



Representations with the same degree

FRANK LÜBECK 

Abstract. In this short note, we show that every connected semisimple simply-connected algebraic group of rank > 1 over the complex numbers has infinitely many pairs of irreducible representations which are not related by an automorphism of the algebraic group and which have the same degree. This answers a question I was asked by J.P. Serre.

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1. Introduction and notation. Let G be a connected semisimple algebraic group defined over the complex numbers, and assume that G is of simply-connected type. We will prove the following theorem.

Theorem 1. *When the rank of G is > 1 , then G has infinitely many pairs of finite dimensional irreducible rational representations ρ_1 and ρ_2 such that there is no automorphism α of G with $\rho_2 = \rho_1 \circ \alpha$ and such that ρ_1 and ρ_2 have the same degree.*

We recall some facts and notation for which we refer to books on linear algebraic groups, for example [4, 6, 9].

Let $B \subseteq G$ be a Borel subgroup and $T \subseteq B$ be a maximal torus of G . To this pair one can associate a root datum $(X, \Phi, Y, \Phi^\vee)$. If l is the rank of G , then $X \cong \mathbb{Z}^l \cong Y$, and the root datum comes with a non-degenerate pairing $\langle \cdot, \cdot \rangle : X \times Y \rightarrow \mathbb{Z}$, Φ is a finite subset of X , called the roots of G , $\Phi^\vee$ is a finite subset of Y , called the coroots of G , and there is a bijection $\Phi \rightarrow \Phi^\vee$, $\alpha \mapsto \alpha^\vee$. The root datum determines G up to isomorphism.

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Each root system is the disjoint union of irreducible root systems, and the irreducible root systems are classified into four infinite series A_l , B_l , C_l , D_l and the exceptional cases G_2 , F_4 , E_l with $6 \leq l \leq 8$; the subscript is the rank l .

A root system contains a subset $\{\alpha_1, \dots, \alpha_l\}$ of l simple roots, the corresponding coroots $\{\alpha_1^\vee, \dots, \alpha_l^\vee\}$ are a set of simple coroots.

Our assumption that G is simply-connected means that the simple coroots are a $\mathbb{Z}$ -basis of the lattice Y . The elements $\{\omega_1, \dots, \omega_l\}$ of the dual basis of X (with respect to the pairing $\langle \cdot, \cdot \rangle$) are called the fundamental weights. The positive coroots $\Phi_+^\vee$ are the elements $\alpha^\vee \in \Phi^\vee$ of the form $\alpha^\vee = b_1 \alpha_1^\vee + \dots + b_l \alpha_l^\vee$ with all $b_i \in \mathbb{Z}_{\geq 0}$ (then $\Phi^\vee = \Phi_+^\vee \cup -\Phi_+^\vee$). So, for any weight $\lambda = a_1 \omega_1 + \dots + a_l \omega_l \in X$, we can compute

$$\langle \lambda, \alpha^\vee \rangle = a_1 b_1 + \dots + a_l b_l.$$

The group G is a direct product of simple simply-connected groups (which correspond to the irreducible subsets of Φ).

The irreducible rational representations of G are parameterized by the set of dominant weights $\{\lambda = a_1 \omega_1 + \dots + a_l \omega_l \mid a_i \in \mathbb{Z}_{\geq 0}\}$. We write $V(\lambda)$ for the module corresponding to the dominant weight λ . We write $\rho = \omega_1 + \dots + \omega_l$ for the sum of the fundamental weights. The Weyl dimension formula describes the degrees of the irreducible representations:

$$\dim(V(\lambda)) = \frac{\prod_{\alpha^\vee \in \Phi_+^\vee} \langle \lambda + \rho, \alpha^\vee \rangle}{\prod_{\alpha^\vee \in \Phi_+^\vee} \langle \rho, \alpha^\vee \rangle}.$$

So, the only information we need for computing the degrees of the irreducible representations of G is the list of positive coroots, expressed as linear combinations of the simple coroots.

Let σ be an automorphism of the algebraic group G . Since the pair $T \subseteq B$ is unique up to conjugation in G , there is an inner automorphism τ of G such that $\sigma \circ \tau$ fixes T and B . This automorphism then induces actions on X and Y which permute the sets of simple coroots and of fundamental weights. It follows that $V(\lambda)$ and $V(\mu)$ cannot be related by an automorphism when the dominant weights λ and μ written as linear combinations of the fundamental weights have different sets of coordinates. (See [5, Ch.11] for a more detailed description of the automorphisms of G .)

Added in proof: Pieter Belmans found out and told me that Andrew Huchala also obtained Theorem 1 in his unpublished B.S. thesis [3] (with somewhat more elaborate computations).

2. Proof of Theorem 1. Let G be a group of rank l as in the previous section.

If for a dominant weight λ we have $\dim(V(\lambda)) = d$, then it is immediate from the Weyl dimension formula that $\dim(V(k(\lambda + \rho) - \rho)) = d \cdot k^N$ for any $k \in \mathbb{Z}_{>0}$, where N is the number of positive (co)roots of G .

Therefore, to show Theorem 1 it is sufficient to find for each G of rank $l > 1$ one pair of dominant weights λ, μ which have different sets of coordinates as linear combinations of fundamental weights such that $\dim(V(\lambda)) = \dim(V(\mu))$.

To prove Theorem 1, we will describe an explicit pair λ, μ in each case.

2.1. The case of rank $l = 1$. In this case, $G \cong SL_2(\mathbb{C})$. Its coroot system of type A_1 has only one simple coroot and this is the only positive coroot. The Weyl dimension formula yields $\dim(V(a\omega_1)) = (a + 1)$ for all $a \in \mathbb{Z}_{\geq 0}$. So, in this case all irreducible representations are determined by their degree. The condition $l > 1$ in Theorem 1 cannot be avoided.

From now on, we assume $l > 1$.

2.2. The case that all simple direct factors have rank 1. Then G is a direct product of two or more copies of $SL_2(\mathbb{C})$. Let $\omega_1, \dots, \omega_l$ be the fundamental weights (one for each simple factor).

The discussion in the previous subsection shows $V(5\omega_1)$ and $V(\omega_1 + 2\omega_2)$ both have dimension 6. This proves Theorem 1 in this case.

If G has a simple direct factor of rank $l > 1$, it is sufficient to show Theorem 1 for this factor (extend by the zero weight corresponding to the trivial representation on the other factors).

From now on, we assume that G is a simple group of rank $l > 1$.

2.3. The case of exceptional types, A_2 and B_2 . In these cases, we just used the Weyl dimension formula to compute explicitly the degrees of $V(\lambda)$ for a range of weights λ with small coordinates until we found examples that prove Theorem 1 in these cases.

Detailed descriptions of irreducible root systems, including a description of all roots as linear combinations of simple roots, can be found in the tables [1, Planches I-IX]. In the results below, we will use the ordering of the simple coroots and corresponding fundamental weights $\omega_1, \dots, \omega_l$ as given in these tables.

For the computations, we used our computer algebra package CHEVIE [2] which contains functions that compute these data.

Here are the smallest dimensional examples for the considered types (we have checked that there are no smaller dimensional examples by enumerating all weights ρ with $\dim(V(\rho)) < d$).

Proposition 2. *The following table gives for G with irreducible root system of exceptional type and of types A_2 and B_2 two dominant weights λ and μ such that $d = \dim(V(\lambda)) = \dim(V(\mu))$.*

type	λ	μ	d
A_2	$\omega_1 + 2\omega_2$	$4\omega_2$	15
B_2	$\omega_1 + 2\omega_2$	$4\omega_2$	35
G_2	$3\omega_1$	$2\omega_2$	77
F_4	$\omega_1 + \omega_4$	$2\omega_1$	1053
E_6	$2\omega_1$	ω_3	351
E_7	$\omega_4 + \omega_5$	$2\omega_6 + 3\omega_7$	1903725824
E_8	$\omega_1 + \omega_3$	$\omega_1 + \omega_7 + \omega_8$	8634368000

In the case E_6 , there is a non-trivial graph automorphism of G which shows that $V(2\omega_6)$ and $V(\omega_5)$ also have degree 351.

3. Cases A_l, B_l, C_l, D_l for large rank l . As in the previous section, we have computed many degrees of representations in these cases up to rank 12 and higher to look for patterns of dominant weights that lead to the same degrees. Interestingly, we could detect in all cases such patterns among weights of the form $a\omega_1 + b\omega_2$.

In the cases A_l, B_l, D_l , we even found examples with $a \in \{0, 1\}$ for all l , and the proof for general l is straight forward.

The more interesting case are groups of type C_l where the smallest degrees which occur twice and the coefficients of the corresponding weights can be small for some l and very large for other l . Here we reduce the existence of examples to the solvability of a Diophantine equation, and show that solutions exist for all l .

Theorem 3. (a) Let G be of type A_l , $l \geq 3$. Then $V((l-1)\omega_2)$ and $V(\omega_1 + (l-2)\omega_2)$ have the same degree

$$\frac{(2l-1) \prod_{k=l+1}^{2l-2} k^2}{(l-1)!^2}.$$

(This is also correct for $l = 2$ but then the two weights are related by a graph isomorphism.)

(b) Let G be of type B_l , $l \geq 3$. Then $V((2l-2)\omega_2)$ and $V(\omega_1 + (2l-3)\omega_2)$ have the same degree

$$\frac{3 \cdot (4l-5)(6l-5)(6l-7) \prod_{k=2l}^{4l-6} k^2}{(2l-3)!^2}.$$

(c) Let G be of type D_l , $l \geq 4$. Then $V((2l-3)\omega_2)$ and $V(\omega_1 + (2l-4)\omega_2)$ have the same degree

$$\frac{3 \cdot (3l-4)(3l-5)(4l-7) \prod_{k=2l-1}^{4l-8} k^2}{(l-2)^2(2l-5)!^2}.$$

(d) Let G be of type C_l , $l \geq 3$. Then for $a \geq 3$, $b \in \mathbb{Z}_{\geq 0}$ the representations $V(a\omega_1 + b\omega_2)$ and $V((a-2)\omega_1 + (b+1)\omega_2)$ have the same degree

$$\frac{(a+1)(a+2b+2l-1) \prod_{k=a+b+2}^{a+b+2l-2} k \prod_{k=b+1}^{b+2l-3} k}{(2l-1)!(2l-3)!}$$

if and only if there exists a positive integer c such that

$$c^2 - (4l-5)a^2 = (2l-3)^2$$

and in that case $b = \frac{1}{2}(c+1-a-2l)$. For every $l \geq 3$, there exist infinitely many pairs (a, b) with this property.

Proof. (a) We use the description in [1, Planche I(II)] which describes all positive coroots as linear combinations of the simple ones:

$$\beta_{i,j} = \sum_{k=i}^j \alpha_k^\vee \text{ for } 1 \leq i \leq j \leq l.$$

Let λ be a dominant weight of the form $\lambda = a\omega_1 + b\omega_2$. Evaluating the numerator in the Weyl dimension formula, the only factors which depend on a and b are:

$$\begin{aligned}\langle \lambda + \rho, \beta_{1,1} \rangle &= a + 1, \\ \langle \lambda + \rho, \beta_{1,j} \rangle &= (a + 1) + (b + 1) + (j - 2) = a + b + j \text{ for } 2 \leq j \leq l, \\ \langle \lambda + \rho, \beta_{2,j} \rangle &= (b + 1) + (j - 2) = b - 1 + j \text{ for } 2 \leq j \leq l.\end{aligned}$$

Substituting in these terms $a = b = 0$, we get the corresponding factors in the denominator of the Weyl dimension formula. All other factors are the same in the numerator and denominator and cancel out.

Now we specialize to $(a, b) = (0, l - 1)$ and $(a, b) = (1, l - 2)$. The factors involving $\beta_{1,j}$ for $j \geq 2$ are the same. And all but the first or last factor involving $\beta_{2,j}$ for $j \geq 2$ are the same. We get

$$\frac{\dim(V((l - 1)\omega_2))}{\dim(V(\omega_1 + (l - 2)\omega_2))} = \frac{1 \cdot (2l - 2)}{2 \cdot (l - 1)} = 1.$$

And for the degree of $V((l - 1)\omega_2)$, we get

$$\dim(V((l - 1)\omega_2)) = \frac{\prod_{k=l+1}^{2l-1} k \prod_{k=l}^{2l-2} k}{\prod_{k=2}^l k \prod_{k=1}^{l-1} k} = \frac{(2l - 1) \prod_{k=l+1}^{2l-2} k^2}{(l - 1)!^2}.$$

- (b) Now let G be of type B_l with $l \geq 3$. Note that the bijection $\Phi \rightarrow \Phi^\vee$, $\alpha \mapsto \alpha^\vee$, maps long roots to short coroots and vice versa. The coroot system is of type C_l , so we need to use [1, Planche III] in this case. We have the following positive coroots:

$$\begin{aligned}\beta_{ij} &= \sum_{k=i}^{j-1} \alpha_k^\vee + 2 \sum_{k=j}^{l-1} \alpha_k^\vee + \alpha_l^\vee \text{ for } 1 \leq i \leq j \leq l, \\ \gamma_{ij} &= \sum_{k=i}^j \alpha_k^\vee \text{ for } 1 \leq i \leq j \leq l - 1.\end{aligned}$$

Let $\lambda = a\omega_1 + b\omega_2$. Only the following factors in the Weyl dimension formula for $V(\lambda)$ depend on a and b :

$$\begin{aligned}\langle \lambda + \rho, \beta_{1,1} \rangle &= 2a + 2b + 2l - 1, \\ \langle \lambda + \rho, \beta_{1,2} \rangle &= a + 2b + 2l - 2, \\ \langle \lambda + \rho, \beta_{1,j} \rangle &= a + b + 2l - j \text{ for } 3 \leq j \leq l, \\ \langle \lambda + \rho, \beta_{2,2} \rangle &= 2b + 2l - 3, \\ \langle \lambda + \rho, \beta_{2,j} \rangle &= b + 2l - 1 - j \text{ for } 3 \leq j \leq l, \\ \langle \lambda + \rho, \gamma_{1,1} \rangle &= a + 1, \\ \langle \lambda + \rho, \gamma_{1,j} \rangle &= a + b + j \text{ for } 2 \leq j \leq l - 1, \\ \langle \lambda + \rho, \gamma_{2,j} \rangle &= b - 1 + j \text{ for } 2 \leq j \leq l - 1.\end{aligned}$$

Specializing to $(a, b) = (0, 2l - 2)$ and $(a, b) = (1, 2l - 3)$, we see again that most of the listed factors are the same in both cases. We get

the following quotient of the corresponding degrees:

$$\frac{\dim(V((2l-2)\omega_2))}{\dim(V(\omega_1 + (2l-3)\omega_2))} = \frac{(6l-6)(6l-7)(4l-6)(3l-4)}{2(6l-7)(6l-9)(3l-4)(2l-2)} = 1.$$

And for the degree, we find after some simplifications

$$\dim(V((2l-2)\omega_2)) = \frac{3 \cdot (4l-5)(6l-5)(6l-7) \prod_{k=2}^{4l-6} k^2}{(2l-3)!^2}.$$

- (c) Now let G be of type D_l . Here we need [1, Planche IV], but note that in this case the positive coroots are not correctly described as linear combinations of simple ones in the reference. We correct this here. The positive coroots are:

$$\begin{aligned}\beta_{ij} &= \sum_{k=i}^j \alpha_k^\vee \text{ for } 1 \leq i \leq j \leq l-1, \\ \eta_i &= \sum_{k=i}^{l-2} \alpha_k^\vee + \alpha_l^\vee \text{ for } 1 \leq i \leq l-1, \\ \gamma_{ij} &= \sum_{k=i}^{j-1} \alpha_k^\vee + 2 \sum_{k=j}^{l-2} \alpha_k^\vee + \alpha_{l-1}^\vee + \alpha_l^\vee.\end{aligned}$$

The rest of the argument is completely analogous to (a) and (b). We skip the details.

- (d) Finally, we consider the case when G is of type C_l with $l \geq 3$. From [1, Planche II] (type B_l), we get the following positive coroots:

$$\begin{aligned}\beta_{ij} &= \sum_{k=i}^j \alpha_k^\vee \text{ for } 1 \leq i \leq j \leq l-1, \\ \eta_i &= \sum_{k=i}^l \alpha_k^\vee \text{ for } 1 \leq i \leq l, \\ \gamma_{ij} &= \sum_{k=i}^{j-1} \alpha_k^\vee + 2 \sum_{k=j}^l \alpha_k^\vee \text{ for } 1 \leq i < j \leq l.\end{aligned}$$

For $\lambda = a\omega_1 + b\omega_2$, the following factors in the Weyl dimension formula depend on a or b :

$$\begin{aligned}\langle \lambda + \rho, \beta_{1,1} \rangle &= a + 1, \\ \langle \lambda + \rho, \beta_{1,j} \rangle &= a + b + j \text{ for } 2 \leq j \leq l-1, \\ \langle \lambda + \rho, \beta_{2,j} \rangle &= b - 1 + j \text{ for } 2 \leq j \leq l-1, \\ \langle \lambda + \rho, \eta_1 \rangle &= a + b + l, \\ \langle \lambda + \rho, \eta_2 \rangle &= b + l - 1, \\ \langle \lambda + \rho, \gamma_{1,2} \rangle &= a + 2b + 2l - 1, \\ \langle \lambda + \rho, \gamma_{1,j} \rangle &= a + b + 2l + 1 - j \text{ for } 3 \leq j \leq l, \\ \langle \lambda + \rho, \gamma_{2,j} \rangle &= b + 2l - j \text{ for } 3 \leq j \leq l.\end{aligned}$$

Taking the product of these factors and substituting $a = b = 0$ in this product yields numerator and denominator of $\dim(V(\lambda))$ as stated in the theorem.

Now assume $a \geq 3$. Substituting in the factors above a by $a - 2$ and b by $b + 1$, and cancelling all common factors, we get

$$\frac{\dim(V(a\omega_1 + b\omega_2))}{\dim(V((a-2)\omega_1 + (b+1)\omega_2))} = \frac{(a+1)(b+1)(a+b+2l-2)}{(a-1)(a+b+1)(b+2l-2)}.$$

Equating numerator and denominator and rearranging terms, we see that this quotient is 1 if and only if

$$b^2 + (a+2l-1)b + \frac{1}{2}((-2l+3)a^2 + (2l-1)a + (4l-4)) = 0.$$

Solving for b yields

$$b = \frac{1}{2} \left(-2l - a + 1 \pm \sqrt{(4l-5)a^2 + (2l-3)^2} \right).$$

So, when b is an integer, then $(4l-5)a^2 + (2l-3)^2$ is the square c^2 of a positive integer c . Conversely, if $(4l-5)a^2 + (2l-3)^2 = c^2$ is the square of a positive integer c , then

$$b = \frac{1}{2}(-2l - a + 1 + c)$$

is also a positive integer: When a is odd, then c is even, and when a is even, then c is odd; and b is positive since $l \geq 3$ and $a \geq 3$.

We have shown that $\dim(V(a\omega_1 + b\omega_2)) = \dim(V((a-2)\omega_1 + (b+1)\omega_2))$ if and only if the Diophantine equation

$$c^2 - (4l-5)a^2 = (2l-3)^2 \quad (*)$$

has a solution (c, a) with $a \geq 3$. And in that case b can be computed as above.

The equation $(*)$ is a *generalized Pell equation*. Obviously, $(c, a) = (2l-3, 0)$ is a solution of $(*)$, but it does not fulfill the condition $a \geq 3$.

Now we use solutions $(x, y) \in \mathbb{Z}_{>0}^2$ of the *Pell equation*

$$x^2 - dy^2 = 1 \text{ with } d = (4l-5). \quad (**)$$

First, note that $(4l-5) \equiv 3 \pmod{4}$ and so d is not a square in $\mathbb{Z}$. In that case, equation $(**)$ has an infinite number of solutions, see [7, Thm. 7.25, 7.26], and they can be found as follows: $\sqrt{d}$ has a continued fraction expansion of some period r . If r is even, let $m = r - 1$ and if r is odd, let $m = 2r - 1$, and compute the m -th continued fraction approximation $\frac{x_1}{y_1}$ of $\sqrt{d}$. Then $(x_1 + y_1\sqrt{d})(x_1 - y_1\sqrt{d}) = x_1^2 - dy_1^2 = 1$ and (x_1, y_1) is called the fundamental solution of $(**)$; all solutions (x_k, y_k) can be found by computing $(x_1 + y_1\sqrt{d})^k = x_k + y_k\sqrt{d}$ for $k \in \mathbb{Z}_{>0}$.

Finally, note that the $(c_k, a_k) = ((2l-3)x_k, (2l-3)y_k)$ are infinitely many solutions of $(*)$, and almost all $a_k \geq 3$. (In general, these are not all solutions of $(*)$.) $\square$

Remarks. (1) For the case C_l in Theorem 3, we remark that the period length of the continued fraction of $4l - 5$ can be large. For example, for $l = 159$, it is 48 and the smallest (a, b) such that $V(a\omega_1 + b\omega_2)$ and $V((a - 2)\omega_1 + (b + 1)\omega_2)$ have the same degree, which we construct in our proof, is

$$(a, b) = (613975804336172576474505, 7404460209629201092363289),$$

and the corresponding degree has 15728 decimal digits. A brute force search for solutions of equation (*) in the previous proof for $l = 159$ yields other solutions, $(a, b) = (87, 902)$ is the smallest; but this is difficult to predict and for other l a brute force search did not yield any solution.

(2) Integer sequences: The sequence of degrees in Theorem 3(a) for type A_l , $l \geq 2$, occurs in numerous mathematical contexts, see the entry A000891 of the Online Encyclopedia of Integer Sequences (OEIS) [8].

The sequence of degrees that occur for multiple dominant weights for type G_2 can be found in [8, A339248].

The corresponding sequence for type E_8 can be found in [8, A181746].

(3) Are there always degrees that occur more than two times?

P. Deligne showed me a proof (using rational points of elliptic curves) that in type A_2 there exists for any positive integer m a degree that occurs for at least m different dominant weights.

Computing many examples, we found that in type A_l one easily finds degrees that occur with high multiplicity. In the other types, this seems to occur less often. We only found some examples where degrees appeared more than two times (say, 3 to 7 times). In the cases G_2 and E_8 , we did not find any degree occurring more than two times (more precisely, we computed that all degrees $< 10^{22}$ for type G_2 and all degrees $< 10^{60}$ for type E_8 occur at most for two dominant weights).

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FRANK LÜBECK

RWTH Aachen University: Rheinisch-Westfälische Technische Hochschule Aachen
Aachen

Germany

e-mail: frank.luebeck@math.rwth-aachen.de

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