Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Structural damage detection using auto correlation functions of vibration response under sinusoidal excitation

Structural damage detection using auto correlation functions of vibration response under sinusoidal excitation

Muyu Zhang[1], Rüdiger Schmidt[1], Bernd Markert[1,2]

[1]Institute of General Mechanics, RWTH Aachen University, Templergraben 64, 52062 Aachen, Germany
[2]Faculty of Engineering and Science, University of Agder, Televeien 9, 4879 Grimstad, Norway

E-mail: muyu@iam.rwth-aachen.de

Abstract. Structural damage detection using time domain vibration responses has attracted more and more researchers in recent years because of its simplicity in calculation and no requirement of a finite element model. This paper proposes a new approach to locate the damage using the auto correlation function of vibration response signals under sinusoidal excitation from different measurement points of the structure, based on which a vector named Auto Correlation Function at Maximum Point Value Vector (AMV) is formulated. A sensitivity analysis of the normalized AMV with respect to the local stiffness shows that under several specific frequency excitations, the normalized AMV has a sharp change around the local stiffness change location, which means that even when the damage is very small, the normalized AMV is a good indicator for the damage. In order to locate the damage, a damage index is defined as the relative change of the normalized AMV before and after damage. Several example cases in stiffness reduction detection of a frame structure valid the results of the sensitivity analysis, demonstrate the efficiency of the normalized AMV in damage localization and the effect of the excitation frequency on its detectability.

1. Introduction

Structural health monitoring and damage detection using time domain vibration response is appealing, because the important health information of the structure is involved in the vibration response signals, which can be easily measured by conventional techniques. Moreover, it has advantages of simplicity in calculation and no finite element analysis is required that can be performed in real-time and online.

In the literature, several approach than use the auto/cross correlation functions of the vibration responses for damage detection are reported. Nichols and Seaver [1] proposed the correlation-function-based transfer entropy, which is used for detecting the presence of damage-induced nonlinearities in structures. Overbuy and Todd [2] modified this transfer entropy and used it for the damage identification of an experimental 8-degrees-of-freedom oscillator. Trendafilova [3] developed a damage indicator from the normalized cross correlation to detect the presence and location of the different kinds of delamination damage in a composite laminate beam that is subjected to random excitation. Li and Law [4] used the matrix of the covariance of covariance of acceleration response signals under white noise excitation to successfully detect the damages of a simply supported plane truss structure. Yang and Yu [5] proposed a vector named Cross Correlation Function Amplitude Vector (CorV) for damage detection. Based on the concept of CorV and Natural Excitation Technique (NExT) [6], Yang and Wang [7] proposed a damage detection method using the Inner Product Vector

(IPV) of the vibration responses. Similar to the IPV, Zhang and Schmidt [8] introduced a damage index named Auto Correlation Function at Maximum Point Value Vector (AMV). The stiffness reduction detection of a 12-story frame structure showed that the AMV-based damage detection method can locate the damage effectively, even in the presence of noise and has a better detectability compared to the other correlation-function-based damage detection methods.

In this paper, the effectiveness of AMV is studied when the sinusoidal excitation is applied on the structure. Firstly, the theory of auto correlation function of vibration responses under sinusoidal excitation is presented, based on which the damage index is formulated. Secondly, sensitivity analysis is used to substantiate the choice of this damage index for damage detection. As an example, damage detection of a 12-story frame structure is used to verify the proposed method, followed by a comparison of the excitation frequency on the detectability. Finally, some conclusions are made.

2. Theory of auto correlation function and damage index

2.1. Auto correlation function

The standard matrix equations of motion is given by

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K}$ is the damping matrix, $\mathbf{f}$ is a vector of the force and $\mathbf{x}(t)$ is the vector of displacement.

Assume real normal modes and the excitation is set at point k, the auto correlation function of the displacement response under sinusoidal excitation at point i when $T=0$ can be expressed as

$$R_{i,i}=\sum_{r=1}^{n}\psi_{ir}[\sum_{s=1}^{n}\psi_{is}\mu_k^{rs}] \tag{2}$$

where ψ_{ir} and ψ_{is} represent the i th component of mode shape $\mathbf{\Psi}_r$ and $\mathbf{\Psi}_s$, respectively, and

$$\mu_k^{rs}=\frac{X_k^2}{2}\frac{\psi_{kr}\psi_{ks}}{m^r m^s L_r L_s}P_{rs} \tag{3}$$

where X_k is the amplitude of the sinusoidal excitation, m^r is the r th modal mass and

$$L_i=[(\omega_n^i)^2+(2\pi f_{0,k})^2]^2-4(2\pi f_{0,k}\omega_d^i)^2 \tag{4}$$

$$P_{rs}=D_r D_s+4(2\pi f_{0,k})^2\zeta^r\omega_n^r\zeta^s\omega_n^s \tag{5}$$

$$D_i=(\omega_n^i)^2-(2\pi f_{0,k})^2 \tag{6}$$

Herein, ω_n^r is the r th modal frequency, $\zeta^r=\frac{1}{2}(\frac{\alpha}{\omega_n^r}+\beta\omega_n^r)$ is the r th modal damping ratio, ω_d^r is the r th damped modal frequency and $f_{0,k}$ is the frequency of the sinusoidal excitation.

2.2. Damage index

Set the values of the auto correlation function of the responses from different measurement points at $T=0$ as a vector

$$\mathbf{R}_{AMV}=[R_{1,1},R_{2,2},\ldots,R_{n,n}]^T \tag{7}$$

where $R_{i,i}$ is the value of the auto correlation function of the response from measurement point i at $T=0$. Since the auto correlation function value at $T=0$ is also its maximum value, we define this vector as Auto Correlation Function at Maximum Point Value Vector (AMV).

In order to eliminate the influence of the excitation, $\mathbf{R}_{AMV}$ is normalized by its root mean square value as follows

$$\overline{\mathbf{R}}_{AMV} = \frac{\mathbf{R}_{AMV}}{rms(\mathbf{R}_{AMV})} = [\overline{R}_{1,1}, \overline{R}_{2,2}, ..., \overline{R}_{n,n}]^T \tag{8}$$

where $rms(\mathbf{R}_{AMV}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} R_{i,i}^2}$.

Then the relative change of the i th element in $\overline{\mathbf{R}}_{AMV}$ is defined as

$$D_{AMV,i} = \frac{\overline{R}_{i,i}^d - \overline{R}_{i,i}^u}{\overline{R}_{i,i}^u} \tag{9}$$

where the superscript d denotes the damaged structural state and u indicates the intact structural state. Thus, the damage index of the AMV method can be defined as $\mathbf{D}_{AMV} = [D_{AMV,1}, D_{AMV,2}, ...D_{AMV,n}]^T$.

3. Sensitivity analysis and damage detection

The sensitivity is defined as the relative change of a variable to another variable z , so the sensitivity of the ith element of the vector $\overline{\mathbf{R}}_{AMV}$ to the local stiffness k_j can be given by [8]

$$\eta(\overline{R}_{i,i} / k_j) = k_j (\frac{\partial R_{i,i} / \partial k_j}{R_{i,i}} - \frac{\partial rms(\mathbf{R}_{AMV}) / \partial k_j}{rms(\mathbf{R}_{AMV})}) \tag{10}$$

which describes how the value $\overline{R}_{i,i}$ will change due to changes of the local stiffness k_j .

Deduced from the basic vibration theory and assume the proportional damping $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$, the partial derivative of the frequency to the local stiffness $\partial\omega_n^r \big/ \partial k_j$ can be obtained from the real part of

$$\frac{\partial\lambda_r}{\partial k_j} = -\frac{1}{2(\lambda_r + \zeta^r\omega_n^r)} \mathbf{\Psi}_r^T (\lambda_r^2 \frac{\partial\mathbf{M}}{\partial k_j} + \lambda_r \frac{\partial\mathbf{C}}{\partial k_j} + \frac{\partial\mathbf{K}}{\partial k_j}) \mathbf{\Psi}_r \tag{11}$$

where $\lambda_r = -\zeta^r\omega_n^r \pm i\omega_d^r$. The partial derivative of the mode shape to the local stiffness $\partial\mathbf{\Psi}_r \big/ \partial k_j$ is given by

$$\frac{\partial\mathbf{\Psi}_r}{\partial k_j} = \sum_{u=1}^{n} a_u \mathbf{\Psi}_u \tag{12}$$

where $a_u = \begin{cases} -\dfrac{1}{(\lambda_r^2 - \lambda_u^2) + (\lambda_r - \lambda_u)[\alpha + \beta(\omega_n^u)^2]} \mathbf{\Psi}_u^T (\lambda_r^2 \dfrac{\partial\mathbf{M}}{\partial k_j} + \lambda_r \dfrac{\partial\mathbf{C}}{\partial k_j} + \dfrac{\partial\mathbf{K}}{\partial k_j}) \mathbf{\Psi}_r, & u \neq r \\ -\dfrac{1}{2} \mathbf{\Psi}_r^T \dfrac{\partial\mathbf{M}}{\partial k_j} \mathbf{\Psi}_r, & u = r \end{cases}$

According to equation (2), $R_{i,i}$ is only related to the frequency and mode shape of the system, so that the partial derivative of $R_{i,i}$ to the local stiffness k_j can be expressed by

$$\frac{\partial R_{i,i}}{\partial k_j} = \frac{\partial f(\psi_{kr}, \psi_{ks}, \psi_{ir}, \psi_{is}, \omega_n^r, \omega_n^s)}{\partial k_j} \tag{13}$$

By substitution of equations (11, 12) into equation (13), the partial derivate $\partial R_{i,i} / \partial k_j$ can be obtained. Then by substituting equation (13) into equation (10), the sensitivity $\eta(\overline{R}_{i,i} / k_j)$ of the normalized correlation function value at $T = 0$ to the local stiffness k_j can be obtained.

3.1. Sensitivity analysis results

As shown in figure 1, a 12-story shear frame structure is adopted as the numerical simulation model. The centred mass of each floor m_i is 1 kg, which is centralized on its floor. The stiffness of each floor is supplied by the braces, and the stiffness in the out-of-the-plane direction y that is perpendicular to the x-z plane is much larger than that in x direction. As a result, only the movement in x direction is considered. The stiffness coefficient k_i of each floor is 20,000 N/m. Based on these assumptions, the mechanical model of the frame structure can be expressed as a 12 degree of freedom (DOF) discrete system, the mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$ can be obtained easily from the basic vibration theory. Besides, proportional damping $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ is adopted.

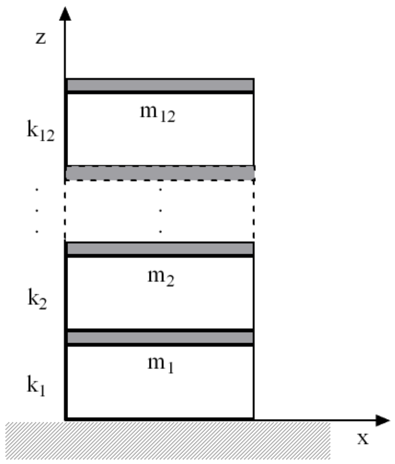

Figure 1. 12-story frame structure

Substituting $[\partial \mathbf{M} / \partial k_j]_{pq} = \partial m_{pq} / \partial k_j$ and $[\partial \mathbf{K} / \partial k_j]_{pq} = \partial k_{pq} / \partial k_j$, where m_{pq} and k_{pq} are the p th row, q th column elements of mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$, respectively, into equation (11) and equation (12), the sensitivity of the normalized AMV with respect to the local stiffness changes can be calculated by equation (10).

Figure 2 to 4 depict the sensitivity of the normalized AMV to the local stiffness k_j, where the local stiffness change location j is chosen from measurement point 1 to measurement point 12. In each figure, the red solid line, the green dotted line, the yellow dash-dotted line and the blue dotted line are the sensitivity of the normalized AMV calculated using the auto correlation function of vibration responses under 1 Hz, 2.5 Hz, 3 Hz and 10 Hz sinusoidal excitation, respectively.

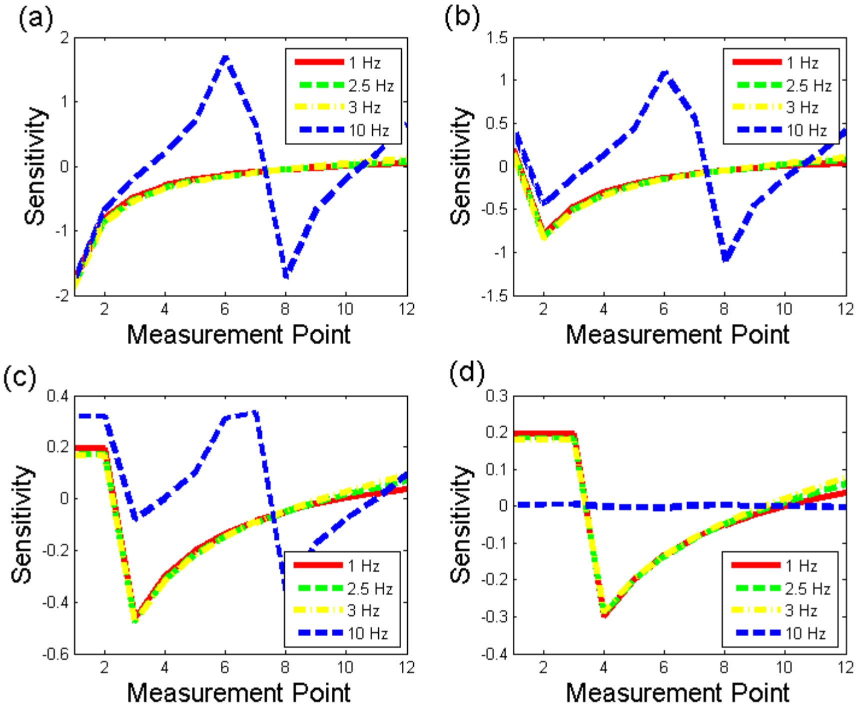

Figure 2. Sensitivity of normalized AMV to the local stiffness k_j:
(a) $j = 1$; (b) $j = 2$; (c) $j = 3$; (d) $j = 4$.

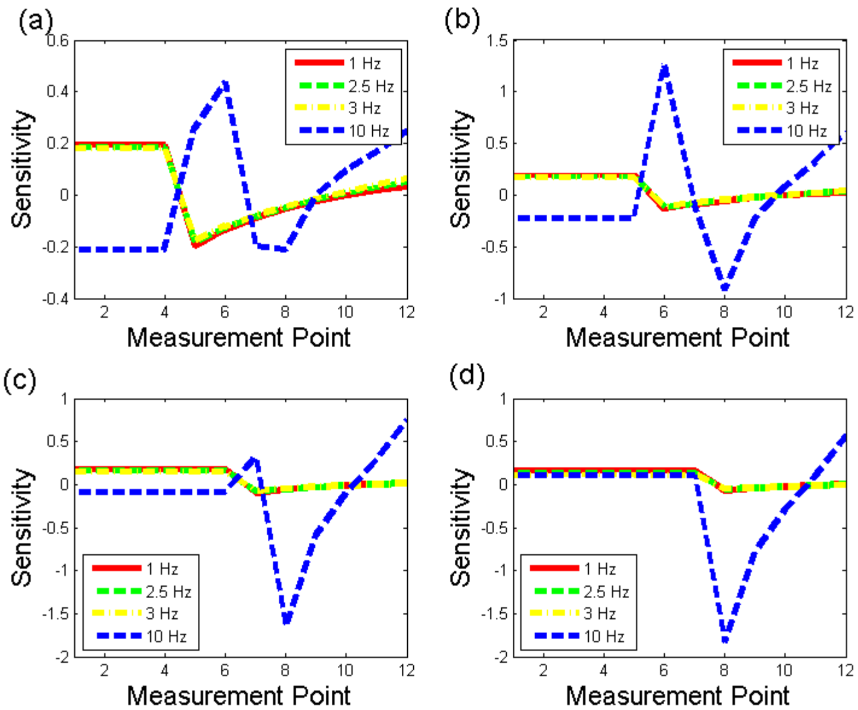

Figure 3. Sensitivity of normalized AMV to the local stiffness k_j:
(a) $j = 5$; (b) $j = 6$; (c) $j = 7$; (d) $j = 8$.

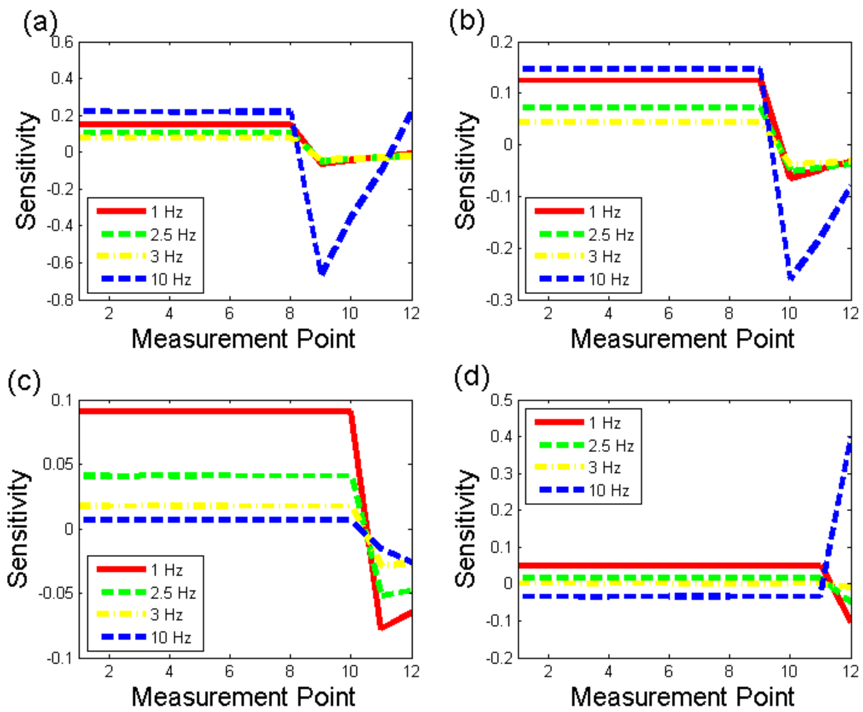

Figure 4. Sensitivity of normalized AMV to the local stiffness k_j:
(a) $j=9$; (b) $j=10$; (c) $j=11$; (d) $j=12$.

From figure 2 to 4, we can observe one common phenomenon for the sensitivities calculated from the responses under 1 Hz, 2.5 Hz and 3 Hz sinusoidal excitation. As the 1st natural frequency of the structure is 2.8 Hz, the excitation frequency is below or a little higher than the 1st natural frequency. The sensitivity of the normalized AMV to the local stiffness k_j has a sharp change at the measurement point j, before point j, it has a positive value while after j, it has a negative value. Consequently, when the local stiffness k_j decreases, the value of $\overline{R}_{i,i}$ also decreases before the measurement point j, but it increases after the measurement point j. Clearly, the relative change of $\overline{R}_{i,i}$ before and after k_j decreases will have a sharp change around the local stiffness change location, i.e. the measurement point j. Therefore, using different measurement point values of $\overline{R}_{i,i}$, local stiffness change locations can be observed, and the damage index calculated from equation (9) can be used for the damage localization.

As for the sensitivity calculated from the response under 10 Hz sinusoidal excitation, the analysis does not reveal the phenomenon as the previous three excitation frequencies. The method just result in random values from measurement point 1 to measurement point 12. Consequently, the damage index calculated from equation (9) cannot be used for damage localization if the excitation frequency is 10Hz.

3.2. Damage detection results

The damage of the frame structure is simulated by reducing the stiffness coefficient of 2nd, 5th, 8th and 11th floor by 5%, respectively. The different damage cases are listed in Table 1.

The excitation is applied on the first floor of the structure. It has duration of 20 s with a sample frequency of 1024 Hz and a magnitude of 5 N. The *Wilson-θ* method is utilized to get the vibration responses of the intact structure and the different damage cases.

Table 1. Damage cases of the frame structure

Damage case	D2	D5	D8	D11
Damage location	2	5	8	11
Damage extent/%		5		

The value of the auto correlation function can be obtained by the inner product of the vibration response. After getting the auto correlation function of the response before and after damage, the damage index can be calculated using the equations (7-9). The results of the different damage cases are shown in figure 5. Therein, the red solid line, the green dotted line, the yellow dash-dotted line and the blue dotted line represent the damage indexes calculated using the auto correlation function of vibration responses under 1 Hz, 2.5 Hz, 3 Hz and 10 Hz, respectively.

The damage indexes coincide well with the sensitivity analysis in section 3.1. Consider the red solid lines in figure 3(a) and figure 5(b) for instance. The red solid line in figure 3(a) is the sensitivity of the normalized AMV to the local stiffness, when the local stiffness change location is set at the 5th floor and the excitation frequency is 1 Hz. The red solid line in figure 5(b) is the damage index value obtained by the vibration responses under 1Hz sinusoidal excitation when the 5th floor has a 5% stiffness reduction. In figure 3(a), the value of $\eta(\overline{R}_{i,i}/k_5)$ has a positive value from measurement point 1 to measurement point 4 and they are almost identical. The value drops sharply at measurement point 5, where it has a negative value. After measurement point 5, it slowly increases to its initial level again. $\eta(\overline{R}_{i,i}/k_5)$ has a minimal value at measurement point 5. Thus, when k_5 decreases, $\overline{R}_{4,4}$ will decrease and $\overline{R}_{5,5}$ will increase. Thereby, the decrease and increase rates are very large. As a consequence, before and after k_5 decreases, the relative change of $\overline{R}_{4,4}$, i.e. $D_{AMV,4}$ is negative while the relative change of $\overline{R}_{5,5}$, i.e. $D_{AMV,5}$ is positive, such that there is a large difference between $D_{AMV,4}$ and $D_{AMV,5}$. This can be seen in figure 5(b). Moreover, for $\eta(\overline{R}_{i,i}/k_5)$, the values at measurement point 1 to measurement point 4 are almost the same and the values at measurement point 5 to measurement point 12 just have a slight difference. So the relative change of $\overline{R}_{1,1} \sim \overline{R}_{4,4}$ before and after k_5 decreases just show a very small change and likewise the relative change of $\overline{R}_{5,5} \sim \overline{R}_{12,12}$. Accordingly, the values of $D_{AMV,1} \sim D_{AMV,4}$ and $D_{AMV,5} \sim D_{AMV,12}$ do not change much, which can also be seen in figure 5(b). Other measurement points shon the same phenomenon. While the sensitivity analysis uses only the modal parameters of the structure and the damage detection uses only the time domain vibration responses of the structure, the good agreement of the results reveals that the damage index proposed in this paper is a good indicator for damage detection.

Similar results can be obtained using the sinusoidal excitation of 2.5 Hz and 3 Hz. For the damage index when the auto correlation function is calculated by the responses under 10 Hz sinusoidal excitation, they also coincide well with the sensitivity results in section 3.1 that the AMV-based damage index cannot locate the damage.

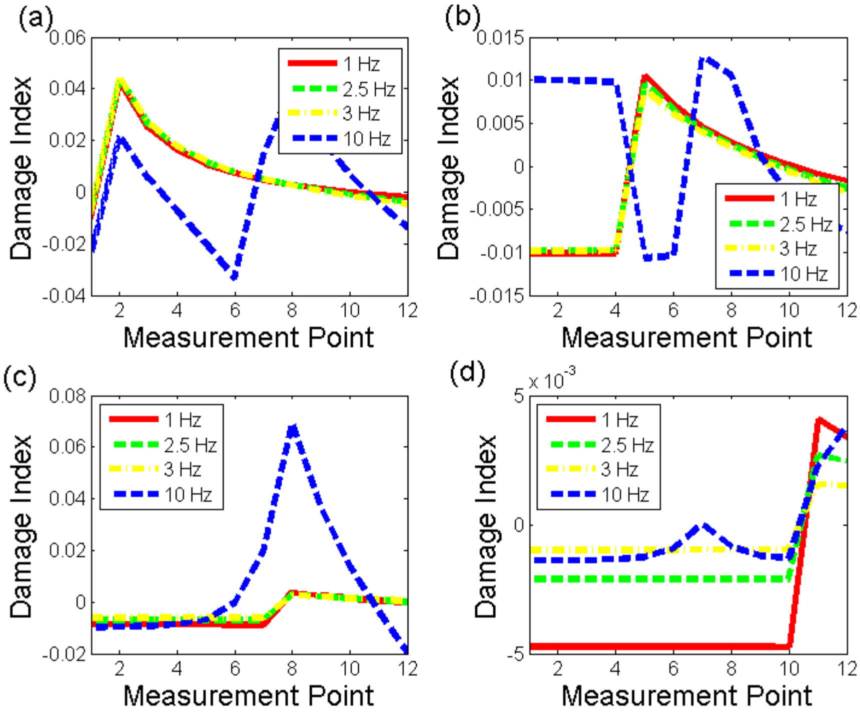

Figure 5. Damage index of different damage cases:
(a) Damage case D2; (b) Damage case D5; (c) Damage case D8; (d) Damage case D11.

4. Conclusions

An AMV-based damage detection method is proposed and studied in this paper. Sensitivity analysis shows that the normalized AMV can be used to locate the damage when the frequency of sinusoidal excitation is below or a little larger than the 1st natural frequency of the structure. Damage detection results of a 12-floor story frame structure prove the results of the sensitivity analysis, showing that the AMV is a good indicator for damage detection.

However, the AMV-based damage index cannot locate the damage for all the frequencies of the sinusoidal excitation. So it is important to choose the suitable excitation frequency when the AMV is applied for damage detection.

References

[1] Nichols J M, Seaver M, Trickey S T, Todd M D, Olson C and Overbey L 2005 *Phys. Rev.* E **72** 046217
[2] Overbey L and Todd M D 2009 *J. Sound. Vib* **322** 438-53
[3] Trendafilova I, Palazzetti R and Zucchelli A 2015 *European Journal of Mechanics-A/SolidsEur. J. Mech* A-Solid **49** 197-204
[4] Li X Y and Law S S 2010 *Mech. Syst. Signal. Pr* **24** 945-56
[5] Yang Z C, Yu Z F and Sun H 2007 *Mech. Syst. Signal. Pr* **21** 2918-32
[6] James G H III and Farrar C R 1997 *J. Sound. Vib* **205** 1-18
[7] Wang L, Yang Z C and Waters T P 2010 *J. Sound. Vib* **329** 5070-86
[8] Zhang M Y and Schmidt R 2014 *J. Sound. Vib* **333** 7352-63