Evolution of networks – An experimental analysis

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It is the main aim of our paper to study network formation in experimental setups in discrete and continuous time. Our design is inspired by the theoretical model on network formation by Bala and Goyal (2000) as well as the experiments by Callander and Plott (2005) and Falk and Kosfeld (2003). In particular, we analyze the role of star-shaped networks which are strict Nash-equilibria of the corresponding network formation game. Our experimental results show that strict Nash networks prove to be a good indicator for predicting network formation, particularly in continuous time. In explaining our results, it turns out that, among others, the complexity in coordinating on stars, the inequity aversion against unequal payoff distribution in the network, and the groups' degrees of activity are the most important determinants for the formation of strict Nash networks.

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1. Introduction

Research on network formation has attracted a lot of attention during the past decade. Substantial empirical evidence has been accumulated emphasizing the important role of social and economic networks in real world economies. Famous examples of network effects can be found in job search (e.g. Rees, 1966; Holzer, 1987; Montgomery, 1991), granting of credit (e.g. McMillan and Woodruff, 1999) and in models of welfare participation (e.g. Bertrand et al., 2000).

The theoretical approaches by Jackson and Wolinsky (1996) and by Bala and Goyal (2000) on network formation turned out to be the most influential. Jackson and Wolinsky introduced the concept of pairwise stability (known from the matching literature) as an equilibrium concept in models of network formation. This approach is sometime called the “cooperative approach.” Pairwise stability requires mutual consent of a pair of agents for link formation, whereas each link can be severed unilaterally.

In the present paper, we base our analysis on the approach by Bala and Goyal in which decision makers in a population select partners as sources of benefits. Players obtain non-rival benefits or information values from being connected with other players, but have to bear connection costs if they hold links to others. Two different types of benefit flow models are distinguished in the literature. In the so called *one-way flow model*, benefits exclusively flow to the player who opened a link. In the *two-way flow model*, benefits flow into both directions, although only one of the two players who participate in the link pays for the connection. As the most prominent example of the two-way flow model, we mention a beneficial telephone call which is paid by the person who opens the connection, while we have a two-way information exchange. Other examples are invitations for dinner where information is exchanged or a meeting where one of the two participants travels to the other and has to bear the costs of travelling. It is one of the most important economic implications of our assumption that agents are able to “subsidize” links.

Both approaches to network formation have their own merits. We prefer the non-cooperative strategic approach, since it is more suitable for running experiments. Furthermore, in this approach, the tension between equilibrium and efficiency is rather low.¹

Our experimental design is based on the two-way flow model. The decisions of agents in a population to sever or open connections with other agents are modeled as strategic decisions in a non-cooperative normal form game called a *network game*. Nash equilibrium in the network game seems to be a weak concept, since the number of weak Nash equilibrium networks is very large even in populations of moderate size. Moreover, we do not observe formation of weak Nash networks in our discrete time experiment at all. In our continuous time experiment, weak Nash networks are reached, but are abandoned after a very short time. In other words, the *strict* Nash concept turns out to be much more important than the weak Nash concept.

Depending on the level of connection costs, Bala and Goyal show that the *empty*

¹For recent overviews of both theoretical approaches see Jackson (2005) and Goyal (2005).

network and the so called *center-sponsored stars* are the only strict Nash networks. The center-sponsored star network can be represented as a directed graph where one player (the center player) holds links to all other players, while no other player holds a link to any other player (see Fig. 1a)). That is, in a center-sponsored star, the center player has to bear all connection costs. Bala and Goyal construct a strategy adaptation process based on best reply dynamics, which guarantees that a population of players will end up with probability one in a center-sponsored star.

In contrast to theoretical work, until recently hardly any experimental work on network formation had been published (see Kosfeld (2004) for an overview). Recent economic experiments that consider network formation are Deck and Johnson (2004), Callander and Plott (2005), and Falk and Kosfeld (2003). The experiment by Deck and Johnson is inspired by the network formation model of Johnson and Gilles (2000), which is different from the theoretical framework we want to utilize in this paper. Callander and Plott (2005) carry out the first network formation experiment. They consider network formation in the one-way flow model of Bala and Goyal (2000) in which the circle networks are the only strict Nash networks. In their experiment, subjects often reach a strict Nash network and stay in it for at least three periods (in 58% of the cases).² Furthermore, it turns out that *real-time link adjustment* facilitates coordination on the strict Nash equilibrium. In their experimental design with costly real-time adjustment of links, 71% of the groups converge to a strict Nash network. We will report on supporting evidence from our own experiments.

The main intention of Falk and Kosfeld (2003) is to test the theoretical model by Bala and Goyal (2000). They consider groups of four players who play the network formation game for five periods. After that, groups are rematched twice. In both repetitions, the network formation game is played again for five periods. As one of the most remarkable results of their experiment strict, Nash networks are reached by many groups (about 50%) only in the one-way flow framework, while they are never reached in the two-way flow framework (the center-sponsored stars). The authors offer some very interesting explanations for their experimental findings. To reach a center-sponsored star in the two-way flow model seems to be a *complex task* for an individual decision maker. One agent in the population has to build up as many links as possible, while the remaining players have to build no links even in a model in which building a single link would generate positive net profits. This is a highly asymmetric decision situation which can be dealt with by solving a complex coordination problem. Another explanation is based on fairness considerations. In the center-sponsored star network, the center player actually has significantly less equilibrium payoff than the remaining players since he subsidizes all connections in the network. Thus, *inequity aversion* might reduce the attractiveness of this kind of network. Therefore, *inequity aversion* may be an issue in network formation.

In our experiments, we use a theoretical model which slightly modifies the network game by Bala and Goyal. In our design, the structure of the network may limit access

²Callander and Plott (2005) distinguish between convergence defined as staying in a network for at least three consecutive periods and reaching a Nash network, i.e. playing it at least in one period. Eleven of twelve groups in their experiment reached Nash networks and nine of them reached strict Nash networks.

to the information values of other population members for an agent. A player can only benefit from his direct neighbors and their direct neighbors, provided he pays for the connection with his direct neighbor. Empirical investigations show (e.g. Granovetter, 1995; Friedkin, 1983) that information will not flow between all neighbors on a path of arbitrary length in the network, as is assumed in the framework of Bala and Goyal. Note that this is also one of the main differences between our experimental design and the design of Falk and Kosfeld who suppose (in accordance with the theoretical model by Bala and Goyal) that a player can benefit from any other player who is reached via a finite number of links.³ In the model of Bala and Goyal with decay for a wide parameter range, center-sponsored, periphery-sponsored⁴, and mixed stars are strict Nash equilibria. In some parameter region, periphery-sponsored stars are the only star networks that are strict Nash equilibria. Hojman and Szeidl (2004) analyze a model of network formation with restricted path lengths for benefit flows in a network and find that periphery-sponsored stars are Nash equilibria. However, their assumption of decreasing returns is not fulfilled in our model.

In our particular framework, the *periphery-sponsored star* turns out to be the only strict Nash network for the two-way flow model. Both star shaped networks, the periphery-sponsored and the center-sponsored star, are illustrated graphically for an example with six players in Fig. 1 below.⁵

Fig 1. a) Center-sponsored star b) Periphery-sponsored star

Our first experiment is conducted in *discrete time*, like the one of Falk and Kosfeld (2003). That is, subjects play the network formation game for a sequence of periods. Our second experiment is carried out in *continuous time (real-time)*.⁶ As far as we know, our experiment is the first network formation experiment conducted completely in continuous time, in which action changes are possible at every point in time and all calculations are adjusted to the continuous time setting. This distinguishes our design

³The so-called truncated connections model by Jackson and Wolinsky (1996) is also a network model with restricted path length for benefit flows.

⁴In a periphery-sponsored star, all periphery players subsidize the connections of the center player who benefits from the information values of all other players without having to pay for it.

⁵For examples of star networks in empirical studies, see the overview in Hojman and Szeidl (2004).

⁶Note that, in computerized experiments, “continuity” cannot be modeled in a strict mathematical sense. More precisely, we suppose that the computer updates incoming information within very short time intervals that are shorter than the players’ reaction times. Individual players perceive these experiments to be conducted in continuous time.

from the design of Callander and Plott (2005), who allow costly action adjustment in real-time, but calculate linking costs and benefits based on the resulting network every two minutes.

In this experimental design, subjects are allowed to select an action, i.e. to open or sever a link to another population member, at any point in time within a given time frame which is known to the participants in advance. The probability that more than one player switches in the same time interval approaches zero when the length of the interval decreases to zero. In this type of experiment, we observe that players change their decisions often. In a discrete time design, in which all players switch actions simultaneously in the same period, one would certainly need a lot of repetitions in order to obtain the same amount of experimental evidence as in continuous time games.

We suppose that it is easier for the players in such a framework to overcome coordination problems than in a framework with simultaneous strategy adaptation in discrete time. Other experiments in continuous time we know of were run, for example, by Selten and Berg (1970), Berninghaus and Ehrhart (2003), Ehrhart (1997), Berninghaus et al. (1999) and by Gueth et al. (2002).⁷

The main aim of our paper is to see how network formation evolves under quite different modes of strategy adaptation (discrete vs. continuous time). Although it is not possible to compare both modes in a strict sense, we observe many similarities in the experimental results. In both modes, strict Nash networks seem to play a significant role as a benchmark in the evolutionary process of network formation. Moreover, our experimental results show that coordination on a periphery-sponsored star is easier in continuous time than in discrete time. While about half of all groups in the discrete time mode approach or even reach the periphery-sponsored star, in the continuous time mode almost all groups do not only reach a periphery-sponsored star but even change the center player several times. Since changing center players shows a tendency to equalize individual payoffs in the long run, our experimental results support the view that fairness and inequity aversion is an issue even in network formation.

Our paper is organized as follows. In the next section we present the particular network game on which our experimental design (in Section 3) is based. Section 4 contains our experimental results for discrete and continuous time experiments. Section 5 concludes.

2. The network game

In this section, we consider the one-shot network game the equilibria of which are considered as benchmark solutions of our experiments. The one-shot game is characterized by the set of players $I = \{1, \dots, n\}$, the strategy sets G_i , and the payoff functions Π_i . An individual strategy g_i of player i is a vector of ones and zeros $g_i = (g_{i1}, \dots, g_{in}) \in G_i := \{0, 1\}^n$ with the following interpretation. If $g_{ij} = 1$, player i establishes a link with player j ; otherwise, we have $g_{ij} = 0$. By convention a player cannot link with himself, that is $g_{ii} := 0$ for all i . A player has to pay for each link that

⁷Continuous time experiments were also conducted in bargaining (for details, see Roth's survey on bargaining experiments (Roth, 1995, Chapter D., p. 323–327)).

he establishes. Note, that a connection between two players in our model is supposed to be already established if at least *one* player wants to open it, i.e. if $g_{ij} + g_{ji} \geq 1$ holds (unilateral link formation). If a link is established, both connected players benefit from the information values even if only one player has to pay for the link. Thus our model belongs to the class of two-way flow models. Each strategy configuration $g = (g_1, \dots, g_n)$ generates a directed graph denoted by $\mathcal{G}_g$, where the vertices represent players and a directed edge from i to j , i.e. $g_{ij} = 1$, indicates that player i holds a link with j . According to the essential assumption of the model by Bala and Goyal, we suppose that player j need not agree when player i wants to open a link with him. This distinguishes our model from another type of network formation models in which forming a link between two players requires bilateral agreement.⁸

In a network generated by the strategy configuration g , each player i may have a number of agents with whom he is connected. We call these agents *neighbors* of i . It is essential for our model to distinguish three types of neighbors. *Actively reached neighbors* are all players i holds links with:

$$N_i^a(g_i) := \{j \in I \mid g_{ij} = 1\}$$

By a slight abuse of language we call players in $N_i^a(g_i)$ *active neighbors* of i . Links of i with players in $N_i^a(g_i)$ are called *active links*. We call *passive neighbors* of i all players who hold a link with i :

$$N_i^p(\mathcal{G}_g) := \{j \in I \mid g_{ji} = 1\}$$

An active link of j to i is called a *passive link* of i . We call *indirect neighbors* of i all active or passive neighbors of all active neighbors of i , i.e.:

$$N_i^{ind}(\mathcal{G}_g) := \{k \in I \mid \exists j \neq i \neq k : g_{ij} = 1 \text{ and } \max\{g_{jk}, g_{kj}\} = 1\}$$

Thus, the set of *all neighbors* of player i is given by:

$$N_i(\mathcal{G}_g) := N_i^a(g_i) \cup N_i^p(\mathcal{G}_g) \cup N_i^{ind}(\mathcal{G}_g)$$

Let $n_i(\mathcal{G}_g)$ denote the number of elements in $N_i(\mathcal{G}_g)$ and $n_i^a(g_i)$ the number of elements in $N_i^a(g_i)$. Note that both active and passive neighbors are direct neighbors (in contrast to indirect neighbors). That is, they are connected in the associated directed graph by one directed edge. Since direct neighbors in our model are treated differently depending on how the links with them are financed, it makes sense to subdivide the class of direct neighbors into active and passive ones.

Costs for opening a link are supposed to be the same for each player and each link and are denoted by c (> 0). The benefit player i can extract from being connected (either actively, passively or indirectly) with player j is the same for all players and supposed to be equal to a (> 0). Given strategy configuration $g = (g_1, \dots, g_n)$, player i 's payoff is given by:

$$\Pi_i(g) := a n_i(\mathcal{G}_g) - c n_i^a(g_i) \tag{1}$$

⁸This type of models was introduced by Jackson and Wolinsky (1996).

We see from this definition that player i may benefit from a connection to j although he does not have to pay for it⁹, which is an implication of our two-way flow model. As another consequence, we find that player i 's payoff is equal to zero if i is an isolated player in the network, that is, if $N_i(\mathcal{G}_g)$ is an empty set.¹⁰

Our framework contrasts with the neighborhood concept applied by Falk and Kosfeld (2003). They assume (according to the theoretical model by Bala and Goyal, 2000) that the set of neighbors of player i consists of all players connected with i by a path in the non-directed graph associated with $\mathcal{G}_g$. We limit access of i , first, to information values of those players j to whom i has an active link (active neighbors) *plus* the information values of all direct neighbors of j . For these active links, i has to pay connection costs. Second, i is connected with those players k who have an active link to him (passive neighbors) but not with the neighbors of k . That is, if i pays for a link with a direct neighbor j , he also has access to the information values of j 's direct neighbors. However, if i does not pay for a link with a direct neighbor k , he does not have access to the information values of k 's neighbors. A player can only benefit once from every other player in the network, i.e. if he is, for example, actively and indirectly connected with the same player he will not get information values twice.

We claim that our neighborhood concept is certainly more realistic for real world networks than the assumption that players have access to the information values of players that are connected with them via a path of arbitrary length in the network. In labor markets, for example, social networks play a significant role in getting a job. The empirical investigation of Granovetter (1995) concentrates on jobs found via social contacts. It reveals that the main part of the people, who got their job via social contacts, heard about it from their friends, relatives or acquaintances (39.1% of the cases), or from acquaintances of those (45.3 %). That is, from a network point of view, the person who is looking for a job has access to the information of his "neighbors" who are at most two "steps" away. Furthermore, 48% of those who heard about their job from their friends were looking for a job, 52% heard of their current job through friends but were not looking for a new job; 72% of those employees who received the information about their job from friends of friends were looking for jobs. Moreover, Friedkin (1983) analyzes communication networks within organizations and finds that observability in these networks tends to be restricted to persons who are either in direct contact or who have at least one contact in common. This supports our interpretation of indirect connections that are only accessible via active links (searching for information) and the restriction of the length of paths on which information values flow.¹¹

For network games $\Gamma = \{I; G_1, \dots, G_n; \Pi_1(\cdot), \dots, \Pi_n(\cdot)\}$ a *Nash network* is defined as a vector of individual link proposals $g^* = (g_1^*, \dots, g_n^*)$ such that no single player i has

⁹For example, if $g_{ij} = 0$, but $g_{ji} = 1$ holds.

¹⁰This assumption contrasts with the framework of Bala and Goyal in which a player consumes his own information value if he is isolated in the network.

¹¹Leaving aside the argument in favor of the higher value of active links and only considering restricted path length (so that players also receive a payoff from all neighbors of all passive neighbors) results in at least 192 strict Nash networks (all kinds of stars). Thus, we assess this model as less adequate to test the convergence to strict Nash equilibria.

an incentive to open additional links different from g_i^* or to sever links prescribed by g_i^* .

Definition 1 A strategy configuration g^* in Γ is a Nash equilibrium if

$$\forall i: \quad \Pi_i(g_{-i}^*, g_i^*) \geq \Pi_i(g_{-i}^*, g_i) \text{ for all } g_i \in G_i, \quad (2)$$

where $g_{-i}^* = (g_1^*, \dots, g_{i-1}^*, g_{i+1}^*, \dots, g_n^*)$. $\mathcal{G}_{g^*}$ is the Nash network generated by g^* .

In a strict Nash network, inequality (2) holds strictly.

To get an overview of all Nash networks in our experimental design, we calculated explicitly all Nash networks for the parameter values of our experiment. The results of our analysis are summarized in Appendix A. Unfortunately, we are not able to give a complete formal characterization of all Nash networks for all possible parameter values in our particular framework.

Because of the large number, strict Nash networks are a reasonable refinement. For the particular parameter constellation of our experiment, strict Nash networks can be *uniquely* characterized as *periphery-sponsored stars* (see Appendix A). In the following, we give sufficient conditions for a periphery-sponsored star being strict Nash in our framework.

Proposition 1 If inequalities $c < (n - 1)a$ and $n > 3$ hold, a periphery-sponsored star is a strict Nash network.

Proof It is sufficient to show that unilateral deviation from a periphery-sponsored network results in payoff reductions. A periphery player in a periphery-sponsored star obtains payoff equal to $a(n - 1) - c$. Opening new links does not add additional information value but increases connection costs and, therefore, reduces his payoff. Moreover, by dropping his only link to the center player, a periphery player reduces his payoff to zero. Combining both deviations, that is, dropping the link to the center player and opening m new links to other periphery players, results in a payoff equal to $(a - c)m + a$ which is for $m \leq (n - 2)$ and $n > 3$ strictly smaller than the status quo payoff. The payoff of the center player in a periphery-sponsored star is equal to $a(n - 1)$. By opening new links, he cannot increase his information values obtained from the other players but has to pay connection costs. Therefore, the payoff of the center player is reduced.

q.e.d.

Remarks:

1. It easily follows from the arguments in the proof of Proposition 1 that for $a < c$ the empty network is also a strict Nash network.
2. In the next section we will see that the parameter values used in our experimental design satisfy the inequalities of Proposition 1. Therefore, periphery-sponsored stars are strict Nash networks in our experimental design (see Appendix A).

3. In the experiments, we consider repeated one-shot network games with $a > c$ in discrete and continuous time. Playing strict Nash networks (periphery-sponsored stars) in all periods (all the time) constitutes a subgame perfect equilibrium of the repeated network games. Note, however, since in our one-shot game there exist two payoff-different Nash equilibria for each player (e.g. center position or periphery position in a periphery-sponsored star), other subgame-perfect equilibria exist (for both the repeated game in discrete time with 15 periods and for our game in continuous time). To give an intuition for the repeated game in discrete time: for the finite number of periods becoming very large, the set of average payoff-vectors that can be reached by subgame-perfect equilibria converges to the set of individual rational and feasible payoffs in which each player receives at least an average payoff of $(n - 1)(a - c)$ ECU and a maximum of $(n - 1)a$ ECU, and the sum of average payoffs is less than or equal to $(n - 1)(na - c)$ ECU (Benoit and Krishna, 1985).¹² The same set of average payoffs can be reached via subgame-perfect equilibria in our repeated game in continuous time (Simon and Stinchcombe, 1989; Ehrhart, 1997, Theorem 5.2).

Bala and Goyal (2000) show that the strict Nash network in their model (the center-sponsored star) will be reached with probability one via a stochastic strategy adaptation process. In this discrete time process, each player is selected with a given probability p to change his action in the network game. In case a player is selected for adapting action, he will choose the *best reply* to the network prevailing in the previous period (myopic best reply). We do not know whether the same strategy adaptation process would also converge to a periphery-sponsored star in our framework. However, we do not consider this a reasonable procedure since overwhelming experimental evidence (by Falk and Kosfeld (2003), Callander and Plott (2005), and own experiments) shows that subjects rarely employ the myopic best reply rule.

3. Experimental designs

3.1. Experiment in discrete time

This experiment was partly conducted at the University of Karlsruhe and partly at the Sonderforschungsbereich 504 (University of Mannheim). Subjects were selected from a pool of students of different faculties.

The experiment was organized in four sessions. In the first and the second sessions respectively, 18 subjects participated, while only twelve subjects participated in sessions 3 and 4. Sessions 1 and 2 were conducted at the University of Karlsruhe, while sessions

¹²The minimax-payoff of a player (i.e. the smallest payoff to which the other players can hold a player) is $(n - 1)(a - c)$ ECU. Here, the other players build no links and the best answer of the player is to build a link to each of the other five players. The individual maximal average payoff is equal to $(n - 1)a$ ECU and is reached by the center player of a periphery-sponsored star. The payoff of $n(n - 1)a - (n - 1)c = (n - 1)(na - c)$ ECU is the maximal sum of payoffs of all players in the network, which is only reached in a periphery-sponsored star.

3 and 4 were conducted at the University of Mannheim. We use the software z-Tree Fischbacher (1999) for the experiment in discrete time. In each session, subjects are divided into groups of six players each who play the network game for 15 periods.¹³ At the beginning of each period, each player has to choose (at most five) active links to other players of his group (first stage of a period). Activated links of a player are marked by mouse clicks as red arrows in a graph appearing on the screen. Once each player in the group has made his decision, the resulting network is shown on the screen (second stage of a period). In this network, each player can easily identify the players with whom he is connected because they are marked in another color than the players with whom he is not connected. In addition, they are informed about their information values, their costs and their payoff in the current period, and about their accumulated payoff. In the next period a new drawing appears on the screen in which each player can again activate links.¹⁴ The graph of the network prevailing in the previous period is also shown on the screen so that players can base their decisions on a one-period history. All groups start with the same *complete network* in a (fictive) period zero. We make the pragmatic assumption that the network in period zero is the complete network since we predict that the groups will reach a periphery-sponsored star, which is the network where all participants are connected with each other with the least number of links. As we try to prevent any hint on that particular network, we do not start with the other symmetric network, the empty network, which is in a sense closer to the periphery-sponsored star (which has five active links) than the complete network (30 active links).

Information values per connected player are set equal to $a = 3$ ECU (Experimental Currency Unit). Costs per link are $c = 2$ ECU. This guarantees that the periphery-sponsored star is the only strict Nash network (see Appendix A). The payoffs for each subject are accumulated over 15 periods and paid out in cash shortly after the experiment is finished.¹⁵ Since all groups in period one make their decisions based on the complete network in the (fictive) previous period, the first period payoff (of period zero) of each player is fixed at 5 ECU. Although the *number* of periods for each group is fixed in advance, *total time* spent in the laboratory is not fixed, since the next period starts only when all subjects in the session have made their decisions. The maximum and minimum payoff was equal to €16.42 resp. €10.08. The average payoff was €13.71.

3.2. Experiment in continuous time

This computerized experiment was conducted at the University of Karlsruhe. Subjects were selected from a pool of students of various faculties. The experiment was organized in four sessions with two groups of six subjects participating in each. Software for the experiment was developed in our laboratory to conduct experiments in continuous time. It has been used several times in other experiments.

¹³In contrast to Falk and Kosfeld (2003) we do not rematch the players during the sessions.

¹⁴Note that a player's links fixed in the previous period are dropped when a new period starts to assure that players really decide in each period on their action and do not just accept their previous decision.

¹⁵The conversion rate is €1 per 12 ECU.

Each game lasts exactly 30 minutes. The game starts when all subjects have made their first decision, i.e. each subject has to decide on his initial active links. Thereafter, subjects can change their actions, i.e., open or sever links at any time. Information is updated by the computer five times per second. The current payoff is computed every fifth of a second and is “integrated” over time. The information presented on the subjects’ computer screens throughout the game includes the player’s current flow of information values, his current cost flow, his current payoff flow, and his current accumulated payoff. As in the discrete time experiment, a subject’s own and the other subjects’ active links are illustrated by arrows. Moreover, the subjects to whom a player is connected have a different color on the computer screen than the remaining ones. The elapsed time is indicated on each screen during the entire course of the experiment. The information values per connected player are set equal to $a = 3$ ECU per minute and the costs per link are $c = 2$ ECU per minute.¹⁶ The net payoffs for each subject are accumulated over 30 minutes and paid out in cash shortly after the experiment is finished.¹⁷ For example, suppose a player extracts a net payoff of 4 ECU per minute from the network for 10 seconds and then switches his action such that he obtains 2 ECU per minute for 50 seconds his total net payoff for this minute is equal to $(4 \text{ ECU}/60 \text{ sec}) * 10 \text{ sec} + (2 \text{ ECU}/60 \text{ sec}) * 50 \text{ sec} = 2.33 \text{ ECU}$.

In contrast to our discrete time experiment, the total time for each group is fixed in advance, but the number of action changes during the course of the experiment is not fixed. The maximum payoff (minimum payoff) earned in the experiment was equal to €17.89 (€12.20). The average payoff was €14.83.

4. Experimental results

4.1. Discrete time experiment

4.1.1. Describing the results

Summarizing the experimental results of our discrete time study, 3 groups (out of 10) reach a periphery-sponsored star (groups 5, 6, 10) and two other groups (4, 8) come very close to a periphery-sponsored star. In one of these groups (group 4), it seems that the center player apparently does not understand that he does not have to pay for the links. He keeps two active links in the final period, although this reduces his payoff by 4 ECU. In the other group, for six periods one periphery player does not understand that he can reach every other player by paying only one link. He is actively linked with three or four players in each period. The center player is actively linked with this player and so the others earn the same amount as in the periphery-sponsored star. Keeping this slight restriction in mind, we can say that about 50% of all groups reach the strict Nash

¹⁶In another paper (Berninghaus et al., 2006) we study the effects of increasing link costs on network formation (in continuous time). When c is large enough, the empty network is also a strict Nash network.

¹⁷The conversion rate is €1 per 24 ECU.

network or at least come very close to it. In the following, let us call these groups the “successful” ones.

We formalize a *distance measure* of a given network from a periphery-sponsored star. Let us regard a network built up by a group of $n \geq 3$ players as a directed graph and denote by $\text{Indeg}(i)$ the in-degree of the vertex represented by player i , i.e. $\text{Indeg}(i)$ is equal to the number of player i 's passive links. We focus on networks in which there is a unique player i^* to whom the majority of the remaining players have established active links. Hence, $\text{Indeg}(i^*) > \text{Indeg}(j)$ for all $j \neq i^*$ and the maximum in-degree $\text{Indeg}(i^*)$ is larger than $(n-1)/2$. These groups seem to be “on the way” to a periphery-sponsored star, with player i^* being the designated central player. Based on these considerations, we define a distance measure $d(g)$ given a strategy configuration g as follows:

$$d(g) := \begin{cases} |\text{Indeg}(i^*) - \max_{j \neq i^*} \{\text{Indeg}(j)\} - (n-1)| & : i^* \text{ exists and } \text{Indeg}(i^*) > \frac{n-1}{2} \\ n-1 & : \text{otherwise} \end{cases}$$

By this distance concept, the maximum in-degree (player i^*) and the second highest in-degree are compared. In our experimental design, we get $d(g) = n-1 = 5$ if the maximum in-degree is not unique and/or the maximum in-degree is smaller than 3. Intuitively, when the difference between the maximum and the second highest in-degree is small, we are still quite far away from a periphery-sponsored star. By subtracting 5 from this difference, the distance measure $d(g)$ is rather large. The following proposition shows that a periphery sponsored star is uniquely characterized by $d(g) = 0$.

Proposition 2 *The network g induces a periphery-sponsored star iff $d(g) = 0$.*

Proof a) Suppose g generates a periphery-sponsored star with center player i^* . Then we have $\text{Indeg}(i^*) = n-1$ and $\max_{j \neq i^*} \text{Indeg}(j) = 0$ which implies $d(g) = 0$.

b) Suppose g does not induce a periphery-sponsored star. Then the resulting network is characterized by $d(g) = n-1$ or by $[\text{Indeg}(i^*) - \max_{j \neq i^*} \text{Indeg}(j)] < n-1$ what implies $d(g) > 0$.

q.e.d.

The evolution of the distance measure over 15 periods for all groups participating in our discrete time experiment is shown in Table 1.

This gives a nice overview of our experimental results in discrete time. Groups 5, 6 and 10 reach a periphery-sponsored star. All three groups show “learning” in the sense that the distance measure d has a strong tendency to decrease to zero. Except for group 5, all groups that reach a strict Nash equilibrium leave it after some periods but return to it before the experiment stops. It is not easy to explain why players in a group leave a strict Nash network because they suffer from strict payoff losses. Groups are said to be “close” to a periphery-sponsored star when $d(g) = 1$. We see from Table 1 that groups 4 and 8 approach the periphery-sponsored star after some periods. Another interesting phenomenon is that a weak Nash network is never reached by any group in any period. This is a surprising result which is not observed in the experiments of Falk and Kosfeld (2003) and Callander and Plott (2005).

Table 1. Evolution of distance measure $d(g_t)$ for $t = 1, \dots, 15$

Group	Period														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	4	4	5	5	5	4	5	4	5	4	5	3	5	4	5
2	5	4	5	5	3	4	4	4	5	3	3	4	3	5	3
3	4	4	5	4	5	3	5	4	4	4	4	4	5	5	4
4	3	2	5	5	5	5	4	5	5	4	4	4	3	1	1
5	5	4	4	2	2	2	1	1	0	0	0	0	0	0	0
6	4	5	4	2	1	3	1	1	0	0	0	2	1	0	0
7	5	4	4	5	5	4	3	5	4	5	4	5	5	5	4
8	4	5	2	3	4	4	2	3	2	1	1	2	1	2	1
9	4	4	5	3	5	5	5	3	4	4	4	5	4	3	2
10	4	5	5	3	3	2	1	1	1	0	0	1	0	0	0

The temporal evolution of the *number of groups* that come close to a strict Nash network or reach a strict equilibrium in a given period is graphically illustrated in Fig. 2, where an almost monotonic increase of the number of groups approaching a periphery-

Fig 2. Number of groups which come close to or reach a strict Nash network

sponsored star can be observed. Participants in the groups seem to “learn” how to reach a strict Nash network, at least on an aggregate level. A closer look at the data in Table 1 shows that a tendency of learning how to reach a periphery-sponsored star can only be observed from the behavior of the successful groups. The remaining groups show a completely different behavior, which is different from learning how to coordinate on a periphery-sponsored star.

We briefly summarize the characteristic features of our experimental results in discrete time as follows.

Result 1 *a) In our discrete time experiment, 50% of all groups reach networks g with*

distance $d(g) \leq 1$ from a strict Nash network. A strict Nash network is reached for the first time in period 9 (groups 5 and 6).

b) Two groups (groups 6 and 10), that reach a strict Nash network, leave it for no more than two consecutive periods. One group (group 5) does not leave the strict Nash network until the end of the game.

c) Weak Nash networks are never observed.

4.1.2. Interpreting the results of the discrete time experiment

In the one-way flow model of Bala and Goyal (2000), the only strict Nash equilibrium is the circle network, where each player maintains exactly one link to another individual such that all individuals together form a circle. The experiments by Falk and Kosfeld (2003) and Callander and Plott (2005) show that the Bala-Goyal model predicts the outcome in the one-way flow model very well. In the Callander and Plott experiment, 75% of the groups reach strict Nash networks. The frequency of strict Nash networks in the one-way flow treatment of Falk and Kosfeld varies between 41% and 59% (depending on the level of connection costs c). However, in their two-way flow treatment, the Bala-Goyal model fails to predict behavior in the experiment. Subjects participating in the two-way flow experiment of Falk and Kosfeld never form strict Nash networks (center-sponsored stars). In our design, the explanatory power of strict Nash networks (periphery-sponsored stars) is significantly improved. This may be explained as follows.

Coordinative complexity and inequity aversion in network experiments We partly base our arguments on Falk and Kosfeld, who explain the difference in their experimental results between the one-way flow and the two-way flow model by the coordinative complexity of the game and inequity aversion. Complexity in coordination on stars in network experiments arises from the fact that the strict Nash network in the two-way flow model is *not symmetric* with respect to strategies. In the Falk-Kosfeld design, exactly one player has to establish links to all other players, while these players have to remain passive, i.e. they form no link with any other player.¹⁸ Assuming that it is easier to understand that forming links may increase payoff than to understand that being passive might be “optimal”, in this setting at least $n - 1$ players would have to understand the equilibrium in the network game.

In our setting, at least one player would have to understand the equilibrium. This player would be a candidate for the center player who has to remain passive (in the periphery-sponsored star). Callander and Plott explain the realization of stable networks in experiments by successful coordination of beliefs: “... at certain critical points in network dynamics, the coordination, bargaining, and free rider aspects of individual decision making become aligned, and stationarity is achieved. At these points, it appears that all decision makers become aware of which network is best for them and are aware

¹⁸Note that the center player subsidizes all connections in the population.

that other agents are aware of this, and so on ad infinitum.” Coordination of beliefs seems to be an easier task in our framework.

Falk and Kosfeld argue that complexity is not the only determinant of the formation of networks in experiments. They argue that inequity aversion may be another obstacle to the formation of center-sponsored stars. Note that in their treatment with two-way flow, there is actually a significant payoff difference between a center player and a periphery player which may prevent players from approaching center-sponsored stars. Indeed, their treatment with the one-way flow model and the experiment of Callander and Plott show that the strict Nash network, that is, the circle network which is characterized by the uniform payoff distribution, is reached fairly often.

In our experimental design, the payoff difference between center player and periphery players is much smaller than in the two-way flow model of Falk and Kosfeld. A center player in a periphery-sponsored star earns 15 ECU, while a periphery player earns 13 ECU.¹⁹ Our experimental results seem to support the arguments of Falk and Kosfeld on the relevance of the inequity aversion motive in network formation. Our results on a continuous time network formation experiment will strengthen this view.

Activity and payoff Intuitively, one would expect that subjects who understand the strategic problem of network formation try to be connected with as many others as possible while having a preferably low number of active links. On the other hand, in a group of six with simultaneous decision making, frequent link changes of all players make it hard to deal with a complex coordination problem such as the problem of network formation.

In order to describe this second kind of behavior more precisely, let us define the *degree of activity* of a group as the number of changing links per player. An individual link change is counted when a player changes a link from one period to the next (e.g. if he opens a link and deletes another link, two changes are counted). Fig. 3 shows the maximum, minimum, and average number of link changes (over 15 periods) of a player in each group. The diagram displays a high variation in total link changes over all participants. Our experimental results show that groups that approach the strict Nash network or even reach it are not very active, i.e. in these groups players do not change their links very often. In particular, the successful groups 4, 5, 6, 8, and 10 are characterized by a low degree of activity (equal to 8.5, 4.0, 6.3, 13.0, and 4.7), whereas the degree of activity of all unsuccessful groups 1, 2, 3, 7, and 9 is higher than 13 (see Fig. 3 and Table 2). We therefore conclude that the number of link changes is a good way of predicting success in building periphery-sponsored stars. An intuitive explanation for this interrelation is the following: many simultaneous link changes in one period probably result in a very different network in the next period such that players are in a completely new situation. Thus, they cannot see any trend or movement to a salient network and, even if they all aimed at such a network, it would be hard to coordinate actions. Therefore, too many simultaneous changes probably lead to mis-coordination.

¹⁹In the Falk-Kosfeld design, the payoff difference between the center player and the periphery players is equal to $3k$, where k can be equal to 5, or 15, depending on the particular treatment.

Fig 3. Degrees of activity of groups

Concluding this section, we analyze the relationship between groups' degrees of activity and their *average payoffs*. Are more active groups more successful in collecting payoffs?

Since periphery-sponsored stars are also efficient networks in the sense that the sum of players' payoffs is maximized, one would intuitively expect from our arguments in the previous paragraphs that more successful groups, i.e. less active groups, would also accumulate a larger amount of payoff. This conjecture actually is supported by our data in Table 2. Spearman's rank correlation coefficient r_s is equal to -0.9515 which coheres with a significant negative relation between the groups' degrees of activity and average payoffs.²⁰

Table 2. Group performance in the experiment in discrete time

Group	1	2	3	4*	5*	6*	7	8*	9	10*	Mean
Degree of activity	17	21	21	9	4	6	23	13	24	5	14
Avg. payoff (ECU/€)	160	153	147	170	184	181	153	170	142	186	165
Degree of efficiency	0.80	0.77	0.74	0.85	0.92	0.91	0.77	0.85	0.71	0.93	0.83

* successful group

Negative correlation between groups' activity levels and profits (payoffs) has been observed recently in experiments with a completely different design. We know of two experiments (Berninghaus et al., 1999; Selten et al., 2006) having similar results: more active subjects obtain on the average less payoff. The degree of efficiency, which gives the fraction of actual average payoff to maximal possible average payoff, is equal to 83% on average.

²⁰Test statistic: $t_8 = r_s \sqrt{\frac{N-2}{1-r_s^2}} = -8.75, p < 0.001$.

4.2. Continuous time experiment

4.2.1. Describing the results

In the previous section, it was shown that, in the discrete time environment, half of all groups reach a strict Nash network (periphery-sponsored star) or at least come close. These results can be improved significantly in the continuous time framework, where seven groups (out of eight) reach the periphery-sponsored star. Moreover, six groups leave the strict Nash network in order to build up further periphery-sponsored stars with a different center player. An overview of our results is given in Table 3.

Table 3. Group performance in the experiment in continuous time

Group	1*	2*	3*	4*	5*	6*	7*	8	Mean
No. of action changes	385	263	100	557	283	278	606	346	352
Degree of activity	64	44	17	93	47	46	101	58	59
No. of stars (different center players)	2	3	1	2	4	3	6	0	3
Time spent in stars:									
Minutes	6	9	20	3	11	7	5	0	8
%	21	31	65	10	36	24	17	0	25
Time until first star:									
Minutes	8	13	3	21	10	18	11	–	12
%	26	42	11	70	33	61	36	–	40
No. of action changes until first star reached	88	63	24	421	82	188	176	–	149
Avg. payoff (ECU)	349	364	381	339	359	360	339	355	356
Avg. payoff (€)	14.54	15.15	15.88	14.14	14.95	15.00	14.13	14.81	14.82
Degree of efficiency	0.87	0.91	0.95	0.85	0.90	0.90	0.85	0.89	0.89

* group that reaches at least one periphery-sponsored star

Typically, in continuous time experiments, subjects change links often. The maximum number of 606 changes is reached by the six members of group 7, while in group 3 we observe only 100 changes during the 30 minutes. The degree of activity is the average number of action changes of a group member. The average payoff does not vary much across the groups, although group behavior is heterogeneous with respect to other aspects. For example, group 7 reaches stars with many different center players, while group 3 coordinates on one star only.²¹ Note that group 7 reaches all six possible strict Nash networks, some of them several times. Variations among groups can also be found with respect to the amount of time it takes to first reach a periphery-sponsored star. It takes group 4 about 70% of the total time, whereas it takes group 3 only 11%. As in the experiment in discrete time, the degree of efficiency gives the fraction of actual

²¹Group 3 plays a periphery-sponsored star in nine separate time segments during the course of the experiment, but always a star with the same center player.

average payoff to maximal possible average payoff. Here, the average degree of efficiency is equal to 89%. We summarize the most interesting features of our results as follows.

- Result 2** *a) In our continuous time experiment, seven out of eight groups reach at least one strict Nash network.*
- b) Except for one group (group 3), all groups that reach a periphery sponsored star leave this configuration after a while and coordinate afterwards on at least one other strict Nash network. The average time the groups spend in strict Nash networks is about 25% of the total time.*
- c) The average time it takes groups to reach a strict Nash network for the first time is about 40% of the total time. This time can be interpreted as the “coordination time” needed to coordinate on a particular star.*
- d) Weak Nash networks are reached by two groups only (groups 1 and 4) and abandoned after a very short time (2 and 18 seconds).*

4.2.2. Discussing the results

Time spent in stars and number of stars reached The graphics in Fig. 4 show how much time the groups spend in a star with a certain center player. Note that most groups leave a star after a while and return to it again or switch to a star with another center player. Group 3 shows “extreme” behavior. Members of this group always return to the same star after they have left. More precisely, subjects in this group reach and leave the same periphery-sponsored star (with center player 5) nine times during 30 minutes without coordinating on another star. Group 8 is also exceptional, since it is the only group that does not reach any periphery-sponsored star network at all.

In Fig. 5 we take a macroscopic view on our data. We consider the number of stars observed during the experiment aggregated over all groups.²² There is a short time interval at the beginning of the experiment in which no group reaches a star (“period of learning how to coordinate”). Then the number increases until it oscillates around 3. Groups apparently need a phase of learning how to coordinate on a star.

“Fair stars” Group 8 was the only one that did not succeed in forming a periphery-sponsored star network. However, this group shows a particular kind of behavior which is related to inequity aversion. Participants of group 8 for a long time stay in a network “close” to a star (see Section 4.1.1). This specific type of network we also call a “fair star” network, since it is a periphery-sponsored star in which the center player holds exactly one active link to another group member. Fair star networks are neither Nash networks nor efficient. They are characterized by the property “uniform distribution of payoffs”. In a periphery sponsored star, the payoff difference between the center player and each periphery player is equal to $c = 2$. By opening exactly one link, the

²²Note that “aggregate view” in this picture simply means counting the *total number* of stars at each point in time.

Fig 4. Stars observed in groups 1–8

center player eliminates his payoff advantage. *Inequity aversion* may be a satisfactory explanation for this kind of behavior.

This type of network seems to be quite attractive for some groups, as is shown in Table 4. Half of the groups (1, 2, 3, 4) stay in fair star networks for less than one minute. Hence, such type of network does not seem attractive to these groups. The other groups (5, 6, 7, 8), however, stay in fair star networks for a significantly longer time. A closer look at the data reveals that the fair star frequently occurs as a temporary pre-stage of a periphery sponsored star. Extreme behavior is shown by group 8, which stays in the fair star network for about 43% of the whole time. A graphical illustration of the behavior of all groups with respect to fair star networks is given in Fig. 6.

Table 4. Group performance with respect to fair stars

Group	1	2	3	4	5	6	7	8	Mean
No. of fair stars	1	2	1	1	2	1	4	1	1.63
Time spent in fair stars:									
Minutes	0.04	0.73	0.05	0.31	4.02	5.79	2.99	12.94	3.36
%	0.15	2.43	0.16	1.03	13.40	19.30	9.96	43.13	11.20

Fig 5. Stars observed and aggregated over all groups

Activity of groups As in our discrete time experiment, we highlight the role of the *degree of activity* of groups in the continuous time experiment. The degree of activity of a group in the context of the continuous time experiment is also measured by the average number of changes per group member (see Table 3). The evolution of the total number of link changes in the groups is displayed graphically in Fig. 9 in Appendix B, exemplary for groups 3 and 7.²³ Note, when a group reaches a strict Nash network, the degree of activity usually drops to zero.

Group 3, which is the only group that stays in the same strict Nash network for 65% of the total time, exhibits a rather low degree of activity compared to the other groups. Group 7, which reaches all possible strict Nash networks within 30 minutes, is the most active one (see Table 3). However, it would be misleading to postulate a positive correlation between the degree of activity of the groups and the number of stars that are reached by this group. Table 5 shows that there is no significant correlation between degree of activity and the number of stars reached.

Although degrees of activity evolve differently in the groups, we get a completely different picture when taking a more macroscopic point of view. The evolution of average degree of activity in the groups in Fig. 7 shows no trend over time.

Another interesting aspect that we regard as an additional expression of group activity is the *number of active links* established in a group. The evolution of the number of active links is illustrated for groups 3 and 7 in Fig. 10 in Appendix B. It is a common feature of all groups that the number of active links suddenly increases when a group leaves a star. Group members seem to look for new center players by testing out new active links. If coordination on a new center player becomes successful, the number of active links drops down to five or to six (fair star network).

We illustrate the temporal evolution of the average number of active links in the

²³We count the number of changes in every time interval of ten seconds. To make these numbers more informative, we multiply them by six, so that the measure shown in the graphic is number of changes per minute. To measure action changes within a time interval of ten seconds is only one of several possibilities. We use these intervals because this proved to be fruitful in producing informative pictures about the temporal evolution of degrees of activity.

Fig 6. Fair stars observed in groups 3, 5, 7, and 8

groups in Fig. 8. The chart shows the total number of links in all groups divided by the number of groups. The curve in Fig. 8 exhibits more “regular” behavior than the respective single group curves in Fig. 10. During the initial coordination phase in the first ten minutes, in which subjects are looking for an appropriate center player, the average number of active links shows a decreasing trend. After that, it oscillates around a value almost equal to six, which is in line with the observation that, in this phase at (almost) all points in time, some groups play a star and, therefore, have five active links (see Fig. 5).

Some correlations between selected aggregate variables from Table 3 complement the previous mainly descriptive analysis. Most interesting are the correlations with respect to the average group payoff, which is an appropriate measure of material success.

Table 5. Spearman rank correlations

No.	Variable 1	Variable 2	Rank correlation coefficient r_s (p -Value)
1	Degree of activity	No. of stars reached	0.24 (0.28)
2	Degree of activity	Time spent in stars	-0.76 (0.02)
3	Degree of activity	Average payoff	-1.00 (0.00)
4	Time spent in stars	Average payoff	0.76 (0.02)
5	No. of stars reached	Average payoff	-0.24 (0.28)

Fig 7. Evolution of average degree of activity

Fig 8. Evolution of average number of active links

Based on a 5% significance level, correlations 2, 3, and 4 are significant (bold numbers). Correlation 3 between groups' degree of activity and average payoffs generated in a group during the experiment is eye-catching because it reveals “perfect” negative correlation. We found a similar phenomenon in our discrete time experiment.

4.3. Interpretation and comparison of the discrete and continuous time results

We regard our continuous time experiment as a natural extension of our discrete time study. Some phenomena, which were indicated already in the discrete time setting, have been observed much more often in the continuous time design.

- In general, we observe the same tendency in both types of experiments: *strict Nash networks are a good indicator* of network formation when the Bala-Goyal model is modified in an appropriate way, that is, when access to neighbors' information value is limited by our assumptions made in Section 2. In our experiments in discrete and continuous time, the periphery-sponsored star is observed in 11% of all periods resp. 25% of the time. We do not know of experiments with two-way flow models in which strict Nash networks have been reached as often.
- Intuitively, one would expect that *complex coordination problems* such as building a network in a non-cooperative game could be better solved in continuous time than in discrete time experiments. It is less complex to solve an asymmetric strategic coordination problem in continuous time which allows for sequential decision making. This is strongly supported by our experimental results. In continuous time it is easier to find rational solutions (equilibria) in the network game, since a single player can always respond to the current strategy configuration in a more adequate way about which all players are perfectly informed. In simultaneous strategy adaptation, it is more difficult to transmit strategic information. We

found similar results in experiments on pure coordination games (see Berninghaus and Ehrhart, 1998).

- Bala and Goyal (2000) propose a kind of *myopic best reply dynamics* to explain how groups reach strict Nash networks. Our experimental observations are not in line with this procedure. Subjects often deviate from strict Nash networks, even though they suffer from severe payoff reductions. In the discrete time experiment, groups which leave a periphery-sponsored star after some periods return to the same star. In the continuous time setting, six groups leave a center-sponsored star network and either return to the same star or build new stars with different center players.
- Concerning the role of *inequity aversion* in network formation, we find, first, that subjects are more willing to form star-shaped networks in our design than in the two-way flow experiment of Falk and Kosfeld (2003) in which no strict Nash network is observed. Inequity aversion may serve as a plausible explanation for this difference because, in our setting, the payoff differences between the subjects playing a strict Nash network are rather small compared to Falk’s and Kosfeld’s design. Second, building fair stars seems to be a type of behavior in network formation that is clearly guided by inequity aversion. It is difficult to find another explanation for staying in such (inefficient) networks. In the continuous time experiment, there is one group (group 8) that stays in a fair star for almost the whole last 13 minutes. Three other groups stay in fair stars for a fairly long time (see Table 4 for details). In the discrete time experiment, there is one successful group (group 5) which plays a fair star in one period (period 7), but leaves it to build a periphery-sponsored star two periods later.

Third, although the payoff difference between center and periphery players in our design is rather small, there is still inequity in payoffs. There seems to be an efficient way to compromise between staying in strict Nash networks *and* restoring equalization of payoffs in the experiment: switching from one star network to another by substituting the respective center players. This behavior can lead to the equalization of the payoffs in a group when the members play all possible periphery-sponsored star networks for the same length of time.

Since it is certainly difficult to communicate in an uncooperative framework, we do not expect this behavior in a simultaneous strategy adaptation environment within 15 periods. However, when subjects, as in our continuous time setting, have more possibilities to activate and deactivate links and to observe other players’ decisions, there will be a larger chance of being successful in the coordination on the periphery-sponsored star. Our experimental results support this view: most groups in the continuous time design (except for groups 3 and 8) reached more than one periphery-sponsored star, what we interpret as an attempt to equalize payoffs, whereas we do not observe this phenomenon in our discrete time experiment.

5. Concluding remarks

It is the main aim of our paper to study the evolution of networks under two different experimental designs, i.e. under discrete time and under continuous time strategy adaptation. We observe that periphery-sponsored stars, which are the unique strict Nash networks in our design, are focal points of our strategy adaptation process. This is the main result of the discrete time network formation process. However, this cannot be the whole story. We observe (in continuous time strategy adaptation) as an additional phenomenon the “changing of center players”, i.e. changing the strict Nash networks, which cannot be explained exclusively by the focal point property of the periphery-sponsored stars.

One plausible explanation of these experimental results is inequity aversion. More precisely, two phenomena observed in our experiments support this view. First, (in the continuous time setting) subjects show a strong tendency to equalize payoff differences in strict Nash networks by alternating the favored position. Second, particular subjects (center players) are often willing to sacrifice even parts of their payoff in order to achieve payoff equalization in the long run (“fair star” phenomenon). However, much more experimental evidence has to be accumulated before further conclusions on the role of the inequity principle in network formation can be drawn.

As a canonical extension of our experiments, we plan to conduct our discrete time experiment with a larger number of periods. It would be interesting to see whether the subjects’ behavior in the discrete time experiment “approaches” the behavior of the subjects in our continuous time experiment, that is, whether subjects also change center players when the number of repetitions in discrete time increases.²⁴

Furthermore, we plan to extend our network experiments by letting subjects select an action in a given two-person game which they play in pairs against all other subjects with whom they choose to be connected. This is a very complex compound strategic decision problem of endogenous network formation and action selection in a matrix game which has recently been dealt with in some theoretical papers (see, for example, Goyal and Vega-Redondo, 2005; Berninghaus and Vogt, 2003). We only know of one experimental approach to this problem by Corbae and Duffy (2003) which, however, is very preliminary. Corbae and Duffy let the subjects choose actions in a pure 2×2 coordination game. Beside pure coordination games, we plan to analyze 2×2 Hawk-Dove games as a base game. We are particularly interested to see how the type of the underlying 2×2 game will determine the evolution of networks.

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²⁴Note that we conducted our discrete time experiment some months before our continuous time experiment, i.e. before we knew the phenomena arising in the continuous time experiment.

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Appendix

A. Calculating all Nash networks for our experimental design

In simulations we found 3666 Nash networks, where six are strict Nash. After determining the equivalence classes²⁵, we obtained eleven classes of different size. The size gives the number of networks in a class, for example, six periphery sponsored stars, as every player can be the center player. We present these classes of Nash networks below. Each class is represented by a non-labeled directed graph. Class 1 consists of all strict Nash networks, i.e. of all periphery sponsored stars. The remaining classes 2–11 are weak Nash networks.

Class 1 of strict Nash equilibria (size 6)

Class 2 of Nash equilibria (size 180)

Class 3 of Nash equilibria (size 360)

²⁵In graph theory, the networks that belong to the same class are called isomorphic graphs. The number of directed networks (labeled) in our experimental design is equal to $2^{n(n-1)} = 1\,073\,741\,824$ (for $n = 6$), the number of different classes is equal to 1 540 944 Oberschelp (1967).

Class 4

Class 4 of Nash equilibria (size 360)

Class 5

Class 5 of Nash equilibria (size 120)

Class 6

Class 6 of Nash equilibria (size 180)

Class 7

Class 7 of Nash equilibria (size 120)

Class 8

Class 8 of Nash equilibria (size 720)

Class 9

Class 9 of Nash equilibria (size 720)

Class 10

Class 10 of Nash equilibria (size 720)

Class 11

Class 11 of Nash equilibria (size 180)

B. Graphical illustrations of selected groups' results

Fig 9. Temporal evolution of degrees of activity (groups 3 and 7)

Fig 10. Temporal evolution of active links (groups 3 and 7)