

Acoustic radiosity simulation with custom bidirectional reflectance distribution implementation in three dimensions^{a)}

João Fatela,^{1,b)}  Anne Heimes,²  Michael Vorländer,²  Massimiliano Masullo,¹  and Luigi Maffei¹ 

¹Department of Architecture and Industrial Design, University of Campania "Luigi Vanvitelli," 81031 Aversa, Caserta, Italy

²Institute for Hearing Technology and Acoustics, RWTH Aachen University, 52074 Aachen, Nordrhein-Westfalen, Germany

ABSTRACT:

Geometrical acoustics methods provide efficient techniques to estimate sound propagation properties in virtual spaces. Advances based on acoustic radiance transfer principles facilitate the simulation of environments with complex scattering distributions via the implementation of bidirectional reflectance distribution functions (BRDF). Though significant work has been developed in the optimization of such methods, the effects of complex BRDF implementation have seldom been studied. In this article, two scattering distribution modeling approaches are compared, based on random incidence and bidirectional scattering coefficients, respectively. Their implementation in an acoustic radiosity (AR) simulation tool is discussed. The simulation of a single façade shows that random-incidence modeling in the radiosity tool aligns with state-of-the-art ray-tracing (RT) simulation. Bidirectional scattering approach results of retro-reflective surface simulations are closest to a reference wave-based simulation output, meaning they capture more of the spatial details of the scattering distribution. The simulation of a street canyon scene further highlights how the choice of scattering modeling leads to significant differences in the energy time curve. The results evidence minor artifacts due to the AR and RT implementations. Major computational performance metrics are also assessed.

© 2026 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>). <https://doi.org/10.1121/10.0043765>

(Received 2 September 2025; revised 6 April 2026; accepted 14 April 2026; published online 8 May 2026)

[Editor: Mlanie Nolan]

Pages: 3925–3940

I. INTRODUCTION

Recent soundscape research indicates that currently implemented noise impact prediction models (European Environment Agency, 2010) lack the detail to accurately characterize noise annoyance and other impacts on humans, such as the qualitative characteristics of the background noise and occurring sound events (Masullo *et al.*, 2024), or the capability to recognize the spatial position of sources, which affects the perceptual and emotional responses of individuals (Yang *et al.*, 2026). This motivates a push for more physically accurate sound propagation models in auralization applications, to capture spatial acoustic nuances of the simulated environments. In urban environments, building façades often feature pronounced and varied geometrical features, which cause complex reflection behaviors. Currently, these behaviors are seldom implemented in auralization, and their impact on street noise and human perception is still under study (Hu *et al.*, 2022).

This paper aims to extend existing simulation models by accounting for such complex reflection effects, motivate the implementation of such effects, and to discuss new opportunities in simulation and auralization for urban planning and soundscape research.

A. Sound scattering modeling

A common way to model sound reflections in acoustic simulation is by separating reflected sound into specular and scattered components. The scattered component is often modeled using Lambert's cosine law, which implies uniform radiation in all directions (Svensson and Savioja, 2024). The amount of scattered energy relative to the total reflected energy is quantified by a scattering coefficient, which is most often determined as a random-incidence (RI) value under diffuse-field conditions according to ISO 17497-1 (ISO, 2004). These coefficients typically assume sufficiently large surfaces, such that scattering behavior can be considered as independent of sample size.

In reality, surfaces with repeating geometrical features scatter energy in distinct directions, depending on the incident angle, the structural wavelength, and the wavelength of sound (Rayleigh, 1896). This can lead to distinct behaviour, such as retro-reflecting façades, as discussed by Cabrera *et al.* (2020). Approaches for the characterization of geometrically complex surfaces are under investigation. This includes conceptual design tools for architects (Tokac *et al.*, 2025) and a fundamental computational and measurement method for the determination of bidirectional scattering coefficients (BSCs) (Heimes and Vorländer, 2025). BSC modeling of a given scattering distribution encodes the reflected energy fraction for given sample incident and outgoing (reflection) directions.

^{a)}This paper is part of a special issue on Sound Absorption and Diffusion: Modeling, Measurement and Application.

^{b)}Email: joao.garrettfatela@unicampania.it

Another approach is to describe the reflectance behavior by bidirectional reflectance distribution functions (BRDFs), which are often used in optics (Nicodemus *et al.*, 1977). In acoustics, this concept can be used in geometrical sound propagation by combining the effects of absorption and scattering. Siltanen *et al.* (2007) and Siltanen *et al.* (2010) derived BRDF from the RI scattering model and absorption coefficients due to a lack of measured BRDF or directional scattering coefficients. Other BRDF implementations have been performed in the context of acoustic radiance methods (Antani *et al.*, 2012; Cao *et al.*, 2016; Scerbo *et al.*, 2024), always featuring results from RI-derived BRDF. Binek *et al.* (2021) estimated BRDF using Phong's reflection model, and later Binek *et al.* (2022) calculated BRDF from diffusor geometries based on reflected sound pressure determined by the Fresnel diffraction method, implementing them in ray tracing simulations.

B. Geometrical acoustics (GA) and sound propagation

In order to study the acoustic properties of virtual scenes, reliable and accurate sound propagation simulations are required. GA methods provide acousticians with a selection of efficient methods to estimate the acoustic energy decay in a given virtual space over time. These methods predict the energy carried by acoustic wavefronts as they interact with the surfaces of the scene. Given the scene's assigned sound propagation properties and the geometrical characteristics of each surface, these simulations compute an impulse response or its energetic representation, the energy time curve (ETC). GA methods feature implicit compromise: complex pressure, phase, or any modal effects of the wavefront are generally not simulated, though work in this area is in development (Marbjerg *et al.*, 2015; Siltanen *et al.*, 2010). Wave-based simulation methods easily account for these effects. However, such methods are significantly less computationally efficient than GA, especially at high frequencies, making them less attractive than GA methods for certain applications.

The two most common GA techniques used today are the image source (IS) method and ray-tracing (RT). These are sometimes combined into hybrid methods (Kuttruff, 2017; Savioja and Svensson, 2015; Vorländer, 2020). IS methods perform extremely well for the simulation of sound reflections at specular reflecting surfaces. It depends exclusively on the principles of specular reflection and has, therefore, very limited applicability for the modeling of diffusely reflecting environments, particularly at high frequencies. RT approaches, on the other hand, model energy propagation by emitting rays into the scene and account for both specular and diffuse reflections using a Monte Carlo-based reflection model.

An alternative GA method is acoustic radiosity (AR) (Kuttruff, 1976; Savioja and Svensson, 2015). This method traces the acoustic energy as it radiates over the surfaces of a scene, instead of estimating the energy at the wavefront like IS or RT. Traditionally, AR depends on the assumption

that all energy is diffusely reflected. This makes AR especially fit for environments with a dominance of diffusely propagated energy. Le Bot and Bocquillet (2000) show how RT and AR schemes provide equivalent approaches to the solution of the acoustic radiance integral in stationary, diffuse environments. This notion remains valid with the introduction of BRDF and time-domain formulations, in the so-called room acoustic rendering equation (Siltanen *et al.*, 2007).

AR has been thoroughly studied in room acoustics (Kuttruff, 1976), in combination with IS methods (Koutsouris *et al.*, 2013), and extended to track wavefront phase (Marbjerg *et al.*, 2015). Kang (2000) evidenced how energy decay estimations differ between conventional IS and AR simulations of a street canyon. In practice, the study compares the effects of perfectly specular reflections to the effects of perfectly diffuse scattering. Both the RT and AR approaches solve the rendering equation using different numerical formulations. In principle, they should yield equivalent solutions under sufficient numerical resolution. In practice, however, discrepancies may arise due to the different nature of both implementations and physical limits on computational resources. Convergence toward the same solution is not always guaranteed in the simulation of ETCs (Hodgson and Nosal, 2006; Nosal *et al.*, 2004b).

Significant computational performance optimizations have been presented for AR/acoustic radiance transfer, e.g., by the introduction of frequency-domain AR formulations (Siltanen *et al.*, 2009); combination of radiance transfer methods with stochastic path tracing (Cao *et al.*, 2016); radiance transfer operator compression (Antani *et al.*, 2012); and delay line implementation (Scerbo *et al.*, 2024). Notably, the work in recent years has relied on methods that inherently result in loss of information, either from their stochastic, iterative, or data-compressive nature. These types of methods are designed such that the information loss is minimized and their results converge to a satisfactory degree. However, these implementations have been tested with BRDF based on RI scattering models, where much of the scattering distribution is uniform. It is uncertain whether a compromise between accuracy and performance can be easily achieved when more complex scattering distributions are introduced.

The accuracy and overall performance of AR, RT, and IS algorithms are generally dependent on specific acoustic and material properties of the simulated scenarios. Generally, IS and RT are found to better emulate specular reflecting boundaries, whereas AR methods seem more fit to model diffuse reflection (Hodgson and Nosal, 2006; Kang, 2000). This means that a simulation combining various approaches is likely a satisfactory approach when modeling RI scattering surfaces (Cao *et al.*, 2016; Koutsouris *et al.*, 2013), since the RI scattering model is a mere superposition of Lambertian and specular reflection. Thus, the implementation of BRDF in radiance transfer simulation models is best motivated by the need to simulate complex scattering patterns.

C. Aims and outline

This article implements a custom AR-based simulation tool (Fatela *et al.*, 2025; Heimes and Fatela, 2025). The classic time-domain AR implementation is extended to support BRDF, following the model proposed by Siltanen *et al.* (2007). Within this tool, the BRDF can be derived from the conventional RI scattering model (Binek *et al.*, 2022; Siltanen *et al.*, 2007). In addition, it can be obtained from the BSC. This enables the reproduction of measured bidirectional scattering characteristics of rough surfaces, thereby overcoming the limitations inherent to the RI Lambertian scattering model. The implementation of the internal radiosity methods, concerning geometrical and energetic relationships between surfaces, relies on explicit numerical algorithms, rather than stochastic methods. These methods are briefly introduced in Sec. IV. Specific implementation details can be consulted in existing literature (Fatela *et al.*, 2025).

In this article, the validity of the implemented AR methods is first validated against the theoretical Lambertian reflection model. Then, both RI and BSC scattering modeling are implemented in the acoustic energy decay simulation of two scenes: a single façade reflection and a street canyon. The scenes feature pronounced surface geometries, which produce complex scattering patterns. The contrasting results highlight the influence of the scattering encoding approach on sound propagation simulation. This serves as a proof of concept for the BSC (and consequently BRDF) encoding of scattering distributions, showing that some spatial effects are not captured by RI scattering modeling. The computational performance of the implemented methods is studied as a function of different simulation parameters.

The algorithms used in this paper are published as an open-source python package called sparrowpy (Heimes and Fatela, 2025).

II. AR

A. Radiosity integral

Radiosity simulations estimate the diffuse radiant energy distribution over the surfaces of a given scene. This is done by integrating the contributions of any radiant sources, as well as the diffusely reflected energy, over the whole geometry. The distribution of irradiance $B(x)$ (energy flux) incident at a given point x of a non-self-intersecting surface S is estimated by the following Eq. (1) (Kuttruff, 1976, 2017; Modest, 2013),

$$B(x) = \iint_S B(x') \cdot \rho(x') \cdot f(x', x) dx' + B_S(x). \quad (1)$$

The main contribution is the integration of the irradiance $B(x')$, distributed over the points $x' \in S$. This is a recursive relationship, since it follows that $B(x')$ depends in turn on the contributions $B(x'')$, and $B(x'')$ on $B(x''')$, and so forth for $\{x'', x''', \dots\} \in S$. As noted by Le Bot and Bocquillet (2000), Eq. (1) represents a Fredholm integral of the second

kind, and its recursive nature is evidenced by its solution via the Neumann series expansion (Antani *et al.*, 2012; Cao *et al.*, 2016). In this integral, $B(x')$ is scaled by $\rho(x')$, the surface's reflection coefficient at x' . In alignment with acoustic convention, ρ can be defined by the surface's absorption coefficient $\alpha(x')$, with $\rho(x') = 1 - \alpha(x')$. An important term of the integral is a geometrical term $f(x', x)$. This term is deeply tied to Lambert's law of diffuse emission and relates to the projection of the infinitesimal surface region $dS' \in S$ surrounding x' onto the region $dS \in S$ surrounding x (Kuttruff, 1976; Modest, 2013),

$$f(x', x) = \frac{\cos \vartheta' \cos \vartheta}{\pi \|r\|^2}. \quad (2)$$

$f(x', x)$ is determined by the distance between the two evaluated points $\|r\| = \|x - x'\|$, as well as angles ϑ and ϑ' . As illustrated in Fig. 1, ϑ and ϑ' represent the angles between the vector r and the surface normal vectors at x and x' , respectively.

The irradiance $B(x)$ naturally depends on the irradiance emitted by energy sources, $B_S(x)$.

These concepts are borrowed for AR, where the irradiance represents the energy carried by wavefronts as they interact with the scene surfaces. However, Eq. (1) can only model stationary, time-independent acoustic fields. Therefore, the radiosity formulation is extended to account for the time-variant nature of acoustic decay. Thus, the propagation time τ of the acoustic interactions is introduced in the AR integral in Eq. (3) (Kuttruff, 2017),

$$B(x, t) = \iint_S \rho(x') B(x', t - \tau) f(x', x) V(x, x') \times A(x, x') dx' + B_S(x, t - \tau_S). \quad (3)$$

Here, τ is estimated based on the speed of sound c and the distance between x and x' ,

$$\tau = \frac{\|r\|}{c}.$$

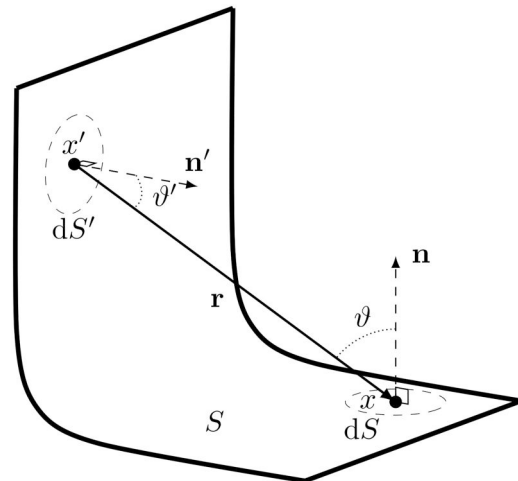


FIG. 1. Illustration of the variables involved in the radiosity integral.

The propagation time τ_s between the sound sources and x is calculated analogously. For generalized applications, a visibility term $V(x, x')$ is included, to account for surface regions shadowed by concave features. The influence of atmospheric absorption is also included in Eq. (3), denoted by $A(x, x')$. Note that both $A(x, x')$ and $\rho(x')$ are frequency dependent. The notation in this article neglects this for simplicity since, in practice, independent simulations are performed for distinct frequency bands.

B. Discrete integral

Few analytical solutions of the AR integral are known, only applicable in basic scene geometries (Kuttruff, 1976, 2017; Modest, 2013). Hence, a generalized AR model relies on the numerical integration of $B(x, t)$. The radiosity integral is discretized both in time and space. The scene surface S is divided into finite planar sub-surfaces S_i , so-called “patches.” At each patch, a simplified version of the radiosity integral can be evaluated, assuming a constant irradiation distribution $B_j(t)$ over each patch j at given points in time t ,

$$B_j(t) = \sum_{i=1}^N F_{ij} \cdot \rho_i \cdot B_i(t - \tau_{ij}) \cdot V_{ij} \cdot A_{ij} + B_j^S(t - \tau_{Sj}). \quad (4)$$

Equation (4) describes the irradiance $B_j(t)$ arriving at a given patch j at time t as a sum of the irradiance contributions B_i from every other patch i . Importantly, B_i contributions are delayed in time by τ_{ij} , corresponding to the mean sound propagation delay between patches i and j . Each B_i contribution is scaled by the respective reflection coefficient ρ_i , and a so-called form factor F_{ij} , defined in Eq. (5),

$$F_{ij} = \frac{1}{\Delta S_j} \int_{S_j} \int_{S_i} f(x_i, x_j) \, dS_i dS_j \\ = \frac{1}{\Delta S_j \pi} \int_{S_j} \int_{S_i} \frac{\cos \vartheta_i \cos \vartheta_j}{||\mathbf{r}_{ij}||^2} \, dS_i dS_j, \quad (5)$$

where ΔS_j represents the area of patch S_j .

The form factor geometrically determines the fraction of the total radiance emitted from patch S_i which irradiates the surface of S_j in perfectly diffuse conditions. Note, that the discrete form factor $F_{ij} = 0$ for $i = j$ following Eq. (5), since the double integral is performed over a flat subsurface, meaning $\cos \vartheta_i \cos \vartheta_j = 0$, $\forall x_i, x_j \in S_i = S_j$.

III. BRDF

A. Definition

The BRDF, originally introduced in the context of geometrical optics by Nicodemus *et al.* (1977), has been adapted for acoustics as described in Binek *et al.* (2022), Siltanen *et al.* (2007), and Siltanen *et al.* (2010). A given BRDF $\mathcal{R}$ is defined by the following equation:

$$\mathcal{R}(x', \mathbf{\Omega}_i, \mathbf{\Omega}_o) = \frac{\partial I_o(x', \mathbf{\Omega}_o)}{B_i(x', \mathbf{\Omega}_i) |\mathbf{n} \cdot \mathbf{\Omega}_i| \partial \mathbf{\Omega}_o}. \quad (6)$$

Here, I_o denotes the outgoing radiance in the direction $\mathbf{\Omega}_o$, and B_i represents the incoming irradiance from direction $\mathbf{\Omega}_i$. The term $\partial \mathbf{\Omega}_o$ is the differential solid angle around the outgoing direction $\mathbf{\Omega}_o$.

A valid BRDF must satisfy two key properties: energy conservation and reciprocity (Binek *et al.*, 2022; Nicodemus *et al.*, 1977).

Reciprocity means that the BRDF $\mathcal{R}$ remains the same if the incident and reflected directions are swapped,

$$\mathcal{R}(\mathbf{\Omega}_i, \mathbf{\Omega}_o) = \mathcal{R}(\mathbf{\Omega}_o, \mathbf{\Omega}_i). \quad (7)$$

Energy conservation ensures that the total reflected energy does not exceed the incoming energy. It is expressed as

$$\int_{\mathbf{\Omega}_o} \mathcal{R}(\mathbf{\Omega}_i, \mathbf{\Omega}_o) |\mathbf{n} \cdot \mathbf{\Omega}_o| d\mathbf{\Omega}_o \leq 1. \quad (8)$$

If the surface does not absorb any energy (e.g., $\alpha = 0$), the result of the integral is exactly 1.

B. Incorporation in the radiosity integral

To incorporate the BRDF into the radiosity formulation, it must be included within the form factor integral from Eq. (5). We denote the modified form factor as $\tilde{F}_{ij,o}$ combining the F_{ij} and ρ_i in Eq. (4),

$$\tilde{F}_{ij,o} = F_{ij} \cdot \rho_i \\ = \frac{1}{\Delta S_j} \int_{S_j} \int_{S_i} \mathcal{R}(\mathbf{\Omega}_i, \mathbf{\Omega}_o) \frac{\cos \vartheta_i \cos \vartheta_j}{||\mathbf{r}_{ij}||^2} \, dS_i dS_j. \quad (9)$$

Note that a factor of $1/\pi$ and reflection coefficient $\rho_i = 1 - \alpha_i$ are included in the BRDF definition. This is consistent with the form factor definition in the diffuse reflection case [Eq. (5)], for which the BRDF is $\mathcal{R}_{\text{diff}} = 1/\pi$.

C. Scattering distributions and BRDF

As shown previously, scattering distributions must be expressed as BRDF for correct implementation in a radiosity framework. The BRDF encode both scattering and absorption behavior of a surface. This, and their particular definition, require specific conversion steps from other scattering modeling approaches.

1. RI scattering to BRDF conversion

A scattering distribution following the RI coefficient model can be represented as a BRDF distribution, after the following equation:

$$\mathcal{R}(\mathbf{\Omega}_i, \mathbf{\Omega}_o) = \frac{(1-s)(1-\alpha)}{\mathbf{\Omega}_i \cdot \mathbf{n}} \frac{1}{w_o} \delta(\mathbf{\Omega}_o - \mathcal{M}(\mathbf{\Omega}_i)) \\ + \frac{s \cdot (1-\alpha)}{\pi}. \quad (10)$$

The first part of Eq. (10) represents the specular reflection, while the second part represents the diffuse scattering.

Ω_i and Ω_o are the incident and reflected directions, respectively, while $\mathbf{n}$ is the normal vector at the point of incidence. w_o , in a discretely sampled BRDF, is the weighting factor of the angular sector sampled by direction Ω_o . The sum of w_o over all angular sectors results in 2π , the area of a unit hemisphere. The Dirac delta function δ ensures that the specular reflection occurs only in the direction of the mirror reflection $\mathcal{M}(\Omega_i)$ of the incoming direction Ω_i .

2. Bidirectional scattering to BRDF conversion

Figure 2 shows a perfectly diffuse reflection as a BSC distribution, as defined by Heimes and Vorländer (2025). It also features a BRDF representation. Note that the BRDF is a continuous function defined as $1/\pi$, whereas the BSC is discretized into specific directions. While the BRDF is normalized by both the cosine of the outgoing angle (i.e., $\Omega_o \cdot \mathbf{n}$) and the sampling weight of the reflection direction, the BSC directly quantifies the energy reflected in a given direction, and therefore, such normalizations are not included. This distinction leads to the following conversion:

$$\mathcal{R}(\Omega_i, \Omega_o) = \frac{(1 - \alpha)}{(\Omega_o \cdot \mathbf{n}) \cdot w_o} s_d(\Omega_i, \Omega_o). \quad (11)$$

This formulation allows the BRDF to be computed from the BSC by appropriately accounting for the geometric and sampling normalization factors.

By inserting Eq. (11) into the energy conservation condition given in Eq. (8), we obtain

$$\begin{aligned} \sum_{\Omega_o} \frac{(1 - \alpha)}{(\Omega_o \cdot \mathbf{n}) \cdot w_o} s_d(\Omega_i, \Omega_o) |\mathbf{n} \cdot \Omega_o| w_o \\ = (1 - \alpha) \sum_{\Omega_o} s_d(\Omega_i, \Omega_o) = 1 - \alpha. \end{aligned} \quad (12)$$

This result confirms the energy conservation property, as the directional scattering distribution satisfies $\sum_{\Omega_o} s_d(\Omega_i, \Omega_o) = 1$, as defined in Heimes and Vorländer (2025).

IV. AR IMPLEMENTATION

The AR implementation employed in this study broadly follows the structure of the AR implementation described by Kang (2000), a conventional time-domain radiosity

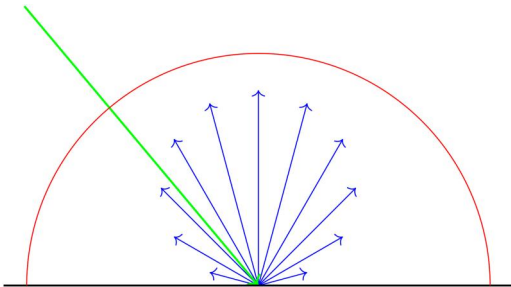


FIG. 2. A perfectly diffuse scatterer represented by the BRDF (red) and the BSC (blue) at a sound incident direction indicated in green.

algorithm. Said implementation relies on three main phases: geometrical preprocessing, radiance exchange, and ETC assembly. Significant extensions are made to said implementation: for one, the implementation supports an arbitrary distribution of patches over arbitrarily oriented surfaces, defined as quadrangles or triangles. Second, the support for scattering distributions in three-dimensional (3D) space is implemented in the form of BRDF, following a similar procedure to Siltanen *et al.* (2007). Implementation details are described in previous work (Fatela *et al.*, 2025).

The geometrical preprocessing step assembles all the geometrical and acoustic propagation data in static data structures, which encode reflectance and temporal relationships between patches. Importantly, a matrix $M_{ij,o} = \tilde{F}_{ij,o} \cdot A_{ij} \cdot V_{ij}$ stores coefficients of the reflectance exchange between an (i,j) patch pair, reflected in a direction o . $\tilde{F}_{ij,o}$ encodes both the geometrical form factor and the directional reflectance behavior in the form of BRDF.

The implementation used in this study relies on two assumptions: First, $\tilde{F}_{ij,o}$ is estimated by assuming a constant BRDF over each patch's surface, and a single incident direction, given by the patch centroid positions. Second, V_{ij} is set as either 1 or 0, based on the mutual visibility of the patch centroids. These assumptions may limit effective simulation of complex scattering patterns or partial patch visibility. Adjusting the resolution of the scene geometries may improve the simulation quality.

A. Form factor integration

The version of the AR tool used in this study (Heimes and Fatela, 2025) implements a combination of approaches to the form factor integration [Eq. (5)], based on patch geometry. One approach implements the Nusselt Analogue (Nusselt, 1928) to evaluate differential form factors dF_{ij} at finite sample points distributed evenly on the surface of a given patch j . The dF_{ij} are integrated in a simple center-point scheme to compute the form factor F_{ij} . The second approach is a contour integral approach (Goral *et al.*, 1984; Modest, 2013). The integrand function is approximated with piecewise 4th order Lagrange polynomials. The energy propagation from source-to-patch and from patch-to-receiver are estimated based on the so-called spherical triangle method (Nosal *et al.*, 2004a). These integration approaches, though numerical, are non-stochastic.

V. VALIDATION BY DIFFUSE REFLECTION SIMULATION

This section aims to validate the base implementation of the methods used in this paper. The accuracy of the simulation of perfectly diffuse reflections can be indicative of the quality of the AR algorithm. Thus, known analytical solutions for diffuse reflection interactions are compared with simulation results to validate it.

The results put forth by Svensson and Savioja (2024) explore the Lambert diffuse reflection model for an omnidirectional point source-receiver pair interacting with an

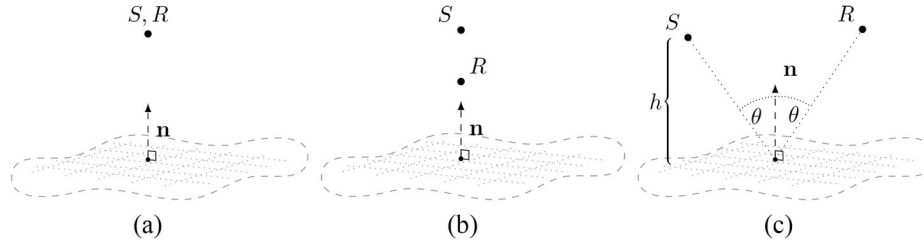


FIG. 3. Illustration of the infinite plane scenarios. (a) Co-located source and receiver. (b) Source and receiver positioned at different heights, in alignment with the plane normal. (c) Source and receiver pair at the same height. The source and receiver positions are represented by S and R , respectively. $\mathbf{n}$ represents the infinite plane normal.

infinite plane surface. In this work, the ratio between the diffusely reflected and specularly reflected radiance in this scenario is analytically derived,

$$\eta = I_{\text{diff}}/I_{\text{spec}},$$

where I_{diff} and I_{spec} represent the radiance at the receiver of a perfectly diffuse and specular reflections, respectively. Svensson and Savioja (2024) derives η in three special cases, as illustrated in Fig. 3:

- (a) a co-located source and receiver, $\eta = 2$;
- (b) a source and receiver aligned with the plane's normal, $\eta = 2$;
- (c) and a source and receiver at the same height h relative to the plane, $\eta = 2 \cdot \cos \theta$;

where θ is the angle between the plane normal and the specular sound path [illustrated in Fig. 3(c)]. A set of AR simulations was performed, replicating these cases.

A. Simulation setup

The infinite plane was emulated by a square 64×64 plane surface. The surface was subdivided into square patches of given side length l . In each case, the receiver was defined at a height of $h = 2$ above the plane. For special case (b), the source was positioned at $h_s = 3$. The simulated

diffuse reflection radiance $\hat{I}_{\text{diff}}$ was estimated by accumulating the total energy of the output ETC. An experimental $\hat{\eta}$ value is calculated by $\hat{\eta} = \hat{I}_{\text{diff}}/I_{\text{spec}}$, where I_{spec} is analytically defined.

B. Results and discussion

The relative error ϵ between η and $\hat{\eta}$ was studied to evaluate the quality of the simulation,

$$\epsilon = \frac{\eta - \hat{\eta}}{\eta}.$$

As shown in Fig. 4, it was found that $|\epsilon|$ decreases as the size of the patches relative to h is decreased. For each of the special cases, $|\epsilon|$ appears to converge quasi-exponentially toward 0 as the ratio l/h approaches 0. Importantly, $|\epsilon| < 2\%$ for $l/h < 1/2$. Hence the simulation error can be minimized by decreasing the patch dimensions relative to their distance to the source and receiver. This appears corroborated by case (b) featuring lower $|\epsilon|$ values than case (a): The source in case (b) is located further from the plane, meaning $l/h_{s(a)} > l/h_{s(b)}$.

Generally, $\epsilon > 0$, meaning $I_{\text{diff}} > \hat{I}_{\text{diff}}$. This aligns with the knowledge that the simulation cannot be performed over a truly infinite surface, and thus cannot account for all the diffusely propagated energy in the special cases. The exception arises in special case (c), where for some l/h values it was found that $\epsilon < 0$ for large θ angles. It is uncertain what may be causing this issue. It is possible that at shallow angles, the F_i^R and F_i^S integration is leading to an overestimation of the propagated energy. Furthermore, as the propagation distances increase and the sound reflects at increasingly shallow angles relative to the surface, the reflected radiance decreases significantly (after Lambert's cosine law), which may magnify the effects of any estimation errors.

VI. APPLICATION EXAMPLES

As an application example, a BRDF was derived from numerically computed BSC of a surface with a repeating triangular cross section, which is known to exhibit retro-reflective behavior (Cabrera *et al.*, 2020). This surface model was applied in two distinct simulation scenarios. The first scenario involved a single façade-like wall with a

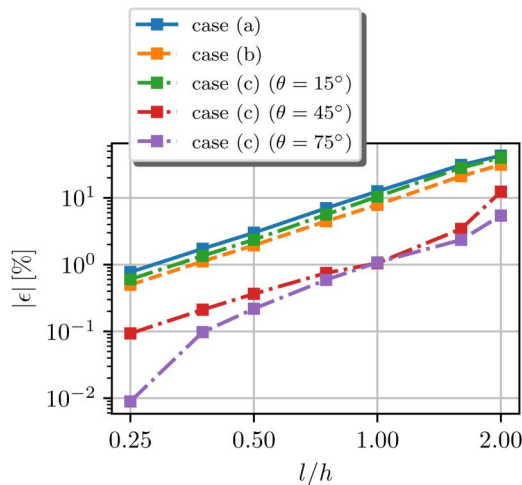


FIG. 4. Relative error modulus $|\epsilon|$ of the simulated and analytical diffuse energy ratios for the different special cases.

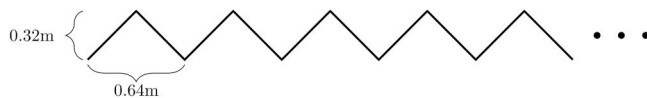


FIG. 5. 2D cross-section sketch of the surface from which the BSC and respective BRDF are derived for the application example scenes.

retro-reflective source-receiver configuration. The second scenario modeled a street canyon geometry with the retro-reflective surfaces on the vertical walls.

The triangular surface profile is illustrated in Fig. 5. First, the BSCs and the equivalent RI scattering coefficients (RISCs) were derived and post-processed as detailed in the Appendix. Then, the respective BRDFs were obtained.

The vertical BRDF was derived from BSC, based on Eq. (11). For the horizontal BRDF, the BSC was first rotated by 90° in azimuth (along the normal of the surface) before the BRDF was calculated based on Eq. (11). The RISC-based BRDF was calculated based on Eq. (10).

The BRDF for the vertical and horizontal orientations and the 125 Hz and 4 kHz octave bands at an incidence angle of 45° and azimuth angle of 0° are shown in Fig. 6. The RISC-based BRDF is identical for both orientations, as it is derived from the same RI scattering coefficient, which does not capture directional scattering information. The BSC-based BRDF, on the other hand, shows strong retro-reflective behavior for the vertical orientation. This is expected, as the triangular profile is oriented in a way that redirects the sound toward the source-receiver position. The horizontal orientation, on the other hand, does not feature such retro-reflective behavior, as the specular reflection is directed away from the source-receiver position. The retro-reflected lobes are stronger at 4 kHz than 125 Hz, which is expected as the frequency increases (Cabrera *et al.*, 2020).

It should be noted that this representation depicts only a cross section; in particular, for the BRDF derived from RI, the diffuse energy is also distributed across other directions.

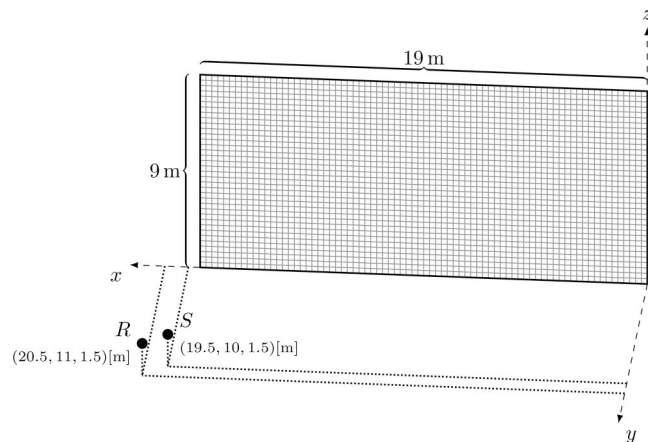


FIG. 7. Sketch of the single façade simulation scene, to scale. The grid on the surface represents the discretization of the surface into square $0.25 \text{ m} \times 0.25 \text{ m}$ patches. Source and receiver are marked by S and R , respectively.

A. Simulation of a retro-reflective façade

In the following section, the retro-reflective BRDF is applied in a simplified simulation scenario featuring a single retro-reflective façade. As detailed in Fig. 7, the source and receiver position are positioned at 1.5 m from ground level, and an incident angle of 45° relative to the center of façade. The façade surface was discretized into patches of size $0.25 \text{ m} \times 0.25 \text{ m}$.

The AR simulation was computed with a temporal resolution of 2 ms and subsequently averaged to 4 ms. This averaging reduces temporal discretization artifacts caused by the patch-wise rounding of energy to the time-window resolution, which is particularly noticeable for single surface reflections. The simulation was performed using a speed of sound of 343.2 m/s and a total ETC duration of 0.5 s. The wall absorption was set to 0, and the air attenuation was neglected. Two simulations of BSC-based BRDF were performed, one with the triangular cross-section oriented

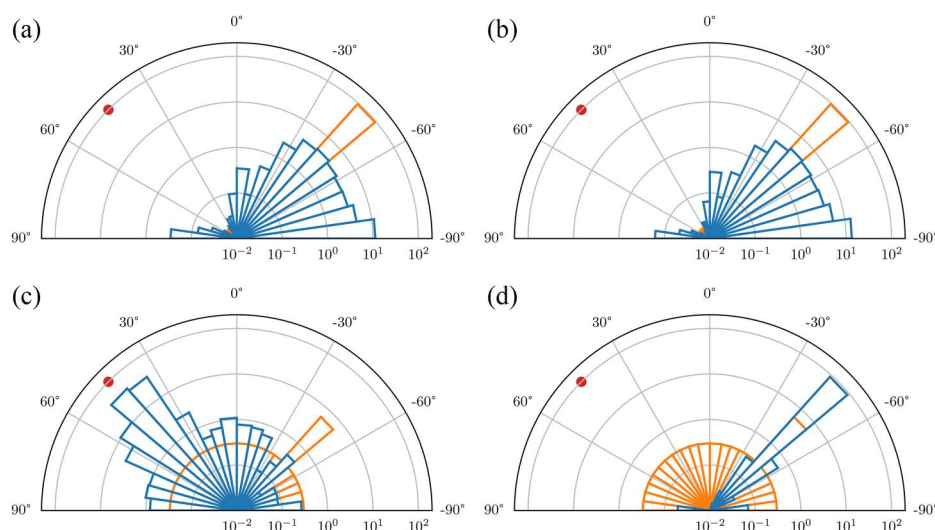


FIG. 6. Octave-band averaged BRDF at incident angle of 45° for (a) vertical orientation at 125 Hz, (b) horizontal orientation at 125 Hz, (c) vertical orientation at 4 kHz, and (d) horizontal orientation at 4 kHz, with the BSC-based BRDF (blue), RISC-based BRDF (orange), and the incidence direction (red).

vertically (along z axis) and another with a horizontal orientation (along the x -axis).

For comparison, the same scene was simulated with a RT approach in RAVEN (Schröder, 2011) using the RISC estimation shown in Fig. 8 and settings analogous to those in the AR simulation. A total of 5 000 000 rays was used.

As a reference for the vertical arrangement, a cross-section of the scene in the xy plane was additionally simulated using a two-dimensional boundary element method (2D-BEM) implemented in COMSOL Multiphysics® software (COMSOL AB, 2026). It is possible to simulate this scene in 2D, since the source and receiver occupy the same horizontal plane and the facade is extendable in the z direction. The structure of the facade was added onto a wall with a thickness of 0.3 m, leading to a maximum thickness of 0.62 m. The contour of the facade was meshed with boundary elements ranging from $\lambda/7$ to $\lambda/6$, where $\lambda = c/f$ denotes the wavelength corresponding to the simulation frequency f . The BEM simulation employed a point source equivalent to the same sound pressure level as in the GA models at a reference distance of 1 m. The simulation was implemented in COMSOL as a user-defined, so-called background pressure field. This is considered a 2.5D simulation, due to the 3D sound source and the infinite extent in the z direction. The atmospheric absorption was ignored in order to match the parameters of the AR and RT simulations. Both the direct sound and the reflected pressure field were evaluated at the receiver position. The frequency range extended from 5 Hz to 6 kHz with a resolution of 5 Hz. The comparison between wave-based and GA methods is inherently challenging due to their different outputs: BEM produces transfer functions, which include wave propagation effects, while GA methods rely on energy propagation leading to ETCs, making a direct comparison difficult. Both results can be converted toward a common representation, such as the ETC, but the process may lose information or introduce artifacts or limitations. Therefore, it is not expected that the BEM simulation will perfectly match the AR and RT simulations, but it can provide a useful reference for the validation of the AR and RT methods, especially in terms of the overall timing and energy of the reflected sound.

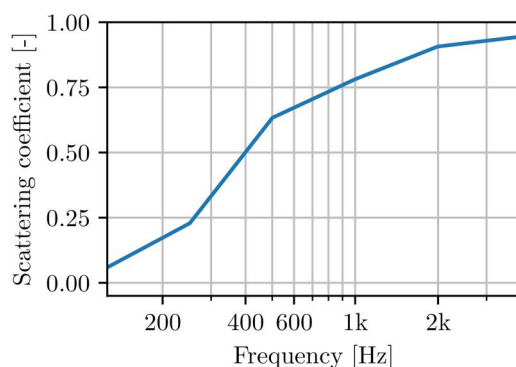


FIG. 8. Octave-band averaged RISC of the triangular surface profile.

For the horizontal arrangement, no BEM simulation was performed, as the computational cost of a corresponding three-dimensional simulation would be prohibitively high.

1. Simulation results

The comparison of the ETC for a low and high frequency band (125 Hz and 4 kHz, respectively) is shown in Figs. 9(a) and 9(b). The ETC obtained from the RISC-based AR and RT simulations shows good agreement across both frequency bands. This is expected, as both methods solve the same diffuse rendering equation in this example using different approaches (Hodgson and Nosal, 2006; Le Bot and Bocquillet, 2000) because no specular reflections propagate toward the receiver.

In contrast, both BSC-based AR simulations differ from the RISC-based results. The vertical orientation exhibits increased reflected energy, while the horizontal orientation shows reduced reflected energy relative to the RISC-based simulations. These differences are more pronounced in the 4 kHz band, while the 125 Hz band shows similar behavior for the two BSC-based simulations. This is because for this frequency range, both orientations are dominated by specular reflection, as scattering starts after the 125 Hz octave band ($f \leq 268$ Hz; Rayleigh, 1896). Therefore, no reflected energy is expected to arrive at the receiver position, and the energy observed in the ETC for both orientations at 125 Hz is likely due to finite-size effects in the BSC determination method (Heimes and Vorländer, 2025). The related BRDF shows similar behavior for the retro-reflection in Fig. 6, where the vertical orientation exhibits stronger retro-reflective lobes than the horizontal orientation, and the retro-reflective lobes are stronger at 4 kHz than at 125 Hz. The ETCs for the 125 Hz band are very similar for the two BSC-based simulations, which is attributed to the low scattering coefficient at this frequency (Fig. 7). This results in mainly a specular reflection and a similar behavior in the BRDF for both orientations [Figs. 6(a) and 6(b)]. Another point is the geometry of the facade: the height of the facade is smaller than its width, and the vectors from the source-receiver

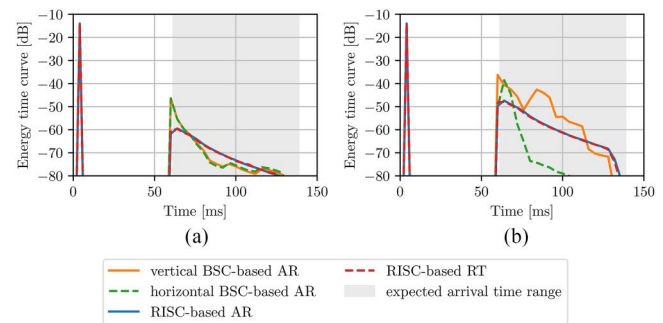


FIG. 9. The ETC for the octave band of 125 Hz (a) and 4 kHz (b) for different scattering modeling approaches. In blue and red, the ETC using an equivalent RISC model in AR and RT simulations, respectively. The output of the synthesized BRDF for the triangular profile surface is plotted in orange and green for vertically and horizontally oriented features, respectively. The expected time of arrival of the reflected sound is highlighted in gray.

positions to the center of the façade lead to an angle of approximately 45° relative to the normal of the façade. This geometry favors retro-reflection in the vertical orientation, as many subsurfaces of the vertical pattern alignment are perpendicular to the incident sound path. In contrast, the horizontal orientation does not benefit from this geometric configuration, resulting in reduced retro-reflective behavior.

Although both the vertical and horizontal BSC-based BRDF are represented by identical RISC-based BRDF, they produce significantly different ETC. This highlights the importance of directional BRDF information in outdoor sound-propagation simulations and for periodic surfaces.

To compare the BEM results with the AR and RT simulations, the BEM transfer function was transformed into the time domain to obtain the impulse response, leading to a sampling rate of 12 kHz. The impulse response was bandpass filtered using a first-order Butterworth filter with cutoff frequencies at 90 Hz and 5.56 kHz, representing the lower cutoff frequency of the lowest octave band and the upper cutoff frequency of the highest octave band used in the AR and RT simulations. Broad-band filtering was chosen over band-wise filtering, as it results in shorter filters and therefore fewer time-domain artifacts. The method proposed by Schröder (2011) was adapted in order to generate an ETC from the filtered BEM impulse response. The ETC was then calculated by determining the energy in each 4 ms time interval of the band-passed impulse response, matching the temporal resolution of the AR and RT simulations. The broadband ETC based on AR and RT simulation was calculated by weighting each octave band ETC with the bandwidth relative to half of the BEM simulation sampling rate (12 kHz). The impulse responses with and without the direct sound, as well as the resulting ETCs, are shown in Fig. 10. The energy of the direct sound in the BEM simulation agrees with the expected energy of a 3D point source [$1/(4\pi r^2) = E_{dir} = 1/(4\pi \cdot 2 \text{ m}^2)$] with a power of 1 W, with an error of less than 0.25 dB. The expected time of arrival, derived from the 2D geometry, is highlighted in gray. The reflected sound energy lies within the expected time window; however, both ETCs show a slight increase before the expected arrival time. As the direct sound energy of the BEM results is validated, the reflected sound energy estimation is also assumed to be reliable. Therefore, the comparison of reflected sound energy between the BEM and AR/RT simulations is considered meaningful.

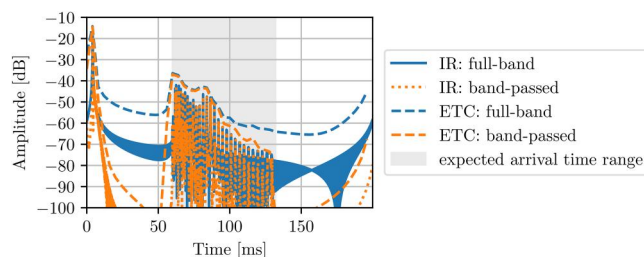


FIG. 10. The complete and bandpass impulse response from the BEM simulation together with its corresponding ETC.

This allows the comparison of the broadband ETC with the reference 2D-BEM simulation [Fig. 11(a)]. All geometrical simulation methods show reflected energy arriving at the expected time of arrival, highlighted in gray. This time is calculated based on the source-receiver positions and the closest and farthest points on the façade in the 3D scene. The direct sound arrives at approximately 9 ms and is correctly predicted by all geometrical simulation methods.

The ETC from the 2D-BEM simulation shows good agreement with the expected time of arrival of the reflected sound, except for the early part of the reflection. This discrepancy is attributed to the ETC calculation method, as it is not observed in the impulse response prior to the ETC processing. The direct sound from the 2D-BEM simulation is also broader than the geometrical simulations, which is attributed to the bandpass filtering of the impulse response. This effect is not present in the geometrical simulations, since no bandpass filtering takes place in those simulations. When comparing with the 2D-BEM reference, it should be noted that the BEM simulation includes diffraction effects at both ends of the 2D façade. These effects primarily occur at the beginning and the end of the expected reflection arrival time and are not present in the geometrical simulations.

When comparing the 2D-BEM reference with the geometrical simulations, the vertical BSC-based AR simulation shows closer agreement than either RISC-based simulations. In particular, the vertical BSC-based AR result closely matches the BEM simulation in the latter part of the reflection. At earlier reflection times, the BEM simulation exhibits slightly higher energy than the vertical BSC-based AR simulation. These differences can be attributed both to diffraction effects present in the 2D-BEM simulation and to limitations of the BSC model itself. Heimes and Vorländer (2025) reported that the BSC produces wider reflection lobes compared to the analytical solution of an infinite surface, particularly at lower frequencies, and tends to underestimate reflected energy into the specular direction across the

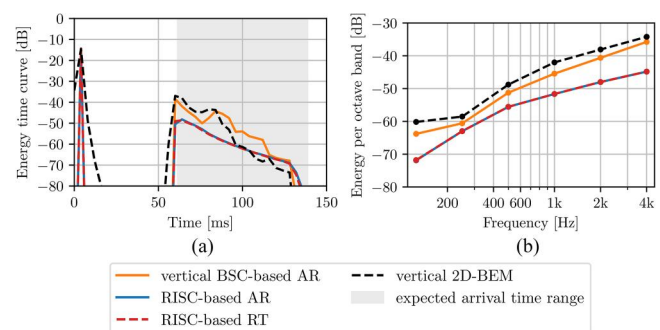


FIG. 11. Broadband ETC (a), and energy of reflection per octave band (b) for different scattering modeling approaches and a reference 2D-BEM simulation, in the vertical façade orientation. In blue and red, the ETC using an equivalent RISC model in AR and RT simulations, respectively. The output of the synthesized BRDF for the triangular profile surface is plotted in orange and green for vertically and horizontally oriented features, respectively. In black, the reference 2D-BEM simulation. The expected time of arrival of the reflected sound is highlighted in gray.

full frequency band. It might be that the BSC leads to an underestimation for non-specular reflections; this is still under investigation. This may explain the higher reflected energy observed in the BEM simulation compared to the BSC-based AR simulation.

In addition, the reflected-sound energy per octave band is compared in Fig. 11(b). For the 2D-BEM reference, the transfer function without the direct sound component was windowed in the frequency domain with a rectangular window for octave bands from 125 Hz to 4 kHz. Based on these windowed transfer functions, the reflected-sound energy was calculated for each octave band. The energy of each filtered signal represents the reflected-sound energy within the corresponding band. For the GA simulations, the ETC excluding the direct sound component was weighted by the bandwidth of each band relative to the half of the BEM simulation sampling rate (12 kHz). The weighted signals were then temporally summed to obtain the reflected-sound energy in each band.

Compared with the 2D-BEM reference, the vertical BSC-based AR simulation shows closer agreement than either RISC-based simulation across all frequency bands. An offset ranging from 1.8 dB to 3.7 dB is observed between the vertical BSC-based AR simulation and the 2D-BEM reference, which may be attributed to the limitations of the BSC model discussed previously. The RISC-based AR and RT simulations yield similar reflected-sound energies across all frequency bands, as both methods solve the same rendering equation. However, the RISC-based simulations exhibit a larger discrepancy with respect to the 2D-BEM reference than the vertical BSC-based AR simulation.

B. Simulation of a retro-reflective street canyon

The urban canyon scene is set up with a single omnidirectional source (S) and receiver (R) pair. Figure 12 illustrates the geometry: The canyon is 60 m long and 9 m wide with 6 m tall walls on each side. S is positioned in (20, 4.5, 1)[m], and R at (21, 4.5, 2)[m] after the (x, y, z) axes of Fig. 12.

The ground surface (in Fig. 12, in dark gray), emulates a surface with a RISC modeled scattering distribution, with $s_{\text{rand}} = 0.1$. The walls (white, vertical surfaces) are assigned a retro-reflective surface as detailed in Sec. VI and illustrated in Fig. 5. The triangular features of the surface are oriented vertically, favoring retro-reflections along the length of the canyon (x -axis in Fig. 12). The scene is assessed in two independent simulations by changing the modeling approach of the vertical walls between the RISC model and the BSC model. The absorption coefficients of the surfaces were set as $\alpha_{\text{ground}} = 0.01$ and $\alpha_{\text{walls}} = 0.07$ for the ground and walls, respectively. The sound speed is set as $c = 343.26$ m/s, and air attenuation effects are ignored.

The ETC length is 0.4 s, with a 2 ms temporal resolution. The surfaces were subdivided into $3 \text{ m} \times 3 \text{ m}$ square patches. The maximum reflection order was manually set at

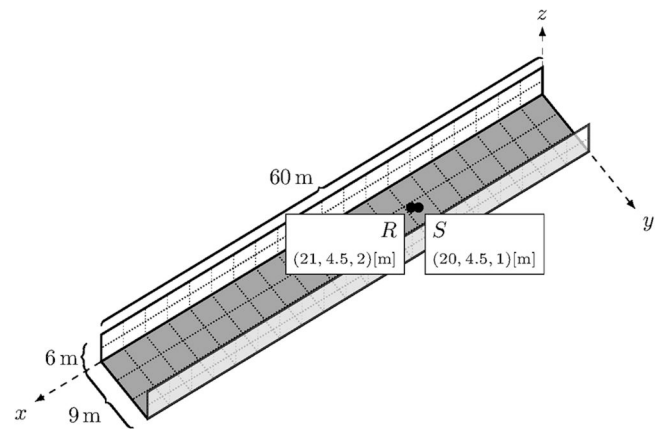


FIG. 12. Sketch of the urban canyon scene geometry. The dotted square pattern represents a subdivision of the surfaces into $3 \text{ m} \times 3 \text{ m}$ patches.

$\mathcal{L} = 30$. Another RT simulation was also run with equivalent parameters and a total of 10×10^6 rays.

1. Simulation results

The simulation of the different modeling approaches resulted in noticeably distinct ETCs. Figure 13 shows the resulting ETC and EDC of the AR simulations, as well as a ray tracing simulation reference, in the vertical orientation of the façade.

In addition, a simple IS simulation of order 5 was run using RAVEN to assess the timing of the specular reflections, so they can be compared to the RISC AR and RT models. All three simulations capture the direct sound ($t = 4$ ms) well, while the first ground reflection was captured accurately by the RT simulation and underestimated in both AR simulations. The specular reflection from the walls arrives at $t = 26$ ms, determined from the IS method. As the ground features a low scattering coefficient, the energy between the specular ground reflection and the first wall reflections should be low. The RISC-based RT simulation reflects the expected behavior, while both AR-based methods present it as two peaks, which contain more energy than the respective sections of the RT-derived ETC.

Both AR simulations feature the same BRDF for the ground, which explains the similar behavior between the two AR-based ETCs in this time range. The underestimation of the specular ground reflection by both AR simulations can be explained by the discretization of the scene geometry. The BRDFs are evaluated at a single point corresponding to the centroid of the patches. This means, unless the specular reflection point neighbors the patch centroids, the BRDF will be evaluated at a sample which does not correctly represent the specular reflection. Thus, some specular reflections may not be handled by the simulation. Figure 14 illustrates the exact issue described previously: The first-order specular ground reflection between source and receiver (blue arrow) is not covered by the patch-wise BRDF distribution, because the specular reflection point does not coincide with the patch centroids (denoted by gray $\times$ marks). Instead, the earliest ground

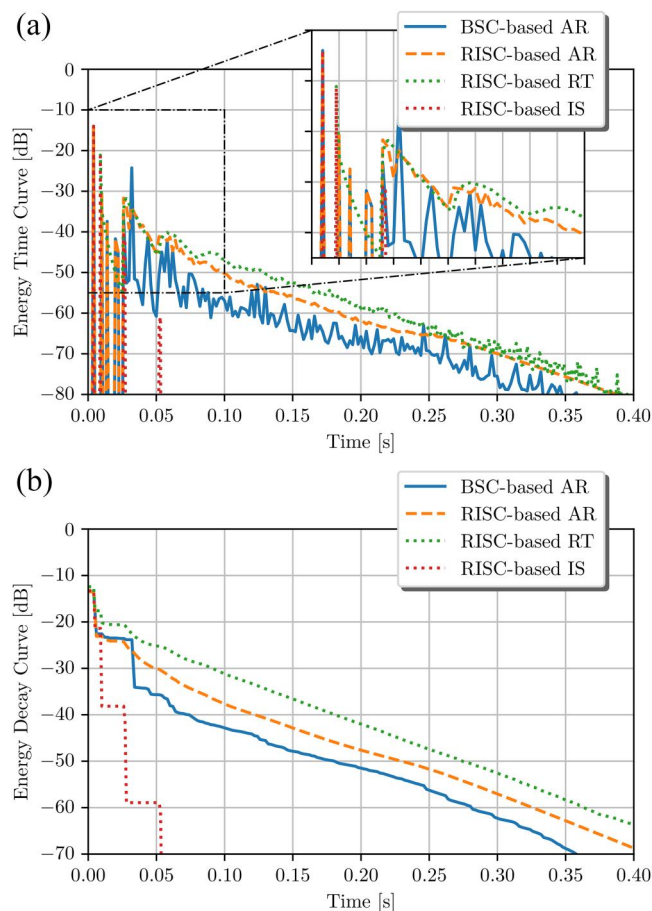


FIG. 13. ETC (a) and energy decay (Schroeder) curve (b) of the urban canyon scenes in the 4 kHz octave band. In the blue line, the AR simulation result using the BSC scattering model on the walls. In the orange dashed line, the ETC for the AR simulation using the RISC scattering model exclusively. The green dotted line shows the ETC computed with an RT simulation. The red dotted line represents a 5th order IS simulation output (perfectly specular propagation).

reflection follows a direction that corresponds to the diffuse scattering region of the BRDF.

Due to the discretization of the BRDFs, the radiance corresponding to a given BRDF sample is not cast in single directions, but towards given angular sectors. In the case of the specular components of the BRDF, these are called pseudo-specular cones (shown in Fig. 14, as a red pyramid). If the resolution of the BRDFs is too high, some effects (e.g., specular reflection, mentioned previously) may not be

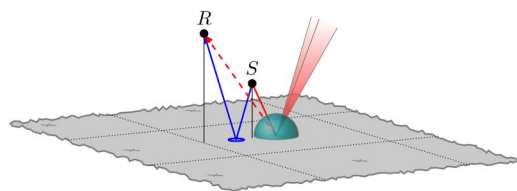


FIG. 14. Illustration of the first-order ground reflections in the street canyon scene. S and R represent the source and receiver, respectively. The cyan hemisphere and red pyramid represent a RISC-based BRDF, where the red pyramid represents the pseudo-specular cone for the source's position. The blue arrow is the specular reflection path, modeled by the IS and RT methods. The red arrow represents the AR reflection path from the nearest patch.

modeled by the simulation. If, on the other hand, the BRDF resolution is too low, specular radiance propagation will be attributed to wide regions, which should not feature the same radiance level.

This represents a major limitation of the AR method if BRDFs are assumed to be uniform over the patch surfaces. These artifacts can be mitigated by increasing the geometrical resolution of the scene or carefully selecting the BRDF discretization. Given the complexity of the simulation and physical hardware limitations, this is often not practical (see Sec. VIB 2).

The jagged behavior of the AR simulations compared to the RT simulation is likely due to the discretization of the surfaces into patches, which leads to a discretization of the reflected energy in time, as the energy is rounded to the nearest time window.

After the first and second order wall reflections ($t \approx 26$ ms), the RISC-based AR and RT simulation show similar behavior, except for the times between 100 and 250 ms, where the RT simulation results in higher energy. The RISC-based models show similar behavior after the first wall reflection to about $t = 200$ ms, after which the RISC-based RT simulation behaves stochastically. This artifact is possibly caused by the stochastic nature of the RT algorithm. The RISC-based AR simulation, on the other hand, features a much smoother decay, which is likely due to the fact that the radiosity algorithm does not rely on stochastic sampling but rather on a deterministic solution of the rendering equation.

The BSC-based AR simulation shows a strong peak after the first wall reflection, which can be attributed to the retro-reflection of the walls. After this, the energy is generally lower than the RISC-based AR and RT simulations, which is likely due to retro-reflection effects. In particular, higher-order reflections contribute less energy than early reflections because retro-reflected paths require the acoustic energy to return toward the source/receiver, typically involving a greater number of reflections and thus increased attenuation. As a result, these paths are attenuated more strongly than, for example, diffusely scattered energy, leading to a lower energy level in the late part of the ETC. The pronounced peaks of the BSC-based ETC can likely be attributed to strong, individual retro-reflections from the walls.

This example highlights the influence that complex scattering distributions may have in the early reflections. It also shows the limitations of the RISC-based AR and RT model, where the scene and BRDF discretization in the AR model might lead to misestimation of the early energy, while the RISC-based RT model leads to stochastic behavior at the late part of the ETC, due to the nature of the RT algorithm. This discrepancy is due to the inclusion of the specular component, as the results were both converging in the diffuse case in Sec. VIA 1. The convergence in the diffuse case is expected, since Le Bot and Bocquillet (2000) showed that both RT and AR methods result in equivalent results, given adequate scene discretization. Additionally, Hodgson

and Nosal (2006) showed that for the purely diffuse case, the ETCs simulated by both radiosity and RT methods can show good agreement, as long as both methods converge. This shows that the AR and RT simulations perform as expected in the diffuse case, but their RISC implementation may accentuate differences caused by their distinct algorithms.

2. Computational performance

A series of simulations of the urban canyon scene was run, in order to assess the computational performance of the AR implementation. Four main simulation properties are investigated: number of ETC samples (tied to ETC length and resolution); reflection order; BRDF resolution; and number of patches. The effects of these properties are studied for the three main steps of the simulation: The geometric preprocessing, the radiance exchange, and the ETC collection at the receiver position.

In order to study the behavior of the simulation for each of the simulation properties individually, a set of baseline conditions is set in Table I. Except for the simulation property in the study at a given time, these are kept constant for the sake of comparability.

The simulations were performed on a desktop computer with an 8-core Intel64 Family model 158 stepping 13 CPU (Intel, Santa Clara, CA). Table II summarizes the machine characteristics.

Generally, it is found that the radiance exchange step accounts for the peak memory consumption, while the geometric preprocessing module generally features a larger run-time for the tested simulation conditions. A short discussion of the computational effects of each of the simulation properties is included in the following.

a. ETC sampling. The number of ETC samples is determined by the length and resolution of the ETC. It is shown in Fig. 15(a) that as the number of time samples grows, so does the run-time of the simulation. There appears to be a significant linear relationship between the run-time of the radiance exchange module and the number of samples. While the geometric preprocessing step generally dominates the run-time of the program, a large enough number of ETC time samples may lead to a larger section of the run-time being dedicated to the radiance exchange. The peak memory consumption [Fig. 15(b)] of the radiance exchange and ETC collection modules grow linearly with the increase in ETC time samples. This is indicative of how the AR implementation stores the radiance exchange data: patch-wise ETCs, collected at the receiver in the collection step.

TABLE I. Baseline simulation properties.

# ETC samples	200
max. reflection order	10
# BRDF samples	4
# patches	140

TABLE II. Machine characteristics for the urban canyon simulation.

Architecture	64 bit
Processor	Intel Core i9-9900k
# CPU cores	8
Clock speed	3.60 GHz (5.00 GHz max.)
OS	Windows 10

b. Reflection order. Predictably, the run-time of the radiance exchange step is linearly proportional to the reflection order of the simulation. Figure 16 reflects how the reflection order does not affect the data structures of the simulation: it has no impact on the run-time of any other simulation step or the memory consumption. Simply, it determines the depth of the recursion of the radiosity integral, computed in the radiance exchange step.

c. BRDF sampling. The number of BRDF samples impacts the run-time of the radiance exchange step linearly. This is an artifact of the specific AR implementation: patch-

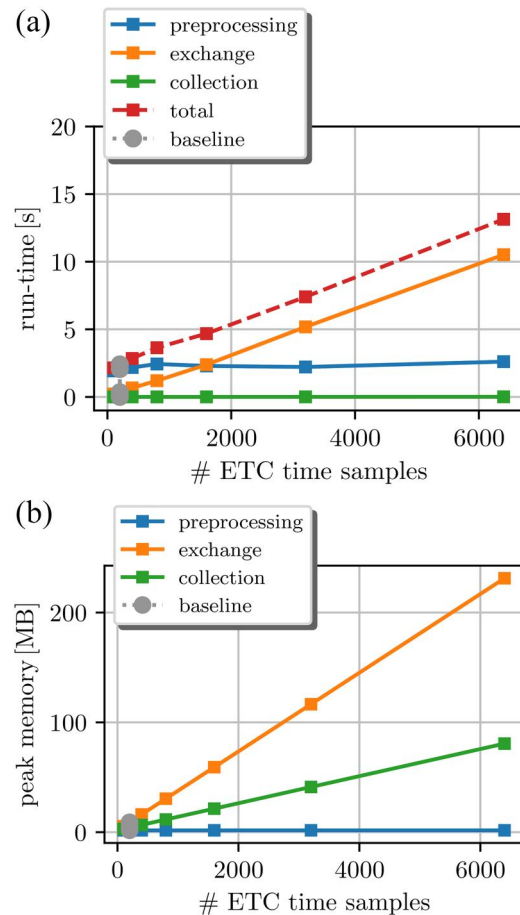


FIG. 15. Major computational measures as a function of the number of ETC samples for the different steps of the simulation. (a) Simulation run-time. (b) Peak memory consumption. In blue, the geometric preprocessing step; in orange, the radiance exchange step; in green, the ETC collection at the receiver. Sub-figure (a) evidences the total simulation run-time in the red dashed line. The gray circles mark the baseline conditions detailed in Table I.

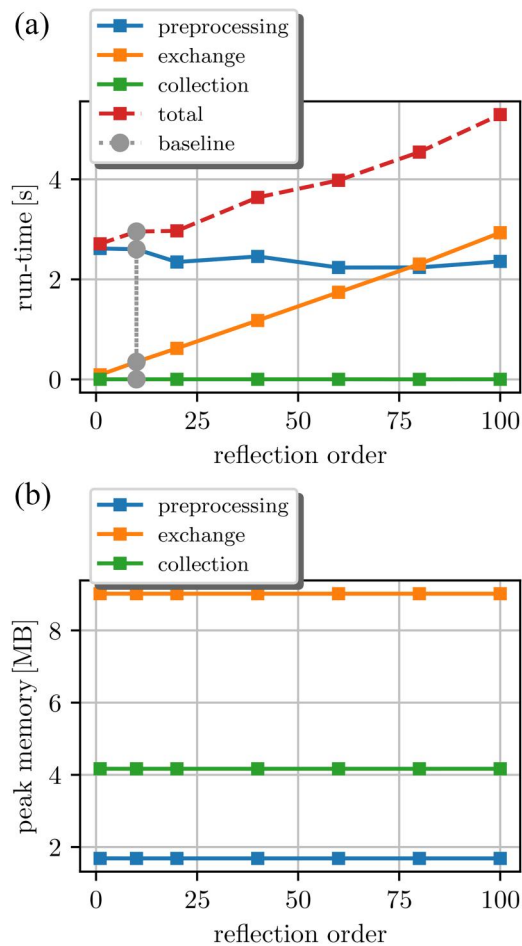


FIG. 16. Major computational measures as a function of the reflection order for the different steps of the simulation. (a) Simulation run-time. (b) Peak memory consumption. In blue, the geometric preprocessing step; in orange, the radiance exchange step; in green, the ETC collection at the receiver. Sub-figure (a) evidences the total simulation run-time in the red dashed line. The gray circles mark the baseline conditions detailed in Table I.

wise ETCs preserve spatial information about the reflection, necessary for the ETC collection step. At the assembly of each patch's ETC, the number of operations grows linearly with the increase in BRDF samples. This is shown by Fig. 17(a). This figure also suggests a small increase in the run-time of the geometric preprocessing step. This is likely due to the initialization of the energy relationship matrix M (see Sec. IV), which grows linearly in size with an increase in the number of BRDF samples. This relationship is evidenced in Fig. 17(b), which shows the peak memory consumption growing linearly with the number of samples for all the steps of the simulation.

d. Geometry discretization. It was found that the discretization of the scene geometry has the most impact in the AR implementation's performance. Given a large enough number of patches, the geometric preprocessing step quickly dominates the run-time [see Fig. 18(a)]. Since the geometric preprocessing algorithm relies on operations between many patch pair combinations, the run-time grows geometrically with the increase in the number of patches. In particular, the

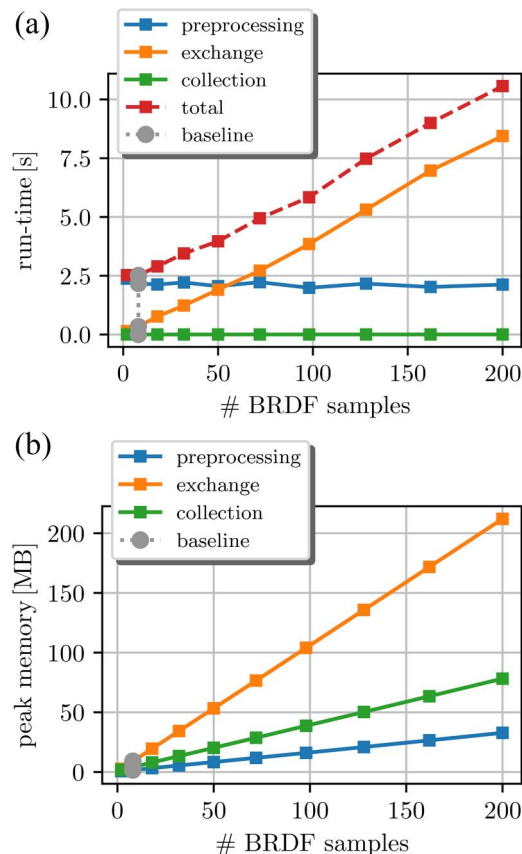


FIG. 17. Major computational measures as a function of the number of BRDF samples for the different steps of the simulation. (a) Simulation run-time. (b) Peak memory consumption. In blue, the geometric preprocessing step; in orange, the radiance exchange step; in green, the ETC collection at the receiver. Sub-figure (a) evidences the total simulation run-time in the red dashed line. The gray circles mark the baseline conditions detailed in Table I.

run-time of the geometric preprocessing step grows quasi-cubically with the number of patches, while the runtimes of the energy exchange and collection steps have a quadratic and quasi-linear relationship with the number of patches used, respectively. As evidenced by Fig. 18(b), the peak memory consumption also features this geometrical growth as the number of patches increases. The cause for the large growth in memory consumption is the assembly of the energy matrix M , which grows quadratically with the number of patches. The profile of the preprocessing confirms this quadratic relationship between memory requirements and patch number.

VII. CONCLUSION

This article presents an investigation into different scattering modeling approaches in the context of acoustic radiance transfer simulation. Two scattering modeling approaches are compared: one based on RISC, and one based on BSC. A time-domain AR implementation simulates the acoustic radiosity of virtual scenarios by encoding both RISC and BSC models in BRDF.

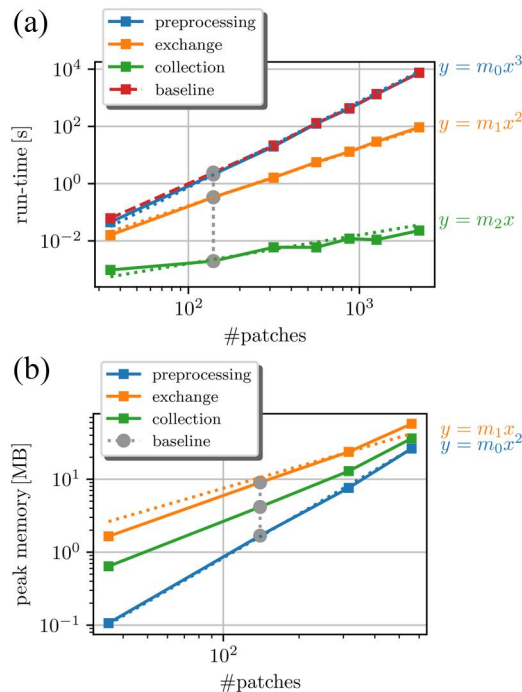


FIG. 18. Major computational measures as a function of the number of patches in the geometry surfaces, for the different steps of the simulation. (a) Simulation run-time. (b) Peak memory consumption. In blue, the geometric preprocessing step; in orange, the radiance exchange step; in green, the ETC collection at the receiver. Sub-figure (a) evidences the total simulation run-time in the red dashed line. The gray circles mark the baseline conditions detailed in Table I. Dotted lines indicate perfectly linear, quadratic, or cubic trends. The memory profile features a reduced number of sample points due to the time-consuming nature of the profiling operation, which became impractical.

The implementation is first shown to align with analytical models for perfectly diffuse (Lambertian) reflection. Then, two application examples were conducted: the first simulating a façade with a retro-reflective surface, and the second a street canyon geometry featuring such retro-reflective façades. In both cases, a retro-reflective source–receiver configuration was employed. This configuration exacerbates the differences between the scattering modeling approaches.

In the single façade application, a wave-based boundary element method (BEM) simulation of the same scene served as a reference. Though the BEM results differed from all the geometrical simulations, the BSC-based ETC matched the BEM reference the closest, both over time and in the frequency spectrum. This highlights how BSC modeling captures more of the scattering distributions’ directional behavior than RISC. This result also shows that capturing such directional behavior, instead of relying on RI Lambertian scattering, can have a large impact on the characteristics of individual sound reflections.

Though the façade results between RISC AR and RT simulations agreed strongly, the same was not found for the street canyon simulation. This is thought to be related to the specific BRDF encoding and geometry discretization used in the simulation. Certain spatial effects, in particular specular reflection, were underestimated due to the sparsity of the BRDF evaluation in the scene. In addition, the RT results

feature some stochastic artifacts toward the end of the acoustic decay, perhaps due to the stochastic nature of the algorithm. In contrast to Hodgson and Nosal (2006), who found good agreement between AR and RT for a purely diffuse case, the results of the street canyon simulation suggest that the inclusion of a specular component in the BRDF may lead to discrepancies between AR and RT simulations. As expected from the results of the previous application, the BSC-based simulation is significantly distinct from the RISC simulations, in particular for the early reflections. The BSC-based simulation captured significant features that are not present in the alternative RISC modeling. These differences motivate further implementation of complex scattering modeling approaches. However, these features may not be as pronounced in reality, since the BSC-based simulation features the same limitations due to the discrete BRDF sampling mentioned before. This is a strong motivation to investigate alternative BRDF encoding strategies.

The computational performance of the current implementation is shortly investigated as a function of different simulation parameters. It is largely dependent on the spatial and temporal discretization of the scene.

A. Outlook

In addition to the numerical analysis performed in this article, it must be investigated whether ETCs resulting from custom BRDF implementation are in fact aligned with real-world measurements of large scale outdoor scenes. Furthermore, it is important to assess whether the computational effort put into such a complex solution translates to noticeable differences by listeners.

A significant improvement will be the integration of the BRDF within the form factor integral. Currently, the BRDF evaluation is separated from the form factor calculation, which is only valid if the BRDF is constant over the patch surfaces. This is not always the case, in particular for neighboring patches and complex BRDF distributions. As seen previously, patchwise evaluations of the BRDF may lead to misestimation of specular reflection contributions (or contributions of otherwise pronounced reflection lobes). By integrating the BRDF over the surface of the patches, these contributions are accounted for, regardless of the scene geometry and BRDF discretization. Analogously to the BRDF integration, the visibility factor calculation can be designed to handle partial patch visibility. This is predicted to significantly increase the accuracy of the radiance exchange. While these changes will likely increase the computational overhead of the simulation, an increase in accuracy translates to lower geometry resolution requirements. This means that some computational trade-off is possible while making the simulation more and more realistic.

At the same time, RT and Monte-Carlo-based AR methods can be designed in a way which handles both BRDF integration and partial visibility effects inherently. This highlights the complexity of the simulation approach choices for radiance transfer methods, especially concerning the trade-off between computational efficiency and output precision. It

warrants comprehensive technical study, to accelerate scientific knowledge in the field of GA and auralization.

Regarding the synthesis of time signals from the ETC, a new paradigm is needed for open geometries. Since typical implementations of reflection statistics rely on the properties of closed rooms, an option is to make predictions about the actual pressure impulse response involved in a single reflection at a discrete patch. In a pressure signal domain, these interactions manifest as a kind of “wavelets.” This investigation is ongoing, since the energy distribution depends on several geometrical properties of the patches and receiver.

SUPPLEMENTARY MATERIAL

See the [supplementary material](#) for the original datasets and code used for the generation of results in this paper. For instructions on how to navigate these materials, please refer to the document named “README.md” included. Note that the scripts used to generate the RT and BEM references for the examples in Sec. VI rely on licensed third-party programs. Some resources to the installation and access to free licensing are linked in the [supplementary material](#) documentation. In any case, the [supplementary material](#) scripts can be run in spite of the ray tracing and BEM simulations, using the datasets simulated by the authors.

ACKNOWLEDGEMENTS

João Fatela and Anne Heimes are co-first authors, given their respective contributions to this project. This research was supported in part by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under the Project No. 456072683. Computations were performed with computing resources granted by RWTH Aachen University under project rwth1245.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts of interest to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available within the article and its [supplementary material](#).

APPENDIX

To determine the BRDF, first, the BSC of a triangular surface was calculated as described by [Heimes and Vorländer \(2025\)](#).

Figures 19(a) and 19(b) show the virtual samples for the calculation of the BSC. The structure was scaled by a factor of 1:8, such that a sample diameter of $d = 0.8$ m with ten repetitions matched the optimal simulation configuration shown in Fig. 19(c), as recommended by [Heimes and Vorländer \(2025\)](#). The sound incident direction was modeled as a plane wave, with incidence directions determined via a 21st-order Gaussian quadrature over a quarter hemisphere. Receiver positions were defined using an equal-area sampling scheme, comprising 23 328 points uniformly distributed on a spherical surface of radius 50 m. This scaling yields a structural height of $h = 40$ mm and a structural wavelength of $\Lambda = 80$ mm.

1. Numerical simulation

The simulations were performed using the BEM solver Mesh2scattering ([Heimes, 2025](#)), which is specifically developed for acoustic scattering problems. This solver is based on Mesh2HRTF ([Brinkmann et al., 2023](#)) and employs the numerical core NumCalc ([Kreuzer et al., 2024](#)). The frequency range for the simulations was set from 1 to 40 kHz, with nine frequencies per octave. This corresponds to a normalized frequency range of Λ/λ from 0.23 to 9, where λ is the wavelength of sound in air, assuming a speed of sound of 343.2 m/s. All simulations were conducted using computing resources provided by RWTH Aachen University.

2. BSC

The BSC was then computed based on the numerical simulation results, leading to a hemispherical Gaussian sampling of order 21 for the incident and reflected directions.

3. RISC

To compute the RISC, the scattering coefficient was first evaluated for each incident angle using Eq. (A1) with s_d being the BSC. Subsequently, Paris' equation [Eq. (A2)] was applied to obtain the RISC,

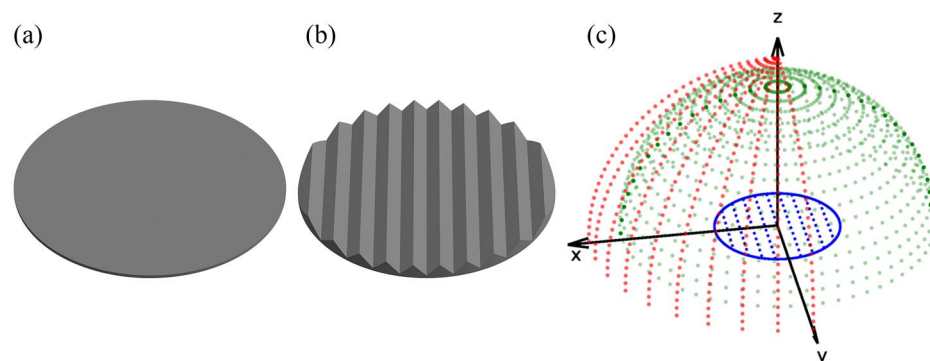


FIG. 19. The samples and setup used for the numerical simulation of the BSC. (a) Reference sample. (b) Sample under investigation. (c) Geometrical setup with the sample in blue, the receiver grid in green, and the incident direction of sound in red.

$$s(\Omega_i) = 1 - s_d(\Omega_i, \mathcal{M}(\Omega_i)), \quad (\text{A1})$$

$$s_{rand} = \sum s(\Omega_i) \cdot |\Omega_i \cdot \mathbf{n}| \cdot w_i. \quad (\text{A2})$$

Finally, the RISC was subsequently averaged over individual octave bands, resulting in the frequency range from 100 Hz to 4 kHz.

The RISC was computed from the BSC by averaging the specularly reflected energy according to Paris' equation, yielding 1-s. The coefficient was subsequently averaged over individual octave bands, covering the frequency range from 100 Hz to 4 kHz.

4. Post-processing

In the following, the BSC will be post-processed to be suitable for sound propagation models as described in Heimes and Vorländer (2025).

First, the surface geometry was scaled by a factor of 8:1, resulting in a desired structural wavelength of $\Lambda = 0.64$ m. The BSC was subsequently averaged over individual octave bands, resulting in a frequency range from 100 Hz to 4 kHz.

Antani, L., Chandak, A., Taylor, M., and Manocha, D. (2012). "Direct-to-indirect acoustic radiance transfer," *IEEE Trans. Visual. Comput. Graph.* **18**(2), 261–269.

Binek, W., Chojak, A., and Kamisiński, T. (2021). "The impact of the sound reflection model on the ray tracing simulation variability," *Vib. Phys. Syst.* **32**(2), 2021203.

Binek, W., Pilch, A., and Kamisiński, T. (2022). "Direct application of the diffusers' reflection patterns in geometrical acoustics simulations," *Appl. Acoust.* **198**, 108949.

Brinkmann, F., Kreuzer, W., Thomsen, J., Dombrovskis, S., Pollack, K., Weinzierl, S., and Majdak, P. (2023). "Recent advances in an open software for numerical HRTF calculation," *J. Audio Eng. Soc.* **71**(7/8), 502–514.

Cabrera, D., Yadav, M., Holmes, J., Fong, O., and Caldwell, H. (2020). "Incidental acoustic retroreflection from building façades: Three instances in Berkeley, Sydney and Hong Kong," *Build. Environ.* **172**, 106733.

Cao, C., Ren, Z., Schissler, C., Manocha, D., and Zhou, K. (2016). "Interactive sound propagation with bidirectional path tracing," *ACM Trans. Graph.* **35**(6), 193.

COMSOL AB (2026). "COMSOL Multiphysics® v. 6.3," www.comsol.com (Last viewed February 27, 2025).

European Environment Agency (2010). "Good practice guide on noise exposure and potential health effects," *EEA Tech. Rep.* **11**, 1–36.

Fatela, J., Heimes, A., and Vorländer, M. (2025). "Efficient acoustic radiosity modelling for precise bi-directional reflectance distribution patterns," in *Proceedings of DAS/DAGA2025*, Deutsche Gesellschaft für Akustik e. V. (DEGA), pp. 1564–1567.

Goral, C. M., Torrance, K. E., Greenberg, D. P., and Battaile, B. (1984). "Modeling the interaction of light between diffuse surfaces," *SIGGRAPH Comput. Graph* **18**(3), 213–222.

Heimes, A. (2025). "mesh2scattering version: 1.0.0 (Python 3.10)," <https://github.com/ahms5/Mesh2scattering> (Last viewed February 27, 2025).

Heimes, A., and Fatela, J. (2025). "Sparrowpy: Sound propagation with acoustic radiosity for realistic outdoor worlds," Zenodo [10.5281/zenodo.17130452](https://zenodo.org/record/17130452).

Heimes, A., and Vorländer, M. (2025). "Bidirectional surface scattering coefficients," *Acta Acust.* **9**, 41.

Hodgson, M., and Nosal, E.-M. (2006). "Experimental evaluation of radiosity for room sound-field prediction," *J. Acoust. Soc. Am.* **120**(2), 808–819.

Hu, Z., Zayed, T., and Cheng, L. (2022). "A critical review of acoustic modelling and research on building façade," *Build. Acoust.* **29**(1), 107–134.

ISO (2004). 17497-1, "Acoustics—Sound-scattering properties of surfaces—Part 1: Measurement of the random-incidence scattering coefficient in a reverberation room" (International Organization for Standardization, Geneva, Switzerland).

Kang, J. (2000). "Sound propagation in street canyons: Comparison between diffusely and geometrically reflecting boundaries," *J. Acoust. Soc. Am.* **107**(3), 1394–1404.

Koutsouris, G. I., Brunskog, J., Jeong, C.-H., and Jacobsen, F. (2013). "Combination of acoustical radiosity and the image source method," *J. Acoust. Soc. Am.* **133**(6), 3963–3974.

Kreuzer, W., Pollack, K., Brinkmann, F., and Majdak, P. (2024). "NumCalc: An open-source BEM code for solving acoustic scattering problems," *Eng. Anal. Boundary Elem.* **161**, 157–178.

Kuttruff, H. (1976). "Reverberation and effective absorption in rooms with diffuse wall reflections," *Acta Acust. united Ac.* **35**(3), 141–153.

Kuttruff, H. (2017). *Room Acoustics*, 6th ed. (CRC Press/Taylor & Francis Group, Boca Raton, FL).

Le Bot, A., and Bocquillet, A. (2000). "Comparison of an integral equation on energy and the ray-tracing technique in room acoustics," *J. Acoust. Soc. Am.* **108**(4), 1732–1740.

Marbjerg, G., Brunskog, J., Jeong, C.-H., and Nilsson, E. (2015). "Development and validation of a combined phased acoustical radiosity and image source model for predicting sound fields in rooms," *J. Acoust. Soc. Am.* **138**(3), 1457–1468.

Masullo, M., Toma, R. A., Yang, M., Brambilla, G., and Maffei, L. (2024). "Noise annoyance and application of the Harmonica index at short time intervals," in *Proceedings of INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, Nantes, France, pp. 6357–6363.

Modest, M. F. (2013). *Radiative Heat Transfer*, 3rd ed. (Academic Press, New York).

Nicodemus, F. E., Richmond, J., Hsia, J., Ginsberg, I., and Limperis, T. (1977). *Geometrical Considerations and Nomenclature for Reflectance* (U.S. Department of Commerce, National Bureau of Standards, Washington, DC).

Nosal, E.-M., Hodgson, M., and Ashdown, I. (2004a). "Improved algorithms and methods for room sound-field prediction by acoustical radiosity in arbitrary polyhedral rooms," *J. Acoust. Soc. Am.* **116**(2), 970–980.

Nosal, E.-M., Hodgson, M., and Ashdown, I. (2004b). "Investigation of the validity of radiosity for sound-field prediction in cubic rooms," *J. Acoust. Soc. Am.* **116**(6), 3505–3514.

Nusselt, W. (1928). "Graphische Bestimmung des Winkelvehältnisses bei der Wärmestrahlung" ("Graphical determination of the angular relationship in thermal radiation"), *VDI Z* **72**, 673.

Rayleigh, J. W. S. B. (1896). *The Theory of Sound*, 11 of New York Dover Publications, 2nd ed. (Macmillan, London, UK), Sec. 272a.

Savioja, L., and Svensson, U. P. (2015). "Overview of geometrical room acoustic modeling techniques," *J. Acoust. Soc. Am.* **138**(2), 708–730.

Scerbo, M., Savioja, L., and De Sena, E. (2024). "Room acoustic rendering networks with control of scattering and early reflections," *IEEE/ACM Trans. Audio. Speech Lang. Process.* **32**, 3745–3758.

Schröder, D. (2011). *Physically Based Real-Time Auralization of Interactive Virtual Environments* (Logos Verlag, Berlin, Germany).

Siltanen, S., Lokki, T., Kiminki, S., and Savioja, L. (2007). "The room acoustic rendering equation," *J. Acoust. Soc. Am.* **122**(3), 1624–1635.

Siltanen, S., Lokki, T., and Savioja, L. (2009). "Frequency domain acoustic radiance transfer for real-time auralization," *Acta Acust. united Ac.* **95**(1), 106–117.

Siltanen, S., Lokki, T., and Savioja, L. (2010). "Room acoustics modeling with acoustic radiance transfer," in *Proceedings of the International Symposium on Room Acoustics, ISRA 2010*, Melbourne, Australia, Paper P5h.

Svensson, U. P., and Savioja, L. (2024). "The Lambert diffuse reflection model revisited," *J. Acoust. Soc. Am.* **156**(6), 3788–3796.

Tokac, I., Heimes, A., Vorländer, M., and Brell-Cokcan, S. (2025). "A rule-based framework for capturing geometric characteristics in design: A study of façade analysis for acoustic behaviour in urban space," *Int. J. Archit. Comput.* **23**(2), 405–425.

Vorländer, M. (2020). *RWTHedition Auralization: Fundamentals of Acoustics, Modelling, Simulation, Algorithms and Acoustic Virtual Reality* (Springer International Publishing, Cham, Switzerland).

Yang, M., Heimes, A., Masullo, M., Maffei, L., Kim, Y.-H., Lee, P.-J., and Vorländer, M. (2026). "Impact of spatial factors of environmental sounds on psychological and physiological responses: A virtual reality study on direction and distance," *Build. Environ.* **287**, 113777.