An Interactive and Immersive Assignment for Teaching Matrix Composition in Secondary Education

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Abstract

Matrix-based transformations are a central topic in mathematics and a foundational concept in computer graphics, yet students often struggle to connect formal algebraic operations with spatial effects. This paper presents a structured 60–70 minute collaborative classroom assignment that introduces translation, rotation, and scaling transformations through a contextualized computer graphics scenario inspired by the rendering pipeline. Students work in small groups where one learner operates in a virtual reality environment while peers observe, guide decisions, and document transformation sequences. By constructing geometric primitives and applying transformation matrices, learners directly experience spatial effects and the non-commutativity of matrix multiplication. The assignment bridges mathematics and computer science education in secondary school and serves as an entry point to undergraduate computer graphics concepts. We describe learning objectives, classroom orchestration, assessment strategies, and practical implementation guidelines. All materials are provided as an open educational resource to support reuse and adaptation.

CCS Concepts

• ***Applied computing*** → ***Education;*** • ***Human-centered computing*** → *Virtual reality;* • ***Computing methodologies*** → *Computer graphics;*

1. Motivation

The rendering pipeline is a central concept in computer graphics education [BWF18]. It provides the structural foundation for understanding how three-dimensional scenes are transformed into two-dimensional images and introduces core ideas such as object representation through primitives, coordinate spaces, and transformation composition. As such, it plays a key role in undergraduate computer graphics curricula and forms an important conceptual backbone for related fields such as visualization, animation, and game development.

However, the abstraction level of the rendering pipeline makes it difficult to adapt directly to secondary education. Many of its components rely on formal mathematical representations, particularly matrix-based transformations. While translation, rotation, and scaling are often part of secondary mathematics curricula in algebraic form, students frequently struggle to connect symbolic matrix operations with their geometric effects in space. The non-commutativity of matrix multiplication, for example, is typically introduced as a formal property, but its spatial implications remain abstract.

Traditional two-dimensional representations—static diagrams on paper or slides—further increase this difficulty. Learners must mentally simulate spatial transformations and composition effects based on symbolic notation and flat visualizations [OHL17]. This separation between algebraic formalism and spatial intuition can impose substantial extraneous cognitive load, as students must translate between representations before reasoning about the underlying concepts.

To address this gap, we frame matrix-based transformations within an interactive and immersive environment that allows students to directly observe and manipulate spatial effects. Selected core concepts (geometric primitives and transformation composition) are used as a contextualized application, a mathematical manipulative [Ler20], to connect mathematics and computer science. By enabling learners to apply and compare transformation sequences in a spatial setting, abstract algebraic properties become observable geometric phenomena.

This approach seeks to provide a structured learning scenario in which spatial interaction supports conceptual understanding and builds a bridge between secondary mathematics and introductory computer graphics education.

2. Learning Context

The assignment is designed for upper secondary education in German Gymnasium (Sekundarstufe II), typically addressing students aged 16–18. At this level, learners are introduced to foundational concepts of analytic geometry and linear algebra, including vectors and basic matrix operations. The depth and explicit treatment of

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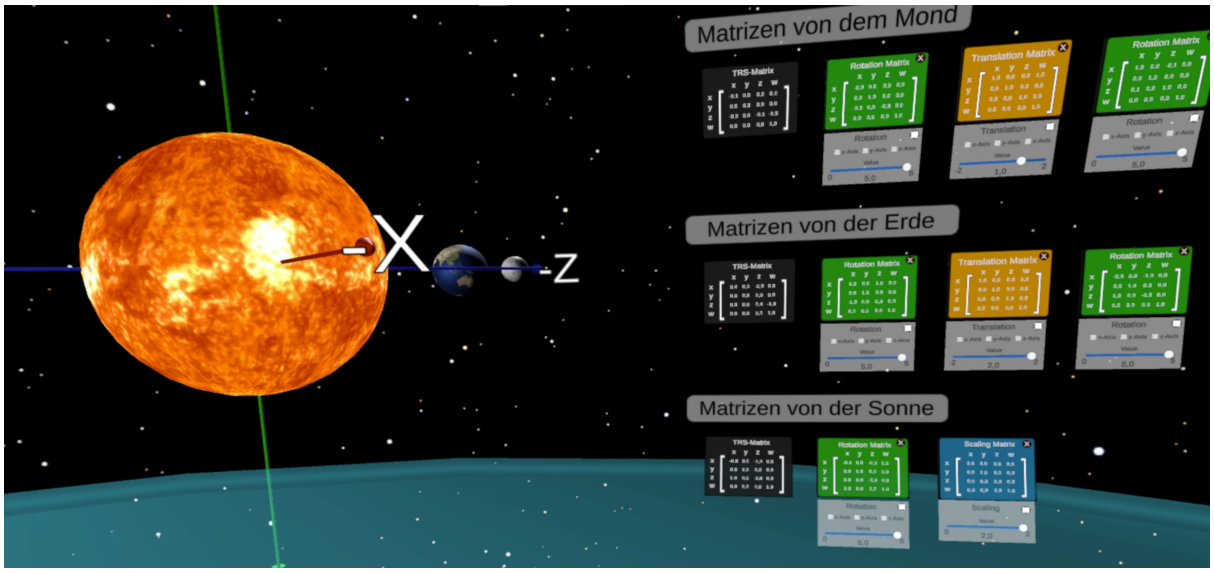


Figure 1: *Close-up from the solar system created in phase 2.*

matrix operations, however, vary across federal states, course levels (basic vs. advanced), and curricular reforms. In some contexts, matrix multiplication and inverse matrices are addressed explicitly, particularly in advanced mathematics courses, while in others they are introduced more implicitly or with reduced emphasis.

The assignment therefore assumes prior exposure to geometric transformations and basic matrix notation but does not require advanced proficiency in matrix algebra. Where necessary, matrix operations can be briefly revisited or introduced conceptually within the session, as the focus lies on understanding the spatial effects of transformation composition rather than on formal proof or algebraic derivation.

The activity is designed for a 60–70 minute classroom session, typically implemented within a double-period structure to allow additional time for technical setup, distribution of devices, and reflection. The core interactive phase fits within one standard lesson unit.

Students work in small groups of three to four learners. Each group adopts defined roles to ensure active participation: a VR operator who performs interactions in the virtual environment; a navigator or analyst who reads task instructions and guides transformation decisions; a recorder who documents transformation sequences and observations; and, optionally, a verifier who checks whether the defined goals are achieved. Roles may rotate between phases to distribute hands-on experience. The role-based structure follows principles of cooperative learning, which emphasize positive interdependence and individual accountability within group work [JJ02].

The hardware setup consists of standalone virtual reality headsets, allowing deployment without external computers or complex installation. Only one headset per group is required, as observation and collaborative reasoning are integral to the assignment design.

The teacher assumes the role of facilitator rather than lecturer, like done in MACE lessons, see [SLK20]. Responsibilities include introducing the mathematical framing, clarifying task goals, moderating group discussions, and leading the final plenary reflection. Reflection and the communication between the peers are promoted by the teacher.

Another important "teacher" is the guide character (robotic tour host "Kyle"), who introduces tasks, formulates predictive questions, and structures transitions between phases. Kyle is structuring the process and supports the cognitive orientation (which also supports the teacher).

3. Assignment Design

The following subsections outline the learning objectives, phase structure, and supporting design elements of the assignment.

3.1. Learning Objectives

The assignment is designed to address learning goals at the intersection of mathematics and computer science. The objectives emphasize conceptual understanding rather than procedural fluency.

Mathematics-oriented objectives

Students should be able to:

- Interpret translation, rotation, and scaling as matrix-based transformations.
- Describe the geometric effect of transformation composition.
- Explain why matrix multiplication is not commutative using spatial reasoning.
- Relate symbolic matrix operations to observable changes in spatial configuration.

Computer science–oriented objectives

Students should be able to:

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- Understand how objects are represented through geometric primitives (triangles).
- Apply ordered transformation sequences to manipulate objects in space.
- Distinguish between object space and world space.
- Recognize how transformation hierarchies enable relative motion (e.g., orbital movement).

The assignment prioritizes conceptual linkage between algebraic representation and spatial intuition. Formal proof or code-level implementation are no learning objectives for this assignment.

3.2. Assignment Phases

The session is structured into three interactive phases followed by plenary reflection. Each phase builds on the previous one, moving from controlled exploration to contextualized application.

3.2.1. Phase 1 – TRS Introduction and Interaction

The assignment begins with the introduction of translation, rotation, and scaling matrices. Guided by the robot Kyle, students explore how individual matrix parameters affect an object's spatial configuration.

Kyle introduces each transformation and poses predictive questions before execution, encouraging groups to anticipate outcomes. Transformation parameters can be manipulated interactively, allowing students to directly observe spatial effects.

Particular emphasis is placed on transformation order. Groups compare sequences such as:

$$T \rightarrow R \quad \text{versus} \quad R \rightarrow T$$

Through experimentation and discussion, students observe that matrix multiplication is not commutative in geometric contexts. The goal of this phase is to externalize the algebraic property as a visible spatial phenomenon.

3.2.2. Phase 2 – Solar System Scenario: Hierarchical Transformation

In the second phase, transformations are embedded within a predefined solar system scenario. Students are tasked with configuring orbital relationships between objects, such as creating circular motion around a central body.

This phase introduces hierarchical transformation: the motion of an object could depend on another's coordinate frame. Students must strategically combine rotation and translation to achieve relative motion.

Kyle provides contextual framing and hints but does not reveal solutions, supporting exploratory reasoning. Multiple solution paths are possible, enabling groups to compare strategies during the subsequent reflection phase.

3.2.3. Phase 3 – Geometry and Triangle Construction

In the final phase, attention shifts to geometric representation. Students construct simple shapes from triangles using a builder interface. By assembling objects from primitives, learners explore how complex forms emerge from basic geometric components.

This phase makes explicit that transformations operate on geometric primitives and their vertices rather than on abstract, indivisible objects. By constructing shapes from triangles, students observe how object representation and transformation are conceptually linked. The activity thus connects the previously explored matrix operations to the underlying geometric data structures used in computer graphics.

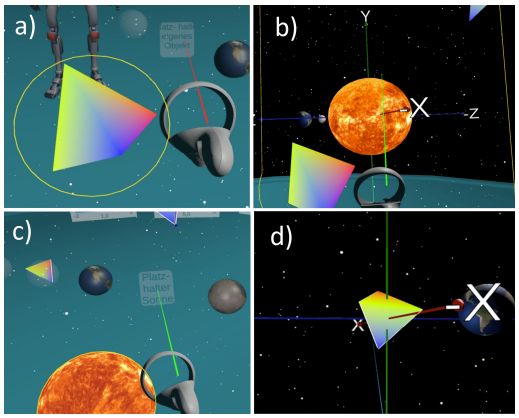


Figure 2: *The objects created in phase 3 can be added to the solar system from the previous phase.*

3.3. Scaffolding and Differentiation

Structural elements commonly found in interactive applications are used to guide progression, including staged task sequences, explicit goal definitions, and immediate visual feedback.

Kyle serves as a narrative and instructional anchor, introducing goals, structuring transitions, and providing directional hints.

As collaborative work naturally progresses at different speeds, the assignment design includes optional extension tasks. Groups that complete the main tasks early are encouraged to construct additional celestial bodies and add them to the solar system (see Fig. 2), experiment with more complex hierarchical transformations, or design alternative orbital configurations.

The open structure of the solar system environment allows increasing complexity without additional technical setup.

The design emphasizes structured activation and collaborative reasoning over reward-based motivation.

4. Classroom Orchestration and Formative Assessment

While the learning context defines the structural framework of the assignment, effective classroom implementation depends on active moderation and structured reflection. The following aspects describe how the session is facilitated in practice.

4.1. Phase Transitions and Moderation

Each phase begins with a short teacher-led framing that clarifies the conceptual focus of the upcoming activity. Rather than re-explaining mathematical procedures, the teacher formulates predictive questions to activate prior knowledge.

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During the TRS phase, for example, students are asked to anticipate spatial outcomes before transformations are executed. This predict–observe–explain pattern encourages deliberate reasoning instead of trial-and-error interaction.

Transitions between phases are explicitly marked to prevent unstructured exploration. The guide character (Kyle) introduces new tasks within the environment, while the teacher could verbally anchor the shift in conceptual focus.

4.2. Facilitating Collaborative Reasoning

Ther roles are important. Non-immersed students are asked to articulate expected results before transformations are applied and to justify decisions regarding transformation order.

The emphasis in the discussions should be led to making reasoning explicit rather than on reaching a predefined "correct" configuration.

4.3. Whole-Class Reflection

The final plenary phase consolidates insights gained during interaction. Groups present differing transformation sequences and compare outcomes. The teacher visualizes selected sequences (e.g., $T \rightarrow R$ versus $R \rightarrow T$) on the board to make non-commutativity explicit.

This reflective discussion connects spatial observation with formal mathematical terminology, supporting transfer between experiential interaction and symbolic reasoning.

4.4. Typical Challenges

Several recurring challenges are observable. Some groups initially rely on trial-and-error interaction rather than prediction. Others struggle with distinguishing between rotation around an object's local origin and rotation within a global coordinate frame.

Active moderation and structured questioning are therefore essential to prevent superficial interaction and to maintain conceptual focus.

4.5. Formative Assessment

Assessment is integrated formatively throughout the assignment. Conceptual understanding is evaluated through prediction, explanation, and comparison of transformation sequences.

During interaction, students are asked to anticipate outcomes before executing transformations. Differences between predicted and observed results serve as immediate indicators of understanding. In the final reflection, groups justify their transformation order and explain non-commutativity in geometric terms.

Optional documentation from the recorder role may be used to further assess reasoning processes.

5. Reflections and Lessons Learned

Implementation of the assignment revealed several practical insights. First, structured moderation is essential. Without explicit prompts for prediction and explanation, some groups tend to rely on trial-and-error interaction rather than deliberate reasoning about transformation order.

Second, the collaborative role model helps to reduce passive observation, particularly when the teacher actively encouraged verbalization from non-immersed students. However, role rotation may be necessary to ensure balanced participation.

Third, the solar system scenario supports intuitive understanding of hierarchical transformations but can also lead to extended exploration. Clear time management and explicit phase transitions are therefore important to maintain conceptual focus.

An additional challenge concerns terminology. Concepts such as transformation composition, or hierarchical transformation introduce subject-specific vocabulary that may exceed students' prior knowledge. Without careful scaffolding and explicit clarification, terminology can become a cognitive barrier rather than a support for abstraction.

Finally, linking spatial interaction to formal mathematical terminology during the plenary reflection is important. The immersive experience alone does not guarantee abstraction; transfer to symbolic reasoning requires structured discussion.

Overall, the assignment demonstrates that immersive interaction can support the connection between matrix-based algebra and spatial reasoning when embedded in guided collaboration and explicit reflection.

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