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GEOMETRIC RADIATIVE INSTABILITY EXPLAINS THE FORMATION OF SNOW PENITENTES

W. Rohlfs[1,2] K. Venner[2] H.G. van der Bijl[2] A. Maßmeyer[1]

[1]Institute of Heat and Mass Transfer, RWTH Aachen University, Augustinerbach 6, 52056 Aachen, Germany
[2]Faculty of Engineering Technology, University of Twente, Drienerlolaan 5, 7522 NB, Enschede, the Netherlands
rohlfs@wsa.rwth-aachen.de

Fig. 1: Photograph of the $\approx 2\,\mathrm{m}$ high blade-like penitente structures in Chile.

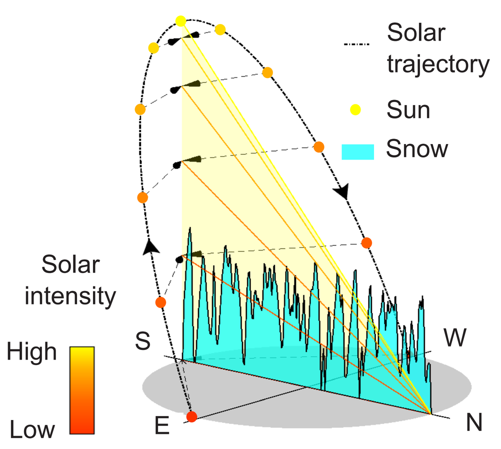


Fig. 2: Solar trajectory and associated intensity change on 2D snow surface profile.

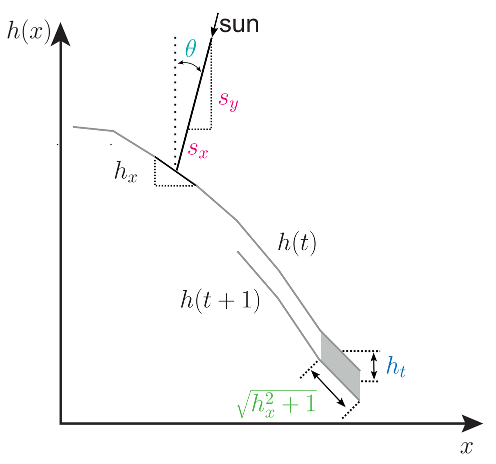


Fig. 3: Geometric relationship between the vertical subsidence rate (h_t), slope gradient (h_x) and the change in volume element on an inclined wall of glacial snow.

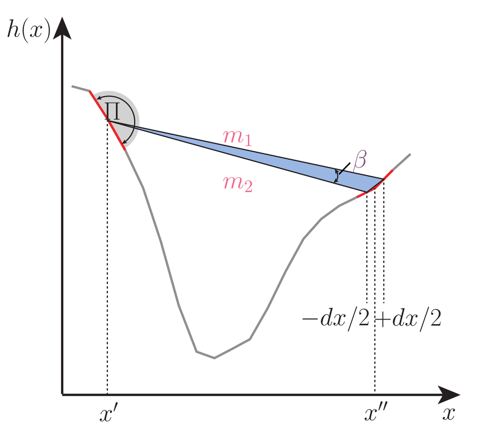


Fig. 4: Geometric setup for the view factor estimation between two surface elements (x' and x'') based on the local viewing angle β and boundary slopes (m_1, m_2).

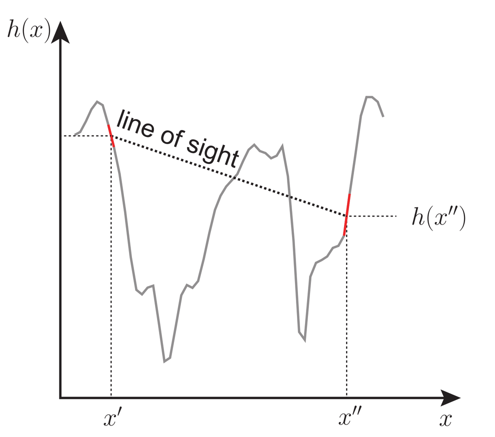


Fig. 5: Diagram of self-shadowing effects where an intermediate snow peak blocks the direct line of sight between surface elements x' and x''.

How much physics needed?

Penitentes are 1–6 meter snow blades observed in high-altitude regions like the Andes or Himalaya.

- **The question:** What is the absolute minimum physics required to describe this phenomenon?
- **The answer:** Surprisingly little.
  - No diffusion
  - No conduction
  - **Just light, shadow, and geometry**

The mechanism

- **Light trapping:** Deep **valleys** (troughs) capture and multiple-reflect sunlight, focusing energy
  → **Accelerated ablation / melting**
- **Self-shadowing:** Sharp **peaks** (crests) remain shaded at certain sun angles
  → **Slower ablation**
- **Result:** Initial surface roughness amplifies spontaneously into giant blades.

The mathematical model

Energy balance & height evolution:
The local height change depends on the ratio of absorbed energy to latent heat:

$$\frac{\partial h}{\partial t} = h_t = (\alpha_{\mathrm{eff}}(x) - 1)\frac{I_{\mathrm{sun}}}{L}$$

h_t: Height change α_{eff}: Effective albedo I_{sun}: Solar intensity
L: Latent heat of sublimation

Wall inclination correction (fig. 3):
Accounting for the geometric volume loss rate on an inclined surface with slope h_x:

$$h_t = -\frac{P(x)}{L\sqrt{h_x^2+1}}$$

$P(x)$: Absorbed power $\sqrt{h_x^2+1}$: Geometric surface expansion

Direct solar radiation:
Projection of the incoming direct solar radiation vector onto the local slope h_x:

$$P_{\mathrm{direct}}(x) = I_{\mathrm{sun}}(1-\alpha)\left|\frac{h_x s_x - s_y}{\sqrt{1+h_x^2}}\right|$$

s_x, s_y: Radiation vector components α: Local material albedo

View factor for mutual reflections (fig. 4):
Diffuse energy fraction between elements x' and x'' approximated via opening angle β:

$$\beta = \arctan\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$$

β: Opening angle ($F_{x' \to x''} \approx \beta/\pi$)
m_1, m_2: Slopes of connecting boundary lines

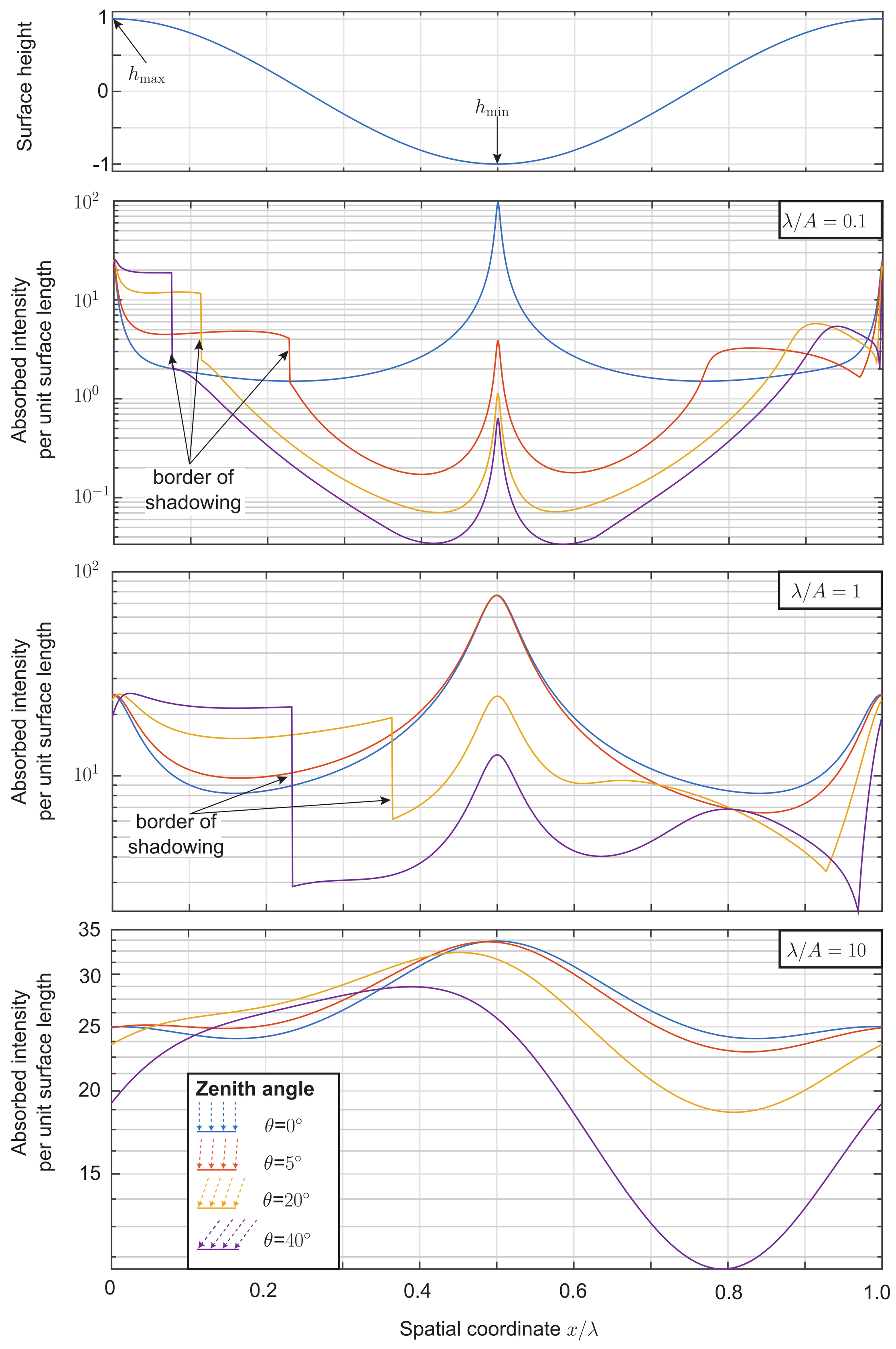
Shadowing & line-of-sight blocking (fig. 5):
If an intermediate snow peak $h(x)$ intersects the line of sight, the view factor F is forced to zero:

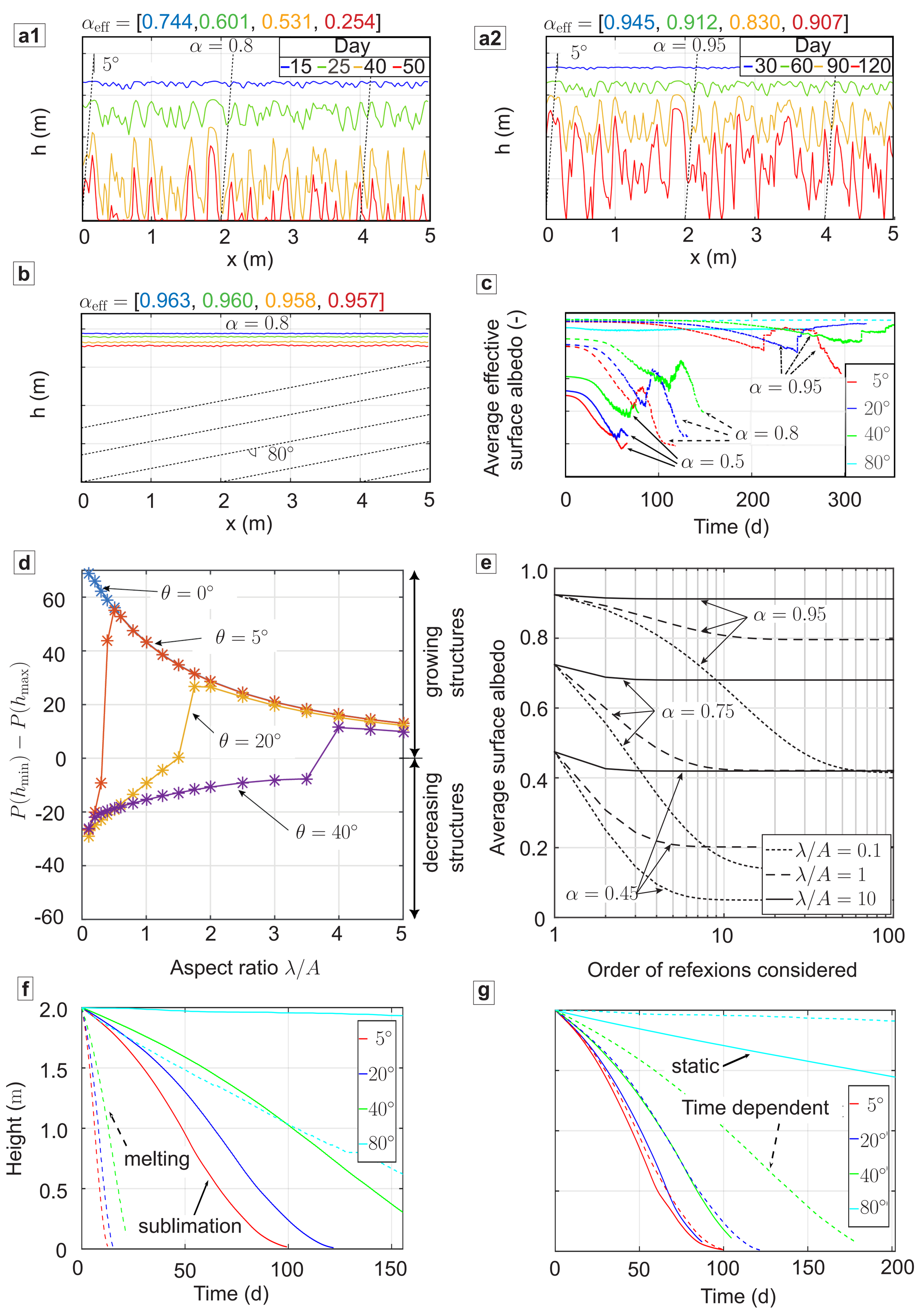
$$\frac{h(x') - h(x'')}{x' - x''}(x - x') + h(x') - h(x) > 0 \quad \text{for } x' < x < x''$$

Total energy balance via unified multiple reflections:
Tracks n reflection orders via a geometric series to avoid a costly matrix system ($i = 0$ seamlessly includes unreflected direct radiation):

$$P(x) = I_{\mathrm{sun}}(1-\alpha)\left|\frac{h_x s_x - s_y}{\sqrt{1+h_x^2}}\right| \sum_{i=0}^{n}\left(\alpha^i F^i_{x \to x'}\right)$$

Results





Discussion

Sinusoidal surface over coordinate x/λ.

High aspect ratio regime:

- **Massive energy spike** concentrated at trough center.
- **Abrupt discontinuities** from neighboring crest shadows.
- Multi-reflection feedback promotes rapid penitente growth.

Intermediate aspect ratio:

- **Transitional behavior** with distinct localized peaks.
- **Geometric tipping point:** Multireflections begin to dominate energy dissipation.

Low aspect ratio:

- **Linear projection dynamics** with smooth profiles.
- **Zero shadowing / trapping** due to shallow topography.

Random surface roughness & influence of initial albedo and sun-angle:

- **Spontaneous amplification:** Rapid growth of minor perturbations into vertical blades (a1, a2)
- **Suppressed instability:** Damping of perturbations at high zenith angles (b)
- **Albedo drop:** Lower effective albedo as deepening geometry traps light (a1, a2, c)

Influence of sun-angle and aspect ratio:

- **Universal stability map:** Strict boundary between growth and decay regimes (d).
- **High convergence:** Up to 100 iterations needed for deep, highly reflective geometries ($\alpha = 0.95$, e)

Melting vs. sublimation and dynamic solar angle:

- **Latent heat:** Melting and sublimation share identical feedback mechanics at different velocities
- **Dynamic path:** Daily solar path shifts shadows and accelerates net vertical growth

Conclusions: It's all about aspect ratio!

- No inherent length scale: penitente formation is determined by a **cut-off aspect ratio**, not a fixed wavelength.
- Too wide or flat structures get damped at a given sun angle.
- If the critical structure matches the aspect ratio, it destabilizes and amplifies into penitentes.

Institute of Heat and Mass Transfer
RWTH AACHEN UNIVERSITY
UNIVERSITY OF TWENTE.