive rate only takes into account events that are solely triggered by this specific line, and not by any other. From these numbers, it is evident that there is a significant overlap between the two-, three-, four-body and V^0 lines, which is to be expected as they are designed to select the same decay signatures. The total expected minimum bias rate calculated from simulation roughly corresponds to a bandwidth of 1 GB/s, which is consistent with the observed bandwidth during data taking [117].

5.2.4 Estimation of selection efficiency

The Hlt2 selection efficiency is estimated relative to the Hlt1 selection stage using Monte Carlo simulation. The trigger efficiency is defined on the subset of events where all final state particles of the signal left reconstructible long tracks in the detector and were selected by the Hlt1 trigger. Since the two-, three-, four-body and V^0 lines do partially select the same signal decays, these lines are combined in groups for each di-lepton type ($\mu\mu$, ee , $e\mu$). The efficiencies quoted in Tab. 5.8 show the selection efficiency of the group corresponding to the decay. An event is counted as accepted by a group if at least one of the associated lines selects it. The efficiencies are calculated for a wide range of decays with dilepton pairs in the final state, with available simulation samples. Due to the PID cuts, the efficiency of lines that do not match the type of the signal di-lepton is negligible and not shown. From the shown efficiencies, it is clear that lines involving electrons are significantly less efficient compared to their muon counterparts. As expected, the efficiencies of the μ^+e^- lines lie between the di-muon and di-electron efficiencies. The reduced selection efficiencies of electrons occur due to the lower resolution of key selection variables used in the preselection and MVA, due to bremsstrahlung effects. Additionally, more stringent PID cuts are applied to electrons to reduce background from pions from the PV.

Table 5.7: Table showing the expected rates of selected minimum bias events of the IDDL Hlt2 trigger for the run conditions of 2025. Inclusive and exclusive rates are listed for each trigger line individually.

Trigger line		Inclusive rate [kHz]	Exclusive rate [kHz]
e^+e^-	(OS)	1.16 ± 0.11	0.52 ± 0.07
e^+e^-	(SS)	0.79 ± 0.09	0.40 ± 0.06
$e^+e^-h^+$	(OS)	1.09 ± 0.10	0.13 ± 0.04
$e^+e^-h^+$	(SS)	0.84 ± 0.09	0.14 ± 0.04
$e^+e^-h^+h^-$	(OS)	0.91 ± 0.10	0.02 ± 0.01
$e^+e^-h^+h^+$	(OS)	0.63 ± 0.08	0.07 ± 0.03
$e^+e^-h^+h^-$	(SS)	0.74 ± 0.09	0.09 ± 0.03
$e^+e^-V^0$	(OS)	1.30 ± 0.11	0.30 ± 0.06
$e^+e^-V^0$	(SS)	1.17 ± 0.11	0.30 ± 0.06
$e^+\mu^-$	(OS)	0.59 ± 0.08	0.38 ± 0.06
$e^+\mu^-$	(SS)	0.29 ± 0.05	0.13 ± 0.04
$e^+\mu^-h^+$	(OS)	0.41 ± 0.06	0.08 ± 0.03
$e^+\mu^-h^+$	(SS)	0.22 ± 0.05	0.04 ± 0.02
$e^+\mu^-h^+h^-$	(OS)	0.42 ± 0.07	0.04 ± 0.02
$e^+\mu^-h^+h^+$	(OS)	0.25 ± 0.05	0.04 ± 0.02
$e^+\mu^-h^+h^-$	(SS)	0.27 ± 0.05	0.03 ± 0.02
$e^+\mu^-V^0$	(OS)	0.42 ± 0.07	0.02 ± 0.01
$e^+\mu^-V^0$	(SS)	0.28 ± 0.05	0.02 ± 0.01
$\mu^+\mu^-$	(OS)	2.49 ± 0.16	1.40 ± 0.12
$\mu^+\mu^-$	(SS)	0.62 ± 0.08	0.31 ± 0.06
$\mu^+\mu^-h^+$	(OS)	1.24 ± 0.11	0.19 ± 0.04
$\mu^+\mu^-h^+$	(SS)	0.52 ± 0.07	0.16 ± 0.04
$\mu^+\mu^-h^+h^-$	(OS)	0.44 ± 0.07	0.03 ± 0.02
$\mu^+\mu^-h^+h^+$	(OS)	0.12 ± 0.04	0.00 ± 0.00
$\mu^+\mu^-h^+h^-$	(SS)	0.16 ± 0.04	0.00 ± 0.00
$\mu^+\mu^-V^0$	(OS)	0.50 ± 0.07	0.07 ± 0.03
$\mu^+\mu^-V^0$	(SS)	0.26 ± 0.05	0.02 ± 0.01
Total		9.05 ± 0.30	9.05 ± 0.30

5.2.5 Comparison to exclusive Hlt2 trigger lines

As mentioned before, there are two fundamentally different approaches to the design of an Hlt2 selection for a large number of signal decay modes. The exclusive trigger approach creates separate lines for each decay, while the Inclusive Detached Di-lepton trigger contains only a few selection lines that should cover a large number of lines. The advantage of the inclusive trigger lines lies in the possibility of using MVA techniques. For the exclusive approach, this is not feasible as the evaluation of a large number of different MVA models would decrease the throughput of the Hlt2 trigger. To compensate for that, the exclusive lines apply cuts on invariant mass windows and more stringent cuts on PID variables. The inclusive lines also apply much looser cuts

Table 5.8: Table showing the total Hlt2 trigger efficiencies of the IDDL lines on a variety of rare mode decays. The efficiencies are quoted for the combination of two-, three-, four-body, and V^0 lines.

Decay	Incl. $\mu\mu$	Incl. ee	Incl. μe
$B^0 \rightarrow \mu^+\mu^-$	0.832 ± 0.016		
$B_s^0 \rightarrow \mu^+\mu^-$	0.825 ± 0.016		
$B^0 \rightarrow (K^{*0}(892) \rightarrow K^+\pi^-)\mu^+\mu^-$	0.802 ± 0.030		
$B_s^0 \rightarrow (\phi \rightarrow K^+K^-)\mu^+\mu^-$	0.721 ± 0.029		
$B^+ \rightarrow \Lambda^0 p^+\mu^+\mu^-$	0.597 ± 0.029		
$\Lambda_b^0 \rightarrow (\Lambda^0 \rightarrow p^+\pi^-)\mu^+\mu^-$	0.840 ± 0.073		
$\Lambda_b^0 \rightarrow p^+K^-\mu^+\mu^-$	0.632 ± 0.032		
$B^+ \rightarrow \pi^+\mu^+\mu^-$	0.744 ± 0.022		
$B^0 \rightarrow p^+p^-\mu^+\mu^-$	0.641 ± 0.031		
$B^0 \rightarrow (\rho^0(770) \rightarrow \pi^+\pi^-)\mu^+\mu^-$	0.728 ± 0.028		
$B_s^0 \rightarrow \gamma\mu^+\mu^-$	0.855 ± 0.027		
$B^0 \rightarrow (K^{*0}(892) \rightarrow K^+\pi^-)e^+e^-$		0.475 ± 0.046	
$B^+ \rightarrow K^+e^+e^-$		0.515 ± 0.032	
$B^+ \rightarrow \pi^+e^+e^-$		0.478 ± 0.030	
$B_s^0 \rightarrow (\phi \rightarrow K^+K^-)e^+e^-$		0.483 ± 0.038	
$\Lambda_b^0 \rightarrow p^+K^-e^+e^-$		0.431 ± 0.040	
$B^0 \rightarrow e^+\mu^-$			0.556 ± 0.033
$B^+ \rightarrow K^+e^+\mu^-$			0.579 ± 0.039
$B_s^0 \rightarrow e^-\mu^+$			0.601 ± 0.034

in the preselection compared to the exclusive lines, which provides the analyst with more possibilities to tune the selection offline. Table 5.9 shows the comparison of Hlt2 selection efficiencies as described in the previous section for different important signal modes¹. The list of specific exclusive Hlt2 selection lines used in the comparison is given in Tab. A.1. The inclusive lines outperform the exclusive lines by a significant margin in all decay modes shown. The inclusive efficiencies shown here neglect the effect of the sprucing that will be applied on top. Since the sprucing selection is aligned to the inclusive trigger preselection, its effect on the final efficiency will be small and does not change the qualitative result of this comparison. The loss in efficiency of the exclusive lines is mostly explained by the very hard PID cuts used to reduce the minimum bias rate, as no MVA selection is available.

5.2.6 Efficiencies of tree-level decays including charmonium resonances

Tree-level b -hadron decays, including charmonium resonances, play an important part in the LHCb physics program. To be able to use the control mode decays to cancel systematic uncertainties on the branching fractions of the rare mode, it is important to align the selection between both modes as much as possible. This also means that both modes should be selected by the same trigger lines. The used MVAs are trained using only rare mode simulation samples since they are the main focus of these trigger

¹Comparison made on Moore master branch (13.08.2025)

Table 5.9: This table compares the selection efficiencies of the IDDL lines to the exclusive Hlt2 lines. The inclusive lines outperform the exclusive lines by a significant margin in all decay modes shown here.

Decay	Incl. $\mu\mu$	Incl. ee	Incl. μe	Excl. line
$B^0 \rightarrow (K^{*0}(892) \rightarrow K^+\pi^-)\mu^+\mu^-$	0.802 ± 0.030			0.395 ± 0.037
$B^0 \rightarrow (K^{*0}(892) \rightarrow K^+\pi^-)e^+e^-$		0.475 ± 0.046		0.229 ± 0.039
$B^+ \rightarrow K^+e^+e^-$		0.515 ± 0.032		0.401 ± 0.032
$B^+ \rightarrow K^+e^+\mu^-$			0.579 ± 0.039	0.428 ± 0.039
$B_s^0 \rightarrow \mu^+\mu^-$	0.825 ± 0.016			0.558 ± 0.021
$B_s^0 \rightarrow e^-\mu^+$			0.601 ± 0.034	0.549 ± 0.034
$B_s^0 \rightarrow (\Phi(1020) \rightarrow K^+K^-)\mu^+\mu^-$	0.721 ± 0.029			0.532 ± 0.033
$B_s^0 \rightarrow (\Phi(1020) \rightarrow K^+K^-)e^+e^-$		0.483 ± 0.038		0.291 ± 0.035
$\Lambda_b^0 \rightarrow p^+K^-\mu^+\mu^-$	0.632 ± 0.032			0.126 ± 0.022
$\Lambda_b^0 \rightarrow p^+K^-e^+e^-$		0.431 ± 0.040		0.124 ± 0.027

Table 5.10: This table shows the trigger efficiencies of tree-level b -hadron decays, including charmonium resonances for the IDDL trigger lines and the corresponding exclusive lines.

Decay	Incl. $\mu\mu$	Incl. ee	Excl. line
$B^0 \rightarrow (J/\psi(1S) \rightarrow \mu^+\mu^-)(K^{*0}(892) \rightarrow K^+\pi^-)$	0.723 ± 0.038		0.277 ± 0.038
$B^0 \rightarrow (J/\psi(1S) \rightarrow e^+e^-)(K^{*0}(892) \rightarrow K^+\pi^-)$		0.396 ± 0.071	0.188 ± 0.056
$B_s^0 \rightarrow (J/\psi(1S) \rightarrow \mu^+\mu^-)(\Phi(1020) \rightarrow K^+K^-)$	0.807 ± 0.030		0.626 ± 0.037
$B_s^0 \rightarrow (J/\psi(1S) \rightarrow e^+e^-)(\Phi(1020) \rightarrow K^+K^-)$		0.500 ± 0.072	0.292 ± 0.066
$B^+ \rightarrow (J/\psi(1S) \rightarrow \mu^+\mu^-)K^+$	0.770 ± 0.026		0.523 ± 0.031
$B^+ \rightarrow (J/\psi(1S) \rightarrow e^+e^-)K^+$		0.500 ± 0.065	0.217 ± 0.053

lines. However, since the rare mode q^2 distribution covers the complete available phase space, the selection should be agnostic about the q^2 distribution of the decay in question. Table 5.10 shows the comparison of the trigger efficiencies between inclusive and exclusive lines for different relevant b -hadron decays, including the J/ψ resonance. The list of specific exclusive Hlt2 selection lines used in the comparison is given in Tab. A.1. The IDDL trigger outperforms the exclusive selection lines specifically designed for the charmonium modes by a significant margin.

5.2.7 First glance at Run 3 data

The LHCb experiment started collecting Run 3 data in 2023. In 2024 alone, an integrated luminosity of 9.56 fb^{-1} has been collected at a center of mass energy of 13.6 TeV. This is more than all previous run periods combined. To verify the functionality of the trigger lines, the reconstructed mass distributions of some important decay modes with sufficiently high abundance in the data are examined. As this is only a qualitative check, only a fraction of the available data is used. To improve the purity of the data sample, a loose offline preselection is applied that includes cuts on the lepton and hadron PID and the χ_{vtx}^2 , χ_{IP}^2 , DIRA, χ_{FD}^2 of the B candidate, as well as the χ_{IP}^2 of the final state particles and the ghost probability of the leptons and hadrons. The exact selection requirements for the offline selection and tree-level decays and rare mode decays are displayed in Tabs. A.2 and A.3, respectively. Separate plots are

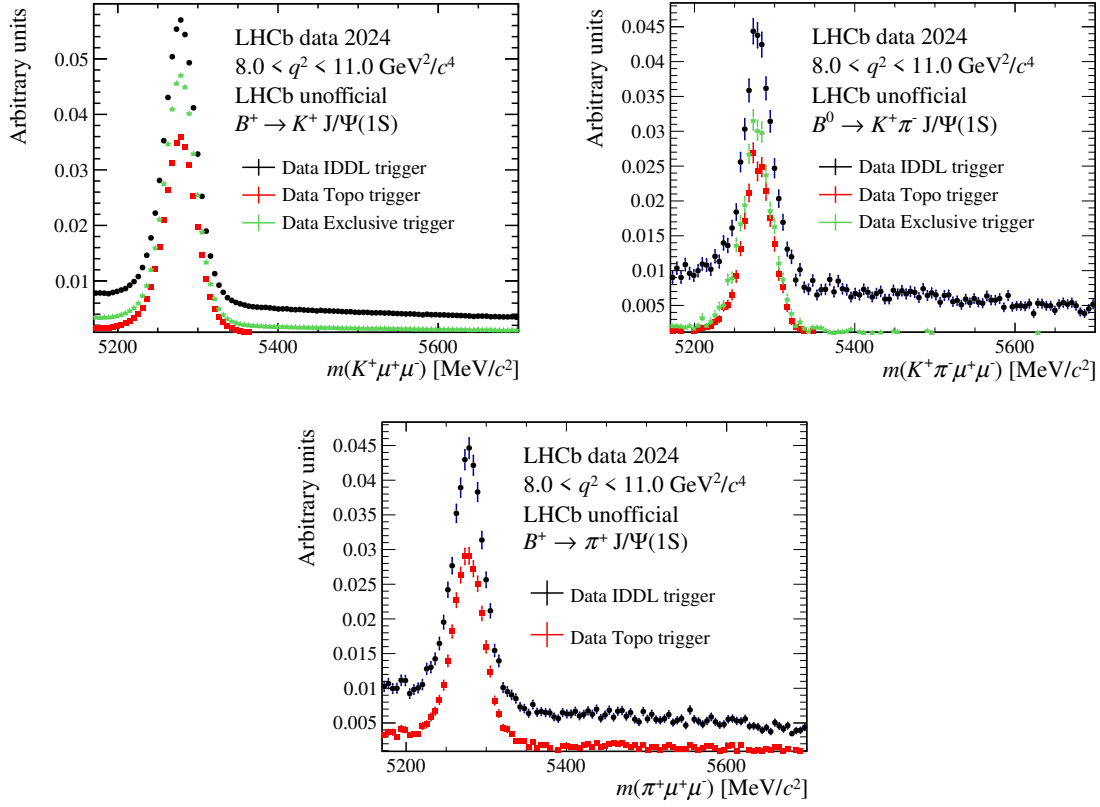


Figure 5.5: Reconstructed invariant mass distributions of the B -meson candidates selected by the IDDL, Topological, and exclusive Hlt2 trigger lines. The J/ψ intermediate resonance is selected by requiring $8.0 < q^2 < 11.0 \text{ GeV}^2/c^4$.

created for the q^2 region of the $J/\Psi(1S)$ resonance ($8.0 < q^2 < 11.0 \text{ GeV}^2/c^4$) and the rare mode region ($q^2 \in [0.06, 0.98] \cup [1.1, 8.0] \cup [11.0, 12.5] \cup [15.0, 19.0] \text{ GeV}^2/c^4$), where the charmonium resonances are vetoed. Invariant mass distributions for B -decays proceeding via the J/ψ resonance selected by the IDDL and Topological trigger lines are displayed in Fig. 5.5, as well as the corresponding exclusive trigger lines if they are available. The comparison of the mass peaks indicates a clear performance advantage of the IDDL trigger with respect to the Topological trigger and also the exclusive trigger lines on the decay modes that the IDDL trigger is designed to select. For the rare decay modes $B^0 \rightarrow K^{*0} \mu^+ \mu^-$, and $B^+ \rightarrow K^+ \mu^+ \mu^-$ the mass peaks are also easily identifiable in Fig. 5.6.

Decays including electrons in the final state exhibit a significantly worse mass resolution compared to the same decays involving muons. For decays involving the J/ψ resonance subsequently decaying into a pair of oppositely charged electrons, it is possible to improve the mass resolution by constraining the di-lepton mass to the J/ψ mass. This constraint mass distribution for the decay $B^+ \rightarrow K^+ J/\psi$ is shown in Fig 5.7. Also, for the mode involving electron pairs, clear B mass signatures are discernible.

In conclusion, the IDDL Hlt2 lines developed as part of this thesis have proven to be very effective in selecting the target decays. They consistently outperform the dedicated exclusive trigger lines as well as the Topological inclusive trigger on decays discussed in this chapter. This has been shown for the tree-level decays involving

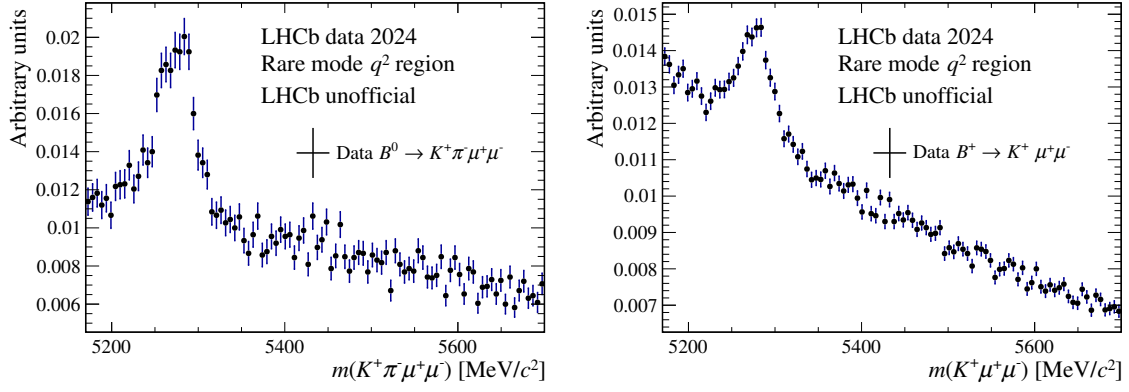


Figure 5.6: Reconstructed invariant mass distributions of the B -meson candidates selected by the IDDM Hlt2 trigger lines. The charmonium resonances are explicitly vetoed.

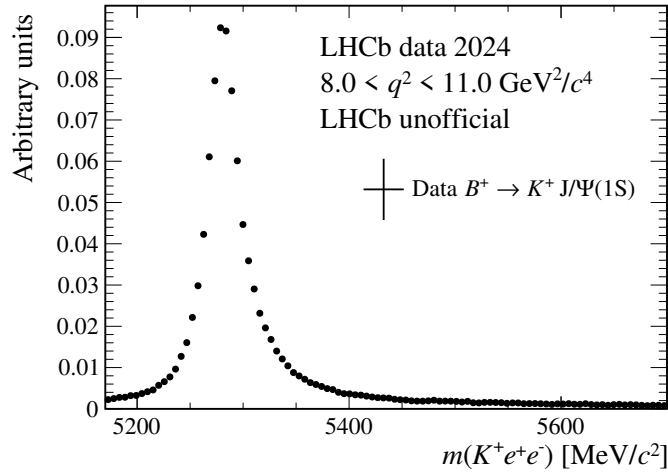


Figure 5.7: Reconstructed invariant mass distributions of the B -meson candidates selected by the IDDE Hlt2 trigger lines. The J/ψ intermediate resonance is selected by requiring $8.0 < q^2 < 11.0 \text{ GeV}^2/c^4$.

charmonium resonances as well as the corresponding rare mode decays for both di-muon and di-electron final states. This inclusive trigger will play an important role in a large number of future analyses, including a future update of the angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ presented in the following chapter.

Chapter 6

Angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

The angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ is one of the key measurements performed at the LHCb experiment. The status of the analysis before this work has been summarized in Sec. 3. This thesis presents the legacy measurement of the q^2 -binned angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ using the Run 1 and Run 2 LHCb data sample corresponding to an integrated luminosity of 8.4 fb^{-1} . A broad overview of the general strategy in this analysis is given in Sec. 6.1. Section 6.2 describes the used data sample and Sec. 6.3 gives a summary of its selection. Sections 6.4 and 6.5 discuss the studies performed in the course of the analysis to develop and validate the fit model. A discussion of systematic effects relevant for this analysis can be found in Sec. 6.6. Finally, the results, including a global analysis of the determined observables, are presented in Sec. 6.7.

6.1 Analysis strategy

The goal of the angular analysis is to measure the full set of angular observables and the branching fraction ($\mathcal{B}$) of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ in bins of q^2 using the data samples collected at the LHCb experiment during the years 2011, 2012, 2016, 2017, and 2018. As described in Sec. 2, the decay rate of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ can be expressed as a function of three decay angles $\cos \theta_\ell$, $\cos \theta_K$, and ϕ , the di-muon invariant mass q^2 , the $K\pi$ invariant mass $m_{K\pi}$, and the angular observables $\vec{I}_i$. By parameterizing the data with the differential decay rate of the signal decay, the angular observables are determined.

This analysis expands on the strategy of the angular analysis of the decay performed by LHCb on the Run 1 and 2016 data samples [24]. The key improvements of the analysis strategy with respect to Ref. [24] are:

1. Dependency on $m_{K\pi}$ parameterized differentially in decay rate
2. S-wave and interference angular observables measured for the first time
3. Simultaneous fit of CP -symmetries and CP -asymmetries
4. Model-independent simultaneous measurement of branching fraction
5. Lepton mass effects taken into account

The decay of interest is selected from the data sample by a series of requirements on the kinematics and topology of the decay. The selection has been completely reoptimized compared to the Ref. [24] by Thomas Oeser [45]. Section 6.3 gives a summary of the selection requirements.

The signal decay topology is characterized by the K^{*0} resonance, which decays into a $K^+\pi^-$ pair. Therefore, the signal is separated from other $K^+\pi^-$ resonances by requiring that the $m_{K\pi}$ of a signal candidate falls within a specific window around the K^{*0} resonance mass ($745.9 < m_{K\pi} < 1095.9 \text{ MeV}/c^2$). This window is significantly larger compared to Ref. [24], leading to an increase in selection efficiency of both P and S-wave candidates in the final data set. The S-wave contribution contains contributions from the spin 0 mesons $K^{*0}(1430)$ and $\kappa(800)$, which both predominantly decay into $K^+\pi^-$. The pole mass of $K^{*0}(1430)$ lies outside of the $m_{K\pi}$ selection window, but due to the large decay width, the tails of its $m_{K\pi}$ spectrum contribute significantly to the population of candidates inside the $m_{K\pi}$ selection window. The $\kappa(800)$, on the other hand, lies inside the $m_{K\pi}$ selection window but is still best described by a nonresonant lineshape [49] due to its large decay width. Signal candidates are limited to the q^2 range of $0.06 \text{ GeV}^2/c^4 < q^2 < 19.0 \text{ GeV}^2/c^4$, because of kinematic restrictions and the decreasing selection efficiency at very low q^2 .

As described in Sec. 2, the signal decay itself is mediated via loop-level diagrams, leading to a continuous q^2 spectrum. However, also tree-level decays of B-mesons into the $K^+\pi^-\mu^+\mu^-$ final state via charmonium resonances are present in the analyzed q^2 range. These tree-level decays have a $\mathcal{O}(100)$ times higher branching fraction compared to the rare mode. Specifically in this case these decay modes are $B^0 \rightarrow (J/\psi \rightarrow \mu^+\mu^-)(K^{*0} \rightarrow K^+\pi^-)$ and $B^0 \rightarrow (\psi(2S) \rightarrow \mu^+\mu^-)(K^{*0} \rightarrow K^+\pi^-)$. To separate them from the rare mode decay, the q^2 regions around the charmonium resonances are explicitly vetoed. In this analysis, $B^0 \rightarrow (J/\psi \rightarrow \mu^+\mu^-)(K^{*0} \rightarrow K^+\pi^-)$ and $B^0 \rightarrow (\psi(2S) \rightarrow \mu^+\mu^-)(K^{*0} \rightarrow K^+\pi^-)$ are used as control modes to cross-check the angular fit procedure. Additionally, the J/ψ mode is used as a reference decay to determine the relative branching fraction of the rare mode.

6.1.1 Binning in q^2

As mentioned in Sec. 2, the angular observables are q^2 dependent. To capture this dependence, the nominal measurements are performed in 10 bins of q^2 . Eight of these are referred to as narrow q^2 bins, which have a width of roughly $2 \text{ GeV}^2/c^4$ or less. The remaining two q^2 bins are referred to as wide bins because they encompass a larger q^2 region. The exact definitions of these bins can be found in Tab. 6.1. The measurement of angular observables in these bins effectively constitutes an averaging over this q^2 range. To prevent pollution from control mode decays in the rare mode fit, the q^2 regions $8.0 \text{ GeV}^2/c^4 < q^2 < 11.0 \text{ GeV}^2/c^4$ and $12.5 \text{ GeV}^2/c^4 < q^2 < 15.0 \text{ GeV}^2/c^4$ are vetoed. Additionally, the very narrow ϕ resonance is vetoed in the $0.98 \text{ GeV}^2/c^4 < q^2 < 1.1 \text{ GeV}^2/c^4$ region. A minor difference to the previous definition of the q^2 bins used in Ref. [24] is the lower q^2 boundary of the first bin, which has been changed from $0.1 \text{ GeV}^2/c^4$ to $0.06 \text{ GeV}^2/c^4$ to incorporate a larger fraction of the photon pole. This is made possible through the proper treatment of lepton mass effects in the differential decay rate, which are particularly strong in the low q^2 regime. The alternative finer binning scheme shown on the right side of Tab. 6.1 is constructed to

improve the resolution of the q^2 dependence. In the following, this scheme is denoted as half-sized q^2 bins.

Table 6.1: The nominal q^2 -binning chosen for this analysis (left) and the alternative finer q^2 -binning scheme (right).

Bin	q^2 range [GeV ² /c ⁴]	Bin	q^2 range [GeV ² /c ⁴]
1	[0.06, 0.98]	1	[0.06, 0.54]
2	[1.1, 2.5]	2	[0.54, 0.98]
3	[2.5, 4.0]	3	[1.1, 1.8]
4	[4.0, 6.0]	4	[1.8, 2.5]
5	[6.0, 8.0]	5	[2.5, 3.25]
6	[11.0, 12.5]	6	[3.25, 4.0]
7	[15.0, 17.0]	7	[4.0, 5.0]
8	[17.0, 19.0]	8	[5.0, 6.0]
9	[1.1, 6.0]	9	[6.0, 7.0]
10	[15.0, 19.0]	10	[7.0, 8.0]
		11	[11.0, 11.75]
		12	[11.75, 12.5]
		13	[15.0, 16.0]
		14	[16.0, 17.0]
		15	[17.0, 18.0]
		16	[18.0, 19.0]

6.1.2 Efficiency parameterization

Detector and selection effects distort the shape of decay angles, $m_{K\pi}$ and q^2 . This effect needs to be accounted for in the determination of the angular observables. This requires a parameterization of the selection efficiency as a function of the five-dimensional phase space. To model the shape as precisely as possible, a set of non-factorizing Legendre polynomials is used. A large Monte Carlo simulation sample, which is re-weighted to be flat in all dimensions before all selection steps and detector effects, is parametrized with this polynomial model using the method of moments [118]. The number of maximum polynomial orders is limited to prevent fast oscillations of the fit function in the border regions, known as Runge's phenomenon [119]. Two separate acceptance functions are determined for the low q^2 region below 0.98 GeV²/c⁴ and the q^2 region above to reduce the necessary maximum order of the q^2 polynomial. Acceptance parameterizations are also fitted separately for decays of B^0 and $\bar{B}^0$ -mesons to capture detection asymmetries between both decay flavors. The flavor of the initial B-meson candidate is determined via the charge of the final state kaon. A detailed discussion on the construction of this parameterization can be found in Sec. 6.5.1.

6.1.3 Maximum log likelihood fit

After the selection of the signal candidates, a maximum log likelihood fit is performed to extract the angular observables. The likelihood is defined as

$$-\ln \mathcal{L} = - \sum_{i=0}^{N_{\text{events}}} \ln (P(\vec{\theta}, \vec{x}_i)). \quad (6.1)$$

The sum runs over all events in the dataset. $P(\vec{\theta}, \vec{x}_i)$ is the probability density function that describes the five-dimensional distribution of the selected set of events as a function of the decay angles, $m_{K\pi}$, q^2 , the angular observables, and additional nuisance parameters. The set of best-fit values of the angular observables is obtained by finding θ such that

$$-\nabla_{\theta} \ln (\mathcal{L}(\vec{\theta})) = 0. \quad (6.2)$$

The minimum of the negative log likelihood is determined with the MIGRAD algorithm [120].

The probability density function

As the data set consists of signal decays and residual combinatorial background, the probability density function (PDF) is also composed of two parts, which describe the signal and background components, respectively.

$$P(\vec{\theta}, \vec{x}_i) = f_{\text{sig}} P_{\text{sig}}(\vec{\theta}_{\text{sig}}, \vec{x}_i) + (1 - f_{\text{sig}}) P_{\text{bkg}}(\vec{\theta}_{\text{bkg}}, \vec{x}_i) \quad (6.3)$$

The signal fraction f_{sig} is defined as

$$f_{\text{sig}} = \frac{n_{\text{sig}}}{n_{\text{sig}} + n_{\text{bkg}}}, \quad (6.4)$$

where n_{sig} and n_{bkg} describe the signal and background yields. The signal PDF is based on the decay rate as a function of the decay angles, $m_{K\pi}$ and q^2 . This function has been described in Sec. 2.5.3. To write down the proper PDF for the signal decay $\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$ the angular observables $\vec{I}_i$ are expressed in terms of the CP -symmetries S_i and CP -asymmetries A_i . These symmetries and asymmetries are normalized to the CP -averaged total integrated decay rate $\frac{d(\Gamma + \bar{\Gamma})}{dq^2}$. P-wave observables are normalized to the P-wave decay rate, while S-wave and interference observables are normalized to the total P+S-wave integrated decay rate. This results in the following expressions for the angular observables $\vec{I}_i$:

$$\begin{aligned} I_i(q^2) &= \left. \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \right|_{\text{P}} \times \frac{1}{2}(S_i + A_i) \\ \bar{I}_i(q^2) &= \left. \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \right|_{\text{P}} \times \frac{1}{2}(S_i - A_i) \end{aligned} \quad (6.5)$$

for the P-wave and

$$\begin{aligned} I_i(q^2) &= \left. \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \right|_{\text{P+S}} \times \frac{1}{2}(S_i + A_i) \\ \bar{I}_i(q^2) &= \left. \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \right|_{\text{P+S}} \times \frac{1}{2}(S_i - A_i) \end{aligned} \quad (6.6)$$

for the S-wave observables. Using the definitions above, the normalized differential decay rate is given by

$$\begin{aligned} \frac{d^5 \vec{\Gamma}_{\text{Norm}}}{dq^2 dm_{K\pi} d\vec{\Omega}} &= \left(1 - \left(\frac{3}{4}F_S - \frac{1}{2}S_{12} \right) \right) \frac{9}{32\pi} \sum_{i \in \text{P-wave}} \frac{1}{2}(S_i \pm A_i) f_i(\vec{\Omega}) |\mathcal{BW}_P(m_{K\pi})|^2 \\ &+ \frac{3}{16\pi} \sum_{i \in 10,12} \frac{1}{2}(S_i \pm A_i) f_i(\vec{\Omega}) |\mathcal{BW}_S(m_{K\pi})|^2 \\ &+ \frac{3}{16\pi} \sum_{i \in 11,13,14,15} \frac{1}{2} f_i(\vec{\Omega}) \left((S_i^{\text{re}} \pm A_i^{\text{re}}) \text{Re}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \right. \\ &\quad \left. - (S_i^{\text{im}} \pm A_i^{\text{im}}) \text{Im}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \right) \\ &+ \frac{3}{16\pi} \sum_{i \in 16,17} \frac{1}{2} f_i(\vec{\Omega}) \left((S_i^{\text{re}} \pm A_i^{\text{re}}) \text{Im}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \right. \\ &\quad \left. + (S_i^{\text{im}} \pm A_i^{\text{im}}) \text{Re}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \right), \end{aligned} \quad (6.7)$$

where $\vec{\Omega}$ denotes the decay angles and $\frac{d^5 \vec{\Gamma}_{\text{Norm}}}{dq^2 dm_{K\pi} d\vec{\Omega}} = \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \bigg|_{\text{P+S}} \frac{d^5 \vec{\Gamma}}{dq^2 dm_{K\pi} d\vec{\Omega}}$. The literature defines a number of established observables, which can be expressed in terms of the angular observables introduced above. The longitudinal polarization fraction F_L and the corresponding CP -asymmetry A_{FL} is defined in the following as

$$F_L = -S_2^c \quad (6.8)$$

$$A_{\text{FL}} = -A_2^c, \quad (6.9)$$

following Ref. [42]. The forward-backward asymmetry is defined as

$$A_{\text{FB}} = \left[\int_0^1 - \int_{-1}^0 \right] d \cos \theta_\ell \frac{d(\Gamma_{\text{Norm}} - \bar{\Gamma}_{\text{Norm}})}{dq^2 d \cos \theta_\ell} = \frac{3}{8}(2S_6^s + S_6^c) \quad (6.10)$$

with the corresponding CP -asymmetry A_{FB}^{CP} being

$$A_{\text{FB}}^{CP} = \left[\int_0^1 - \int_{-1}^0 \right] d \cos \theta_\ell \frac{d(\Gamma_{\text{Norm}} + \bar{\Gamma}_{\text{Norm}})}{dq^2 d \cos \theta_\ell} = \frac{3}{8}(2A_6^s + A_6^c). \quad (6.11)$$

Finally, the total P-wave CP -asymmetry A_{CP} can be expressed as

$$A_{CP} = \frac{3}{4}(2A_1^s + A_1^c) - \frac{1}{4}(2A_2^s + A_2^c) \quad (6.12)$$

The complete signal probability density function, not neglecting lepton masses (and not accounting for acceptance effects), is therefore given by

$$\begin{aligned}
\mathcal{P}_{B^0/\bar{B}^0}^{\text{sig}} = & \left(\frac{9}{32\pi} (1 - F_S) \left[\frac{1}{2} (S_1^s \mp A_1^s) \sin^2 \theta_K + \frac{1}{2} (S_1^c \mp A_1^c) \cos^2 \theta_K \right. \right. \\
& + \left(\frac{1}{2} (S_2^s \mp A_2^s) \sin^2 \theta_K - \frac{1}{2} (F_L \mp A_{FL}) \cos^2 \theta_K \right) \cos 2\theta_\ell \\
& + \frac{1}{2} (S_3 \mp A_3) \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + \frac{1}{2} (S_4 \mp A_4) \sin 2\theta_K \sin 2\theta_\ell \cos \phi \\
& + \frac{1}{2} (S_5 \mp A_5) \sin 2\theta_K \sin \theta_\ell \cos \phi + \left(\frac{1}{2} (S_6^s \mp A_6^s) \sin^2 \theta_K \right. \\
& + \left. \frac{1}{2} (S_6^c \mp A_6^c) \cos^2 \theta_K \right) \cos \theta_\ell \\
& + \frac{1}{2} (S_7 \mp A_7) \sin 2\theta_K \sin \theta_\ell \sin \phi + \frac{1}{2} (S_8 \mp A_8) \sin 2\theta_K \sin 2\theta_\ell \sin \phi \\
& \left. + \frac{1}{2} (S_9 \mp A_9) \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right] \times |\mathcal{BW}_P(m_{K\pi})|^2 \\
& + \frac{3}{16\pi} \left[\left(\frac{1}{2} (F_S \mp A_{FS}) + \frac{1}{2} (S_{12} \mp A_{12}) \cos 2\theta_\ell \right) \times |\mathcal{BW}_S(m_{K\pi})|^2 \right. \\
& + \frac{1}{2} (S_{11}^{\text{re}} \mp A_{11}^{\text{re}}) \cos \theta_K \times \text{Re}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& - \frac{1}{2} (S_{11}^{\text{im}} \mp A_{11}^{\text{im}}) \cos \theta_K \times \text{Im}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& + \frac{1}{2} (S_{13}^{\text{re}} \mp A_{13}^{\text{re}}) \cos \theta_K \cos 2\theta_\ell \times \text{Re}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& - \frac{1}{2} (S_{13}^{\text{im}} \mp A_{13}^{\text{im}}) \cos \theta_K \cos 2\theta_\ell \times \text{Im}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& + \frac{1}{2} (S_{14}^{\text{re}} \mp A_{14}^{\text{re}}) \sin \theta_K \sin 2\theta_\ell \cos \phi \times \text{Re}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& - \frac{1}{2} (S_{14}^{\text{im}} \mp A_{14}^{\text{im}}) \sin \theta_K \sin 2\theta_\ell \cos \phi \times \text{Im}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& + \frac{1}{2} (S_{15}^{\text{re}} \mp A_{15}^{\text{re}}) \sin \theta_K \sin \theta_\ell \cos \phi \times \text{Re}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& - \frac{1}{2} (S_{15}^{\text{im}} \mp A_{15}^{\text{im}}) \sin \theta_K \sin \theta_\ell \cos \phi \times \text{Im}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& + \frac{1}{2} (S_{16}^{\text{re}} \mp A_{16}^{\text{re}}) \sin \theta_K \sin \theta_\ell \sin \phi \times \text{Im}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& + \frac{1}{2} (S_{16}^{\text{im}} \mp A_{16}^{\text{im}}) \sin \theta_K \sin \theta_\ell \sin \phi \times \text{Re}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& + \frac{1}{2} (S_{17}^{\text{re}} \mp A_{17}^{\text{re}}) \sin \theta_K \sin 2\theta_\ell \sin \phi \times \text{Im}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \\
& \left. + \frac{1}{2} (S_{17}^{\text{im}} \mp A_{17}^{\text{im}}) \sin \theta_K \sin 2\theta_\ell \sin \phi \times \text{Re}[\mathcal{BW}_S(m_{K\pi}) \mathcal{BW}_P^*(m_{K\pi})] \right] \bigg) / \bar{\mathcal{N}}_{\text{sig}}^{\zeta_{\text{sig}}^2}
\end{aligned} \tag{6.13}$$

The factors $\mathcal{N}_{\text{sig}}$ and $\bar{\mathcal{N}}_{\text{sig}}$ ensure that the signal PDF is normalized. When neglecting the acceptance effect, these normalization factors are simply equal to one. The only differences between the PDFs of B^0 and $\bar{B}^0$ are the signs in front of the CP -asymmetries. This allows to perform a simultaneous fit of the B^0 and $\bar{B}^0$ samples to determine both symmetries and asymmetries.

Assumption of massless leptons

The terms in Eq. 6.13 are the most general expressions without further simplifying assumptions. The large number of observables can be problematic due to the low expected number of candidates in the signal channel. Therefore, it is useful to reduce the number of observables by making approximately valid assumptions about the effect of the lepton mass on the value of the angular observables. As indicated in Tab. 2.3, the definition of the angular observables as a function of the transversity amplitudes generally depends on the lepton mass m_μ . However, m_μ mostly occurs in ratios over q^2 . If this ratio is sufficiently suppressed in a given q^2 range, it is a valid assumption to neglect the lepton mass. Figure 6.1 shows that this is mostly the case for q^2 bins above $0.98 \text{ GeV}^2/c^4$. Only for the angular observable F_L , a small lepton mass effect is visible up to $4 \text{ GeV}^2/c^4$ in q^2 . Assuming massless leptons leads to the relations between angular observables displayed in Tab. 6.2.

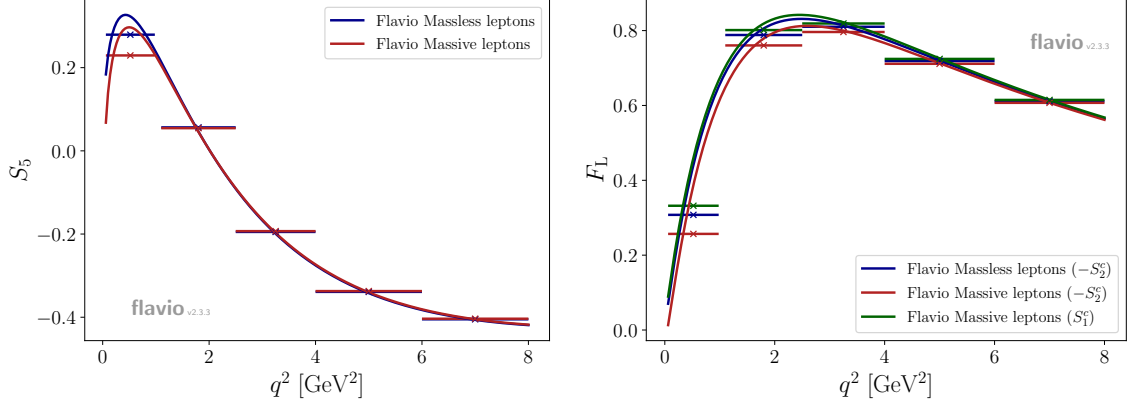


Figure 6.1: Standard model predictions for the observables S_5 and F_L under the assumption of negligible and non-negligible lepton masses. The SM predictions are calculated using the Flavio package [57].

Table 6.2: Relations between angular observables which simplify the signal PDF in the case of the assumption of massless leptons.

CP-symmetries	CP-asymmetries
$S_1^s = 3S_2^s$	$A_1^s = 3A_2^s$
$S_1^c = -S_2^c$	$A_1^c = -A_2^c$
$S_{10} = -S_{12}$	$A_{10} = -A_{12}$
$S_{11} = -S_{13}$	$A_{11} = -A_{13}$

Inserting these relations in Eq. 6.13 reduces the degrees of freedom significantly and therefore simplifies the determination of the angular observables. Figure 6.1 shows that the small lepton mass effect in F_L coincides with the relation $S_1^c = -S_2^c$ being slightly broken in the q^2 region up to $4 \text{ GeV}^2/c^4$. This effect can be accounted for by dropping the assumption $S_1^c = -S_2^c$, while still enforcing the remaining relations in Tab. 6.2. In the following, this is called the partially massive model. Depending on the desired configuration of the angular fit in a specific q^2 bin, leptons can be assumed as fully massive, partially massive, or massless.

Normalization condition and A_{CP} relation

From Eq. 6.13, it is possible to derive two more relations between the angular symmetries and asymmetries. They arise from the following integral equations that reflect the fact that the PDF is normalized to the CP -averaged integrated decay rate and the definition of the total decay asymmetry A_{CP} .

$$\begin{aligned}
\int \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \bigg|_P \frac{d^5(\Gamma + \bar{\Gamma})}{dq^2 d\vec{\Omega} dm_{K\pi}} d\vec{\Omega} dm_{K\pi} &= 1 \\
\int \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \bigg|_{P+S} \frac{d^5(\Gamma - \bar{\Gamma})}{dq^2 d\vec{\Omega} dm_{K\pi}} d\vec{\Omega} dm_{K\pi} &= A_{CP}^{\text{PW+SW}}
\end{aligned} \tag{6.14}$$

Here, $A_{CP}^{\text{PW+SW}}$ denotes the total CP -asymmetry including both P-wave and S-wave contributions. Equation 6.14 leads to the following relations for the CP -symmetries

$$1 = \frac{3}{2}S_1^s - \frac{1}{2}S_2^s - \frac{1}{4}S_2^c + \frac{3}{4}S_1^c \quad (6.15)$$

and for the CP -asymmetries

$$A_{CP}^{\text{PW+SW}} = \left(1 - \left(\frac{3}{2}S_{10} - \frac{1}{2}S_{12}\right)\right) \left(\frac{3}{4}(2A_1^s + A_1^c) - \frac{1}{4}(2A_2^s + A_2^c)\right) + \left(\frac{3}{2}A_{10} - \frac{1}{2}A_{12}\right). \quad (6.16)$$

These relations are crucial as they further reduce the degrees of freedom of the fit, also for the case where leptons are considered to be massive. While Eq. 6.15 reduces the number of independent observables directly, Eq. 6.16 relates the CP -asymmetries with the total CP -asymmetry $A_{CP}^{\text{PW+SW}}$, which in turn can be constrained using an extended fit. However, this analysis measures the total CP -asymmetry related to the P-wave part of the decay rate, which in the following is denoted as A_{CP} and defined in Eq. 6.12.

$m_{K\pi}$ description

The P- and S-wave contributions to the signal decay can mainly be distinguished by their distributions in $m_{K\pi}$. Therefore, it is important to describe both distributions accurately in order to ensure a clean separation in the fit, especially in light of the wider $m_{K\pi}$ selection window. The P-wave contribution is described as a relativistic Breit-Wigner function, [121], which can be expressed as

$$\begin{aligned} \mathcal{BW}'_P(m_{K\pi}) = & \sqrt{k p} \times F^{(L_{K^*0})}(p, p_0, d_{K^*}) \left(\frac{p}{m_{K^*0}}\right)^{L_{K^*0}} \times F^{(L_{B^0})}(k, k_0, d_B) \left(\frac{k}{m_{B^0}}\right)^{L_{B^0}} \\ & \times \frac{1}{m_{K^*0}^2 - m_{K\pi}^2 - i m_{K^*0} \Gamma(m_{K\pi})}. \end{aligned} \quad (6.17)$$

The definitions of the used parameters are given in Tab. 6.3. The function is peaking sharply around the K^{*0} mass. The physical width of the resonance dominates the mass resolution due to the transition of K^{*0} into $K^+\pi^-$ being a strong decay. The running width of the K^{*0} is given by

$$\Gamma(m_{K\pi}) = \Gamma_{892} \times \left(F^{(L_{K^*0})}(k, k_0, d_{K^*0})\right)^2 \times \left(\frac{k}{k_0}\right)^{2L_{K^*0}+1} \times \left(\frac{m_{K^*0}}{m_{K\pi}}\right), \quad (6.18)$$

where Γ_{892} describes the resonance width of the K^{*0} and $F^{(L_X)}(q, q_0, d_X)$ denotes the Blatt-Weisskopf angular barrier factors [122]. For the P-wave, the angular momentum L_{K^*0} is equal to one, while L_{B^0} is zero for the B^0 meson. The relevant Blatt-Weisskopf form factors are given by

$$\begin{aligned} F^{(L_X=0)}(q, q_0, d_X) &= 1 \\ F^{(L_X=1)}(q, q_0, d_X) &= \sqrt{\frac{1 + d_X^2 q_0^2}{1 + d_X^2 q^2}} \end{aligned} \quad (6.19)$$

Table 6.3: Definitions of the variables included in the analytical expression of the signal $m_{K\pi}$ distribution.

param.	Definition
k	K^+ momentum in K^{*0} r.f.
p	K^{*0} momentum in B^0 r.f.
k^0	K^+ momentum in K^{*0} r.f. at resonance mass
p^0	K^{*0} momentum in B^0 r.f. at resonance mass
L_{B^0}	relative angular momentum between K^{*0} and $\mu^+\mu^-$
$L_{K^{*0}}$	relative angular momentum between K^+ and π^-
m_{B^0}	B^0 resonance mass
$m_{K^{*0}}$	K^{*0} resonance mass
$\Gamma(m_{K\pi})$	$m_{K\pi}$ -dependent running width of the K^{*0}
d_{K^*}	hadron scale of the K^* resonance
d_B	hadron scale of the B meson

The S-wave inside the $m_{K\pi}$ selection window consists of contributions from two different $K^+\pi^-$ resonances. The $K^+\pi^-$ pair is produced either via the decay of $K^{*0}(1430)$ or $\kappa(800)$ mesons. While the decay of the $K^{*0}(1430)$ resembles the shape of the K^{*0} with significantly higher resonance mass and larger width, the $\kappa(800)$ is better described by a non-resonant shape. Since the $K^{*0}(1430)$ peak lies outside the $m_{K\pi}$ selection window, only the lower tail of the resonance contributes to the S-wave distribution, which results in a slowly varying shape. The two contributions are jointly described by the LASS model [49] in the nominal fit setup. Its algebraic definition is given by

$$\begin{aligned} \mathcal{BW}'_{\text{S,LASS}}(m_{K\pi}) &= \sqrt{kp} \times \frac{m_{K\pi}}{p} \times F^{(L_{B^0})}(k, k_0, d_B) \left(\frac{k}{m_{B^0}} \right)^{L_{B^0}} \\ &\times F^{(L_{1430})}(p, p_0, d_{K^*}) \left(\frac{p}{m_{1430}} \right)^{L_{1430}} \times \left(\frac{1}{\cot \delta_B - i} + e^{2i\delta_B} \frac{1}{\cot \delta_R - i} \right). \end{aligned} \quad (6.20)$$

The non-resonant part of the S-wave is described by the term

$$\cot \delta_B = \frac{1}{ap} + \frac{1}{2}rp, \quad (6.21)$$

where a is the scattering length and r is the effective range. The resonant part is described by the term

$$\cot \delta_R = \frac{m_{1430}^2 - m_{K\pi}^2}{m_{1430}\Gamma_{1430}(m_{K\pi})}, \quad (6.22)$$

which peaks around the $K^{*0}(1430)$ resonant mass and includes the running width $\Gamma_{1430}(m_{K\pi})$, defined as

$$\Gamma_{1430}(m_{K\pi}) = \Gamma_{1430} \times \left(\frac{p}{p_0} \right) \times \left(\frac{m_{1430}}{m_{K\pi}} \right). \quad (6.23)$$

The PDG [26] cites the parameter values as $m_{1430} = 1425 \pm 50 \text{ MeV}/c^2$ and $\Gamma_{1430} = 270 \pm 80 \text{ MeV}/c^2$. The values for the scattering length a and effective range r are fixed to

$$\begin{aligned} a &= 3.253 \text{ GeV}^{-1}c \\ r &= 1.7 \text{ GeV}^{-1}c, \end{aligned}$$

which are determined from a fit to the control mode $B^0 \rightarrow K^{*0} J/\psi$ in combination with results from Ref. [123].

Efficiency correction

The equations in the previous section that discussed the signal PDF only described the signal model without taking into account the fact that the angular distributions are distorted by selection and detection effects. They can be accounted for using two different approaches that use the parameterization of the acceptance effect described in Sec. 6.1.2. First, there is the possibility of incorporating the acceptance function directly into the signal PDF. This is done by multiplying the uncorrected PDF by the acceptance parameterization function.

$$\mathcal{P}_{\text{sig}}^{\text{eff}}(\vec{\Omega}, m_{K\pi}, q_{\text{fix}}^2, \vec{\theta}) = \epsilon(\vec{\Omega}, m_{K\pi}, q_{\text{fix}}^2) \mathcal{P}_{\text{sig}}(\vec{\Omega}, m_{K\pi}, q_{\text{fix}}^2, \vec{\theta}) \quad (6.24)$$

Before including the acceptance parametrization into the signal PDF, the normalization factors from Eq. 6.13 were equal to one. When considering the acceptance effect, this is not the case any longer. The five-dimensional integral over $\mathcal{P}_{\text{sig}}^{\text{eff}}$ needs to be performed explicitly and used to normalize the PDF.

$$\begin{aligned} \mathcal{N}_{B^0} &= \int \epsilon(\vec{\Omega}, m_{K\pi}, q_{\text{fix}}^2) \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \Big|_{\text{P+S}} \frac{d^5\Gamma}{dq^2 d\vec{\Omega} dm_{K\pi}} d\vec{\Omega} dm_{K\pi} \\ \mathcal{N}_{\bar{B}^0} &= \int \epsilon(\vec{\Omega}, m_{K\pi}, q_{\text{fix}}^2) \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \Big|_{\text{P+S}} \frac{d^5\bar{\Gamma}}{dq^2 d\vec{\Omega} dm_{K\pi}} d\vec{\Omega} dm_{K\pi}. \end{aligned} \quad (6.25)$$

This results in the properly normalized probability density functions

$$\mathcal{P}_{B^0} = \epsilon(\vec{\Omega}, m_{K\pi}, q_{\text{fix}}^2) \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \Big|_{\text{P+S}} \frac{d^5\Gamma}{dq^2 d\vec{\Omega} dm_{K\pi}} \Big/ \mathcal{N}_{B^0} \quad (6.26)$$

and

$$\mathcal{P}_{\bar{B}^0} = \epsilon(\vec{\Omega}, m_{K\pi}, q_{\text{fix}}^2) \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \Big|_{\text{P+S}} \frac{d^5\bar{\Gamma}}{dq^2 d\vec{\Omega} dm_{K\pi}} \Big/ \mathcal{N}_{\bar{B}^0} \quad (6.27)$$

Since this analysis evaluates the angular observables averaged over bins in q^2 , it is necessary to evaluate the acceptance function at a fixed point in q^2 denoted as q_{fix}^2 when using this method. This is a valid approximation as long as the q^2 distribution and the value of the acceptance do not vary strongly across the q^2 bin. This is generally the case in the narrow q^2 bins above $0.98 \text{ GeV}^2/c^4$ and below $17.0 \text{ GeV}^2/c^4$. In the first narrow q^2 bin $0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$, the q^2 distribution and the acceptance vary strongly

due to the photon pole at very low q^2 , while in the q^2 bin $17.0 < q^2 < 19.0 \text{ GeV}^2/c^4$ the phase space boundary leads to a quick drop off in the q^2 distribution. The assumption of negligible q^2 variation of the acceptance is also not valid in the wide q^2 bins due to the large q^2 range that is covered. The alternative option is to perform a weighted fit. Using this approach, every event is weighted with the inverse value of the acceptance function to effectively obtain the angular distribution of the signal decay before the selection. This unfolded distribution is then fitted with the PDF given in Eq. 6.13. This method can describe strongly varying q^2 dependencies better, as the acceptance is evaluated at the precise reconstructed q^2 value of each event.

Model of the $m_{K\pi\mu\mu}$ line shape

The $m_{K\pi\mu\mu}$ invariant mass shape of the signal decay is described by a double crystal ball function [124] with one shared mean.

$$\mathcal{P}_{\text{DS CB}}(m \mid \mu, \sigma_1, \sigma_2, \alpha_1, \alpha_2, n_1, n_2, \rho) = \rho \times \mathcal{P}_{\text{CB}}(m \mid \mu, \sigma_1, \alpha_1, n_1) + (1 - \rho) \times \mathcal{P}_{\text{CB}}(m \mid \mu, \sigma_2, \alpha_2, n_2) \quad (6.28)$$

The parameters of the signal model are fixed by fits to simulation samples. The shape of the Gaussian core of the crystal ball function is determined on control mode simulation, while the shape of the tails is subsequently determined for each q^2 bin on rare mode simulation. Corrections to the Gaussian core widths and mean of the crystal ball functions correcting for data-simulation differences in the mass resolution are derived from control mode data. An exponential function is used to describe the $m_{K\pi\mu\mu}$ distribution of the combinatorial background contribution.

$$\mathcal{P}_{\text{bkg}}^{\text{comb}}(m) \propto e^{-m/\tau_m} \quad (6.29)$$

Angular description of the combinatorial background

The angular distribution of the combinatorial background arises from random combinations of tracks from the PV. Therefore, it is necessary to use a general model that can describe a broad range of background shapes. In this analysis, this is done with the help of Chebyshev polynomials $T_i(x)$ [125] for the decay angles $\cos \theta_\ell$, $\cos \theta_K$, ϕ , and $m_{K\pi}$. It is assumed that the distributions of the combinatorial background factorize in the decay angles and $m_{K\pi}$. This means that the background probability density function is parameterized by

$$\mathcal{P}'_{\text{bkg}} = \left(\sum_{i=0}^n c_i T_i(\cos \theta_K) \right) \left(\sum_{j=0}^n d_j T_j(\cos \theta_\ell) \right) \left(\sum_{k=0}^n e_k T_k(\phi) \right) \left(\sum_{l=0}^n f_l T_l(m_{K\pi}) \right), \quad (6.30)$$

where c_i , d_j , e_k , and f_l are real coefficients that are floated in the fit to data. As the expected combinatorial background yield in the q^2 region below $4.0 \text{ GeV}^2/c^4$ is low, a first-order polynomial is used to describe the angular background shape in this region. For q^2 bins above $4.0 \text{ GeV}^2/c^4$ a second-order polynomial is used.

The decay $B^+ \rightarrow K^+ \mu^+ \mu^-$ is a physical background contribution, which mimics the signal in combination with a random pion from the PV. A veto against this background,

which uses the $m_{K\mu\mu}$ invariant mass, is designed to remove this background with a high signal efficiency. It is discussed in more detail in Sec. 6.3. However, an unwanted side-effect of this veto is the shaping of the combinatorial background at high $\cos\theta_K$ by this selection. The shape can not be properly described by the polynomial background model described above. Therefore, the shape of the veto is modeled explicitly by expressing it as a function of q^2 , $m_{K\pi}$, $m_{K\pi\mu\mu}$, and $\cos\theta_K$. The $m_{K\mu\mu}$ invariant mass can be written down as

$$m_{K\mu\mu} = \sqrt{(p_K + p_{\mu\mu})^2} = \sqrt{m_K^2 + q^2 + 2p_K^\lambda p_{\mu\mu,\lambda}}. \quad (6.31)$$

With the help of the definition of $\cos\theta_K$ the four momentum product $p_K^\lambda p_{\mu\mu,\lambda}$ can be rewritten as

$$\begin{aligned} 2p_K p_{\mu\mu} &= 2p_K(p_B - p_{K\pi}) \\ &= 2E_K^{K\pi}(E_B^{K\pi} - m_{K\pi}) + 2|\vec{p}_K^{K\pi}||\vec{p}_B^{K\pi}|\cos\theta_K \end{aligned} \quad (6.32)$$

in the rest frame of the $K\pi$ system. In this reference frame, the kaon momentum and energy are fully defined by the particle masses.

$$|\vec{p}_K^{K\pi}| = \frac{1}{2m_{K\pi}} \sqrt{(m_{K\pi}^2 - (m_K + m_\pi)^2)(m_{K\pi}^2 - (m_K - m_\pi)^2)} \quad \text{and} \quad (6.33)$$

$$E_K^{K\pi} = \sqrt{|\vec{p}_K^{K\pi}|^2 + m_K^2}. \quad (6.34)$$

Using four-momentum conservation, the momentum and energy of the B in the rest frame of the $K\pi$ system are determined to be

$$E_B^{K\pi} = \frac{1}{2m_{K\pi}} (m_{K\pi\mu\mu}^2 - q^2 + m_{K\pi}^2) \quad \text{and} \quad (6.35)$$

$$|\vec{p}_B^{K\pi}| = \sqrt{(E_B^{K\pi})^2 - m_{K\pi\mu\mu}^2}. \quad (6.36)$$

The calculation above is used to express the shape of the veto analytically as follows:

$$\varepsilon(m_{K\pi\mu\mu}, q^2, m_{K\pi}, \cos\theta_K) = \begin{cases} 1 & |m_{K\mu\mu} - m_{B^+}^{\text{PDG}}| \geq 50 \text{ MeV}/c^2 \\ 0 & |m_{K\mu\mu} - m_{B^+}^{\text{PDG}}| < 50 \text{ MeV}/c^2 \end{cases} \quad (6.37)$$

The total background PDF can then be expressed as

$$\begin{aligned} \mathcal{P}_{\text{bkg}}(m_{K\pi\mu\mu}, q^2, m_{K\pi}, \cos\theta_\ell, \phi, \cos\theta_K) &= \varepsilon(m_{K\pi\mu\mu}, q^2, m_{K\pi}, \cos\theta_K) \times e^{-m_{K\pi\mu\mu}/\tau_m} \\ &\times \mathcal{P}'_{\text{bkg}}(\cos\theta_K, \cos\theta_\ell, \phi, m_{K\pi})/\mathcal{N}(q^2), \end{aligned} \quad (6.38)$$

with $\mathcal{N}$ being the appropriate normalization factor.

6.1.4 Determination of A_{CP} and $\mathcal{B}(B^0 \rightarrow K^{*0}\mu^+\mu^-)$

From Eq. 6.16 it is clear that the values of the CP -asymmetries are constrained by $A_{CP}^{\text{PW+SW}}$. This constraint is realized in the fit by introducing an extended term to the negative log likelihood $\mathcal{L}$, which connects the number of events in the data set to the signal yields and A_{CP} . The full extended negative log likelihood is given as

$$-\log(\mathcal{L}_{B^0}^{\text{ext}}) = -2(N_{\text{obs}} \log(n_{\text{sig}} + n_{\text{bkg}}) - (n_{\text{sig}} + n_{\text{bkg}})) - \log(\mathcal{L}_{B^0}) \quad (6.39)$$

$$-\log(\mathcal{L}_{\bar{B}^0}^{\text{ext}}) = -2(\bar{N}_{\text{obs}} \log(\bar{n}_{\text{sig}} + \bar{n}_{\text{bkg}}) - (\bar{n}_{\text{sig}} + \bar{n}_{\text{bkg}})) - \log(\mathcal{L}_{\bar{B}^0}) \quad (6.40)$$

$\bar{N}_{\text{obs}}$ refers to the observed number of events in each data set and $\bar{n}_{\text{sig}}/\bar{n}_{\text{bkg}}$ are the signal and background yields for the B^0 and $\bar{B}^0$ decays. The extended fit also makes a fit of the differential branching fraction of the rare mode decay in bins of q^2 relative to the control mode branching fraction possible. This is done by expressing n_{sig} and $\bar{n}_{\text{sig}}$ as a function of the relative branching fraction $\mathcal{B}^{\text{rel}}$ and several other auxiliary parameters.

$$\bar{n}_{\text{sig}} = \frac{\bar{n}_{J/\psi}^{B^0+\bar{B}^0}}{1 - F_S} \frac{\int d\vec{\Omega} dm_{K\pi} \bar{\Gamma}(\vec{\Omega}, m_{K\pi}) \epsilon(\vec{\Omega}, m_{K\pi})}{\int d\vec{\Omega} dm_{K\pi} \bar{\Gamma}^{J/\psi}(\vec{\Omega}, m_{K\pi}) \epsilon^{J/\psi}(\vec{\Omega}, m_{K\pi})} \alpha_\epsilon \mathcal{B}^{\text{rel}}, \quad (6.41)$$

where $\mathcal{B}^{\text{rel}}(B^0 \rightarrow K^{*0}\mu^+\mu^-)$ is defined as

$$\mathcal{B}^{\text{rel}} = \frac{d\mathcal{B}(B^0 \rightarrow K^{*0}\mu^+\mu^-)/dq^2}{\mathcal{B}(B^0 \rightarrow K^+\pi^- J/\psi) \cdot \mathcal{B}(J/\psi(\rightarrow \mu^+\mu^-))}. \quad (6.42)$$

The yield of $B^0 \rightarrow K^{*0}J/\psi$ candidates after the selection $\bar{n}_{J/\psi}^{B^0+\bar{B}^0}$ is extracted from an extended angular fit to the control mode data set ($8.0 < q^2 < 11.0 \text{ GeV}^2/c^4$). The integrals over the product of the normalized decay rate and the acceptance function in the control mode $\int \Gamma^{J/\psi}(\vec{\Omega}, m_{K\pi}) \epsilon(\vec{\Omega}, m_{K\pi})$ and $\int \bar{\Gamma}^{J/\psi}(\vec{\Omega}, m_{K\pi}) \bar{\epsilon}(\vec{\Omega}, m_{K\pi})$ are also extracted from the same fit. The relative branching fraction $\mathcal{B}^{\text{rel}}$ and the S-wave fraction of the rare mode F_S are floated in the angular fit to rare mode data.

Fixed q^2 acceptance evaluation

The values of the integrals over $\Gamma(\vec{\Omega}, m_{K\pi})$ and $\bar{\Gamma}(\vec{\Omega}, m_{K\pi})$ in the rare mode are evaluated on the fly during the rare mode angular fit as they depend on the angular observables. The ratio of the integral terms between rare and control mode is equivalent to the efficiency ratio between the rare and control modes in regular branching fraction analysis, usually determined on simulation samples generated with a defined amplitude model. In the case of the fits where the acceptance is evaluated at a fixed point in q^2 , the integrals can be evaluated analytically for the decay angles and numerically for $m_{K\pi}$. However, since the integral can only be evaluated inside the $m_{K\pi}$ and $m_{K\pi\mu\mu}$ selection window, an additional correction factor α_ϵ needs to be applied to correct for efficiency differences of these selection windows between rare and control mode. Another contribution to the correction factor stems from the fact that the control mode integral is evaluated at $q^2 = m_{J/\psi}^2$ and thus does not account for signal loss due to the J/ψ q^2 selection window. This is not the case for the rare mode, as the differential branching fraction is evaluated for a specific bin.

Weighted fit

In the case of the weighted fit, $\epsilon(\vec{\Omega}, m_{K\pi})$ is set to one since the acceptance effect has already been corrected by the reweighting of the candidates with the inverse value of the acceptance parameterization $1/\epsilon(\vec{\Omega}, m_{K\pi})$. Since $\Gamma(\vec{\Omega}, m_{K\pi})$ is normalized to one, the integrals will also be equal to one if they are performed over the complete phase space. The rare mode efficiency enters the PDF via $\vec{N}_{\text{obs}}$, which turns into the sum of weights $\sum_{i=0}^N 1/\epsilon$. The values of the correction factor α_ϵ remain identical to the fixed q^2 evaluation point method.

Control mode branching fraction

The measurement is performed relative to the control mode $B^0 \rightarrow K^+ \pi^- J/\psi$. The spectrum of this decay includes irreducible contributions from the S-wave and other higher K^* resonances like $K^*(1410)$, $K_2^*(1430)$, $K^*(1680)$, $K_3^*(1780)$, $K_0^*(1950)$, $K_2^*(1980)$ and $K_4^*(2045)$ [126]. Additionally to that, the control mode region includes candidates from exotic resonances like $Z_c^+(4430)$. Thus, the rare mode branching fraction is measured relative to the inclusive branching ratio $B^0 \rightarrow K^+ \pi^- J/\psi$ that has been measured by Belle [126]. As the Belle measurement of the branching fraction is performed over the complete available $m_{K\pi}$ spectrum, the result needs to be adapted for the $m_{K\pi}$ selection window used in this analysis. A correction factor is derived by calculating the fraction of events falling into the $m_{K\pi}$ selection window using a pseudo-event generator that utilized the full amplitude model of the decay $B^0 \rightarrow K^+ \pi^- J/\psi$ [126]. The uncertainty on this correction factor is obtained using bootstrapping. The effective branching fraction is found to be

$$\mathcal{B}(B^0 \rightarrow K^+ \pi^- (J/\psi \rightarrow \mu^+ \mu^-)) = (4.86 \pm 0.22) \times 10^{-5}. \quad (6.43)$$

6.1.5 Optimized observables

As discussed in Sec. 2, the theoretical expression of the angular observables depends on a multitude of parameters. The hadronic form factors that describe the transition of the B -meson into the K^{*0} meson can be calculated from theory using light cone sum rules or lattice QCD. They are connected to significant theory uncertainties. This effect can be mitigated by redefining the angular observables in such a way that they are less dependent on the hadronic form factors. These optimized angular observables P_i [23] are constructed as ratios of the observables in the S_i basis such that the hadronic form factors cancel in the ratio to leading order. In the limit of large hadronic recoil (*i.e.* $q^2 \ll m_{B^0}^2$), the soft contributions to the hadronic form factors are related via

$$\begin{aligned} \frac{m_{B^0}}{m_{B^0} + m_{K^{*0}}} V(q^2) &= \frac{m_{B^0} + m_{K^{*0}}}{2E} A_1(q^2) = T_1(q^2) = \frac{m_{B^0}}{2E} T_2(q^2) = \xi_{\perp}(E) \quad (6.44) \\ \frac{m_{K^{*0}}}{E} A_0(q^2) &= \frac{m_{B^0} + m_{K^{*0}}}{2E} A_1(q^2) - \frac{m_{B^0} - m_{K^{*0}}}{m_{B^0}} A_2(q^2) \\ &= \frac{m_{B^0}}{2E} T_2(q^2) - T_3(q^2) = \xi_{\parallel}(E), \end{aligned}$$

neglecting corrections of order $\frac{1}{m_B}$ and α_s [127]. In this limit, the transversity amplitudes $A_{\parallel}^{L/R}$ and $A_{\perp}^{L/R}$ are directly proportional to $\xi_{\perp}$ and $A_0^{L/R}$ is directly proportional to $\xi_{\parallel}$ [128]. Given the definitions of S_{2s} and S_{2c} in Tab. 2.3 it follows that S_{2s} is proportional to $\xi_{\perp}^2$ and S_{2c} is proportional to $\xi_{\parallel}^2$. This means that S_{2s} and S_{2c} can be used to cancel the dependence on the form factors for any other angular observables to leading order in the limit of large hadronic recoil. This analysis aims to extract the optimized observables for all P-wave symmetries. A full list of the observables and their definitions in terms of the S_i basis is given in Tab. 6.4. Separate fits are performed for the observables in the S_i and P_i basis, and both sets of results are provided.

Table 6.4: Complete list of all optimized angular observables and their definitions in terms of the S_i basis [23].

Opt. symmetries	Definition	Opt. symmetries	Definition
P_1	$\frac{S_3}{2S_{2s}}$	P_5	$\frac{S_5}{2\sqrt{-S_{2s}S_{2c}}}$
P_2	$\frac{S_{6s}}{8S_{2s}}$	P_6	$\frac{S_7}{2\sqrt{-S_{2s}S_{2c}}}$
P_3	$-\frac{S_9}{4S_{2s}}$	P_8	$\frac{S_8}{2\sqrt{-S_{2s}S_{2c}}}$
P_4	$\frac{S_4}{2\sqrt{-S_{2s}S_{2c}}}$	P_{6c}	$\frac{S_{6c}}{S_{2c}}$

6.1.6 List of performed fit setups

Fits in five different configurations are performed on the same data sample to extract as much information from the data as possible. These setups differ in the number and type of observables floated in the fit. Explicitly, the fits performed during this analysis are:

(i) Angular fit with floated S_{1c} in S_i basis

This fit is performed with the partially massive lepton configuration described in Sec. 6.1.3 in the q^2 region above $1.1 \text{ GeV}^2/c^4$. This improves the description of the residual lepton mass effect in the q^2 region from $1.1 \text{ GeV}^2/c^4$ to $4.0 \text{ GeV}^2/c^4$, which can have a small effect on F_L . The increased complexity of the fit is compensated for by not floating the CP-asymmetries. The fit in the lowest q^2 bin below $0.98 \text{ GeV}^2/c^4$ is performed in the massive lepton configuration. In the following, this fit setup is also referred to as the default fit setup.

(ii) Angular fit with floated S_{1c} in P_i basis

This fit setup is equivalent to the default fit setup, with the only exception being the P-wave angular observables that are replaced by their optimized counterparts as defined in Tab. 6.4.

(iii) Angular fit assuming massless leptons and floated CP-asymmetries

This fit setup assumes the lepton mass to be negligible compared to q^2 in the q^2 region above $1.1 \text{ GeV}^2/c^4$. The relations between the angular observables F_L , S_1^c , S_1^s , and S_2^s are detailed in Tab. 6.2. Of these observables, only F_L is floated in the fit. The fit in the lowest q^2 bin below $0.98 \text{ GeV}^2/c^4$ is performed in the massive lepton configuration. Additionally, the CP asymmetries discussed in Sec. 6.1.3 are floated along with A_{CP} . This fit is performed in the S_i basis only.

(iv) Angular fit assuming massive leptons in P-wave

In order to maximize the sensitivity to scalar and tensor amplitudes, the lepton masses are assumed to be non-negligible for the P-wave part of the PDF, also above $q^2 = 1.1 \text{ GeV}^2/c^4$. The CP-asymmetries are fixed to zero and the fit is performed in the S_i basis.

(v) Angular fit in half-sized q^2 bins

The angular fit is repeated in the alternative half-sized q^2 binning scheme given in Tab. 6.1. All other settings are equivalent to setup (i).

6.2 Data samples

This analysis uses the LHCb data samples from 2011/2012 (Run 1), 2016 (Run 2p1), and 2017/2018 (Run 2p2). The Run 1 data sample corresponds to a luminosity of $\mathcal{L} = 3 \text{ fb}^{-1}$ and was produced at a center of mass energy of $\sqrt{s} = 7 \text{ TeV}$ for 2011 and $\sqrt{s} = 8 \text{ TeV}$ for 2012. The 2015 data set is excluded from the analysis due to its small size in comparison to the effort necessary to incorporate it into the analysis. The 2016 data set was produced with a center of mass energy of $\sqrt{s} = 13 \text{ TeV}$, which increased the B^0 production cross section compared to Run 1 such that the data sample contains roughly the same expected number of signal decays as the complete Run 1 sample even though the integrated luminosity only amounts to $\mathcal{L} = 1.67 \text{ fb}^{-1}$. Adding the Run 2p2 data set, which was produced at the same center of mass energy as the 2016 sample, doubles the expected number of signal decays again compared to the Run 1 and 2016 data samples. Table 6.5 lists all used data samples with their integrated luminosity, $b\bar{b}$ cross-section, and center of mass energy.

Table 6.5: Integrated luminosity $\int \mathcal{L}$ [129], $b\bar{b}$ pair production cross-section $\sigma_{b\bar{b}}$ [101], and center-of-mass energy $\sqrt{s}$ for each year of data taking considered in this analysis.

Year	$\int \mathcal{L} [\text{fb}^{-1}]$	$\sigma_{b\bar{b}} [\mu\text{b}]$	$\sqrt{s} [\text{TeV}]$
2011	1.11	251.8	7.0
2012	2.08	291.6	8.0
2016	1.67	488.1	13.0
2017	1.71	488.1	13.0
2018	2.19	488.1	13.0

6.3 Selection

The following section gives a summary of the requirements applied to select the signal candidates for the angular analysis. The selection requirements have been developed by Thomas Oeser and yield a significantly higher selection efficiency and background reduction than in Ref. [24]. More details on the selection and the corresponding studies can be found in Ref. [45].

6.3.1 Trigger selection

The first selection steps are already performed by the LHCb trigger system, which has been described in detail in Sec. 4. Table 6.6 lists the names of the trigger requirements that need to be fulfilled by each signal candidate. For each of the trigger stages, at least one of the trigger decisions needs to be positive for the specific candidate to retain it for further selection. The L0 muon trigger imposes a p_T requirement on the muons of the candidate, which needs to be fulfilled by at least one of them. The L0 di-muon trigger instead requires that the product of transverse muon momenta is larger than a certain threshold. The p_T thresholds of the L0 trigger differ between the years. They are listed in Tab. 6.7. The Hlt1 software trigger applies a selection to single tracks or

pairs of tracks that are reconstructed using a simplified reconstruction algorithm. The trigger decisions combine multiple variables of the track topology into a single MVA decision that is optimized for the efficient selection of a wide range of decays [108]. The Hlt2 stage applies further MVA selections on fully reconstructed signal candidates. These MVAs exploit the distinctive detached decay topology of the signal decay to train efficient Boosted Decision Trees (BDT) that use a large number of decay topology parameters to distinguish signal from combinatorial background [108, 130, 131].

Table 6.6: Trigger requirements for the three separate data taking periods (Run 1, 2016, 2017+2018) considered in this analysis. On each trigger stage, at least one of the triggers is required to accept the event [45].

	Run 1	2016	2017 + 2018
L0	Muon DiMuon	Muon DiMuon	Muon DiMuon
HLT1	TrackAllL0 TrackMuon	TrackMVA TrackMuon TwoTrackMVA	TrackMVA TrackMuon TwoTrackMVA
HLT2	Topo [2, 3, 4] BodyBBDT TopoMu [2, 3, 4] BodyBBDT DiMuonDetached SingleMuon	Topo [2, 3, 4] Body TopoMu [2, 3, 4] Body	Topo [2, 3, 4] Body TopoMu [2, 3, 4] Body DiMuonDetached

Table 6.7: L0 trigger threshold for the Muon and DiMuon trigger lines used for the majority of data taking in each year [45].

	Year	2011	2012	2016	2017	2018
Muon p_T threshold [GeV/ c]		1.48	1.76	1.80	1.40	1.75
DiMuon $p_T \times p_T$ threshold [GeV ² / c^2]		1.68	2.56	2.25	1.69	3.24

6.3.2 Stripping selection

Events that are selected by the LHCb trigger system are written to a permanent data storage system. The amount of data that is collected over time is too large to be handled effectively by the analyst. Therefore, a first loose preselection (Stripping selection) is applied in order to reduce the size of the data files. This selection requires that these candidates fall inside the expected mass windows and that the secondary vertex is significantly detached from the primary vertex. Loose cuts on PID variables reduce background from misidentified particles originating from the PV, which are mostly pions. Also, candidates are selected for their reconstruction quality of vertices and tracks. A full list of selection requirements can be found in Tab. B.1.

Table 6.8: Preselection requirements imposed on all signal candidates [45].

Candidate	Requirement
Track pairs	Angular separation $\theta > 1$ mrad
Hadrons	Hits in the RICH subsystem (<code>hasRich</code>)
B^0	Vertex $\chi^2/\text{DOF} < 6$
K^{*0}	$m_{K\pi} \in [745.9, 1095.9]$ MeV/ c^2
μ^-, μ^+	<code>isMuon</code> decision
K^+	$p_z(K^+) > 2000.0$ MeV/ c $ p_x(K^+) < 0.33(p_z(K^+) - 2000.0 \text{ MeV}/c)$

6.3.3 Preselection requirements

The preselection is the first selection step that is performed offline. Its goal is to tighten the Stripping selection and add more cuts to remove more combinatorial and physical backgrounds. This includes cuts on the angular separation of tracks and the $m_{K\pi}$ invariant mass. Combined requirements on the K^+ absolute and transverse momenta are designed to remove phase space regions near the edge of the detector that contain large detection asymmetries. These large asymmetries are caused by the magnetic field that sweeps out kaons of a particular charge out of the detector acceptance for a given magnet polarity. Kaons of opposite charge, on the other hand, remain inside the detector acceptance, leading to a maximal detection asymmetry in this region. As the flavor of the B -candidate is inferred from the charge of the final state kaon, this effect could influence the measurement of CP-asymmetries and is therefore mitigated by these cuts [132]. A full list of selection requirements can be found in Tab. 6.8.

6.3.4 Particle identification selection

Particle identification is one of the key capabilities of the LHCb detector. PID variables allow precise tuning of the misidentification background rejection and signal efficiency. As described in Sec. 6.1, physical backgrounds which may peak below the signal $m_{K\pi\mu\mu}$ distribution may have a significant impact on the measurement of physics observables if they are not removed beforehand. Most of these potentially dangerous physical backgrounds have different final state configurations compared to the signal. If final state particles of these backgrounds are misidentified, the decay can be wrongly reconstructed as a signal candidate, which might distort the invariant mass and angular distributions. By applying cuts on the aforementioned PID variables, the rate of these misidentifications is suppressed to the point where it is negligible while simultaneously preserving a high signal efficiency. While some selection requirements target general misidentifications of final state particles, other selections target specific physical decay modes and exploit the possibility of reconstructing the invariant masses of the b -hadron or intermediate resonances of the background decay in combinations with PID variables.

General requirements

First of all, general requirements on the neural network-based **ProbNN** variables are applied. These suppress misidentification of the final state hadrons by imposing minimum values on **ProbNNk**(K) and **ProbNNpi**(π), which describe the probability of a kaon being a true kaon and a pion being a true pion. These simple cuts are relatively loose and are designed to remove a sizable part of the misidentified particles, while preserving signal decays with high efficiency. The exact requirements for all years are listed in Tab. 6.9.

Table 6.9: General requirements on PID variables of the hadrons [45].

Candidate	Year		
	2011/2012	2016	2017/2018
K	ProbNNk (K) > 0.02	ProbNNk (K) > 0.03	ProbNNk (K) > 0.05
π	ProbNNpi (π) > 0.02	ProbNNpi (π) > 0.03	ProbNNpi (π) > 0.05

In a second step, more specialized cuts on specific misidentifications between kaons and pions are applied. These involve multiple combinations of **ProbNNk**(π), **ProbNNpi**(K) and the **ProbNN** variables mentioned above. These cuts remove a large amount of $K^+ \leftrightarrow \pi^-$ misidentifications from the data sample. The specific requirements can be found in Tab. 6.10. These general requirements do not include cuts on the muon PID variables as the preselection already applies the isMuon criterion and **ProbNNmu**(μ) is also used in the MVA selection, which is discussed in Sec. 6.3.5.

Table 6.10: Hadron misidentification requirements for the three periods of data taking. These requirements ensure that a large portion of $K\pi$ misidentifications are excluded from the dataset [45].

Year	Requirement
2011/2012	ProbNNk (π) $\times$ ProbNNpi (K) < 0.08
2016	(1 - ProbNNk (K)) $\times$ ProbNNk (π) < 0.10
2017/2018	(1 - ProbNNk (K)) $\times$ ProbNNk (π) < 0.12

$B_s^0 \rightarrow \phi(1020)\mu^+\mu^-$ veto

The decay $B_s^0 \rightarrow (\phi(1020) \rightarrow K^+K^-)\mu^+\mu^-$ might be mistaken as the signal decay if one of the kaons in the final state is misidentified as a pion. The mass of the B_s^0 -meson is 87.28 MeV/ c^2 above the mass of the B^0 -meson. If the kaon is misidentified as a significantly lighter pion, the reconstructed $m_{K\pi\mu\mu}$ mass of the $B_s^0 \rightarrow \phi(1020)\mu^+\mu^-$ decay is peaking below the invariant mass peak of the signal. The original mass of the $\phi(1020)$ can be reconstructed by assigning the reconstructed pion the mass of a kaon ($m(K\pi \rightarrow K)$). Inside a tight $m(K\pi \rightarrow K)$ window around the $\phi(1020)$ mass, hard **ProbNNk**(π) cuts are applied to reject the background. As these cuts are only applied to the small subset of the events falling inside this mass window, the signal efficiency is still high. An event is vetoed if it fulfills $m(K\pi \rightarrow K\mu\mu) \in [5321, 5411]$ MeV/ c^2 and either

$$m(K\pi_{\rightarrow K}) \in [1010, 1030] \text{ MeV}/c^2$$

$$\text{AND ProbNNk}(\pi) > \begin{cases} 0.005 & 2011/2012/2016 \\ 0.01 & 2017/2018 \end{cases}$$

or

$$m(K\pi_{\rightarrow K}) \in [1030, 1075] \text{ MeV}/c^2$$

$$\text{AND} \begin{cases} \text{ProbNNk}(\pi) \times (1 - \text{ProbNNpi}(\pi)) > 0.4 & 2011/2012 \\ \text{ProbNNk}(\pi) > 0.75 & 2016 \\ \text{ProbNNk}(\pi) > 0.7 & 2017/2018 \end{cases}$$

$\Lambda_b^0 \rightarrow pK^-\mu^+\mu^-$ veto

Another decay that can mimic the signal through a single misidentification of one hadron is the mode $\Lambda_b^0 \rightarrow pK^-\mu^+\mu^-$. In this case, the proton is misidentified as a pion. The difference between the kaon and pion mass is much larger compared to the background discussed before. However, since the Λ_b^0 has a significantly higher mass compared to the B^0 , the reconstructed $m_{K\pi\mu\mu}$ invariant mass peaks in the vicinity of the signal. To veto this decay mode, the corrected $m(K\pi_{\rightarrow p}\mu\mu)$ mass is calculated where the mass of the reconstructed pion is replaced this that of the proton. The $p \rightarrow \pi$ misidentification background is vetoed by discarding events fulfilling the criterion

$$m(K\pi_{\rightarrow p}\mu\mu) \in [5575, 5665] \text{ MeV}/c^2$$

$$\text{AND} \begin{cases} (1 - \text{ProbNNpi}(\pi)) \times \text{ProbNNp}(\pi) > 0.002 & 2011/2012 \\ \text{ProbNNp}(\pi) > 0.06 & 2016 \\ \text{ProbNNp}(\pi) > 0.07 & 2017/2018 \end{cases}$$

The same decay can also be reconstructed as the signal with two hadron misidentifications. In that case, the proton is misidentified as a kaon, and the kaon is misidentified as a pion. The equivalent approach is applied to calculate the corrected Λ_b^0 mass, and cuts on $\text{ProbNNp}(K)$ and $\text{ProbNNk}(\pi)$ are applied inside a mass window around the Λ_b^0 peak.

$$m(K_{\rightarrow p}\pi_{\rightarrow K}\mu\mu) \in [5575, 5665] \text{ MeV}/c^2$$

$$\text{AND ProbNNp}(K) \times \text{ProbNNk}(\pi) > \begin{cases} 0.002 & 2011/2012 \\ 0.01 & 2016 \\ 0.015 & 2017/2018 \end{cases}$$

$B^0 \rightarrow J/\psi K^{*0}$ and $B^0 \rightarrow \psi(2S)K^{*0}$ double misidentification veto

The control mode decays $B^0 \rightarrow J/\psi K^{*0}$ and $B^0 \rightarrow \psi(2S)K^{*0}$ are usually separated from the rare mode by excluding the corresponding q^2 regions. Due to the size of the control mode branching fraction, double misidentifications of one muon from the charmonium resonance and one hadron from the K^{*0} resonance can contribute significantly to the rare mode. The misidentification causes candidates to leak outside of the control mode q^2 region. The swap masses $m(\pi_{\rightarrow\mu}^+\mu^-)$ and $m(K_{\rightarrow\mu}^+\mu^-)$ are calculated, and candidates close to the charmonium resonance masses that do not pass

stringent PID requirements are rejected. The requirements to reject the $\pi \leftrightarrow \mu$ double misidentification of the J/ψ mode are

$$m(\pi_{\rightarrow\mu}^+\mu^-) \in [3036, 3156] \text{ MeV}/c^2$$

$$\text{AND}$$

$$\begin{cases} \text{isMuon}(\pi) \text{ OR ProbNNmu}(\mu) < 0.15 & 2011/2012 \\ \text{isMuon}(\pi) \text{ OR ProbNNpi}(\pi) < 0.3 & 2016/2017/2018 \end{cases}$$

In the q^2 region between 4.0 and 8.0 GeV^2/c^4 this cut is required to be tighter to remove all background candidates. To remove any residual peaking background in this region, the cut for the J/ψ $\pi \leftrightarrow \mu$ double swap is tightened to remove all events that fulfill the requirement

$$m(\pi_{\rightarrow\mu}\mu) \in [3036, 3156] \text{ MeV}/c^2.$$

This concerns the q^2 bins 4-6 GeV^2/c^4 , 6-8 GeV^2/c^4 , and 1.1-6.0 GeV^2/c^4 . Furthermore, the cut

$$m(K_{\rightarrow\mu}\mu) \in [3036, 3156] \text{ MeV}/c^2$$

$$\text{AND}$$

$$\begin{cases} \text{isMuon}(K) \text{ OR ProbNNmu}(\mu) < 0.15 & 2011/2012 \\ \text{isMuon}(K) \text{ OR ProbNNk}(\mu) > 0.4 & 2016 \\ \text{isMuon}(K) \text{ OR ProbNNk}(\mu) > 0.5 & 2017/2018 \end{cases}$$

is designed to remove the $K \leftrightarrow \mu$ double misidentification of the J/ψ mode, while the corresponding background from the $\psi(2S)$ resonance is vetoed by the requirements

$$m(\pi_{\rightarrow\mu}\mu) \in [3626, 3746] \text{ MeV}/c^2$$

$$\text{AND isMuon}(\pi)$$

and

$$m(K_{\rightarrow\mu}\mu) \in [3626, 3746] \text{ MeV}/c^2$$

$$\text{AND isMuon}(K).$$

$B^+ \rightarrow K^+\mu^+\mu^-$ background veto

There are also background modes that do not require misidentified particles to resemble the signal. One example is the decay mode $B^+ \rightarrow K^+\mu^+\mu^-$, where one random pion from the PV is added to form the candidate. The random pion smears the $m_{K\pi\mu\mu}$ distribution such that it does not peak below the signal mass peak. The shape is similar to the combinatorial background. The angular shape of the background, however, can distort the angular distribution of the data. The $m_{K\mu\mu}$ mass is used to reconstruct the B^+ , and a simple cut on the mass is applied. This cut can shape the $\cos\theta_K$ distribution of the combinatorial background, which needs to be modeled in the PDF. Candidates are vetoed if they fall into the mass window defined by

$$m_{K\mu\mu} \in [5220, 5340] \text{ MeV}/c^2.$$

6.3.5 Combinatorial background suppression

The largest background contribution is randomly combined tracks that do not originate from the same decay. To access the angular distribution of the signal, the background obscuring it must be removed. A lot of the randomly combined tracks originate directly from a PV. Thus, parameters like B^0 proper decay time, $B^0 \chi^2_{\text{vtx}}$ and $B^0 p_T$ yield high separation power between the signal and background candidates since the signal is high p_T and detached from the PV, while the background is predominantly not. To reduce the combinatorial background efficiently, a boosted decision tree is trained that combines the information from the parameters listed in Tab. B.2. The hyperparameters used in the training of the BDT are also listed in the same table. Separate BDTs are trained for the data samples from the years 2011/12, 2016, and 2017/18 to account for differences in the run conditions between the years. As a signal proxy, sWeighted [133] data from the control mode region is used. Data from the upper mass sideband ($m_{K\pi\mu\mu} > 5500 \text{ MeV}/c^2$) are used as a proxy for the background. To prevent training bias from affecting data events in the control or rare mode angular fit, k-folding with 10 folds is used as a cross-validation technique [134]. The cut on the regression result is optimized to maximize the signal significance.

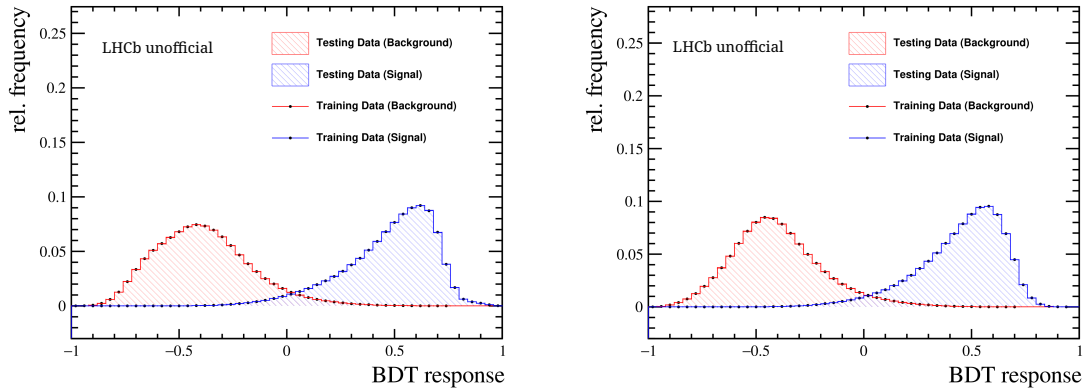


Figure 6.2: Signal and background BDT classifier output distributions from the years 2012 (left) and 2017 (right) overlayed for training and testing samples. The figures have been produced by Thomas Oeser [45].

6.3.6 Semi-leptonic backgrounds

Another potential background contribution arises from semi-leptonic decays of B-mesons involving D-mesons. These include multiple decay modes described by the topology $B \rightarrow (D \rightarrow K\mu\nu_\mu X)\mu\nu X$, where X denotes possible additional pions in the decay. Due to the loss of energy through the neutrinos, this background is only relevant in $q^2 < 8 \text{ GeV}^2/c^4$ and it also does not peak below the B^0 -mass. Instead, these semi-leptonic decays peak at high $\cos\theta_\ell$, which can theoretically impact the background model in the fit. A boosted decision tree is trained to remove this background using semi-leptonic background-enriched data from outside the signal region ($m_{K\pi\mu\mu} > 5350 \text{ MeV}/c^2$, $m_{K+\mu^-} < 1900 \text{ MeV}/c^2$) as background proxy. Similar to the BDT classifier against combinatorial background, k-folding is used as a cross-

validation technique. A cut on the BDT classifier output is placed to remove the semi-leptonic background to a negligible level [45].

6.3.7 Background from $B^+ \rightarrow J/\psi K^+$ decays

Candidates from $B^+ \rightarrow J/\psi K^+$ decays can contribute to the data sample if a random pion from the underlying event is combined with the final state particles of this decay. Usually, this kind of background is vetoed by the charmonium veto, but due to the large branching fraction of this decay, the lower tail of the reconstructed J/ψ resonance can leak into the signal region. This mainly impacts the q^2 bin below the J/ψ resonance. Due to the reconstructed q^2 being in the lower tail of the resonance, the background evades the veto designed to remove backgrounds from $B^+ \rightarrow K^+ \mu^+ \mu^-$ decays described in Sec. 6.3.4. Due to the combinatorial nature of this background mode, it does not peak below the signal shape, but rather follows the shape of an exponential function. A boosted decision tree is trained on simulated signal and $B^+ \rightarrow J/\psi K^+$ background samples to remove this background. Again, k-folding is used as a cross-validation technique. A requirement on the response of the BDT classifier reduces the background to a negligible level.

6.3.8 Invariant mass distributions on fully selected data

After the complete selection is applied, the combinatorial background is suppressed to a level where the signal decays are no longer obscured and are accessible for statistical analysis. Physical background contributions that peak below the signal in the invariant B mass distribution are reduced to a sub-percent level [45] and are neglected in the angular fit model. Systematic effects arising from small residual physical background contributions are studied in Sec. 6.6.11 and found to be small with respect to the expected statistical uncertainty.

Figure 6.3 shows two-dimensional histograms displaying the invariant mass distributions $m_{K\pi\mu\mu}$ and q^2 of fully selected LHCb data. All four plots show the same characteristic structure. A large number of candidates are concentrated around the B^0 mass, which originate from $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ decays. The different contributions can be distinguished using the q^2 distribution. The control mode decays $B^0 \rightarrow J/\psi K^{*0}$ and $B^0 \rightarrow \psi(2S) K^{*0}$ peak sharply at the J/ψ and $\psi(2S)$ masses. This is clearly visible in the plots since the tree-level decays dominate the signal due to their much larger branching fraction. The signal is characterized by a continuous distribution in q^2 , which is visible as a broad vertical band around the value of the B^0 mass. Candidates from residual combinatorial background are scattered over the displayed phase space region. The horizontal bands around the J/ψ and $\psi(2S)$ masses are also combinatorial background candidates, where a J/ψ or $\psi(2S)$ from the PV has been combined with random kaons and pions. Due to the abundance of the charmonium resonances, the combinatorial pollution is distinctly visible around the J/ψ and $\psi(2S)$ mass. Figure 6.4 displays the two-dimensional histograms of the invariant mass distributions $m_{K\pi\mu\mu}$ and $m_{K\pi}$ in the signal region, where the charmonium resonances are vetoed. The peak of the K^{*0} resonance is clearly visible around 892 MeV/ c^2 .

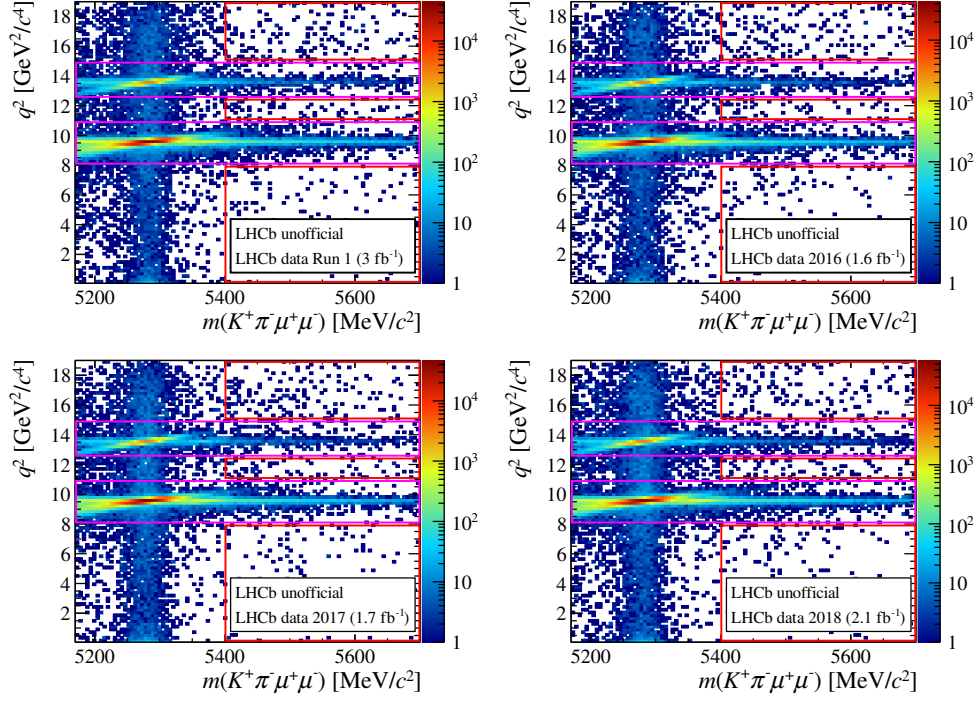


Figure 6.3: Two-dimensional histograms displaying the invariant mass distributions $m_{K\pi\mu\mu}$ and q^2 of fully selected LHCb data from Run 1 (top left), 2016 (top right), 2017 (bottom left), and 2018 (bottom right).

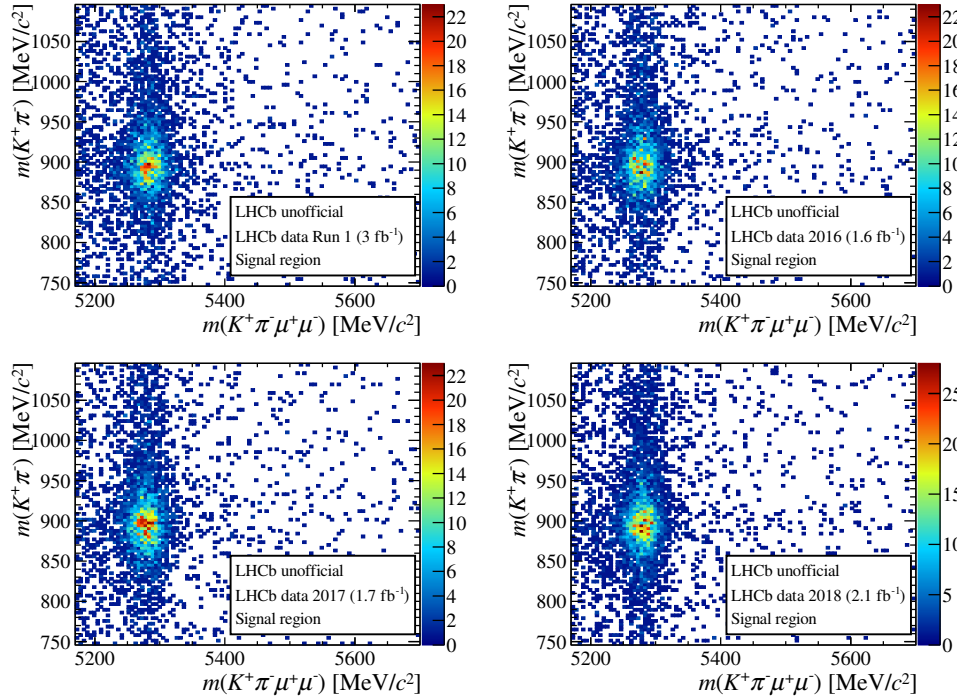


Figure 6.4: Two-dimensional histograms displaying the invariant mass distributions $m_{K\pi\mu\mu}$ and $m_{K\pi}$ of fully selected LHCb data in the signal region from Run 1 (top left), 2016 (top right), 2017 (bottom left), and 2018 (bottom right).

6.4 Mass fits

The $m_{K\pi\mu\mu}$ distribution is fitted simultaneously to the decay angles and $m_{K\pi}$ to separate signal from combinatorial background. The signal B^0 mass model needs to be fully determined before the final fit to the rare mode data due to the small number of candidates. The signal $m_{K\pi\mu\mu}$ model consists of two crystal ball functions [124] with opposite tail directions and shared means μ . The complete model is given in Eq. 6.45.

$$\mathcal{P}_{\text{DSCB}}(m_{K\pi\mu\mu} | m_{B^0}, \sigma_1, \sigma_2, \alpha_1, \alpha_2, n_1, n_2, \rho) = \rho \times \mathcal{P}_{\text{CB}}(m_{K\pi\mu\mu} | m_{B^0}, \sigma_1, \alpha_1, n_1) + (1 - \rho) \times \mathcal{P}_{\text{CB}}(m_{K\pi\mu\mu} | m_{B^0}, \sigma_2, \alpha_2, n_2). \quad (6.45)$$

6.4.1 Fit to control mode simulation

In a first step to determine the parameters of the mass model, a fit to fully selected Monte Carlo simulation of the control mode $B^0 \rightarrow J/\psi K^{*0}$ is performed for the Run 1, 2016, 2017, and 2018 data samples each.

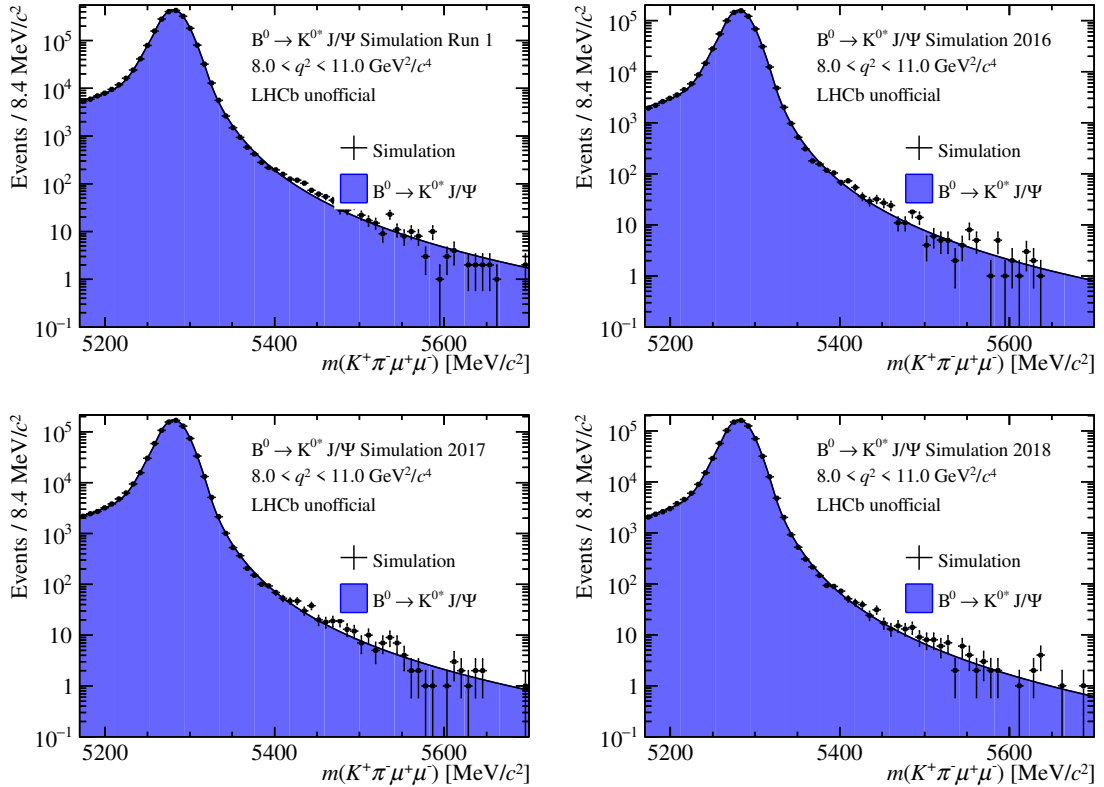


Figure 6.5: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow J/\psi K^{*0}$ Monte Carlo simulation samples. These fits of Run 1, 2016, 2017, and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

The fit projections in Fig. 6.5 describe the simulation well. In Sec. 6.6, a systematic uncertainty study is described that covers the effect of the assumption that the double crystal ball function accurately describes the shape of the signal $m_{K\pi\mu\mu}$ distribution. The resulting fit parameters for all run periods are listed in Tab. 6.11.

Table 6.11: Table of the mass model parameters resulting from the fit to control mode simulation.

Parameter	Run 1	2016	2017	2018
m_{B^0}	5280.425 ± 0.017	5281.040 ± 0.052	5281.013 ± 0.026	5281.05 ± 0.03
$f_{m,1}^{\text{sig}}$	0.566 ± 0.009	0.493 ± 0.023	0.555 ± 0.013	0.524 ± 0.015
$\sigma_{m,1}$	12.972 ± 0.056	12.63 ± 0.17	12.949 ± 0.084	12.8 ± 0.1
$\sigma_{m,2}$	19.36 ± 0.12	18.46 ± 0.25	19.08 ± 0.15	18.57 ± 0.16
α_{CB}	1.656 ± 0.016	1.502 ± 0.048	1.595 ± 0.022	1.512 ± 0.028
n_{CB}	1.049 ± 0.017	1.124 ± 0.054	1.095 ± 0.027	1.17 ± 0.03
$\alpha_{\text{CB},2}$	-2.143 ± 0.014	-2.198 ± 0.041	-2.217 ± 0.022	-2.188 ± 0.022
$n_{\text{CB},2}$	3.610 ± 0.069	3.44 ± 0.19	3.3 ± 0.1	3.54 ± 0.11

6.4.2 Fit to control mode data

After the B^0 mass signal model from Tab. 6.11 is obtained using $B^0 \rightarrow J/\psi K^{*0}$ simulation, it is used to perform a fit to LHCb data in the control mode region. It is known that the resolution and mean of the B^0 mass peak show differences between data and simulation. These disagreements are also present in rare mode data and therefore need to be accounted for. To achieve this, a common width scale for the Gaussian widths σ_1 and σ_2 is introduced.

$$\sigma_i \rightarrow s_{\sigma}^{J/\psi} \times \sigma_i \quad (6.46)$$

It is floated alongside the m_{B^0} in the mass fits to control mode data. The fitted parameter m_{B^0} and the width scale are also used in the fits to rare mode data. The data include more contributions than the control mode decay $B^0 \rightarrow K^{*0} J/\psi$ but also combinatorial background that originates from randomly combined tracks and an irreducible contribution from $B_s^0 \rightarrow K^{*0} J/\psi$ decays. As mentioned in Sec. 6.1.3, the combinatorial background contribution is described by an exponential function. The $B_s^0 \rightarrow K^{*0} J/\psi$ contribution is modeled in the same way as the control mode model. The parameters between both models are identical except for the Gaussian mean μ . A shift Δm_{B_s} is introduced relative to m_{B^0} , which is Gaussian constrained to the expected difference between m_{B^0} and $m_{B_s^0}$ from the PDG [26].

Table 6.12: Table of the mass model parameters resulting from the fit to control mode data.

Parameter	Run 1	2016	2017	2018
m_{B^0}	5284.015 ± 0.033	5279.763 ± 0.033	5279.507 ± 0.031	5280.341 ± 0.028
$s_m^{J/\psi}$	1.145 ± 0.002	1.155 ± 0.002	1.1263 ± 0.0018	1.1247 ± 0.0017
N_{bkg}	17735.7 ± 232.4	14115.0 ± 220.1	13308.8 ± 217.8	15752.4 ± 237.8
$N_{\overline{\text{bkg}}}$	17694.8 ± 231.5	13940.7 ± 219.0	13115.5 ± 214.9	15595.3 ± 234.9
τ_m	157.7 ± 1.3	163.7 ± 1.7	163.6 ± 1.8	163.6 ± 1.6
fb_s	0.01086 ± 0.00028	0.01185 ± 0.00028	0.01154 ± 0.00026	0.01204 ± 0.00024
Δm_{B_s}	87.30 ± 0.16	87.12 ± 0.15	87.13 ± 0.15	87.15 ± 0.15

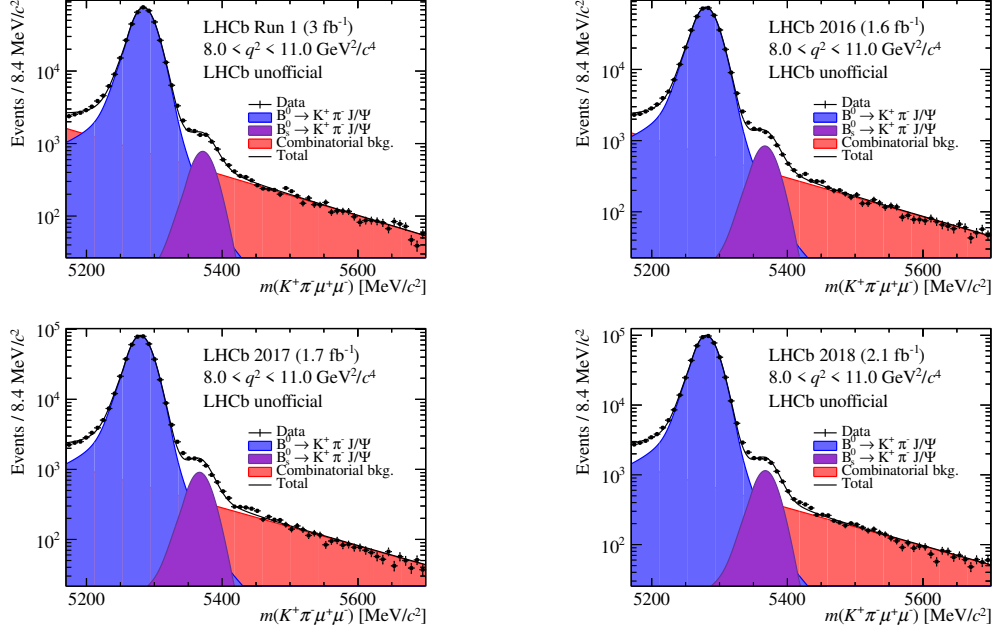


Figure 6.6: Projections of the $m_{K\pi\mu\mu}$ mass fits to $B^0 \rightarrow K^{*0} J/\psi$ data samples. These fits of Run 1, 2016, 2017 and 2018 data samples are used to determine corrections for the widths and mean of the $m_{K\pi\mu\mu}$ signal mass model.

The mass projections of these fits are displayed in Fig. 6.6 for all years separately and show good agreement between the data and the fit model. The results of the fit for all years are listed in Tab. 6.12. The resolution of the B^0 mass is $\mathcal{O}(10\%)$ larger than on Monte Carlo simulation. The B^0 mass m_{B^0} is fitted to be less than 5 MeV larger compared to the values from simulation.

6.4.3 Fit to rare mode simulation

Lastly, fits are performed to rare mode simulation where the Gaussian widths σ_1, σ_2 and relative fractions $f_{\text{sig}}^{m,1}$ of the crystal ball functions are fixed to the values obtained from the control mode fits. The parameters describing the tails $\alpha_1, \alpha_2, n_1, n_2$ are floated as their exact shape does depend on q^2 . Additionally, another shared scale factor for the kernel widths σ_1 and σ_2 is introduced and fitted. It accounts for the fact that the B^0 mass resolution depends on q^2 . These fits are performed separately for each q^2 bin, and the resulting parameters are used in the rare mode data fits.

$$\sigma_i(q^2) = s_\sigma(q^2) \times s_\sigma^{J/\psi} \times \sigma_i, \quad (6.47)$$

The projections of these mass fits for the years 2017/18 are shown as an example in Fig. 6.7. All additional projections for other years can be found in Sec. B.2. The fit results for all crystal ball parameters for each q^2 bin are given in Tab. B.3.

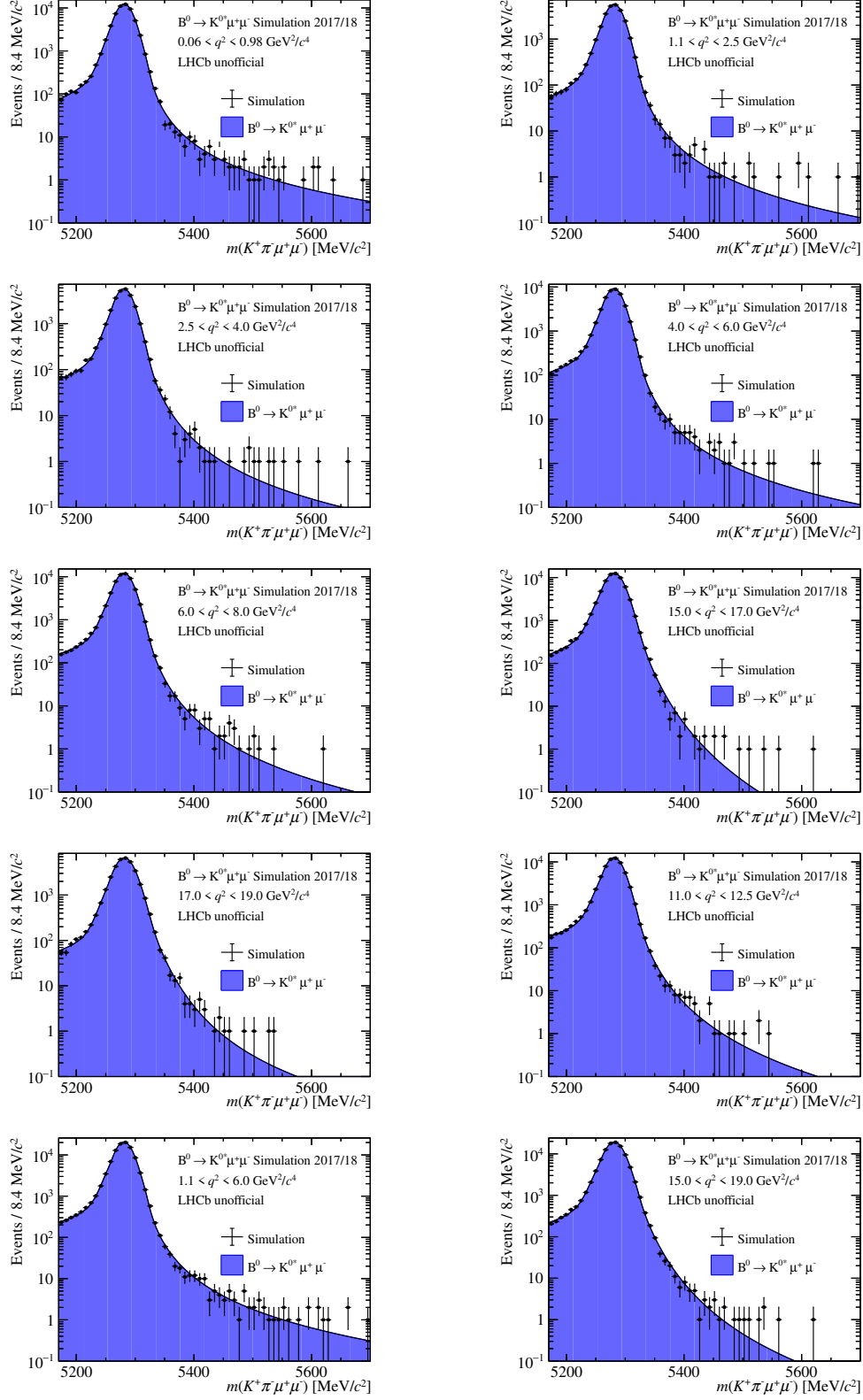


Figure 6.7: Projections of the $m_{K\pi\mu\mu}$ mass fits to $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ Monte Carlo simulation samples. These plots show fits to simulation samples from the years 2017 and 2018 for all ten q^2 bins.

Table 6.13: This table displays the signal yields of the mass fit to rare mode data in all 10 q^2 bins.

q^2 [GeV ² /c ⁴]	Signal yield N_{sig}			
	Run 1	2016	2017	2018
[0.06, 0.98]	414.7 ± 21.6	455.2 ± 22.6	481.3 ± 23.1	509.3 ± 23.9
[1.1, 2.5]	211.2 ± 16.3	223.7 ± 17.1	247.5 ± 17.5	290.9 ± 19.0
[2.5, 4.0]	203.8 ± 16.9	216.7 ± 17.2	230.8 ± 17.2	263.4 ± 18.3
[4.0, 6.0]	350.7 ± 21.9	318.2 ± 20.9	387.1 ± 22.5	398.4 ± 22.6
[6.0, 8.0]	411.1 ± 24.5	403.9 ± 24.6	433.2 ± 24.5	499.3 ± 26.7
[15.0, 17.0]	565.4 ± 28.0	508.4 ± 26.1	554.1 ± 27.2	721.0 ± 30.6
[17.0, 19.0]	357.7 ± 22.7	337.0 ± 21.0	348.0 ± 21.6	444.1 ± 24.2
[11.0, 12.5]	390.1 ± 24.3	372.6 ± 23.6	499.0 ± 25.9	563.5 ± 27.7
[1.0, 6.0]	765.5 ± 32.1	758.4 ± 32.0	864.5 ± 33.4	947.9 ± 34.6
[15.0, 19.0]	922.6 ± 35.9	845.9 ± 33.5	900.5 ± 34.6	1156.8 ± 38.8

6.4.4 Fit to rare mode data

For the final mass model, the mean B^0 mass m_{B^0} from the fit to control mode data in Tab. 6.12 is used along with the Gaussian core width from the fit to control mode simulation in Tab. 6.11. The correction factors for the core widths obtained in the fit to control mode data and rare mode simulation are applied. The tail parameters $\alpha_{CB,x}$ and $n_{CB,x}$ are taken from the fit to rare mode simulation in bins of q^2 displayed in Tab. B.3.

After the $m_{K\pi\mu\mu}$ signal mass model has been fully determined with fits to simulation and control mode data, all the parameters are fixed and used to perform an extended fit to rare mode data. Similar to the control mode data fits, the combinatorial background contribution is modeled with an exponential function. Contrary to the control mode fit a potential contribution $B_s^0 \rightarrow K^{*0} \mu^+ \mu^-$ is not modeled due to the negligible statistics. Fig. 6.8 shows the $m_{K\pi\mu\mu}$ projections of these fits combined for the complete data sample. The plots show good agreement between the fitted model and the data sample across all q^2 bins. The resulting signal yields from these data fits can be found in Tab. 6.13. The remaining fit parameters are listed in Sec. B.4.

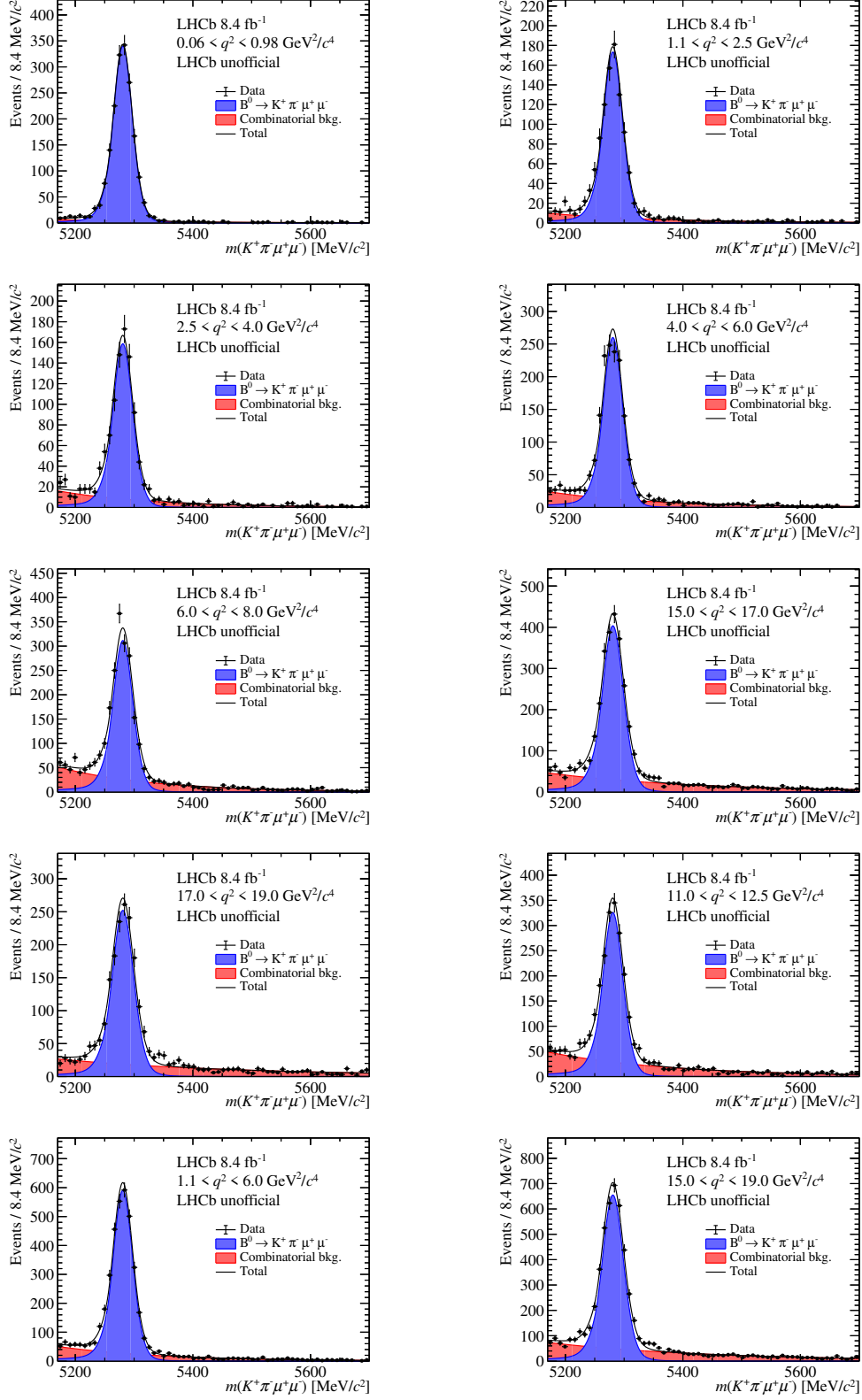


Figure 6.8: Projections of the $m_{K^+\pi^-\mu^+\mu^-}$ invariant mass fits to the full rare mode data set. The plots show separate fits for all 10 q^2 bins. The projections show good agreement between the fit model and the data distribution.

6.5 Angular fit

6.5.1 Acceptance effect

The shape of the decay angles $\cos\theta_K$, $\cos\theta_\ell$, ϕ , and invariant masses $m_{K\pi}$, q^2 are distorted by detector and selection effects. In order to perform a proper fit to the fully selected data sample, it is of paramount importance to understand and parameterize these effects fully. This parameterization is called the acceptance function, and its determination is described in the following section.

Parametrization of the acceptance

To be able to describe a five-dimensional acceptance shape, a general polynomial function is used, which constitutes a complete basis. Explicitly, the acceptance function can be written as

$$\varepsilon(\cos\theta_K, \cos\theta_\ell, \phi, q^2, m_{K\pi}) = \sum_{k,l,m,n,o} c_{klmno} P_k(\cos\theta_\ell) P_l(\cos\theta_K) P_m(\phi) P_n(q^2) P_o(m_{K\pi}). \quad (6.48)$$

$P_i(x)$ is a Legendre polynomial of order i as a function of the decay angles, $m_{K\pi}$ or q^2 . These polynomials are defined on a range of -1 to 1 . Therefore, the input variables need to be rescaled to this range before the acceptance is determined. The parameterization is fully defined by the set of parameters c_{klmno} . The shape of the acceptance effect can be simulated with $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ Monte Carlo simulation, which is generated flat in the decay angles and q^2 at the generator-level, meaning before any detector interactions or selection effects have been taken into account. Thus, the shape of the simulation after reconstruction and selection only originates from the acceptance effect. A minor complication is posed by the fact that the simulation is not flat in the invariant mass $m_{K\pi}$ as the K^{*0} resonance has the form of a relativistic Breit-Wigner function. Thus, the simulation needs to be re-weighted at generator-level to match this requirement. Compared to Ref. [24], this analysis operates on a significantly wider $m_{K\pi}$ selection window. This poses an additional challenge for the determination of the acceptances as the kinematic boundary, defined by the relation

$$m_B = m_{K\pi} + \sqrt{q_{\max}^2}, \quad (6.49)$$

lies within the considered phase space region. Above this boundary, no events are present as this region is kinematically forbidden. This mimics an artificial efficiency drop of 100%. This effect is displayed in Fig. 6.9 on the left side. Since the number of events in the data sample above this kinematic boundary is negligible and the signal PDF is zero in this region, the actual value of the acceptance does not impact the fit. However, a steep decline in the simulation distribution in this region could have undesired border effects on the acceptance function close to the phase space boundary. To avoid this effect, events close to the boundary are mirrored into the kinematically forbidden region along the boundary to ensure a fully populated phase space, as shown in Fig. 6.9 on the right side. The coefficients of the acceptance parameterization are determined using the method of moments [118]. They can be expressed as a sum of

Legendre polynomials of the order given by the coefficient index over all events in the simulation sample.

$$c_{klmno} = \frac{1}{N'} \sum_{i=1}^N w_i \left[\left(\frac{2k+1}{2} \right) \left(\frac{2l+1}{2} \right) \left(\frac{2m+1}{2} \right) \left(\frac{2n+1}{2} \right) \left(\frac{2o+1}{2} \right) \right. \\ \left. \times P_k(\cos \theta_K) P_l(\cos \theta_\ell) P_m(\phi) P_n(q^2) P_o(m_{K\pi}) \right]. \quad (6.50)$$

w_i denotes the total weight of event i in the simulation sample, composed of the weights obtained from the flatness re-weighting mentioned in this section. The normalization constant N' is given by the sum of weights of all events, *i.e.*

$$N' = \sum_{i=1}^N w_i. \quad (6.51)$$

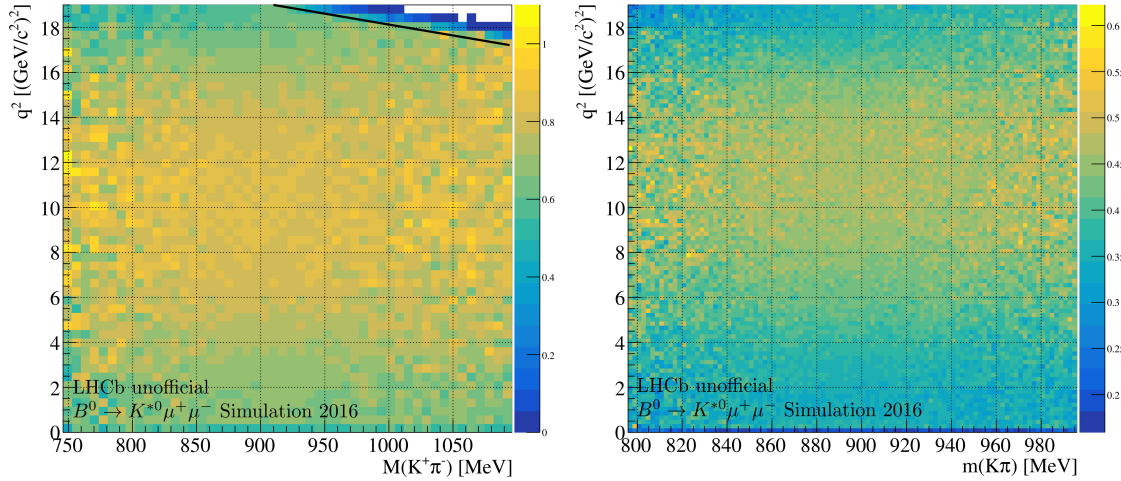


Figure 6.9: Two dimensional distribution of q^2 and $m_{K\pi}$ from simulation after q^2 - and $m_{K\pi}$ -flatness re-weighting. Left: The kinematic boundary is visible at high values of both q^2 and $m_{K\pi}$ in the upper right corner. This boundary artificially mimics a drop in efficiency. Right: Distribution after mirroring of events along the kinematic boundary.

It is necessary to limit the maximum number of polynomial orders for each dimension to a finite number. In general, the maximum number of orders needs to be chosen such that the model is sufficiently complex to describe the shape of the acceptance fully. On the other hand, using maximum orders that are too high can induce fast oscillations in the boundary regions of the acceptance function. Such oscillations might have a negative impact on the accuracy of the fit as they might lead to negative acceptance values. Due to the symmetry of the detector, it is expected that the shape of the acceptance is symmetric in ϕ around zero. Thus, only even polynomial orders are considered for this decay angle. It is found that the shape of the acceptance varies strongly at low q^2 . Therefore, two separate acceptance parameterizations are calculated in the q^2 regions from $0.058 \text{ GeV}^2/c^4$ to $1.2 \text{ GeV}^2/c^4$ and $0.98 \text{ GeV}^2/c^4$ to $19.1 \text{ GeV}^2/c^4$.

Table 6.14: This table shows the maximum polynomial order used to describe the acceptance shape in the q^2 region $0.9 < q^2 < 19.1 \text{ GeV}^2$.

	Run 1	2016	Run 2p2
$\cos(\theta_L)$	4	4	4
$\cos(\theta_K)$	7	7	8
Φ	6	6	6
$m_{K\pi}$	3	3	3
q^2	4	4	5

Table 6.15: This table shows the maximum polynomial order used to describe the acceptance shape in the q^2 region $0.058 < q^2 < 1.2 \text{ GeV}^2$.

	Run 1	2016	Run 2p2
$\cos(\theta_L)$	4	4	4
$\cos(\theta_K)$	4	4	7
Φ	0	0	0
$m_{K\pi}$	2	2	2
q^2	2	2	2

This is done to limit the required maximum polynomial order in q^2 , which is necessary for a sufficiently precise parameterization. Another effect that requires consideration is the potential difference between B^0 and $\bar{B}^0$ decays and the resulting detection asymmetry. To account for this effect, separate acceptance functions are determined for both decay flavors. The maximum order of Legendre polynomials used to describe the acceptance can be found in Tab. 6.14 and Tab. 6.15.

Acceptance projections

One-dimensional projections of the four determined acceptance functions per run period are displayed in Figs. 6.11 to 6.14 and Figs. B.21 to B.28. They show good agreement between the parameterizations and the simulated distributions across all years, q^2 regions, and B -flavors. However, since these plots integrate over the remaining angles and invariant masses, they are not sufficient to determine the quality of the fit. In the following sections, multiple ways of verifying the accuracy of the description are discussed. From the one-dimensional $m_{K\pi}$ projection and Fig. 6.10, it is evident that the description of the acceptance effect in $m_{K\pi}$ is non-trivial. A slight positive slope is visible in the acceptance shape from low to high $m_{K\pi}$. Also, Fig. 6.10 shows that the $m_{K\pi}$ acceptance effect does not factorize from decay angles like ϕ . Therefore, it is clear that the five-dimensional description of the acceptance is more accurate compared to Ref. [24], where the $m_{K\pi}$ acceptance was assumed to be flat and factorized with respect to the decay angles. A systematic uncertainty was assigned to cover this effect, which is no longer necessary.

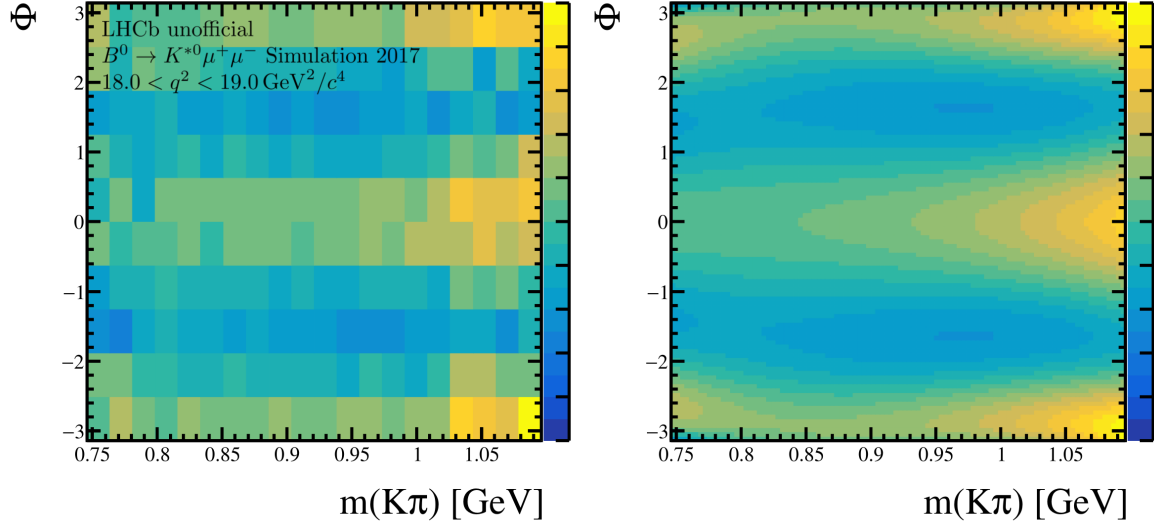


Figure 6.10: Two-dimensional acceptance projection on $m_{K\pi}$ versus ϕ , obtained for 2017 data taking conditions, in the q^2 region of $18.0 < q^2 < 19.0 \text{ GeV}^2/c^4$. It is evident that the acceptance effect of $m_{K\pi}$ does vary with ϕ and therefore does not fully factorize, making a five-dimensional description of the acceptance necessary.

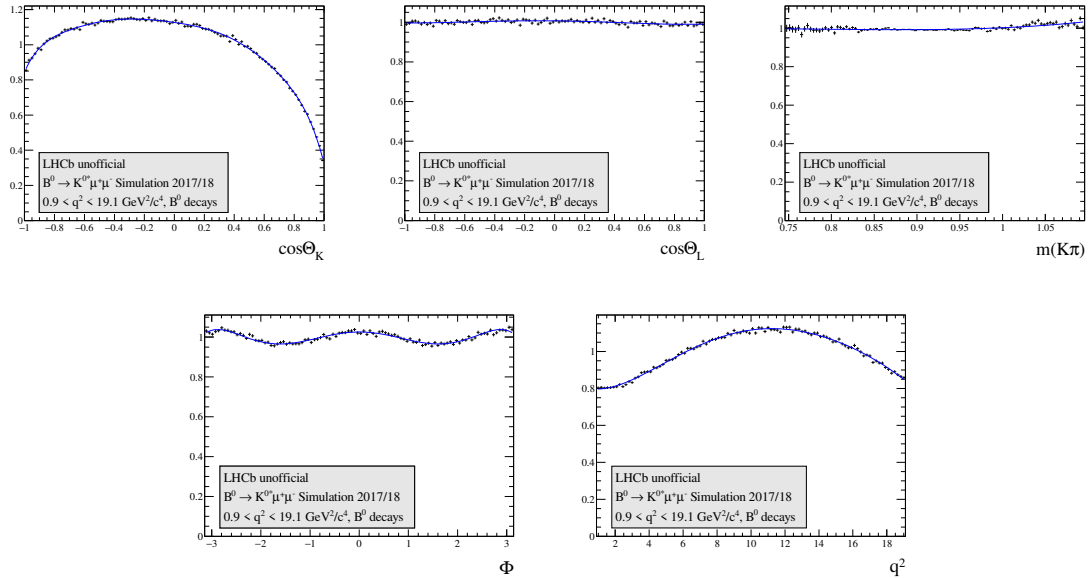


Figure 6.11: One-dimensional acceptance projections for B^0 events between $0.98 < q^2 < 19.1 \text{ GeV}^2$ determined simulation. Projections are shown for $\cos\theta_K$ (top left), $\cos\theta_\ell$ (top center), $m_{K\pi}$ (top right), ϕ (bottom left), and q^2 (bottom right).

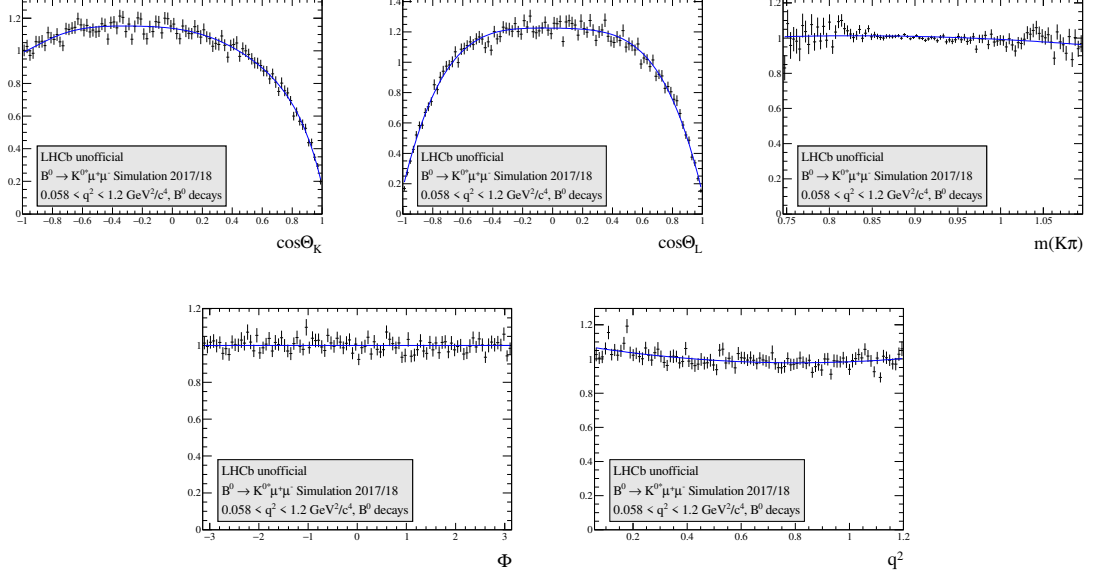


Figure 6.12: One-dimensional acceptance projections for B^0 events between $0.058 < q^2 < 1.2 \text{ GeV}^2$ determined from simulation. Projections are shown for $\cos\theta_K$ (top left), $\cos\theta_\ell$ (top center), $m_{K\pi}$ (top right), ϕ (bottom left), and q^2 (bottom right).

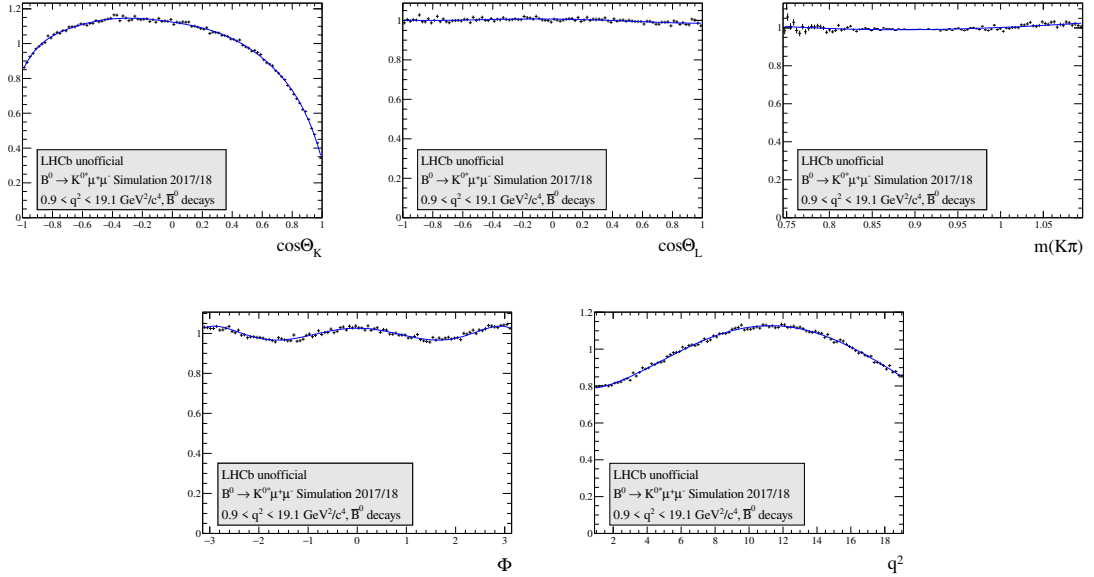


Figure 6.13: One-dimensional acceptance projections for $\bar{B}^0$ events between $0.98 < q^2 < 19.1 \text{ GeV}^2$ determined from simulation. Projections are shown for $\cos\theta_K$ (top left), $\cos\theta_\ell$ (top center), $m_{K\pi}$ (top right), ϕ (bottom left), and q^2 (bottom right).

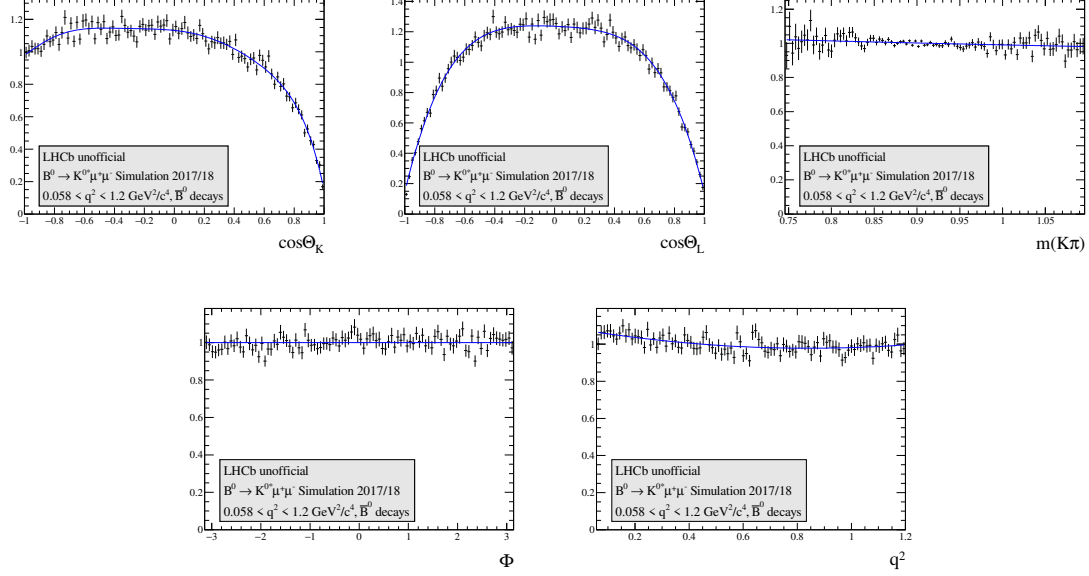


Figure 6.14: One-dimensional acceptance projections for $\bar{B}^0$ events between $0.058 < q^2 < 1.2 \text{ GeV}^2$ determined from simulation. Projections are shown for $\cos\theta_K$ (top left), $\cos\theta_\ell$ (top center), $m_{K\pi}$ (top right), ϕ (bottom left), and q^2 (bottom right).

Direct calculation of normalization integrals

For the non-weighted fits performed in the narrow q^2 bins, the acceptance enters via the normalization, where the product of the acceptance and signal pdf is integrated over the decay angles $\vec{\Omega}$ and $m_{K\pi}$. These integrals are of the form

$$\xi_j = \int f_j(\vec{\Omega}) \mathcal{BW}(m_{K\pi}) \epsilon(\vec{\Omega}, m_{K\pi}) d\vec{\Omega} dm_{K\pi}. \quad (6.52)$$

The definitions of the angular terms f_i are given in Tab. 2.3. In the nominal fit setup $\epsilon(\vec{\Omega}, m_{K\pi})$ is given by the acceptance function described in Sec. 6.5.1. The integrals over the decay angles $\cos\theta_K$, $\cos\theta_\ell$, and ϕ are performed analytically, while the integral over $m_{K\pi}$ is performed numerically.

As a cross-check, these integrals can alternatively be calculated numerically using the simulation sample by Monte Carlo integration. This avoids the necessity to determine an explicit expression for the acceptance. The integrals ξ_i can be determined using the formula

$$\xi_j = \frac{1}{N_{\text{gen}}} \sum_e w_e 8\pi (m_{K\pi}^{\text{max}} - m_{K\pi}^{\text{min}}) \cdot f_j(\vec{\Omega}^e, m_{K\pi}^e), \quad (6.53)$$

where the sum runs over all events of the simulation sample in the specific q^2 bin under study. These integrals can be compared directly to the ones determined from the acceptance parameterization. Eight example integrals for the years 2017/18 can be found as a function of q^2 in Fig. 6.15. Both methods show good agreement.

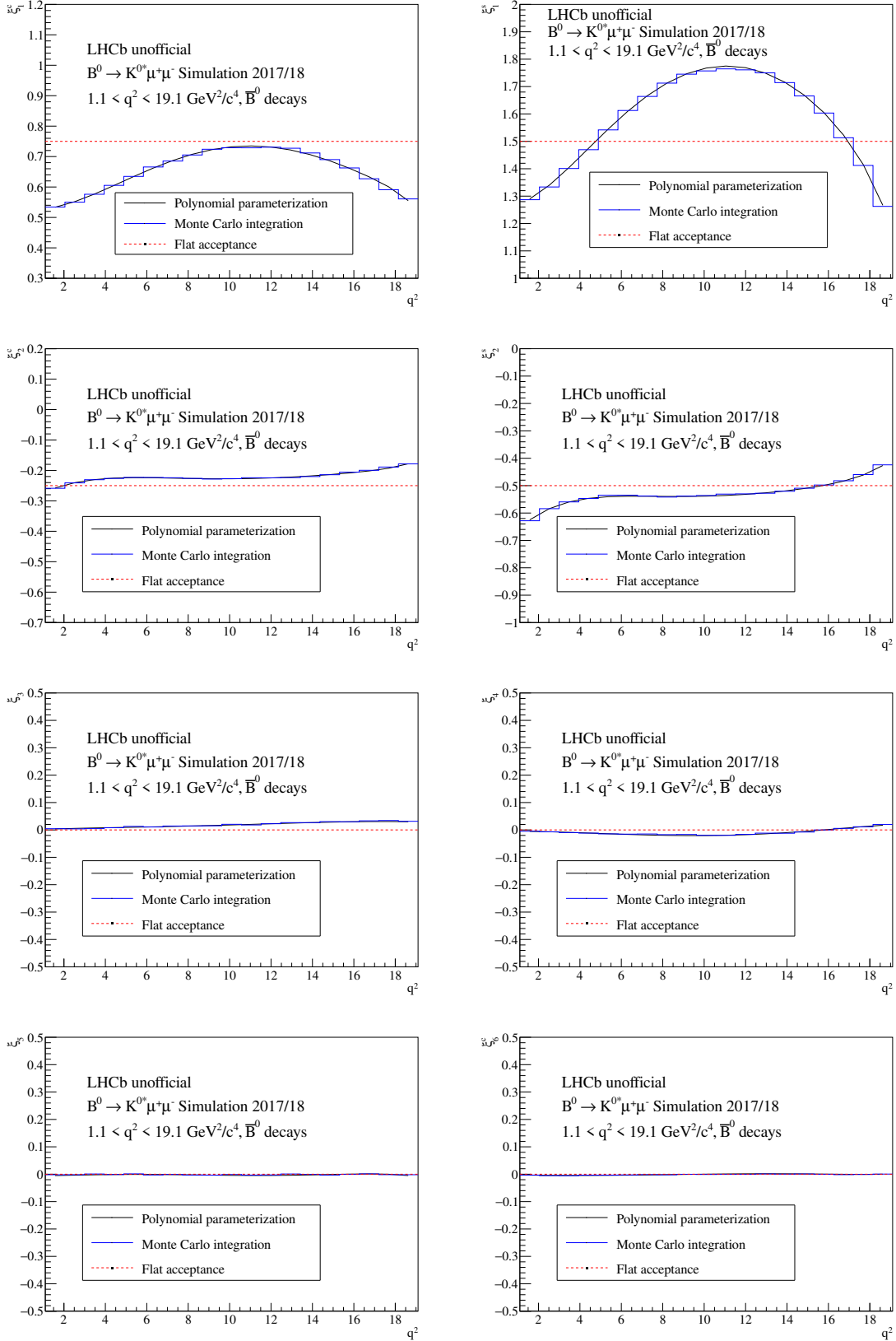


Figure 6.15: Comparison of ξ_1^s , ξ_1^c , ξ_2^s , ξ_2^c , ξ_3 , ξ_4 , ξ_5 and ξ_6^s above $q^2 = 1.1$ GeV²/c⁴ for Run 2p2 calculated using the nominal acceptance parameterization using legendre polynomials (black) and Monte Carlo integration (blue). This crosscheck uses the $\bar{B}^0$ acceptance parameterization and $\bar{B}^0$ simulation.

Acceptance validation through fits to $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ simulation

The validity of the acceptance parameterization is also cross-checked by fits to fully selected $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ simulation. The angular distribution of this simulation is generated based on an amplitude model using SM theory calculations as input [135]. The S-wave and interference parameters are fixed to zero because the simulation only contains P-wave $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays. The fits are performed separately for each run period. The theoretical values of the angular observables are extracted from fits to rare mode simulation before any reconstruction or selection with a flat acceptance function. If the parameterization describes the acceptance effect accurately, the fit results of the fully selected and the generator-level simulation should agree within their statistical and systematic uncertainties. Figures 6.17 and 6.18 show the comparison of the generator-level and fully simulated fit for Run 2p2. The comparison plots for the remaining run periods can be found in Sec. B.5. The uncertainties displayed in these plots and used to determine the pulls are a combination of the statistical uncertainty of the selected simulation and the systematic uncertainty covering the statistical uncertainty on the acceptance parameterization and the acceptance evaluation point in q^2 . The fit results show good agreement across all years. An important result of this study is that if the lepton masses are assumed to be non-negligible in the q^2 region below $0.98 \text{ GeV}^2/c^4$, it is necessary to perform a weighted fit to get sufficiently accurate results. The weighted fit takes into account the q^2 dependence of the acceptance and the q^2 distribution of the simulation, which both vary strongly in this region.

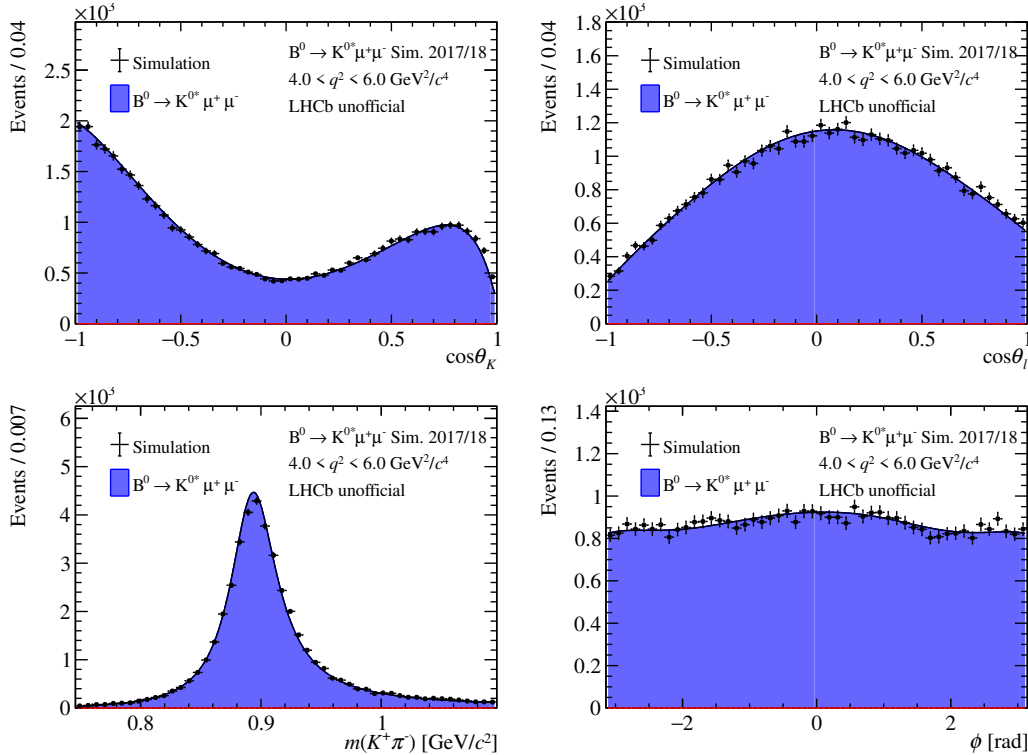


Figure 6.16: Projections of the angular fit to simulated $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ events after the full selection in $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$ (2017/18). These fits are used to validate the acceptance parameterization by comparing the fit results to the parameters of the underlying physics model.

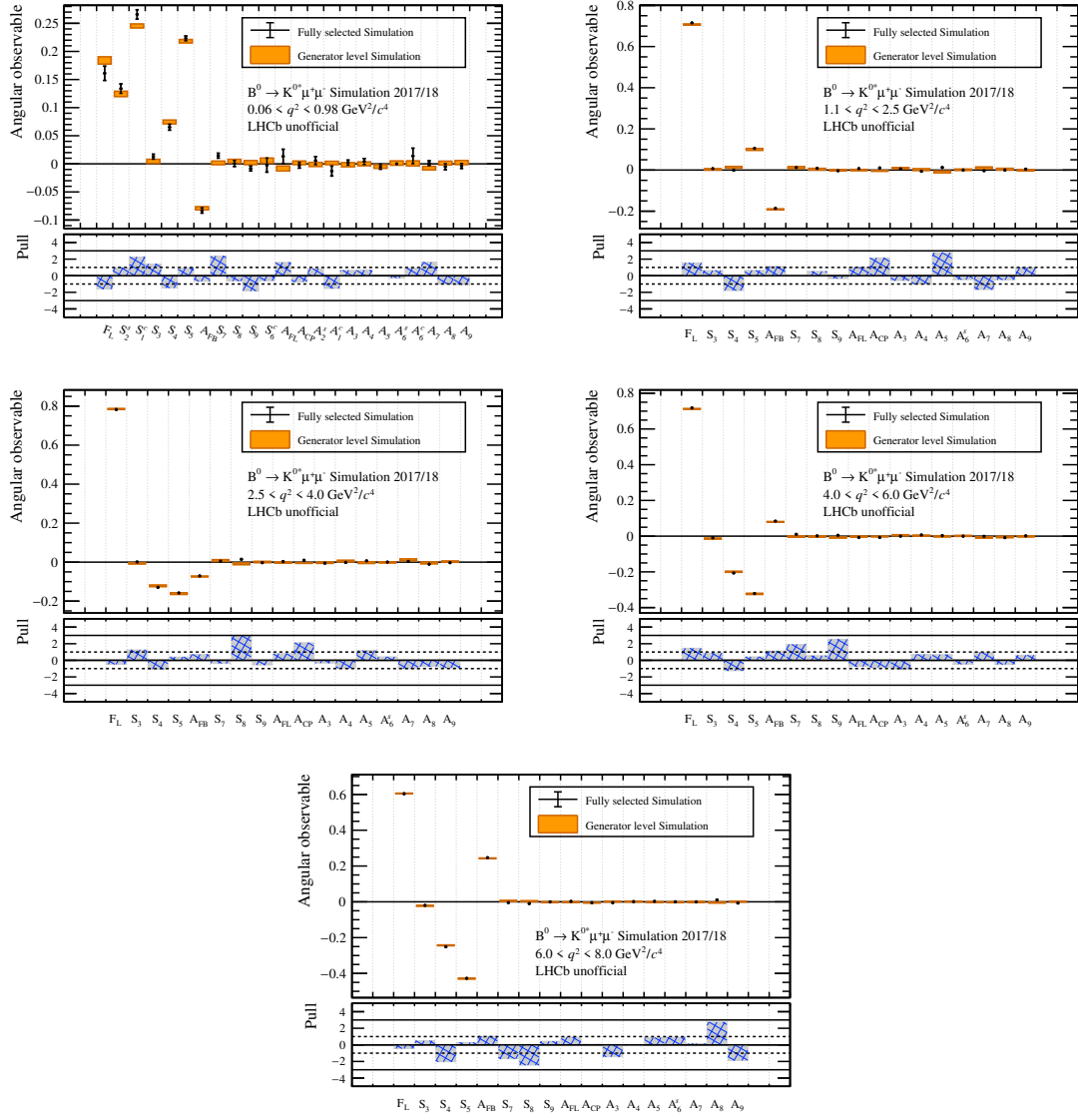


Figure 6.17: Comparison between fits to generator-level simulation and fully selected simulation from 2017. For the fit to selected and generator-level simulation, the nominal and flat acceptance functions are used, respectively. The plots compare the P-wave angular observables for the q^2 bins from $0.06 - 6.0 \text{ GeV}^2/c^4$.

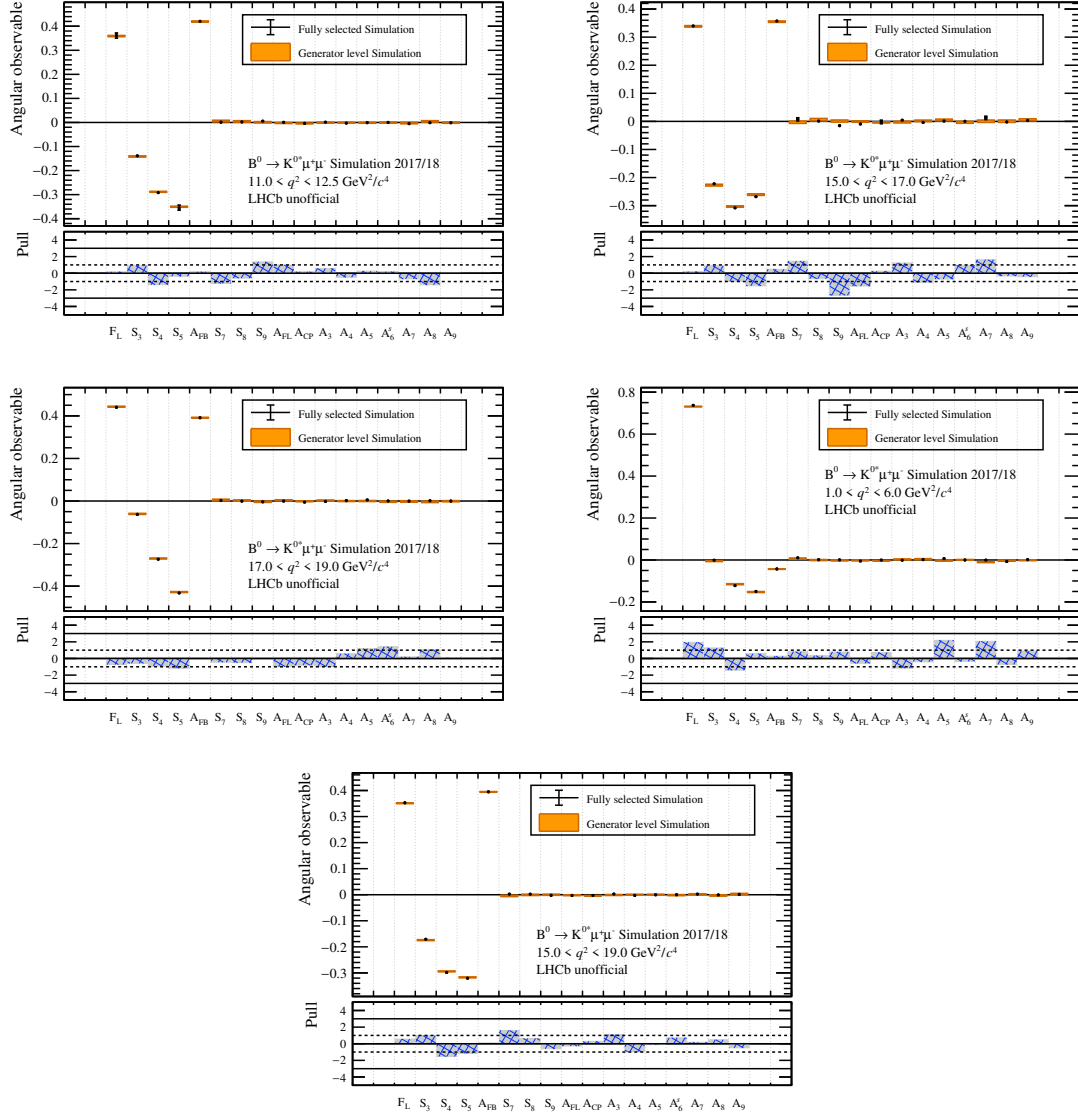


Figure 6.18: Comparison between fits to generator-level simulation and fully selected simulation from 2017. For the fit to selected and generator-level simulation, the nominal and flat acceptance functions are used, respectively. The plots compare the P-wave angular observables for the q^2 bins from 6.0 – 19.0 GeV^2/c^4 .

Simulation fits with Monte-Carlo integrated ξ_i

Using the method introduced in Sec. 6.5.1, it is possible to calculate the integrals ξ_i necessary for the normalization of the signal PDF, just using simulation. Since this method is independent of any assumptions about a parameterization function, such as the maximum polynomial order, it can serve as a cross-check of the validity of these assumptions. The integrals ξ_i are calculated as an average over the respective q^2 bin under study. To take into account the non-flat q^2 distribution, the simulation events are re-weighted according to the q^2 distribution from simulation generated with an underlying amplitude model. This is important for the q^2 region below $0.98 \text{ GeV}^2/c^4$ where the acceptance function and q^2 distribution vary strongly. Figures 6.19 and 6.20 show the fit results for the acceptance parameterization using Legendre polynomials and the Monte-Carlo integrated ξ_i for Run 2p2. The results of the generator-level fit are shown for reference. Comparison plots for the remaining run periods can be found in the appendix Sec. B.6. The pulls show good agreement between the method that uses the acceptance parameterization and the one that calculates the integrals using Monte Carlo integration. This confirms that the acceptance parameterization indeed describes the shape of the simulation sufficiently well despite the fact that the maximum order of polynomials is limited.

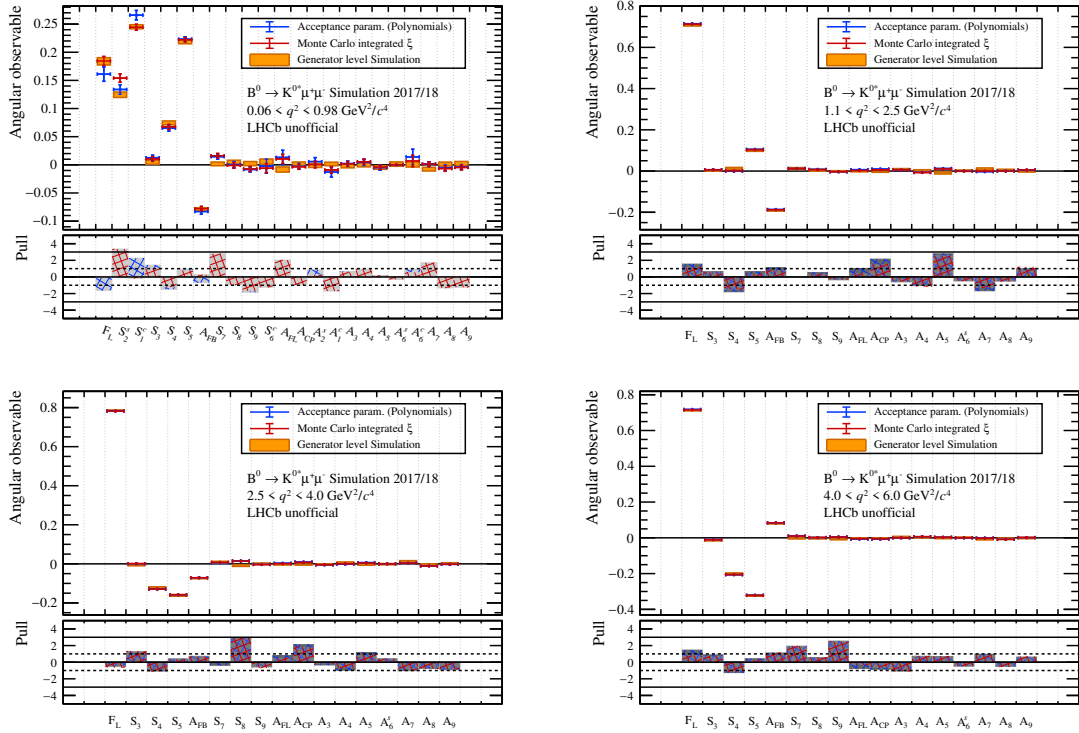


Figure 6.19: Comparison between fits to generator-level simulation and fully selected simulation from 2017. For the fit to the selected simulation, the acceptance parameterization using Legendre polynomials or the Monte Carlo-integrated ξ_i are employed. For the generator-level fit, the acceptance is flat. The plots compare the P-wave angular observables for the narrow q^2 bins from $0.06 - 6.0 \text{ GeV}^2/c^4$.

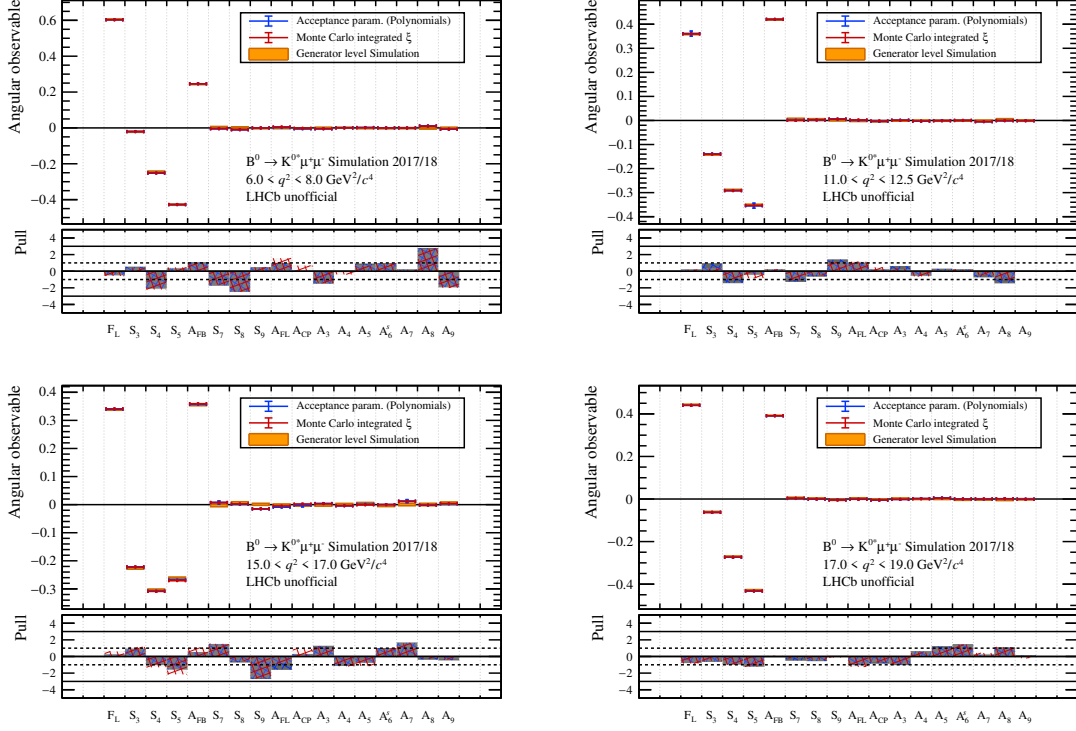


Figure 6.20: Comparison between fits to generator-level simulation and fully selected simulation from 2017. For the fit to the selected simulation, the acceptance parameterization using Legendre polynomials or the Monte Carlo-integrated ξ_i are employed. For the generator-level fit, the acceptance is flat. The plots compare the P-wave angular observables for the narrow q^2 bins from $6.0 - 19.0 \text{ GeV}^2/c^4$.

6.5.2 Fit validation on data using $B^0 \rightarrow J/\psi K^{*0}$

In addition to the checks on simulation, the acceptance parameterization is also verified on data. For this purpose, the control mode data sample containing a large amount of $B^0 \rightarrow K^{*0} J/\psi$ decays is used. These events are located in the q^2 region between $8.0 \text{ GeV}^2/c^4$ and $11.0 \text{ GeV}^2/c^4$. Fits are performed simultaneously to all data sets as well as separately for the Run 1, 2016, 2017, and 2018 data sets to check the consistency between the years. To confirm the validity of the acceptance and PDF, the resulting angular observables are compared to the results of a previous amplitude analysis performed by LHCb [136]. The results of this amplitude analysis can be found in Tab. 6.16. Since the amplitude analysis only allows for inference of the P-wave CP -symmetries from its results, the CP -asymmetries as well as S-wave and interference observables can not be cross-checked. The paper quotes the amplitudes $|A_{0,\perp,\parallel}|$ and the strong phases $\delta_{\perp,\parallel}$ directly. Thus, they are translated into the angular observables used in this analysis using the definition in Tab. 2.3. Due to the high yield of the control mode, it is possible to float some parameters that need to be fixed in the rare mode fits. This includes the parameters describing the P- and S-wave $m_{K\pi}$ models. The results are subsequently used in the angular fits to the rare mode.

Table 6.16: Results for the P -wave observables obtained from the results for the amplitudes $|A_{0,\perp,\parallel}|$ and the strong phases $\delta_{\perp,\parallel}$ reported in the LHCb analysis of $B^0 \rightarrow J/\psi K^{*0}$ [136].

Parameter	LHCb result
F_L	0.572 ± 0.008
S_3	-0.013 ± 0.010
S_4	-0.250 ± 0.006
S_5	0
A_{FB}	0
S_7	0
S_8	-0.048 ± 0.007
S_9	-0.084 ± 0.006

The values of the angular observables obtained from the single-year fits as well as the simultaneous fit to all data samples are reported in Tab. 6.17. The mass and background parameters are listed in Tab. B.5.

The crosscheck finds good agreement with the results of the dedicated amplitude analysis shown in Tab. 6.16. The uncertainties shown for this cross-check are of statistical nature only, which is relevant as the uncertainties are heavily systematically dominated due to the large size of the control mode sample. Overall, good agreement across the run periods is found for the absolute majority of angular observables, considering only the statistical uncertainty. Exceptions are F_L and S_{11}^{re} , which are the angular observables that are most affected by the systematic uncertainties. Changes in the run conditions between the years can induce systematic differences in angular observables between run periods. The differences in F_L and S_{11}^{re} between years displayed in Tab. 6.17 are small compared to the expected statistical uncertainty on the rare mode (Tab. 6.19). All CP -asymmetries are compatible with zero, which is in line with the expectation for the control mode decay [137]. The total P -wave CP -asymmetry A_{CP} is also consistent with zero. This is an artifact of the effective correction of $A_{CP}^{K^{*0}\mu^+\mu^-,PW+SW}$ with $A_{CP}^{K^{*0}J/\psi,PW+SW}$ to account for the expected detection and production asymmetries discussed in Sec. 6.1.4. This correction is also applied here as a cross-check.

Figures 6.21 and 6.22 show the one-dimensional projections in $m_{K\pi\mu\mu}$, $m_{K\pi}$, and the decay angles of the control mode fit for each run period. They show an overall good fit quality. However, there are some expected systematic structures in $\cos\theta_K$. The control mode q^2 region contains contributions from exotic decays [126]. These have a different angular structure compared to the control mode and can not be removed from the data sample. The effect of these exotic decays is evident from a slight mismodeling in the lower region of the $\cos\theta_K$ projections. These exotic decays are only present inside the charmonium regions and thus are not modeled in the signal model as they are not relevant for the rare mode fit. Overall, the results of the control mode fit confirm that the acceptance and PDF are able to replicate previous results within the expected uncertainties. The results are also overall consistent between years. Small differences in angular observables between run periods are negligible compared to the expected statistical rare mode uncertainty.

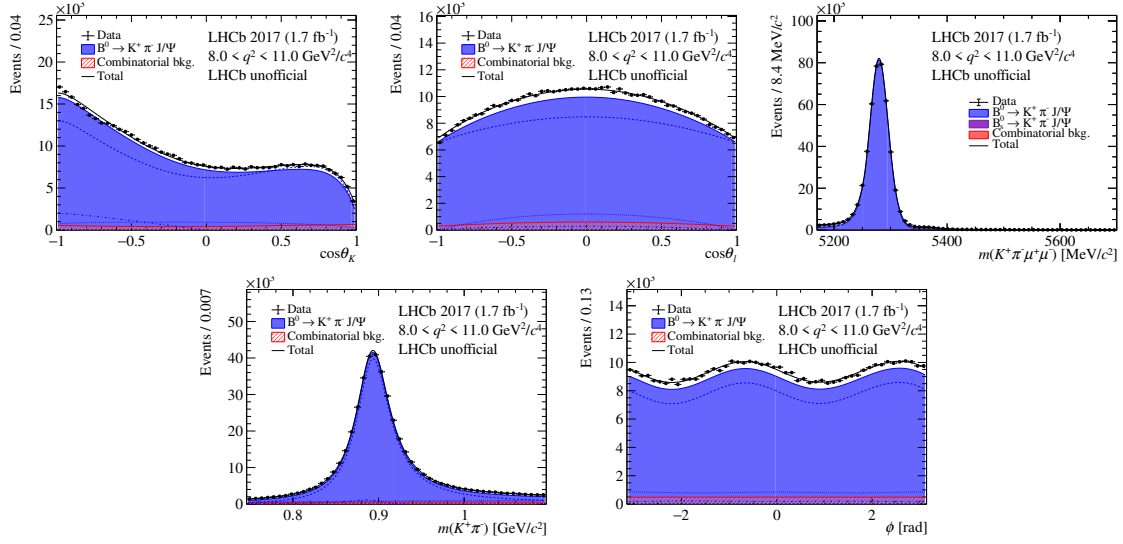


Figure 6.21: Fit projections of the control mode fit to LHCb data from the year 2017 in $\cos \theta_\ell$, $\cos \theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$.

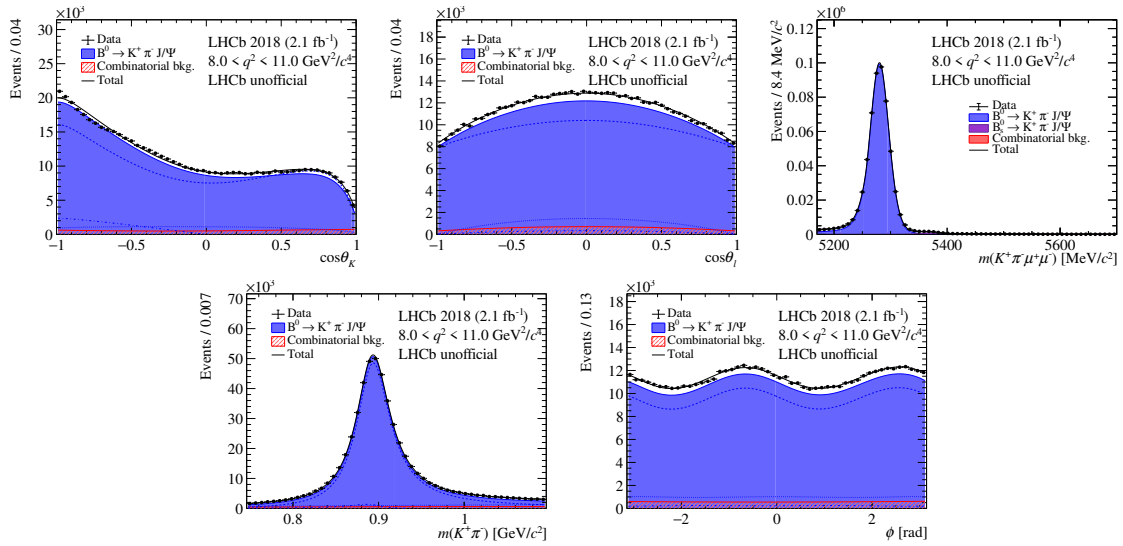


Figure 6.22: Fit projections of the control mode fit to LHCb data from the year 2018 in $\cos \theta_\ell$, $\cos \theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$.

Table 6.17: Results for the angular observables from single-year fits as well as a simultaneous fit to all data. The shown uncertainties are of statistical nature only.

Observable	Run 1	2016	2017	2018	Simultaneous fit
F_L	0.5554 ± 0.0012	0.5650 ± 0.0012	0.55992 ± 0.00081	0.55992 ± 0.00081	0.5600 ± 0.0006
S_3	-0.0023 ± 0.0017	-0.0020 ± 0.0016	-0.0026 ± 0.0011	-0.0026 ± 0.0011	-0.00240 ± 0.00079
S_4	-0.2483 ± 0.0018	-0.2421 ± 0.0018	-0.2427 ± 0.0012	-0.2427 ± 0.0012	-0.24382 ± 0.00086
S_5	0.00049 ± 0.00179	0.0035 ± 0.0018	0.0015 ± 0.0012	0.0015 ± 0.0012	0.00171 ± 0.00086
A_{FB}	0.00018 ± 0.00112	0.0013 ± 0.0011	-0.00091 ± 0.00072	-0.00091 ± 0.00072	-0.00015 ± 0.00053
S_7	0.0015 ± 0.0019	0.0016 ± 0.0019	-0.00047 ± 0.00120	-0.00047 ± 0.00120	0.00044 ± 0.00089
S_8	-0.0490 ± 0.0019	-0.0529 ± 0.0019	-0.0506 ± 0.0012	-0.0506 ± 0.0012	-0.0508 ± 0.0009
S_9	-0.0845 ± 0.0017	-0.0861 ± 0.0016	-0.0851 ± 0.0011	-0.0851 ± 0.0011	-0.08521 ± 0.00078
A_{FL}	-0.0000 ± 0.0017	-0.0033 ± 0.0017	-0.00032 ± 0.00113	-0.00032 ± 0.00113	-0.00093 ± 0.00083
A_3	0.0014 ± 0.0017	-0.0017 ± 0.0016	-0.00098 ± 0.00107	-0.00098 ± 0.00107	-0.00062 ± 0.00079
A_4	-0.0010 ± 0.0018	0.0019 ± 0.0018	0.00076 ± 0.00117	0.00076 ± 0.00117	0.00062 ± 0.00086
A_5	-0.0016 ± 0.0018	0.0015 ± 0.0018	0.00004 ± 0.00116	0.00004 ± 0.00116	-0.00001 ± 0.00086
A_{6s}	-0.0015 ± 0.0015	0.00038 ± 0.00145	0.00042 ± 0.00095	0.00042 ± 0.00095	-0.00003 ± 0.00070
A_7	0.0023 ± 0.0019	-0.00048 ± 0.00185	-0.00052 ± 0.00120	-0.00052 ± 0.00120	0.00014 ± 0.00089
A_8	0.00088 ± 0.00189	-0.00055 ± 0.00189	-0.0012 ± 0.0012	-0.0012 ± 0.0012	-0.00057 ± 0.00090
A_9	-0.00016 ± 0.00166	0.0023 ± 0.0016	-0.00015 ± 0.00106	-0.00015 ± 0.00106	0.00043 ± 0.00078
F_S	0.0931 ± 0.0013	0.0929 ± 0.0013	0.09255 ± 0.00082	0.09255 ± 0.00082	0.09277 ± 0.00061
S_{11}^{re}	-0.3632 ± 0.0032	-0.3539 ± 0.0032	-0.3484 ± 0.0021	-0.3484 ± 0.0021	-0.3531 ± 0.0015
S_{11}^{im}	0.0025 ± 0.0068	-0.0012 ± 0.0064	0.0041 ± 0.0040	0.0041 ± 0.0040	0.0025 ± 0.0030
S_{14}^{re}	0.1397 ± 0.0044	0.1309 ± 0.0043	0.1387 ± 0.0028	0.1387 ± 0.0028	0.1371 ± 0.0021
S_{14}^{im}	-0.0633 ± 0.0038	-0.0653 ± 0.0037	-0.0612 ± 0.0024	-0.0612 ± 0.0024	-0.0626 ± 0.0018
S_{15}^{re}	-0.0046 ± 0.0039	-0.0044 ± 0.0038	0.0008 ± 0.0025	0.0008 ± 0.0025	-0.0016 ± 0.0018
S_{15}^{im}	0.0016 ± 0.0028	-0.0017 ± 0.0027	0.0000 ± 0.0018	0.0000 ± 0.0018	-0.00002 ± 0.00131
S_{16}^{re}	0.0012 ± 0.0028	-0.0024 ± 0.0028	0.0007 ± 0.0018	0.0007 ± 0.0018	0.00008 ± 0.00134
S_{16}^{im}	0.00029 ± 0.00395	0.0030 ± 0.0039	-0.0011 ± 0.0025	-0.0011 ± 0.0025	0.00017 ± 0.00187
S_{17}^{re}	0.175 ± 0.003	0.1645 ± 0.0029	0.1721 ± 0.0019	0.1721 ± 0.0019	0.1710 ± 0.0014
S_{17}^{im}	-0.0371 ± 0.0053	-0.0274 ± 0.0051	-0.0375 ± 0.0033	-0.0375 ± 0.0033	-0.0351 ± 0.0025
A_{FS}	0.00046 ± 0.00108	0.0013 ± 0.0011	0.00012 ± 0.00069	0.00012 ± 0.00069	0.00047 ± 0.00051
A_{11}^{re}	-0.0026 ± 0.0030	-0.0068 ± 0.0030	-0.0025 ± 0.0019	-0.0025 ± 0.0019	-0.0035 ± 0.0014
A_{11}^{im}	0.00011 ± 0.00215	-0.0016 ± 0.0022	-0.0031 ± 0.0014	-0.0031 ± 0.0014	-0.002 ± 0.001
A_{14}^{re}	0.0047 ± 0.0041	-0.0012 ± 0.0041	-0.00039 ± 0.00265	-0.00039 ± 0.00265	0.00055 ± 0.00195
A_{14}^{im}	-0.00005 ± 0.00289	0.0014 ± 0.0029	0.0039 ± 0.0019	0.0039 ± 0.0019	0.0024 ± 0.0014
A_{15}^{re}	0.0047 ± 0.0038	-0.0025 ± 0.0038	-0.0017 ± 0.0024	-0.0017 ± 0.0024	-0.00041 ± 0.00181
A_{15}^{im}	0.00044 ± 0.00273	-0.0043 ± 0.0027	-0.0035 ± 0.0018	-0.0035 ± 0.0018	-0.0028 ± 0.0013
A_{16}^{re}	0.00037 ± 0.00280	0.0034 ± 0.0028	0.0014 ± 0.0018	0.0014 ± 0.0018	0.0016 ± 0.0013
A_{16}^{im}	-0.00038 ± 0.00393	-0.0022 ± 0.0039	-0.0026 ± 0.0025	-0.0026 ± 0.0025	-0.0020 ± 0.0019
A_{17}^{re}	-0.0008 ± 0.0030	0.0061 ± 0.0029	0.00022 ± 0.00191	0.00022 ± 0.00191	0.0013 ± 0.0014
A_{17}^{im}	-0.00093 ± 0.00425	-0.0014 ± 0.0042	0.0012 ± 0.0027	0.0012 ± 0.0027	0.00011 ± 0.00202
A_{CP}	0.00038 ± 0.00166	-0.00033 ± 0.00166	0.00046 ± 0.00108	0.00046 ± 0.00108	0.00026 ± 0.00079

6.5.3 Fit validation on data using $B^0 \rightarrow K^{*0}\psi(2S)$

In addition to the control mode decay $B^0 \rightarrow K^{*0}J/\psi$, it is useful to study the other charmonium resonance in the analyzed q^2 range. This is the decay $B^0 \rightarrow K^{*0}\psi(2S)$, which allows the validation of the relative branching fraction fit and the determination of A_{CP} . As the yield of the $B^0 \rightarrow K^{*0}J/\psi$ decay mode $N_{J/\psi}$ and $A_{CP}^{J/\psi, \text{PW}+\text{SW}}$ are input parameters for the fit, a cross-check needs to be performed in an independent q^2 region to be conclusive. The expected value of the relative branching fraction for the $B^0 \rightarrow K^{*0}\psi(2S)$ decay can be calculated using the formula

$$\mathcal{B}^{\text{rel}} = \frac{\mathcal{B}(B^0 \rightarrow K^{*0}\psi(2S)) \times \mathcal{B}(\psi(2S) \rightarrow \mu^+\mu^-)}{\mathcal{B}(B^0 \rightarrow K^{*0}J/\psi) \times \mathcal{B}(J/\psi \rightarrow \mu^+\mu^-)} = (0.064 \pm 0.007) . \quad (6.54)$$

The branching fractions used in this formula are taken from the PDG [26]. The fit is performed as similar to the rare mode as possible to emulate the procedure.

To emulate the fit to the rare mode as closely as possible, all parameters describing the $m_{K\pi}$ and $m_{K\pi\mu\mu}$ distributions are fixed in this fit. However, due to the high statistic of the $\psi(2S)$ mode, it is necessary to model the contribution of $B_s^0 \rightarrow K^{*0}\psi(2S)$ in the reconstructed B^0 mass, similar to the control mode fit. The B_s^0 fraction $f_{B_s^0}$ and the B mass difference between B^0 and B_s^0 called $\delta m_{B_s^0}$, are floated in the fit. The fit results are listed in Tab. 6.18 for the angular observables and Tab. B.6 for the nuisance parameters. The one-dimensional projections for the year 2017/18 are shown in Fig. 6.23. The results show generally good agreement across the run periods. Furthermore, the value obtained for the relative branching fraction is in good agreement with the value from the PDG quoted in Eq. 6.54. Also, the determined value for A_{CP} is in agreement with 0. Similar to the control mode fit the lower regions of the $\cos\theta_K$ distribution show a systematic structure due to the irreducible contribution from exotic decays. The other projections show good agreement with the data.

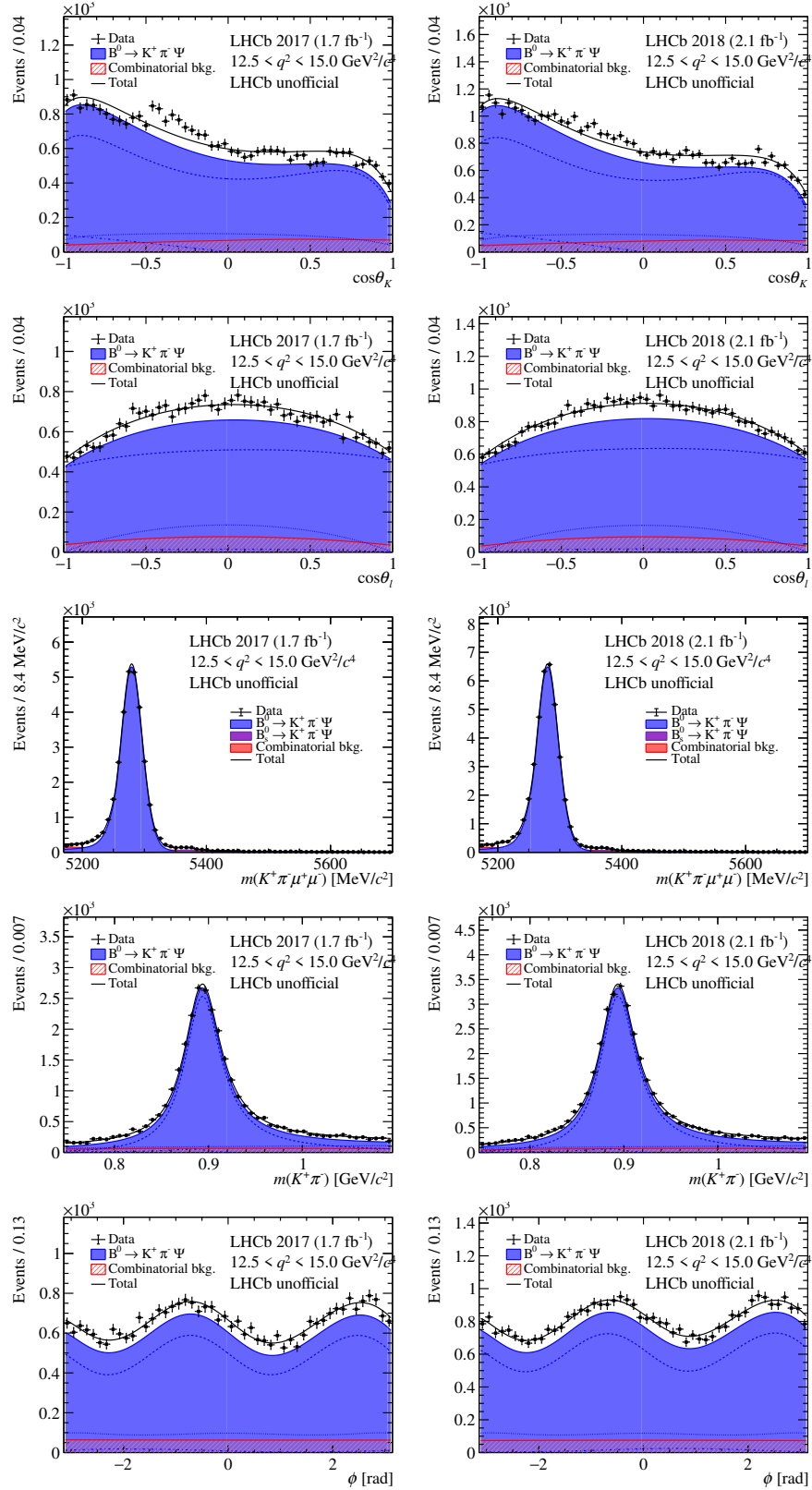


Figure 6.23: Fit projections of the $B^0 \rightarrow K^{*0} \psi(2S)$ fit to LHCb data from the years 2017 and 2018 in $\cos \theta_\ell$, $\cos \theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$.

Table 6.18: Fit results for the angular observables of a full angular fit to $B^0 \rightarrow K^{*0}\psi(2S)$ data in the single periods as well as a simultaneous fit to all data. The agreement across all years is good, and the CP -asymmetry A_{CP} as well as the relative branching fraction $\mathcal{B}^{\text{rel}}$ are in good agreement with previous measurements.

Observable	Run 1	2016	2017	2018	Simultaneous fit
F_L	0.5158 ± 0.0051	0.5202 ± 0.0052	0.5181 ± 0.0034	0.5181 ± 0.0034	0.5180 ± 0.0025
S_3	-0.0085 ± 0.0069	-0.0253 ± 0.0068	-0.0076 ± 0.0045	-0.0076 ± 0.0045	-0.0119 ± 0.0033
S_4	-0.1863 ± 0.0073	-0.1925 ± 0.0073	-0.1820 ± 0.0048	-0.1820 ± 0.0048	-0.1855 ± 0.0035
S_5	-0.0173 ± 0.0071	-0.0225 ± 0.0072	-0.0090 ± 0.0047	-0.0090 ± 0.0047	-0.0140 ± 0.0035
A_{FB}	0.0139 ± 0.0047	0.0085 ± 0.0047	0.011 ± 0.003	0.011 ± 0.003	0.0108 ± 0.0022
S_7	-0.0056 ± 0.0075	-0.0111 ± 0.0077	-0.0026 ± 0.0049	-0.0026 ± 0.0049	-0.0051 ± 0.0036
S_8	-0.0895 ± 0.0077	-0.0731 ± 0.0078	-0.0719 ± 0.0051	-0.0719 ± 0.0051	-0.0762 ± 0.0037
S_9	-0.1732 ± 0.0067	-0.1815 ± 0.0065	-0.1792 ± 0.0043	-0.1792 ± 0.0043	-0.1784 ± 0.0032
A_{FL}	0.0050 ± 0.0068	0.0048 ± 0.0070	-0.0016 ± 0.0045	-0.0016 ± 0.0045	0.0015 ± 0.0033
A_3	0.0025 ± 0.0069	0.0042 ± 0.0068	0.0013 ± 0.0045	0.0013 ± 0.0045	0.0023 ± 0.0033
A_4	-0.0083 ± 0.0074	-0.0034 ± 0.0074	-0.0073 ± 0.0049	-0.0073 ± 0.0049	-0.0067 ± 0.0036
A_5	0.00024 ± 0.00715	0.00035 ± 0.00718	0.0020 ± 0.0047	0.0020 ± 0.0047	0.0013 ± 0.0035
A_{6s}	-0.0057 ± 0.0061	-0.0067 ± 0.0062	0.00051 ± 0.00400	0.00051 ± 0.00400	-0.0026 ± 0.0029
A_7	0.0105 ± 0.0075	-0.0050 ± 0.0077	-0.00025 ± 0.00494	-0.00025 ± 0.00494	0.0012 ± 0.0036
A_8	0.00099 ± 0.00775	0.00095 ± 0.00786	0.00069 ± 0.00509	0.00069 ± 0.00509	0.00086 ± 0.00374
A_9	0.0068 ± 0.0067	-0.0078 ± 0.0066	0.0078 ± 0.0043	0.0078 ± 0.0043	0.0040 ± 0.0032
F_S	0.1572 ± 0.0046	0.1505 ± 0.0046	0.157 ± 0.003	0.157 ± 0.003	0.1553 ± 0.0022
S_{11}^{re}	-0.319 ± 0.011	-0.336 ± 0.012	-0.3123 ± 0.0076	-0.3123 ± 0.0076	-0.3190 ± 0.0056
S_{11}^{im}	0.0281 ± 0.0088	0.032 ± 0.009	0.0394 ± 0.0058	0.0394 ± 0.0058	0.0351 ± 0.0043
S_{14}^{re}	0.019 ± 0.016	0.026 ± 0.016	0.025 ± 0.010	0.025 ± 0.010	0.0241 ± 0.0077
S_{14}^{im}	-0.204 ± 0.011	-0.213 ± 0.011	-0.2037 ± 0.0075	-0.2037 ± 0.0075	-0.2059 ± 0.0055
S_{15}^{re}	-0.014 ± 0.015	0.021 ± 0.015	0.0032 ± 0.0097	0.0032 ± 0.0097	0.0034 ± 0.0071
S_{15}^{im}	-0.0031 ± 0.0111	0.016 ± 0.011	0.0092 ± 0.0072	0.0092 ± 0.0072	0.0078 ± 0.0053
S_{16}^{re}	0.0078 ± 0.0116	0.0046 ± 0.0117	0.0117 ± 0.0074	0.0117 ± 0.0074	0.0092 ± 0.0055
S_{16}^{im}	0.0085 ± 0.0154	-0.0049 ± 0.0156	0.014 ± 0.010	0.014 ± 0.010	0.0082 ± 0.0074
S_{17}^{re}	0.187 ± 0.012	0.196 ± 0.012	0.1872 ± 0.0078	0.1872 ± 0.0078	0.1894 ± 0.0058
S_{17}^{im}	-0.081 ± 0.017	-0.079 ± 0.017	-0.075 ± 0.011	-0.075 ± 0.011	-0.077 ± 0.008
A_{FS}	0.0035 ± 0.0047	-0.0050 ± 0.0047	0.00009 ± 0.00301	0.00009 ± 0.00301	-0.00025 ± 0.00223
A_{11}^{re}	0.0064 ± 0.0112	0.0063 ± 0.0115	-0.0084 ± 0.0074	-0.0084 ± 0.0074	-0.0014 ± 0.0054
A_{11}^{im}	-0.0120 ± 0.0087	-0.0032 ± 0.0089	-0.00039 ± 0.00571	-0.00039 ± 0.00571	-0.0037 ± 0.0042
A_{14}^{re}	0.017 ± 0.016	-0.019 ± 0.016	-0.00011 ± 0.01050	-0.00011 ± 0.01050	-0.00071 ± 0.00770
A_{14}^{im}	0.0017 ± 0.0115	-0.0032 ± 0.0116	-0.0056 ± 0.0075	-0.0056 ± 0.0075	-0.0036 ± 0.0055
A_{15}^{re}	-0.0077 ± 0.0147	-0.019 ± 0.015	0.0091 ± 0.0096	0.0091 ± 0.0096	-0.0013 ± 0.0071
A_{15}^{im}	0.010 ± 0.011	-0.0089 ± 0.0109	0.0144 ± 0.0071	0.0144 ± 0.0071	0.0083 ± 0.0052
A_{16}^{re}	-0.0043 ± 0.0115	0.0016 ± 0.0116	-0.0052 ± 0.0074	-0.0052 ± 0.0074	-0.0034 ± 0.0055
A_{16}^{im}	-0.016 ± 0.015	-0.00065 ± 0.01544	0.0102 ± 0.0099	0.0102 ± 0.0099	0.0021 ± 0.0073
A_{17}^{re}	-0.0032 ± 0.0121	-0.025 ± 0.012	-0.0011 ± 0.0078	-0.0011 ± 0.0078	-0.0070 ± 0.0058
A_{17}^{im}	-0.0048 ± 0.0166	0.024 ± 0.017	-0.0097 ± 0.0108	-0.0097 ± 0.0108	-0.0011 ± 0.0080
A_{CP}	0.0072 ± 0.0064	0.0112 ± 0.0065	0.00028 ± 0.00418	0.00028 ± 0.00418	0.0044 ± 0.0031
R	$2.1218(\text{fixed})$	$2.1218(\text{fixed})$	$2.1218(\text{fixed})$	$2.1218(\text{fixed})$	$2.1218(\text{fixed})$
$\mathcal{B}^{\text{rel}}$	0.05991 ± 0.00048	0.06000 ± 0.00049	0.06025 ± 0.00032	0.06025 ± 0.00032	0.06011 ± 0.00023

6.5.4 Fit validation using pseudo-experiments generated from SM expectations

The analysis validation on simulation and control mode data is based on single fits to samples with a large number of events. However, the signal region of the data sample is expected to contain significantly fewer candidates. Therefore, it is additionally important to validate the fit on samples of the same expected size as the rare mode data sample. Maximum likelihood fits on small data samples face a series of challenges that can not be studied with the validation methods mentioned above. These challenges include statistically biased fit results, incorrect coverage of the statistical uncertainties, and potentially lacking fit stability. In the case of validation approaches based on small samples, it is necessary to study large ensembles of fits to access the statistical distribution of observables. The only way to achieve this is through performing pseudo-experiments. This method generates a set of pseudo events according to the fit PDF defined by a given set of input parameters. These can, for example, be the SM predictions of the angular observables. An event is fully defined by the values of the decay angles $\cos\theta_K$, $\cos\theta_\ell$, ϕ , and the invariant masses $m_{K\pi}$, $m_{K\pi\mu\mu}$ as well as q^2 . Additionally, one bool is required that defines the flavor of the B hadron. The events are generated using the Accept/Reject method. The target number of events for signal and background with B^0 and $\bar{B}^0$ tags are randomly generated from Poisson distributions, which use the expected yields for each type of event as the mean. One advantage of this method is that the input parameters used to generate the pseudo experiments are known exactly and do not need to be determined with additional fits or taken from previous analyses. Also, since the pseudo-experiments are exactly generated according to the signal and background model used in the subsequent fit, no systematic effects like mismodeling or residual backgrounds play a role. For each q^2 bin, an ensemble of 1000 pseudo-experiments is generated. The angular observables are initialized with SM predictions from Flavio [57]. Each of these pseudo data sets is fitted with the nominal fit setup.

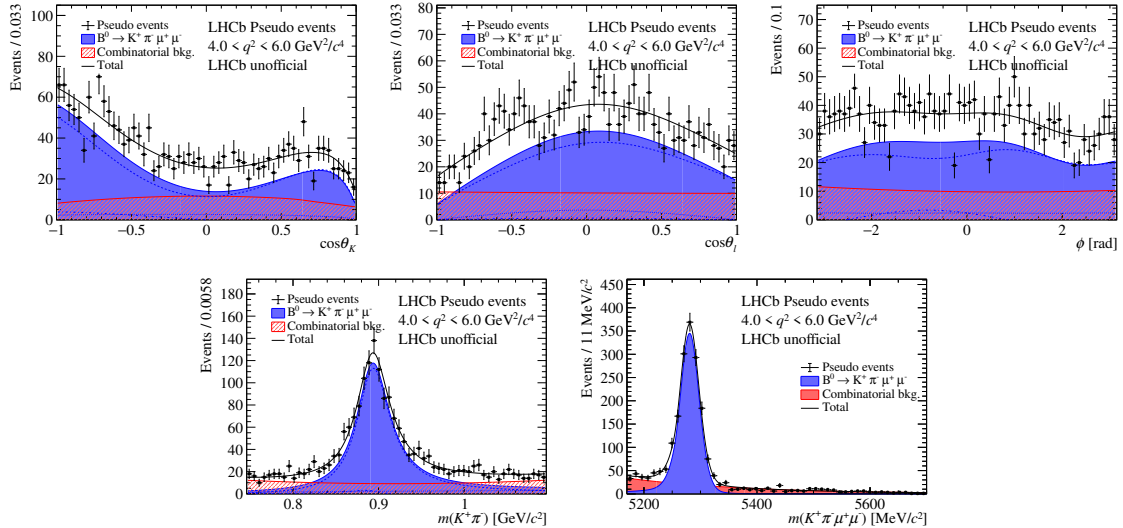


Figure 6.24: Example fit projections from a fit to a pseudo experiment: Top row: $\cos\theta_K$ (left), $\cos\theta_\ell$ (center), ϕ (right) projections for the B^0 sub-sample. Bottom row: $m_{K\pi}$ projection (left) and $m_{K\pi\mu\mu}$ projection (right).

For every angular observable the pull $\frac{x_i - x_i^{\text{gen}}}{\sigma_i}$ is calculated. The pull distribution is fitted with a Gaussian function to extract the pull mean and pull width.

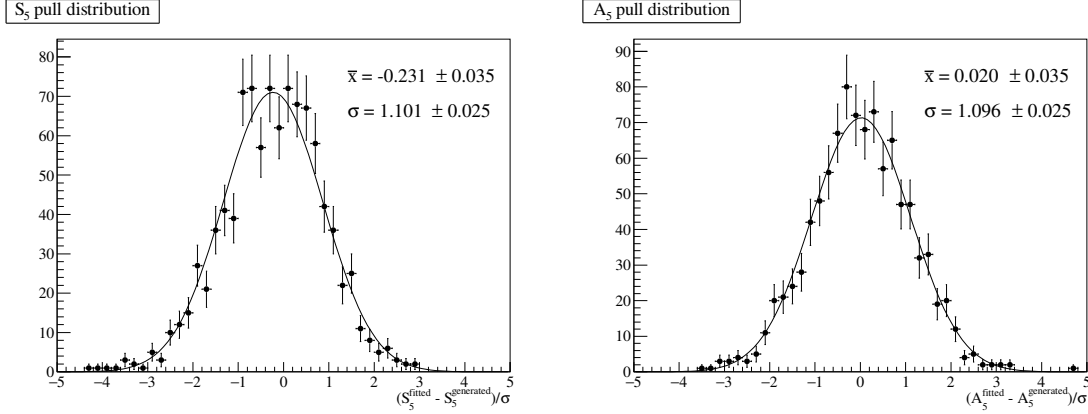


Figure 6.25: Pull distributions of the observables S_5 and A_5 from one thousand fits to SM pseudo experiments in the q^2 region $[6, 8] \text{ GeV}^2/c^4$.

Ideally, the pull distribution is Gaussian and centered around zero with a width of one. A Gaussian mean that deviates from zero is a sign for a biased fit, which can occur if an unbinned maximum likelihood fit is performed on a data sample with low signal and background yields and a large number of degrees of freedom. Bias can also occur if there are negative regions of the signal PDF close to the physical set of angular observables. This may lead to asymmetric pull distributions since results in a negative PDF region are discouraged. Furthermore, small data samples may lead to an incorrect coverage of the statistical uncertainties determined by the fit. Undercoverage, which is possible in the case of low statistics, is indicated by a pull width larger than one. If this is the case, a coverage correction method like the Neyman construction [138] needs to be employed. Coverage correction methods can also correct for asymmetric pull distributions.

The results for 1000 pseudo-experiments in fit configurations (i) and (ii) for the different q^2 bins are shown in Fig. 6.26 to Fig. 6.29. From these plots, it is evident that indeed there are significant signs of bias and undercoverage for a small subset of angular observables. The expected sensitivities for the angular observables in fit configuration (i) are shown in Tab. 6.19. They are calculated by fitting the value distribution of the pseudo-experiments using a Gaussian function, where the width corresponds to the expected sensitivity, *i.e.* the expected statistical uncertainty on the observables.

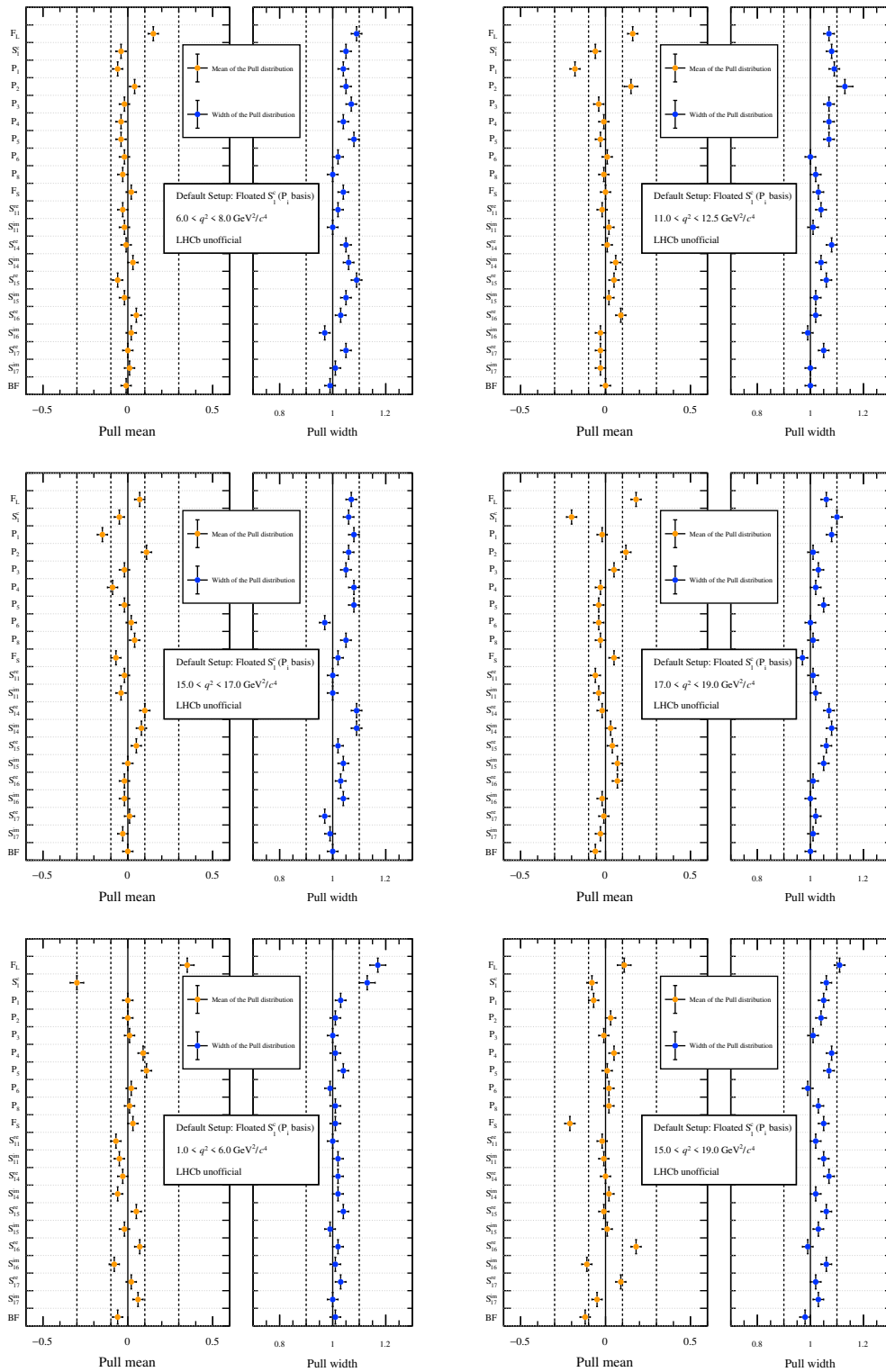


Figure 6.27: Results from 1000 pseudoexperiments corresponding to the combined Run 1, 2016 and 2017/18 data. The plots show the means and widths of Gaussian functions fitted to the pull distributions for the q^2 bins 5-10 for fit setup (ii).

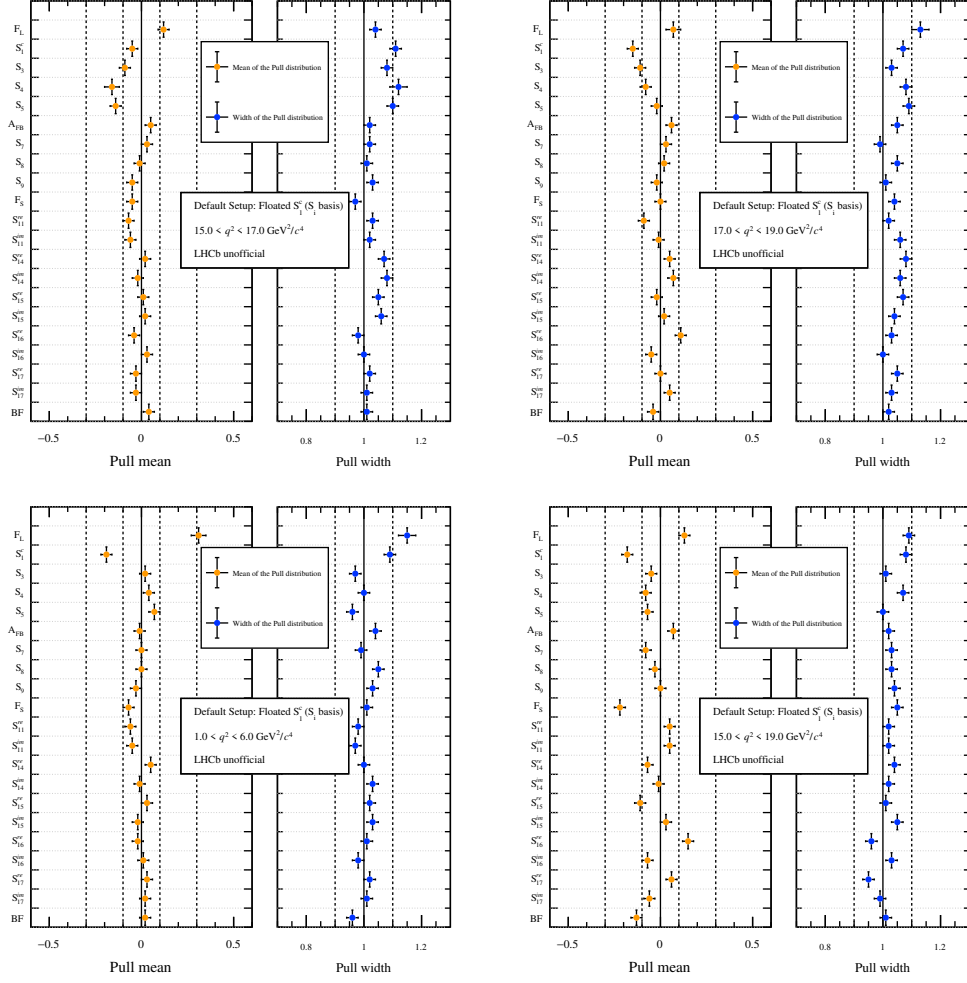


Figure 6.29: Results from 1000 pseudoexperiments corresponding to the combined Run 1, 2016 and 2017/18 data. The plots show the means and widths of Gaussian functions fitted to the pull distributions for the q^2 bins 7-10 for fit setup (i).

Table 6.19: Table of the expected sensitivities of the angular observables determined with the default fit setup (i).

param.	q^2 range [GeV ² /c ⁴]									
	[0.06, 0.98]	[1.1, 2.5]	[2.5, 4.0]	[4.0, 6.0]	[6.0, 8.0]	[11.0, 12.5]	[15.0, 17.0]	[17.0, 19.0]	[1.1, 6.0]	[15.0, 19.0]
F_L	0.083	0.089	0.071	0.049	0.040	0.040	0.033	0.050	0.042	0.027
S_2^*	0.041									
S_2^c	0.048	0.051	0.040	0.032	0.026	0.026	0.024	0.033	0.025	0.018
S_3	0.024	0.033	0.030	0.025	0.022	0.024	0.021	0.030	0.015	0.016
S_4	0.030	0.046	0.041	0.030	0.026	0.025	0.021	0.030	0.022	0.016
S_5	0.026	0.043	0.037	0.029	0.025	0.024	0.020	0.028	0.020	0.015
A_{FB}	0.028	0.029	0.022	0.017	0.015	0.017	0.015	0.023	0.013	0.012
S_7	0.026	0.041	0.040	0.031	0.028	0.028	0.025	0.033	0.021	0.020
S_8	0.032	0.048	0.042	0.032	0.026	0.027	0.024	0.036	0.024	0.020
S_9	0.023	0.032	0.030	0.024	0.022	0.024	0.021	0.030	0.016	0.018
S_6^c	0.073									
F_S	0.037	0.022	0.023	0.019	0.018	0.019	0.015	0.022	0.013	0.013
S_{12}	0.028									
S_{11}^c	0.104	0.072	0.071	0.056	0.047	0.047	0.035	0.041	0.038	0.026
S_{11}^{im}	0.073	0.050	0.050	0.039	0.033	0.033	0.027	0.050	0.027	0.024
S_{13}^c	0.153									
S_{13}^{im}	0.109									
S_{14}^c	0.073	0.102	0.087	0.067	0.060	0.062	0.050	0.064	0.044	0.038
S_{14}^{im}	0.050	0.071	0.064	0.049	0.042	0.046	0.041	0.075	0.034	0.034
S_{15}^c	0.062	0.082	0.073	0.058	0.054	0.059	0.047	0.058	0.039	0.034
S_{15}^{im}	0.042	0.059	0.055	0.042	0.041	0.043	0.038	0.066	0.031	0.033
S_{16}^c	0.042	0.060	0.056	0.047	0.042	0.048	0.043	0.083	0.031	0.037
S_{16}^{im}	0.058	0.084	0.074	0.063	0.058	0.067	0.057	0.069	0.040	0.045
S_{17}^c	0.052	0.074	0.064	0.051	0.043	0.049	0.046	0.091	0.034	0.038
S_{17}^{im}	0.076	0.108	0.087	0.068	0.060	0.069	0.057	0.074	0.047	0.044

6.5.5 Fit validation using pseudo-experiments generated from best-fit values

The pseudo-experiment studies performed in Sec. 6.5.4 use SM predictions of the angular observables to generate the toy data sets. This works very well for the P-wave observables, where numerous precise predictions exist. This is due to the fact that the P-wave observables have been a key interest of the community for a long time. Calculations of S-wave form factors come with significantly larger theory uncertainties [139], which translate to the SM predictions for the S-wave and interference observables.

Another way to find useful initial values for the toy generation that does not rely on SM predictions is to use the best-fit values from the rare mode data. A random blinding delta is used to blind the results of the fit to data from the signal region to avoid experimenters' bias. A complication of this method is that, due to the large expected statistical uncertainty of the rare mode fit, the fit result can contain phase space regions with negative signal PDF values. Negative phase space regions can pose a problem in the toy generation process, as these regions are not populated with signal events even though the true underlying signal distribution allows it. The depleted phase space regions can lead to non-physical biases, which would distort the results of the toy study. Thus, instead of using the exact values of the fit result, we use the closest set of values that results in a physical signal PDF, *i.e.* that has no negative regions in phase space. Here, closest is defined as the physical set of parameters with the smallest χ^2 with respect to the fit result, where χ^2 is defined as

$$\chi^2 = \sum_{i \in \{obs\}} \frac{(x_i^{phys} - x_i^{fit})^2}{\sigma_i^{fit}}. \quad (6.55)$$

Due to the complex structure of the signal PDF and the large number of free parameters,

the optimal set of parameters is determined using a Monte Carlo sampling method rather than analytically. In a first step, an initial physical start point of the algorithm is searched by randomly sampling parameter sets using Gaussian distributions around the central values of the fit result. To check if a set of central values is physical, one million uniformly distributed points in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , and $m_{K\pi}$ phase space are sampled, and the angular PDF is evaluated for each of these points. If all PDF values are found to be positive, the set of central values is accepted as physical. Using this initial physical point as a seed, the Gaussian sampling around the current set of central values is repeated until a new physical set of parameters with a smaller χ^2 is found, and the current set of parameters is updated to the newly found set. This algorithm is repeated until the reduction in χ^2 per step becomes negligible.

Using the final set of physical central values, a large set of one thousand toys is generated for each q^2 bin. The results are analyzed in the same way as described in Sec. 6.5.4. Figures 6.30–6.33 show the pull means and pull widths of these sets of toys. The results confirm the conclusions of the toy studies with SM generated values. A Neyman correction method will be required to correct the coverage of the uncertainty and bias.

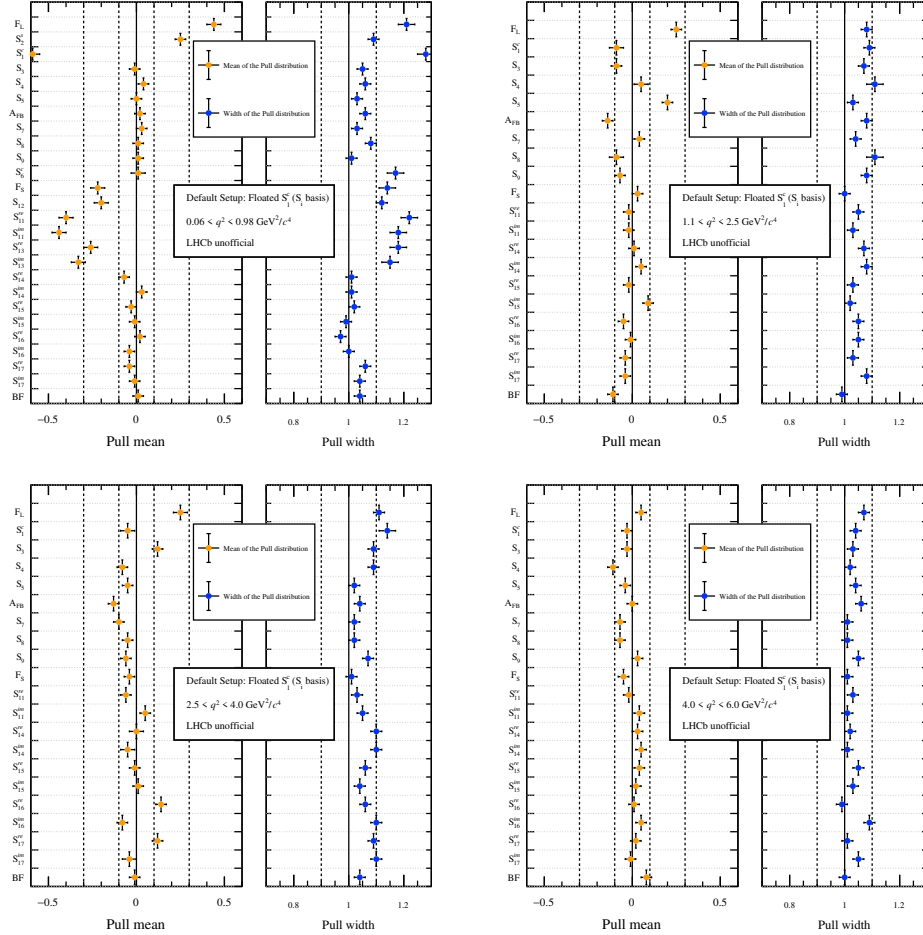
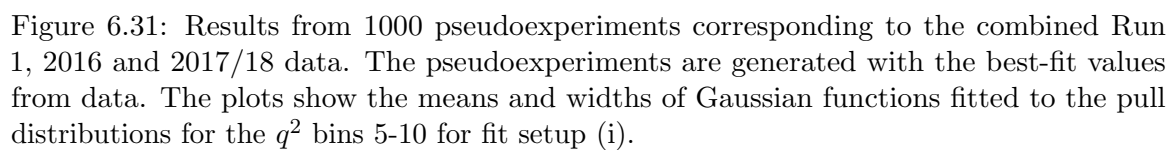


Figure 6.30: Results from 1000 pseudoexperiments corresponding to the combined Run 1, 2016 and 2017/18 data. The pseudoexperiments are generated with the best-fit values from data. The plots show the means and widths of Gaussian functions fitted to the pull distributions for the q^2 bins 1–4 for fit setup (i).



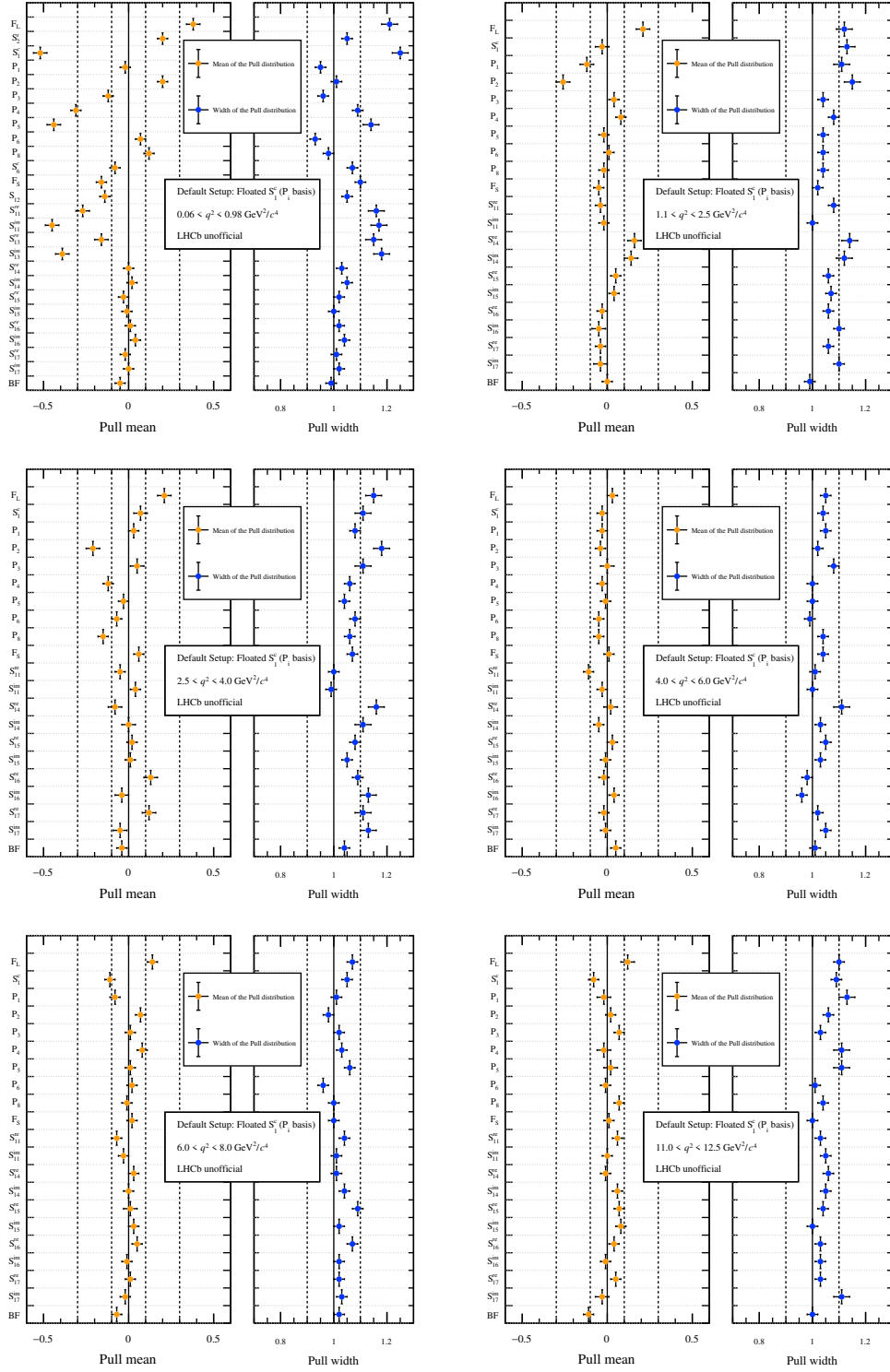


Figure 6.32: Results from 1000 pseudoexperiments corresponding to the combined Run 1, 2016 and 2017/18 data. The pseudoexperiments are generated with the best-fit values from data. The plots show the means and widths of Gaussian functions fitted to the pull distributions for the q^2 bins 1-6 for fit setup (ii).

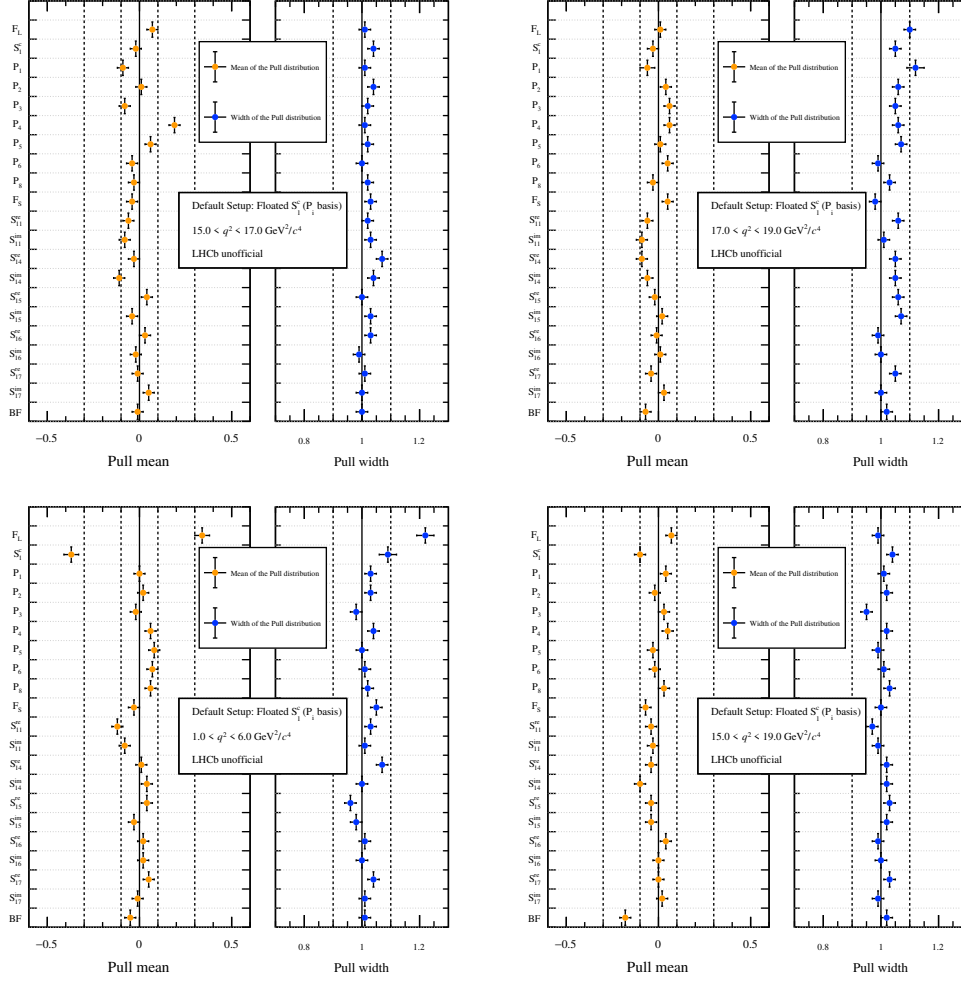


Figure 6.33: Results from 1000 pseudoexperiments corresponding to the combined Run 1, 2016 and 2017/18 data. The pseudoexperiments are generated with the best-fit values from data. The plots show the means and widths of Gaussian functions fitted to the pull distributions for the q^2 bins 6-10 for fit setup (ii).

6.5.6 Coverage correction

As discussed in Sec. 6.5.4, the small yield of the signal decay and the large number of physics observables to be determined lead to undercoverage of the uncertainties and statistical bias in some observables. This means that the uncertainties returned by the maximum likelihood fit, which are calculated using the Hesse method [120], are underestimating the actual uncertainty of the measurement. Statistical bias as well as incorrect coverage require correction, which is achieved by the construction of Neyman intervals for observables that display an undercoverage or statistical bias of more than 10% of the statistical uncertainty. As the Feldman Cousins method [140] does not correct for statistical bias, the classical Neyman construction is used [138]. Ideally, a full simultaneous construction of all observables would be performed. However, due to the high dimensionality of the problem, the required computational resources would be prohibitively large. Therefore, for each of the observables requiring correction, a one-dimensional scan of 100 steps in the $\pm 2\sigma$ region around the best fit value

is performed. In each step, an angular fit to the data is performed in which the scanned parameter is fixed. The result of this fit is used to generate a set of 1000 pseudo-experiments, which are used to determine the 68.3% acceptance region for each step. As the fit to data is not required to return a set of angular observables leading to a strictly positive signal PDF over the full phase, the nuisance parameters are varied to find the closest point in parameter space with a strictly positive signal PDF. This is done to prevent unphysical bias introduced in the generation process through negative PDF values. The variation of the nuisance parameters is found to have a negligible impact on the final result as long as the signal PDF is positive. Likelihood fits to each of the generated pseudo-experiments are performed in which all angular observables are floated. The acceptance region is constructed as an asymmetric interval with equal probability content of 34.15% around the median. The bias-corrected central value and the confidence interval are obtained by evaluating the confidence belt and median of the toy distributions at the best fit value from the log likelihood fit. This approach guarantees a correct coverage of the statistical uncertainties and an unbiased central value. Examples for these constructions for the observable P'_5 on data are displayed in Fig. 6.34.

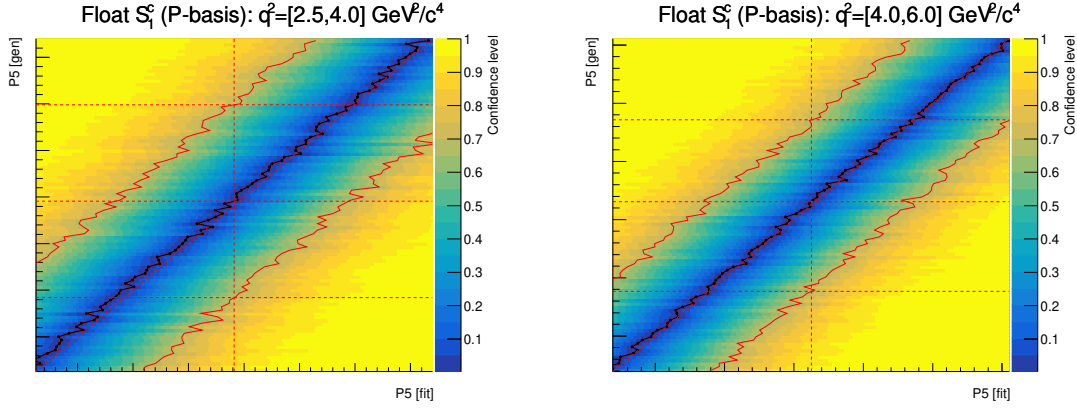


Figure 6.34: Example plot showing the result of the classical Neyman construction of the well-known observable P'_5 on data. The 68.3% confidence belt is indicated by the red lines, while the median of the toy distribution is displayed in black.

6.5.7 Goodness of fit for $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

Due to the high dimensionality of the fit, the goodness of fit is not trivial to evaluate. χ^2 tests suffer from the small yield of the rare mode, as the size of phase space volumes in which the χ^2 is evaluated needs to be increased to a point where the sensitivity degrades significantly. Also, the pulls of the fit projections only provide limited information as fit deficiencies might be integrated out. An alternative is the point-to-point dissimilarity method [141]. The model obtained by the fit is compared to the data by generating toy data sets from the fit model and evaluating the similarity between both sets of events. A metric for the dissimilarity between the event sets is

given by

$$T = \frac{1}{n_d^2} \sum_{i,j>i}^{n_d} \Psi(|x_i^d - x_j^d|) - \frac{1}{n_d n_{\text{toy}}} \sum_{i,j}^{n_d, n_{\text{toy}}} \Psi(|x_i^d - x_j^{\text{toy}}|), \quad (6.56)$$

where n_d and n_{toy} correspond to the number of data and toy events, respectively. The function Ψ is defined as a Gaussian function where the width is chosen to be $\sigma = 0.1$. The argument of Ψ is the normalized distance between two events. It is defined as

$$|x - y| = \sqrt{\sum_{n=0}^d \frac{x_n - y_n}{\theta_n^{\text{high}} - \theta_n^{\text{low}}}}, \quad (6.57)$$

where $\theta_n^{\text{high}} - \theta_n^{\text{low}}$ is the size of the definition space of dimension n . In order to eliminate the statistical uncertainty arising from the statistics of the toy representing the fit model, n_{Toy} is set to 100 times the statistics of the data set.

The distribution of the test statistic is obtained by generating a set of 500 pseudo-experiments according to the best-fit values from the data. The number of candidates in each pseudo-experiment is set to be equal to the number of candidates in the corresponding data sample. The test statistic is evaluated for each of the pseudo-experiments in place of the rare mode data.

Given that larger values of T correspond to a larger dissimilarity between model and data, a one-sided test with respect to the toy distribution is performed for each year separately. All q^2 bins pass the goodness of fit test, returning a p-value larger than the three σ equivalent.

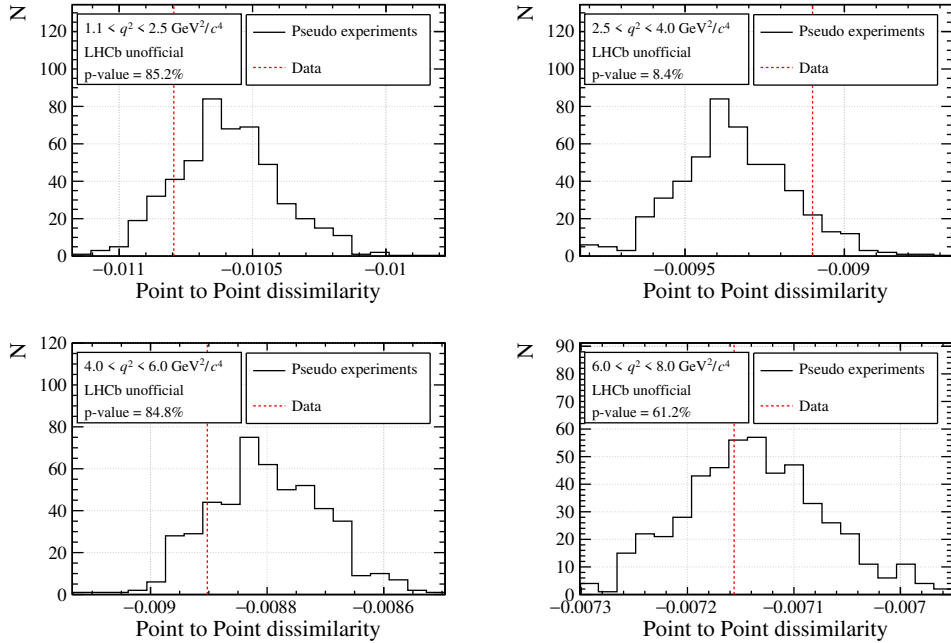


Figure 6.35: Distribution of the goodness of fit test statistic in several example q^2 bins calculated from toys. The observed test statistic from data is displayed as a red line.

6.6 Systematic uncertainties

Systematic uncertainties on the angular observables can originate from different sources. All auxiliary inputs to the fit are, in principle, related to a systematic uncertainty. This includes the acceptance parameterization, fixed parameters of the signal model, and the choice of models in certain variables for the signal as well as the background description. Additionally, residual contributions to the data sample that are not modeled in the signal or background sample can have an effect on the final result. All of these effects are studied using pseudo-experiments with very high signal and background yields. This is done in order to mitigate any statistical effect on the results of the study. These high-statistic toys are generated with a model that simulates a scenario in which the nominal fit setup does not properly describe the underlying reality in the data. These pseudo-experiments, generated with an alternative model, are subsequently fitted with the nominal fit setup. As the fit is unbiased with a sufficiently high number of candidates, a statistically significant difference between the generated values of the angular observables and the fit result must be due to the effect of the mismodeling. An alternative approach is to additionally fit the toy with the alternative fit model that corresponds to the model used to generate the toy. In this case, both fit results are compared to evaluate the systematic effect. The advantage of this approach is that residual statistical effects cancel in the difference between the fits. This reduces the number of pseudo-experiments that are required to achieve a satisfactory precision. The distribution of the systematic differences for each observable is determined and analyzed with a large number of pseudo-experiments. The distribution of either the difference between both fit results or the difference between the fit result and the generated value is fitted with a Gaussian function. Depending on the studied systematic effect, the bias of the Gaussian mean or the Gaussian width of the distribution is used to estimate the systematic effect. A full list of the individual contributions for fit configuration (i) can be found in Tab. B.7-B.16.

6.6.1 Data simulation agreement in hadron p/p_T distribution

The acceptance parameterization is determined using large samples of simulated signal events. Mismatches between the relevant kinematic distributions of the signal decays on data and on simulation can have an impact on the extracted angular observables. Such a mismatch is introduced by a mismodeling of the reconstruction efficiency of the final state hadrons, leading to discrepancies between the distributions of momentum p and transverse momentum p_T for kaons and pions in data and simulation, respectively. Examples for the differences between these distributions are shown in Fig. 6.36. The remaining plots can be found in Fig. B.36 and Fig. B.35. The MC distribution is taken from fully simulated and corrected control mode simulation. It is compared to sWeighted control mode data [133]. Since the data contains an irreducible contribution from the S-wave decays, the simulation needs to be reweighted to correctly describe the angular distribution of the data. Since p_T and p are strongly correlated, two two-dimensional isopopulated correction weight maps are determined for kaons and pions separately. These maps are used to determine alternative acceptance coefficients on hadron momentum corrected simulation samples. This acceptance is used to generate the high statistics toys, which are then fitted with both the nominal and

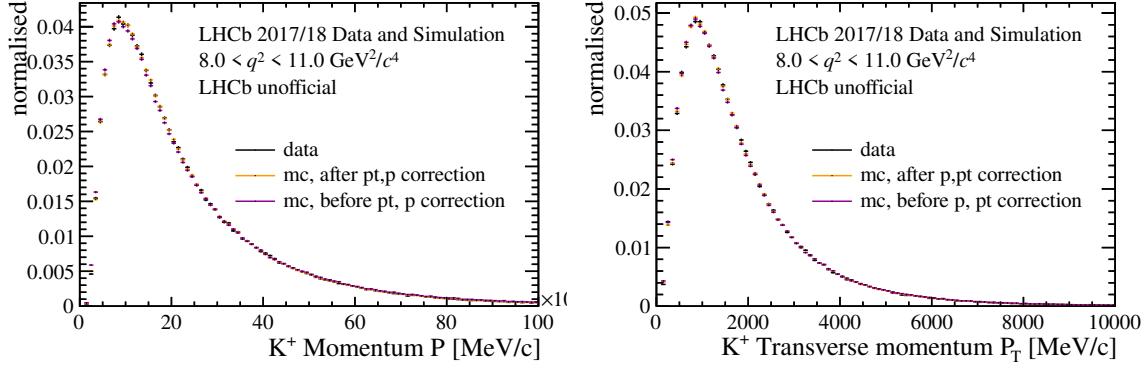


Figure 6.36: Comparison between the $K^+ p$ (left) and p_T (right) distribution in non-reweighted simulation, reweighted simulation, and sWeighted data for the 2018 run period.

hadron momentum corrected acceptance. The difference between the two fit results is Gaussian distributed. The distribution is fitted with an unbinned maximum likelihood fit. The resulting bias for each observable is used as an estimation for the two separate systematic uncertainties for kaons and pions.

6.6.2 Data simulation agreement in $\eta(B^0)$ and $p_T(B^0)$

As mentioned before, the data simulation agreement is corrected for using reweighting and resampling of the MC samples. One set of weights corrects for the mismatch in the pseudo-rapidity η and p_T distribution of the B^0 . Angular observables that are contributing significantly to the normalization of the PDF from Eq. 6.25, like F_L , can be strongly affected by this correction, as the acceptance parameterization directly impacts the normalization constants. To evaluate the systematic uncertainty related to the correction of the data simulation agreement in $B^0 \eta$ and p_T , an alternative correction map is derived using the decay mode $B^+ \rightarrow K^+ J/\psi$ instead of the control mode $B^0 \rightarrow K^{*0} J/\psi$. The kinetic properties and mass of the B^+ are slightly different compared to the B^0 . Additionally, the decay $B^+ \rightarrow K^+ J/\psi$ does not come with an S-wave contribution like the nominal control mode. A correction of the angular distribution of the $B^+ \rightarrow K^+ J/\psi$ simulation according to an amplitude model including the S-wave and exotic contribution to match the distribution on data, as performed for the $B^0 \rightarrow K^{*0} J/\psi$ control mode, is thus not necessary. A new acceptance is derived from simulation corrected with the alternative correction maps to replace the nominal acceptance parameterization. Similar to the correction in Sec. 6.6.1, the generated pseudo-experiments are fitted with both the nominal and alternative acceptance. The bias, determined from a fit to the Gaussian distribution of the differences between nominal and alternative fits, is used as systematic uncertainty.

6.6.3 Data simulation agreement in $\chi_{\text{IP}}^2(B^0)$ and $\chi_{\text{vtx}}^2(B^0)$

The χ_{IP}^2 and χ_{vtx}^2 of the B^0 show slight mismodeling between data and simulation. Similar to the approach described in Sec. 6.6.2, a two-dimensional correction map is created based on the differences between control mode simulation and sWeighted control mode data. Since this correction only has a small effect on the angular

observables, it is not used for the calculation of the nominal acceptance. A systematic uncertainty to cover the mismodeling is calculated by deriving an alternative acceptance parameterization using signal simulation that is reweighted according to the correction map mentioned above. The value of the systematic uncertainty is evaluated from pseudo-experiments in the same way as discussed in Sec. 6.6.2.

6.6.4 Data simulation agreement in particle identification

Particle identification at LHCb is performed using information from different subdetectors. This includes the Ring Imaging Cherenkov detectors, the muon chambers, the tracking stations, and the calorimeters. The response of these detectors is condensed into neural network-based PID classifiers. The responses of these classifiers are known to display differences between data and simulation. This is mostly due to difficulties in the simulation of low-energy RICH photons. To correct the PID distributions, clean data samples are used to create PID templates as a function of the kinematic properties of the final state particle. These templates are used to either sample a new value or transform the existing value of all PID variables according to the template function for each simulated candidate, given its kinematic properties [142]. The dependence of the PID variables on the kinematic variables is parameterized by constructing a multidimensional isopopulated binning in those kinematic variables. The binning scheme and limited statistics in those bins are sources of uncertainty that propagate to the acceptance and, therefore, the measurement of the angular observables.

In order to quantify the effect related to the binning scheme, an alternative scheme is used to sample new values for the corrected PID distributions. An alternative acceptance parameterization is calculated from simulation using the alternative PID re-sampling. This acceptance is then used to generate high-statistics pseudo-experiments, and the systematic uncertainties are derived as described before.

Another systematic uncertainty related to the limited statistics of the calibration samples is evaluated by bootstrapping these samples and resampling the corresponding PID variables five times. For each of the bootstraps, an alternative acceptance parameterization is determined. In analogy to the variation of the binning scheme, the resulting bias of the angular observable is determined on high-statistics pseudo-experiments for every alternative acceptance, separately. Given that the statistical spread of the biases originates from the statistical variation of the bootstraps, the standard deviation of the five biases is used as an estimation of the systematic uncertainty.

The size of both systematic effects is found to be negligible.

6.6.5 Simulation of trigger efficiencies

The simulation samples used to calculate the acceptance parameterization usually only contain one consistent set of muon p_T thresholds for the L0 trigger. This set of thresholds is labeled by a hexadecimal key, the trigger configuration key TCK. However, over one run period, the trigger thresholds typically change, and the data sample contains multiple TCKs. Thus, the simulated trigger efficiencies do not accurately describe the true efficiency. A correction weight map is derived using the TISTOS method [143] in bins of the $\max(p_T(\mu))$. The events selected by the trigger can be grouped into three different categories: TIS, TOS, and TISTOS. TIS stands for Triggered Independent of Signal, which means that the trigger selection would select the event if the signal candidate and all its related detector hits were removed from the event. TOS stands for Triggered On Signal, meaning that the signal candidate is sufficient to trigger the event positively. An event falls into the TISTOS category if it fulfills the TIS as well as the TOS conditions. It should be noted that there are also events that fall in neither the TIS nor the TOS category. The TOS trigger efficiency is calculated on a fully selected control mode sample. For each $\max(p_T(\mu))$ bin the L0 TOS trigger efficiency is given by

$$\epsilon_{\text{TOS}|\text{Sel}} = \frac{N_{\text{TOS}|\text{Sel}}}{N_{\text{Sel}}} = \frac{N_{\text{TOS}|\text{Sel}}}{N_{\text{TIS}|\text{Sel}}} \cdot \frac{N_{\text{TIS}|\text{Sel}}}{N_{\text{Sel}}} = \frac{N_{\text{TOS}|\text{Sel}}}{N_{\text{TIS}|\text{Sel}}} \epsilon_{\text{TIS}}. \quad (6.58)$$

It is not generally possible to determine ϵ_{TIS} on data since there is no information on N_{Sel} , which is the number of signal events passing the complete selection chain without applying the L0 trigger requirement. However, it is possible to determine

$$\epsilon_{\text{TIS}|\text{TOS}} = \frac{N_{\text{TISTOS}|\text{Sel}}}{N_{\text{TOS}|\text{Sel}}}. \quad (6.59)$$

It is reasonable to assume that the L0 trigger responses for the signal and the rest of the event are largely uncorrelated within a $\max(p_T(\mu))$ bin. Thus, the trigger efficiency in a specific phase space bin can be written as

$$\epsilon_{\text{TOS}|\text{Sel}} = \frac{N_{\text{TOS}|\text{Sel}}}{N_{\text{TIS}|\text{Sel}}} \epsilon_{\text{TIS}|\text{TOS}}. \quad (6.60)$$

A corresponding efficiency histogram is calculated for the control mode MC simulation and data. The ratio of both histograms is used as a correction map for the rare mode simulation to determine the nominal acceptance function. To obtain an alternative acceptance function, this correction weight is removed from the signal simulation. The resulting parameterization is used to generate a set of high-statistics pseudo-experiments, and the systematic uncertainty is determined in the same fashion as in earlier studies involving alternative acceptances. The size of the systematic uncertainty is found to be small.

6.6.6 Tracking efficiency

The efficiency of the track reconstruction is modeled in the simulation samples used to determine the acceptance parameterization. To account for potential differences between data and simulation, the tracking efficiency can also be determined on a clean sample of $J/\psi \rightarrow \mu^+ \mu^-$ decays from B -decays using a tag and probe approach [110]. A combination of a fully reconstructed long track and an at least partially reconstructed track, which point to the same SV and fall into an invariant mass window around the J/ψ mass, is used. The long track identified as a muon is used as a tag, while the at least partially reconstructed track is used as a probe. The tracking efficiency is calculated as the ratio of probe tracks that can be matched to a long track in bins of the track momentum p and pseudorapidity η . The ratio of the data efficiency histogram and a similar histogram from simulation is used to derive a weight for each event for each final state track. These weights are then used in the nominal simulation samples that are used to calculate the acceptance function. An alternative acceptance parameterization is calculated without applying the tracking efficiency correction weights. The systematic uncertainty is determined in the same way as the systematic uncertainty on the L0 trigger efficiency. The size of the systematic uncertainty is found to be small.

6.6.7 Limited statistics of the MC simulation

The rare mode simulation samples used to calculate the acceptance parameterizations contain a large number of events. However, due to the fact that the efficiency is determined in five dimensions, the limited amount of simulated events has a non-negligible effect on the angular observables. This effect is quantified again using pseudo-experiments. For each pseudo-experiment, a new acceptance function is calculated from a bootstrap of the signal simulation sample used to calculate the original acceptance parameterization. This way, the statistical variation of the simulation sample is propagated to the acceptance that is used in the generation of the pseudo-experiments. Two separate fits are performed with both the nominal and bootstrapped acceptance. Similar to before, the distribution of the differences is fitted for each angular observable. Since the bootstrapping results in acceptances that statistically vary around the nominal acceptance, the mean bias is expected to be zero. However, the spread of the distribution parameterized by the width of the fitted Gaussian is a measure of the systematic uncertainty covering the limited statistics of the simulation sample.

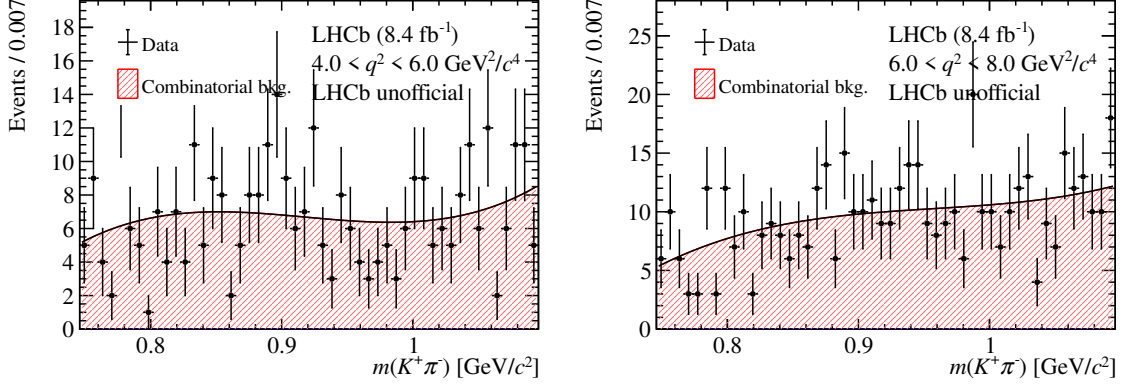


Figure 6.37: Fourth order Chebyshev polynomial $m_{K\pi}$ parameterization of upper mass sideband data for the q^2 regions between 4-6 GeV^2/c^4 and 6-8 GeV^2/c^4 . The plots show the events for the full data sample combined.

6.6.8 Shape of the $m_{K\pi}$ background

The combinatorial background contribution is described by Chebyshev polynomials in each decay angle and $m_{K\pi}$. The maximum order is restricted depending on the q^2 bin due to the low number of background events in the rare mode. The exact model for each bin is described in Sec. 6.1.3. Higher-order models could lead to overfitting and bias due to the increased number of fitted parameters. The uncertainty related to the maximum polynomial order is estimated by extracting an $m_{K\pi}$ background model from data. The mass region between $5450 \text{ MeV}/c^2 < m_{K\pi\mu\mu} < 5700 \text{ MeV}/c^2$, called the upper mass sideband, provides a clean combinatorial background sample. To increase the number of candidates in the data sample, the BDT cut against the combinatorial background is loosened. For each q^2 bin, the $m_{K\pi}$ distribution of the background is fitted with polynomials of maximum order four. Example plots displaying those fits can be found in Fig 6.37. This background model is subsequently used in the toy generation process. The toys are then fitted twice with the fourth order and with the nominal polynomial background models. The distribution of the differences between the fit results is fitted, and the resulting bias is taken as a systematic uncertainty. In general, the impact on the observables is small. The impact on the angular observables is more relevant in the lower q^2 regime, where only a first-order polynomial is used to parameterize the background.

6.6.9 Shape of the angular background

Similar to the uncertainty described in Sec. 6.6.8, the angular background shape is also described with Chebyshev polynomials. In the same manner as in the $m_{K\pi}$ case, an alternative background model is derived by fitting fourth-order polynomials to the $\cos\theta_K$, $\cos\theta_\ell$, and ϕ distributions of the upper mass sideband. Again, to increase the number of candidates in the fit, the BDT cut against the combinatorial background is loosened. Example projections of the mentioned background fits can be found in Fig. 6.38. The event deficit in the distribution at $\cos\theta_K > 0.8$ is caused by the $B^+ \rightarrow K^+\mu^+\mu^-$ veto, which is properly parameterized in the fits as described in

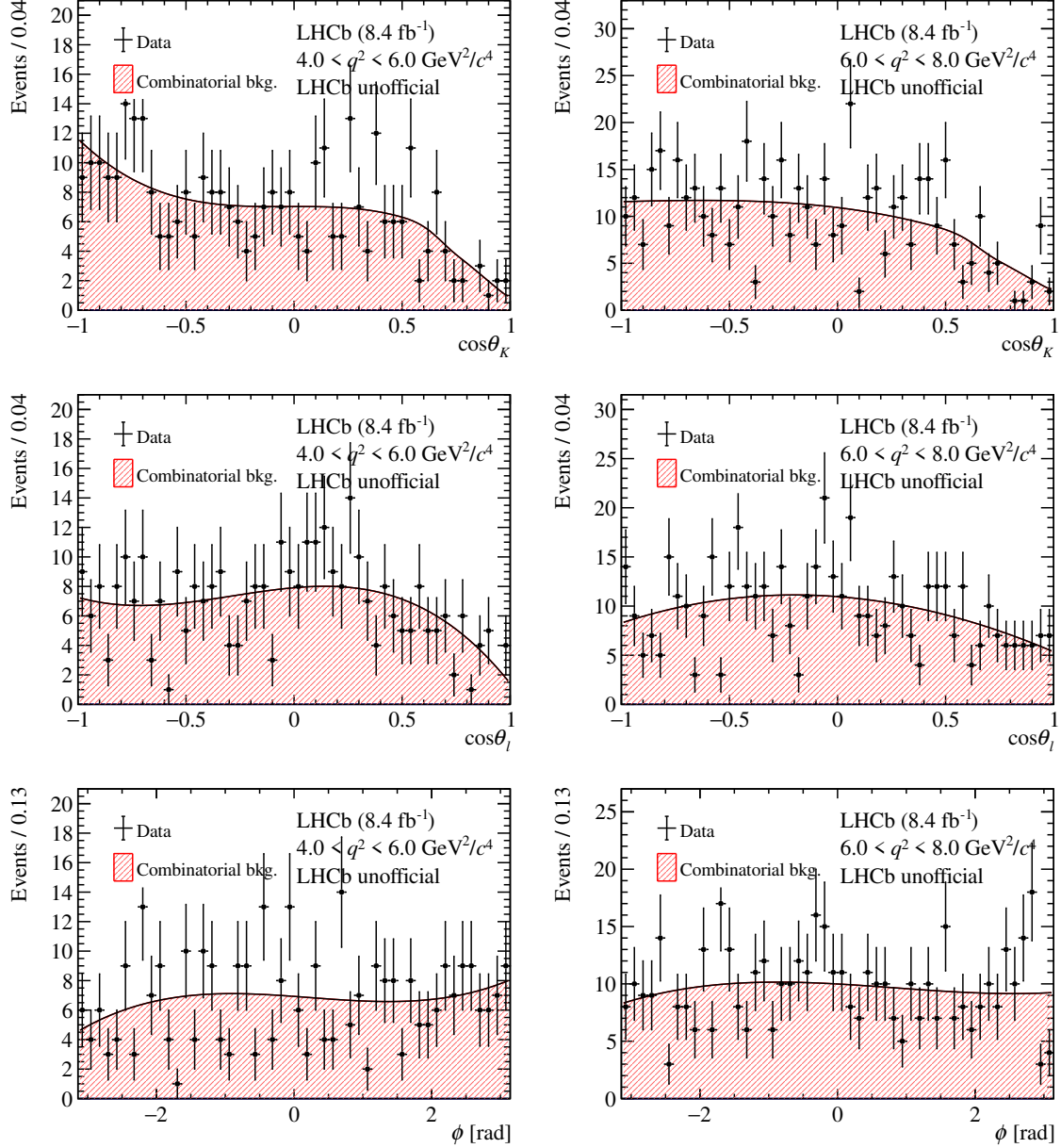


Figure 6.38: Fourth order Chebyshev polynomial angular parameterization of upper mass sideband data for $\cos(\theta_K)$ (top), $\cos(\theta_l)$ (middle) and ϕ (bottom) in the q^2 regions between 4-6 GeV^2/c^4 and 6-8 GeV^2/c^4 . The plots show the events for the full data sample combined.

Sec. 6.1.3. The parameterization of this veto, which is included in the fit, describes the shapes well. The evaluation of the systematic uncertainty is identical to the $m_{K\pi}$ background shape. The size of uncertainty is found to be small compared to the expected statistical uncertainty. The impact on the angular observables is slightly larger in the lower q^2 regime, where a first-order polynomial is used to parameterize the angular background as described in Sec. 6.1.3.

6.6.10 Modeling of the S-wave $m_{K\pi}$ shape

The nominal $m_{K\pi}$ model of the S-wave contribution to the signal PDF is described by the LASS function as described in Sec. 6.1.3. The parameters of this model are partly taken from external measurements and angular fits to the control mode channel. As the interference parameters of the model are strongly dependent on the complex phase of the S-wave $m_{K\pi}$ parameterization, a systematic uncertainty needs to be assigned for the chosen values. The parameters from Ref. [144] are used as an alternative set of values. Pseudo-experiments with high yields are generated using the alternative LASS parameters. The events are fitted twice with the nominal and alternative sets of values. The average bias extracted from the fit to the resulting distribution of deviations is taken as a systematic uncertainty. The systematic uncertainty of the S-wave $m_{K\pi}$ model is the dominant systematic uncertainty for the angular observables parameterizing the interference between P- and S-wave. In some observables, it even exceeds the size of the statistical uncertainty.

6.6.11 Peaking backgrounds

The PID cuts and vetoes of the selection are designed to efficiently remove physical background sources from the data sample. However, no selection can be one hundred percent effective against all physical backgrounds without severely reducing the sensitivity of the analysis. Therefore, a systematic uncertainty is assigned for the residual peaking background sources. The expected relative background yield can be calculated from simulation. To estimate the impact on the angular observables, it is necessary to inject events with the correct mass and angular distribution into high-statistics pseudo-experiments. Simulated samples of the backgrounds are used to describe their angular shape. Due to the relatively large number of events that are needed in total for the pseudo-experiments, the angular distributions are parameterized with a Gaussian kernel density estimator in five dimensions. This also preserves correlations between different angles and masses. For each residual background contribution, the background events are randomly generated from the appropriate kernels using the accept-reject method. The number of generated events is randomly drawn from a Poisson distribution with the expected yield as the mean. For some samples, the full selection removes all or most events from the simulation sample due to its limited statistics. For these decay modes, the selection is loosened in order to obtain enough events to model the shape. To quantify the systematic effect of peaking backgrounds, a normal high-statistics toy is generated and fitted with the nominal setup. Afterward, the generated toy peaking background events are injected into the toy, and the fit is repeated. The difference between the fit results is fitted with a Gaussian function, and the mean bias is used as an estimation of the systematic uncertainty for each observable. The systematic uncertainty on the residual physical backgrounds is found to be one of the leading systematic uncertainties. However, it is still small compared to the expected statistical uncertainty.

6.6.12 Ghost tracks

During the reconstruction of the tracks, described in Sec. 4, a set of random hits in the tracking detectors that don't originate from the same final state particle can be mistakenly combined into a reconstructed track. In the LHCb framework, these types of tracks are commonly referred to as ghost tracks. When combined with tracks originating from a genuine signal decay, these ghost tracks can form candidates that pass the selection and distort the angular distribution. The abundance of these kinds of tracks is reduced by requirements on the output of certain classifiers that quantify the probability of a track being a ghost during the selection step. However, a small amount of ghost tracks is still retained in the data sample, and a systematic uncertainty is assigned to cover the effect on the extracted observables. To obtain the shape of the angular distributions of events including this phenomenon, simulated events of the signal decay are used. Simulated signal candidates, including at least one track classified as a ghost track, are selected and used to construct a kernel density estimate of their angular shape. This model is then used to sample pseudo-events that are injected into high-statistics pseudo-experiments, similar to the study on residual peaking backgrounds discussed in Sec. 6.6.11. The pseudo-experiments are fitted using the nominal fit setup, and the bias introduced by the presence of the injected events is taken as systematic uncertainty. The size of the effect is found to be small.

6.6.13 Peaking backgrounds in the Control mode

As described in Sec. 6.1.4, the result of the control mode fit enters in the determination of the branching fraction via several nuisance parameters. Similar to the rare mode, a small contribution of physical backgrounds remains in the control mode data after the application of the selection. To determine the effect on the rare mode branching fraction, peaking background events generated from a Gaussian density estimation on the angular distribution from simulated samples are injected into pseudo-experiments representing the control mode. The approach is analogous to Sec. 6.6.11. The effect on the nuisance parameters $\vec{N}_{\text{sig}}^{J/\psi}$ and $\vec{\Gamma}^{J/\psi}$ of the branching fraction fit that are determined from the fit to control mode data is evaluated using high-statistics toys modeled from the angular observables displayed in Tab. 6.17. The relative uncertainty on the normalization factor in the extended term of the rare mode fit propagated from $\vec{N}_{\text{sig}}^{J/\psi}$ and $\vec{\Gamma}^{J/\psi}$ is assumed as a systematic uncertainty for all rare mode branching fraction results. This results in a relative uncertainty of approximately 2% on the relative branching fraction.

6.6.14 Detection asymmetry

In addition to the potential physical CP -asymmetries, additional asymmetries originate from the production of the $B^0/\bar{B}^0$ mesons and the detection of the final state particles. This is due to the fact that the charge sign of kaons and pions flips between B^0 and $\bar{B}^0$. While the material interaction cross-section for pions does not vary strongly for π^+ and π^- , this is not the case for kaons. This results in an $\mathcal{O}(1\%)$ asymmetry between B^0 and $\bar{B}^0$ tags because K^- have a higher interaction probability with the detector material and therefore a significantly reduced tracking efficiency [145]. As described in Sec. 6.1.4,

Table 6.20: The expected detection asymmetry in bins of q^2 calculated from rare and control mode simulation.

$q^2[\text{GeV}^2]$	Detection asymmetry A_{CP}^{det}			
	Run 1	2016	2017	2018
[0.06, 0.98]	-0.0114	-0.0097	-0.0103	-0.0103
[1.1, 2.5]	-0.0116	-0.0075	-0.0106	-0.0106
[2.5, 4.0]	-0.0111	-0.0084	-0.0086	-0.0086
[4.0, 6.0]	-0.0111	-0.0102	-0.0090	-0.0090
[6.0, 8.0]	-0.0115	-0.0100	-0.0084	-0.0084
[15.0, 17.0]	-0.0129	-0.0099	-0.0130	-0.0130
[17.0, 19.0]	-0.0115	-0.0111	-0.0095	-0.0095
[11.0, 12.5]	-0.0126	-0.0107	-0.0090	-0.0090
[1.1, 6.0]	-0.0112	-0.0089	-0.0093	-0.0093
[15.0, 19.0]	-0.0123	-0.0104	-0.0114	-0.0114
Control mode	-0.0109	-0.0095	-0.0101	-0.0101

a data-driven correction of the detection and production asymmetry is derived from the control mode fit. To evaluate the systematic effect of potential kinematic differences between the control and rare mode on the detection asymmetry that is not properly covered by the q^2 dependence of the acceptance, pseudo experiments are generated with the expected detection asymmetry in each q^2 bin from simulation. The values of the detection asymmetry displayed in Tab. 6.20 are taken from rare mode simulation for each q^2 bin separately. The detection asymmetry expected for the control mode is similarly taken from simulation. The control mode CP asymmetry $A_{CP}^{J/\psi}$ is set to the expected detection asymmetry of the control mode. The distribution of the difference between the fit results and the initial generation values is fitted with a Gaussian function, and the mean bias is taken as a systematic uncertainty. The systematic uncertainty is found to be relevant for the CP -asymmetries and negligible for the CP -symmetries. However, the systematic uncertainty is still small compared to the expected statistical uncertainty on the asymmetry.

6.6.15 Production asymmetry

At the LHCb experiment B^0 and $\bar{B}^0$ mesons are produced with a small initial asymmetry. This production asymmetry is defined as

$$\mathcal{A}_{\text{prod}} = \frac{N(\bar{B}^0) - N(B^0)}{N(\bar{B}^0) + N(B^0)}, \quad (6.61)$$

where N is the number of B -mesons produced in the proton-proton collisions. It is measured to be $(-0.35 \pm 0.76 \pm 0.28)\%$ [146]. Due to the mixing of b -mesons, the production asymmetry is diluted by a factor κ which is defined as

$$\kappa = \frac{\int_0^\infty \epsilon(t) e^{-\Gamma t} \cos(\Delta m_d t) dt}{\int_0^\infty \epsilon(t) e^{-\Gamma t} dt} \quad (6.62)$$

Table 6.21: This table shows the dilution factor κ for each year.

	run 1	2016	2017	2018
κ	0.369006	0.364941	0.35346	0.36074

where $\epsilon(t)$ is the B^0 decay time acceptance calculated from simulation for each year separately. The effective production asymmetry is given by $\kappa\mathcal{A}_{\text{prod}}$. Table 6.21 lists the dilution factor κ for each year. Since this effect is not dependent on q^2 , it is effectively canceled in through the detection and production asymmetry correction described in Sec. 6.1.4. The resulting systematic uncertainty, which is determined analogously to Sec. 6.6.14, is therefore negligibly small.

6.6.16 Mismodeling of the $m_{K\pi\mu\mu}$ shape

As discussed in Sec. 6.4, the $m_{K\pi\mu\mu}$ signal distribution is modeled with a double crystal ball function. As described in Sec. 6.4, the parameters of this model are determined on Monte Carlo simulation samples of the control and rare mode. Even though the model describes the simulation well, in principle, another model could be chosen to describe the $m_{K\pi\mu\mu}$ shape of the signal decays. The Hypatia function [147] is a generalized form of the crystal ball function. To evaluate the effect of the chosen $m_{K\pi\mu\mu}$ model on the angular observables, the Hypatia function is assumed as an alternative model. The parameters of this function are determined from simulation, similarly to the nominal model. The alternative model is used to generate high statistics pseudo-experiments, which are fitted with both the Hypatia model as well as the Double Crystal Ball model. The resulting systematic uncertainty is found to be negligible.

6.6.17 q^2 evaluation point

As mentioned in Sec. 6.1, the acceptance is usually evaluated at a single fixed point in the narrow q^2 bins, neglecting the q^2 dependence of the acceptance within the bin. This assumption is justified by the fact that the widths of the bins are small and the variation of the acceptance in q^2 is negligible. As described in Sec. 6.1.3, the weighted fit properly takes into account this q^2 dependence by evaluating the acceptance for each event individually. In the low q^2 region $0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$ and in the high q^2 region $17.0 < q^2 < 19.0 \text{ GeV}^2/c^4$, it was discovered that the variation of the acceptance across the bin can have a significant effect on the angular observables and branching fraction. In these q^2 regions, a weighted fit is used to properly account for the shape of the acceptance over the complete width of the q^2 bin. For the remaining narrow q^2 bins, a systematic uncertainty is assigned to account for the slight q^2 variation of the acceptance. High statistics pseudo-experiments are generated according to the SM q^2 distribution obtained from the Flavio prediction [57], taking into account the q^2 dependence of the acceptance. The pseudo-experiments are subsequently fitted using both the weighted and fixed q^2 approach. The weighted fit takes the q^2 shape into account properly, and the difference in the angular observables to the fixed q^2 acceptance evaluation approach is used to evaluate the systematic effect of q^2 variation of the acceptance. The systematic uncertainty is found to be small.

6.6.18 Angular resolution

The angular signal model is derived from the theoretical description of the differential branching fraction. The theoretical distribution of the decay angles is fully described by combinations of trigonometric functions. In practice, the LHCb detector is only able to reconstruct the momenta of the final state particles and thereby the decay angles with a finite precision. These angular resolution effects are not accounted for in the angular PDF, and thus a systematic uncertainty must be assigned. The angular resolution is described in the simulation of the signal mode and can thus be estimated from it. The distribution of the resolution is fitted from simulation for the decay angles $\cos\theta_K$, $\cos\theta_\ell$, ϕ , and the invariant masses $m_{K\pi}$ and q^2 . The resolution of the $m_{K\pi\mu\mu}$ invariant mass is already taken into account in the PDF by the various width scale factors determined from simulation and data. The fit of the resolution is performed using a double Gaussian function with a shared mean. As an additional safety margin that accounts for differences between data and simulation, an additional factor of 10% is added to the resolution determined from simulation.

To evaluate the systematic effect of the angular and invariant mass resolution, a set of pseudo-experiments with a high number of events is produced, where each decay angle, q^2 , and $m_{K\pi}$ value of the event is smeared in the generation with the corresponding resolution function. The fit results are compared to the generated observable values, and the difference is taken as a systematic uncertainty. The impact of the angular resolution on the angular observables is found to be negligible.

6.6.19 q^2 dependent PDF in the weighted fit

In some q^2 regions, it is necessary to perform a weighted fit to properly take into account the q^2 variation over the width of the bins, which can affect the acceptance correction. Additionally, the $m_{K\pi}$ description of the signal also depends on the value of q^2 due to the phase-space constraints at high q^2 values. However, the PDF does not explicitly model the shape of q^2 . In general, not modeling the complete distribution of all input parameters, which might change on an event-by-event basis in a multi-component fit, can lead to a bias in the final fit result. In literature, this is known as the Punzi bias [148], and a correction term to be included in the signal PDF is proposed. However, the calculation of this correction term requires knowledge about the q^2 shape of the signal decays and the combinatorial background, which is only available in pseudo-experiments. To evaluate the size of this effect, pseudo-experiments are generated with the appropriate SM q^2 shape. The fit results with and without the Punzi correction terms are compared, and a systematic uncertainty is evaluated for all observables extracted using a weighted fit. The effect is found to be negligibly small.

6.6.20 Combination of the systematic uncertainties

The size of the systematic uncertainties heavily depends on the observable and q^2 bin. In general, the size of the systematic uncertainties is still small compared to the size of the statistical uncertainty for the vast majority of the angular observables. The observables most affected by the systematic uncertainties are F_L and the branching fraction for the P-wave, as well as the interference observables. The total

systematic uncertainty is calculated by adding the single contributions in quadrature. Figures 6.39-6.42 and Fig. B.37-B.41 show the size of all the systematic uncertainties relative to the expected statistical uncertainty. The size of the single contributions to the total error budget is displayed separately. The total systematic uncertainty on the P-wave angular observables, except for F_L , in general does not exceed 30% of the statistical uncertainty. The size of the systematic uncertainty on F_L can reach 40% of the statistical uncertainty. Depending on the q^2 bin, the total systematic uncertainty on the branching fraction can be at the order of the statistical uncertainty. As mentioned before, the size of the systematic uncertainty on the interference parameters is dominated by the uncertainty on the $m_{K\pi}$ model of the S-wave. In some interference observables, it exceeds the expected statistical uncertainty. A full list of the individual contributions for fit configuration (i) can be found in Tab. B.7-B.16. The total systematic uncertainty for each observable is listed as the second uncertainty in Tab. 6.22-6.23 alongside the fit result.

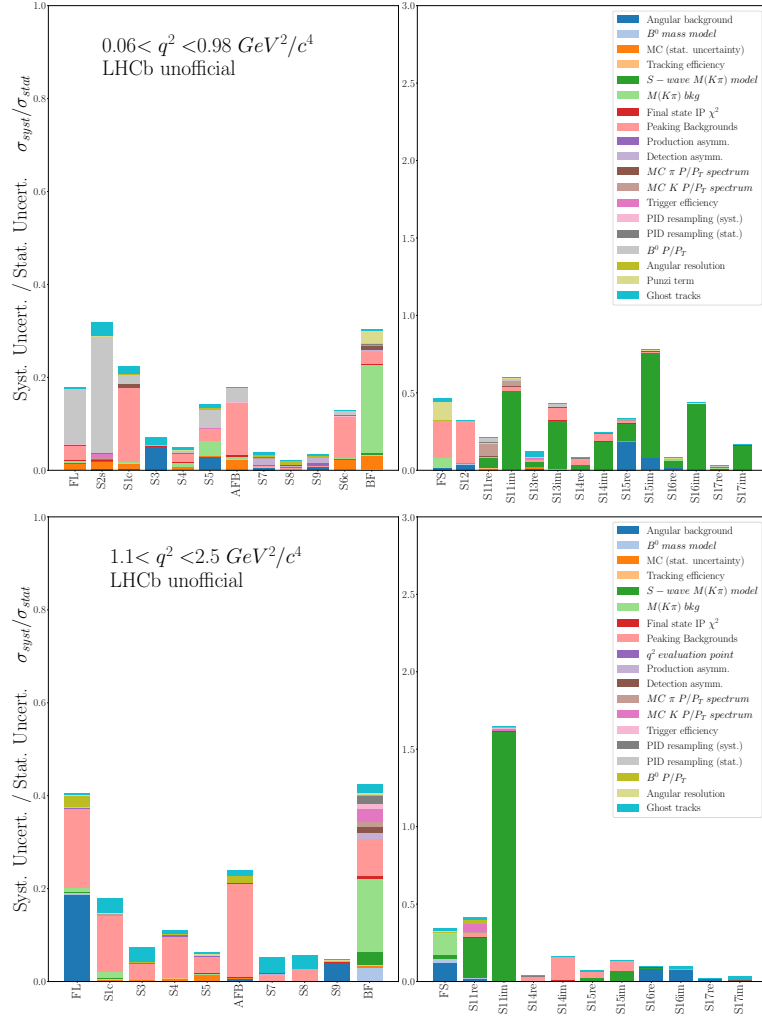


Figure 6.39: Systematic uncertainties for all extracted observables from fit configuration (i) in the bins of $0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$ and $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$. The total systematic uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are stacked according to their relative contributions to the total systematic uncertainty.

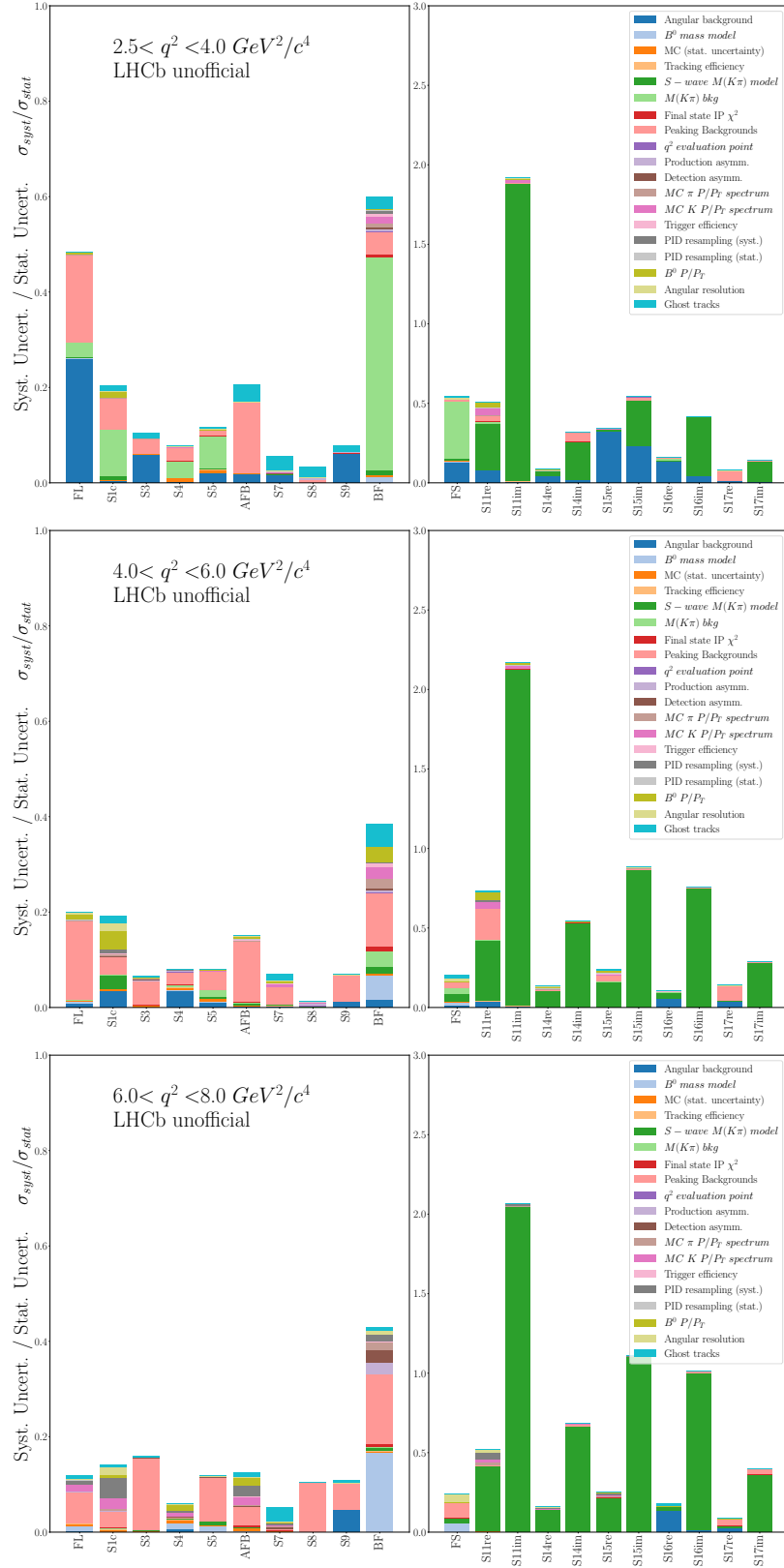


Figure 6.40: Systematic uncertainties for all extracted observables from fit configuration (i) in the bins of $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$, $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$, and $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$. The total systematic uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are stacked according to their relative contributions to the total systematic uncertainty.

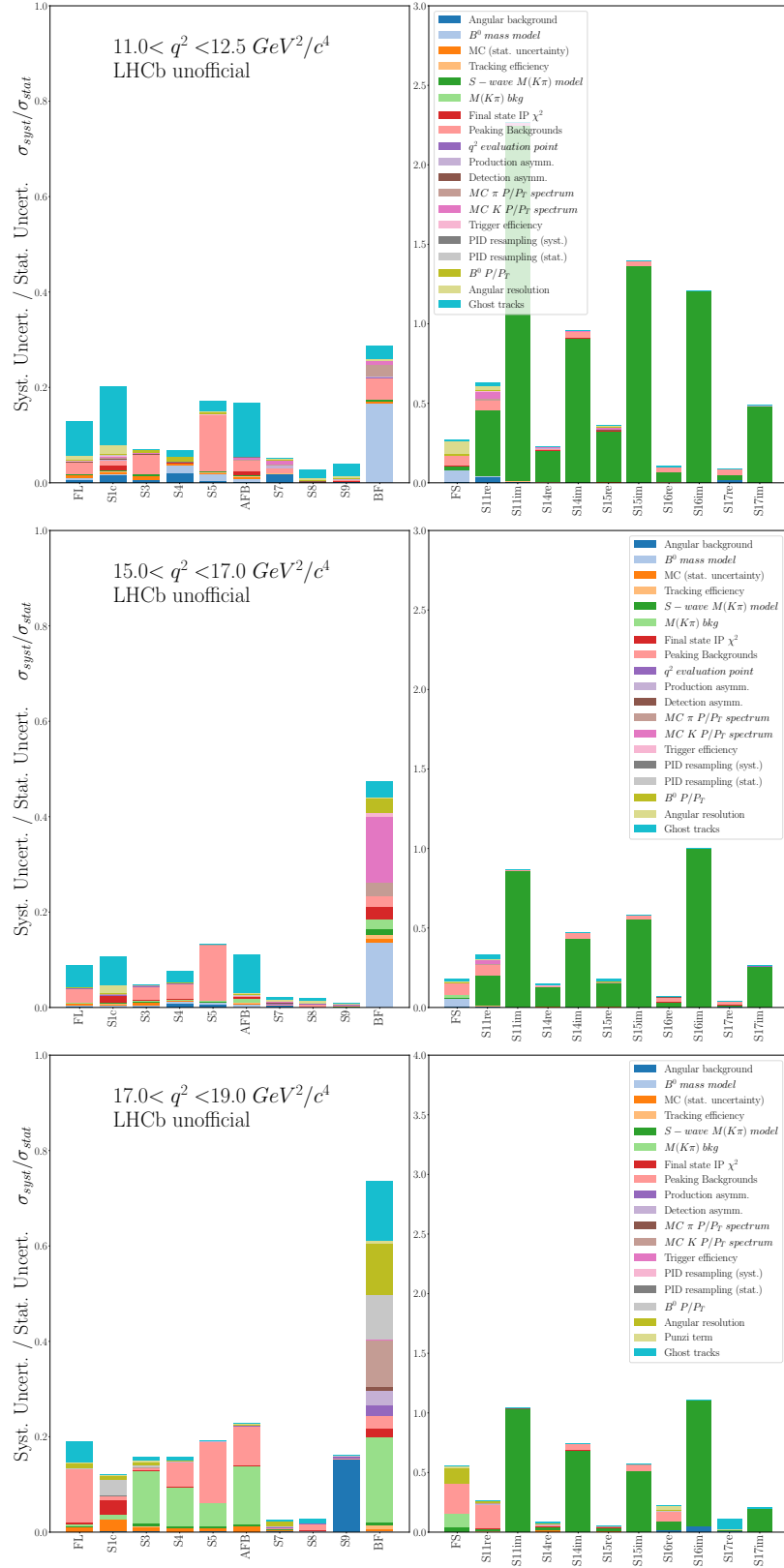


Figure 6.41: Systematic uncertainties for all extracted observables from fit configuration (i) in the bins of $11.0 < q^2 < 12.5 \text{ GeV}^2/c^4$, $15.0 < q^2 < 17.0 \text{ GeV}^2/c^4$, and $17.0 < q^2 < 19.0 \text{ GeV}^2/c^4$. The total systematic uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are stacked according to their relative contributions to the total systematic uncertainty.

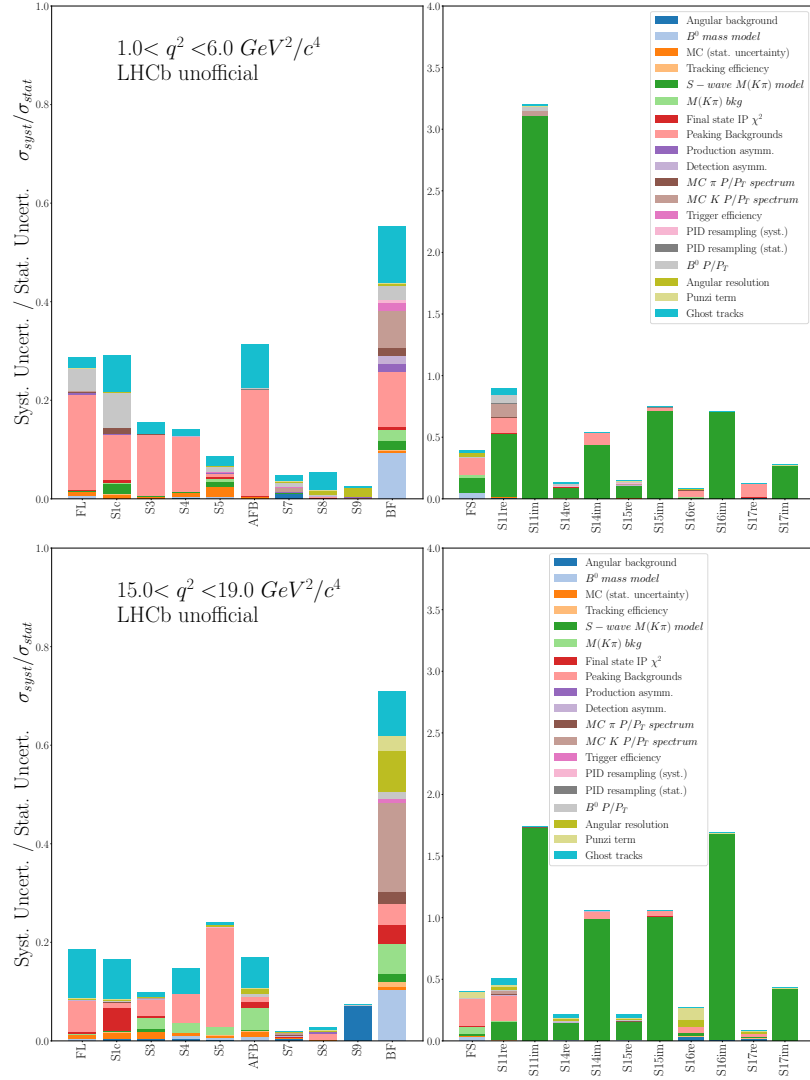


Figure 6.42: Systematic uncertainties for all extracted observables from fit configuration (i) in the bins of $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$ and $15.0 < q^2 < 19.0 \text{ GeV}^2/c^4$. The total systematic uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are stacked according to their relative contributions to the total systematic uncertainty.

6.7 Results

To determine the angular observables, likelihood fits are performed for every q^2 bin separately. The one-dimensional projections of the fits are displayed in Figs. B.42 to B.51. The results of the analysis consist of full sets of angular observables in bins of q^2 for each of the five fit configurations detailed in Sec. 6.1.6. Tables 6.22 to 6.25 display the numerical results for the default configuration in the S_i basis (i) as well as the optimized P_i basis (ii), respectively. The fit results for the remaining fit configurations can be found in Tab. B.17 to B.25. The total correlation matrices including systematic and statistical contributions for all fit configurations are given in Tab. B.36-B.81. A comparison of the fitted observables to their respective SM expectation is shown in Figs. 6.43 to 6.50, where both the fit results and SM predictions, including their theoretical uncertainties, are overlayed. Multiple SM predictions using different sets of hadronic form factors from the two different flavor physics frameworks, Flavio (BSZ) [57] and EOS (GRvDV) [58] are used as well as the SM predictions for the optimized observables from Ref. [149] (ABCDMN). The hadronic form factor parameterizations used for the predictions can be found in Refs. [50] and [53], respectively.

6.7.1 Configuration (i)+(ii) - Floated S_1^c and fixed A_i

Figures 6.44 and 6.43 show the measurements of the P-wave angular observables in the S_i basis for the default configuration. Figure 6.45 displays the corresponding optimized observables in the P_i basis. The SM predictions provided by the Flavio and EOS flavor physics frameworks are only valid below a q^2 value of $8 \text{ GeV}^2/c^4$.

The optimized observable P_5' displays local tensions with the BSZ (GRvDV) SM predictions of $2.8, 2.6$, and 3.1σ ($2.7, 2.6$, and 2.7σ) in the q^2 bins $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$, $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$ and $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$, respectively. For the ABCDMN SM prediction, the local tensions in P_5' are slightly smaller at $1.6, 2.1$, and 2.4σ . Overall, the measured value of P_5' is consistently above all discussed SM predictions in the intermediate q^2 range from 1.1 to $8.0 \text{ GeV}^2/c^4$. Similarly, the forward-backward asymmetry A_{FB} is measured to be consistently smaller than the SM predictions. The discrepancy in the $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$ and $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$ bins is found to be at the level of 2.5 and 1.9σ (2.2 and 1.7σ) with respect to the BSZ (GRvDV) SM predictions. The same tensions are visible in the corresponding observables S_5 and P_2 . Furthermore, the differential branching fraction is found to be below the SM predictions in the same intermediate q^2 range. The discrepancy with respect to the SM prediction amounts to 2.0σ (1.7σ) in the wide q^2 bin $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$. Slight additional local tensions are visible for the observables S_1^c in $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$ and $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$ as well as S_7 in the intermediate q^2 range. All other observables show good agreement with the SM predictions. For the S-wave and P-/S-wave interference observables, no SM predictions are available. Therefore, the plots in Fig. B.54 to B.55 only show the measured results.

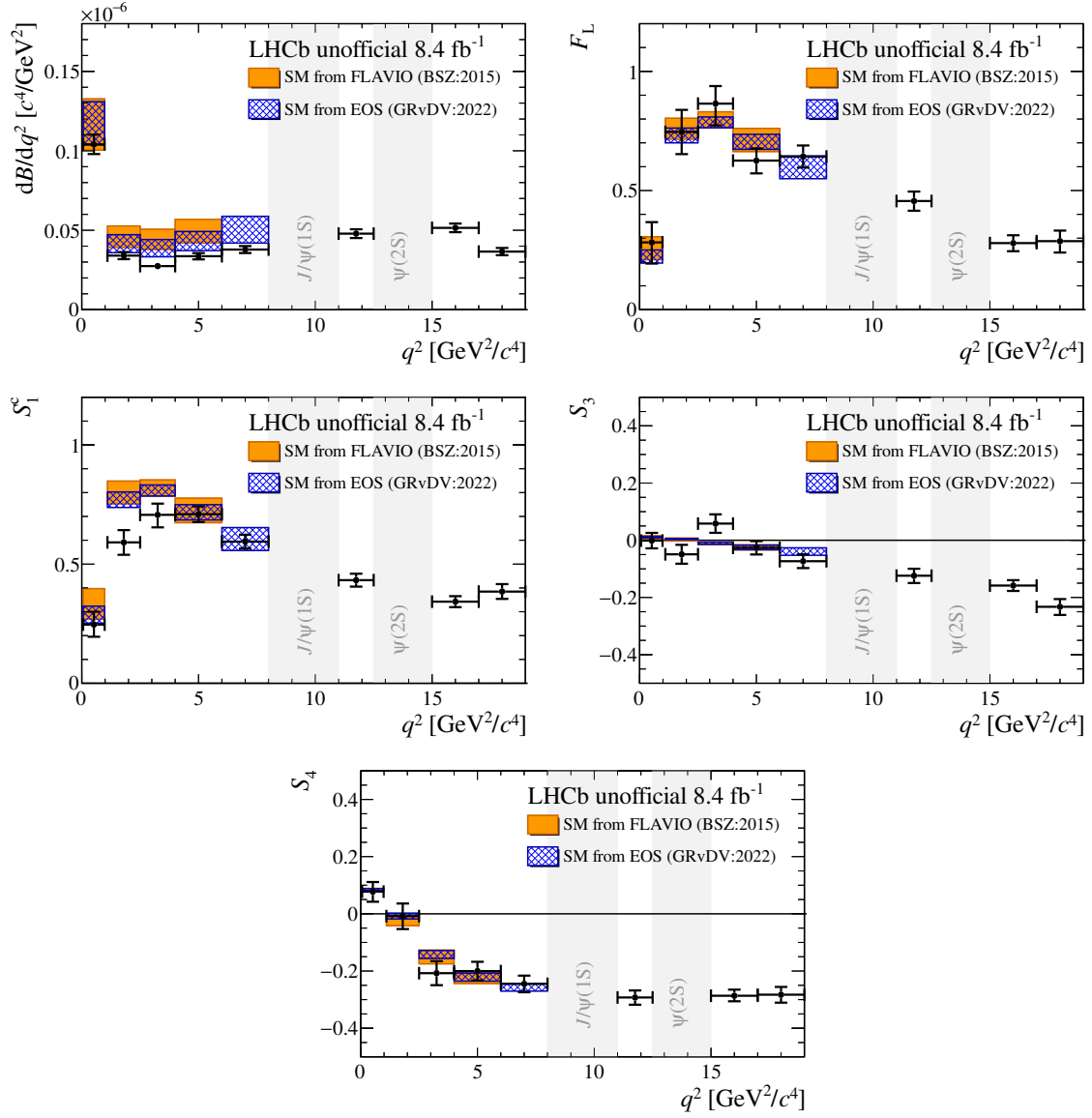


Figure 6.43: Comparison of the results from fit configuration (i) and two different SM predictions of the differential branching fraction dB/dq^2 , F_L , S_1^c , S_3 , and S_4 .

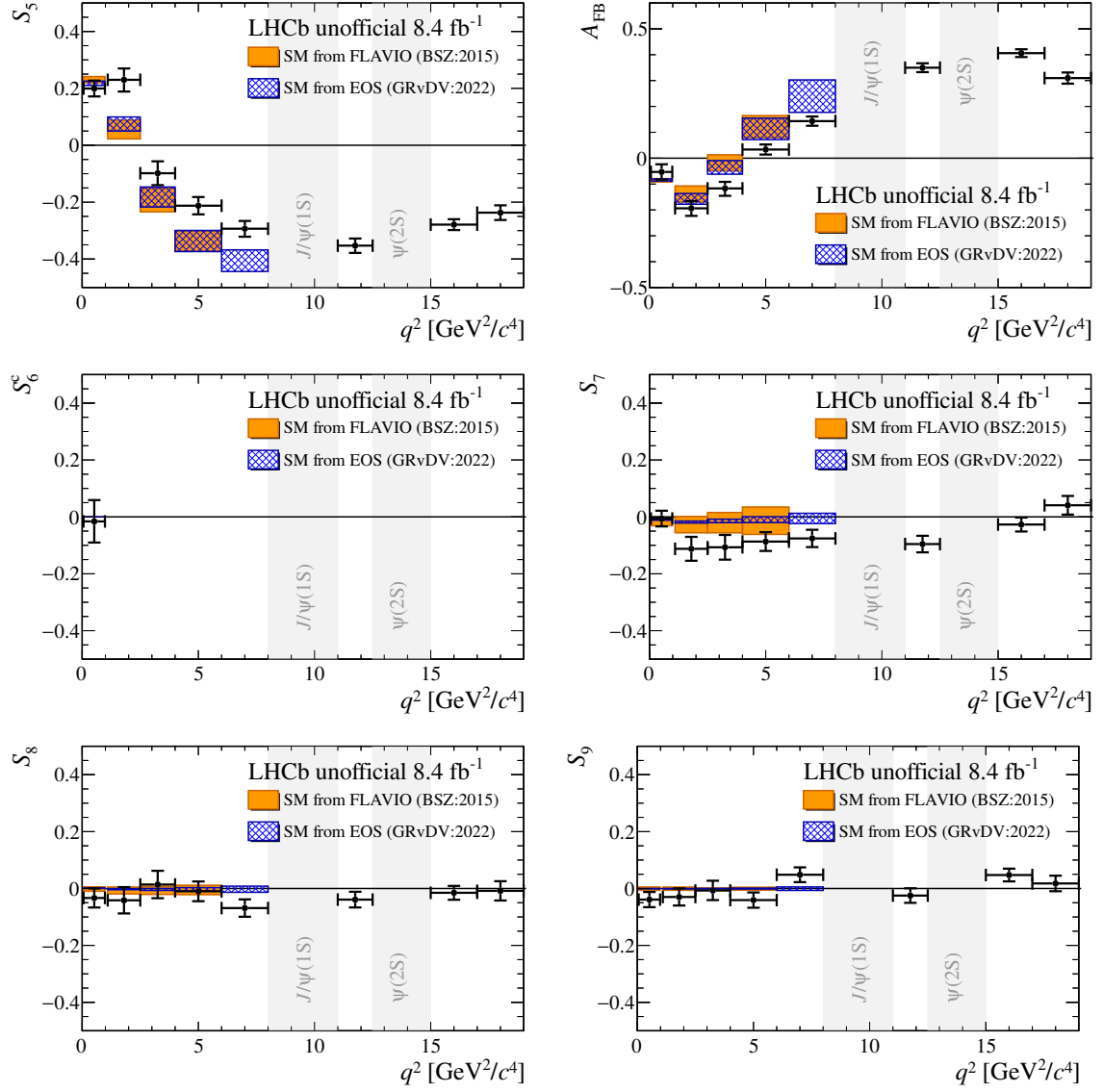


Figure 6.44: Comparison of the results from fit configuration (i) in the S_i -basis and two different SM predictions of the angular observables S_5 , A_{FB} , S_6^c , and S_7 - S_9 .

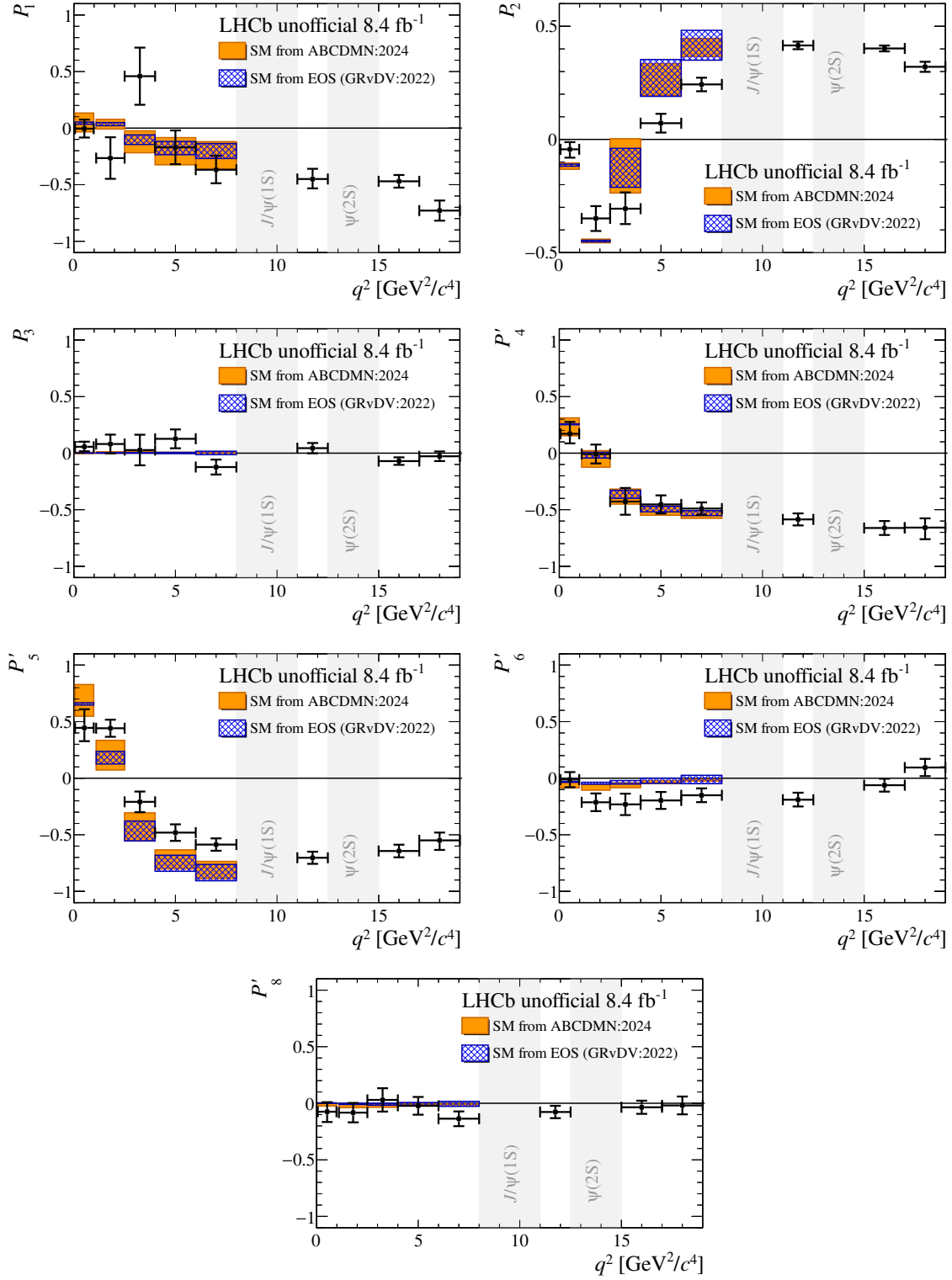


Figure 6.45: Comparison of the results from fit configuration (ii) in the P_i -basis and two different SM predictions of the angular observables.

Table 6.22: Measured values of the angular observables including statistical and systematic uncertainties from the default configuration (i) in the S_i basis.

$0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$			$1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$			$2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$		
F_L	$0.282^{+0.084}_{-0.088}^{+0.014}_{-0.014}$		F_L	$0.746^{+0.086}_{-0.086}^{+0.035}_{-0.035}$		F_L	$0.865^{+0.066}_{-0.085}^{+0.032}_{-0.032}$	
S_2^s	$0.175^{+0.047}_{-0.046}^{+0.012}_{-0.012}$		S_1^c	$0.591^{+0.051}_{-0.051}^{+0.009}_{-0.009}$		S_1^c	$0.707^{+0.046}_{-0.052}^{+0.008}_{-0.008}$	
S_1^c	$0.246^{+0.053}_{-0.050}^{+0.010}_{-0.010}$		S_3	$-0.049^{+0.033}_{-0.033}^{+0.002}_{-0.002}$		S_3	$0.058^{+0.032}_{-0.032}^{+0.003}_{-0.003}$	
S_3	$-0.001^{+0.027}_{-0.027}^{+0.002}_{-0.002}$		S_4	$-0.008^{+0.044}_{-0.045}^{+0.005}_{-0.005}$		S_4	$-0.207^{+0.042}_{-0.042}^{+0.003}_{-0.003}$	
S_4	$0.078^{+0.034}_{-0.035}^{+0.001}_{-0.001}$		S_5	$0.230^{+0.040}_{-0.041}^{+0.002}_{-0.002}$		S_5	$-0.098^{+0.042}_{-0.042}^{+0.004}_{-0.004}$	
S_5	$0.199^{+0.028}_{-0.028}^{+0.004}_{-0.004}$		A_{FB}	$-0.194^{+0.027}_{-0.028}^{+0.007}_{-0.007}$		A_{FB}	$-0.117^{+0.025}_{-0.028}^{+0.004}_{-0.004}$	
A_{FB}	$-0.053^{+0.029}_{-0.029}^{+0.005}_{-0.005}$		S_7	$-0.112^{+0.042}_{-0.042}^{+0.002}_{-0.002}$		S_7	$-0.107^{+0.043}_{-0.043}^{+0.002}_{-0.002}$	
S_7	$-0.006^{+0.028}_{-0.028}^{+0.001}_{-0.001}$		S_8	$-0.041^{+0.046}_{-0.046}^{+0.003}_{-0.003}$		S_8	$0.014^{+0.048}_{-0.048}^{+0.001}_{-0.001}$	
S_8	$-0.033^{+0.033}_{-0.033}^{+0.001}_{-0.001}$		S_9	$-0.030^{+0.030}_{-0.030}^{+0.001}_{-0.001}$		S_9	$-0.007^{+0.034}_{-0.034}^{+0.002}_{-0.002}$	
S_9	$-0.039^{+0.027}_{-0.027}^{+0.001}_{-0.001}$		F_S	$0.121^{+0.023}_{-0.023}^{+0.008}_{-0.008}$		F_S	$0.146^{+0.025}_{-0.025}^{+0.013}_{-0.013}$	
S_6^c	$-0.016^{+0.074}_{-0.074}^{+0.009}_{-0.009}$		S_{11}^{re}	$-0.400^{+0.065}_{-0.065}^{+0.028}_{-0.028}$		S_{11}^{re}	$-0.534^{+0.071}_{-0.071}^{+0.034}_{-0.034}$	
F_S	$0.078^{+0.043}_{-0.043}^{+0.017}_{-0.017}$		S_{11}^{sim}	$-0.084^{+0.049}_{-0.049}^{+0.084}_{-0.084}$		S_{11}^{sim}	$-0.031^{+0.054}_{-0.054}^{+0.091}_{-0.091}$	
S_{12}	$-0.000^{+0.000}_{-0.029}^{+0.009}_{-0.009}$		S_{14}^{re}	$0.007^{+0.094}_{-0.091}^{+0.004}_{-0.004}$		S_{14}^{re}	$0.070^{+0.091}_{-0.091}^{+0.007}_{-0.007}$	
S_{11}^{re}	$-0.072^{+0.108}_{-0.102}^{+0.020}_{-0.020}$		S_{14}^{sim}	$-0.065^{+0.067}_{-0.068}^{+0.011}_{-0.011}$		S_{14}^{sim}	$0.022^{+0.073}_{-0.073}^{+0.020}_{-0.020}$	
S_{11}^{sim}	$-0.108^{+0.082}_{-0.080}^{+0.037}_{-0.037}$		S_{15}^{re}	$-0.094^{+0.073}_{-0.073}^{+0.006}_{-0.006}$		S_{15}^{re}	$0.123^{+0.082}_{-0.082}^{+0.025}_{-0.025}$	
S_{13}^{re}	$0.183^{+0.152}_{-0.151}^{+0.018}_{-0.018}$		S_{15}^{sim}	$0.165^{+0.058}_{-0.061}^{+0.007}_{-0.007}$		S_{15}^{sim}	$0.132^{+0.065}_{-0.065}^{+0.030}_{-0.030}$	
S_{13}^{sim}	$-0.181^{+0.120}_{-0.114}^{+0.041}_{-0.041}$		S_{16}^{re}	$-0.165^{+0.062}_{-0.062}^{+0.006}_{-0.006}$		S_{16}^{re}	$0.123^{+0.064}_{-0.064}^{+0.010}_{-0.010}$	
S_{14}^{re}	$-0.087^{+0.081}_{-0.081}^{+0.006}_{-0.006}$		S_{16}^{sim}	$0.074^{+0.080}_{-0.080}^{+0.008}_{-0.008}$		S_{16}^{sim}	$0.014^{+0.081}_{-0.081}^{+0.031}_{-0.031}$	
S_{14}^{sim}	$0.057^{+0.061}_{-0.061}^{+0.012}_{-0.012}$		S_{17}^{re}	$-0.013^{+0.068}_{-0.072}^{+0.001}_{-0.001}$		S_{17}^{re}	$0.161^{+0.071}_{-0.074}^{+0.005}_{-0.005}$	
S_{15}^{re}	$-0.177^{+0.066}_{-0.066}^{+0.018}_{-0.018}$		S_{17}^{sim}	$-0.090^{+0.099}_{-0.101}^{+0.003}_{-0.003}$		S_{17}^{sim}	$0.022^{+0.095}_{-0.094}^{+0.013}_{-0.013}$	
S_{15}^{sim}	$0.017^{+0.047}_{-0.047}^{+0.032}_{-0.032}$		$d\mathcal{B}/dq^2/10^{-7}$	$0.340 \pm 0.015 \pm 0.017$		$d\mathcal{B}/dq^2/10^{-7}$	$0.274 \pm 0.013 \pm 0.015$	
S_{16}^{re}	$-0.095^{+0.047}_{-0.047}^{+0.004}_{-0.004}$							
S_{16}^{sim}	$0.024^{+0.069}_{-0.069}^{+0.026}_{-0.026}$							
S_{17}^{re}	$0.069^{+0.059}_{-0.059}^{+0.001}_{-0.001}$							
S_{17}^{sim}	$-0.043^{+0.088}_{-0.088}^{+0.011}_{-0.011}$							
$d\mathcal{B}/dq^2/10^{-7}$	$1.041 \pm 0.037 \pm 0.049$							

$4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$			$6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$		
F_L	$0.626^{+0.050}_{-0.053}^{+0.010}_{-0.010}$		F_L	$0.643^{+0.046}_{-0.046}^{+0.005}_{-0.005}$	
S_1^c	$0.709^{+0.032}_{-0.032}^{+0.006}_{-0.006}$		S_1^c	$0.594^{+0.029}_{-0.029}^{+0.004}_{-0.004}$	
S_3	$-0.026^{+0.023}_{-0.023}^{+0.002}_{-0.002}$		S_3	$-0.073^{+0.024}_{-0.024}^{+0.004}_{-0.004}$	
S_4	$-0.200^{+0.032}_{-0.032}^{+0.002}_{-0.002}$		S_4	$-0.245^{+0.029}_{-0.029}^{+0.001}_{-0.001}$	
S_5	$-0.212^{+0.030}_{-0.030}^{+0.003}_{-0.003}$		S_5	$-0.293^{+0.027}_{-0.028}^{+0.003}_{-0.003}$	
A_{FB}	$0.034^{+0.019}_{-0.019}^{+0.003}_{-0.003}$		A_{FB}	$0.143^{+0.018}_{-0.018}^{+0.002}_{-0.002}$	
S_7	$-0.087^{+0.033}_{-0.033}^{+0.002}_{-0.002}$		S_7	$-0.076^{+0.030}_{-0.030}^{+0.001}_{-0.001}$	
S_8	$-0.010^{+0.035}_{-0.035}^{+0.000}_{-0.000}$		S_8	$-0.069^{+0.030}_{-0.030}^{+0.003}_{-0.003}$	
S_9	$-0.041^{+0.027}_{-0.027}^{+0.002}_{-0.002}$		S_9	$0.048^{+0.026}_{-0.026}^{+0.002}_{-0.002}$	
F_S	$0.088^{+0.019}_{-0.019}^{+0.004}_{-0.004}$		F_S	$0.067^{+0.018}_{-0.018}^{+0.005}_{-0.005}$	
S_{11}^{re}	$-0.526^{+0.060}_{-0.060}^{+0.040}_{-0.040}$		S_{11}^{re}	$-0.337^{+0.052}_{-0.052}^{+0.025}_{-0.025}$	
S_{11}^{sim}	$0.030^{+0.038}_{-0.038}^{+0.088}_{-0.088}$		S_{11}^{sim}	$-0.040^{+0.037}_{-0.037}^{+0.074}_{-0.074}$	
S_{14}^{re}	$0.149^{+0.079}_{-0.079}^{+0.009}_{-0.009}$		S_{14}^{re}	$0.181^{+0.064}_{-0.064}^{+0.010}_{-0.010}$	
S_{14}^{sim}	$0.003^{+0.049}_{-0.049}^{+0.027}_{-0.027}$		S_{14}^{sim}	$0.050^{+0.046}_{-0.046}^{+0.032}_{-0.032}$	
S_{15}^{re}	$0.178^{+0.056}_{-0.056}^{+0.014}_{-0.014}$		S_{15}^{re}	$0.194^{+0.060}_{-0.060}^{+0.015}_{-0.015}$	
S_{15}^{sim}	$0.030^{+0.046}_{-0.046}^{+0.041}_{-0.041}$		S_{15}^{sim}	$0.116^{+0.042}_{-0.042}^{+0.049}_{-0.049}$	
S_{16}^{re}	$0.234^{+0.048}_{-0.048}^{+0.005}_{-0.005}$		S_{16}^{re}	$0.269^{+0.049}_{-0.049}^{+0.008}_{-0.008}$	
S_{16}^{sim}	$0.065^{+0.062}_{-0.062}^{+0.048}_{-0.048}$		S_{16}^{sim}	$0.011^{+0.068}_{-0.068}^{+0.060}_{-0.060}$	
S_{17}^{re}	$0.080^{+0.052}_{-0.052}^{+0.007}_{-0.007}$		S_{17}^{re}	$0.031^{+0.052}_{-0.052}^{+0.004}_{-0.004}$	
S_{17}^{sim}	$0.145^{+0.080}_{-0.080}^{+0.020}_{-0.020}$		S_{17}^{sim}	$0.018^{+0.072}_{-0.072}^{+0.026}_{-0.026}$	
$d\mathcal{B}/dq^2/10^{-7}$	$0.337 \pm 0.012 \pm 0.016$		$d\mathcal{B}/dq^2/10^{-7}$	$0.378 \pm 0.012 \pm 0.018$	

Table 6.23: Measured values of the angular observables, including statistical and systematic uncertainties from the default configuration (i) in the S_i basis.

15.0 < q^2 < 17.0 GeV^2/c^4			17.0 < q^2 < 19.0 GeV^2/c^4			11.0 < q^2 < 12.5 GeV^2/c^4		
F_L	0.279	$^{+0.033}_{-0.034} \text{ } ^{+0.004}_{-0.004}$	F_L	0.288	$^{+0.044}_{-0.048} \text{ } ^{+0.005}_{-0.005}$	F_L	0.456	$^{+0.039}_{-0.040} \text{ } ^{+0.007}_{-0.007}$
S_1^c	0.342	$^{+0.023}_{-0.023} \text{ } ^{+0.005}_{-0.005}$	S_1^c	0.385	$^{+0.031}_{-0.031} \text{ } ^{+0.004}_{-0.004}$	S_1^c	0.433	$^{+0.027}_{-0.027} \text{ } ^{+0.003}_{-0.003}$
S_3	-0.158	$^{+0.019}_{-0.019} \text{ } ^{+0.001}_{-0.001}$	S_3	-0.233	$^{+0.027}_{-0.028} \text{ } ^{+0.002}_{-0.002}$	S_3	-0.124	$^{+0.024}_{-0.025} \text{ } ^{+0.004}_{-0.004}$
S_4	-0.286	$^{+0.021}_{-0.019} \text{ } ^{+0.001}_{-0.001}$	S_4	-0.283	$^{+0.027}_{-0.028} \text{ } ^{+0.002}_{-0.002}$	S_4	-0.292	$^{+0.024}_{-0.025} \text{ } ^{+0.004}_{-0.004}$
S_5	-0.279	$^{+0.019}_{-0.019} \text{ } ^{+0.003}_{-0.003}$	S_5	-0.237	$^{+0.025}_{-0.025} \text{ } ^{+0.004}_{-0.004}$	S_5	-0.353	$^{+0.025}_{-0.025} \text{ } ^{+0.005}_{-0.005}$
A_{FB}	0.406	$^{+0.015}_{-0.015} \text{ } ^{+0.003}_{-0.003}$	A_{FB}	0.310	$^{+0.021}_{-0.021} \text{ } ^{+0.003}_{-0.003}$	A_{FB}	0.350	$^{+0.017}_{-0.017} \text{ } ^{+0.004}_{-0.004}$
S_7	-0.027	$^{+0.024}_{-0.024} \text{ } ^{+0.001}_{-0.001}$	S_7	0.041	$^{+0.033}_{-0.033} \text{ } ^{+0.001}_{-0.001}$	S_7	-0.096	$^{+0.029}_{-0.029} \text{ } ^{+0.001}_{-0.001}$
S_8	-0.015	$^{+0.024}_{-0.025} \text{ } ^{+0.001}_{-0.001}$	S_8	-0.008	$^{+0.034}_{-0.034} \text{ } ^{+0.001}_{-0.001}$	S_8	-0.039	$^{+0.028}_{-0.028} \text{ } ^{+0.001}_{-0.001}$
S_9	0.047	$^{+0.022}_{-0.022} \text{ } ^{+0.001}_{-0.001}$	S_9	0.018	$^{+0.027}_{-0.027} \text{ } ^{+0.000}_{-0.000}$	S_9	-0.025	$^{+0.025}_{-0.025} \text{ } ^{+0.004}_{-0.004}$
F_S	0.020	$^{+0.015}_{-0.015} \text{ } ^{+0.004}_{-0.004}$	F_S	0.051	$^{+0.019}_{-0.019} \text{ } ^{+0.004}_{-0.004}$	F_S	0.054	$^{+0.018}_{-0.018} \text{ } ^{+0.011}_{-0.011}$
S_{11}^{re}	-0.158	$^{+0.038}_{-0.038} \text{ } ^{+0.022}_{-0.022}$	S_{11}^{re}	-0.113	$^{+0.039}_{-0.039} \text{ } ^{+0.014}_{-0.014}$	S_{11}^{re}	-0.280	$^{+0.050}_{-0.050} \text{ } ^{+0.013}_{-0.013}$
S_{11}^{im}	0.046	$^{+0.027}_{-0.027} \text{ } ^{+0.058}_{-0.058}$	S_{11}^{im}	-0.083	$^{+0.045}_{-0.045} \text{ } ^{+0.042}_{-0.042}$	S_{11}^{im}	0.015	$^{+0.033}_{-0.033} \text{ } ^{+0.036}_{-0.036}$
S_{14}^{re}	0.091	$^{+0.056}_{-0.056} \text{ } ^{+0.012}_{-0.012}$	S_{14}^{re}	-0.000	$^{+0.057}_{-0.055} \text{ } ^{+0.010}_{-0.010}$	S_{14}^{re}	0.189	$^{+0.053}_{-0.053} \text{ } ^{+0.005}_{-0.005}$
S_{14}^{im}	-0.058	$^{+0.037}_{-0.037} \text{ } ^{+0.038}_{-0.038}$	S_{14}^{im}	-0.093	$^{+0.070}_{-0.070} \text{ } ^{+0.037}_{-0.037}$	S_{14}^{im}	0.051	$^{+0.044}_{-0.044} \text{ } ^{+0.035}_{-0.035}$
S_{15}^{re}	0.177	$^{+0.051}_{-0.051} \text{ } ^{+0.016}_{-0.016}$	S_{15}^{re}	0.081	$^{+0.054}_{-0.054} \text{ } ^{+0.010}_{-0.010}$	S_{15}^{re}	0.341	$^{+0.058}_{-0.058} \text{ } ^{+0.003}_{-0.003}$
S_{15}^{im}	-0.042	$^{+0.036}_{-0.036} \text{ } ^{+0.054}_{-0.054}$	S_{15}^{im}	0.034	$^{+0.063}_{-0.063} \text{ } ^{+0.039}_{-0.039}$	S_{15}^{im}	0.072	$^{+0.044}_{-0.044} \text{ } ^{+0.026}_{-0.026}$
S_{16}^{re}	0.116	$^{+0.046}_{-0.046} \text{ } ^{+0.005}_{-0.005}$	S_{16}^{re}	0.258	$^{+0.078}_{-0.078} \text{ } ^{+0.006}_{-0.006}$	S_{16}^{re}	0.313	$^{+0.050}_{-0.050} \text{ } ^{+0.010}_{-0.010}$
S_{16}^{im}	-0.008	$^{+0.061}_{-0.061} \text{ } ^{+0.070}_{-0.070}$	S_{16}^{im}	-0.074	$^{+0.065}_{-0.065} \text{ } ^{+0.068}_{-0.068}$	S_{16}^{im}	-0.060	$^{+0.074}_{-0.074} \text{ } ^{+0.076}_{-0.076}$
S_{17}^{re}	-0.012	$^{+0.048}_{-0.048} \text{ } ^{+0.004}_{-0.004}$	S_{17}^{re}	0.014	$^{+0.085}_{-0.085} \text{ } ^{+0.004}_{-0.004}$	S_{17}^{re}	0.109	$^{+0.047}_{-0.047} \text{ } ^{+0.005}_{-0.005}$
S_{17}^{im}	0.041	$^{+0.064}_{-0.064} \text{ } ^{+0.026}_{-0.026}$	S_{17}^{im}	-0.040	$^{+0.065}_{-0.065} \text{ } ^{+0.020}_{-0.020}$	S_{17}^{im}	-0.142	$^{+0.068}_{-0.068} \text{ } ^{+0.014}_{-0.014}$
$d\mathcal{B}/dq^2/10^{-7}$	0.515	$\pm 0.013 \pm 0.024$	$d\mathcal{B}/dq^2/10^{-7}$	0.366	$\pm 0.013 \pm 0.019$	$d\mathcal{B}/dq^2/10^{-7}$	0.479	$\pm 0.015 \pm 0.023$

1.1 < q^2 < 6.0 GeV^2/c^4			15.0 < q^2 < 19.0 GeV^2/c^4		
F_L	0.743	$^{+0.044}_{-0.043} \text{ } ^{+0.011}_{-0.011}$	F_L	0.268	$^{+0.026}_{-0.028} \text{ } ^{+0.005}_{-0.005}$
S_1^c	0.643	$^{+0.027}_{-0.029} \text{ } ^{+0.007}_{-0.007}$	S_1^c	0.364	$^{+0.018}_{-0.018} \text{ } ^{+0.003}_{-0.003}$
S_3	-0.012	$^{+0.018}_{-0.018} \text{ } ^{+0.002}_{-0.002}$	S_3	-0.194	$^{+0.015}_{-0.015} \text{ } ^{+0.002}_{-0.002}$
S_4	-0.147	$^{+0.024}_{-0.024} \text{ } ^{+0.003}_{-0.003}$	S_4	-0.285	$^{+0.016}_{-0.016} \text{ } ^{+0.002}_{-0.002}$
S_5	-0.050	$^{+0.022}_{-0.022} \text{ } ^{+0.002}_{-0.002}$	S_5	-0.254	$^{+0.015}_{-0.015} \text{ } ^{+0.004}_{-0.004}$
A_{FB}	-0.086	$^{+0.014}_{-0.014} \text{ } ^{+0.004}_{-0.004}$	A_{FB}	0.364	$^{+0.012}_{-0.012} \text{ } ^{+0.002}_{-0.002}$
S_7	-0.115	$^{+0.024}_{-0.024} \text{ } ^{+0.001}_{-0.001}$	S_7	0.001	$^{+0.020}_{-0.020} \text{ } ^{+0.000}_{-0.000}$
S_8	0.004	$^{+0.025}_{-0.025} \text{ } ^{+0.001}_{-0.001}$	S_8	-0.007	$^{+0.021}_{-0.021} \text{ } ^{+0.000}_{-0.000}$
S_9	-0.033	$^{+0.018}_{-0.018} \text{ } ^{+0.000}_{-0.000}$	S_9	0.028	$^{+0.017}_{-0.017} \text{ } ^{+0.001}_{-0.001}$
F_S	0.102	$^{+0.013}_{-0.013} \text{ } ^{+0.005}_{-0.005}$	F_S	0.041	$^{+0.012}_{-0.012} \text{ } ^{+0.005}_{-0.005}$
S_{11}^{re}	-0.475	$^{+0.043}_{-0.043} \text{ } ^{+0.034}_{-0.034}$	S_{11}^{re}	-0.127	$^{+0.027}_{-0.027} \text{ } ^{+0.013}_{-0.013}$
S_{11}^{im}	-0.022	$^{+0.029}_{-0.029} \text{ } ^{+0.090}_{-0.090}$	S_{11}^{im}	-0.005	$^{+0.023}_{-0.023} \text{ } ^{+0.039}_{-0.039}$
S_{14}^{re}	0.076	$^{+0.053}_{-0.053} \text{ } ^{+0.006}_{-0.006}$	S_{14}^{re}	0.055	$^{+0.039}_{-0.039} \text{ } ^{+0.009}_{-0.009}$
S_{14}^{im}	-0.019	$^{+0.036}_{-0.036} \text{ } ^{+0.019}_{-0.019}$	S_{14}^{im}	-0.050	$^{+0.032}_{-0.032} \text{ } ^{+0.036}_{-0.036}$
S_{15}^{re}	0.101	$^{+0.044}_{-0.044} \text{ } ^{+0.006}_{-0.006}$	S_{15}^{re}	0.113	$^{+0.036}_{-0.036} \text{ } ^{+0.008}_{-0.008}$
S_{15}^{im}	0.090	$^{+0.034}_{-0.034} \text{ } ^{+0.023}_{-0.023}$	S_{15}^{im}	-0.009	$^{+0.030}_{-0.030} \text{ } ^{+0.033}_{-0.033}$
S_{16}^{re}	0.085	$^{+0.033}_{-0.033} \text{ } ^{+0.002}_{-0.002}$	S_{16}^{re}	0.167	$^{+0.038}_{-0.038} \text{ } ^{+0.011}_{-0.011}$
S_{16}^{im}	0.045	$^{+0.044}_{-0.044} \text{ } ^{+0.030}_{-0.030}$	S_{16}^{im}	-0.048	$^{+0.045}_{-0.045} \text{ } ^{+0.072}_{-0.072}$
S_{17}^{re}	0.085	$^{+0.037}_{-0.037} \text{ } ^{+0.004}_{-0.004}$	S_{17}^{re}	-0.017	$^{+0.041}_{-0.041} \text{ } ^{+0.003}_{-0.003}$
S_{17}^{im}	0.028	$^{+0.055}_{-0.055} \text{ } ^{+0.013}_{-0.013}$	S_{17}^{im}	0.003	$^{+0.045}_{-0.045} \text{ } ^{+0.018}_{-0.018}$
$d\mathcal{B}/dq^2/10^{-7}$	0.319	$\pm 0.008 \pm 0.015$	$d\mathcal{B}/dq^2/10^{-7}$	0.444	$\pm 0.009 \pm 0.021$

Table 6.24: Measured values of the angular observables, including statistical and systematic uncertainties from the default configuration (ii) in the P_i basis.

$0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$			$1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$			$2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$		
F_L	$0.282^{+0.088}_{-0.087}^{+0.014}_{-0.014}$		F_L	$0.748^{+0.085}_{-0.089}^{+0.036}_{-0.036}$		F_L	$0.863^{+0.069}_{-0.081}^{+0.032}_{-0.032}$	
S_2^s	$0.176^{+0.045}_{-0.046}^{+0.012}_{-0.012}$		S_1^c	$0.591^{+0.052}_{-0.052}^{+0.010}_{-0.010}$		S_1^c	$0.706^{+0.046}_{-0.052}^{+0.008}_{-0.008}$	
S_1^c	$0.246^{+0.055}_{-0.052}^{+0.010}_{-0.010}$		P_1	$-0.265^{+0.183}_{-0.182}^{+0.020}_{-0.020}$		P_1	$0.459^{+0.251}_{-0.251}^{+0.031}_{-0.031}$	
P_1	$-0.005^{+0.080}_{-0.078}^{+0.008}_{-0.008}$		P_2	$-0.349^{+0.049}_{-0.051}^{+0.023}_{-0.023}$		P_2	$-0.306^{+0.070}_{-0.066}^{+0.015}_{-0.015}$	
P_2	$-0.044^{+0.030}_{-0.036}^{+0.009}_{-0.009}$		P_3	$0.081^{+0.083}_{-0.083}^{+0.007}_{-0.007}$		P_3	$0.027^{+0.135}_{-0.134}^{+0.012}_{-0.012}$	
P_3	$0.056^{+0.045}_{-0.040}^{+0.001}_{-0.001}$		P'_4	$-0.007^{+0.082}_{-0.082}^{+0.013}_{-0.013}$		P'_4	$-0.428^{+0.119}_{-0.116}^{+0.014}_{-0.014}$	
P'_4	$0.171^{+0.105}_{-0.084}^{+0.009}_{-0.009}$		P'_5	$0.442^{+0.075}_{-0.075}^{+0.005}_{-0.005}$		P'_5	$-0.209^{+0.090}_{-0.090}^{+0.016}_{-0.016}$	
P'_5	$0.445^{+0.163}_{-0.115}^{+0.026}_{-0.026}$		P'_6	$-0.213^{+0.078}_{-0.078}^{+0.005}_{-0.005}$		P'_6	$-0.231^{+0.095}_{-0.095}^{+0.005}_{-0.005}$	
P'_6	$-0.012^{+0.066}_{-0.068}^{+0.003}_{-0.003}$		P'_8	$-0.083^{+0.086}_{-0.086}^{+0.006}_{-0.006}$		P'_8	$0.030^{+0.103}_{-0.103}^{+0.003}_{-0.003}$	
P'_8	$-0.075^{+0.084}_{-0.091}^{+0.001}_{-0.001}$		F_S	$0.121^{+0.023}_{-0.023}^{+0.008}_{-0.008}$		F_S	$0.146^{+0.024}_{-0.024}^{+0.013}_{-0.013}$	
P_6^c	$0.060^{+0.222}_{-0.222}^{+0.028}_{-0.028}$		S_{11}^{re}	$-0.400^{+0.064}_{-0.064}^{+0.028}_{-0.028}$		S_{11}^{re}	$-0.534^{+0.071}_{-0.071}^{+0.034}_{-0.034}$	
F_S	$0.077^{+0.043}_{-0.041}^{+0.017}_{-0.017}$		S_{11}^{sim}	$-0.084^{+0.049}_{-0.049}^{+0.084}_{-0.084}$		S_{11}^{sim}	$-0.027^{+0.054}_{-0.054}^{+0.091}_{-0.091}$	
S_{12}	$-0.002^{+0.002}_{-0.025}^{+0.009}_{-0.009}$		S_{14}^{re}	$0.007^{+0.094}_{-0.090}^{+0.004}_{-0.004}$		S_{14}^{re}	$0.070^{+0.091}_{-0.091}^{+0.007}_{-0.007}$	
S_{11}^{re}	$-0.073^{+0.109}_{-0.103}^{+0.020}_{-0.020}$		S_{14}^{sim}	$-0.065^{+0.067}_{-0.068}^{+0.011}_{-0.011}$		S_{14}^{sim}	$0.024^{+0.066}_{-0.066}^{+0.020}_{-0.020}$	
S_{11}^{sim}	$-0.111^{+0.083}_{-0.080}^{+0.037}_{-0.037}$		S_{15}^{re}	$-0.097^{+0.079}_{-0.076}^{+0.006}_{-0.006}$		S_{15}^{re}	$0.123^{+0.082}_{-0.082}^{+0.025}_{-0.025}$	
S_{13}^{re}	$0.180^{+0.153}_{-0.150}^{+0.018}_{-0.018}$		S_{15}^{sim}	$0.166^{+0.058}_{-0.061}^{+0.007}_{-0.007}$		S_{15}^{sim}	$0.132^{+0.065}_{-0.065}^{+0.030}_{-0.030}$	
S_{13}^{sim}	$-0.184^{+0.120}_{-0.113}^{+0.041}_{-0.041}$		S_{16}^{re}	$-0.163^{+0.063}_{-0.062}^{+0.006}_{-0.006}$		S_{16}^{re}	$0.124^{+0.063}_{-0.065}^{+0.010}_{-0.010}$	
S_{14}^{re}	$-0.087^{+0.081}_{-0.081}^{+0.006}_{-0.006}$		S_{16}^{sim}	$0.074^{+0.080}_{-0.080}^{+0.008}_{-0.008}$		S_{16}^{sim}	$0.015^{+0.079}_{-0.082}^{+0.031}_{-0.031}$	
S_{14}^{sim}	$0.057^{+0.061}_{-0.061}^{+0.012}_{-0.012}$		S_{17}^{re}	$-0.016^{+0.069}_{-0.069}^{+0.001}_{-0.001}$		S_{17}^{re}	$0.161^{+0.070}_{-0.074}^{+0.005}_{-0.005}$	
S_{15}^{re}	$-0.177^{+0.066}_{-0.066}^{+0.018}_{-0.018}$		S_{17}^{sim}	$-0.090^{+0.098}_{-0.100}^{+0.003}_{-0.003}$		S_{17}^{sim}	$0.022^{+0.094}_{-0.092}^{+0.013}_{-0.013}$	
S_{15}^{sim}	$0.017^{+0.047}_{-0.047}^{+0.032}_{-0.032}$		$d\mathcal{B}/dq^2/10^{-7}$	$0.340 \pm 0.015 \pm 0.017$		$d\mathcal{B}/dq^2/10^{-7}$	$0.274 \pm 0.013 \pm 0.015$	
S_{16}^{re}	$-0.095^{+0.047}_{-0.047}^{+0.004}_{-0.004}$							
S_{16}^{sim}	$0.024^{+0.069}_{-0.069}^{+0.026}_{-0.026}$							
S_{17}^{re}	$0.069^{+0.059}_{-0.059}^{+0.001}_{-0.001}$							
S_{17}^{sim}	$-0.043^{+0.088}_{-0.088}^{+0.011}_{-0.011}$							
$d\mathcal{B}/dq^2/10^{-7}$	$1.041 \pm 0.037 \pm 0.049$							

$4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$			$6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$		
F_L	$0.625^{+0.052}_{-0.054}^{+0.010}_{-0.010}$		F_L	$0.641^{+0.044}_{-0.045}^{+0.005}_{-0.005}$	
S_1^c	$0.709^{+0.032}_{-0.032}^{+0.006}_{-0.006}$		S_1^c	$0.594^{+0.028}_{-0.028}^{+0.004}_{-0.004}$	
P_1	$-0.170^{+0.149}_{-0.149}^{+0.013}_{-0.013}$		P_1	$-0.368^{+0.123}_{-0.118}^{+0.019}_{-0.019}$	
P_2	$0.072^{+0.041}_{-0.041}^{+0.006}_{-0.006}$		P_2	$0.243^{+0.029}_{-0.030}^{+0.004}_{-0.004}$	
P_3	$0.127^{+0.083}_{-0.083}^{+0.006}_{-0.006}$		P_3	$-0.123^{+0.066}_{-0.066}^{+0.007}_{-0.007}$	
P'_4	$-0.453^{+0.080}_{-0.080}^{+0.007}_{-0.007}$		P'_4	$-0.490^{+0.054}_{-0.054}^{+0.003}_{-0.003}$	
P'_5	$-0.481^{+0.073}_{-0.073}^{+0.006}_{-0.006}$		P'_5	$-0.587^{+0.055}_{-0.055}^{+0.004}_{-0.004}$	
P'_6	$-0.197^{+0.075}_{-0.075}^{+0.005}_{-0.005}$		P'_6	$-0.151^{+0.061}_{-0.061}^{+0.003}_{-0.003}$	
P'_8	$-0.023^{+0.078}_{-0.078}^{+0.001}_{-0.001}$		P'_8	$-0.136^{+0.064}_{-0.066}^{+0.006}_{-0.006}$	
F_S	$0.089^{+0.019}_{-0.019}^{+0.004}_{-0.004}$		F_S	$0.067^{+0.018}_{-0.018}^{+0.005}_{-0.005}$	
S_{11}^{re}	$-0.526^{+0.060}_{-0.060}^{+0.040}_{-0.040}$		S_{11}^{re}	$-0.337^{+0.052}_{-0.052}^{+0.025}_{-0.025}$	
S_{11}^{sim}	$0.030^{+0.038}_{-0.038}^{+0.088}_{-0.088}$		S_{11}^{sim}	$-0.040^{+0.037}_{-0.037}^{+0.074}_{-0.074}$	
S_{14}^{re}	$0.148^{+0.068}_{-0.068}^{+0.009}_{-0.009}$		S_{14}^{re}	$0.181^{+0.064}_{-0.064}^{+0.010}_{-0.010}$	
S_{14}^{sim}	$0.003^{+0.049}_{-0.049}^{+0.027}_{-0.027}$		S_{14}^{sim}	$0.050^{+0.046}_{-0.046}^{+0.032}_{-0.032}$	
S_{15}^{re}	$0.178^{+0.056}_{-0.056}^{+0.014}_{-0.014}$		S_{15}^{re}	$0.194^{+0.059}_{-0.059}^{+0.015}_{-0.015}$	
S_{15}^{sim}	$0.031^{+0.046}_{-0.047}^{+0.041}_{-0.041}$		S_{15}^{sim}	$0.116^{+0.042}_{-0.042}^{+0.049}_{-0.049}$	
S_{16}^{re}	$0.233^{+0.047}_{-0.050}^{+0.005}_{-0.005}$		S_{16}^{re}	$0.269^{+0.049}_{-0.049}^{+0.008}_{-0.008}$	
S_{16}^{sim}	$0.065^{+0.062}_{-0.062}^{+0.048}_{-0.048}$		S_{16}^{sim}	$0.011^{+0.068}_{-0.068}^{+0.060}_{-0.060}$	
S_{17}^{re}	$0.080^{+0.052}_{-0.052}^{+0.007}_{-0.007}$		S_{17}^{re}	$0.031^{+0.052}_{-0.052}^{+0.004}_{-0.004}$	
S_{17}^{sim}	$0.145^{+0.080}_{-0.080}^{+0.020}_{-0.020}$		S_{17}^{sim}	$0.018^{+0.072}_{-0.072}^{+0.026}_{-0.026}$	
$d\mathcal{B}/dq^2/10^{-7}$	$0.337 \pm 0.012 \pm 0.016$		$d\mathcal{B}/dq^2/10^{-7}$	$0.378 \pm 0.012 \pm 0.018$	

Table 6.25: Measured values of the angular observables, including statistical and systematic uncertainties from the default configuration (ii) in the P_i basis.

15.0 < q^2 < 17.0 GeV^2/c^4			17.0 < q^2 < 19.0 GeV^2/c^4			11.0 < q^2 < 12.5 GeV^2/c^4		
F_L	0.279 ^{+0.033} _{-0.034}	+0.004 -0.004	F_L	0.286 ^{+0.046} _{-0.046}	+0.005 -0.005	F_L	0.456 ^{+0.040} _{-0.040}	+0.007 -0.007
S_1^c	0.342 ^{+0.024} _{-0.024}	+0.004 -0.004	S_1^c	0.385 ^{+0.031} _{-0.031}	+0.002 -0.002	S_1^c	0.433 ^{+0.026} _{-0.026}	+0.005 -0.005
P_1	-0.470 ^{+0.056} _{-0.056}	+0.006 -0.006	P_1	-0.729 ^{+0.089} _{-0.089}	+0.006 -0.006	P_1	-0.450 ^{+0.090} _{-0.082}	+0.011 -0.011
P_2	0.402 ^{+0.013} _{-0.013}	+0.001 -0.001	P_2	0.321 ^{+0.022} _{-0.022}	+0.001 -0.001	P_2	0.415 ^{+0.016} _{-0.016}	+0.004 -0.004
P_3	-0.070 ^{+0.033} _{-0.033}	+0.001 -0.001	P_3	-0.028 ^{+0.042} _{-0.042}	+0.001 -0.001	P_3	0.044 ^{+0.045} _{-0.045}	+0.006 -0.006
P_4'	-0.662 ^{+0.062} _{-0.062}	+0.003 -0.003	P_4'	-0.658 ^{+0.082} _{-0.103}	+0.003 -0.003	P_4'	-0.586 ^{+0.053} _{-0.053}	+0.007 -0.007
P_5'	-0.643 ^{+0.056} _{-0.056}	+0.005 -0.005	P_5'	-0.549 ^{+0.069} _{-0.084}	+0.006 -0.006	P_5'	-0.703 ^{+0.053} _{-0.053}	+0.007 -0.007
P_6'	-0.062 ^{+0.057} _{-0.057}	+0.002 -0.002	P_6'	0.095 ^{+0.076} _{-0.076}	+0.001 -0.001	P_6'	-0.190 ^{+0.062} _{-0.060}	+0.001 -0.001
P_8'	-0.035 ^{+0.058} _{-0.058}	+0.001 -0.001	P_8'	-0.019 ^{+0.079} _{-0.079}	+0.002 -0.002	P_8'	-0.077 ^{+0.055} _{-0.055}	+0.001 -0.001
F_S	0.020 ^{+0.015} _{-0.015}	+0.004 -0.004	F_S	0.047 ^{+0.030} _{-0.030}	+0.004 -0.004	F_S	0.054 ^{+0.018} _{-0.018}	+0.011 -0.011
S_{11}^{re}	-0.158 ^{+0.037} _{-0.037}	+0.022 -0.022	S_{11}^{re}	-0.113 ^{+0.039} _{-0.039}	+0.014 -0.014	S_{11}^{re}	-0.280 ^{+0.050} _{-0.050}	+0.013 -0.013
S_{11}^{im}	0.046 ^{+0.027} _{-0.027}	+0.058 -0.058	S_{11}^{im}	-0.083 ^{+0.045} _{-0.045}	+0.042 -0.042	S_{11}^{im}	0.015 ^{+0.033} _{-0.033}	+0.036 -0.036
S_{14}^{re}	0.091 ^{+0.056} _{-0.056}	+0.012 -0.012	S_{14}^{re}	0.000 ^{+0.055} _{-0.057}	+0.010 -0.010	S_{14}^{re}	0.189 ^{+0.053} _{-0.053}	+0.005 -0.005
S_{14}^{im}	-0.058 ^{+0.037} _{-0.037}	+0.038 -0.038	S_{14}^{im}	-0.089 ^{+0.068} _{-0.069}	+0.037 -0.037	S_{14}^{im}	0.051 ^{+0.044} _{-0.044}	+0.035 -0.035
S_{15}^{re}	0.177 ^{+0.051} _{-0.051}	+0.016 -0.016	S_{15}^{re}	0.081 ^{+0.054} _{-0.054}	+0.010 -0.010	S_{15}^{re}	0.332 ^{+0.063} _{-0.061}	+0.003 -0.003
S_{15}^{im}	-0.042 ^{+0.036} _{-0.036}	+0.054 -0.054	S_{15}^{im}	0.033 ^{+0.058} _{-0.059}	+0.039 -0.039	S_{15}^{im}	0.072 ^{+0.044} _{-0.044}	+0.026 -0.026
S_{16}^{re}	0.116 ^{+0.046} _{-0.046}	+0.005 -0.005	S_{16}^{re}	0.258 ^{+0.078} _{-0.078}	+0.006 -0.006	S_{16}^{re}	0.313 ^{+0.050} _{-0.050}	+0.010 -0.010
S_{16}^{im}	-0.008 ^{+0.061} _{-0.061}	+0.070 -0.070	S_{16}^{im}	-0.074 ^{+0.065} _{-0.065}	+0.068 -0.068	S_{16}^{im}	-0.060 ^{+0.074} _{-0.074}	+0.076 -0.076
S_{17}^{re}	-0.012 ^{+0.048} _{-0.048}	+0.004 -0.004	S_{17}^{re}	0.014 ^{+0.085} _{-0.085}	+0.004 -0.004	S_{17}^{re}	0.109 ^{+0.047} _{-0.047}	+0.005 -0.005
S_{17}^{im}	0.041 ^{+0.064} _{-0.064}	+0.026 -0.026	S_{17}^{im}	-0.040 ^{+0.066} _{-0.066}	+0.020 -0.020	S_{17}^{im}	-0.142 ^{+0.067} _{-0.067}	+0.014 -0.014
$d\mathcal{B}/dq^2/10^{-7}$	0.515 ± 0.013 ± 0.024		$d\mathcal{B}/dq^2/10^{-7}$	0.366 ± 0.020 ± 0.019		$d\mathcal{B}/dq^2/10^{-7}$	0.479 ± 0.015 ± 0.023	

1.1 < q^2 < 6.0 GeV^2/c^4			15.0 < q^2 < 19.0 GeV^2/c^4		
F_L	0.742 ^{+0.046} _{-0.044}	+0.011 -0.011	F_L	0.268 ^{+0.027} _{-0.028}	+0.005 -0.005
S_1^c	0.643 ^{+0.027} _{-0.029}	+0.007 -0.007	S_1^c	0.364 ^{+0.019} _{-0.019}	+0.003 -0.003
P_1	-0.073 ^{+0.105} _{-0.105}	+0.020 -0.020	P_1	-0.588 ^{+0.044} _{-0.044}	+0.006 -0.006
P_2	-0.171 ^{+0.028} _{-0.028}	+0.011 -0.011	P_2	0.367 ^{+0.011} _{-0.011}	+0.001 -0.001
P_3	0.097 ^{+0.055} _{-0.055}	+0.001 -0.001	P_3	-0.043 ^{+0.026} _{-0.026}	+0.002 -0.002
P_4'	-0.294 ^{+0.049} _{-0.049}	+0.011 -0.011	P_4'	-0.677 ^{+0.054} _{-0.054}	+0.003 -0.003
P_5'	-0.100 ^{+0.045} _{-0.045}	+0.006 -0.006	P_5'	-0.603 ^{+0.046} _{-0.046}	+0.005 -0.005
P_6'	-0.233 ^{+0.051} _{-0.049}	+0.003 -0.003	P_6'	0.003 ^{+0.047} _{-0.047}	+0.001 -0.001
P_8'	0.008 ^{+0.051} _{-0.051}	+0.002 -0.002	P_8'	-0.016 ^{+0.049} _{-0.049}	+0.001 -0.001
F_S	0.102 ^{+0.013} _{-0.014}	+0.005 -0.005	F_S	0.041 ^{+0.012} _{-0.012}	+0.005 -0.005
S_{11}^{re}	-0.475 ^{+0.043} _{-0.043}	+0.034 -0.034	S_{11}^{re}	-0.127 ^{+0.027} _{-0.027}	+0.013 -0.013
S_{11}^{im}	-0.022 ^{+0.029} _{-0.029}	+0.090 -0.090	S_{11}^{im}	-0.005 ^{+0.023} _{-0.023}	+0.039 -0.039
S_{14}^{re}	0.076 ^{+0.053} _{-0.053}	+0.006 -0.006	S_{14}^{re}	0.055 ^{+0.039} _{-0.039}	+0.009 -0.009
S_{14}^{im}	-0.019 ^{+0.036} _{-0.036}	+0.019 -0.019	S_{14}^{im}	-0.050 ^{+0.032} _{-0.032}	+0.036 -0.036
S_{15}^{re}	0.101 ^{+0.044} _{-0.044}	+0.006 -0.006	S_{15}^{re}	0.113 ^{+0.036} _{-0.036}	+0.008 -0.008
S_{15}^{im}	0.090 ^{+0.034} _{-0.034}	+0.023 -0.023	S_{15}^{im}	-0.009 ^{+0.030} _{-0.030}	+0.033 -0.033
S_{16}^{re}	0.085 ^{+0.033} _{-0.033}	+0.002 -0.002	S_{16}^{re}	0.167 ^{+0.038} _{-0.038}	+0.011 -0.011
S_{16}^{im}	0.045 ^{+0.044} _{-0.044}	+0.030 -0.030	S_{16}^{im}	-0.048 ^{+0.045} _{-0.045}	+0.072 -0.072
S_{17}^{re}	0.085 ^{+0.037} _{-0.037}	+0.004 -0.004	S_{17}^{re}	-0.017 ^{+0.041} _{-0.041}	+0.003 -0.003
S_{17}^{im}	0.028 ^{+0.055} _{-0.055}	+0.013 -0.013	S_{17}^{im}	0.003 ^{+0.045} _{-0.045}	+0.018 -0.018
$d\mathcal{B}/dq^2/10^{-7}$	0.319 ± 0.008 ± 0.015		$d\mathcal{B}/dq^2/10^{-7}$	0.444 ± 0.009 ± 0.021	

6.7.2 Configuration (iii) - Floated CP asymmetries

The P-wave CP -asymmetries determined from fit configuration (iii) are displayed in Fig. 6.46 and Fig. 6.47. The results include the angular asymmetries as well as the total CP asymmetry A_{CP} of the decay. As discussed in Sec. 6.7.1, two sets of SM predictions for these asymmetries are displayed in the plots. In the SM, the CP -asymmetries are expected to be small and only originate from the imaginary part of the CKM matrix in conjunction with strong phases from the decay. Unlike for the CP -symmetries, the associated theoretical uncertainties are negligibly small compared to the experimental uncertainties. Significant deviations from the SM would be a clear sign of New Physics. The figures show no tension between the measured results and the SM prediction. In fit configuration (iii), the CP -symmetries are determined simultaneously to the CP -asymmetries discussed above under the assumption of massless leptons above q^2 values of $1.1 \text{ GeV}^2/c^4$. The results for the CP -symmetries as well as the CP -asymmetries in the S-wave and P-/S-wave interference can be found in Figs. 6.46 to 6.47 and Figs. B.56 to B.62 as well as Tab. B.17 to B.20. The CP -symmetries extracted from the fit with floated asymmetries exhibit the same tensions with the SM in A_{FB} and S_5 as discussed in Sec. 6.7.1.

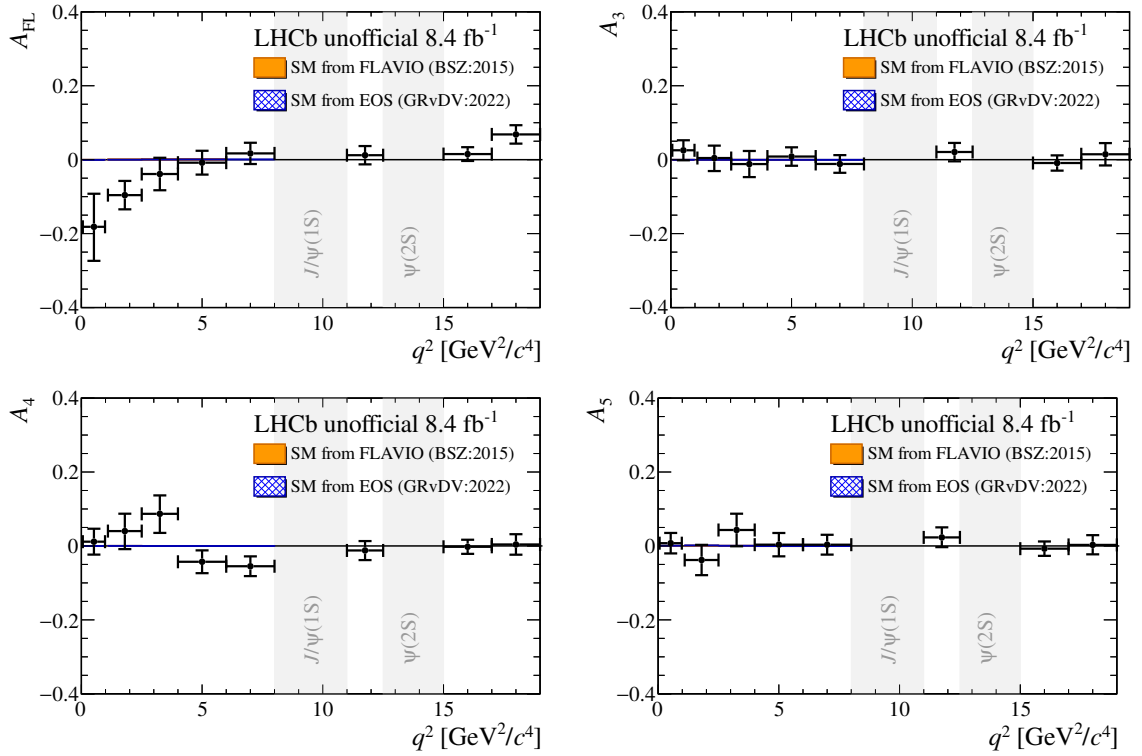


Figure 6.46: Comparison of the final fit results of fit configuration (iii) in the S_i -basis and two different SM predictions of the angular observables. The measurement is in good agreement with the SM expectation.

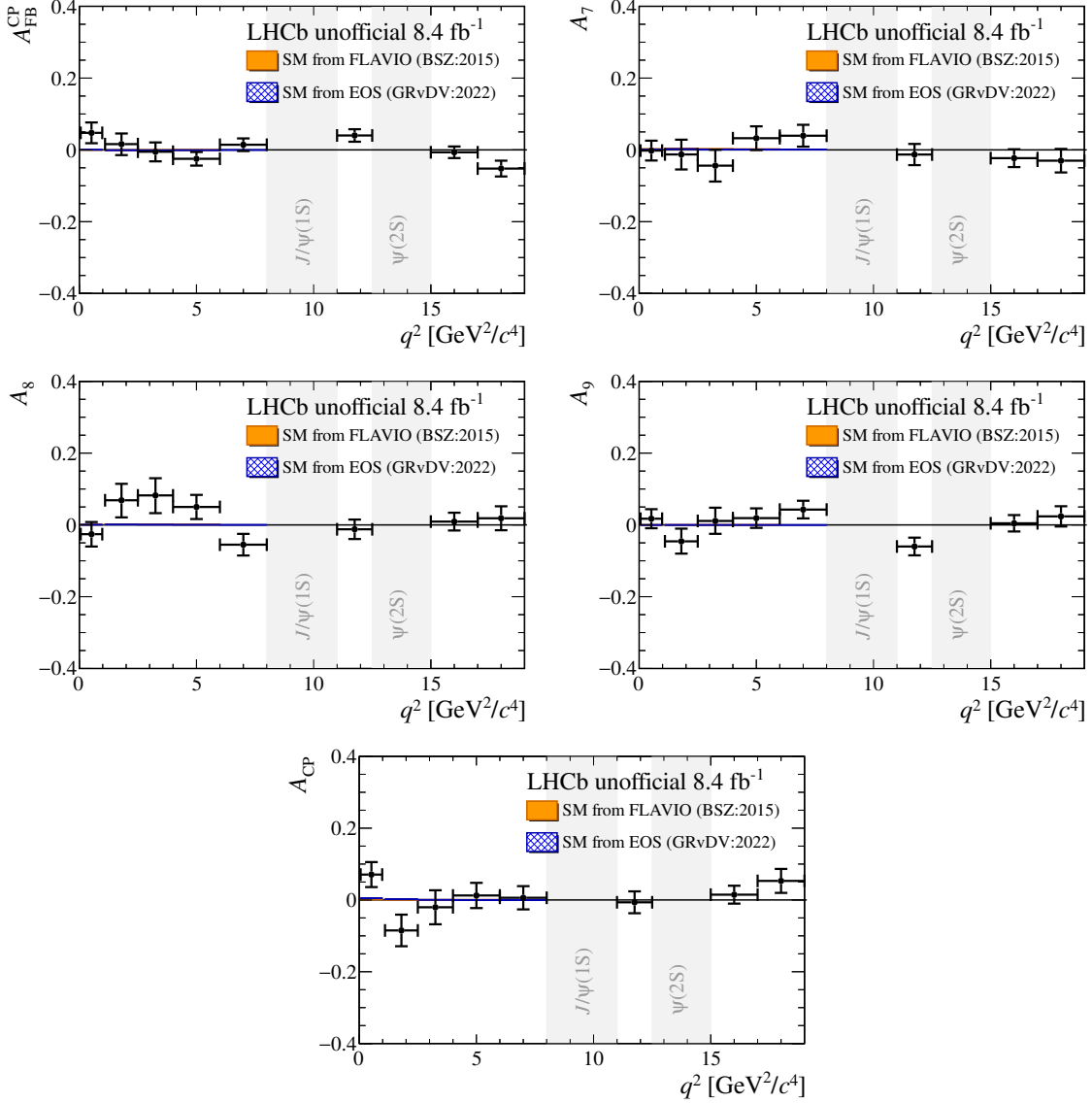


Figure 6.47: Comparison of the final fit results of fit configuration (iii) in the S_i -basis and two different SM predictions of the angular observables.

6.7.3 Configuration (iv) - Massive lepton configuration

The massive lepton configuration assumes non-negligible lepton masses for the P-wave part of the angular PDF in all regions of q^2 . This allows for maximal sensitivity to potential scalar, pseudoscalar, and tensor contributions to the decay. Compared to the default configuration, two additional observables are floated, S_2^s and S_6^c . S_6^c is expected to be strictly zero in the SM. Therefore, any significant deviation would be a theoretically clean sign of NP. The plots comparing the measured values of S_2^s and S_6^c to the SM predictions can be found in Fig. 6.48. The results of these observables are in good agreement with the SM predictions. The complete set of observables from this fit configuration can be found in Figs. B.63 to B.66 as well as Tabs. B.21 to B.22. The remaining CP symmetries determined in this configuration display the same behavior as already discussed in the default configuration.

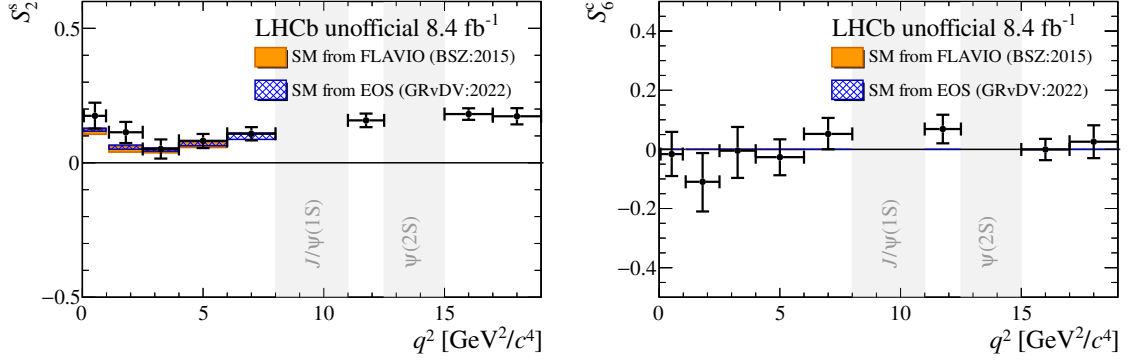


Figure 6.48: Comparison of the final fit results of the default configuration (Non-negligible lepton masses, Fixed CP -asymmetries) in the S_i -basis and two different SM predictions of the angular observables.

6.7.4 Configuration (v) - Half-sized q^2 bins

In this fit configuration, the finer q^2 -binning scheme defined in Tab. 6.1 is chosen to achieve a higher q^2 resolution of the q^2 dependence of the observables. This comes at the cost of a significantly increased statistical uncertainty. The plots displaying the results for the P-wave observables are shown in Fig. 6.49 and Fig. 6.50. The remaining results are shown in Fig. B.67 to B.70 as well as Tab. B.23 to B.25. The same trends as demonstrated in Sec. 6.7.1 are reproduced. The angular observable S_5 is measured to be consistently larger than the SM prediction in the q^2 region between 1.1 and 8.0 GeV^2/c^4 . A_{FB} and S_7 are consistently smaller than the SM prediction in the same q^2 range. Although the local significances of these tensions are smaller due to the larger statistical uncertainty, the tensions prove to be consistent over a larger number of q^2 bins.

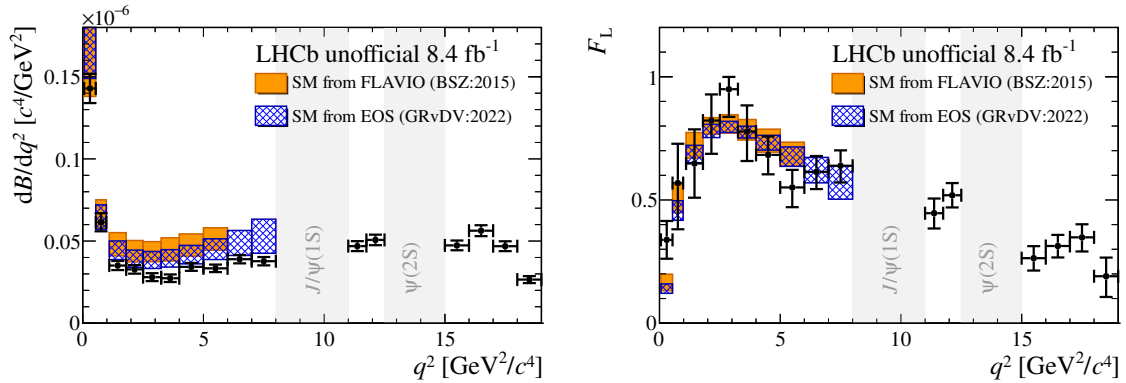


Figure 6.49: Comparison of the final fit result of the half-sized q^2 bin configuration (v) and two different SM predictions of the differential branching fraction dB/dq^2 and F_L .

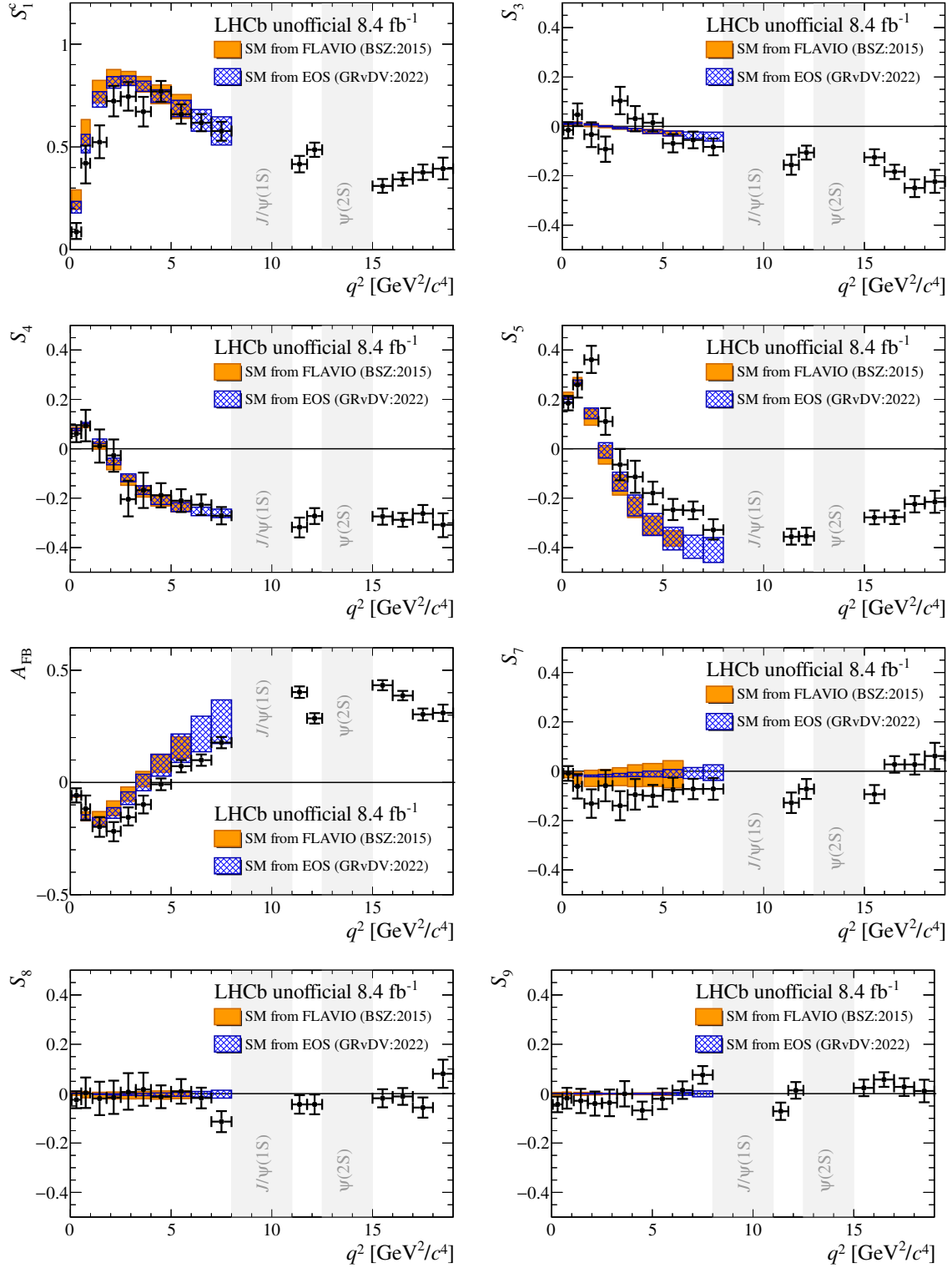


Figure 6.50: Comparison of the final fit results of the half-sized q^2 bin configuration (v) in the S_i -basis and two different SM predictions of the angular observables S_1^c , S_3 - S_5 , A_{FB} and S_7 - S_9 .

6.7.5 Comparison to LHCb and CMS measurements

As discussed in Sec. 3, the most recent measurements of the angular observables of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ have been published by the LHCb and CMS collaborations [24, 72]. Both analyses fit the angular observables in the massless lepton approximation for the complete q^2 range. While the LHCb measurement quotes the results for the S_i as well as the optimized P_i basis, the CMS measurement only provides the results in the P_i basis. Figure 6.51 displays the results for the observables S_5 , A_{FB} , P_2 , and P'_5 of fit configurations (i) and (ii) discussed in Sec. 6.7.1 as well as the results from Refs. [24] and [72]. The measurement of the angular observables presented in this thesis displays excellent agreement with the LHCb measurement using 4.7 fb^{-1} of integrated luminosity. A pseudo-experiment study accounting for the overlap of the data samples between Ref. [24] and this analysis shows that any differences between the angular observables in both analyses are consistent with the expected statistical fluctuations. The precision of the angular analysis presented in this thesis is significantly larger compared to Ref. [24] due to the larger data sample and the improvements in the analysis strategy discussed in Sec. 6.1.

The analysis conducted by the CMS collaboration chooses a different binning scheme compared to this analysis, displayed in Tab. 6.1. Nevertheless, the visual comparison of the results of P_2 and P'_5 in Fig. 6.51 also shows excellent agreement. In particular, the patterns of the deviations from the SM predictions are found to be consistent.

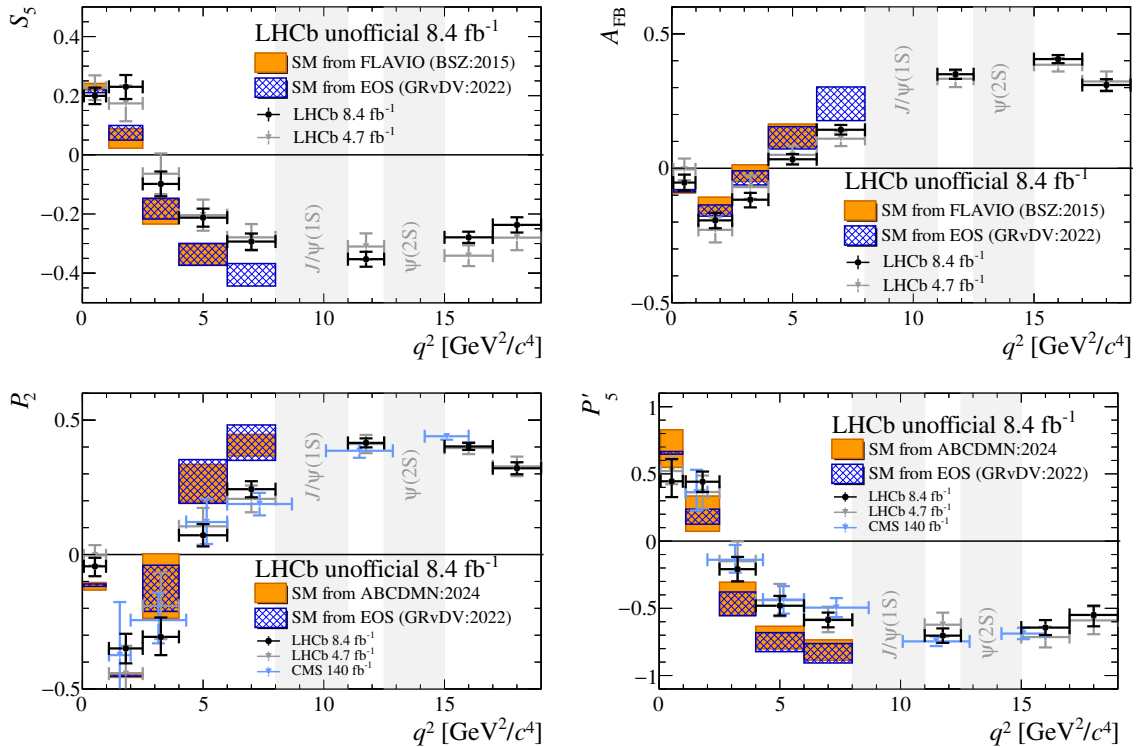


Figure 6.51: Comparison of the results from the configurations (i) and (ii) to the measurement of the angular observables S_5 , A_{FB} , P'_5 , and P_2 from Ref. [24] and Ref. [72].

6.7.6 Inference of Wilson Coefficients

Flavor physics theory frameworks like EOS and Flavio can calculate the expected values of the angular observables given a set of hadronic form factors and values of the underlying Wilson coefficients. Using the experimental results discussed in Sec. 6.7, it is also possible to infer the corresponding values and uncertainties of the Wilson coefficients that describe the data set best. This allows condensing the large number of observables in bins of q^2 into a small set of parameters and their uncertainties. The determined values of the WC can be compared to the SM expectation, and a p-value can be assigned to accept or reject the standard model hypothesis.

However, the determination of the Wilson coefficients depends on the underlying hadronic form factors and is therefore model-dependent. The WC and the corresponding p-values are determined for two sets of hadronic form factors from Ref. [50] using Flavio and Ref. [53] using EOS. Both theory frameworks use different parameterizations of the non-local form factors as described in Sec. 2. Furthermore, different statistical methods are available to determine the WC. Flavio employs the FastLikelihood approach, where a χ^2 between the theory predictions and experimental results is constructed using a combined covariance matrix of the theory and experimental uncertainties. This explicitly assumes the experimental and theoretical uncertainties to be Gaussian. Also, it assumes the dependence of the theory uncertainties on the NP part of the WC to be negligible. This approach was first described in Ref. [93]. EOS performs a full Bayesian marginalization of the nuisance parameters using Markov Chain Monte Carlo sampling methods [58].

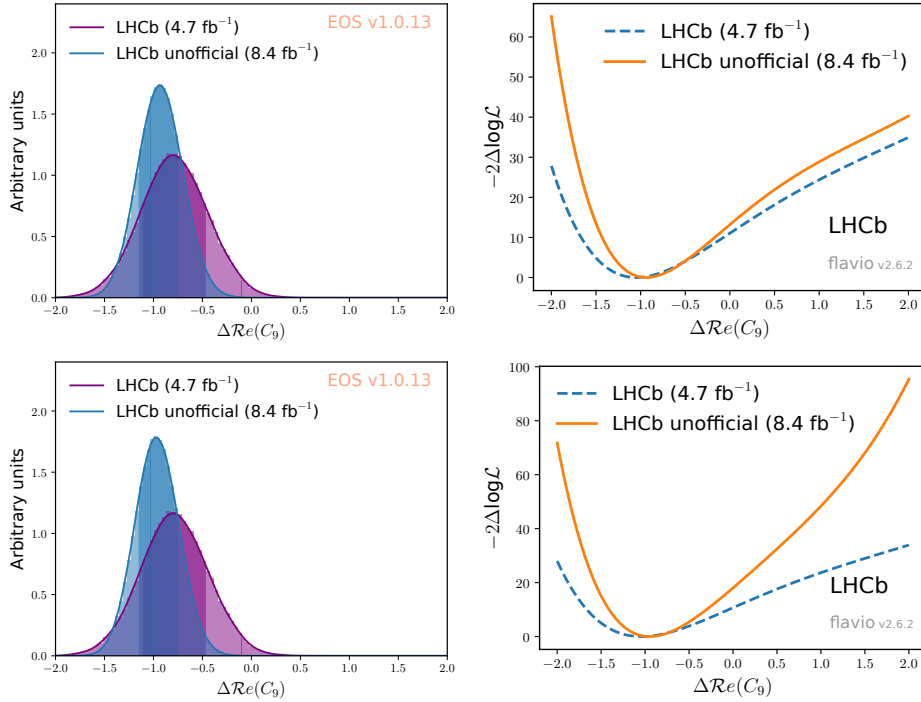


Figure 6.52: One dimensional determination of the Wilson coefficient $\Delta\mathcal{R}e(C_9)$ with EOS and Flavio. Both approaches prefer a clear negative shift of $\mathcal{R}e(C_9)$. The coefficients are extracted twice; first using the results of the angular observables only (top) and secondly, adding the branching fraction to the set of constraints from the angular observables (bottom).

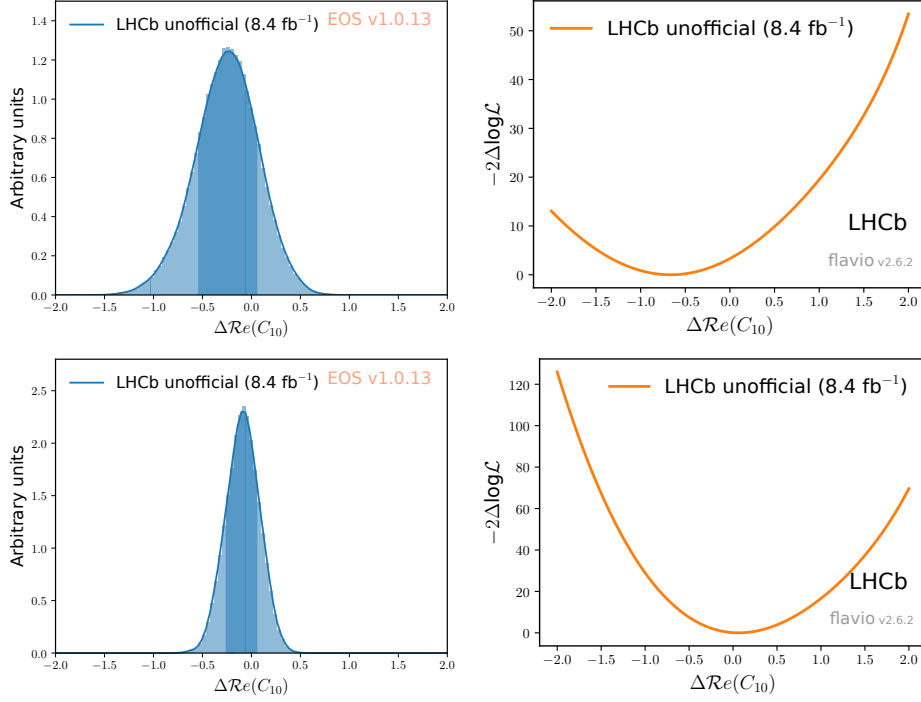


Figure 6.53: One dimensional determination of the Wilson coefficient $\Delta\mathcal{R}e(C_{10})$ with EOS and Flavio. The results from both approaches are in good agreement with the SM expectation. The coefficients are extracted twice: first using the results of the angular observables only (top) and secondly, adding the branching fraction to the set of constraints from the angular observables (bottom).

The measurement of the CP -averaged angular observables gives access to the real parts of the Wilson coefficients C_9 and C_{10} . In a first step, the value of $\Delta\mathcal{R}e(C_9)$ is determined, while the value of $\Delta\mathcal{R}e(C_{10})$ is kept fixed at the SM value. Figure 6.52 shows the inferred posterior distribution of $\Delta\mathcal{R}e(C_9)$ from EOS and the delta log-likelihood shape of $\Delta\mathcal{R}e(C_9)$ from Flavio. The determination of the Wilson coefficients is performed twice; first using the angular observables only and second, adding the result of the branching fraction to the set of constraints. The resulting values and uncertainties are listed in Tab. 6.26. The branching fraction leads to a slight increase in sensitivity compared to the determination using angular observables only. A clear negative shift of $\Delta\mathcal{R}e(C_9) \approx -1$ is preferred by the data. Using the results of the angular observables, the significance of this shift from the SM is evaluated to be 3.65σ and 3.75σ from Flavio and EOS, respectively. Adding the branching fraction to the constraints, these significances of the tension with the SM increase to 4.11σ and 4.20σ , respectively. The central values of $\Delta\mathcal{R}e(C_9)$ displayed in Tab. 6.26 with and without constraint from the branching fraction are in excellent agreement, indicating that the shifts of the branching fraction and the angular observables with respect to the SM predictions are consistent.

A similar one-dimensional determination of the Wilson coefficient $\Delta\mathcal{R}e(C_{10})$ is performed, while the value of C_9 is fixed to its SM value. The results are displayed in Fig. 6.53 and Tab. 6.26. $\Delta\mathcal{R}e(C_{10})$ is found to be consistent with the SM expectation. Adding the branching fraction to the set of constraints improves the sensitivity to $\Delta\mathcal{R}e(C_{10})$ significantly.

Additionally to the one-dimensional determination of $\Delta\mathcal{R}e\mathcal{C}_9$ and $\Delta\mathcal{R}e(\mathcal{C}_{10})$, a two-dimensional determination of the real parts of $\Delta\mathcal{R}e(\mathcal{C}_9)$ and $\Delta\mathcal{R}e(\mathcal{C}_{10})$ is performed. The likelihood contours (Flavio) and credibility intervals (EOS) are shown in Fig. 6.54. The results are consistent with the one-dimensional analysis and display a clear preference for a shift in $\Delta\mathcal{R}e(\mathcal{C}_9)$ to negative values, while $\Delta\mathcal{R}e(\mathcal{C}_{10})$ remains consistent with the SM expectation.

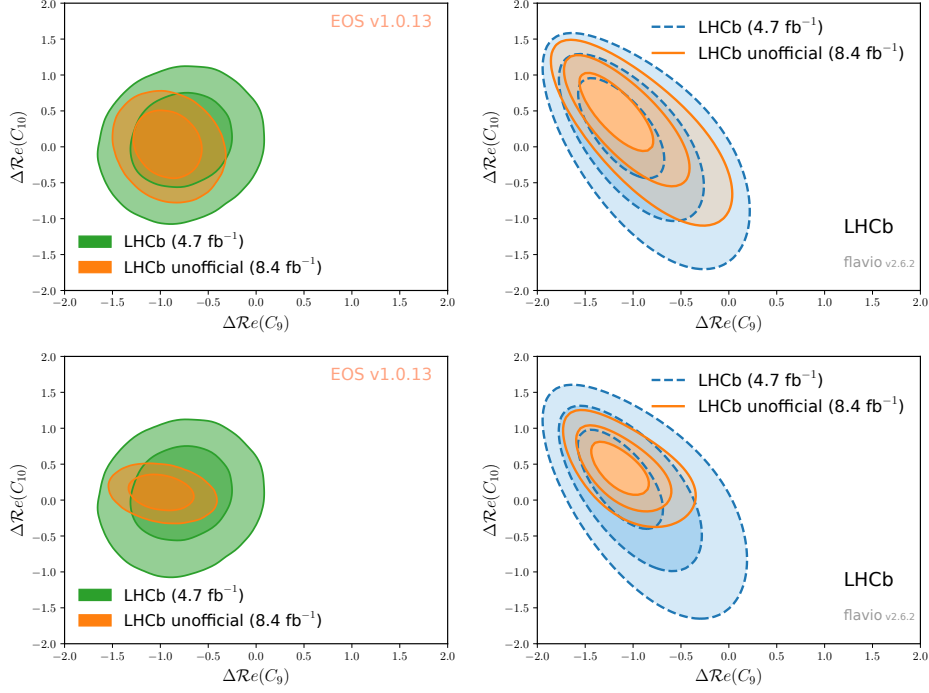


Figure 6.54: Likelihood contours (Flavio) and credibility intervals (EOS) of the Wilson coefficients $\Delta\mathcal{R}e(\mathcal{C}_9)$ and $\Delta\mathcal{R}e(\mathcal{C}_{10})$. The coefficients have been extracted twice: first using the results of the angular observables only (top) and second, adding the branching fraction to the set of constraints from the angular observables (bottom).

The fit of the CP -asymmetries yields sensitivity to the imaginary parts of $\mathcal{C}_9$ and $\mathcal{C}_{10}$ as significant CP -violation in the signal decay can only arise from non-zero imaginary parts of the Wilson coefficients. The imaginary part of a WC is determined together with its corresponding real part in two dimensions, effectively constituting a comprehensive determination of the Wilson coefficients shifts $\Delta\mathcal{C}_9$ or $\Delta\mathcal{C}_{10}$, respectively. As this analysis extracts the CP -symmetries and CP -asymmetries simultaneously, the correlations between both sets of observables are taken into account, which was not possible with previous extractions of the CP -asymmetries. The likelihood contours (Flavio) and credibility intervals (EOS) for the inferred coefficients are displayed in Fig. 6.55. The central values can be found in Tab. 6.26. The imaginary parts $\Delta\mathcal{I}m(\mathcal{C}_9)$ and $\Delta\mathcal{I}m(\mathcal{C}_{10})$ are found to be in good agreement with the SM.

In the SM the right-handed Wilson coefficients $\Delta\mathcal{R}e(\mathcal{C}'_9)$ and $\Delta\mathcal{R}e(\mathcal{C}'_{10})$ are expected to be zero as weak interactions can only occur between particles of left-handed chirality. Any deviation of these Wilson coefficients from zero would be a clear sign of new physics. The Wilson coefficients $\Delta\mathcal{R}e(\mathcal{C}'_9)$ and $\Delta\mathcal{R}e(\mathcal{C}'_{10})$ are extracted simultaneously to their left-handed counterparts in two dimensions. The resulting likelihood contours and credibility intervals contours are shown in Fig. 6.56. The results for the

right-handed coefficients are found to be in good agreement with the SM.

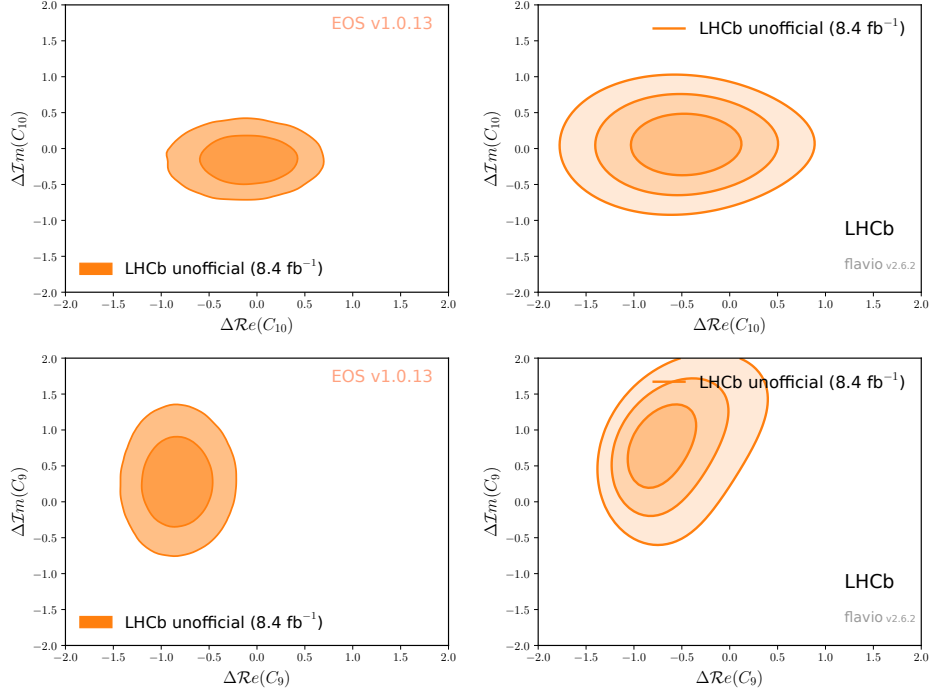


Figure 6.55: Likelihood contours (Flavio) and credibility intervals (EOS) of the Wilson coefficient pairs $\Delta\mathcal{R}e(C_9), \Delta\mathcal{I}m(C_9)$ and $\Delta\mathcal{R}e(C_{10}), \Delta\mathcal{I}m(C_{10})$. $\Delta\mathcal{I}m(C_9)$ and $\Delta\mathcal{I}m(C_{10})$ are found to be in good agreement with the SM expectations.

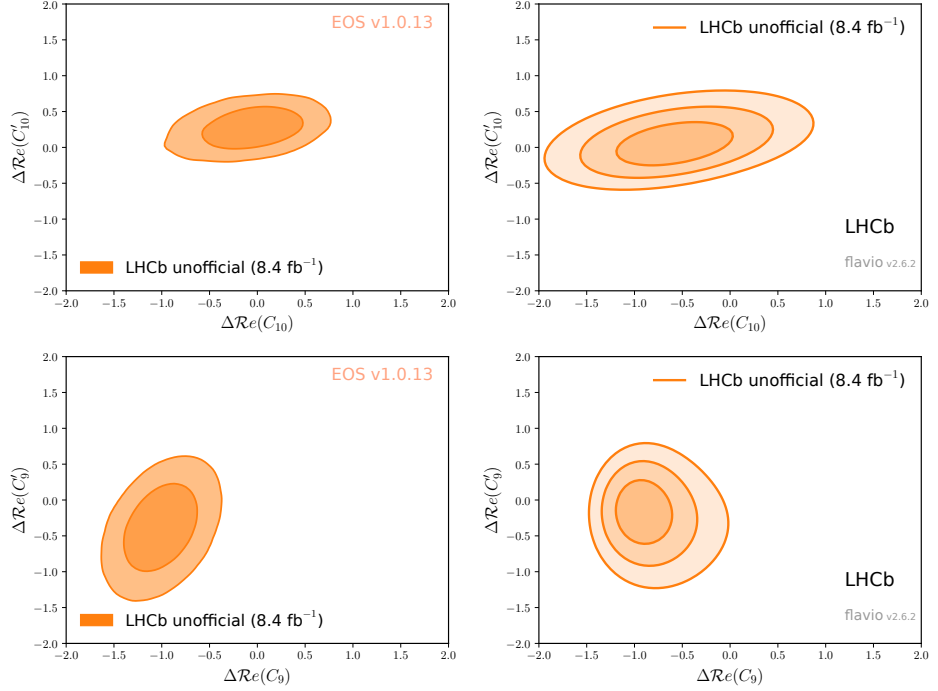


Figure 6.56: Likelihood contours (Flavio) and credibility intervals (EOS) of the Wilson coefficient pairs $\Delta\mathcal{R}e(C_9), \Delta\mathcal{R}e(C'_9)$ and $\Delta\mathcal{R}e(C_{10}), \Delta\mathcal{R}e(C'_{10})$. $\Delta\mathcal{R}e(C'_9)$ and $\Delta\mathcal{R}e(C'_{10})$ are found to be in good agreement with the SM expectations.

The best fit values of the Wilson Coefficients for the combinations discussed above are listed in Tab. 6.26, along with the significance of the deviation from the SM hypothesis in terms of the equivalent to standard deviations σ . The Flavio software returns the shape of the negative log likelihood as a function of the scanned Wilson coefficient profiled over all nuisance parameters. The p-value of the SM hypothesis is evaluated using Wilks' theorem [150]. It states that the difference between the negative log likelihoods of the best fit and SM point $-(\ln(\mathcal{L}_{\text{SM}}) - \ln(\mathcal{L}_{\text{best fit}}))$ is χ^2 -distributed with the number of degrees of freedom equal to the number of Wilson coefficients scanned. The p-value is transformed into the significance in σ using the two-sided cumulative distribution function of the normal distribution. The EOS package, on the other hand, allows sampling from the posterior distribution of the Wilson Coefficients. The p-value is determined by performing a two-sided integral over the posterior distribution between the best-fit value and the SM point.

Overall, the determination of the Wilson Coefficients with two different software frameworks shows that $\mathcal{R}e(C_9)$ shows a clear preference for a negative shift, while other Wilson Coefficients are in good agreement with the SM. As discussed in Sec. 3, this is in accordance with the results of other angular and branching fraction analyses of $b \rightarrow s \ell^+ \ell^-$ transitions at LHCb and other experiments.

An additional cross-check has been performed where the Wilson coefficient fit ($\mathcal{R}e(C_9)$ w. BF) is repeated using Flavio for each q^2 bin below $8 \text{ GeV}^2/c^4$ separately to check the consistency across the q^2 region. As discussed in Sec. 2, effects from non-local contributions can be q^2 dependent, while a heavy NP contribution would produce a consistent q^2 independent shift of the WC because it can only contribute as virtual particles. A plot of $\Delta \mathcal{R}e(C_9)$ in bins of q^2 can be found in Fig. 6.57. The shifts $\Delta \mathcal{R}e(C_9)$ are consistent across the q^2 range, which would be expected if a heavy NP effect induced the shift. The value of $\Delta \mathcal{R}e(C_9)$ for the lowest q^2 bin in $0.06 < q^2 < 0.98$ prefers a value closer to 0, but also has a significantly larger uncertainty compared to other bins.

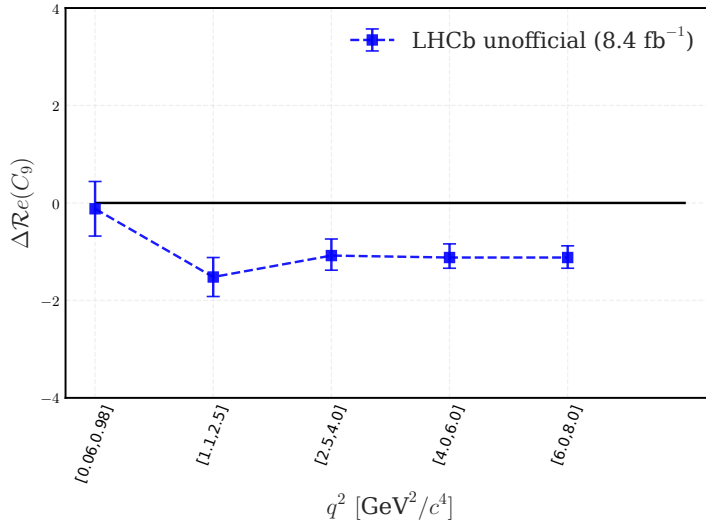


Figure 6.57: Separate WC fits of $\Delta \mathcal{R}e(C_9)$ for each q^2 bin below $8 \text{ GeV}^2/c^4$ in fit setup (i) using the Flavio framework. The used constraints include the angular observables and the branching fraction.

Table 6.26: Wilson coefficient results for all New Physics scenarios, including the significance of their deviation from the SM. Uncertainties are only shown for the one-dimensional determinations of the WC due to the slightly non-Gaussian shape of the two-dimensional intervals.

NP scenario	Flavio						significance
	$\Delta\mathcal{R}e\mathcal{C}_9$	$\Delta\mathcal{R}e\mathcal{C}_{10}$	$\Delta\mathcal{I}m\mathcal{C}_9$	$\Delta\mathcal{I}m\mathcal{C}_{10}$	$\Delta\mathcal{R}e\mathcal{C}'_9$	$\Delta\mathcal{R}e\mathcal{C}'_{10}$	
$\Delta\mathcal{R}e\mathcal{C}_9$	$-0.94^{+0.20}_{-0.17}$						3.65σ
$\Delta\mathcal{R}e\mathcal{C}_9$ w. BF	$-0.92^{+0.18}_{-0.16}$						4.11σ
$\Delta\mathcal{R}e\mathcal{C}_{10}$		$-0.62^{+0.37}_{-0.37}$					1.68σ
$\Delta\mathcal{R}e\mathcal{C}_{10}$ w. BF		$-0.05^{+0.23}_{-0.20}$					0.25σ
$\Delta\mathcal{R}e\mathcal{C}_9, \Delta\mathcal{R}e\mathcal{C}_{10}$	-1.26	0.58					3.49σ
$\Delta\mathcal{R}e\mathcal{C}_9, \Delta\mathcal{R}e\mathcal{C}_{10}$ w. BF	-1.17	0.48					4.18σ
$\Delta\mathcal{R}e\mathcal{C}_9, \Delta\mathcal{I}m\mathcal{C}_9$	-0.79		0.76				3.39σ
$\Delta\mathcal{R}e\mathcal{C}_{10}, \Delta\mathcal{I}m\mathcal{C}_{10}$		-0.37		0.06			0.48σ
$\Delta\mathcal{R}e\mathcal{C}_9, \Delta\mathcal{R}e\mathcal{C}'_9$	-0.93				-0.10		3.12σ
$\Delta\mathcal{R}e\mathcal{C}_{10}, \Delta\mathcal{R}e\mathcal{C}'_{10}$		0.14				0.19	0.52σ
NP scenario	EOS						significance
	$\Delta\mathcal{R}e\mathcal{C}_9$	$\Delta\mathcal{R}e\mathcal{C}_{10}$	$\Delta\mathcal{I}m\mathcal{C}_9$	$\Delta\mathcal{I}m\mathcal{C}_{10}$	$\Delta\mathcal{R}e\mathcal{C}'_9$	$\Delta\mathcal{R}e\mathcal{C}'_{10}$	
$\Delta\mathcal{R}e\mathcal{C}_9$	$-0.93^{+0.23}_{-0.23}$						3.75σ
$\Delta\mathcal{R}e\mathcal{C}_9$ w. BF	$-0.97^{+0.23}_{-0.22}$						4.20σ
$\Delta\mathcal{R}e\mathcal{C}_{10}$		$-0.26^{+0.32}_{-0.32}$					0.78σ
$\Delta\mathcal{R}e\mathcal{C}_{10}$ w. BF		$-0.09^{+0.17}_{-0.18}$					0.51σ
$\Delta\mathcal{R}e\mathcal{C}_9, \Delta\mathcal{R}e\mathcal{C}_{10}$	-0.91	0.04					3.56σ
$\Delta\mathcal{R}e\mathcal{C}_9, \Delta\mathcal{R}e\mathcal{C}_{10}$ w. BF	-1.01	0.12					4.03σ
$\Delta\mathcal{R}e\mathcal{C}_9, \Delta\mathcal{I}m\mathcal{C}_9$	-0.84		0.32				3.03σ
$\Delta\mathcal{R}e\mathcal{C}_{10}, \Delta\mathcal{I}m\mathcal{C}_{10}$		-0.06		-0.16			0.31σ
$\Delta\mathcal{R}e\mathcal{C}_9, \Delta\mathcal{R}e\mathcal{C}'_9$	-1.02				-0.35		4.04σ
$\Delta\mathcal{R}e\mathcal{C}_{10}, \Delta\mathcal{R}e\mathcal{C}'_{10}$		-0.02				0.26	1.09σ

6.7.7 Publication of the results

The analysis has been conducted by two independent groups at LHCb simultaneously. Extensive cross-checks have been performed to ensure the compatibility of the results. Both analyses rely on identical selection criteria described in Sec. 6.3 and on the same fit model described in Sec. 6.1. Small differences between the results arise from different control modes that have been used for data-simulation agreement corrections and a different parameterization of the veto against $B^+ \rightarrow K^+ \mu^+ \mu^-$ background in the combinatorial background model. The systematic uncertainties and coverage corrections evaluated by both groups have been combined. The set of angular fit results from one group was chosen at random through a coin toss, corrected with the combined coverage corrections, and will be published alongside the combined systematic uncertainties in Ref. [151]. The fit results, systematic uncertainties, and coverage corrections presented here have been obtained in the course of this thesis and correspond to the results of one of these groups. Therefore, the results presented in this thesis are not necessarily identical to the results in Ref. [151].

Chapter 7

Conclusions and outlook

This thesis presents the legacy angular analysis of the rare B -decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ using the LHCb data set collected in the years 2011-2012 and 2016-2018. In the course of this thesis, a five-dimensional angular maximum likelihood fit has been developed that parametrizes the distributions of the decay angles $\cos \theta_\ell$, $\cos \theta_K$, and ϕ as well as the invariant masses $m_{K\pi}$ and $m_{K\pi\mu\mu}$ to determine the complete set of angular observables describing both the P- and S-wave contributions to the signal decay in bins of q^2 . The angular observables are extracted in the S_i as well as the optimized P_i basis, where the local form factor uncertainties cancel to leading order. This fit directly includes the $m_{K\pi}$ dependence of the angular decay rate into the PDF for the first time. The treatment of the $m_{K\pi}$ dependence allows a more sophisticated description of the interference between P- and S-wave of the decay, leading to additional angular observables that are also measured in this analysis for the first time. The acceptance effect is parameterized using five-dimensional Legendre polynomials by applying the method of moments to a sample of fully selected simulated signal events. Separation power between signal and combinatorial background contributions is provided by a simultaneous fit to the $m_{K\pi\mu\mu}$ spectrum. Furthermore, a simultaneous fit of the CP -symmetries and the CP -asymmetries is implemented that provides access to the full set of CP -asymmetric angular observables and correlations. The fit is able to take lepton mass effects into account, which can have a significant impact on the angular observables in the low q^2 regime. The additional angular observables arising from the parameterization of the lepton mass effects (S_1^c , S_2^s , S_6^c , etc.) are measured for the first time. Lastly, an extended term is added to the likelihood, which allows a model-independent fit of the total CP -asymmetry A_{CP} as well as the branching fraction by exploiting information from the angular PDF and the acceptance parameterization. The selection reoptimized by Thomas Oeser compared to Ref. [24] leads to improved selection efficiencies and signal purity. This allows the $m_{K\pi}$ selection window to be widened significantly to facilitate a more precise determination of the S-wave contribution.

To extract as much information from the data as possible, the analysis includes five different fit configurations that have been summarized in Sec. 6.1.6. These configurations include fits of a partially massive lepton model to account for small residual lepton mass effects on the angular observable F_L in the q^2 region from 1.1 to 4.0 GeV²/c⁴. Additionally, a dedicated fit configuration is designed to fit the CP -symmetries and CP -asymmetries simultaneously. To maximize the sensitivity to scalar

and pseudo-scalar NP contributions, a fit configuration that considers leptons as fully massive in the P-wave decay rate also above $q^2 = 0.98 \text{ GeV}^2/c^4$ is employed. Lastly, a configuration with a finer q^2 binning scheme defined in Tab. 6.1 is used to measure the angular observables with an improved resolution of the q^2 shape.

The fit procedure and acceptance parameterization are validated extensively on simulation, data from the tree-level decays $B^0 \rightarrow K^{*0} J/\psi$ and $B^0 \rightarrow K^{*0} \psi(2S)$, as well as pseudo-experiments. A classical Neyman construction is employed to correct statistical bias and undercoverage that are found to be prevalent in fits to pseudo-experiments.

A broad range of systematic effects on the angular observables and branching fraction is studied. Amongst others, these studies include effects from residual peaking backgrounds, the impact of data simulation agreement corrections on the acceptance parameterization, the modeling of the invariant mass distributions of signal and background candidates, angular resolution effects and the limited size of available simulation samples.

The numerical results for the partially massive default fit configurations in the S_i and P_i basis, including the full uncertainties, are listed in Tabs. 6.22-6.25, while the results for the remaining fit configurations are displayed in Tabs. B.17-B.25. The results for the well-known angular observables P'_5 and A_{FB} as well as the branching fraction and A_{CP} are again summarized in Tab. 7.1.

The angular observables P'_5 and A_{FB} display clear tensions with the SM prediction across multiple q^2 bins. The experimentally observed value of P'_5 is found to be consistently larger than the SM predictions in the intermediate q^2 region from 1.1 to $8.0 \text{ GeV}^2/c^4$. The local tension with respect to the SM predictions calculated using the Flavio (EOS) framework, corresponds to 2.8 , 2.6 and 3.1σ (2.7 , 2.6 and 2.7σ) in the q^2 bins $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$, $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$ and $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$, respectively. Similarly, the angular observable A_{FB} is found to be in tension with the SM prediction in the same intermediate q^2 range. The discrepancy in the $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$ and $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$ bins is found to be at the level of 2.5 and 1.9σ (2.2 and 1.7σ) with respect to the Flavio (EOS) SM predictions. Also, the measured value of the differential branching fraction is found to be consistently lower than expected in the SM. The tension corresponds to 2.0σ (1.7σ) with respect to the SM prediction in the wide q^2 bin $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$. For the total CP -asymmetry A_{CP} and the CP -asymmetric angular observables in general, no significant deviation from the SM prediction is found.

The results are found to be in excellent agreement with the most recent measurements of the angular observables in $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays performed by LHCb [24] and CMS [72]. The measurement presented here constitutes the most precise determination of the angular observables of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ to date.

The results of the angular observables are used to determine the best-fit values of the Wilson coefficients for different New Physics scenarios. For the WC $Re(C_9)$, the data clearly indicate a negative shift from the SM expectation. When determining $\Delta Re(C_9)$ in one dimension, only considering constraints from the angular observables, the shifts $\Delta Re(C_9) = -0.94^{+0.20}_{-0.17}$ and $\Delta Re(C_9) = -0.93^{+0.23}_{-0.23}$ are obtained using Flavio and EOS, respectively. These values correspond to significances of 3.65σ and 3.75σ with respect to the SM expectation. When additionally including the measured values of the branching fraction, the best fit values are $\Delta Re(C_9) = -0.92^{+0.18}_{-0.16}$ using Flavio

Table 7.1: Summary of the results for the observables P'_5 , A_{FB} , A_{CP} and $d\mathcal{B}/dq^2$. The displayed uncertainties are the statistical and the systematic uncertainty.

	q^2 bin [GeV^2/c^4]		
	0.06–0.98	1.1–2.5	2.5–4.0
P'_5	$0.445^{+0.163+0.026}_{-0.115-0.026}$	$0.442^{+0.075+0.005}_{-0.075-0.005}$	$-0.209^{+0.090+0.016}_{-0.090-0.016}$
A_{FB}	$-0.053^{+0.029+0.005}_{-0.029-0.005}$	$-0.194^{+0.027+0.007}_{-0.028-0.007}$	$-0.117^{+0.025+0.004}_{-0.028-0.004}$
A_{CP}	$0.071^{+0.035+0.005}_{-0.035-0.005}$	$-0.085^{+0.044+0.005}_{-0.044-0.005}$	$-0.020^{+0.047+0.006}_{-0.047-0.006}$
$d\mathcal{B}/dq^2$ [10^{-7}]	$1.041 \pm 0.037 \pm 0.049$	$0.340 \pm 0.015 \pm 0.017$	$0.274 \pm 0.013 \pm 0.015$
	q^2 bin [GeV^2/c^4]		
	4.0–6.0	6.0–8.0	15.0–17.0
P'_5	$-0.481^{+0.073+0.006}_{-0.073-0.006}$	$-0.587^{+0.055+0.004}_{-0.055-0.004}$	$-0.643^{+0.056+0.005}_{-0.056-0.005}$
A_{FB}	$0.034^{+0.019+0.003}_{-0.019-0.003}$	$0.143^{+0.018+0.002}_{-0.018-0.002}$	$0.406^{+0.015+0.003}_{-0.015-0.003}$
A_{CP}	$0.012^{+0.035+0.003}_{-0.035-0.003}$	$0.006^{+0.032+0.004}_{-0.032-0.004}$	$0.015^{+0.025+0.002}_{-0.025-0.002}$
$d\mathcal{B}/dq^2$ [10^{-7}]	$0.337 \pm 0.012 \pm 0.016$	$0.378 \pm 0.012 \pm 0.018$	$0.515 \pm 0.013 \pm 0.024$
	q^2 bin [GeV^2/c^4]		
	17.0–19.0	11.0–12.5	1.1–6.0
P'_5	$-0.549^{+0.069+0.006}_{-0.084-0.006}$	$-0.703^{+0.053+0.007}_{-0.053-0.007}$	$-0.100^{+0.045+0.006}_{-0.045-0.006}$
A_{FB}	$0.310^{+0.021+0.003}_{-0.021-0.003}$	$0.350^{+0.017+0.004}_{-0.017-0.004}$	$-0.086^{+0.014+0.004}_{-0.014-0.004}$
A_{CP}	$0.053^{+0.033+0.004}_{-0.033-0.004}$	$-0.007^{+0.030+0.003}_{-0.030-0.003}$	$-0.026^{+0.026+0.004}_{-0.026-0.004}$
$d\mathcal{B}/dq^2$ [10^{-7}]	$0.366 \pm 0.013 \pm 0.019$	$0.479 \pm 0.015 \pm 0.023$	$0.319 \pm 0.008 \pm 0.015$
	q^2 bin [GeV^2/c^4]		
	15.0–19.0		
P'_5	$-0.603^{+0.046+0.005}_{-0.046-0.005}$		
A_{FB}	$0.364^{+0.012+0.002}_{-0.012-0.002}$		
A_{CP}	$0.033^{+0.020+0.003}_{-0.020-0.003}$		
$d\mathcal{B}/dq^2$ [10^{-7}]	$0.444 \pm 0.009 \pm 0.021$		

and $\Delta\text{Re}(C_9) = -0.97^{+0.23}_{-0.22}$ using EOS. These values are in tension with the SM expectation at the level of 4.11σ and 4.20σ , respectively.

Overall, the results of this angular analysis are clearly in tension with the predictions from the SM, with high global significances of up to the level of 4σ , depending on the observables included in the determination of the Wilson Coefficients. It is exceedingly unlikely that these deviations from the SM are due to statistical fluctuations. The observed values in P'_5 are found to be systematically larger compared to the available SM predictions in the q^2 region from 1.1–8.0 GeV^2/c^4 , not only in single bins. As discussed in Sec. 3, these results are consistent with results of $b \rightarrow s\ell^+\ell^-$ studies in different decay modes, not only obtained by LHCb but also by other experiments that are able to study these kinds of rare decays [72]. However, the indirect nature of this type of precision measurement gives no clear indication of the origin of the observed tension. It is particularly difficult to definitely rule out non-local SM effects like the charm loops discussed in Sec. 2, as a source of at least part of the observed

tension. There is an ongoing discussion in the theory community about the size of these non-local contributions. Thus, it is not possible at this point to clearly identify these tensions as a sign of New Physics.

From the experimental side, the future prospects of this analysis are very promising. During the current data-taking period, the LHCb experiment collected a data sample corresponding to an integrated luminosity of 9.56 fb^{-1} at a center of mass energy of 13.6 TeV during the year 2024 alone, which is comparable to the size of the data set analyzed in this thesis. During 2025, a data sample corresponding to another 10 fb^{-1} at the same center of mass energy has been collected up until this point. Until the end of Run 4, the LHCb experiment is expected to collect a total of 50 fb^{-1} of integrated luminosity, and until the end of the lifetime of LHCb Upgrade II, the total collected integrated luminosity is expected to exceed 300 fb^{-1} , at which point the statistical uncertainties are anticipated to be comparable to the systematic uncertainties. However, before this goal is reached, the LHCb experiment will have to undergo major upgrades that will completely transform the detector.

Apart from LHCb, other collider experiments also aim to significantly improve their precision on the relevant $b \rightarrow s \ell^+ \ell^-$ observables in the future. The Belle 2 B-factory at the SuperKEKB collider aims to collect 50 ab^{-1} at the $\Upsilon(4S)$ resonance until the end of its run time. A determination of the angular observables of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ at an electron collider with high precision would constitute an interesting cross-check in a non-proton-proton environment.

To draw decisive conclusions on the nature of the observed tensions and make statements on the validity of specific New Physics models, a significant improvement in the precision of the theoretical predictions is necessary. The evolution of these uncertainties is hard to predict, as significant advances in the theoretical understanding of the local and non-local hadronic form factors involving complex QCD calculations would be required.

In conclusion, the topic of flavor anomalies is more relevant than ever, and a concerted effort of the experimental and theory community is ongoing to systematically solve the open questions posed by the results of this and other $b \rightarrow s \ell^+ \ell^-$ analyses.

Appendix A

Inclusive Detached Dilepton Trigger

A.1 Exclusive trigger comparison

Table A.1: Exclusive Hlt2 trigger decisions used for the comparison to the IDDL lines on rare decays (left) and tree-level decays (right).

Exclusive trigger decision RD	Exclusive trigger decision B2CC
Hlt2RD_B0ToKpPimMuMuDecision	Hlt2B2CC_BdToJpsiKstar_JpsiToMuMu_DetachedDecision
Hlt2RD_B0ToKpPimEEDecision	Hlt2B2CC_BdToJpsiKstar_JpsiToEE_DetachedDecision
Hlt2RD_BuToKpEEDecision	Hlt2B2CC_BsToJpsiPhi_DetachedDecision
Hlt2RD_BuToKpEMuDecision	Hlt2B2CC_BsToJpsiPhi_JpsiToEE_DetachedDecision
Hlt2RD_BToMuMuDecision	Hlt2B2CC_BuToJpsiKplus_JpsiToMuMu_DetachedDecision
Hlt2RD_BToMuEDecision	Hlt2B2CC_BuToJpsiKplus_JpsiToEE_DetachedDecision
Hlt2RD_B0ToKpKmMuMuDecision	
Hlt2RD_B0ToKpKmEEDecision	
Hlt2RD_LbTopKMuMu_MVADecision	
Hlt2RD_LbTopKEE_MVADecision	

Table A.2: Loose offline selection requirements applied to the output of the IDDL, Topological, and exclusive trigger lines for tree-level B -decays.

Candidate	Requirement
B^0	Inv. mass $\in [5170.0, 5700.0]$ MeV/ c^2
B^0	End-vertex χ^2 per DOF < 16.0
B^0	$\chi_{\text{IP}}^2 < 30.0$
B^0	DIRA > 0.9995
B^0	Flight distance $\chi^2 > 20.0$
K^{*0}	Inv. mass $\in [745.9, 1095.0]$ MeV/ c^2
$\mu^+\mu^-$	$8.0 < q^2 < 11.0$ GeV $^2/c^4$
K, π	Minimum $\chi_{\text{IP}}^2 > 6.0$
K	DLL $_{\pi K} > -3.0$
π	DLL $_{\pi K} < 3.0$
π	DLL $_{\pi K} < -30.0$ ($B^+ \rightarrow \pi^+\mu^+\mu^-$)
μ^+, μ^-	Minimum $\chi_{\text{IP}}^2 > 6.0$
μ^+, μ^-	DLL $_{\pi\mu} > -3.0$
Tracks	Ghost probability < 0.5

Table A.3: Loose offline selection requirements applied to the output of the IDDL trigger lines for rare mode B -decays.

Candidate	Requirement
B^0	Inv. mass $\in [5170.0, 5700.0]$ MeV/ c^2
B^0	End-vertex χ^2 per DOF < 6.0
B^0	$\chi_{\text{IP}}^2 < 16.0$
B^0	DIRA > 0.9999
B^0	Flight distance $\chi^2 > 64.0$
K^{*0}	Inv. mass $\in [745.9, 1095.0]$ MeV/ c^2
$\mu^+\mu^-$	$8.0 < q^2 < 11.0$ GeV $^2/c^4$
K, π	Minimum $\chi_{\text{IP}}^2 > 9.0$
K	DLL $_{\pi K} > 0.0$
π	DLL $_{\pi K} < 0.0$
μ^+, μ^-	Minimum $\chi_{\text{IP}}^2 > 9.0$
μ^+, μ^-	DLL $_{\pi\mu} > 0.0$
Tracks	Ghost probability < 0.5

Appendix B

Angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

B.1 Selection

Table B.1: Stripping requirements applied to the signal candidates [45].

Candidate	Requirement
B^0	Measured mass $\in [4600.0, 7100.0]$ MeV/ c^2
B^0	Mass $\in [4700.0, 7000.0]$ MeV/ c^2
B^0	End-vertex χ^2 per DOF < 8.0
B^0	$\chi_{\text{IP}}^2 < 16.0$
B^0	DIRA > 0.9999
B^0	Flight distance $\chi^2 > 64.0$
B^0	Maximum daughter $\chi_{\text{IP}}^2 > 9.0$
K^{*0}	Measured mass $\in [0.0, 6200.0]$ MeV/ c^2
K^{*0}	Mass $\in [0.0, 6200.0]$ MeV/ c^2
K^{*0}	End-vertex χ^2 per DOF < 12.0
K^{*0}	DIRA > -0.9
K^{*0}	Flight distance $\chi^2 > 16.0$
K^{*0}	Minimum $\chi_{\text{IP}}^2 > 0.0$
K^{*0}	Maximum daughter $\chi_{\text{IP}}^2 > 6.0$
$\mu^+ \mu^-$	Measured mass < 7100.0 MeV
$\mu^+ \mu^-$	End-vertex χ^2 per DOF < 12.0
$\mu^+ \mu^-$	DIRA > -0.9
$\mu^+ \mu^-$	Flight distance $\chi^2 > 9.0$
$\mu^+ \mu^-$	Maximum χ_{IP}^2 of either μ^+ or $\mu^- > 6.0$
K, π	Minimum $\chi_{\text{IP}}^2 > 6.0$
K, π	Hits in the RICH subsystem
μ^+, μ^-	Minimum $\chi_{\text{IP}}^2 > 6.0$
μ^+, μ^-	$\text{DLL}_{\pi\mu} > -3.0$
μ^+, μ^-	Positive isMuon decision, based on hits in muon chambers
Tracks	Ghost probability < 0.5
Event	Maximum hits in SPD sub-detector < 600

BDT input variables	Hyper-parameter	Setting
B^0 proper decay time ct	nCuts	30
B^0 (transverse) momentum p (p_T)	nTrees	2000
B^0 DIRA	MinNodeSize	0.25%
B^0 decay vertex χ^2	MaxDepth	4
ProbNNpi(π)	BoostType	AdaBoost
ProbNNk(K)	AdaBoostBeta	0.05
ProbNNmu(μ^-), ProbNNmu(μ^+)	SeparationType	GiniIndex
Isolation of μ^+ , μ^- and π , K	PruneMethod	NoPruning

Table B.2: Input variables (left) and optimized hyper-parameters (right) used in training of the BDT against combinatorial background. The same set of variables and hyperparameters is used for all years [45].

B.2 Simulation mass fits

Table B.3: Table of the mass model parameters from the fit to rare mode simulation.

q^2 [GeV ² /c ⁴]	Parameter	Run 1	2016	2017	2018
[0.06, 0.98]	s_m	0.9625 ± 0.0032	0.9511 ± 0.0036	0.957 ± 0.005	0.9699 ± 0.0053
[1.1, 2.5]	s_m	0.9842 ± 0.0048	0.9719 ± 0.0057	0.9819 ± 0.0077	0.992 ± 0.008
[2.5, 4.0]	s_m	0.9835 ± 0.0049	1.0037 ± 0.0058	0.9786 ± 0.0077	0.9989 ± 0.0082
[4.0, 6.0]	s_m	0.9870 ± 0.0039	0.9822 ± 0.0045	0.9689 ± 0.0062	1.0004 ± 0.0065
[6.0, 8.0]	s_m	0.9938 ± 0.0034	0.993 ± 0.004	0.9957 ± 0.0055	1.0048 ± 0.0057
[15.0, 17.0]	s_m	1.0711 ± 0.0034	1.0659 ± 0.0041	1.0611 ± 0.0056	1.0724 ± 0.0057
[17.0, 19.0]	s_m	1.0835 ± 0.0046	1.0817 ± 0.0055	1.0979 ± 0.0079	1.0962 ± 0.0078
[11.0, 12.5]	s_m	1.0183 ± 0.0034	1.001 ± 0.004	1.0115 ± 0.0056	1.0207 ± 0.0057
[1.0, 6.0]	s_m	0.9854 ± 0.0026	0.986 ± 0.003	0.9752 ± 0.0041	0.9977 ± 0.0043
[15.0, 19.0]	s_m	1.0768 ± 0.0028	1.0720 ± 0.0033	1.0744 ± 0.0046	1.0812 ± 0.0046
[0.06, 0.98]	α_{CB}	1.757 ± 0.035	1.493 ± 0.039	1.562 ± 0.045	1.62 ± 0.05
[1.1, 2.5]	α_{CB}	1.744 ± 0.047	1.482 ± 0.052	1.558 ± 0.055	1.582 ± 0.071
[2.5, 4.0]	α_{CB}	1.644 ± 0.045	1.61 ± 0.06	1.611 ± 0.065	1.606 ± 0.092
[4.0, 6.0]	α_{CB}	1.733 ± 0.026	1.491 ± 0.038	1.603 ± 0.052	1.541 ± 0.055
[6.0, 8.0]	α_{CB}	1.737 ± 0.035	1.467 ± 0.035	1.598 ± 0.045	1.550 ± 0.043
[15.0, 17.0]	α_{CB}	1.762 ± 0.031	1.560 ± 0.038	1.563 ± 0.044	1.551 ± 0.047
[17.0, 19.0]	α_{CB}	1.723 ± 0.041	1.469 ± 0.048	1.716 ± 0.053	1.637 ± 0.038
[11.0, 12.5]	α_{CB}	1.664 ± 0.033	1.501 ± 0.033	1.573 ± 0.053	1.57 ± 0.05
[1.0, 6.0]	α_{CB}	1.715 ± 0.025	1.528 ± 0.024	1.592 ± 0.033	1.569 ± 0.036
[15.0, 19.0]	α_{CB}	1.760 ± 0.025	1.534 ± 0.035	1.613 ± 0.042	1.579 ± 0.043
[0.06, 0.98]	n_{CB}	1.387 ± 0.078	1.7 ± 0.1	1.78 ± 0.13	1.48 ± 0.12
[1.1, 2.5]	n_{CB}	1.12 ± 0.09	1.37 ± 0.11	1.30 ± 0.12	1.26 ± 0.15
[2.5, 4.0]	n_{CB}	1.160 ± 0.086	1.0 ± 0.1	1.14 ± 0.12	1.14 ± 0.17
[4.0, 6.0]	n_{CB}	0.936 ± 0.043	1.14 ± 0.07	1.053 ± 0.092	1.17 ± 0.11
[6.0, 8.0]	n_{CB}	0.873 ± 0.055	1.173 ± 0.067	1.093 ± 0.084	1.041 ± 0.077
[15.0, 17.0]	n_{CB}	0.887 ± 0.051	1.07 ± 0.07	1.20 ± 0.09	1.165 ± 0.094
[17.0, 19.0]	n_{CB}	1.25 ± 0.09	1.57 ± 0.13	1.28 ± 0.12	1.344 ± 0.085
[11.0, 12.5]	n_{CB}	0.985 ± 0.057	1.054 ± 0.058	1.037 ± 0.094	0.994 ± 0.084
[1.0, 6.0]	n_{CB}	1.038 ± 0.044	1.156 ± 0.045	1.141 ± 0.063	1.186 ± 0.071
[15.0, 19.0]	n_{CB}	0.976 ± 0.044	1.20 ± 0.07	1.22 ± 0.09	1.22 ± 0.09
[0.06, 0.98]	$\alpha_{CB,2}$	-2.192 ± 0.042	-2.16 ± 0.05	-2.303 ± 0.084	-2.282 ± 0.076
[1.1, 2.5]	$\alpha_{CB,2}$	-2.332 ± 0.058	-2.16 ± 0.07	-2.27 ± 0.12	-2.3 ± 0.1
[2.5, 4.0]	$\alpha_{CB,2}$	-2.373 ± 0.065	-2.379 ± 0.073	-2.38 ± 0.14	-2.19 ± 0.11
[4.0, 6.0]	$\alpha_{CB,2}$	-2.285 ± 0.052	-2.310 ± 0.069	-2.204 ± 0.083	-2.47 ± 0.11
[6.0, 8.0]	$\alpha_{CB,2}$	-2.19 ± 0.05	-2.27 ± 0.05	-2.233 ± 0.076	-2.184 ± 0.075
[15.0, 17.0]	$\alpha_{CB,2}$	-2.043 ± 0.061	-2.20 ± 0.07	-2.05 ± 0.05	-2.15 ± 0.05
[17.0, 19.0]	$\alpha_{CB,2}$	-2.10 ± 0.06	-1.925 ± 0.076	-1.795 ± 0.056	-2.19 ± 0.82
[11.0, 12.5]	$\alpha_{CB,2}$	-2.172 ± 0.056	-2.148 ± 0.049	-2.218 ± 0.071	-2.208 ± 0.074
[1.0, 6.0]	$\alpha_{CB,2}$	-2.326 ± 0.037	-2.290 ± 0.039	-2.275 ± 0.057	-2.31 ± 0.06
[15.0, 19.0]	$\alpha_{CB,2}$	-2.088 ± 0.047	-2.092 ± 0.048	-1.965 ± 0.075	-2.166 ± 0.057
[0.06, 0.98]	$n_{CB,2}$	2.73 ± 0.17	2.52 ± 0.18	2.28 ± 0.25	2.52 ± 0.27
[1.1, 2.5]	$n_{CB,2}$	2.19 ± 0.18	3.02 ± 0.32	2.39 ± 0.42	2.51 ± 0.37
[2.5, 4.0]	$n_{CB,2}$	2.2 ± 0.2	2.27 ± 0.24	2.31 ± 0.44	3.16 ± 0.53
[4.0, 6.0]	$n_{CB,2}$	2.6 ± 0.2	2.64 ± 0.26	2.98 ± 0.38	2.24 ± 0.34
[6.0, 8.0]	$n_{CB,2}$	3.26 ± 0.27	2.78 ± 0.21	2.93 ± 0.34	3.63 ± 0.46
[15.0, 17.0]	$n_{CB,2}$	6.20 ± 0.91	4.0 ± 0.5	6.40 ± 0.22	4.7 ± 0.3
[17.0, 19.0]	$n_{CB,2}$	4.02 ± 0.42	4.86 ± 0.73	9.18 ± 0.32	3.3 ± 4.4
[11.0, 12.5]	$n_{CB,2}$	4.0 ± 0.4	3.83 ± 0.33	3.29 ± 0.38	3.69 ± 0.48
[1.0, 6.0]	$n_{CB,2}$	2.34 ± 0.12	2.62 ± 0.15	2.58 ± 0.21	2.58 ± 0.22
[15.0, 19.0]	$n_{CB,2}$	4.89 ± 0.46	4.30 ± 0.37	6.9 ± 1.3	4.02 ± 0.41

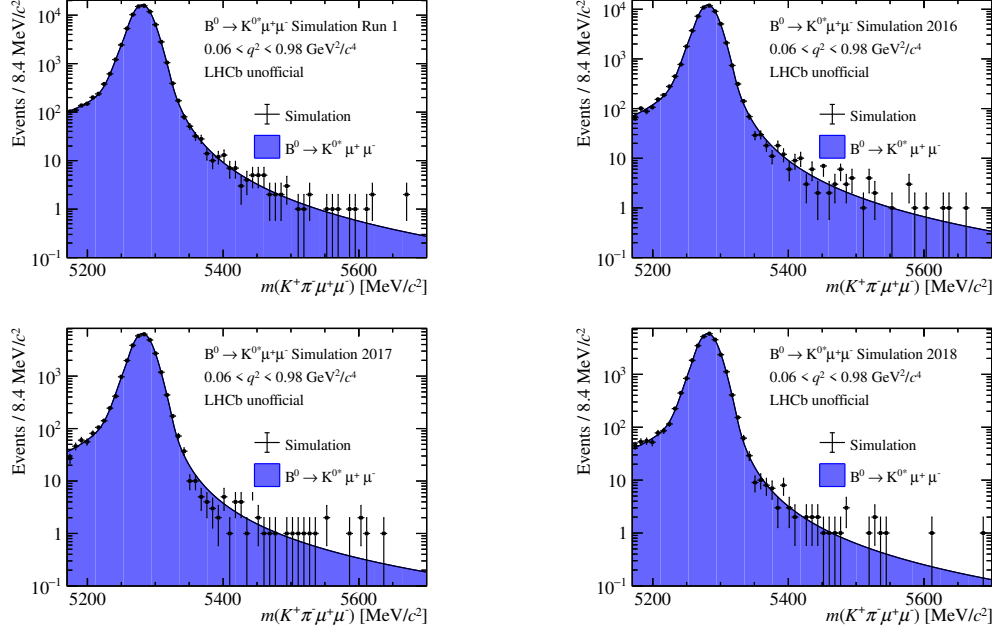


Figure B.1: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

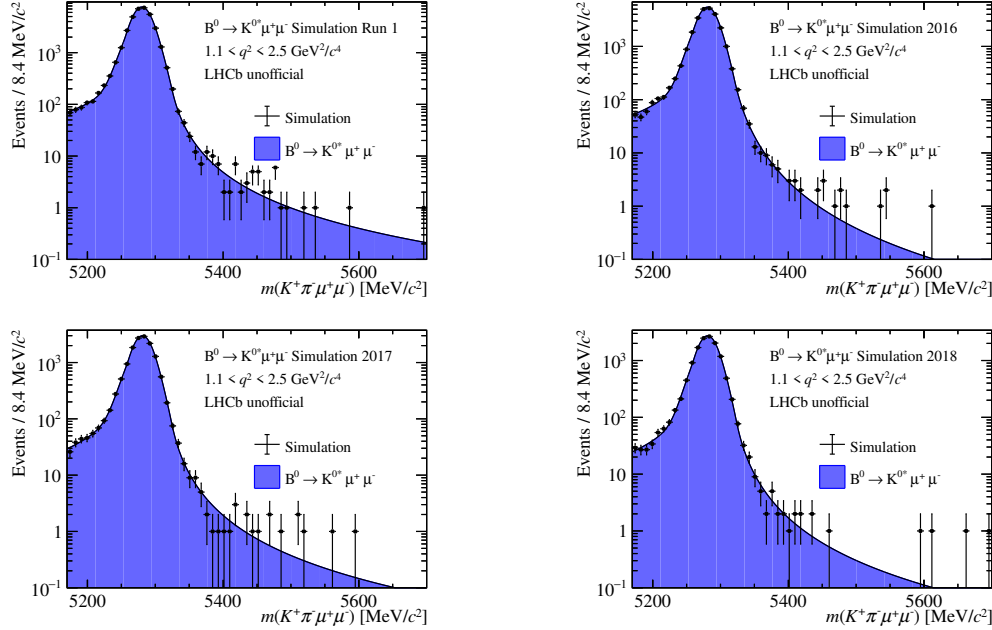


Figure B.2: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

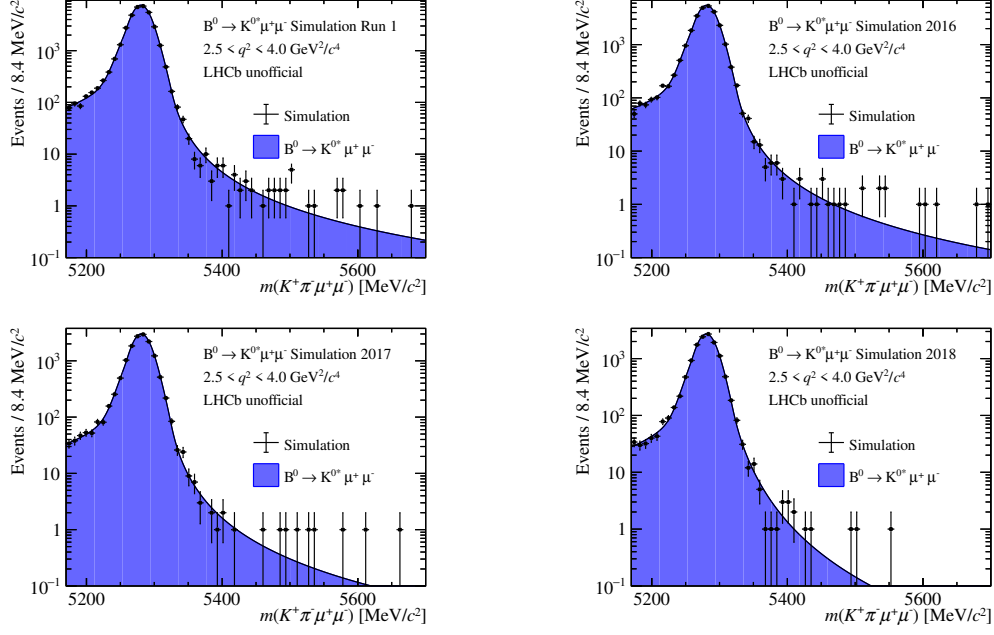


Figure B.3: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

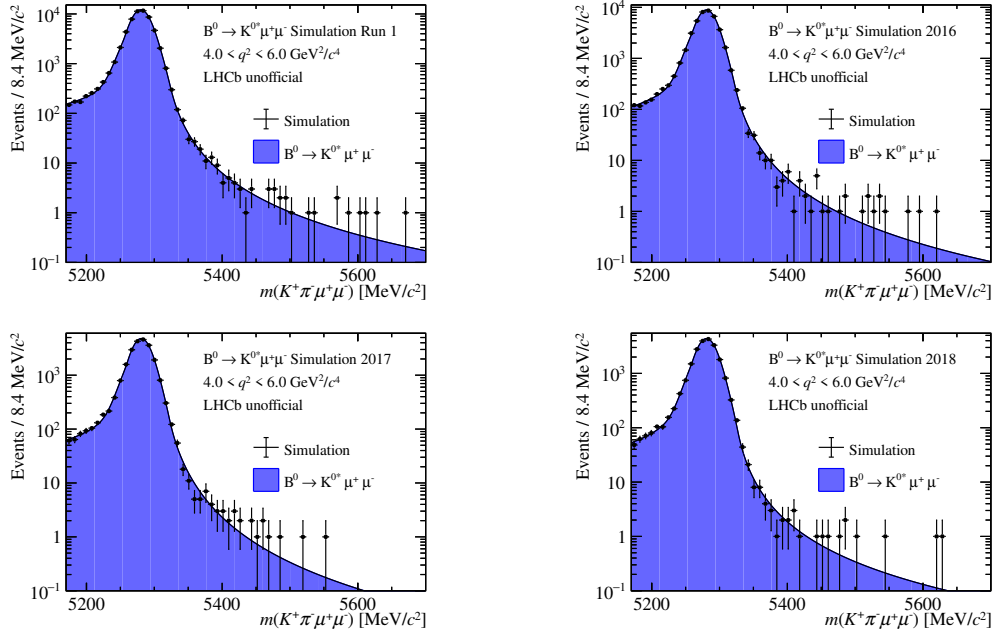


Figure B.4: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

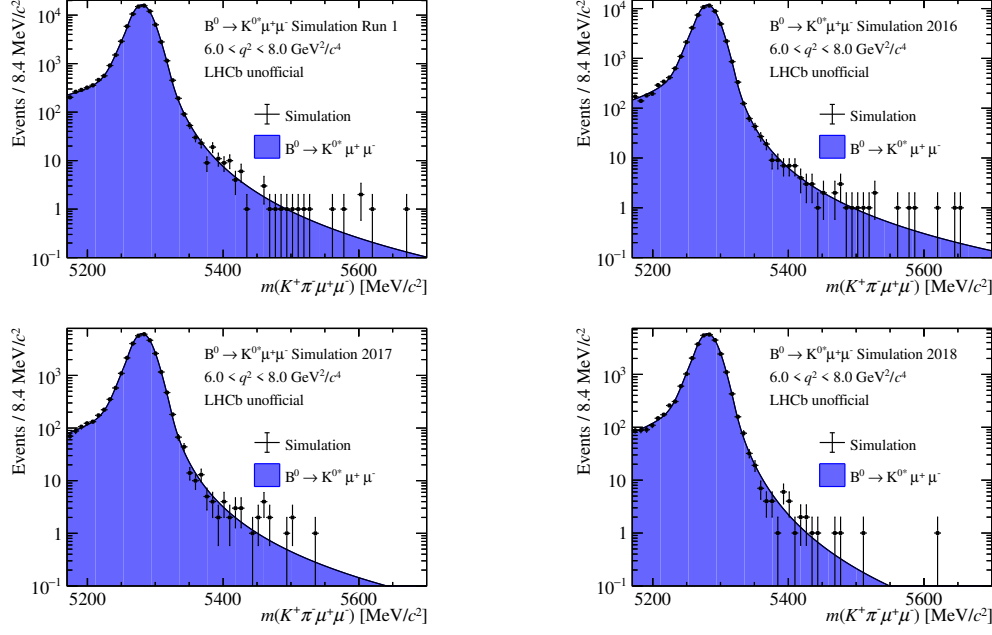


Figure B.5: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

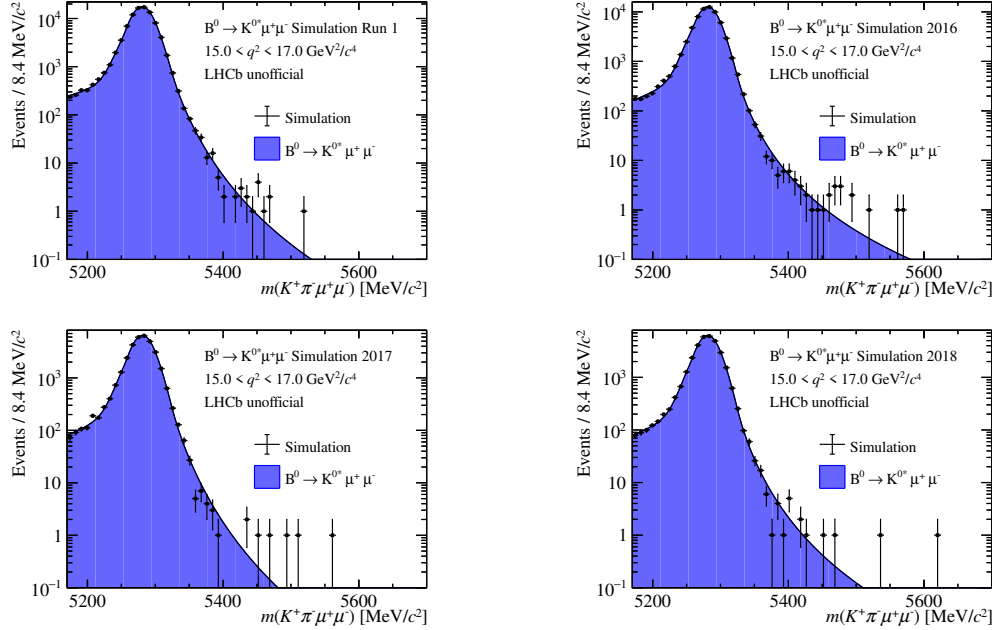


Figure B.6: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

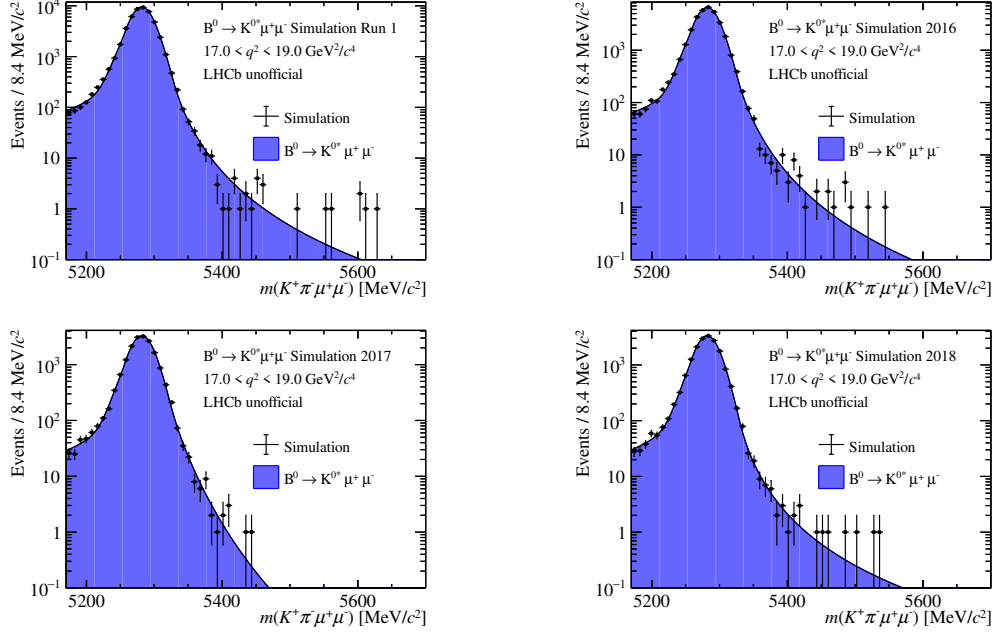


Figure B.7: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

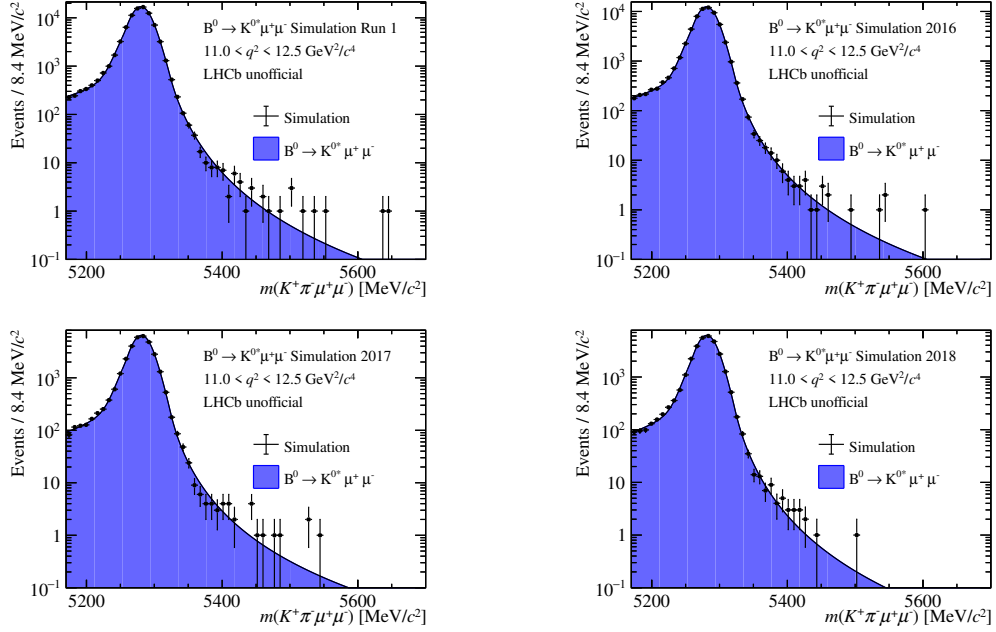


Figure B.8: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

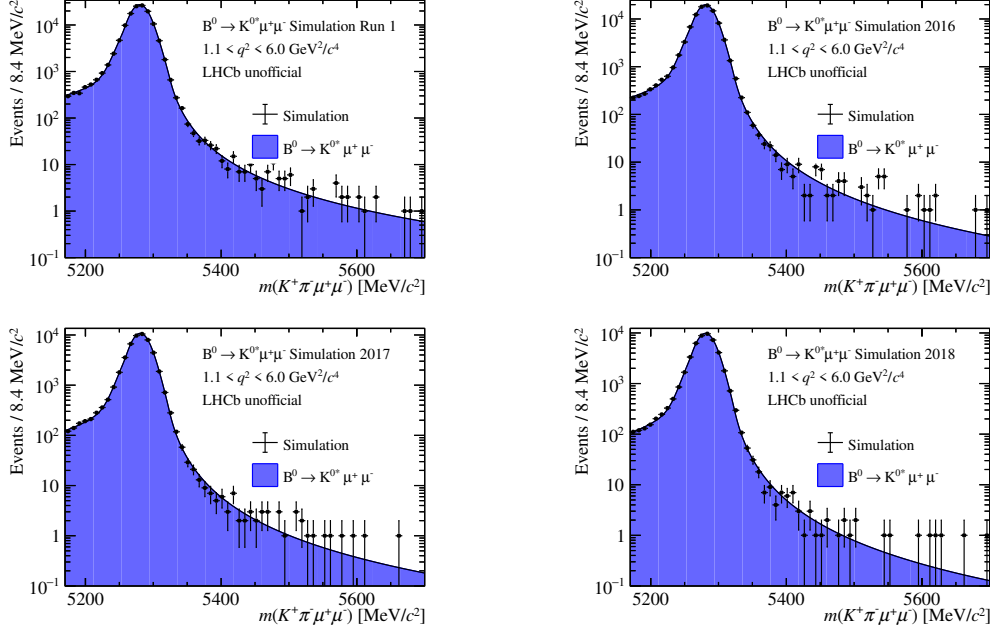


Figure B.9: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017 and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

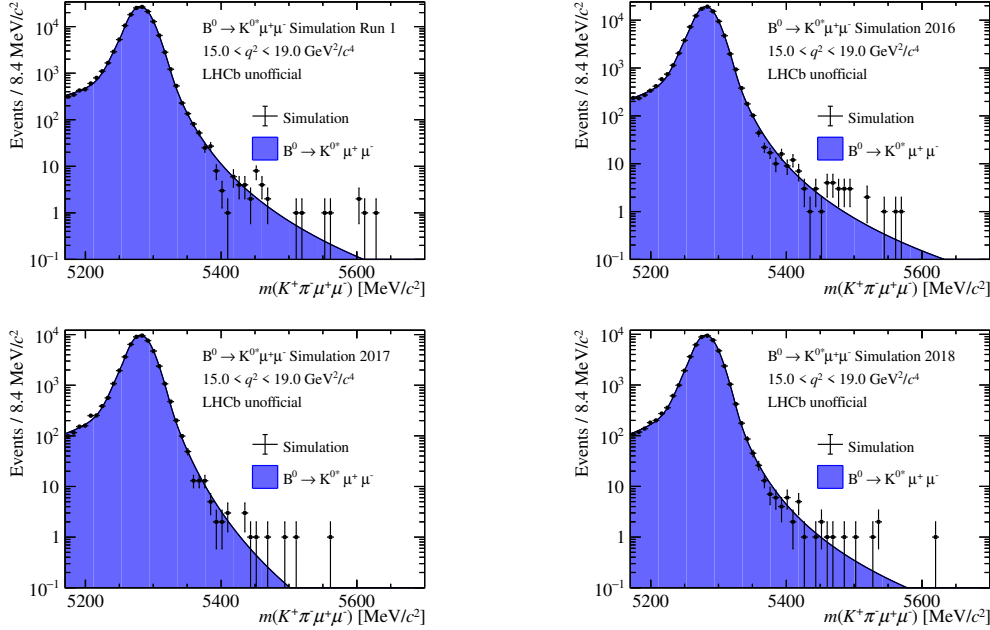


Figure B.10: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^{*0}\mu^+\mu^-$ Monte Carlo simulation samples. The fits to Run 1, 2016, 2017, and 2018 simulation samples are used to fix B^0 mass model parameters in the rare mode data fits.

B.3 Rare mode mass fits

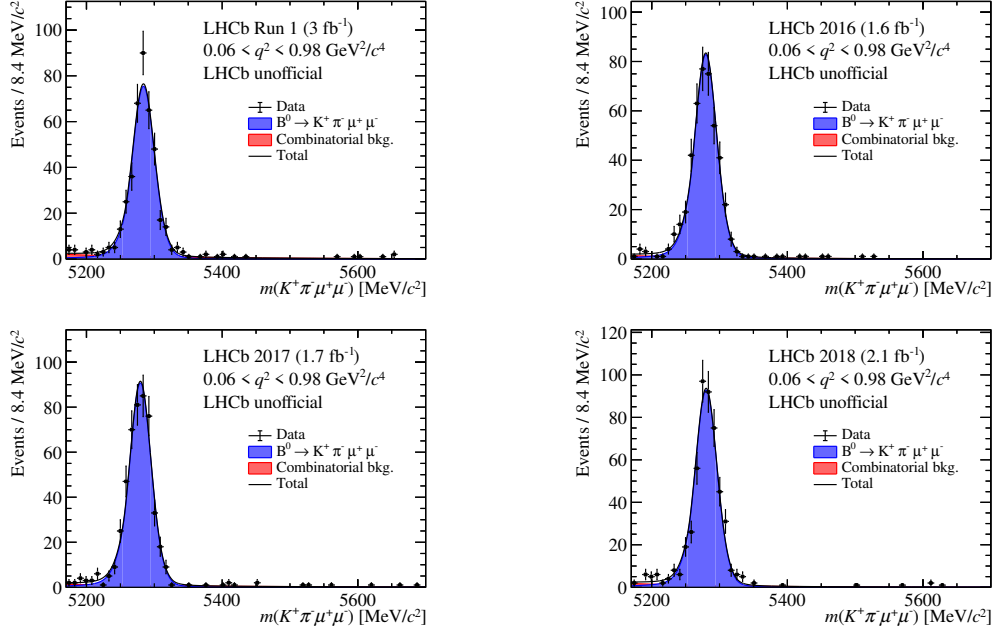


Figure B.11: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ data samples.

Table B.4: Table displaying the background parameters of the $m_{K\pi\mu\mu}$ fit to rare mode data in all 10 q^2 bins.

q^2 [GeV $^2/c^4$]	Parameter	Run 1	2016	2017	2018
[0.06, 0.98]	N_{bkg}	31.3 ± 7.6	16.8 ± 6.3	26.0 ± 7.3	26.3 ± 7.8
[1.1, 2.5]	N_{bkg}	53.7 ± 9.2	50.1 ± 9.0	49.1 ± 9.1	48.0 ± 9.2
[2.5, 4.0]	N_{bkg}	86.4 ± 11.3	61.2 ± 10.0	46.3 ± 8.8	66.6 ± 10.2
[4.0, 6.0]	N_{bkg}	114.2 ± 13.3	96.2 ± 12.3	83.8 ± 11.7	95.1 ± 12.3
[6.0, 8.0]	N_{bkg}	186.8 ± 16.6	181.5 ± 16.6	149.3 ± 15.3	189.8 ± 16.9
[15.0, 17.0]	N_{bkg}	161.9 ± 16.5	138.7 ± 14.8	158.0 ± 15.7	158.8 ± 16.2
[17.0, 19.0]	N_{bkg}	135.6 ± 14.5	83.0 ± 11.0	95.0 ± 12.4	126.0 ± 13.9
[11.0, 12.5]	N_{bkg}	181.4 ± 16.9	146.7 ± 15.2	137.6 ± 14.5	186.4 ± 16.7
[1.0, 6.0]	N_{bkg}	254.7 ± 19.7	207.6 ± 18.3	178.9 ± 17.2	211.9 ± 18.5
[15.0, 19.0]	N_{bkg}	298.7 ± 21.9	220.4 ± 18.4	253.8 ± 20.0	288.9 ± 21.4
[0.06, 0.98]	$N_{\text{bkg}}^{\text{sig}}$	33.0 ± 7.6	31.0 ± 7.6	17.6 ± 5.8	24.4 ± 6.9
[1.1, 2.5]	$N_{\text{bkg}}^{\text{sig}}$	36.1 ± 7.6	57.2 ± 9.6	27.4 ± 7.1	29.0 ± 7.6
[2.5, 4.0]	$N_{\text{bkg}}^{\text{sig}}$	71.9 ± 10.6	68.1 ± 10.2	51.0 ± 8.9	56.0 ± 9.4
[4.0, 6.0]	$N_{\text{bkg}}^{\text{sig}}$	106.0 ± 12.9	104.6 ± 12.8	81.2 ± 11.8	84.5 ± 11.6
[6.0, 8.0]	$N_{\text{bkg}}^{\text{sig}}$	190.1 ± 16.9	176.6 ± 16.6	141.5 ± 14.8	180.9 ± 16.9
[15.0, 17.0]	$N_{\text{bkg}}^{\text{sig}}$	168.7 ± 16.1	140.9 ± 14.8	148.9 ± 15.3	136.2 ± 15.1
[17.0, 19.0]	$N_{\text{bkg}}^{\text{sig}}$	121.6 ± 14.0	101.0 ± 12.4	72.0 ± 10.9	88.0 ± 12.1
[11.0, 12.5]	$N_{\text{bkg}}^{\text{sig}}$	170.6 ± 16.0	155.7 ± 15.6	134.4 ± 14.7	172.1 ± 16.3
[1.0, 6.0]	$N_{\text{bkg}}^{\text{sig}}$	213.9 ± 18.3	230.1 ± 18.9	160.6 ± 16.5	172.2 ± 16.8
[15.0, 19.0]	$N_{\text{bkg}}^{\text{sig}}$	289.7 ± 21.3	242.8 ± 19.3	221.7 ± 18.7	228.3 ± 19.4
[0.06, 0.98]	τ_m	192.2 ± 39.7	193.7 ± 49.8	267.2 ± 100.5	191.1 ± 47.2
[1.1, 2.5]	τ_m	201.9 ± 35.6	216.5 ± 37.2	190.4 ± 35.6	169.4 ± 29.1
[2.5, 4.0]	τ_m	179.5 ± 21.4	187.4 ± 26.0	231.1 ± 45.0	199.6 ± 29.9
[4.0, 6.0]	τ_m	160.5 ± 15.0	184.3 ± 20.1	228.0 ± 34.0	202.2 ± 25.6
[6.0, 8.0]	τ_m	160.9 ± 11.5	177.6 ± 14.0	180.2 ± 16.1	182.3 ± 14.5
[15.0, 17.0]	τ_m	266.0 ± 31.0	256.2 ± 31.7	246.1 ± 28.0	231.0 ± 25.8
[17.0, 19.0]	τ_m	236.9 ± 27.7	566.8 ± 185.3	307.3 ± 58.5	352.5 ± 68.1
[11.0, 12.5]	τ_m	228.5 ± 22.2	215.5 ± 21.7	232.5 ± 26.9	238.1 ± 23.9
[1.0, 6.0]	τ_m	173.9 ± 11.8	192.5 ± 14.8	219.1 ± 21.9	192.5 ± 16.0
[15.0, 19.0]	τ_m	252.4 ± 20.8	333.2 ± 40.7	264.7 ± 25.8	264.8 ± 25.0

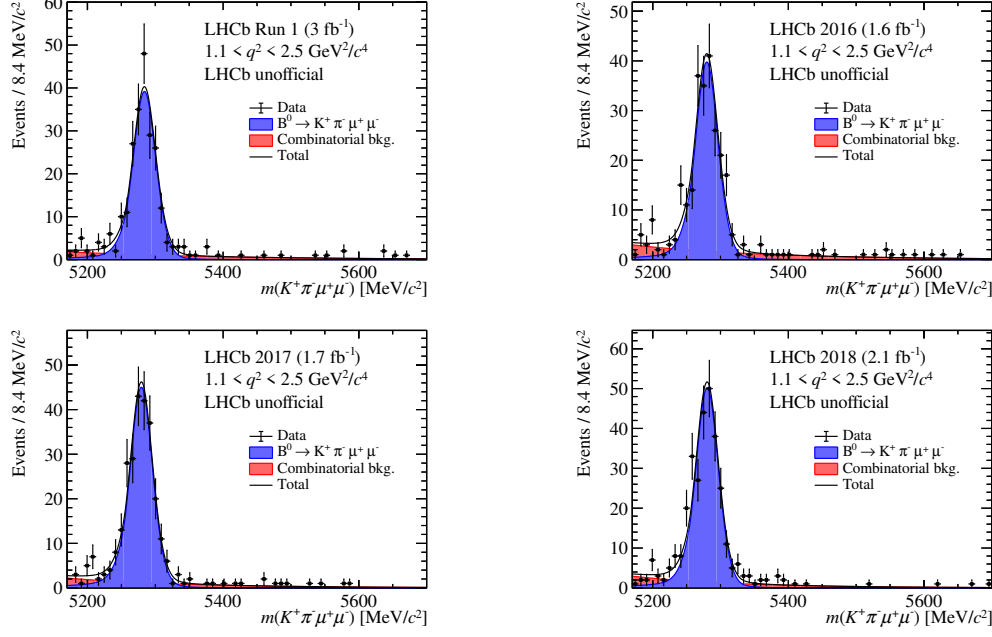


Figure B.12: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ data samples.

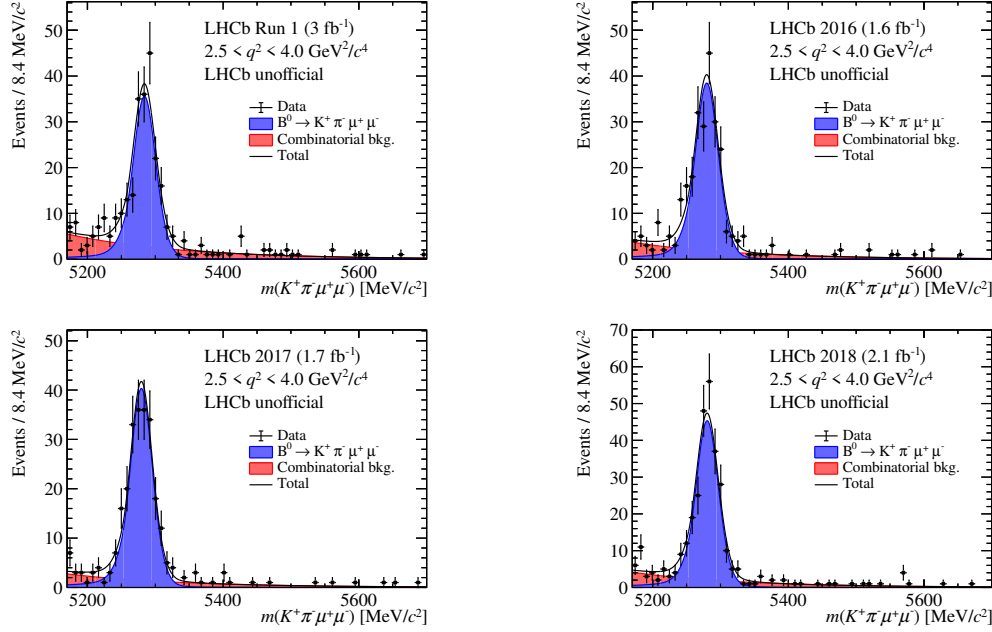


Figure B.13: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ data samples.

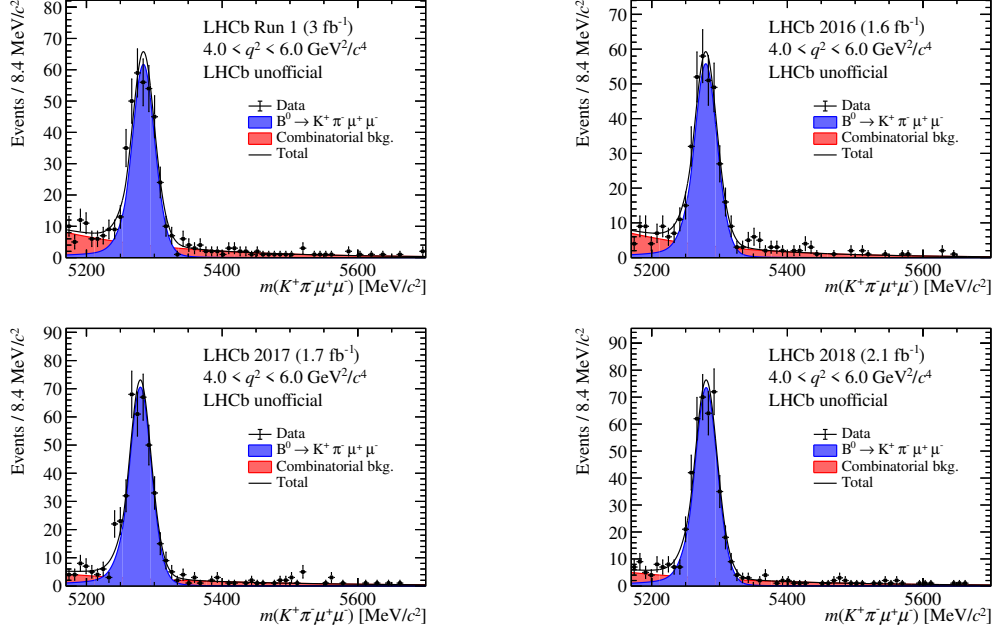


Figure B.14: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+\pi^-\mu^+\mu^-$ data samples.

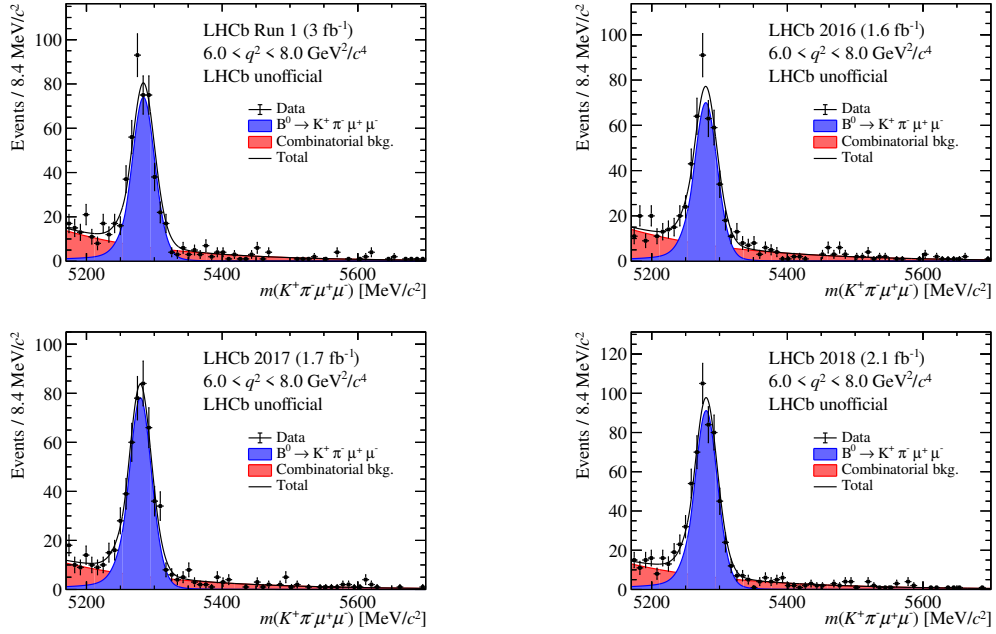


Figure B.15: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+\pi^-\mu^+\mu^-$ data samples.

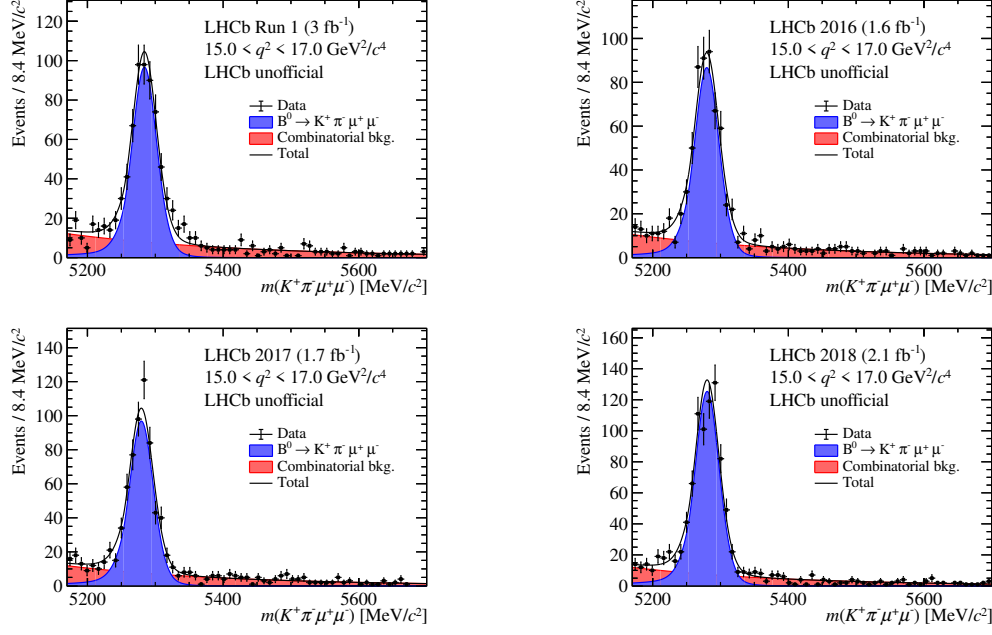


Figure B.16: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ data samples.

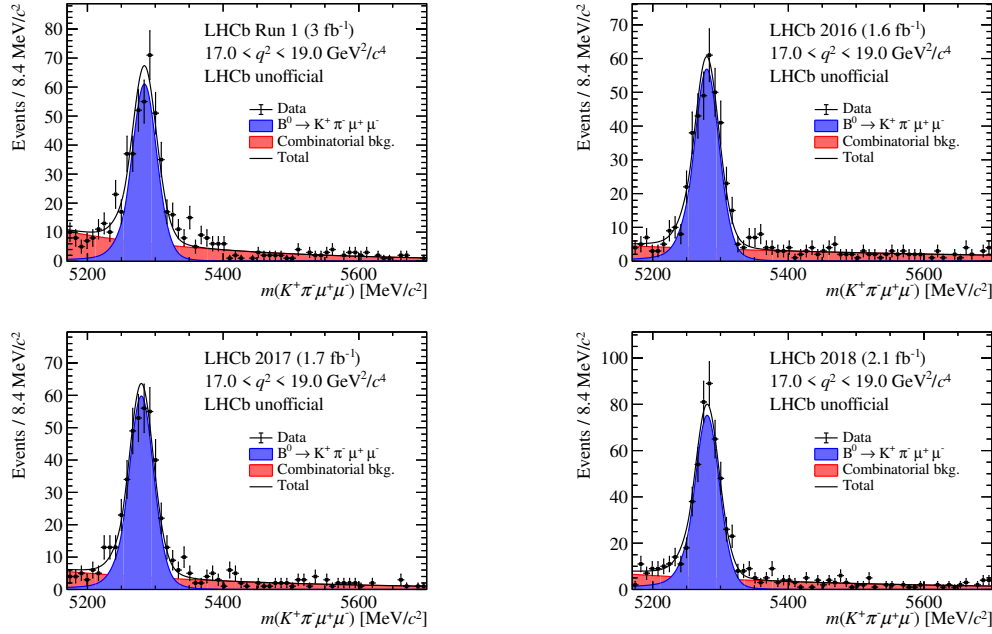


Figure B.17: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ data samples.

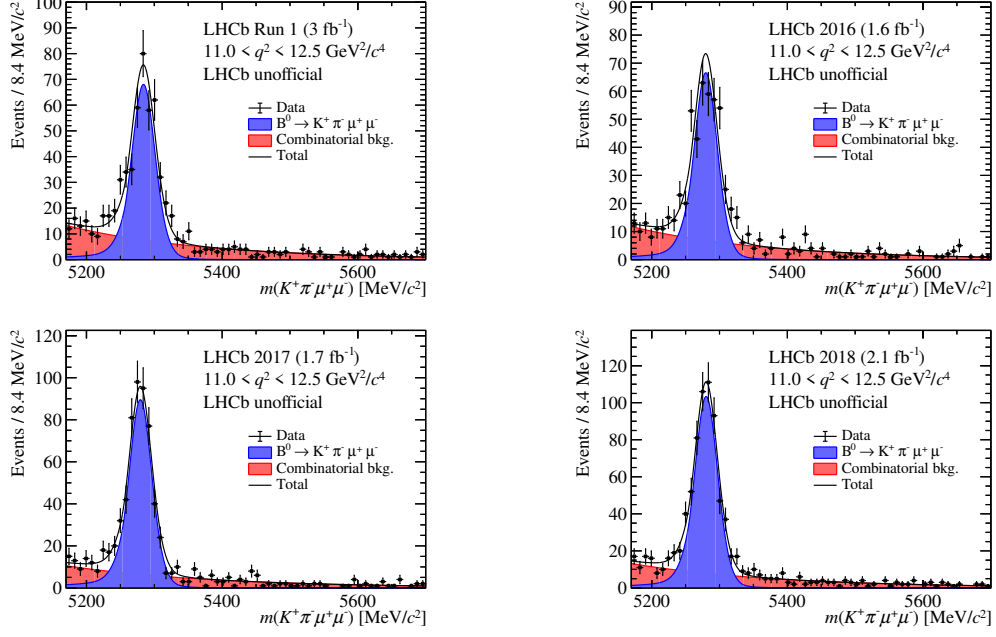


Figure B.18: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+\pi^-\mu^+\mu^-$ data samples.

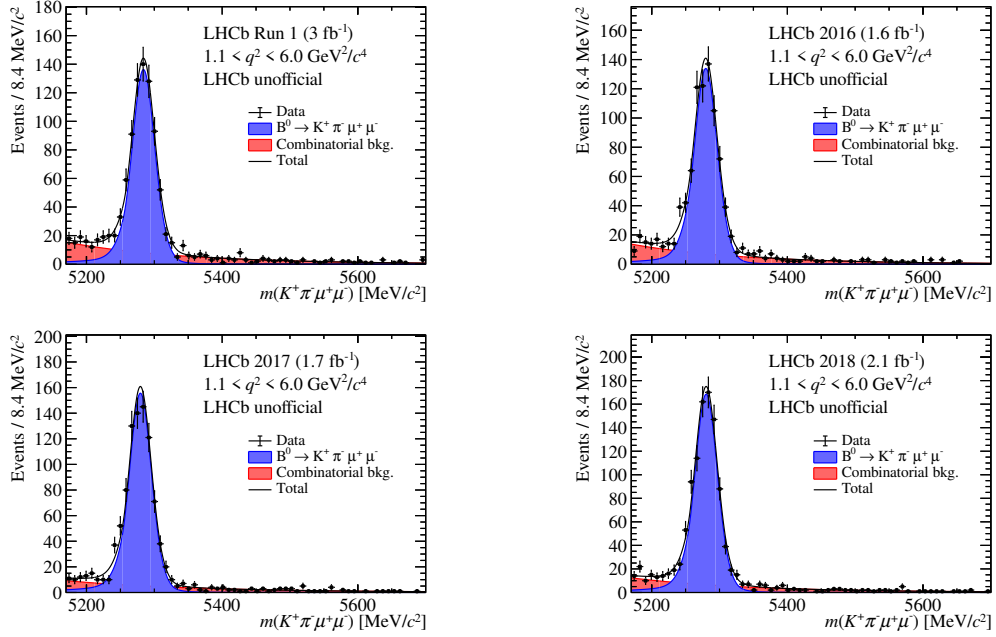


Figure B.19: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+\pi^-\mu^+\mu^-$ data samples.

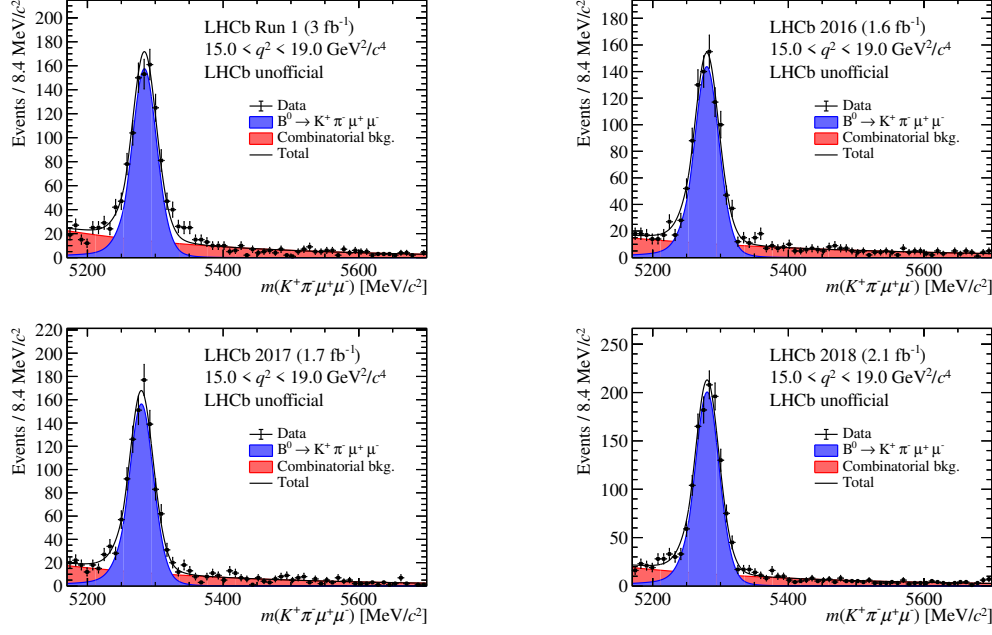


Figure B.20: $m_{K\pi\mu\mu}$ projections of invariant mass fits to $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ data samples.

B.4 Acceptance

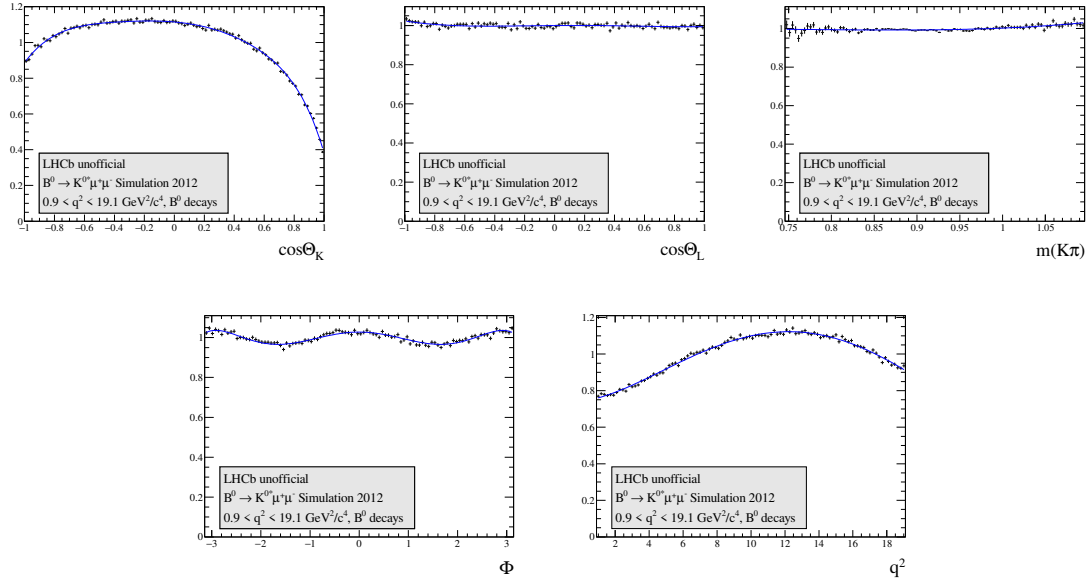


Figure B.21: One-dimensional acceptance projections for B^0 events between $0.9 < q^2 < 19.1 \text{ GeV}^2$ determined from 2012 simulation in the wide $m_{K\pi}$ window. Projection on $\cos \theta_K$ (top left), $\cos \theta_L$ (top center), ϕ (top right), q^2 (bottom left), and $m_{K\pi}$ (bottom right).

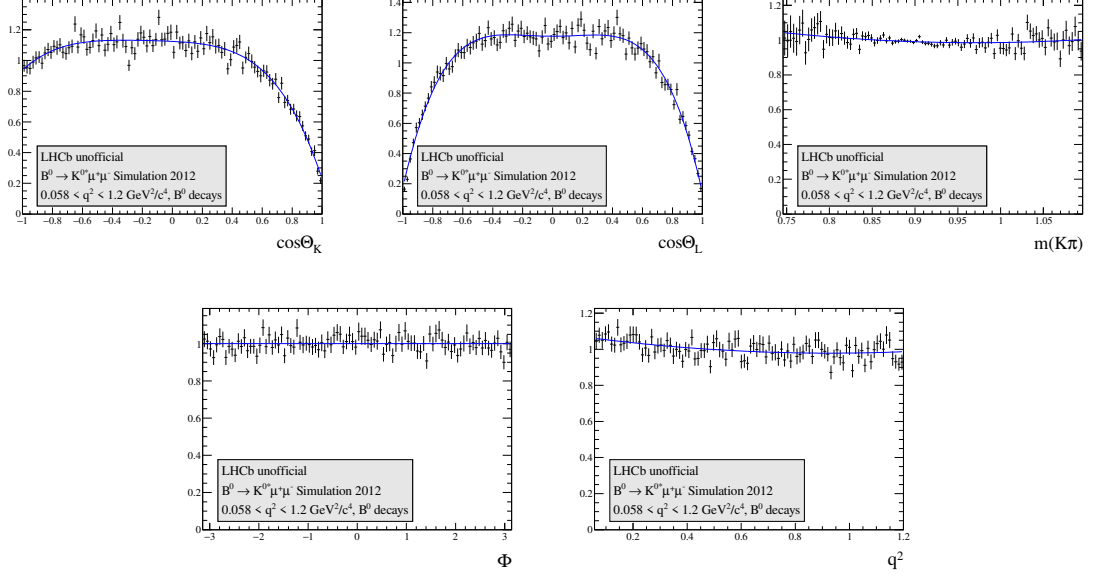


Figure B.22: One-dimensional acceptance projections for B^0 events between $0.058 < q^2 < 1.2 \text{ GeV}^2$ determined from 2012 simulation in the wide $m_{K\pi}$ window. Projection on $\cos\theta_K$ (top left), $\cos\theta_\ell$ (top center), ϕ (top right), q^2 (bottom left), and $m_{K\pi}$ (bottom right).

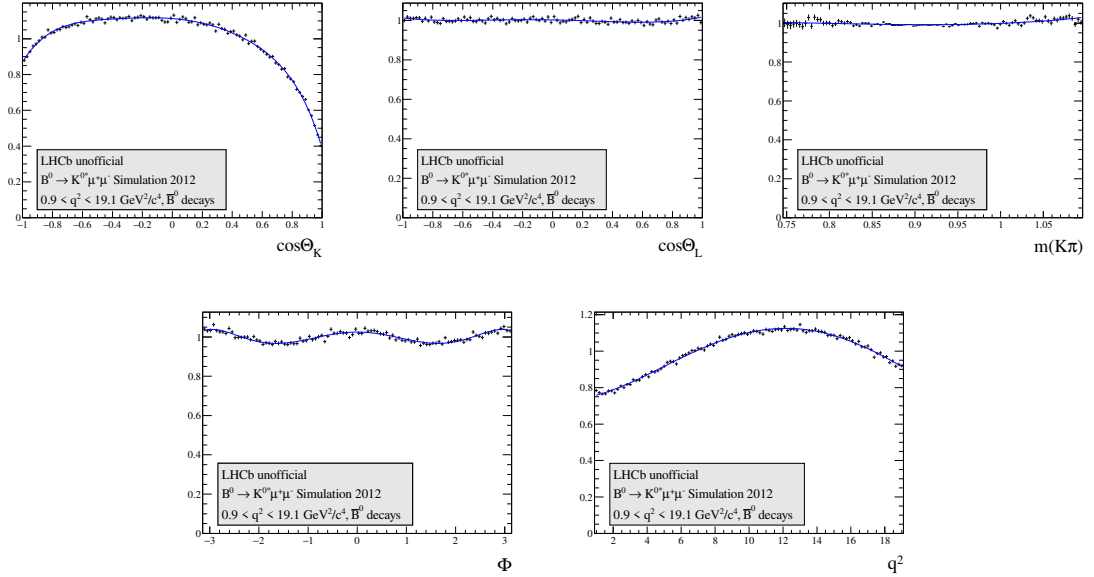


Figure B.23: One-dimensional acceptance projections for $\bar{B}^0$ events between $0.9 < q^2 < 19.1 \text{ GeV}^2$ determined from 2012 simulation in the wide $m_{K\pi}$ window. Projection on $\cos\theta_K$ (top left), $\cos\theta_\ell$ (top center), ϕ (top right), q^2 (bottom left), and $m_{K\pi}$ (bottom right).

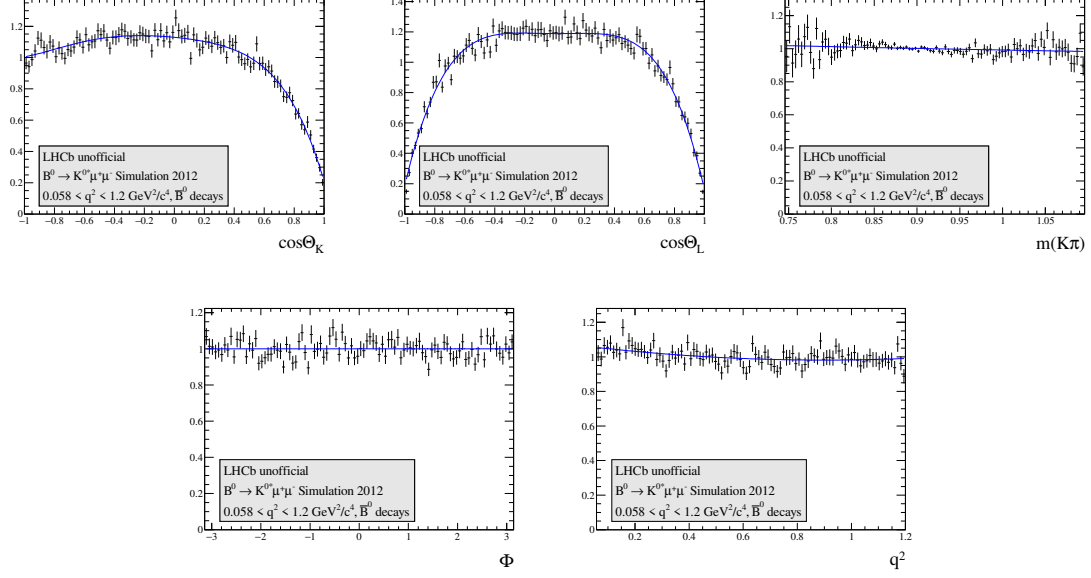


Figure B.24: One-dimensional acceptance projections for $\bar{B}^0$ events between $0.058 < q^2 < 1.2 \text{ GeV}^2$ determined from 2012 simulation in the wide $m_{K\pi}$ window. Projection on $\cos \theta_K$ (top left), $\cos \theta_\ell$ (top center), ϕ (top right), q^2 (bottom left), and $m_{K\pi}$ (bottom right).

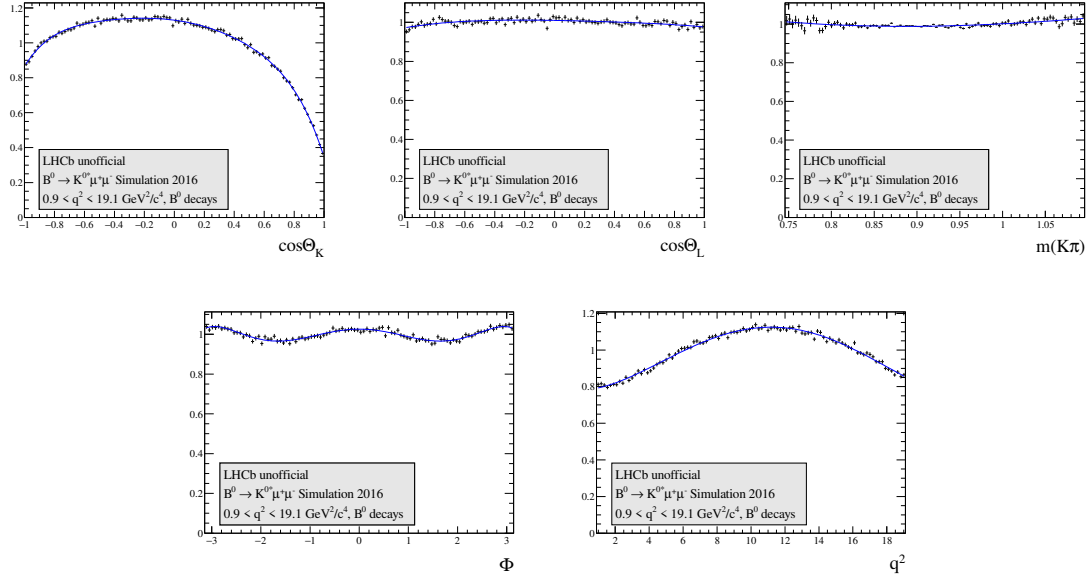


Figure B.25: One-dimensional acceptance projections for B^0 events between $0.9 < q^2 < 19.1 \text{ GeV}^2$ determined from 2016 simulation in the wide $m_{K\pi}$ window. Projection on $\cos \theta_K$ (top left), $\cos \theta_\ell$ (top center), ϕ (top right), q^2 (bottom left), and $m_{K\pi}$ (bottom right).

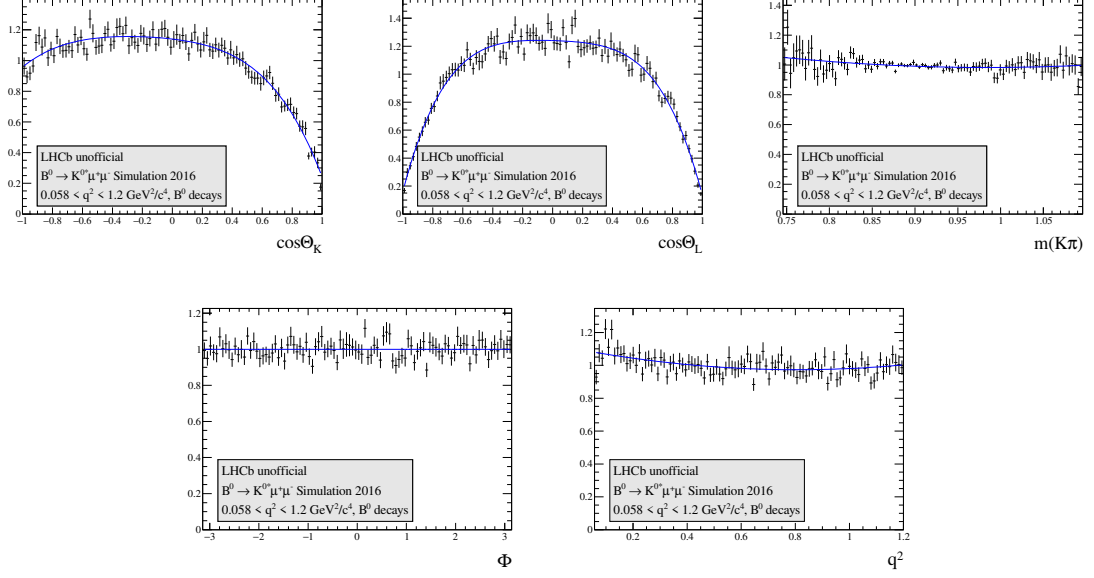


Figure B.26: One-dimensional acceptance projections for B^0 events between $0.058 < q^2 < 1.2 \text{ GeV}^2$ determined from 2016 simulation in the wide $m_{K\pi}$ window. Projection on $\cos \theta_K$ (top left), $\cos \theta_\ell$ (top center), ϕ (top right), q^2 (bottom left), and $m_{K\pi}$ (bottom right).

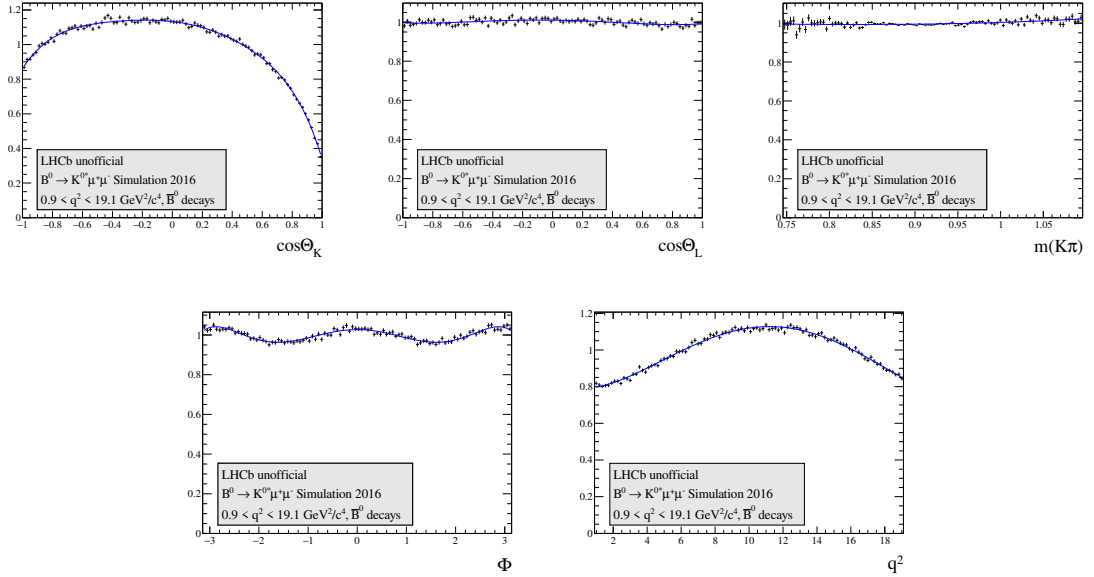


Figure B.27: One-dimensional acceptance projections for $\bar{B}^0$ events between $0.9 < q^2 < 19.1 \text{ GeV}^2$ determined from 2016 simulation in the wide $m_{K\pi}$ window. Projection on $\cos \theta_K$ (top left), $\cos \theta_\ell$ (top center), ϕ (top right), q^2 (bottom left), and $m_{K\pi}$ (bottom right).

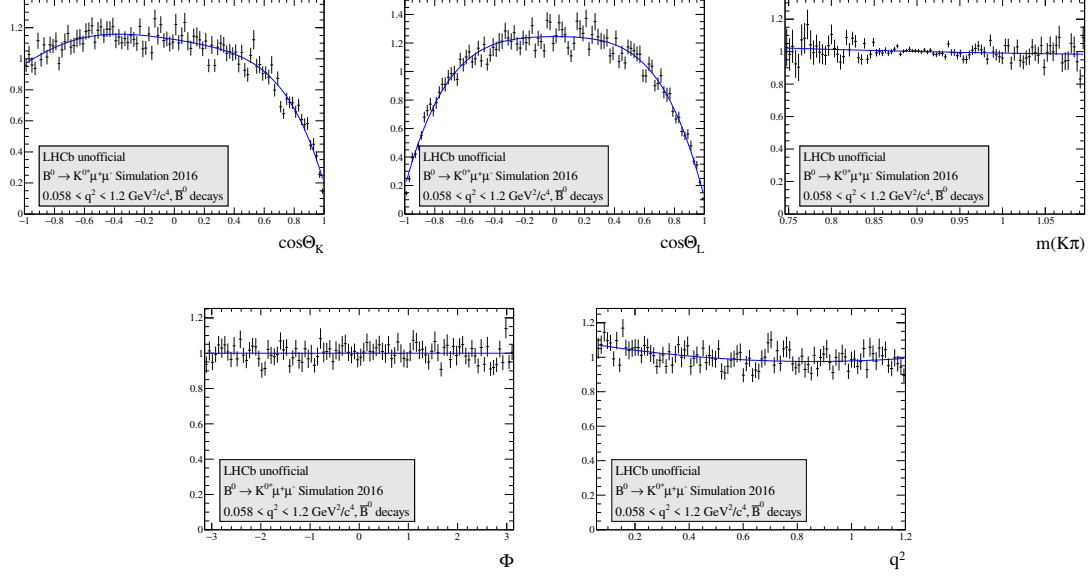


Figure B.28: One-dimensional acceptance projections for $\overline{B}^0$ events between $0.058 < q^2 < 1.2 \text{ GeV}^2$ determined from 2016 simulation in the wide $m_{K\pi}$ window. Projection on $\cos \theta_K$ (top left), $\cos \theta_\ell$ (top center), ϕ (top right), q^2 (bottom left), and $m_{K\pi}$ (bottom right).

B.5 Simulation fits

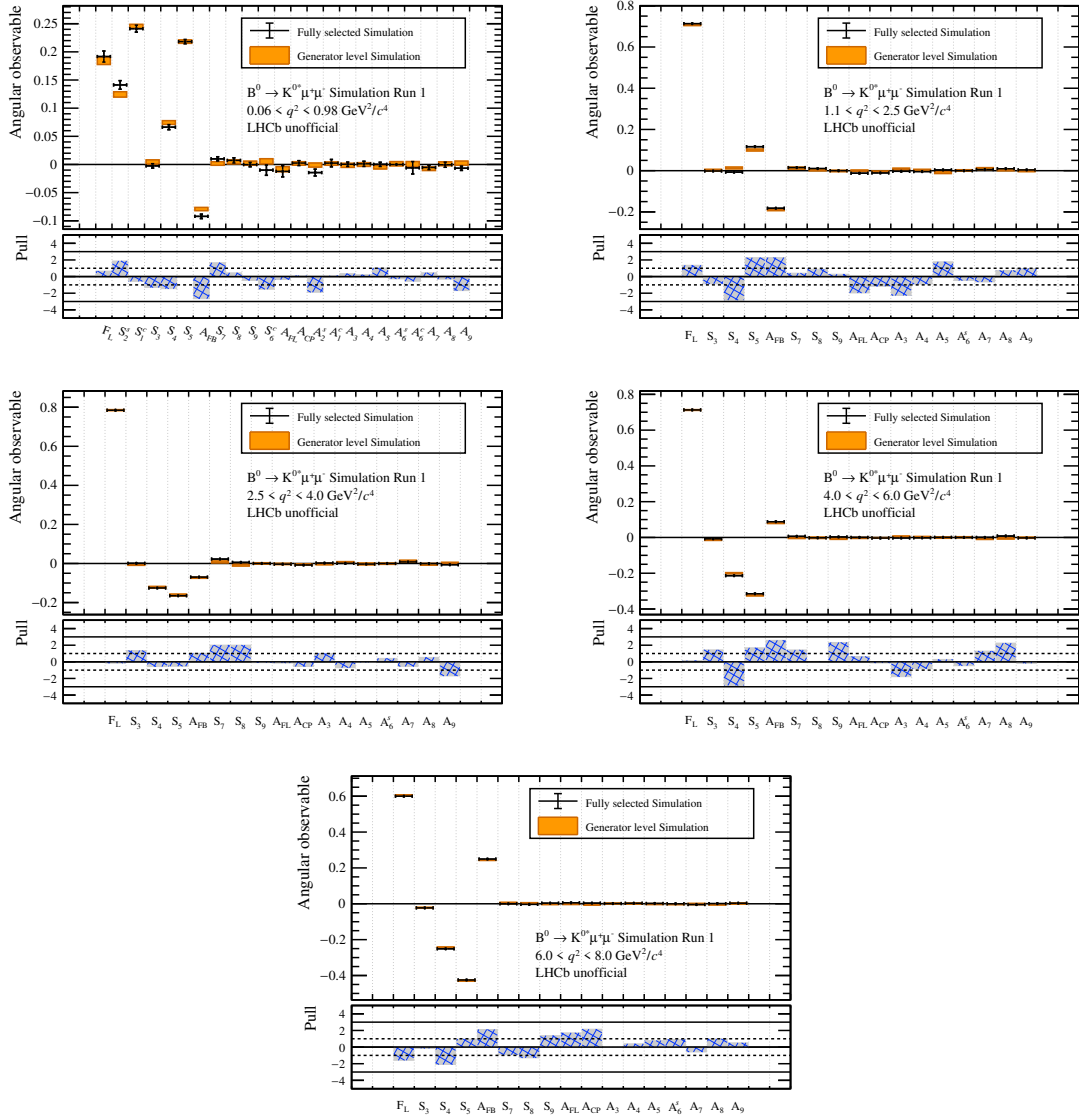


Figure B.29: Comparison between fits to generator-level simulation and fully selected simulation from 2012. For the fit to selected and generator-level simulation, the nominal and flat acceptance functions are used, respectively. The plots compare the P-wave angular observables for the first 5 q^2 bins.

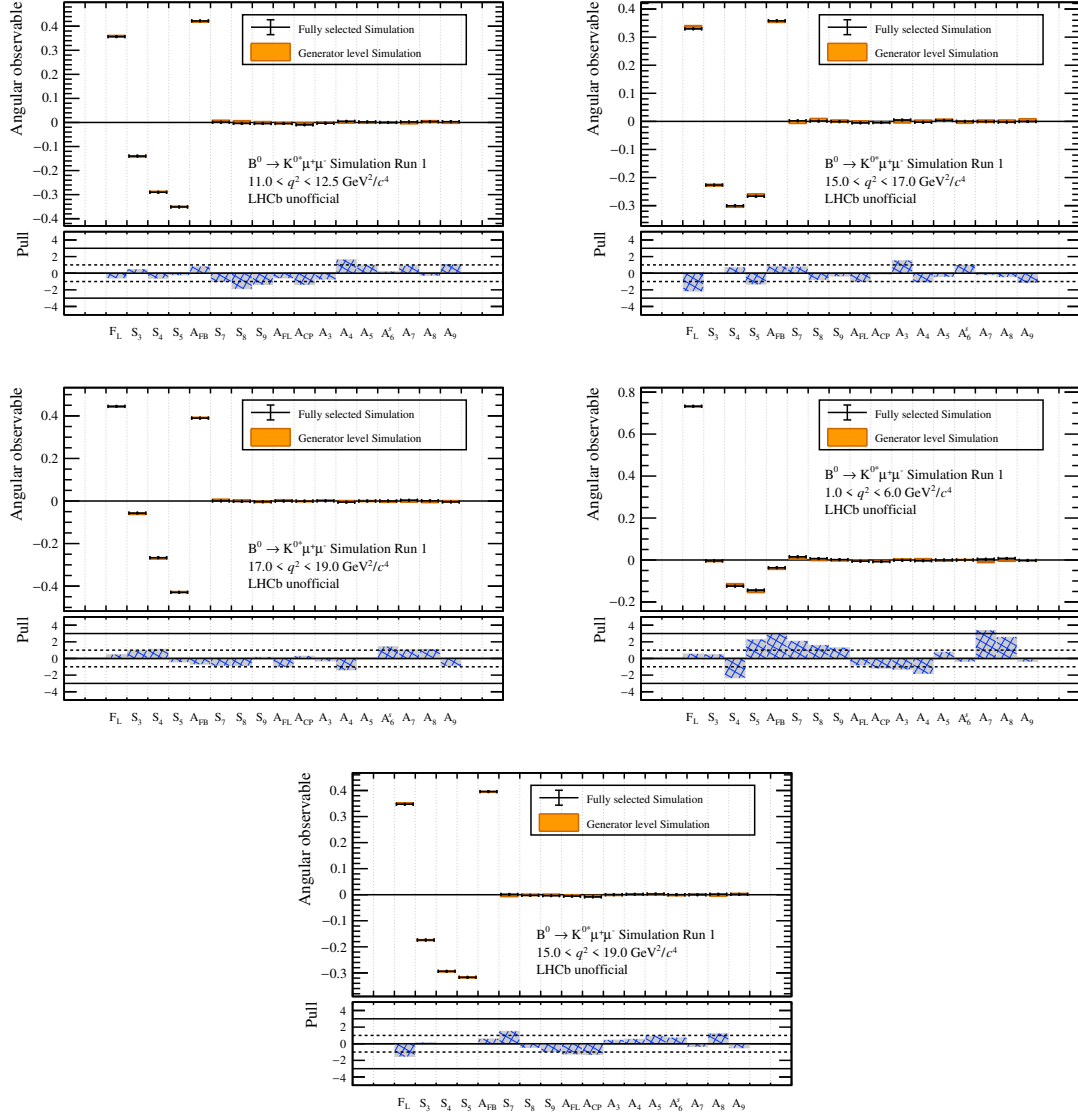


Figure B.30: Comparison between fits to generator-level simulation and fully selected simulation from 2012. For the fit to selected and generator-level simulation, the nominal and flat acceptance functions are used, respectively. The plots compare the P-wave angular observables for the last 5 q^2 bins.

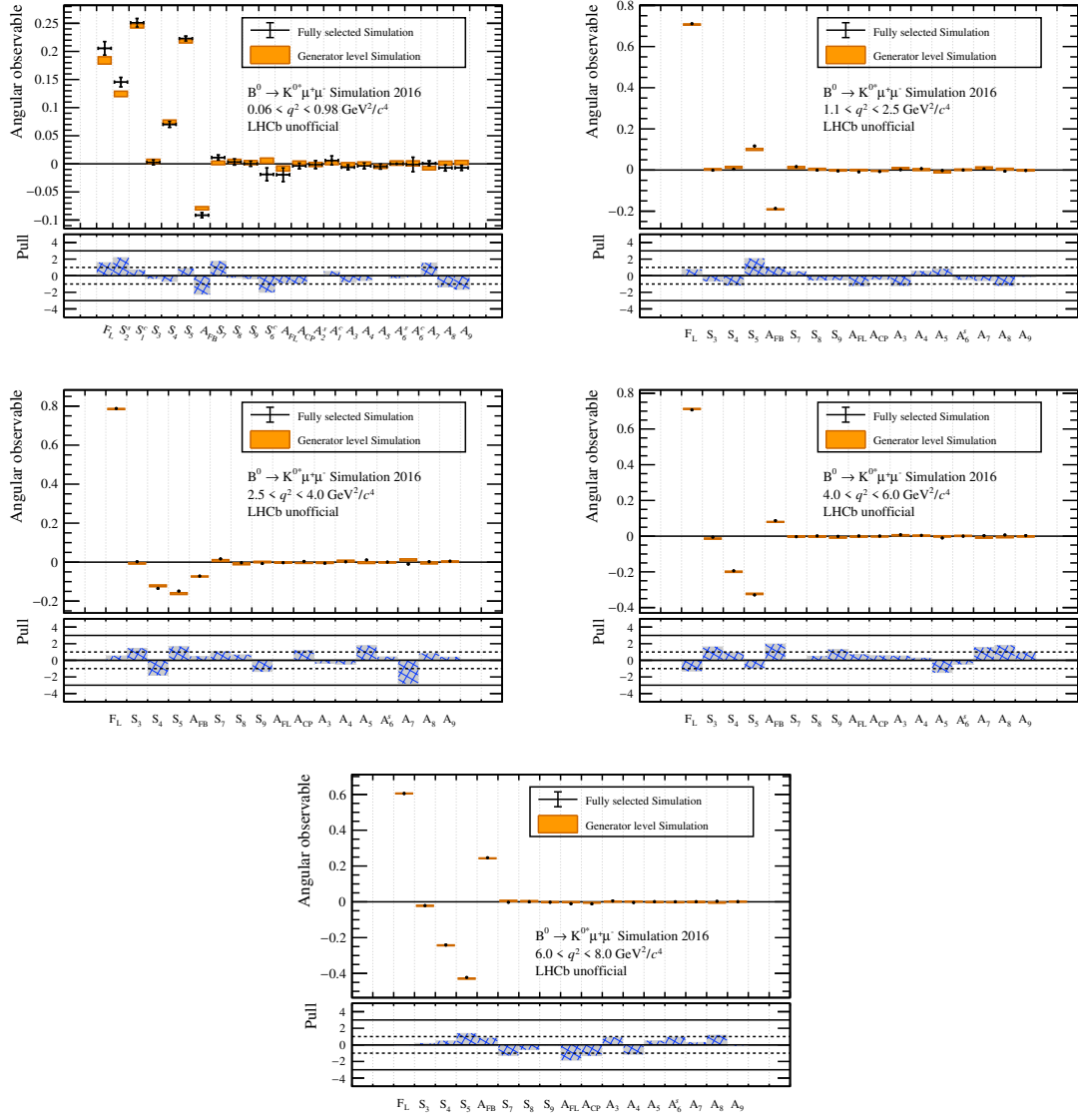


Figure B.31: Comparison between fits to generator-level simulation and fully selected simulation from 2016. For the fit to selected and generator-level simulation, the nominal and flat acceptance functions are used, respectively. The plots compare the P-wave angular observables for the first 5 q^2 bins.

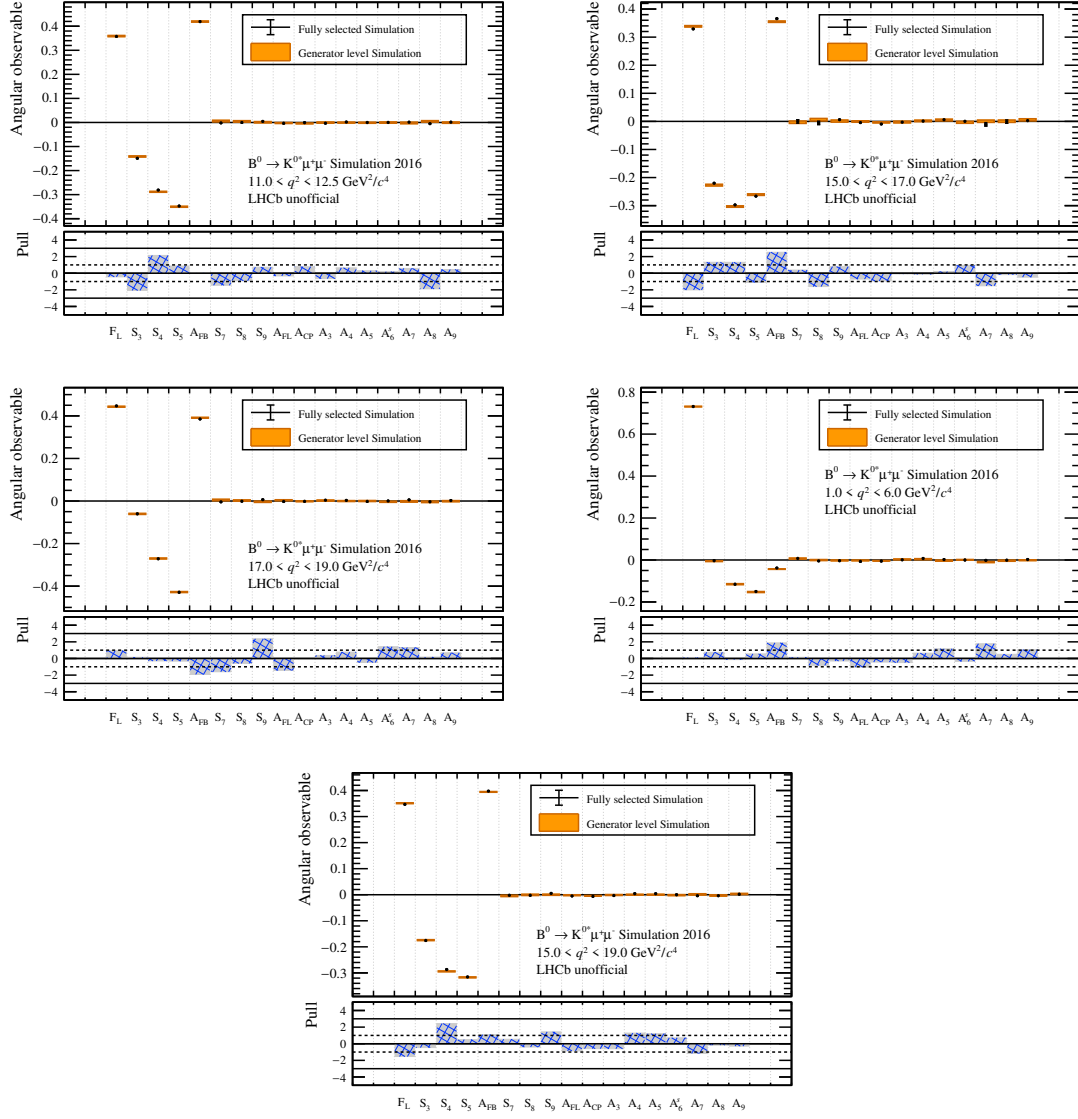


Figure B.32: Comparison between fits to generator-level simulation and fully selected simulation from 2016. For the fit to selected and generator-level simulation, the nominal and flat acceptance functions are used, respectively. The plots compare the P-wave angular observables for the last 5 q^2 bins.

B.6 Simulation fits with Monte Carlo integrated ξ_i

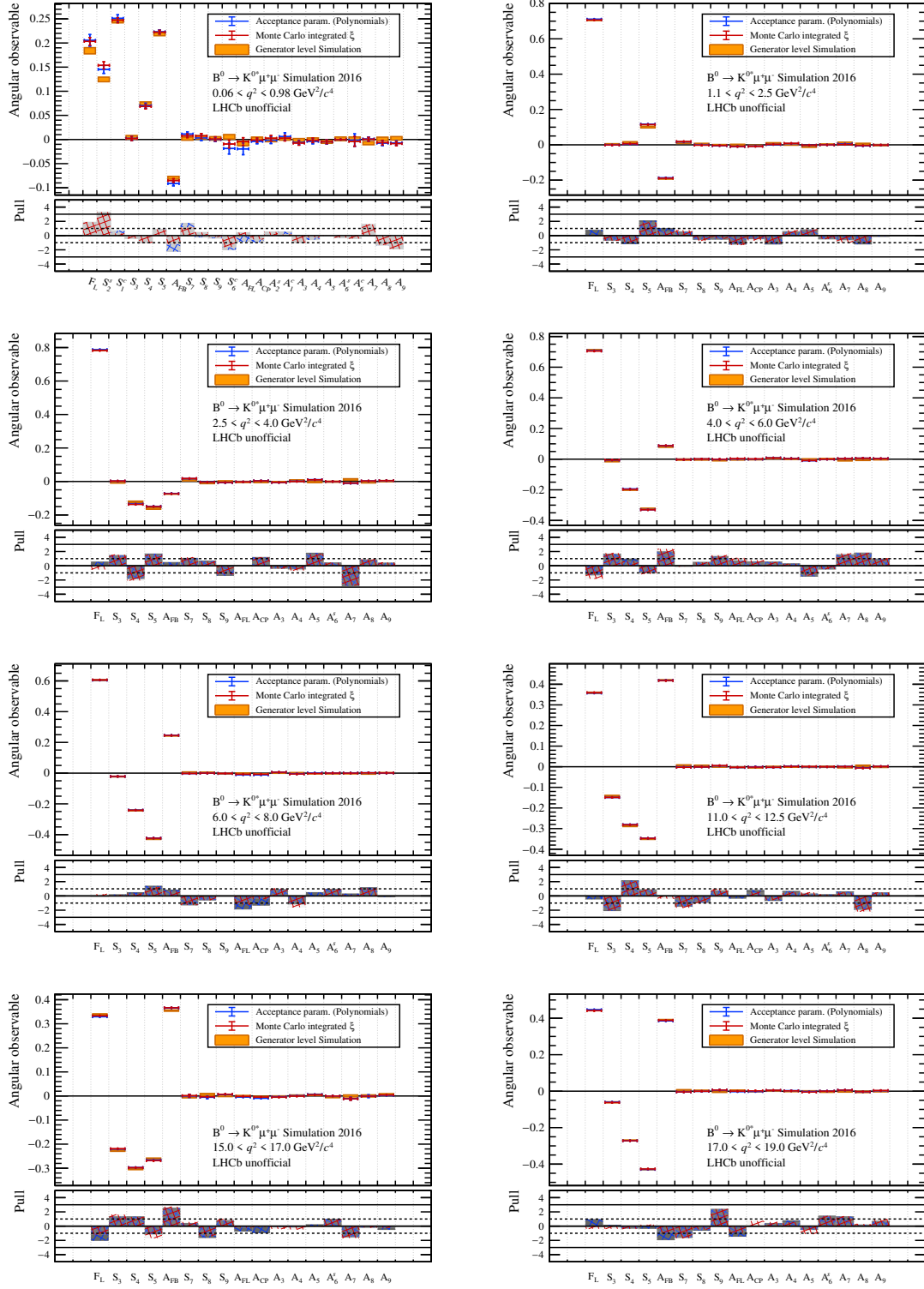


Figure B.33: Comparison between fits to generator-level simulation and fully selected simulation from 2012. For the fit to selected simulation, the acceptance parameterization using Legendre polynomials or the Monte Carlo integrated ξ_i are used. For the generator-level, the acceptance is flat.

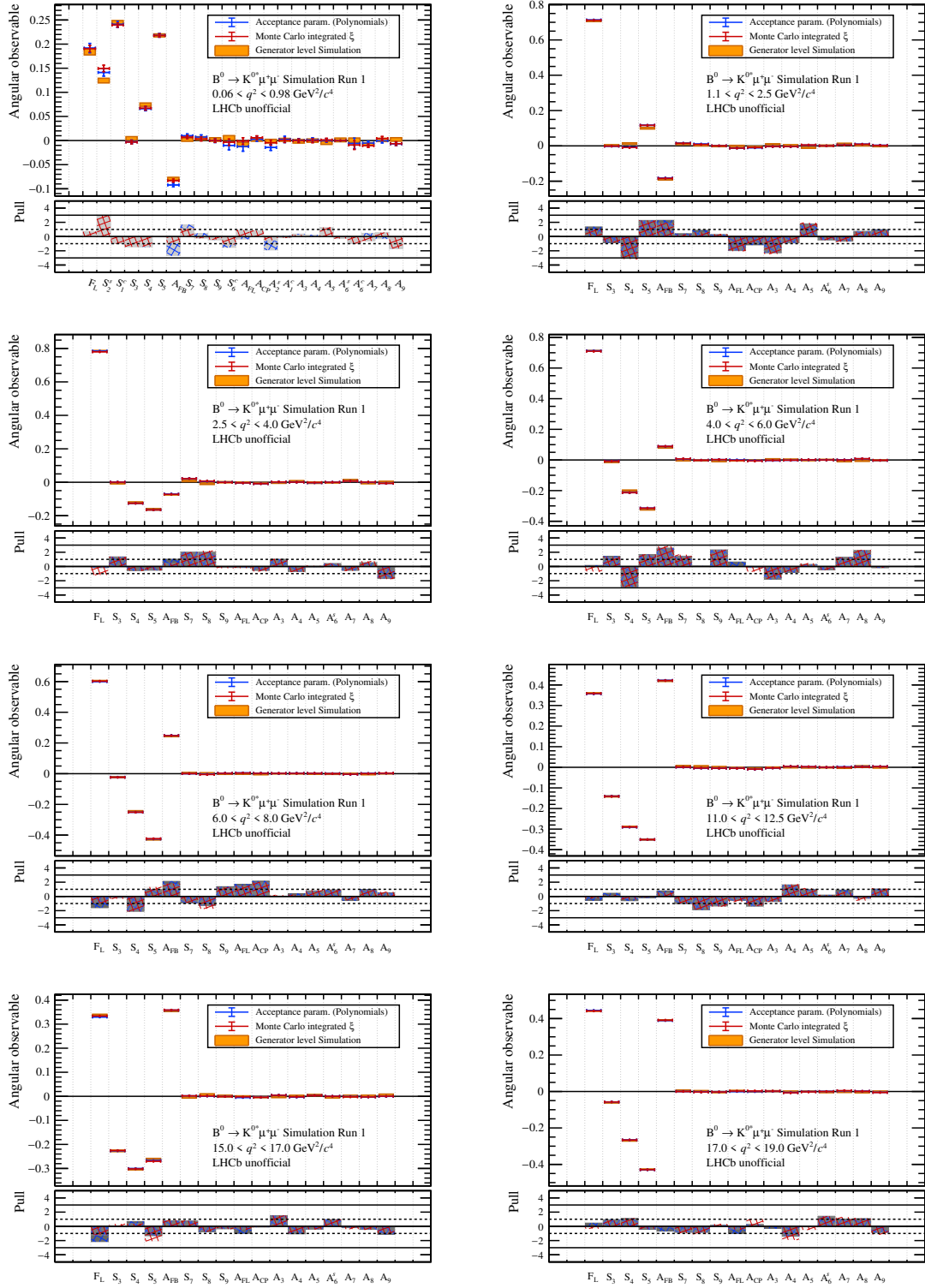


Figure B.34: Comparison between fits to generator-level simulation and fully selected simulation from 2016. For the fit to selected simulation, the acceptance parameterization using Legendre polynomials or the Monte Carlo integrated ξ_i are used. For the generator-level, the acceptance is flat.

B.7 Control channel fit

Table B.5: Nuisance parameters from single-year control channel fits.

Observable	Run 1	2016	2017	2018
N_{bkg}	19281.1 ± 209.4	15366.7 ± 195.0	14849.4 ± 189.1	14849.4 ± 189.1
$N_{\overline{\text{bkg}}}$	19396.2 ± 208.5	15308.2 ± 193.8	14604.4 ± 187.3	14604.4 ± 187.3
$m_{B^0} [\text{MeV}/c^2]$	5284.049 ± 0.032	5279.788 ± 0.032	5279.538 ± 0.031	5279.538 ± 0.031
$s_{\sigma}^{J/\psi}$	1.1379 ± 0.0019	1.1493 ± 0.0019	1.1202 ± 0.0018	1.1202 ± 0.0018
$\tau_m [c^2/\text{MeV}]$	167.0 ± 1.4	171.8 ± 1.6	170.7 ± 1.7	170.7 ± 1.7
$f_{B_s^0}$	0.00815 ± 0.00024	0.00934 ± 0.00025	0.00905 ± 0.00023	0.00905 ± 0.00023
$\Delta(m_{B^0} - m_{B_s^0}) [\text{MeV}/c^2]$	87.33 ± 0.16	87.15 ± 0.16	87.14 ± 0.15	87.14 ± 0.15
$d_{B^0} [c^2/\text{GeV}]$	$1.6000(\text{fixed})$	$1.6000(\text{fixed})$	$1.6000(\text{fixed})$	$1.6000(\text{fixed})$
$a [c^2/\text{GeV}]$	3.33 ± 0.15	3.29 ± 0.14	3.309 ± 0.089	3.309 ± 0.089
$r [c^2/\text{GeV}]$	$1.7000(\text{fixed})$	$1.7000(\text{fixed})$	$1.7000(\text{fixed})$	$1.7000(\text{fixed})$
$\Gamma(K^{*0}) [\text{MeV}/c^2]$	0.04853 ± 0.00015	0.04839 ± 0.00015	0.04828 ± 0.00013	0.04828 ± 0.00013
$m(K^{*0}) [\text{MeV}/c^2]$	0.89503 ± 0.00006	0.89501 ± 0.00006	0.89482 ± 0.00005	0.89482 ± 0.00005
$\Gamma_{1440} [\text{GeV}/c^2]$	$0.2700(\text{fixed})$	$0.2700(\text{fixed})$	$0.2700(\text{fixed})$	$0.2700(\text{fixed})$
$m(K^*(1440)) [\text{GeV}/c^2]$	$1.4250(\text{fixed})$	$1.4250(\text{fixed})$	$1.4250(\text{fixed})$	$1.4250(\text{fixed})$
$c_{\text{bkg}}^1(\cos\theta_l)$	-0.024 ± 0.012	-0.044 ± 0.014	-0.036 ± 0.014	-0.036 ± 0.014
$c_{\text{bkg}}^2(\cos\theta_l)$	-0.456 ± 0.014	-0.481 ± 0.016	-0.399 ± 0.016	-0.399 ± 0.016
$c_{\text{bkg}}^1(\cos\theta_K)$	0.387 ± 0.012	0.297 ± 0.014	0.337 ± 0.014	0.337 ± 0.014
$c_{\text{bkg}}^2(\cos\theta_K)$	0.3555 ± 0.0094	0.256 ± 0.012	0.254 ± 0.012	0.254 ± 0.012
$c_{\text{bkg}}^1(\phi)$	0.00077 ± 0.01215	-0.015 ± 0.014	-0.0035 ± 0.0144	-0.0035 ± 0.0144
$c_{\text{bkg}}^2(\phi)$	-0.008 ± 0.012	0.0022 ± 0.0138	-0.0051 ± 0.0141	-0.0051 ± 0.0141
$c_{\text{bkg}}^1(m_{K\pi})$	0.086 ± 0.011	0.138 ± 0.013	0.111 ± 0.013	0.111 ± 0.013
$c_{\text{bkg}}^2(m_{K\pi})$	-0.082 ± 0.012	-0.077 ± 0.014	-0.103 ± 0.015	-0.103 ± 0.015

Table B.6: Nuisance parameters from single-year $B^0 \rightarrow K^{*0}\psi(2S)$ fits.

Observable	Run 1	2016	2017	2018
N_{bkg}	2063.2 ± 65.0	1921.4 ± 62.3	1646.2 ± 58.7	1646.2 ± 58.7
$N_{\overline{\text{bkg}}}$	2001.8 ± 63.8	1800.7 ± 61.0	1549.5 ± 57.8	1549.5 ± 57.8
$m_{B^0} [\text{MeV}/c^2]$	$5284.0200(\text{fixed})$	$5279.7600(\text{fixed})$	$5279.5100(\text{fixed})$	$5279.5100(\text{fixed})$
$s_{\sigma}^{J/\psi}$	$1.1450(\text{fixed})$	$1.1545(\text{fixed})$	$1.1263(\text{fixed})$	$1.1263(\text{fixed})$
$\tau_m [c^2/\text{MeV}]$	210.4 ± 6.2	188.5 ± 5.3	199.6 ± 6.5	199.6 ± 6.5
$f_{B_s^0}$	0.0099 ± 0.0010	0.008 ± 0.001	0.0122 ± 0.0011	0.0122 ± 0.0011
$\Delta(m_{B^0} - m_{B_s^0}) [\text{MeV}/c^2]$	87.40 ± 0.16	87.40 ± 0.16	87.40 ± 0.16	87.40 ± 0.16
$d_{B^0} [c^2/\text{GeV}]$	$1.6000(\text{fixed})$	$1.6000(\text{fixed})$	$1.6000(\text{fixed})$	$1.6000(\text{fixed})$
$a [c^2/\text{GeV}]$	$3.2925(\text{fixed})$	$3.2925(\text{fixed})$	$3.2925(\text{fixed})$	$3.2925(\text{fixed})$
$r [c^2/\text{GeV}]$	$1.7000(\text{fixed})$	$1.7000(\text{fixed})$	$1.7000(\text{fixed})$	$1.7000(\text{fixed})$
$\Gamma(K^{*0}) [\text{MeV}/c^2]$	$0.0474(\text{fixed})$	$0.0474(\text{fixed})$	$0.0474(\text{fixed})$	$0.0474(\text{fixed})$
$m(K^{*0}) [\text{MeV}/c^2]$	$0.8950(\text{fixed})$	$0.8950(\text{fixed})$	$0.8950(\text{fixed})$	$0.8950(\text{fixed})$
$\Gamma_{1440} [\text{GeV}/c^2]$	$0.2700(\text{fixed})$	$0.2700(\text{fixed})$	$0.2700(\text{fixed})$	$0.2700(\text{fixed})$
$m(K^*(1440)) [\text{GeV}/c^2]$	$1.4250(\text{fixed})$	$1.4250(\text{fixed})$	$1.4250(\text{fixed})$	$1.4250(\text{fixed})$
$c_{\text{bkg}}^1(\cos\theta_l)$	0.074 ± 0.036	0.026 ± 0.037	-0.014 ± 0.042	-0.014 ± 0.042
$c_{\text{bkg}}^2(\cos\theta_l)$	-0.40 ± 0.04	-0.500 ± 0.043	-0.390 ± 0.046	-0.390 ± 0.046
$c_{\text{bkg}}^1(\cos\theta_K)$	0.424 ± 0.036	0.404 ± 0.038	0.341 ± 0.042	0.341 ± 0.042
$c_{\text{bkg}}^2(\cos\theta_K)$	0.066 ± 0.034	-0.060 ± 0.038	-0.059 ± 0.042	-0.059 ± 0.042
$c_{\text{bkg}}^1(\phi)$	-0.051 ± 0.036	0.023 ± 0.038	-0.026 ± 0.042	-0.026 ± 0.042
$c_{\text{bkg}}^2(\phi)$	-0.043 ± 0.036	0.031 ± 0.036	0.0095 ± 0.0403	0.0095 ± 0.0403
$c_{\text{bkg}}^1(m_{K\pi})$	0.107 ± 0.033	0.143 ± 0.035	0.018 ± 0.039	0.018 ± 0.039
$c_{\text{bkg}}^2(m_{K\pi})$	0.0069 ± 0.0353	-0.169 ± 0.038	-0.068 ± 0.042	-0.068 ± 0.042

B.8 Systematic uncertainties

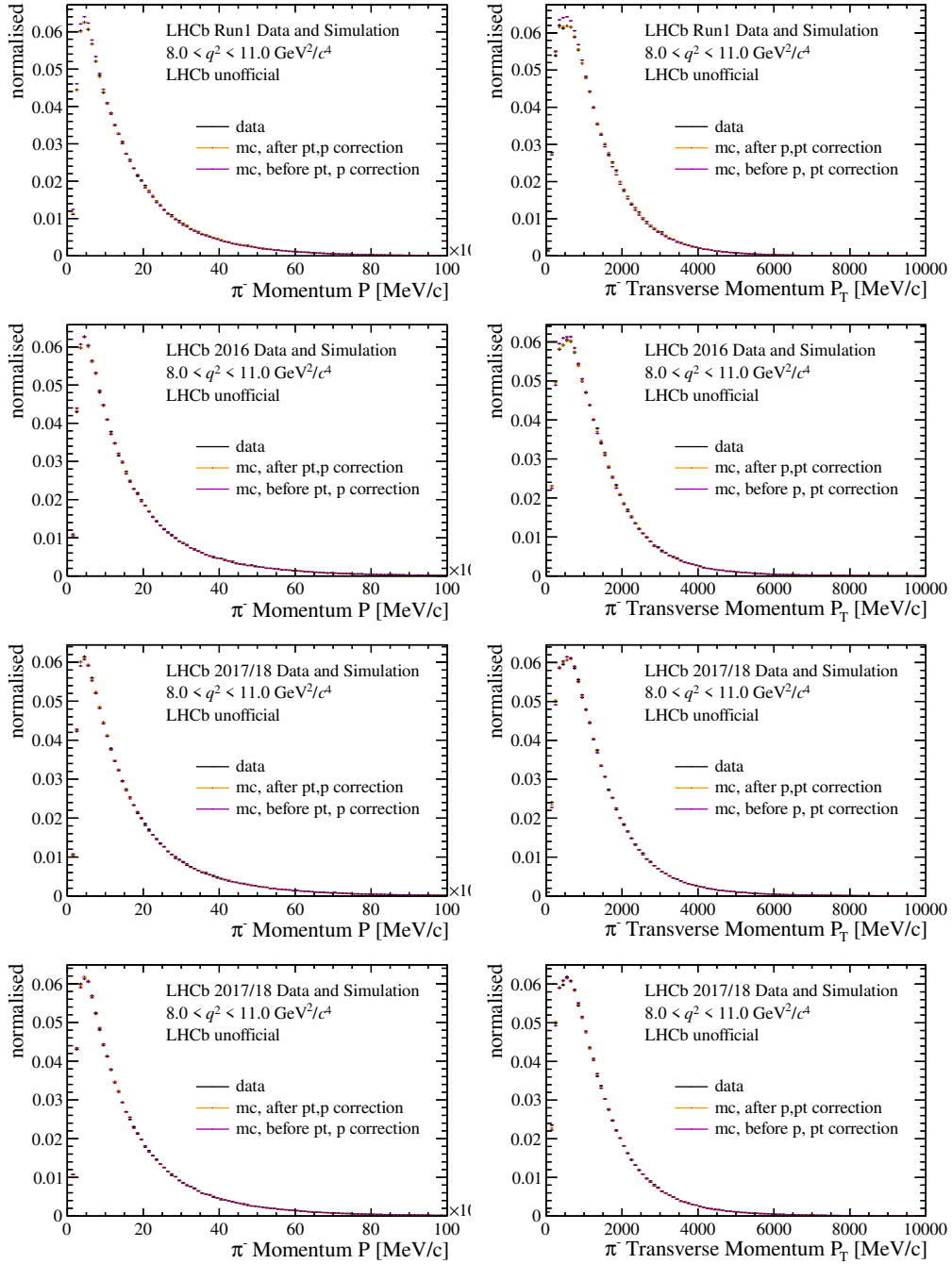


Figure B.35: Comparison between the pion p (left) as well as p_T (right) distributions on non-reweighted simulation, reweighted simulation, and sWeighted data for Run 1 (top), 2016 (second row), 2017 (third row), 2018 (bottom).

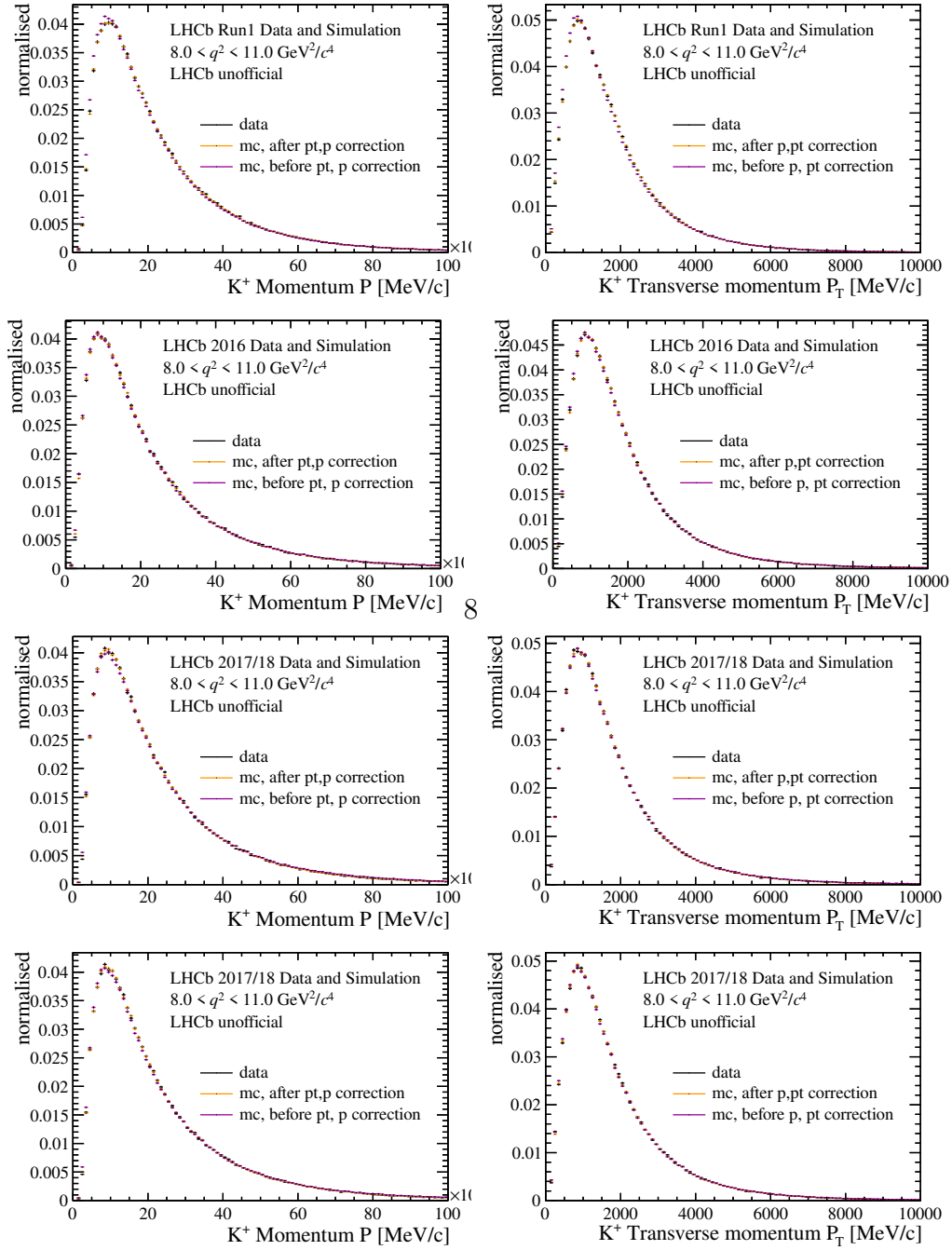


Figure B.36: Comparison between the kaon p (left) as well as p_T (right) distributions on non-reweighted simulation, reweighted simulation, and sWeighted data for Run 1 (top), 2016 (second row), 2017 (third row), 2018 (bottom).

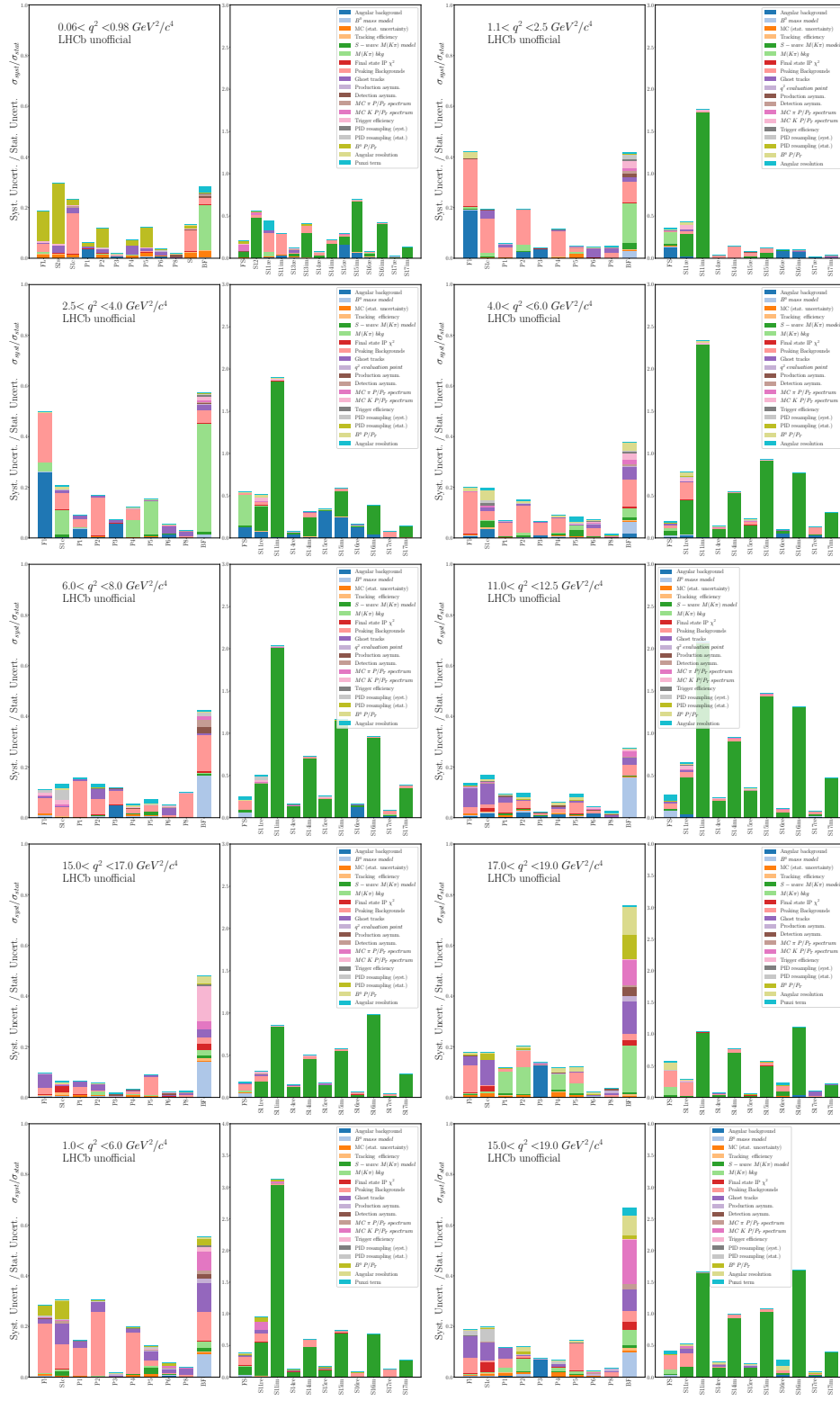


Figure B.37: Systematic uncertainties for all extracted observables in fit configuration (ii) (floated S_1^c , P_i basis). The total uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are combined according to their relative contributions to the total systematic uncertainty.



Figure B.38: Systematic uncertainties for all extracted observables in fit configuration (iii) (floated A_i , S_i basis). The total uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are combined according to their relative contributions to the total systematic uncertainty.

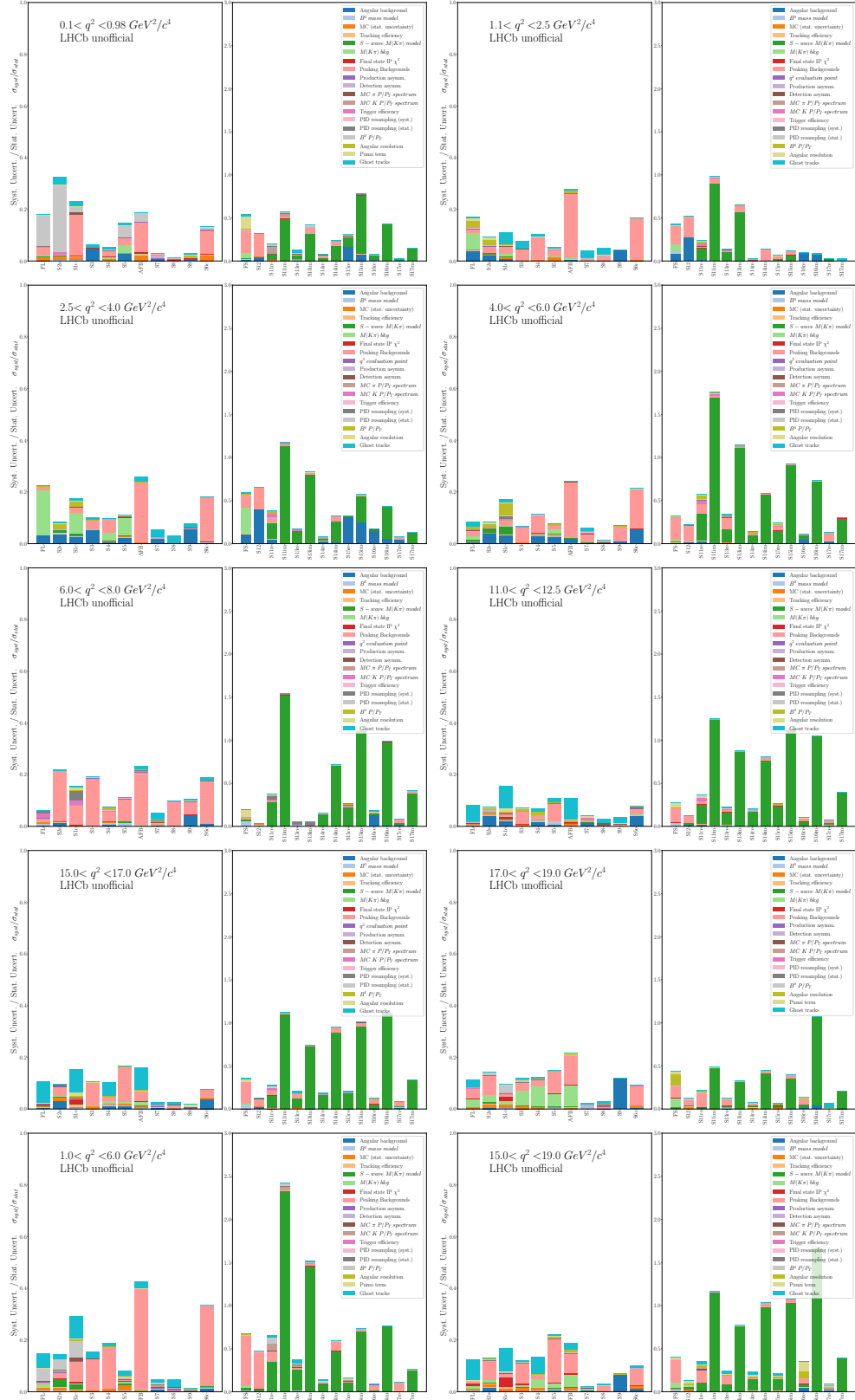


Figure B.39: Systematic uncertainties for all extracted observables in fit configuration (iv) ($m_\ell \neq 0$ S_i basis). The total uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are combined according to their relative contributions to the total systematic uncertainty.

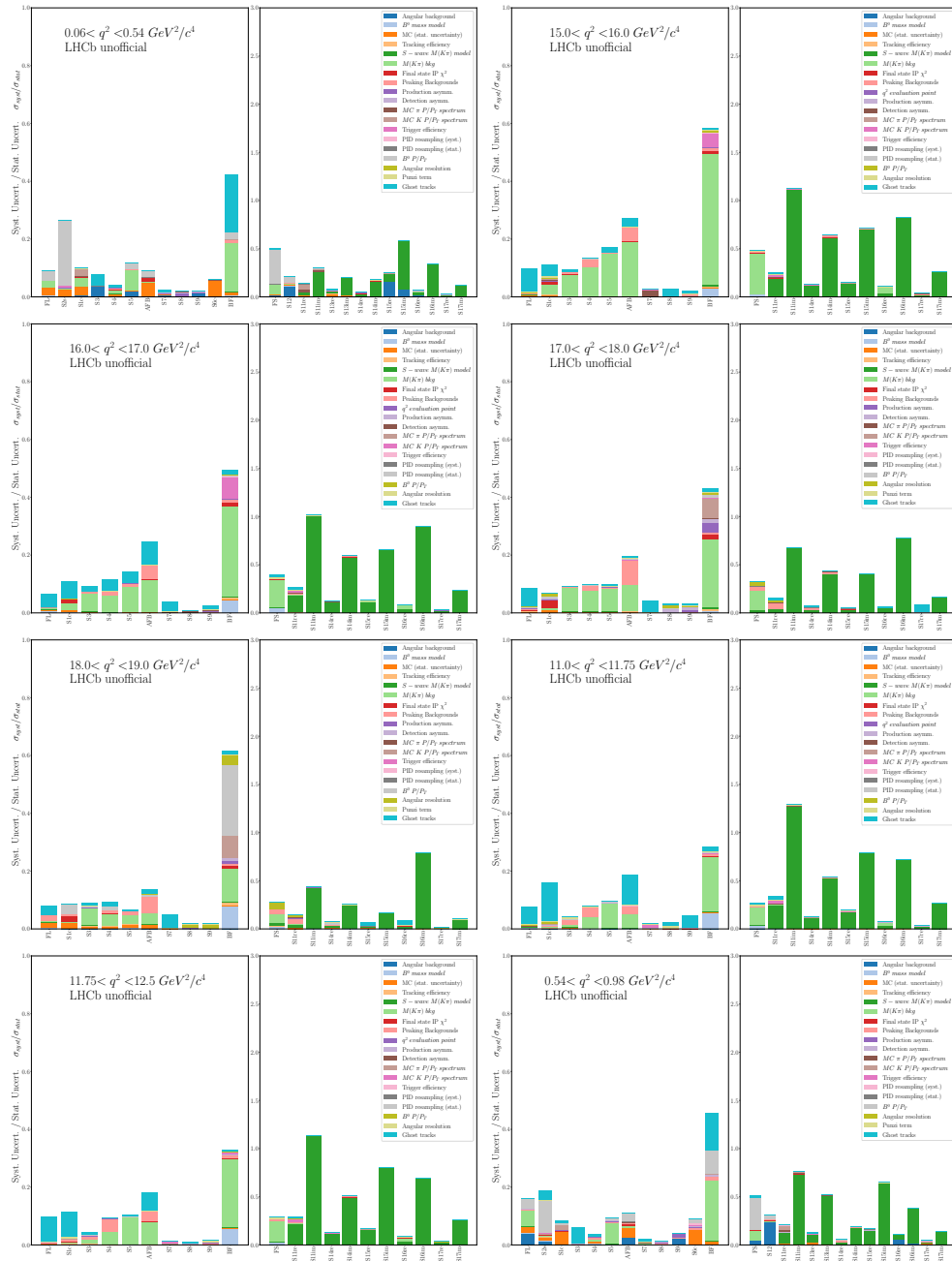


Figure B.40: Systematic uncertainties for all extracted observables in fit configuration (v) (Floated S_1^c , S_i basis). The total uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are combined according to their relative contributions to the total systematic uncertainty.

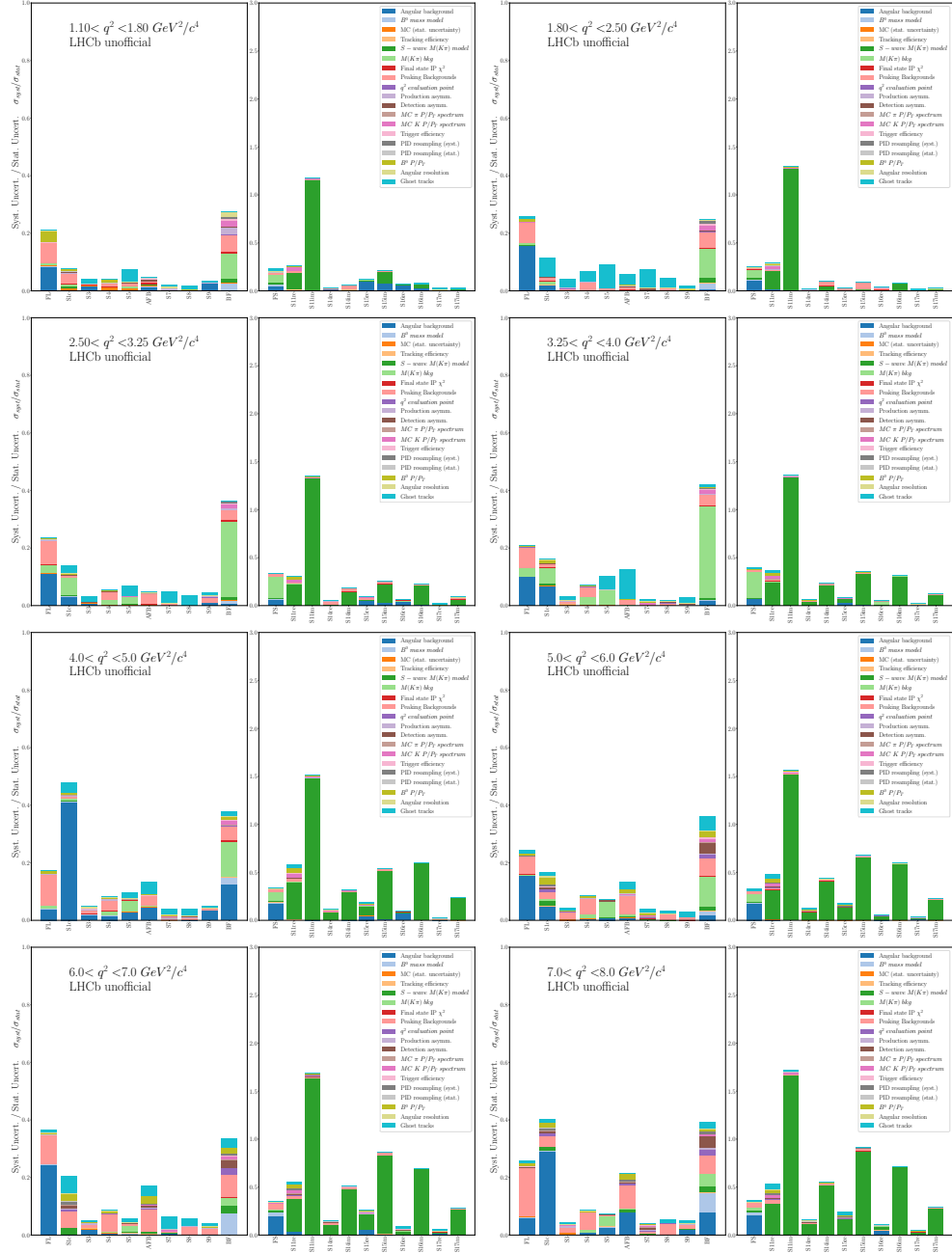


Figure B.41: Systematic uncertainties for all extracted observables in fit configuration (v) (Floated S_1^c , S_i basis). The total uncertainty is shown relative to the expected statistical uncertainty. The contributions of individual sources of uncertainty are combined according to their relative contributions to the total systematic uncertainty.

Table B.7: Systematic uncertainties in q^2 bin $[0.06, 0.98]\text{GeV}^2/c^4$ for fit configuration (i).

q^2 range: $[0.06, 0.98] \text{ GeV}^2/c^4$																				
param.	ang. bkg	B^0 mass	MC stats.	Tracking	LASS model	$m_{K\pi}$ bkg.	w_{kin}	Peaking bkg.	Peaking bkg. J/ψ	$\mathcal{A}_{prod}$	$\tau_{p/\mu}$	$K_{p/\mu}$	L0	PID B kinematics	Ang. Res.	Punzi term	Ghost tracks	Total syst.		
F_L	-0.0013	-0.0001	0.0035	0.0009	0.0016	0.0020	-0.0016	0.0058	0.0000	0.0003	-0.0002	0.0004	-0.0004	0.0012	0.0012	0.0006	-0.0003	-0.0017	0.0137	
S_2^s	0.0011	-0.0000	0.0027	-0.0005	0.0008	0.0008	0.0009	0.0006	0.0000	0.0005	-0.0005	0.0004	-0.0022	0.0006	-0.0108	0.0005	-0.0000	-0.0037	0.0121	
S_1^c	-0.0013	0.0000	0.0023	0.0011	0.0006	-0.0004	-0.0008	-0.0088	0.0000	-0.0003	-0.0000	-0.0021	0.0002	-0.0000	0.0006	0.0031	-0.0003	0.0000	-0.0028	0.0104
S_3	-0.0014	-0.0000	0.0001	0.0000	0.0000	0.0001	0.0000	-0.0002	0.0000	-0.0001	0.0002	-0.0000	-0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.0008	0.0016	
S_4	0.0003	-0.0000	0.0005	0.0000	0.0001	0.0006	0.0003	0.0009	0.0000	0.0003	0.0001	-0.0001	0.0000	-0.0001	0.0001	-0.0004	-0.0001	0.0005	0.0015	
S_5	-0.0016	-0.0001	0.0004	0.0002	0.0001	0.0017	-0.0001	0.0015	0.0000	-0.0001	0.0002	0.0000	0.0000	0.0003	-0.0000	0.0019	0.0006	-0.0002	0.0036	
A_{FB}	-0.0004	-0.0000	0.0017	0.0001	0.0005	-0.0006	0.0008	-0.0037	0.0000	-0.0003	-0.0002	-0.0001	-0.0001	0.0002	0.0004	0.0019	-0.0001	-0.0001	0.0047	
S_7	0.0004	-0.0000	0.0002	0.0000	0.0000	-0.0001	-0.0000	-0.0001	0.0000	-0.0003	0.0005	-0.0000	0.0003	-0.0000	-0.0000	0.0002	0.0003	-0.0000	0.0010	
S_8	0.0000	0.0000	0.0002	0.0000	0.0000	-0.0000	-0.0000	0.0003	0.0000	-0.0002	0.0002	-0.0000	0.0001	0.0000	-0.0001	0.0001	-0.0004	-0.0002	0.0007	
S_9	-0.0004	-0.0000	0.0001	-0.0000	0.0000	-0.0000	-0.0000	-0.0002	0.0000	-0.0004	0.0004	0.0000	0.0000	-0.0000	-0.0000	0.0000	-0.0003	-0.0003	0.0008	
S_6^c	0.0001	-0.0001	0.0036	-0.0005	0.0012	0.0001	0.0007	-0.0071	0.0000	-0.0001	-0.0002	-0.0000	-0.0003	-0.0001	0.0012	-0.0023	-0.0002	0.0007	0.0085	
F_S	0.0031	0.0014	0.0018	0.0002	0.0012	0.0037	0.0005	-0.0121	0.0000	-0.0003	0.0000	0.0003	0.0001	-0.0002	-0.0004	-0.0008	0.0018	0.0086	0.0169	
S_{12}	0.0030	-0.0000	0.0013	-0.0000	0.0005	-0.0001	0.0002	-0.0078	0.0000	-0.0004	0.0001	-0.0000	0.0000	-0.0003	0.0001	-0.0000	-0.0000	0.0004	0.0085	
S_{11}^c	-0.0001	0.0002	0.0048	0.0014	0.0111	-0.0002	-0.0002	-0.0005	0.0000	0.0005	-0.0015	-0.0023	0.0121	-0.0000	-0.0038	0.0074	-0.0005	0.0012	0.0194	
S_{11}^{im}	-0.0004	0.0000	0.0029	0.0011	0.0342	0.0002	-0.0009	0.0077	0.0000	-0.0005	0.0003	-0.0011	0.0090	0.0001	-0.0018	0.0058	-0.0003	0.0001	0.0369	
S_{13}^c	-0.0047	-0.0002	0.0062	0.0001	0.0090	0.0002	0.0007	0.0055	0.0000	0.0006	-0.0025	0.0013	-0.0044	0.0019	-0.0045	0.0002	-0.0025	-0.0015	0.0180	
S_{13}^{im}	-0.0033	-0.0000	0.0046	-0.0002	0.0352	-0.0002	0.0012	0.0181	0.0000	-0.0010	0.0013	0.0010	-0.0033	0.0008	0.0000	-0.0083	-0.0018	-0.0001	0.0412	
S_{14}^c	-0.0009	0.0001	0.0008	0.0002	0.0037	-0.0000	-0.0003	-0.0038	0.0000	-0.0009	0.0007	-0.0003	0.0015	0.0001	-0.0005	0.0013	-0.0003	0.0004	0.0060	
S_{14}^{im}	-0.0005	0.0000	0.0005	0.0002	0.0105	0.0001	-0.0001	-0.0054	0.0000	0.0001	0.0004	-0.0001	0.0011	0.0000	-0.0002	0.0007	0.0003	0.0001	0.0119	
S_{15}^c	0.0136	0.0002	0.0010	0.0004	0.0106	-0.0002	-0.0003	-0.0042	0.0000	0.0001	-0.0004	-0.0008	0.0032	-0.0003	-0.0002	0.0017	0.0007	0.0012	0.0182	
S_{15}^{im}	0.0103	0.0000	0.0007	0.0003	0.0298	0.0002	-0.0003	-0.0033	0.0000	-0.0005	-0.0004	-0.0003	0.0023	-0.0001	-0.0004	0.0021	-0.0000	0.0000	0.0320	
S_{16}^c	0.0017	-0.0000	0.0004	0.0000	0.0025	-0.0010	-0.0001	-0.0010	0.0000	0.0003	-0.0001	-0.0003	0.0003	-0.0002	0.0001	-0.0002	0.0004	0.0003	0.0010	
S_{16}^{im}	-0.0018	-0.0000	0.0004	-0.0001	0.0258	-0.0001	0.0000	0.0007	0.0000	0.0003	-0.0000	0.0001	-0.0005	0.0000	0.0001	-0.0001	0.0007	-0.0000	0.0260	
S_{17}^c	-0.0002	-0.0000	0.0004	0.0000	0.0008	-0.0005	-0.0003	-0.0005	0.0000	0.0004	0.0006	-0.0001	0.0001	0.0001	-0.0000	0.0004	0.0001	-0.0000	0.0015	
S_{17}^{im}	0.0001	-0.0000	0.0005	-0.0000	0.0111	-0.0001	-0.0000	0.0014	0.0000	-0.0004	-0.0012	0.0000	-0.0001	-0.0000	-0.0001	0.0001	-0.0000	-0.0003	0.0112	
B^{red}	-0.0002	0.0012	0.0030	-0.0000	0.0011	-0.0077	-0.0007	-0.0028	0.0020	0.0005	0.0011	0.0018	0.0001	-0.0002	0.0000	-0.0001	0.0001	-0.0000	0.0099	

Table B.8: Systematic uncertainties in q^2 bin $[1.1, 2.5]\text{GeV}^2/c^4$ for fit configuration (i).

q^2 range: [1.1,2.5] GeV ² /c ⁴																				
param.	ang. bkg	B^0 mass	MC stats.	Tracking	LASS model	m_{K^*} bkg.	w_{kin}	Peaking bkg.	Peaking bkg. J/ψ	q^2 eval.	$\mathcal{A}_{prod}$	$\mathcal{A}_{det}$	π_p/μ	K_p/μ	L0	PID B kinematics	Ang. Res.	Ghost tracks	Total syst.	
F_L	-0.0236	-0.0032	0.0024	-0.0002	0.0014	0.0056	-0.0003	0.0226	0.0000	-0.0014	-0.0006	-0.0004	-0.0004	-0.0013	0.0013	0.0003	0.0087	0.0026	-0.0033	0.0349
S_1^e	0.0009	0.0005	0.0010	0.0006	0.0013	0.0024	-0.0002	-0.0075	0.0000	-0.0002	0.0004	0.0002	-0.0010	-0.0001	-0.0003	0.0006	0.0003	0.0004	-0.0038	0.0091
S_3	0.0000	-0.0000	0.0005	-0.0000	0.0000	0.0000	0.0000	-0.0015	0.0000	0.0002	0.0003	0.0001	-0.0000	-0.0000	0.0000	-0.0000	-0.0002	0.0000	0.0015	0.0022
S_4	0.0001	0.0001	0.0012	-0.0001	0.0003	-0.0003	0.0003	0.0044	0.0000	-0.0002	0.0006	0.0004	-0.0001	-0.0000	-0.0000	-0.0003	0.0008	-0.0002	0.0013	0.0049
S_5	-0.0000	-0.0001	0.0012	-0.0000	0.0001	0.0005	-0.0003	0.0019	0.0000	0.0001	-0.0004	-0.0003	0.0001	0.0001	0.0002	-0.0001	0.0003	-0.0002	0.0006	0.0025
A_{FB}	-0.0011	-0.0002	0.0005	0.0001	0.0002	-0.0004	0.0001	-0.0061	0.0000	-0.0003	0.0002	0.0001	-0.0004	-0.0004	0.0000	0.0002	0.0016	0.0005	-0.0014	0.0066
S_7	-0.0000	0.0000	0.0000	0.0000	0.0001	-0.0002	-0.0000	-0.0011	0.0000	-0.0001	0.0006	-0.0004	-0.0000	-0.0000	0.0000	0.0000	-0.0000	-0.0001	-0.0017	0.0022
S_8	-0.0001	-0.0000	0.0001	-0.0000	0.0000	0.0000	-0.0000	-0.0019	0.0000	-0.0004	-0.0001	-0.0002	0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000	-0.0019	0.0027
S_9	-0.0014	-0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0000	-0.0002	0.0000	-0.0001	-0.0003	-0.0002	0.0000	-0.0000	-0.0000	-0.0000	-0.0000	0.0002	0.0001	0.0014
F_5	-0.0048	0.0020	0.0006	-0.0001	0.0020	0.0051	0.0002	0.0007	0.0000	-0.0002	-0.0000	-0.0001	0.0002	0.0002	0.0001	-0.0000	0.0011	0.0014	0.0018	0.0080
S_{11}^e	-0.0064	0.0014	0.0022	0.0014	0.0224	-0.0021	-0.0004	-0.0078	0.0000	-0.0005	0.0007	0.0001	-0.0019	0.0105	-0.0002	0.0013	0.0066	0.0017	-0.0052	0.0284
S_{11}^m	-0.0029	0.0002	0.0012	0.0006	0.0831	0.0003	-0.0005	0.0030	0.0000	-0.0003	-0.0001	-0.0002	-0.0012	0.0086	0.0000	0.0013	0.0052	-0.0003	-0.0032	0.0839
S_{14}^e	0.0003	-0.0000	0.0016	-0.0003	0.0002	-0.0001	0.0001	-0.0032	0.0000	-0.0002	0.0007	-0.0002	0.0000	0.0000	-0.0001	0.0005	-0.0006	0.0001	-0.0001	0.0038
S_{14}^m	0.0002	0.0000	0.0011	-0.0002	0.0025	-0.0000	0.0002	-0.0104	0.0000	-0.0001	-0.0006	-0.0004	0.0000	0.0000	-0.0001	-0.0002	-0.0003	0.0002	0.0002	0.0108
S_{15}^e	-0.0008	0.0001	0.0016	0.0002	0.0033	-0.0001	0.0000	-0.0041	0.0000	0.0003	-0.0006	0.0004	-0.0000	0.0004	0.0001	0.0005	0.0004	-0.0001	-0.0021	0.0060
S_{15}^m	-0.0005	0.0000	0.0010	0.0001	0.0054	0.0002	-0.0002	-0.0050	0.0000	0.0002	-0.0001	0.0001	0.0000	0.0004	0.0001	0.0001	0.0003	0.0006	-0.0001	0.0075
S_{16}^e	0.0056	-0.0000	0.0001	0.0000	0.0016	0.0001	-0.0000	-0.0009	0.0000	0.0000	0.0001	-0.0007	-0.0000	0.0002	-0.0000	0.0000	0.0001	0.0003	0.0007	0.0060
S_{16}^m	-0.0069	-0.0000	0.0001	-0.0000	0.0014	0.0000	0.0000	-0.0001	0.0000	0.0001	-0.0003	0.0008	0.0000	-0.0002	0.0000	-0.0000	-0.0001	0.0013	-0.0034	0.0080
S_{17}^e	-0.0011	0.0000	0.0001	0.0000	0.0003	-0.0001	0.0000	-0.0004	0.0000	0.0001	0.0001	0.0008	-0.0000	0.0001	-0.0000	0.0000	0.0000	-0.0003	-0.0000	0.0015
S_{17}^m	0.0014	-0.0000	0.0002	-0.0000	0.0001	0.0000	0.0000	-0.0004	0.0000	0.0001	0.0005	-0.0012	0.0000	-0.0001	-0.0000	-0.0000	-0.0001	0.0012	0.0028	0.0037
BF^{red}	0.0009	0.0047	0.0021	0.0006	0.0047	-0.0111	0.0023	-0.0080	0.0020	-0.0007	0.0030	0.0033	0.0029	0.0046	0.0028	0.0039	-0.0007	0.0020	-0.0039	0.0184

Table B.9: Systematic uncertainties in q^2 bin $[2.5, 4.0]\text{GeV}^2/c^4$ for fit configuration (i).

q^2 range: [2.5,4.0] GeV ² /c ²																				
param.	ang. bkg	B^0 mass	MC stats.	Tracking	LASS model	m_{K^+} bkg.	w_{kin}	Peaking bkg.	Peaking bkg. J/ψ	q^2 eval.	$\mathcal{A}_{prod}$	$\mathcal{A}_{det}$	π_p/μ	K_p/μ	L0	PID B kinematics	Ang. Res.	Ghost tracks	Total syst.	
F_L	-0.0236	-0.0018	0.0014	-0.0000	0.0009	0.0083	-0.0002	0.0198	0.0000	-0.0001	0.0003	0.0007	-0.0003	-0.0011	0.0003	-0.0000	0.0029	0.0001	-0.0012	0.0322
S_1^e	-0.0011	0.0002	0.0008	0.0005	0.0016	0.0057	-0.0004	-0.0046	0.0000	-0.0002	0.0000	0.0002	-0.0009	-0.0002	-0.0000	-0.0001	0.0019	0.0011	-0.0019	0.0083
S_3	-0.0022	-0.0001	0.0005	-0.0001	0.0001	-0.0000	-0.0000	-0.0016	0.0000	0.0000	-0.0003	-0.0002	-0.0001	-0.0001	-0.0000	0.0000	0.0000	-0.0001	0.0010	0.0030
S_4	-0.0007	0.0000	0.0009	-0.0000	0.0002	-0.0021	-0.0000	0.0019	0.0000	-0.0002	0.0000	0.0001	-0.0002	-0.0001	-0.0001	-0.0001	0.0000	0.0005	-0.0000	0.0031
S_5	-0.0018	0.0001	0.0010	0.0001	0.0006	-0.0033	-0.0003	-0.0013	0.0000	0.0002	0.0002	0.0001	-0.0001	-0.0002	0.0001	0.0002	0.0006	-0.0004	0.0006	0.0043
A_{FB}	-0.0013	-0.0001	0.0004	0.0000	0.0000	0.0000	0.0002	-0.0037	0.0000	-0.0001	0.0000	0.0001	-0.0001	-0.0001	-0.0000	-0.0001	0.0003	0.0002	-0.0018	0.0044
S_7	-0.0012	0.0000	0.0001	-0.0000	0.0002	-0.0002	0.0000	-0.0001	0.0000	-0.0001	-0.0001	-0.0005	0.0000	-0.0005	0.0000	0.0000	-0.0003	0.0004	0.0016	0.0022
S_8	-0.0000	-0.0000	0.0001	-0.0000	0.0001	-0.0000	-0.0000	0.0007	0.0000	0.0001	-0.0003	0.0002	0.0000	-0.0002	0.0000	0.0000	-0.0001	-0.0002	-0.0011	0.0014
S_9	0.0019	0.0000	0.0001	-0.0000	0.0000	-0.0000	0.0000	0.0003	0.0000	-0.0001	-0.0002	0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	-0.0001	-0.0009	0.0022
F_5	-0.0063	0.0016	0.0005	-0.0001	0.0021	0.0104	0.0001	0.0021	0.0000	0.0000	0.0002	0.0001	0.0003	0.0003	0.0000	-0.0002	0.0008	0.0014	0.0018	0.0128
S_{11}^e	-0.0133	0.0012	0.0017	0.0014	0.0256	-0.0060	-0.0004	-0.0088	0.0000	-0.0003	0.0007	-0.0013	-0.0021	0.0103	-0.0000	-0.0006	0.0088	0.0014	-0.0027	0.0339
S_{11}^m	-0.0066	0.0002	0.0011	0.0007	0.0898	0.0001	-0.0005	0.0050	0.0000	0.0002	0.0007	-0.0002	-0.0013	0.0085	-0.0000	-0.0004	0.0067	-0.0005	-0.0029	0.0909
S_{14}^e	-0.0051	-0.0002	0.0011	-0.0002	0.0041	0.0000	0.0004	-0.0010	0.0000	0.0000	-0.0006	0.0004	0.0005	-0.0010	-0.0000	0.0004	-0.0021	-0.0002	0.0005	0.0073
S_{14}^m	-0.0048	-0.0000	0.0009	-0.0001	0.0173	-0.0002	0.0002	-0.0084	0.0000	-0.0001	-0.0012	0.0004	0.0003	-0.0009	-0.0000	-0.0001	-0.0009	-0.0002	-0.0016	0.0200
S_{15}^e	0.0241	-0.0003	0.0011	-0.0003	0.0047	0.0013	0.0001	-0.0012	0.0000	0.0000	-0.0003	-0.0004	0.0004	-0.0015	-0.0000	0.0003	-0.0016	0.0008	0.0010	0.0248
S_{15}^m	0.0199	-0.0001	0.0008	-0.0001	0.0221	0.0001	-0.0001	-0.0062	0.0000	0.0000	-0.0003	0.0001	0.0002	-0.0013	0.0000	-0.0000	-0.0012	0.0006	0.0010	0.0305
S_{16}^e	-0.0088	0.0001	0.0003	-0.0001	0.0011	0.0030	0.0002	0.0018	0.0000	-0.0000	0.0006	0.0007	0.0003	0.0003	0.0000	-0.0001	-0.0001	0.0001	-0.0011	0.0097
S_{16}^m	0.0103	0.0000	0.0001	0.0000	0.0295	0.0016	0.0000	0.0001	0.0000	-0.0001	0.0002	-0.0006	-0.0000	0.0000	0.0000	-0.0001	0.0001	0.0005	0.0001	0.0313
S_{17}^e	0.0020	0.0000	0.0002	-0.0000	0.0004	0.0006	0.0002	0.0044	0.0000	-0.0000	0.0007	0.0001	0.0001	0.0001	-0.0000	-0.0001	-0.0002	0.0008	0.0013	0.0052
S_{17}^m	-0.0031	0.0000	0.0002	0.0000	0.0122	0.0007	-0.0000	-0.0010	0.0000	-0.0002	0.0007	0.0002	-0.0000	0.0000	0.0000	-0.0000	-0.0000	-0.0003	0.0019	0.0128
BF^{red}	-0.0005	0.0037	0.0017	0.0007	0.0034	-0.0218	0.0022	-0.0072	0.0020	-0.0009	0.0021	0.0021	0.0029	0.0040	0.0027	0.0027	0.0017	0.0005	-0.0053	0.0254

Table B.10: Systematic uncertainties in q^2 bin $[4.0, 6.0]\text{GeV}^2/c^4$ for fit configuration (i).

q^2 range: $[4.0, 6.0]$ GeV^2/c^4																				
param.	ang. bkg	B^0 mass	MC stats.	Tracking	LASS model	m_{K^*} bkg.	w_{kin}	Peaking bkg.	Peaking bkg. J/ψ	q^2 eval.	$\mathcal{A}_{prod}$	$\mathcal{A}_{det}$	$\pi_{p/\mu}$	$K_{p/\mu}$	L0	PID B kinematics	Ang. Res.	Ghost tracks	Total syst.	
F_L	0.0021	-0.0012	0.0010	0.0002	0.0003	0.0010	-0.0003	0.0089	0.0000	-0.0002	-0.0001	-0.0003	-0.0004	-0.0009	-0.0000	-0.0006	0.0024	-0.0012	-0.0008	0.0098
S_1^e	-0.0025	0.0001	0.0006	0.0005	0.0022	0.0006	-0.0005	-0.0025	0.0000	-0.0003	0.0003	0.0002	-0.0009	-0.0005	-0.0001	-0.0012	0.0026	0.0018	-0.0016	0.0058
S_3	-0.0001	-0.0001	0.0004	-0.0000	0.0002	-0.0001	-0.0000	-0.0015	0.0000	-0.0000	0.0001	-0.0004	-0.0001	-0.0002	0.0000	-0.0001	0.0002	0.0002	0.0004	0.0017
S_4	-0.0017	0.0003	0.0006	0.0000	0.0003	-0.0006	-0.0001	0.0014	0.0000	-0.0002	-0.0005	-0.0000	-0.0002	-0.0001	-0.0001	-0.0002	-0.0001	-0.0001	0.0000	0.0025
S_5	-0.0009	0.0005	0.0006	0.0001	0.0006	-0.0011	-0.0002	-0.0017	0.0000	-0.0001	0.0002	-0.0002	-0.0001	-0.0003	-0.0000	0.0001	0.0005	-0.0002	0.0000	0.0025
A_{FB}	0.0003	-0.0001	0.0003	-0.0001	0.0004	0.0003	0.0003	-0.0024	0.0000	0.0001	-0.0001	-0.0001	0.0002	0.0003	0.0000	0.0003	-0.0005	-0.0002	-0.0002	0.0026
S_7	-0.0004	0.0000	0.0001	-0.0000	0.0005	0.0001	0.0000	0.0015	0.0000	0.0000	0.0000	0.0001	0.0001	-0.0006	0.0000	0.0003	-0.0005	0.0002	-0.0009	0.0020
S_8	-0.0001	-0.0000	0.0000	-0.0000	0.0001	0.0000	-0.0000	0.0001	0.0000	0.0001	0.0002	-0.0001	0.0000	-0.0002	0.0000	0.0001	-0.0001	-0.0000	0.0001	0.0004
S_9	-0.0007	0.0000	0.0001	0.0000	0.0001	0.0000	0.0000	0.0015	0.0000	0.0000	0.0001	-0.0001	-0.0000	0.0000	-0.0000	-0.0000	0.0000	0.0001	0.0001	0.0017
F_S	0.0009	0.0013	0.0004	-0.0001	0.0019	0.0016	0.0001	-0.0017	0.0000	0.0001	-0.0000	0.0000	0.0002	0.0002	0.0000	-0.0002	0.0004	0.0011	0.0013	0.0039
S_{11}^e	-0.0088	0.0015	0.0014	0.0013	0.0284	-0.0031	-0.0002	-0.0202	0.0000	-0.0000	-0.0006	0.0005	-0.0022	0.0005	0.0001	-0.0048	0.0100	0.0017	-0.0041	0.0393
S_{11}^m	-0.0058	0.0002	0.0008	0.0006	0.0868	-0.0006	-0.0002	0.0037	0.0000	-0.0001	-0.0007	0.0005	-0.0014	0.0076	-0.0000	-0.0037	0.0074	0.0000	-0.0022	0.0879
S_{14}^e	0.0006	-0.0004	0.0009	-0.0003	0.0079	0.0009	0.0004	0.0023	0.0000	0.0000	-0.0000	-0.0007	0.0007	-0.0015	-0.0000	0.0013	-0.0026	0.0001	0.0018	0.0092
S_{14}^m	-0.0011	-0.0000	0.0006	-0.0001	0.0271	-0.0000	0.0001	-0.0026	0.0000	0.0000	0.0001	-0.0001	0.0004	-0.0013	-0.0000	0.0004	-0.0016	-0.0000	0.0013	0.0274
S_{15}^e	0.0003	-0.0006	0.0010	-0.0005	0.0115	0.0015	0.0000	0.0055	0.0000	0.0000	-0.0007	-0.0005	0.0009	-0.0027	-0.0000	0.0018	-0.0038	-0.0006	0.0024	0.0141
S_{15}^m	-0.0013	-0.0000	0.0007	-0.0002	0.0408	0.0000	-0.0000	-0.0035	0.0000	0.0000	-0.0000	-0.0004	0.0004	-0.0022	0.0000	0.0011	-0.0024	-0.0005	0.0015	0.0412
S_{16}^e	0.0035	-0.0002	0.0003	-0.0001	0.0029	-0.0009	0.0002	0.0008	0.0000	0.0002	-0.0001	0.0004	0.0004	0.0003	0.0000	0.0003	-0.0005	-0.0001	-0.0005	0.0048
S_{16}^m	-0.0017	0.0001	0.0001	0.0000	0.0480	0.0013	0.0000	-0.0014	0.0000	0.0001	0.0012	0.0001	-0.0000	0.0002	0.0000	-0.0002	0.0001	-0.0016	0.0006	0.0481
S_{17}^e	0.0038	-0.0002	0.0002	-0.0001	0.0013	-0.0004	0.0002	0.0060	0.0000	-0.0000	0.0002	-0.0005	0.0002	0.0002	0.0000	0.0001	-0.0002	-0.0004	0.0005	0.0073
S_{17}^m	0.0014	0.0000	0.0001	0.0000	0.0202	0.0005	-0.0000	-0.0026	0.0000	0.0000	0.0004	0.0012	-0.0000	0.0000	0.0000	-0.0001	-0.0001	-0.0001	-0.0001	0.0205
BF^{red}	-0.0028	0.0048	0.0013	0.0006	0.0025	-0.0039	0.0020	-0.0071	0.0020	-0.0013	0.0011	0.0015	0.0031	0.0033	0.0021	-0.0006	0.0039	0.0003	-0.0047	0.0134

Table B.11: Systematic uncertainties in q^2 bin $[6.0, 8.0]\text{GeV}^2/c^4$ for fit configuration (i).

q^2 range: [6.0,8.0] GeV ² /c ⁴																				
param.	ang. bkg	B^0 mass	MC stats.	Tracking	LASS model	m_{K^*} bkg.	w_{kin}	Peaking bkg.	Peaking bkg. J/ψ	q^2 eval.	$\mathcal{A}_{prod}$	$\mathcal{A}_{det}$	$\pi_{p/\mu}$	$K_{p/\mu}$	L0	PID B kinematics	Ang. Res.	Ghost tracks	Total syst.	
F_L	0.0007	-0.0014	0.0009	0.0002	0.0003	0.0000	-0.0001	0.0036	0.0000	-0.0001	0.0006	0.0003	-0.0000	0.0016	-0.0002	-0.0014	-0.0003	-0.0007	-0.0012	0.0048
S_1^e	-0.0002	0.0003	0.0005	0.0004	0.0004	0.0000	-0.0004	-0.0018	0.0000	-0.0003	-0.0002	-0.0002	-0.0006	0.0015	-0.0001	-0.0020	-0.0007	0.0013	-0.0007	0.0037
S_3	0.0000	-0.0002	0.0005	0.0000	0.0003	-0.0000	0.0000	-0.0035	0.0000	-0.0001	0.0001	-0.0002	-0.0001	-0.0000	0.0000	-0.0001	0.0001	-0.0002	0.0005	0.0036
S_4	-0.0005	0.0006	0.0005	0.0001	0.0003	-0.0000	-0.0001	-0.0002	0.0000	-0.0002	0.0001	0.0002	-0.0001	0.0006	-0.0001	-0.0003	-0.0007	0.0003	0.0000	0.0015
S_5	-0.0001	0.0009	0.0005	0.0001	0.0007	-0.0001	-0.0001	-0.0026	0.0000	0.0000	-0.0000	0.0003	-0.0001	-0.0000	-0.0000	-0.0001	-0.0000	-0.0002	-0.0003	0.0029
A_{FB}	0.0002	-0.0003	0.0004	-0.0002	0.0002	0.0000	0.0003	-0.0011	0.0000	0.0001	0.0001	-0.0001	0.0003	-0.0007	0.0001	0.0008	0.0007	-0.0002	-0.0005	0.0019
S_7	0.0003	-0.0000	0.0001	-0.0000	0.0002	0.0000	-0.0000	0.0004	0.0000	0.0000	-0.0001	0.0004	-0.0003	-0.0003	0.0000	0.0004	0.0001	-0.0003	0.0011	0.0014
S_8	-0.0001	0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0000	0.0027	0.0000	0.0000	0.0001	-0.0001	-0.0001	-0.0001	-0.0000	0.0001	0.0000	0.0001	0.0003	0.0028
S_9	-0.0016	0.0000	0.0001	-0.0000	0.0001	-0.0000	0.0000	0.0018	0.0000	-0.0000	-0.0000	-0.0001	0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0003	0.0006	0.0025
F_S	0.0002	0.0023	0.0003	-0.0000	0.0017	0.0001	0.0001	0.0029	0.0000	-0.0001	-0.0001	-0.0002	0.0002	0.0002	-0.0000	-0.0001	0.0004	0.0022	0.0006	0.0046
S_{11}^e	0.0004	0.0021	0.0015	0.0007	0.0214	-0.0002	0.0004	0.0031	0.0000	0.0002	-0.0002	0.0003	0.0040	0.0049	0.0001	-0.0069	-0.0008	0.0039	-0.0010	0.0241
S_{11}^m	0.0000	0.0003	0.0007	0.0003	0.0738	-0.0000	0.0003	-0.0012	0.0000	-0.0002	0.0002	-0.0003	0.0032	0.0034	-0.0000	-0.0052	-0.0012	0.0004	0.0003	0.0741
S_{14}^e	0.0005	-0.0007	0.0009	-0.0002	0.0087	0.0001	0.0001	-0.0002	0.0000	-0.0002	-0.0000	-0.0002	-0.0004	-0.0021	0.0001	0.0021	0.0004	-0.0004	-0.0010	0.0094
S_{14}^m	-0.0004	-0.0000	0.0006	-0.0000	0.0320	-0.0000	-0.0001	-0.0046	0.0000	-0.0001	0.0002	0.0000	-0.0004	-0.0012	0.0000	0.0008	-0.0001	0.0012	-0.0003	0.0324
S_{15}^e	0.0004	-0.0012	0.0013	-0.0003	0.0131	0.0002	-0.0002	-0.0019	0.0000	-0.0002	-0.0006	-0.0004	-0.0014	-0.0026	0.0000	0.0035	0.0004	-0.0010	-0.0008	0.0142
S_{15}^m	-0.0005	-0.0001	0.0007	-0.0001	0.0493	0.0000	-0.0002	-0.0009	0.0000	0.0000	-0.0001	0.0000	-0.0013	-0.0015	0.0000	0.0021	0.0004	0.0005	-0.0004	0.0494
S_{16}^e	0.0068	-0.0005	0.0003	-0.0001	0.0027	-0.0001	0.0001	0.0007	0.0000	-0.0000	-0.0003	-0.0000	0.0002	-0.0004	0.0000	0.0007	0.0008	-0.0001	-0.0021	0.0078
S_{16}^m	-0.0070	0.0002	0.0001	0.0000	0.0596	0.0001	0.0000	0.0059	0.0000	0.0000	0.0001	-0.0005	0.0001	0.0002	0.0000	-0.0002	-0.0001	-0.0006	0.0024	0.0604
S_{17}^e	0.0022	-0.0005	0.0002	-0.0000	0.0017	-0.0000	0.0001	0.0029	0.0000	-0.0000	-0.0000	0.0001	0.0001	-0.0002	0.0000	0.0003	0.0005	-0.0007	-0.0011	0.0043
S_{17}^m	-0.0004	0.0000	0.0001	-0.0000	0.0245	0.0000	-0.0000	0.0076	0.0000	0.0000	-0.0002	-0.0007	0.0000	0.0000	0.0000	-0.0000	-0.0001	-0.0010	0.0008	0.0258
BF^{red}	0.0001	0.0083	0.0012	-0.0004	0.0018	-0.0002	0.0017	-0.0078	0.0020	0.0001	0.0031	0.0033	-0.0026	-0.0004	0.0009	-0.0024	-0.0002	0.0017	-0.0017	0.0135

Table B.12: Systematic uncertainties in q^2 bin $[11.0, 12.5]\text{GeV}^2/c^4$ for fit configuration (i).

q^2 range: [11.0,12.5] GeV^2/c^4																				
param.	ang. bkg	B^0 mass	MC stats.	Tracking	LASS model	m_{K^*} bkg.	w_{kin}	Peaking bkg.	Peaking bkg.	J/ψ	q^2 eval.	$\mathcal{A}_{prod}$	$\mathcal{A}_{det}$	$\pi_{p/\mu}$	$K_{p/\mu}$	L0	PID B kinematics	Ang. Res.	Ghost tracks	Total syst.
F_L	-0.0010	-0.0009	0.0008	0.0004	0.0004	-0.0000	-0.0003	0.0019	0.0000	-0.0001	0.0003	0.0001	-0.0004	-0.0004	-0.0004	-0.0002	-0.0004	-0.0011	-0.0032	0.0043
S_1^e	-0.0013	0.0005	0.0005	0.0006	0.0005	0.0000	-0.0010	-0.0011	0.0000	-0.0001	-0.0001	-0.0005	-0.0006	-0.0005	-0.0005	-0.0000	-0.0004	0.0006	0.0014	0.0046
S_3	-0.0004	-0.0002	0.0004	0.0001	0.0004	0.0000	0.0001	-0.0011	0.0000	0.0001	0.0001	-0.0001	0.0000	-0.0003	0.0001	0.0002	0.0004	0.0002	-0.0002	0.0015
S_4	0.0007	0.0007	0.0003	0.0000	0.0002	0.0000	-0.0000	0.0002	0.0000	-0.0001	0.0000	-0.0000	-0.0001	0.0001	-0.0000	-0.0001	-0.0005	0.0000	0.0006	0.0014
S_5	0.0005	0.0010	0.0004	-0.0001	0.0004	0.0000	-0.0000	-0.0027	0.0000	-0.0000	-0.0001	-0.0001	-0.0001	0.0000	0.0000	-0.0000	-0.0002	-0.0005	-0.0004	0.0033
A_{FB}	0.0003	-0.0005	0.0004	-0.0003	0.0003	-0.0000	0.0005	0.0009	0.0000	0.0000	0.0001	0.0001	0.0001	0.0003	0.0003	0.0001	0.0003	0.0001	0.0021	0.0026
S_7	-0.0007	-0.0000	0.0001	-0.0001	0.0002	-0.0000	0.0001	0.0006	0.0000	-0.0000	0.0003	-0.0001	0.0002	-0.0005	0.0000	0.0001	-0.0001	0.0003	-0.0001	0.0012
S_8	-0.0000	0.0000	0.0000	-0.0000	0.0001	-0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0001	0.0002	0.0000	-0.0001	0.0000	-0.0000	-0.0000	0.0003	-0.0005	0.0006
S_9	-0.0002	0.0000	0.0001	-0.0000	0.0001	0.0000	-0.0000	0.0002	0.0000	-0.0000	-0.0002	-0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0000	-0.0003	-0.0006	0.0008
F_S	-0.0006	0.0023	0.0003	-0.0000	0.0013	-0.0001	0.0001	-0.0021	0.0000	-0.0001	0.0002	0.0001	0.0000	0.0000	0.0001	-0.0000	0.0000	0.0007	0.0024	0.0043
S_{11}^e	-0.0055	0.0016	0.0011	0.0012	0.0179	0.0001	-0.0017	-0.0069	0.0000	-0.0005	0.0002	-0.0004	-0.0026	0.0061	-0.0001	-0.0001	-0.0019	0.0023	0.0039	0.0221
S_{11}^m	-0.0037	0.0002	0.0007	0.0006	0.0573	0.0000	-0.0008	0.0003	0.0000	-0.0003	0.0004	0.0002	-0.0020	0.0045	-0.0002	-0.0009	0.0009	0.0010	-0.0005	0.0577
S_{14}^e	-0.0009	-0.0008	0.0011	-0.0004	0.0110	-0.0000	0.0006	0.0026	0.0000	-0.0002	-0.0007	-0.0001	0.0012	-0.0018	0.0000	0.0009	-0.0006	-0.0008	0.0013	0.0118
S_{14}^m	-0.0006	-0.0000	0.0007	-0.0001	0.0373	0.0000	0.0003	-0.0079	0.0000	-0.0000	-0.0007	-0.0008	0.0008	-0.0013	0.0000	0.0005	-0.0004	-0.0002	0.0000	0.0382
S_{15}^e	-0.0009	-0.0013	0.0013	-0.0006	0.0153	-0.0001	0.0011	0.0016	0.0000	0.0001	-0.0003	0.0007	0.0016	-0.0031	0.0001	0.0014	-0.0010	-0.0019	0.0016	0.0163
S_{15}^m	-0.0012	-0.0000	0.0009	-0.0003	0.0535	-0.0000	0.0005	-0.0073	0.0000	0.0001	-0.0005	-0.0001	0.0011	-0.0023	0.0001	0.0007	-0.0005	0.0001	0.0000	0.0541
S_{16}^e	0.0002	-0.0008	0.0003	-0.0001	0.0037	0.0000	0.0003	0.0024	0.0000	-0.0001	-0.0004	0.0004	0.0002	0.0002	-0.0000	0.0001	0.0007	0.0001	-0.0012	0.0048
S_{16}^m	-0.0010	0.0001	0.0002	0.0000	0.0705	-0.0001	-0.0000	0.0013	0.0000	-0.0001	0.0006	0.0001	-0.0000	0.0001	-0.0000	0.0000	0.0001	-0.0011	0.0009	0.0705
S_{17}^e	-0.0016	-0.0008	0.0002	-0.0000	0.0021	-0.0000	0.0001	0.0025	0.0000	-0.0000	-0.0000	0.0001	0.0001	0.0000	0.0000	0.0000	0.0001	0.0003	-0.0004	0.0038
S_{17}^m	-0.0007	-0.0000	0.0001	0.0000	0.0262	-0.0000	0.0000	0.0014	0.0000	0.0000	0.0008	-0.0003	0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0005	0.0006	0.0263
BF^{red}	-0.0005	0.0075	0.0010	0.0005	0.0011	0.0001	-0.0002	-0.0039	0.0020	0.0009	0.0004	0.0005	0.0029	-0.0016	-0.0002	0.0001	-0.0004	-0.0010	-0.0030	0.0100

Table B.13: Systematic uncertainties in q^2 bin $[15.0, 17.0]\text{GeV}^2/c^4$ for fit configuration (i).

q^2 range: $[15.0, 17.0] \text{ GeV}^2/c^4$																				
param.	ang. bkg	B^0 mass	MC stats.	Tracking	LASS model	$m_{K^\pm}$ bkg.	w_{kin}	Peaking bkg.	Peaking bkg. J/ψ	q^2 eval.	$\mathcal{A}_{prod}$	$\mathcal{A}_{det}$	$\pi_{p/\mu}$	$K_{p/\mu}$	L0	PID B kinematics	Ang. Res.	Ghost tracks	Total syst.	
F_L	0.0007	-0.0007	0.0009	0.0005	0.0003	-0.0000	-0.0006	0.0026	0.0000	-0.0003	-0.0000	0.0003	-0.0001	-0.0001	-0.0001	0.0003	-0.0004	-0.0006	-0.0033	0.0046
S_1^e	-0.0004	0.0005	0.0006	0.0005	0.0003	-0.0003	-0.0013	-0.0003	0.0000	-0.0006	0.0000	0.0001	-0.0002	-0.0002	0.0000	0.0002	-0.0006	0.0013	-0.0027	0.0036
S_3	0.0004	-0.0001	0.0006	0.0000	0.0002	-0.0004	-0.0002	-0.0012	0.0000	-0.0001	0.0003	-0.0001	0.0000	-0.0002	0.0000	0.0000	-0.0001	0.0001	0.0003	0.0015
S_4	-0.0008	0.0005	0.0004	-0.0001	0.0001	-0.0005	0.0001	-0.0015	0.0000	-0.0000	-0.0004	0.0001	-0.0001	0.0000	0.0000	-0.0001	-0.0002	-0.0003	0.0013	0.0024
S_5	-0.0008	0.0006	0.0005	-0.0001	0.0001	-0.0006	0.0001	-0.0036	0.0000	-0.0001	-0.0000	-0.0001	-0.0001	0.0000	-0.0000	0.0000	-0.0001	-0.0002	0.0003	0.0038
A_{FB}	0.0002	-0.0005	0.0005	-0.0003	0.0002	0.0007	0.0006	-0.0001	0.0000	0.0003	0.0000	-0.0000	0.0001	0.0001	-0.0000	-0.0001	0.0005	-0.0002	0.0022	0.0026
S_7	0.0003	0.0000	0.0001	-0.0001	0.0001	0.0000	-0.0000	-0.0001	0.0000	-0.0003	0.0001	-0.0001	0.0002	-0.0002	0.0000	-0.0000	-0.0000	-0.0003	0.0004	0.0007
S_8	-0.0000	0.0000	0.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0004	0.0000	-0.0001	-0.0001	0.0002	0.0000	-0.0000	-0.0000	-0.0000	-0.0000	0.0003	0.0003	0.0006
S_9	0.0001	0.0000	0.0001	-0.0000	0.0001	0.0000	0.0000	0.0001	0.0000	0.0001	0.0001	-0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0001	-0.0001	0.0001	0.0003
F_5	0.0006	0.0021	0.0004	0.0000	0.0003	0.0013	0.0002	-0.0025	0.0000	0.0004	0.0001	-0.0001	0.0001	-0.0000	0.0001	-0.0000	-0.0002	0.0010	0.0011	0.0040
S_{11}^e	0.0019	0.0010	0.0011	0.0013	0.0105	-0.0008	-0.0006	-0.0062	0.0000	-0.0004	-0.0001	-0.0002	-0.0023	0.0035	-0.0003	0.0004	0.0014	0.0015	-0.0039	0.0139
S_{11}^m	0.0013	0.0000	0.0008	0.0007	0.0414	-0.0000	0.0001	-0.0017	0.0000	0.0008	-0.0002	0.0001	-0.0016	0.0024	-0.0002	0.0003	0.0002	-0.0006	-0.0021	0.0417
S_{14}^e	0.0012	-0.0006	0.0013	-0.0005	0.0087	0.0007	0.0004	0.0018	0.0000	0.0000	0.0001	-0.0005	0.0010	-0.0013	0.0001	-0.0002	-0.0007	-0.0010	0.0027	0.0097
S_{14}^m	0.0007	-0.0001	0.0009	-0.0003	0.0355	-0.0000	0.0000	-0.0105	0.0000	-0.0003	0.0002	0.0003	0.0006	-0.0009	0.0001	0.0001	0.0003	0.0003	0.0021	0.0372
S_{15}^e	0.0010	-0.0007	0.0012	-0.0007	0.0090	0.0008	0.0006	0.0004	0.0000	0.0000	0.0000	0.0000	0.0011	-0.0016	0.0001	-0.0004	-0.0006	-0.0010	0.0027	0.0099
S_{15}^m	0.0006	0.0000	0.0009	-0.0003	0.0385	-0.0001	0.0000	-0.0079	0.0000	-0.0004	0.0002	0.0002	0.0007	-0.0011	0.0001	0.0000	0.0003	0.0009	0.0012	0.0394
S_{16}^e	0.0002	-0.0008	0.0003	-0.0001	0.0039	-0.0002	0.0004	0.0036	0.0000	0.0007	-0.0003	-0.0005	0.0001	-0.0000	0.0000	-0.0001	0.0004	-0.0011	0.0008	0.0057
S_{16}^m	0.0004	0.0001	0.0002	0.0001	0.0684	0.0007	0.0000	0.0012	0.0000	0.0002	0.0004	0.0003	-0.0000	0.0001	0.0000	-0.0000	-0.0005	-0.0005	-0.0000	0.0684
S_{17}^e	0.0017	-0.0007	0.0002	-0.0000	0.0016	0.0002	0.0001	0.0028	0.0000	0.0003	-0.0005	-0.0002	0.0000	0.0000	-0.0000	-0.0000	0.0001	-0.0004	0.0001	0.0038
S_{17}^m	0.0005	-0.0001	0.0002	-0.0000	0.0196	0.0000	-0.0000	0.0010	0.0000	-0.0001	0.0008	-0.0001	-0.0000	0.0000	0.0000	-0.0000	-0.0001	0.0003	-0.0004	0.0197
BF^{red}	-0.0008	0.0094	0.0021	0.0023	0.0027	-0.0038	-0.0041	-0.0037	0.0020	0.0001	-0.0001	0.0002	0.0043	-0.0005	-0.0023	0.0007	0.0045	-0.0007	-0.0047	0.0177

Table B.15: Systematic uncertainties in q^2 bin $[1.1, 6.0]\text{GeV}^2/c^4$ for fit configuration (i).

q^2 range: [1.1,6.0] GeV^2/c^4																					
param.	ang. bkg	B^0 mass	MC stats.	Tracking	LASS model	$m_{K\pi}$	bkg.	w_{kin}	Peaking bkg.	J/ψ	$\mathcal{A}_{prod}$	$\mathcal{A}_{det}$	$\pi_{p/\mu}$	$K_{p/\mu}$	$L0$	PID	B kinematics	Ang. Res.	Punzi term	Ghost tracks	Total syst.
F_L	0.0005	-0.0016	0.0019	0.0001	0.0005	0.0009	-0.0004	0.0093	0.0000	0.0013	0.0006	-0.0007	-0.0009	0.0003	-0.0000	0.0045	0.0003	0.0002	-0.0031	0.0113	
S_1^c	-0.0002	-0.0001	0.0011	0.0007	0.0017	0.0008	-0.0009	-0.0038	0.0000	-0.0004	-0.0005	-0.0013	0.0002	-0.0001	-0.0003	0.0033	0.0008	0.0000	-0.0034	0.0068	
S_3	-0.0000	-0.0000	0.0005	-0.0000	0.0001	-0.0000	-0.0000	-0.0022	0.0000	0.0000	-0.0002	-0.0001	-0.0001	0.0000	-0.0001	0.0001	-0.0001	-0.0000	0.0009	0.0024	
S_4	-0.0004	0.0002	0.0008	0.0000	0.0003	-0.0003	0.0001	0.0029	0.0000	-0.0003	0.0002	-0.0002	0.0000	-0.0001	-0.0001	0.0001	-0.0000	-0.0000	0.0010	0.0032	
S_5	-0.0001	0.0003	0.0008	0.0000	0.0006	-0.0005	-0.0004	-0.0005	0.0000	-0.0002	-0.0001	-0.0001	-0.0002	0.0001	0.0002	0.0006	-0.0003	-0.0000	0.0008	0.0017	
A_{FB}	-0.0001	-0.0000	0.0004	-0.0000	0.0001	0.0000	0.0003	-0.0033	0.0000	0.0000	0.0001	0.0000	0.0000	-0.0000	-0.0001	0.0003	-0.0000	0.0000	-0.0021	0.0039	
S_7	-0.0005	0.0000	0.0001	-0.0000	0.0002	-0.0000	0.0000	-0.0000	0.0000	0.0001	-0.0000	0.0001	-0.0004	0.0000	0.0001	-0.0004	-0.0003	-0.0000	-0.0005	0.0010	
S_8	-0.0001	0.0000	0.0002	-0.0000	0.0001	-0.0000	0.0000	0.0003	0.0000	-0.0000	0.0002	0.0000	-0.0002	0.0000	0.0000	-0.0001	0.0005	-0.0000	-0.0010	0.0012	
S_9	-0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	-0.0000	0.0000	0.0000	0.0001	0.0001	-0.0000	0.0000	-0.0000	-0.0000	0.0000	-0.0004	-0.0000	-0.0001	0.0004	
F_S	0.0005	0.0017	0.0004	-0.0001	0.0029	0.0014	0.0002	-0.0030	0.0000	-0.0001	0.0001	0.0003	0.0001	0.0001	-0.0003	0.0008	0.0014	-0.0002	0.0012	0.0052	
S_{11}^{re}	-0.0038	0.0015	0.0016	0.0018	0.0254	-0.0019	-0.0010	-0.0127	0.0000	-0.0001	-0.0004	-0.0029	0.0117	-0.0002	-0.0018	0.0090	0.0017	-0.0002	-0.0084	0.0337	
S_{11}^{im}	-0.0030	-0.0000	0.0011	0.0009	0.0882	0.0000	-0.0011	0.0014	0.0000	-0.0001	-0.0000	-0.0020	0.0098	-0.0001	-0.0018	0.0095	0.0004	0.0000	-0.0055	0.0896	
S_{14}^{re}	-0.0003	-0.0003	0.0011	-0.0004	0.0048	0.0004	0.0006	-0.0002	0.0000	0.0003	-0.0003	0.0007	-0.0010	-0.0000	0.0005	-0.0022	-0.0001	0.0000	0.0019	0.0060	
S_{14}^{im}	-0.0007	-0.0000	0.0010	-0.0001	0.0172	0.0000	0.0003	-0.0080	0.0000	0.0004	0.0001	0.0004	-0.0007	-0.0000	-0.0001	-0.0011	-0.0002	0.0000	0.0010	0.0191	
S_{15}^{re}	-0.0009	-0.0004	0.0011	-0.0003	0.0051	0.0005	0.0002	0.0008	0.0000	-0.0003	0.0000	0.0006	-0.0018	0.0000	0.0007	-0.0020	-0.0003	0.0001	0.0014	0.0063	
S_{15}^{im}	-0.0011	-0.0000	0.0009	-0.0001	0.0222	-0.0000	0.0001	-0.0044	0.0000	-0.0004	0.0000	0.0003	-0.0014	0.0001	0.0002	-0.0016	-0.0004	-0.0000	0.0011	0.0228	
S_{16}^{re}	0.0004	-0.0001	0.0003	-0.0001	0.0008	-0.0004	0.0003	0.0019	0.0000	-0.0006	0.0000	0.0003	0.0002	0.0000	-0.0000	-0.0003	0.0008	0.0000	-0.0002	0.0025	
S_{16}^{im}	0.0005	0.0000	0.0002	-0.0000	0.0296	0.0006	0.0000	-0.0003	0.0000	0.0003	0.0001	-0.0000	0.0001	0.0000	-0.0001	0.0001	-0.0003	0.0000	-0.0011	0.0296	
S_{17}^{re}	0.0013	-0.0001	0.0002	-0.0001	0.0005	-0.0003	0.0002	0.0041	0.0000	-0.0000	-0.0000	0.0002	0.0001	-0.0000	-0.0000	-0.0002	-0.0003	0.0000	0.0007	0.0044	
S_{17}^{im}	0.0006	0.0001	0.0003	-0.0000	0.0124	0.0003	-0.0000	-0.0004	0.0000	-0.0005	-0.0002	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0007	0.0000	0.0014	0.0125	
BF^{red}	-0.0002	0.0056	0.0013	0.0009	0.0025	-0.0027	0.0015	-0.0061	0.0020	0.0025	0.0024	0.0022	0.0051	0.0024	0.0016	0.0031	0.0013	-0.0002	-0.0063	0.0139	

B.9 Angular projections

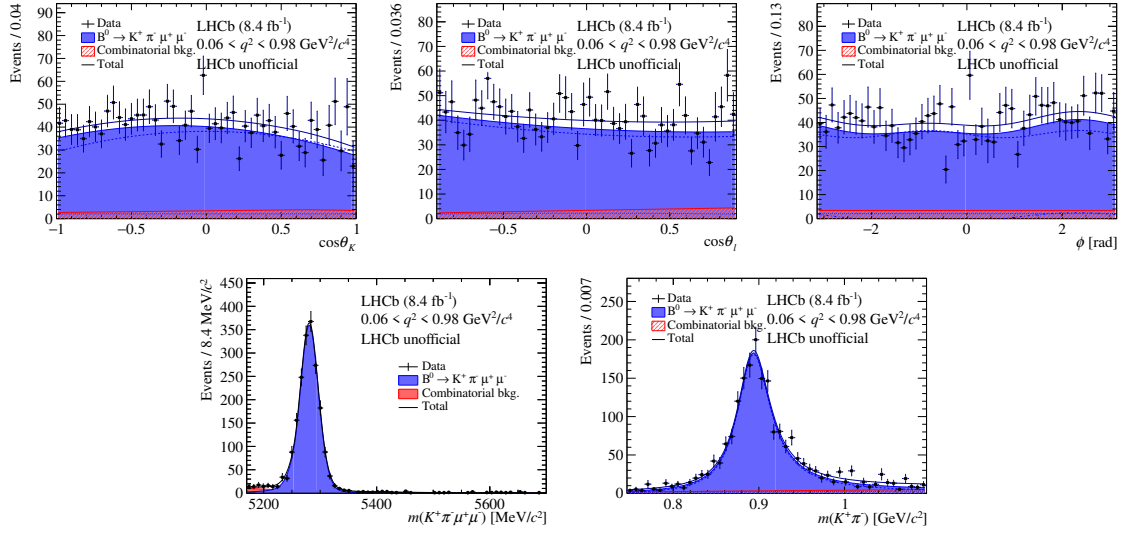


Figure B.42: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$.

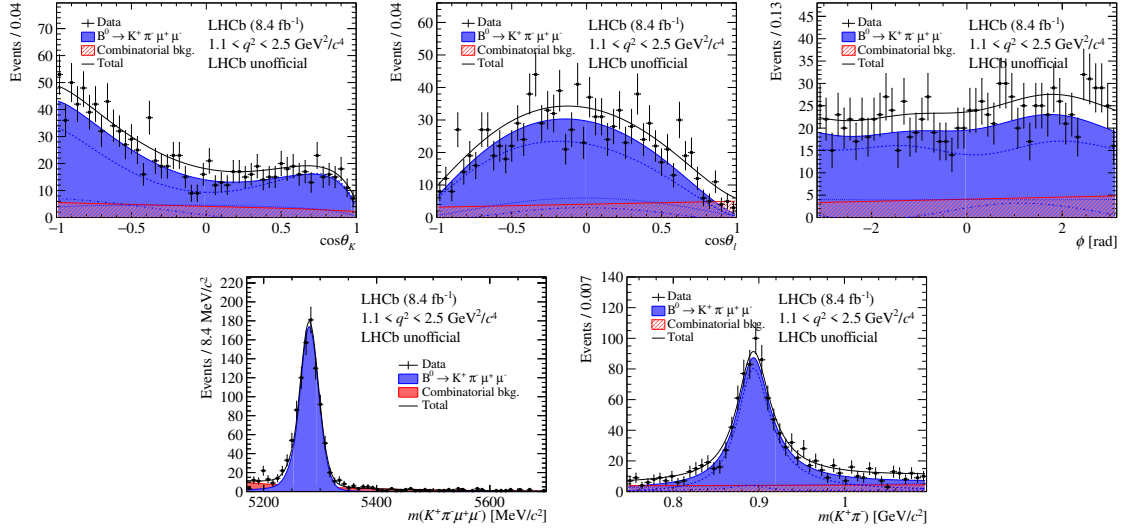


Figure B.43: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$.

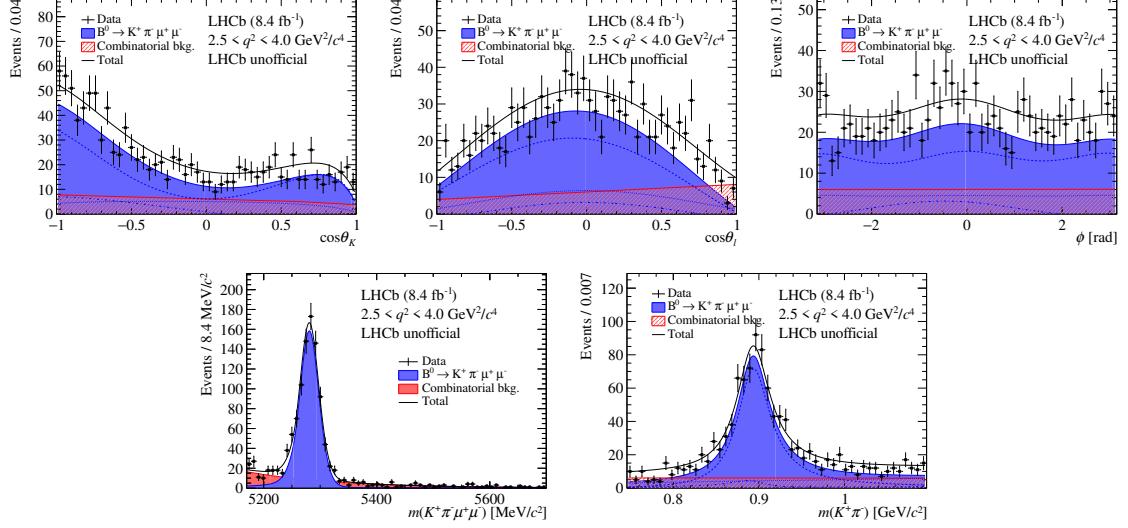


Figure B.44: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$.

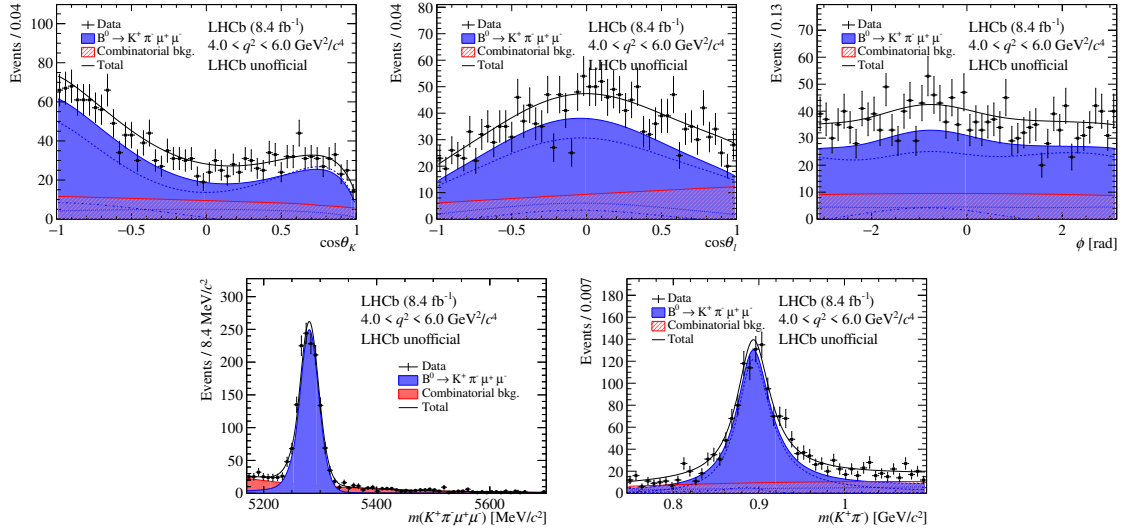


Figure B.45: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$.

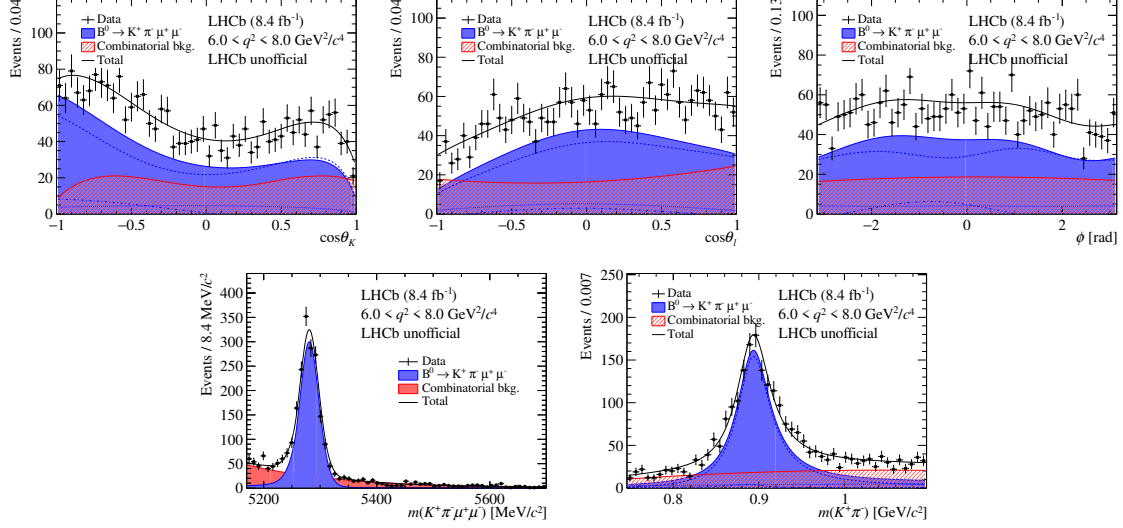


Figure B.46: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$.

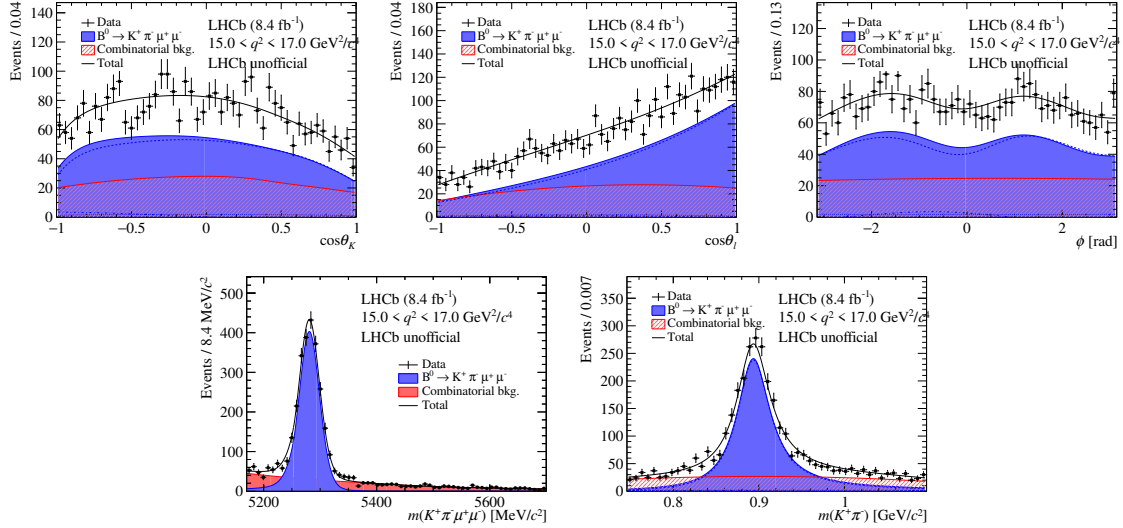


Figure B.47: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $15.0 < q^2 < 17.0 \text{ GeV}^2/c^4$.

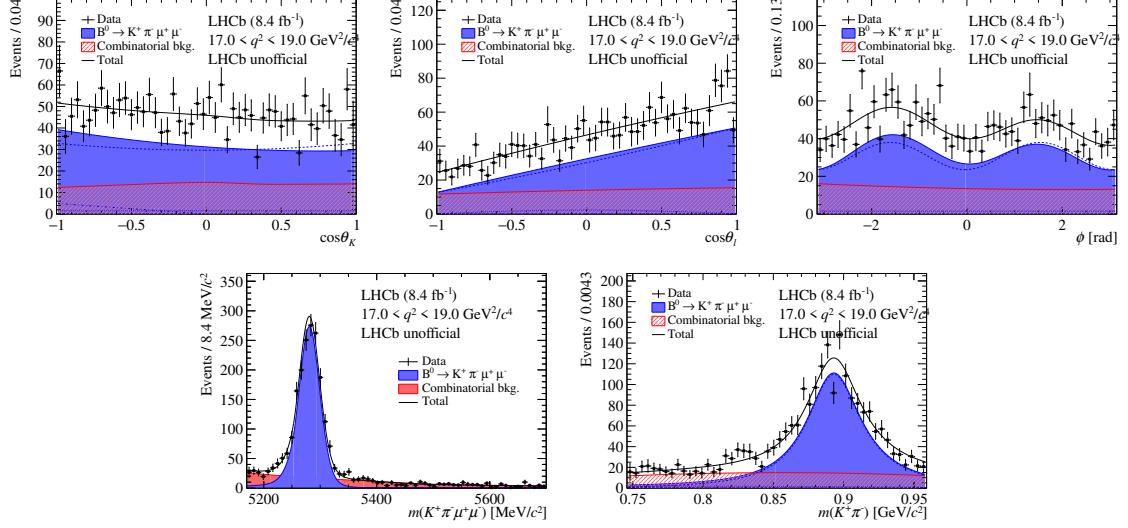


Figure B.48: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $17.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

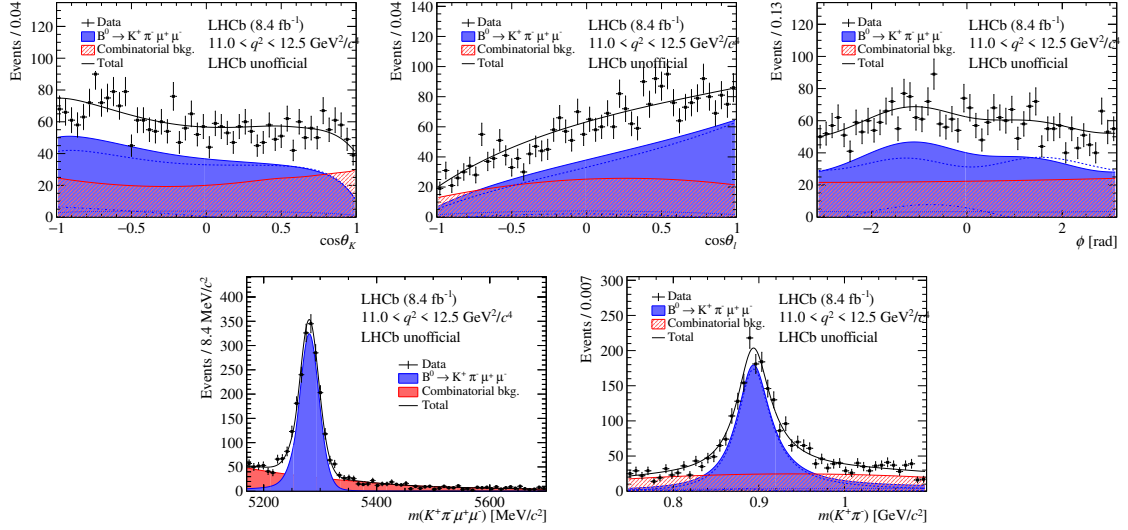


Figure B.49: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $11.0 < q^2 < 12.5 \text{ GeV}^2/c^4$.

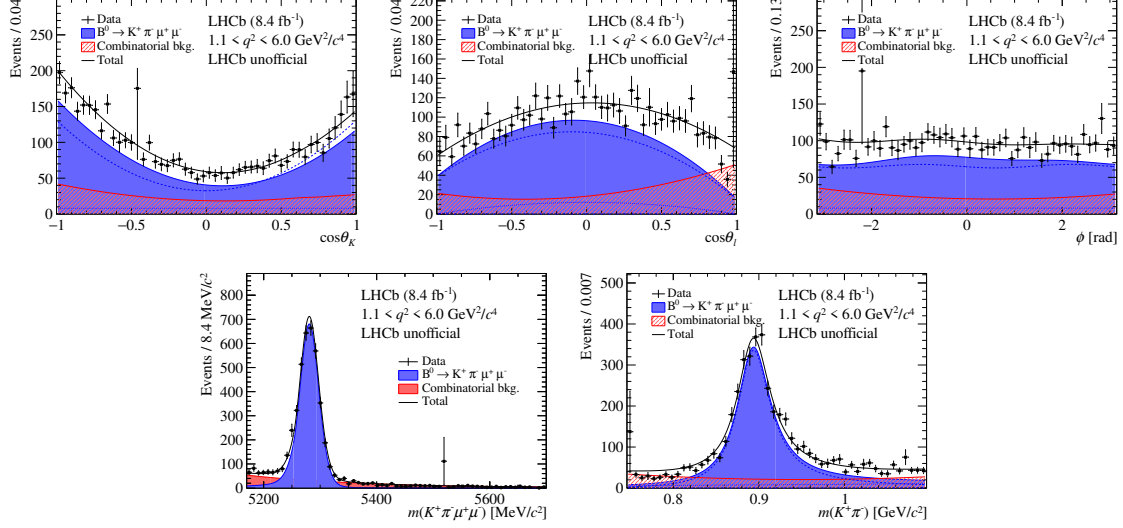


Figure B.50: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$.

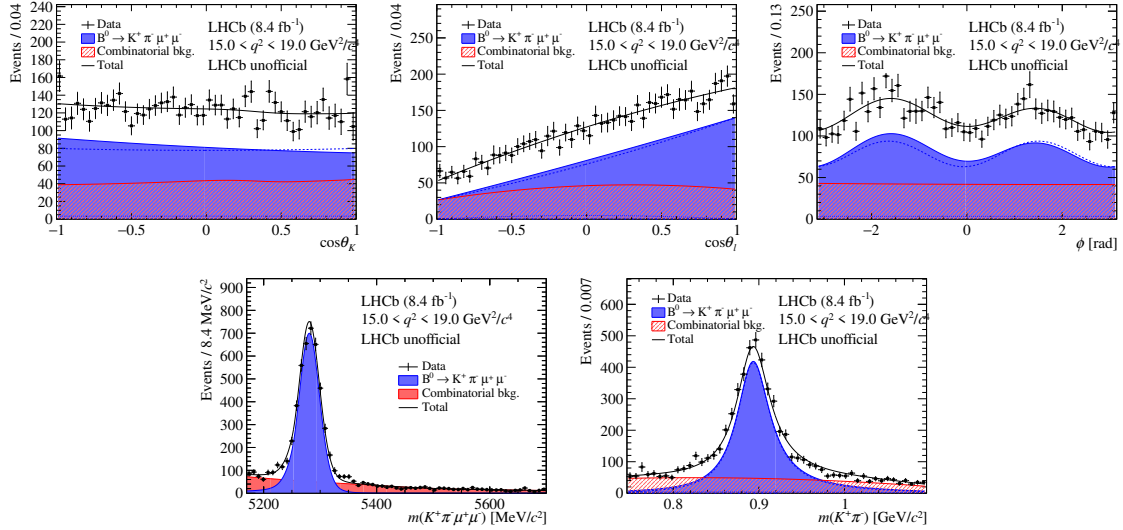


Figure B.51: Combined fit projections of the angular rare mode fits to data in $\cos\theta_\ell$, $\cos\theta_K$, ϕ , $m_{K\pi}$ and $m_{K\pi\mu\mu}$ in $15.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

B.10 Angular rare mode fit results

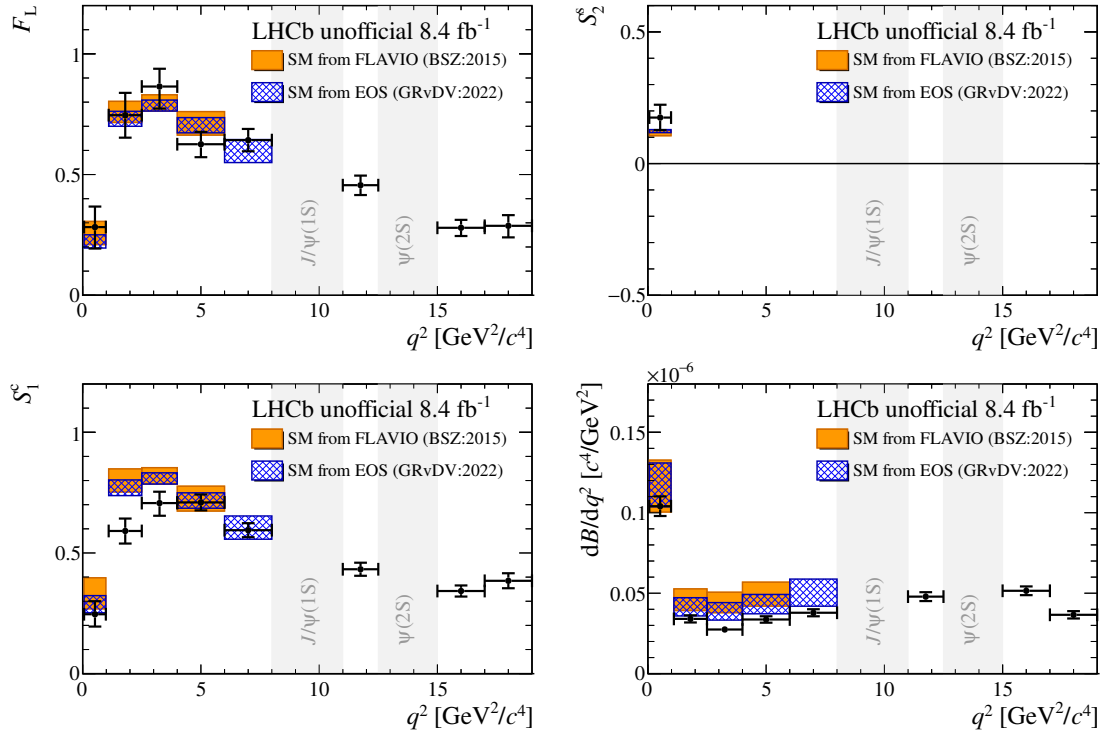


Figure B.52: Results for the angular observables F_L , S_2^s , S_1^c , and dB/dq^2 extracted from fit configuration (i) in bins of q^2 .

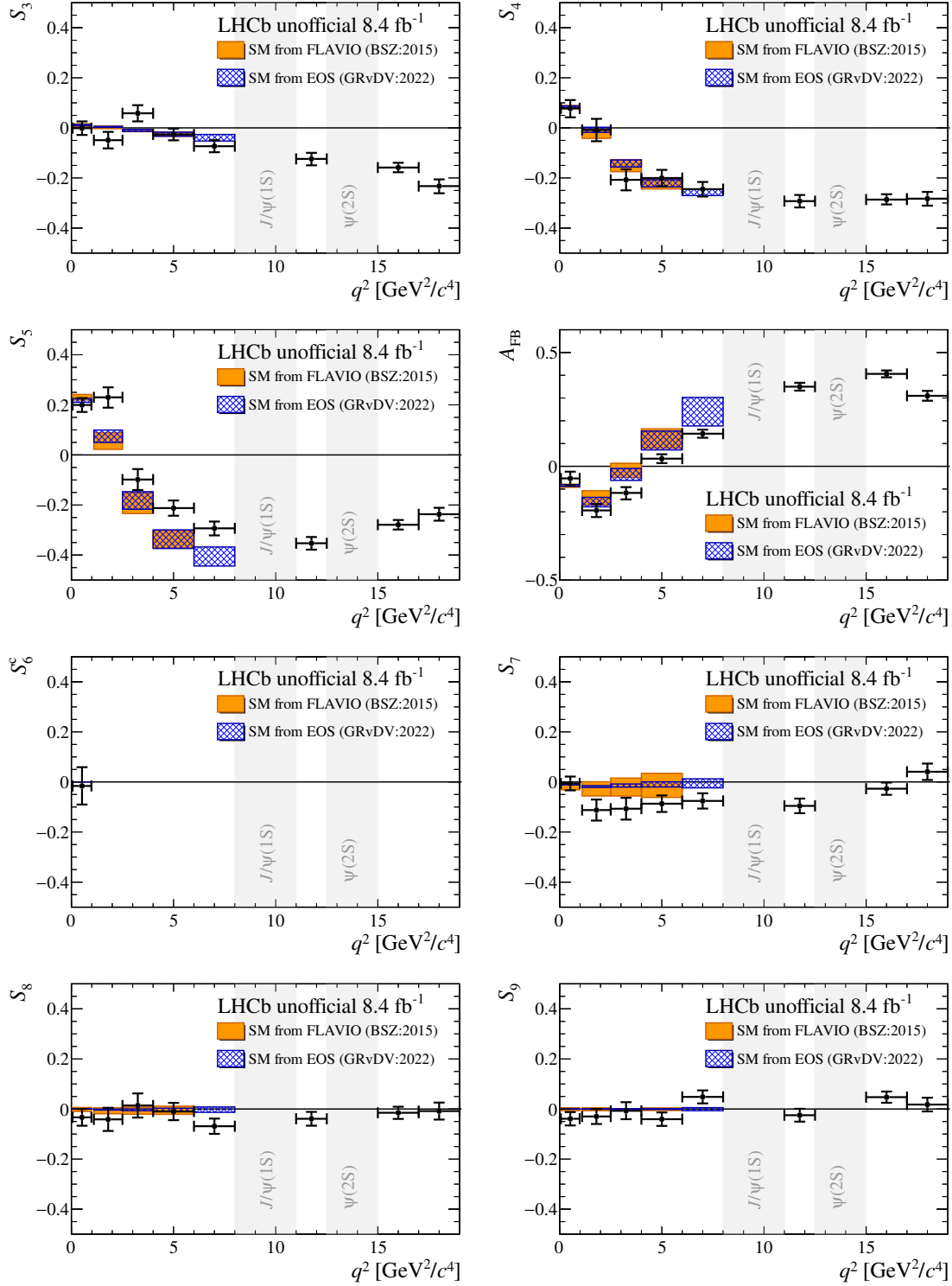


Figure B.53: Results for the angular observables A_{FB} and S_3 - S_9 extracted from fit configuration (i) in bins of q^2 .

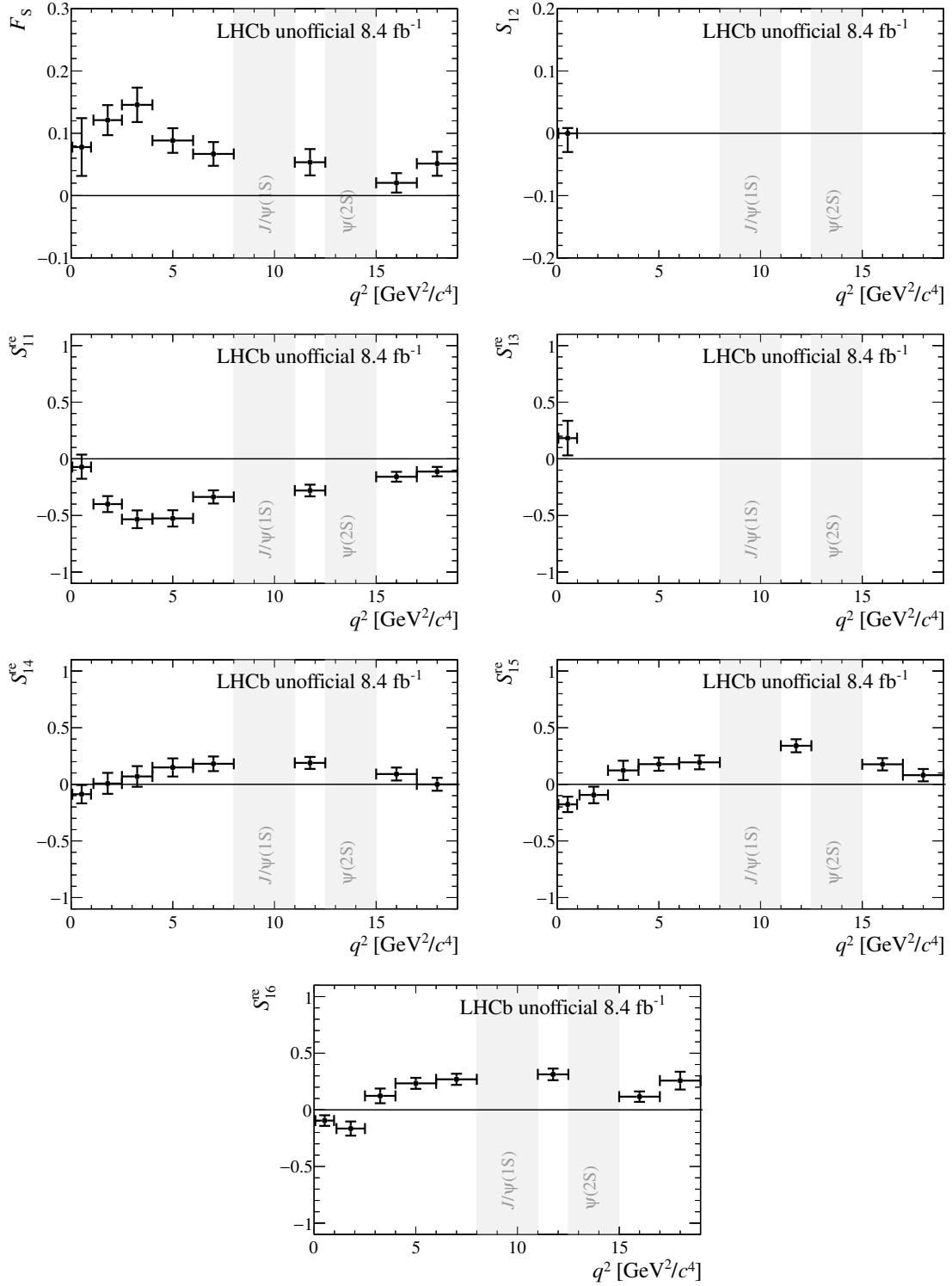


Figure B.54: Results for the angular observables F_S, S_{12} and $S_{11}^{re}, S_{14}^{re}, S_{15}^{re}, S_{16}^{re}$ extracted from fit configuration (i) in bins of q^2 .

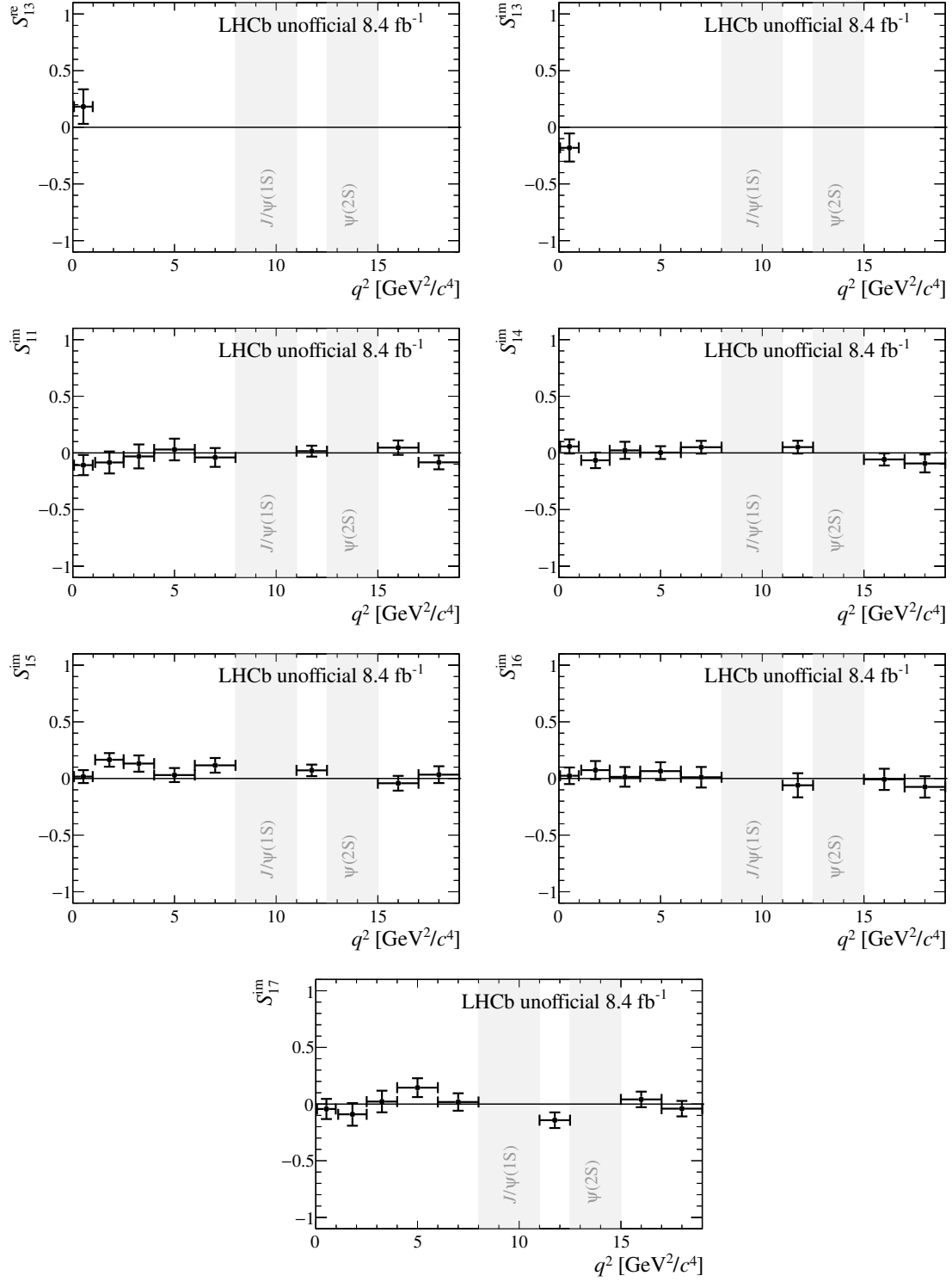


Figure B.55: Results for the angular observables S_{13}^{re} , S_{13}^{im} and $S_{11}^{im}, S_{14}^{im}-S_{17}^{im}$ extracted from fit configuration (i) in bins of q^2 .

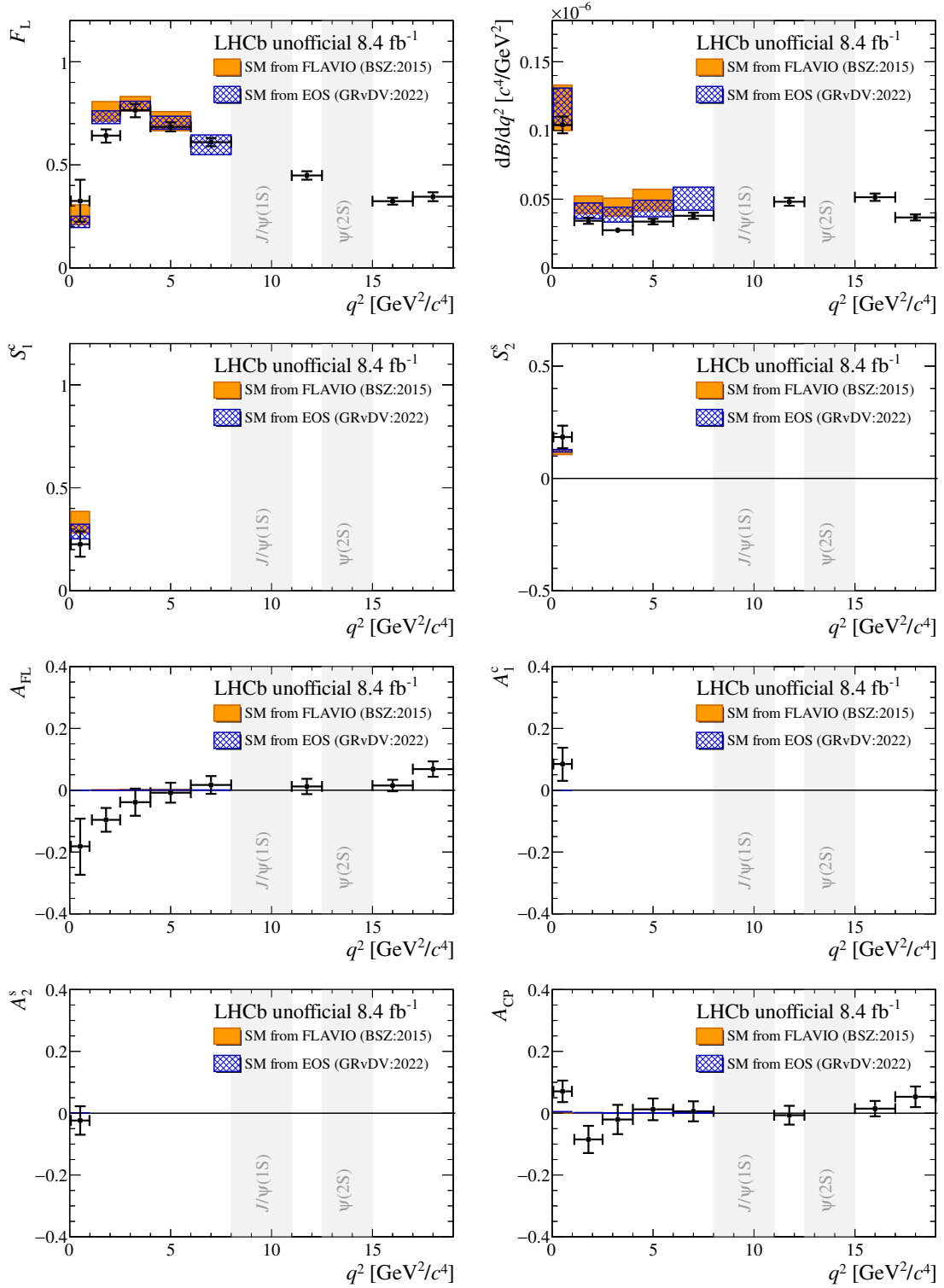


Figure B.56: Results for the angular observables F_L , S_2^s , S_1^c , dB/dq^2 , A_{FL} , A_2^s , A_1^c and A_{CP} extracted from fit configuration (iii) in bins of q^2 .

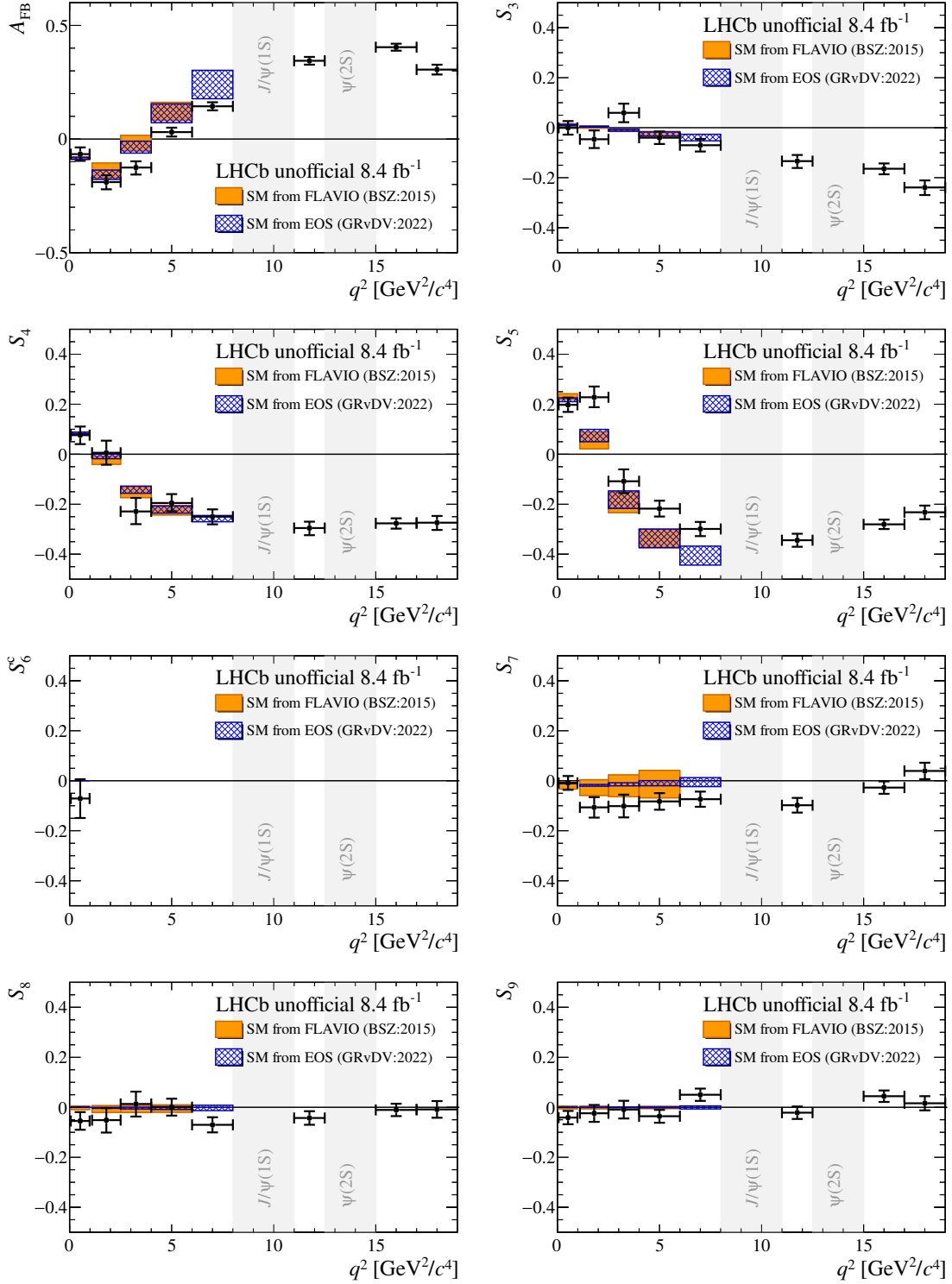


Figure B.57: Results for the angular observables A_{FB} and S_3 - S_9 extracted from fit configuration (iii) in bins of q^2 .

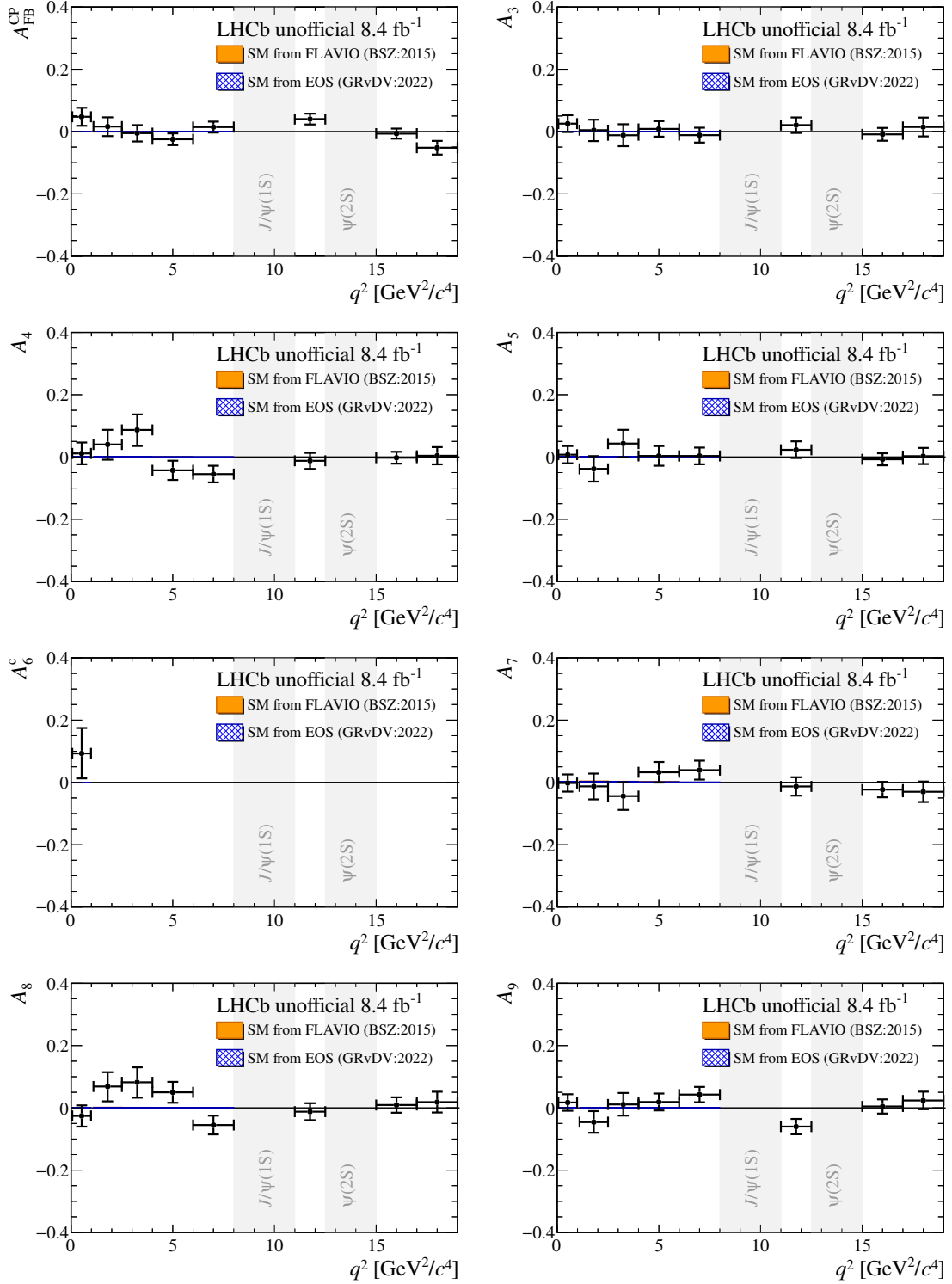


Figure B.58: Results for the angular observables $A_{\text{FB}}^{\text{CP}}$ and A_3 - A_9 extracted from fit configuration (iii) in bins of q^2 .

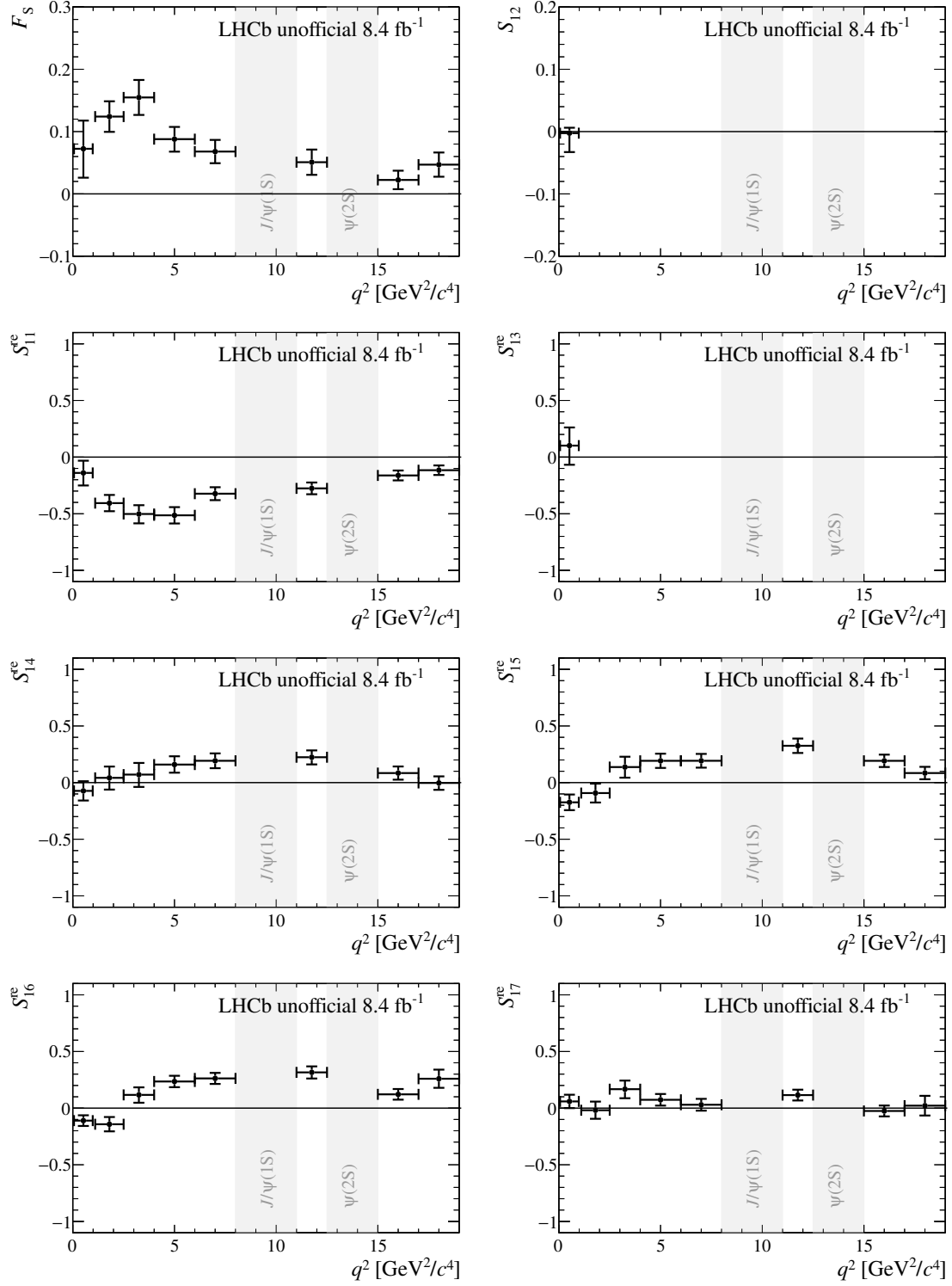


Figure B.59: Results for the angular observables F_S , S_{12} , and $S_{11}^{\text{re}}-S_{17}^{\text{re}}$ extracted from fit configuration (iii) in bins of q^2 .

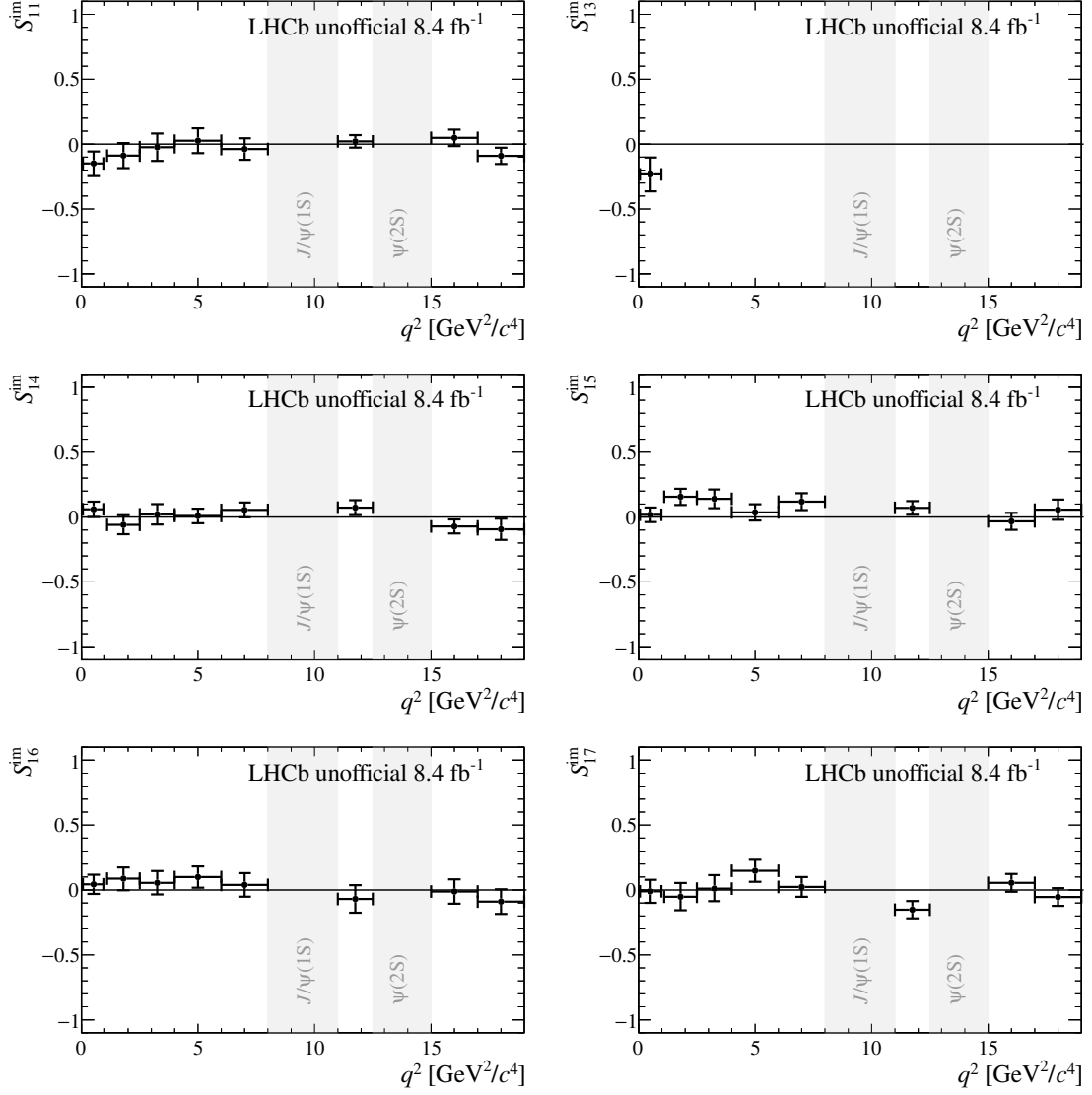


Figure B.60: Results for the angular observables S_{11}^{im} - S_{17}^{im} extracted from fit configuration (iii) in bins of q^2 .

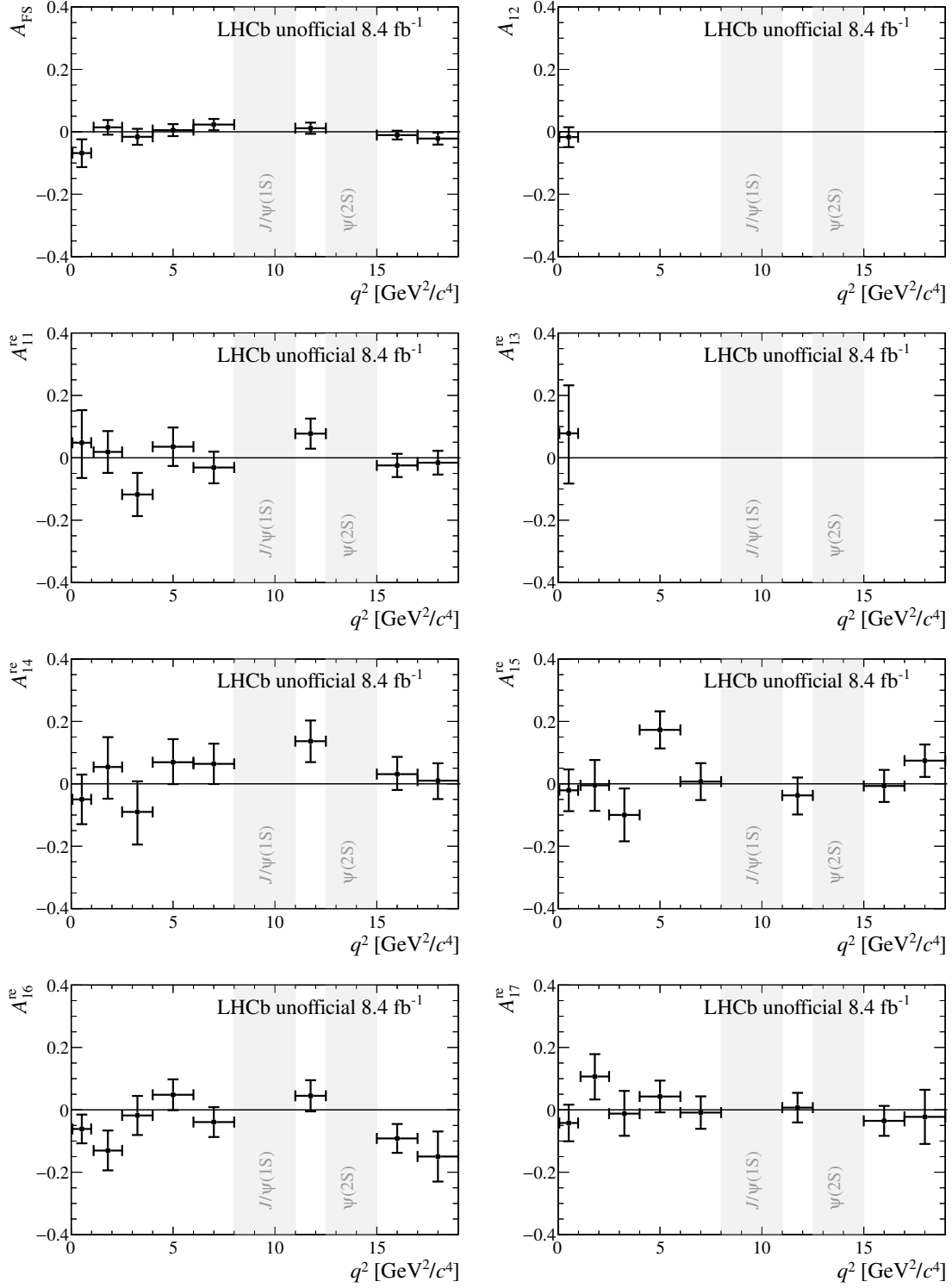


Figure B.61: Results for the angular observables A_{FS} , A_{12} and A_{11}^{re} - A_{17}^{re} extracted from fit configuration (iii) in bins of q^2 .

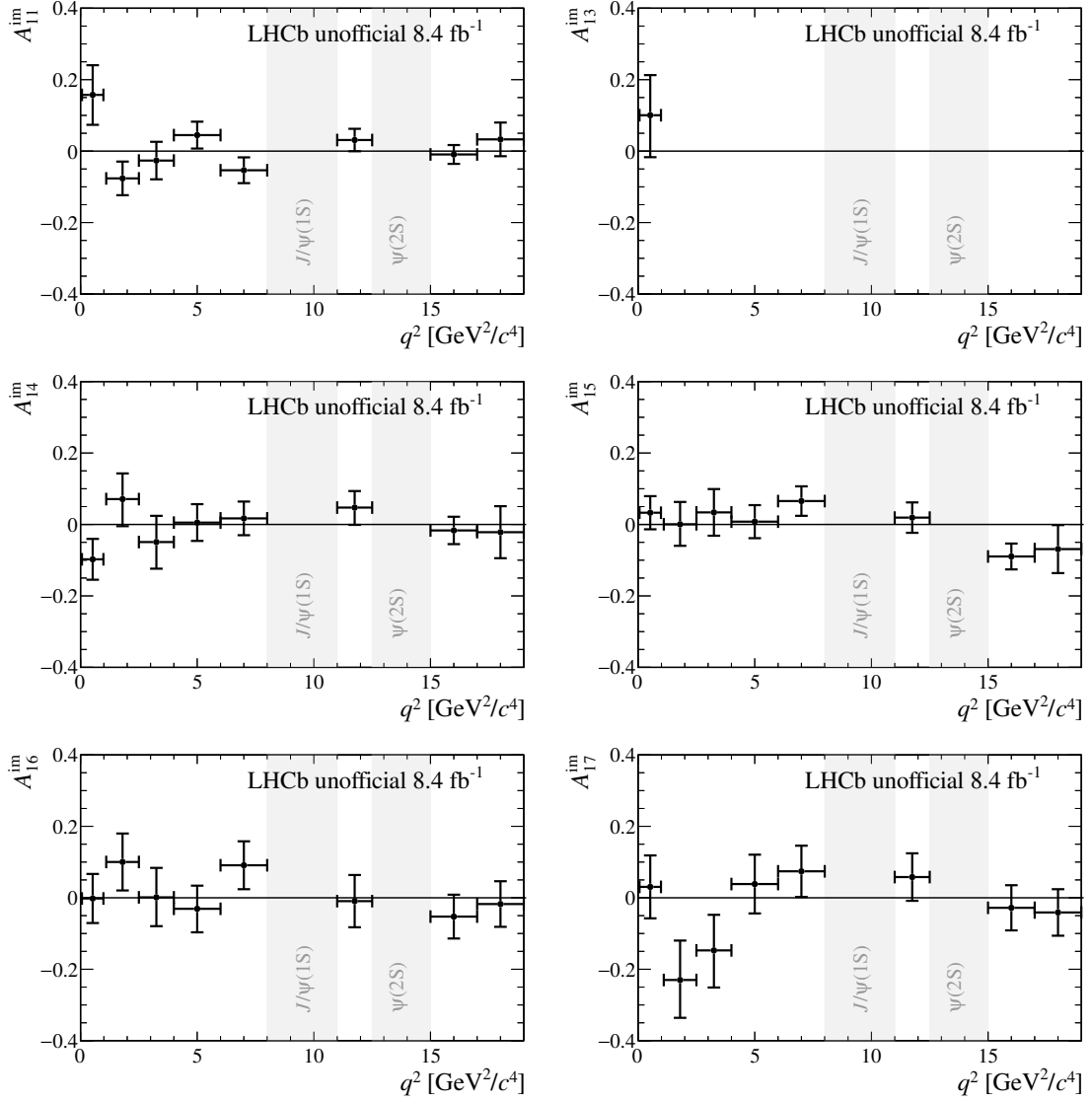


Figure B.62: Results for the angular observables A_{11}^{im} - A_{17}^{im} extracted from fit configuration (iii) in bins of q^2 .

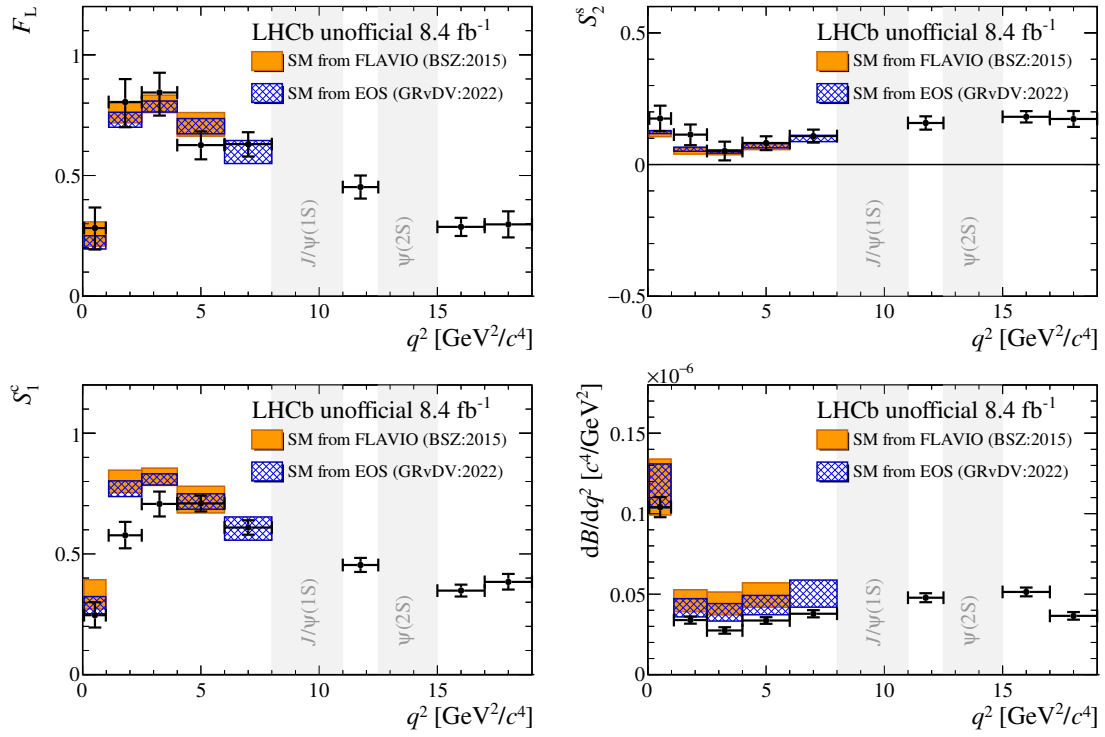


Figure B.63: Results for the angular observables F_L , S_2^s , S_1^c , and dB/dq^2 extracted from fit configuration (iv) in bins of q^2 .

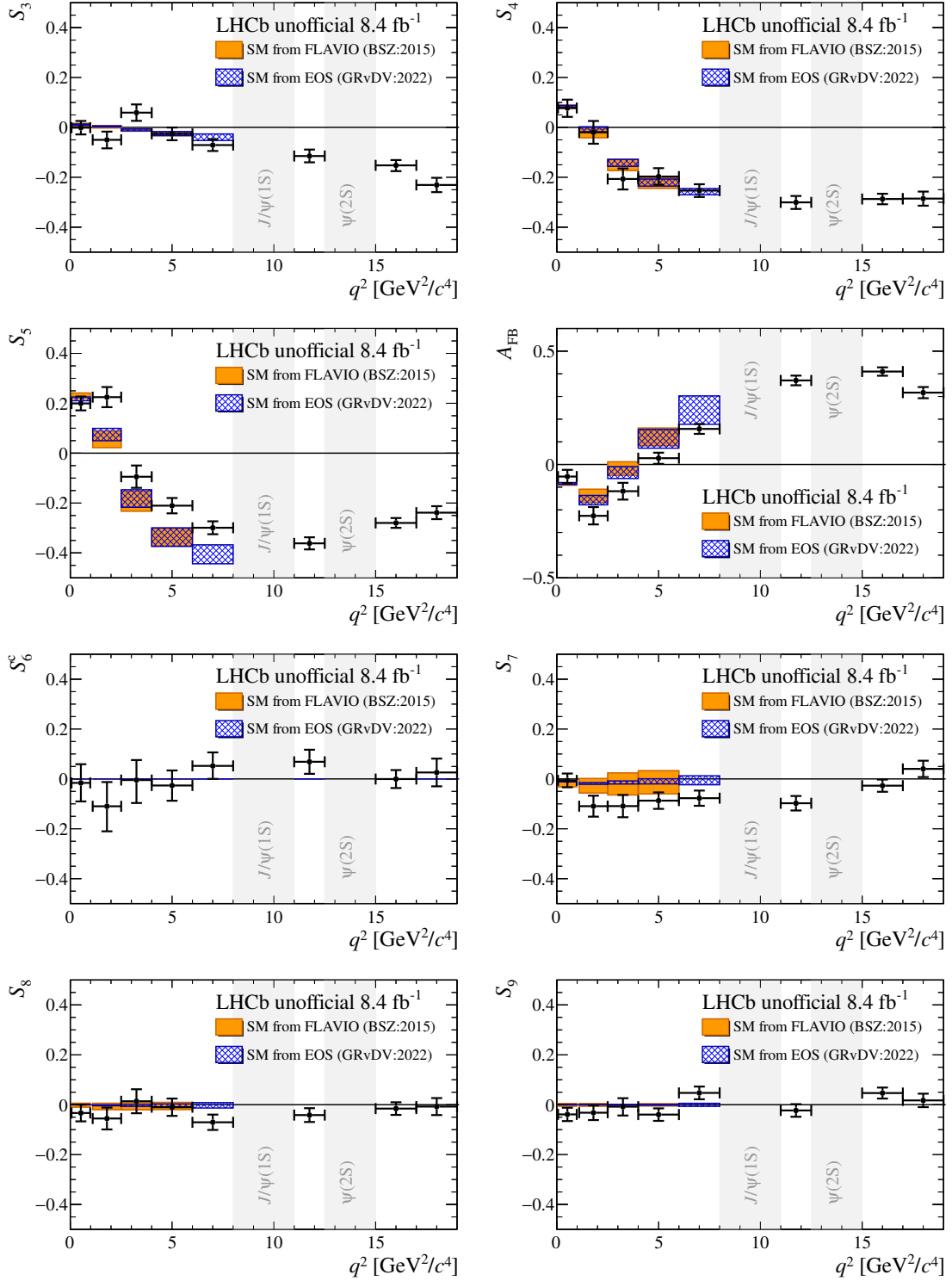


Figure B.64: Results for the angular observables A_{FB} and S_3 - S_9 extracted from fit configuration (iv) in bins of q^2 .

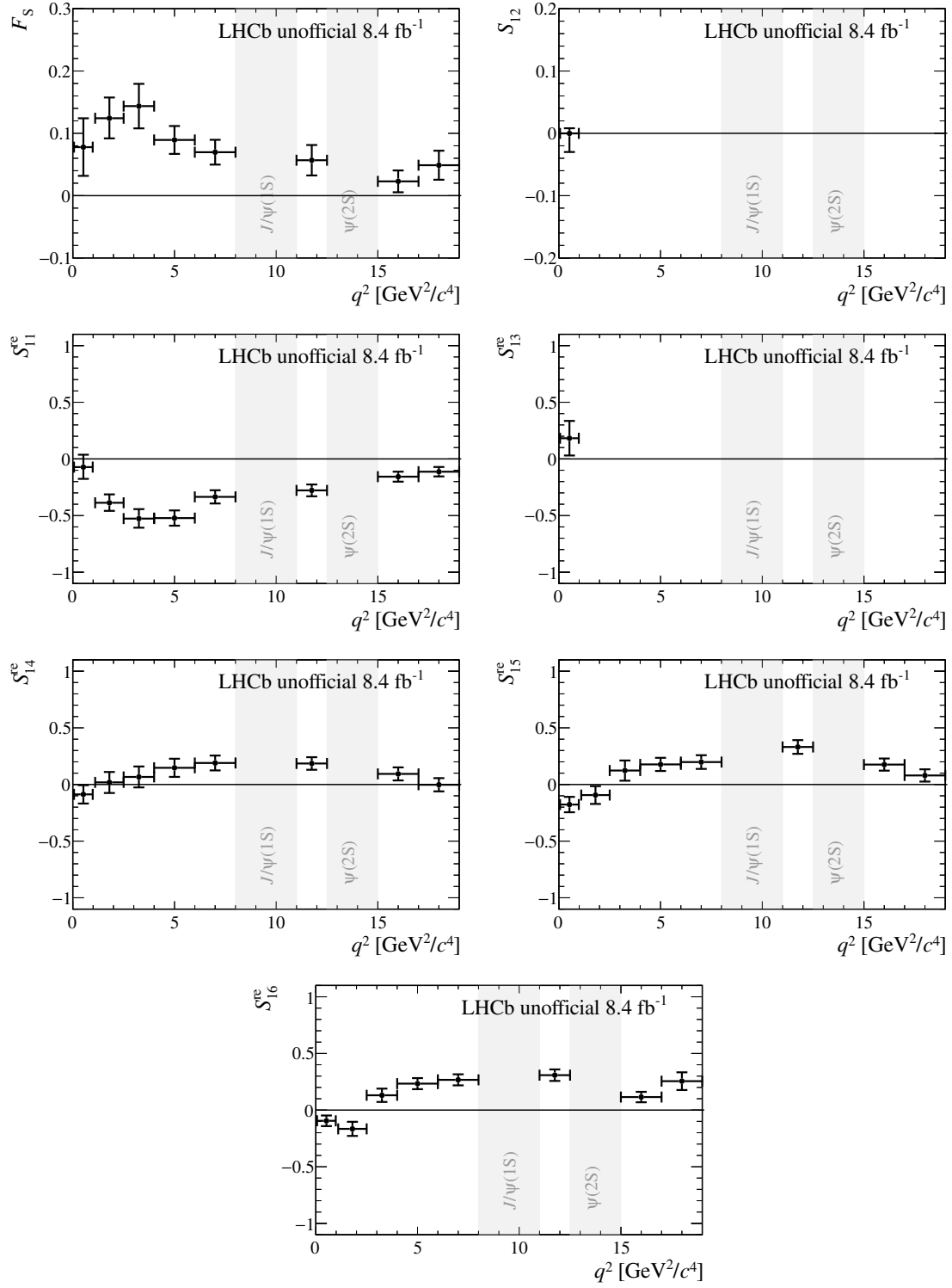


Figure B.65: Results for the angular observables F_S, S_{12} and $S_{11}^{re}, S_{14}^{re}-S_{17}^{re}$ extracted from fit configuration (iv) in bins of q^2 .

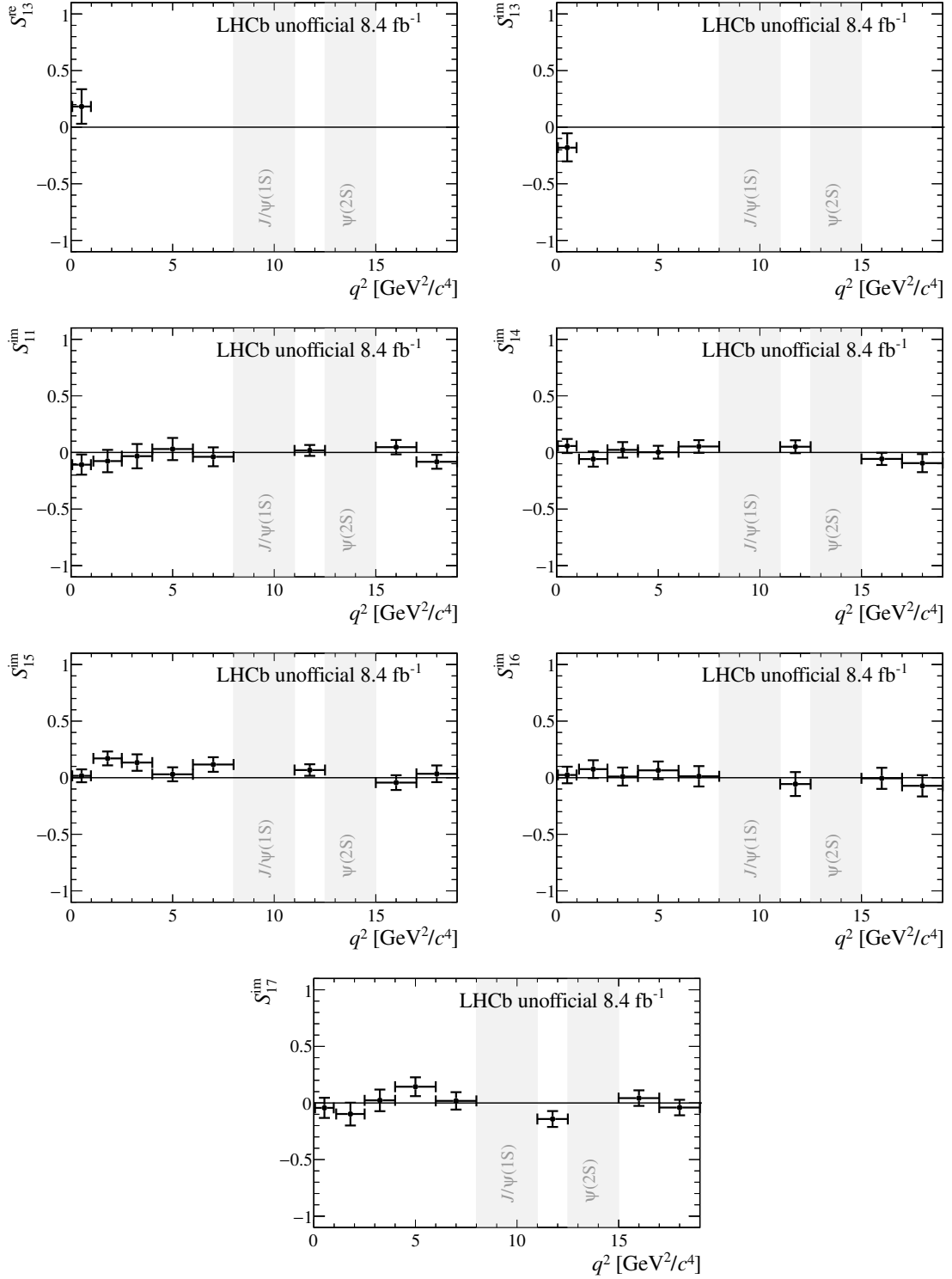


Figure B.66: Results for the angular observables S_{13}^{re} , S_{13}^{im} and S_{11}^{im} , S_{14}^{im} , S_{15}^{im} , S_{16}^{im} , S_{17}^{im} extracted from fit configuration (iv) in bins of q^2 .

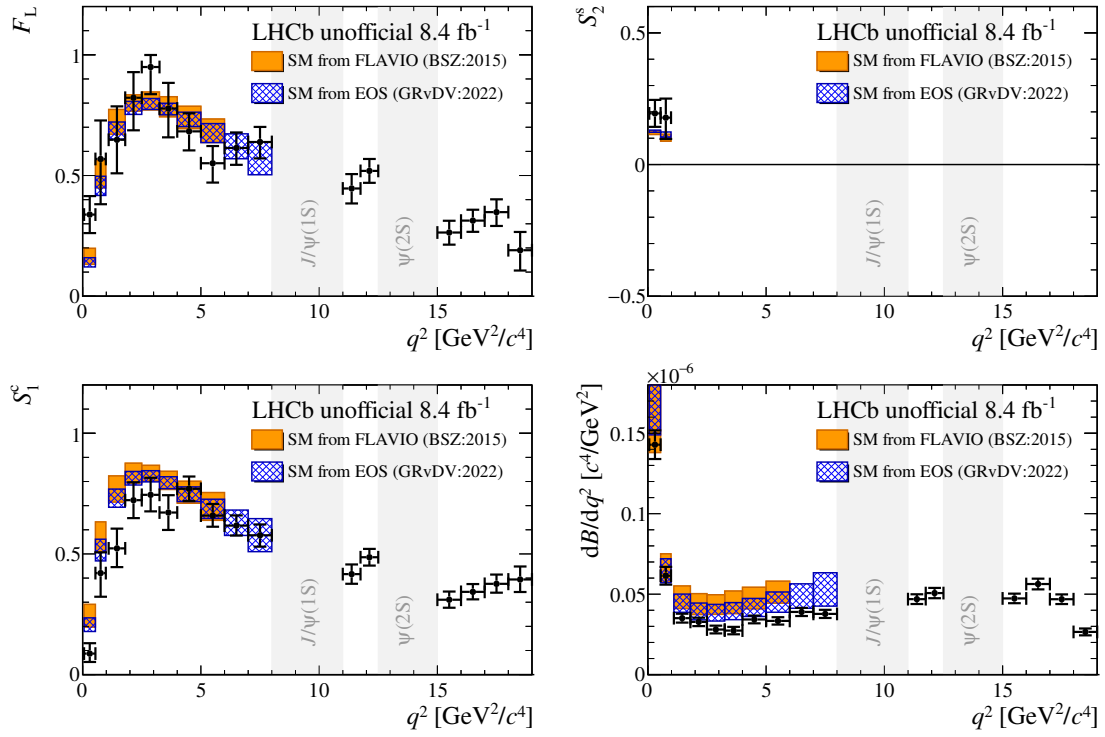


Figure B.67: Results for the angular observables F_L , S_2^s , S_1^c , and dB/dq^2 extracted from fit configuration (v) in bins of q^2 .

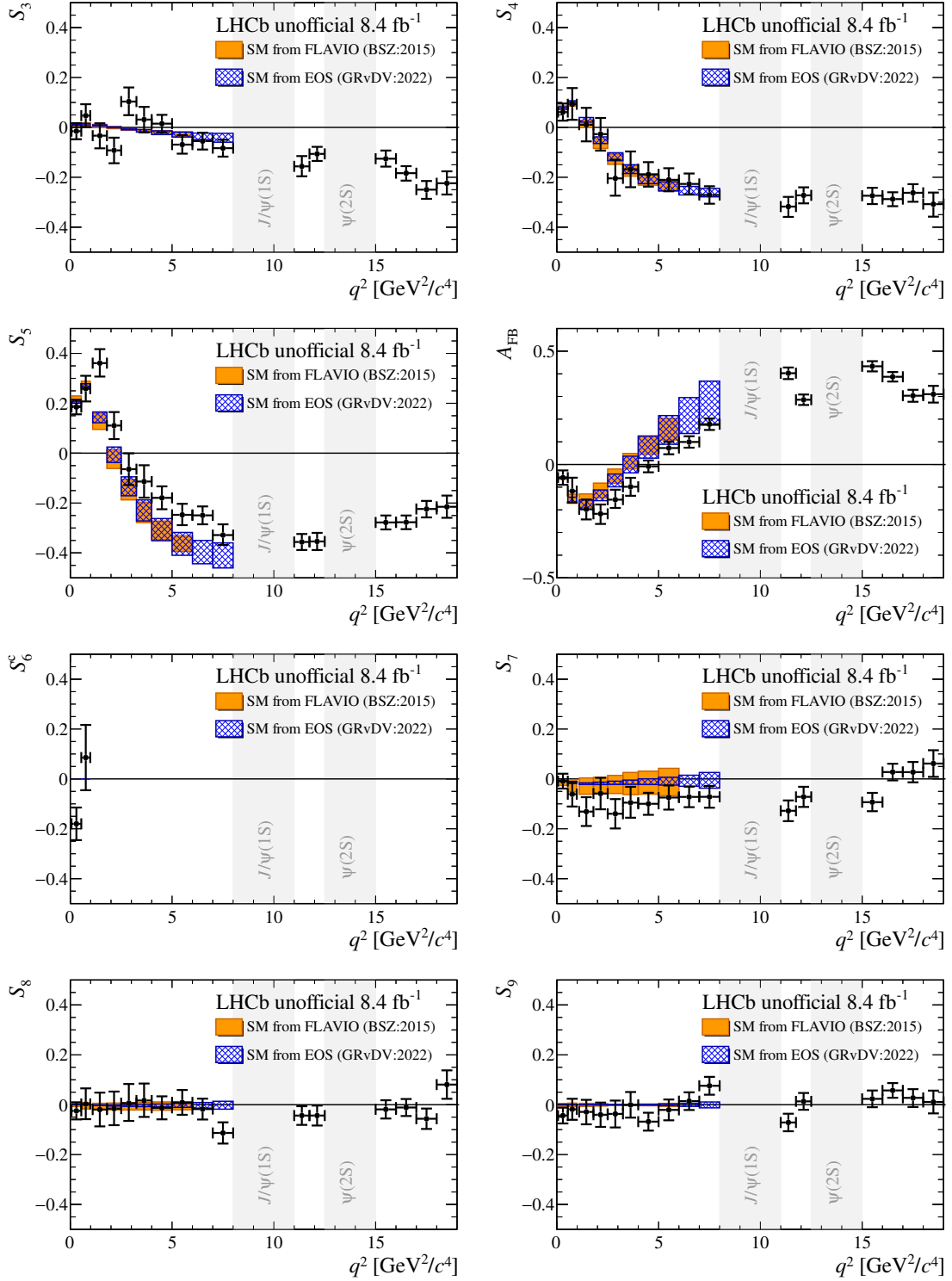


Figure B.68: Results for the angular observables A_{FB} and S_3 - S_9 extracted from fit configuration (v) in bins of q^2 .

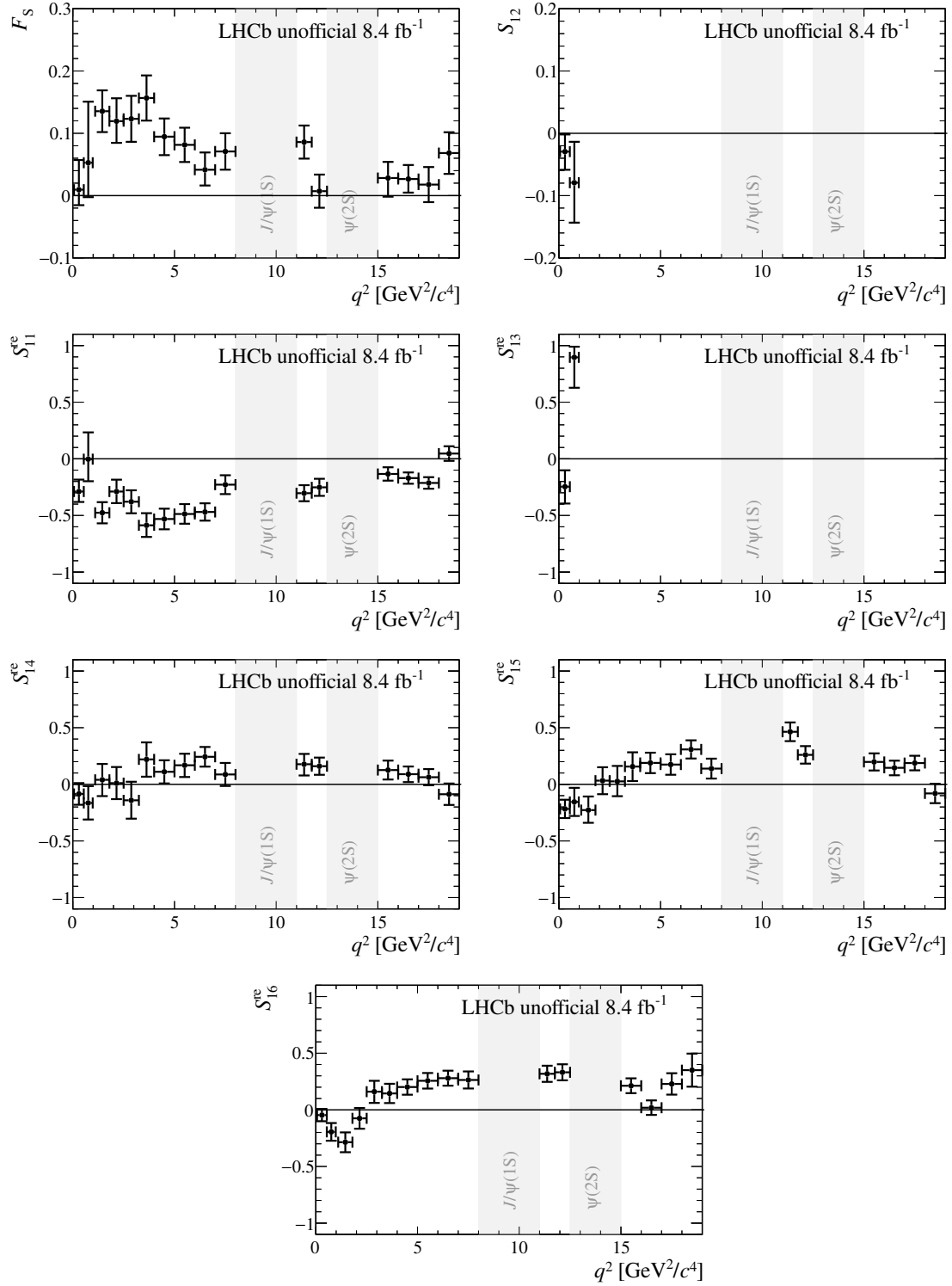


Figure B.69: Results for the angular observables F_S, S_{12} and $S_{11}^{re}, S_{13}^{re}, S_{14}^{re}, S_{15}^{re}, S_{16}^{re}$ extracted from fit configuration (v) in bins of q^2 .

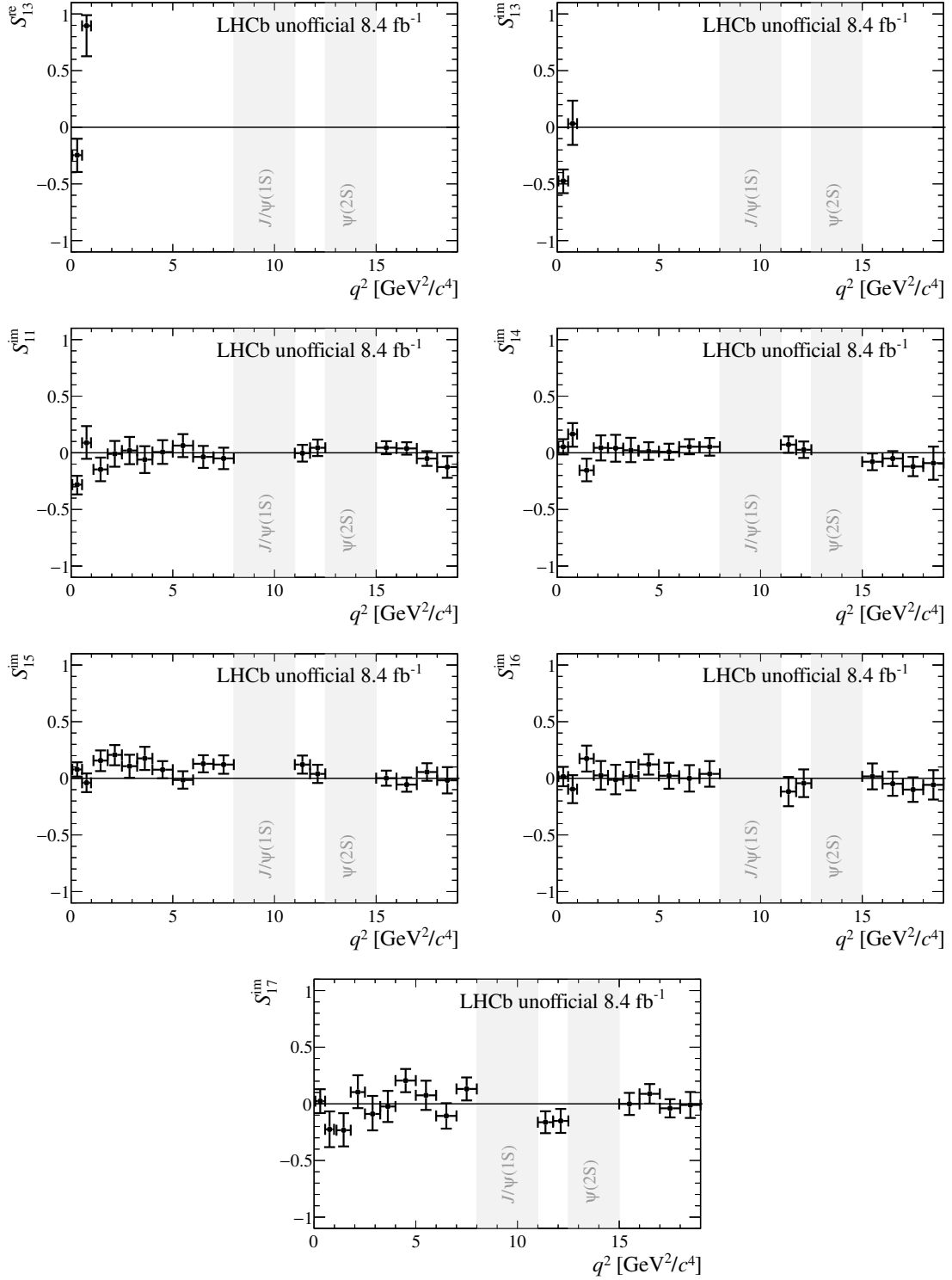


Figure B.70: Results for the angular observables S_{13}^{re} , S_{13}^{im} and $S_{11}^{im}, S_{14}^{im}, S_{15}^{im}, S_{16}^{im}, S_{17}^{im}$ extracted from fit configuration (v) in bins of q^2 .

Table B.17: Measured values of the angular observables including statistical and systematic uncertainties from configuration (iii) in the S_i basis.

$0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$			$1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$			$2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$		
F_L	$0.325^{+0.102}_{-0.100} \text{ }^{+0.013}_{-0.013}$		F_L	$0.642^{+0.029}_{-0.033} \text{ }^{+0.010}_{-0.010}$		F_L	$0.765^{+0.028}_{-0.032} \text{ }^{+0.012}_{-0.012}$	
S_2^s	$0.184^{+0.049}_{-0.048} \text{ }^{+0.012}_{-0.012}$		S_3	$-0.046^{+0.035}_{-0.035} \text{ }^{+0.002}_{-0.002}$		S_3	$0.059^{+0.037}_{-0.037} \text{ }^{+0.003}_{-0.003}$	
S_1^c	$0.226^{+0.062}_{-0.060} \text{ }^{+0.010}_{-0.010}$		S_4	$0.006^{+0.048}_{-0.048} \text{ }^{+0.005}_{-0.005}$		S_4	$-0.229^{+0.054}_{-0.051} \text{ }^{+0.003}_{-0.003}$	
S_3	$-0.001^{+0.027}_{-0.027} \text{ }^{+0.002}_{-0.002}$		S_5	$0.228^{+0.043}_{-0.040} \text{ }^{+0.002}_{-0.002}$		S_5	$-0.108^{+0.047}_{-0.047} \text{ }^{+0.004}_{-0.004}$	
S_4	$0.076^{+0.034}_{-0.036} \text{ }^{+0.002}_{-0.002}$		A_{FB}	$-0.189^{+0.029}_{-0.031} \text{ }^{+0.007}_{-0.007}$		A_{FB}	$-0.125^{+0.027}_{-0.031} \text{ }^{+0.004}_{-0.004}$	
S_5	$0.197^{+0.029}_{-0.028} \text{ }^{+0.004}_{-0.004}$		S_7	$-0.106^{+0.041}_{-0.041} \text{ }^{+0.002}_{-0.002}$		S_7	$-0.101^{+0.046}_{-0.045} \text{ }^{+0.002}_{-0.002}$	
A_{FB}	$-0.066^{+0.029}_{-0.029} \text{ }^{+0.005}_{-0.005}$		S_8	$-0.051^{+0.048}_{-0.050} \text{ }^{+0.003}_{-0.003}$		S_8	$0.013^{+0.049}_{-0.050} \text{ }^{+0.001}_{-0.001}$	
S_7	$-0.009^{+0.028}_{-0.028} \text{ }^{+0.001}_{-0.001}$		S_9	$-0.024^{+0.033}_{-0.034} \text{ }^{+0.001}_{-0.001}$		S_9	$-0.009^{+0.035}_{-0.036} \text{ }^{+0.002}_{-0.002}$	
S_8	$-0.055^{+0.036}_{-0.035} \text{ }^{+0.001}_{-0.001}$		A_{FL}	$-0.096^{+0.038}_{-0.038} \text{ }^{+0.003}_{-0.003}$		A_{FL}	$-0.039^{+0.044}_{-0.044} \text{ }^{+0.005}_{-0.005}$	
S_9	$-0.041^{+0.027}_{-0.027} \text{ }^{+0.001}_{-0.001}$		A_3	$0.004^{+0.033}_{-0.035} \text{ }^{+0.002}_{-0.002}$		A_3	$-0.012^{+0.035}_{-0.035} \text{ }^{+0.002}_{-0.002}$	
S_6^c	$-0.071^{+0.077}_{-0.078} \text{ }^{+0.008}_{-0.008}$		A_4	$0.040^{+0.047}_{-0.049} \text{ }^{+0.003}_{-0.003}$		A_4	$0.087^{+0.050}_{-0.052} \text{ }^{+0.002}_{-0.002}$	
A_{FL}	$-0.182^{+0.089}_{-0.092} \text{ }^{+0.008}_{-0.008}$		A_5	$-0.038^{+0.041}_{-0.041} \text{ }^{+0.001}_{-0.001}$		A_5	$0.043^{+0.044}_{-0.044} \text{ }^{+0.001}_{-0.001}$	
A_{2s}	$-0.024^{+0.046}_{-0.046} \text{ }^{+0.004}_{-0.004}$		A_{FB}^{CP}	$0.016^{+0.030}_{-0.031} \text{ }^{+0.001}_{-0.001}$		A_{FB}^{CP}	$-0.005^{+0.026}_{-0.027} \text{ }^{+0.001}_{-0.001}$	
A_{1c}	$0.085^{+0.053}_{-0.054} \text{ }^{+0.004}_{-0.004}$		A_7	$-0.013^{+0.041}_{-0.042} \text{ }^{+0.000}_{-0.000}$		A_7	$-0.044^{+0.044}_{-0.044} \text{ }^{+0.001}_{-0.001}$	
A_3	$0.025^{+0.027}_{-0.027} \text{ }^{+0.001}_{-0.001}$		A_8	$0.069^{+0.046}_{-0.048} \text{ }^{+0.001}_{-0.001}$		A_8	$0.082^{+0.048}_{-0.049} \text{ }^{+0.001}_{-0.001}$	
A_4	$0.012^{+0.035}_{-0.035} \text{ }^{+0.002}_{-0.002}$		A_9	$-0.046^{+0.035}_{-0.034} \text{ }^{+0.001}_{-0.001}$		A_9	$0.011^{+0.037}_{-0.036} \text{ }^{+0.001}_{-0.001}$	
A_5	$0.007^{+0.028}_{-0.028} \text{ }^{+0.002}_{-0.002}$		F_S	$0.124^{+0.023}_{-0.023} \text{ }^{+0.008}_{-0.008}$		F_S	$0.155^{+0.025}_{-0.025} \text{ }^{+0.013}_{-0.013}$	
A_{FB}^{CP}	$0.047^{+0.029}_{-0.029} \text{ }^{+0.004}_{-0.004}$		A_{FS}	$0.014^{+0.023}_{-0.023} \text{ }^{+0.001}_{-0.001}$		A_{FS}	$-0.016^{+0.026}_{-0.026} \text{ }^{+0.001}_{-0.001}$	
A_{6c}	$0.093^{+0.081}_{-0.080} \text{ }^{+0.006}_{-0.006}$		S_{11}^{re}	$-0.406^{+0.067}_{-0.066} \text{ }^{+0.028}_{-0.028}$		S_{11}^{re}	$-0.502^{+0.070}_{-0.076} \text{ }^{+0.034}_{-0.034}$	
A_7	$-0.002^{+0.028}_{-0.028} \text{ }^{+0.000}_{-0.000}$		A_{11}^{re}	$0.019^{+0.067}_{-0.067} \text{ }^{+0.003}_{-0.003}$		A_{11}^{re}	$-0.118^{+0.069}_{-0.069} \text{ }^{+0.003}_{-0.003}$	
A_8	$-0.026^{+0.034}_{-0.034} \text{ }^{+0.001}_{-0.001}$		S_{11}^{im}	$-0.089^{+0.048}_{-0.048} \text{ }^{+0.084}_{-0.084}$		S_{11}^{im}	$-0.024^{+0.053}_{-0.053} \text{ }^{+0.091}_{-0.091}$	
A_9	$0.017^{+0.027}_{-0.027} \text{ }^{+0.000}_{-0.000}$		A_{11}^{im}	$-0.076^{+0.047}_{-0.047} \text{ }^{+0.003}_{-0.003}$		A_{11}^{im}	$-0.026^{+0.052}_{-0.052} \text{ }^{+0.001}_{-0.001}$	
F_S	$0.072^{+0.043}_{-0.044} \text{ }^{+0.014}_{-0.014}$		S_{14}^{re}	$0.042^{+0.100}_{-0.104} \text{ }^{+0.003}_{-0.003}$		S_{14}^{re}	$0.071^{+0.103}_{-0.109} \text{ }^{+0.007}_{-0.007}$	
A_{FS}	$-0.068^{+0.044}_{-0.045} \text{ }^{+0.002}_{-0.002}$		A_{14}^{re}	$0.054^{+0.096}_{-0.102} \text{ }^{+0.003}_{-0.003}$		A_{14}^{re}	$-0.090^{+0.098}_{-0.105} \text{ }^{+0.001}_{-0.001}$	
S_{12}	$-0.003^{+0.003}_{-0.029} \text{ }^{+0.008}_{-0.008}$		S_{14}^{im}	$-0.060^{+0.073}_{-0.071} \text{ }^{+0.010}_{-0.010}$		S_{14}^{im}	$0.021^{+0.076}_{-0.076} \text{ }^{+0.019}_{-0.019}$	
A_{12}	$-0.017^{+0.032}_{-0.032} \text{ }^{+0.002}_{-0.002}$		A_{14}^{im}	$0.071^{+0.072}_{-0.076} \text{ }^{+0.003}_{-0.003}$		A_{14}^{im}	$-0.049^{+0.073}_{-0.075} \text{ }^{+0.002}_{-0.002}$	
S_{11}^{re}	$-0.140^{+0.106}_{-0.109} \text{ }^{+0.019}_{-0.019}$		S_{15}^{re}	$-0.092^{+0.083}_{-0.083} \text{ }^{+0.006}_{-0.006}$		S_{15}^{re}	$0.137^{+0.088}_{-0.090} \text{ }^{+0.025}_{-0.025}$	
A_{11}^{re}	$0.048^{+0.104}_{-0.113} \text{ }^{+0.007}_{-0.007}$		A_{15}^{re}	$-0.005^{+0.081}_{-0.082} \text{ }^{+0.003}_{-0.003}$		A_{15}^{re}	$-0.100^{+0.085}_{-0.084} \text{ }^{+0.002}_{-0.002}$	
S_{11}^{im}	$-0.149^{+0.084}_{-0.090} \text{ }^{+0.037}_{-0.037}$		S_{15}^{im}	$0.156^{+0.061}_{-0.063} \text{ }^{+0.007}_{-0.007}$		S_{15}^{im}	$0.140^{+0.065}_{-0.065} \text{ }^{+0.031}_{-0.031}$	
A_{11}^{im}	$0.157^{+0.083}_{-0.084} \text{ }^{+0.004}_{-0.004}$		A_{15}^{im}	$0.000^{+0.063}_{-0.060} \text{ }^{+0.001}_{-0.001}$		A_{15}^{im}	$0.034^{+0.065}_{-0.065} \text{ }^{+0.002}_{-0.002}$	
S_{13}^{re}	$0.101^{+0.159}_{-0.168} \text{ }^{+0.020}_{-0.020}$		S_{16}^{re}	$-0.142^{+0.063}_{-0.062} \text{ }^{+0.006}_{-0.006}$		S_{16}^{re}	$0.117^{+0.066}_{-0.069} \text{ }^{+0.010}_{-0.010}$	
A_{13}^{re}	$0.078^{+0.154}_{-0.161} \text{ }^{+0.009}_{-0.009}$		A_{16}^{re}	$-0.131^{+0.064}_{-0.063} \text{ }^{+0.001}_{-0.001}$		A_{16}^{re}	$-0.018^{+0.063}_{-0.063} \text{ }^{+0.002}_{-0.002}$	
S_{13}^{im}	$-0.233^{+0.124}_{-0.124} \text{ }^{+0.041}_{-0.041}$		S_{16}^{im}	$0.087^{+0.086}_{-0.089} \text{ }^{+0.008}_{-0.008}$		S_{16}^{im}	$0.055^{+0.086}_{-0.083} \text{ }^{+0.031}_{-0.031}$	
A_{13}^{im}	$0.100^{+0.112}_{-0.117} \text{ }^{+0.008}_{-0.008}$		A_{16}^{im}	$0.100^{+0.080}_{-0.080} \text{ }^{+0.002}_{-0.002}$		A_{16}^{im}	$0.001^{+0.082}_{-0.081} \text{ }^{+0.002}_{-0.002}$	
S_{14}^{re}	$-0.073^{+0.085}_{-0.085} \text{ }^{+0.006}_{-0.006}$		S_{17}^{re}	$-0.019^{+0.076}_{-0.076} \text{ }^{+0.002}_{-0.002}$		S_{17}^{re}	$0.167^{+0.076}_{-0.080} \text{ }^{+0.005}_{-0.005}$	
A_{14}^{re}	$-0.050^{+0.079}_{-0.079} \text{ }^{+0.002}_{-0.002}$		A_{17}^{re}	$0.107^{+0.072}_{-0.074} \text{ }^{+0.001}_{-0.001}$		A_{17}^{re}	$-0.012^{+0.073}_{-0.071} \text{ }^{+0.001}_{-0.001}$	
S_{14}^{im}	$0.060^{+0.057}_{-0.057} \text{ }^{+0.012}_{-0.012}$		S_{17}^{im}	$-0.052^{+0.106}_{-0.104} \text{ }^{+0.003}_{-0.003}$		S_{17}^{im}	$0.010^{+0.103}_{-0.096} \text{ }^{+0.013}_{-0.013}$	
A_{14}^{im}	$-0.098^{+0.057}_{-0.057} \text{ }^{+0.001}_{-0.001}$		A_{17}^{im}	$-0.230^{+0.110}_{-0.106} \text{ }^{+0.004}_{-0.004}$		A_{17}^{im}	$-0.147^{+0.100}_{-0.104} \text{ }^{+0.001}_{-0.001}$	
S_{15}^{re}	$-0.174^{+0.067}_{-0.067} \text{ }^{+0.018}_{-0.018}$		A_{CP}	$-0.085^{+0.044}_{-0.044} \text{ }^{+0.005}_{-0.005}$		A_{CP}	$-0.020^{+0.047}_{-0.047} \text{ }^{+0.006}_{-0.006}$	
A_{15}^{re}	$-0.021^{+0.067}_{-0.067} \text{ }^{+0.002}_{-0.002}$		$BF/10^{-7}$	$0.343 \pm 0.015 \pm 0.017$		$BF/10^{-7}$	$0.274 \pm 0.013 \pm 0.015$	
S_{15}^{im}	$0.017^{+0.046}_{-0.046} \text{ }^{+0.032}_{-0.032}$							
A_{15}^{im}	$0.033^{+0.046}_{-0.046} \text{ }^{+0.002}_{-0.002}$							
S_{16}^{re}	$-0.110^{+0.047}_{-0.047} \text{ }^{+0.004}_{-0.004}$							
A_{16}^{re}	$-0.061^{+0.046}_{-0.046} \text{ }^{+0.001}_{-0.001}$							
S_{16}^{im}	$0.044^{+0.069}_{-0.069} \text{ }^{+0.026}_{-0.026}$							
A_{16}^{im}	$-0.002^{+0.069}_{-0.069} \text{ }^{+0.002}_{-0.002}$							
S_{17}^{re}	$0.059^{+0.059}_{-0.059} \text{ }^{+0.002}_{-0.002}$							
A_{17}^{re}	$-0.042^{+0.059}_{-0.059} \text{ }^{+0.002}_{-0.002}$							
S_{17}^{im}	$-0.010^{+0.088}_{-0.088} \text{ }^{+0.011}_{-0.011}$							
A_{17}^{im}	$0.031^{+0.088}_{-0.088} \text{ }^{+0.002}_{-0.002}$							
A_{CP}	$0.071^{+0.035}_{-0.035} \text{ }^{+0.005}_{-0.005}$							
$BF/10^{-7}$	$1.040 \pm 0.036 \pm 0.049$							

Table B.18: Measured values of the angular observables including statistical and systematic uncertainties from the floated A_i in the S_i basis.

$4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$			$6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$		
F_L	$0.684^{+0.022}_{-0.022} +0.004_{-0.004}$		F_L	$0.610^{+0.020}_{-0.020} +0.002_{-0.002}$	
S_3	$-0.040^{+0.025}_{-0.025} +0.002_{-0.002}$		S_3	$-0.070^{+0.025}_{-0.025} +0.003_{-0.003}$	
S_4	$-0.195^{+0.036}_{-0.033} +0.002_{-0.002}$		S_4	$-0.250^{+0.029}_{-0.031} +0.001_{-0.001}$	
S_5	$-0.218^{+0.032}_{-0.032} +0.002_{-0.002}$		S_5	$-0.299^{+0.028}_{-0.029} +0.003_{-0.003}$	
A_{FB}	$0.031^{+0.019}_{-0.019} +0.003_{-0.003}$		A_{FB}	$0.144^{+0.018}_{-0.018} +0.002_{-0.002}$	
S_7	$-0.083^{+0.033}_{-0.033} +0.002_{-0.002}$		S_7	$-0.074^{+0.030}_{-0.030} +0.001_{-0.001}$	
S_8	$0.001^{+0.034}_{-0.034} +0.000_{-0.000}$		S_8	$-0.070^{+0.030}_{-0.030} +0.003_{-0.003}$	
S_9	$-0.036^{+0.025}_{-0.026} +0.002_{-0.002}$		S_9	$0.050^{+0.024}_{-0.024} +0.003_{-0.003}$	
A_{FL}	$-0.008^{+0.032}_{-0.032} +0.003_{-0.003}$		A_{FL}	$0.017^{+0.029}_{-0.029} +0.003_{-0.003}$	
A_3	$0.009^{+0.025}_{-0.025} +0.002_{-0.002}$		A_3	$-0.012^{+0.024}_{-0.024} +0.001_{-0.001}$	
A_4	$-0.043^{+0.031}_{-0.031} +0.003_{-0.003}$		A_4	$-0.055^{+0.027}_{-0.027} +0.002_{-0.002}$	
A_5	$0.003^{+0.031}_{-0.031} +0.003_{-0.003}$		A_5	$0.003^{+0.027}_{-0.027} +0.003_{-0.003}$	
A_{FB}^{CP}	$-0.025^{+0.019}_{-0.019} +0.001_{-0.001}$		A_{FB}^{CP}	$0.014^{+0.018}_{-0.018} +0.001_{-0.001}$	
A_7	$0.033^{+0.033}_{-0.033} +0.001_{-0.001}$		A_7	$0.039^{+0.030}_{-0.030} +0.002_{-0.002}$	
A_8	$0.050^{+0.034}_{-0.034} +0.001_{-0.001}$		A_8	$-0.055^{+0.030}_{-0.030} +0.001_{-0.001}$	
A_9	$0.019^{+0.027}_{-0.027} +0.003_{-0.003}$		A_9	$0.043^{+0.024}_{-0.024} +0.002_{-0.002}$	
F_S	$0.088^{+0.019}_{-0.020} +0.004_{-0.004}$		F_S	$0.068^{+0.018}_{-0.018} +0.004_{-0.004}$	
A_{FS}	$0.005^{+0.019}_{-0.019} +0.001_{-0.001}$		A_{FS}	$0.023^{+0.018}_{-0.018} +0.002_{-0.002}$	
S_{11}^{re}	$-0.514^{+0.061}_{-0.061} +0.039_{-0.039}$		S_{11}^{re}	$-0.323^{+0.052}_{-0.052} +0.023_{-0.023}$	
A_{11}^{re}	$0.036^{+0.060}_{-0.060} +0.013_{-0.013}$		A_{11}^{re}	$-0.031^{+0.051}_{-0.051} +0.002_{-0.002}$	
S_{11}^{im}	$0.026^{+0.038}_{-0.038} +0.088_{-0.088}$		S_{11}^{im}	$-0.038^{+0.037}_{-0.037} +0.074_{-0.074}$	
A_{11}^{im}	$0.045^{+0.038}_{-0.038} +0.001_{-0.001}$		A_{11}^{im}	$-0.054^{+0.036}_{-0.036} +0.004_{-0.004}$	
S_{14}^{re}	$0.159^{+0.072}_{-0.071} +0.009_{-0.009}$		S_{14}^{re}	$0.193^{+0.065}_{-0.065} +0.009_{-0.009}$	
A_{14}^{re}	$0.069^{+0.074}_{-0.070} +0.002_{-0.002}$		A_{14}^{re}	$0.064^{+0.065}_{-0.065} +0.002_{-0.002}$	
S_{14}^{im}	$0.008^{+0.049}_{-0.049} +0.027_{-0.027}$		S_{14}^{im}	$0.055^{+0.047}_{-0.047} +0.032_{-0.032}$	
A_{14}^{im}	$0.005^{+0.052}_{-0.051} +0.002_{-0.002}$		A_{14}^{im}	$0.017^{+0.047}_{-0.047} +0.005_{-0.005}$	
S_{15}^{re}	$0.192^{+0.061}_{-0.061} +0.014_{-0.014}$		S_{15}^{re}	$0.193^{+0.059}_{-0.059} +0.014_{-0.014}$	
A_{15}^{re}	$0.173^{+0.059}_{-0.059} +0.008_{-0.008}$		A_{15}^{re}	$0.007^{+0.059}_{-0.059} +0.006_{-0.006}$	
S_{15}^{im}	$0.036^{+0.047}_{-0.047} +0.041_{-0.041}$		S_{15}^{im}	$0.119^{+0.042}_{-0.042} +0.050_{-0.050}$	
A_{15}^{im}	$0.008^{+0.046}_{-0.046} +0.002_{-0.002}$		A_{15}^{im}	$0.065^{+0.041}_{-0.041} +0.004_{-0.004}$	
S_{16}^{re}	$0.235^{+0.050}_{-0.050} +0.005_{-0.005}$		S_{16}^{re}	$0.262^{+0.048}_{-0.048} +0.008_{-0.008}$	
A_{16}^{re}	$0.048^{+0.049}_{-0.049} +0.001_{-0.001}$		A_{16}^{re}	$-0.039^{+0.047}_{-0.047} +0.006_{-0.006}$	
S_{16}^{im}	$0.100^{+0.066}_{-0.066} +0.048_{-0.048}$		S_{16}^{im}	$0.039^{+0.067}_{-0.067} +0.061_{-0.061}$	
A_{16}^{im}	$-0.031^{+0.064}_{-0.065} +0.009_{-0.009}$		A_{16}^{im}	$0.091^{+0.067}_{-0.067} +0.005_{-0.005}$	
S_{17}^{re}	$0.073^{+0.051}_{-0.051} +0.007_{-0.007}$		S_{17}^{re}	$0.030^{+0.052}_{-0.052} +0.004_{-0.004}$	
A_{17}^{re}	$0.043^{+0.051}_{-0.051} +0.002_{-0.002}$		A_{17}^{re}	$-0.009^{+0.052}_{-0.052} +0.004_{-0.004}$	
S_{17}^{im}	$0.148^{+0.082}_{-0.082} +0.021_{-0.021}$		S_{17}^{im}	$0.024^{+0.072}_{-0.072} +0.026_{-0.026}$	
A_{17}^{im}	$0.038^{+0.082}_{-0.082} +0.005_{-0.005}$		A_{17}^{im}	$0.074^{+0.072}_{-0.072} +0.005_{-0.005}$	
A_{CP}	$0.012^{+0.035}_{-0.035} +0.003_{-0.003}$		A_{CP}	$0.006^{+0.032}_{-0.032} +0.004_{-0.004}$	
$BF/10^{-7}$	$0.337 \pm 0.012 \pm 0.017$		$BF/10^{-7}$	$0.379 \pm 0.012 \pm 0.019$	

Table B.19: Measured values of the angular observables including statistical and systematic uncertainties from configuration (iii) in the S_i basis.

15.0 < q^2 < 17.0 GeV^2/c^4			17.0 < q^2 < 19.0 GeV^2/c^4			11.0 < q^2 < 12.5 GeV^2/c^4		
F_L	0.323 ^{+0.016} _{-0.016}	+0.004 -0.004	F_L	0.345 ^{+0.021} _{-0.021}	+0.004 -0.004	F_L	0.448 ^{+0.020} _{-0.020}	+0.003 -0.003
S_3	-0.164 ^{+0.021} _{-0.022}	+0.001 -0.001	S_3	-0.239 ^{+0.028} _{-0.030}	+0.001 -0.001	S_3	-0.134 ^{+0.024} _{-0.027}	+0.004 -0.004
S_4	-0.277 ^{+0.020} _{-0.021}	+0.001 -0.001	S_4	-0.274 ^{+0.027} _{-0.029}	+0.002 -0.002	S_4	-0.295 ^{+0.025} _{-0.028}	+0.005 -0.005
S_5	-0.280 ^{+0.019} _{-0.019}	+0.003 -0.003	S_5	-0.232 ^{+0.026} _{-0.028}	+0.004 -0.004	S_5	-0.344 ^{+0.025} _{-0.026}	+0.005 -0.005
A_{FB}	0.404 ^{+0.015} _{-0.015}	+0.003 -0.003	A_{FB}	0.306 ^{+0.021} _{-0.021}	+0.003 -0.003	A_{FB}	0.344 ^{+0.017} _{-0.017}	+0.004 -0.004
S_7	-0.027 ^{+0.025} _{-0.025}	+0.001 -0.001	S_7	0.039 ^{+0.033} _{-0.033}	+0.001 -0.001	S_7	-0.098 ^{+0.029} _{-0.029}	+0.001 -0.001
S_8	-0.011 ^{+0.025} _{-0.025}	+0.001 -0.001	S_8	-0.009 ^{+0.033} _{-0.033}	+0.001 -0.001	S_8	-0.043 ^{+0.027} _{-0.027}	+0.001 -0.001
S_9	0.044 ^{+0.023} _{-0.023}	+0.001 -0.001	S_9	0.016 ^{+0.028} _{-0.028}	+0.000 -0.000	S_9	-0.022 ^{+0.024} _{-0.025}	+0.004 -0.004
A_{FL}	0.015 ^{+0.019} _{-0.019}	+0.002 -0.002	A_{FL}	0.068 ^{+0.025} _{-0.025}	+0.002 -0.002	A_{FL}	0.012 ^{+0.025} _{-0.025}	+0.002 -0.002
A_3	-0.009 ^{+0.021} _{-0.021}	+0.001 -0.001	A_3	0.015 ^{+0.030} _{-0.030}	+0.001 -0.001	A_3	0.021 ^{+0.024} _{-0.025}	+0.001 -0.001
A_4	-0.002 ^{+0.019} _{-0.019}	+0.001 -0.001	A_4	0.004 ^{+0.028} _{-0.028}	+0.002 -0.002	A_4	-0.012 ^{+0.025} _{-0.026}	+0.002 -0.002
A_5	-0.007 ^{+0.019} _{-0.019}	+0.002 -0.002	A_5	0.003 ^{+0.026} _{-0.025}	+0.002 -0.002	A_5	0.023 ^{+0.027} _{-0.026}	+0.001 -0.001
A_{FB}^{CP}	-0.007 ^{+0.016} _{-0.016}	+0.002 -0.002	A_{FB}^{CP}	-0.052 ^{+0.022} _{-0.022}	+0.002 -0.002	A_{FB}^{CP}	0.040 ^{+0.017} _{-0.017}	+0.002 -0.002
A_7	-0.023 ^{+0.025} _{-0.025}	+0.001 -0.001	A_7	-0.030 ^{+0.033} _{-0.033}	+0.001 -0.001	A_7	-0.013 ^{+0.029} _{-0.029}	+0.001 -0.001
A_8	0.009 ^{+0.025} _{-0.025}	+0.001 -0.001	A_8	0.019 ^{+0.033} _{-0.033}	+0.000 -0.000	A_8	-0.012 ^{+0.027} _{-0.027}	+0.001 -0.001
A_9	0.005 ^{+0.023} _{-0.023}	+0.000 -0.000	A_9	0.024 ^{+0.028} _{-0.028}	+0.001 -0.001	A_9	-0.060 ^{+0.025} _{-0.025}	+0.001 -0.001
F_S	0.022 ^{+0.014} _{-0.014}	+0.003 -0.003	F_S	0.047 ^{+0.019} _{-0.019}	+0.003 -0.003	F_S	0.051 ^{+0.018} _{-0.018}	+0.009 -0.009
A_{FS}	-0.011 ^{+0.014} _{-0.014}	+0.001 -0.001	A_{FS}	-0.022 ^{+0.019} _{-0.019}	+0.001 -0.001	A_{FS}	0.011 ^{+0.018} _{-0.018}	+0.001 -0.001
S_{11}^{re}	-0.162 ^{+0.038} _{-0.038}	+0.022 -0.022	S_{11}^{re}	-0.116 ^{+0.040} _{-0.040}	+0.014 -0.014	S_{11}^{re}	-0.276 ^{+0.050} _{-0.050}	+0.012 -0.012
A_{11}^{re}	-0.024 ^{+0.037} _{-0.037}	+0.001 -0.001	A_{11}^{re}	-0.016 ^{+0.038} _{-0.038}	+0.002 -0.002	A_{11}^{re}	0.077 ^{+0.048} _{-0.048}	+0.002 -0.002
S_{11}^{im}	0.048 ^{+0.027} _{-0.027}	+0.058 -0.058	S_{11}^{im}	-0.090 ^{+0.047} _{-0.047}	+0.042 -0.042	S_{11}^{im}	0.021 ^{+0.032} _{-0.032}	+0.036 -0.036
A_{11}^{im}	-0.009 ^{+0.026} _{-0.026}	+0.001 -0.001	A_{11}^{im}	0.033 ^{+0.047} _{-0.047}	+0.001 -0.001	A_{11}^{im}	0.031 ^{+0.031} _{-0.031}	+0.003 -0.003
S_{14}^{re}	0.084 ^{+0.057} _{-0.057}	+0.012 -0.012	S_{14}^{re}	-0.003 ^{+0.057} _{-0.060}	+0.010 -0.010	S_{14}^{re}	0.224 ^{+0.060} _{-0.063}	+0.006 -0.006
A_{14}^{re}	0.031 ^{+0.055} _{-0.051}	+0.001 -0.001	A_{14}^{re}	0.010 ^{+0.056} _{-0.059}	+0.002 -0.002	A_{14}^{re}	0.137 ^{+0.066} _{-0.067}	+0.002 -0.002
S_{14}^{im}	-0.072 ^{+0.038} _{-0.038}	+0.038 -0.038	S_{14}^{im}	-0.094 ^{+0.073} _{-0.072}	+0.037 -0.037	S_{14}^{im}	0.072 ^{+0.046} _{-0.046}	+0.035 -0.035
A_{14}^{im}	-0.017 ^{+0.038} _{-0.038}	+0.001 -0.001	A_{14}^{im}	-0.022 ^{+0.073} _{-0.073}	+0.002 -0.002	A_{14}^{im}	0.047 ^{+0.046} _{-0.048}	+0.002 -0.002
S_{15}^{re}	0.192 ^{+0.052} _{-0.052}	+0.016 -0.016	S_{15}^{re}	0.084 ^{+0.054} _{-0.054}	+0.010 -0.010	S_{15}^{re}	0.325 ^{+0.064} _{-0.063}	+0.004 -0.004
A_{15}^{re}	-0.007 ^{+0.051} _{-0.051}	+0.001 -0.001	A_{15}^{re}	0.074 ^{+0.052} _{-0.052}	+0.002 -0.002	A_{15}^{re}	-0.037 ^{+0.057} _{-0.061}	+0.001 -0.001
S_{15}^{im}	-0.033 ^{+0.037} _{-0.037}	+0.054 -0.054	S_{15}^{im}	0.057 ^{+0.067} _{-0.067}	+0.039 -0.039	S_{15}^{im}	0.071 ^{+0.045} _{-0.046}	+0.026 -0.026
A_{15}^{im}	-0.090 ^{+0.036} _{-0.036}	+0.002 -0.002	A_{15}^{im}	-0.069 ^{+0.067} _{-0.067}	+0.002 -0.002	A_{15}^{im}	0.019 ^{+0.042} _{-0.042}	+0.003 -0.003
S_{16}^{re}	0.122 ^{+0.047} _{-0.047}	+0.005 -0.005	S_{16}^{re}	0.259 ^{+0.080} _{-0.080}	+0.005 -0.005	S_{16}^{re}	0.315 ^{+0.052} _{-0.053}	+0.012 -0.012
A_{16}^{re}	-0.092 ^{+0.046} _{-0.046}	+0.001 -0.001	A_{16}^{re}	-0.150 ^{+0.080} _{-0.080}	+0.002 -0.002	A_{16}^{re}	0.045 ^{+0.050} _{-0.050}	+0.004 -0.004
S_{16}^{im}	-0.012 ^{+0.062} _{-0.062}	+0.070 -0.070	S_{16}^{im}	-0.089 ^{+0.066} _{-0.066}	+0.068 -0.068	S_{16}^{im}	-0.069 ^{+0.073} _{-0.073}	+0.076 -0.076
A_{16}^{im}	-0.053 ^{+0.061} _{-0.061}	+0.003 -0.003	A_{16}^{im}	-0.017 ^{+0.064} _{-0.064}	+0.001 -0.001	A_{16}^{im}	-0.009 ^{+0.073} _{-0.073}	+0.003 -0.003
S_{17}^{re}	-0.025 ^{+0.048} _{-0.048}	+0.004 -0.004	S_{17}^{re}	0.022 ^{+0.087} _{-0.087}	+0.004 -0.004	S_{17}^{re}	0.115 ^{+0.048} _{-0.048}	+0.004 -0.004
A_{17}^{re}	-0.035 ^{+0.048} _{-0.048}	+0.001 -0.001	A_{17}^{re}	-0.023 ^{+0.087} _{-0.087}	+0.001 -0.001	A_{17}^{re}	0.007 ^{+0.047} _{-0.047}	+0.002 -0.002
S_{17}^{im}	0.055 ^{+0.063} _{-0.063}	+0.026 -0.026	S_{17}^{im}	-0.054 ^{+0.065} _{-0.065}	+0.020 -0.020	S_{17}^{im}	-0.152 ^{+0.065} _{-0.065}	+0.014 -0.014
A_{17}^{im}	-0.028 ^{+0.063} _{-0.063}	+0.001 -0.001	A_{17}^{im}	-0.041 ^{+0.065} _{-0.065}	+0.001 -0.001	A_{17}^{im}	0.058 ^{+0.066} _{-0.066}	+0.004 -0.004
A_{CP}	0.015 ^{+0.025} _{-0.025}	+0.002 -0.002	A_{CP}	0.053 ^{+0.033} _{-0.033}	+0.004 -0.004	A_{CP}	-0.007 ^{+0.030} _{-0.030}	+0.003 -0.003
$BF/10^{-7}$	0.514 ± 0.013 ± 0.024		$BF/10^{-7}$	0.367 ± 0.013 ± 0.018		$BF/10^{-7}$	0.481 ± 0.015 ± 0.025	

Table B.20: Measured values of the angular observables including statistical and systematic uncertainties from configuration (iii) in the S_i basis.

$1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$			$15.0 < q^2 < 19.0 \text{ GeV}^2/c^4$		
F_L	$0.678^{+0.016}_{-0.016} +0.006_{-0.006}$		F_L	$0.330^{+0.013}_{-0.013} +0.003_{-0.003}$	
S_3	$-0.013^{+0.017}_{-0.017} +0.002_{-0.002}$		S_3	$-0.203^{+0.016}_{-0.016} +0.002_{-0.002}$	
S_4	$-0.152^{+0.027}_{-0.025} +0.003_{-0.003}$		S_4	$-0.276^{+0.015}_{-0.015} +0.003_{-0.003}$	
S_5	$-0.055^{+0.022}_{-0.022} +0.002_{-0.002}$		S_5	$-0.255^{+0.015}_{-0.015} +0.004_{-0.004}$	
A_{FB}	$-0.084^{+0.014}_{-0.014} +0.004_{-0.004}$		A_{FB}	$0.361^{+0.012}_{-0.012} +0.002_{-0.002}$	
S_7	$-0.108^{+0.023}_{-0.023} +0.001_{-0.001}$		S_7	$0.001^{+0.020}_{-0.020} +0.000_{-0.000}$	
S_8	$-0.002^{+0.026}_{-0.026} +0.001_{-0.001}$		S_8	$-0.002^{+0.020}_{-0.020} +0.001_{-0.001}$	
S_9	$-0.029^{+0.018}_{-0.018} +0.000_{-0.000}$		S_9	$0.024^{+0.017}_{-0.017} +0.001_{-0.001}$	
A_{FL}	$-0.047^{+0.024}_{-0.024} +0.003_{-0.003}$		A_{FL}	$0.038^{+0.015}_{-0.015} +0.001_{-0.001}$	
A_3	$-0.006^{+0.017}_{-0.017} +0.002_{-0.002}$		A_3	$-0.006^{+0.016}_{-0.016} +0.002_{-0.002}$	
A_4	$0.004^{+0.024}_{-0.024} +0.002_{-0.002}$		A_4	$-0.018^{+0.016}_{-0.016} +0.001_{-0.001}$	
A_5	$-0.018^{+0.022}_{-0.022} +0.001_{-0.001}$		A_5	$-0.004^{+0.016}_{-0.016} +0.001_{-0.001}$	
A_{FB}^{CP}	$-0.014^{+0.014}_{-0.014} +0.001_{-0.001}$		A_{FB}^{CP}	$-0.025^{+0.013}_{-0.013} +0.002_{-0.002}$	
A_7	$-0.018^{+0.023}_{-0.023} +0.000_{-0.000}$		A_7	$-0.031^{+0.020}_{-0.020} +0.001_{-0.001}$	
A_8	$0.082^{+0.026}_{-0.026} +0.001_{-0.001}$		A_8	$0.010^{+0.020}_{-0.020} +0.001_{-0.001}$	
A_9	$0.001^{+0.018}_{-0.018} +0.001_{-0.001}$		A_9	$0.014^{+0.017}_{-0.017} +0.000_{-0.000}$	
F_S	$0.106^{+0.013}_{-0.013} +0.005_{-0.005}$		F_S	$0.039^{+0.012}_{-0.012} +0.005_{-0.005}$	
A_{FS}	$0.004^{+0.013}_{-0.013} +0.001_{-0.001}$		A_{FS}	$-0.018^{+0.012}_{-0.012} +0.001_{-0.001}$	
S_{11}^{re}	$-0.472^{+0.042}_{-0.042} +0.034_{-0.034}$		S_{11}^{re}	$-0.130^{+0.027}_{-0.027} +0.013_{-0.013}$	
A_{11}^{re}	$-0.017^{+0.041}_{-0.041} +0.002_{-0.002}$		A_{11}^{re}	$-0.011^{+0.027}_{-0.027} +0.002_{-0.002}$	
S_{11}^{im}	$-0.020^{+0.029}_{-0.029} +0.090_{-0.090}$		S_{11}^{im}	$-0.006^{+0.023}_{-0.023} +0.039_{-0.039}$	
A_{11}^{im}	$-0.023^{+0.028}_{-0.028} +0.002_{-0.002}$		A_{11}^{im}	$0.006^{+0.023}_{-0.023} +0.001_{-0.001}$	
S_{14}^{re}	$0.089^{+0.053}_{-0.053} +0.006_{-0.006}$		S_{14}^{re}	$0.046^{+0.039}_{-0.039} +0.009_{-0.009}$	
A_{14}^{re}	$-0.004^{+0.053}_{-0.053} +0.002_{-0.002}$		A_{14}^{re}	$0.023^{+0.039}_{-0.039} +0.002_{-0.002}$	
S_{14}^{im}	$-0.013^{+0.036}_{-0.036} +0.019_{-0.019}$		S_{14}^{im}	$-0.062^{+0.033}_{-0.033} +0.036_{-0.036}$	
A_{14}^{im}	$-0.006^{+0.036}_{-0.036} +0.001_{-0.001}$		A_{14}^{im}	$-0.010^{+0.033}_{-0.033} +0.002_{-0.002}$	
S_{15}^{re}	$0.102^{+0.043}_{-0.043} +0.006_{-0.006}$		S_{15}^{re}	$0.118^{+0.036}_{-0.036} +0.007_{-0.007}$	
A_{15}^{re}	$0.028^{+0.043}_{-0.043} +0.002_{-0.002}$		A_{15}^{re}	$0.021^{+0.036}_{-0.036} +0.002_{-0.002}$	
S_{15}^{im}	$0.095^{+0.033}_{-0.033} +0.023_{-0.023}$		S_{15}^{im}	$0.003^{+0.031}_{-0.031} +0.033_{-0.033}$	
A_{15}^{im}	$-0.009^{+0.033}_{-0.033} +0.001_{-0.001}$		A_{15}^{im}	$-0.069^{+0.030}_{-0.030} +0.002_{-0.002}$	
S_{16}^{re}	$0.079^{+0.033}_{-0.033} +0.002_{-0.002}$		S_{16}^{re}	$0.172^{+0.039}_{-0.039} +0.010_{-0.010}$	
A_{16}^{re}	$-0.032^{+0.033}_{-0.033} +0.001_{-0.001}$		A_{16}^{re}	$-0.094^{+0.038}_{-0.038} +0.003_{-0.003}$	
S_{16}^{im}	$0.043^{+0.044}_{-0.044} +0.030_{-0.030}$		S_{16}^{im}	$-0.054^{+0.045}_{-0.045} +0.072_{-0.072}$	
A_{16}^{im}	$-0.004^{+0.044}_{-0.044} +0.002_{-0.002}$		A_{16}^{im}	$-0.036^{+0.045}_{-0.045} +0.002_{-0.002}$	
S_{17}^{re}	$0.084^{+0.037}_{-0.037} +0.005_{-0.005}$		S_{17}^{re}	$-0.022^{+0.040}_{-0.040} +0.003_{-0.003}$	
A_{17}^{re}	$0.050^{+0.037}_{-0.037} +0.001_{-0.001}$		A_{17}^{re}	$-0.003^{+0.040}_{-0.040} +0.001_{-0.001}$	
S_{17}^{im}	$0.020^{+0.054}_{-0.054} +0.012_{-0.012}$		S_{17}^{im}	$0.005^{+0.045}_{-0.045} +0.018_{-0.018}$	
A_{17}^{im}	$-0.074^{+0.054}_{-0.054} +0.002_{-0.002}$		A_{17}^{im}	$-0.034^{+0.045}_{-0.045} +0.002_{-0.002}$	
A_{CP}	$-0.026^{+0.026}_{-0.026} +0.004_{-0.004}$		A_{CP}	$0.033^{+0.020}_{-0.020} +0.003_{-0.003}$	
$BF/10^{-7}$	$0.319 \pm 0.008 \pm 0.015$		$BF/10^{-7}$	$0.444 \pm 0.009 \pm 0.022$	

Table B.21: Measured values of the angular observables, including statistical and systematic uncertainties from configuration (iv) in the S_i basis.

$0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$			$1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$			$2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$		
F_L	$0.282^{+0.084}_{-0.088}$	$^{+0.014}_{-0.014}$	F_L	$0.805^{+0.093}_{-0.103}$	$^{+0.017}_{-0.017}$	F_L	$0.844^{+0.080}_{-0.094}$	$^{+0.019}_{-0.019}$
S_2^s	$0.175^{+0.047}_{-0.046}$	$^{+0.013}_{-0.013}$	S_2^s	$0.114^{+0.038}_{-0.040}$	$^{+0.004}_{-0.004}$	S_2^s	$0.051^{+0.035}_{-0.035}$	$^{+0.003}_{-0.003}$
S_1^c	$0.246^{+0.053}_{-0.050}$	$^{+0.010}_{-0.010}$	S_1^c	$0.577^{+0.055}_{-0.053}$	$^{+0.006}_{-0.006}$	S_1^c	$0.707^{+0.051}_{-0.051}$	$^{+0.007}_{-0.007}$
S_3	$-0.001^{+0.027}_{-0.027}$	$^{+0.002}_{-0.002}$	S_3	$-0.050^{+0.033}_{-0.034}$	$^{+0.002}_{-0.002}$	S_3	$0.059^{+0.033}_{-0.033}$	$^{+0.003}_{-0.003}$
S_4	$0.078^{+0.034}_{-0.035}$	$^{+0.002}_{-0.002}$	S_4	$-0.020^{+0.045}_{-0.045}$	$^{+0.005}_{-0.005}$	S_4	$-0.207^{+0.042}_{-0.042}$	$^{+0.004}_{-0.004}$
S_5	$0.199^{+0.028}_{-0.028}$	$^{+0.004}_{-0.004}$	S_5	$0.225^{+0.040}_{-0.040}$	$^{+0.002}_{-0.002}$	S_5	$-0.095^{+0.045}_{-0.044}$	$^{+0.004}_{-0.004}$
A_{FB}	$-0.053^{+0.029}_{-0.029}$	$^{+0.005}_{-0.005}$	A_{FB}	$-0.226^{+0.037}_{-0.036}$	$^{+0.011}_{-0.011}$	A_{FB}	$-0.118^{+0.036}_{-0.036}$	$^{+0.008}_{-0.008}$
S_7	$-0.006^{+0.028}_{-0.028}$	$^{+0.001}_{-0.001}$	S_7	$-0.109^{+0.042}_{-0.042}$	$^{+0.002}_{-0.002}$	S_7	$-0.109^{+0.045}_{-0.045}$	$^{+0.002}_{-0.002}$
S_8	$-0.033^{+0.033}_{-0.033}$	$^{+0.000}_{-0.000}$	S_8	$-0.055^{+0.044}_{-0.044}$	$^{+0.002}_{-0.002}$	S_8	$0.014^{+0.048}_{-0.048}$	$^{+0.001}_{-0.001}$
S_9	$-0.039^{+0.027}_{-0.027}$	$^{+0.001}_{-0.001}$	S_9	$-0.032^{+0.029}_{-0.029}$	$^{+0.001}_{-0.001}$	S_9	$-0.008^{+0.034}_{-0.035}$	$^{+0.002}_{-0.002}$
S_6^c	$-0.016^{+0.074}_{-0.074}$	$^{+0.009}_{-0.009}$	S_6^c	$-0.110^{+0.096}_{-0.099}$	$^{+0.017}_{-0.017}$	S_6^c	$-0.004^{+0.079}_{-0.091}$	$^{+0.014}_{-0.014}$
F_S	$0.078^{+0.043}_{-0.043}$	$^{+0.016}_{-0.016}$	F_S	$0.124^{+0.025}_{-0.023}$	$^{+0.022}_{-0.022}$	F_S	$0.144^{+0.025}_{-0.025}$	$^{+0.025}_{-0.025}$
S_{12}	$-0.000^{+0.000}_{-0.029}$	$^{+0.008}_{-0.008}$	S_{11}^{re}	$-0.387^{+0.067}_{-0.064}$	$^{+0.031}_{-0.031}$	S_{11}^{re}	$-0.527^{+0.074}_{-0.071}$	$^{+0.038}_{-0.038}$
S_{11}^{re}	$-0.072^{+0.108}_{-0.102}$	$^{+0.020}_{-0.020}$	S_{11}^{im}	$-0.076^{+0.048}_{-0.047}$	$^{+0.086}_{-0.086}$	S_{11}^{im}	$-0.033^{+0.054}_{-0.054}$	$^{+0.093}_{-0.093}$
S_{11}^{im}	$-0.108^{+0.082}_{-0.080}$	$^{+0.037}_{-0.037}$	S_{14}^{re}	$0.019^{+0.092}_{-0.094}$	$^{+0.003}_{-0.003}$	S_{14}^{re}	$0.067^{+0.092}_{-0.092}$	$^{+0.007}_{-0.007}$
S_{13}^{re}	$0.183^{+0.152}_{-0.151}$	$^{+0.019}_{-0.019}$	S_{14}^{im}	$-0.059^{+0.066}_{-0.066}$	$^{+0.010}_{-0.010}$	S_{14}^{im}	$0.023^{+0.066}_{-0.066}$	$^{+0.020}_{-0.020}$
S_{13}^{im}	$-0.181^{+0.120}_{-0.114}$	$^{+0.041}_{-0.041}$	S_{15}^{re}	$-0.093^{+0.078}_{-0.078}$	$^{+0.006}_{-0.006}$	S_{15}^{re}	$0.123^{+0.085}_{-0.086}$	$^{+0.024}_{-0.024}$
S_{14}^{re}	$-0.087^{+0.081}_{-0.081}$	$^{+0.006}_{-0.006}$	S_{15}^{im}	$0.170^{+0.061}_{-0.061}$	$^{+0.007}_{-0.007}$	S_{15}^{im}	$0.134^{+0.066}_{-0.066}$	$^{+0.031}_{-0.031}$
S_{14}^{im}	$0.057^{+0.061}_{-0.061}$	$^{+0.012}_{-0.012}$	S_{16}^{re}	$-0.166^{+0.062}_{-0.062}$	$^{+0.006}_{-0.006}$	S_{16}^{re}	$0.131^{+0.058}_{-0.058}$	$^{+0.011}_{-0.011}$
S_{15}^{re}	$-0.177^{+0.066}_{-0.066}$	$^{+0.018}_{-0.018}$	S_{16}^{im}	$0.076^{+0.078}_{-0.078}$	$^{+0.008}_{-0.008}$	S_{16}^{im}	$0.010^{+0.073}_{-0.073}$	$^{+0.032}_{-0.032}$
S_{15}^{im}	$0.017^{+0.047}_{-0.047}$	$^{+0.032}_{-0.032}$	S_{17}^{re}	$-0.010^{+0.073}_{-0.070}$	$^{+0.001}_{-0.001}$	S_{17}^{re}	$0.170^{+0.063}_{-0.063}$	$^{+0.006}_{-0.006}$
S_{16}^{re}	$-0.095^{+0.047}_{-0.047}$	$^{+0.004}_{-0.004}$	S_{17}^{im}	$-0.097^{+0.100}_{-0.102}$	$^{+0.003}_{-0.003}$	S_{17}^{im}	$0.023^{+0.095}_{-0.095}$	$^{+0.013}_{-0.013}$
S_{16}^{im}	$0.024^{+0.069}_{-0.069}$	$^{+0.026}_{-0.026}$	$BF/10^{-7}$	$0.339 \pm 0.015 \pm 0.017$		$BF/10^{-7}$	$0.275 \pm 0.013 \pm 0.016$	
S_{17}^{re}	$0.069^{+0.059}_{-0.059}$	$^{+0.001}_{-0.001}$						
S_{17}^{im}	$-0.043^{+0.088}_{-0.088}$	$^{+0.011}_{-0.011}$						
$BF/10^{-7}$	$1.041 \pm 0.037 \pm 0.051$							

$4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$			$6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$		
F_L	$0.626^{+0.056}_{-0.059}$	$^{+0.005}_{-0.005}$	F_L	$0.630^{+0.049}_{-0.051}$	$^{+0.003}_{-0.003}$
S_2^s	$0.082^{+0.025}_{-0.027}$	$^{+0.002}_{-0.002}$	S_2^s	$0.108^{+0.024}_{-0.024}$	$^{+0.005}_{-0.005}$
S_1^c	$0.709^{+0.032}_{-0.033}$	$^{+0.005}_{-0.005}$	S_1^c	$0.609^{+0.030}_{-0.029}$	$^{+0.005}_{-0.005}$
S_3	$-0.026^{+0.025}_{-0.025}$	$^{+0.002}_{-0.002}$	S_3	$-0.071^{+0.023}_{-0.023}$	$^{+0.004}_{-0.004}$
S_4	$-0.197^{+0.033}_{-0.033}$	$^{+0.004}_{-0.004}$	S_4	$-0.254^{+0.025}_{-0.025}$	$^{+0.002}_{-0.002}$
S_5	$-0.211^{+0.031}_{-0.031}$	$^{+0.002}_{-0.002}$	S_5	$-0.299^{+0.026}_{-0.026}$	$^{+0.003}_{-0.003}$
A_{FB}	$0.028^{+0.023}_{-0.024}$	$^{+0.006}_{-0.006}$	A_{FB}	$0.157^{+0.021}_{-0.021}$	$^{+0.005}_{-0.005}$
S_7	$-0.087^{+0.033}_{-0.033}$	$^{+0.002}_{-0.002}$	S_7	$-0.077^{+0.030}_{-0.030}$	$^{+0.001}_{-0.001}$
S_8	$-0.010^{+0.035}_{-0.035}$	$^{+0.000}_{-0.000}$	S_8	$-0.071^{+0.031}_{-0.031}$	$^{+0.003}_{-0.003}$
S_9	$-0.040^{+0.025}_{-0.025}$	$^{+0.002}_{-0.002}$	S_9	$0.047^{+0.025}_{-0.025}$	$^{+0.002}_{-0.002}$
S_6^c	$-0.026^{+0.059}_{-0.060}$	$^{+0.014}_{-0.014}$	S_6^c	$0.052^{+0.053}_{-0.051}$	$^{+0.010}_{-0.010}$
F_S	$0.089^{+0.020}_{-0.020}$	$^{+0.010}_{-0.010}$	F_S	$0.070^{+0.019}_{-0.019}$	$^{+0.005}_{-0.005}$
S_{11}^{re}	$-0.522^{+0.054}_{-0.053}$	$^{+0.041}_{-0.041}$	S_{11}^{re}	$-0.335^{+0.052}_{-0.052}$	$^{+0.026}_{-0.026}$
S_{11}^{im}	$0.030^{+0.040}_{-0.040}$	$^{+0.089}_{-0.089}$	S_{11}^{im}	$-0.039^{+0.037}_{-0.037}$	$^{+0.075}_{-0.075}$
S_{14}^{re}	$0.147^{+0.080}_{-0.080}$	$^{+0.009}_{-0.009}$	S_{14}^{re}	$0.190^{+0.065}_{-0.065}$	$^{+0.009}_{-0.009}$
S_{14}^{im}	$0.002^{+0.049}_{-0.049}$	$^{+0.028}_{-0.028}$	S_{14}^{im}	$0.053^{+0.046}_{-0.046}$	$^{+0.032}_{-0.032}$
S_{15}^{re}	$0.177^{+0.056}_{-0.056}$	$^{+0.015}_{-0.015}$	S_{15}^{re}	$0.198^{+0.059}_{-0.059}$	$^{+0.014}_{-0.014}$
S_{15}^{im}	$0.030^{+0.046}_{-0.046}$	$^{+0.041}_{-0.041}$	S_{15}^{im}	$0.117^{+0.041}_{-0.041}$	$^{+0.050}_{-0.050}$
S_{16}^{re}	$0.233^{+0.049}_{-0.049}$	$^{+0.005}_{-0.005}$	S_{16}^{re}	$0.267^{+0.048}_{-0.048}$	$^{+0.008}_{-0.008}$
S_{16}^{im}	$0.065^{+0.062}_{-0.062}$	$^{+0.048}_{-0.048}$	S_{16}^{im}	$0.013^{+0.067}_{-0.067}$	$^{+0.060}_{-0.060}$
S_{17}^{re}	$0.082^{+0.052}_{-0.052}$	$^{+0.007}_{-0.007}$	S_{17}^{re}	$0.030^{+0.052}_{-0.052}$	$^{+0.004}_{-0.004}$
S_{17}^{im}	$0.144^{+0.080}_{-0.080}$	$^{+0.021}_{-0.021}$	S_{17}^{im}	$0.018^{+0.072}_{-0.072}$	$^{+0.026}_{-0.026}$
$BF/10^{-7}$	$0.336 \pm 0.012 \pm 0.017$		$BF/10^{-7}$	$0.378 \pm 0.013 \pm 0.019$	

Table B.22: Measured values of the angular observables, including statistical and systematic uncertainties from configuration (iv) in the S_i basis.

15.0 < q^2 < 17.0 GeV^2/c^4			17.0 < q^2 < 19.0 GeV^2/c^4			11.0 < q^2 < 12.5 GeV^2/c^4		
F_L	0.287 ^{+0.037} _{-0.038}	+0.004 -0.004	F_L	0.298 ^{+0.054} _{-0.054}	+0.004 -0.004	F_L	0.452 ^{+0.047} _{-0.047}	+0.006 -0.006
S_2^s	0.181 ^{+0.021} _{-0.021}	+0.002 -0.002	S_2^s	0.173 ^{+0.030} _{-0.030}	+0.002 -0.002	S_2^s	0.158 ^{+0.025} _{-0.025}	+0.005 -0.005
S_1^c	0.348 ^{+0.024} _{-0.024}	+0.005 -0.005	S_1^c	0.384 ^{+0.033} _{-0.031}	+0.004 -0.004	S_1^c	0.454 ^{+0.029} _{-0.029}	+0.003 -0.003
S_3	-0.152 ^{+0.021} _{-0.023}	+0.002 -0.002	S_3	-0.231 ^{+0.029} _{-0.029}	+0.002 -0.002	S_3	-0.115 ^{+0.026} _{-0.025}	+0.004 -0.004
S_4	-0.287 ^{+0.021} _{-0.021}	+0.002 -0.002	S_4	-0.286 ^{+0.028} _{-0.028}	+0.002 -0.002	S_4	-0.301 ^{+0.025} _{-0.026}	+0.004 -0.004
S_5	-0.280 ^{+0.019} _{-0.019}	+0.003 -0.003	S_5	-0.239 ^{+0.026} _{-0.026}	+0.003 -0.003	S_5	-0.362 ^{+0.024} _{-0.024}	+0.004 -0.004
A_{FB}	0.410 ^{+0.018} _{-0.018}	+0.002 -0.002	A_{FB}	0.317 ^{+0.024} _{-0.024}	+0.003 -0.003	A_{FB}	0.371 ^{+0.021} _{-0.021}	+0.006 -0.006
S_7	-0.027 ^{+0.024} _{-0.024}	+0.001 -0.001	S_7	0.040 ^{+0.033} _{-0.033}	+0.001 -0.001	S_7	-0.098 ^{+0.029} _{-0.029}	+0.001 -0.001
S_8	-0.016 ^{+0.025} _{-0.025}	+0.001 -0.001	S_8	-0.008 ^{+0.034} _{-0.034}	+0.001 -0.001	S_8	-0.042 ^{+0.028} _{-0.028}	+0.001 -0.001
S_9	0.047 ^{+0.022} _{-0.022}	+0.001 -0.001	S_9	0.017 ^{+0.027} _{-0.027}	+0.000 -0.000	S_9	-0.023 ^{+0.025} _{-0.025}	+0.004 -0.004
S_6^c	-0.001 ^{+0.036} _{-0.036}	+0.004 -0.004	S_6^c	0.026 ^{+0.055} _{-0.055}	+0.003 -0.003	S_6^c	0.069 ^{+0.048} _{-0.048}	+0.006 -0.006
F_S	0.023 ^{+0.016} _{-0.016}	+0.008 -0.008	F_S	0.049 ^{+0.022} _{-0.022}	+0.008 -0.008	F_S	0.057 ^{+0.019} _{-0.019}	+0.016 -0.016
S_{11}^{re}	-0.157 ^{+0.038} _{-0.038}	+0.023 -0.023	S_{11}^{re}	-0.113 ^{+0.039} _{-0.039}	+0.014 -0.014	S_{11}^{re}	-0.278 ^{+0.050} _{-0.050}	+0.014 -0.014
S_{11}^{im}	0.047 ^{+0.027} _{-0.027}	+0.058 -0.058	S_{11}^{im}	-0.083 ^{+0.045} _{-0.045}	+0.042 -0.042	S_{11}^{im}	0.018 ^{+0.033} _{-0.033}	+0.036 -0.036
S_{14}^{re}	0.094 ^{+0.056} _{-0.056}	+0.012 -0.012	S_{14}^{re}	-0.002 ^{+0.058} _{-0.058}	+0.010 -0.010	S_{14}^{re}	0.185 ^{+0.055} _{-0.055}	+0.005 -0.005
S_{14}^{im}	-0.057 ^{+0.038} _{-0.038}	+0.038 -0.038	S_{14}^{im}	-0.094 ^{+0.071} _{-0.071}	+0.037 -0.037	S_{14}^{im}	0.050 ^{+0.045} _{-0.045}	+0.035 -0.035
S_{15}^{re}	0.176 ^{+0.051} _{-0.051}	+0.016 -0.016	S_{15}^{re}	0.080 ^{+0.053} _{-0.053}	+0.010 -0.010	S_{15}^{re}	0.331 ^{+0.060} _{-0.060}	+0.004 -0.004
S_{15}^{im}	-0.043 ^{+0.036} _{-0.036}	+0.054 -0.054	S_{15}^{im}	0.034 ^{+0.063} _{-0.063}	+0.039 -0.039	S_{15}^{im}	0.068 ^{+0.044} _{-0.044}	+0.026 -0.026
S_{16}^{re}	0.115 ^{+0.046} _{-0.046}	+0.005 -0.005	S_{16}^{re}	0.255 ^{+0.078} _{-0.078}	+0.006 -0.006	S_{16}^{re}	0.308 ^{+0.050} _{-0.050}	+0.011 -0.011
S_{16}^{im}	-0.006 ^{+0.061} _{-0.061}	+0.070 -0.070	S_{16}^{im}	-0.072 ^{+0.065} _{-0.065}	+0.068 -0.068	S_{16}^{im}	-0.056 ^{+0.073} _{-0.073}	+0.076 -0.076
S_{17}^{re}	-0.013 ^{+0.049} _{-0.049}	+0.004 -0.004	S_{17}^{re}	0.016 ^{+0.086} _{-0.086}	+0.004 -0.004	S_{17}^{re}	0.110 ^{+0.047} _{-0.047}	+0.005 -0.005
S_{17}^{im}	0.043 ^{+0.064} _{-0.064}	+0.026 -0.026	S_{17}^{im}	-0.040 ^{+0.066} _{-0.066}	+0.020 -0.020	S_{17}^{im}	-0.142 ^{+0.069} _{-0.069}	+0.014 -0.014
$BF/10^{-7}$	0.514 ± 0.013 ± 0.024		$BF/10^{-7}$	0.365 ± 0.013 ± 0.020		$BF/10^{-7}$	0.478 ± 0.015 ± 0.024	

1.1 < q^2 < 6.0 GeV^2/c^4			15.0 < q^2 < 19.0 GeV^2/c^4		
F_L	0.755 ^{+0.050} _{-0.049}	+0.007 -0.007	F_L	0.273 ^{+0.030} _{-0.031}	+0.004 -0.004
S_2^s	0.102 ^{+0.020} _{-0.021}	+0.003 -0.003	S_2^s	0.175 ^{+0.018} _{-0.018}	+0.003 -0.003
S_1^c	0.647 ^{+0.027} _{-0.029}	+0.007 -0.007	S_1^c	0.369 ^{+0.019} _{-0.019}	+0.003 -0.003
S_3	-0.012 ^{+0.017} _{-0.017}	+0.003 -0.003	S_3	-0.192 ^{+0.016} _{-0.016}	+0.002 -0.002
S_4	-0.140 ^{+0.023} _{-0.023}	+0.004 -0.004	S_4	-0.287 ^{+0.016} _{-0.016}	+0.002 -0.002
S_5	-0.048 ^{+0.022} _{-0.022}	+0.002 -0.002	S_5	-0.255 ^{+0.015} _{-0.015}	+0.003 -0.003
A_{FB}	-0.098 ^{+0.017} _{-0.017}	+0.007 -0.007	A_{FB}	0.368 ^{+0.014} _{-0.014}	+0.003 -0.003
S_7	-0.114 ^{+0.023} _{-0.023}	+0.001 -0.001	S_7	0.001 ^{+0.020} _{-0.020}	+0.000 -0.000
S_8	-0.000 ^{+0.025} _{-0.025}	+0.001 -0.001	S_8	-0.007 ^{+0.021} _{-0.021}	+0.001 -0.001
S_9	-0.031 ^{+0.018} _{-0.018}	+0.000 -0.000	S_9	0.028 ^{+0.017} _{-0.017}	+0.001 -0.001
S_6^c	-0.059 ^{+0.052} _{-0.056}	+0.015 -0.015	S_6^c	0.011 ^{+0.030} _{-0.030}	+0.003 -0.003
F_S	0.107 ^{+0.013} _{-0.013}	+0.014 -0.014	F_S	0.042 ^{+0.013} _{-0.013}	+0.008 -0.008
S_{11}^{re}	-0.474 ^{+0.041} _{-0.040}	+0.036 -0.036	S_{11}^{re}	-0.127 ^{+0.029} _{-0.027}	+0.014 -0.014
S_{11}^{im}	-0.022 ^{+0.029} _{-0.029}	+0.091 -0.091	S_{11}^{im}	-0.005 ^{+0.023} _{-0.022}	+0.039 -0.039
S_{14}^{re}	0.078 ^{+0.053} _{-0.053}	+0.006 -0.006	S_{14}^{re}	0.056 ^{+0.040} _{-0.040}	+0.009 -0.009
S_{14}^{im}	-0.019 ^{+0.037} _{-0.037}	+0.020 -0.020	S_{14}^{im}	-0.050 ^{+0.032} _{-0.032}	+0.036 -0.036
S_{15}^{re}	0.098 ^{+0.043} _{-0.043}	+0.007 -0.007	S_{15}^{re}	0.113 ^{+0.036} _{-0.036}	+0.007 -0.007
S_{15}^{im}	0.088 ^{+0.033} _{-0.033}	+0.023 -0.023	S_{15}^{im}	-0.009 ^{+0.030} _{-0.030}	+0.033 -0.033
S_{16}^{re}	0.083 ^{+0.033} _{-0.033}	+0.003 -0.003	S_{16}^{re}	0.166 ^{+0.038} _{-0.038}	+0.011 -0.011
S_{16}^{im}	0.049 ^{+0.043} _{-0.043}	+0.029 -0.029	S_{16}^{im}	-0.046 ^{+0.045} _{-0.045}	+0.072 -0.072
S_{17}^{re}	0.090 ^{+0.037} _{-0.037}	+0.004 -0.004	S_{17}^{re}	-0.018 ^{+0.041} _{-0.041}	+0.003 -0.003
S_{17}^{im}	0.027 ^{+0.054} _{-0.054}	+0.012 -0.012	S_{17}^{im}	0.004 ^{+0.045} _{-0.045}	+0.018 -0.018
$BF/10^{-7}$	0.318 ± 0.008 ± 0.019		$BF/10^{-7}$	0.443 ± 0.009 ± 0.023	

Table B.23: Measured values of the angular observables, including statistical and systematic uncertainties from configuration (v) in the S_i basis.

$0.06 < q^2 < 0.54 \text{ GeV}^2/c^4$			$0.54 < q^2 < 0.98 \text{ GeV}^2/c^4$			$1.1 < q^2 < 1.8 \text{ GeV}^2/c^4$		
F_L	$0.338^{+0.076}_{-0.076}$	$^{+0.008}_{-0.008}$	F_L	$0.569^{+0.158}_{-0.186}$	$^{+0.021}_{-0.021}$	F_L	$0.648^{+0.136}_{-0.136}$	$^{+0.028}_{-0.028}$
S_2^s	$0.194^{+0.049}_{-0.049}$	$^{+0.013}_{-0.013}$	S_2^s	$0.178^{+0.072}_{-0.080}$	$^{+0.010}_{-0.010}$	S_1^c	$0.523^{+0.082}_{-0.078}$	$^{+0.006}_{-0.006}$
S_1^c	$0.087^{+0.043}_{-0.034}$	$^{+0.006}_{-0.006}$	S_1^c	$0.421^{+0.086}_{-0.098}$	$^{+0.005}_{-0.005}$	S_3	$-0.033^{+0.050}_{-0.051}$	$^{+0.002}_{-0.002}$
S_3	$-0.015^{+0.032}_{-0.032}$	$^{+0.002}_{-0.002}$	S_3	$0.047^{+0.046}_{-0.046}$	$^{+0.001}_{-0.001}$	S_4	$0.011^{+0.068}_{-0.067}$	$^{+0.003}_{-0.003}$
S_4	$0.061^{+0.034}_{-0.034}$	$^{+0.001}_{-0.001}$	S_4	$0.093^{+0.065}_{-0.062}$	$^{+0.003}_{-0.003}$	S_5	$0.361^{+0.056}_{-0.054}$	$^{+0.004}_{-0.004}$
S_5	$0.185^{+0.029}_{-0.029}$	$^{+0.003}_{-0.003}$	S_5	$0.259^{+0.051}_{-0.051}$	$^{+0.003}_{-0.003}$	A_{FB}	$-0.198^{+0.043}_{-0.045}$	$^{+0.002}_{-0.002}$
A_{FB}	$-0.058^{+0.032}_{-0.032}$	$^{+0.003}_{-0.003}$	A_{FB}	$-0.117^{+0.059}_{-0.050}$	$^{+0.004}_{-0.004}$	S_7	$-0.131^{+0.058}_{-0.058}$	$^{+0.001}_{-0.001}$
S_7	$-0.009^{+0.030}_{-0.030}$	$^{+0.001}_{-0.001}$	S_7	$-0.061^{+0.049}_{-0.049}$	$^{+0.001}_{-0.001}$	S_8	$-0.020^{+0.068}_{-0.067}$	$^{+0.001}_{-0.001}$
S_8	$-0.025^{+0.034}_{-0.034}$	$^{+0.000}_{-0.000}$	S_8	$0.003^{+0.062}_{-0.061}$	$^{+0.001}_{-0.001}$	S_9	$-0.029^{+0.048}_{-0.050}$	$^{+0.002}_{-0.002}$
S_9	$-0.044^{+0.032}_{-0.032}$	$^{+0.000}_{-0.000}$	S_9	$-0.018^{+0.042}_{-0.042}$	$^{+0.001}_{-0.001}$	F_S	$0.135^{+0.033}_{-0.033}$	$^{+0.008}_{-0.008}$
S_6^c	$-0.180^{+0.065}_{-0.065}$	$^{+0.005}_{-0.005}$	S_6^c	$0.086^{+0.131}_{-0.131}$	$^{+0.008}_{-0.008}$	S_{11}^{re}	$-0.476^{+0.090}_{-0.090}$	$^{+0.027}_{-0.027}$
F_S	$0.009^{+0.041}_{-0.008}$	$^{+0.024}_{-0.024}$	F_S	$0.053^{+0.095}_{-0.050}$	$^{+0.023}_{-0.023}$	S_{11}^{im}	$-0.147^{+0.068}_{-0.067}$	$^{+0.080}_{-0.080}$
S_{12}	$-0.029^{+0.027}_{-0.028}$	$^{+0.007}_{-0.007}$	S_{12}	$-0.079^{+0.065}_{-0.063}$	$^{+0.011}_{-0.011}$	S_{14}^{re}	$0.040^{+0.140}_{-0.144}$	$^{+0.005}_{-0.005}$
S_{11}^{re}	$-0.289^{+0.104}_{-0.090}$	$^{+0.017}_{-0.017}$	S_{11}^{re}	$-0.003^{+0.236}_{-0.194}$	$^{+0.018}_{-0.018}$	S_{14}^{im}	$-0.155^{+0.102}_{-0.096}$	$^{+0.007}_{-0.007}$
S_{11}^{im}	$-0.281^{+0.074}_{-0.083}$	$^{+0.025}_{-0.025}$	S_{11}^{im}	$0.088^{+0.145}_{-0.139}$	$^{+0.026}_{-0.026}$	S_{15}^{re}	$-0.227^{+0.119}_{-0.111}$	$^{+0.015}_{-0.015}$
S_{13}^{re}	$-0.246^{+0.144}_{-0.148}$	$^{+0.016}_{-0.016}$	S_{13}^{re}	$0.896^{+0.093}_{-0.268}$	$^{+0.016}_{-0.016}$	S_{15}^{im}	$0.157^{+0.088}_{-0.091}$	$^{+0.018}_{-0.018}$
S_{13}^{im}	$-0.474^{+0.098}_{-0.105}$	$^{+0.025}_{-0.025}$	S_{13}^{im}	$0.032^{+0.201}_{-0.186}$	$^{+0.027}_{-0.027}$	S_{16}^{re}	$-0.286^{+0.086}_{-0.088}$	$^{+0.006}_{-0.006}$
S_{14}^{re}	$-0.085^{+0.096}_{-0.096}$	$^{+0.005}_{-0.005}$	S_{14}^{re}	$-0.165^{+0.149}_{-0.146}$	$^{+0.005}_{-0.005}$	S_{16}^{im}	$0.174^{+0.114}_{-0.114}$	$^{+0.011}_{-0.011}$
S_{14}^{im}	$0.053^{+0.066}_{-0.066}$	$^{+0.011}_{-0.011}$	S_{14}^{im}	$0.165^{+0.097}_{-0.111}$	$^{+0.011}_{-0.011}$	S_{17}^{re}	$0.069^{+0.103}_{-0.103}$	$^{+0.003}_{-0.003}$
S_{15}^{re}	$-0.217^{+0.079}_{-0.079}$	$^{+0.019}_{-0.019}$	S_{15}^{re}	$-0.155^{+0.124}_{-0.124}$	$^{+0.012}_{-0.012}$	S_{17}^{im}	$-0.233^{+0.150}_{-0.143}$	$^{+0.006}_{-0.006}$
S_{15}^{im}	$0.078^{+0.054}_{-0.054}$	$^{+0.032}_{-0.032}$	S_{15}^{im}	$-0.039^{+0.077}_{-0.077}$	$^{+0.030}_{-0.030}$	$BF/10^{-7}$	$0.351 \pm 0.022 \pm 0.017$	
S_{16}^{re}	$-0.046^{+0.054}_{-0.054}$	$^{+0.003}_{-0.003}$	S_{16}^{re}	$-0.195^{+0.078}_{-0.078}$	$^{+0.005}_{-0.005}$			
S_{16}^{im}	$0.016^{+0.081}_{-0.081}$	$^{+0.026}_{-0.026}$	S_{16}^{im}	$-0.096^{+0.120}_{-0.121}$	$^{+0.027}_{-0.027}$			
S_{17}^{re}	$0.077^{+0.068}_{-0.068}$	$^{+0.001}_{-0.001}$	S_{17}^{re}	$-0.097^{+0.106}_{-0.101}$	$^{+0.003}_{-0.003}$			
S_{17}^{im}	$0.024^{+0.105}_{-0.105}$	$^{+0.011}_{-0.011}$	S_{17}^{im}	$-0.226^{+0.158}_{-0.156}$	$^{+0.011}_{-0.011}$			
$BF/10^{-7}$	$1.429 \pm 0.056 \pm 0.069$		$BF/10^{-7}$	$0.614 \pm 0.041 \pm 0.037$				

$1.8 < q^2 < 2.5 \text{ GeV}^2/c^4$			$2.5 < q^2 < 3.25 \text{ GeV}^2/c^4$		
F_L	$0.822^{+0.102}_{-0.132}$	$^{+0.028}_{-0.028}$	F_L	$0.949^{+0.044}_{-0.110}$	$^{+0.024}_{-0.024}$
S_1^c	$0.723^{+0.074}_{-0.074}$	$^{+0.008}_{-0.008}$	S_1^c	$0.745^{+0.070}_{-0.068}$	$^{+0.008}_{-0.008}$
S_3	$-0.092^{+0.051}_{-0.052}$	$^{+0.002}_{-0.002}$	S_3	$0.104^{+0.057}_{-0.054}$	$^{+0.001}_{-0.001}$
S_4	$-0.027^{+0.065}_{-0.066}$	$^{+0.005}_{-0.005}$	S_4	$-0.204^{+0.075}_{-0.069}$	$^{+0.003}_{-0.003}$
S_5	$0.111^{+0.054}_{-0.054}$	$^{+0.005}_{-0.005}$	S_5	$-0.064^{+0.063}_{-0.063}$	$^{+0.004}_{-0.004}$
A_{FB}	$-0.218^{+0.041}_{-0.045}$	$^{+0.003}_{-0.003}$	A_{FB}	$-0.155^{+0.044}_{-0.036}$	$^{+0.002}_{-0.002}$
S_7	$-0.059^{+0.063}_{-0.063}$	$^{+0.004}_{-0.004}$	S_7	$-0.140^{+0.059}_{-0.059}$	$^{+0.003}_{-0.003}$
S_8	$-0.016^{+0.068}_{-0.066}$	$^{+0.003}_{-0.003}$	S_8	$0.006^{+0.077}_{-0.071}$	$^{+0.002}_{-0.002}$
S_9	$-0.040^{+0.049}_{-0.049}$	$^{+0.001}_{-0.001}$	S_9	$-0.037^{+0.053}_{-0.054}$	$^{+0.002}_{-0.002}$
F_S	$0.119^{+0.036}_{-0.034}$	$^{+0.009}_{-0.009}$	F_S	$0.123^{+0.035}_{-0.035}$	$^{+0.012}_{-0.012}$
S_{11}^{re}	$-0.289^{+0.101}_{-0.098}$	$^{+0.029}_{-0.029}$	S_{11}^{re}	$-0.379^{+0.096}_{-0.097}$	$^{+0.030}_{-0.030}$
S_{11}^{im}	$-0.010^{+0.072}_{-0.071}$	$^{+0.088}_{-0.088}$	S_{11}^{im}	$0.020^{+0.079}_{-0.079}$	$^{+0.091}_{-0.091}$
S_{14}^{re}	$0.011^{+0.142}_{-0.141}$	$^{+0.003}_{-0.003}$	S_{14}^{re}	$-0.141^{+0.164}_{-0.162}$	$^{+0.007}_{-0.007}$
S_{14}^{im}	$0.044^{+0.109}_{-0.111}$	$^{+0.011}_{-0.011}$	S_{14}^{im}	$0.041^{+0.116}_{-0.119}$	$^{+0.016}_{-0.016}$
S_{15}^{re}	$0.032^{+0.118}_{-0.119}$	$^{+0.004}_{-0.004}$	S_{15}^{re}	$0.026^{+0.136}_{-0.130}$	$^{+0.010}_{-0.010}$
S_{15}^{im}	$0.207^{+0.087}_{-0.091}$	$^{+0.007}_{-0.007}$	S_{15}^{im}	$0.107^{+0.099}_{-0.101}$	$^{+0.020}_{-0.020}$
S_{16}^{re}	$-0.075^{+0.091}_{-0.092}$	$^{+0.003}_{-0.003}$	S_{16}^{re}	$0.160^{+0.096}_{-0.099}$	$^{+0.006}_{-0.006}$
S_{16}^{im}	$0.026^{+0.127}_{-0.125}$	$^{+0.010}_{-0.010}$	S_{16}^{im}	$-0.013^{+0.130}_{-0.126}$	$^{+0.025}_{-0.025}$
S_{17}^{re}	$-0.107^{+0.107}_{-0.113}$	$^{+0.003}_{-0.003}$	S_{17}^{re}	$0.352^{+0.108}_{-0.122}$	$^{+0.002}_{-0.002}$
S_{17}^{im}	$0.104^{+0.149}_{-0.142}$	$^{+0.005}_{-0.005}$	S_{17}^{im}	$-0.089^{+0.159}_{-0.145}$	$^{+0.012}_{-0.012}$
$BF/10^{-7}$	$0.328 \pm 0.021 \pm 0.016$		$BF/10^{-7}$	$0.280 \pm 0.019 \pm 0.015$	

Table B.24: Measured values of the angular observables, including statistical and systematic uncertainties from configuration (v) in the S_i basis.

$3.25 < q^2 < 4.0 \text{ GeV}^2/c^4$			$4.0 < q^2 < 5.0 \text{ GeV}^2/c^4$			$5.0 < q^2 < 6.0 \text{ GeV}^2/c^4$		
F_L	$0.777^{+0.104}_{-0.118} +0.020_{-0.020}$		F_L	$0.683^{+0.074}_{-0.078} +0.012_{-0.012}$		F_L	$0.551^{+0.069}_{-0.078} +0.017_{-0.017}$	
S_1^c	$0.672^{+0.071}_{-0.072} +0.009_{-0.009}$		S_1^c	$0.770^{+0.047}_{-0.047} +0.021_{-0.021}$		S_1^c	$0.659^{+0.047}_{-0.046} +0.007_{-0.007}$	
S_3	$0.031^{+0.051}_{-0.051} +0.001_{-0.001}$		S_3	$0.015^{+0.035}_{-0.035} +0.002_{-0.002}$		S_3	$-0.069^{+0.037}_{-0.036} +0.001_{-0.001}$	
S_4	$-0.168^{+0.071}_{-0.072} +0.004_{-0.004}$		S_4	$-0.188^{+0.049}_{-0.049} +0.004_{-0.004}$		S_4	$-0.210^{+0.046}_{-0.046} +0.004_{-0.004}$	
S_5	$-0.114^{+0.065}_{-0.064} +0.005_{-0.005}$		S_5	$-0.179^{+0.046}_{-0.045} +0.004_{-0.004}$		S_5	$-0.247^{+0.044}_{-0.041} +0.003_{-0.003}$	
A_{FB}	$-0.098^{+0.039}_{-0.040} +0.004_{-0.004}$		A_{FB}	$-0.008^{+0.026}_{-0.026} +0.003_{-0.003}$		A_{FB}	$0.073^{+0.027}_{-0.027} +0.003_{-0.003}$	
S_7	$-0.095^{+0.064}_{-0.066} +0.001_{-0.001}$		S_7	$-0.100^{+0.044}_{-0.044} +0.002_{-0.002}$		S_7	$-0.075^{+0.048}_{-0.048} +0.002_{-0.002}$	
S_8	$0.017^{+0.068}_{-0.066} +0.001_{-0.001}$		S_8	$-0.013^{+0.046}_{-0.046} +0.002_{-0.002}$		S_8	$0.009^{+0.050}_{-0.050} +0.001_{-0.001}$	
S_9	$-0.001^{+0.052}_{-0.051} +0.001_{-0.001}$		S_9	$-0.068^{+0.036}_{-0.036} +0.002_{-0.002}$		S_9	$-0.021^{+0.042}_{-0.042} +0.001_{-0.001}$	
F_S	$0.157^{+0.034}_{-0.034} +0.012_{-0.012}$		F_S	$0.095^{+0.028}_{-0.028} +0.010_{-0.010}$		F_S	$0.081^{+0.026}_{-0.026} +0.009_{-0.009}$	
S_{11}^{re}	$-0.587^{+0.102}_{-0.097} +0.033_{-0.033}$		S_{11}^{re}	$-0.531^{+0.084}_{-0.084} +0.034_{-0.034}$		S_{11}^{re}	$-0.487^{+0.080}_{-0.078} +0.036_{-0.036}$	
S_{11}^{im}	$-0.060^{+0.076}_{-0.076} +0.091_{-0.091}$		S_{11}^{im}	$0.007^{+0.055}_{-0.055} +0.089_{-0.089}$		S_{11}^{im}	$0.064^{+0.053}_{-0.053} +0.087_{-0.087}$	
S_{14}^{re}	$0.220^{+0.150}_{-0.153} +0.007_{-0.007}$		S_{14}^{re}	$0.111^{+0.101}_{-0.102} +0.009_{-0.009}$		S_{14}^{re}	$0.168^{+0.101}_{-0.103} +0.012_{-0.012}$	
S_{14}^{im}	$0.023^{+0.108}_{-0.103} +0.021_{-0.021}$		S_{14}^{im}	$0.014^{+0.075}_{-0.073} +0.026_{-0.026}$		S_{14}^{im}	$0.009^{+0.065}_{-0.065} +0.030_{-0.030}$	
S_{15}^{re}	$0.157^{+0.126}_{-0.127} +0.009_{-0.009}$		S_{15}^{re}	$0.189^{+0.089}_{-0.089} +0.014_{-0.014}$		S_{15}^{re}	$0.174^{+0.089}_{-0.089} +0.016_{-0.016}$	
S_{15}^{im}	$0.176^{+0.098}_{-0.098} +0.029_{-0.029}$		S_{15}^{im}	$0.076^{+0.065}_{-0.065} +0.038_{-0.038}$		S_{15}^{im}	$-0.014^{+0.062}_{-0.062} +0.046_{-0.046}$	
S_{16}^{re}	$0.145^{+0.085}_{-0.085} +0.004_{-0.004}$		S_{16}^{re}	$0.201^{+0.068}_{-0.068} +0.007_{-0.007}$		S_{16}^{re}	$0.256^{+0.069}_{-0.069} +0.004_{-0.004}$	
S_{16}^{im}	$0.019^{+0.119}_{-0.120} +0.035_{-0.035}$		S_{16}^{im}	$0.123^{+0.079}_{-0.079} +0.044_{-0.044}$		S_{16}^{im}	$0.023^{+0.102}_{-0.102} +0.052_{-0.052}$	
S_{17}^{re}	$0.059^{+0.106}_{-0.102} +0.002_{-0.002}$		S_{17}^{re}	$-0.077^{+0.075}_{-0.076} +0.002_{-0.002}$		S_{17}^{re}	$0.212^{+0.070}_{-0.070} +0.002_{-0.002}$	
S_{17}^{im}	$-0.023^{+0.137}_{-0.136} +0.015_{-0.015}$		S_{17}^{im}	$0.205^{+0.101}_{-0.101} +0.019_{-0.019}$		S_{17}^{im}	$0.075^{+0.127}_{-0.127} +0.022_{-0.022}$	
$BF/10^{-7}$	$0.274 \pm 0.018 \pm 0.015$		$BF/10^{-7}$	$0.343 \pm 0.017 \pm 0.017$		$BF/10^{-7}$	$0.334 \pm 0.016 \pm 0.016$	

$6.0 < q^2 < 7.0 \text{ GeV}^2/c^4$			$7.0 < q^2 < 8.0 \text{ GeV}^2/c^4$		
F_L	$0.614^{+0.060}_{-0.066} +0.023_{-0.023}$		F_L	$0.639^{+0.061}_{-0.067} +0.013_{-0.013}$	
S_1^c	$0.617^{+0.041}_{-0.040} +0.008_{-0.008}$		S_1^c	$0.577^{+0.043}_{-0.045} +0.014_{-0.014}$	
S_3	$-0.055^{+0.034}_{-0.034} +0.002_{-0.002}$		S_3	$-0.083^{+0.034}_{-0.034} +0.001_{-0.001}$	
S_4	$-0.227^{+0.042}_{-0.041} +0.003_{-0.003}$		S_4	$-0.271^{+0.035}_{-0.035} +0.003_{-0.003}$	
S_5	$-0.249^{+0.036}_{-0.036} +0.002_{-0.002}$		S_5	$-0.329^{+0.043}_{-0.039} +0.003_{-0.003}$	
A_{FB}	$0.099^{+0.025}_{-0.025} +0.004_{-0.004}$		A_{FB}	$0.178^{+0.025}_{-0.025} +0.005_{-0.005}$	
S_7	$-0.072^{+0.041}_{-0.041} +0.002_{-0.002}$		S_7	$-0.072^{+0.044}_{-0.044} +0.001_{-0.001}$	
S_8	$-0.018^{+0.042}_{-0.042} +0.002_{-0.002}$		S_8	$-0.113^{+0.043}_{-0.043} +0.002_{-0.002}$	
S_9	$0.014^{+0.035}_{-0.035} +0.001_{-0.001}$		S_9	$0.076^{+0.036}_{-0.036} +0.002_{-0.002}$	
F_S	$0.042^{+0.026}_{-0.024} +0.009_{-0.009}$		F_S	$0.071^{+0.028}_{-0.028} +0.009_{-0.009}$	
S_{11}^{re}	$-0.469^{+0.067}_{-0.067} +0.037_{-0.037}$		S_{11}^{re}	$-0.228^{+0.075}_{-0.075} +0.036_{-0.036}$	
S_{11}^{im}	$-0.036^{+0.051}_{-0.051} +0.083_{-0.083}$		S_{11}^{im}	$-0.049^{+0.053}_{-0.053} +0.078_{-0.078}$	
S_{14}^{re}	$0.243^{+0.086}_{-0.086} +0.013_{-0.013}$		S_{14}^{re}	$0.087^{+0.101}_{-0.101} +0.014_{-0.014}$	
S_{14}^{im}	$0.054^{+0.058}_{-0.058} +0.033_{-0.033}$		S_{14}^{im}	$0.053^{+0.070}_{-0.070} +0.035_{-0.035}$	
S_{15}^{re}	$0.308^{+0.078}_{-0.078} +0.020_{-0.020}$		S_{15}^{re}	$0.139^{+0.085}_{-0.085} +0.021_{-0.021}$	
S_{15}^{im}	$0.128^{+0.056}_{-0.056} +0.051_{-0.051}$		S_{15}^{im}	$0.121^{+0.060}_{-0.060} +0.054_{-0.054}$	
S_{16}^{re}	$0.280^{+0.066}_{-0.066} +0.006_{-0.006}$		S_{16}^{re}	$0.264^{+0.075}_{-0.075} +0.006_{-0.006}$	
S_{16}^{im}	$-0.000^{+0.102}_{-0.102} +0.057_{-0.057}$		S_{16}^{im}	$0.039^{+0.095}_{-0.095} +0.062_{-0.062}$	
S_{17}^{re}	$0.064^{+0.070}_{-0.070} +0.004_{-0.004}$		S_{17}^{re}	$0.047^{+0.075}_{-0.075} +0.003_{-0.003}$	
S_{17}^{im}	$-0.107^{+0.110}_{-0.110} +0.024_{-0.024}$		S_{17}^{im}	$0.132^{+0.098}_{-0.098} +0.026_{-0.026}$	
$BF/10^{-7}$	$0.390 \pm 0.017 \pm 0.019$		$BF/10^{-7}$	$0.377 \pm 0.017 \pm 0.018$	

Table B.25: Measured values of the angular observables, including statistical and systematic uncertainties from configuration (v) in the S_i basis.

15.0 < q^2 < 16.0 GeV^2/c^4			16.0 < q^2 < 17.0 GeV^2/c^4			17.0 < q^2 < 18.0 GeV^2/c^4		
F_L	0.264 ^{+0.049} _{-0.050}	^{+0.005} _{-0.005}	F_L	0.313 ^{+0.045} _{-0.046}	^{+0.003} _{-0.003}	F_L	0.348 ^{+0.053} _{-0.057}	^{+0.006} _{-0.006}
S_1^c	0.310 ^{+0.034} _{-0.033}	^{+0.003} _{-0.003}	S_1^c	0.343 ^{+0.032} _{-0.031}	^{+0.003} _{-0.003}	S_1^c	0.376 ^{+0.038} _{-0.037}	^{+0.003} _{-0.003}
S_3	-0.126 ^{+0.033} _{-0.032}	^{+0.003} _{-0.003}	S_3	-0.184 ^{+0.028} _{-0.030}	^{+0.003} _{-0.003}	S_3	-0.250 ^{+0.034} _{-0.037}	^{+0.004} _{-0.004}
S_4	-0.274 ^{+0.032} _{-0.033}	^{+0.004} _{-0.004}	S_4	-0.288 ^{+0.027} _{-0.028}	^{+0.004} _{-0.004}	S_4	-0.262 ^{+0.034} _{-0.036}	^{+0.004} _{-0.004}
S_5	-0.278 ^{+0.028} _{-0.028}	^{+0.005} _{-0.005}	S_5	-0.277 ^{+0.027} _{-0.028}	^{+0.004} _{-0.004}	S_5	-0.224 ^{+0.032} _{-0.034}	^{+0.004} _{-0.004}
A_{FB}	0.433 ^{+0.022} _{-0.022}	^{+0.005} _{-0.005}	A_{FB}	0.387 ^{+0.020} _{-0.020}	^{+0.005} _{-0.005}	A_{FB}	0.303 ^{+0.025} _{-0.026}	^{+0.006} _{-0.006}
S_7	-0.093 ^{+0.037} _{-0.036}	^{+0.000} _{-0.000}	S_7	0.028 ^{+0.033} _{-0.033}	^{+0.001} _{-0.001}	S_7	0.027 ^{+0.041} _{-0.041}	^{+0.002} _{-0.002}
S_8	-0.019 ^{+0.037} _{-0.037}	^{+0.001} _{-0.001}	S_8	-0.011 ^{+0.034} _{-0.034}	^{+0.000} _{-0.000}	S_8	-0.056 ^{+0.041} _{-0.041}	^{+0.001} _{-0.001}
S_9	0.023 ^{+0.034} _{-0.034}	^{+0.001} _{-0.001}	S_9	0.057 ^{+0.030} _{-0.030}	^{+0.001} _{-0.001}	S_9	0.027 ^{+0.035} _{-0.037}	^{+0.001} _{-0.001}
F_S	0.028 ^{+0.024} _{-0.028}	^{+0.010} _{-0.010}	F_S	0.027 ^{+0.021} _{-0.020}	^{+0.008} _{-0.008}	F_S	0.018 ^{+0.026} _{-0.026}	^{+0.010} _{-0.010}
S_{11}^{re}	-0.134 ^{+0.057} _{-0.057}	^{+0.013} _{-0.013}	S_{11}^{re}	-0.170 ^{+0.048} _{-0.048}	^{+0.012} _{-0.012}	S_{11}^{re}	-0.213 ^{+0.050} _{-0.050}	^{+0.009} _{-0.009}
S_{11}^{im}	0.045 ^{+0.039} _{-0.039}	^{+0.042} _{-0.042}	S_{11}^{im}	0.038 ^{+0.036} _{-0.036}	^{+0.041} _{-0.041}	S_{11}^{im}	-0.051 ^{+0.054} _{-0.054}	^{+0.037} _{-0.037}
S_{14}^{re}	0.126 ^{+0.083} _{-0.083}	^{+0.010} _{-0.010}	S_{14}^{re}	0.090 ^{+0.067} _{-0.069}	^{+0.010} _{-0.010}	S_{14}^{re}	0.063 ^{+0.071} _{-0.070}	^{+0.007} _{-0.007}
S_{14}^{im}	-0.079 ^{+0.063} _{-0.066}	^{+0.036} _{-0.036}	S_{14}^{im}	-0.050 ^{+0.054} _{-0.056}	^{+0.036} _{-0.036}	S_{14}^{im}	-0.121 ^{+0.078} _{-0.078}	^{+0.035} _{-0.035}
S_{15}^{re}	0.198 ^{+0.075} _{-0.075}	^{+0.011} _{-0.011}	S_{15}^{re}	0.146 ^{+0.063} _{-0.065}	^{+0.009} _{-0.009}	S_{15}^{re}	0.188 ^{+0.064} _{-0.064}	^{+0.005} _{-0.005}
S_{15}^{im}	0.001 ^{+0.053} _{-0.053}	^{+0.041} _{-0.041}	S_{15}^{im}	-0.055 ^{+0.051} _{-0.051}	^{+0.037} _{-0.037}	S_{15}^{im}	0.056 ^{+0.072} _{-0.072}	^{+0.029} _{-0.029}
S_{16}^{re}	0.213 ^{+0.065} _{-0.065}	^{+0.006} _{-0.006}	S_{16}^{re}	0.019 ^{+0.065} _{-0.063}	^{+0.006} _{-0.006}	S_{16}^{re}	0.229 ^{+0.094} _{-0.094}	^{+0.006} _{-0.006}
S_{16}^{im}	0.017 ^{+0.092} _{-0.092}	^{+0.068} _{-0.068}	S_{16}^{im}	-0.048 ^{+0.081} _{-0.081}	^{+0.070} _{-0.070}	S_{16}^{im}	-0.100 ^{+0.080} _{-0.080}	^{+0.074} _{-0.074}
S_{17}^{re}	-0.016 ^{+0.067} _{-0.067}	^{+0.002} _{-0.002}	S_{17}^{re}	-0.003 ^{+0.070} _{-0.070}	^{+0.002} _{-0.002}	S_{17}^{re}	0.029 ^{+0.099} _{-0.099}	^{+0.007} _{-0.007}
S_{17}^{im}	-0.001 ^{+0.095} _{-0.095}	^{+0.021} _{-0.021}	S_{17}^{im}	0.088 ^{+0.084} _{-0.084}	^{+0.019} _{-0.019}	S_{17}^{im}	-0.040 ^{+0.079} _{-0.079}	^{+0.016} _{-0.016}
$BF/10^{-7}$	0.474 ± 0.018 ± 0.024		$BF/10^{-7}$	0.563 ± 0.019 ± 0.027		$BF/10^{-7}$	0.469 ± 0.020 ± 0.022	

18.0 < q^2 < 19.0 GeV^2/c^4			11.0 < q^2 < 11.75 GeV^2/c^4			11.75 < q^2 < 12.5 GeV^2/c^4		
F_L	0.191 ^{+0.075} _{-0.084}	^{+0.005} _{-0.005}	F_L	0.446 ^{+0.059} _{-0.062}	^{+0.005} _{-0.005}	F_L	0.519 ^{+0.049} _{-0.049}	^{+0.005} _{-0.005}
S_1^c	0.394 ^{+0.054} _{-0.052}	^{+0.004} _{-0.004}	S_1^c	0.416 ^{+0.040} _{-0.040}	^{+0.006} _{-0.006}	S_1^c	0.486 ^{+0.034} _{-0.034}	^{+0.004} _{-0.004}
S_3	-0.224 ^{+0.048} _{-0.045}	^{+0.004} _{-0.004}	S_3	-0.156 ^{+0.041} _{-0.040}	^{+0.002} _{-0.002}	S_3	-0.106 ^{+0.028} _{-0.028}	^{+0.002} _{-0.002}
S_4	-0.308 ^{+0.046} _{-0.050}	^{+0.004} _{-0.004}	S_4	-0.318 ^{+0.039} _{-0.041}	^{+0.003} _{-0.003}	S_4	-0.273 ^{+0.032} _{-0.032}	^{+0.003} _{-0.003}
S_5	-0.215 ^{+0.045} _{-0.044}	^{+0.003} _{-0.003}	S_5	-0.356 ^{+0.032} _{-0.032}	^{+0.004} _{-0.004}	S_5	-0.354 ^{+0.034} _{-0.034}	^{+0.004} _{-0.004}
A_{FB}	0.310 ^{+0.037} _{-0.037}	^{+0.004} _{-0.004}	A_{FB}	0.403 ^{+0.025} _{-0.026}	^{+0.004} _{-0.004}	A_{FB}	0.286 ^{+0.023} _{-0.023}	^{+0.004} _{-0.004}
S_7	0.062 ^{+0.053} _{-0.053}	^{+0.002} _{-0.002}	S_7	-0.128 ^{+0.042} _{-0.042}	^{+0.001} _{-0.001}	S_7	-0.072 ^{+0.040} _{-0.040}	^{+0.001} _{-0.001}
S_8	0.081 ^{+0.057} _{-0.057}	^{+0.001} _{-0.001}	S_8	-0.044 ^{+0.037} _{-0.037}	^{+0.001} _{-0.001}	S_8	-0.044 ^{+0.040} _{-0.040}	^{+0.000} _{-0.000}
S_9	0.011 ^{+0.046} _{-0.046}	^{+0.001} _{-0.001}	S_9	-0.071 ^{+0.035} _{-0.035}	^{+0.002} _{-0.002}	S_9	0.013 ^{+0.034} _{-0.034}	^{+0.001} _{-0.001}
F_S	0.068 ^{+0.033} _{-0.033}	^{+0.007} _{-0.007}	F_S	0.086 ^{+0.026} _{-0.026}	^{+0.007} _{-0.007}	F_S	0.007 ^{+0.025} _{-0.025}	^{+0.008} _{-0.008}
S_{11}^{re}	0.046 ^{+0.064} _{-0.064}	^{+0.008} _{-0.008}	S_{11}^{re}	-0.304 ^{+0.067} _{-0.067}	^{+0.022} _{-0.022}	S_{11}^{re}	-0.252 ^{+0.073} _{-0.073}	^{+0.020} _{-0.020}
S_{11}^{im}	-0.125 ^{+0.089} _{-0.089}	^{+0.033} _{-0.033}	S_{11}^{im}	-0.004 ^{+0.044} _{-0.044}	^{+0.059} _{-0.059}	S_{11}^{im}	0.044 ^{+0.047} _{-0.047}	^{+0.056} _{-0.056}
S_{14}^{re}	-0.087 ^{+0.094} _{-0.094}	^{+0.007} _{-0.007}	S_{14}^{re}	0.177 ^{+0.090} _{-0.099}	^{+0.012} _{-0.012}	S_{14}^{re}	0.160 ^{+0.075} _{-0.075}	^{+0.012} _{-0.012}
S_{14}^{im}	-0.091 ^{+0.142} _{-0.143}	^{+0.033} _{-0.033}	S_{14}^{im}	0.072 ^{+0.062} _{-0.062}	^{+0.038} _{-0.038}	S_{14}^{im}	0.027 ^{+0.062} _{-0.062}	^{+0.038} _{-0.038}
S_{15}^{re}	-0.080 ^{+0.085} _{-0.086}	^{+0.006} _{-0.006}	S_{15}^{re}	0.464 ^{+0.081} _{-0.081}	^{+0.017} _{-0.017}	S_{15}^{re}	0.260 ^{+0.076} _{-0.076}	^{+0.016} _{-0.016}
S_{15}^{im}	-0.017 ^{+0.115} _{-0.115}	^{+0.019} _{-0.019}	S_{15}^{im}	0.121 ^{+0.058} _{-0.058}	^{+0.055} _{-0.055}	S_{15}^{im}	0.039 ^{+0.060} _{-0.060}	^{+0.053} _{-0.053}
S_{16}^{re}	0.350 ^{+0.146} _{-0.146}	^{+0.012} _{-0.012}	S_{16}^{re}	0.318 ^{+0.071} _{-0.071}	^{+0.006} _{-0.006}	S_{16}^{re}	0.331 ^{+0.071} _{-0.071}	^{+0.006} _{-0.006}
S_{16}^{im}	-0.058 ^{+0.106} _{-0.106}	^{+0.075} _{-0.075}	S_{16}^{im}	-0.117 ^{+0.108} _{-0.108}	^{+0.070} _{-0.070}	S_{16}^{im}	-0.043 ^{+0.100} _{-0.100}	^{+0.071} _{-0.071}
S_{17}^{re}	-0.002 ^{+0.169} _{-0.169}	^{+0.002} _{-0.002}	S_{17}^{re}	0.122 ^{+0.067} _{-0.067}	^{+0.003} _{-0.003}	S_{17}^{re}	0.086 ^{+0.067} _{-0.067}	^{+0.002} _{-0.002}
S_{17}^{im}	-0.010 ^{+0.114} _{-0.114}	^{+0.012} _{-0.012}	S_{17}^{im}	-0.162 ^{+0.093} _{-0.093}	^{+0.026} _{-0.026}	S_{17}^{im}	-0.151 ^{+0.103} _{-0.103}	^{+0.026} _{-0.026}
$BF/10^{-7}$	0.265 ± 0.015 ± 0.015		$BF/10^{-7}$	0.469 ± 0.021 ± 0.022		$BF/10^{-7}$	0.507 ± 0.021 ± 0.024	

B.11 Correlation matrices

B.11.1 Fit configuration (i)

Table B.26: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.29	1.00										
S_1^c	-0.71	-0.43	1.00									
S_3	-0.03	-0.02	0.03	1.00								
S_4	-0.23	-0.10	0.24	0.06	1.00							
S_5	-0.01	-0.09	0.09	0.12	0.07	1.00						
A_{FB}	-0.07	-0.06	0.12	0.02	0.25	0.10	1.00					
S_7	0.01	-0.02	0.01	-0.05	0.00	-0.02	-0.01	1.00				
S_8	-0.05	-0.03	0.09	0.01	-0.02	0.00	-0.01	-0.02	1.00			
S_9	0.01	0.02	-0.00	-0.01	-0.02	-0.01	-0.00	0.07	0.01	1.00		
S_6^c	-0.06	-0.03	0.08	0.02	0.23	0.11	0.45	0.05	0.03	-0.03	1.00	
dB/dq^2	-0.11	0.14	0.13	0.01	0.03	-0.04	0.04	0.01	0.01	0.03	0.03	1.00

Table B.27: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.23	1.00								
S_3	-0.00	0.06	1.00							
S_4	-0.02	0.11	-0.01	1.00						
S_5	0.04	-0.04	0.10	-0.21	1.00					
A_{FB}	0.00	0.36	-0.03	0.14	-0.01	1.00				
S_7	-0.20	0.14	0.09	0.03	-0.01	-0.00	1.00			
S_8	0.26	-0.19	-0.00	-0.09	0.02	-0.03	-0.23	1.00		
S_9	0.05	0.14	-0.04	0.05	-0.00	-0.10	0.20	0.03	1.00	
dB/dq^2	-0.09	0.02	0.00	0.02	-0.04	0.07	0.05	-0.00	-0.02	1.00

Table B.28: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.03	1.00								
S_3	-0.04	-0.18	1.00							
S_4	-0.05	0.09	-0.12	1.00						
S_5	-0.06	-0.07	-0.07	-0.17	1.00					
A_{FB}	0.12	0.25	0.10	-0.03	-0.14	1.00				
S_7	-0.03	0.04	0.05	-0.11	0.02	-0.01	1.00			
S_8	-0.16	0.09	0.04	0.04	-0.08	-0.10	-0.12	1.00		
S_9	0.06	0.08	0.05	-0.04	-0.07	-0.10	0.08	-0.06	1.00	
dB/dq^2	-0.11	-0.04	-0.07	0.06	0.04	0.06	-0.00	-0.03	-0.01	1.00

Table B.29: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.42	1.00								
S_3	-0.02	0.05	1.00							
S_4	0.17	-0.11	-0.05	1.00						
S_5	0.03	0.02	-0.15	-0.05	1.00					
A_{FB}	-0.08	-0.07	0.03	-0.13	-0.10	1.00				
S_7	0.01	0.05	0.04	-0.01	-0.01	-0.01	1.00			
S_8	-0.02	0.04	0.00	0.00	0.00	-0.06	0.04	1.00		
S_9	0.01	0.09	-0.07	-0.00	-0.11	-0.04	-0.12	-0.07	1.00	
dB/dq^2	-0.02	0.03	0.01	0.05	0.05	0.00	0.03	0.02	0.01	1.00

Table B.30: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.33	1.00								
S_3	-0.01	0.06	1.00							
S_4	0.17	-0.10	-0.09	1.00						
S_5	-0.00	0.00	-0.21	-0.04	1.00					
A_{FB}	-0.13	-0.21	-0.01	-0.11	-0.11	1.00				
S_7	0.03	0.00	0.04	-0.06	-0.01	-0.02	1.00			
S_8	0.02	-0.02	0.03	0.00	-0.04	-0.08	0.15	1.00		
S_9	-0.01	-0.06	-0.09	0.01	-0.02	-0.00	-0.09	-0.05	1.00	
dB/dq^2	0.00	0.05	0.03	0.03	0.08	-0.04	0.01	0.02	-0.00	1.00

Table B.31: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $15.0 < q^2 < 17.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.29	1.00								
S_3	-0.01	0.21	1.00							
S_4	0.12	-0.20	-0.22	1.00						
S_5	-0.09	-0.07	-0.13	0.05	1.00					
A_{FB}	-0.29	-0.37	0.06	0.06	-0.12	1.00				
S_7	-0.02	-0.03	-0.01	-0.06	-0.02	0.02	1.00			
S_8	0.03	-0.07	-0.02	0.00	-0.07	0.01	0.30	1.00		
S_9	-0.02	-0.01	-0.01	0.01	-0.03	0.04	-0.14	-0.07	1.00	
dB/dq^2	0.04	0.05	0.03	0.02	0.05	-0.09	-0.00	-0.01	-0.02	1.00

Table B.32: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $17.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.44	1.00								
S_3	-0.07	0.16	1.00							
S_4	0.05	-0.07	-0.21	1.00						
S_5	-0.06	-0.01	-0.14	0.31	1.00					
A_{FB}	-0.12	-0.22	0.01	-0.03	-0.13	1.00				
S_7	0.03	0.01	0.02	0.02	0.04	0.00	1.00			
S_8	0.01	-0.03	-0.01	0.07	0.03	-0.02	0.22	1.00		
S_9	-0.00	-0.03	0.06	-0.03	0.06	-0.01	-0.09	-0.12	1.00	
dB/dq^2	0.02	0.06	0.04	0.04	0.04	-0.08	-0.00	0.01	-0.00	1.00

Table B.33: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $11.0 < q^2 < 12.5 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.25	1.00								
S_3	0.01	0.19	1.00							
S_4	0.06	-0.01	0.01	1.00						
S_5	-0.08	-0.02	-0.25	0.03	1.00					
A_{FB}	-0.27	-0.45	-0.11	-0.12	-0.05	1.00				
S_7	-0.01	-0.03	-0.04	-0.11	-0.11	0.02	1.00			
S_8	0.04	-0.02	-0.01	-0.04	-0.11	-0.03	0.24	1.00		
S_9	-0.02	-0.00	0.09	-0.04	-0.11	-0.05	-0.11	-0.11	1.00	
dB/dq^2	0.04	0.09	0.05	0.08	0.07	-0.14	0.00	-0.00	0.01	1.00

Table B.34: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.33	1.00								
S_3	-0.01	-0.01	1.00							
S_4	0.12	-0.08	-0.05	1.00						
S_5	0.04	-0.05	-0.01	-0.09	1.00					
A_{FB}	0.10	0.13	0.02	-0.05	-0.07	1.00				
S_7	0.03	0.01	0.07	-0.04	0.00	0.02	1.00			
S_8	-0.12	0.07	-0.02	0.02	-0.03	-0.10	-0.11	1.00		
S_9	0.01	0.06	0.00	-0.03	-0.09	-0.04	-0.01	-0.06	1.00	
dB/dq^2	-0.02	0.04	-0.02	0.03	0.02	0.02	0.01	0.01	0.00	1.00

Table B.35: Total correlation matrix of the S_i observables obtained from fit configuration (i) in q^2 bin $15.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.37	1.00								
S_3	-0.03	0.25	1.00							
S_4	0.08	-0.15	-0.20	1.00						
S_5	-0.08	-0.05	-0.18	0.12	1.00					
A_{FB}	-0.20	-0.29	0.09	-0.00	-0.15	1.00				
S_7	0.00	-0.01	0.01	-0.03	0.01	0.02	1.00			
S_8	0.03	-0.05	-0.02	0.04	-0.02	-0.02	0.27	1.00		
S_9	-0.01	-0.01	-0.02	0.03	0.00	0.02	-0.11	-0.11	1.00	
dB/dq^2	0.03	0.05	0.03	0.02	0.04	-0.08	-0.00	0.00	-0.01	1.00

B.11.2 Fit configuration (ii)

Table B.36: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00										
S_2^s	0.30	1.00									
S_1^c	-0.72	-0.43	1.00								
P_1	-0.02	-0.02	0.03	1.00							
P_2	0.06	0.37	-0.12	-0.01	1.00						
P_3	-0.12	-0.34	0.14	0.01	-0.13	1.00					
P_4	-0.49	-0.39	0.47	0.06	-0.10	0.15	1.00				
P_5	-0.63	-0.70	0.61	0.09	-0.25	0.24	0.48	1.00			
P_6	0.04	0.02	-0.02	-0.05	-0.04	-0.07	-0.02	-0.04	1.00		
P_8	0.10	0.12	-0.05	0.01	0.01	-0.05	-0.10	-0.11	-0.01	1.00	
dB/dq^2	-0.11	0.14	0.13	0.01	0.06	-0.07	0.02	-0.03	0.01	0.01	1.00

Table B.37: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.23	1.00								
P_1	-0.03	-0.03	1.00							
P_2	-0.11	-0.15	-0.04	1.00						
P_3	-0.02	-0.04	0.06	0.21	1.00					
P_4	-0.01	0.12	-0.02	0.08	-0.05	1.00				
P_5	-0.11	0.22	0.09	-0.02	-0.01	-0.18	1.00			
P_6	-0.13	0.03	0.09	-0.02	-0.20	0.01	0.01	1.00		
P_8	0.23	-0.20	-0.00	0.01	-0.05	-0.10	-0.06	-0.19	1.00	
dB/dq^2	-0.08	0.02	0.00	0.10	0.02	0.01	-0.03	0.05	-0.00	1.00

Table B.38: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.03	1.00								
P_1	0.05	0.03	1.00							
P_2	-0.08	-0.22	0.15	1.00						
P_3	-0.04	-0.03	-0.08	0.14	1.00					
P_4	-0.02	-0.15	-0.12	-0.02	0.06	1.00				
P_5	-0.05	-0.19	-0.09	-0.07	0.07	-0.14	1.00			
P_6	-0.02	-0.09	0.06	0.01	-0.07	-0.10	0.04	1.00		
P_8	-0.18	0.10	0.04	-0.11	0.06	0.02	-0.09	-0.13	1.00	
dB/dq^2	-0.11	-0.03	-0.08	0.07	0.01	0.07	0.06	-0.00	-0.03	1.00

Table B.39: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.41	1.00								
P_1	-0.02	-0.02	1.00							
P_2	-0.05	0.06	0.03	1.00						
P_3	0.01	0.02	0.08	0.03	1.00					
P_4	0.35	-0.36	-0.04	-0.14	0.00	1.00				
P_5	0.26	-0.29	-0.13	-0.11	0.11	0.10	1.00			
P_6	0.10	-0.07	0.03	-0.01	0.13	0.03	0.03	1.00		
P_8	-0.02	0.03	-0.00	-0.06	0.07	-0.01	-0.00	0.04	1.00	
dB/dq^2	-0.02	0.03	0.01	0.01	-0.01	0.03	0.03	0.03	0.02	1.00

Table B.40: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.32	1.00								
P_1	-0.04	-0.07	1.00							
P_2	-0.02	0.17	-0.01	1.00						
P_3	-0.01	-0.02	0.09	0.01	1.00					
P_4	0.43	-0.38	-0.07	-0.15	-0.01	1.00				
P_5	0.31	-0.34	-0.18	-0.16	0.02	0.17	1.00			
P_6	0.10	-0.07	0.04	-0.02	0.09	0.00	0.04	1.00		
P_8	0.09	-0.10	0.03	-0.09	0.06	0.05	0.01	0.16	1.00	
dB/dq^2	0.00	0.05	0.02	-0.02	-0.00	0.02	0.06	0.01	0.01	1.00

Table B.41: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $15.0 < q^2 < 17.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.28	1.00								
P_1	-0.05	0.06	1.00							
P_2	-0.15	0.28	0.24	1.00						
P_3	0.01	-0.03	0.00	-0.04	1.00					
P_4	0.69	-0.43	-0.18	-0.16	0.01	1.00				
P_5	0.59	-0.38	-0.13	-0.33	0.04	0.49	1.00			
P_6	0.03	-0.05	-0.01	-0.02	0.15	-0.01	0.01	1.00		
P_8	0.04	-0.07	-0.01	-0.05	0.08	0.04	-0.02	0.30	1.00	
dB/dq^2	0.04	0.05	0.01	-0.04	0.01	0.03	0.05	-0.00	-0.01	1.00

Table B.42: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $17.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.45	1.00								
P_1	-0.07	-0.12	1.00							
P_2	-0.10	0.26	-0.03	1.00						
P_3	0.00	0.01	-0.06	0.02	1.00					
P_4	0.67	-0.47	-0.13	-0.16	0.02	1.00				
P_5	0.57	-0.42	-0.08	-0.25	-0.05	0.61	1.00			
P_6	-0.08	0.08	0.01	0.04	0.09	-0.05	-0.02	1.00		
P_8	0.03	-0.04	0.01	-0.04	0.12	0.08	0.05	0.22	1.00	
dB/dq^2	-0.02	0.07	-0.19	0.18	-0.01	-0.14	-0.14	0.03	-0.04	1.00

Table B.43: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $11.0 < q^2 < 12.5 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.23	1.00								
P_1	-0.02	0.00	1.00							
P_2	-0.11	0.26	0.01	1.00						
P_3	0.02	0.04	-0.10	0.08	1.00					
P_4	0.45	-0.27	-0.01	-0.19	0.03	1.00				
P_5	0.40	-0.33	-0.23	-0.20	0.09	0.24	1.00			
P_6	0.10	-0.11	-0.03	-0.04	0.11	-0.03	-0.03	1.00		
P_8	0.08	-0.05	-0.01	-0.06	0.10	0.00	-0.05	0.25	1.00	
dB/dq^2	0.04	0.09	0.03	-0.07	-0.01	0.07	0.06	0.00	-0.00	1.00

Table B.44: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.33	1.00								
P_1	-0.02	-0.04	1.00							
P_2	0.02	-0.11	0.04	1.00						
P_3	0.01	0.01	-0.01	0.05	1.00					
P_4	0.24	-0.26	-0.04	-0.02	0.03	1.00				
P_5	0.09	-0.12	-0.01	-0.05	0.08	-0.05	1.00			
P_6	0.12	-0.13	0.07	0.02	0.01	0.01	0.02	1.00		
P_8	-0.12	0.07	-0.02	-0.10	0.06	-0.01	-0.04	-0.12	1.00	
dB/dq^2	-0.02	0.05	-0.01	0.02	-0.00	0.03	0.02	0.00	0.01	1.00

Table B.45: Total correlation matrix of the P_i observables obtained from fit configuration (ii) in q^2 bin $15.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	P_1	P_2	P_3	P_4	P_5	P_6	P_8	dB/dq^2
F_L	1.00									
S_1^c	-0.37	1.00								
P_1	-0.06	0.03	1.00							
P_2	-0.13	0.25	0.21	1.00						
P_3	0.01	-0.02	0.01	-0.02	1.00					
P_4	0.69	-0.47	-0.16	-0.17	-0.01	1.00				
P_5	0.61	-0.42	-0.17	-0.29	0.01	0.53	1.00			
P_6	-0.00	-0.01	0.01	0.02	0.11	-0.02	0.01	1.00		
P_8	0.04	-0.06	-0.01	-0.05	0.11	0.06	0.01	0.27	1.00	
dB/dq^2	0.03	0.05	0.01	-0.04	0.01	0.02	0.04	-0.00	0.00	1.00

B.11.3 Fit configuration (iii)

Table B.46: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$.

	F_L	S_2^*	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	A_{FL}	A_{2s}	A_{1c}	A_3	A_4	A_5	A_{fb}^{CP}	A_{6c}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																							
S_2^*	0.34	1.00																						
S_1^c	-0.76	-0.44	1.00																					
S_3	-0.04	-0.03	0.05	1.00																				
S_4	-0.16	-0.09	0.17	0.06	1.00																			
S_5	0.01	-0.10	0.06	0.11	-0.00	1.00																		
A_{FB}	-0.11	-0.06	0.13	0.02	0.25	0.09	1.00																	
S_7	0.04	-0.01	-0.01	-0.05	-0.02	-0.04	-0.02	1.00																
S_8	-0.12	-0.05	0.14	0.03	-0.04	-0.01	-0.01	-0.03	1.00															
S_9	0.02	0.02	-0.00	-0.01	-0.02	-0.00	-0.01	0.07	0.01	1.00														
S_6^c	-0.09	0.00	0.08	0.02	0.21	0.08	0.44	0.05	0.03	-0.03	1.00													
A_{FL}	-0.05	0.04	-0.02	0.02	-0.08	-0.00	-0.01	0.04	0.14	-0.03	0.01	1.00												
A_{2s}	-0.01	0.07	-0.02	0.03	-0.07	-0.06	-0.02	0.04	0.02	0.00	0.01	0.39	1.00											
A_{1c}	-0.05	-0.06	0.11	-0.02	0.09	-0.01	0.06	-0.05	-0.14	0.01	0.07	-0.72	-0.34	1.00										
A_3	0.03	0.02	-0.03	0.06	0.01	0.04	0.00	-0.01	-0.02	-0.00	-0.00	-0.04	-0.03	0.05	1.00									
A_4	-0.08	-0.07	0.09	0.02	0.17	0.15	-0.01	-0.03	0.04	-0.00	-0.03	-0.15	-0.05	0.19	0.05	1.00								
A_5	-0.02	-0.06	0.00	0.05	0.15	0.04	0.02	-0.01	-0.02	0.02	0.04	-0.02	-0.01	0.14	0.11	0.02	1.00							
A_{fb}^{CP}	-0.01	-0.06	0.04	0.01	-0.00	0.02	0.09	-0.04	-0.03	-0.03	0.07	-0.08	-0.06	0.08	0.01	0.24	0.09	1.00						
A_{6c}	0.01	-0.00	0.04	-0.00	-0.03	0.03	0.06	-0.07	-0.11	0.02	0.11	-0.06	0.01	0.05	0.02	0.21	0.11	0.42	1.00					
A_7	0.04	0.04	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	0.10	0.03	-0.06	0.04	-0.01	-0.02	-0.05	-0.03	-0.04	-0.02	0.05	1.00				
A_8	0.15	0.04	-0.14	-0.02	0.04	-0.01	-0.04	0.10	0.09	-0.01	-0.11	-0.12	-0.06	0.12	0.03	-0.04	-0.00	-0.01	0.03	-0.03	1.00			
A_9	-0.02	-0.01	-0.00	-0.00	-0.01	0.02	-0.03	-0.00	0.04	0.01	0.02	-0.00	-0.02	-0.01	-0.02	-0.01	-0.01	-0.03	0.07	0.02	1.00			
A_{CP}	-0.09	-0.00	0.07	0.03	0.01	0.06	-0.01	-0.05	0.02	0.06	-0.02	0.37	0.33	0.01	0.11	0.18	-0.02	0.03	-0.02	-0.01	-0.02	1.00		
dB/dq^2	-0.12	0.12	0.14	0.01	0.02	-0.05	0.05	-0.01	0.03	0.02	0.05	0.00	0.02	0.01	-0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.02	-0.01	0.02	1.00

Table B.47: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	0.06	1.00																
S_4	0.10	0.02	1.00															
S_5	0.03	0.12	-0.16	1.00														
A_{FB}	0.31	-0.01	0.10	-0.02	1.00													
S_7	-0.00	0.09	0.03	-0.02	-0.03	1.00												
S_8	-0.02	-0.01	-0.10	0.01	-0.01	-0.23	1.00											
S_9	0.10	-0.10	-0.01	-0.01	-0.10	0.19	0.00	1.00										
A_{FL}	-0.10	0.04	0.05	-0.04	-0.02	0.03	-0.01	-0.02	1.00									
A_3	0.05	-0.02	-0.06	0.03	-0.03	-0.09	-0.07	-0.08	-0.00	1.00								
A_4	0.04	-0.06	0.08	0.03	-0.04	-0.01	-0.01	0.01	0.07	0.02	1.00							
A_5	-0.04	0.03	0.03	0.02	-0.03	-0.04	-0.02	-0.06	0.15	0.09	-0.15	1.00						
A_{fb}^{CP}	0.00	-0.04	-0.04	-0.02	-0.14	0.04	0.00	0.01	0.07	0.00	0.10	-0.06	1.00					
A_7	0.04	-0.09	-0.01	-0.04	0.04	-0.04	0.02	0.05	-0.04	0.10	0.03	-0.03	-0.02	1.00				
A_8	-0.03	-0.07	-0.01	-0.02	0.00	0.02	-0.10	-0.01	-0.02	-0.01	-0.10	0.01	-0.02	-0.23	1.00			
A_9	0.00	-0.08	0.01	-0.06	0.01	0.05	-0.01	0.11	0.02	-0.10	-0.02	-0.02	-0.08	0.19	0.00	1.00		
A_{CP}	-0.02	0.01	0.03	-0.02	-0.03	-0.01	0.02	-0.03	0.69	-0.06	0.00	0.19	-0.23	-0.06	-0.01	-0.07	1.00	
dB/dq^2	-0.06	0.00	-0.01	-0.06	0.08	0.04	0.03	-0.02	0.01	0.01	-0.01	0.00	-0.01	0.01	-0.03	0.03	-0.01	1.00

Table B.48: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	-0.08	1.00																
S_4	-0.00	-0.10	1.00															
S_5	-0.09	-0.10	-0.10	1.00														
A_{FB}	0.24	-0.00	-0.00	-0.10	1.00													
S_7	0.01	0.01	-0.09	0.01	0.01	1.00												
S_8	-0.04	0.03	0.03	-0.06	-0.10	-0.10	1.00											
S_9	0.08	0.09	-0.05	-0.06	-0.02	0.11	-0.07	1.00										
A_{FL}	-0.02	0.03	0.06	0.02	0.03	-0.06	0.04	-0.01	1.00									
A_3	0.05	0.16	0.01	0.04	-0.03	-0.02	-0.00	-0.02	-0.08	1.00								
A_4	0.06	0.02	-0.02	0.09	-0.05	0.03	-0.02	0.02	-0.10	-0.09	1.00							
A_5	0.03	0.05	0.07	-0.01	0.01	-0.03	0.02	-0.03	-0.12	-0.09	-0.10	1.00						
A_{fb}^{CP}	0.01	-0.04	-0.04	0.02	0.07	0.03	0.06	-0.16	0.05	0.02	-0.01	-0.09	1.00					
A_7	-0.05	-0.03	0.02	-0.03	0.02	-0.06	0.05	-0.08	-0.09	0.02	-0.08	0.03	0.03	1.00				
A_8	0.02	0.01	-0.03	0.02	0.06	0.06	-0.01	-0.00	-0.05	0.03	0.03	-0.05	-0.10	-0.10	1.00			
A_9	-0.05	-0.02	0.01	-0.03	-0.16	-0.07	-0.01	-0.10	0.01	0.09	-0.04	-0.05	-0.02	0.12	-0.07	1.00		
A_{CP}	-0.02	-0.01	0.03	0.01	0.03	-0.03	0.04	0.03	0.77	-0.03	-0.14	-0.09	-0.14	-0.12	-0.02	-0.05	1.00	
dB/dq^2	-0.13	-0.08	0.08	0.05	0.07	-0.00	-0.02	-0.02	0.01	0.01	-0.04	-0.02	0.03	0.02	-0.03	0.01	-0.00	1.00

Table B.49: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	0.03	1.00																
S_4	0.00	0.01	1.00															
S_5	0.04	-0.08	-0.06	1.00														
A_{FB}	-0.12	0.06	-0.14	-0.08	1.00													
S_7	0.06	0.05	-0.04	0.00	0.00	1.00												
S_8	0.02	-0.00	-0.04	-0.02	-0.07	0.04	1.00											
S_9	0.11	-0.08	0.01	-0.13	-0.04	-0.13	-0.05	1.00										
A_{FL}	0.02	0.00	-0.05	-0.01	-0.01	0.04	0.05	-0.01	1.00									
A_3	-0.04	0.03	-0.00	-0.00	-0.00	-0.01	-0.08	-0.04	0.00	1.00								
A_4	-0.01	-0.02	-0.05	-0.10	0.01	-0.02	-0.04	0.02	-0.11	0.01	1.00							
A_5	0.00	-0.02	-0.10	-0.00	-0.03	-0.00	-0.00	-0.02	-0.11	-0.08	-0.05	1.00						
A_{fb}^{CP}	0.02	-0.00	0.01	-0.03	0.11	0.02	-0.01	-0.01	-0.02	0.05	-0.16	-0.09	1.00					
A_7	0.02	-0.01	-0.02	0.00	0.02	0.00	-0.05	-0.01	0.02	0.05	-0.04	0.00	-0.00	1.00				
A_8	0.01	-0.07	-0.03	0.01	-0.02	-0.05	0.02	-0.01	0.03	-0.00	-0.05	-0.03	-0.06	0.04	1.00			
A_9	-0.03	-0.04	0.02	-0.02	-0.01	-0.02	-0.02	0.13	0.04	-0.08	0.02	-0.12	-0.05	-0.14	-0.05	1.00		
A_{CP}	-0.04	0.04	-0.05	-0.00	-0.03	0.03	0.06	0.01	0.75	-0.02	-0.15	-0.17	0.07	-0.03	0.03	-0.05	1.00	
dB/dq^2	0.01	0.02	0.06	0.05	0.00	0.03	0.02	0.00	0.01	0.02	-0.00	-0.01	0.01	-0.00	0.00	-0.01	0.02	1.00

Table B.50: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	0.06	1.00																
S_4	0.05	-0.08	1.00															
S_5	-0.01	-0.21	0.07	1.00														
A_{FB}	-0.29	0.00	-0.09	-0.11	1.00													
S_7	0.03	0.03	-0.03	-0.02	-0.03	1.00												
S_8	0.01	0.03	-0.02	-0.02	-0.07	0.16	1.00											
S_9	-0.06	-0.09	0.00	-0.04	-0.00	-0.10	-0.07	1.00										
A_{FL}	-0.00	0.00	-0.04	0.01	0.01	0.02	-0.03	-0.02	1.00									
A_3	0.02	-0.05	-0.03	-0.00	0.04	-0.01	0.04	-0.04	-0.00	1.00								
A_4	0.00	-0.02	-0.01	0.07	0.02	0.03	0.04	-0.02	-0.15	-0.08	1.00							
A_5	-0.00	0.01	0.08	0.03	-0.05	0.02	0.02	0.04	-0.19	-0.21	0.11	1.00						
A_{fb}^{CP}	0.01	0.04	0.01	-0.05	-0.04	-0.03	0.06	0.02	-0.02	-0.01	-0.13	-0.15	1.00					
A_7	0.00	-0.01	0.04	0.02	-0.04	0.00	-0.02	0.06	-0.03	0.04	-0.02	-0.01	-0.04	1.00				
A_8	0.02	0.03	0.03	0.00	0.07	-0.03	-0.06	-0.02	-0.02	0.03	-0.01	-0.01	-0.07	0.16	1.00			
A_9	-0.07	-0.04	-0.01	0.05	0.01	0.06	-0.03	-0.04	-0.00	-0.09	-0.00	-0.05	0.01	-0.10	-0.07	1.00		
A_{CP}	0.01	-0.02	-0.06	0.02	0.00	0.03	-0.06	0.03	0.74	-0.05	-0.24	-0.24	0.21	-0.06	-0.04	0.05	1.00	
dB/dq^2	0.04	0.03	0.04	0.08	-0.04	0.01	0.02	-0.01	-0.00	-0.00	0.01	0.01	-0.01	-0.01	0.00	-0.01	0.00	1.00

Table B.51: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $15.0 < q^2 < 17.0 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	0.15	1.00																
S_4	-0.12	-0.20	1.00															
S_5	-0.14	-0.13	0.08	1.00														
A_{FB}	-0.54	0.01	0.09	-0.12	1.00													
S_7	-0.04	-0.00	-0.05	-0.01	0.02	1.00												
S_8	-0.04	-0.00	-0.04	-0.05	0.01	0.31	1.00											
S_9	-0.03	-0.01	0.01	-0.02	0.04	-0.14	-0.05	1.00										
A_{FL}	0.01	0.04	-0.02	-0.07	-0.01	-0.02	0.01	-0.00	1.00									
A_3	0.05	-0.09	0.06	-0.06	0.05	-0.01	-0.02	0.06	0.05	1.00								
A_4	-0.02	0.05	-0.23	-0.06	0.07	-0.05	0.05	-0.00	-0.32	-0.16	1.00							
A_5	-0.07	-0.06	-0.06	-0.05	-0.01	-0.01	-0.08	-0.04	-0.30	-0.09	0.17	1.00						
A_{fb}^{CP}	-0.01	0.05	0.04	-0.03	0.01	0.01	-0.07	-0.04	-0.09	-0.04	-0.09	-0.22	1.00					
A_7	-0.01	-0.02	-0.06	-0.02	0.02	0.03	-0.02	0.04	-0.05	0.00	-0.04	-0.01	0.00	1.00				
A_8	0.00	-0.01	0.06	-0.07	-0.09	-0.02	0.05	-0.01	-0.05	0.00	-0.03	-0.04	-0.00	0.31	1.00			
A_9	-0.00	0.05	-0.01	-0.04	-0.04	0.04	-0.01	-0.05	-0.01	-0.01	0.00	-0.02	0.04	-0.14	-0.06	1.00		
A_{CP}	0.00	-0.00	-0.01	-0.03	-0.02	-0.02	0.02	-0.00	0.56	-0.13	-0.36	-0.30	0.47	-0.03	-0.02	0.02	1.00	
dB/dq^2	0.08	0.03	0.01	0.04	-0.09	-0.00	-0.00	-0.02	-0.01	0.01	-0.01	-0.01	-0.00	-0.00	0.00	-0.00	-0.01	1.00

Table B.52: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $17.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	0.12	1.00																
S_4	-0.00	-0.16	1.00															
S_5	-0.09	-0.18	0.23	1.00														
A_{FB}	-0.32	-0.00	-0.02	-0.12	1.00													
S_7	0.04	0.02	0.03	0.06	0.00	1.00												
S_8	-0.02	0.03	0.04	0.03	-0.01	0.23	1.00											
S_9	-0.01	0.03	0.02	0.07	-0.02	-0.06	-0.13	1.00										
A_{FL}	0.12	-0.02	-0.00	0.02	-0.03	-0.00	0.01	0.00	1.00									
A_3	-0.06	-0.02	-0.10	0.00	-0.03	0.01	0.01	-0.02	-0.03	1.00								
A_4	-0.03	-0.10	0.07	-0.11	-0.03	-0.02	-0.08	0.05	-0.17	-0.09	1.00							
A_5	0.00	-0.01	-0.11	-0.08	0.01	-0.03	-0.03	-0.08	-0.22	-0.13	0.28	1.00						
A_{fb}^{CP}	0.05	-0.03	-0.05	-0.01	0.06	-0.00	-0.03	-0.01	-0.04	-0.07	-0.10	-0.19	1.00					
A_7	0.02	-0.00	-0.03	-0.04	0.01	0.10	-0.08	0.02	0.06	0.02	0.02	0.05	0.01	1.00				
A_8	0.00	0.02	-0.08	-0.02	-0.04	-0.08	0.15	0.00	-0.01	0.02	0.03	0.02	0.00	0.23	1.00			
A_9	-0.01	-0.01	0.05	-0.08	-0.01	0.02	0.00	0.16	-0.00	0.02	0.01	0.07	-0.02	-0.06	-0.14	1.00		
A_{CP}	0.07	0.03	0.01	0.00	-0.09	-0.04	0.01	0.01	0.57	-0.22	-0.28	-0.24	0.34	0.03	0.01	0.01	1.00	
dB/dq^2	0.07	0.03	0.03	0.04	-0.08	-0.00	0.01	0.00	-0.00	-0.00	0.01	-0.01	0.01	-0.00	-0.00	-0.01	-0.00	1.00

Table B.53: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $11.0 < q^2 < 12.5 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	0.13	1.00																
S_4	-0.00	0.02	1.00															
S_5	-0.09	-0.24	0.04	1.00														
A_{FB}	-0.65	-0.12	-0.09	-0.02	1.00													
S_7	-0.05	-0.04	-0.11	-0.11	0.03	1.00												
S_8	-0.01	-0.02	-0.01	-0.10	-0.04	0.26	1.00											
S_9	-0.01	0.05	-0.08	-0.09	-0.02	-0.10	-0.12	1.00										
A_{FL}	0.05	0.05	0.03	0.01	-0.05	-0.05	0.00	-0.01	1.00									
A_3	0.04	-0.20	0.02	0.06	-0.08	-0.01	0.00	0.07	0.01	1.00								
A_4	0.02	-0.00	0.08	0.04	0.04	-0.04	-0.06	-0.07	-0.21	0.03	1.00							
A_5	-0.01	0.05	0.03	0.05	-0.01	-0.08	-0.04	-0.01	-0.32	-0.23	0.08	1.00						
A_{fb}^{CP}	-0.05	-0.02	0.03	0.00	-0.09	0.00	0.04	0.02	-0.08	-0.12	-0.16	-0.12	1.00					
A_7	-0.05	-0.02	-0.04	-0.09	0.01	-0.02	0.06	0.02	-0.11	-0.02	-0.08	-0.02	-0.02	1.00				
A_8	-0.01	-0.00	-0.06	-0.03	0.03	0.07	-0.14	-0.04	-0.04	-0.02	-0.00	-0.09	-0.05	0.26	1.00			
A_9	0.03	0.06	-0.08	-0.02	0.06	0.01	-0.04	-0.06	-0.01	0.07	-0.07	-0.09	-0.02	-0.10	-0.12	1.00		
A_{CP}	0.04	0.05	-0.01	-0.00	0.06	-0.01	0.01	-0.05	0.69	-0.13	-0.26	-0.34	0.42	-0.10	-0.04	-0.01	1.00	
dB/dq^2	0.13	0.05	0.08	0.07	-0.16	-0.00	0.01	0.01	0.02	0.01	0.01	-0.02	-0.01	-0.00	0.01	0.01	0.01	1.00

Table B.54: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	-0.01	1.00																
S_4	0.02	-0.04	1.00															
S_5	-0.01	-0.01	-0.07	1.00														
A_{FB}	0.20	0.02	-0.04	-0.07	1.00													
S_7	0.04	0.06	-0.03	-0.01	0.00	1.00												
S_8	-0.01	-0.03	0.01	-0.02	-0.10	-0.09	1.00											
S_9	0.06	0.00	-0.03	-0.09	-0.04	-0.01	-0.06	1.00										
A_{FL}	-0.04	0.01	0.01	-0.04	0.01	-0.02	0.06	-0.01	1.00									
A_3	0.02	0.05	-0.01	0.01	-0.01	-0.00	-0.07	-0.04	-0.05	1.00								
A_4	0.01	-0.01	0.03	0.01	-0.02	0.01	0.00	0.03	-0.06	-0.03	1.00							
A_5	-0.01	0.01	0.00	0.02	-0.00	-0.01	0.01	-0.05	-0.04	-0.01	-0.07	1.00						
A_{fb}^{CP}	0.02	-0.02	-0.02	0.00	0.05	0.09	-0.05	-0.02	0.06	0.03	-0.03	-0.07	1.00					
A_7	0.03	-0.00	0.00	-0.01	0.08	0.01	-0.01	-0.02	-0.05	0.07	-0.02	-0.01	0.02	1.00				
A_8	-0.03	-0.07	0.02	0.01	-0.04	-0.00	0.04	-0.00	0.01	-0.02	0.00	-0.02	-0.11	-0.09	1.00			
A_9	-0.02	-0.04	0.03	-0.05	-0.02	-0.01	-0.01	0.11	0.01	0.00	-0.03	-0.09	-0.04	-0.01	-0.06	1.00		
A_{CP}	-0.06	-0.01	0.01	-0.04	-0.00	-0.04	0.11	0.01	0.74	-0.06	-0.09	-0.03	-0.12	-0.10	0.03	-0.04	1.00	
dB/dq^2	0.03	-0.02	0.04	0.02	0.01	0.01	0.01	0.00	-0.01	0.00	0.00	-0.01	0.01	-0.01	0.01	0.00	0.00	1.00

Table B.55: Total correlation matrix of the S_i observables obtained from fit configuration (iii) in q^2 bin $15.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	A_{FL}	A_3	A_4	A_5	A_{fb}^{CP}	A_7	A_8	A_9	A_{CP}	dB/dq^2
F_L	1.00																	
S_3	0.17	1.00																
S_4	-0.08	-0.18	1.00															
S_5	-0.10	-0.18	0.15	1.00														
A_{FB}	-0.42	0.02	0.02	-0.14	1.00													
S_7	-0.00	0.01	-0.04	0.01	0.03	1.00												
S_8	-0.03	0.01	0.01	-0.03	-0.02	0.27	1.00											
S_9	-0.02	0.00	0.04	0.01	0.00	-0.11	-0.11	1.00										
A_{FL}	0.04	0.02	-0.03	-0.02	-0.03	-0.02	0.01	-0.01	1.00									
A_3	0.01	-0.04	0.00	-0.03	0.03	0.01	-0.00	0.03	0.03	1.00								
A_4	-0.03	-0.00	-0.03	-0.07	0.01	-0.02	-0.00	0.01	-0.25	-0.11	1.00							
A_5	-0.03	-0.04	-0.07	-0.00	-0.00	-0.01	-0.03	-0.05	-0.25	-0.13	0.21	1.00						
A_{fb}^{CP}	0.02	0.02	-0.01	-0.02	0.05	-0.00	-0.04	0.00	-0.07	-0.05	-0.10	-0.22	1.00					
A_7	-0.00	0.00	-0.03	-0.01	0.01	0.06	-0.05	0.02	-0.00	0.01	-0.03	0.01	0.02	1.00				
A_8	0.00	0.00	-0.00	-0.02	-0.05	-0.05	0.10	-0.01	-0.03	0.01	0.01	-0.03	-0.02	0.27	1.00			
A_9	-0.01	0.04	0.01	-0.05	0.00	0.02	-0.01	0.03	-0.01	0.00	0.04	0.01	0.00	-0.11	-0.11	1.00		
A_{CP}	0.03	0.00	-0.01	-0.01	-0.06	-0.03	0.01	0.00	0.57	-0.19	-0.32	-0.27	0.41	-0.00	0.00	0.01	1.00	
dB/dq^2	0.07	0.03	0.03	0.05	-0.09	-0.00	0.00	-0.01	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.00	1.00

B.11.4 Fit configuration (iv)

Table B.56: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $0.06 < q^2 < 0.98 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.29	1.00										
S_1^c	-0.71	-0.43	1.00									
S_3	-0.03	-0.02	0.03	1.00								
S_4	-0.23	-0.10	0.24	0.06	1.00							
S_5	-0.01	-0.09	0.09	0.13	0.07	1.00						
A_{FB}	-0.07	-0.06	0.11	0.02	0.25	0.11	1.00					
S_7	0.01	-0.02	0.01	-0.05	0.00	-0.02	-0.01	1.00				
S_8	-0.05	-0.03	0.09	0.01	-0.02	0.00	-0.01	-0.02	1.00			
S_9	0.01	0.02	-0.00	-0.01	-0.02	-0.01	-0.00	0.07	0.01	1.00		
S_6^c	-0.06	-0.03	0.08	0.02	0.23	0.11	0.45	0.05	0.03	-0.03	1.00	
dB/dq^2	-0.11	0.14	0.12	0.01	0.03	-0.04	0.04	0.01	0.01	0.03	0.03	1.00

Table B.57: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $1.1 < q^2 < 2.5 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.28	1.00										
S_1^c	-0.30	-0.32	1.00									
S_3	-0.02	0.02	0.08	1.00								
S_4	-0.14	-0.11	0.18	-0.02	1.00							
S_5	-0.04	-0.11	-0.01	0.10	-0.23	1.00						
A_{FB}	-0.19	-0.20	0.28	-0.00	0.22	0.04	1.00					
S_7	-0.13	0.03	0.12	0.10	0.02	-0.02	-0.04	1.00				
S_8	-0.01	-0.04	-0.05	0.01	-0.07	0.02	0.13	-0.29	1.00			
S_9	0.00	-0.02	0.14	-0.04	0.04	-0.01	-0.07	0.19	0.02	1.00		
S_6^c	-0.32	-0.04	0.02	0.04	0.15	0.07	0.54	-0.03	0.22	0.05	1.00	
dB/dq^2	-0.10	0.03	0.02	0.01	0.03	-0.04	0.08	0.05	0.02	-0.02	0.04	1.00

Table B.58: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $2.5 < q^2 < 4.0 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.33	1.00										
S_1^c	-0.06	-0.07	1.00									
S_3	-0.07	-0.04	-0.16	1.00								
S_4	-0.07	-0.05	0.09	-0.11	1.00							
S_5	-0.08	-0.02	-0.07	-0.07	-0.17	1.00						
A_{FB}	0.18	-0.08	0.10	0.07	-0.03	-0.13	1.00					
S_7	-0.01	-0.01	0.02	0.05	-0.11	0.02	0.01	1.00				
S_8	-0.13	-0.00	0.07	0.03	0.04	-0.08	-0.06	-0.12	1.00			
S_9	0.07	0.04	0.07	0.04	-0.04	-0.07	-0.10	0.07	-0.06	1.00		
S_6^c	0.23	0.12	-0.25	-0.05	-0.02	-0.01	0.48	0.05	0.06	0.00	1.00	
dB/dq^2	-0.14	-0.01	-0.03	-0.07	0.07	0.04	0.05	-0.00	-0.03	-0.01	0.01	1.00

Table B.59: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $4.0 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.35	1.00										
S_1^c	-0.41	-0.28	1.00									
S_3	0.02	0.07	0.04	1.00								
S_4	0.15	-0.02	-0.12	-0.05	1.00							
S_5	0.06	0.07	0.01	-0.14	-0.04	1.00						
A_{FB}	0.03	0.12	-0.10	0.03	-0.20	-0.13	1.00					
S_7	0.03	0.03	0.05	0.05	-0.02	-0.01	0.02	1.00				
S_8	-0.01	-0.00	0.03	0.00	-0.00	0.00	-0.04	0.03	1.00			
S_9	0.05	0.09	0.08	-0.06	-0.01	-0.10	-0.01	-0.11	-0.06	1.00		
S_6^c	0.05	-0.02	-0.03	0.01	-0.17	-0.11	0.59	0.03	0.01	0.02	1.00	
dB/dq^2	-0.04	-0.05	0.03	0.00	0.05	0.05	0.02	0.03	0.02	0.00	0.03	1.00

Table B.60: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $6.0 < q^2 < 8.0 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.32	1.00										
S_1^c	-0.28	-0.20	1.00									
S_3	0.02	0.08	0.07	1.00								
S_4	0.18	-0.04	-0.13	-0.11	1.00							
S_5	0.06	0.06	-0.03	-0.22	0.05	1.00						
A_{FB}	-0.29	0.07	0.05	0.02	-0.20	-0.17	1.00					
S_7	0.04	0.01	0.00	0.04	-0.04	-0.00	-0.04	1.00				
S_8	0.05	0.01	-0.05	0.03	0.02	-0.03	-0.08	0.18	1.00			
S_9	-0.04	-0.06	-0.06	-0.09	0.02	-0.02	-0.02	-0.09	-0.06	1.00		
S_6^c	-0.41	-0.17	0.36	0.03	-0.20	-0.15	0.53	-0.05	-0.04	-0.00	1.00	
dB/dq^2	-0.02	-0.08	0.05	0.02	0.04	0.08	-0.03	0.01	0.02	0.00	0.03	1.00

Table B.61: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $15.0 < q^2 < 17.0 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.32	1.00										
S_1^c	-0.29	-0.24	1.00									
S_3	0.11	0.25	0.18	1.00								
S_4	0.09	-0.07	-0.23	-0.24	1.00							
S_5	-0.06	-0.03	-0.12	-0.14	0.11	1.00						
A_{FB}	-0.18	0.40	-0.22	0.18	-0.08	-0.23	1.00					
S_7	-0.01	-0.00	-0.04	-0.01	-0.05	-0.00	-0.01	1.00				
S_8	0.03	-0.00	-0.08	-0.02	0.01	-0.05	-0.02	0.30	1.00			
S_9	-0.03	-0.04	-0.02	-0.02	0.02	-0.02	0.01	-0.14	-0.07	1.00		
S_6^c	-0.19	-0.08	0.23	-0.02	-0.19	-0.24	0.42	-0.05	-0.04	-0.02	1.00	
dB/dq^2	0.01	-0.09	0.06	0.00	0.02	0.05	-0.10	-0.00	-0.01	-0.01	0.01	1.00

Table B.62: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $17.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.40	1.00										
S_1^c	-0.45	-0.30	1.00									
S_3	0.06	0.25	0.13	1.00								
S_4	-0.00	-0.09	-0.06	-0.24	1.00							
S_5	-0.06	-0.01	-0.02	-0.15	0.34	1.00						
A_{FB}	-0.03	0.24	-0.15	0.10	-0.09	-0.18	1.00					
S_7	0.01	-0.02	0.02	0.01	0.03	0.05	-0.01	1.00				
S_8	0.02	0.03	-0.02	0.00	0.07	0.03	-0.00	0.24	1.00			
S_9	-0.04	-0.07	-0.02	0.04	-0.03	0.06	-0.02	-0.10	-0.12	1.00		
S_6^c	-0.04	-0.04	0.13	0.06	-0.10	-0.14	0.41	-0.01	0.02	0.02	1.00	
dB/dq^2	-0.01	-0.07	0.06	0.03	0.04	0.06	-0.09	-0.00	0.01	0.00	0.01	1.00

Table B.63: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $11.0 < q^2 < 12.5 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.41	1.00										
S_1^c	-0.38	-0.35	1.00									
S_3	0.15	0.32	0.15	1.00								
S_4	0.05	-0.12	-0.06	-0.07	1.00							
S_5	-0.05	-0.01	-0.05	-0.26	0.10	1.00						
A_{FB}	-0.28	0.32	-0.12	0.06	-0.20	-0.11	1.00					
S_7	-0.02	0.01	-0.03	-0.03	-0.10	-0.10	0.00	1.00				
S_8	0.09	0.03	-0.07	-0.01	-0.03	-0.10	-0.05	0.29	1.00			
S_9	-0.01	0.03	0.00	0.08	-0.05	-0.11	-0.02	-0.12	-0.11	1.00		
S_6^c	-0.44	-0.22	0.46	0.03	-0.12	-0.11	0.43	-0.03	-0.06	0.03	1.00	
dB/dq^2	-0.00	-0.11	0.10	0.01	0.09	0.07	-0.14	0.00	-0.00	0.01	0.02	1.00

Table B.64: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.29	1.00										
S_1^c	-0.24	-0.28	1.00									
S_3	-0.01	-0.00	-0.01	1.00								
S_4	0.09	-0.00	-0.05	-0.05	1.00							
S_5	0.03	-0.00	-0.03	-0.01	-0.14	1.00						
A_{FB}	0.35	-0.02	-0.14	0.02	-0.11	-0.08	1.00					
S_7	0.05	0.04	0.01	0.07	-0.06	-0.01	0.03	1.00				
S_8	-0.08	0.01	0.04	-0.02	-0.04	-0.05	-0.11	-0.15	1.00			
S_9	0.06	0.08	0.05	0.01	-0.03	-0.09	-0.03	0.01	-0.05	1.00		
S_6^c	0.55	0.17	-0.42	-0.01	-0.14	-0.05	0.52	0.04	-0.07	0.02	1.00	
dB/dq^2	-0.02	0.01	0.06	-0.00	0.02	0.02	0.04	0.01	0.01	0.00	0.00	1.00

Table B.65: Total correlation matrix of the S_i observables obtained from fit configuration (iv) in q^2 bin $15.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.36	1.00										
S_1^c	-0.39	-0.26	1.00									
S_3	0.11	0.28	0.20	1.00								
S_4	0.04	-0.08	-0.17	-0.22	1.00							
S_5	-0.05	-0.00	-0.09	-0.18	0.16	1.00						
A_{FB}	-0.12	0.30	-0.15	0.16	-0.11	-0.22	1.00					
S_7	0.00	-0.01	-0.02	0.00	-0.02	0.01	-0.00	1.00				
S_8	0.03	0.02	-0.06	-0.01	0.04	-0.02	-0.02	0.27	1.00			
S_9	-0.02	-0.03	-0.01	-0.03	0.04	0.01	-0.00	-0.11	-0.11	1.00		
S_6^c	-0.17	-0.10	0.24	-0.02	-0.16	-0.18	0.42	-0.03	-0.02	-0.02	1.00	
dB/dq^2	0.00	-0.07	0.05	0.01	0.03	0.06	-0.10	-0.00	0.00	-0.01	0.01	1.00

B.11.5 Fit configuration (v)

Table B.66: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $0.06 < q^2 < 0.54 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.32	1.00										
S_1^c	-0.68	-0.36	1.00									
S_3	-0.02	0.00	0.02	1.00								
S_4	0.01	0.03	0.00	0.04	1.00							
S_5	0.12	-0.05	-0.03	0.14	-0.10	1.00						
A_{FB}	0.03	0.02	-0.05	0.00	0.14	0.04	1.00					
S_7	0.01	-0.00	-0.01	-0.02	0.01	-0.00	-0.01	1.00				
S_8	-0.08	-0.01	0.04	0.03	-0.03	-0.01	0.01	-0.11	1.00			
S_9	0.01	0.00	0.01	-0.02	0.02	-0.04	0.03	0.08	0.01	1.00		
S_6^c	0.09	0.09	-0.17	0.00	0.09	0.01	0.29	-0.02	0.01	-0.02	1.00	
dB/dq^2	-0.08	0.06	0.15	0.01	-0.00	-0.05	0.02	-0.00	0.02	0.01	0.02	1.00

Table B.67: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $0.54 < q^2 < 0.98 \text{ GeV}^2/c^4$.

	F_L	S_2^s	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	S_6^c	dB/dq^2
F_L	1.00											
S_2^s	0.39	1.00										
S_1^c	-0.76	-0.53	1.00									
S_3	-0.02	-0.06	-0.01	1.00								
S_4	-0.09	-0.06	0.08	0.07	1.00							
S_5	0.00	-0.05	-0.01	0.17	-0.10	1.00						
A_{FB}	-0.04	-0.06	0.08	0.02	0.31	0.07	1.00					
S_7	0.02	0.06	-0.03	0.02	-0.02	-0.01	-0.02	1.00				
S_8	0.03	0.08	-0.01	0.02	-0.04	-0.02	-0.11	-0.14	1.00			
S_9	0.04	0.02	-0.03	0.11	-0.07	-0.06	-0.07	0.07	0.06	1.00		
S_6^c	0.05	0.05	-0.11	0.03	0.17	0.03	0.52	0.03	-0.09	0.01	1.00	
dB/dq^2	-0.17	0.11	0.10	-0.00	-0.04	-0.13	0.03	0.02	0.00	0.02	-0.02	1.00

Table B.68: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $1.1 < q^2 < 1.8 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.08	1.00								
S_3	0.04	0.11	1.00							
S_4	0.05	0.10	-0.01	1.00						
S_5	0.01	0.03	0.10	-0.17	1.00					
A_{FB}	-0.02	0.30	0.01	0.15	-0.01	1.00				
S_7	-0.26	0.14	0.11	-0.06	0.09	0.02	1.00			
S_8	0.30	-0.27	-0.00	0.00	-0.05	-0.03	-0.26	1.00		
S_9	0.09	0.17	0.10	0.11	-0.03	-0.11	0.21	0.02	1.00	
dB/dq^2	-0.06	0.02	0.00	-0.02	-0.13	0.08	0.07	-0.00	-0.03	1.00

Table B.69: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $1.8 < q^2 < 2.5 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.52	1.00								
S_3	-0.05	-0.03	1.00							
S_4	0.01	0.09	0.05	1.00						
S_5	0.08	-0.15	0.15	-0.39	1.00					
A_{FB}	-0.04	0.27	0.09	0.12	-0.05	1.00				
S_7	-0.02	0.09	0.06	0.08	-0.14	-0.01	1.00			
S_8	-0.01	0.10	0.03	-0.18	0.06	-0.13	-0.34	1.00		
S_9	-0.00	0.07	-0.27	-0.09	0.07	-0.13	0.10	-0.02	1.00	
dB/dq^2	-0.14	0.01	0.07	0.04	-0.02	0.12	0.03	0.03	-0.04	1.00

Table B.70: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $2.5 < q^2 < 3.25 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.17	1.00								
S_3	0.04	-0.02	1.00							
S_4	-0.23	0.13	-0.14	1.00						
S_5	0.01	-0.20	-0.05	-0.12	1.00					
A_{FB}	0.12	0.28	-0.14	0.03	-0.17	1.00				
S_7	-0.07	0.03	0.00	-0.12	0.00	0.03	1.00			
S_8	0.08	-0.13	0.07	0.01	-0.06	-0.13	-0.34	1.00		
S_9	0.02	0.12	-0.02	-0.12	-0.18	0.13	0.12	-0.17	1.00	
dB/dq^2	-0.14	-0.04	-0.14	0.09	0.02	0.07	-0.02	-0.09	0.01	1.00

Table B.71: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $3.25 < q^2 < 4.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.12	1.00								
S_3	-0.05	-0.13	1.00							
S_4	-0.06	0.07	-0.06	1.00						
S_5	-0.09	0.01	-0.11	-0.21	1.00					
A_{FB}	0.05	0.13	0.08	-0.02	-0.11	1.00				
S_7	-0.01	-0.01	0.11	-0.06	0.04	0.00	1.00			
S_8	-0.16	0.09	0.05	0.11	-0.05	-0.05	-0.08	1.00		
S_9	0.02	0.04	0.11	0.01	-0.05	-0.13	0.09	-0.01	1.00	
dB/dq^2	-0.10	-0.00	-0.04	0.03	0.06	0.07	0.01	-0.02	-0.01	1.00

Table B.72: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $4.0 < q^2 < 5.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.45	1.00								
S_3	-0.01	0.03	1.00							
S_4	0.08	-0.08	-0.05	1.00						
S_5	0.02	-0.00	-0.11	-0.00	1.00					
A_{FB}	-0.06	0.02	0.01	-0.09	-0.07	1.00				
S_7	0.04	0.07	0.07	-0.05	-0.04	-0.03	1.00			
S_8	0.01	0.04	-0.01	-0.02	-0.02	-0.17	-0.03	1.00		
S_9	0.00	0.07	-0.10	-0.01	-0.08	-0.06	-0.01	-0.06	1.00	
dB/dq^2	-0.04	0.05	0.00	0.05	0.07	0.03	0.05	0.02	0.03	1.00

Table B.73: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $5.0 < q^2 < 6.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.42	1.00								
S_3	-0.02	0.06	1.00							
S_4	0.25	-0.13	-0.02	1.00						
S_5	0.06	0.02	-0.21	-0.07	1.00					
A_{FB}	-0.07	-0.13	0.06	-0.20	-0.10	1.00				
S_7	0.01	0.01	0.00	0.01	0.06	0.00	1.00			
S_8	-0.01	0.02	0.02	-0.00	0.03	0.02	0.08	1.00		
S_9	0.03	0.05	0.05	0.06	-0.07	0.00	-0.17	-0.06	1.00	
dB/dq^2	0.00	0.03	0.03	0.04	0.07	-0.00	0.04	0.02	0.01	1.00

Table B.74: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $6.0 < q^2 < 7.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.40	1.00								
S_3	0.02	0.02	1.00							
S_4	0.18	-0.11	-0.03	1.00						
S_5	-0.03	-0.03	-0.11	0.00	1.00					
A_{FB}	-0.07	-0.14	0.02	-0.10	-0.10	1.00				
S_7	0.01	-0.03	0.02	-0.04	-0.00	0.01	1.00			
S_8	0.01	-0.02	0.01	0.05	-0.04	-0.06	0.09	1.00		
S_9	-0.02	-0.03	-0.13	0.02	-0.03	0.02	-0.05	-0.05	1.00	
dB/dq^2	-0.01	0.04	0.04	0.02	0.06	-0.01	0.01	-0.00	0.00	1.00

Table B.75: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $7.0 < q^2 < 8.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.37	1.00								
S_3	-0.07	0.11	1.00							
S_4	0.18	-0.14	-0.14	1.00						
S_5	0.08	-0.03	-0.30	0.04	1.00					
A_{FB}	-0.14	-0.19	-0.05	-0.10	-0.08	1.00				
S_7	0.11	-0.02	0.06	-0.11	-0.05	-0.04	1.00			
S_8	0.08	-0.05	0.05	-0.12	-0.06	-0.08	0.19	1.00		
S_9	0.02	-0.08	-0.06	0.03	-0.02	-0.01	-0.10	-0.09	1.00	
dB/dq^2	0.02	0.07	0.04	0.05	0.11	-0.03	0.00	0.02	-0.01	1.00

Table B.76: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $15.0 < q^2 < 16.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.31	1.00								
S_3	0.00	0.20	1.00							
S_4	0.07	-0.22	-0.24	1.00						
S_5	-0.07	-0.07	-0.11	-0.10	1.00					
A_{FB}	-0.26	-0.39	0.10	0.01	-0.06	1.00				
S_7	-0.04	-0.04	0.01	-0.07	-0.01	0.04	1.00			
S_8	0.02	-0.05	-0.01	-0.05	-0.05	0.00	0.33	1.00		
S_9	0.01	0.01	-0.07	0.04	-0.10	0.05	-0.12	-0.06	1.00	
dB/dq^2	0.05	0.09	0.04	-0.03	0.09	-0.04	-0.00	-0.01	-0.01	1.00

Table B.77: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $16.0 < q^2 < 17.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.22	1.00								
S_3	-0.01	0.17	1.00							
S_4	0.16	-0.16	-0.22	1.00						
S_5	-0.06	-0.03	-0.15	0.22	1.00					
A_{FB}	-0.33	-0.24	-0.00	0.02	-0.14	1.00				
S_7	-0.00	-0.03	-0.01	-0.05	-0.00	0.01	1.00			
S_8	0.07	-0.10	-0.02	0.05	-0.07	0.00	0.31	1.00		
S_9	-0.03	-0.05	0.13	-0.02	0.05	-0.01	-0.17	-0.09	1.00	
dB/dq^2	0.07	0.07	0.06	0.01	0.08	-0.06	-0.01	-0.00	-0.02	1.00

Table B.78: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $17.0 < q^2 < 18.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.32	1.00								
S_3	-0.04	0.16	1.00							
S_4	0.11	-0.11	-0.06	1.00						
S_5	-0.01	-0.10	-0.09	0.17	1.00					
A_{FB}	-0.15	-0.24	-0.02	-0.09	-0.00	1.00				
S_7	0.02	-0.01	-0.00	-0.05	0.07	0.00	1.00			
S_8	0.07	-0.11	0.03	0.12	-0.03	-0.00	0.22	1.00		
S_9	0.01	0.00	0.00	0.06	0.02	-0.06	-0.02	-0.09	1.00	
dB/dq^2	0.06	0.06	0.07	0.04	0.06	-0.06	-0.01	0.03	-0.01	1.00

Table B.79: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $18.0 < q^2 < 19.0 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.54	1.00								
S_3	-0.04	0.19	1.00							
S_4	-0.01	0.00	-0.27	1.00						
S_5	-0.11	0.09	-0.19	0.33	1.00					
A_{FB}	-0.14	-0.14	0.02	0.05	-0.23	1.00				
S_7	0.05	0.05	-0.04	0.07	0.03	0.02	1.00			
S_8	-0.02	0.04	-0.00	0.04	0.08	-0.08	0.17	1.00		
S_9	0.01	-0.04	-0.05	-0.00	-0.01	0.00	-0.13	-0.18	1.00	
dB/dq^2	0.02	0.05	0.08	-0.00	0.03	-0.08	-0.00	-0.01	0.00	1.00

Table B.80: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $11.0 < q^2 < 11.75 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.25	1.00								
S_3	-0.04	0.12	1.00							
S_4	-0.07	0.05	-0.13	1.00						
S_5	-0.11	-0.07	-0.18	0.07	1.00					
A_{FB}	-0.35	-0.49	-0.09	-0.04	0.01	1.00				
S_7	0.05	-0.06	-0.02	-0.05	-0.07	0.03	1.00			
S_8	0.02	-0.03	0.02	-0.04	-0.04	-0.09	0.31	1.00		
S_9	0.00	0.03	0.12	0.04	-0.09	-0.11	-0.06	-0.12	1.00	
dB/dq^2	0.07	0.12	0.06	0.07	0.09	-0.17	0.02	0.00	0.05	1.00

Table B.81: Total correlation matrix of the S_i observables obtained from fit configuration (v) in q^2 bin $11.75 < q^2 < 12.5 \text{ GeV}^2/c^4$.

	F_L	S_1^c	S_3	S_4	S_5	A_{FB}	S_7	S_8	S_9	dB/dq^2
F_L	1.00									
S_1^c	-0.21	1.00								
S_3	0.05	0.22	1.00							
S_4	0.09	0.03	0.03	1.00						
S_5	-0.02	-0.00	-0.26	-0.10	1.00					
A_{FB}	-0.26	-0.42	-0.10	-0.20	-0.04	1.00				
S_7	-0.03	-0.01	-0.03	-0.13	-0.13	-0.00	1.00			
S_8	0.06	-0.01	-0.03	-0.11	-0.13	-0.00	0.20	1.00		
S_9	-0.02	-0.05	0.04	-0.05	-0.09	-0.04	-0.10	-0.09	1.00	
dB/dq^2	0.05	0.10	0.05	0.06	0.07	-0.08	-0.00	0.00	0.01	1.00

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