

Aspects of Hadronic B Decays in and Beyond the Standard Model

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Abstract

In this thesis we address various issues of hadronic B decays, in the Standard Model and beyond. Concerning the first aspect, we focus on the problem of understanding better low energy strong interactions in these decays. We consider in particular B decays into a charmonium state and a light meson. We develop a complete treatment of low energy QCD interaction in the context of QCD factorization, treating the charmonia as non-relativistic bound states. This allows us to demonstrate that, in the heavy-quark limit, a perturbative treatment of these decays is possible, even in case of decays into P -waves, which were found to be non-factorizing in previous studies. We achieve this, including in the analysis the bound state scales of charmonium, which in turn requires to consider charmonium production through colour-octet operators. Although there are very large uncertainties, we find reasonable parameter choices, where the main features of the data – large corrections to (naive) factorization and suppression of the χ_{c2} and h_c final states – are reproduced though the suppression of χ_{c2} is not as strong as seen in the data. Our results also provide an example, where an endpoint divergence in hard spectator-scattering factorizes and is absorbed into colour-octet operator matrix elements.

The second part of the thesis is devoted to a series of analyses of non-leptonic B decays in extensions of the Standard Model. The aim of these studies is twofold: on one hand we are interested in testing the sensitivity of these decays to new physics; on the other hand, we look for actual discrepancies between theory predictions and experimental results, trying to explain them in the context of a new physics model. Concerning the first aspect, we consider two well-motivated new physics scenarios, in which large deviations from the Standard Model are expected, *i.e.* the MSSM with large $\tan\beta$, and a supersymmetric GUT in which the large neutrino mixing angles give rise to a large mixing between the right-handed quarks. We find that, in both cases, effects in non-leptonic B decays are small, when constraints from other flavour observables, like leptonic B decays, are taken into account. Concerning the second issue, we focus on the discrepancies between theory and experiment, which point to new physics in the electroweak penguin sector of the theory. We consider a modified Z^0 penguin scenario, where this possibility is realized, and we fit the couplings of the model from the $B \rightarrow \pi K$ decays. We show that, in this class of models, a sizeable enhancement of the $B \rightarrow \phi\pi, \phi\rho$ decays is expected, even if this is reduced, when constraints from semi-leptonic B decays are considered.

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Chapter 1

Introduction

1.1 Non-leptonic B decays: overview

The study of “non-leptonic”, or “hadronic” B decays usually refers to the analysis of B decays into lighter mesons, such as pions, kaons, η mesons, D mesons, *etc.*. At the quark level, these decays are driven by the transitions of the b quark inside the B meson into lighter quarks, which subsequently are rearranged into mesons by the strong interaction. In the Standard Model, these transitions occur by means of the electroweak interaction and they are regulated by the Cabibbo-Kobayashi-Maskawa (CKM) [1, 2] quark mixing matrix.

These decays represent one of the most interesting phenomena in particle physics, since they allow us to test nearly all the aspects of the Standard Model, and beyond. Firstly, they can be used to study the flavour structure of the theory, which is the aspect of the Standard Model still known with least precision¹, and which contains the largest number of free parameters. The flavour structure of the Standard Model is encoded in the CKM matrix, and one of the most interesting aspects of this sector is the phenomenon of CP violation, *i.e.* the non-invariance of the weak interaction with respect to a combined charge-conjugation (C) and parity (P) transformation, which is made possible by the existence of three different quark families with different masses and which has cosmological implications, since it is a key ingredient for baryogenesis [3]. The actual observation of CP violation is nontrivial, since it manifests only in processes made up from the interference of several competing amplitudes, in which weak and strong-interaction phases appear. This is exactly what happens in case of non-leptonic B decays. Indeed, one of the biggest achievement of the B factories was the observation of CP violation in the $B_d \rightarrow J/\psi K_S$ decay mode in 2001, [4, 5], which represented the first observation of CP violation outside the kaon system. This particular kind of CP violation originates from the interference between the B_d^0 - $\bar{B}_d^0$ mixing and the $B_d^0 \rightarrow J/\psi K_S$, $\bar{B}_d^0 \rightarrow J/\psi K_S$ decays, and it is referred

¹For comparison, the experimental value of the muon anomalous magnetic moment is known with a precision of $\sim$ one part over 10^{-8} , $a_\mu^{\text{exp}} = 11659208.0(5.4)(3.3) \cdot 10^{-10}$ [8–11], and compared with the theoretical prediction, $a_\mu^{\text{SM}} = 11659178.8(0.2)(4.6)(3.5) \cdot 10^{-10}$ [8], the difference between theory and experiment is $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 292(63)(58) \cdot 10^{-11}$; on the other hand, the CKM matrix element known with best precision is known up to $\sim$ one part over 10^{-4} , $|V_{ud}| = 0.97418 \pm 0.00027$, [8, 12].

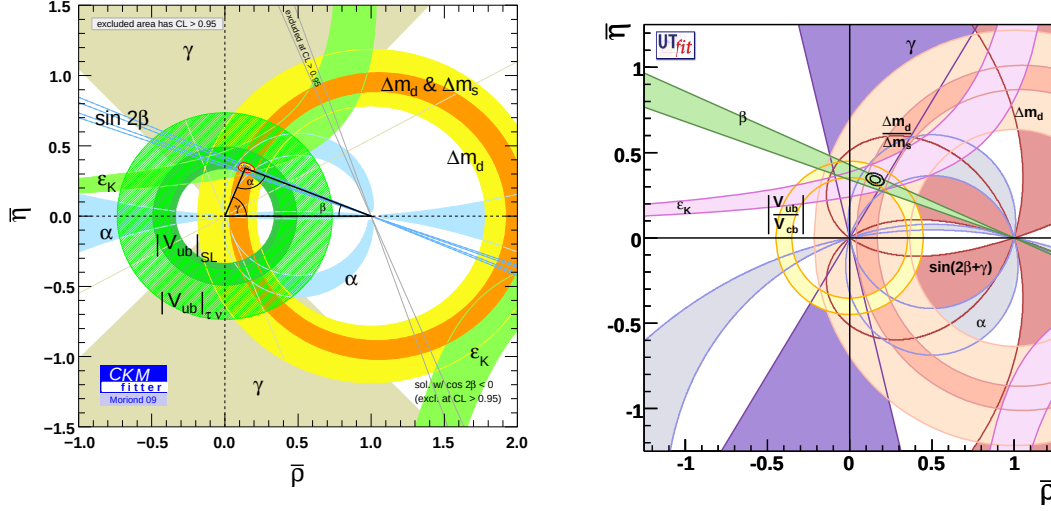


Figure 1.1: Global fit of the unitarity triangle, as obtained by the CKMfitter (left) and the UTfit (right) collaborations.

to as “mixing induced” CP violation; later in 2004, also “direct” CP violation was observed in the $B_d \rightarrow \pi^\mp K^\pm$ decays [6, 7], complementing the picture.

In the Standard Model, field redefinitions can be used to reduce the number of phases in the quark mixing matrix to only one. The unitarity of the CKM matrix is used to construct “unitarity triangles”, encoded for instance in the relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$. The angles involved in these unitarity triangles measure the CP violation in the Standard Model, and they can be extracted from various non-leptonic B decays. With the standard choice of phase conventions, one defines the weak phases $\beta = -\arg(V_{td})$ and $\gamma = \arg(V_{ub}^*)$, as well as $\alpha = 180^\circ - \beta - \gamma$. For instance, the angle β can be extracted in a theoretical clean way by measuring the time-dependent rates for the $B_d^0, \bar{B}_d^0 \rightarrow J/\psi K_S$ decays. There are then various methods of extracting the angles γ and α from non-leptonic decays, even if these measurements are more difficult because of the hadronic dynamics involved in these decays, so that specific methods have to be developed in order to gain control over it and reduce hadronic uncertainties as much as possible. As an example, the angle γ can be extracted from the charged decay modes $B^\pm \rightarrow (\pi K)^\pm$ and $B^\pm \rightarrow \pi^\pm \pi^0$ [13–20], as well as from the time-dependent $B_d^0, \bar{B}_d^0 \rightarrow \pi^+ \pi^-$, $B_s^0, \bar{B}_s^0 \rightarrow K^+ K^-$ modes [21], or from $B \rightarrow D\pi$, $B \rightarrow DK$ decays [22–33]; the angle α can instead be measured *e.g.* from the time dependent CP asymmetry in the decays $B_d^0, \bar{B}_d^0 \rightarrow \pi^+ \pi^-$, and in a similar way from the related $B \rightarrow \rho\pi$, $B \rightarrow \rho\rho$ decays [34–41]. Thanks to these results and together with the measurement obtained from semi-leptonic and leptonic B decays, as well as radiative decays, meson mixing, kaon and D physics, we have today a good understanding of the flavour structure of the Standard Model, which can be summarized with the unitarity triangle fits obtained by the CKMfitter [42] and UTfit [43] collaborations, shown in picture 1.1. More extensive reviews of the experimental status of non-leptonic B decays, and more in general of flavour physics can be found in [44–49].

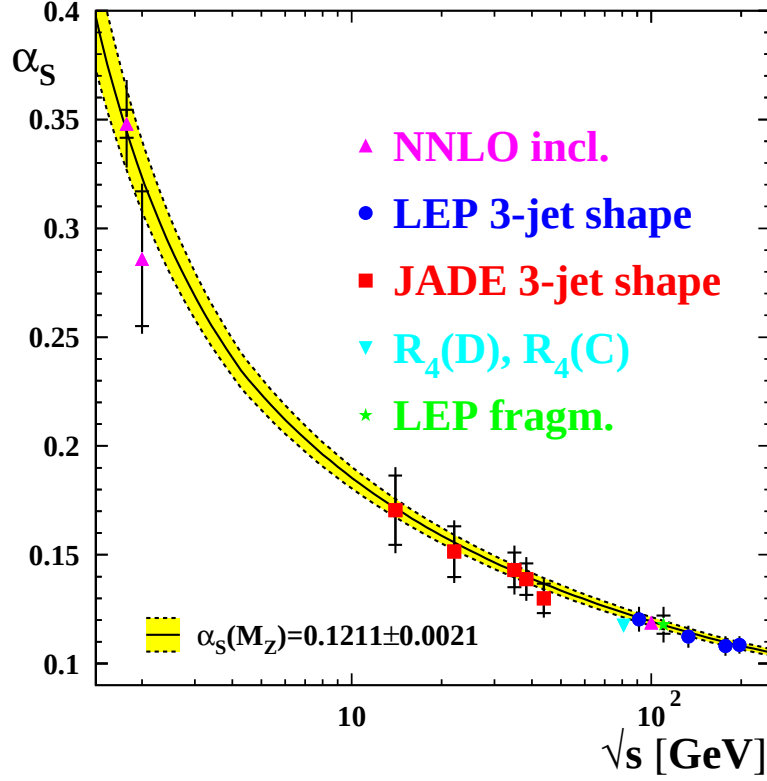


Figure 1.2: Running of the strong coupling constant, taken from [52].

Since quarks take also part in the strong interactions, non-leptonic B decays are dominated by the effects of this force, which is the largest source of uncertainties and prevents us from reaching the same level of precision obtained in other sectors of the Standard Model. The reason for this is the phenomenon of confinement [50, 51]: while at high energy the strong interaction becomes weaker, allowing for a perturbative treatment of high-energy processes, at the hadronic scale $\sim \Lambda_{\text{QCD}}$ involved in the B meson decays the strong coupling constant α_s becomes large enough that the perturbative expansion is no more reliable (see figure 1.2). On one hand, this is an obstacle on the route of extracting the relevant flavour information of the weak transition; on the other hand, it can be regarded as a good framework in which we can improve our knowledge about the strong interaction as well.

The effects of the strong interaction have to be taken into account from the high scale $\mu_{\text{EW}} \simeq M_W = 80.4$ GeV, which is the typical energy scale at which the electroweak transition takes place, down to the scale at which the strong interaction cannot be treated perturbatively any more, $\Lambda_{\text{QCD}} \sim 500$ MeV. The decays of the B meson in this respect are special, because the large mass of the decaying meson, $m_B \simeq 5$ GeV, introduces an intermediate scale, at which the strong interaction is still perturbative. QCD interactions above this scale can be treated perturbatively, and their effects are now well understood. At the scale m_b the electroweak transitions with exchange of W and Z bosons (as well as

photons with a comparable energy) are almost point-like interactions, and the decaying meson is described more realistically by a series of effective four-quark operators, in which the heavy bosons have been integrated out. These operators are multiplied by “effective coupling constants”, which include the effects of the strong interaction above m_b as a perturbative expansion in α_s . The problem of the appearance of large logarithms in the “fixed order” perturbation theory, related to the presence of the two scales $\mu_{EW} \simeq M_W$ and m_b , has been solved by the development of the “renormalization group”-improved perturbative expansion, which is now a standard textbook topic. Good reviews of this subject in the context of flavour physics are *e.g.* [53, 54], to which we refer the reader for further details.

The main problem one has to face in calculating the decay amplitudes is therefore the evaluation of the operator matrix elements, which include strong interaction effects below the scale m_b . Various strategies have been developed in the past for this purpose. For instance, the isospin and flavour symmetries of QCD were considered in order to relate matrix elements of different decay amplitudes [13, 16, 20, 21, 34, 55, 56, 300]. In this way it is possible to derive the matrix elements from decays which are easier to measure experimentally, and then the information can be used to estimate other decay modes. Even if good precision can be reached, depending on the experimental results and the accuracy of the flavour symmetries, this method does not allow one to calculate the matrix elements from first principles, and it is in practice limited to a handful of decays. A true resolute approach should therefore address the issue of understanding the structure of QCD interaction below the scale m_b . Given the ultimate non-perturbative nature of the strong interaction, this means in practice to understand what can be calculated perturbatively and what cannot. In this respect, properties of the QCD interaction such as the “colour transparency” [57, 58], the presence of different colours (“large N_c hypothesis” [59]), and the large mass of the decaying meson, which reflects into the production of light energetic partons, suggest that it should be possible to describe these interactions by means of a few “universal” non-perturbative objects, while the largest effects of the strong interaction should occur at the scale m_b , and therefore it should be possible to treat them in a perturbative framework.

The attempts of evaluating the operator matrix elements exploiting these ideas go under the name of “factorization”, since one tries to factorize the perturbative part of the interaction from the soft physics. Historically, the first approach was to assume simply that strong interactions below m_b could be neglected. This assumption, which is referred to as “naive factorization”, [60–62], has now been overtaken by more elaborate theories, which implement systematically the ideas mentioned above. These theories are named the “QCD factorization” approach, [63–68], and the “perturbative QCD” approach, [71–75]. While the underlying goal is in both cases the separation of the soft and hard physics, the two approaches differ in the implementation of this idea. According to the perturbative QCD approach, the leading contribution to the amplitude is expected to come solely from the exchange of hard gluons between the spectator and the quarks of the emission meson², (“perturbative hard-scattering”, [69, 70]), while soft gluon exchanges are expected to be

²The meson that does not pick up the spectator quark.

suppressed by “Sudakov effects”. As a consequence, the operator matrix elements start at order α_s , and no form factors are involved in the amplitude. In the QCD factorization approach, however, the “Sudakov suppression” is not expected to be relevant for realistic B meson masses, so there are soft gluon exchanges which are of the same order as the hard ones, and they are parameterized by means of form factors. At the technical level factorization is achieved performing a systematic expansion of the amplitude in the small ratio Λ_{QCD}/m_b , and proving that strong interaction effects are either perturbative, or they can be included in the definition of non-perturbative parameters, which correspond to form factors, meson decay constants and light-cone distribution amplitudes. QCD factorization has been proved at leading order in the Λ_{QCD}/m_b expansion, but so far it fails at the subleading order. Since there are a number of subleading amplitudes which are numerically important, this constitutes the main limitation of the approach.

Besides this still-ongoing problem, the attempts to explain the strong interaction dynamics in non-leptonic (and semi-leptonic) B decays have also pushed significantly the development of effective field theories [78], in order to put on solid theoretical grounds the expansion of the amplitudes in the relevant momentum regions [76, 77]. Thanks to this effort, QCD factorization has now its basis in effective field theories such as the heavy quark effective field theory [79, 80] and the soft collinear effective field theory [81–88]; analysis of non-leptonic B decays within this theory were performed in [89–95]. In recent years it was realized that the field of application of these theories can be much larger and they are now being applied to the physics of colliders [96–116], a development that can be seen as a byproduct of the studies in flavour physics.

Needless to say, non-leptonic B decays can be useful also for searches for new physics. Many hints, nowadays, suggest that there should be new physics beyond the Standard Model, since the latter cannot explain, for instance, the existence of dark matter and neutrino masses. On the theoretical side, the Higgs boson mass is sensitive to quadratic corrections, so that it is very unlikely to obtain the predicted small mass of $\mathcal{O}(100 \text{ GeV})$ for it without invoking some new symmetry, which should protect it from acquiring a large mass. All these phenomena suggest that the Standard Model is probably only the low energy approximation of a more fundamental theory. Both the existence of dark matter and the naturalness problem of the Higgs boson mass suggest that new physics should appear in the TeV range, and the Large Hadron Collider (LHC) at CERN has been constructed to test this expectation. Many theories have in the meantime been elaborated, which are able to accommodate the problems mentioned above. Among these, supersymmetry is probably the most studied, but many possible extensions of the Standard Model have been considered, like (supersymmetric) grand unified theories, Higgsless models and models of extra dimensions, all predicting new particles in the TeV range.

While the direct production of new particles will have to wait until the start of this machine, we already know many features that the new theory should possess, and in particular we can constrain its flavour structure, thanks to low energy processes like the non-leptonic B decays. The new heavy particles in fact are expected to appear as virtual intermediate states and they should therefore modify the amplitudes of the processes, which, in the Standard Model, receive contributions from the exchange of virtual elec-

troweak gauge bosons only. Thanks to this effect, low energy processes are sensitive to energies much higher than those reached in collider experiments, so that they allow us to shed light on energies at which many fundamental phenomena, like the generations of the hierarchy between fermion masses, or the unification of the known forces, are expected to have their origin.

The flavour sector of the theories quoted above has been studied intensively in the last ten years, and the literature in this field is probably $O(10^4)$ papers. Non-leptonic B decays were considered too, even if the strong interaction uncertainties limit considerably these analyses. If, on one hand, this claims for further improvements in the comprehension of the low-energy QCD interaction involved in these processes, on the other hand, tensions between the Standard Model prediction and the experimental results were found, raising our expectations for the next generation of flavour experiments [45, 117–120].

1.2 Outline

In this thesis we study aspects of non-leptonic B decays, both in the Standard Model and beyond. Concerning the first issue, we focus on the problem of understanding better the strong interaction dynamics involved in these decays, considering some specific interesting decay modes, where it was possible to develop a perturbative analysis. Regarding the second topic, we study some specific well-motivated scenarios of physics beyond the Standard Model, looking for possible effects in non-leptonic B decays.

We felt it was necessary to start our discussion with an exhaustive introduction to the theory of QCD factorization, which is the method used throughout the thesis for the analysis of non-leptonic B decays. This review is presented in chapter 2 and its purpose is to render the whole thesis self-contained. The first section of this chapter is devoted to a qualitative analysis of factorization for all types of non-leptonic B decays, with the aim of giving a transparent description of the concepts behind the theoretical framework. This will set the context and the motivation for the study described in chapter 3. In the second part of chapter 2 we provide a technical introduction to the actual calculation of the decay amplitudes, in the case of B decays into two light mesons. On one hand, this is done in order to illustrate the theoretical methods that will be used in the analysis reported in chapter 3; on the other hand, it will provide the building blocks of the decay amplitudes considered in the phenomenological analyses described in chapters 4, 5 and 6.

In chapter 3 we present our work concerning the inspection of the low-energy strong interaction dynamics of non-leptonic B decays within the Standard Model. This chapter is an extension of the study performed in [125] and focusses on the decays of the B meson into a charmonium state and a light meson. These decay modes are interesting because of the presence of the quarkonium in the final state, which is best described in the context of potential non-relativistic QCD [121–124]. As a consequence, the usual QCD factorization framework has to be reconsidered and extended, in order to include the formalism necessary to treat the non-relativistic bound states. This work was motivated by the failure of the standard QCD factorization formalism, already at leading order in the Λ_{QCD}/m_b expansion, when considering P -wave states [191, 192]. We find that the proper

treatment of the non-relativistic bound states requires the introduction of two additional scales in the evaluation of the matrix element, namely the scale of the inverse Bohr radius of charmonium, and its binding energy, which are perturbative in the heavy-quark limit. This, in turn, implies that one has to consider the contribution of colour-octet operators; the inclusion of their matrix elements to the total amplitude makes the computation of the latter consistent and restores its finiteness at leading order in the Λ_{QCD}/m_b expansion.

In chapter 4, 5 and 6 we present results of our analyses of non-leptonic B decays in various extension of the Standard Model. The aim of these studies was twofold: on one hand, we wanted to test the sensitivity of these decays to new physics; on the other, we were interested in analysing if it was already possible to derive hints about the nature of new physics contributing, studying the current deviations of experimental results from the predictions of the Standard Model.

As of the first point, the analysis is motivated because one has to take into account that the large QCD uncertainties limit the predictive power of these decays, so that usually one gets stronger constraints on new physics from other flavour observables, like leptonic, semi-leptonic and radiative decays, or from kaon physics [45]. In chapters 4 and 5 we considered therefore two well motivated models, in which large deviations from the Standard Model are expected, and we looked for possible effects in non-leptonic decays. Because of the motivation above, an important point of the analysis was the correlation with the decay amplitudes of other processes, in order to check if one gets more insight from the non-leptonic decays. In chapter 4 we study the minimal supersymmetric Standard Model [129–131] with large $\tan\beta$, a minimal flavour violating scenario [132,133] which nevertheless can exhibit large deviations from the Standard Model [240,248,251], thanks to flavour-changing neutral-currents mediated by the neutral and charged Higgs bosons, which are enhanced by powers of $\tan\beta$. This analysis is a result of the work presented in [126], of which chapter 4 represents a more detailed presentation. In chapter 5 we report an analysis, still in progress [127], of non-leptonic decays in a grand unified theory [134,135], namely the Chang-Masiero-Murayama model [278]. This scenario has the interesting feature of relating in a realistic way the observed large neutrino mixing angles with the right-handed quark sector of the theory. Despite the intrinsic interest of the models considered, we find that in both cases the predicted effects in non-leptonic B decays are small. In the large $\tan\beta$ scenario this happens because of the strong constraints one obtains from the leptonic B decays, while in case of the Chang-Masiero-Murayama model the constraints arise mainly from the predicted particle spectrum. Other observables, including flavour physics in the leptonic sector, give weaker constraints once the bounds from the sparticle masses are taken into account.

The second type of analysis [128] was motivated by actual discrepancies between theory and experiment, which were found in recent years. Needless to say, these studies are also limited by theoretical uncertainties, and the discrepancies found could be accommodated as well within the Standard Model. Nevertheless, in chapter 6 we pursue the route of considering these discrepancies effective, according to recent works [300], which found that these discrepancies point to new physics in the electroweak penguin sector of the theory. Chapter 6 is an exposition of this still-ongoing work, where we focus more specifically on

effects in non-leptonic B_s decays.

The thesis is completed by a number of appendices, which collect some background material that we felt necessary to include, in order to make the work self contained. In appendix A we give a short introduction to the effective field theories involved in the analysis of non-leptonic decays, *i.e.* the heavy-quark effective field theory, the soft collinear effective field theory and the non-relativistic QCD. The study of these theories was necessary to perform the analysis presented in chapter 3. In appendix B we provide some more details about the minimal supersymmetric Standard Model with large $\tan\beta$. We describe briefly how the mechanism of $\tan\beta$ enhancement arises, and we provide the flavour-changing couplings originating through the exchange of Higgs bosons, which are the starting point for the analysis of chapter 4. Similarly, a brief description of the grand-unified Chang-Masiero-Murayama model is given in appendix C, where, in light of the study reported in chapter 5, we provide again expressions for the flavour changing couplings arising in the model.

Chapter 2

QCD Factorization for Non-leptonic B Decays

2.1 Introduction

The idea of factorization concerns the separation of physics occurring at different lengths, or momentum scales, and is a widely applied concept in particle physics. The reduction of a calculation into a series of single-scale problems simplifies a lot the understanding of a process, and this is particularly useful when the strong interaction is involved, since it allows to separate short- from long-distance physics, determining what can be calculated in perturbation theory and the basic “universal” quantities that need a non-perturbative treatment.

Here we focus on the factorization of QCD interactions in non-leptonic B decays, below the scale m_b . QCD effects which involve virtualities above this scale are in fact well understood and included in the renormalization of the coefficients multiplying the local operators Q_i in the weak effective Hamiltonian [53]. As a consequence, the amplitude for the decay $B \rightarrow M_1 M_2$ is given by

$$\mathcal{A}(B \rightarrow M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) \langle M_1 M_2 | Q_i | B \rangle(\mu), \quad (2.1)$$

where G_F is the Fermi constant. Each term in the sum is the product of a Cabibbo-Kobayashi-Maskawa (CKM) factor λ_i times the short-distance coefficient function $C_i(\mu)$ and the matrix elements of the four-fermion operators Q_i . The most difficult theoretical problem is the calculation of these matrix elements, or their reduction to simpler non-perturbative objects.

The “naive factorization” approach recalled in chapter 1 replaces the matrix element of the four-quark operators by the product of two currents [60–62], for example

$$\langle \pi^+ \pi^- | (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} | \bar{B}_d \rangle \rightarrow \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle \langle \pi^+ | (\bar{u}b)_{V-A} | \bar{B}_d \rangle; \quad (2.2)$$

this is the simplest approach, which assumes that the exchange of “non-factorizable” gluons, *i.e.* gluons with virtuality equal to or below m_b , can be neglected. This however

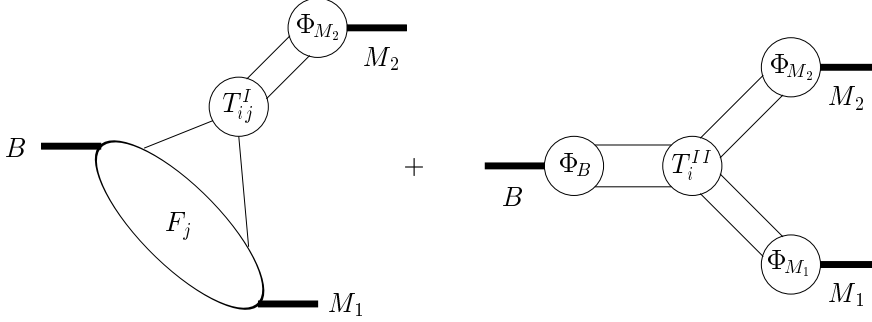


Figure 2.1: Graphical representation of the factorization formula, taken from [64]. Only one of the two form-factor terms is shown for simplicity.

is in general not justified: it does not take into account any physical mechanism for rescattering in the final state and for the generation of a strong phase shift between different amplitudes. Moreover, “non-factorizable” corrections must exist, because the scale dependence of the two sides of (2.2) is different: since “non-factorizable” corrections at scales larger than μ are taken into account in the short distance coefficients of the weak effective Hamiltonian, it appears arbitrary to leave them out below μ .

The *QCD factorization* approach (QCDF) developed by Beneke, Buchalla, Neubert and Sachrajda in a series of papers starting from the end of the 90s, [63–68], provides a framework where such effects can be computed systematically. The method relies on the existence of the intermediate scale m_b , and on the concept of “colour transparency” [57, 58]. The first feature assures that, in the heavy-quark limit, *i.e.* $m_b \gg \Lambda_{\text{QCD}}$, QCD interactions at the scales m_b can be computed perturbatively; the second idea is essential in understanding how short- and long-distance effects can be disentangled. The concept of colour transparency in fact guarantees that the light, energetic meson M_2 , that does not pick up the spectator quark from the B meson, escapes the interaction region as a compact object of small transverse size, which remains invisible to soft-gluon interactions. Non-perturbative soft effects (*i.e.*, of order Λ_{QCD}) are therefore confined to the $B \rightarrow M_1$ and the M_2 subsystems, while interactions between the two occur only at the hard scale m_b and can be computed perturbatively.

This qualitative picture can be translated into a *factorization formula*, whose pictorial representation is given in Fig. 2.1, according to which to leading power in Λ_{QCD}/m_b , but to all order in α_s , the matrix elements of the local operators Q_i in the effective Hamiltonian read

$$\begin{aligned} \langle M_1 M_2 | Q_i | \bar{B} \rangle &= (F^{B \rightarrow M_1} T_i^I \star f_{M_2} \Phi_{M_2} + [M_1 \leftrightarrow M_2]) \\ &\quad + T_i^{II} \star f_B \Phi_B \star f_{M_1} \Phi_{M_1} \star f_{M_2} \Phi_{M_2}, \end{aligned} \quad (2.3)$$

where Φ_i are leading twist light-cone distribution amplitudes (LCDA), and $F^{B \rightarrow M_i}$ are form factors; the star products imply an integration over the light-cone momentum fractions of the constituent quarks inside the mesons. $T_i^{\text{I,II}}$ are hard scattering kernels, calculable in perturbation theory. T_i^{I} starts at tree level and, at higher order in α_s , contains

“non-factorizable” corrections from hard-gluon exchange or quark loops. Hard, “non-factorizable” interactions involving the spectator quark are part of T_i^{II} , which start at order α_s . The hard-scattering kernels and the light-cone distribution amplitudes depend on a factorization scale and scheme, that is not shown explicitly in the notation of (2.3).

It is important to note that the factorization formula does not imply that hadronic B decays are perturbative in nature. Dominant soft contributions to the decay amplitudes exist, which cannot be controlled in a hard-scattering approach. However, at leading power in Λ_{QCD}/m_b all these effects are contained in the semileptonic form factors and the light-cone distribution amplitudes; the advantage of the QCD factorization approach is that these non-perturbative parameters are simpler in structure than the original matrix elements, or they are process independent and can be either extracted from experiment, or calculated with other theoretical methods, like lattice QCD or QCD sum rules.

While at leading order in α_s the result of naive factorization is recovered, at higher order in α_s the kernels $T_i^{\text{I,II}}$ contain imaginary parts, which determine the strong rescattering phases of the decay amplitudes. The rescattering phases are therefore either perturbative, or power suppressed. Currently, both kernels $T_i^{\text{I,II}}$ are known at order α_s^2 , which means at two-loops in case of T_i^{I} [141–145], and at one-loop in case of T_i^{II} [136–140].

The factorization formula is derived rigorously in the framework of effective field theories (EFTs), in particular in soft-collinear effective field theory (SCET) in case of B decays into two light mesons. EFTs are a powerful method to deal with problems involving multiple scales. The short-distance kernels $T_i^{\text{I,II}}$ can be determined by extracting the hard and hard-collinear momentum regions from quark decay amplitudes, according to the strategy of expanding Feynman diagrams by regions [76, 77]. The calculation is more transparent, when it is organized as an operator matching calculation in SCET [91, 94, 136, 137]. In particular, because of the presence of an additional scale $\sqrt{m_b \Lambda_{\text{QCD}}}$, the kernel T_i^{II} factorizes further according to [91, 92]

$$T_i^{\text{II}} = H_i^{\text{II}} \star J, \quad (2.4)$$

which involves a convolution of the hard scattering kernels H_i^{II} with a universal jet function J . In the context of SCET it is possible to give a rigorous operatorial definition of these quantities, including the form factors and the light-cone distributions amplitudes.

One should note that the factorization formula (2.3) is valid at leading order in Λ_{QCD}/m_b . Reliable phenomenological analyses in fact require the inclusion of some sub-leading terms, which have large normalization factors; *e.g.*, in case of the pion

$$r_\chi^\pi(\mu) = \frac{2m_\pi^2}{m_b(\mu)(m_u + m_d)(\mu)} \sim \frac{\Lambda_{\text{QCD}}}{m_b}; \quad (2.5)$$

for realistic b quark masses these “chirally enhanced” terms are not much suppressed numerically, and have to be included in phenomenological analyses [63, 65, 67]. This is important, *e.g.*, to obtain the large branching fractions of penguin-dominated decays like $B \rightarrow \pi K$, but it causes some conceptual problems. Factorization in fact is not expected to hold beyond leading power in Λ_{QCD}/m_b , and indeed some of these chirally enhanced terms violate it. The violation appears explicitly as an endpoint divergence in the convolution integrals of twist-3 distribution amplitudes with the kernels T_i^{II} ; this implies a

non-factorizable soft interaction with the spectator quark, while M_1 is formed in a highly asymmetric configuration, in which one of the quarks carries almost all the momentum of the meson. Similar factorization-breaking effects occur in weak annihilation contributions, which correspond to configurations in which the b quark annihilates with the spectator antiquark of the B meson, and which are also suppressed by at least one power of Λ_{QCD}/m_b . These corrections are usually parameterized in an ad hoc way, and the variation of the corresponding parameters introduces a considerable uncertainty in the estimates of branching ratios and other phenomenological observables. Nevertheless, it has been proven that these factorization-breaking terms do not render the framework unproductive; in particular, so far no indication that these corrections should be outside the range specified by the error estimate has come from experimental data [67].

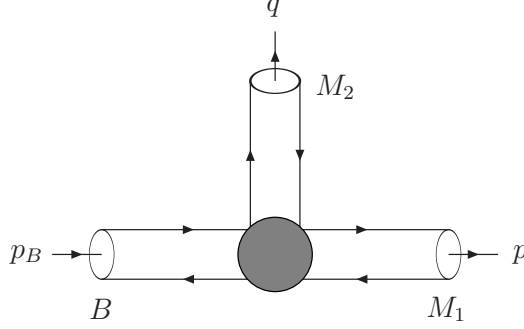
Non-leptonic B decays have a rich phenomenology, because of the large number of final states and observables. Having an overall understanding of them in the context of QCD factorization increases our confidence in the reliability of the approach, and sets limits for it. The colour transparency argument in fact does not apply to all types of non-leptonic decays, *e.g.* in case of a heavy meson M_2 , so that there is no factorization for these classes of decays; similarly, endpoint divergences appear already at leading order in case of the transverse amplitudes of B decays into two vector mesons, rendering the calculations of these decays amplitudes more uncertain. Further developments in the application of QCD factorization to the phenomenology of non-leptonic B decays, including decays into final states containing scalar and axial-vector mesons can be found in [146, 147, 182–187]

2.2 Aspects of QCD Factorization

In this section we provide a qualitative description of factorization in non-leptonic B decays, analysing the relevant diagrams contributing to the decays with the method of expansion by regions [76, 77], according to which the various contributions are separated into their hard, soft, and collinear part. The qualitative evaluation of these contributions leads to the power-counting arguments that justify the factorized structure of (2.3), and motivates the description of the amplitudes in terms of the non-perturbative objects entering in the factorization formula.

2.2.1 Non-perturbative parameters

We start our discussion with a short description of the non-perturbative parameters entering in the factorization formula, motivating their use in non-leptonic B decays. In the following pictures the two lines directed above represent the meson M_2 , that will be assumed to be a π^- , for definiteness. The two lines on the left represent the $\bar{B}_d$ meson, the lower line being the spectator ($\bar{d}$) quark. The two lines on the right represent the meson M_1 , that will be assumed to be a D^+ or a π^+ in case of a heavy or light meson, respectively. The black square represents the weak decay vertex for $b \rightarrow q\bar{u}d$ transitions, with $q = u, c$. In order to set notation, we assign momenta p_B to the B meson, p to the meson M_1 and q to the meson M_2 , according to Fig. 2.2. One has $q_+ = q \cdot n_- \sim m_b$, where $n_{\pm} = (1, 0, 0, \pm 1)$ are light-like vectors defining the directions of the decaying mesons, so

**Figure 2.2:** Kinematics and notation.

that $p = m_B n_-/2$, $q = m_B n_+/2$ when meson masses are neglected (in which case M_1 is light). Moreover, we define the quark and antiquark momenta in the meson M_2 as

$$l_q = uq + l_\perp - \frac{l_\perp^2}{2u m_B} n_-, \quad l_{\bar{q}} = \bar{u}q - l_\perp - \frac{l_\perp^2}{2\bar{u} m_B} n_-, \quad (2.6)$$

where the notation $\bar{u} = 1 - u$ is adopted; note that $q \neq l_q + l_{\bar{q}}$, but the off-shellness $(l_q + l_{\bar{q}})^2$ is of the same order as the light meson mass, which can be neglected at leading power in $1/m_B$. A similar decomposition is used for M_1 in terms of v , p and $l'_\perp$. l_s will denote the momentum of the spectator quark.

The $B \rightarrow M_1$ form factor

We follow here the description given in [64] and the analyses performed in [148, 150]. Further details and references can be found in [149, 151–156]. The form factors $F_j^{B \rightarrow M}(q^2)$ arise in the decomposition of matrix elements of the form

$$\langle M(p) | \bar{q} \Gamma b | \bar{B}(p_B) \rangle, \quad (2.7)$$

where Γ represents any irreducible Dirac matrix. As an example we show here the decomposition of the matrix element of the vector current in case of a pseudoscalar meson $M = P$, that is conventionally parameterized in terms of two scalar form factors:

$$\begin{aligned} \langle P(p) | \bar{q} \gamma^\mu b | \bar{B}(p_B) \rangle &= F_+^{B \rightarrow P}(q^2) \left(p_B^\mu + p^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right) \\ &+ F_0^{B \rightarrow P} \frac{m_B^2 - m_P^2}{q^2} q^\mu, \end{aligned} \quad (2.8)$$

with $q = p_B - p$; m_P and m_B are the masses of the pseudoscalar meson P and the B meson respectively. For $q^2 = 0$ the two form factors coincide, $F_+^{B \rightarrow P}(0) = F_0^{B \rightarrow P}(0)$.

The evaluation of matrix elements (2.7) requires (at order α_s) the calculation of the diagrams appearing in Fig 2.3. The diagrams (a) and (b) represent a soft contribution, while (c) and (d) can give an hard or a soft contribution, depending on the energy of the

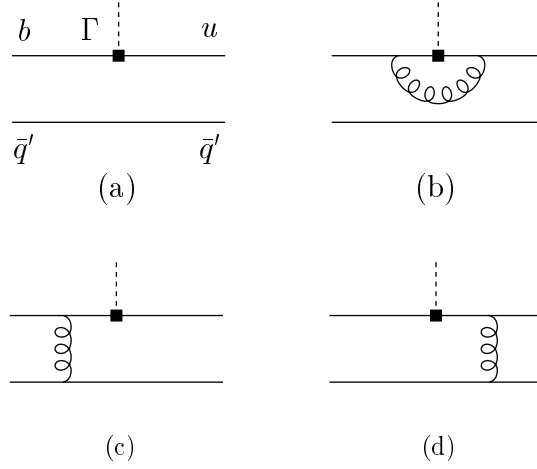


Figure 2.3: Various contributions to the $B \rightarrow M_1$ form factor, taken from [148]. The dashed line represents the weak current. The two lines to the left belong to the B meson, the ones to the right to the recoil meson M_1 . (a) Soft contribution; (b) hard vertex renormalization; (c,d) hard spectator interaction.

exchanged gluon. In order to understand the relevance of the form factor in the factorization formula, one has to determine the scaling behaviour of the two types of contributions. A leading contribution coming solely from the hard gluon exchange would imply that non-leptonic B decays could be treated completely in the hard-scattering approach, *i.e.* only the second term in the factorization formula would appear. It is possible to show, however, that this is not the case, *i.e.* the form factor receives a leading contribution from the soft gluon exchange too, which does not admit a perturbative treatment. In the case of a $B \rightarrow M$ transition where M is heavy, as if $M = D$, this is actually the only leading contribution. The exchange of a hard gluon in fact makes the final spectator quark to have momentum of order m_b and this has no overlap with a heavy meson (where the spectator quark is expected to move “softly” around the heavy quark). As a consequence, in the case of a heavy meson M_1 , like in the $\bar{B}_d \rightarrow D^+ \pi^-$ decay, only the first term appears in the factorization formula. In case of M_1 light, *e.g.* $M_1 = \pi$, the form factor receives leading contributions from both the exchange of a hard and a soft gluon; a simple power counting argument [148] shows that the two contributions are of order¹

$$F_{+,0;\text{hard}}^{B \rightarrow \pi}(0) \sim \alpha_s \left(\sqrt{m_b \Lambda_{\text{QCD}}} \right) \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^{3/2} \quad (2.9)$$

for the hard region, and of order

$$F_{+,0;\text{soft}}^{B \rightarrow \pi}(0) \sim \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^{3/2}. \quad (2.10)$$

¹We will not deal here with the argument according to which the soft contribution could be suppressed by a Sudakov form factor, if logarithmic effects in m_b were taken into account. We remind only that this suppression should be negligible for realistic a b quark mass, $m_b \sim 5\text{GeV}$.

for the soft region. Strictly speaking, the first contribution should appear as part of T_i^{II} ; however, an unambiguous separation of the two terms above cannot be achieved, because of the appearance of endpoint divergences in the hard region, in the limit in which the gluon becomes soft². The solution adopted here is to bypass the problem, considering the $F_j^{B \rightarrow M}(q^2)$ appearing in the parametrization of (2.8) and (2.3) as the *physical* form factor [64, 148, 150]. One has therefore to omit the hard contribution to the form factor in the hard scattering kernels T_i^{II} . A direct advantage of using a physical form factor is that they are directly related to measurable quantities, or to the form factors obtained from lattice QCD or QCD sum rules.

Light-cone distribution amplitudes

We follow the discussion given in [64]; further details on the precise definition of the meson distribution amplitudes can be found in [148, 150]. Light-cone distribution amplitudes (LCDAs) play the same role for hard exclusive processes that parton distributions play for inclusive ones. The leading-twist distribution amplitudes that appear at leading order in the Λ_{QCD}/m_b expansion are given by two-particle operators with a certain helicity structure, which is determined by the angular momentum of the meson and the fact that the spinor of an energetic quark has only two large components [161, 162]. As an example, the leading-twist LCDA of a pseudoscalar and a longitudinally polarized vector mesons with flavour content $\bar{q}q'$ read

$$\begin{aligned} \langle P(q) | \bar{q}(y)_\alpha q'(x)_\beta | 0 \rangle |_{(x-y)^2=0} &= \frac{if_P}{4} (\not{x} \gamma_5)_{\beta\alpha} \int_0^1 du e^{i(\bar{u}qx + uqy)} \Phi_P(u, \mu), \\ \langle V_\parallel(q) | \bar{q}(y)_\alpha q'(x)_\beta | 0 \rangle |_{(x-y)^2=0} &= -\frac{if_V}{4} \not{x} \beta_\alpha \int_0^1 du e^{i(\bar{u}qx + uqy)} \Phi_\parallel(u, \mu), \end{aligned} \quad (2.11)$$

where the equality sign is understood as “equal up to higher-twist terms”. The operator on the left is a colour singlet. The distribution amplitudes are normalized as $\int_0^1 du \Phi_P(u, \mu) = 1$. f_P , f_V are decay constants, and they refer to the normalization in which $f_\pi = 131 \text{ MeV}$. In (2.11) a path-ordered exponential is implicitly understood, in order to connect the two quark fields at different positions and to render the light-cone operators gauge invariant. The LCDAs are scale dependent. While an exact definition will be given later, their basic properties can be derived from their asymptotic form, that is defined as the limit in which the renormalization scale is sent to infinity. In this case one has *e.g.*

$$\Phi_X(u, \mu) \stackrel{\mu \rightarrow \infty}{=} 6u\bar{u}, \quad (2.12)$$

for both $X = P, V_\parallel$. The representation of a meson in terms of LCDA assumes that it can be described entirely by its longitudinal momentum components and that its preferred configuration corresponds to a state, in which the total momentum of the meson is shared equally between the valence quarks. As a consequence, the use of LCDAs in non-leptonic B decays is justified if one can prove that there is no leading contribution in $1/m_b$ from

²Endpoint divergences arise when one calculates the form factor in a light-cone distribution amplitudes model.

the endpoint regions $u \sim \Lambda_{\text{QCD}}/m_b$ and $\bar{u} \sim \Lambda_{\text{QCD}}/m_b$, and that one can set $l_\perp = 0$ in the amplitude. These points have to be investigated in each diagram contributing to the decay amplitude. It is useful to note here that neglecting transverse momenta requires to consider them to be of order Λ_{QCD} when comparing terms of different magnitude in the scattering amplitude. In QCD this is only an approximation, since transverse momenta are distributed logarithmically up to the hard scale. One can prove however that contributions that violate the assumption of limited transverse momenta can be absorbed into the meson LCDAs; the statement that transverse momenta count as Λ_{QCD} is to be understood after these subtractions have been performed.

The proof of these points requires the knowledge of the power-counting rules for the LCDAs, which can be derived from their asymptotic form. From (2.12) one derives

$$\Phi(u) \sim \begin{cases} 1, & \text{generic } u, \\ \Lambda_{\text{QCD}}/m_b & u, \bar{u} \sim \Lambda_{\text{QCD}}/m_b. \end{cases} \quad (2.13)$$

Endpoint regions are defined as the regions where $u(\bar{u})$ is of order Λ_{QCD}/m_b , such that the quark(antiquark) momentum is of order Λ_{QCD} . The endpoint region has size of order Λ_{QCD}/m_b , so that the endpoint suppression is of order $(\Lambda_{\text{QCD}}/m_b)^2$; this suppression has to be weighted against the potential enhancement of the partonic amplitude when one of the propagator approaches the mass-shell. In case of the B meson LCDA³, see *e.g.* [164–167], the scaling is different, and it is given by

$$\Phi_B(\xi) \sim \begin{cases} m_b/\Lambda_{\text{QCD}}, & \xi \sim \Lambda_{\text{QCD}}/m_b, \\ 0 & \xi \sim 1. \end{cases} \quad (2.14)$$

2.2.2 Non-leptonic decay amplitudes

To complete the proof of the factorization formula, one has to evaluate diagram by diagram, checking if the QCD interaction can be separated into the convolution of the hard scattering kernels with LCDA. According to the illustrative purposes of this section, we give a qualitative discussion of the lowest order and one-gluon exchange diagrams, following [64].

Lowest order

There is a single diagram contributing at lowest order, shown in figure 2.4. The spectator quark in the B meson is soft and it is absorbed as soft in the meson M_1 . Such a contribution belongs to the first term in the factorization formula, and involves the form factor. The hard subprocess is just given by the insertion of a four-quark operator and it does not depend on the longitudinal momentum fraction u of the two quarks that form the meson M_2 . At lowest order therefore T_i^{I} is independent of u and the convolution integral reduces to the normalization condition for the wave-function of the meson M_2 . The power counting for the lowest order diagram gives

$$\mathcal{A}(\bar{B}_d \rightarrow D^+ \pi^-) \sim G_F m_b^2 F^{B \rightarrow D}(0) f_\pi \sim G_F m_b^2 \Lambda_{\text{QCD}} \quad (2.15)$$

³the precise definition in terms of matrix element of SCET operators will be given in section 2.3.3

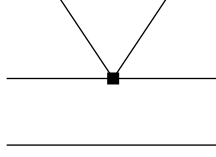


Figure 2.4: Leading-order contribution to the hard scattering kernel $T_i^I(u)$. Figures 2.4-2.9 are taken from [64].

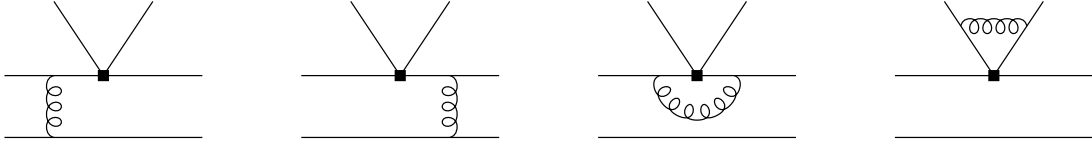


Figure 2.5: Diagrams at order α_s that need not to be calculated.

in case of a heavy meson $M_1 = D$, and

$$\mathcal{A}(\bar{B}_d \rightarrow \pi^+ \pi^-) \sim G_F m_b^2 F^{B \rightarrow \pi}(0) f_\pi \sim G_F m_b^{1/2} \Lambda_{\text{QCD}}^{5/2} \quad (2.16)$$

in case of a light meson $M_1 = \pi$. Higher-order contribution (in α_s or Λ_{QCD}/m_b) must be compared with these scaling rules.

One-gluon exchange

The lowest order contribution discussed above corresponds to the naive factorization approximation. Its accuracy can be proven in QCD factorization, if we are able to show that higher order terms in the α_s and Λ_{QCD}/m_b expansions are either suppressed in one of these two parameters, or already contained in the definition of the form factor and the decay constant of M_2 .

Diagrams with one-gluon exchange are shown in figures 2.5 - 2.9. Fig. 2.5 involves only “factorizable” gluons, since no exchange occurs between the $B \rightarrow M_1$ and the M_2 subsystem. It is easy to show that these diagrams need not to be calculated: they renormalize the form factor (first three), which however is the physical form factor in our conventions, and the $(\bar{u}d)$ light-quark $V - A$ current (last diagram), which is conserved. In order to analyse the factorization properties of the decay amplitude we have to focus on the diagrams appearing in the successive figures, which contain “non-factorizable” gluons that connect the $B \rightarrow M_1$ and M_2 system. These gluons violate the naive factorization ansatz; the purpose of the factorization formula in (2.3) is precisely to show that their contribution can be calculated and included as an α_s correction into the hard scattering kernels $T_i^{\text{I,II}}(u)$.

The diagrams in figure 2.6 represent a “vertex correction”, and should be included into the scattering kernel T_i^{I} . This is proven if (a) the transverse momentum of the quarks that form M_2 can be neglected at leading power, *i.e.* the two momenta in (2.6) can be

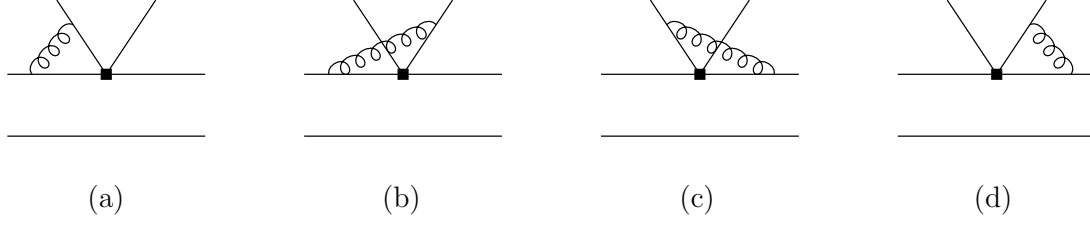


Figure 2.6: “Non-factorizable” vertex correction.

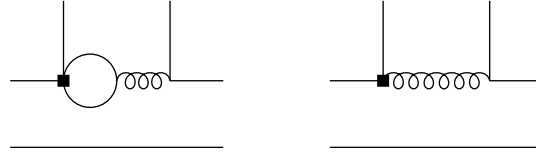


Figure 2.7: Diagram with a “penguin” contraction. The second diagram represents a contribution from the chromomagnetic dipole operator in the weak Hamiltonian.

approximated by uq and $\bar{u}q$, respectively, and if (b) the contribution from the soft and collinear gluons is power suppressed. The first condition guarantees that only a convolution in the longitudinal momentum fraction u appears in the factorization formula (*i.e.*, that LCDAs can be used), while the second implies that no infrared (collinear) divergences should appear in the sum of the four diagrams, *i.e.*, that one can treat the contribution perturbatively.

Neither of the two conditions is satisfied by the diagrams individually, which exhibit infrared and collinear divergences. However, an explicit calculation shows that all the divergences cancel in the sum of the four diagrams. This is a technical manifestation of colour transparency, according to which soft-gluon interactions with the emitted colour-singlet $\bar{u}d$ pair are suppressed, because they interact only with the colour dipole moment of the compact light-quark pair. The argument however applies only if the $\bar{u}d$ pair is compact. Factorization would fail if the emitted pion was formed in a highly asymmetric configuration, in which one of the quark would carry almost all of the pion’s momentum. Such asymmetric configurations however are suppressed by a probability of $(\Lambda_{\text{QCD}}/m_b)^2$, and would be important only if the hard-scattering amplitude favoured the production of asymmetric pairs, *i.e.* if $T_i^{\text{I}} \sim 1/u^2$ for $u \rightarrow 0$, ($T_i^{\text{I}} \sim 1/\bar{u}^2$ for $u \rightarrow 1$); this is not the case for the vertex correction.

Since the hard gluons transfer large momentum to the quarks that form the emission pair, the hard-scattering amplitude results in a nontrivial convolution with the pion distribution amplitude, implying that “non-factorizable” contributions are not universal, *i.e.* they depend on the type of meson M_2 considered.

The “penguin contraction” diagrams in Fig. 2.7 exist for $\bar{B}_d \rightarrow \pi^+\pi^-$ but not for $\bar{B}_d \rightarrow D^+\pi^-$. In order to show that these contributions are part of $T_i^{\text{I}}(u)$ too, one has to verify that all the internal lines are hard. This is proven, noting that the quark that goes

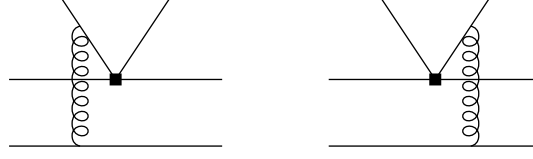


Figure 2.8: “Non-factorizable” spectator interaction.

into the recoil meson π^+ must be energetic, in order to form an energetic pion, since the spectator quark is soft; on the other hand, configurations in which the other quark is soft are power suppressed by the endpoint behaviour of the pion distribution amplitude. This means that the gluon in Fig. 2.7 must have a large virtuality $\sim \bar{u}m_b$. This argument alone does not guarantee that both the quarks in the loop are hard, but a comparison with the structure of the vacuum polarization diagram shows that a configuration in which one quark carries a large momentum, while the other is soft, is power suppressed by $(\Lambda_{\text{QCD}}/m_b)^2$ with respect to the one in which both quarks are hard.

Contributions to the second term in the factorization formula arise because of the “hard spectator interaction” diagrams in Fig. 2.8. As in case of the vertex correction, a leading contribution from soft gluon exchange would prevent factorization. The mechanism of colour transparency however guarantees also in this case the cancellation of soft contributions in the sum of the two diagrams in Fig. 2.8.

More in detail, in case of the $\bar{B} \rightarrow D^+\pi^-$ decay, the sum of the two diagrams when the gluon is soft reads

$$\begin{aligned} \mathcal{A}(\bar{B} \rightarrow D^+\pi^-)_{\text{h.s.i}} &\sim G_F f_\pi f_D f_B \alpha_s \int_0^1 \frac{d\xi}{\xi} \Phi_B(\xi) \int_0^1 \frac{d\eta}{\eta} \Phi_D(\eta) \int_0^1 \frac{du}{u} \Phi_\pi(u) \\ &\sim G_F \alpha_s m_b \Lambda_{\text{QCD}}^2, \end{aligned} \quad (2.17)$$

that is power suppressed with respect to (2.15). Strictly speaking, since the gluon has virtuality of order $\xi \eta m_b^2 \sim \Lambda_{\text{QCD}}^2$, the analysis in terms of LCDA is not justified, but it can be used to estimate the size of the soft contribution. Considering now a hard gluon exchange, this makes the spectator quark energetic. As already discussed for the form factor, this configuration has no overlap with $M_1 = D$, so that in case of an heavy meson M_1 the second term in the factorization formula is absent. In case of a decay into two light mesons, such as $\bar{B} \rightarrow \pi^+\pi^-$, one obtains

$$\begin{aligned} \mathcal{A}(\bar{B} \rightarrow \pi^+\pi^-)_{\text{h.s.i}} &\sim G_F f_\pi^2 f_B \alpha_s \int_0^1 \frac{d\xi}{\xi} \Phi_{B1}(\xi) \int_0^1 \frac{dv}{v} \Phi_\pi(v) \int_0^1 \frac{du}{u} \Phi_\pi(u) \\ &\sim \begin{cases} G_F \alpha_s m_b^{1/2} \Lambda_{\text{QCD}}^{5/2}, & \text{hard gluon,} \\ G_F \alpha_s m_b^{-1/2} \Lambda_{\text{QCD}}^{7/2}, & \text{soft gluon.} \end{cases} \end{aligned} \quad (2.18)$$

The soft contribution is suppressed; the hard contribution has the same power counting as the leading order (2.16), times an additional factor of α_s , and therefore it has to be included in T_i^{II} .

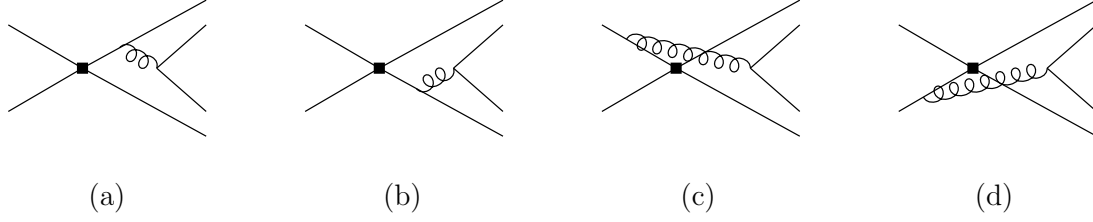


Figure 2.9: *Annihilation diagrams.*

So far we have proven factorization for the vertex, the penguin and the hard spectator interaction corrections, which in the heavy-quark limit are all of the same order in Λ_{QCD}/m_b as the leading contribution, but are suppressed by one power of α_s . It remains to analyse the *annihilation* diagrams in Fig. 2.9, whose hard contribution, if leading, should contribute to T_i^{II} , while a soft one would violate factorization. One finds however that the annihilation diagrams are always suppressed by at least one power of Λ_{QCD}/m_b .

In case of the $\bar{B} \rightarrow \pi^+\pi^-$ decay, this happens because there is only one leading configuration, in which the gluon has virtuality of order m_b^2 and all the quarks are energetic. This configuration scales as

$$G_F f_\pi^2 f_B \alpha_s \sim G_F \alpha_s m_b^{-1/2} \Lambda_{\text{QCD}}^{7/2}, \quad (2.19)$$

which is one power of Λ_{QCD}/m_b smaller than (2.16). Other configurations, in which the gluon has virtuality of order $m_b \Lambda_{\text{QCD}}$, and one of the quarks is soft, give an amplitude that is enhanced by one power of m_b/Λ_{QCD} , but this is overcompensated by a suppression of order $(\Lambda_{\text{QCD}}/m_b)^2$ from the endpoint behaviour of the LCDA, and from the small region of phase space considered. Configurations with two final-state soft quarks are even more suppressed.

In case of $\bar{B} \rightarrow D^+\pi^-$ decay the power counting is different because the light quark that goes into the D meson must be soft. Hence, the virtuality of the gluon cannot be larger than $m_b \Lambda_{\text{QCD}}$. It can be shown that the contribution to the amplitude is still suppressed, noting that it scales like

$$\mathcal{A}(\bar{B} \rightarrow D^+\pi^-)_{\text{ann}} \sim G_F f_\pi f_D f_B \alpha_s \int_0^1 d\xi d\eta du \Phi_B(\xi) \Phi_D(\eta) \Phi_\pi(u) T_{\text{ann}}(\xi, \eta, u), \quad (2.20)$$

where the dimensionless function $T^{\text{ann}}(\xi, \eta, u)$ is a product of propagators and vertices. The product of decay constants scales like $\Lambda_{\text{QCD}}^4/m_b$; since $d\xi \Phi_B(\xi)$ and $d\eta \Phi_D(\eta)$ scale like 1, while $du \Phi_\pi(u)$ is never larger than one, $T^{\text{ann}}(\xi, \eta, u)$ should be of order larger than $(m_b/\Lambda_{\text{QCD}})^3$ to give a contribution scaling like (2.15). Since $T^{\text{ann}}(\xi, \eta, u)$ contains only two propagators, this can be achieved only if both quarks in which the gluon split are soft, in which case $T^{\text{ann}}(\xi, \eta, u) \sim (m_b/\Lambda_{\text{QCD}})^4$. But then $du \Phi_\pi(u) \sim (\Lambda_{\text{QCD}}/m_b)^2$ and the whole contribution is power suppressed.

Non-leptonic decays when M_2 is not light

So far we have analysed the diagrams contributing to non-leptonic B decays in case of two light mesons, or a heavy M_1 and a light M_2 mesons. It is interesting to consider also the case in which M_2 is a quarkonium (“onium”), like a J/ψ , or a heavy meson like a D . While we are going to explain why factorization does not hold for the latter, the decay into an onium M_2 and a light meson M_1 is particularly interesting. In fact, even if a qualitative power counting analysis confirms factorization for these decays, the presence of an additional scale makes factorization to work differently, depending on the onium angular momentum state considered. This point constitutes the motivation for the analysis performed in chapter 3.

We start considering a decay into a heavy M_2 , like in case of $\bar{B}_d \rightarrow \pi^0 D^0$, $\bar{B}_d \rightarrow D^+ D^-$ decays. Factorization is problematic in this case, because the heavy D meson has large overlap with the $\bar{B} \rightarrow \pi$ or $\bar{B} \rightarrow D$ transitions. The “technical” reason is that, in this case, the cancellation of infrared and collinear divergencies in the diagrams of figures 2.6 and 2.8 does not happen. Explicitly, the coupling of the gluon to the quark-antiquark pair in the meson $M_2 = D$ reads

$$J_\lambda = \frac{\gamma_\lambda(l_q + \not{k} + m_c)\Gamma}{2l_q \cdot k + k^2} - \frac{\Gamma(l_{\bar{q}} + \not{k})\gamma_\lambda}{2l_{\bar{q}} \cdot k + k^2}, \quad (2.21)$$

from which one can see that the cancellation of this contribution, for k soft, occurs in case of a light meson (in which case the m_c term is absent) due to the symmetry between l_q and $l_{\bar{q}}$, which are dominated by their longitudinal components. In case of a heavy M_2 however this cancellation cannot happen, because l_q and $l_{\bar{q}}$ are now asymmetric, since all the components of the light antiquark momentum $l_{\bar{q}}$ are soft, *i.e.* of order Λ_{QCD}/m_b , while the zero component of the heavy-quark momentum l_q is of order of $m_c \sim m_b$. One obtains

$$J_\lambda \sim \frac{\delta_{\lambda 0}\Gamma}{k_0} - \frac{\Gamma(l_{\bar{q}} + \not{k})\gamma_\lambda}{2l_{\bar{q}} \cdot k + k^2} \sim \frac{1}{\Lambda_{\text{QCD}}}, \quad (2.22)$$

and the soft cancellation does not occur. The emitted D meson does not factorize from the rest of the process, and no factorization formula applies in this case. In particular, this implies that naive factorization itself is not expected to work. There are decays into a heavy meson that receive contributions from both the configurations in which the D meson is either M_1 , or M_2 , as in case of the $B^- \rightarrow D^0 \pi^-$ decay. It is possible to show that, for this type of decay, the non-factorizing part of the amplitude is power-suppressed with respect to the factorizing one, and in this sense these decays can be considered factorizable, since the non-factorizing part constitutes a subleading correction.

The case where M_2 is a heavy quarkonium (H) is special, because of the appearance of additional scales beyond m_b , *i.e.* $m_c \sim m_b \rightarrow \infty$, the binding energy $m_c v^2$ and the typical momentum scale of a c quark inside the meson, $m_c v$, where $v \ll 1$ is the velocity of the quark.

In case of an onium state the coupling to the gluon analogous to (2.21) is

$$J_\lambda = \frac{\gamma_\lambda(l_q + \not{k} + m_c)\Gamma}{(l_q + k)^2 - m_c^2} - \frac{\Gamma(l_{\bar{q}} + \not{k} - m_c)\gamma_\lambda}{(l_{\bar{q}} + k)^2 - m_c^2}, \quad (2.23)$$

where $l_q = q/2 + p$, $l_{\bar{q}} = q/2 - p$ in the heavy quark limit, with $p = m_c v \sim m_c \alpha_s$ representing the momenta of the charm-anticharm pair inside the onium, of order of the inverse size of the meson. In this case the quark-antiquark pair is off-shell by an amount of order $\delta \sim (m_c \alpha_s)^2$. When k is soft, $k \sim \Lambda_{\text{QCD}}$, the denominators in (2.23) are dominated by the off-shellness δ , and the current simplifies to

$$J_\lambda \sim \frac{1}{\delta} (4p_\lambda \Gamma + (-\not{l}_q + m_c) \gamma_\lambda \Gamma - \Gamma \gamma_\lambda (-\not{l}_{\bar{q}} + m_c)) \sim \frac{4p_\lambda \Gamma}{\delta} \sim \frac{1}{m_c \alpha_s}. \quad (2.24)$$

This result is obtained noting that the two terms $-\not{l}_q + m_c$, $-\not{l}_{\bar{q}} + m_c$ give a contribution of order $m_c \alpha_s^2$. (2.24) shows that factorization holds, since the soft contribution is suppressed with respect to the leading contribution. The naive argument above however does not allow one to discuss decays into different charmonium angular momentum states. Indeed, it was found that the estimation above works for S -wave states (*e.g.*, a J/ψ) [188–190], but not for P -wave states (*e.g.*, a χ_{c0}) [191–194, 196, 197], motivating for further analysis, which has been developed in our work [125], reported in chapter 3. The key point is to fully exploit the presence of the charmonium scales $m_c v$, $m_c v^2$. This is realized constructing an effective field theory in which one scale is integrated out at each step. In this framework, one finds that gluons decouple at a scale m_b in case of S -waves, being soft gluon exchange suppressed by a factor of $\alpha_s v^2$, while in case of P -waves gluons decouple only at a scale $m_c v^2$, being contributions from soft gluon of this energy of the same order as the contributions from gluons of energy m_b .

Non-leptonic decays into two vector mesons

QCD factorization for the decays into two vector meson has been discussed in [176–181]. These decays obey the factorization formula (2.3), but they have a much richer structure due to their polarization states, so that the decay amplitude can be decomposed into three helicity amplitudes. A QCD factorization analysis shows that the different helicity amplitudes have quite different patterns, already at the naive factorization level. At leading order one finds a hierarchy of helicity amplitudes, given by

$$\bar{\mathcal{A}}_0 : \bar{\mathcal{A}}_- : \bar{\mathcal{A}}_+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^2, \quad (2.25)$$

that is a consequence of the left-handedness of the weak interactions and the fact that QCD interactions conserve helicity. This is particularly interesting in view of the search for new physics, because different helicity structures of the primary transition could result in a different helicity hierarchy in the decay amplitudes. Despite this possibility, one should note that the calculation of the transverse helicity amplitudes in QCD factorization is more involved than the longitudinal one, the intuitive reason being the fact that they are already subleading effects.

As a short survey, we point out the main qualitative differences that one finds in the calculation of the transverse helicity amplitudes with respect to the longitudinal one, which follows the same pattern as the decay amplitudes into a pseudoscalar and a vector meson [179]:

- The spectator scattering is divergent even at leading order (in case of a decay into two pseudoscalar mesons, divergences arise only at subleading order.). Because of the uncertainties related to the estimate of the divergent contribution, in some cases one can only estimate the order of magnitude of the transverse amplitudes (*e.g.* in case of colour-suppressed tree amplitudes).
- The positive helicity amplitude in the Standard Model is even power suppressed with respect to the negative one. A reliable calculation therefore requires to keep the transverse momenta of the quarks that are collinear to both mesons non-zero, and such a calculation has not yet been performed. In general, because of its double power-suppression, it is consistent to set it to zero in the Standard Model.
- The helicity hierarchy in (2.25) is violated at subleading order by electromagnetic effects [178]. A transverse polarization amplitude in fact can be generated by a short-distance transition to a vector meson and a photon with small virtuality, which subsequently converts into a vector meson. This transition is enhanced by a factor $(m_b/\Lambda_{\text{QCD}})^2$ due to the large photon propagator, resulting in the parametric relation

$$\bar{\mathcal{A}}_0 : \bar{\mathcal{A}}_- : \bar{\mathcal{A}}_+ = 1 : \frac{\alpha_{\text{em}} m_b}{\Lambda_{\text{QCD}}} : \alpha_{\text{em}}, \quad (2.26)$$

so that formally the negative helicity amplitude is leading in the heavy-quark limit. As a consequence, the colour-allowed electroweak penguin amplitude is completely different from its naive-factorization value.

- The weak annihilation contribution to the QCD penguin amplitude in the negative helicity amplitude is numerically larger than in case of $B \rightarrow PP$, PV decays. This is another additional source of uncertainties in the calculation of the transverse decay amplitudes.

Non-leading Fock states

A complete proof of the factorization formula requires to analyse the possibility of having leading contributions from non-leading Fock states, *i.e.* final states formed by more particles than the valence quark-antiquark pair described by the LCDAs in (2.3), such as a quark-antiquark-gluon ($q\bar{q}g$) state.

It is easy to show that none of these contributions can be of the same order as the leading term in (2.15), (2.16). Additional valence gluons in the meson M_2 , in fact, must be collinear and carry a finite fraction of the meson momenta, or they must be soft in case of the B meson. In both cases this increases the number of off-shell propagators in a given diagram, even for the additional soft gluons in the B meson, because when connected to the hard scattering amplitudes, the virtuality of the internal propagators is in any case of order $m_b\Lambda_{\text{QCD}}$.

One is left with the possibility of having a B meson state formed by additional gluons, which are absorbed as additional gluons in the meson M_1 . There is no suppression for such contributions, but once more these contributions are adsorbed in the physical form factor.

One has instead to show the power suppression of diagrams where one of these (soft) gluons from the soft-overlap goes into the meson M_2 , that would violate factorization. Such a configuration would be formed by a hard quark-antiquark pair in an octet state, together with a soft gluon. One can show by means of a non-local “operator product expansion” that the same cancellation as in case of the vertex correction arises, so that this contribution is power suppressed too. This covers all possible non-leading Fock state contributions to the decay amplitude.

2.2.3 Limitations of the factorization formula

So far we have discussed the factorization properties of non-leptonic B decays, showing the cases in which the factorization formula in (2.3) is valid or not. An important point to keep in mind is that the overall description is valid in the heavy-quark limit $m_b \rightarrow \infty$. In the real world m_b is fixed to $m_b \simeq 5\text{GeV}$, and “non-factorizable” corrections to (2.3) are expected to be of order Λ_{QCD}/m_b . This has to be taken into account, especially if one wants to perform *e.g.* new physics analyses, as we will do in chapters 4, 5 and 6. Therefore, we collect here the relevant cases in which large uncertainties can arise. This is usually the case when the leading factorizable contribution becomes somehow suppressed, due to the presence of small parameters.

- One of the achievement of QCD factorization is that strong phases are either perturbative, or power suppressed. They are important phenomenologically, because they correspond to rescattering processes in the final state, and their knowledge allows the calculation of CP asymmetries. One should note, however, that the perturbative strong phases arise at order α_s . In realistic B mesons, α_s is not much larger than Λ_{QCD}/m_b , so that non-perturbative strong phases could play an important role, too. As a result, the calculation of strong phases has to be thought more as an order-of-magnitude estimate than a precise evaluation; the important point is that strong phases are predicted to be small, since they are of order α_s .
- Depending on the topology of the transition, the leading-order contribution can be colour suppressed with respect to the order- α_s one, or viceversa. In the latter case, that occurs *e.g.* for $\bar{B}_d \rightarrow D^+\pi^-$ and $\bar{B}_d \rightarrow \pi^+\pi^-$, first-order corrections to naive factorization are small, and often QCD predictions are in good agreement with experimental results; eventually, second-order corrections may become as important as the first-order one, even if both are small. However, in the first case, that occurs *e.g.* in case of $\bar{B}_d \rightarrow J/\psi\bar{K}$ and $\bar{B}_d \rightarrow \pi^0\pi^0$, perturbative corrections are large, and the strong phases become more important. Because of the discussion in the item above, however, also power-suppressed contributions become more important, so that larger uncertainties are expected in these decays.
- The weak Hamiltonian contains only one large Wilson coefficient, $C_1 \sim 1$, while all the other short-distance coefficients, especially the QCD and electroweak penguin, are considerably smaller. This pattern has to be kept in mind, because there are decays where power-suppressed contributions come with the larger Wilson coefficient

C_1 . These decays are annihilation dominated, and in this case factorization is not expected to be reliable.

- Some amplitudes can be suppressed by a small combination of CKM matrix elements. This is the case *e.g.* for the $B \rightarrow \pi K$ decays, that receive a large contribution from the QCD penguin operators, despite the small Wilson coefficients, because this part of the amplitude is enhanced by a large CKM factor, while the “tree amplitude” that contains the larger C_1 is CKM suppressed.
- There are a few cases in which power-suppressed contributions are numerically enhanced. This is the case, *e.g.* for contribution of the penguin operator $Q_6 = (\bar{d}_i b_j)_{V-A}(\bar{u}_j u_i)_{V+A}$ to the decay amplitude of $\bar{B}_d \rightarrow \pi^+ \pi^-$, whose leading contribution, corresponding to Fig. 2.4 is

$$\langle \pi^+ \pi^- | (\bar{d}_i b_j)_{V-A} (\bar{u}_j u_i)_{V+A} | \bar{B}_d \rangle = i m_b^2 F_+^{B \rightarrow \pi}(0) f_\pi \times \frac{2\mu_\pi}{m_b}, \quad (2.27)$$

which is formally a Λ_{QCD}/m_b power correction compared to (2.16), but it is numerically large, due to the factor

$$2\mu_\pi = \frac{2m_\pi^2}{m_u + m_d} = -\frac{4\langle \bar{q}q \rangle}{f_\pi^2} \simeq 3\text{GeV}, \quad (2.28)$$

which is much larger than the naive scaling Λ_{QCD} . ($\langle \bar{q}q \rangle = \langle 0 | \bar{u}u | 0 \rangle = \langle 0 | \bar{d}d | 0 \rangle$ is the quark condensate.) There would be no problem if those terms could be included in the factorization formula. However, such terms arise when considering two-particle twist-3 distribution amplitudes, to which the factor μ_π is related. Twist-3 distribution amplitudes have a different endpoint behaviour (they approach a constant), so that when higher α_s corrections are considered, the resulting convolution integrals are divergent. This implies that α_s corrections to (2.27) are of the type $\alpha_s \times \text{logarithmic divergence}$, and therefore they should be considered as leading. The non-factorization of these “chirally-enhanced” terms may introduce a substantial uncertainties in some decay modes.

This concludes our survey of the factorization approach to non-leptonic B decays. Having in mind an overall picture about the arguments in favour of factorization and its limitation, we focus in the rest of the chapter to non-leptonic B decays into two light mesons.

2.3 Non-leptonic B decays into two light mesons

Non-leptonic B decays into two light mesons involve 130 different decay channels. Due to the large number of final states, it is beyond the scope of this introductory chapter to give a complete derivation of all the decay amplitudes in QCD factorization. Rather, the aim of this section is to introduce the quantities in terms of which the decay amplitudes are expressed, and to explain, in some cases, how they are calculated in QCD factorization. We will provide the NLO expression for them, including the relevant power-suppressed contributions. The results described have been derived in the literature in [65–67, 179];

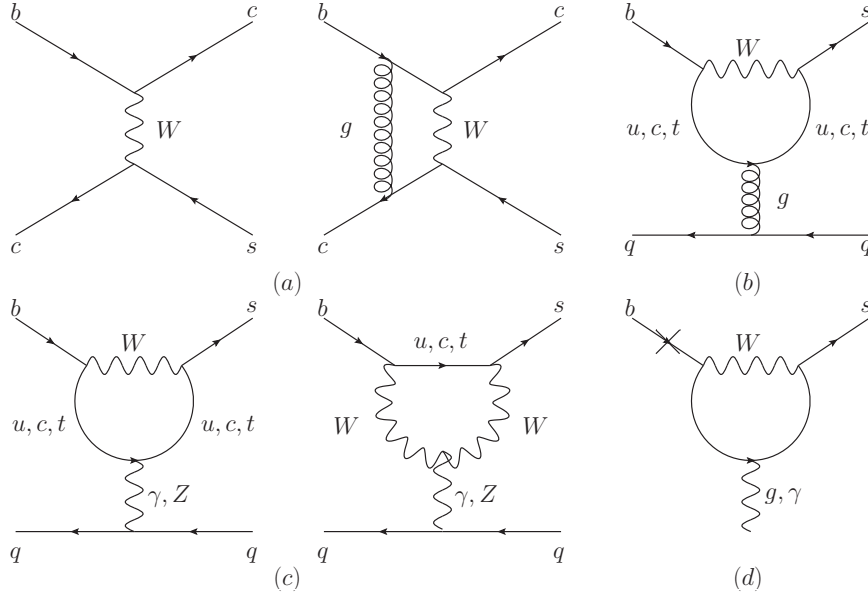


Figure 2.10: Electroweak transitions at the basis of the four-quark operators in the effective Hamiltonian.

the explanation on how these quantities are derived will be given following the more modern approach developed in [136, 137], according to which the evaluation is organized as a series of matching calculations in the framework of the effective field theories (EFT).

2.3.1 Decay amplitudes

Effective weak Hamiltonian

In Fig. 2.10 we show the electroweak transitions responsible for the b -quark decay into lighter quarks. These involve the electroweak bosons W , Z and photons, so that the typical scale of the momenta exchanged is of order M_W . This is much larger than the typical hadronic scale, that is of order of the mass of the decaying hadron, *i.e.* $\sim m_B$. The true picture of a decaying hadron is therefore described by effective point-like vertices, which are represented by local four-fermion operators Q_i multiplied by short-distance coefficients C_i , that have to be regarded as effective coupling constants. The derivation of these effective operators is known as operator product expansion, (OPE); the series of operators is collected into an effective weak Hamiltonian, which in case of non-leptonic decays reads

$$\mathcal{H}_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}, \quad (2.29)$$

where $\lambda_p^{(D)} = V_{pb} V_{pD}^*$ represents a product of elements of the quark mixing (CKM) matrix, and $p = u, c$, while $D = d, s$ depending on the transition. One obtains the effective Weak Hamiltonian above using the unitarity relation $\lambda_u + \lambda_c + \lambda_t = 0$ to eliminate the λ_t term

in favour of λ_u and λ_c . $Q_{1,2}^p$ are the left-handed current-current operators arising from W exchange, fig 2.10 (a), $Q_{3,\dots,6}$, fig 2.10 (b) and $Q_{7,\dots,10}$, fig 2.10 (c) are QCD and electroweak penguin operators respectively. $Q_{7\gamma}$ and Q_{8g} , fig 2.10 (d) are the electromagnetic and chromomagnetic dipole operators. Explicitly, the four-quark operators read

$$\begin{aligned}
Q_1^p &= (\bar{p}b)_{V-A}(\bar{D}p)_{V-A} & Q_2^p &= (\bar{p}_i b_j)_{V-A}(\bar{D}_j p_i)_{V-A} \\
Q_3 &= (\bar{D}b)_{V-A} \sum_q (\bar{q}q)_{V-A} & Q_4 &= (\bar{D}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \\
Q_5 &= (\bar{D}b)_{V-A} \sum_q (\bar{q}q)_{V+A} & Q_6 &= (\bar{D}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \\
Q_7 &= (\bar{D}b)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}q)_{V+A} & Q_8 &= (\bar{D}_i b_j)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}_j q_i)_{V+A} \\
Q_9 &= (\bar{D}b)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}q)_{V-A} & Q_{10} &= (\bar{D}_i b_j)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}_j q_i)_{V-A} \\
Q_{7\gamma} &= -\frac{e}{8\pi^2} m_b \bar{D} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b & Q_{8g} &= -\frac{g_s}{8\pi^2} m_b \bar{D} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b,
\end{aligned} \tag{2.30}$$

where $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$, i, j are colour indices, e_q are electric charges of the quarks in unit of $|e|$, and the summation is over $q = u, d, s, c, b$. The definition of the dipole operators corresponds to the sign convention $iD^\mu = i\partial^\mu + g_s A_a^\mu t_a$ for the gauge covariant derivative. The short-distance coefficients contain effects associated with the electroweak scale, or above (see section 2.3.2); they are therefore the elements where possible new physics effects enter. The decay amplitude is obtained taking the matrix element of the effective Hamiltonian between the initial and final states:

$$\mathcal{A} \sim \langle M'_1 M'_2 | \mathcal{H}_{\text{eff}} | \bar{B} \rangle. \tag{2.31}$$

Transition Operators

Depending on the decay considered, different operators appear in the matrix element of (2.31). According to the QCD factorization formula, their calculation can be organized by matching the effective Hamiltonian onto two transition operators, such that its matrix element is given by

$$\langle M'_1 M'_2 | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = \sum_{p=u,c} \lambda_p^{(D)} \langle M'_1 M'_2 | \mathcal{T}_A^p + \mathcal{T}_B^p | \bar{B} \rangle; \tag{2.32}$$

the two operators reflect the two terms appearing in the factorization formula, the form factor and the hard-scattering terms. In the literature [65], the transition operators are organized in terms of their Dirac and flavour structure. Focusing first on $\mathcal{T}_A^p$, one has

$$\begin{aligned}
\mathcal{T}_A^p &= a_1(M_1 M_2) \delta_{pu} (\bar{p}b)_{V-A} \otimes (\bar{D}p)_{V-A} + a_2(M_1 M_2) \delta_{pu} (\bar{D}b)_{V-A} \otimes (\bar{p}p)_{V-A} \\
&+ a_3(M_1 M_2) \sum_q (\bar{D}b)_{V-A} \otimes (\bar{q}q)_{V-A} + a_4(M_1 M_2) \sum_q (\bar{q}b)_{V-A} \otimes (\bar{D}q)_{V-A} \\
&+ a_5(M_1 M_2) \sum_q (\bar{D}b)_{V-A} \otimes (\bar{q}q)_{V+A} + a_6(M_1 M_2) \sum_q (-2)(\bar{q}b)_{S-P} \otimes (\bar{D}q)_{S+P} \\
&+ a_7(M_1 M_2) \sum_q (\bar{D}b)_{V-A} \otimes \frac{3}{2} e_q (\bar{q}q)_{V+A}
\end{aligned}$$

$$\begin{aligned}
& +a_8(M_1M_2) \sum_q (-2)(\bar{q}b)_{S-P} \otimes \frac{3}{2}e_q(\bar{D}q)_{S+P} \\
& +a_9(M_1M_2) \sum_q (\bar{D}b)_{V-A} \otimes \frac{3}{2}e_q(\bar{q}q)_{V-A} \\
& +a_{10}(M_1M_2) \sum_q (\bar{q}b)_{V-A} \otimes \frac{3}{2}e_q(\bar{D}q)_{V-A},
\end{aligned} \tag{2.33}$$

where $(\bar{q}_1q_2)_{S\pm P} = \bar{q}_1(1 \pm \gamma_5)q_2$, and a summation over $q = u, d$ is implied. The symbol $\otimes$ indicates that the matrix elements of the operators in $\mathcal{T}_A^p$ must be evaluated in the factorized form $\langle M'_1M'_2|\mathcal{T}_A^p|\bar{B}\rangle = \langle M'_1M'_2|j_1 \otimes j_2|\bar{B}\rangle = \langle M'_1|j_1|\bar{B}\rangle\langle M'_2|j_2|0\rangle$, as appropriate. “Non-factorizable” corrections are therefore included in the coefficients a_i .

Flavour amplitudes

With the notation employed in (2.33) the terms in the transition operator are in direct correspondence with the operators appearing in the effective weak Hamiltonian. In order to treat in a unified way decays into PP , PV and VV final states, however, it is more suitable to introduce another notation, whose purpose is to keep track only of the flavour structure of an operator [67]; the reason for this is that operators with different Dirac-, but identical flavour-structure, always contribute in the same combination to a particular set of final states.

Fixing the flavour convention for the meson flavour wave-function as

$$B^- \sim \bar{u}b, \quad \bar{B}_d \sim \bar{d}b, \quad \bar{B}_s \sim \bar{s}b \tag{2.34}$$

for the B meson states;

$$\begin{aligned}
\pi^0 & \sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d), & \pi^- & \sim \bar{u}d, & \pi^+ & \sim \bar{d}u, \\
\bar{K}^0 & \sim \bar{d}s, & K^0 & \sim \bar{s}d, & K^- & \sim \bar{u}s, & K^+ & \sim \bar{s}u,
\end{aligned} \tag{2.35}$$

for the non-singlet members of the light nonet of pseudoscalar mesons, with analogous expressions for the corresponding vector mesons, and assuming

$$\eta_q \sim \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d), \quad \eta_s \sim \bar{s}s; \tag{2.36}$$

in case of flavour-singlet mesons⁴, the transition operator $\mathcal{T}_A^p$ can be rewritten in terms of six flavour topologies, *i.e.*

$$\begin{aligned}
\mathcal{T}_A^p & = \delta_{pu}\alpha_1(M_1M_2)A([\bar{q}_sp][\bar{p}D]) + \delta_{pu}\alpha_2(M_1M_2)A([\bar{q}_sD][\bar{p}p]) \\
& + \alpha_3^p(M_1M_2) \sum_q A([\bar{q}_sD][\bar{q}q]) + \alpha_4^p(M_1M_2) \sum_q A([\bar{q}_sq][\bar{q}D])
\end{aligned}$$

⁴We follow the convention of treating the η meson (and similarly the η' , the ω and the ϕ mesons) as a coherent superposition of the flavour component η_q, η_s ; moreover, in case of the ω and ϕ mesons ideal mixing is assumed.

$$+ \alpha_{3,\text{EW}}^p(M_1 M_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s D][\bar{q} q]) + \alpha_{4,\text{EW}}^p(M_1 M_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s q][\bar{q} D]), \quad (2.37)$$

where the sum extends now on $q = u, d, s$, and q_s denotes the spectator antiquark. The operators $A([\bar{q}_{M_1} q_{M_1}][\bar{q}_{M_2} q_{M_2}])$ also contain an implicit sum over $q_s = u, d, s$ to cover all possible B -meson final states. The coefficients α_i^p contain the dynamical information, while the arguments of A encode the flavour composition of the final state. The utility of this notation is that the coefficients α_i introduced in (2.37) are in close correspondence with the “graphical” or “topological” amplitudes defined in the literature: α_1 (α_2) with the colour-allowed (colour-suppressed) tree amplitude; α_4^p with the QCD penguin amplitude; α_3^p with the QCD flavour-singlet penguin amplitude; $\alpha_{3,\text{EW}}^p$, ($\alpha_{4,\text{EW}}^p$) with the colour-allowed (colour-suppressed) electroweak penguin amplitude. The matrix elements of these operators are defined as

$$\langle M'_1 M'_2 | \alpha_i^p(M_1 M_2) A([\dots][\dots]) | \bar{B} \rangle \equiv c \alpha_i^p(M'_1 M'_2) A_{M'_1 M'_2} \quad (2.38)$$

whenever the quark flavours of the first and the second square brackets match those of M'_1 and M'_2 , respectively, and

$$\langle M'_1 M'_2 | \alpha_i^p(M_1 M_2) A([\dots][\dots]) | \bar{B} \rangle \equiv c \alpha_i^p(M'_2 M'_1) A_{M'_2 M'_1},$$

whenever the quark flavours of the first and the second square brackets match those of M'_2 and M'_1 . The factor c denotes a Clebsch-Gordan coefficient $1, \pm 1/\sqrt{2}$, from the flavour composition of the mesons. Since the coefficients α_i contain the non-factorizable corrections, the operator $A([\bar{q}_{M_1} q_{M_1}][\bar{q}_{M_2} q_{M_2}])$ are intended to be evaluated in the factorized form, so that⁵

$$A_{M'_1 M'_2} = \frac{G_F}{\sqrt{2}} \langle M'_1 | (\bar{q}_s \Gamma b) | \bar{B}_{q_s} \rangle \langle M'_2 | (\bar{q} \Gamma q') | 0 \rangle, \quad (2.39)$$

i.e. more explicitly,

$$A_{M_1 M_2} = i \frac{G_F}{\sqrt{2}} \begin{cases} m_B^2 f_+^{BM_1}(0) f_{M_2}, & \text{if } M_1 M_2 = PP, \\ -m_B^2 A_0^{BM_1}(0) f_{M_2}, & \text{if } M_1 M_2 = VP, \\ -m_B^2 f_+^{BM_1}(0) f_{M_2}, & \text{if } M_1 M_2 = PV, \\ m_B^2 A_0^{BM_1}(0) f_{M_2}, & \text{if } M_1 M_2 = V^0 V^0, \\ m_B m_{V_2} f_{\pm}^{BM_1}(0) f_{M_2}, & \text{if } M_1 M_2 = V^{\pm} V^{\pm}. \end{cases} \quad (2.40)$$

Here $f_{\pm}$, A_0 denote pseudoscalar and vector meson form factors, while f_{M_2} are the longitudinal decay constants, normalized according to

$$\langle \pi^-(q) | \bar{d} \gamma_{\mu} \gamma_5 u | 0 \rangle = -i f_{\pi} q_{\mu}, \quad \langle \rho^-(q) | \bar{d} \gamma_{\mu} u | 0 \rangle = -i f_{\rho} m_{\rho} \epsilon_{\mu}^*. \quad (2.41)$$

With this normalization, at leading order the α coefficients are simply given by a combination of short-distance coefficients. This allows us to recover in (2.40) the helicity hierarchy

⁵since $\bar{q}_{M_1} = \bar{q}_s$ in (2.39), the flavour structure of the operator $A([\bar{q}_{M_1} q_{M_1}][\bar{q}_{M_2} q_{M_2}])$ is determined unambiguously from that of the operator in (2.39).

that was described in par. 2.2.2, (2.25): the further suppression of the positive helicity amplitude with respect to the negative one follows from the fact that $f_+^{BV_1}(0) \sim \Lambda_{\text{QCD}}/m_B$ in the heavy-quark limit [179].

In order to explain the notation we consider *e.g.* the decay $\bar{B}_d \rightarrow \pi^0 \rho^0$, for which $q_s = d$, $D = d$. The spectator quark can go to either one of the two mesons, so M_1 can be π^0 or ρ^0 , and, for instance,

$$\langle \pi^0 \rho^0 | \alpha_4^p(M_1 M_2) \sum_q A([\bar{d}q][\bar{q}d]) | \bar{B}_d \rangle = -\frac{1}{2} [\alpha_4^p(\pi^0 \rho^0) A_{\pi^0 \rho^0} + \alpha_4^p(\rho^0 \pi^0) A_{\rho^0 \pi^0}] ; \quad (2.42)$$

on the other side, according to the flavour definitions in 2.35,

$$\langle \pi^0 \rho^0 | \alpha_3^p(M_1 M_2) \sum_q A([\bar{d}d][\bar{q}q]) | \bar{B}_d \rangle = 0, \quad (2.43)$$

because $q = u, d$ contribute equally but with opposite sign for the mesons π^0 and ρ^0 .

Since the contributions of the “non-factorizable” corrections is usually given in terms of the a_i coefficients, we provide here the relation between them and the coefficients α_i :

$$\begin{aligned} \alpha_1(M_1 M_2) &= a_1(M_1 M_2), \\ \alpha_2(M_1 M_2) &= a_2(M_1 M_2), \\ \alpha_3^p(M_1 M_2) &= \begin{cases} a_3^p(M_1 M_2) - a_5^p(M_1 M_2), & \text{if } M_1 M_2 = PP, VP, \\ a_3^p(M_1 M_2) + a_5^p(M_1 M_2), & \text{if } M_1 M_2 = PV, VV, \end{cases} \\ \alpha_4^p(M_1 M_2) &= \begin{cases} a_4^p(M_1 M_2) + r_\chi^{M_2} a_6^p(M_1 M_2), & \text{if } M_1 M_2 = PP, PV, \\ a_4^p(M_1 M_2) - r_\chi^{M_2} a_6^p(M_1 M_2), & \text{if } M_1 M_2 = VP, VV, \end{cases} \\ \alpha_{3,\text{EW}}^p(M_1 M_2) &= \begin{cases} a_9^p(M_1 M_2) - a_7^p(M_1 M_2), & \text{if } M_1 M_2 = PP, VP, \\ a_9^p(M_1 M_2) + a_7^p(M_1 M_2), & \text{if } M_1 M_2 = PV, VV, \end{cases} \\ \alpha_{4,\text{EW}}^p(M_1 M_2) &= \begin{cases} a_{10}^p(M_1 M_2) + r_\chi^{M_2} a_8^p(M_1 M_2), & \text{if } M_1 M_2 = PP, PV, \\ a_{10}^p(M_1 M_2) - r_\chi^{M_2} a_8^p(M_1 M_2), & \text{if } M_1 M_2 = VP, VV, \end{cases} \end{aligned} \quad (2.44)$$

where the ratios $r_\chi^{M_2}$ are defined *e.g.* as

$$r_\chi^\pi(\mu) = \frac{2m_\pi^2}{m_b(\mu)2m_q(\mu)}, \quad r_\chi^K(\mu) = \frac{2m_K^2}{m_b(\mu)(m_q + m_s)(\mu)}, \quad (2.45)$$

or as

$$r_\chi^V(\mu) = \frac{2m_V}{m_b(\mu)} \frac{f_V^\perp(\mu)}{f_V}, \quad (2.46)$$

and the scale-dependent transverse decay constant reads

$$\langle V(p, \epsilon^*) | \bar{q} \sigma_{\mu\nu} q' | 0 \rangle = f_V^\perp (p_\mu \epsilon_\nu^* - p_\nu \epsilon_\mu^*). \quad (2.47)$$

It remains to analyse the flavour structure of the transition operator $\mathcal{T}_B^P$. Since it describes hard-scattering interactions where all the six quarks are involved, it has to be of

the form $B([\bar{q}_{M_1} q_{M_1}][\bar{q}_{M_2} q_{M_2}][\bar{q}_s b])$, with possible sum over quarks from penguin operators or flavour-singlet conversion $g \rightarrow \bar{q}q$. We define the matrix elements of the B operators as

$$\langle M'_1 M'_2 | B([\dots][\dots][\dots]) | \bar{B} \rangle \equiv c B_{M'_1 M'_2}, \quad (2.48)$$

with

$$B_{M_1 M_2} = \pm i \frac{G_F}{\sqrt{2}} f_B f_{M_1} f_{M_2}, \quad (2.49)$$

whenever the quark flavours of the three brackets match those of M_1 , M_2 and $\bar{B}$. The constant c is the same appearing in (2.38). The upper sign in the definition of $B_{M_1 M_2}$ applies when both mesons are pseudoscalar or vector, while the lower sign in the remaining two cases.

The most important case of spectator scattering is when the spectator-quark line goes from the B meson to the meson M_1 . This transition involves a hard spectator interaction, in which (at NLO level) a hard gluon is exchanged between the spectator quark and the quarks of the meson M_2 (Fig. 2.8), and it is the only spectator interaction appearing at leading order in the heavy-quark limit. In this case $\bar{q}_s = \bar{q}_{M_1}$, so that there are six possible different flavour amplitudes, in direct correspondence with the six A -operators, according to

$$B([\bar{q}_s q_{M_1}][\bar{q}_{M_2} q_{M_2}][\bar{q}_s b]) = \frac{B_{M_1 M_2}}{A_{M_1 M_2}} A([\bar{q}_s q_{M_1}][\bar{q}_{M_2} q_{M_2}]); \quad (2.50)$$

It is therefore usual to absorb the spectator contribution into the coefficients α_i . In this way, the operator $\mathcal{T}_A^p$ contains all the leading-power contributions in the heavy-quark limit; $\mathcal{T}_B^p$ contains only the power-suppressed weak annihilation contribution, and therefore it can be parameterized as

$$\begin{aligned} \mathcal{T}_B^p = & \delta_{pu} b_1(M_1 M_2) \sum_{q'} B([\bar{u}q'][\bar{q}'u][\bar{D}b]) + \delta_{pu} b_2(M_1 M_2) \sum_{q'} B([\bar{u}q'][\bar{q}'D][\bar{u}b]) \\ & + b_3^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'D][\bar{q}b]) + b_4^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{D}b]) \\ & + b_{3,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q'][\bar{q}'D][\bar{q}b]) \\ & + b_{4,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q'][\bar{q}'q][\bar{D}b]) \\ & + \delta_{pu} b_{S1}(M_1 M_2) \sum_{q'} B([\bar{u}u][\bar{q}'q'][\bar{D}b]) + \delta_{pu} b_{S2}(M_1 M_2) \sum_{q'} B([\bar{u}D][\bar{q}'q'][\bar{u}b]) \\ & + b_{S3}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}D][\bar{q}'q'][\bar{q}b]) + b_{S4}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q][\bar{q}'q'][\bar{D}b]) \\ & + b_{S3,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}D][\bar{q}'q'][\bar{q}b]) \\ & + b_{S4,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q][\bar{q}'q'][\bar{D}b]), \end{aligned} \quad (2.51)$$

where the sum extends over $q, q' = u, d, s$. The sum over q' arises because a quark-antiquark pair must be created by $g \rightarrow \bar{q}'q$ after the spectator quark is annihilated. The operators with the flavour structure $\sum_{q,q'} B([\bar{q}q'][\bar{q}'D][\bar{q}b])$ are redundant, because they are equivalent to $\sum_{q,q'} A([\bar{q}_s q][\bar{q}D])$, which includes an implicit sum over q_s . The six coefficients with subscript “ S ” contribute only to final states containing flavour-singlet mesons or neutral vector mesons. We will write the amplitudes in terms of

$$\beta_i^p \equiv \frac{B_{M_1 M_2}}{A_{M_1 M_2}} b_i^p(M_1 M_2), \quad (2.52)$$

whenever $A_{M_1 M_2}$ does not vanish; otherwise, we will express them in terms of the original $b_i^p(M_1 M_2)$.

To conclude the section we give, as an example, the expression of the amplitude of two decays, written in the notation described: the $\Delta D = 1$ decay

$$\begin{aligned} \mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ \rho^-} = \sum_p \lambda_p^{(d)} \left\{ A_{\pi \rho} \left[\delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,\text{EW}} + \beta_3^p + \beta_4^p - \frac{1}{2} \beta_{3,\text{EW}}^p - \frac{1}{2} \beta_{4,\text{EW}}^p \right] \right. \\ \left. + A_{\rho \pi} \left[\delta_{pu} \beta_1 + \beta_4^p + \beta_{4,\text{EW}}^p \right] \right\}, \end{aligned} \quad (2.53)$$

and the $\Delta S = 1$ decay

$$\begin{aligned} \sqrt{2} \mathcal{A}_{B^- \rightarrow \pi^0 K^-} = \sum_p \lambda_p^{(d)} \left\{ A_{\pi \bar{K}} \left[\delta_{pu} (\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,\text{EW}}^p + \beta_{3,\text{EW}}^p \right] \right. \\ \left. + A_{\bar{K} \pi} \left[\delta_{pu} \alpha_2 + \frac{3}{2} \alpha_{3,\text{EW}}^p \right] \right\}. \end{aligned} \quad (2.54)$$

In the phenomenological application of chapters 4, 5 and 6 we will use NLO expressions for the coefficients α and β . In section 2.3.3 we show an example on how they can be derived using EFTs techniques, while in section 2.3.4 we will provide the NLO result for them.

2.3.2 Short-distance coefficients

The short-distance coefficients are obtained matching the amplitudes calculated in the full theory to the amplitudes obtained in the effective theory, described by the weak Hamiltonian (2.29). The full amplitudes involve the diagrams shown in Fig. 2.10, which contain the exchange of virtual heavy particles like the W and Z bosons, and the top quark. In extensions of the Standard Model, one has to consider the possibility of having flavour-changing currents mediated by new heavy particles. This will be the case, indeed, in the analyses performed in chapters 4, 5, and 6. The short distance coefficients depend on the masses of the exchanged heavy particles and involve virtualities of order $\sim M_W$.

When the QCD interaction is taken into account, the short-distance coefficients acquire a scale dependence, which match that of the matrix elements of the effective operators. QCD effects must be evaluated at the hadronic scale $\sim m_b$, which is still large enough to allow for a perturbative treatment. Because of the large hierarchy $m_b \ll M_W$, however,

large logarithms $\ln M_W/m_b$ appear, which compensate the smallness of $\alpha_s(m_b)$ and spoil the reliability of the perturbative expansion. The solution is to consider the renormalization group improved perturbative expansion, which allows to resum large logarithms of the type $\alpha_s^n (\ln M_W/\mu)^n$ and $\alpha_s^n (\ln M_W/\mu)^{n-1}$ to all order in α_s . The procedure consist in calculating the short-distance coefficients at the scale $\mu \sim M_W$, $\vec{C}(M_W)$, where large logarithms do not appear. We calculate then the anomalous dimension of the corresponding operators, and derive a renormalization group equation for the short-distance coefficients, whose solution allows us to derive an expression for them at the low scale, according to $\vec{C}(\mu) = \mathbf{U}(\mu, M_W) \vec{C}(M_W)$, where $\mathbf{U}(\mu, M_W)$ is the “evolution operator”.

Consistency in the perturbative expansion requires all the terms appearing in the decay amplitude to be homogeneous in the coupling constant α_s . At LO one has therefore to consider both the short distance coefficients and the matrix elements of the operators at leading order in α_s ; at NLO the leading order matrix elements of the operators needs to be multiplied with the NLO expression of the short-distance coefficients, and vice versa.

At NLO we use the evolution operator adopted in [65], which reads

$$\vec{C}(\mu) = \left[\mathbf{U}_0 + \frac{\alpha_s(\mu)}{4\pi} \mathbf{J} \mathbf{U}_0 - \frac{\alpha_s(M_W)}{4\pi} \mathbf{U}_0 \mathbf{J} + \frac{\alpha_{\text{em}}}{4\pi} \left(\frac{4\pi}{\alpha_s(\mu)} \mathbf{R}_0 + \mathbf{R}_1 \right) \right] \vec{C}(M_W), \quad (2.55)$$

where the matrices $\mathbf{U}_0$, $\mathbf{J}$, $\mathbf{R}_0$, $\mathbf{R}_1$ depend on the ratio $\alpha_s(\mu)/\alpha_s(M_W)$ and on the anomalous dimension and β functions. It corresponds to consider some electroweak effects of order α_{em} as leading; in fact, in (2.55) the term $\mathbf{U}_0 + (\alpha_{\text{em}}/\alpha_s) \mathbf{R}_0$ represents the leading order result, being the remaining ones the NLO contribution.

In the Standard Model, only one short-distance coefficient, C_1 , is $\mathcal{O}(1)$, the other coefficients being significantly smaller: the QCD penguin short distance coefficients are of order of some per cent, while the electroweak penguin ones are of order of some per mill. This hierarchy is important in phenomenological analysis, and it has to be kept in mind in particular when comparing with possible contributions from new physics. In table 2.1 we show the LO and NLO values of the Standard Model short-distance coefficients.

2.3.3 Calculation of the matrix elements in SCET

The purpose of this section is to show how effective field theory techniques can be used to calculate the flavour amplitudes α_i , β_i defined in section 2.3.1. Since this is needed only as an example, in order to introduce the techniques used later in chapter 3, we will focus on the general case, leaving out more subtle discussions such as the derivation of the flavour-singlet QCD penguin amplitude α_3 , the discussion of flavour-singlet final-states, and higher order corrections in Λ_{QCD}/m_b . These results are included without derivation in section 2.3.4.

Power counting

The evaluation of the diagrams containing non-factorizable gluon exchanges can be organized as a series of matching steps between different effective field theories [91, 93, 94, 136–138, 150]. In this way the evaluation is more transparent, and the structure of the

NLO	C_1	C_2	C_3	C_4	C_5	C_6
$\mu = m_b/2$	1.137	-0.295	0.021	-0.051	0.010	-0.065
$\mu = m_b$	1.081	-0.190	0.014	-0.036	0.009	-0.042
$\mu = 2m_b$	1.045	-0.113	0.009	-0.025	0.007	-0.027
	C_7/α_{em}	C_8/α_{em}	C_9/α_{em}	$C_{10}/\alpha_{\text{em}}$	$C_{7\gamma}^{\text{eff}}$	C_{8g}^{eff}
$\mu = m_b/2$	-0.024	0.096	-1.325	0.331	—	—
$\mu = m_b$	-0.011	0.060	-1.254	0.223	—	—
$\mu = 2m_b$	0.011	0.039	-1.195	0.144	—	—
LO	C_1	C_2	C_3	C_4	C_5	C_6
$\mu = m_b/2$	1.185	-0.387	0.018	-0.038	0.010	-0.053
$\mu = m_b$	1.117	-0.268	0.012	-0.027	0.008	-0.034
$\mu = 2m_b$	1.074	-0.181	0.008	-0.019	0.006	-0.022
	C_7/α_{em}	C_8/α_{em}	C_9/α_{em}	$C_{10}/\alpha_{\text{em}}$	$C_{7\gamma}^{\text{eff}}$	C_{8g}^{eff}
$\mu = m_b/2$	-0.012	0.045	-1.358	0.418	-0.364	-0.169
$\mu = m_b$	-0.001	0.029	-1.276	0.288	-0.318	-0.151
$\mu = 2m_b$	0.018	0.019	-1.212	0.193	-0.281	-0.136

Table 2.1: Wilson coefficients C_i in the NDR scheme, based on (2.55).

factorization formula naturally emerges. In case of exclusive non-leptonic B decays two kinds of short-distance modes are successively integrated out: hard modes, with virtualities of order m_b^2 , and hard collinear modes, with virtualities of order $\sqrt{m_b \Lambda_{\text{QCD}}}$. The first matching step leads from QCD to the so called SCET_I, while the second one leads from SCET_I to SCET_{II}. The effective theory SCET_{II} only contains long-distance modes with virtualities of order Λ_{QCD} . In the B meson rest frame these are denoted as “soft” (all momentum component of order Λ_{QCD}) and “collinear” (one momentum components scales like m_b). Fields and operators in the effective theory have a definite power-counting in terms of the expansion parameter $\lambda^2 \equiv \Lambda_{\text{QCD}}/m_b$. A short introduction to SCET_I and SCET_{II}, with their field content and scaling is given in appendix A. The purpose of this section is to show how the operators that contribute at leading order in the Λ_{QCD}/m_b expansion can be singled out, following the strategy discussed in [93, 150].

Firstly, one needs to single out the operators contributing in SCET_I. For this purpose, fields in the collinear sector defined by the direction of the meson M_1 (that picks up the spectator antiquark) and therefore moving in the direction of the vector n_- , will be denoted by $\xi_{c,\text{hc}}$ ⁶; fields moving in the opposite direction, n_+ , belong to the collinear-2 sector (meson M_2) and will be denoted by $\chi_{c,\text{hc}}$; the heavy quark field will be labelled by

⁶we label here the hard-collinear fields in SCET_I with the index “hc”, even if this is a bit improper, since in SCET_I no distinction between hard-collinear and collinear-modes is done; this choice reflects the subsequent matching to SCET_{II}.

the time-like vector $v = (n_- + n_+)/2$, with $v^2 = 1$. We will not consider here the flavour content of these operators (as well as Dirac or Lorentz indices), which can be determined matching them with the corresponding operators in the effective weak Hamiltonian; we will focus on this in the next section. In the second matching step, one has to generate the minimal field content

$$[\bar{q}_s \dots h_v][\bar{\xi}_c \dots \xi_c][\bar{\chi}_c \dots \chi_c] \quad (2.56)$$

that is necessary to build up the initial and final state quantum numbers (the dots stand for additional $q\bar{q}$ pairs or gluons fields of the same kind); we do not consider flavour-singlet meson in this example. The generation of soft and collinear fields from hard-collinear ones costs a certain power of the small expansion parameters λ , which can be read off the corresponding interaction terms in the SCET_I Lagrangian. Examples are

$$\begin{aligned} \xi_{hc} &\xrightarrow{\lambda} \xi_c, \\ \xi_{hc} &\xrightarrow{\lambda^2} \xi_c A_{hc}^\perp, \\ A_{hc}^\perp &\xrightarrow{\lambda} \bar{q}_s \xi_{hc}, \\ A_{hc}^\perp &\xrightarrow{\lambda^2} \bar{q}_s \xi_c. \end{aligned} \quad (2.57)$$

Not considering three-body SCET_I operators, which give in the end subleading contributions, we focus instead on the four-body operator

$$[\bar{\chi}_{hc} \chi_{hc}][\bar{\xi}_{hc} h_v] \sim \lambda^6, \quad (2.58)$$

that constitutes a leading contribution to T_i^I . In order to see this, we first analyse its matching to SCET_{II} operators. The conversion of the hard-collinear quark-antiquark pair to a collinear one costs twice a power of λ (see (2.57)), and the conversion of ξ_{hc} costs a factor of λ^4 , for instance through the chain

$$\bar{\xi}_{hc} \xrightarrow{\lambda^2} \bar{\xi}_c A_{hc}^\perp \xrightarrow{\lambda^2} \bar{\xi}_c [\bar{q}_s \xi_c]; \quad (2.59)$$

these four-quark operators therefore match onto 6-body operators in SCET_{II}, which scale as λ^{12} . To understand if these operators can give a leading contribution to the amplitude, their matrix elements must be compared with the naive factorization result, whose scaling is $\sim f_{M_2} f_+^{B M_1} \sim \lambda^5$, *e.g.* in case of $B \rightarrow PP$ transitions. Defining the mesons with respect to the exact soft-collinear Hamiltonian, so that the meson states are the same as in full QCD, one has the conventional normalization of hadronic states:

$$|B(p_B)\rangle \sim \lambda^{-3}, \quad |M(p)\rangle \sim \lambda^{-2}, \quad (2.60)$$

with M one of the light mesons. The matrix elements of the six-body SCET_{II} operators scale therefore as λ^5 , and constitute a leading contribution to the decay amplitude. In a similar way one can prove that the five-body operator

$$[\bar{\chi}_{hc} \chi_{hc}][\bar{\xi}_{hc} A_{hc}^\perp h_v] \sim \lambda^7 \quad (2.61)$$

matches onto a leading-power six-quark operator in SCET_{II}, which correspond to the factorizable hard-spectator diagrams in the QCD factorization approach, and that the operator

$$[\bar{\xi}_{hc}\chi_{hc}][\bar{\chi}_{hc}\xi_{hc}][\bar{q}_s h_v] \quad (2.62)$$

matches onto a 6-body operator of order λ^{14} , and therefore subleading, corresponding to the $1/m_b$ -suppressed annihilation graphs of Fig. 2.9.

Following the same strategy, a complete analysis can be performed, and all the operators contributing at leading- and higher orders can be singled out. Here we have shown how the two leading-order operators arise, and we have given an example of a subleading operator. In the next paragraph we are going to explain in more detail the matching process for the two leading operators.

From m_b to $\sqrt{m_b \Lambda_{\text{QCD}}}$: SCET_I

We consider only final states containing non-singlet PP , VP , PV or longitudinally polarized VV mesons. The Lorentz and Dirac structures of the leading SCET_I operators individuated in (2.58) and (2.61) is fixed by chirality conservation (with respect to the operators Q_i in the effective weak Hamiltonian) and the requirements that the operator be a Lorentz scalar. It follows that the structure of the collinear-2 sector bilinears can be only of the type $(\chi W_{c2})(tn_-)\not{n}_-(1 \pm \gamma_5)(W_{c2}^\dagger \chi)(0)$; the two operators are then reproduced multiplying this with an A0- and B1-type current from the $\bar{B} \rightarrow M_1$ transition [150], defined in appendix A.1, (A.29). One has a set of four operators of the form

$$\begin{aligned} O_{L,R}^{\text{I}}(t) &= \left[(\bar{\chi} W_{c2})(tn_-) \frac{\not{n}_-}{2} (1 \mp \gamma_5) (W_{c2}^\dagger \chi) \right] \left[\tilde{C}_{f_+}^{(A0)} (\bar{\xi} W_{c1}) \not{n}_+ (1 - \gamma_5) h_v \right. \\ &\quad \left. - \frac{1}{m_b} \int d\hat{s} \tilde{C}_{f_+}^{(B1)}(\hat{s}) (\bar{\xi} W_{c1}) \not{n}_+ [W_{c1}^\dagger i \not{D}_{\perp c1} W_{c1}](sn_+) (1 + \gamma_5) h_v \right], \\ O_{L,R}^{\text{II}}(t, s) &= \left[(\bar{\chi} W_{c2})(tn_-) \frac{\not{n}_-}{2} (1 \mp \gamma_5) (W_{c2}^\dagger \chi) \right] \\ &\quad \times \left[(\bar{\chi} W_{c1}) \frac{\not{n}_+}{2} [W_{c1}^\dagger i \not{D}_{\perp c1} W_{c1}](sn_+) (1 + \gamma_5) h_v \right]. \end{aligned} \quad (2.63)$$

The operators $O_{L,R}^{\text{I}}$ include the short-distance coefficients $\tilde{C}_{f_+}^{(A0)}$, $\tilde{C}_{f_+}^{(B1)}(\hat{s})$ such that their matrix elements are proportional to the form factor $f_+^{BM_1}(0)$ ($A_0^{BM_1}(0)$ for vector mesons) in QCD. In (2.63) fields without position argument are at $x = 0$, and the field products within the large brackets are colour-singlets. In (2.63) W_{ci} are Wilson lines, as explained in appendix A. The diagrams corresponding to these operators are shown in Fig. 2.12.

In order to determine the flavour structures of the operators, one has to consider all the possible contractions of the operators in the effective weak Hamiltonian that give rise to a given final state. In figure 2.11 we show the various contractions of the effective vertex with the external quark-lines. Initial and final state mesons are denoted by their flavour content, so that, for instance, in Fig. 2.11 (a) the $\bar{B}$ meson has flavour quantum number $[\bar{q}_s b]$, the meson M_2 is represented by the up-going quark lines, with flavour content $[\bar{u} D]$,

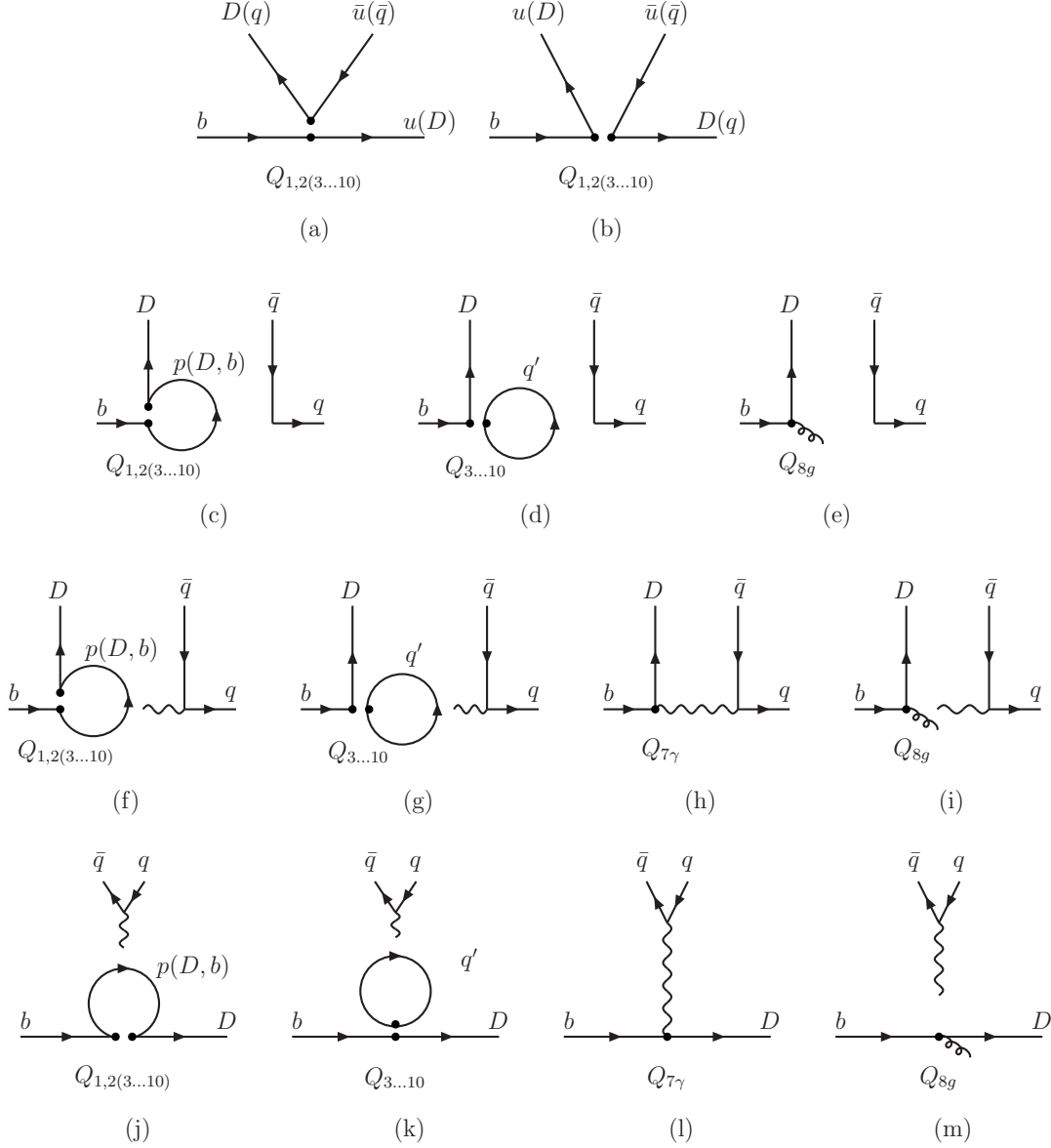


Figure 2.11: The various contractions of the external lines with the weak effective vertex, excluding the exchange of arbitrarily many gluons. The connected fermion lines indicate the contraction of the spinor indices. Picture taken from [137].

and the meson M_1 has flavour content $[\bar{q}_s u]$. The picture does not include arbitrarily many gluons, that may connect the quark lines, or originate from a quark line and connect with the spectator quark. It is possible to see that many different contractions are possible, that in the end are in correspondence with the flavour amplitudes introduced so far in section 2.3.1. A detailed discussion about the various ways a given operator Q_i appears in the contractions is not needed here, and we refer for this to [137]. We note only that the “right” (a) and “wrong” (b) insertion in the first line of Fig. 2.11 can contribute to any of the topological amplitudes α_i , while the contractions in the second line contributes to α_4^P ;

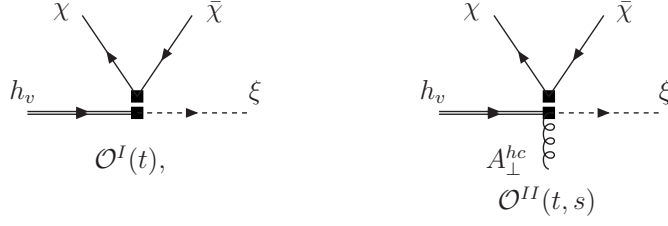


Figure 2.12: Vertices corresponding to the $SCET_I$ operators $O_{L,R}^I$, $O_{L,R}^{II}$.

contractions from (h) to (m) contribute to the electroweak penguin amplitudes. This can be seen by comparing the flavour labels of Fig. 2.11 with (2.37). In the end, the flavour structure of the operator in (2.63) can be taken into account adding the following labels to them:

$$O_{L,R}^{I,II} \rightarrow O_{L,R}^{I,II}([\bar{q}_s q_{M_1}][\bar{q}_{M_2} q_{M_2}]), \quad (2.64)$$

where the labels $q_{M_{1,2}}$ and $\bar{q}_{M_2}$ give the flavours of the fields $\bar{\xi}$, $\bar{\chi}$, and χ , respectively; the label $\bar{q}_s$ is redundant, and it is added to match the notation in (2.37). At leading power in Λ_{QCD}/m_b then, the complete weak Hamiltonian in (2.29) can be accounted for in $SCET_I$ (not counting flavour-singlet final states) by

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \left(\delta_{pu} \{ T_1^I \star O_L^I([\bar{q}_s u][\bar{u} D]) + H_1^{II} \star O_L^{II}([\bar{q}_s u][\bar{u} D]) \right. \\ & + T_2^I \star O_L^I([\bar{q}_s D][\bar{u} u]) + H_2^{II} \star O_L^{II}([\bar{q}_s D][\bar{u} u]) \} \\ & + \sum_{k=L,R} \left\{ T_{3k}^{I,p} \star \sum_q O_k^I([\bar{q}_s D][\bar{q} q]) + H_{3k}^{II,p} \star \sum_q O_k^{II}([\bar{q}_s D][\bar{q} q]) \right. \\ & + T_{3k,\text{EW}}^{I,p} \star \sum_q \frac{3}{2} e_q O_k^I([\bar{q}_s D][\bar{q} q]) + H_{3k,\text{EW}}^{II,p} \star \sum_q \frac{3}{2} e_q O_k^{II}([\bar{q}_s D][\bar{q} q]) \} \\ & + \sum_{k=L,R} \left\{ T_{4k}^{I,p} \star \sum_q O_k^I([\bar{q}_s q][\bar{q} D]) + H_{4k}^{II,p} \star \sum_q O_k^{II}([\bar{q}_s q][\bar{q} D]) \right. \\ & \left. + T_{4k,\text{EW}}^{I,p} \star \sum_q \frac{3}{2} e_q O_k^I([\bar{q}_s q][\bar{q} D]) + H_{4k,\text{EW}}^{II,p} \star \sum_q \frac{3}{2} e_q O_k^{II}([\bar{q}_s q][\bar{q} D]) \right\} \Bigg), \quad (2.65) \end{aligned}$$

where we employed the notation

$$T_{ik}^I \star O_k^I = \int d\hat{t} \tilde{T}_{ik}^I(\hat{t}) O_k^I(t), \quad H_{ik}^{II} \star O_k^{II} = \int d\hat{t} d\hat{s} \tilde{H}_{ik}^{II}(\hat{t}, \hat{s}) O_k^{II}(t, s), \quad (2.66)$$

where t and s depend on the integration variables via $\hat{s} = n_+ p s = m_B s$, $\hat{t} = n_- q t = m_B t$. The sums over q extend over the light quarks u, d, s , as in (2.37). The coefficients

$$T_{ik}^I(u) = \int d\hat{t} e^{i u \hat{t}} \tilde{T}_{ik}^I(\hat{t}), \quad H_{i,k}^{II}(u, v) = \int d\hat{t} d\hat{s} e^{i(u \hat{t} + v \hat{s})} \tilde{H}_{ik}^{II}(\hat{t}, \hat{s}) \quad (2.67)$$

correspond to the hard scattering kernels appearing in the factorization formula. Since no expansion in m_c/m_b is performed, these coefficients are functions of the ratio $s_c = m_c/m_b$, whenever diagrams with internal charm-quark loops contribute.

The individual terms in (2.65) are in close correspondence with the α_i amplitude parameters. The precise connection follows by evaluating the matrix elements of (2.65). Because the SCET Lagrangian contains no leading-power interactions between the collinear-2 and collinear-1 fields, the matrix elements of O_k^I , O_k^{II} fall apart into two factors each. For a pseudoscalar $M_2 = P$,

$$\langle P | (\bar{\xi} W_{c2})(tn_-) \frac{\not{n}_-}{2} (1 \pm \gamma_5) (W_{c2}^\dagger \chi) | 0 \rangle = \mp \frac{if_P m_B}{2} \int_0^1 du e^{iut} \Phi_P(u), \quad (2.68)$$

while for a vector $M_2 = V$ with polarization vector ϵ_μ we have

$$\langle V | (\bar{\xi} W_{c2})(tn_-) \frac{\not{n}_-}{2} (1 \pm \gamma_5) (W_{c2}^\dagger \chi) | 0 \rangle = -\frac{if_V m_V}{2} n_- \cdot \epsilon^* \int_0^1 du e^{iut} \Phi_V(u), \quad (2.69)$$

such that only the longitudinal polarization state contributes. Here $\Phi_P(u)$ ($\Phi_V(u)$) denotes the leading-twist light-cone distribution amplitude of a pseudoscalar (longitudinally polarized vector) meson. Using [156]

$$\langle P | (\bar{\xi} W_{c1}) \frac{\not{n}_+}{2} [W_{c1}^\dagger i \not{D}_{\perp c} W_{c1}] (sn_+) (1 + \gamma_5) h_v | \bar{B} \rangle = -m_b m_B \int_0^1 d\tau e^{i\tau \hat{s}} \Xi_P(\tau), \quad (2.70)$$

$$\langle V | (\bar{\xi} W_{c1}) \frac{\not{n}_+}{2} [W_{c1}^\dagger i \not{D}_{\perp c} W_{c1}] (sn_+) (1 + \gamma_5) h_v | \bar{B} \rangle = m_b m_V (n_+ \cdot \epsilon^*) \int_0^1 d\tau e^{i\tau \hat{s}} \frac{m_B}{2m_V} \Xi_{\parallel}(\tau), \quad (2.71)$$

and defining

$$\hat{\Xi}_{M_1}(t) = \begin{cases} \Xi_P(\tau) & M_1 = P, \\ \frac{m_B}{2m_V} \Xi_{\parallel}(\tau) & M_1 = V, \end{cases} \quad (2.72)$$

we can write the matrix elements of the terms in (2.65) as

$$\begin{aligned} \alpha_{ik}^p(M_1 M_2) &= \int_0^1 du T_{ik}^{I,p}(u) \Phi_{M_2}(u) \\ &\quad - \frac{1}{2F^{BM_1}(0)} \int_0^1 du dv H_{ik}^{II,p}(u, v) \hat{\Xi}_{M_1}(1-v) \Phi_{M_2}(u) \end{aligned} \quad (2.73)$$

with $F^{BM_1}(0) = f_+^{BM_1}(0)$ when $M_1 = P$, and $F^{BM_1}(0) = A_0^{BM_1}(0)$ for $M_1 = V$. The index i applies to $i = 1, 2, 3, 4, 3EW, 4EW$, and $k = L, R$ except for $i = 1, 2$, where k is empty (as is p). The two coefficients $\alpha_1(M_1 M_2)$, $\alpha_2(M_1 M_2)$ correspond to the “tree” flavour amplitudes in (2.37), while the four “penguin” amplitudes are given by

$$\begin{aligned} \alpha_3^p(M_1 M_2) &= \alpha_{3L}^p(M_1 M_2) \mp \alpha_{3R}^p(M_1 M_2), \\ \alpha_4^p(M_1 M_2) &= \alpha_{4L}^p(M_1 M_2) \mp \alpha_{4R}^p(M_1 M_2), \\ \alpha_{3,EW}^p(M_1 M_2) &= \alpha_{3L,EW}^p(M_1 M_2) \mp \alpha_{3R,EW}^p(M_1 M_2), \\ \alpha_{4,EW}^p(M_1 M_2) &= \alpha_{4L,EW}^p(M_1 M_2) \mp \alpha_{4R,EW}^p(M_1 M_2), \end{aligned} \quad (2.74)$$

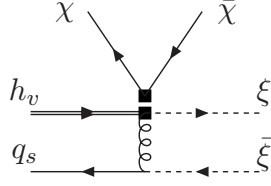


Figure 2.13: Hard spectator scattering in SCET_I .

where the upper (lower) signs correspond to the case $M_2 = P(V)$. In terms of the amplitudes a_i , the coefficients $a_{3L;4L;3L,\text{EW};4L,\text{EW}}$ correspond to $a_{3;4;5;10}$ respectively, while $a_{3R;3R,\text{EW}}$ to $a_{5;7}$. The remaining $a_{4R;4R,\text{EW}}$ vanish, while $a_{6;8}$ correspond to power suppressed contributions and therefore do not appear in this derivation.

From $\sqrt{m_b} \Lambda_{\text{QCD}}$ to Λ_{QCD} : SCET_II

Hard-collinear momentum regions appear only in the spectator scattering, which convert the soft spectator antiquark into the collinear antiquark in the meson M_1 . The matching onto SCET_II therefore leaves unchanged the first term on the r.h.s of (2.73), while the SCET_I form factor Ξ_{M_1} related to the matrix element (2.70), (2.71) must be matched onto SCET_II . This involves the matching of the B-type current appearing in O_k^II onto a four-quark SCET_II operator, corresponding to the evaluation of the diagram in Fig. 2.13. In the end, the form factor Ξ_{M_1} is expressed in terms of a jet function $J_\parallel(\tau; u, \omega)$, that represents the matching coefficient of the four-quark SCET_II operator, and light-cone distribution amplitudes, in terms of which the matrix element of the four-quark SCET_II operator is expressed, since it splits into its soft (B meson) and collinear (meson M_1) part, according to

$$\hat{\Xi}(\tau) = \frac{m_B}{4m_b} \int_0^\infty \frac{d\omega}{\omega} \hat{f}_B \Phi_{B+}(\omega) \int_0^1 dw f_{M_1} \Phi_{M_1}(w) J(\tau; w, \omega), \quad (2.75)$$

because no leading soft-collinear interactions appear in the SCET_II Lagrangian. In more detail, the result in (2.75) is obtained defining the matrix element of the bilinear composed of the collinear quark-antiquark pair that forms the meson M_1 as in (2.68), (2.69), while for the soft b - and spectator-antiquark bilinear that form the B meson one has

$$\langle 0 | (\bar{q}_s Y)(t n_-) (Y^\dagger h_v) | 0 \rangle = -\frac{i f_B m_B}{4} \left(\frac{1 + \psi}{2} \left[n_- v \not{\epsilon}_+ \tilde{\Phi}_{B+}(t) + n_+ v \not{\epsilon}_- \tilde{\Phi}_{B-}(t) \right] \gamma_5 \right), \quad (2.76)$$

with

$$\tilde{\Phi}_{B\pm}(t) = \int_0^\infty d\omega e^{-i\omega t} \Phi_{B\pm}(\omega) \quad (2.77)$$

the Fourier transform of the B meson distribution amplitude. This step allows to rewrite (2.73) in a form similar to the factorization formula

$$\alpha_{ik}^p(M_1 M_2) = \int_0^1 du T_{ik}^{\text{I},p}(u) \Phi_{M_2}(u) - \frac{1}{2F^{BM_1}(0)} \int_0^\infty d\omega \int_0^1 du dv$$

$$\cdot T_{ik}^{\text{II},p}(u, v) \hat{f}_B \Phi_{B+}(\omega) f_{M_1} \Phi_{M_1}(v) f_{M_2} \Phi_{M_2}(u), \quad (2.78)$$

with

$$T_{ik}^{\text{II},p}(u, v) = -\frac{m_B}{8m_b} \int_0^1 dz H_{ik}^{\text{II},p}(u, z) J_{\parallel}(\bar{z}; v, \omega). \quad (2.79)$$

The jet function is unique, *i.e.* all the hard-scattering functions $H_{ik}^{\text{II},p}$ are convoluted with the same $J_{\parallel}(\bar{z}; v, \omega)$. The jet function has been calculated at order α_s^2 in [153–156]; at order α_s , consistently with the NLO results presented below, it reads

$$J_{\parallel}(\bar{z}; v, \omega) = -\frac{4\pi\alpha_s C_F}{N_c} \frac{1}{m_B \omega \bar{v}} \delta(\bar{z} - \bar{v}). \quad (2.80)$$

To conclude this section, we note that because of the two matching steps, the spectator scattering involves two scales $\mu \sim m_b$, $\mu_h \sim \sqrt{m_b \Lambda_{\text{QCD}}}$, so that large logarithms involving the ratio of the two scales may appear, and they must be resummed, according to

$$\begin{aligned} & C(\mu) H^{II}(\mu) \star J_{\parallel}(\mu) \star [\hat{f}_b \Phi_{B+}](\mu) \star \Phi_{M_1}(\mu) \star \Phi_{M_2}(\mu) \\ & \Rightarrow C(\mu) H^{II}(\mu) U(\mu, \mu_h) \star J_{\parallel}(\mu_h) \star [\hat{f}_b \Phi_{B+}](\mu_h) \star \Phi_{M_1}(\mu_h) \star \Phi_{M_2}(\mu_h), \end{aligned} \quad (2.81)$$

where $U(\mu, \mu_h)$ is the evolution kernel for the SCET_I operator O^{II} . While in the first line either $H^{II}(\mu)$ or $J_{\parallel}(\mu)$ contains large logarithms, neither $H^{II}(\mu)$ or $J_{\parallel}(\mu_h)$ does. The evolution kernel for O^{II} factorizes into U_{BL} related to the Brodsky-Lepage kernel [69, 70] in the collinear-2 sector and the evolution kernel $U_{\parallel}(\mu, \mu_h)$ for a B1-type current. Since $U_{\text{BL}}(\mu, \mu_h) \Phi_{M_2}(\mu_h) = \Phi_{M_2}(\mu)$, we can write the previous expression as

$$C(\mu) H^{II}(\mu) U_{\parallel}(\mu, \mu_h) \star J_{\parallel}(\mu_h) \star [\hat{f}_b \Phi_{B+}](\mu_h) \star \Phi_{M_1}(\mu_h) \star \Phi_{M_2}(\mu_h). \quad (2.82)$$

In our NLO analysis however this feature is not considered, because no distinction is made between $\Phi_{M_2}(\mu_h)$ and $\Phi_{M_2}(\mu)$, that is beyond the NLO precision. Moreover, $U_{\parallel}(\mu, \mu_h)$ is not calculated, so that we will use

$$C(\mu) H^{II}(\mu) \star J_{\parallel}(\mu_h) \star [\hat{f}_b \Phi_{B+}](\mu_h) \star \Phi_{M_1}(\mu_h) \star \Phi_{M_2}(\mu_h). \quad (2.83)$$

2.3.4 NLO results for the topological amplitudes

We collect in this section the NLO expression of the topological amplitudes, summarizing results obtained in [65–67, 179]. These expression are needed to calculate the decay amplitudes in the phenomenological analyses of chapter 4, 5 and 6. We provide the expression of the coefficients a_i ; results for the topological amplitudes α_i can be obtained using the relations in (2.44). The results shown in this section rely on the following approximations:

- *Electroweak penguin effects and electromagnetic corrections.* Electromagnetic corrections to the QCD coefficients α_1 , α_2 , α_3^p and α_4^p are neglected, since they are much smaller than the NLO QCD corrections; similarly, corrections of order $\alpha_s C_{7,\dots,10}$ to the same coefficients are neglected, too, because the short-distance coefficients $C_{7,\dots,10}$ are proportional to the electromagnetic coupling α_{em} . On the other hand,

since the LO expressions for the electroweak penguin amplitudes $\alpha_{3,\text{EW}}^p$, $\alpha_{4,\text{EW}}^p$ involve only $C_{7,\dots,10}$, we include in this case corrections of order $\alpha_s C_{7,\dots,10}$; electromagnetic corrections are included when they are proportional to the large short-distance coefficients $C_{1,2,7\gamma}$, *i.e.*, we include corrections of order $\alpha_{\text{em}} C_{1,2,7\gamma}$, but we neglect those proportional to $\alpha_{\text{em}} C_{3,\dots,10}$.

- *Weak annihilation.* We neglect weak annihilation mechanisms involving photons ($\gamma \rightarrow \bar{q}q$), that were not considered in the flavour structure of (2.51) either. In practice, only annihilation terms proportional to $\alpha_s C_i$ are considered, while we neglect $\alpha_{\text{em}} C_i$ terms. The reason for this is that the latter can never be CKM-enhanced when one of the large short-distance coefficients C_1 , C_2 is involved. We neglect also all the singlet weak annihilation contributions except β_{S3} .

The general form of the coefficients a_i at NLO in α_s reads

$$a_i^p(M_1 M_2) = \left(C_i + \frac{C_{i\pm 1}}{N_c} \right) N_i(M_2) + \frac{C_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \left[V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1 M_2) \right] + P_i^p(M_2), \quad (2.84)$$

where the upper (lower) signs apply when i is odd (even). The superscript p has to be omitted for $i = 1, 2$. The quantities $N_i(M_2)$, $V_i(M_2)$, $H_i(M_1 M_2)$, $P_i^p(M_2)$ stands respectively for the tree-level result, the one loop vertex correction, the spectator scattering and the penguin contractions. Explicitly, they are given as follows:

Leading order (Fig. 2.4). It is given simply by the normalization of the LCDA $\Phi_{P,V}$ or $\Phi_{p,v}$; P, V refer obviously to pseudoscalar or vector meson. Capital letters refer to leading-twist LCDA, small letters to twist-3 LCDA. The latter appear only in $a_{6,8}$, in correspondence with the scalar Dirac structure $(S - P) \otimes (S + P)$, and are responsible for the vanishing of the leading contribution in case of vector mesons, since $\int_0^1 dx \Phi_v(x) = 0$. One obtains

$$N_i^{0,-}(M_2) = \begin{cases} 0, & \text{for } i = 6, 8 \text{ and } M_2 = V, \\ 1, & \text{otherwise,} \end{cases} \quad (2.85)$$

where we have indicated explicitly, with the superscripts $0, -$, the polarization of the vectors appearing in the final state.

Vertex terms (Fig. 2.6). One has

$$V_i^0(M_2) = \begin{cases} \int_0^1 dx \Phi_{M_2} \left[12 \ln \frac{m_b}{\mu} - 18 + g(x) \right], & \text{for } i = 1, \dots, 4, 9, 10, \\ \int_0^1 dx \Phi_{M_2} \left[-12 \ln \frac{m_b}{\mu} + 6 - g(\bar{x}) \right], & \text{for } i = 5, 7, \\ \int_0^1 dx \Phi_{m_2} [-6 + h(x)], & \text{for } i = 6, 8, \end{cases} \quad (2.86)$$

in case of $M_2 = P$ or a longitudinally polarized vector meson $M_2 = V$, while

$$V_i^-(V_2) = \begin{cases} \int_0^1 dx \phi_{b_{V_2}} \left[12 \ln \frac{m_b}{\mu} - 18 + g_T(x) \right], & \text{for } i = 1, \dots, 4, 9, 10, \\ \int_0^1 dx \phi_{a_{V_2}} \left[-12 \ln \frac{m_b}{\mu} + 6 - g_T(\bar{x}) \right], & \text{for } i = 5, 7, \\ 0, & \text{for } i = 6, 8, \end{cases} \quad (2.87)$$

in case of a vector meson V_2 with negative polarization. (we recall that $\bar{x} \equiv 1 - x$). The functions $g(x)$, $g_T(x)$, $h(x)$ contain the finite corrections from the loop integration, and read

$$\begin{aligned} g(x) &= 3 \left(\frac{1-2x}{\bar{x}} \ln x - i\pi \right) + \left[2\text{Li}_2(x) - \ln^2 x + \frac{2\ln x}{\bar{x}} - (3 + 2i\pi) \ln x - (x \leftrightarrow \bar{x}) \right], \\ g_T(x) &= g(x) + \frac{\ln x}{\bar{x}}, \\ h(x) &= 2\text{Li}_2(x) - \ln^2 x - (1 + 2i\pi) \ln x - (x \leftrightarrow \bar{x}). \end{aligned} \quad (2.88)$$

Φ_M , Φ_m , ϕ_a , ϕ_b are LCDA that will be defined in par. 2.3.5. The constants $-18, 6, -6$ are scheme dependent, and correspond to the use of the NDR scheme for γ_5 . This constant does not appear in $a_{6,8}$ in case of vector mesons, as expected, because there is no tree level contribution that could cause a scheme dependence at NLO.

Penguin terms (ig. 2.7). At order α_s corrections are present only for $i = 4, 6$. For $i = 4$ we have

$$\begin{aligned} P_4^{0,p}(M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left\{ C_1 \left[\frac{4}{3} \log \frac{m_b}{\mu} + \frac{2}{3} - G_{M_2}(s_p) \right] \right. \\ &\quad + C_3 \left[\frac{8}{3} \log \frac{m_b}{\mu} + \frac{4}{3} - G_{M_2}(0) - G_{M_2}(1) \right] \\ &\quad + (C_4 + C_6) \left[\frac{4n_f}{3} \log \frac{m_b}{\mu} - (n_f - 2)G_{M_2}(0) \right. \\ &\quad \quad \left. - G_{M_2}(s_c) - G_{M_2}(1) \right] \\ &\quad \left. - 2C_{8g}^{eff} \int_0^1 \frac{dx}{1-x} \Phi_{M_2}(x) \right\}, \end{aligned} \quad (2.89)$$

if M_2 is a pseudoscalar or a longitudinally polarized vector meson, while

$$\begin{aligned} P_4^{-,p}(V_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left\{ C_1 \left[\frac{4}{3} \log \frac{m_b}{\mu} + \frac{2}{3} - G_{V_2}^-(s_p) \right] \right. \\ &\quad + C_3 \left[\frac{8}{3} \log \frac{m_b}{\mu} + \frac{4}{3} - G_{V_2}^-(0) - G_{V_2}^-(1) \right] \\ &\quad + (C_4 + C_6) \left[\frac{4n_f}{3} \log \frac{m_b}{\mu} - (n_f - 2)G_{V_2}^-(0) \right. \\ &\quad \quad \left. - G_{M_2}^-(s_c) - G_{M_2}^-(1) \right] \left. \right\}, \end{aligned} \quad (2.90)$$

in case of a vector meson with transverse polarizations. $n_f = 5$ indicates the number of light fermions, while $s_u = 0$, $s_c = (m_c/m_b)^2$ represent the mass ratios involved in the

evaluation of the penguin diagrams. The functions $G_{M_2}(s)$, $G_{V_2}^-(s)$ are given by

$$\begin{aligned} G_{M_2}(s) &= \int_0^1 dx G(s - i\epsilon, \bar{x}) \Phi_{M_2}(x), \\ G_{V_2}^-(s) &= \int_0^1 dx G(s - i\epsilon, \bar{x}) \phi_{b_{V_2}}(x), \end{aligned} \quad (2.91)$$

where

$$\begin{aligned} G(s, \bar{x}) &= -4 \int_0^1 du u \bar{u} \ln[s - u \bar{u} x] \\ &= \frac{2(12s + 5x - 3x \ln s)}{9x} - \frac{4\sqrt{4s - x}(2s + x)}{3x^{3/2}} \arctan \sqrt{\frac{x}{4s - x}}. \end{aligned} \quad (2.92)$$

For $i = 6$ the result is

$$\begin{aligned} P_6^p(M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left\{ C_1 \left[\frac{4}{3} \log \frac{m_b}{\mu} + \frac{2}{3} - \hat{G}_{M_2}(s_p) \right] \right. \\ &\quad + C_3 \left[\frac{8}{3} \log \frac{m_b}{\mu} + \frac{4}{3} - \hat{G}_{M_2}(0) - \hat{G}_{M_2}(1) \right] \\ &\quad + (C_4 + C_6) \left[\frac{4n_f}{3} \log \frac{m_b}{\mu} - (n_f - 2) \hat{G}_{M_2}(0) \right. \\ &\quad \left. \left. - \hat{G}_{M_2}(s_c) - \hat{G}_{M_2}(1) \right] - 2C_{8g}^{eff} \right\}, \end{aligned} \quad (2.93)$$

if M_2 is a pseudoscalar meson, and

$$\begin{aligned} P_6^{0,p}(V_2) &= -\frac{C_F \alpha_s}{4\pi N_c} \left\{ C_1 \hat{G}_{M_2}(s_p) + C_3 \left[\hat{G}_{M_2}(0) + \hat{G}_{M_2}(1) \right] \right. \\ &\quad \left. + (C_4 + C_6) \left[(n_f - 2) \hat{G}_{M_2}(0) + \hat{G}_{M_2}(s_c) + \hat{G}_{M_2}(1) \right] \right\}, \end{aligned} \quad (2.94)$$

if M_2 is a longitudinally polarized vector meson; $P_6^{-,p}(M_2) = 0$ for transversely polarized vector mesons. In this case

$$\hat{G}_{M_2}(s) = \int_0^1 dx G(s - i\epsilon, \bar{x}) \Phi_{m_2}(x). \quad (2.95)$$

It remains to consider electromagnetic penguin corrections, according to the approximation discussed at the beginning of the paragraph. These corrections appears for $i = 8, 10$, and considering only the contribution of $C_{1,2,\gamma}$ they read

$$P_8^p(M_2) = \frac{\alpha_{em}}{4\pi N_c} \left\{ (C_1 + N_c C_2) \left[\frac{4}{3} \log \frac{m_b}{\mu} + \frac{2}{3} - \hat{G}_{M_2}(s_p) \right] - 3C_{7\gamma}^{eff} \right\}, \quad (2.96)$$

if M_2 is a pseudoscalar meson, and

$$P_8^{0,p}(V_2) = -\frac{\alpha_{em}}{4\pi N_c} \left\{ (C_1 + N_c C_2) \hat{G}_{M_2}(s_p) \right\}, \quad (2.97)$$



Figure 2.14: Additional contributions to the penguin terms when $M_2 = \rho^0, \omega, \phi$, taken from [67]. Wavy lines denote a photon.

for a longitudinally polarized vector meson, while $P_8^{-,p}(M_2) = 0$ for transversely polarized vector mesons. Then

$$P_{10}^{0,p}(M_2) = \frac{\alpha_{\text{em}}}{4\pi N_c} \left\{ (C_1 + N_c C_2) \left[\frac{4}{3} \log \frac{m_b}{\mu} + \frac{2}{3} - \hat{G}_{M_2}(s_p) \right] - 3C_{7\gamma}^{\text{eff}} \int_0^1 \frac{dx}{1-x} \Phi_{M_2}(x) \right\},$$

for M_2 pseudoscalar and longitudinally polarized vector meson and

$$P_{10}^{-,p}(M_2) = \frac{\alpha_{\text{em}}}{4\pi N_c} \left\{ (C_1 + N_c C_2) \left[\frac{4}{3} \log \frac{m_b}{\mu} + \frac{2}{3} - \hat{G}_{M_2}^{\pm}(s_p) \right] \right\}, \quad (2.98)$$

for transversely polarized vector mesons.

It is interesting to discuss at this point another contribution, which corresponds to the exchange of a photon, as in Fig. 2.14, with the subsequent production of a $\bar{q}q$ pair. Such diagrams can contribute only to final states with a neutral vector meson V_2 . It is beyond the purpose of this introduction to discuss in detail the conceptual issues behind the transition; however, it is interesting to note that, while in case of PV , or longitudinally polarized VV final states these contributions are very small, in case of a final states VV with transversely polarized vector mesons they give rise to an enhanced contribution [178], that is at the origin of the modified hierarchy (2.26) among the helicity amplitudes. In case of PV , or longitudinally polarized VV final states the additional contribution is usually included in an ad-hoc term

$$\Delta\alpha_{3,\text{EW}}^p(M_1 V) = \frac{2\alpha_{\text{em}}}{27\pi} \left\{ 4(C_1 + N_c C_2) \left(\frac{1}{2} - \delta_{pc} \ln \frac{\mu}{m_c} - \delta_{pu} \ln \frac{\mu}{\nu} \right) - 9C_{7\gamma}^{\text{eff}} \right\}; \quad (2.99)$$

In case of VV final states with negative polarization the new contribution is written as

$$P_7^{-,p}(V_2) = P_9^{-,p}(V_2) = -\frac{\alpha_{\text{em}}}{3\pi} C_{7\gamma}^{\text{eff}} \frac{m_B m_b}{m_{V_2}^2} + \frac{2\alpha_{\text{em}}}{27\pi} (C_1 + N_c C_2) \left[\delta_{pc} \ln \frac{m_c^2}{\mu^2} + \delta_{pu} \ln \frac{\nu^2}{\mu^2} + 1 \right], \quad (2.100)$$

where the appearance of the $(m_b/\Lambda_{\text{QCD}})^2$ enhancement proportional to $C_{7\gamma}^{\text{eff}}$ is evident, due to the factor $m_B m_b/m_{V_2}^2$.

Hard spectator terms (Fig. 2.8). The correction from hard gluon exchange between M_2 and the spectator quark is given by

$$H_i(M_1 M_2) = + \frac{B_{M_1 M_2}}{A_{M_1 M_2}} \frac{m_B}{\lambda_B} \int_0^1 dx dy \left[\frac{\Phi_{M_2}(x) \Phi_{M_1}(y)}{\bar{x} \bar{y}} + r_\chi^{M_1} \frac{\Phi_{M_2}(x) \Phi_{m_1}(y)}{x \bar{y}} \right], \quad (2.101)$$

for $i = 1, \dots, 4, 9, 10$,

$$H_i(M_1 M_2) = - \frac{B_{M_1 M_2}}{A_{M_1 M_2}} \frac{m_B}{\lambda_B} \int_0^1 dx dy \left[\frac{\Phi_{M_2}(x) \Phi_{M_1}(y)}{x \bar{y}} + r_\chi^{M_1} \frac{\Phi_{M_2}(x) \Phi_{m_1}(y)}{\bar{x} \bar{y}} \right], \quad (2.102)$$

for $i = 5, 7$, and

$$H_i(M_1 M_2) = 0, \quad (2.103)$$

for $i = 6, 8$. These results are valid for PP , PV , VP , or longitudinally polarized VV final states, while in case of transversely polarized VV final states one has

$$H_i^-(V_1 V_2) = - \frac{2f_B f_{V_1}^\perp}{m_B m_b f_-^{BV_1}(0)} \frac{m_b}{\lambda_B} \int_0^1 dx dy \frac{\phi_{bV_2}(x) \Phi_1^\perp(y)}{x \bar{y}^2}, \quad (2.104)$$

for $i = 1, \dots, 4, 9, 10$,

$$H_i^-(V_1 V_2) = + \frac{2f_B f_{V_1}^\perp}{m_B m_b f_-^{BV_1}(0)} \frac{m_b}{\lambda_B} \int_0^1 dx dy \frac{\phi_{aV_2}(x) \Phi_1^\perp(y)}{\bar{x} \bar{y}^2}, \quad (2.105)$$

for $i = 5, 7$ and

$$H_i^-(V_1 V_2) = - \frac{f_B f_{V_1}}{m_B m_b f_-^{BV_1}(0)} \frac{m_b m_{V_1}}{m_{V_2}^2} \frac{m_b}{\lambda_B} \int_0^1 dx dy \frac{\Phi_2^\perp(x) \phi_{aV_1}(y)}{x \bar{x} \bar{y}}, \quad (2.106)$$

for $i = 6, 8$.

In these results λ_B represents the first inverse moment of the B meson distribution amplitude appearing in eq. (2.75), and it is defined as

$$\int_0^\infty d\omega \frac{\Phi_{B^+}(\omega)}{\omega} \equiv \frac{1}{\lambda_B}. \quad (2.107)$$

The main feature of the spectator scattering is the inclusion of some subleading terms that are proportional to $r_\chi^{M_1}$ in (2.101)-(2.102). These terms are formally suppressed by Λ_{QCD}/m_b in the heavy-quark limit, but they are included because they are numerically large. As anticipated in section 2.2.3, these terms give rise to conceptual problems, because the twist-3 distribution amplitude Φ_{p_1} does not vanish at the endpoint $y = 1$, and the power-suppressed term is divergent. Since the aim of this section is to provide the phenomenological amplitudes used in the analyses of chapter 4, 5 and 6, we do not address this problem in detail here; some discussion about it will be given in chapter 3, where a similar problem is studied and solved in the context of a non-relativistic system. We adopt here the standard solution, that consists of isolating the divergence by defining a parameter

$$\begin{aligned} \int_0^1 \frac{dy}{\bar{y}} \Phi_{m_1}(y) &= \Phi_{m_1}(1) \int_0^1 \frac{dy}{\bar{y}} + \int_0^1 \frac{dy}{\bar{y}} [\Phi_{m_1}(y) - \Phi_{m_1}(1)] \\ &\equiv \Phi_{m_1}(1) X_H^{M_1} + \int_0^1 \frac{dy}{[\bar{y}]_+} \Phi_{m_1}(y). \end{aligned} \quad (2.108)$$

The remaining integral is finite, and the $X_H^{M_1}$ introduced is an unknown parameter representing a soft-gluon interaction with the spectator quark. The endpoint divergence appears in an attempt to calculate the soft interaction perturbatively, and therefore it should be regulated by soft physics occurring at the scale Λ_{QCD} . For this reason $X_H^{M_1}$ is expected to be of order $X_H^{M_1} \sim \ln(m_b/\Lambda_{\text{QCD}})$, however with a potential complex coefficient, since multiple soft scattering can introduce a strong rescattering phase. A consequence of this is that spectator scattering beyond leading power in Λ_{QCD}/m_b is necessary model dependent. In phenomenological analyses, $X_H^{M_1}$ is treated as a parameter, that is varied inside a certain range, as we will define in the next section. Usually it is also assumed to be universal, *i.e.* independent of M_1 and the index i of $H_i(M_1 M_2)$.

Weak annihilation (Fig. 2.9). Within the approximation described at the beginning of the section, the weak annihilation coefficients b_i can be expressed in term of some building blocks A_i . Explicitly

$$\begin{aligned} b_1 &= \frac{C_F}{N_c^2} C_1 A_1^i, & b_3^p &= \frac{C_F}{N_c^2} [C_3 A_1^i + C_5 (A_3^i + A_3^f) + N_c C_6 A_3^f], \\ b_2 &= \frac{C_F}{N_c^2} C_2 A_1^i, & b_4^p &= \frac{C_F}{N_c^2} [C_4 A_1^i + C_6 A_2^i], \\ b_{3,\text{EW}}^p &= \frac{C_F}{N_c^2} [C_9 A_1^i + C_7 (A_3^i + A_3^f) + N_c C_8 A_3^f], \\ b_{4,\text{EW}}^p &= \frac{C_F}{N_c^2} [C_{10} A_1^i + C_8 A_2^i], \end{aligned} \quad (2.109)$$

where the superscript i and f refers to gluon emission from initial and final state, respectively. The subscript “ k ” in $A_k^{i,f}$ instead refers to one of the three possible Dirac structures $\Gamma_1 \otimes \Gamma_2$, which arise when the four-quark operators in the effective Hamiltonian are Fierz-transformed into the form $(\bar{q}_1 b)_{\Gamma_1} (\bar{q}_2 q_3)_{\Gamma_2}$, such that the quarks in the first bracket refer to the constituents of the $\bar{B}$ meson. Specifically, one has $k = 1$ for $(V - A) \otimes (V - A)$, $k = 2$ for $(V - A) \otimes (V + A)$, and $k = 3$ for $(-2)(S - P) \otimes (S + P)$.

The weak annihilation contributions contain endpoint divergences, too, which are treated in the same way as in case of the spectator scattering, *i.e.* introducing a complex parameter X_A (a different X_L is necessary in case of VV final states, too), of order $\ln(m_b/\Lambda_{\text{QCD}})$. Specifically, they are introduced to regularize the following integrals:

$$\int_0^1 \frac{dy}{y} \rightarrow X_A^{M_1}, \quad \int_0^1 dy \frac{\ln y}{y} \rightarrow -\frac{1}{2} (X_A^{M_1})^2, \quad \int_0^1 dy \frac{1}{y^2} \rightarrow -X_L^{M_1}. \quad (2.110)$$

The dependence on M_1 here is left explicit, because in the phenomenological scenario discussed in the next section a certain dependence on M_1 is considered. In literature it is possible to find exact expressions for the coefficients $A_k^{i,f}$, before the distribution amplitude momentum fraction integration [67,179]. Since the endpoint divergences introduce sizeable uncertainties, it is sufficient to consider approximate expressions, that are intended as an estimate of the weak annihilation contribution, rather than a precise result. These are obtained by evaluating the convolution integral with the asymptotic expressions for the

LCDA, with the results

$$\begin{aligned}
A_1^i &\approx A_2^i \approx 2\pi\alpha_s \left[9 \left(X_A - 4 + \frac{\pi^2}{3} \right) + r_\chi^{M_1} r_\chi^{M_2} X_A^2 \right], \\
A_1^f &= A_2^f = 0, \\
A_3^i &\approx 6\pi\alpha_s (r_\chi^{M_1} - r_\chi^{M_2}) \left(X_A^2 - 2X_A + \frac{\pi^2}{3} \right), \\
A_3^f &\approx 6\pi\alpha_s (r_\chi^{M_1} - r_\chi^{M_2}) (2X_A^2 - X_A),
\end{aligned} \tag{2.111}$$

when both the final state mesons are pseudoscalar, and

$$\begin{aligned}
A_1^i &\approx -A_2^i \approx 6\pi\alpha_s \left[6 \left(X_A - 4 + \frac{\pi^2}{3} \right) + r_\chi^{M_1} r_\chi^{M_2} (X_A^2 - 2X_A) \right], \\
A_1^f &= A_2^f = 0, \\
A_3^i &\approx 6\pi\alpha_s \left[-3r_\chi^{M_1} \left(X_A^2 - 2X_A - \frac{\pi^2}{3} + 4 \right) + r_\chi^{M_2} \left(X_A^2 - 2X_A + \frac{\pi^2}{3} \right) \right], \\
A_3^f &\approx 6\pi\alpha_s [3r_\chi^{M_1} (2X_A - 1)(2 - X_A) - r_\chi^{M_2} (2X_A^2 - X_A)],
\end{aligned} \tag{2.112}$$

when M_1 is a vector and M_2 a pseudoscalar. We consider here as M_1 the meson containing the antiquark from the weak vertex. In the opposite case in which M_1 is a pseudoscalar and M_2 a vector one has to exchange $r_\chi^{M_1} \leftrightarrow r_\chi^{M_2}$ and to change the sign of A_3^f . In case of a final state with longitudinally polarized vector meson one obtains

$$\begin{aligned}
A_1^{f,0} &\approx A_2^{f,0} \approx 18\pi\alpha_s \left[\left(X_A - 4 + \frac{\pi^2}{3} \right) + r_\chi^{V_1} r_\chi^{V_2} (X_A - 2)^2 \right], \\
A_3^{i,0} &\approx 18\pi\alpha_s (r_\chi^{V_1} - r_\chi^{V_2}) \left(-X_A^2 + 2X_A - 4 + \frac{\pi^2}{3} \right) \\
A_3^{f,0} &\approx 18\pi\alpha_s (r_\chi^{V_1} + r_\chi^{V_2}) (2X_A - 1)(2 - X_A),
\end{aligned} \tag{2.113}$$

while the case of vector mesons with negative helicity reads

$$\begin{aligned}
A_1^{i,-} &\approx A_2^{i,-} \approx 18\pi\alpha_s \frac{m_{V_1} m_{V_2}}{m_B^2} \left(\frac{1}{2} X_L + \frac{5}{2} - \frac{\pi^2}{3} \right), \\
A_3^{i,-} &\approx 18\pi\alpha_s \left(\frac{m_{V_1}}{m_{V_2}} r_\chi^{V_2} - \frac{m_{V_2}}{m_{V_1}} r_\chi^{V_1} \right) (X_A^2 - 2X_A + 2), \\
A_3^{f,-} &\approx 18\pi\alpha_s \left(\frac{m_{V_1}}{m_{V_2}} r_\chi^{V_2} + \frac{m_{V_2}}{m_{V_1}} r_\chi^{V_1} \right) (2X_A^2 - 5X_A + 3).
\end{aligned} \tag{2.114}$$

We conclude the paragraph giving the result for the singlet weak annihilation term b_{S3} , that in our approximation is the only one considered. b_{S1} and b_{S1} are neglected not because they are supposed to be small, but because they always appear with the large

coefficients α_1, α_2 . Its effect is limited to final states containing an η or an η' meson and it can be written as

$$b_{S3}(M_1 P_{q,s}) = \frac{C_F}{N_c^2} (C_5 + N_c C_6) 5\pi \alpha_s B_2^{P_g}(\mu_h) X_A^{M_1}, \quad (2.115)$$

where $P_{q,s}$ refer to the η state, and $B_2^{P_g}(\mu)$ comes from the Gegenbauer expansion of a two-gluon distribution amplitude truncated at leading order, for which we assume $B_2^{P_g}(1\text{GeV}) = 2 \pm 3$.

2.3.5 Phenomenology

Input parameters

The expressions for the $a_i(M_1 M_2)$ coefficients quoted in section 2.3.4 depend on many parameters. There are Standard Model parameters, like the CKM matrix elements, the quark masses, the strong and electromagnetic coupling constant α_s and α_{em} , and hadronic parameters, like the form factors, the decay constants and so on. In table 2.2 we give a summary of the most relevant parameters needed for the numerical evaluation of the non-leptonic B decay amplitudes. The aim of this section is to discuss briefly the origin of these parameters, and their importance in the evaluation of the decay amplitudes.

Starting from the Standard Model parameters, We consider $G_F = 1.16639 \cdot 10^{-5} \text{ GeV}^{-2}$ [8]. Next, we define the quark masses as running masses in the $\overline{\text{MS}}$ scheme, while the value of the QCD scale parameter corresponds to $\alpha_s(M_Z) = 0.118$ for the two-loop running coupling in the $\overline{\text{MS}}$ scheme. The QCD scale is $\Lambda_{\overline{\text{MS}}}^{(5)} = 225 \text{ MeV}$, ($\overline{\text{MS}}$ scheme, five quark flavours). The ratio $s_c = (m_c^2/m_b^2)$ needed for the calculation of the penguin contribution is scale independent; the values of the light quark masses are defined such that $r_\chi^K = r_\chi^\pi$. Among the Standard Model parameters, the decay amplitudes are most sensitive to the value of m_s , which sets the scale for the ratio r_χ for pseudoscalar mesons, defined in (2.45). Some branching ratios, and in particular the direct CP asymmetries, are also very sensitive to $|V_{ub}|$ and its phase, $\gamma = \arg(V_{ub}^*)$.

Larger uncertainties come from the hadronic parameters. While some of these could be determined from experiment, like the decay constants and the transition form factors, in many cases we use theoretical results derived in lattice QCD or with QCD sum rules. Only the experimental results from the leptonic decays $\pi^- \rightarrow \mu \bar{\nu}_\mu$, $K^- \rightarrow \mu \bar{\nu}_\mu$, the semi-leptonic $\tau \rightarrow \rho^- \nu_\tau$ decay, and the electromagnetic decays $V \rightarrow e^+ e^-$ with $V = \rho^0, \omega, \phi$ are used to determine the values of the decay constants of pseudoscalar and longitudinally polarized vector mesons [157], that are known with a good accuracy, so that they can be considered without uncertainties in the QCD factorization approach. We consider here the values given in [67]. Transition form factors instead are taken from QCD sum rules [158–160] or lattice QCD results, and have larger uncertainties, that often reflect in large uncertainties in the decay amplitudes, too. We employ here the value used in [67, 179]; in case of $f^\perp$ we take the updated values from [160].

A fundamental role in the numerical evaluation is played by the LCDAs and their integration over the momentum fractions. The LCDA are usually expanded in Gegenbauer polynomials, since the Gegenbauer moments are multiplicatively renormalized. We collect

here the definitions of the LCDA entering in the equations of section 2.3.4. Starting from the leading-twist distribution amplitudes, we have

$$\Phi_{P,V,\perp}(x, \mu) = 6x\bar{x} \left[1 + \sum_{n=1}^{\infty} \alpha_{n,(\perp)}^{P,V}(\mu) C_n^{(3/2)}(2x-1) \right], \quad (2.116)$$

where $P, V, \perp$ refer to a pseudoscalar, a longitudinally polarized vector or a transverse vector meson; $C_n^{(3/2)}(x)$ are the Gegenbauer polynomials, while $\alpha_{n,(\perp)}^{P,V}(\mu)$ are Gegenbauer moments. In case of twist-3 distribution amplitudes one has⁷

$$\begin{aligned} \Phi_p(x) &= 1, \\ \Phi_v(x) &= 3 \sum_{n=0}^{\infty} \alpha_{n,\perp}^V(\mu) P_{n+1}(2x-1), \end{aligned} \quad (2.117)$$

where $\alpha_{0,\perp}^V = 1$, and $P_n(x)$ are the Legendre polynomials. Like $\Phi_p(x)$, $\Phi_v(x)$ does not vanish at the endpoints; one has, however,

$$\int_0^1 dx \Phi_v(x) = 0, \quad (2.118)$$

that is at the basis of the result for $a_{6,8}$ in (2.85). Then, in case of decays into transverse vector mesons, one needs also

$$\phi_a(u) = \int_u^1 dv \frac{\Phi(v)}{v}, \quad \phi_b(u) = \int_0^u dv \frac{\Phi(v)}{\bar{v}}. \quad (2.119)$$

The LCDA expansion is usually truncated at the second Gegenbauer moment. These parameters are still rather poorly known, and experiments can put only indirect constraints on them. Fortunately, the sensitivity of the decay amplitudes to these parameters turns out to be small, except in case of the colour-suppressed tree amplitude α_2 , which shows a considerable dependence on the first inverse Gegenbauer moment of the B -meson distribution amplitude (λ_B) and the second Gegenbauer moment of the light mesons through the hard-spectator interaction. Gegenbauer moments are calculated by means of QCD sum rules [163, 168–173], or on the lattice [174, 175]

As explained in section 2.3.4, the convolution integrals of twist-3 distribution amplitudes involve endpoint divergences, that are parameterized by the unknown parameters $X_{H,A,L}$ [65, 67, 179]. For phenomenological analyses these parameters are parameterized as

$$\begin{aligned} X_i &= \left(1 + \varrho_i e^{i\varphi_i}\right) \ln \frac{m_B}{\lambda_h}, \text{ for } i = H, A, \\ X_L &= \left(1 + \varrho_L e^{i\varphi_L}\right) \frac{m_B}{\lambda_h}, \end{aligned} \quad (2.120)$$

⁷at twist-3 more LCDAs than shown here appear. Since we don't consider three-particle distribution amplitudes, the twist-3 distributions are related by the equations of motion. We show here only the ones that appear explicitly in the formulas of section 2.3.4, and we refer to [65, 67] for more details.

where φ_i are arbitrary strong-interaction phases, and $\Lambda_h = 0.5\text{GeV}$. In the subsequent chapters we will assume $\varrho_{H,L} = 0$, while ϱ_A is varied in the range $\varrho_A \leq 1$ (by default, $\varrho = 1$ for PP , PV , VP final states and $\varrho = 0.6$ for VV final states) and

$$\varphi_A = \begin{cases} -55^\circ, & \text{for } M_1 M_2 = PP, \\ -20^\circ, & \text{for } M_1 M_2 = PV, \\ -70^\circ & \text{for } M_1 M_2 = VP, \\ -40^\circ & \text{for } M_1 M_2 = VV, \end{cases} \quad (2.121)$$

Since the signs of these phases are not predicted, the signs of the CP asymmetries in these parametrization must be taken with caution. Finally, according to the discussion at the end of section 2.3.3, we will evaluate α_s appearing in the spectator scattering and in the annihilation amplitudes at the scale $\mu_h \sim \sqrt{m_b \Lambda_{\text{QCD}}} = \sqrt{\mu \Lambda_h}$, while the Wilson coefficients will be evaluated at the scale μ .

In case of decays into final states containing η , η' states, one needs more inputs about their decay constants and form factors. Since this requires a throughout description of the conventions adopted for their mixing, which is not needed here, we refer the reader to the treatment given in the literature [66,67]. In table 2.2 some results are quoted without derivation.

Finally, we mention that, because of the suppression of the positive helicity amplitude with respect to the negative one, in case of decays into two vectors, we will assume for this amplitude the naive factorization result, and we allow the corresponding form factor to vary within the range $F_+^{BV_1} = 0 \pm 0.06$, following [179].

Survey of theoretical predictions in the Standard Model

Non-leptonic B decays present a huge phenomenology due to the large number of final states and observables that can be studied. A complete analysis of QCD factorization predictions for these quantities is beyond the scope of this review chapter. Anyway, the purpose of this section is to give a qualitative picture of the most important phenomenological aspects of these decays, and of the strategies that are used to avoid the theoretical uncertainties appearing in QCD factorization.

Usually, the aim of phenomenological analyses of non-leptonic B decays is to extract the CKM matrix elements, or to point out possible new physics contributions. This is possible if a successful prediction of the branching ratio and CP asymmetries can be performed, and in turn it requires a theoretical control over the magnitudes and relative strong-interaction phases of tree topologies, penguin topologies, electroweak penguin topologies, and annihilation topologies. The fact that generically a given decay amplitude receives several interfering contributions complicates the interpretation of the experimental data in terms of flavour topologies. The largest theoretical uncertainties are expected for the annihilation topology, whose strong phases are incalculable in QCD factorization. This problem afflicts in general the whole prediction of strong phases, because, as anticipated in section 2.2.3, they are either of order α_s , and calculable, or of order Λ_{QCD}/m_b , and incalculable; but since Λ_{QCD}/m_b numerically is not much smaller than α_s , in practice

QCD scale and running quark masses (GeV)					
$\Lambda_{\overline{\text{MS}}}^{(5)}$	$m_b(m_b)$	$m_c(m_b)$	$m_s(2\text{GeV})$	m_q/m_s	
0.225	4.2	1.3 ± 0.2	0.090 ± 0.020	0.0413	
CKM parameters					
λ	$ V_{cb} $	$ V_{ub}/V_{cb} $	γ	$\sin(2\beta)$	
0.225	0.0415 ± 0.0010	$0.085^{+0.025}_{-0.015}$	$(70 \pm 20)^\circ$	0.673 ± 0.23	
B meson parameters					
		B^-	$\bar{B}^0$	$\bar{B}_s^0$	
Lifetime	$\tau[\text{ps}]$	1.638	1.530	1.470	
Decay constant	$f_B[\text{MeV}]$	210 ± 20		240 ± 20	
	$\lambda_B[\text{MeV}]$	200^{+250}_{-0}		200^{+250}_{-0}	
Pseudoscalar-meson decay constants and Gegenbauer moments					
	π	K	η	η'	
$f[\text{MeV}]$	131	160			
$a_1, a_{1,\perp}[\text{MeV}]$	0	0.06 ± 0.06	0	0	
$a_2, a_{2,\perp}[\text{MeV}]$	0.20 ± 0.15	0.20 ± 0.15	0 ± 0.3	0 ± 0.3	
Vector-meson decay constants and Gegenbauer moments					
	ρ	K^*	ω	ϕ	
$f[\text{MeV}]$	209 ± 1	218 ± 4	187 ± 3	221 ± 3	
$f^\perp[\text{MeV}]$	165 ± 9	185 ± 10	151 ± 9	186 ± 9	
$a_1, a_{1,\perp}[\text{MeV}]$	0	0.06 ± 0.06	0	0	
$a_2, a_{2,\perp}[\text{MeV}]$	0.1 ± 0.2	0.1 ± 0.2	0 ± 0.3	0 ± 0.3	
Pseudoscalar-meson form factor at $q^2 = 0$					
	$B \rightarrow \pi$	$B \rightarrow K$	$B_s \rightarrow \bar{K}$	$B \rightarrow \eta$	$B \rightarrow \eta'$
$f_+[\text{MeV}]$	0.25 ± 0.05	0.34 ± 0.05	0.31 ± 0.05	~ 0.23	~ 0.19
Vector-meson form factor at $q^2 = 0$					
	$B \rightarrow \rho$	$B \rightarrow K^*$	$B \rightarrow \omega$	$B_s \rightarrow \bar{K}^*$	$B_s \rightarrow \phi$
$A_0[\text{MeV}]$	$0.30^{+0.07}_{-0.03}$	0.39 ± 0.06	$0.25^{+0.07}_{-0.03}$	0.33 ± 0.05	$0.38^{+0.10}_{-0.02}$
$f_+[\text{MeV}]$	0.00 ± 0.06	0.00 ± 0.06	0.00 ± 0.06	0.00 ± 0.06	0.00 ± 0.06
$f_-[\text{MeV}]$	0.55 ± 0.06	0.68 ± 0.07	0.50 ± 0.05	0.53 ± 0.05	$0.65^{+0.14}_{-0.00}$

Table 2.2: Summary of the theoretical input parameters for non-leptonic B meson decays into two light mesons. All scale-dependent quantities refer to $\mu = 2\text{GeV}$ unless otherwise stated. The values are taken from the papers cited in the text, or from [8] if not indicated otherwise.

the two contribution can be of the same order, and a large uncertainty arises. Given this limitation, it is important to test the reliability of QCD factorization prediction in decays that are dominated by only one flavour topology, so that no interference with other topologies arise. This can probe the magnitude of the single topologies, while leaving the relative strong phases undetermined. From a practical point of view, the strategy to achieve this is often to consider ratios of branching ratios and CP asymmetries, in order to extract a single flavour topology or to cancel, in the ratio, the dependence on hadronic parameters like the form factors, that can be the source of other uncertainties.

Considering now more in detail specific decay channels, the $\Delta S = 1$ decays are always penguin dominated, and typically have “large” branching ratios of order few times 10^{-6} to few times 10^{-5} . To this categories belong decay modes like $\bar{B} \rightarrow \pi \bar{K}$, $\bar{B} \rightarrow \bar{K} \omega$, $\bar{B} \rightarrow \bar{K} \phi$ decays. The theoretical predictions for these modes often suffer from large uncertainties due to the s -quark mass and due to power corrections contributing to the weak annihilation coefficient β_3^c , which is part of the dominant penguin amplitude $\hat{\alpha}_4^c = \alpha_4^c + \beta_3^c$. These two sources of uncertainties are almost fully correlated among the various decay channels, since they always contribute to the penguin coefficients $\hat{\alpha}_4^c$. The standard strategy to reduce these uncertainties is to extract from data the magnitude and strong phases of the dominant penguin contributions. Besides these decay modes, there are also a few $\Delta D = 1$ decays which are penguin or annihilation dominated, like the $\bar{B} \rightarrow K \bar{K}$ decays; they show in general large theoretical uncertainties, but the experimental data available can be used to constrain the annihilation parameters.

Most of the $\Delta D = 1$ decays are instead tree dominated. This implies that they do not suffer from a large sensitivity to light quark masses or chirally-enhanced power corrections. In many cases the dominant theoretical uncertainties arise from the variation of the CKM parameters or form factors and decay constants. While the latter source of uncertainty can be reduced, as long as better data on semileptonic or leptonic B decays becomes available, the sensitivity to CKM parameters is not a theoretical limitation, but rather provides access to $|V_{ub}|$ and γ .

Decays into final states containing η or η' mesons require particular care, because they are characterized by a complicated interplay of many different flavour topologies, and suffer from additional large theoretical uncertainties due to $\eta - \eta'$ mixing, the two-gluon component in the $\eta^{(\prime)}$ wave-functions, and the annihilation contribution to the $B \rightarrow \eta^{(\prime)}$ semileptonic form factor.

In view of the analyses on possible new physics contributions to these decays, performed in chapters 4, 5 and 6, we conclude this survey by pointing out the somewhat unpleasant situation, according to which most of the new physics models predict new physics in the $\Delta S = 1$ decay channels, which present the largest uncertainties. Indeed, most of the $\Delta D = 1$ decay modes are in good agreement with experiment, while in a number of cases deviations from the theoretical predictions arise, at the $\sim 2\sigma$ level, in the $\Delta S = 1$ decay modes, so that a clearer picture can arise only from increased experimental statistics, and from an improvement of the theoretical methods.

Chapter 3

QCD factorization for Exclusive $b \rightarrow c\bar{c}D$ decays

3.1 Introduction

$B \rightarrow c\bar{c}D$ transitions, with $D = d, s$, are phenomenologically very interesting. They are responsible for decays into charmonium states and a light meson, such as *e.g.* $B \rightarrow J/\psi K_S$, which are dominated by a single CKM factor and therefore offer a phenomenologically clean way to extract the CKM angle β . Moreover, the dominant contribution to the amplitude comes from the colour-suppressed tree topology, so that they can give more insight into the features of this flavour amplitude, in which non-factorizable corrections play an important role.

Here we are mainly concerned with the factorization properties of these decays, that are also very interesting, because of the many scales involved in the process: indeed, a proper theoretical treatment of these decays involves an interplay between various effective field theories, the heavy-quark effective field theory for the description of the B meson, the NRQCD for the description of the charmonium states, and the soft-collinear effective field theory for the light meson; these decays offer therefore a unique system where the ideas behind these theories, and behind factorization, can be tested.

As explained in chapter 2, the idea at the basis of factorization, in case of exclusive B decays into two light mesons, $B \rightarrow M_1 M_2$, is that the quark-antiquark progenitor of the “emitted” meson escapes the decay region as an object of small transverse size, which remains invisible to soft-gluon interactions with the $B \rightarrow M_1$ transition. In section 2.2.2 we have shown that the argument extends to any colour-singlet object M_2 that is small compared to the inverse strong interaction scale, $1/\Lambda$, and therefore it should hold for decays into a charmonium state and a light meson, $\bar{B} \rightarrow H M_1$, [64], because in the heavy-quark and non-relativistic limit an onium is still a compact object, whose Bohr radius is of order $1/(m_c v) \ll 1/\Lambda$.

This expectation has been confirmed for decays to the S -wave charmonia J/ψ and η_c by explicit next-to-leading order (NLO) calculations [188–190].¹ However, when the

¹Some of these papers use a light-cone rather than non-relativistic description of the charmonium, but

$B \rightarrow M_1$	J/ψ	$\psi(2S)$	η_c	$\eta_c(2S)$
$B^- \rightarrow K^-$	10.26 ± 0.37	6.32 ± 0.37	9.8 ± 1.3	3.4 ± 1.8
$\bar{B}_d^0 \rightarrow \bar{K}^0$	8.63 ± 0.35	6.55 ± 0.66	8.7 ± 1.9	—
$B^- \rightarrow \pi^-$	0.48 ± 0.04	—	—	—
$\bar{B}_d^0 \rightarrow \pi^0$	0.20 ± 0.02	—	—	—
	χ_{c0}	χ_{c1}	χ_{c2}	h_c
$B^- \rightarrow K^-$	1.88 ± 0.30	5.01 ± 0.37	< 0.30	< 0.38
$\bar{B}_d^0 \rightarrow \bar{K}^0$	< 12.4	3.96 ± 0.43	< 0.41	—
$B^- \rightarrow \pi^-$	< 0.61	0.22 ± 0.05	—	—
$\bar{B}_d^0 \rightarrow \pi^0$	—	—	—	—

Table 3.1: Averages of $B \rightarrow$ charmonium K, π branching fraction data in units of 10^{-4} , taken from the Heavy Flavour Averaging Group [44]. The original experimental results can be found in [204–225]. The limit for h_c assumes $Br(h_c \rightarrow \eta_c \gamma) = 0.5$.

formalism was applied to P -wave charmonium states [191–194, 196, 197] infrared (IR) divergences appeared that seem to violate factorization².

In [125] we have revisited this problem and shown that factorization is recovered, if one includes the charmonium bound-state scales $m_c v$, $m_c v^2$ (with v a characteristic velocity of the charm quark in the bound state, $v \ll 1$) into the theoretical framework. These scales are assumed to be intermediate between the heavy quark masses m_b , m_c and the strong interaction scale Λ . The divergence structure described above bears resemblance with inclusive charmonium decay or production in the colour-singlet model. As is well-known, the IR divergence problem in the P -wave colour-singlet amplitudes is resolved by the introduction of colour-octet operators [200], and a similar mechanism is at work for $B \rightarrow HM_1$. However, there is an important difference between P -wave charmonium production in inclusive B decay [201, 202] and exclusive decays. While in the former the charmonium decouples from the inclusive $B \rightarrow X$ transition below the heavy quark mass scale, or at least is assumed to in previous treatments, this decoupling takes place for exclusive decays only below the scale of the binding energy $m_c v^2$, since gluons with this energy can reconnect to the BK system. In the (formal) heavy-quark limit this effect is perturbatively calculable since $m_c v^2 \gg \Lambda$; in the real world, however, $m_c v^2 \sim \Lambda$, and hence reliable calculations appear to be hard to come by for decays to charmonium, contrary to decays to light M_2 .

this does not affect the conclusion.

² B decays into charmonium states and a kaon were studied also in [195], in the context of perturbative QCD

In Table 3.1 we collect the current branching fraction measurements for some of the decays in question, [204–225], including the S -wave final states. Considering in particular the case $M_1 = K$, and comparing χ_{c1} to the S -waves, we conclude that the P -wave suppression is almost absent. The pattern of P -wave results then is even more striking, if one recall that in naive factorization only the χ_{c1} state is produced. All these considerations suggest the existence of significant decay amplitudes beyond naive factorization and, indeed, the colour-octet contributions that we have identified in [125] may well be the dominant decay amplitudes, although they turn out to be hard to calculate for real-world charmonium. Nevertheless, it is interesting to see whether something can be said from theory that may help to understand the pattern of experimental data.

The infrared divergence in hard spectator-scattering is actually an endpoint divergence in a momentum-fraction convolution integral. Such endpoint divergences prohibit hard-scattering factorization of power-suppressed effects in non-leptonic B decays to light mesons [63], and of B to light meson form factors even at leading power [148, 226]. Understanding whether and how such endpoint divergences can be factorized remains a major challenge to theory. In B decays to P -wave charmonia the endpoint singularity also arises at leading order in the Λ/m_b expansion. In [125] we have shown that it can be factorized into the matrix elements of colour-octet operators. This is of some conceptual interest, since it is not known in general how to factorize endpoint divergences.

Factorization of B decays to charmonium have been discussed recently also in [198, 199]. The first of these papers deals with the leading order in the non-relativistic velocity expansion applicable to decays to S -wave charmonia, but does not address P -waves. The second considers the leading term in an expansion in m_c/m_b , which is identical to the case of two light final state mesons [63], but the limit $m_c/m_b \rightarrow 0$ is different from the assumption m_c/m_b fixed (while m_b becomes large) that we make in [125] and here. While it is evident that B decays to P -wave charmonia do not factorize at the heavy-quark mass scale due to the infrared divergences mentioned above, the concern in was to show that perturbative factorization is recovered when $m_c v^2 \gg \Lambda$ as conjectured in [64]. Corrections to naive factorization for B decays to charmonium have also been estimated with light-cone QCD sum rules [227–229], but with this method the issue of infrared singularities in the QCD factorization result is not addressed.

The purpose of this chapter is to review and extend the analysis we performed in [125]. Our attention, there, was in fact centered on the factorization properties of decays into charmonium P wave states and a kaon, in order to address the problem raised in [191–194, 196, 197]. We find however more interesting to organize this chapter as a more comprehensive analysis of the whole class of $B \rightarrow HM_1$ decays, with M_1 a pseudoscalar non-singlet meson, *i.e.* $M_1 = K, \pi$. This allows us to include a phenomenological discussion about decays into S -wave states and a light meson, whose NLO (exclusive) amplitude, that we present here, has not yet appeared elsewhere in terms of a non-relativistic formulation for the treatment of the charmonium states. Despite the absence of infrared divergences, *i.e.* of a leading contribution from colour-octet operators, these decays, too, turn out to be difficult to be computed precisely, and it may be interesting to compare their features with respect to the P -wave case. Also, the inclusion of decays in which $M_1 = \pi$ can offer

some more insight in the analysis of the spectator scattering contribution to these decays.

3.2 Operator definitions and tree-level results

3.2.1 Effective Hamiltonian and kinematics

Since we are interested to study the factorization properties of these decays, it is sufficient to consider the following effective weak-interaction Hamiltonian for the $b \rightarrow c\bar{c}D$ transition:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cD}^* (C_1 Q_1 + C_2 Q_2), \quad (3.1)$$

with

$$Q_1 = (\bar{c}_i b_i)_{V-A} (\bar{D}_j c_j)_{V-A}, \quad Q_2 = (\bar{c}_i b_j)_{V-A} (\bar{D}_j c_i)_{V-A}, \quad (3.2)$$

neglecting the contributions from the penguin operators. This is a very good approximation, given that their Wilson coefficients are one order of magnitude smaller than the large C_1 coefficient. In fact, it has been shown in [188–190, 196] that penguin contributions constitute at most a few per cent of the total amplitude, and, as we will see in section 3.6, this is in any case beyond the precision one can reach in the calculation of these decays.

The following notation is adopted for the kinematics of the two-body decay process $B \rightarrow HM_1$: p_B is the momentum of the B meson, $w^\mu \equiv p_B^\mu/M_B$ its velocity with $w^2 = 1$; q is the momentum of the charmonium, with $v^\mu \equiv q^\mu/M_H$ the charmonium 4-velocity (M_H being the charmonium mass) with $v^2 = 1$; l^μ defines the relative momentum of the c quark inside the charmonium, so that $p_c = q/2 + l$, $\bar{p}_c = q/2 - l$ with $q \cdot l = 0$; p is the momentum of the meson M_1 . Since kaon and pion mass effects can be neglected in the heavy quark limit, the vector $n_-^\mu = p^\mu/E_{M_1}$ (E_{M_1} being the meson M_1 energy in the B rest frame) is light-like. The opposite-pointing light-like vector is denoted by n_+^μ with $n_- \cdot n_+ = 2$. We also define $z = 4m_c^2/m_b^2$, equal to M_H^2/M_B^2 up to corrections of order³ Λ/m_b and v^2 . From energy-momentum conservation

$$E_{M_1} = \frac{n_+ \cdot p}{2} = \frac{M_B^2 - M_H^2}{2M_B}, \quad n_- \cdot v = \frac{1}{n_+ \cdot v} = \frac{M_B}{M_H}. \quad (3.3)$$

For any vector a (or Lorentz index μ) the components transverse to n_- and n_+ are denoted by $a_\perp$ ($\mu_\perp$), for those orthogonal to v we use $a_\top$ ($\mu_\top$). Thus

$$a^\mu = a_\perp^\mu + \frac{n_+ \cdot a}{2} n_-^\mu + \frac{n_- \cdot a}{2} n_+^\mu = a_\top^\mu + v \cdot a v^\mu. \quad (3.4)$$

³Note the dual use of v : in power-counting estimates v denotes the small *relative* velocity of the heavy quarks in the charmonium in the rest frame of the charmonium. In kinematical relations v is the charmonium velocity vector.

3.2.2 SCET/NRQCD operator definitions

In the following we assume the bottom and charm quark masses to be heavy, with m_c/m_b fixed in the heavy-quark limit. Integrating out the heavy quark mass scales m_b, m_c leads to an effective theory, in which the b quark is static as in heavy-quark effective theory, the charm quarks are non-relativistic (in their center-of-mass frame) as in NRQCD and the light quarks are collinear (or soft) as in soft-collinear effective theory (SCET). The situation is similar to the corresponding one for charmless B decays, [91,92,136] except that the meson that does not absorb the spectator quark is now described by non-relativistic rather than collinear fields.

The construction of operators representing the four-quark transitions in the effective theory below the heavy quark scale can be done following the same procedure explained in section 2.3.3. The only difference is that the quark-antiquark bilinear necessary for the creation of the light meson M_2 in the collinear-2 sector, $(\chi W_{c2})(tn_-)\not{n}_-(1 \pm \gamma_5)(W_{c2}^\dagger \chi)(0)$, has now to be substituted with non-relativistic bilinears, responsible for the creation of a heavy quark-antiquark pair in a definite angular momentum state. Here we need to introduce the following colour-singlet bilinears,

$$\begin{aligned}
\mathcal{O}(^1S_0^{(0)}) &\equiv \psi_v^\dagger [\gamma_5] \chi_v, \\
\mathcal{O}(^3S_1^{(1)}) &\equiv \psi_v^\dagger [n_{-\mu} \gamma_\top^\mu] \chi_v, \\
\mathcal{O}(^1P_1^{(1)}) &\equiv \psi_v^\dagger \left[n_{-\mu} \left(-\frac{i}{2} \right) \overleftrightarrow{D}_\top^\mu \gamma_5 \right] \chi_v, \\
\mathcal{O}(^3P_0^{(1)}) &\equiv \psi_v^\dagger \left[-\frac{1}{\sqrt{3}} \left(-\frac{i}{2} \right) \overleftrightarrow{\not{D}}_\top \right] \chi_v, \\
\mathcal{O}(^3P_1^{(1)}) &\equiv \psi_v^\dagger \left[\frac{n_{-\mu}}{2\sqrt{2}} \left(-\frac{i}{2} \right) \left[\overleftrightarrow{\not{D}}_\top, \gamma_\top^\mu \right] \gamma_5 \right] \chi_v, \\
\mathcal{O}(^3P_2^{(1)}) &\equiv \psi_v^\dagger \left[n_{-\mu} n_{-\nu} \left(-\frac{i}{2} \right) \overleftrightarrow{D}_\top^{(\mu} \gamma_\top^{\nu)} \right] \chi_v,
\end{aligned} \tag{3.5}$$

where ψ and χ are non-relativistic quark and anti-quark spinor fields, respectively,⁴ and we have used the spectroscopy notation $^{2S+1}L_J$, with S indicating the spin, L the orbital angular momentum, and J the total angular momentum. Beside these bilinears, charmonium (P -wave) production through colour-octet operators requires to consider also the following colour-octet bilinears:

$$\begin{aligned}
\mathcal{O}(^1S_0^{(8)}) &\equiv \psi_v^\dagger [\gamma_5 T^A] \chi_v, \\
\mathcal{O}(^3S_1^{(8)}) &\equiv \psi_v^\dagger [n_{-\mu} \gamma_\top^\mu T^A] \chi_v, \\
\mathcal{O}^\mu(^1S_0^{(8)}) &\equiv \psi_v^\dagger [v^\mu \gamma_5 T^A] \chi_v,
\end{aligned}$$

⁴ $\overleftrightarrow{D}_\top^{(\mu} \gamma_\top^{\nu)}$ denotes the symmetric, traceless part of the tensor $\overleftrightarrow{D}_\top^\mu \gamma_\top^\nu$.

$$\mathcal{O}^\mu(^3S_1^{(8)}) \equiv \psi_v^\dagger [\gamma_\top^\mu T^A] \chi_v. \quad (3.6)$$

In the following it is convenient to use a covariant generalization of the NRQCD Lagrangian, in order to describe the charm-anticharm pair in the B meson rest frame; thus, the non-relativistic fields introduced so far are four-component spinors satisfying $\not{v}\psi_v = \psi_v$ and $\not{v}\chi_v = -\chi_v$. In the charmonium rest frame $v = (1, \mathbf{0})$ the non-zero spinor components reduce to the familiar non-relativistic two-spinors, and the $\top$ components of a contravariant index equal the spatial components. For more detail on the NRQCD conventions used we refer to the appendix A.3.

The $B \rightarrow M_1$ transition instead is completely equivalent to the one occurring in charmless B decays, so that we can construct the heavy-to-light transition in terms of the A0- and B1-type SCET_I currents, as explained in section 2.3.3. Merging the two pieces together, we obtain the following colour-singlet operators

$$\begin{aligned} \mathcal{O}^A(^{2S+1}L_J^{(1)}) &= \left[\tilde{C}_{f_+}^{(A0)}(\bar{\xi}W_c)\not{v}_+(1 - \gamma_5)h_w - \frac{1}{m_b} \int d\hat{s} \tilde{C}_{f_+}^{(B1)}(\hat{s}) \right. \\ &\quad \left. (\bar{\xi}W_c)\not{v}_+[W_c^\dagger i\not{D}_\perp W_c](sn_+)(1 + \gamma_5)h_w \right] \mathcal{O}(^{2S+1}L_J^{(1)}), \end{aligned} \quad (3.7)$$

$$\mathcal{O}^B(^{2S+1}L_J^{(1)})(s) = \frac{1}{m_b}(\bar{\xi}W_c)\frac{\not{v}_+}{2}[W_c^\dagger i\not{D}_\perp W_c](sn_+)(1 + \gamma_5)h_w \mathcal{O}(^{2S+1}L_J^{(1)}), \quad (3.8)$$

with $L = S, P$, associated with the vertex and spectator-scattering amplitudes that have been considered in previous works [191–194, 196, 197]. The operators $\mathcal{O}^A(^{2S+1}L_J^{(1)})$ are written in such a way that their tree-level matrix elements are proportional to the form factor $f_+^{BM_1}(M_H^2)$ times the derivative of the quarkonium wave function at the origin, since the expression in square brackets in (3.7) is the SCET representation of the full QCD form factor [156]. The effective vertices generated by the A- and B-type operators (in light-cone gauge, where $W_c = 1$) are shown in Figure 3.1. New and central for the discussion of P -wave production are the colour-octet operators

$$\begin{aligned} \mathcal{O}_\parallel^A(^{2S+1}S_J^{(8)}) &= (\bar{\xi}W_c)\not{v}_+(1 - \gamma_5)T^A h_w \mathcal{O}(^{2S+1}S_J^{(8)}), \\ \mathcal{O}_\perp^A(^{2S+1}S_J^{(8)}) &= (\bar{\xi}W_c)\gamma_{\perp\mu}(1 - \gamma_5)T^A h_w \mathcal{O}^\mu(^{2S+1}S_J^{(8)}), \end{aligned} \quad (3.9)$$

constructed from an A-type SCET_I current and the colour-octet bilinears in (3.5). According to the assumption that charmonium is a Coulomb bound state, there exist two sets of low-energy scales, $m_c v, m_c v^2 \gg \Lambda$ related to the non-relativistic expansion and $\sqrt{m_b \Lambda}, \Lambda$, related to the collinear expansion and the strong-interaction scale. In the case of charmless decays the matrix elements of colour-octet operators can be non-zero only due to power-suppressed soft-gluon interactions, where soft means momentum of order Λ , thus they can be neglected at leading order in the Λ/m_b expansion. For charmonium, however, the decoupling of gluons with small momentum holds only when the momentum is much smaller than $m_c v^2$; gluons with momentum $m_c v^2$ contribute to the octet operator matrix elements even at leading order in Λ/m_b . These contributions are subleading in v ,

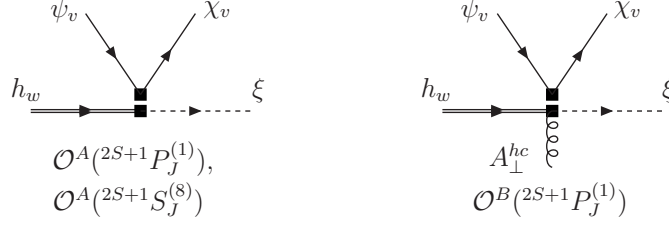


Figure 3.1: Vertices corresponding to the A- and B-type operators.

but so are the P -wave operators due to the extra derivative in $\mathcal{O}(^{2S+1}P_J^{(1)})$, hence the gluon-exchange contribution to the S -wave octet operators is relevant at leading order in the velocity expansion to P -wave charmonium production.

Concentrating on the terms relevant to P -wave production at leading (non-vanishing) order in the v and Λ/m_b expansion, the effective weak-interaction Hamiltonian below the heavy quark mass scale is therefore given by

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{(P)} = & \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_{S,J} \left\{ \mathcal{C}^A(^{2S+1}P_J^{(1)}) \mathcal{O}^A(^{2S+1}P_J^{(1)}) \right. \\ & + \int d\hat{s} \tilde{\mathcal{C}}^B(^{2S+1}P_J^{(1)})(\hat{s}) \mathcal{O}^B(^{2S+1}P_J^{(1)})(s) \\ & \left. + \mathcal{C}_{\parallel}^A(^{2S+1}S_J^{(8)}) \mathcal{O}_{\parallel}^A(^{2S+1}S_J^{(8)}) + \mathcal{C}_{\perp}^A(^{2S+1}S_J^{(8)}) \mathcal{O}_{\perp}^A(^{2S+1}S_J^{(8)}) \right\}, \end{aligned} \quad (3.10)$$

where we introduced the short-distance coefficients $\mathcal{C}$ and $\hat{s} \equiv n_+ \cdot p s = M_B s$.

In principle, it is possible to consider a similar production process for charmonium S -wave states, *i.e.* their production through colour octet P -wave operators, that subsequently are converted into an S -wave state through the exchange of a soft gluon. In this case however the process is suppressed by two powers of v , one due to the soft-gluon exchange, and the other from the suppression of P -wave operators with respect to the S -wave ones. These corrections are therefore subleading and we will not consider them in this work, even if their contribution to the total amplitude may be of some importance, and we will clarify why later. Therefore, to first power in v , the effective Hamiltonian for S -wave production reads

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{(S)} = & \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_{S,J} \left\{ \mathcal{C}^A(^{2S+1}S_J^{(1)}) \mathcal{O}^A(^{2S+1}S_J^{(1)}) \right. \\ & \left. + \int d\hat{s} \tilde{\mathcal{C}}^B(^{2S+1}S_J^{(1)})(\hat{s}) \mathcal{O}^B(^{2S+1}S_J^{(1)})(s) \right\}. \end{aligned} \quad (3.11)$$

3.2.3 Tree-level matching of A-type operators

The leading-order matching coefficients $\mathcal{C}^A$ are found by comparing the $b \rightarrow c(p_c)\bar{c}(p_{\bar{c}})D$ tree amplitude (topology as in the left diagram of Figure 3.1) computed with the effective

Hamiltonian (3.1) to the corresponding amplitude from (3.10), (3.11). The result for the colour-singlet operators is:

$$\begin{aligned} \mathcal{C}_0^A(^1S_0^{(1)}) &= -\frac{1}{2\sqrt{z}} \left(C_2 + \frac{C_1}{N_c} \right), \\ \mathcal{C}_0^A(^3S_1^{(1)}) &= \frac{1}{2} \left(C_2 + \frac{C_1}{N_c} \right), \\ \mathcal{C}_0^A(^3P_1^{(1)}) &= -\frac{\sqrt{2}}{m_b\sqrt{z}} \left(C_2 + \frac{C_1}{N_c} \right), \\ \mathcal{C}_0^A(^1P_1^{(1)}) &= \mathcal{C}_0^A(^3P_0^{(1)}) = \mathcal{C}_0^A(^3P_2^{(1)}) = 0, \end{aligned} \quad (3.12)$$

while in case of the S -wave colour octet operators one finds

$$\begin{aligned} \mathcal{C}_\parallel^A(^1S_0^{(8)}) &= -\frac{1}{2\sqrt{z}} (2C_1), & \mathcal{C}_\perp^A(^1S_0^{(8)}) &= -(2C_1), \\ \mathcal{C}_\parallel^A(^3S_1^{(8)}) &= \frac{1}{2} (2C_1), & \mathcal{C}_\perp^A(^3S_1^{(8)}) &= (2C_1). \end{aligned} \quad (3.13)$$

Since the matrix elements of the colour-octet operators and spectator-scattering are both suppressed by a factor of α_s , this reproduces the well-known result from naive factorization that only the S -wave and the χ_{c1} states are produced at leading order.

3.2.4 Estimate of the branching fraction

The leading-order decay amplitude is now given by the expression

$$\mathcal{A}_{B \rightarrow H(^{2S+1}L_J)M_1}^{\text{LO}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \mathcal{C}_0^A(^{2S+1}L_J^{(1)}) \langle H(^{2S+1}L_J)M_1 | \mathcal{O}^A(^{2S+1}L_J^{(1)}) | \bar{B} \rangle. \quad (3.14)$$

The hadronic matrix element factorizes at leading order in the expansion in Λ/m_b and α_s according to

$$\begin{aligned} &\langle H(^{2S+1}L_J)M_1 | \mathcal{O}^A(^{2S+1}L_J^{(1)}) | \bar{B} \rangle \\ &= \langle M_1 | \left[\tilde{C}_{f_+}^{(A0)} (\bar{\xi} W_c) \not{h}_+ (1 - \gamma_5) h_w - \frac{1}{m_b} \int d\hat{s} \tilde{C}_{f_+}^{(B1)}(\hat{s}) \right. \\ &\quad \left. \times (\bar{\xi} W_c) \not{h}_+ [W_c^\dagger i \not{D}_\perp W_c] (s n_+) (1 + \gamma_5) h_w \right] | \bar{B} \rangle \langle H(^{2S+1}L_J) | \mathcal{O}^{(2S+1}L_J^{(1)}) | 0 \rangle, \end{aligned} \quad (3.15)$$

where the first factor reduce to the QCD $B \rightarrow M_1$ form factor,

$$\begin{aligned} &\langle M_1 | \left[\tilde{C}_{f_+}^{(A0)} (\bar{\xi} W_c) \not{h}_+ (1 - \gamma_5) h_w - \frac{1}{m_b} \int d\hat{s} \tilde{C}_{f_+}^{(B1)}(\hat{s}) \right. \\ &\quad \left. \times (\bar{\xi} W_c) \not{h}_+ [W_c^\dagger i \not{D}_\perp W_c] (s n_+) (1 + \gamma_5) h_w \right] | \bar{B} \rangle \\ &= 2M_B \left(1 - \frac{M_H^2}{M_B^2} \right) f_+^{BM_1}(M_H^2), \end{aligned} \quad (3.16)$$

and the second factor reduce to the charmonium wave function at the origin, or its derivative, depending if one consider S - or P -wave production:

$$\begin{aligned}\langle \eta_c | \mathcal{O}(^1S_0^{(1)}) | 0 \rangle &= \langle \mathcal{O}(^1S_0^{(1)}) \rangle, \\ \langle J/\psi | \mathcal{O}(^3S_1^{(1)}) | 0 \rangle &= n_- \cdot \varepsilon \langle \mathcal{O}(^1S_0^{(1)}) \rangle,\end{aligned}\quad (3.17)$$

$$\begin{aligned}\langle h_c | \mathcal{O}(^1P_1^{(1)}) | 0 \rangle &= n_- \cdot \varepsilon \langle \mathcal{O}(^3P_0^{(1)}) \rangle, \\ \langle \chi_{c0} | \mathcal{O}(^3P_0^{(1)}) | 0 \rangle &= \langle \mathcal{O}(^3P_0^{(1)}) \rangle, \\ \langle \chi_{c1} | \mathcal{O}(^3P_1^{(1)}) | 0 \rangle &= n_- \cdot \varepsilon \langle \mathcal{O}(^3P_0^{(1)}) \rangle, \\ \langle \chi_{c2} | \mathcal{O}(^3P_2^{(1)}) | 0 \rangle &= n_-^\mu n_-^\nu \varepsilon_{\mu\nu} \langle \mathcal{O}(^3P_0^{(1)}) \rangle,\end{aligned}\quad (3.18)$$

with

$$\begin{aligned}\langle \mathcal{O}(^1S_0^{(1)}) \rangle &= \sqrt{2N_c} \sqrt{2M_{\eta_c}} \sqrt{\frac{1}{4\pi}} R_{10}(0), \\ \langle \mathcal{O}(^3P_0^{(1)}) \rangle &= \sqrt{2N_c} \sqrt{2M_{\chi_{c0}}} (-i) \sqrt{\frac{3}{4\pi}} R'_{21}(0).\end{aligned}\quad (3.19)$$

Here spin symmetry of the leading non-relativistic interactions has been used to write the six matrix elements in terms of $\langle \mathcal{O}(^1S_0^{(1)}) \rangle$ and $\langle \mathcal{O}(^3P_0^{(1)}) \rangle$, or equivalently $R_{10}(0)$ and $R'_{21}(0)$, where $R_{nl}(r)$ denotes the radial Schrödinger wave function of the n -th, l -th state, and the prime denotes a derivative.

Squaring the amplitude, integrating over the two-body phase space, where we neglect the light meson M_1 mass, and summing over the charmonium polarizations with the help of

$$\text{PolSum}[H(^{2S+1}L_J)] = \begin{cases} 1 & H = \eta_c, \chi_{c0} \\ \sum_{\lambda=-1}^1 |n_- \cdot \varepsilon(\lambda)|^2 = \frac{M_B^2}{M_H^2} & H = J/\psi, h_c, \chi_{c1} \\ \sum_{\lambda=-2}^2 |n_-^\mu n_-^\nu \varepsilon_{\mu\nu}(\lambda)|^2 = \frac{2M_B^4}{3M_{\chi_{c2}}^4} & H = \chi_{c2} \end{cases} \quad (3.20)$$

we obtain the branching fraction

$$\begin{aligned}\text{Br}(\bar{B} \rightarrow H(^{2S+1}L_J)M_1) &= \tau_B \frac{G_F^2}{2} |V_{cb}V_{cs}^*|^2 \frac{M_B^2 - M_H^2}{16\pi M_B^3} \left| C_0^A(^{2S+1}L_J^{(1)}) \right|^2 \\ &\times \text{PolSum}[H(^{2S+1}L_J)] \left[2M_B \left(1 - \frac{M_H^2}{M_B^2} \right) f_+^{BM_1}(M_H^2) \langle \mathcal{O}(^{2S+1}L_J^{(1)}) \rangle \right]^2, \quad (3.21)\end{aligned}$$

where, using the relations (3.17), (3.18), $\langle \mathcal{O}(^{2S+1}L_J^{(1)}) \rangle = \langle \mathcal{O}(^1S_0^{(1)}) \rangle$ for S -waves and $\langle \mathcal{O}(^{2S+1}L_J^{(1)}) \rangle = \langle \mathcal{O}(^3P_0^{(1)}) \rangle$ for P -waves.

The branching ratio in (3.21) at leading order are different from zero only for the S -wave and the χ_{c1} states, due to (3.12). With parameters as given in section 3.6, varying

$B \rightarrow M_1$	J/ψ	$\psi(2S)$	η_c	$\eta_c(2S)$	χ_{c1}
$\bar{B}_d^0 \rightarrow \bar{K}^0$	1.19 – 6.75	0.67 – 3.82	0.77 – 6.37	0.62 – 5.09	0.13 – 1.06
$\bar{B}_d^0 \rightarrow \pi^0$	0.04 – 0.22	0.02 – 0.12	0.03 – 0.21	0.02 – 0.16	0.004 – 0.03

Table 3.2: Theoretical predictions for the tree level branching ratio, according to (3.21). The two values for each decay mode represent the results for $\mu = 2 \text{ GeV}$, $m_c = 1.75 \text{ GeV}$ and $\mu = 4.8 \text{ GeV}$, $m_c = 1.45 \text{ GeV}$ respectively.

the renormalization scale μ between 2 and 4.8 GeV, and m_c between 1.45 and 1.75 GeV, we find the results in table 3.2. The largest uncertainty arises from the scale-dependence of the “colour-suppressed” Wilson coefficient $C_2 + C_1/N_c$. In case of S -wave production the naive factorization prediction is around two times smaller than the experimental result, indicating the importance of the NLO corrections, as expected for colour-suppressed decay modes. In case of P wave, however, the difference is at least a factor of four smaller than the experimental result; this analysis shows that (naively) non-factorizable dynamics, such as radiative corrections, spectator-scattering, and the colour-octet contributions constitute even the dominant contribution to the decay amplitude, in this case.

3.2.5 Overview of next-to-leading order terms: P -waves

Due to the different power-counting for S - and P -wave production, resulting in the two different Hamiltonians (3.10), (3.11), we prefer to discuss the two cases separately, starting from P -waves that involve the new production mechanism through colour-octet operators introduced in [125].

The leading contribution to (3.10) comes from the colour-singlet P -wave A-type operators $\mathcal{O}^A(2P+1P_J^{(1)})$. Not including the fields themselves in the power-counting estimate, this contribution is of order $\alpha_s^0 v$, where the factor v arises from the derivatives in the non-relativistic P -wave operators (3.5) and the factor α_s^0 is from the tree-level coefficient function. Corrections to the leading order start at order $\alpha_s v$ and in [125] we have shown that they can be consistently computed, when $m_c v^2 \gg \Lambda_{\text{QCD}}$, if the following contributions are included:

1. One-loop corrections to the short-distance coefficients $\mathcal{C}^A(2S+1P_J^{(1)})$ of the P -wave colour-singlet operators (“vertex corrections”):

$$\mathcal{A}_{B \rightarrow H(2S+1P_J)M_1}^{\text{hard vertex}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \mathcal{C}^A(2S+1P_J^{(1)})|_{\text{one-loop}} \langle \mathcal{O}^A(2S+1P_J^{(1)}) \rangle|_{\text{tree}}, \quad (3.22)$$

where $\langle \dots \rangle \equiv \langle H(2S+1P_J^{(1)})M_1 | \dots | \bar{B} \rangle$ and $\langle \mathcal{O}^A(2S+1P_J^{(1)}) \rangle|_{\text{tree}}$ equals the factorized matrix element (3.15).

Loop corrections to the matrix elements of these operators need not be considered. In particular, corrections to the factorization relation (3.15) are suppressed by powers

of Λ/m_b , v or α_s^2 , where the two powers of the coupling arise from the vanishing colour-projection of one-gluon exchange.

2. The tree-level matrix element of the B-type P -wave colour-singlet operators $\mathcal{O}^B(2S+1P_J^{(1)})$ is of order α_s , since one factor of g_s is in the definition of the operator and another is provided by the coupling of the collinear gluon to the spectator-quark line (“spectator-scattering”). The B-type form factor is of the same order as $f_+^{BM_1}$ [150], hence this contribution is also of order $\alpha_s v$. The hard spectator-scattering amplitude is

$$\mathcal{A}_{B \rightarrow H(2S+1P_J)M_1}^{\text{hard spectator}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \int d\hat{s} \tilde{\mathcal{C}}^B(2S+1P_J^{(1)})(\hat{s})|_{\text{tree}} \langle \mathcal{O}^B(2S+1P_J^{(1)})(s) \rangle|_{\text{tree}}. \quad (3.23)$$

3. The colour-octet operators $\mathcal{O}^A(2S+1S_J^{(8)})$ are of order v^0 , but their tree-level matrix elements vanish. The matrix element is non-zero due to soft gluon exchange between the charm-quark lines and the b -quark and light-quark lines, including the spectator quark. At order $\alpha_s v$, one must include the one-loop matrix element with exchange of a gluon of momentum $m_c v^2$:

$$\mathcal{A}_{B \rightarrow H(2S+1P_J)M_1}^{\text{soft vertex}} + \mathcal{A}_{B \rightarrow H(2S+1P_J)M_1}^{\text{soft spectator}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_{a=\parallel, \perp} \sum_{J'=0,1} \mathcal{C}_a^A(2S+1S_{J'}^{(8)})|_{\text{tree}} \langle \mathcal{O}_a^A(2S+1S_{J'}^{(8)}) \rangle|_{\text{one-gluon}}. \quad (3.24)$$

Loop corrections to the short-distance coefficients $\mathcal{C}^A(2S+1S_{J'}^{(8)})$ need not to be considered, since the tree-level matrix element vanishes.

Colour-singlet S -wave operators contribute only at higher orders due to the vanishing colour projection in the one-gluon exchange contribution to their matrix elements.

The inclusion of the last contribution was missed in literature and we have pointed out its importance in [125]. Because “non-factorizable gluons” do not decouple below m_b in case of P -wave production, as signal the infrared divergences found in the evaluation of the first two contributions described here, its inclusion is necessary for a consistent treatment of radiative corrections down to the scale Λ_{QCD} .

3.2.6 Overview of next-to-leading order terms: S -waves

In case of S -wave production the leading contribution to (3.11) comes from the colour-singlet A-type operators $\mathcal{O}^A(2S+1S_J^{(1)})$, and is of order $\alpha_s^0 v^0$, because the S -wave operators are leading in the v expansion, as one can see from (3.5), while the factor α_s^0 is from the tree-level coefficient function as before. Corrections to the leading order start in this case at order $\alpha_s v^0$, and only two contributions are involved, corresponding to the first two items of section 3.2.5:

1. the one-loop corrections to the short-distance coefficients $\mathcal{C}^A(2S+1S_J^{(1)})$ of the S -wave colour-singlet operators:

$$\mathcal{A}_{B \rightarrow H(2S+1S_J)M_1}^{\text{hard vertex}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \mathcal{C}^A(2S+1S_J^{(1)})|_{\text{one-loop}} \langle \mathcal{O}^A(2S+1S_J^{(1)}) \rangle|_{\text{tree}}, \quad (3.25)$$

2. and the tree-level matrix element of the B-type S -wave colour-singlet operators $\mathcal{O}^B(2S+1S_J^{(1)})$

$$\mathcal{A}_{B \rightarrow H(2S+1S_J)M_1}^{\text{hard spectator}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \int d\hat{s} \tilde{\mathcal{C}}^B(2S+1S_J^{(1)})(\hat{s})|_{\text{tree}} \langle \mathcal{O}^B(2S+1S_J^{(1)})(s) \rangle|_{\text{tree}}. \quad (3.26)$$

Again, loop corrections to the matrix elements of these operators need not to be considered, because they are suppressed by powers of Λ/m_b , v or α_s^2 .

In this case, colour octet operators contribute only starting at higher order in v , *i.e.* at order $\alpha_s v^2$. This is because first, one has to consider the P -wave colour-octet operators $\mathcal{O}^A(2S+1P_J^{(8)})$, that are of order v . Moreover, since their tree-level matrix elements vanish, one has to consider the one-loop matrix element with exchange of a gluon of momentum $m_c v^2$,

$$\mathcal{A}_{B \rightarrow H(2S+1S_J)M_1}^{\text{soft vertex}} + \mathcal{A}_{B \rightarrow H(2S+1S_J)M_1}^{\text{soft spectator}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_{a=\parallel, \perp} \sum_{J'=0,1} \mathcal{C}_a^A(2S+1P_{J'}^{(8)})|_{\text{tree}} \langle \mathcal{O}_a^A(2S+1P_{J'}^{(8)}) \rangle|_{\text{one-gluon}}, \quad (3.27)$$

whose total contribution is therefore of order $\alpha_s v^2$.

Because of the colour-suppression of the tree-level amplitude, that involves the short-distance coefficient $C_2 + C_1/N_c$, radiative corrections are in general important, and the contribution in (3.27) could still be relevant for a precise evaluation of the decay amplitude. Nevertheless, because of the relation $\alpha_s(M_H v) \sim v$, a consistent evaluation of the decay amplitude at this level require to consider as well the two-loop corrections to the short-distance coefficients $\mathcal{C}^A(2S+1S_J^{(1)})$,

$$\mathcal{A}_{B \rightarrow H(2S+1S_J)M_1}^{\alpha_s^2 \text{ vertex}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \mathcal{C}^A(2S+1S_J^{(1)})|_{\text{two-loop}} \langle \mathcal{O}^A(2S+1S_J^{(1)}) \rangle|_{\text{tree}}, \quad (3.28)$$

and the one loop correction to the short-distance coefficients $\mathcal{C}^B(2S+1S_J^{(1)})$,

$$\mathcal{A}_{B \rightarrow H(2S+1S_J)M_1}^{\alpha_s^2 \text{ hard spectator}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \int d\hat{s} \tilde{\mathcal{C}}^B(2S+1S_J^{(1)})(\hat{s})|_{\text{one-loop}} \langle \mathcal{O}^B(2S+1S_J^{(1)})(s) \rangle|_{\text{tree}}. \quad (3.29)$$

that are of order $\alpha_s^2 v^0$, thus formally larger than the $\alpha_s v^2$ corrections because of the relation above. In [202] it was shown in the inclusive case that the contributions (3.27), (3.28) and (3.29) are indeed important, with (3.27) resulting numerically the larger, because of its large colour-octet short-distance coefficients. They should therefore be included in a reliable phenomenological analysis. Here we focus on the factorization properties of these decays; therefore, the inclusion of (3.27), (3.28) and (3.29) is beyond the purpose of this work.

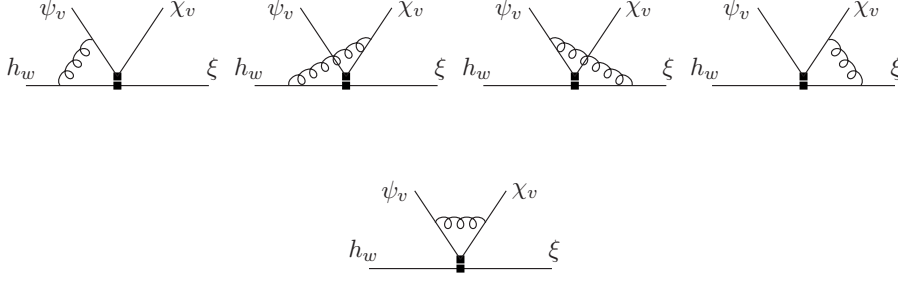


Figure 3.2: One-loop corrections to the short-distance coefficients $\mathcal{C}^A(2S+1L_J^{(1)})$ (vertex correction).

3.3 Short-distance contributions

In this section we compute the short-distance contributions related to items (3.22), (3.23) and (3.25), (3.26).

3.3.1 One-loop short-distance coefficients

The one-loop correction to the short-distance coefficients of the A-type colour-singlet P - and S -wave operators (3.7) are obtained from matching the QCD diagrams in Figure 3.2 to the expression (3.22) and (3.25), respectively.

In case of decays into the η_{c1} and η_{c2} states one has to take into account an additional contribution, that comes from the diagrams (shown in Fig. 2.5) with gluon exchange between the b - and the spectator-quark line, as well as the s - and the spectator-quark, the b - and the s -quark line. This is a consequence of the form factor parametrization considered here. In full QCD in fact the matrix element $\langle M_1 | \bar{q} \gamma^\mu (1 - \gamma_5) b | \bar{B} \rangle$ is parameterized in terms of two form factors, $f_+^{BM_1}$ and $f_0^{BM_1}$, according to (2.8). At leading order, in full QCD, one finds that the decay amplitudes into the η_{c1} and η_{c2} states are proportional to $f_0^{BM_1}$, while the remaining decays are proportional to $f_+^{BM_1}$. However, since $E_{M_1} \sim M_B/2$, we are in the kinematical regime of large recoil, which allows us to perform the calculation within the SCET formalism and which implies symmetries that reduce the number of independent form factors: $f_0^{BM_1}$ and $f_+^{BM_1}$ are related by

$$f_0^{BM_1} = \left(1 - \frac{M_H^2}{M_B^2}\right) f_+^{BM_1}, \quad (3.30)$$

so that the relation (3.16) holds and all the amplitudes can be expressed in terms of only $f_+^{BM_1}$, according to (3.21). Starting at NLO however there are symmetry breaking corrections and (3.30) is modified [148]. These corrections, which correspond exactly to the evaluation of the first three diagrams in Fig. 2.5, have therefore to be taken into account when expressing the amplitudes in terms of only $f_+^{BM_1}$, as in (3.21). The “vertex correction” (third diagram in Fig. 2.5) gives rise to an additional contribution to the short-distance coefficient $\mathcal{C}_1^A(1S_0^{(1)})$, while the “spectator scattering correction” (first two diagrams in Fig. 2.5) gives an additional term to $\mathcal{C}_1^B(1S_0^{(1)})$.

The loop correction is expected to be large, since it comes with the large Wilson coefficient C_1 , while the tree amplitudes are either zero or colour-suppressed. In case of S -wave production we obtain

$$\begin{aligned} \mathcal{C}_1^A(^1S_0^{(1)}) &= -\frac{1}{2\sqrt{z}} \frac{\alpha_s C_F}{4\pi} \frac{C_1}{N_c} \left(f[^1S_0] + \frac{N_c C_2 + C_1}{C_1} (-6 + \delta f^{BM_1}) \right), \\ \mathcal{C}_1^A(^3S_1^{(1)}) &= \frac{1}{2} \frac{\alpha_s C_F}{4\pi} \frac{C_1}{N_c} \left(f[^3S_1] - 8 \frac{N_c C_2 + C_1}{C_1} \right), \end{aligned} \quad (3.31)$$

while for P -wave we have

$$\begin{aligned} \mathcal{C}_1^A(^1P_1^{(1)}) &= \frac{1}{m_b \sqrt{z}} \frac{\alpha_s C_F}{4\pi} \frac{C_1}{N_c} f[^1P_1], \\ \mathcal{C}_1^A(^3P_0^{(1)}) &= -\frac{1}{\sqrt{3}m_b} \frac{\alpha_s C_F}{4\pi} \frac{C_1}{N_c} f[^3P_0], \\ \mathcal{C}_1^A(^3P_1^{(1)}) &= -\frac{\sqrt{2}}{m_b \sqrt{z}} \frac{\alpha_s C_F}{4\pi} \frac{C_1}{N_c} \left(f[^3P_1] - 4 \frac{N_c C_2 + C_1}{C_1} \right), \\ \mathcal{C}_1^A(^3P_2^{(1)}) &= -\frac{1}{m_b z} \frac{\alpha_s C_F}{4\pi} \frac{C_1}{N_c} f[^3P_2]. \end{aligned} \quad (3.32)$$

The extra term proportional to $(N_c C_2 + C_1)/C_1$ in $\mathcal{C}_1^A(^1S_0^{(1)})$, $\mathcal{C}_1^A(^3S_1^{(1)})$ and $\mathcal{C}_1^A(^3P_1^{(1)})$ arises from the $c\bar{c}$ vertex correction in the second line of Figure 3.2, and was derived from [230]. The term proportional to δf^{BM_1} appearing only in case of the 1S_0 coefficient correspond to the (vertex) correction factor between $f_0^{BM_1}$ and $f_+^{BM_1}$, as explained above. It reads [148]

$$\delta f^{BM_1} = 2 + 2 \frac{1-z}{z} \ln[1-z]. \quad (3.33)$$

The loop functions $f[^{2S+1}L_J]$ are extracted from the $b(m_b w) \rightarrow c(p_c)\bar{c}(\bar{p}_c)D$ amplitude expanded to first order in the relative momentum l . The expansion is done in the integrand to extract the hard momentum region, and the integration is performed after expansion. The decomposition into the six angular momentum states is done according to the operators in (3.5). We use dimensional regularization with $d = 4 - 2\epsilon$ for both ultraviolet and infrared singularities, and the NDR scheme (naive anti-commuting γ_5) for the treatment of γ_5 and the definition of the weak effective Hamiltonian (3.1). Ultraviolet divergences are subtracted according to the $\overline{\text{MS}}$ prescription. An ultraviolet divergence is present only in $f[^1S_0]$, $f[^3S_1]$ and $f[^3P_1]$, since for 1S_0 , 3S_1 and 3P_1 states there exists a non-zero tree amplitude, while infrared divergences appear for the other P -wave operators but not for 3P_1 . This can be understood from the fact that the infrared divergences are related to the 1S_0 and 3S_1 colour-octet matrix elements, and that soft gluons change angular momentum by one unit, but do not change spin. As expected, no infrared divergences arise for S -waves, and in fact no compensation could arise from the $^{2S+1}P_J$ colour-octet matrix elements, that contribute at higher orders.

Defining the auxiliary function

$$f_{I0} = -\left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_b^2} \right) \left(\frac{8}{1-z} + \frac{8 \ln z}{(1-z)^2} \right) + \frac{16 \left(\text{Li}_2 \left[\frac{-1}{1-z} \right] - \text{Li}_2 \left[\frac{-z}{2(1-z)} \right] \right)}{(1-z)^2}$$

$$-\frac{8\ln^2 2}{(1-z)^2} + \frac{4\ln^2 z}{(1-z)^2} - \frac{16}{(1-z)^2} \ln \frac{2-z}{1-z} \ln \frac{z}{2}, \quad (3.34)$$

and the ratio $z = 4m_c^2/m_b^2$ of heavy quark pole masses, we find

$$\begin{aligned} f[^1S_0] &= -6\ln \frac{\mu^2}{m_b^2} - 18 - \frac{2z}{2-z} + \frac{4(3-4z+z^2)}{(2-z)^2} (\ln[1-z] - i\pi), \\ f[^3S_1] &= -6\ln \frac{\mu^2}{m_b^2} - 18 + \frac{2z}{2-z} + \frac{4(3-5z+2z^2)}{(2-z)^2} (\ln[1-z] - i\pi), \end{aligned} \quad (3.35)$$

for S -waves, and

$$\begin{aligned} f[^1P_1] &= z f_{I0} - \frac{4(2-3z+2z^2)}{(2-z)^2} - \frac{4(8-8z+z^2+z^3)}{(2-z)^3} (\ln[1-z] - i\pi) \\ &\quad - \frac{4(6+16z-19z^2+5z^3)}{(1-z)(2-z)^2} \ln 2 - \frac{2z(8+13z-18z^2+5z^3)}{(1-z)^2(2-z)^2} \ln z, \\ f[^3P_0] &= f_{I0} - \frac{4(2-7z+2z^2)}{z(2-z)^2} - \frac{8(4-z-3z^2+z^3)}{z(2-z)^3} (\ln[1-z] - i\pi) \\ &\quad - \frac{4(6+22z-27z^2+7z^3)}{z(1-z)(2-z)^2} \ln 2 + \frac{2(20-73z+58z^2-13z^3)}{(1-z)^2(2-z)^2} \ln z, \\ f[^3P_1] &= -6\ln \frac{\mu^2}{m_b^2} - 18 + \frac{2z}{2-z} + \frac{4(3-5z+2z^2)}{(2-z)^2} (\ln[1-z] - i\pi) \\ &\quad - \frac{8z}{2-z} \ln 2 - \frac{4z^2}{(1-z)(2-z)} \ln z, \\ f[^3P_2] &= z^2 f_{I0} - \frac{4z(2-z-z^2)}{(2-z)^2} - \frac{4z(8-8z+3z^2-z^3)}{(2-z)^3} (\ln[1-z] - i\pi) \\ &\quad - \frac{8z(3+8z-9z^2+2z^3)}{(1-z)(2-z)^2} \ln 2 - \frac{2z^2(16-5z-4z^2+z^3)}{(1-z)^2(2-z)^2} \ln z \end{aligned} \quad (3.36)$$

for P -waves. The infrared $1/\epsilon$ pole, which violates factorization at the heavy quark mass scale, is exhibited explicitly in f_{I0} . The singularity arises here as a consequence of the expansion to first order in the relative momentum l , and signals that – unsurprisingly – the colour-transparency argument does not hold at the heavy-quark mass scale, for the P -wave decay channels.

Results for the hard vertex correction, corresponding to the first four diagrams in figure 3.2, have been obtained previously, for decays into P -wave states, in [191–194, 196, 197], where the infrared divergences were noted for the first time. These papers use a gluon mass rather than space-time dimension as infrared regulator, while the finite part is given only in parametric form. This makes difficult to compare these result with ours, except for the infrared divergent part, where we agree, and for the χ_{c1} case, where the finite contribution is given explicitly [196] and there is no infrared regulator dependence, and where we also agree. The fifth diagram in Figure 3.2 was not taken into account in literature, before [125]. Results for S -waves were presented in literature only within the LCDA approach [188–190], while we present them here for the first time within the non-relativistic approach.

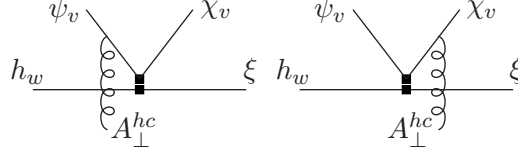


Figure 3.3: Hard spectator-scattering – tree diagrams for the coefficient functions $\mathcal{C}_1^B(2S+1L_J)(y)$.

3.3.2 Short-distance spectator scattering

The tree contribution to the short-distance coefficients of the B -type colour-singlet P - and S -wave operators (3.8) is obtained from matching the QCD amplitude for the process $b(m_b w) \rightarrow c(p_c)\bar{c}(p_{\bar{c}})D(y p)g(\bar{y} p)$ shown in Figure 3.3 to the expression (3.23) and (3.26), respectively. In this way, we calculate directly the momentum-space coefficient function

$$\mathcal{C}^B(2S+1L_J^{(1)})(y) = \int d\hat{s} e^{i\bar{y}\hat{s}} \tilde{\mathcal{C}}^B(2S+1L_J^{(1)})(\hat{s}), \quad (3.37)$$

where $\bar{y} \equiv 1 - y$ is the fraction of longitudinal momentum $n_+ p = m_b(1 - z)$ carried by the hard-collinear gluon. We find

$$\begin{aligned} \mathcal{C}_1^B(1S_0^{(1)})(y) &= -\frac{2C_1}{N_c} \frac{1}{\sqrt{z}} (1 + \delta f_s^{BM_1}), \\ \mathcal{C}_1^B(3S_1^{(1)})(y) &= \frac{2C_1}{N_c} \end{aligned} \quad (3.38)$$

for S -wave states, and

$$\begin{aligned} \mathcal{C}_1^B(1P_1^{(1)})(y) &= -\frac{2C_1}{N_c} \frac{2}{m_b \sqrt{z}}, \\ \mathcal{C}_1^B(3P_0^{(1)})(y) &= \frac{2C_1}{N_c} \frac{2}{\sqrt{3}m_b z} \left(1 - \frac{2z}{1-z} \frac{1}{\bar{y}}\right), \\ \mathcal{C}_1^B(3P_1^{(1)})(y) &= -\frac{2C_1}{N_c} \frac{2\sqrt{2}}{m_b \sqrt{z}} \left(1 + \frac{z}{1-z} \frac{1}{\bar{y}}\right), \\ \mathcal{C}_1^B(3P_2^{(1)})(y) &= \frac{2C_1}{N_c} \frac{2}{m_b} \left(1 + \frac{z}{1-z} \frac{1}{\bar{y}}\right). \end{aligned} \quad (3.39)$$

For P -wave states. The coefficient $\delta f_s^{BM_1}$ is the (spectator scattering) correction factor between $f_0^{BM_1}$ and $f_+^{BM_1}$, which reads [148]

$$\delta f_s^{BM_1} = \frac{z}{1-z} \frac{N_c C_2 + C_1}{C_1}. \quad (3.40)$$

For later purposes it will be useful to express these results as

$$\mathcal{C}^B(2S+1L_J^{(1)})(y) = \frac{2C_1}{N_c} \left(\mathcal{C}^B[2S+1L_J] + \frac{B[2S+1L_J]}{\bar{y}} \right) \quad (3.41)$$

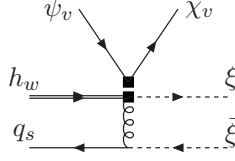


Figure 3.4: Hard spectator-scattering – matrix element.

with y -independent coefficients $C^B[{}^{2S+1}L_J]$, $B[{}^{2S+1}L_J]$, which follow by comparison with (3.38), (3.39).

The hard spectator-scattering amplitude (3.23), (3.26) requires the evaluation of the convolution

$$\int d\hat{s} \tilde{C}^B({}^{2S+1}L_J^{(1)})(\hat{s})|_{\text{tree}} \langle \mathcal{O}^B({}^{2S+1}L_J^{(1)})(s) \rangle|_{\text{tree}}, \quad (3.42)$$

which is done following the method described in [136], which is explained in section 2.3.3. The corresponding diagram is shown in Figure 3.4. First, as in the case of the A-type operators, the tree matrix element factorizes according to

$$\begin{aligned} \langle H({}^{2S+1}L_J)M_1 | \mathcal{O}^B({}^{2S+1}L_J^{(1)})(s) | \bar{B} \rangle = \\ \langle M_1 | \frac{1}{m_b} (\bar{\xi} W_c) \frac{\not{h}_+}{2} [W_c^\dagger i \not{D}_\perp W_c](sn_+) (1 + \gamma_5) h_w | \bar{B} \rangle \\ \times \langle H({}^{2S+1}L_J) | \mathcal{O}({}^{2S+1}L_J^{(1)}) | 0 \rangle. \end{aligned} \quad (3.43)$$

The heavy-to-light matrix element is the same as appears in non-leptonic decay to two light mesons; comparing with section 2.3.3, (2.70), (2.75) one derive

$$\begin{aligned} \langle M_1 | \frac{1}{m_b} (\bar{\xi} W_c) \frac{\not{h}_+}{2} [W_c^\dagger i \not{D}_\perp W_c](sn_+) (1 + \gamma_5) h_w | \bar{B} \rangle = \\ - \frac{2E_{M_1} M_B}{4m_b} \int_0^1 d\tau \int_0^\infty d\omega \int_0^1 dy e^{i\bar{\tau}\hat{s}} J_\parallel(\bar{\tau}; y, \omega) f_B \Phi_{B+}(\omega) f_{M_1} \Phi_{M_1}(y). \end{aligned} \quad (3.44)$$

Here $\Phi_{B+}(\omega)$ and $\Phi_{M_1}(y)$ are the leading-twist light-cone distribution amplitudes of the B and the M_1 meson, respectively, while the hard-collinear “jet function” $J_\parallel(\bar{\tau}; y, \omega)$ is equivalent to (2.80), and at leading order is given explicitly by

$$J_\parallel^{(0)}(\bar{\tau}; y, \omega) = - \frac{4\pi\alpha_s C_F}{N_c} \frac{1}{2E_{M_1}\omega\bar{y}} \delta(\bar{\tau} - \bar{y}). \quad (3.45)$$

Using (2.107), the tree-level matrix elements reads

$$\begin{aligned} \langle M_1 | \frac{1}{m_b} (\bar{\xi} W_c) \frac{\not{h}_+}{2} [W_c^\dagger i \not{D}_\perp W_c](sn_+) (1 + \gamma_5) h_w | \bar{B} \rangle_{\text{tree}} = \\ \frac{\pi\alpha_s C_F}{N_c} \frac{f_B M_B}{m_b \lambda_B} \int_0^1 dy e^{i\bar{y}\hat{s}} \frac{1}{\bar{y}} f_{M_1} \Phi_{M_1}(y). \end{aligned} \quad (3.46)$$

Inserting the Fourier representation of the coefficient function into (3.42), we find the amplitude

$$\begin{aligned} \mathcal{A}_{B \rightarrow H(^{2S+1}L_J)M_1}^{\text{hard spectator}} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \frac{\pi \alpha_s C_F}{N_c} \langle H(^{2S+1}L_J) | \mathcal{O}(^{2S+1}L_J^{(1)}) | 0 \rangle \\ &\times \frac{f_{M_1} f_B M_B}{m_b \lambda_B} \int_0^1 dy \mathcal{C}^B(^{2S+1}L_J^{(1)})(y) \frac{\Phi_{M_1}(y)}{\bar{y}}. \end{aligned} \quad (3.47)$$

This result has been obtained previously, for P -waves, in [191–194, 196], by direct evaluation of the spectator-scattering amplitude.

The problematic aspect of this expression is the endpoint divergence of the convolution integral when $y \rightarrow 1$. The meson M_1 light-cone distribution amplitude behaves as $\Phi_{M_1}(y) \propto \bar{y}$ for small $\bar{y}$. But contrary to the situation for decays to two light mesons or an S -wave charmonium and a light meson, the coefficient function $\mathcal{C}^B(^{2S+1}P_J^{(1)})(y)$ contains a piece proportional to $1/\bar{y}$, resulting in a logarithmically divergent integral. We regularize this integral by introducing a cutoff that replaces the upper limit by $1 - \mu$ with $\mu \ll 1$. Using

$$\int_0^{1-\mu} dy \frac{\Phi_{M_1}(y)}{\bar{y}^2} = \Phi'_{M_1}(1) \ln \mu + \int_0^1 dy \frac{\Phi_{M_1}(y) + \bar{y} \Phi'_{M_1}(1)}{\bar{y}^2} + O(\mu) \quad (3.48)$$

the convolution integral in (3.47) takes the final form

$$\begin{aligned} \int_0^{1-\mu} dy \mathcal{C}^B(^{2S+1}P_J^{(1)})(y) \frac{\Phi_{M_1}(y)}{\bar{y}} &= \\ &\frac{2C_1}{N_c} \left\{ B[^{2S+1}P_J] \Phi'_{M_1}(1) \ln \mu + B[^{2S+1}P_J] \int_0^1 dy \frac{\Phi_{M_1}(y) + \bar{y} \Phi'_{M_1}(1)}{\bar{y}^2} \right. \\ &\left. + C^B[^{2S+1}P_J] \int_0^1 dy \frac{\Phi_{M_1}(y)}{\bar{y}} \right\}. \end{aligned} \quad (3.49)$$

The regulator-dependent $\ln \mu$ term appears to violate factorization even at leading power in the heavy quark expansion. We shall show below, however, that this dependence is cancelled by a corresponding ultraviolet divergence in the colour-octet matrix elements.

3.4 Colour-octet matrix elements

The important new element in our treatment of B decays to P -wave charmonia are the colour-octet contributions (3.10), (3.24) to the decay amplitude. In this section we show how to compute these matrix elements when the charmonium is a Coulomb bound state and demonstrate that the infrared singularities in the vertex correction and hard spectator-scattering can be absorbed into a renormalization of the colour-octet matrix elements. In the Coulomb limit the octet matrix elements can be computed in perturbation theory, and we provide results at order α_s , corresponding to the accuracy of the short-distance terms. In general, and more realistically, the colour-octet matrix elements may be introduced as new non-perturbative parameters, but their scale dependence still cancels

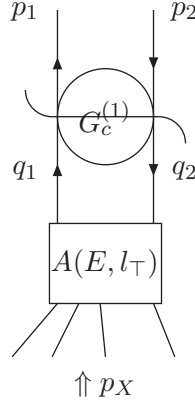


Figure 3.5: Amplitude exhibiting a quarkonium pole after summing Coulomb gluon exchanges in $G_c^{(1)}$.

the factorization scale dependence of the hard-scattering terms. However, factorization in the sense of separating the $B \rightarrow M_1$ transition from the vacuum to charmonium matrix element holds only in the Coulomb limit, since otherwise the octet matrix elements of the SCET/NRQCD four-quark operators contain strongly interacting gluon exchanges between the charmonium and the BM_1 system.

3.4.1 Reduction formula for quarkonium matrix elements

We briefly review the formalism for the calculation of quarkonium matrix elements

$$\langle HX | \mathcal{O} | Y \rangle, \quad (3.50)$$

where $\mathcal{O}$ is some operator, H the quarkonium with momentum $q = M_H v$ in some polarization state, and X, Y denote arbitrary other particles.

The quarkonium matrix element is identified in the standard way as part of the residue factor of the pole of a suitable Green function. We pick a Green function with two external charm quark fields. The quarkonium bound state poles appear after summing ladder diagrams with colour-singlet Coulomb (potential) gluon exchange as the bound state poles of the Coulomb Green function $G_c^{(1)}$ as indicated in Figure 3.5. For external momenta p_1, p_2 with $(p_1 + p_2)^2$ near $4m_c^2$, the bound state poles appear in the integration region where $q_1 = m_c v + \ell_1$, $q_2 = m_c v + \ell_2$, with $E \equiv v \cdot (q_1 + q_2) - 2m_c = v \cdot (\ell_1 + \ell_2)$ and $\ell_{1\top}, \ell_{2\top} = -\ell_{1\top} \equiv -\ell_\top$ small, of order $m_c \alpha_s^2$ and $m_c \alpha_s$, respectively. A specific charmonium state is extracted by performing the following steps: (1) we insert the spectral representation of the Green function and use that near a bound state pole

$$\tilde{G}_c^{(1)}(\mathbf{k}, \ell; E) \xrightarrow{E \rightarrow E_n} \sum_{l, m} \frac{\Psi_{nlm}(\mathbf{k}) \Psi_{nlm}^*(\ell)}{E_n - E - i\epsilon}, \quad (3.51)$$

where $\Psi_{nlm}(\ell)$ is the momentum-space wave-function of the Schrödinger operator in the nlm basis. The three-vectors are introduced by writing four-vectors $a_\top^\mu$ orthogonal to v as

$a_\perp^\mu = L^\mu{}_\nu(v) \hat{a}^\nu$, where $\hat{a}^\mu = (0, \mathbf{a})$ and $L^\mu{}_\nu(v)$ is the Lorentz boost from the quarkonium rest frame to the frame where its momentum is $M_H v$. (2) To separate the degenerate states with different angular momentum and spin, we first introduce the spherical decomposition

$$\Psi_{nlm}(\ell) = \tilde{R}_{nl}(\ell) Y_{lm}(\Omega) \quad (3.52)$$

defining $\ell = |\ell|$. Then we perform a Fierz transformation in the Dirac indices of the two intermediate charm propagators in Figure 3.5 to obtain the projection on the spin zero and spin 1 components. For $l = 1$, the $J = 0, 1, 2$ states follow from the standard Clebsch-Gordon relations.

The final result can be expressed in terms of the on-shell $c\bar{c}$ matrix element corresponding to (3.50), which we write in the form

$$\langle c(q_1) \bar{c}(q_2) X | \mathcal{O} | Y \rangle = \bar{u}_c(q_1) A(E, \ell_\perp) v_c(q_2). \quad (3.53)$$

Defining the matrices

$$\Lambda[H] = \begin{cases} \frac{1}{\sqrt{3}} \gamma_5 & H = \eta_c \\ \frac{1}{\sqrt{3}} \epsilon^*(\lambda) \cdot \gamma_\perp & H = J/\psi \\ -\frac{\epsilon^*(\lambda) \cdot \ell_\perp}{\ell} \gamma_5 & H = h_c \\ \frac{1}{\sqrt{3}} \frac{\not{\ell}_\perp}{\ell} & H = \chi_{c0} \\ \frac{1}{2\sqrt{2}} \left[\frac{\not{\ell}_\perp}{\ell}, \not{\epsilon}^*(\lambda) \right] \gamma_5 & H = \chi_{c1} \\ -\epsilon_{\alpha\beta}^*(\lambda) \frac{\ell_\perp^\alpha}{\ell} \gamma_\perp^\beta & H = \chi_{c2} \end{cases} \quad (3.54)$$

in Dirac-index space (and diagonal in colour space), the desired quarkonium matrix element is

$$\langle H X | \mathcal{O} | Y \rangle = \sqrt{2M_H} \frac{1}{\sqrt{2N_c}} \int \frac{d^3\ell}{(2\pi)^3} \sqrt{\frac{3}{4\pi}} \tilde{R}_{nl}(\ell) \text{tr} \left(\Lambda[H] \frac{1+\not{v}}{2} A(E, \ell_\perp) \frac{1-\not{v}}{2} \right). \quad (3.55)$$

This is valid in a leading-order treatment of the non-relativistic bound state dynamics. Beyond this approximation, corrections to the wave-function and trace expression are required. Eq. (3.55) can be used to calculate the first non-vanishing contribution to a quarkonium matrix element, and this will be sufficient for the colour-octet terms considered below.

The momentum-space radial wave function follows from the Fourier transform of the position-space expression $R_{nl}(r) Y_{lm}(\Omega)$ and is given by

$$\tilde{R}_{nl}(k) = (-i)^l \int_0^\infty dr 4\pi r^2 j_l(kr) R_{nl}(r). \quad (3.56)$$

The case of interest here correspond to $n = 1, l = 0$, (S-waves), and $n = 2, l = 1$ (P -waves). Using the spherical Bessel function

$$j_0(kr) = \frac{\sin kr}{kr},$$

$$j_1(kr) = \frac{\sin kr - kr \cos kr}{(kr)^2}, \quad (3.57)$$

the integral evaluates to

$$\begin{aligned} \tilde{R}_{10}(k) &= \frac{8\pi\gamma_B}{(k^2 + \gamma_B^2)^2} R_{10}(0), \\ \tilde{R}_{21}(k) &= (-i) \frac{1024\pi\gamma_B k}{(4k^2 + \gamma_B^2)^3} R'_{21}(0), \end{aligned} \quad (3.58)$$

where $R'_{21}(0)$ denotes the derivative of the position-space wave function at the origin, and

$$\gamma_B = \frac{m_c \alpha_s C_F}{2} \quad (3.59)$$

is the inverse Bohr radius of the charmonium.

As an example of using (3.55) we calculate $\langle \chi_{c1} | \mathcal{O}(^3P_1^{(1)}) | 0 \rangle$ with $\mathcal{O}(^3P_1^{(1)})$ given in (3.5). The tree amplitude is

$$A(E, \ell_\top) = \frac{n_- \cdot \ell}{2\sqrt{2}} [\not{\ell}_\top, \gamma_\top^\mu] \gamma_5 \quad (3.60)$$

resulting in

$$\text{tr} \left(\Lambda[\chi_{c1}] \frac{1+\not{\ell}}{2} A(E, \ell_\top) \frac{1-\not{\ell}}{2} \right) = \frac{2N_c}{3} \ell n_- \cdot \epsilon(\lambda). \quad (3.61)$$

It follows that

$$\begin{aligned} \langle \chi_{c1} | \mathcal{O}(^3P_1^{(1)}) | 0 \rangle &= \sqrt{2N_c} \sqrt{2M_{\chi_{c1}}} n_- \cdot \epsilon(\lambda) \frac{1}{3} \int \frac{d^3\ell}{(2\pi)^3} \sqrt{\frac{3}{4\pi}} \tilde{R}_{21}(\ell) \ell \\ &= n_- \cdot \epsilon(\lambda) \sqrt{2N_c} \sqrt{2M_{\chi_{c1}}} \sqrt{\frac{3}{4\pi}} (-i) R'_{21}(0), \end{aligned} \quad (3.62)$$

which is consistent with (3.18), (3.19). The other relations in (3.18), (3.19) can be derived in a similar way.

3.4.2 Soft vertex correction

We proceed to the calculation of the colour-octet matrix elements $\langle \chi_{cJ} M_1 | \mathcal{O}_a^A(^3S_1^{(8)}) | \bar{B} \rangle$ and $\langle h_c \bar{M}_1 | \mathcal{O}_a^A(^1S_0^{(8)}) | \bar{B} \rangle$ ($a = \parallel, \perp$). Since this involves only the production of P -wave, we follow closely the treatment given in [125].

At order α_s the interactions are spin-symmetric, so there is no contribution of the $^3S_1^{(8)}$ ($^1S_0^{(8)}$) operator to the h_c (χ_{cJ}) final state. Each matrix element receives contributions from vertex diagrams (first four diagrams in Figure 3.2) and spectator-scattering diagrams (Figure 3.3 with gluon attached to the undisplayed spectator quark line), except that now the gluon virtuality is small, of order $(m_c v^2)^2$ for the vertex diagrams.

We begin by writing down the soft gluon coupling to the $c\bar{c}$ pair. The leading interactions of dynamical gluons with momentum of order $m_c v^2$ to the heavy charm quarks are provided by the (P)NRQCD effective Lagrangian, that we review in appendix A.3. They

read $g_s\psi^\dagger(x)(A^0(t, \mathbf{0}) - \mathbf{x} \cdot \mathbf{E}(t, \mathbf{0}))\psi(x)$ (A.51) together with a similar term for the anti-quark field. The contribution from the A^0 coupling cancels in the sum of the attachments to the c and the $\bar{c}$ line (or can be gauged away), leaving the chromoelectric dipole interaction. The dipole interaction provides the additional factor of velocity v which renders the octet S -wave operator matrix element of the same order in v as the singlet P -wave operator.

The part of the amplitude involving the charm quark lines can now be expressed in the form

$$A_{c\bar{c}}(E, \ell_\top) = \int \frac{d^3\ell'_\top}{(2\pi)^3} \Gamma_a[{}^{2S+1}S_J^{(8)}] iG_c^{(8)}(\ell'_\top, k_\top + \ell_\top; E + v \cdot k) \delta U(k + \ell, \ell), \quad (3.63)$$

see Figure 3.6. The various items in this equation are as follows: k denotes the outgoing soft gluon momentum; $\Gamma_a[{}^{2S+1}S_J^{(8)}]$ comes from the $c\bar{c}$ part of the colour-octet operator as given by the contents of square brackets in (3.5); $\delta U(k + \ell, \ell)$ is the momentum-space soft gluon interaction vertex

$$\delta U(k + \ell, \ell) = -ig_s T^B \frac{\overleftarrow{\partial}}{\partial \ell_\top^\lambda} \left(v^\rho k_\top^\lambda - g_\top^{\rho\lambda} v \cdot k \right), \quad (3.64)$$

where ρ and “B” are, respectively, the Lorentz and colour index of the soft gluon; $G_c^{(8)}(\ell'_\top, k_\top + \ell_\top; E + v \cdot k)$ is the Coulomb Green function that sums an infinite number of gluon exchanges “between” the operator and the soft gluon vertex. Here we need the colour-octet Green function, since the $c\bar{c}$ pair is produced at the operator vertex in a colour-octet state. The calculation of the vertex diagrams with the full Coulomb Green function is quite involved (see [203] for the calculation of colour-octet inclusive quarkonium production matrix elements), but turns out to be unnecessary to good approximation. Similar calculations involving colour-octet Coulomb Green functions [231, 232] find that the numerically largest term arises from the no-gluon exchange term, since every colour-octet exchange is suppressed by the small colour factor $-1/(2N_c)$. We therefore simplify (3.63) by approximating

$$iG_c^{(8)}(\ell_\top, \ell'_\top; E) = \frac{-i}{E + \frac{\ell_\top^2}{m_c} + i\epsilon} (2\pi)^3 \delta^{(3)}(\ell_\top - \ell'_\top), \quad (3.65)$$

obtaining

$$A_{c\bar{c}}(E, \ell_\top) = i^2 g_s \Gamma_a[{}^{2S+1}S_J^{(8)}] \left(-\frac{2}{m_c} \right) \frac{\left(v^\rho k_\top^\lambda - g_\top^{\rho\lambda} v \cdot k \right) \ell_{\top\lambda}}{\left(E + v \cdot k + \frac{\ell_\top^2}{m_c} + i\epsilon \right)^2} T^B. \quad (3.66)$$

Attaching the gluon to the bottom quark and the strange quark line and making use of (3.55), we arrive at

$$\begin{aligned} \langle HM_1 | \mathcal{O}_\parallel^A({}^{2S+1}S_J^{(8)}) | \bar{B} \rangle_{\text{vertex}} = \\ \sqrt{2M_H} \frac{1}{\sqrt{2N_c}} \int \frac{d^3\ell}{(2\pi)^3} \sqrt{\frac{3}{4\pi}} \tilde{R}_{21}(\ell) \tilde{\mu}^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{(-i)}{k^2 + i\epsilon} \end{aligned}$$

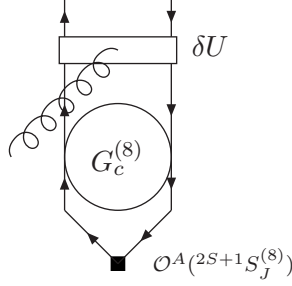


Figure 3.6: Soft gluon attachment to the charm quark lines.

$$\begin{aligned}
& \times \text{tr} \left(\Lambda[H] \frac{1+\not{p}}{2} A_{c\bar{c}}(E_H, \ell_\top) \frac{1-\not{p}}{2} \right) \\
& \times i^2 g_s \left(\frac{p_B \rho}{p_B \cdot k} - \frac{p_\rho}{p \cdot k} \right) \langle M_1 | (\bar{\xi} W_c) \not{n}_+ (1 - \gamma_5) T^A T^B h_w | \bar{B} \rangle, \quad (3.67)
\end{aligned}$$

where we used that k is soft to simplify the bottom and strange quark propagators, and denote the binding energy by

$$E_H = M_H - 2m_c + i\epsilon. \quad (3.68)$$

The colour index “A” is contracted with an index hidden in the definition of $\Gamma_a[{}^{2S+1}S_J^{(8)}]$, while ρ and “B” are contracted with the corresponding indices in (3.66). The order of the colour matrices $T^A T^B$, which is different for the attachment to the bottom and strange line plays no role, since the colour part of the trace evaluates to $\frac{1}{2}\delta^{AB}$. The corresponding matrix elements of the operators $\mathcal{O}_\perp^A({}^{2S+1}S_J^{(8)})$ have $\gamma_{\perp\mu}$ instead of $\not{n}_+$ in the $\langle M_1 | \dots | \bar{B} \rangle$ matrix element. The absence of a transverse vector implies that the matrix element vanishes, so at this order

$$\langle H M_1 | \mathcal{O}_\perp^A({}^{2S+1}S_J^{(8)}) | \bar{B} \rangle_{\text{vertex}} = 0. \quad (3.69)$$

The integrations appearing in (3.67) can be performed exactly. The loop integral is of the general form

$$\tilde{\mu}^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2 P \cdot k (v \cdot k + \omega)^2} = \frac{i}{(4\pi)^2} (I_P v_\mu + J_P P_\mu). \quad (3.70)$$

Only I_P enters the final result, and can be calculated directly by introducing Feynman parameters. In contrast to the hard vertex correction, this loop integral is infrared finite. The sensitivity to very soft gluon momenta is cut off at the scale of the binding energy E_H since $\omega = E_H - \ell_\top^2/m_c$. On the other hand, there is a logarithmic ultraviolet divergence, which we regulate dimensionally to be consistent with the calculation of the short-distance correction. The angular ℓ -integral is also easily done using⁵

$$\int d\Omega \ell_\top^\alpha \ell_\top^\beta = -\frac{4\pi}{3} \ell^2 g_\top^{\alpha\beta}. \quad (3.71)$$

⁵Note $\ell_\top^2 = -\ell^2 = -\ell^2$.

The remaining ℓ -integral is of the type

$$\int_0^\infty d\ell \ell^3 \tilde{R}_{21}(\ell) \left[A + B \ln \left(-\frac{2(E_H - \ell^2/m_c)}{\mu} \right) \right] \quad (3.72)$$

with ℓ -independent constants A , B , and is evaluated using the explicit form (3.58) of the momentum-space radial wave function. Including the colour and spin traces we find

$$\begin{aligned} \langle h_c M_1 | \mathcal{O}_\parallel^A(^1S_0^{(8)}) | \bar{B} \rangle_{\text{vertex}} &= -\frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \mathcal{D}_{sv}(^1P_1^{(1)}) F_{sv} \langle h_c M_1 | \mathcal{O}^A(^1P_1^{(1)}) | \bar{B} \rangle_{\text{tree}}, \\ \langle \chi_{cJ} M_1 | \mathcal{O}_\parallel^A(^3S_1^{(8)}) | \bar{B} \rangle_{\text{vertex}} &= -\frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \mathcal{D}_{sv}(^3P_J^{(1)}) F_{sv} \langle \chi_{cJ} M_1 | \mathcal{O}^A(^3P_J^{(1)}) | \bar{B} \rangle_{\text{tree}}, \end{aligned} \quad (3.73)$$

where we have re-expressed the product of the derivative of the wave function at the origin and the $B \rightarrow M_1$ form factor in terms of the factorized matrix element (3.15) to facilitate the comparison with the hard vertex amplitude (3.22). Eq. (3.73) contains the spin-dependent coefficients $\mathcal{D}_{sv}(^1P_1^{(1)}) = 1$ and $\mathcal{D}_{sv}(^3P_J^{(1)}) = \{1/(\sqrt{3}z), 0, 1\}$ (for $J = 0, 1, 2$), and the loop function

$$\begin{aligned} F_{sv} &= \frac{8z}{m_b(1-z)^2} \left((1-z+\ln z) \left[\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_b^2} + \ln z - 2\Delta F \right] \right. \\ &\quad \left. + 2\text{Li}_2(z) - \frac{1}{2} \ln^2 z + 2 \ln z \ln(1-z) - 2 \ln z - \frac{\pi^2}{3} \right), \\ \Delta F &= \ln \frac{(2\sqrt{-m_c E_H} + \gamma_B)^2}{m_b^2} + \frac{4}{3} \frac{\gamma_B(5\sqrt{-m_c E_H} + 2\gamma_B)}{(2\sqrt{-m_c E_H} + \gamma_B)^2}. \end{aligned} \quad (3.74)$$

This includes the ultraviolet divergence explicitly, as well as dependence on the bound state parameters γ_B (3.59) and E_H (3.68) in ΔF . The colour-octet matrix elements are complex, since they contain soft-rescattering phases. However, as can be seen from the expression for ΔF or from (3.72), in the present one-loop approximation a rescattering phase exists only for positive binding energy.

Consistency of the approach requires that the infrared singularities in the coefficient functions of the P -wave colour-singlet operators cancel with the $1/\epsilon$ pole in (3.74). In this case we may interpret the singularities and corresponding μ -dependence as factorization scale dependence that cancels in the unambiguous sum of the two contributions. Including the tree-level matching coefficients for the colour-octet operators (3.13) and making use of (3.32), the cancellation condition reads

$$\mathcal{C}_1^A(^{2S+1}P_J^{(1)}) - \mathcal{C}_\parallel^A(^{2S+1}S_1^{(1)}) \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \mathcal{D}_{sv}(^{2S+1}P_J^{(1)}) F_{sv} = \text{finite}. \quad (3.75)$$

Inserting results from (3.32), (3.34), (3.74), one checks that the $1/\epsilon$ terms do indeed cancel. In particular, the vanishing of $\mathcal{D}_{sv}(^3P_1^{(1)})$, hence the complete absence of a soft vertex contribution, for the χ_{c1} case is consistent with the absence of an IR divergence in the loop coefficient $f[^3P_1]$.

3.4.3 Soft spectator-scattering

Next we calculate the spectator-scattering contribution to the colour-octet matrix elements. We first consider the part of the amplitude shown in Figure 3.3, before attaching the gluon to the spectator quark line. In the present tree approximation, the gluon momentum k is the difference between the momentum of the antiquark in the meson M_1 , $p_q^{M_1}$ and the spectator-antiquark momentum p_q^B in the B meson. All components of these momenta involve factors of Λ , except for $n_+ \cdot p_q^{M_1} = \bar{y}n_+p = 2\bar{y}E_{M_1}$. Since $m_c v^2 \gg \Lambda$ by assumption, we may drop all small components (except in the denominator of the gluon propagator, which would be exactly zero) and approximate k by

$$k^\mu = n_+ \cdot k \frac{n_-^\mu}{2} \quad (3.76)$$

with $n_+ \cdot k \sim m_c v^2$, since k is soft. This implies that $\bar{y} \sim v^2$, so soft-spectator scattering corresponds to an endpoint configuration, in which very little momentum is transferred to the spectator antiquark. Almost all of the meson M_1 momentum is carried by the quark generated at the $b \rightarrow s$ vertex. The gluon virtuality is given by $k^2 = -n_+ \cdot p_q^{M_1} n_- \cdot p_q^B = -2E_{M_1}\bar{y}\omega$.

The starting expression for the matrix element is

$$\begin{aligned} \langle HM_1 | \mathcal{O}_\perp^A(^{2S+1}S_J^{(8)}) | \bar{B} \rangle_{\text{spect}} = & \sqrt{2M_H} \frac{1}{\sqrt{2N_c}} \int \frac{d^3\ell}{(2\pi)^3} \sqrt{\frac{3}{4\pi}} \tilde{R}_{21}(\ell) \frac{1}{N_c} \int_0^\infty d\omega f_B \Phi_{B+}(\omega) \\ & \times \frac{1}{N_c} \int_0^1 dy f_{M_1} \Phi_{M_1}(y) \frac{(-i)g_{\rho\alpha}}{-2E_{M_1}\bar{y}\omega} \text{tr} \left(\Lambda[H] \frac{1+\not{y}}{2} A_{c\bar{c}}(E_H, \ell_\top) \frac{1-\not{y}}{2} \right) \\ & \times (-i)g_s \text{tr} (T^A T^B) \frac{i}{4} \left(-\frac{i}{4} \right) M_B \text{tr} \left(\not{p} \gamma_5 \gamma_\perp^\mu (1 - \gamma_5) \frac{1+\not{p}}{2} \not{p}_+ \gamma_5 \gamma_\alpha \right), \quad (3.77) \end{aligned}$$

where the second trace arises from the gluon coupling to the spectator quark and the projections on the leading-twist light-cone distribution amplitudes of the meson M_1 and the B meson. The attachment of the soft gluon to the $c\bar{c}$ lines, $A_{c\bar{c}}(E_H, \ell_\top)$, is the same for the vertex and spectator contribution. Substituting (3.76) into (3.63) shows that the gluon index ρ must be transverse. The corresponding matrix elements of the operators $\mathcal{O}_\parallel^A(^{2S+1}S_J^{(8)})$ have $\not{p}_+$ instead of $\gamma_{\perp\mu}$ in the second trace. The trace then vanishes (also for the projection on $\Phi_{B-}(\omega)$, since α must be transverse), so at this order

$$\langle HM_1 | \mathcal{O}_\parallel^A(^{2S+1}S_J^{(8)}) | \bar{B} \rangle_{\text{spect}} = 0. \quad (3.78)$$

Further evaluation of (3.77) is straightforward: perform the traces; convert the ω -integral into $1/\lambda_B$ (2.107); do the ℓ angular and then the radial integral. To facilitate the comparison with (3.47), we provide the final result for the partial amplitude

$$\mathcal{A}_{B \rightarrow H(^{2S+1}P_J)M_1}^{\text{soft spectator}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \mathcal{C}_\perp^A(^{2S+1}S_S^{(8)}) \langle H \bar{M}_1 | \mathcal{O}_\perp^A(^{2S+1}S_S^{(8)}) | \bar{B} \rangle_{\text{spect}} \quad (3.79)$$

rather than the matrix element itself:

$$\begin{aligned} \mathcal{A}_{B \rightarrow H^{(2S+1)P_J} M_1}^{\text{soft spectator}} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \frac{\pi \alpha_s C_F}{N_c} \langle H^{(2S+1)P_J} | \mathcal{O}^{(2S+1)P_J^{(1)}} | 0 \rangle \\ &\times \frac{f_{M_1} f_B M_B}{m_b \lambda_B} \frac{2C_1}{N_c} B[{}^{2S+1}P_J^{(1)}] \\ &\times \int_0^1 dy \Phi_{M_1}(y) \left(\sqrt{-\left(\bar{y} + \frac{2\sqrt{z}E_H}{m_b(1-z)}\right)} + \frac{\gamma_B}{m_b\sqrt{1-z}} \right)^{-4} \end{aligned} \quad (3.80)$$

Here $B[{}^{2S+1}P_J^{(1)}]$ is given by the same expression as defined in (3.39), (3.41). Notice that there is no soft spectator-scattering contribution to the h_c final state, as $B[{}^1P_1^{(1)}] = 0$, which will be important in the numerical analysis.

We compare the integral over the meson M_1 distribution amplitude to (3.47). While the integrand there was applicable to y not near 1 and exhibited a logarithmic endpoint divergence as $y \rightarrow 1$, the present integrand is appropriate only to $1 - y \sim v^2$, *i.e.* in the endpoint region. There is no divergence here as $y \rightarrow 1$. However, for $\bar{y} \gg v^2$ the integrand has the same logarithmic behaviour $\int dy \Phi_{M_1}(y)/\bar{y}^2$ as does the hard-spectator contribution for $\bar{y} \ll 1$. In (3.48), (3.49) we regulated the endpoint divergence in hard spectator-scattering by cutting off the y integral above $1 - \mu$. This corresponds to a hard factorization scale in the energy of the gluon that connects to the spectator quark. The spectator-scattering contribution to the colour-octet matrix element originates precisely from the energy region that was left out in (3.49), thus the correct interpretation of the y -integral in (3.80) is $\int_0^1 dy \rightarrow \int_{1-\mu}^1 dy$. To combine with (3.49) we must evaluate the regularized version of (3.80) up to terms of order v^2/μ . This allows us to approximate $\Phi_{M_1}(y) \approx -\bar{y} \Phi'_{M_1}(1)$, resulting in

$$\begin{aligned} &\int_{1-\mu}^1 dy \Phi_{M_1}(y) \left(\sqrt{-\left(\bar{y} + \frac{2\sqrt{z}E_H}{m_b(1-z)}\right)} + \frac{\gamma_B}{m_b\sqrt{1-z}} \right)^{-4} \\ &\approx -\Phi'_{M_1}(1) \int_0^\mu dy y \left(\sqrt{-\left(y + \frac{2\sqrt{z}E_H}{m_b(1-z)}\right)} + \frac{\gamma_B}{m_b\sqrt{1-z}} \right)^{-4} \\ &\approx \Phi'_{M_1}(1) (-\ln \mu + F_{ss}), \end{aligned} \quad (3.81)$$

where

$$F_{ss} = -\ln(1-z) + 1 + i\pi + \Delta F, \quad (3.82)$$

and ΔF is given in (3.74). Comparing (3.47) to (3.80), together with (3.49), the previous equation demonstrates that the regulator-dependent $\ln \mu$ terms cancel. We may therefore conclude that the endpoint singularity in hard spectator-scattering does not violate factorization, since it can be factorized into the colour-octet matrix elements.

3.4.4 Further remarks on the endpoint singularity

A rigorous understanding of endpoint singularities in convolution integrals would enhance the predictivity of QCD factorization approaches for exclusive B decays considerably. It

would also provide meaning to formal factorization “theorems” derived in soft-collinear effective theory, which generically result in ill-defined convolutions, with exceptions in many leading-power applications. Despite several attempts [87, 150, 236] there is currently no satisfactory framework for factorizing endpoint divergences and for associating them with well-defined operator matrix elements.

The calculation of the $B \rightarrow \chi_{cJ} M_1$ decay amplitudes elaborated in [125] and reported here provides the first example, where an endpoint singularity in a hard-scattering convolution integral can be factored consistently (at least at the leading order) into precisely defined objects, the colour-octet matrix elements. The example does not quite represent what is required for other cases, such as the $B \rightarrow \pi$ form factor, since in $B \rightarrow \chi_{cJ} M_1$ the endpoint divergence arises from factorization at the scale $m_c v^2$, not Λ .⁶ Nevertheless, it is worthwhile to collect some observations on the structure of the endpoint contribution.

- The endpoint contribution is proportional to $\Phi'_{M_1}(1)$, the derivative of the distribution amplitude at the endpoint.⁷ This is because the endpoint region is of size v^2 rather than Λ/m_b , hence it is justified to describe the quarks in the kaon by collinear quark fields. In general, we do not expect the distribution amplitudes to be relevant in the endpoint region, since one of the quarks does not carry a collinear momentum.
- The factor $1/\bar{y}$ that renders the hard-spectator convolution integral divergent, originates from the expansion of the charm propagators. It is therefore not possible to associate the endpoint contribution with a matrix element involving only the meson M_1 state, as in the case of the light-cone distribution amplitude. Rather it reflects a large non-factorizing contribution to the entire process from the scale $m_c v^2$.
- The endpoint contribution contains a large rescattering phase as seen from (3.82). This observation casts doubt on the correctness of a claim made in [95] that the power-suppressed weak annihilation contributions to charmless decays are real in first approximation. In fact, applying the prescription of endpoint subtraction used in [95] to $B \rightarrow \chi_{cJ} M_1$ would simply set the soft-spectator scattering contribution to zero. This implies an uncanceled subtraction scale dependence proportional to $\ln \mu$, which is also present in the result of [95]. More important to the claim, it would miss the soft spectator-rescattering phase.

3.5 Twist-3 spectator scattering contribution

In section 2.3.4 we have explained that, despite being a power-suppressed effect in the Λ_{QCD}/m_b expansion, the twist-3 contribution to the spectator scattering correction is numerically large and therefore important for reliable estimates of B decays into two light mesons. In $B \rightarrow H M_1$ decays, because of the destructive interference between the leading amplitude and the vertex correction, the spectator scattering may be even the dominant

⁶This aspect is similar to the discussion of the $B \rightarrow \eta_c$ form factor in [233], which also exhibits a calculable endpoint logarithm.

⁷As in the expression for the $B \rightarrow \pi$ form factor in the heavy quark limit of its light-cone QCD sum rule representation [234].

contribution to the decay amplitude [125]. For this reason, it is mandatory to include the chirally enhanced twist-3 contribution in the analysis. As we will show in section 3.6, its inclusion is interesting also for a better understanding of the nontrivial interplay among the different scales involved in the problem, in particular $m_c v^2$ and Λ_{QCD} .

The evaluation of the twist-3 spectator scattering correction has not yet been formulated consistently in the framework of SCET_{II} because of the appearance of endpoint singularities that prevent factorization. However, this contribution can be evaluated within a phenomenological approach, [63–65, 67]. When considering $B \rightarrow HM_1$ decays, one has to take into account a hard contribution from the colour-singlet operators, and, for P -wave, a soft contribution from the colour-octet operators, as in case of the leading-twist spectator scattering. The two contributions can be cast into a form similar to (3.47) and (3.80). The twist-3 hard spectator scattering correction reads

$$\begin{aligned} \mathcal{A}_{B \rightarrow H(2S+1)L_J M_1}^{\text{hard spectator} - \text{twist-3}} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \frac{\pi \alpha_s C_F}{N_c} \langle H(2S+1)L_J | \mathcal{O}(2S+1)L_J^{(1)} | 0 \rangle \\ &\times \frac{f_{M_1} f_B M_B}{m_b \lambda_B} \frac{r_\chi^{M_1}}{1-z} \int_0^1 dy \mathcal{C}_{t3}^B(2S+1)L_J^{(1)}(y) \frac{\Phi_{m_1}(y)}{\bar{y}}, \end{aligned} \quad (3.83)$$

where $\Phi_{m_1}(y)$ is the twist-3 distribution amplitude of the meson M_1 , and $r_\chi^{M_1}$ is defined as in (2.45). The twist-3 short-distance coefficients $\mathcal{C}_{t3}^B(2S+1)L_J^{(1)}(y)$ read

$$\begin{aligned} \mathcal{C}_{t3,1}^B(1S_0^{(1)})(y) &= -\frac{2C_1}{N_c} \frac{1}{\sqrt{z}}, \\ \mathcal{C}_{t3,1}^B(3S_1^{(1)})(y) &= \frac{2C_1}{N_c}, \end{aligned} \quad (3.84)$$

for S -wave states, and

$$\begin{aligned} \mathcal{C}_{t3,1}^B(1P_1^{(1)})(y) &= \frac{2C_1}{N_c} \frac{2}{m_b \sqrt{z}} \left(\frac{1+z}{1-z} - \frac{2z}{1-z} \frac{1}{\bar{y}} \right), \\ \mathcal{C}_{t3,1}^B(3P_0^{(1)})(y) &= -\frac{2C_1}{N_c} \frac{2}{\sqrt{3} m_b z} \left(1 - \frac{2z}{1-z} \frac{1}{\bar{y}} \right), \\ \mathcal{C}_{t3,1}^B(3P_1^{(1)})(y) &= -\frac{2C_1}{N_c} \frac{2\sqrt{2}}{m_b \sqrt{z}} \left(\frac{1+z}{1-z} - \frac{z}{1-z} \frac{1}{\bar{y}} \right), \\ \mathcal{C}_{t3,1}^B(3P_2^{(1)})(y) &= -\frac{2C_1}{N_c} \frac{2}{m_b} \left(\frac{1+2z}{1-z} - \frac{2z}{1-z} \frac{1}{\bar{y}} \right), \end{aligned} \quad (3.85)$$

in case of P waves. The results for the decays into the χ_{c0} , χ_{c2} and h_c states have been presented in literature in [193, 197], and we agree with these papers. The results for the other decays however are new, and are presented here for the first time. As expected for the twist-3 contribution, each term in the short-distance coefficients $\mathcal{C}_{t3}^B(2S+1)L_J^{(1)}(y)$ has endpoint divergences, because $\Phi_{m_1}(y) = 1$. However, since the terms proportional to $\bar{y}^{-2}$ are in direct correspondence with the colour-octet contribution, it is useful to introduce, as in (3.41), the y -independent coefficients $C_{t3}^B[2S+1L_J]$, $B_{t3}[2S+1L_J]$, such that

$$\mathcal{C}_{t3}^B(2S+1)L_J^{(1)}(y) = \frac{2C_1}{N_c} \left(C_{t3}^B[2S+1L_J] + \frac{B_{t3}[2S+1L_J]}{\bar{y}} \right). \quad (3.86)$$

There is no colour octet contribution corresponding to the term proportional to $C_{t3}^B[{}^{2S+1}L_J]$. The logarithmic endpoint divergence contained in these terms correspond to the standard divergence appearing at twist-3 in B decays into two light mesons, and signals the non-decoupling of the gluons up to the scale Λ_{QCD} . We use for these divergences the standard treatment defined by (2.108), introducing a phenomenological parameter $X_H^{M_1} \sim \ln(m_b/\Lambda_{\text{QCD}})$. The terms proportional to $B_{t3}[{}^{2S+1}L_J]$ contain more severe linear endpoint divergences, in this case however one has to add the soft contribution from the colour octet operators before getting to the conclusion. We regularize this integral as in section 3.3.2, introducing a cut-off that replaces the upper limit by $1 - \mu$ with $\mu \ll 1$. In this way, with $\Phi_{m_1} = 1$, we obtain

$$\int_0^{1-\mu} dy \frac{\Phi_{m_1}(y)}{\bar{y}^2} = -1 + \frac{1}{\mu}. \quad (3.87)$$

The result of the convolution in (3.83) is therefore expressed as

$$\begin{aligned} & \int_0^{1-\mu} dy C_{t3}^B({}^{2S+1}L_J^{(1)})(y) \frac{\Phi_{m_1}(y)}{\bar{y}} \\ &= \frac{2C_1}{N_c} \left\{ B_{t3}[{}^{2S+1}L_J] \left(-1 + \frac{1}{\mu} \right) + C_{t3}^B[{}^{2S+1}L_J] X_H^{M_1} \right\}, \end{aligned} \quad (3.88)$$

where the $B_{t3}[{}^{2S+1}L_J]$ term is present only for P -waves.

The twist-3 spectator scattering contribution to the colour octet matrix element can be evaluated following the methods illustrated in section 3.4.3, the only difference being that the leading-twist distribution amplitude in (3.77) has to be replaced with its twist-3 counterpart, and the trace appearing in the last line has to be replaced with

$$m_b \frac{r_\chi^{M_1}}{2} \frac{1}{p_{\bar{q}}^{M_1} \cdot p_q^{M_1}} \text{tr} \left(\gamma_5 \not{p}_{\bar{q}}^{M_1} \not{p}_q^{M_1} \gamma_\perp^\mu (1 - \gamma_5) \frac{1 + \not{y}}{2} \not{y} + \gamma_5 \gamma_\alpha \right), \quad (3.89)$$

where $p_{\bar{q}}^{M_1}$, $p_q^{M_1}$ are the momenta of the antiquark and the quark inside the meson M_1 , respectively. Again, we provide the final result for the partial amplitude, that reads (restricting to P -waves)

$$\begin{aligned} \mathcal{A}_{B \rightarrow H({}^{2S+1}P_J)M_1}^{\text{soft spectator} - \text{twist-3}} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \frac{\pi \alpha_s C_F}{N_c} \langle H({}^{2S+1}P_J) | \mathcal{O}({}^{2S+1}P_J^{(1)}) | 0 \rangle \\ &\times \frac{f_{M_1} f_B M_B}{m_b \lambda_B} \frac{r_\chi^{M_1}}{1-z} \frac{2C_1}{N_c} B_{t3}[{}^{2S+1}P_J^{(1)}] \\ &\times \int_0^1 dy \Phi_{m_1}(y) \left(\sqrt{-\left(\bar{y} + \frac{2\sqrt{z}E_H}{m_b(1-z)} \right)} + \frac{\gamma_B}{m_b \sqrt{1-z}} \right)^{-4}. \end{aligned} \quad (3.90)$$

The present integrand is again appropriate only in the endpoint region $1 - y \sim v^2$. There is no divergence as $y \rightarrow 1$, but for $\bar{y} \gg v^2$ the integrand has the same linear behaviour $\int dy 1/\bar{y}^2$ as does the hard contribution for $\bar{y} \ll 1$. As before one has to interpret the y -integral in (3.90) as $\int_0^1 dy \rightarrow \int_{1-\mu}^1 dy$. To combine with (3.88) we must evaluate the

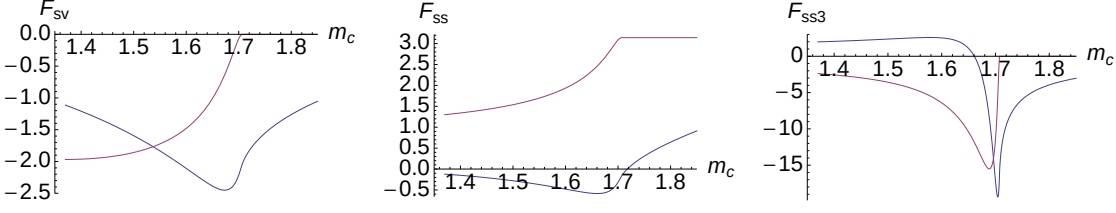


Figure 3.7: Real (blue) and imaginary (red) part of F_{sv} , F_{ss} and F_{ss3} as a function of m_c .

regularized version of (3.90) up to terms of order v^2/μ . Remembering that $\Phi_{m1}(y) = 1$, we obtain

$$\begin{aligned}
 & \int_{1-\mu}^1 dy \Phi_{m1}(y) \left(\sqrt{-\left(\bar{y} + \frac{2\sqrt{z}E_H}{m_b(1-z)}\right)} + \frac{\gamma_B}{m_b\sqrt{1-z}} \right)^{-4} \\
 & \approx \int_0^\mu dy \left(\sqrt{-\left(y + \frac{2\sqrt{z}E_H}{m_b(1-z)}\right)} + \frac{\gamma_B}{m_b\sqrt{1-z}} \right)^{-4} \\
 & \approx \left(-\frac{1}{\mu} + F_{ss3} \right), \tag{3.91}
 \end{aligned}$$

where

$$F_{ss3} = -\frac{m_b^2(1-z)(\gamma_B + 6\sqrt{-m_c E_H})}{3(\gamma_B + 2\sqrt{-m_c E_H})^3}. \tag{3.92}$$

Comparing (3.88) with (3.91), it is possible to see that the regulator dependent term cancels, while a large phase arises from the finite contribution, but only for positive binding energy, as in case of the vertex correction. Part of the twist-3 endpoint singularities therefore does not violate factorization, because they can be absorbed in the colour octet matrix element as in case of the leading twist contribution.

In Fig. 3.7 we compare the real (blue) and imaginary (red) part of the functions F_{sv} , F_{ss} and F_{ss3} . At threshold, *i.e.* for $E_H = 0$, the three functions are approximated by

$$F_{sv} \sim \frac{1}{m_b} \left(\ln \frac{m_b^2}{\gamma_B^2} + \mathcal{O}(1) \right), \quad F_{ss} \sim \ln \frac{m_b^2}{\gamma_B^2} + \mathcal{O}(1), \quad F_{ss3} \sim \frac{m_b^2}{\gamma_B^2}, \tag{3.93}$$

which explains the large value of F_{ss3} at threshold.

From a phenomenological point of view, this leads to an interesting picture, according to which one has interference between the subleading (in Λ_{QCD}/m_b) unknown phase in X_H^{M1} , and the phase arising from the soft spectator scattering contribution. We will return to this point in section 3.6.

3.6 Estimates of branching ratios

Before discussing the numerical results, we present an expression for the sum of the different contributions to the amplitude:

$$\mathcal{A}_{B \rightarrow H(2S+1L_J)M_1} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \langle H(2S+1L_J) | \mathcal{O}(2S+1P_J^{(1)}) | 0 \rangle \left[A_V + A_S + A_{S_3} \right], \quad (3.94)$$

where separating the vertex corrections from the spectator-scattering leading- and twist-3 corrections we have

$$\begin{aligned} A_V &= 2M_B \left(1 - \frac{M_H^2}{M_B^2} \right) f_+^{BM_1}(M_H^2) \left[\left(\mathcal{C}_0^A(2S+1L_J^{(1)}) \right. \right. \\ &\quad \left. \left. + \mathcal{C}_1^A(2S+1L_J^{(1)}) \right) - \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \mathcal{D}_{sv}(2S+1L_J^{(1)}) F_{sv} \mathcal{C}_{\parallel}^A(2S+1S_S^{(1)}) \right], \\ A_S &= \frac{\pi\alpha_s C_F}{N_c} \frac{f_{M_1} f_B M_B}{m_b \lambda_B} \frac{2C_1}{N_c} \left\{ C^B[2S+1L_J] \int_0^1 dy \frac{\Phi_{M_1}(y)}{\bar{y}} \right. \\ &\quad \left. + B[2S+1L_J] \left(\Phi'_{M_1}(1) F_{ss} + \int_0^1 dy \frac{\Phi_{M_1}(y) + \bar{y} \Phi'_{M_1}(1)}{\bar{y}^2} \right) \right\}, \\ A_{S_3} &= \frac{\pi\alpha_s C_F}{N_c} \frac{f_{M_1} f_B M_B}{m_b \lambda_B} \frac{r_\chi^{M_1}}{1-z} \frac{2C_1}{N_c} \left\{ C_{t3}^B[2S+1L_J] X_H^{M_1} \right. \\ &\quad \left. + B_{t3}[2S+1L_J] (-1 + F_{ss3}) \right\}, \end{aligned} \quad (3.95)$$

where the colour-octet contributions enter only in the P -wave amplitudes; all quantities have been defined previously. The branching fraction, extending the leading-order expression (3.21), is given by

$$\begin{aligned} \text{Br}(\bar{B} \rightarrow H(2S+1L_J)M_1) &= \tau_B \frac{G_F^2}{2} |V_{cb} V_{cs}^*|^2 \frac{M_B^2 - M_H^2}{16\pi M_B^3} \text{PolSum}[H(2S+1L_J)] \\ &\quad \times \left[\langle \mathcal{O}(2S+1L_J^{(1)}) \rangle \right]^2 \left| A_V + A_S + A_{S_3} \right|^2, \end{aligned} \quad (3.96)$$

with $\langle \mathcal{O}(2S+1L_J^{(1)}) \rangle = \langle \mathcal{O}(1S_0^{(1)}) \rangle$ for S -waves and $\langle \mathcal{O}(2S+1L_J^{(1)}) \rangle = \langle \mathcal{O}(3P_0^{(1)}) \rangle$ for P -waves.

3.6.1 Parameters

The numerical result depends on various parameters. For many of them the input values are collected in table 2.1, or given in section 2.3.5. According to it, we have *e.g.* $|V_{cb} V_{cs}| = 40.4 \cdot 10^{-3}$. We use next-to-leading logarithmic running of the strong coupling and Wilson coefficients. At $\mu = 3 \text{ GeV}$: $\alpha_s(3 \text{ GeV}) = 0.2503$, $C_1(3 \text{ GeV}) = 1.105$, $C_2(3 \text{ GeV}) = -0.2366$. The renormalization scale for these quantities is denoted by μ . However, in the strong coupling that multiplies the spectator scattering term we use the

intermediate scale $\mu_h = \sqrt{0.5 \text{ GeV}} \mu$, and in the expression for the inverse Bohr radius (3.59) we imagine choosing the scale of α_s (or m_c) such that $\gamma_B = 500 \text{ MeV}$. The values of the quark masses m_b, m_c will be discussed below.

The meson masses are $M_B = 5.279 \text{ GeV}$, $M_{\eta_c} = 2.980 \text{ GeV}$, $M_{\eta'_c} = 3.638 \text{ GeV}$, $M_{J/\psi} = 3.097 \text{ GeV}$, $M_{\psi'} = 3.686 \text{ GeV}$, $M_{\chi_{c0}} = 3.415 \text{ GeV}$, $M_{\chi_{c1}} = 3.511 \text{ GeV}$, $M_{\chi_{c2}} = 3.556 \text{ GeV}$, $M_{h_c} = 3.525 \text{ GeV}$. The derivative of the wave function at the origin can be determined from χ_{cJ} decays and takes the value $|R'_{21}(0)|^2 = 0.050 \text{ GeV}^5$, while $|R_{10}|^2 = 0.810 \text{ GeV}^3$, $|R_{20}|^2 = 0.529 \text{ GeV}^3$ [202, 235]. We will assume the moment of the B -meson distribution amplitude to take the small value $\lambda_B = 200 \text{ MeV}$ that is favoured by the large rates of colour-suppressed charmless B decays [136], or alternatively the larger value $\lambda_B = 350 \text{ MeV}$ that is expected with QCD sum rules and models of the B meson distribution amplitude, [164–167], where values twice as large as $\lambda_B = 200 \text{ MeV}$ are not excluded. The $B \rightarrow M_1$ form factor is parameterized following [158]. In case of $B \rightarrow K$ transition we have

$$f_+^{BK}(q^2) = \frac{0.162}{1 - q^2/M_{B_s}^2} + \frac{0.173}{(1 - q^2/M_{B_s}^2)^2} \quad (M_{B_s} = 5.41 \text{ GeV}), \quad (3.97)$$

while for the $B \rightarrow \pi$ transition the parametrization reads

$$f_+^{B\pi}(q^2) = \frac{0.744}{1 - q^2/M_{B_d}^2} - \frac{0.486}{(1 - q^2/M_{\text{fit}}^2)^2} \quad (M_{B_s} = 5.32 \text{ GeV}, M_{\text{fit}} = 6.38 \text{ GeV}). \quad (3.98)$$

Many of these parameters have significant theoretical errors, but in view of other uncertainties discussed below, they are less relevant, except for the parameter λ_B . Finally, the meson M_1 light-cone distribution amplitude is expanded into Gegenbauer polynomials, according to (2.116), and we truncate the expansion at order $n = 2$. The first two Gegenbauer moments for pions and kaons are collected in table 2.1. [168–171, 174, 175], while using “asymptotic” distribution amplitudes amounts to setting the Gegenbauer moments to zero. In terms of Gegenbauer moments the expressions appearing in (3.95) read

$$\begin{aligned} \Phi'_{M_1}(1) &= (-6) \left(1 + 3a_1^{M_1} + 6a_2^{M_1} \right), \\ I_1 &\equiv \int_0^1 dy \frac{\Phi_{M_1}(y)}{\bar{y}} = 3 \left(1 + a_1^{M_1} + a_2^{M_1} \right), \\ I_2 &\equiv \int_0^1 dy \frac{\Phi_{M_1}(y) + \bar{y} \Phi'_{M_1}(1)}{\bar{y}^2} = (-6) \left(1 + 6a_1^{M_1} + 16a_2^{M_1} \right). \end{aligned} \quad (3.99)$$

While I_1 is well-behaved, $\Phi'_{M_1}(1)$ and I_2 exhibit a very large sensitivity to the higher Gegenbauer moments. Comparing the maximal value of I_2 to its asymptotic one, we find $I_2^{\text{max}}/I_2^{\text{as}} = 7.32$ for kaons and 6.6 for pions, and it is not clear whether the Gegenbauer expansion is converging at all. A consequence of this is that the size of the spectator-scattering amplitude is uncertain by a factor of several (including the uncertainty in λ_B) for χ_{cJ} , where $B[{}^{2S+1}P_J]$ is not vanishing.

3.6.2 Results

Given the large ambiguities mentioned above, but also the fact that our calculation relies on the unrealistic assumption that charmonium is a Coulomb bound state, we do not

expect reliable quantitative results for the $B \rightarrow HM_1$ branching fractions. The aim of this phenomenological analysis is instead to investigate, whether the QCD factorization model elaborated so far, with inclusion of the colour octet operators, is able to catch the main features of the experimental results. This implies to address mainly the following questions:

- 1) Are large corrections to naive factorization expected theoretically? This is necessary, given the small tree-level branching ratios obtained in table 3.2, to explain the large J/ψ , η_c , χ_{c1} and χ_{c0} experimental branching ratios in table 3.1.
- 3) Why are the $\chi_{c2}K$ and h_cK final states suppressed relative to $\chi_{c0}K$, $\chi_{c1}K$?

Our calculation results in exactly the same decay rates for charged and neutral B decay. Thus, branching fractions of pairs of related decays differ only by the lifetime ratio τ_{B_u}/τ_{B_d} . In the following we consider only $\bar{B}_d^0$ decay using $\tau_{B_d} = 1.53 \cdot 10^{-12}$ s. The analysis will focus mainly on the final states with $M_1 = \bar{K}$, since these are the decays modes for which we have more complete experimental data.

We begin by discussing the dependence of the branching fractions on the various inputs, when we neglect the spectator-scattering term entirely. The scale-dependence, adopting the quark masses values $m_b = 4.8$ GeV, $m_c = 1.4$ GeV, is shown in the plots of Figure 3.8; the plot on the left contains the scale dependence of the leading and next-to-leading order amplitude for decays into S -waves, and the results for the χ_{c1} has been included, for comparison, being the only P -wave state produced at leading order. The plot on the right shows results for the P -wave final states. There is a large cancellation between the tree level and one-loop contribution to the coefficient function relevant to the J/ψ , η_c and χ_{c1} final states, resulting in a very small branching fraction. The branching fractions for the other final states are also quite small, not exceeding a few times 10^{-5} with h_c and χ_{c2} being even smaller than the other two. At this point we can already conclude that corrections to naive factorization are order one effects, providing a positive answer to the first question above, as is in fact expected for colour-suppressed decay modes. The final-state dependence might be similar to the data, but this cannot be the complete story, since the J/ψ , η_c , χ_{c0} and χ_{c1} branching fractions fall short of the data by about an order of magnitude.

The scale dependence of the NLO result remains significant, simply because the LO term for J/ψ , η_c and χ_{c1} is cancelled, and there is no LO term for the other final states. The scale dependence is exactly of the form $(\alpha_s C_1)^2$ for χ_{c0} , χ_{c2} , and h_c and approximately so for J/ψ , η_c and χ_{c1} . This causes an uncertainty of a factor of 2 when μ is varied between 2 GeV and 5 GeV, and larger if one allows smaller scales. However, below about 1.5 GeV the scale-dependence blows up as seen in the Figure. In the following we fix $\mu = 3$ GeV. Results for other choices of μ can be obtained approximately by multiplying with $(\alpha_s C_1)^2(\mu)/(\alpha_s C_1)^2(3 \text{ GeV})$. (This remains true, when the spectator-scattering terms are added back.)

Other significant parameter dependencies arise from the quark mass values. The dependence on m_c is more important than the one on m_b , so we fix $m_b = 4.8$ GeV in the following. The charm quark mass dependence of the branching fractions, still omitting

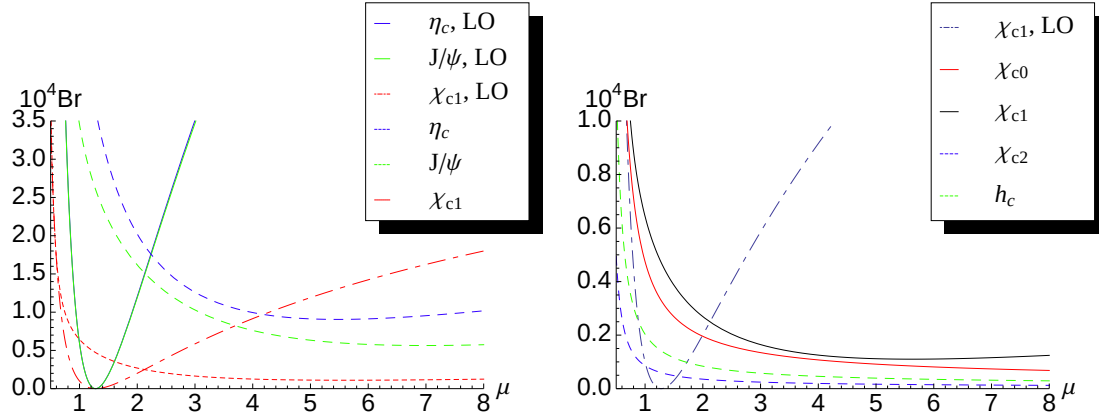


Figure 3.8: Branching fractions in units of 10^{-4} , when the spectator-scattering contribution is set to zero: renormalization scale dependence for S -waves and χ_{c1} , (left) and for P -waves (right).

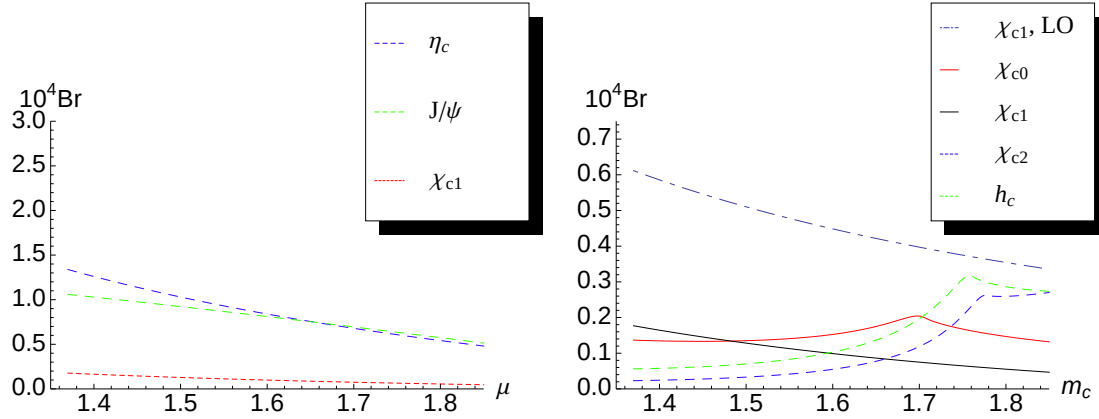


Figure 3.9: Branching fractions in units of 10^{-4} , when the spectator-scattering contribution is set to zero: charm quark mass dependence for S -waves and χ_{c1} , (left) and for P -waves (right).

spectator scattering, is shown in the plots of Figure 3.9. The m_c dependence is mild for the S -wave states (left plot), though a bit more pronounced for the η_c final state. (at leading order, the J/ψ final state is independent from m_c , while the η_c state depends on m_c through $1/\sqrt{z}$.) The m_c dependence is more important for P -waves, since the size of the χ_{c0} , χ_{c1} branching fractions versus χ_{c2} , h_c reverses as m_c increases (right plot in Figure 3.9). The charm quark mass here is the pole mass, which is a poorly defined quantity in perturbation theory, due to large radiative corrections. Typical values are $m_c = 1.4 \dots 1.7 \text{ GeV}$. There may be good reason to choose larger values here, since the charm quark pole mass controls the binding energy $E_H = M_H - 2m_c$, which should be negative in the approximation of charmonium as a non-relativistic bound state, and is negative in reality when measured relative to the $D\bar{D}$ threshold. From the Figure it is evident that with the NLO vertex correction alone it is not possible to explain the experimental data, since the J/ψ , η_c , χ_{c0} , and χ_{c1} branching fractions are too small, even

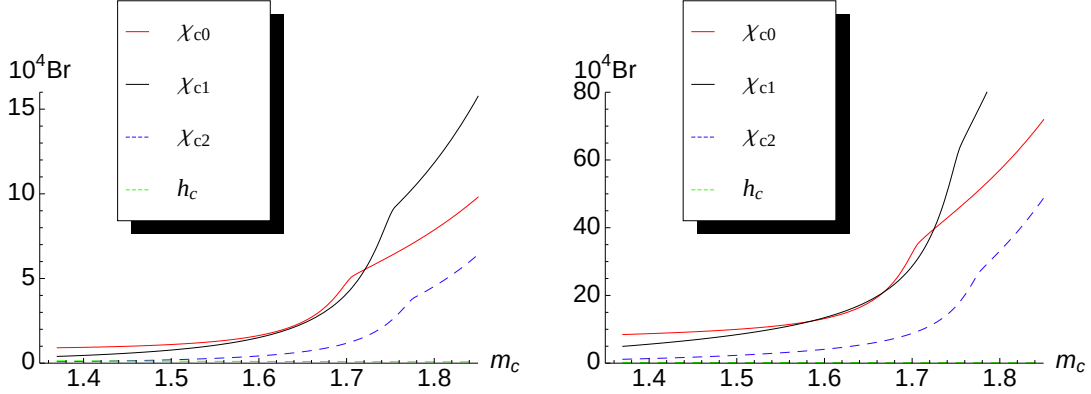


Figure 3.10: Charm quark mass dependence of the branching fractions in units of 10^{-4} from spectator scattering only. Left for asymptotic kaon distribution amplitude ($a_1^{\bar{K}} = a_2^{\bar{K}} = 0$), right for $a_1^{\bar{K}} = 0.06$ and $a_2^{\bar{K}} = 0.2$. Only P -waves final states are shown.

allowing for theoretical uncertainties in the $B \rightarrow K$ form factor or the charmonium wave function.

Next we imagine that the branching fractions are given by the leading-twist spectator-scattering term alone. The largest parameter dependencies now arise from the charm quark mass, the Gegenbauer moments of the kaon light-cone distribution amplitude, and the B meson distribution amplitude parameter λ_B . Focusing on the P -wave, in Figure 3.10 we show the m_c dependence for asymptotic distribution amplitudes (left) and for $a_1^{\bar{K}} = 0.06$, $a_2^{\bar{K}} = 0.2$ (right). In these plots we take $\lambda_B = 200$ MeV. The branching fractions grow rapidly with m_c , when the spectator amplitude becomes dominated by the imaginary part from the colour-octet contributions. We also observe a huge dependence on the Gegenbauer moments, confirming the expectation that the expansion may be invalid. Even $a_2^{\bar{K}} = 0.2$ leads to unacceptably large branching fractions. These effects are less pronounced when λ_B is larger, since the spectator-scattering branching fraction shown in Figure 3.10 is proportional to $1/\lambda_B^2$. Independent of these uncertainties, we always find that spectator-scattering is a small effect for h_c , and larger for χ_{c0} , χ_{c1} than for the other two final states.

When we now add both contributions together, including the interference term, but neglecting for the moment the twist-3 spectator scattering, we obtain the result shown in Figure 3.11 for asymptotic distribution amplitudes and $\lambda_B = 200$ MeV. For m_c in the range from 1.5 GeV to 1.7 GeV, this suggests the interpretation that the χ_{c0} , χ_{c1} final states are dominated by spectator scattering, more precisely by the spectator-scattering contribution to the colour-octet matrix element. The smallness of the h_c branching fraction is explained by the absence of such a contribution (at leading order) for this final state. The χ_{c2} case is intermediate with a rapidly rising branching fraction in the interesting charm-quark mass window. Numerical results for some values of m_c are provided in Table 3.3. We emphasize that in addition to the charm-quark mass dependence displayed explicitly there are further large theoretical uncertainties related to scale-dependence, which shifts all branching fractions uniformly as described above, to m_b -dependence, to

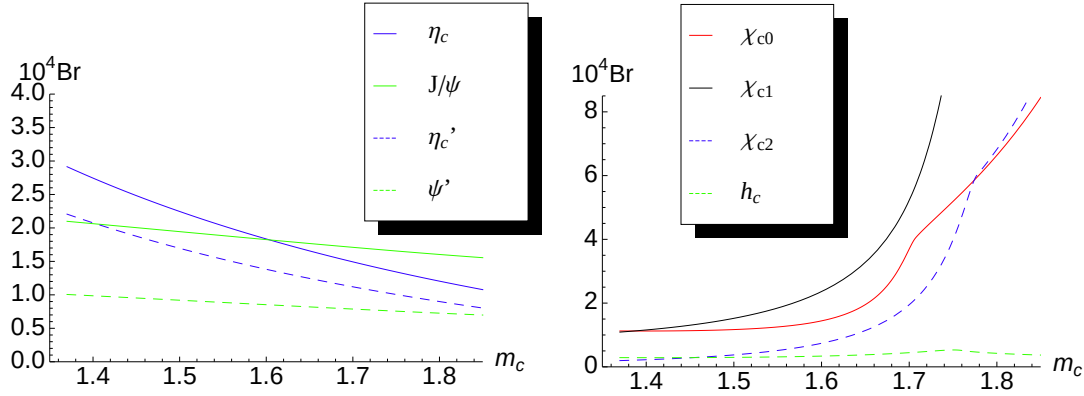


Figure 3.11: Charm quark mass dependence of the branching fractions in units of 10^{-4} including all contributions except twist-3 spectator scattering. Left, S -waves and right, P -waves.

the Gegenbauer moments, and to λ_B . There are some parameter degeneracies that allow making $a_2^{\bar{K}}$ and λ_B simultaneously larger. In view of these uncertainties, the results in Table 3.3 and Figure 3.11 show anyway that there are reasonable regions of parameter space ($m_c \approx 1.65$ GeV, small λ_B , and asymptotic kaon distribution amplitude), where the theoretical calculations in our model for the colour-octet matrix elements are in qualitative agreement with the experimental data for P -wave production, namely the existence of large contributions beyond naive factorization, and the suppression of the $\chi_{c2}K$ and h_cK modes. From Table 3.3 we see that the small h_c branching fraction is a robust feature of our results, but we find it difficult to explain the strong χ_{c2} suppression seen in the data compiled in Table 3.1, while maintaining the sizeable χ_{c1} branching fraction. At the same time, predictions for S -waves are not so good, too, because their branching ratios are predicted to be about four times smaller than the experimental results. This reflects the importance that the subleading contribution pointed out in section 3.2.6 could have, in particular if there existed an enhanced correction from colour-octet operators. Since these terms were not calculated, the present result for S -waves has to be intended as an estimate of the relative importance of the leading (in powers of v) contribution versus the power suppressed corrections.

The phenomenological analysis performed in [125] did not take into account the twist-3 contribution, that we consider here for the first time. Results for the P -wave branching ratios, assuming that only the twist-3 term contribute to them are shown in figure 3.12, for $\lambda_B = 200$ MeV. The twist-3 correction is strongly peaked around the threshold point $2m_c = M_H$. Comparing A_S and A_{S_3} one realizes that at threshold the twist three contribution is formally of order $m_b \Lambda_{\text{QCD}} / \gamma_B^2 \sim \Lambda_{\text{QCD}} / m_b v^2$ with respect to the leading twist spectator, since $\gamma_B \sim m_b v$. This is because of $F_{ss3} \sim m_b^2 / \gamma_B^2$ at threshold, while $r_\chi^{M_1} \sim \Lambda_{\text{QCD}} / m_b$. Remembering that the latter relation is only formal, being $r_\chi^{M_1} \sim \mathcal{O}(1)$ numerically, one explains the large values taken by the twist-3 contribution at threshold. The main contribution to the twist-3 spectator scattering comes again from the colour-octet production mechanism, which can interfere destructively (χ_{c0} and χ_{c1} states) or

m_c/GeV	$J/\psi \bar{K}^0$	$\psi' \bar{K}^0$	$\eta_c \bar{K}^0$	$\eta'_c \bar{K}^0$	$\chi_{c0} \bar{K}^0$	$\chi_{c1} \bar{K}^0$	$\chi_{c2} \bar{K}^0$	$h_c \bar{K}^0$
1.45	2.00	0.95	2.48	1.87	1.13	1.31	0.28	0.29
1.50	1.94	0.92	2.25	1.70	1.17	1.52	0.37	0.30
1.55	1.88	0.89	2.03	1.53	1.25	1.85	0.51	0.31
1.60	1.83	0.85	1.84	1.38	1.44	2.36	0.74	0.34
1.65	1.77	0.82	1.66	1.25	1.95	3.28	1.14	0.38
1.70	1.71	0.79	1.49	1.12	3.70	5.23	1.95	0.44
1.75	1.60	0.73	1.20	0.90	5.19	10.31	3.98	0.53

Table 3.3: $\bar{B}_d^0 \rightarrow H \bar{K}^0$ branching fractions in units of 10^{-4} for various charm quark mass inputs. Asymptotic distribution amplitudes, $\lambda_B = 200 \text{ MeV}$ and $m_b = 4.8$ are fixed.

constructively (χ_{c2}) with the corresponding colour octet contribution in the leading twist spectator amplitude (there is no spectator scattering colour octet production for the h_c state). As a consequence of these patterns, and because of its large contribution, the twist-3 correction cannot be accommodated in the “optimal” scenario developed in [125] and described above, where asymptotic distribution amplitudes are considered. Here instead one is lead to consider the full expression of the leading twist-distribution amplitudes, with $a_1^{\bar{K}} = 0.06$ and $a_2^{\bar{K}} = 0.2$. Because of the convergence problems mentioned above, together with the strong dependence on m_c of the twist-3 correction, the present evaluation of the branching ratios is even more uncertain, and therefore has to be considered as an estimate, from which one can infer hints about the importance of subleading (in Λ_{QCD}/m_b) contributions to the decay amplitudes.

The interesting point about including subleading (though chirally enhanced) corrections in the analysis is that they may lead to identify a different set of parameters, that are able to catch the main experimental features of the decay modes considered. In Figure 3.13 the m_c dependence of the branching ratio is shown, when $a_1^{\bar{K}} = 0.06$ and $a_2^{\bar{K}} = 0.2$ is used for the leading-twist distribution amplitudes, and the twist-3 correction is included, for a larger $\lambda_B = 350 \text{ MeV}$. Even though in the interesting window $m_c = 1.5, \dots, 1.7 \text{ GeV}$ the hierarchy between the χ_{c0} and χ_{c1} amplitudes is inverted, and the χ_{c2} amplitude is far too large, this is not the end of the story. The S -wave amplitude, for instance, becomes a little larger, because of the constructive interference of the leading- and twist-3 spectator scattering. Moreover, the graphs in Figure 3.13 are obtained in the standard scenario for which $\varrho_H = 0$, and this misses the possibility of having different interference patterns in the twist-3 contribution, between the soft-phase of the colour-octet contribution and the soft phase φ_H in X_H . The variation of this phase is shown in Figure 3.14 for P -waves, for different values of m_c and $\varrho_H = 1$. The χ_{c1} and χ_{c2} shows a larger sensitivity to this

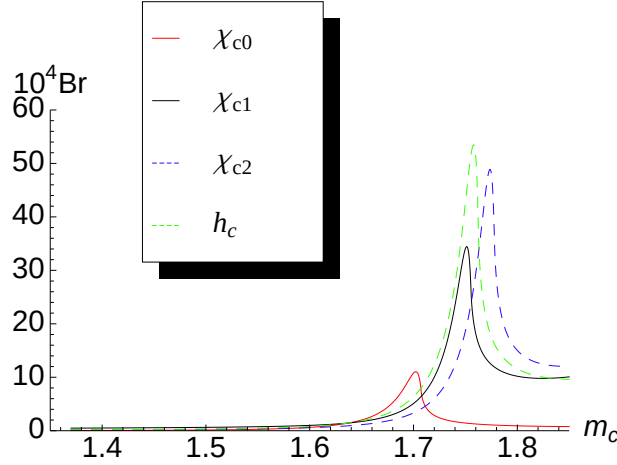


Figure 3.12: Charm quark mass dependence of the branching fractions in units of 10^{-4} , including only the twist-3 spectator scattering.

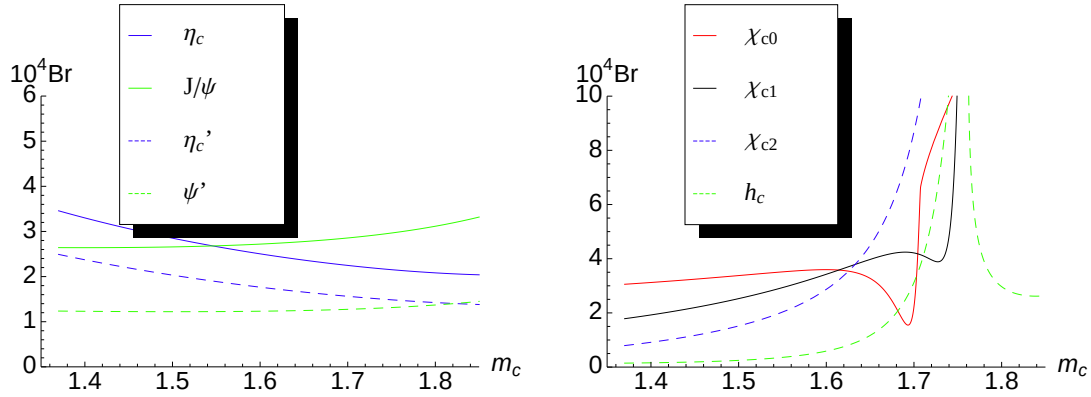


Figure 3.13: Charm quark mass dependence of the branching fractions in units of 10^{-4} including all contributions. Left, S -waves and right, P -waves.

parameter than the χ_{c0} , h_c states but, more importantly, the interference pattern is opposite among the χ_{c0} , χ_{c1} and the h_c , χ_{c2} states, so that to a large branching ratio for the χ_{c1} , χ_{c0} states correspond a suppressed one for the h_c , χ_{c2} states, in the range in which $240^\circ \lesssim \varphi \lesssim 300^\circ$. At the same time it is interesting to note that, comparing with Figure 3.15, the branching ratio of the decay modes containing S -waves in the final states are also enhanced in the same range of φ , giving a value closer to the experimental results. In table 3.4 we show some value of the branching ratios corresponding to the set of parameters that give the results more close to the experiment. One can see that the suppression of the h_c state is still effective and consistent with the experimental data, however it is still difficult to explain the suppression observed for χ_{c2} state and the prediction for the χ_{c0} state is in this case a bit too large. All in all, we find that the set of parameters that better describes the experimental data is given by $m_c = 1.55$ GeV, a bit lower than in

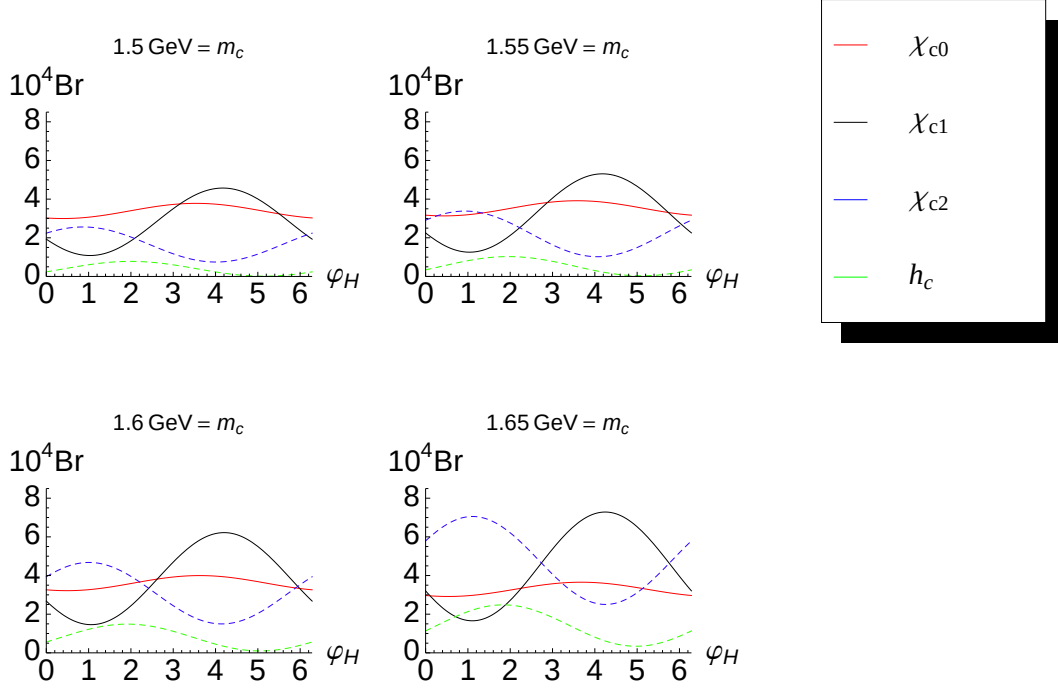


Figure 3.14: Dependence of the P -waves branching fractions, in units of 10^{-4} , from the unknown non-perturbative phase φ_H , for different values of m_c

the previous scenario, $\lambda_B = 400$ MeV and the soft phase in the range $240^\circ \lesssim \varphi \lesssim 270^\circ$. For this set of parameters, however, the branching ratios for the S -waves are too low by a factor 2. The lesson from this is that subleading corrections in Λ_{QCD}/m_B are important, too, but difficult to evaluate precisely, because they receive contributions mainly from the $m_c v^2$ and the Λ_{QCD} regions.

Before concluding our analysis we briefly report results for decays into a charmonia state and a pion. The dependence of the total branching ratio (excluding the twist-3 contribution) on m_c is shown in Figure 3.16, while numerical results for the same set of parameters used in table 3.3 are shown in table 3.5. Because of their smaller branching ratio, many of these decays are not yet observed experimentally, and our prediction may indicate which of them could be observed in the future. However, they may be of some interest from the point of view of testing the factorization properties of the approach developed here. In fact, thanks to the lower decay constant of the pion, $f_\pi = 131$ MeV vs. $f_K = 160$ MeV, together with the smaller Gegenbauer moments, $\alpha_1^\pi = 0$, $\alpha_2^\pi = 0.02$ vs. $a_1^{\bar{K}} = 0.06$ and $a_2^{\bar{K}} = 0.2$, the contribution of the spectator scattering, which is the source of the largest uncertainties, is expected to be slightly smaller. Also interesting, in this respect, is the ratio between the branching fraction of decays into a charmonium state and a pion, and a charmonium state and a kaon. In table 3.6 we report the experimental ratio, when it is available, together with the ratio predicted at leading order in α_s , that is given simply by the ratio of the CKM factors and form factors and at NLO, with the best

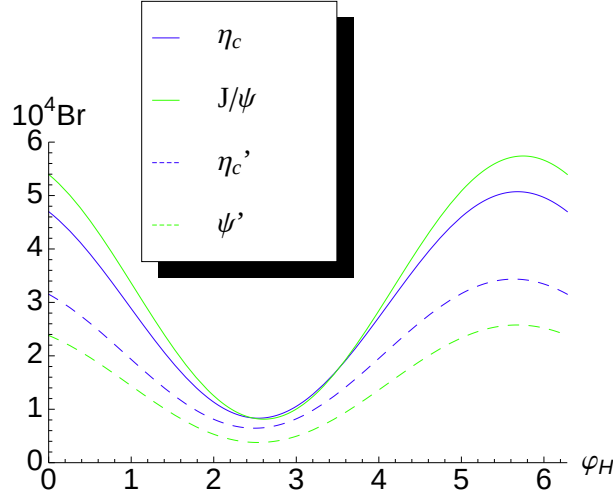


Figure 3.15: Dependence of the S -waves branching fractions, in units of 10^{-4} , from the unknown non-perturbative phase φ_H , for $m_c = 1.6$ GeV.

choice of input parameters individuated previously in the leading-twist approximation. The experimental ratio are generally larger than the predicted one. It is possible to see that the NLO results slightly improve the prediction, making it closer to the experimental data. One can note, however, the result for the neutral B decay ratio, that deviates sensibly from the charged B decay ratios. Since in our model the difference between the two decays is given only by the different lifetimes, this trait cannot be explained and may signal the presence of additional non-perturbative contribution to the decay amplitude in the neutral decay modes, a feature already observed for other neutral B decays like $\bar{B} \rightarrow \pi^0 \pi^0$ or $\bar{B} \rightarrow \pi^0 \bar{K}^0$.

3.7 Conclusion

This chapter has been devoted to a treatment of B decays into a charmonium state and a light meson, in view of the analysis we performed for P -wave charmonia in [125], motivated by previous studies [191–194, 196, 197] of these decays in the QCD factorization framework, that reported a violation of factorization.

Our main result is the inclusion of the colour-octet operators in the calculation of the decay amplitudes into P -wave. The colour octet operators are in fact not suppressed by Λ/m_b , contrary to the case of charmless decays, due to the existence of the charmonium binding energy scale. After the addition of this contribution factorization is recovered, at least at order α_s . The infrared divergences found in previous calculations can be subtracted consistently into the matrix elements of these operators. This includes the endpoint divergence that is found in the unsubtracted coefficient function associated with spectator-scattering. Our calculations demonstrate that the endpoint contribution, now contained in the colour-octet matrix element, can lead to a large rescattering phase. These observations may be of conceptual interest, since it is presently still unclear in the general

$[m_c, \lambda_B]/\text{GeV}, \varphi_H$	$J/\psi \bar{K}^0$	$\psi' \bar{K}^0$	$\eta_c \bar{K}^0$	$\eta'_c \bar{K}^0$	$\chi_{c0} \bar{K}^0$	$\chi_{c1} \bar{K}^0$	$\chi_{c2} \bar{K}^0$	$h_c \bar{K}^0$
1.55, 0.350, 240°	3.28	1.56	3.31	2.36	3.84	5.30	1.02	0.22
1.55, 0.350, 270°	4.43	2.07	4.35	3.03	3.69	5.02	1.26	0.07
1.55, 0.350, 300°	5.25	2.40	5.06	3.47	3.49	4.27	1.73	0.04
1.60, 0.350, 240°	3.31	1.56	3.12	2.21	3.94	6.21	1.50	0.34
1.60, 0.350, 270°	4.53	2.10	4.15	2.88	3.78	5.91	1.74	0.14
1.60, 0.350, 300°	5.42	2.47	4.87	3.33	3.59	5.05	2.35	0.11
1.55, 0.400, 240°	2.74	1.33	2.79	2.00	3.01	4.23	0.81	0.20
1.55, 0.400, 270°	3.62	1.71	3.58	2.52	2.88	4.01	0.98	0.07
1.55, 0.400, 300°	4.20	1.94	4.08	2.83	2.73	3.43	1.34	0.04
1.60, 0.400, 240°	2.75	1.32	2.62	1.87	3.06	4.93	1.18	0.30
1.60, 0.400, 270°	3.68	1.73	3.40	2.38	2.95	4.70	1.35	0.13
1.60, 0.400, 300°	4.31	1.99	3.91	2.69	2.80	4.02	1.81	0.11

Table 3.4: $\bar{B}_d^0 \rightarrow H \bar{K}^0$ branching fractions in units of 10^{-4} for various charm quark mass inputs. Asymptotic distributions amplitudes, $\lambda_B = 200 \text{ MeV}$ and $m_b = 4.8 \text{ GeV}$ are fixed.

case, whether and how endpoint divergences that often appear in convolutions in collinear factorization formulas can be absorbed into well-defined non-perturbative objects and what these objects are. We find it plausible that factorization of B decays to P -wave charmonium extends to higher orders in the coupling expansion when $m_c v^2 \gg \Lambda$, in view of the argument presented in [64]; nonetheless, it would be of great interest to verify the factorization of endpoint divergences beyond the tree-approximation to the hard-scattering sub-graph.

In this chapter we have extended the results of [125], including the twist-3 spectator scattering correction, and considering also decays into S -waves at leading order in v . Contrary to the previous numerical estimates of the branching fractions, that relied on ad hoc treatments of the infrared regulator dependence, in the present framework this is unnecessary, but an estimate of the colour-octet operator matrix elements is needed, which may even be the largest contribution to the decay amplitude. To this end we adopted a description of charmonium as a Coulomb bound state, which corresponds to the formal heavy quark limit. In practice, this limit is probably unreliable, and our results do indeed exhibit large theoretical uncertainties. Nevertheless, we find that for plausible theoretical inputs it is possible to reproduce qualitatively what we consider to be the

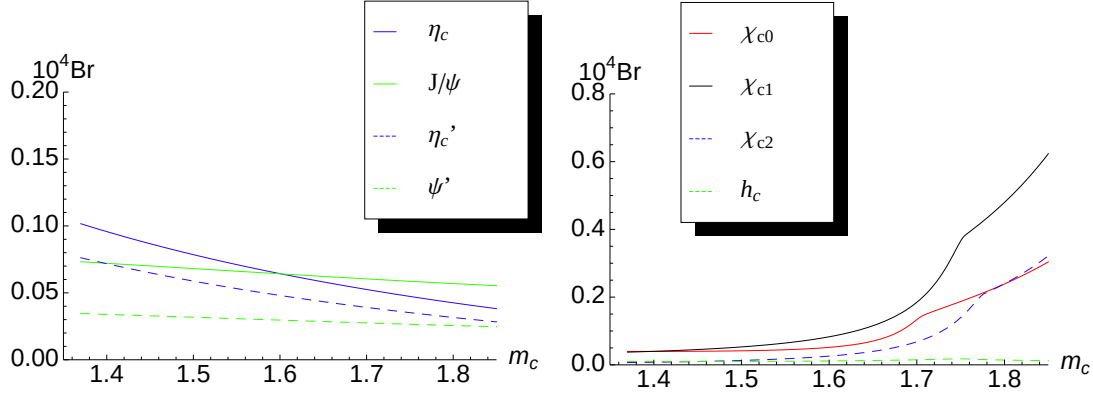


Figure 3.16: Charm quark mass dependence of the branching fractions in units of 10^{-4} including all contributions. Left, S-waves and right, P-waves.

most interesting features of current experimental data: suppression of the $\chi_{c2}K$ and h_cK final states and amplitudes that must be dominated by terms beyond naive factorization, though the suppression of $\chi_{c2}K$ is not as strong as observed. An interesting avenue to pursue in the future might be to consider the colour-octet matrix elements as unknown non-perturbative parameters, which is more realistic in view of $m_c v^2 \sim \Lambda$, and to exploit the constraints imposed by spin-symmetry on the leading contributions to these matrix elements.

m_c/GeV	$J/\psi \bar{K}^0$	$\psi' \bar{K}^0$	$\eta_c \bar{K}^0$	$\eta'_c \bar{K}^0$	$\chi_{c0} \bar{K}^0$	$\chi_{c1} \bar{K}^0$	$\chi_{c2} \bar{K}^0$	$h_c \bar{K}^0$
1.45	2.00	0.95	2.48	1.87	1.13	1.31	0.28	0.29
1.50	1.94	0.92	2.25	1.70	1.17	1.52	0.37	0.30
1.55	1.88	0.89	2.03	1.53	1.25	1.85	0.51	0.31
1.60	1.83	0.85	1.84	1.38	1.44	2.36	0.74	0.34
1.65	1.77	0.82	1.66	1.25	1.95	3.28	1.14	0.38
1.70	1.71	0.79	1.49	1.12	3.70	5.23	1.95	0.44
1.75	1.60	0.73	1.20	0.90	5.19	10.31	3.98	0.53

Table 3.5: $\bar{B}_d^0 \rightarrow H \bar{K}^0$ branching fractions in units of 10^{-4} for various charm quark mass inputs. Asymptotic distributions amplitudes, $\lambda_B = 200 \text{ MeV}$ and $m_b = 4.8 \text{ GeV}$ are fixed.

	J/ψ	ψ'	η_c	η'_c	χ_{c0}	χ_{c1}	χ_{c2}	h_c
Experiment, B^-	0.047	—	—	—	—	0.044	—	—
Experiment, $\bar{B}^0$	0.023	—	—	—	—	—	—	—
Theory, $\bar{B}^0$, LO	0.033	0.032	0.033	0.032	—	0.032	—	—
Theory, $\bar{B}^0$, NLO	0.035	0.035	0.035	0.035	0.036	0.035	0.035	0.034

Table 3.6: Ratio $(\bar{B} \rightarrow H\pi)/(\bar{B} \rightarrow H\bar{K})$ for $m_c = 1.65 \text{ GeV}$, $\lambda_B = 200 \text{ MeV}$. Asymptotic distribution amplitudes are used in the NLO results.

Chapter 4

Non-leptonic B decays in the MSSM with large $\tan \beta$

4.1 Introduction

If new particles exist at the TeV scale, then the striking absence of evidence so far for their virtual effects in B or K meson mixing and decay suggests that the pattern of flavour-changing interactions is governed by the Standard-Model Yukawa coupling matrices even at the TeV scale. The minimal supersymmetric Standard Model (MSSM) with large ratio $\tan \beta$ of the Higgs bosons vacuum expectation values and no new sources of flavour violation in the supersymmetry-breaking Lagrangian is an example of such a minimally flavour-violating (MFV) theory, which nevertheless may exhibit sizeable differences from the Standard Model due to Higgs bosons exchange [237–262]. The reason for this is that, at the one-loop level, a coupling of the down-type quarks with the “wrong” up-type Higgs boson is generated, that is forbidden by supersymmetry at tree level. However, due to the large ratio of the vacuum expectation values (VEVs), the down-type quark masses receive a large $\tan \beta$ enhanced correction to their tree-level value, and similarly happens for the $Hd_I d_I$ couplings, when the corresponding Yukawa couplings are eliminated in favour of the down-type masses. The loop effect is not large by itself, but it provides large corrections to couplings or observables, which are suppressed by the small down-type VEV, or which vanish at tree level. This includes in particular flavour-changing neutral-currents (FCNC), like $b \rightarrow s$ transitions, which can affect significantly processes like the $B_s \rightarrow \mu^+ \mu^-$ decay, or the meson mixing $\bar{B}_s - B_s$, whose supersymmetric contributions are enhanced respectively by a factor of $\tan^6 \beta$ and $\tan^4 \beta$, respectively. Large corrections arise also in case of charged currents, which affect *e.g.* the $B^+ \rightarrow \tau^+ \nu_\tau$ decay. These processes have been extensively studied in [238, 239, 241–245, 248, 250–252, 255, 260], and indeed large deviations from the Standard Model prediction were found, which can increase by orders of magnitudes the $B_s \rightarrow \mu^+ \mu^-$ decay, while a significant decrease of the ΔM_s mass difference and of the $B^+ \rightarrow \tau^+ \nu_\tau$ decay is expected. These correlations are typical of this model, and make it an easily distinguishable scenario.

Higgs exchange also generates scalar four-quark operators, which contribute to non-

leptonic B decays. These transitions may give interesting effects, too, because some of the $b \rightarrow sd_I d_I$ penguin-like operators are enhanced by $\tan^3\beta$; moreover, each transition is proportional to the mass of the $d_I d_I$ pair produced, and this provides a clear source of isospin violation. The effects of scalar operators on non-leptonic B decays have been studied in the MSSM (not necessarily minimally flavour-violating) and a general two Higgs doublet model in [266–274], mostly in connection with transverse polarization in B decays to two vector mesons (VV), and for specific decay modes. Some of these studies found large deviations from Standard Model expectations for non-leptonic decays.

In this chapter we describe the results obtained in our work [126], that was motivated by the question whether, given the present strong constraints from the leptonic decays, further insight on the MFV MSSM at large $\tan\beta$ can be derived from charmless non-leptonic B decays. To this end, extending previous analyses, we have related directly the Wilson coefficients of the leptonic to the relevant hadronic scalar operators, including charged Higgs exchange effects, and we have calculated the hadronic matrix elements in QCD factorization. We have also studied observables related to the helicity amplitudes of $B \rightarrow VV$, which exhibit an enhanced sensitivity to the Higgs-induced scalar operators.

Our results show that the present limit on the $B_s \rightarrow \mu^+ \mu^-$ branching fraction, and the observation of $B^+ \rightarrow \tau^+ \nu_\tau$ with a branching fraction close to the Standard Model expectation, exclude any visible effects in hadronic decays, but for an academic exception: the positive-helicity amplitude of $\bar{B} \rightarrow VV$ modes may receive order one modifications relative to the Standard Model. However, this amplitude is too small to be detected at present or planned B factories.

In the following we report in detail the results from [126], starting from the derivation of the effective four-quark operators, and the corresponding short-distance coefficients. Subsequently we show how these short-distance coefficients can be related to the leptonic decay observables; the evaluation of the matrix elements of the new operators in QCD factorization is described afterwards. A short discussion of the phenomenological consequences completes the chapter.

4.2 Scalar four-quark operators in the MSSM with large $\tan\beta$

The Standard Model effective Weak Hamiltonian for charmless B decays has been described in section 2.3.1, to which we refer for further details. Here we only note that all the four-quark “current-current” and “penguin” operators $Q_{1,2}^p, Q_{3-10}$ are all of the $(V - A) \times (V \mp A)$ form.

In the MSSM new four-quark operators are generated and the coefficients of the Standard Model operators are modified. We consider the large- $\tan\beta$ scenario in a set-up, where the superpartner particles are somewhat heavier than the electroweak gauge bosons and the Higgs bosons (the “decoupling limit”), such that the leading effect is due to Higgs exchange not only for the neutral but also for the charged current interactions, as shown in figure 4.1. Of particular interest are the flavour-changing neutral Higgs couplings to fermions, which originate from a loop-induced coupling of the “wrong” Higgs

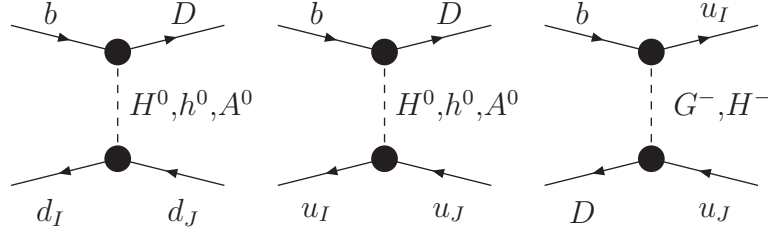


Figure 4.1: Four-quark interactions mediated by neutral and charged Higgs bosons.

field H_u to the down-type quarks, since these couplings are enhanced by several powers of $\tan \beta$ [239, 248, 251]. In appendix B we give explicit expressions for these couplings, showing how the short-distance coefficients of the scalar four-quark operators are derived. The coefficients are then evolved from the electroweak scale to the bottom mass scale m_b by the renormalization group equations. We focus here only on the relevant Higgs-induced terms in the effective Hamiltonian, that can be written as

$$\mathcal{H}_{\text{eff}}^{\text{Higgs}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \left(C_{11}^D Q_{11}^p + C_{12}^D Q_{12}^p + \sum_{i=13}^{14} \sum_{q=d,s,b} C_i^q Q_i^q \right) + \text{h.c.} \quad (4.1)$$

The “current-current” operators

$$Q_{11}^p = (\bar{p}_i b_i)_{S+P} (\bar{D}_j p_j)_{S-P}, \quad Q_{12}^p = (\bar{p}_i b_j)_{S+P} (\bar{D}_j p_i)_{S-P}, \quad (p = u, c) \quad (4.2)$$

originate from charged Higgs exchange; the “penguin” operators

$$Q_{13}^q = (\bar{D}_i b_i)_{S+P} (\bar{q}_j q_j)_{S-P}, \quad Q_{14}^q = (\bar{D}_i b_j)_{S+P} (\bar{q}_j q_i)_{S-P}, \quad (q = d, s, b) \quad (4.3)$$

from the loop-induced neutral Higgs-fermion vertices. Here i, j denote colour indices and $(\bar{q}q)_{S\pm P} = \bar{q}(1 \pm \gamma_5)q$. The CKM factors $\lambda_p^{(D)}$ in (2.1), (4.1) are now assumed to be composed of the effective CKM matrix elements V_{ij}^{eff} that correspond to the low-energy couplings.

Neutral Higgs exchange: $b \rightarrow D\bar{q}q$ transitions

It is straightforward to assemble the short-distance coefficients from tree-level Higgs exchange in terms of the effective neutral Higgs couplings given in appendix B. Combining a flavour-changing and a flavour-conserving coupling, we find in the large- $\tan \beta$ limit, where $\sin \beta \approx 1$, $1/\cos \beta \approx \tan \beta$:

$$C_{13}^{d_J}(\mu_H) = \frac{1}{2} \frac{\bar{m}_{d_J} \bar{m}_b \epsilon_Y y_t^2 \tan^3 \beta}{(1 + \tilde{\epsilon}_3 \tan \beta)(1 + \epsilon_0 \tan \beta)(1 + \tilde{\epsilon}_J \tan \beta)} \mathcal{F}_{2,J}^-, \quad C_{14}^{d_J}(\mu_H) = 0. \quad (4.4)$$

Here

$$\mathcal{F}_{2,J}^- = \frac{s_{\alpha-\beta}(c_\alpha + \tilde{\epsilon}_J s_\alpha)}{M_{H^0}^2} + \frac{c_{\alpha-\beta}(-s_\alpha + \tilde{\epsilon}_J c_\alpha)}{M_{h^0}^2} - \frac{1}{M_{A^0}^2} \approx -\frac{2}{M_{A^0}^2}, \quad (4.5)$$

with $c_\alpha \equiv \cos\alpha, \dots$. The ϵ -coefficients appearing in (4.4) are defined in [251] and denote the loop-induced Higgs-fermion couplings. In the large- $\tan\beta$ MSSM products $\epsilon \times \tan\beta$ can be of order one. Just as in the $b \rightarrow D\ell_J\ell_J$ transitions, the coefficients of the hadronic Higgs penguin operators are strongly enhanced by the factor $\tan^3\beta$. The quark masses $\bar{m}_q$ are the $\overline{\text{MS}}$ masses in the low-energy effective theory at the matching scale μ_H .

We briefly mention here that Higgs exchange generates $(\bar{D}b)_{S-P}(\bar{q}q)_{S+P}$ operators as well, but in this case the factor $\bar{m}_b$ in (4.4) is replaced by $\bar{m}_D$, which is at most $\bar{m}_s$. Moreover, there are other four operators, arising from the remaining two helicity combinations, that have short-distance coefficients multiplied by a function $\mathcal{F}_{2,J}^+$, which vanishes in the large- $\tan\beta$ limit. Thus, it is sufficient to consider the operators $Q_{13,14}^q$. The neutral Higgs coupling to up-type quarks (second diagram in figure 4.1) is suppressed at large $\tan\beta$ relative to the down-type quarks, thus $q = d_J = d, s, b$. In fact, the operators $Q_{13,14}^d$ might also be dropped due to the small down-quark mass. The operator $Q_{13,14}^b$ has the largest coefficient, but it contributes to non-leptonic decays only through loops. Finally, we note that the double Higgs penguin diagrams (first diagram in figure 4.1 with flavour change at both vertices) are irrelevant to non-leptonic decays due to their extra CKM suppression. We therefore conclude that in the MFV MSSM with large $\tan\beta$, only a small set of scalar penguin operators $Q_{13,14}^{s,b}$ is relevant. Of these $Q_{14}^{s,b}$ is absent at tree-level, but it is kept for the moment, since it may be generated by renormalization group evolution, as we are going to discuss below.

Charged Higgs exchange

The operators $Q_{11,12}^p$ arise from the third diagram in figure 4.1. Once again only the $(S+P) \times (S-P)$ Dirac structure is dominant at large $\tan\beta$. For charmless decays we need only the cases $u_I = u_J = p = u, c$, and obtain

$$C_{11}^D(\mu_H) = -\frac{\bar{m}_b\bar{m}_D}{M_{H^+}^2} \frac{\tan^2\beta}{(1 + \epsilon_0 \tan\beta)^2}, \quad C_{12}^D(\mu_H) = 0. \quad (4.6)$$

Although C_{11}^D is enhanced only by $\tan^2\beta$, there is no loop suppression factor ϵ_Y . Due to the factor $\bar{m}_D$, charged Higgs exchange is relevant in practice only for $b \rightarrow s$ transitions.

Renormalization group evolution

We first discuss the evolution of the short-distance coefficients from a typical Higgs mass scale, which we assume to be $\mu_H = 200 \text{ GeV}$, to the bottom mass scale $m_b = 4.2 \text{ GeV}$, when penguin diagrams are neglected. Then each pair of operators (Q_{11}^p, Q_{12}^p) , (Q_{13}^q, Q_{14}^q) evolves independently in leading logarithmic (LL) accuracy with anomalous dimension matrix (in units of $\alpha_s/(4\pi)$)

$$\gamma_{2 \times 2} = \begin{pmatrix} -6N_c + \frac{6}{N_c} & 0 \\ -6 & \frac{6}{N_c} \end{pmatrix}, \quad (4.7)$$

where $N_c = 3$ is the number of colours. With $\alpha_s(m_b)/\alpha_s(\mu_H) \approx 2.13$, this results in

$$C_{11}^D(m_b)/C_{11}^D(\mu_H) = C_{13}^q(m_b)/C_{13}^q(\mu_H) \approx 2.20, \quad (4.8)$$

while $C_{12}^D(\mu)$, $C_{14}^q(\mu)$ remain zero. Using the 2-loop NDR scheme anomalous dimension matrix (ADM) [263]

$$\gamma_{2 \times 2}^{2\text{-loops}} = \begin{pmatrix} -\frac{203}{6}N_c^2 + \frac{479}{6} + \frac{15}{2N_c^2} + \frac{10}{3}N_cn_f - \frac{22}{3N_c}n_f & -\frac{71}{2}N_c - \frac{18}{N_c} + 4n_f \\ -\frac{100}{3}N_c + \frac{3}{N_c} + \frac{22}{3}n_f & \frac{137}{6} + \frac{15}{2N_c^2} - \frac{22}{3N_c}n_f \end{pmatrix}, \quad (4.9)$$

where $n_f = 5$ is the number of active flavour¹, we obtain 2.35 instead and $C_{12}^D(m_b)/C_{11}^D(\mu_H) = C_{14}^q(m_b)/C_{13}^q(\mu_H) \approx 0.088$, but since we do not have the 1-loop correction to the initial condition of the scalar operators at μ_H , the next-to-leading logarithmic (NLL) evolution is not fully consistent. In any case, we conclude that the operators Q_{12}^p, Q_{14}^q can be neglected to first approximation, since their coefficient functions are suppressed by a factor 25.

Including penguin diagrams requires to enlarge the operator basis, since the scalar operators mix at the LL level into the Standard Model penguin operators as well as their “mirror” copies, defined by a global exchange of left- and right chiralities of the quark fields. For the following discussion we neglect the electroweak penguin operators, so we deal with the six Standard Model operators $Q_{1,2}^p$, Q_{3-6} , their mirror copies $\tilde{Q}_{1,2}^p$, $\tilde{Q}_{3-6}$, and the six scalar operators $Q_{11,12}^p$, $Q_{13,14}^{D,b}$. The structure of the ADM reads

$$\gamma = \begin{pmatrix} \gamma_{6 \times 6} & 0_{6 \times 6} & 0_{6 \times 6} \\ 0_{6 \times 6} & \tilde{\gamma}_{6 \times 6} & 0_{6 \times 6} \\ \gamma_{6 \times 6}^{\text{sc-p}} & \tilde{\gamma}_{6 \times 6}^{\text{sc-p}} & \gamma_{6 \times 6}^{\text{sc}} \end{pmatrix}, \quad (4.10)$$

where $\gamma_{6 \times 6} = \tilde{\gamma}_{6 \times 6}$ is the ADM for the Standard Model current-current and QCD penguin operators (equal for the mirror operators) and $\gamma_{6 \times 6}^{\text{sc}}$ is a block-diagonal matrix with three identical 2×2 blocks given by $\gamma_{2 \times 2}$ in (4.7): one for $Q_{11,12}^p$, one for $Q_{13,14}^D$, depending on the transition, and one for $Q_{13,14}^b$. The matrices $\gamma_{6 \times 6}^{\text{sc-p}}$, $\tilde{\gamma}_{6 \times 6}^{\text{sc-p}}$ describe the mixing of the scalar operators into the penguin operators. We find that $Q_{11,12}^p$ and $Q_{13,14}^D$ mix into the mirror penguin operators, while only $Q_{13,14}^b$ mixes into the Standard Model penguins. Thus $[\gamma_{6 \times 6}^{\text{sc-p}}]^T = (0|0|\Gamma^T)$ and $[\tilde{\gamma}_{6 \times 6}^{\text{sc-p}}]^T = (\Gamma^T|\Gamma^T|0)$, where

$$\Gamma = \begin{pmatrix} 0 & 0 & \frac{1}{3N_c} & -\frac{1}{3} & \frac{1}{3N_c} & -\frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (4.11)$$

Solving the RGE equations leaves (4.8) unchanged, generates the mirror QCD penguin operators with coefficient functions

$$\tilde{C}_i^D(m_b) \approx -0.71 C_i^{\text{SM}}(m_b) \times [C_{11}^D(\mu_H) + C_{13}^D(\mu_H)], \quad i = 3 \dots 6, \quad (4.12)$$

and modifies the Standard Model penguin coefficients according to $C_i = C_i^{\text{SM}} + \delta C_i$ with

$$\delta C_i(m_b) \approx -0.71 C_i^{\text{SM}}(m_b) \times C_{13}^b(\mu_H), \quad i = 3 \dots 6. \quad (4.13)$$

Since the Standard Model penguin coefficients $C_i^{\text{SM}}(m_b)$ are small numbers, the penguin-mixing effects are small, unless the coefficient functions of the scalar operators are of

¹we consider $\mu_t \simeq m_t \simeq 200 \text{ GeV}$, so that the top quark is integrated out and the number of active flavour below 200 GeV is $n_f = 5$.

order one. However, due to their different chiral structure, the mirror penguin operators contribute differently from the standard ones to the transverse helicity amplitudes in $B \rightarrow VV$ decays as discussed below.

4.3 Constraints from $B_s \rightarrow \mu^+ \mu^-$ and $B^+ \rightarrow \tau^+ \nu_\tau$

The natural size of the loop-induced neutral Higgs couplings $\epsilon_0, \epsilon_Y, \tilde{\epsilon}_J$ is of order 0.01, the precise values depending on MSSM parameters. Assuming $M_{A^0} = 200 \text{ GeV}$ and $\tan \beta = 50$, this allows the scalar penguin operators to have coefficients of order $C_{13}^s \simeq 0.01$, $C_{13}^b \simeq 0.5$, which are comparable to Standard Model penguin coefficients. However, the non-observation of $B_s \rightarrow \mu^+ \mu^-$ implies much stronger limits on the size of the scalar four-quark operator coefficient functions.

The decay $B_s \rightarrow \mu^+ \mu^-$ proceeds via an interaction similar to the first diagram of figure 4.1, except that the lower legs are replaced by a muon pair. Since the lower vertex is a tree-level neutral Higgs coupling, the leptonic and hadronic decay are closely related. For large $\tan \beta$, a single scalar operator $(\bar{D}b)_{S+P}(\bar{\mu}\mu)_{S-P}$, similar in structure to Q_{13}^q , dominates the $B_s \rightarrow \mu^+ \mu^-$ decay amplitude. Its coefficient function is given by

$$C_{\mu\mu}(\mu_H) = -\frac{1}{2} \frac{\bar{m}_b m_\mu \epsilon_Y y_t^2 \tan^3 \beta}{(1 + \tilde{\epsilon}_3 \tan \beta)(1 + \epsilon_0 \tan \beta)} \mathcal{F}_{2l}^-, \quad (4.14)$$

with

$$\mathcal{F}_{2l}^- = \frac{s_{\alpha-\beta}(c_\alpha)}{M_{H^0}^2} + \frac{c_{\alpha-\beta}(-s_\alpha)}{M_{h^0}^2} - \frac{1}{M_{A^0}^2} \approx \mathcal{F}_{2,J}^-. \quad (4.15)$$

For large $\tan \beta$, and at the level of the present experimental limit, the Standard Model contribution to the decay amplitude is negligible, and the branching ratio is given by

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) = \frac{G_F^2 f_{B_s}^2 m_{B_s}^5 \tau_{B_s}}{8\pi(\bar{m}_b + \bar{m}_s)^2} \left| \lambda_t^{(s)} \right|^2 |C_{\mu\mu}|^2. \quad (4.16)$$

Comparing (4.4) to (4.14), we see that we can eliminate $C_{\mu\mu}$ in favour of C_{13}^q in the previous equation and turn it into

$$(1 + \tilde{\epsilon}_J \tan \beta) |C_{13}^{dJ}(\mu_H)| = \frac{2\sqrt{2}\pi(\bar{m}_b + \bar{m}_s)(\mu_H)}{G_F f_{B_s} m_{B_s}^{5/2} \tau_{B_s}^{1/2} |\lambda_t^{(s)}|} \frac{\bar{m}_{dJ}(\mu_H)}{m_\mu} \left[\text{Br}(B_s \rightarrow \mu^+ \mu^-) \right]^{1/2}. \quad (4.17)$$

The present experimental upper limit on the $B_s \rightarrow \mu^+ \mu^-$ branching fraction is $\text{Br}(B_s \rightarrow \mu^+ \mu^-) \leq 5.8 \cdot 10^{-8}$ at 95% C.L. [275]. Using for f_{B_s} , $\bar{m}_s(2 \text{ GeV})$ and $\bar{m}_b(\bar{m}_b)$ the values given in table 2.1, and evolving both quark masses to the common scale $\mu_H = 200 \text{ GeV}$, we obtain

$$(1 + \epsilon_0 \tan \beta) |C_{13}^s(\mu_H)| \leq 1.4 \cdot 10^{-4}, \quad (1 + \tilde{\epsilon}_3 \tan \beta) |C_{13}^b(\mu_H)| \leq 7.9 \cdot 10^{-3}. \quad (4.18)$$

When ϵ_0 and/or $\tilde{\epsilon}_3$ are negative, the coefficient functions can be larger than the values on the right-hand side. However, the brackets multiplying the coefficient functions enter the relation between the quark masses and the down-type Yukawa couplings, and hence

$(1 + \tilde{\epsilon}_3 \tan \beta)$ cannot become very small, if the bottom Yukawa coupling is to remain perturbative. We allow a factor of three enhancement of the coefficient functions to be conservative (that is, the brackets are required to be larger than $1/3$). Including the factor (4.8) from evolution to the scale m_b leads to

$$|C_{13}^s(m_b)| \leq 0.001, \quad |C_{13}^b(m_b)| \leq 0.05, \quad (4.19)$$

while $C_{13}^d(m_b)$ is a factor $\bar{m}_d/\bar{m}_s$ smaller than $C_{13}^s(m_b)$ and therefore negligible. Thus, the coefficient functions of the hadronic flavour-changing neutral Higgs penguin operators are constrained to be a factor of 10 smaller than the above estimates derived from $M_{A^0} = 200 \text{ GeV}$ and $\tan \beta = 50$.

The short-distance coefficient $C_{11}^D(\mu_H)$, arising from charged Higgs exchange, can be related to $B^+ \rightarrow \tau^+ \nu_\tau$ in a similar way. Using (4.6) the ratio [237, 252]

$$R_{\tau\nu_\tau} \equiv \frac{\text{Br}(B^+ \rightarrow \tau^+ \nu_\tau)_{\text{MSSM}}}{\text{Br}(B^+ \rightarrow \tau^+ \nu_\tau)_{\text{SM}}} = \left(1 - \frac{m_B^2}{m_{H^+}^2} \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta} \right)^2, \quad (4.20)$$

is expressed in terms of $C_{11}^D(\mu_H)$ as

$$R_{\tau\nu_\tau} = \left(1 + C_{11}^D(\mu_H) \frac{m_B^2(1 + \epsilon_0 \tan \beta)}{\bar{m}_D(\mu_H)\bar{m}_b(\mu_H)} \right)^2. \quad (4.21)$$

The present average of the Babar and Belle measurements of the branching fraction is $\text{Br}(B^+ \rightarrow \tau^+ \nu_\tau) = (1.51 \pm 0.33) \cdot 10^{-4}$ [44, 276, 277]. Employing the central value $|V_{ub}| f_{B_d} = 7.4 \cdot 10^{-4} \text{ GeV}$ and assigning a conservative 50% uncertainty to the Standard Model prediction of the branching fraction, the measurement constrains $R_{\tau\nu_\tau}$ to lie in the range

$$0.72 < R_{\tau\nu_\tau} < 2.40. \quad (4.22)$$

Concentrating on the case $D = s$ this implies the allowed ranges

$$\begin{aligned} -0.012 &< (1 + \epsilon_0 \tan \beta) C_{11}^s(\mu_H) < -0.009, \\ -0.001 &< (1 + \epsilon_0 \tan \beta) C_{11}^s(\mu_H) < 0.003. \end{aligned} \quad (4.23)$$

The first range corresponds to the situation, where the charged Higgs contribution is about twice as large as the Standard Model one, and opposite in sign. Requiring $1 + \epsilon_0 \tan \beta > 1/3$ and including the RG evolution (4.8) results in

$$-0.08 < C_{11}^s(m_b) < -0.06, \quad \text{or} \quad -0.005 < C_{11}^s(m_b) < 0.018. \quad (4.24)$$

The constraint from $B^+ \rightarrow \tau^+ \nu_\tau$ on C_{11}^s is not as stringent as the one from $B_s \rightarrow \mu^+ \mu^-$ on C_{13}^s , but one must remember that the charged Higgs contribution to hadronic charmless decays must compete with the Standard Model tree operators rather than the penguin operators. In addition, since $|C_{11}^s| \ll 1$, the contribution (4.12) to the mirror penguin coefficients remains small. These conclusions hold *a fortiori* for C_{11}^d , which is a factor of $\bar{m}_d/\bar{m}_s$ smaller than C_{11}^s .

4.4 Parameter space analysis

Besides the constraints on C_{11}^s and C_{13}^s derived from the correlation with the $B^+ \rightarrow \tau^+ \nu_\tau$ and $B_s \rightarrow \mu^+ \mu^-$ decays, we performed also a MSSM parameter space scan, whose result agrees with the limits derived in 4.3, and therefore was not discussed in details in [126]. We provide here some more information, because while no more stringent constraints can be derived, a parameter space scan can be useful to obtain a more clear picture of the allowed range for the fundamental parameters of the model, in particular of $\tan\beta$ versus the mass of the neutral CP-odd Higgs boson A^0 . In this framework the loop-induced ϵ parameters can be calculated explicitly, and this allows to verify also some of the hypothesis at the basis of the analysis performed in the previous paragraph, *e.g.* that $|\epsilon_i| \simeq 0.01$ for large $\tan\beta$, in the allowed region of the parameter space.

We consider a MFV scenario with 8 fundamental parameters: the squark mass matrices appearing in the soft Lagrangian are assumed to be proportional to the unit matrix, with the three parameters m_Q , m_U , m_D being the proportionality constants; similarly, the trilinear soft terms are assumed to be proportional to the corresponding Yukawa couplings, with proportionality constant A , so that $\mathbf{A}_u = A \mathbf{y}_u$, $\mathbf{A}_d = A \mathbf{y}_d$, $\mathbf{A}_l = A \mathbf{y}_l$; given a common gaugino mass m_{12} , we assume the relations $M_1 = 1/2 m_{12}$, $M_2 = m_{12}$, $M_3 = m_{\tilde{g}} = 3 m_{12}$. The parameters are extracted at the scale $\mu_H = 2M_W$, *i.e.*, no running from the grand unified scale to the electroweak scale is considered. In practice, the scenario is minimal supergravity inspired, and we assume that the usual renormalization group evolution from the high scale to the electroweak scale does not introduce relevant sources of flavour violation. The minimal supergravity scenario depends only on 5 parameters, *i.e.* a common mass m_0 for squark, slepton, and Higgs, a common gaugino mass m_{12} , the soft trilinear parameter A , $\tan\beta$, and $\text{sign}(\mu)$. We parameterize our ignorance about the renormalization group evolution by splitting the common mass m_0 into four different parameters m_Q , m_U , m_D , and M_{A^0} , while keeping a common soft trilinear coupling A and a common gaugino mass m_{12} , since the relations for these parameters given above are, at first approximation, renormalization group invariant.

Since we are interested in those regions of the parameter space where the short-distance coefficients considered above are large, we generate the parameters randomly with a probability distribution proportional to C_{13}^s (this is a simplified approach with respect to [264, 265]); we consider then the processes $B_s \rightarrow \mu^+ \mu^-$, $B^+ \rightarrow \tau^+ \nu_\tau$, and in addition $B \rightarrow X_s \gamma$ and $\Delta M_{d,s}$: we calculate the corresponding observables at each point of the parameters space extracted, and reject those that are already ruled out by experimental constraints on these processes. Beside the limits for $B^+ \rightarrow \tau^+ \nu_\tau$ and $B_s \rightarrow \mu^+ \mu^-$, we assume

$$2.15 \times 10^{-4} < \text{Br}(\bar{B} \rightarrow X_s \gamma) < 4.89 \times 10^{-4},$$

$$-1 < \frac{\Delta M_{d,s}^{\text{MSSM}}}{\Delta M_{d,s}^{\text{SM}}} < 1. \quad (4.25)$$

We consider also constraints from the sparticle spectrum, requiring

$$50 < m_{\tilde{\chi}^0} < 5000 \text{ GeV}, \quad 90 < m_{\tilde{\chi}^\pm} < 5000 \text{ GeV},$$

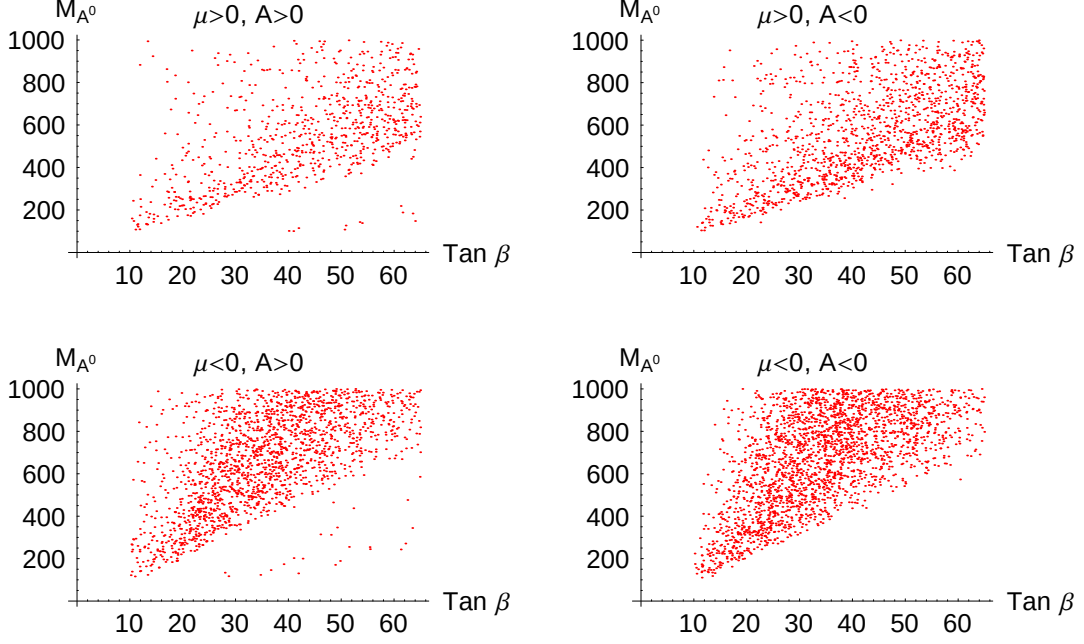


Figure 4.2: Parameter space in the plane $\tan \beta$, M_{A^0} .

$$100 < m_{\tilde{t}, \tilde{b}} < 10^4 \text{ GeV}, \quad 115 < m_{h^0} < 500 \text{ GeV}; \quad (4.26)$$

for the calculation of M_{h^0} we employ the one loop definition. We extract the free parameters in the following range:

$$\begin{aligned} 200 < m_{Q,U,D} < 2000 \text{ GeV}, \quad 200 < m_{12} < 2000 \text{ GeV}, \quad 200 < |A| < 2000 \text{ GeV}, \\ 100 < M_{A^0} < 1000 \text{ GeV}, \quad 10 < \tan \beta < 65, \quad 200 < |\mu| < 2000 \text{ GeV}. \end{aligned} \quad (4.27)$$

In figure 4.2 we show the selected parameter space. The graphs show explicitly which regions of the parameter space are ruled out by leptonic and radiative B decays. $B_s \rightarrow \mu^+ \mu^-$ removes most of the points at low M_{A^0} with large $\tan \beta$, while $B^+ \rightarrow \tau^+ \nu_\tau$ is responsible for removing a stripe of points for low-moderate values of M_{A^0} , through the whole $\tan \beta$ region. Below this stripe, the $\tan \beta$ enhanced MSSM contribution to $B^+ \rightarrow \tau^+ \nu_\tau$ is larger than the Standard Model one, *i.e.*, it corresponds to the first range individuated in (4.23). The combined result of these two constraints rules out most of the point in the triangular region with low-moderate M_{A^0} , through the whole $\tan \beta$ region. $\bar{B} \rightarrow X_s \gamma$ removes points at low $\tan \beta$ and m_{A^0} ; the other observables considered here give weaker constraints.

In figure 4.3 we show the values assumed by $C_{13}^s(m_b)$ in the selected parameter space, *i.e.*, the result shown takes into account the renormalization group factor (4.8). The parameter space analysis confirms the constraints derived in 4.3. A qualitative difference, however, is that the largest values for $C_{13}^s(m_b)$ are reached already for $\tan \beta \sim 30 - 40$, because larger values of $C_{13}^s(m_b)$ for larger value of $\tan \beta$ are ruled out by the other

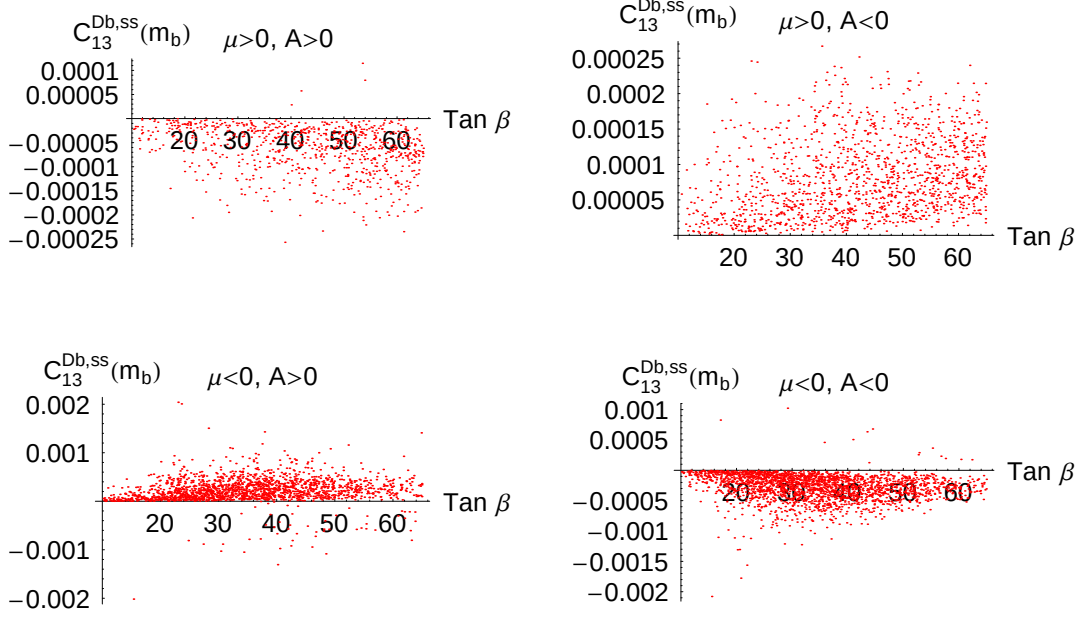


Figure 4.3: The short-distance coefficient C_{13}^s ($= C_{13}^{Db,ss}$ in the picture) as a function of $\tan \beta$.

constraints (a similar outcome is found for $C_{11}^s(m_b)$, not shown.). The main result of the parameter space analysis is therefore that it is not necessary to hypothesize extremely large values of $\tan \beta$, (i.e $\tan \beta \geq 50$), to fill the limits (4.19) and (4.24).

4.5 Hadronic matrix elements for $B \rightarrow M_1 M_2$

To calculate the decay amplitudes of non-leptonic, charmless B decays, we employ the QCD factorization (QCDF) framework described in chapter 2. Given the constraints on the coefficient functions, it is important to emphasize that a leading-order treatment, where QCDF is equivalent to naive factorization, would suffice. The full NLO analysis performed in [126] was developed with the aim of giving explicit expression for the first-order radiative corrections of the mirror and scalar operators, that can have a more general application besides the specific model of new physics considered in this chapter.

The new mirror and scalar operators can be taken into account in QCD factorization generalizing the transition operators in (2.31). The expression for the operator $\mathcal{T}_A$ in 2.37 becomes

$$\begin{aligned}
 \mathcal{T}_A^p &= \delta_{pu} [\alpha_1(M_1 M_2) + \alpha_{11}^D(M_1 M_2)] A([\bar{q}_s u][\bar{u} D]) \\
 &+ \delta_{pu} [\alpha_2(M_1 M_2) + \alpha_{12}^D(M_1 M_2)] A([\bar{q}_s D][\bar{u} u]) \\
 &+ [\alpha_3^p(M_1 M_2) + \tilde{\alpha}_3^{pD}(M_1 M_2)] \sum_{q=u,d,s} A([\bar{q}_s D][\bar{q} q])
 \end{aligned}$$

$$\begin{aligned}
& + [\alpha_4^p(M_1 M_2) + \tilde{\alpha}_4^{pD}(M_1 M_2)] \sum_{q=u,d,s} A([\bar{q}_s q][\bar{q} D]) \\
& + \alpha_{3,\text{EW}}^p(M_1 M_2) \sum_{q=u,d,s} \frac{3}{2} e_q A([\bar{q}_s D][\bar{q} q]) \\
& + \alpha_{4,\text{EW}}^p(M_1 M_2) \sum_{q=u,d,s} \frac{3}{2} e_q A([\bar{q}_s q][\bar{q} D]) \\
& + \sum_{q=d,s} \alpha_{3q}^p(M_1 M_2) A([\bar{q}_s D][\bar{q} q]) \\
& + \sum_{q=d,s} \alpha_{4q}^p(M_1 M_2) A([\bar{q}_s q][\bar{q} D]). \tag{4.28}
\end{aligned}$$

The new contributions are encoded in $\alpha_{11,12}^D$ (charged Higgs effects), $\tilde{\alpha}_{3,4}^{pD}$ (mirror QCD penguins) and $\alpha_{3q,4q}^p$ (neutral Higgs effects), as well as modifications of the standard QCD penguin amplitudes $\alpha_{3,4}^p$. A similar generalization of (2.51) applies to the annihilation amplitudes, namely

$$\begin{aligned}
\mathcal{T}_B^p = & \delta_{pu} [b_1(M_1 M_2) + b_{11}^D(M_1 M_2)] \sum_{q'} B([\bar{u} q'][\bar{q}' u][\bar{D} b]) \\
& + \delta_{pu} [b_2(M_1 M_2) + b_{12}^D(M_1 M_2)] \sum_{q'} B([\bar{u} q'][\bar{q}' D][\bar{u} b]) \\
& + [b_3^p(M_1 M_2) + \tilde{b}_3^{pD}(M_1 M_2)] \sum_{q,q'} B([\bar{q} q'][\bar{q}' D][\bar{q} b]) \\
& + [b_4^p(M_1 M_2) + \tilde{b}_4^{pD}(M_1 M_2)] \sum_{q,q'} B([\bar{q} q'][\bar{q}' q][\bar{D} b]) \\
& + b_{3,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q} q'][\bar{q}' D][\bar{q} b]) \\
& + b_{4,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q} q'][\bar{q}' q][\bar{D} b]) \\
& + \delta_{pu} [b_{S1}(M_1 M_2) + b_{S11}^D(M_1 M_2)] \sum_{q'} B([\bar{u} u][\bar{q}' q'][\bar{D} b]) \\
& + \delta_{pu} [b_{S2}(M_1 M_2) + b_{S11}^D(M_1 M_2)] \sum_{q'} B([\bar{u} D][\bar{q}' q'][\bar{u} b]) \\
& + [b_{S3}^p(M_1 M_2) + \tilde{b}_{S3}^{pD}(M_1 M_2)] \sum_{q,q'} B([\bar{q} D][\bar{q}' q'][\bar{q} b]) \\
& + [b_{S4}^p(M_1 M_2) + \tilde{b}_{S4}^{pD}(M_1 M_2)] \sum_{q,q'} B([\bar{q} q][\bar{q}' q'][\bar{D} b]) \\
& + b_{S3,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q} D][\bar{q}' q'][\bar{q} b])
\end{aligned}$$

$$\begin{aligned}
& + b_{S4,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q][\bar{q}'q'][\bar{D}b]) \\
& + b_{3q}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'D][\bar{q}b]) + b_{4q}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{D}b]) \\
& + b_{S3q}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'D][\bar{q}b]) + b_{S4q}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{D}b]), \quad (4.29)
\end{aligned}$$

where the sum over q' is over u, d, s always, while the sum over q is over u, d, s except for $b_{(S)3,4q}^p(M_1 M_2)$ where it runs over d, s . Our aim is to compare the new coefficients to those present in the Standard Model for PP, PV, VP, VV (P pseudoscalar, V vector meson) final states. Note that for VV, (2.37), (4.28) and (4.29) apply separately to each of the three independent helicity amplitudes $h = 0, -, +$, but the helicity label is suppressed in our notation.

Since $V \pm A$ and $S \pm P$ operators contribute differently to pseudoscalar and vector final states, the relation between the $\tilde{a}_i$ and the $\tilde{\alpha}_i$ coefficients, as well as the one between $a_{13q,14q}$ and $\alpha_{3q,4q}^p$ is different with respect to the relation in (2.44). Here we obtain²

$$\begin{aligned}
\tilde{\alpha}_3^p(M_1 M_2) &= \begin{cases} -\tilde{a}_3^p(M_1 M_2) + \tilde{a}_5^p(M_1 M_2), & \text{if } M_1 M_2 = PP, \\ \tilde{a}_3^p(M_1 M_2) + \tilde{a}_5^p(M_1 M_2), & \text{if } M_1 M_2 = PV, \\ \tilde{a}_3^p(M_1 M_2) - \tilde{a}_5^p(M_1 M_2), & \text{if } M_1 M_2 = VP, \\ -\tilde{a}_3^p(M_1 M_2) - \tilde{a}_5^p(M_1 M_2), & \text{if } M_1 M_2 = V^0 V^0, \\ -f_{\pm}^{M_1} (\tilde{a}_3^p(M_1 M_2) + \tilde{a}_5^p(M_1 M_2)), & \text{if } M_1 M_2 = V^{\pm} V^{\pm}, \end{cases} \\
\tilde{\alpha}_4^p(M_1 M_2) &= \begin{cases} -\tilde{a}_4^p(M_1 M_2) - r_{\chi}^{M_2} \tilde{a}_6^p(M_1 M_2), & \text{if } M_1 M_2 = PP, \\ \tilde{a}_4^p(M_1 M_2) + r_{\chi}^{M_2} \tilde{a}_6^p(M_1 M_2), & \text{if } M_1 M_2 = PV, \\ \tilde{a}_4^p(M_1 M_2) - r_{\chi}^{M_2} \tilde{a}_6^p(M_1 M_2), & \text{if } M_1 M_2 = VP, \\ -\tilde{a}_4^p(M_1 M_2) + r_{\chi}^{M_2} \tilde{a}_6^p(M_1 M_2), & \text{if } M_1 M_2 = V^0 V^0, \\ f_{\pm}^{M_1} (-\tilde{a}_4^p(M_1 M_2) + r_{\chi}^{M_2} \tilde{a}_6^p(M_1 M_2)), & \text{if } M_1 M_2 = V^{\pm} V^{\pm}, \end{cases} \\
\alpha_{3q}^p(M_1 M_2) &= \frac{r_{\chi}^{M_2}}{2} \begin{cases} a_{13q}^p(M_1 M_2), & \text{if } M_1 M_2 = PP, VP, \\ -a_{13q}^p(M_1 M_2), & \text{if } M_1 M_2 = PV, V^0 V^0, \\ -f_{\pm}^{M_1} a_{13q}^p(M_1 M_2), & \text{if } M_1 M_2 = V^{\pm} V^{\pm}, \end{cases} \\
\alpha_{4q}^p(M_1 M_2) &= \frac{1}{2} \begin{cases} -a_{14q}^p(M_1 M_2), & \text{if } M_1 M_2 = PP, PV, \\ a_{14q}^p(M_1 M_2), & \text{if } M_1 M_2 = VP, V^0 V^0, \\ f_{\pm}^{M_1} a_{14q}^p(M_1 M_2), & \text{if } M_1 M_2 = V^{\pm} V^{\pm}; \end{cases} \quad (4.30)
\end{aligned}$$

the same relations as the last two hold between α_{11}^D and a_{11D} , and between α_{12}^D and a_{12D} , respectively. We denote by $f_{\pm}^{M_1} = F_{\mp}^{B \rightarrow M_1}(0)/F_{\pm}^{B \rightarrow M_1}(0)$ a ratio of form factors, such

²In the following we drop the superscript “D” (referring to $b \rightarrow D$ transitions) on the amplitude parameters and Wilson coefficients of the mirror penguin contributions.

that $f_+^{M_1} \sim m_B/\Lambda_{\text{QCD}}$ and $f_-^{M_1} \sim \Lambda_{\text{QCD}}/m_B$ in the heavy-quark limit. It follows that for the transverse helicity amplitudes of $\bar{B} \rightarrow VV$ decay modes the contributions from the new operators obey a different hierarchy in the heavy-quark limit. While in the Standard Model

$$\bar{\mathcal{A}}_0 : \bar{\mathcal{A}}_- : \bar{\mathcal{A}}_+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \frac{\Lambda_{\text{QCD}}^2}{m_b^2} \quad (4.31)$$

(up to certain electromagnetic effects [178]), the Higgs contributions to the amplitude satisfy

$$\bar{\mathcal{A}}_0 : \bar{\mathcal{A}}_- : \bar{\mathcal{A}}_+ = 1 : \frac{\Lambda_{\text{QCD}}^2}{m_b^2} : \frac{\Lambda_{\text{QCD}}}{m_b}. \quad (4.32)$$

This effect, noted first in [270], is interesting, since it increases the sensitivity of certain polarization observables to the new short-distance coefficients by a factor $f_+^{M_1} \approx 10$. On the other hand, the absence of tensor operators implies that the formal dominance of the longitudinal amplitude is preserved by the Higgs contributions.

In QCDF the $\tilde{a}_i^p$ coefficients introduced in (4.30) can be written at next-to-leading order (NLO) in the form

$$\begin{aligned} \tilde{a}_i^p(M_1 M_2) &= \left(\tilde{C}_i + \frac{\tilde{C}_{i\pm 1}}{N_c} \right) \tilde{N}_i(M_2) \\ &+ \frac{\tilde{C}_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \left[\tilde{V}_i(M_2) + \frac{4\pi^2}{N_c} \tilde{H}_i(M_1 M_2) \right] + \tilde{P}_i^p(M_2), \end{aligned} \quad (4.33)$$

where the upper (lower) signs apply when i is odd (even). The quantities $\tilde{N}_i(M_2)$, $\tilde{V}_i(M_2)$, $\tilde{H}_i(M_1 M_2)$, $\tilde{P}_i^p(M_2)$ stand, respectively, for the tree-level result (“naive factorization”), the 1-loop vertex correction, spectator scattering, and the penguin diagrams. In case of a_{iq}^p , setting C_{12}^D and C_{14}^q to zero (see section 4.2), and using the Fierz symmetry of the NDR renormalization scheme for the scalar operators [263], we obtain

$$\begin{aligned} a_{11D}(M_1 M_2) &= C_{11}^D N_{11}, \\ a_{12D}(M_1 M_2) &= \frac{C_{11}^D}{N_c} N_{12} + \frac{C_{11}^D}{N_c} \frac{C_F \alpha_s}{4\pi} \left[V_{12}(M_2) + \frac{4\pi^2}{N_c} H_{12}(M_1 M_2) \right], \end{aligned} \quad (4.34)$$

$$\begin{aligned} a_{13q}^p(M_1 M_2) &= C_{13}^q N_{13q}, \\ a_{14q}^p(M_1 M_2) &= \frac{C_{13}^q}{N_c} N_{14q} + \frac{C_{13}^q}{N_c} \frac{C_F \alpha_s}{4\pi} \left[V_{14}(M_2) + \frac{4\pi^2}{N_c} H_{14}(M_1 M_2) \right]. \end{aligned} \quad (4.35)$$

The leading-order (naive factorization) terms in (4.33), (4.34) and (4.35) are simply a combination of short-distance coefficients, except for cases where a vector meson couples to a scalar current, where it is zero. This is summarized by

$$N_i(M_2) = \begin{cases} 1, & \text{for } i = 3, 4, 5, 12, 14q, \\ 1, & \text{for } i = 6, 11, 13q \text{ and } M_2 = P, \\ 0, & \text{for } i = 6, 11, 13q \text{ and } M_2 = V, \end{cases} \quad (4.36)$$

and the same relation holds for the $\tilde{N}_i$ coefficients. The NLO coefficients in (4.33), (4.33) and (4.35) can mostly be expressed in terms of those already known from the Standard Model operators, collected in section 2.3.4. For the mirror QCD penguin operators, we find that they are almost identical to the Standard Model QCD penguins, that is

$$\tilde{V}_i(M_2) = V_i(M_2), \text{ for } M_2 = P, V^0, \quad (4.37)$$

while in case of transversely polarized vectors we find

$$\tilde{V}_i^\mp(V_2) = \begin{cases} \int_0^1 dx \phi_{a_{V_2}/b_{V_2}} \left[12 \ln \frac{m_b}{\mu} - 18 + g_T(x) \right], & \text{for } i = 3, 4, \\ \int_0^1 dx \phi_{b_{V_2}/a_{V_2}} \left[-12 \ln \frac{m_b}{\mu} + 6 - g_T(\bar{x}) \right], & \text{for } i = 5, \\ 0, & \text{for } i = 6, \end{cases} \quad (4.38)$$

i.e., $\phi_{a_{V_2}}, \phi_{b_{V_2}}$ are exchanged with respect to the QCD penguins. The vertex correction in case of the scalar operator can be read too from the Standard Model expressions, namely

$$V_{11}(M_2) = V_{13}(M_2) = V_6(M_2), \quad V_{12}(M_2) = V_{14}(M_2) = V_5(M_2). \quad (4.39)$$

(the first expression is given for completeness, even though it does not enter (4.35), (4.36) because of $C_{12}^D = C_{14}^q = 0$). In (4.39) the same exchange of $\phi_{a_{V_2}}, \phi_{b_{V_2}}$ is understood, as in (4.38), in the case of transverse helicity amplitudes. In case of the spectator scattering we find

$$\tilde{H}_i(M_1 M_2) = H_i(M_1 M_2), \text{ for } M_2 = P, V^0, \quad (4.40)$$

while for transversely polarized vector meson³

$$\tilde{H}_i^-(V_1 V_2) = 0, \quad \tilde{H}_i^+(V_1 V_2) = H_i^-(V_1 V_2) \quad (4.41)$$

Effectively, the expressions are exchanged between the negative and positive helicity with respect to the Standard Model QCD penguin, for which $H_i^+(V_1 V_2) = 0$. The result for the scalar operators read

$$\begin{aligned} H_{11}(M_1 M_2) &= H_{13}(M_1 M_2) = H_6(M_1 M_2), \\ H_{12}(M_1 M_2) &= H_{14}(M_1 M_2) = H_5(M_1 M_2), \end{aligned} \quad (4.42)$$

with the exception of the transverse helicity amplitudes, in which case

$$\begin{aligned} H_{11}^\pm(M_1 M_2) &= H_{13}^\pm(M_1 M_2) = H_6^\mp(M_1 M_2), \\ H_{12}^\pm(M_1 M_2) &= H_{14}^\pm(M_1 M_2) = H_5^\mp(M_1 M_2). \end{aligned} \quad (4.43)$$

For the penguin contribution $\tilde{P}_{4,6}^p(M_2)$ one replaces $C_i \rightarrow \tilde{C}_i$ in the Standard Model expression and exchanges the function $G_{V_2}^-(s)$ with the function

$$G_{V_2}^+(s) = \int_0^1 dy \phi_{a_2}(y) G(s - i\epsilon, 1 - y), \quad (4.44)$$



Figure 4.4: Penguin contractions. Due to colour only the second diagram contributes to insertions of Q_{11}^p , Q_{13}^q .

in case of the negative helicity amplitudes, while the opposite is true in case of the positive helicity amplitudes.

There are no penguin contributions to (4.34), (4.35). However, as discussed in 4.2, the insertion of scalar operators into the penguin diagrams shown in figure 4.4 modifies the evolution of the (mirror) QCD penguin operators. Accordingly, it also contributes to the penguin terms $\tilde{P}_{4,6}^p(M_2)$ in $\tilde{a}_{4,6}^p(M_1 M_2)$. The correction terms are proportional to the coefficient functions of the scalar operators and read

$$\begin{aligned}\delta P_4^p(M_1 M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left(-\frac{1}{2}\right) C_{13}^b \left[\frac{4}{3} \log \frac{m_b}{\mu} - G_{M_2}^f(1) \right], \\ \delta P_6^p(M_1 M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left(-\frac{1}{2}\right) C_{13}^b \left[N_6(M_2) \frac{4}{3} \log \frac{m_b}{\mu} - \hat{G}_{M_2}^f(1) \right],\end{aligned}\quad (4.45)$$

where $G_{M_2}^f(s)$ equals $G_{M_2}(s)$ (2.91) for $M_1 M_2 = PP, PV, VP, V^0 V^0$, and $G_{M_2}^\pm(s)$ for $V^\pm V^\pm$, while $\hat{G}_{M_2}^f(s)$ equals $\hat{G}_{M_2}(s)$ 2.95 for $M_1 M_2 = PP, PV, VP, V^0 V^0$, and is zero for $V^\pm V^\pm$. Similarly, for the mirror penguin coefficients

$$\begin{aligned}\delta \tilde{P}_4^p(M_1 M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left(-\frac{1}{2}\right) \left\{ C_{13}^D \left[\frac{4}{3} \log \frac{m_b}{\mu} - G_{M_2}^f(0) \right] \right. \\ &\quad \left. + C_{11}^D \left[\frac{4}{3} \log \frac{m_b}{\mu} - G_{M_2}^f(s_p) \right] \right\}, \\ \delta \tilde{P}_6^p(M_1 M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left(-\frac{1}{2}\right) \left\{ C_{13}^D \left[N_6(M_2) \frac{4}{3} \log \frac{m_b}{\mu} - \hat{G}_{M_2}^f(0) \right] \right. \\ &\quad \left. + C_{11}^D \left[N_6(M_2) \frac{4}{3} \log \frac{m_b}{\mu} - \hat{G}_{M_2}^f(s_p) \right] \right\},\end{aligned}\quad (4.46)$$

where $s_u = 0$, $s_c = (m_c/m_b)^2$, and now $G_{M_2}^f(s)$ equals $G_{M_2}^\mp(s)$ for $V^\pm V^\pm$ and else as above. Note that the explicit scale dependence in $\delta P_{4,6}^p(M_2)$, $\delta \tilde{P}_{4,6}^p(M_2)$ cancels the extra scale dependence of the (mirror) QCD penguin coefficients at LL accuracy. At this point we should mention that the constant terms in the real part of the NLO matrix elements should, strictly speaking, only be considered at the NLL order. At this order, our calculation is, however, incomplete, since we do not consider the 1-loop QCD correction to the initial condition of the scalar operators, and the 2-loop mixing into the penguin operators, as well as the small contributions from C_{12}^D and C_{14}^q . Since we do not need precise results

³to be more precise, $\tilde{H}_i^-(V_1 V_2)$ and $H_i^+(V_1 V_2)$ are suppressed by one power of m_{V_1}/m_b with respect to the opposite helicity amplitude.

for the NLO terms, as will be seen below, the present approximation is adequate for our purpose. However, the complete NLO results for the matrix elements of scalar and mirror penguin operators given above might be of more general interest.

It remains to analyse the new weak annihilation terms appearing in 4.29. Numerically, these terms can in some cases be as large as the corresponding α_i amplitudes, but in general they are not relevant for a precise estimate of the decay amplitudes, and were not included in [126]. We report here the expressions for the b_i terms corresponding to the scalar and QCD penguin mirror operators, because of their more general interest for other extension of the Standard Model, like *e.g.* the one discussed in chapter 5.

All the new b_i coefficients in 4.29 can be expressed in terms of the Standard Model b_i coefficients, collected in section 2.3.4. Starting from the mirror QCD operators, we find

$$\tilde{b}_{3,4}^{pD}(M_1 M_2) = \begin{cases} -b_{3,4}^{pD}(M_1 M_2), & \text{if } M_1 M_2 = PP, \\ b_{3,4}^{pD}(M_1 M_2), & \text{if } M_1 M_2 = PV, VP, \\ -b_{3,4}^{pD}(M_1 M_2), & \text{if } M_1 M_2 = V^0 V^0, \\ -b_{3,4}^{-,pD}(M_1 M_2), & \text{if } M_1 M_2 = V^+ V^+, \\ -b_{3,4}^{+,pD}(M_1 M_2), & \text{if } M_1 M_2 = V^- V^-, \end{cases} \quad (4.47)$$

where it is intended that the short-distance coefficients C_i appearing in $b_{3,4}^{pD}(M_1 M_2)$ have to be replaced with $\tilde{C}_i$. In the last two lines we have written explicitly the helicity index, to indicate that the expressions for the $b_{3,4}^{\pm,pD}(M_1 M_2)$ coefficients have the same expressions of the corresponding Standard Model coefficients, with the helicities exchanged. The same result holds formally for the singlet coefficients $b_{3,4}^{pD}(M_1 M_2)$, too. In our evaluation, however, because of their small value, we set all the singlet coefficients to zero. The b_i coefficients originated from the scalar operators can be written in terms of the Standard Model coefficients $A_{2,3}^{i,f}$ defined in (2.111)-(2.114). We find

$$b_{3q}^p(M_1 M_2) = \frac{C_F}{2N_c^2} \begin{cases} C_{13}^q A_2^i, & \text{if } M_1 M_2 = PP, \\ -C_{13}^q A_2^i, & \text{if } M_1 M_2 = PV, VP \\ C_{13}^q A_2^{i,0}, & \text{if } M_1 M_2 = V^0 V^0, \\ C_{13}^q A_2^{i,-}, & \text{if } M_1 M_2 = V^+ V^+, \\ C_{13}^q A_2^{i,+}, & \text{if } M_1 M_2 = V^- V^-, \end{cases}$$

$$b_{4q}^p(M_1 M_2) = \frac{C_F}{2N_c^2} \begin{cases} N_c C_{13}^q A_3^f, & \text{if } M_1 M_2 = PP, \\ -N_c C_{13}^q A_3^f, & \text{if } M_1 M_2 = PV, VP \\ N_c C_{13}^q A_3^{f,0}, & \text{if } M_1 M_2 = V^0 V^0, \\ N_c C_{13}^q A_3^{f,-}, & \text{if } M_1 M_2 = V^+ V^+, \\ 0, & \text{if } M_1 M_2 = V^- V^-. \end{cases} \quad (4.48)$$

$b_{11}^D(M_1 M_2)$ and $b_{12}^D(M_1 M_2)$ have the same expressions, with C_{13}^q replaced by C_{11}^D . In the relations (4.48) we have dropped the terms proportional to $C_{11}^D = C_{14}^q = 0$.

4.6 Non-leptonic decays

We are now ready to discuss the question whether there are observable effects on non-leptonic, charmless decays due to Higgs exchange in the MSSM with large $\tan\beta$. To this end, we compare the new amplitudes to those present in the Standard Model. The essential features can be deduced from (4.28).

- Charged Higgs exchange ($\alpha_{11,12}^D$) contributes directly to tree-dominated decays (such as $B \rightarrow \pi\pi, \pi\rho, \rho\rho$), but must compete with the sizeable Standard Model tree amplitudes $\alpha_{1,2}$. However, since $\alpha_{11,12}^D \propto \bar{m}_D$, only the case of $b \rightarrow s\bar{u}u$ transitions is of interest. But there are no tree-dominated decays of this type, since for $D = s$ the tree amplitudes are doubly CKM-suppressed, $\lambda_u^{(s)} \ll \lambda_c^{(s)}$.
- The effects from the mirror QCD penguin operators ($\alpha_{3,4}'^p$) must compete with the Standard Model penguin amplitudes, which according to (4.12) requires the scalar operator Wilson coefficients to be of order 1 in general, and of order 0.1 in case of the plus-helicity amplitude in $\bar{B} \rightarrow VV$.
- The direct contribution from the FCNC Higgs couplings ($\alpha_{3q,4q}^p$) is an isospin-violating effect that must compete only with the small Standard Model electroweak penguins, and is therefore most likely to lead to an observable effect. Since $\alpha_{3q,4q}^p \propto \bar{m}_q$, only the case $q = s$ is of interest. For the case of $D = s$, the $b \rightarrow s\bar{s}s$ transition leads to final states with flavour content $M_1 = \bar{q}_s s$, $M_2 = \bar{s} s$ with $\bar{q}_s$ the flavour of the $\bar{B}$ meson spectator antiquark. This singles out the decay modes $\bar{B} \rightarrow \bar{K}^{(*)}(\eta^{(\prime)}, \phi)$ and $\bar{B}_s \rightarrow (\eta^{(\prime)}, \phi)(\eta^{(\prime)}, \phi)$. For the case of $D = d$, the potentially interesting modes are $\bar{B} \rightarrow \bar{K}^{(*)}K^{(*)}$ and $\bar{B}_s \rightarrow K^{(*)}\phi$. However, in all these decays it is impossible to extract the EW penguin amplitude, so the new contributions must in fact be compared to the larger Standard Model QCD penguins.

We now proceed to a more detailed discussion. The numerical amplitude values given below are calculated using the input parameters defined in table 2.2 and in section 2.3.5. The Wilson coefficients are evaluated at the scale $\mu = m_b = 4.2 \text{ GeV}$.

$B \rightarrow PP, PV$

In table 4.1 we show the numerical results of the α_i amplitude coefficients defined in (4.28) for the decay modes $\bar{B} \rightarrow \bar{K}\eta, \bar{K}^*\eta, \bar{K}\phi$. (η_s in the table refers to the strange component of η , see (2.36) and [66].) To evaluate the Higgs contributions we assume the largest values of the coefficient functions allowed by the constraints from leptonic decays derived in section 4.3, in detail: $C_{11}^s(m_b) = -0.08$, $C_{13}^s(m_b) = 0.001$, $C_{13}^b(m_b) = 0.05$.

Among the Higgs penguin amplitudes α_{3q}^p is the larger of $\alpha_{3q,4q}^p$, since α_{4q}^p is colour-suppressed and is further reduced by the radiative correction given in (4.34), (4.35). However, the strong constraint on C_{13}^s renders α_{3q}^p always negligible, in particular as it should be compared to the QCD penguin amplitude α_4^p rather than the electroweak penguin. This remains true for VP amplitudes despite the fact that the Standard Model penguin amplitude is smaller for these final states, and for PV amplitudes, where α_{3q}^p vanishes.

	$\bar{K}\eta_s$	$\bar{K}^*\eta_s$	$\bar{K}\phi$
α_1	$0.966 + 0.021i$ [$\pi\bar{K}$]	$0.981 + 0.021i$ [$\rho\bar{K}$]	$0.973 + 0.021i$ [$\pi\bar{K}^*$]
α_2	$0.351 - 0.084i$ [$\bar{K}\pi$]	$0.260 - 0.084i$ [$\bar{K}^*\pi$]	$0.323 - 0.084i$ [$\bar{K}\rho$]
α_{11}^s	-0.059 [$\pi\bar{K}$]	-0.059 [$\rho\bar{K}$]	0 [$\pi\bar{K}^*$]
α_{12}^s	$0.003 + 0.003i$ [$\bar{K}\pi$]	$-0.006 - 0.003i$ [$\bar{K}^*\pi$]	$0.004 + 0.003i$ [$\bar{K}\rho$]
α_3^u	$-0.0013 + 0.0046i$	$0.0027 + 0.0046i$	$0.0006 - 0.0005i$
α_4^u	$-0.095 - 0.040i$	$0.038 + 0.008i$	$-0.031 - 0.017i$
δP_4^u	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$
δP_6^u	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$	$-1.5 \cdot 10^{-5}$
$\tilde{\alpha}_3^u$	$7.8 \cdot 10^{-5} - 0.0001i$	$2.8 \cdot 10^{-5} + 0.0001i$	$(1.4 - 1.3i) \cdot 10^{-5}$
$\tilde{\alpha}_4^u$	$0.0035 + 0.0015i$	$0.0011 + 0.0003i$	$-0.0013 - 0.0006i$
$\delta \tilde{P}_4^u$	$-0.0005 - 0.0007i$	$-0.0005 - 0.0007i$	$-0.0005 - 0.0007i$
$\delta \tilde{P}_6^u$	$-0.0006 - 0.0007i$	$-0.0006 - 0.0007i$	-0.0003
$\alpha_{3,\text{EW}}^u$	$-0.0089 - 0.0002i$	$-0.0091 - 0.0002i$	$-0.0082 - 0.0001i$
$\alpha_{4,\text{EW}}^u$	$-0.0016 + 0.0006i$	$-0.0025 + 0.0008i$	$-0.0024 + 0.0007i$
α_{3s}^u	0.00078	0.00078	0
α_{4s}^u	$(-6.3 - 3.7i) \cdot 10^{-5}$	$(9.7 + 3.7i) \cdot 10^{-5}$	$(-6.3 - 3.7i) \cdot 10^{-5}$

Table 4.1: Numerical results for the α_i coefficients of some representative $\Delta S = 1$ decay channels. The value of α_4^u corresponds to the Standard Model contribution only. The final states $\bar{K}\eta_s$, $\bar{K}^*\eta_s$, $\bar{K}\phi$ do not receive tree-amplitude contributions. For $\alpha_{1,2}$ and $\alpha_{11,12}^s$, we therefore provide results for the final states in square brackets.

	$(\bar{K}^*\phi)^{00}$	$(\bar{K}^*\phi)^{--}$	$(\bar{K}^*\phi)^{++}$
$\alpha_1 [\rho\bar{K}^*]$	$0.987 + 0.021i$	$1.101 + 0.041i$	1.018
$\alpha_2 [\bar{K}^*\rho]$	$0.240 - 0.084i$	$-0.173 - 0.169i$	0.170
$\alpha_{11}^s [\rho\bar{K}^*]$	0	0	0
$\alpha_{12}^s [\bar{K}^*\rho]$	$-0.007 - 0.003i$	-0.002	$-0.247 - 0.068i$
α_3^u	$0.0001 - 0.0005i$	$-0.0023 - 0.0010i$	-0.0035
α_4^u	$-0.026 - 0.015i$	$-0.044 - 0.017i$	-0.031
δP_4^u	$1.4 \cdot 10^{-5}$	$0.7 \cdot 10^{-5}$	$2.2 \cdot 10^{-5}$
δP_6^u	$-1.5 \cdot 10^{-5}$	0	0
$\tilde{\alpha}_3^u$	$(-0.2 + 1.3i) \cdot 10^{-5}$	$(5.1 + 2.3i) \cdot 10^{-6}$	0.0010
$\tilde{\alpha}_4^u$	$0.0011 + 0.0006i$	$0.0001 + 5.5 \cdot 10^{-5}i$	$0.0173 + 0.0074i$
$\delta \tilde{P}_4^u$	$-0.0005 - 0.0007i$	$-0.0004 - 0.0007i$	$-0.0007 - 0.0007i$
$\delta \tilde{P}_6^u$	-0.0003	0	0
$\alpha_{3,\text{EW}}^u$	$-0.0084 - 0.0001i$	$0.0044 - 0.0003i$	-0.009
$\alpha_{4,\text{EW}}^u$	$-0.0017 + 0.0007i$	$0.0015 + 0.0014i$	-0.0015
α_{3s}^u	0	0	0
α_{4s}^u	$(9.7 + 3.7i) \cdot 10^{-5}$	$2.0 \cdot 10^{-5}$	$0.0031 + 0.0008i$

Table 4.2: Numerical results for the α_i coefficients pertaining to the three helicity amplitudes of $\bar{B} \rightarrow VV$ decays. The value of α_4^u corresponds to the Standard Model contribution only. To compare the absolute values of the helicity amplitudes the numbers for the $(00, --, ++)$ parameters must be multiplied by $A_{K^*\phi} = \frac{iG_F}{\sqrt{2}} m_B f_\phi (m_B A_0^{B \rightarrow K^*}, m_\phi F_-^{B \rightarrow K^*}, m_\phi F_+^{B \rightarrow K^*})$. The estimates above use $F_+^{B \rightarrow V_1} = 0.06$ in order to compare with the maximal Standard Model $++$ amplitude. The final state $\bar{K}^*\phi$ does not receive tree contributions. For $\alpha_{1,2}$ and $\alpha_{11,12}^s$, we therefore provide results for the final states in square brackets.

$B \rightarrow VV$

The effect of Higgs exchange is also negligible in case of the longitudinal amplitude in $\bar{B} \rightarrow VV$ decays, since it follows the same pattern as for the PV decays with $M_2 = V$. Due to the inverted hierarchy of the transverse polarization amplitudes, see (4.32), the minus-helicity amplitude is suppressed, while the plus-helicity amplitude is enhanced by a factor of m_b/Λ_{QCD} relative to the Standard Model. To compare the Higgs contributions to the plus-helicity amplitude in the Standard Model, in table 4.2 we show the α_i coefficients assuming $F_+^{B \rightarrow V_1} = 0.06$, which is the upper limit allowed in [179]. It is evident that the magnitude of the Higgs-induced α_i coefficients is now larger for the plus amplitude than for PP, PV final states and the other polarization amplitudes. In fact, α_{3s}^u would now be comparable to the Standard Model penguin amplitude, if it were not annihilated by the projection on the vector meson at tree level, see (4.36). Thus, among amplitudes with the same flavour topology, we find that only α_{12}^s is larger than the corresponding Standard Model colour-suppressed tree amplitude α_2 . The mirror QCD penguin amplitude α_4^u amounts to a substantial fraction of the standard penguin amplitude that may reach one if $F_+^{B \rightarrow V_1}$ is smaller than the assumed value. This would affect the azimuthal angular distribution of $\Delta S = 1$ decays; in practice, however, the effect is unobservable. Not only is the amplitude very small in absolute terms, but the tree amplitudes are also subleading to the penguin amplitudes in $\Delta S = 1$ decays.

It is straightforward to compute branching fractions, CP asymmetries and polarization observables including the Higgs-exchange contributions. However, since the α_i parameters discussed above form the basic constituents of observables, it follows that any modification of the Standard Model predictions will be invisible within theoretical uncertainties.

4.7 Conclusion

Motivated by the interest in the minimally flavour-violating MSSM with large $\tan\beta$ owing to its potentially large impact on leptonic B decays, we analysed non-leptonic B decays in this model. The hadronic and leptonic flavour-changing interactions are closely related, which allows us to translate the present limit on the $B_s \rightarrow \mu^+ \mu^-$ branching fraction, and the observation of $B^+ \rightarrow \tau^+ \nu_\tau$ into a constraint on the Wilson coefficients of the relevant scalar four-quark operators. We then calculated the matrix elements of scalar operators and mirror QCD penguin operators at next-to-leading order in the framework of QCD factorization and find that the limits on leptonic B decay branching fractions exclude any visible effects in hadronic decays, but for an academic exception: the positive-helicity amplitude of $\bar{B} \rightarrow VV$ may receive order one modifications relative to the Standard Model, but this amplitude is too small to be detected at present or planned B factories.

Chapter 5

Non-leptonic B decays in a supersymmetric grand unified theory

5.1 Introduction

Grand Unified theories (GUT) constitute an interesting extension of the Standard Model. The running of the strong, the weak and the electromagnetic coupling suggests that they could unify in a unique force at some high energy scale M_{GUT} and, indeed, this is realized with good accuracy in the context of the MSSM, the latter providing also the necessary stabilization for the electroweak symmetry breaking scale. The basic hypothesis is that the Standard Model gauge group, $G_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$, is embedded into a simple group at M_{GUT} , such as $\text{SU}(5)$ or $\text{SO}(10)$. The latter is probably the most natural GUT group: both the Standard Model gauge and matter fields are unified, introducing only one additional matter particle, the right-handed neutrino. It is an anomaly-free theory and therefore explains the nontrivial cancellations of the anomalies in the Standard Model. Moreover, it contains $B - L$ as a local symmetry, B and L being the baryon and lepton number, respectively. The breaking of $B - L$ provides light neutrino masses through the seesaw mechanism. In this way, neutrino masses are linked to the breaking of the GUT symmetry itself. Remarkably, $M_{\text{GUT}} \approx 10^{16}$ GeV is of the right order of magnitude to generate neutrino masses in the sub-eV range.

As in case of the MSSM with large $\tan \beta$, in order to satisfy the strong constraints imposed by low energy processes on the flavour structure of the theory, one is forced to assume a minimal flavour violating scenario for the quark sector, *i.e.*, one considers the Yukawa coupling as the only source of flavour violation; moreover, the supersymmetry breaking terms are assumed to be universal at a fundamental scale, that here we assume to be $M_{\text{Pl}} \simeq 10^{19}$ GeV.

On the other side, the observation of neutrino masses and oscillations introduces the possibility of having flavour violation in the lepton sector, too. This is usually encoded in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [289, 290]. In contrast

to the quark sector of the Standard Model, where the mixing is realized in terms of small mixing angles, the PMNS matrix contains two large angles, the neutrino solar and atmospheric mixing angles, with the latter being close to maximal.

In the Standard Model, the effects of the quark and lepton mixing are confined to the two sector of the theory. In GUTs however, the separation of quarks and leptons is removed, since they are unified at high energy. As a consequence, their masses and mixing are related to each other. While there are many possible ways to relate the two sectors, it is natural to expect imprints of the lepton mixing into the quark sector, and viceversa. In particular, it arises the interesting possibility to observe effects of the two large neutrino mixing angles in the down-type right-handed quark mixing. In the Standard Model these effects would be unobservable, due to the absence of right-handed flavour-changing currents at the weak scale. In the context of supersymmetry, however, flavour changing right-handed currents arise through loop effects, because of the mixing of the corresponding scalar partners of quarks and leptons. This involves in particular the $b_R - s_R$ quark mixing, so that large effects are expected in B_s physics.

These ideas were investigated in a particular GUT model [278], constructed with the specific aim of accommodating in a consistent model the observed large solar and atmospheric neutrino mixing angles. Subsequently, the flavour structure of the model was analysed in [279–287], where a full renormalization group analysis of the model was developed. In these works it was found the possibility of having a large supersymmetric contribution to the $B_s - \bar{B}_s$ mixing, comparable in magnitude with the Standard Model one. Large effects were found in the $b \rightarrow s\gamma$ transition and in the $\tau \rightarrow \mu\gamma$ decay, too. Analysing the predicted particle spectrum, however, it was found that strong constraints come from the LEP bound on the lightest Higgs boson, namely $M_{h_0} \geq 114.4$ GeV, which forces the gluino mass to be larger than ~ 300 GeV, and the down-type squark masses to be larger than ~ 1 TeV. Despite these constraints, however, sizeable flavour effects were found to be still possible.

In this chapter we propose to investigate further the Chang-Masiero-Murayama (CMM) model, in the context of non-leptonic decays, to see if the large effects found in the $\bar{B}_s - B_s$ can be observed in these processes too, and if further insight can be gained. The analysis is motivated by the naive expectation of finding sizeable effects in non-leptonic B decays, because the operators contributing to these decays have the same structure as those contributing to the $\bar{B}_s - B_s$ mixing. Moreover, because of the vectorial nature of the flavour-changing currents, one expects relevant contributions not only in case of $B \rightarrow PP$ decays, but also in case of $B \rightarrow PV$, $B \rightarrow VV$ decays, in contrast to the large $\tan\beta$ scenario. The new contributions are expected to show different patterns with respect to the Standard Model amplitudes because of the right-handed nature of the flavour changing currents. Besides these features, the model exhibit isospin violation as well. Finally, the presence of a new weak phase related to the phases in the PMNS matrix, identified with ϕ_{B_s} , could alter significantly some CP asymmetries.

The results of our analysis shows that while all these effects are realized at the level of the decay amplitudes, their magnitude is in general too small to affect significantly the branching ratio and other experimental observables. Even if the largest deviations

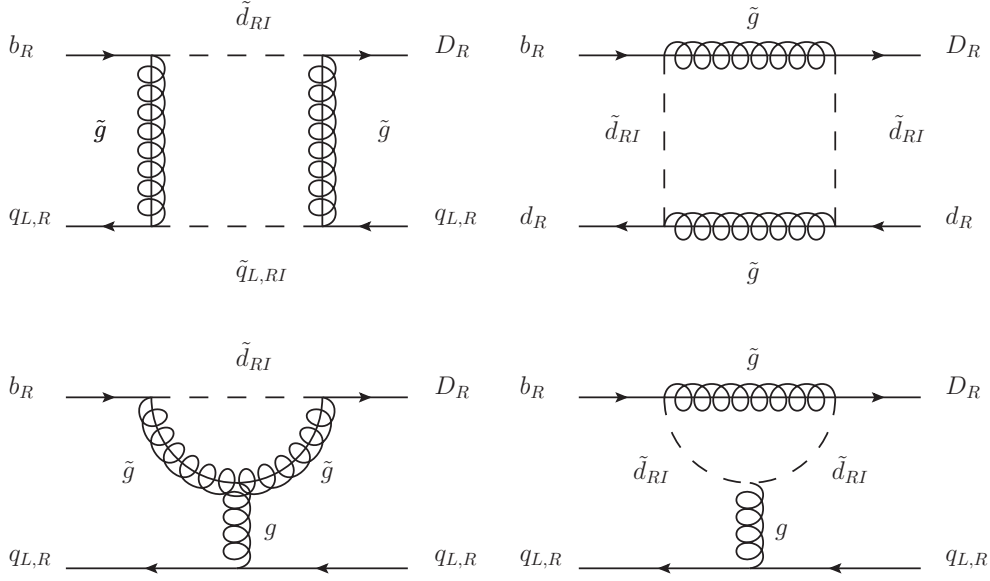


Figure 5.1: Four-quark interactions in the CMM model.

observed are somehow larger than the one expected in the large $\tan \beta$ scenario, especially in $B \rightarrow PV$ decays, they are still too small, in order to emerge from the present theoretical and experimental uncertainties.

5.2 Four-quark operators in the Chang-Masiero-Murayama model

In supersymmetric theories, new four-quark operators arise, if one considers penguin and box diagrams with the exchange of gluinos and squarks in the loop, as in figure 5.1. These diagrams give usually small contributions, and therefore they are less important *e.g.* in the large $\tan \beta$ scenario, with respect to the $\tan \beta$ enhanced scalar operators. They become relevant in scenarios where a large mass splitting between the squark masses is generated and $\mathcal{O}(1)$ flavour changing quark-squark-gluinos couplings are present. These conditions are indeed satisfied in the CMM model. We refer to appendix C for a brief review of the model, that we consider here in the set-up elaborated in [287]. In view of the requirements above, the important features to be considered can be summarized as follows:

- because of the renormalization group equation, the universality of the soft terms assumed at M_{Pl} is no longer present at the electroweak scale. For instance, in case of the $\tilde{d}_R$ soft breaking terms one has

$$\mathbf{M}_{\tilde{d}_R}^2(M_Z) = \text{diag} \left(m_{\tilde{d}}^2, m_{\tilde{d}}^2, m_{\tilde{d}}^2(1 - \Delta_{\tilde{d}}) \right). \quad (5.1)$$

The important point here is to consider M_{Pl} as the fundamental scale, rather than M_{GUT} , as usually assumed. The reason to take M_{Pl} as the fundamental scale is

simply motivated by the fact that, in absence of a specific grand unified model, the analysis of the region between M_{Pl} and M_{GUT} cannot be performed. This, however, misses potentially important flavour effects. In the CMM model a complete chain of grand-unified theories is constructed up to M_{Pl} , so that the renormalization group analysis can be performed up to this scale, and this allows for rather large values of $\Delta_{\tilde{d}}$, because of the fast renormalization group evolution between M_{Pl} and v_0 , the vev of the $\text{SU}(5)$ singlet 45_H Higgs field (see appendix C for the definition of the GUT chain and fields).

- In the basis where the down quarks are mass eigenstates, the so-called super-CKM basis, the soft mass matrix (5.1) is no longer diagonal; in particular, all the elements of the 2-3 block are of comparable size:

$$\tilde{\mathbf{M}}_{\tilde{d}}^2 = \mathbf{R}_d^\dagger \mathbf{M}_{\tilde{d}}^2 \mathbf{R}_d = m_{\tilde{d}}^2 \begin{pmatrix} 1 - \sin^2 \theta \Delta_{\tilde{d}}/2 & \sin(2\theta) e^{-i\phi_K} \Delta_{\tilde{d}}/4 & \sin \theta e^{-i\phi_{B_d}} \Delta_{\tilde{d}}/2 \\ \sin(2\theta) e^{i\phi_K} \Delta_{\tilde{d}}/4 & 1 - \cos^2 \theta \Delta_{\tilde{d}}/2 & -\cos \theta e^{-i\phi_{B_s}} \Delta_{\tilde{d}}/2 \\ \sin \theta e^{i\phi_{B_d}} \Delta_{\tilde{d}}/2 & -\cos \theta e^{i\phi_{B_s}} \Delta_{\tilde{d}}/2 & 1 - \Delta_{\tilde{d}}/2 \end{pmatrix}, \quad (5.2)$$

where the matrix $\mathbf{R}_d$ is defined in appendix C. This is a direct consequence of the GUT relation between the right-handed down-quark and the left-handed lepton Yukawa couplings, as explained in appendix C¹.

The second item guarantees the existence of flavour changing $d_{RI} \tilde{d}_{RJ} \tilde{g}$ couplings, whose Feynman rules are collected in appendix C. The first one guarantees that smaller cancellations occur, when loop processes like those in figure 5.1 are considered: having degenerate squark masses would result in a “GIM” mechanism for the superpartners of the Standard Model particles, suppressing flavour changing processes.

The relevant new operators can be collected in the effective Hamiltonian

$$\mathcal{H}_{eff}^{\text{CMM}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \left(\sum_{i=3}^6 \tilde{C}_i^{q,D} \tilde{Q}_i^q + \tilde{C}_{8g} \tilde{Q}_{8g} \right), \quad (5.3)$$

where the $\tilde{Q}_i^q$ are defined as²

$$\begin{aligned} \tilde{Q}_3^q &= (\bar{D}_i b_i)_{V+A} (\bar{q}_j q_j)_{V+A}, & \tilde{Q}_4^q &= (\bar{D}_i b_j)_{V+A} (\bar{q}_j q_i)_{V+A}, \\ \tilde{Q}_5^q &= (\bar{D}_i b_i)_{V+A} (\bar{q}_j q_j)_{V-A}, & \tilde{Q}_6^q &= (\bar{D}_i b_j)_{V+A} (\bar{q}_j q_i)_{V-A}, \end{aligned} \quad (5.4)$$

¹The full squark mass matrix is a 6×6 matrix, with the left-handed down squark in the upper 3×3 block, and the right-handed ones in the 3×3 lower block. The diagonalization of the full matrix gives six mass eigenstates. Here we neglect the effects of the two 3×3 blocks outside the diagonal, (the “left-right” and “right-left” blocks), so that one has 3 mass eigenvalues for the right-handed squarks and 3 for the left-handed ones. This is a good approximation in the decoupling limit, assumed in this chapter, too, because the “left-right” and “right-left” blocks are of order $\mu_{\text{EW}} \times \mu_S$, with μ_S the typical scale of the soft supersymmetry breaking terms, while the blocks on the diagonal are of order μ_S^2 .

²Actually, two operators more, $\tilde{Q}_7^q = (\bar{q}_i b_i)_{V+A} (\bar{D}_j q_j)_{V-A}$, $\tilde{Q}_8^q = (\bar{q}_i b_j)_{V+A} (\bar{D}_j q_i)_{V-A}$ could be constructed. We don’t consider them here, because for $q = D = s$ they are Fierz equivalent to $\tilde{Q}_{5,6}^s$, while for $q = d, b$ they are very small.

Note also the different notation with respect to [288] = “GKN”, *i.e.*, we have $\tilde{Q}_{3,4}^q = \tilde{Q}_{3,4,\text{GKN}}^q$, $\tilde{Q}_{5,6}^q = \tilde{Q}_{1,2,\text{GKN}}^q$, $\tilde{Q}_{7,8}^q = \tilde{Q}_{5,6,\text{GKN}}^q$. Our choice is motivated by the preference of labelling with the same number operators with the same (mirror) Dirac structure.

where i, j are colour indices and $D = d, s$. The new operators have the same Dirac structure as the corresponding QCD mirror penguin operators, but here no sum over q appears, and they depend on q itself, thus allowing for isospin violation. The corresponding short-distance coefficients at the scale $\mu_S = \mathcal{O}(m_{\tilde{q}_{LI}}, m_{\tilde{d}_{RI}}, m_{\tilde{u}_{RI}}, m_{\tilde{g}})$ were calculated in a generic MSSM model in [288], and in the context of the CMM Model in [280]. Our aim is to provide expressions for them based on the parametrization of the mixing matrix $\mathbf{R}_d$ proposed in [287], that is slightly more general than the one in [280]. Moreover, we find it useful to give two intermediate results, which help to better understand the structure of the short distance coefficients, and they can be used in contexts more generic than the CMM model itself.

The first is derived directly from [288], and it is based on a generic expression for the mixing matrix $\mathbf{R}_d$:

$$\begin{aligned}
\tilde{C}_3^{u,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* \delta_{Ju} L_3(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{u}_{RJ}\tilde{g}}) \right. \\
&\quad \left. + [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* P_0(x_{\tilde{d}_{RI}\tilde{g}}) \right\}, \\
\tilde{C}_3^{d,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* [\mathbf{R}_d]_{Jd} [\mathbf{R}_d]_{Jd}^* L_3(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{d}_{RJ}\tilde{g}}) \right. \\
&\quad + [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{JD}^* [\mathbf{R}_d]_{Jd} [\mathbf{R}_d]_{Id}^* L_4(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{u}_{RJ}\tilde{g}}) \\
&\quad \left. + [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* P_0(x_{\tilde{d}_{RI}\tilde{g}}) \right\}, \\
\tilde{C}_4^{u,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* \delta_{Ju} L_4(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{u}_{RJ}\tilde{g}}) \right. \\
&\quad \left. + [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* P_0(x_{\tilde{d}_{RI}\tilde{g}}) \right\}, \\
\tilde{C}_4^{d,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* [\mathbf{R}_d]_{Jd} [\mathbf{R}_d]_{Jd}^* L_4(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{d}_{RJ}\tilde{g}}) \right. \\
&\quad + [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{JD}^* [\mathbf{R}_d]_{Jd} [\mathbf{R}_d]_{Id}^* L_3(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{d}_{RJ}\tilde{g}}) \\
&\quad \left. - 3[\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* P_0(x_{\tilde{d}_{RI}\tilde{g}}) \right\}, \\
\tilde{C}_5^{q,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* \delta_{Jq} L_5(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{q}_{LJ}\tilde{g}}) \right. \\
&\quad \left. + [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* P_0(x_{\tilde{d}_{RI}\tilde{g}}) \right\},
\end{aligned}$$

$$\begin{aligned}
\tilde{C}_6^{q,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* \delta_{Jq} L_6(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{q}_{LJ}\tilde{g}}) \right. \\
&\quad \left. + [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* P_0(x_{\tilde{d}_{RI}\tilde{g}}) \right\}, \\
\tilde{C}_{8g}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\pi\alpha_s(\mu_S)}{\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{Ib} [\mathbf{R}_d]_{ID}^* G_8(x_{\tilde{d}_{RI}\tilde{g}}) \right\}, \tag{5.5}
\end{aligned}$$

where a sum over the flavour indices I, J is understood. The loop functions $L_{3,4,5,6}(x, y)$ arise in the calculation of the box diagrams and can be expressed in terms of two functions $F(x, y)$ and $G(x, y)$:

$$\begin{aligned}
L_3(x, y) &= -\frac{5}{9} F(x, y) + \frac{1}{36} G(x, y), \\
L_4(x, y) &= \frac{1}{3} F(x, y) - \frac{7}{12} G(x, y), \\
L_5(x, y) &= \frac{1}{18} F(x, y) - \frac{5}{18} G(x, y), \\
L_6(x, y) &= \frac{7}{6} F(x, y) + \frac{1}{6} G(x, y), \tag{5.6}
\end{aligned}$$

which in turn read

$$\begin{aligned}
F(x, y) &= -\frac{x \ln x}{(x-y)(x-1)^2} - \frac{y \ln y}{(y-x)(y-1)^2} - \frac{1}{(x-1)(y-1)}, \\
G(x, y) &= \frac{x^2 \ln x}{(x-y)(x-1)^2} + \frac{y^2 \ln y}{(y-x)(y-1)^2} + \frac{1}{(x-1)(y-1)}. \tag{5.7}
\end{aligned}$$

The function $P_0(x)$ arises in the computation of the penguin diagram and it is expressed as a function of $A(x)$, $B(x)$:

$$P_0(x) = \frac{1}{2} A(x) + \frac{2}{9} B(x), \tag{5.8}$$

which in turn read

$$\begin{aligned}
A(x) &= \frac{1}{2(1-x)} + \frac{(1+2x) \ln x}{6(1-x)^2}, \\
B(x) &= -\frac{11-7x+2x^2}{18(1-x)^3} - \frac{\ln x}{3(1-x)^4}. \tag{5.9}
\end{aligned}$$

The loop function appearing in $\tilde{C}_{8g}$ reads

$$G_8(x) = -3C(x) + \frac{4}{3}D(x), \tag{5.10}$$

with

$$\begin{aligned}
C(x) &= \frac{1-x^2+2x \ln x}{4(x-1)^3}, \\
D(x) &= \frac{-2-3x+6x^2-x^3-6x \ln x}{6(x-1)^4}. \tag{5.11}
\end{aligned}$$

In (5.5), the argument of the loop functions refers to ratios of the supersymmetric particles, namely $x_{\tilde{q}_{LI}\tilde{g}} = m_{\tilde{q}_{LI}}^2/m_{\tilde{g}}^2$, $x_{\tilde{d}_{RI}\tilde{g}} = m_{\tilde{d}_{RI}}^2/m_{\tilde{g}}^2$, $x_{\tilde{u}_{RI}\tilde{g}} = m_{\tilde{u}_{RI}}^2/m_{\tilde{g}}^2$, where $m_{\tilde{q}_{LI}}^2$, $m_{\tilde{d}_{RI}}^2$, $m_{\tilde{u}_{RI}}^2$ are the mass eigenvalues of the squark soft bilinear terms.

The expressions in (5.5) can be made more explicit simply by exploiting the unitarity of the mixing matrix $\mathbf{R}_d$ and the mass degeneracy of the first two squark states (5.1)³, without any further assumption on $\mathbf{R}_d$. The result is

$$\begin{aligned}
\tilde{C}_3^{u,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3D}^* \left\{ \left[L_3(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{u}_{R\tilde{g}}}) - L_3(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{u}_{R\tilde{g}}}) \right] \right. \\
&\quad \left. + P_0(x_{\tilde{b}_{R\tilde{g}}}) - P_0(x_{\tilde{d}_{R\tilde{g}}}) \right\}, \\
\tilde{C}_3^{d,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3D}^* \left[L_3(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - L_3(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) \right] \right. \\
&\quad + [\mathbf{R}_d]_{3d} [\mathbf{R}_d]_{3d}^* \left(L_3(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - 2L_3(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) + L_3(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{b}_{R\tilde{g}}}) \right) \\
&\quad \left. + P_0(x_{\tilde{b}_{R\tilde{g}}}) - P_0(x_{\tilde{d}_{R\tilde{g}}}) \right] \\
&\quad + \delta_{Dd} [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3d}^* \left[L_4(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - L_4(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) \right] \\
&\quad + [\mathbf{R}_d]_{3d} [\mathbf{R}_d]_{3D}^* \left[\delta_{bd} \left(L_4(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - L_4(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) \right) \right. \\
&\quad \left. + [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3d}^* \left(L_4(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - 2L_4(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) + L_4(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{b}_{R\tilde{g}}}) \right) \right] \Big\}, \\
\tilde{C}_4^{u,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3D}^* \left\{ \left[L_4(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{u}_{R\tilde{g}}}) - L_4(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{u}_{R\tilde{g}}}) \right] \right. \\
&\quad \left. + 3 \left[P_0(x_{\tilde{b}_{R\tilde{g}}}) - P_0(x_{\tilde{d}_{R\tilde{g}}}) \right] \right\}, \\
\tilde{C}_4^{d,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} \left\{ [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3D}^* \left[L_4(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - L_4(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) \right] \right. \\
&\quad + [\mathbf{R}_d]_{3d} [\mathbf{R}_d]_{3d}^* \left(L_4(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - 2L_4(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) + L_4(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{b}_{R\tilde{g}}}) \right) \\
&\quad \left. - 3 \left(P_0(x_{\tilde{b}_{R\tilde{g}}}) - P_0(x_{\tilde{d}_{R\tilde{g}}}) \right) \right] \\
&\quad + \delta_{Dd} [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3d}^* \left[L_3(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - L_3(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) \right]
\end{aligned}$$

³this is valid also for $M_{\tilde{u}}^2$ and $M_{\tilde{q}}^2$, provided one substitutes $d \rightarrow u, q$ in (5.1)

$$\begin{aligned}
& + [\mathbf{R}_d]_{3d} [\mathbf{R}_d]_{3D}^* \left[\delta_{bd} \left(L_3(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - L_3(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) \right) \right. \\
& \quad \left. + [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3d}^* \left(L_3(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - 2L_3(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) + L_3(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{b}_{R\tilde{g}}}) \right) \right] \Bigg\}, \\
\tilde{C}_5^{q,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3D}^* \left\{ \left[L_5(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{q}_{L\tilde{g}}}) - L_5(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{q}_{L\tilde{g}}}) \right] \right. \\
& \quad \left. + P_0(x_{\tilde{b}_{R\tilde{g}}}) - P_0(x_{\tilde{d}_{R\tilde{g}}}) \right\}, \\
\tilde{C}_6^{q,D}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\alpha_s^2(\mu_S)}{2\sqrt{2}G_F m_{\tilde{g}}^2} [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3D}^* \left\{ \left[L_6(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{q}_{L\tilde{g}}}) - L_6(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{q}_{L\tilde{g}}}) \right] \right. \\
& \quad \left. + 3 \left[P_0(x_{\tilde{b}_{R\tilde{g}}}) - P_0(x_{\tilde{d}_{R\tilde{g}}}) \right] \right\} \\
\tilde{C}_{8g}(\mu_S) &= \frac{1}{-\lambda_t^{(D)}} \frac{\pi\alpha_s(\mu_S)}{\sqrt{2}G_F m_{\tilde{g}}^2} [\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3D}^* \left\{ G_8(x_{\tilde{b}_{R\tilde{g}}}) - G_8(x_{\tilde{d}_{R\tilde{g}}}) \right\}. \tag{5.12}
\end{aligned}$$

Expressions specific for the CMM model can be obtained substituting in (5.12) the explicit expression for the products of the mixing matrix $\mathbf{R}_d$:

$$\begin{aligned}
[\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3d}^* &= -\frac{1}{2} e^{-i\phi_{B_d}} \sin \theta, \\
[\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3s}^* &= \frac{1}{2} e^{-i\phi_{B_s}} \cos \theta, \\
[\mathbf{R}_d]_{3s} [\mathbf{R}_d]_{3d}^* &= -\frac{1}{2} e^{-i(\phi_{B_d} - \phi_{B_s})} \cos \theta \sin \theta, \\
[\mathbf{R}_d]_{3d} [\mathbf{R}_d]_{3d}^* &= \frac{1}{2} \sin^2 \theta, \\
[\mathbf{R}_d]_{3s} [\mathbf{R}_d]_{3s}^* &= \frac{1}{2} \cos^2 \theta, \\
[\mathbf{R}_d]_{3b} [\mathbf{R}_d]_{3b}^* &= \frac{1}{2}. \tag{5.13}
\end{aligned}$$

From (5.5) and (5.12) we deduce already many features of the new short distance coefficients. First, using the approximate relation $(\alpha_s/\alpha_{\text{em}})(M_W^2/\mu_S^2) \sim 1$, one can estimate $\tilde{C}_{3,\dots,6}^q$ to be of the same order as the electroweak penguin short-distance coefficients times the loop functions. This immediately tells us that no large effects can be expected in non-leptonic B decays, unless sizeable isospin violation is generated. Then, looking at (5.5) and (5.12), one notes that there are indeed various sources of isospin violation: in $\tilde{C}_3^q$, $\tilde{C}_4^q$ it originates between the up- and down-type short-distance coefficients, due to the additional contribution of the diagram in figure 5.1 (b) to $\tilde{C}_{3,4}^d$. A second source of isospin

violation between the up- and down type short-distance coefficients arises because of the different arguments in the box functions $L_i(x, y)$, and depends on the ratio $m_{\tilde{d}_R}^2/m_{\tilde{u}_R}^2$. Other sources of isospin violation depend specifically on the different matrix elements of $\mathbf{R}_d$. Needless to say, one has to look more closely at each contribution, in order to estimate the real magnitude of the isospin violation, and in the end all these effects tend to be small.

In order to see why, we start looking at the matrix elements in (5.13). As described in appendix C, the angle θ governs the mixing between the first two down squark families and it is strongly constrained by K and B_d mixing. In [287] it was found $\theta^{\max} \sim 1^\circ$, so that in (5.13) it is safe to set $\sin \theta \simeq \sin^2 \theta = 0$, $\cos \theta \simeq \cos^2 \theta = 1$. This immediately rules out the possibility of having new contributions to $\Delta D = 1$ transitions, leaving the $\Delta S = 1$ case. At the same time, the additional contributions to $\tilde{C}_{3,4}^d$ with respect to $\tilde{C}_{3,4}^u$ drop out in case of the first family.

Numerical evaluation and parameter space

More insight into the features of $\tilde{C}_{3,\dots,6}^q$ requires a numerical estimate of the penguin and box functions $P_0(x)$, $L_i(x, y)$, which depend directly on the particle spectrum. In the CMM model (and, more generally, in GUTs) the various mass parameters are related at the high scales by unification conditions, so that the only way to perform a precise evaluation is to calculate the low energy spectrum by means of a renormalization group evolution from the high scales. Here we use the *Mathematica* code written by the authors of [283–286], which implements the relations between the set of basic input parameters and the remaining ones at the electroweak scale. In the CMM model the large number of free MSSM parameters shrinks to six input parameters at the electroweak scale, in addition to θ and the phases ϕ_K , ϕ_{B_d} , ϕ_{B_s} . According to [283–286], they can be chosen to be the gluino mass $m_{\tilde{g}}$, the first generation $\tilde{d}_R$ and $\tilde{u}_R$ soft masses $m_{\tilde{d}}$ and $m_{\tilde{u}}$, the ratio of the (11)-element of the trilinear and Yukawa couplings $a_d^1 = (A_d)_{11}/(Y_d)_{11}$, the phase of the μ parameter in the Higgs potential $\arg(\mu)$ and the ratio of the two Higgs-doublet VEVs, $\tan \beta$. Note that similar inputs are usually considered in other specific scenarios such as minimal supergravity or the constrained MSSM. Nevertheless, the absence of grand unification there leads to very different phenomenologies, because quark and lepton parameters are not related to each other. For instance, in non-unified models the fields can be rotated independently and the large lepton mixing angles do not become visible in the quark sector.

The short-distance coefficients in (5.5), (5.12) are most sensitive to the gluino mass $m_{\tilde{g}}$ and the mass splitting $\Delta_{\tilde{d}}$, because of the loop function difference $f(x_{\tilde{b}_R}, \dots) - f(x_{\tilde{d}_R}, \dots)$ appearing almost everywhere in $\tilde{C}_{3,\dots,6}^q$. We therefore perform a more detailed analysis of the dependence of the short-distance coefficients on $m_{\tilde{g}}$ and $m_{\tilde{d}}$, while we fix the other parameters to some values, as suggested by phenomenological considerations. The parameter a_d^1 , for instance, is constrained by the requirement of avoiding the scalar fermion acquiring VEVs, because this would lead to charge and colour breaking. One finds [284] $-2.5 < a_d^1/m_{\tilde{d}} < 2.5$, and here we will choose $a_d^1/m_{\tilde{d}} = 1.8$, following [287]. Moreover, we assume $\arg(\mu) = 0$. These conditions give a rather large allowed space for $m_{\tilde{g}}$ and $m_{\tilde{d}}$

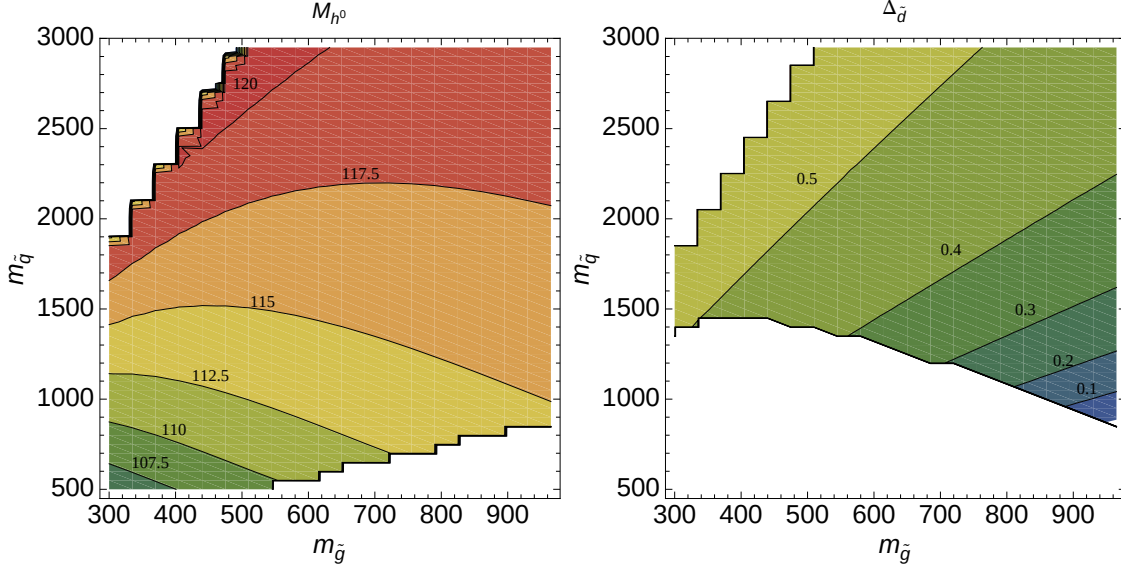


Figure 5.2: Value of the lightest Higgs boson mass in the $m_{\tilde{g}}$ - $m_{\tilde{d}}$ plane, left, and of the mass splitting $\Delta_{\tilde{d}}$, in the same plane, right. In the left plot excluded regions correspond to negative squark and sleptons masses; in the right plot excluded regions include the LEP bound on the lightest Higgs boson mass.

around 1 TeV. Finally, we take $m_{\tilde{u}} = m_{\tilde{d}}$ and $\tan \beta = 5$. This value is justified because too small values of $\tan \beta$, $\tan \beta \leq 3$, are forbidden by the bounds on the light Higgs boson mass, $M_{h^0} \geq 114.4$ GeV; on the other side, $\tan \beta$ cannot be too large in this model, in order to have a large top Yukawa coupling, and to preserve the hierarchy between the top and bottom Yukawa couplings.

In [287] it was found that a strong constraint on the parameter space is given by the bounds on the light Higgs boson mass; other processes reflecting the large atmospheric neutrino mixing angles, like $\tau \rightarrow \mu \gamma$, $b \rightarrow s \gamma$ give some additional constraints. In figure 5.2 we show in the left plot the predicted value of the light Higgs boson mass in the plane $m_{\tilde{d}}$ - $m_{\tilde{g}}$, for $(300 < m_{\tilde{g}} < 1000)$ GeV, $(500 < m_{\tilde{d}} < 3000)$ GeV: the constraint $M_{h^0} \geq 114.4$ GeV forbids gluino masses $m_{\tilde{g}} \leq 300$ GeV, and down squark masses $m_{\tilde{d}} \leq 1$ TeV. The excluded region on the bottom right corner and on the left upper corner are excluded, because in these regions the squark masses become negative. In the right plot we show the corresponding mass splitting $\Delta_{\tilde{d}}$, without the region excluded by the constraints. Large values of $\Delta_{\tilde{d}}$ are possible for small values of the gluino mass, and medium - large values of the down quark masses. These results agrees with [287].

The analysis of the loop functions entering $\tilde{C}_{3,\dots,6}^q$ gives the following results: first, $L_{3,\dots,6}(x, y)$ and $P_0(x)$ range from some per mill to some per cent, in the parameter space region considered. Most important, the functions $L_{3,4,5}(x, y)$ and $L_6(x, y)$, $P_0(x)$, have opposite sign. As a result, $L_{3,5}(x, y)$ interfere destructively with $P_0(x)$ in $\tilde{C}_{3,5}^q$, $L_6(x, y)$ interferes destructively with $-3P_0(x)$ in $\tilde{C}_6^q$, while $L_4(x, y)$ interferes constructively with $-3P_0(x)$ in $\tilde{C}_4^q$. Because of these patterns, $\tilde{C}_{3,5}^q$ turn out to be very small; the cancellation

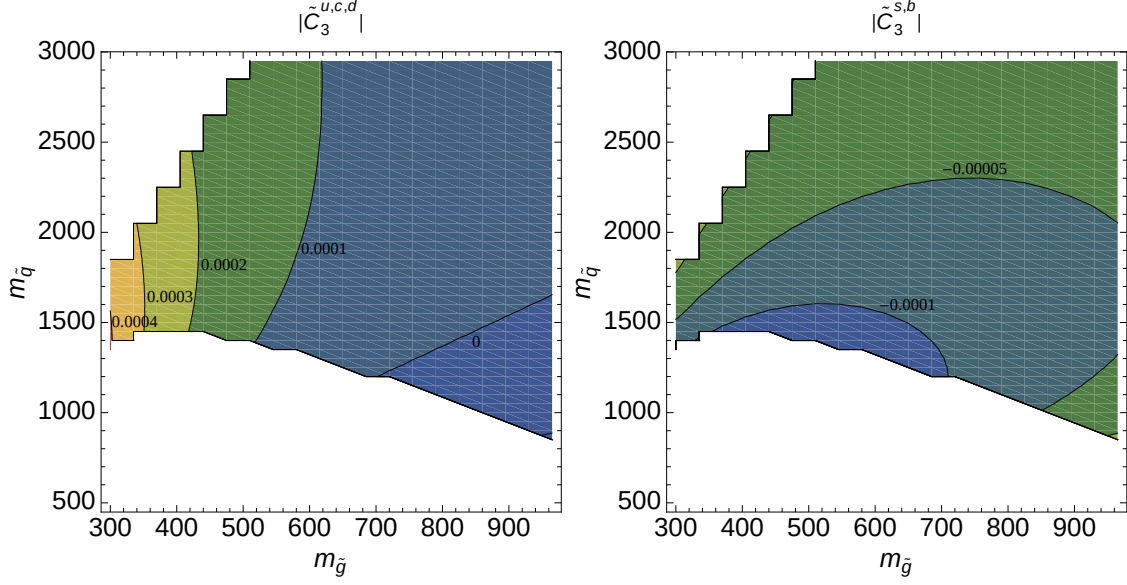


Figure 5.3: Absolute value of the short-distance coefficients $\tilde{C}_3^q$, at the high scale M_Z .

in $\tilde{C}_6^q$ is partially counterbalanced by the factor 3 multiplying the penguin function, so that still $\tilde{C}_6^q \gg \tilde{C}_{3,5}^q$. $\tilde{C}_4^q$ is the largest short-distance coefficient.

With these elements at hands we can return to the the issue of isospin violation. It is now clear that this is expected to be a minor effect: because of the assumption $\theta = 0$, in fact, there is only one additional contribution to $\tilde{C}_{3,4}^{s,b}$ with respect to $\tilde{C}_{3,4}^d$, which is anyway small; on the other hand, because of the assumption $m_{\tilde{u}} = m_{\tilde{d}}$ at the high scale, the box functions $L_i(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{u}_{RJ}\tilde{g}})$ and $L_i(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{d}_{RJ}\tilde{g}})$ are almost equal, eliminating the possibility of having sizeable isospin violation between the up-type short-distance coefficients $\tilde{C}_i^u$ and the down type ones $\tilde{C}_i^d$. Of course, the condition $m_{\tilde{u}} = m_{\tilde{d}}$ is suggested by the idea of choosing a universal soft quark mass at the high scale, but it is not imposed by any GUT relation, or by phenomenological constraints. It would therefore be interesting to investigate the effect of having $m_{\tilde{u}} \neq m_{\tilde{d}}$. Because this possibility is not (yet) fully implemented in the *Mathematica* code of [283–286], we leave the analysis for a future work. Here we note only that, because of the interference pattern between the penguin and the box function, the effect of having $m_{\tilde{u}} \neq m_{\tilde{d}}$ should be not too sizeable. On the other hand, an analysis of a similar scenario, in which one has new physics with isospin violation between the up and down type short-distance coefficients, is analysed in chapter 6, where we consider the possibility of having new physics in the electroweak penguin sector.

This analysis shows that, within the standard conditions usually considered in minimal supergravity scenarios, like the assumption of a universal soft mass for the squark at high energy, there is no room for a large isospin violation in the model. Since the various $\tilde{C}_{iq}$ with same i but different q are almost equal, their effects sum up, giving a total contribution which has the structure of the mirror QCD penguins. They have therefore to compete with the Standard Model QCD penguin, whose magnitude is one order of

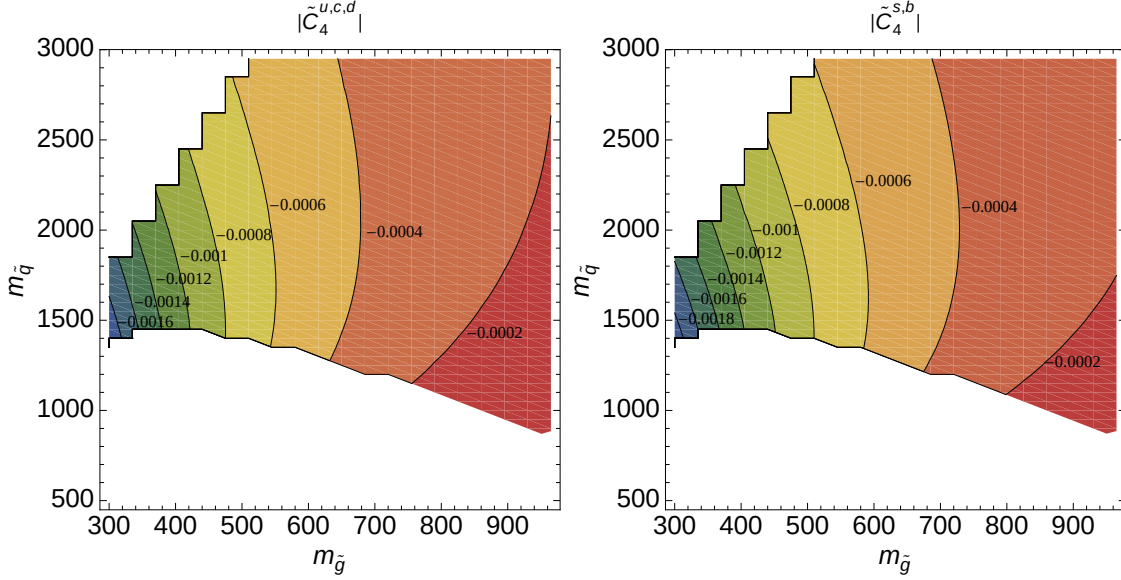


Figure 5.4: Absolute value of the short-distance coefficients $\tilde{C}_4^q$, at the high scale M_Z .

magnitude larger. One has however to keep in mind the possibility of having a large new weak phase associated with the new short-distance coefficients, which is most likely to reveal its presence in CP asymmetry observables. As a visual summary, in pictures 5.3 - 5.7 we show the absolute value of the short-distance coefficients in the $m_{\tilde{d}} - m_{\tilde{g}}$ plane.

Renormalization group evolution

The short distance coefficients, calculated so far at the high scale $\mu_S = M_Z^4$ must be evaluated at the scale $\mu = m_b$ by means of the renormalization group equations. Because of the small isospin violation, it would be a good approximation to combine $\tilde{C}_i^q$ into QCD-like mirror operators; then, one could use immediately the anomalous dimension (up to the NLL level) of the corresponding QCD penguin operators to evolve $\tilde{C}_i$ to the low scale. We prefer however to provide a more general result, deriving the leading logarithmic (LL) anomalous dimension for $\tilde{Q}_{3,\dots,6}^q$. This will be useful in view of an analysis where a larger isospin violation is present, *e.g.* in case of $m_{\tilde{u}} \neq m_{\tilde{d}}$.

Each pair of operators (Q_3^q, Q_4^q) , (Q_5^q, Q_6^q) evolves independently under the renormalization group evolution. Their anomalous dimension at LL reads (in units of $\alpha_s/(4\pi)$)

$$\gamma_{2 \times 2}^{3,4} = \begin{pmatrix} -\frac{6}{N_c} & 6 \\ 6 & -\frac{6}{N_c} \end{pmatrix}, \quad (5.14)$$

⁴In fact, it would be more correct to consider $\mu_S = \mathcal{O}(m_{\tilde{q}_{LI}}, m_{\tilde{d}_{RI}}, m_{\tilde{u}_{RI}}, m_{\tilde{g}}) \sim 500$ GeV, as stated in the previous section. We assume here $\mu_S = M_Z$, following the *Mathematica* code of [283–286], which gives as output the MSSM parameters at the scale $\mu_S = M_Z$. This choice leaves out only a potential mixing of the operators between $\mu = 500$ GeV and $\mu = M_Z$, which however should be small, since $\alpha_s(M_Z)/\alpha_s(500 \text{ GeV}) \simeq 1.26$.

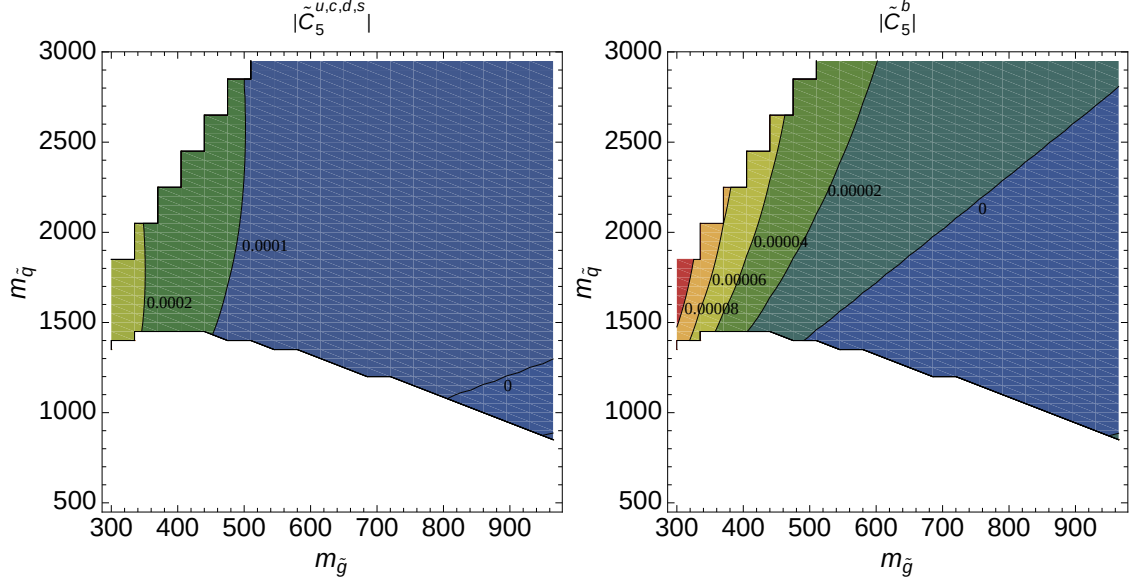


Figure 5.5: Absolute value of the short-distance coefficients $\tilde{C}_5^q$, at the high scale M_Z .

for (Q_3^q, Q_4^q) , while in case of (Q_5^q, Q_6^q) we have

$$\gamma_{2 \times 2}^{5,6} = \begin{pmatrix} \frac{6}{N_c} & -6 \\ 0 & -6N_c + \frac{6}{N_c} \end{pmatrix}. \quad (5.15)$$

With $\alpha_s(m_b)/\alpha_s(M_Z) \simeq 1.88$, we obtain

$$\begin{aligned} \tilde{C}_3^q(m_b) &\simeq 1.12 \tilde{C}_3^q(M_Z) - 0.28 \tilde{C}_4^q(M_Z), \\ \tilde{C}_4^q(m_b) &\simeq -0.28 \tilde{C}_3^q(M_Z) + 1.12 \tilde{C}_4^q(M_Z), \\ \tilde{C}_5^q(m_b) &\simeq 0.92 \tilde{C}_5^q(M_Z), \\ \tilde{C}_6^q(m_b) &\simeq 0.35 \tilde{C}_5^q(M_Z) + 1.96 \tilde{C}_6^q(M_Z). \end{aligned} \quad (5.16)$$

As in chapter 5 we can use the 2-loop NDR scheme anomalous dimension matrix (ADM) [53, 263],

$$\gamma_{2 \times 2}^{3,4;2\text{-loops}} = \begin{pmatrix} -\frac{21}{2} - \frac{2f}{9} & \frac{7}{2} + \frac{2f}{3} \\ \frac{7}{2} + \frac{2f}{3} & -\frac{21}{2} - \frac{2f}{9} \end{pmatrix}, \quad (5.17)$$

$$\gamma_{2 \times 2}^{5,6;2\text{-loops}} = \begin{pmatrix} \frac{137}{6} + \frac{15}{2N_c^2} - \frac{22}{3N_c}n_f & -\frac{100}{3}N_c + \frac{3}{N_c} + \frac{22}{3}n_f \\ -\frac{71}{2}N_c - \frac{18}{N_c} + 4n_f & -\frac{203}{6}N_c^2 + \frac{479}{6} + \frac{15}{2N_c^2} + \frac{10}{3}N_cn_f - \frac{22}{3N_c}n_f \end{pmatrix}, \quad (5.18)$$

to obtain

$$\begin{aligned} \tilde{C}_3^q(m_b) &\simeq 1.11 \tilde{C}_3^q(M_Z) - 0.25 \tilde{C}_4^q(M_Z), \\ \tilde{C}_4^q(m_b) &\simeq -0.25 \tilde{C}_3^q(M_Z) + 1.11 \tilde{C}_4^q(M_Z), \end{aligned}$$

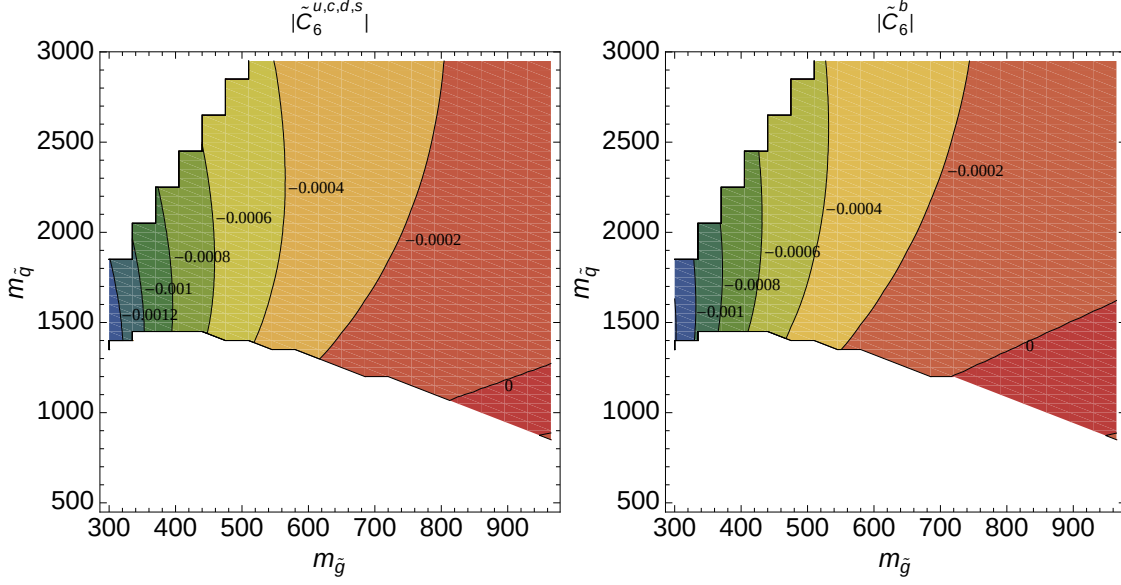


Figure 5.6: Absolute value of the short-distance coefficients $\tilde{C}_6^q$, at the high scale M_Z .

$$\begin{aligned}\tilde{C}_5^q(m_b) &\simeq 0.93 \tilde{C}_5^q(M_Z) + 0.07 \tilde{C}_6^q(M_Z), \\ \tilde{C}_6^q(m_b) &\simeq 0.37 \tilde{C}_5^q(M_Z) + 2.04 \tilde{C}_6^q(M_Z).\end{aligned}\tag{5.19}$$

The 2-loop result differs less than 10% from the one loop result. Since it is not completely consistent, because we do not have the 1-loop correction to the initial conditions at the scale M_Z , we will not consider it. Next, we have to take into account that the new operators mix with the mirror QCD penguin operators. The structure of the ADM reads

$$\gamma = \begin{pmatrix} \gamma_{6 \times 6} & 0_{6 \times 6} & 0_{10 \times 6} & 0_{10 \times 6} \\ 0_{6 \times 6} & \tilde{\gamma}_{6 \times 6} & 0_{10 \times 6} & 0_{10 \times 6} \\ 0_{6 \times 10} & \tilde{\gamma}_{6 \times 10}^{34-p} & \gamma_{10 \times 10}^{34} & 0_{10 \times 10} \\ 0_{6 \times 10} & \tilde{\gamma}_{6 \times 10}^{56-p} & 0_{10 \times 10} & \gamma_{10 \times 10}^{56} \end{pmatrix}, \tag{5.20}$$

where we used the same notation as in (4.10). In this case $\gamma_{10 \times 10}^{34}$ is block diagonal, with five identical 2×2 blocks given by $\gamma_{2 \times 2}^{3,4}$; similarly, $\gamma_{10 \times 10}^{56}$ is block diagonal, with five identical 2×2 blocks given by $\gamma_{2 \times 2}^{5,6}$. We have 5 copies of each 2×2 matrix, because we have to take into account the possible 5 operators $\tilde{Q}_{3,\dots,6}^q$ with $q = u, d, s, c, b$. The matrices $\tilde{\gamma}_{6 \times 10}^{34-p}$ and $\tilde{\gamma}_{6 \times 10}^{56-p}$ describe the mixing of the operators $\tilde{Q}_{3,\dots,6}^q$ into the mirror QCD penguin operators. We find in this case that no mixing with the Standard Model QCD penguin arises. One has $[\tilde{\gamma}_{6 \times 10}^{34-p}]^T = (\Gamma_{34}^T | \Gamma_{34}^T | \Gamma_{34}^T | \Gamma_{34}^T | \Gamma_{34}^T)$, $[\tilde{\gamma}_{6 \times 10}^{56-p}]^T = (\Gamma_{56}^T | \Gamma_{56}^T | \Gamma_{56}^T | \Gamma_{56}^T | \Gamma_{56}^T)$, with

$$\Gamma_{34} = \begin{pmatrix} 0 & 0 & -\frac{4}{3N_c} & -\frac{4}{3} & -\frac{4}{3N_c} & -\frac{4}{3} \\ 0 & 0 & -\frac{2}{3N_c} & -\frac{2}{3} & -\frac{2}{3N_c} & -\frac{2}{3} \end{pmatrix}, \tag{5.21}$$

$$\Gamma_{56} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3N_c} & -\frac{1}{3} & \frac{1}{3N_c} & -\frac{1}{3} \end{pmatrix}. \tag{5.22}$$

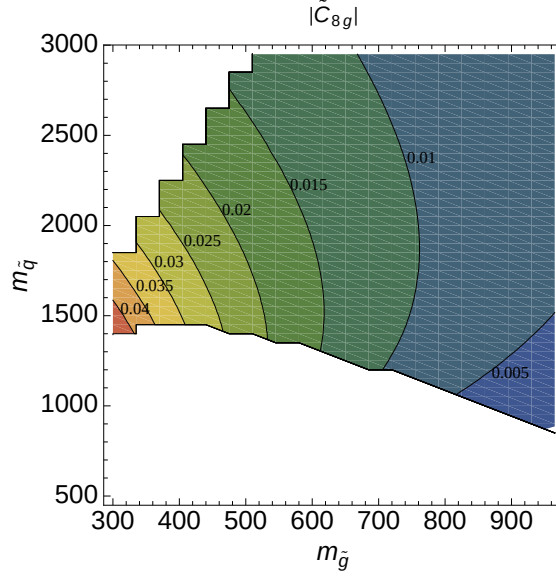


Figure 5.7: Absolute value of the short-distance coefficients $\tilde{C}_{8g}^q$, at the high scale M_Z .

Solving the full RGE leaves (5.16) unchanged and generates the mirror QCD penguin operators with short-distance coefficients

$$\begin{aligned} \tilde{C}_i(m_b) \approx C_i^{\text{SM}}(m_b) & \left(1.87 \sum_q \tilde{C}_3^q(M_Z) + 0.75 \sum_q \tilde{C}_4^q(M_Z) \right. \\ & \left. - 0.07 \sum_q \tilde{C}_5^q(M_Z) - 0.67 \sum_q \tilde{C}_6^q(M_Z) \right), \quad i = 3 \dots 6. \end{aligned} \quad (5.23)$$

Since both the Standard Model QCD penguin coefficients and $\tilde{C}_{3,\dots,6}^q(M_Z)$ are small, the penguin mixing effects are small. We include them here in order to have cancellation of the scale dependence with the corresponding NLO contribution to the matrix elements of the QCD mirror operators.

We conclude this section considering the evolution equation for $\tilde{C}_{7\gamma}$ and $\tilde{C}_{8g}$. We set $\tilde{C}_{7\gamma}$ to zero at the high scale, because we didn't calculate the photon penguin, and in the literature evaluations exist only in the mass insertion approximation, which may not be good here, due to the large mixing among squarks. Even if a precise evaluation of $\tilde{C}_{7\gamma}$ would be interesting in view of the enhanced contribution to $B \rightarrow VV$ decays, we leave this calculation for a future work. We note however that, in general, $\tilde{C}_{7\gamma}$ has a small effect on hadronic B decays, since it gets suppressed by $\alpha_{\text{em}}/\alpha_s$.

Because of the mixing with $\tilde{C}_{8g}$, and since $\tilde{C}_1(M_Z) = 0$, we get at the low scale [53]

$$\tilde{C}_{7\gamma}(m_b) = \frac{8}{3} \left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}} \right) \tilde{C}_{8g}(M_Z), \quad (5.24)$$

with $\eta \equiv \frac{\alpha_s(M_Z)}{\alpha_s(m_b)}$. Similarly, we obtain

$$\tilde{C}_{8g}(m_b) = \eta^{\frac{14}{23}} \tilde{C}_{8g}(M_Z), \quad (5.25)$$

so that $\tilde{C}_{8g}(m_b) < \tilde{C}_{8g}(M_Z)$.

Constraints from ΔM_s

In the CMM model large flavour effects are already constrained by the bounds on the particle spectrum [279–281, 283–287]. Once these limits are taken into account, the predicted flavour violation in processes like the $b \rightarrow s\gamma$ and $\tau \rightarrow \mu\gamma$ decays can be reduced below the current experimental level by considering minor cuts in the parameter space. In this context, we concluded that effects in non-leptonic decays are expected to be small. It is therefore important to check whether constraints from other flavour processes reduce further modifications in these decays. We focus here only on the $B_s - \bar{B}_s$ mass difference, because this process involves diagrams similar to those appearing in non-leptonic B decays. We leave for a future work a more thorough comparison with the $b \rightarrow s\gamma$ and $\tau \rightarrow \mu\gamma$ decays. Correlation with other processes is not immediate as in the large $\tan\beta$ scenario. Here we cannot relate directly the short distance coefficients of non-leptonic and other flavour processes, because diagrams with gluinos and squarks in the loops do not enter *e.g.* in leptonic or semi-leptonic decays. It is however possible to compare results of different processes by means of the RGE, starting from the same set of fundamental parameters. In case of the $b \rightarrow s\gamma$ and $\tau \rightarrow \mu\gamma$ decays we take into account the constraints obtained in [279–281, 283–285, 287] whose study is still in progress [286].

For a comprehensive review of the $B_s - \bar{B}_s$ mixing we refer to [291, 292]. We provide here only the basic elements to calculate the mass difference. We follow the analysis developed in [287]. The oscillation of a $P^0 - \bar{P}^0$ meson system can be described by a Schrödinger-type equation

$$i\frac{d}{dt} \begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} = \left[M^P - \frac{i}{2}\Gamma^P \right] \begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix}, \quad (5.26)$$

where M^P and Γ^P are two 2×2 hermitian matrices. They describe the four transitions $P^0/\bar{P}^0 \rightarrow P^0/\bar{P}^0$ via virtual and physical intermediate states, respectively. The physical states $|P_1^0\rangle$ and $|P_2^0\rangle$ are obtained by diagonalizing $M^P - \frac{i}{2}\Gamma^P$. Here we are interested in the local contribution to the off-diagonal element of M^P ,

$$M_{12}^P = \frac{1}{2M_P} \langle P^0 | \mathcal{H}_{\text{eff}}^{\Delta F=2} | \bar{P}^0 \rangle, \quad (5.27)$$

because defining the B_s mass difference as $\Delta M_s = M_H^s - M_L^s$, one has

$$\Delta M_s = 2|M_{12}^{B_s}|; \quad (5.28)$$

in 5.27 M_P indicates the average meson mass $(M_{P_L} + M_{P_H})/2$.

The effective Hamiltonian $\mathcal{H}_{\text{eff}}^{\Delta F=2}$ is given by a sum over effective operators and in general it can be written in terms of eight effective operators,

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \frac{G_F^2 M_W^2}{16\pi^2} \sum_{i=1}^8 C_P^i(\mu_P) Q_P^i(\mu_P), \quad (5.29)$$

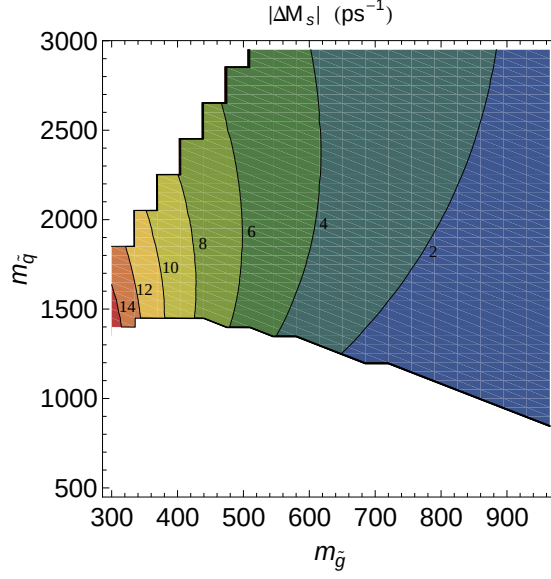


Figure 5.8: Absolute value of the new contribution to ΔM_s^{CMM} in the CMM model.

defined *e.g.* in [251]. Here $\mu_P \sim M_P$.

In the Standard Model only one of these operator contributes,

$$Q_{B_s}^{\text{VLL}} = (\bar{q}_L \gamma_\mu b_L)(\bar{q}_L \gamma_\mu b_L), \quad (5.30)$$

that originates from box diagrams with the exchange of a W bosons and top quarks. The corresponding short-distance coefficient reads

$$C_{B_s}^{\text{VLL}}(\mu_{B_s}) = 4U_{B_s}(\mu_{B_s})(\lambda_t^s)^2 \eta_B S_0(x_t), \quad (5.31)$$

where the factor

$$U_{B_s}(\mu) = [\alpha_s(\mu)]^{-6/23} \left[1 + \frac{\alpha_s(\mu)}{4\pi} J_5 \right] \quad (5.32)$$

encodes the (μ_{B_s}) -dependent part of the short-distance QCD correction up to NLO, while η_B accounts for their (μ_{B_s}) -independent contribution. For the definition of all the other parameters entering in (5.32), including their numerical value and the loop function $S_0(x_t)$, we refer to [287], according to which we choose our inputs. The matrix element of $Q_{B_s}^{\text{VLL}}$ is given by

$$\langle B_s^0 | Q_{B_s}^{\text{VLL}} | \bar{B}_s^0 \rangle = \frac{2}{3} M_{B_s}^2 f_{B_s}^2 B_{B_s}(\mu), \quad (5.33)$$

where f_{B_s} is the decay constant of the meson. The scale dependence of $U_{B_s}(\mu)$ cancels that of $B_{B_s}(\mu)$; one introduces therefore the renormalization group invariant parameters $\hat{B}_{B_s} = U_{B_s}(\mu) B_{B_s}(\mu)$. This results in

$$(M_{12}^{B_s})^{\text{SM}} = \frac{G_F^2 M_W^2}{12\pi^2} M_{B_s} f_{B_s}^2 \hat{B}_{B_s} (\lambda_t^s)^2 \eta_B S_0(x_t). \quad (5.34)$$

In the CMM model the dominant new contribution is given by gluino box diagrams, like the ones in figure 5.1 (a) and (b) where the s_R and b_R quarks appear as external states in the lower line. This contribution gives rise to the operator

$$Q_{B_s}^{\text{VRR}} = (\bar{q}_R \gamma_\mu b_R)(\bar{q}_R \gamma_\mu b_R). \quad (5.35)$$

The corresponding short-distance coefficient at the high scale μ_S reads

$$C_{B_s}^{\text{CMM}}(\mu_S) = \frac{16\pi^2}{G_F^2 M_W^2} \frac{\alpha_s^2(M_S)}{2m_{\tilde{g}}^2} \sum_{I,J=1}^3 (\mathbf{R}_d)_{Ib} (\mathbf{R}_d)_{Id}^* (\mathbf{R}_d)_{Jb} (\mathbf{R}_d)_{Jb}^* L_0(x_{\tilde{d}_{RI}\tilde{g}}, x_{\tilde{d}_{RJ}\tilde{g}}), \quad (5.36)$$

and the loop function reads

$$L_0(x, y) = \frac{11}{18} G(x, y) - \frac{2}{9} F(x, y). \quad (5.37)$$

Because of the absence of a penguin contribution and the different box function it is not possible to correlate directly ΔM_{B_s} with the non-leptonic short-distance coefficients. Therefore, we compare them using the same set of basic parameters. The short-distance coefficients in (5.36) can be rearranged as in (5.12), exploiting the mass degeneracy of the first two generations, and the unitarity of $\mathbf{R}_d$:

$$C_{B_s}^{\text{CMM}}(\mu_S) = \frac{16\pi^2}{G_F^2 M_W^2} \frac{\alpha_s^2(M_S)}{2m_{\tilde{g}}^2} [(\mathbf{R}_d)_{3b} (\mathbf{R}_d)_{3d}^*]^2 \cdot \left\{ L_0(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - 2L_0(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{b}_{R\tilde{g}}}) + L_0(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{b}_{R\tilde{g}}}) \right\}, \quad (5.38)$$

then the new contribution to ΔM_s in the CMM model reads

$$(M_{12}^{B_s})^{\text{CMM}} = \frac{\alpha_s^2(M_S)}{6m_{\tilde{g}}^2} M_{B_s} f_{B_s}^2 \hat{B}_{B_s} \frac{e^{-2i\phi_{B_s}} \cos^2 \theta}{4} \frac{\eta_B \eta_6}{r} \cdot \left\{ L_0(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{d}_{R\tilde{g}}}) - 2L_0(x_{\tilde{d}_{R\tilde{g}}}, x_{\tilde{b}_{R\tilde{g}}}) + L_0(x_{\tilde{b}_{R\tilde{g}}}, x_{\tilde{b}_{R\tilde{g}}}) \right\}, \quad (5.39)$$

where the additional factor $\eta_B \eta_6 / r$ arises when expressing the short-distance coefficient at the scale m_{B_s} ,

$$C_{B_s}^{\text{CMM}}(\mu_{B_s}) = U_{B_s}(\mu_{B_s}) \frac{1}{r} \eta_6 C_{B_s}^{\text{CMM}}(\mu_S), \quad (5.40)$$

see [287] for further details. In figure 5.8 we show ΔM_s in the $m_{\tilde{d}} - m_{\tilde{g}}$ plane, assuming that only $(M_{12}^{B_s})^{\text{CMM}}$ contributes to it, *i.e.* $\Delta M_s^{\text{CMM}} = 2|(M_{12}^{B_s})^{\text{CMM}}|$. In the region where the non-leptonic short-distance coefficients are large, ΔM_s^{CMM} is comparable in size with the Standard Model contribution, $\Delta M_s^{\text{SM}} = 18.42 \text{ ps}^{-1}$. Despite the strong experimental constraint on ΔM_s , a new contribution as large as the Standard model one is still possible, because of the relative phase between the two terms. Depending on the set of parameters used, only a constraint on the phase $|2\phi_{B_s}|$ can arise. Here we consider the experimental results

$$\Delta M_s^{\text{exp}} = (17.78 \pm 0.12) \text{ ps}^{-1}, \quad \phi_s^{\text{exp}} = (-0.77_{-0.37}^{+0.29}) \cup (-2.36_{-0.29}^{+0.37}) \text{ rad}, \quad (5.41)$$

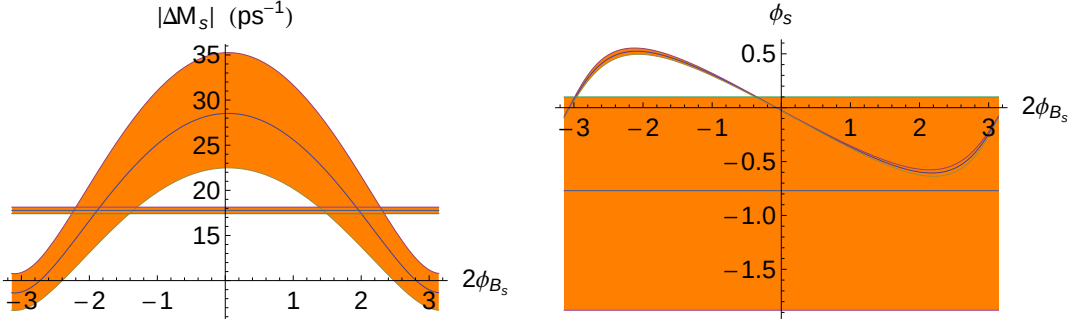


Figure 5.9: Comparison between theoretical prediction and experimental result for ΔM_s and ϕ_s . The band around the experimental result correspond to the 3σ uncertainty; the theoretical error band represents the 1σ uncertainty.

where the phase is defined as

$$\phi_s = \arg \frac{M_{12}^{B_s}}{(M_{12}^{B_s})^{\text{SM}}}, \quad (5.42)$$

and

$$\Delta M_s = 2|(M_{12}^{B_s})^{\text{SM}} + (M_{12}^{B_s})^{\text{CMM}}|. \quad (5.43)$$

In figure 5.9 we show ΔM_s and ϕ_s as a function of $2\phi_{B_s}$, compared with the experimental results within its 3 sigma range. In figure 5.9 we fix $m_{\tilde{g}} = 370$ GeV, $m_{\tilde{d}} = 1800$ GeV. The very narrow experimental uncertainty on ΔM_s is evident, which implies the constraint

$$(1.4 \lesssim |2\phi_{B_s}| \lesssim 2.4) \text{ rad}. \quad (5.44)$$

The phase ϕ_s gives a weaker constraint. These results agree with [287], when compared with the values of $m_{\tilde{g}}$, $m_{\tilde{d}}$ used there. We will take the constraint (5.44) into account in our phenomenological analysis in section 5.4. Our conclusion is that there are no large additional constraints on the non-leptonic short-distance coefficients, at least from the ΔM_s mass difference. We note also that since the loop function L_0 appearing in $C_{B_s}^{\text{CMM}}$ does not depend on the up-type squark masses, ΔM_s is not able to give additional constraints in case of $m_{\tilde{u}} \neq m_{\tilde{d}}$. Since this condition affects only the quark sector, non-leptonic B decays remain the only low energy process that could test this possibility.

We conclude this section by giving, in table 5.1, the numerical values for the new short distance coefficient, at the low scale m_b , compatible with the constraint on the light Higgs boson mass. The value chosen corresponds, at the high scale, to the point $m_{\tilde{g}} = 370$ GeV, $m_{\tilde{d}} = 1800$ GeV considered above.

5.3 Hadronic matrix elements for $B \rightarrow M_1 M_2$

The matrix elements of the operators in (5.3) can be derived easily from the results obtained in chapter 4. In the following we assume $\theta = 0$, so that only $\Delta S = 1$ transitions are modified. Accordingly, the index D will be always $D = s$, and therefore we suppress

	$ \tilde{C}_3^{u,c,d} $	$ \tilde{C}_4^{u,c,d} $	$ \tilde{C}_5^{u,c,d,s} $	$ \tilde{C}_6^{u,c,d,s} $	$ \tilde{C}_{7\gamma} $
$\mu = m_b$	0.00062	-0.00134	0.00017	-0.00168	-0.0028
	$ \tilde{C}_3^{s,b} $	$ \tilde{C}_4^{s,b} $	$ \tilde{C}_5^b $	$ \tilde{C}_6^b $	$ \tilde{C}_{8g} $
$\mu = m_b$	0.00030	-0.00138	0.00005	-0.00157	-0.0198

Table 5.1: Absolute value of the short distance coefficients $\tilde{C}_i^q$ at the scale m_b . The full short-distance coefficients are obtained multiplying the modulus listed here by $e^{-i\phi_{B_s}}$.

it from the notation. Inserting it back in the equations is straightforward. The inclusion of the new current-like and the mirror QCD penguin operators can be taken into account defining the following transition operators:

$$\begin{aligned}
\mathcal{T}_A^p = & \delta_{pu} [\alpha_1(M_1 M_2) A([\bar{q}_s u][\bar{u} D]) + \alpha_2(M_1 M_2) A([\bar{q}_s D][\bar{u} u])] \\
& + [\alpha_3^p(M_1 M_2) + \tilde{\alpha}_3^p(M_1 M_2)] \sum_{q=u,d,s} A([\bar{q}_s D][\bar{q} q]) \\
& + [\alpha_4^p(M_1 M_2) + \tilde{\alpha}_4^p(M_1 M_2)] \sum_{q=u,d,s} A([\bar{q}_s q][\bar{q} D]) \\
& + \alpha_{3,\text{EW}}^p(M_1 M_2) \sum_{q=u,d,s} \frac{3}{2} e_q A([\bar{q}_s D][\bar{q} q]) \\
& + \alpha_{4,\text{EW}}^p(M_1 M_2) \sum_{q=u,d,s} \frac{3}{2} e_q A([\bar{q}_s q][\bar{q} D]) \\
& + \sum_{q=u,d,s} \tilde{\alpha}_{3q}^p(M_1 M_2) A([\bar{q}_s D][\bar{q} q]) \\
& + \sum_{q=u,d,s} \tilde{\alpha}_{4q}^p(M_1 M_2) A([\bar{q}_s q][\bar{q} D]), \tag{5.45}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{T}_B^p = & \delta_{pu} \left[b_1(M_1 M_2) \sum_{q'} B([\bar{u} q'][\bar{q}' u][\bar{D} b]) + b_2(M_1 M_2) \sum_{q'} B([\bar{u} q'][\bar{q}' D][\bar{u} b]) \right] \\
& + \left[b_3^p(M_1 M_2) + \tilde{b}_3^p(M_1 M_2) \right] \sum_{q,q'} B([\bar{q} q'][\bar{q}' D][\bar{q} b]) \\
& + \left[b_4^p(M_1 M_2) + \tilde{b}_4^p(M_1 M_2) \right] \sum_{q,q'} B([\bar{q} q'][\bar{q}' q][\bar{D} b]) \\
& + b_{3,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q} q'][\bar{q}' D][\bar{q} b]) \\
& + b_{4,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q} q'][\bar{q}' q][\bar{D} b])
\end{aligned}$$

$$\begin{aligned}
& + \delta_{pu} \left[b_{S1}(M_1 M_2) \sum_{q'} B([\bar{u}u][\bar{q}'q'][\bar{D}b]) + b_{S2}(M_1 M_2) \sum_{q'} B([\bar{u}D][\bar{q}'q'][\bar{u}b]) \right] \\
& + \left[b_{S3}^p(M_1 M_2) + \tilde{b}_{S3}^p(M_1 M_2) \right] \sum_{q,q'} B([\bar{q}D][\bar{q}'q'][\bar{q}b]) \\
& + \left[b_{S4}^p(M_1 M_2) + \tilde{b}_{S4}^p(M_1 M_2) \right] \sum_{q,q'} B([\bar{q}q][\bar{q}'q'][\bar{D}b]) \\
& + b_{S3,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}D][\bar{q}'q'][\bar{q}b]) \\
& + b_{S4,\text{EW}}^p(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q][\bar{q}'q'][\bar{D}b]) \\
& + \tilde{b}_{3q}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'D][\bar{q}b]) + \tilde{b}_{4q}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{D}b]) \\
& + \tilde{b}_{S3q}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'D][\bar{q}b]) + \tilde{b}_{S4q}^p(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{D}b]), \quad (5.46)
\end{aligned}$$

where the sum over q, q' is over u, d, s . The expression for the coefficients $\tilde{\alpha}_{3,4}^p(M_1 M_2)$ has been given in (4.30). The expression for $\tilde{\alpha}_{3q,4q}^p(M_1 M_2)$ is identical to (4.30), provided one substitutes $\tilde{\alpha}_{3,4}^p(M_1 M_2)$ with $\tilde{\alpha}_{3q,4q}^p(M_1 M_2)$. Comparing with (4.30), we observe that the matrix elements of the operators $\tilde{Q}_i^q$ obey the same modified helicity hierarchy (4.32) as the mirror QCD penguin ones, when $M_1 M_2 = VV$; this is a consequence of the fact that the two types of operators have the same Dirac structure, but only different flavour content.

The NLO expression of the coefficients $\tilde{a}_i^p$ has been given in (4.33); the one of the coefficients $\tilde{a}_{iq}^p$ is similar, but there are no penguin contraction contributions. One has

$$\begin{aligned}
\tilde{a}_{iq}^p(M_1 M_2) &= \left(\tilde{C}_i^q + \frac{\tilde{C}_{i\pm 1}^q}{N_c} \right) \tilde{N}_i(M_2) \\
&+ \frac{\tilde{C}_{i\pm 1}^q}{N_c} \frac{C_F \alpha_s}{4\pi} \left[\tilde{V}_i(M_2) + \frac{4\pi^2}{N_c} \tilde{H}_i(M_1 M_2) \right], \quad (5.47)
\end{aligned}$$

where the definition of the quantities $\tilde{N}_i(M_2)$, $\tilde{V}_i(M_2)$, $\tilde{H}_i(M_1 M_2)$ has been given in section 4.5. The insertion of the operators $\tilde{Q}_i^q$ into the diagrams of figure 4.4 gives an additional contribution to the penguin correction of the mirror QCD penguin operators, whose scale dependence cancels the one from the evolution of the mirror short-distance coefficient in (5.23). We find

$$\begin{aligned}
\delta \tilde{P}_4^p(M_1 M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left\{ \sum_{q=s,b} \tilde{C}_3^q \left[\frac{4}{3} \log \frac{m_b}{\mu} + \frac{2}{3} - G_{M_2}^f(s_q) \right] \right. \\
&\quad \left. + \sum_{q=u,d,s,c,b} \left(\tilde{C}_4^q + \tilde{C}_6^q \right) \left[\frac{4}{3} \log \frac{m_b}{\mu} - G_{M_2}^f(s_q) \right] \right\},
\end{aligned}$$

$$\delta\tilde{P}_6^p(M_1M_2) = \frac{C_F\alpha_s}{4\pi N_c} \left\{ \sum_{q=s,b} \tilde{C}_3^q \left[N_6(M_2) \frac{4}{3} \log \frac{m_b}{\mu} + \frac{2}{3} - \hat{G}_{M_2}^f(s_q) \right] \right. \\ \left. + \sum_{q=u,d,s,c,b} \left(\tilde{C}_4^q + \tilde{C}_6^q \right) \left[N_6(M_2) \frac{4}{3} \log \frac{m_b}{\mu} - \hat{G}_{M_2}^f(s_q) \right] \right\}, \quad (5.48)$$

where $s_q = 0$ for $q = u, d, s$, while $s_c = (m_c/m_b)^2$ and $s_b = 1$. $G_{M_2}^f(s)$ equals $G_{M_2}(s)$ of (2.91) for $M_1M_2 = PP, PV, VP, V^0V^0$, and $G_{M_2}^\mp(s)$ for $V^\pm V^\pm$, while $\hat{G}_{M_2}^f(s)$ equals $\hat{G}_{M_2}(s)$ of (2.95) for $M_1M_2 = PP, PV, VP, V^0V^0$, and is zero for $V^\pm V^\pm$. The annihilation contribution of the mirror QCD penguin operator has been given in (4.47); the annihilation contribution of the operators $\tilde{Q}_i^q$ has the same structure, provided one substitutes the coefficients $\tilde{b}_{3,4}^p(M_1M_2)$ with $\tilde{b}_{3q,4q}^p(M_1M_2)$, where the index q is added, to intend that the short-distance coefficients appearing there are the $\tilde{C}_{iq}$.

5.4 Non-leptonic decays

With the results of sections 5.2 and 5.3 at hand, we are now able to face the question whether the new contributions arising in the CMM model are observable in non-leptonic B decays. To this end, we compare the new amplitudes with those present in the Standard Model. The essential features can be deduced from (5.45), (5.46).

- In principle, each $\tilde{\alpha}_{3q,4q}^p$ is an isospin violating contribution, and it should compete with the electroweak penguin amplitude of the Standard Model. However, since the various $\tilde{\alpha}_{iq}^p$ with same i but different q are numerically very similar, their effect sums up, and in the end they give a contribution similar in structure to the mirror QCD penguin amplitudes. This is a direct consequence of the results found for the short-distance coefficients in section 5.2. Consequently, they have to be compared with the larger Standard Model QCD penguin amplitude α_3^p and α_4^p .
- The same is true for the “proper” mirror QCD penguin amplitudes $\tilde{\alpha}_{3,4}^p$, so that they have to be compared with α_3^p and α_4^p as well.
- Given the results illustrated in the two items above, one expects modifications mainly in QCD penguin dominated B decays. Recalling that the new contributions are present only in case of $\Delta S = 1$ transitions, this means that modifications could arise in case of $\bar{B} \rightarrow \pi \bar{K}$, $\bar{B} \rightarrow \bar{K} \eta^{(\prime)}$, $\bar{B} \rightarrow \bar{K} K$ decays, and the corresponding PV , VP and VV modes.
- The sensitivity to the new contributions could be enhanced by the right-handed nature of the transition involved. Because of this feature, in fact, one expects a different correlation between *e.g.* the branching ratios of decays into PP , PV , VP and VV final states, with respect to the opposite case in which the new physics would have the same helicity as the Standard Model operators. Moreover, the new contribution to the positive helicity amplitude in case of decays into VV final states is expected to be of the same order as the new contribution to the negative one, and one has to check if this can affect the angular observables of these decays.

- The important element to take into account in the analysis is the potentially large new weak phase. While the (CP averaged) branching ratios are not sensitive to a new phase, its effects are more likely to appear in the CP asymmetries. Because of the large uncertainties associated with the direct CP asymmetry, more interesting here is the time dependent CP asymmetry⁵ [282], which has a relatively small theoretical uncertainty, since it is proportional to the real part of the hadronic amplitude (6.12).

In the following we comment in more on detail the numerical results collected in tables 5.2 and 5.3 for the α_i coefficients. Numerical results are obtained using the parameters listed in table 2.2, or as otherwise explained in chapter 2. The short-distance coefficients are evaluated at the scale $\mu = m_b = 4.2$ GeV.

$B \rightarrow PP, PV$

In table 5.2 we show the numerical results of the α_i amplitude coefficients defined in (5.45) for the decay modes $\bar{B} \rightarrow K^{(*)}\eta^{(\prime)}, K\phi$, evaluated using the values of the short-distance coefficients collected in table 5.1. The values of the mirror QCD penguin short-distance coefficients can be derived using (5.23). Similar results can be found for the other relevant decay modes $\bar{B} \rightarrow \pi\bar{K}, \rho\bar{K}, \pi\bar{K}^*$. Since $|\tilde{C}_{4,6}^q| \gg |\tilde{C}_{3,5}^q|$, we obtain $\tilde{\alpha}_{4q}^p \gg \tilde{\alpha}_{3q}^p$. The effect of $\tilde{\alpha}_{3q}^p$ on the total amplitude is therefore negligible, being two orders of magnitude smaller than α_3^p , the Standard Model amplitude with the same flavour structure, and 3 orders of magnitude smaller than the dominant penguin amplitude α_4^p . The new contribution $\tilde{\alpha}_{4q}^p$ is of the same order of magnitude as the electroweak penguin short-distance coefficients, even if it is always smaller than the large electroweak coefficient $\alpha_{3,\text{EW}}^p$. As explained above, however, $\tilde{\alpha}_{4q}$ has to be compared with α_4 , which is always one order of magnitude larger. For this reason, small modifications are expected in the branching ratios, because the new amplitudes have to compete with the Standard Model amplitudes α_i^c , which in turn are multiplied by the large CKM factor $\lambda_c^{(s)}$. In case of time dependent CP asymmetries, however, because of the presence of the new weak phase, the new amplitudes $\tilde{\alpha}_i^c$ have to compete with the α_i^u amplitudes, which in turn are multiplied by the small CKM factor $\lambda_u^{(s)}$. This suppression compensates the difference between $\tilde{\alpha}_{4q}^p$ and α_4^p , making the time dependent CP asymmetries quite sensitive to the new amplitudes.

Since the factorization properties of the $\tilde{\alpha}_{4q}$ and α_4 are similar, the ratio between the two amplitudes stays fairly constant between the different PP, PV and VV final states, so that the same level of modification is expected in the various decay modes. In addition, we also note that the mirror QCD penguin amplitudes are always very small and negligible, as expected from (5.23), with the exception of $\tilde{\alpha}_4$ which is of the same order as $\tilde{\alpha}_{4q}$.

$B \rightarrow VV$

Results for the amplitude coefficients of the decays $\bar{B} \rightarrow K^*\phi$ ($\bar{B} \rightarrow \rho K^*$) are collected in table 5.3. In general, the new contributions follow the same patterns as for the PV decays. It is therefore more interesting to inspect the inverted helicity hierarchy of the

⁵both the direct and the time dependent CP asymmetries are defined in chapter 6, (6.7), (6.11) and (6.12).

	$\bar{K}\eta_s$	$\bar{K}^*\eta_s$	$\bar{K}\phi$
α_1	$0.966 + 0.021i$ [$\pi\bar{K}$]	$0.981 + 0.021i$ [$\rho\bar{K}$]	$0.973 + 0.021i$ [$\pi\bar{K}^*$]
α_2	$0.351 - 0.084i$ [$\bar{K}\pi$]	$0.260 - 0.084i$ [$\bar{K}^*\pi$]	$0.323 - 0.084i$ [$\bar{K}\rho$]
α_3^u	$-0.0013 + 0.0046i$	$0.0027 + 0.0046i$	$0.0006 - 0.0005i$
α_4^u	$-0.095 - 0.040i$	$0.038 + 0.008i$	$-0.031 - 0.017i$
$\alpha_{3,\text{EW}}^u$	$-0.0089 - 0.0002i$	$-0.0091 - 0.0002i$	$-0.0082 - 0.0001i$
$\alpha_{4,\text{EW}}^u$	$-0.0016 + 0.0006i$	$-0.0025 + 0.0008i$	$-0.0024 + 0.0007i$
$\tilde{\alpha}_3^u/e^{-i\phi_{B_s}}$	$0.00024 + 8.3 \cdot 10^{-7}i$	$0.00020 - 8.3 \cdot 10^{-7}i$	$-9.5 \cdot 10^{-7} + 4.9 \cdot 10^{-7}i$
$\tilde{\alpha}_4^u/e^{-i\phi_{B_s}}$	$-0.00190 - 0.00044i$	$0.00034 - 0.00009i$	$0.00115 + 0.00016i$
$\tilde{\alpha}_{3u}^u/e^{-i\phi_{B_s}}$	$-0.00006 - 0.00023i$	$0.00027 + 0.00023i$	$-0.00003 - 0.00003i$
$\tilde{\alpha}_{4u}^u/e^{-i\phi_{B_s}}$	$0.00360 + 0.00005i$	$0.00142 - 0.00005i$	$-0.00106 - 0.00006i$
$\tilde{\alpha}_{3d}^u/e^{-i\phi_{B_s}}$	$-0.00006 - 0.00023i$	$0.00027 + 0.00023i$	$-0.00003 - 0.00003i$
$\tilde{\alpha}_{4d}^u/e^{-i\phi_{B_s}}$	$0.00360 + 0.00005i$	$0.00142 - 0.00005i$	$-0.00106 - 0.00006i$
$\tilde{\alpha}_{3s}^u/e^{-i\phi_{B_s}}$	$0.00027 - 0.00023i$	$-0.00006 + 0.00023i$	$-0.00036 - 0.00002i$
$\tilde{\alpha}_{4s}^u/e^{-i\phi_{B_s}}$	$0.00378 + 0.00002i$	$0.00127 - 0.00002i$	$-0.00124 - 0.00003i$

Table 5.2: Numerical results for the α_i coefficients of some representative $\Delta S = 1$ decay channels. In case the amplitude coefficient does not exist for the decay considered, as in case e.g. of the tree amplitude α_1 , we provide results from the $\bar{B} \rightarrow \pi\bar{K}, \rho\bar{K}, \pi\bar{K}^*$ decays.

new transverse polarization amplitudes. In order to estimate the plus-helicity amplitude we take the upper limit of the form factor, $F_+^{BV_1} = 0.06$, as in section 4.6. We obtain that the coefficients $\tilde{\alpha}_i^+$, $\tilde{\alpha}_{iq}^+$ are larger than the corresponding ones in the PP , PV modes and the other helicity amplitudes. Moreover, $\tilde{\alpha}_{4q}^+$ is now of the same order as α_4^+ ; with the assumption above on $F_+^{BV_1}$, $\tilde{\alpha}_{4q}^+ \leq \alpha_4^+$. However, if a smaller form factor is taken, $\tilde{\alpha}_{4q}^+$ can easily become the largest penguin contribution to the amplitude. This would affect the azimuthal angular distribution of the $\Delta S = 1$ $B \rightarrow VV$ decays. However, since the transverse amplitudes are small in absolute terms and have large uncertainties, these effects are small, and probably unobservable. We leave a more precise analysis of the angular observables to a future work.

One can easily derive expressions for the branching ratios, CP asymmetries and other observables. In Fig. 5.10 and 5.11 we provide a few examples, which support the considerations written above. Modifications to the branching ratios are small, and always much smaller than the theoretical uncertainties, so that it is unlikely to be able to observe such effects in experiments. Larger modifications arise in case of the time dependent CP asymmetries. The golden channels here are the decays $\bar{B}^0 \rightarrow \bar{K}^0\eta'$, $\bar{B}^0 \rightarrow \bar{K}^0\phi$, whose time dependent CP asymmetry has the smallest theoretical uncertainty, and the modification due to the new contribution is expected to be of the same order, or even larger. The graphs represent the observables as a function of ϕ_{B_s} . One should note that, because of the constraints from ΔM_s , ϕ_{B_s} is constrained to lie in the range $(0.7 \lesssim |\phi_{B_s}| \lesssim 1.2) \cup (1.9 \lesssim |\phi_{B_s}| \lesssim 2.4)$

	$(\bar{K}^*\phi)^{00}$	$(\bar{K}^*\phi)^{--}$	$(\bar{K}^*\phi)^{++}$
$\alpha_1 [\rho\bar{K}^*]$	$0.987 + 0.021i$	$1.101 + 0.041i$	$1.077 + 0.0005$
$\alpha_2 [\bar{K}^*\rho]$	$0.240 - 0.084i$	$-0.173 - 0.169i$	$-0.082 + 0.001i$
α_3^u	$0.0001 - 0.0005i$	$-0.0023 - 0.0010i$	-0.0017
α_4^u	$-0.026 - 0.015i$	$-0.044 - 0.017i$	-0.034
$\alpha_{3,\text{EW}}^u$	$-0.0084 - 0.0001i$	$0.0044 - 0.0003i$	-0.0095
$\alpha_{4,\text{EW}}^u$	$-0.0017 + 0.0007i$	$0.0015 + 0.0014i$	0.0007
$\tilde{\alpha}_3^u$	$5.04 \cdot 10^{-7} - 4.89 \cdot 10^{-7}i$	$-1.10 \cdot 10^{-7}$	$-0.00002 - 0.00001i$
$\tilde{\alpha}_4^u$	$-0.00109 - 0.00018i$	$-0.00001 - 0.00002i$	$-0.00248 - 0.00174i$
$\tilde{\alpha}_{3u}^u$	$0.00005 + 0.00003i$	0.00001	$0.00192 + 0.00057i$
$\tilde{\alpha}_{4u}^u$	$0.00112 + 0.00004I$	0.00011	$0.01470 + 0.00104I$
$\tilde{\alpha}_{3d}^u$	$0.00005 + 0.00003I$	0.00001	$0.00192 + 0.00057I$
$\tilde{\alpha}_{4d}^u$	$0.00112 + 0.00004I$	0.00011	$0.01470 + 0.00104I$
$\tilde{\alpha}_{3s}^u$	$0.00038 + 0.00002I$	0.00004	$0.00553 + 0.00051I$
$\tilde{\alpha}_{4s}^u$	$0.00127 + 0.00001I$	0.00012	$0.01523 + 0.00051I$

Table 5.3: Numerical results for the α_i coefficients pertaining to the three helicity amplitudes of $\bar{B} \rightarrow VV$ decays. To compare the absolute values of the helicity amplitudes the numbers for the $(00, --, ++)$ parameters must be multiplied by $A_{K^*\phi} = \frac{iG_F}{\sqrt{2}} m_B f_\phi (m_B A_0^{B \rightarrow K^*}, m_\phi F_-^{B \rightarrow K^*}, m_\phi F_+^{B \rightarrow K^*})$. The estimates above use $F_+^{B \rightarrow V_1} = 0.06$ in order to compare with the maximal Standard Model $++$ amplitude. In case the amplitude coefficient does not exist for the decay considered we provide results from the $\bar{B} \rightarrow \rho\bar{K}^*$ decays. The Standard Model coefficients are equal to that in table 4.2, except in case of the positive helicity. In table 4.2 we provided the leading order expression for it, while here the full NLO result is considered, according to the definitions given in section 4.5.

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5.5 Conclusion

In this chapter we have considered a well motivated supersymmetric grand unified scenario, in which the large neutrino mixing angles give rise to a large mixing between the right-handed quarks. In the model, supersymmetry is a key ingredient, not only for the stabilization of the electroweak scale, but also because it allows the mixing among right-handed quarks to become physical, thanks to the flavour-changing couplings arising between quarks, squarks and gluinos. The construction of a whole chain of grand unified theories up to the Plank scale allows to describe the low energy observables in terms of a few fundamental parameters, so that the model is predictive and gives specific signatures and correlations. In this scenario, bounds from the particle spectrum, such as the lightest Higgs boson M_{h^0} , limit the possibility of having large flavour violation; however, sizeable effects are still expected, in particular in case of $\tau \rightarrow \mu\gamma$, $b \rightarrow s\gamma$ and ΔM_s . The large

$b_R - s_R$ mixing suggests possible large effects in non-leptonic decays too, in particular since a new weak phase arises. This motivation is at the basis of the present analysis. We compared non-leptonic decays with $\bar{B}_s$ meson mixing, and found that the latter does not constrain significantly new contributions in these decays. On the basis of the results found in [279–281, 283–287] we conclude that also $\tau \rightarrow \mu\gamma$, $b \rightarrow s\gamma$ decays do not give (yet) strong additional constraints, once the Higgs boson mass bound is taken into account. We find that the new contributions to non-leptonic B decays are predicted to be small. This is because the new short-distance coefficients appearing in the model can reach at most the order of magnitude of the electroweak penguin coefficients, but they exhibit a small isospin violation, so that they have to compete with the larger short-distance QCD penguin coefficients. We found that a larger isospin violation could arise when relaxing the condition $m_{\tilde{u}} = m_{\tilde{d}}$. We leave however the analysis of the case $m_{\tilde{u}} \neq m_{\tilde{d}}$ for a future work. Here we were interested in deriving results within the “standard” minimal supergravity scenario, which assumes a common soft mass for all the squark. In this set-up we expect small modifications of non-leptonic branching ratios, though we find larger deviations in case of the time dependent CP asymmetries, which are in reach of the future LHCb and B factories. Order one modifications can also arise for the $\bar{B} \rightarrow VV$ positive helicity amplitudes. In this case however the positive helicity amplitudes are themselves a small effect compared with the longitudinal helicity amplitude, and they are affected by large uncertainties, so that it is more difficult to find observables sensitive to this effect. We reserve to perform a more detailed analysis of the $\bar{B} \rightarrow VV$ decays angular observables in a future work.

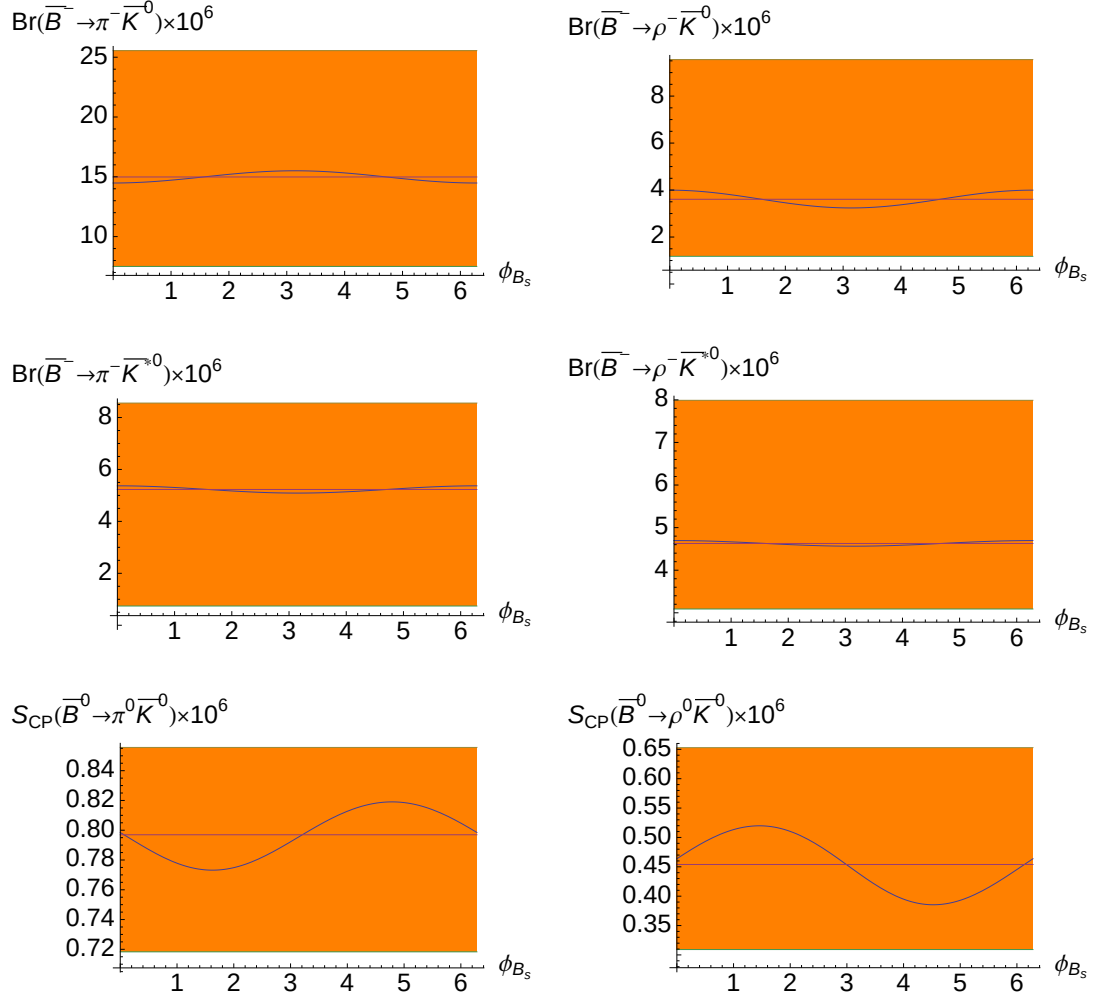


Figure 5.10: The first four pictures represent the branching ratios of the QCD penguin dominated modes $\bar{B} \rightarrow \pi^- \bar{K}^0, \rho^- \bar{K}^0, \pi^- \bar{K}^{*0}, \rho^- \bar{K}^{*0}$; the last two pictures represent the time dependent CP asymmetries of $\bar{B}^0 \rightarrow \pi^0 \bar{K}^0, \rho^0 \bar{K}^0$. The coloured band represent the theoretical error; the central line the Standard Model prediction; the wavy line the prediction in the CMM model.

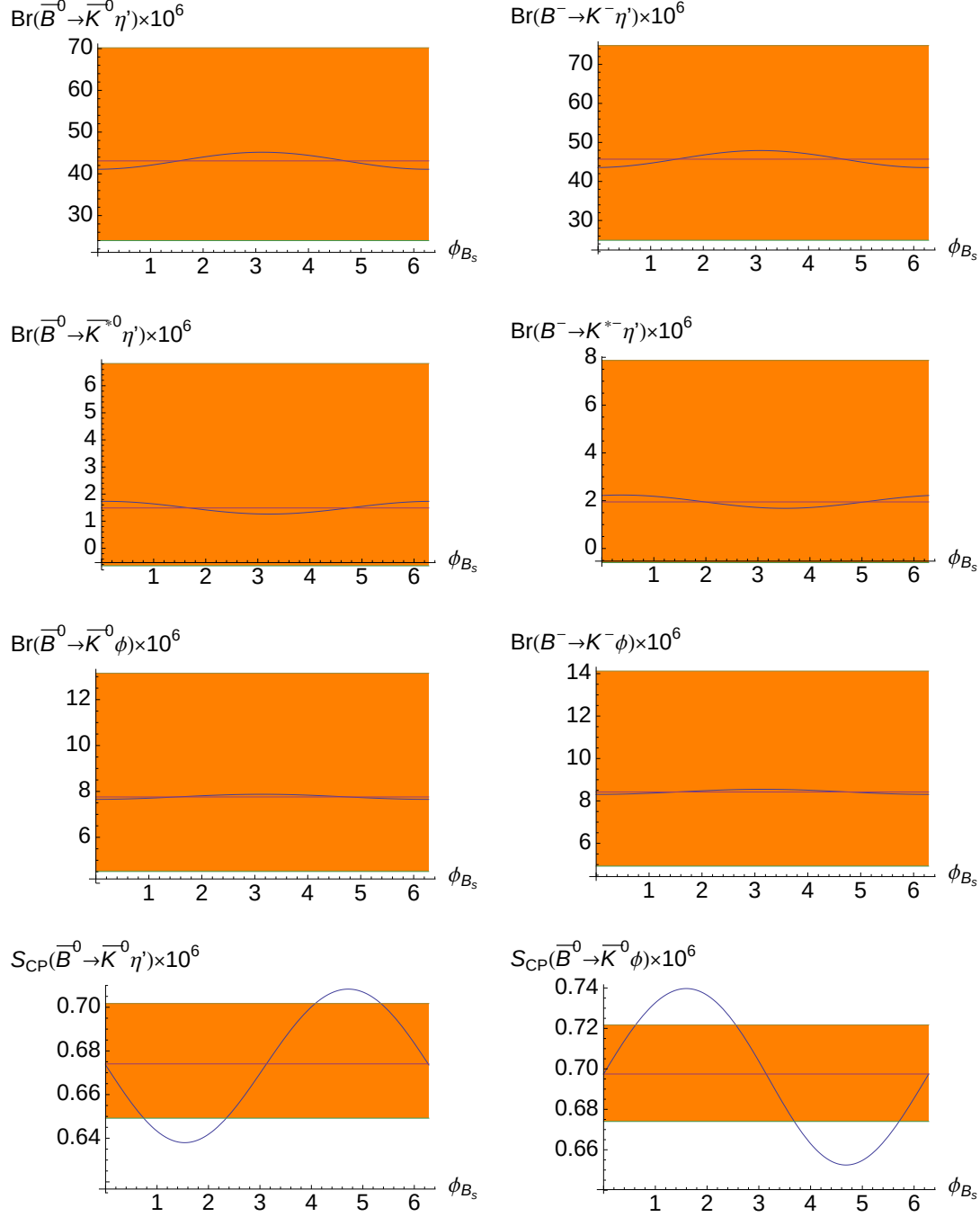


Figure 5.11: Branching ratios of the QCD penguin dominated modes $\bar{B}^0 \rightarrow \bar{K}^0 \eta', \bar{K}^{*0} \eta', \bar{K}^0 \phi$, left, and the corresponding charged modes $B^- \rightarrow K^- \eta', K^{*-} \eta', K^- \phi$, right. The last two pictures represent the time dependent CP asymmetries of $\bar{B}^0 \rightarrow \bar{K}^0 \eta', \bar{K}^0 \phi$, which show the largest deviation from the Standard Model prediction.

Chapter 6

Non-leptonic B decays with new physics in the electroweak penguin sector

6.1 Introduction

Another approach to the search for new physics in non-leptonic B decays is to look for actual discrepancies between the experimental data and the theoretical predictions. In this case one proceeds in a model independent way, trying to find patterns in the data, which may point to a particular new physics scenario. The main limitations of the approach are on one hand the statistics of the experimental data, which however is expected to improve as long as more data becomes available; on the other hand, the uncertainties on the theoretical predictions may set limits on the predictive power of the methods.

To clarify the last issue, we recall from chapter 2 that the most difficult part of the decay amplitude to be evaluated is the hadronic matrix element, which involves QCD interactions at or below the scale m_b . To overcome this problem, various methods have been developed. One of the most used in this type of analysis is the exploitation of QCD flavour symmetries, which was described briefly in chapter 1. Within this approach, one tries to relate the matrix elements of different decays, in order to extrapolate them from decays which are measured with greater precision, and that are supposed to receive no (or very small) new physics contributions, like the tree level dominated $\bar{B} \rightarrow \pi\pi$ decays. As anticipated in the introduction, the drawback of this method is that the matrix elements are not calculated from first principles: consequently, only a few sets of decay modes can be predicted, since, for instance, matrix elements of transitions to PP , PV and VV states cannot be related.

On the other hand, the method of QCD factorization used in this thesis has some limitations, too, as explained in chapter 2, because of the presence of large subleading contributions, which are not calculable within the method and need to be parameterized phenomenologically. Despite this problem, the QCD factorization approach allows one to make reliable predictions, because the orderliness of the method itself tell us in which

cases one should be careful and interpret the result with more regard, because *e.g.* a soft phase could arise, which would fake the presence of a new weak phase.

Taking these considerations into account, the purpose of this chapter is to perform a model independent analysis [128], using as a starting point the discrepancies between experimental data and theoretical predictions recently found in the $\bar{B} \rightarrow \pi \bar{K}$ decays. Many analyses were performed on these modes, using both SU(3) flavour symmetries and QCD factorization for the evaluation of the hadronic matrix elements, which point in the direction of new physics in the electroweak penguin sector [293–310]. However, the present experimental results do not allow us to draw firm conclusions. It is therefore important to consider other decays, which could also be affected by new physics in the electroweak penguin sector. To this class belongs *e.g.* the PV , VP , VV modes corresponding to the $\bar{B} \rightarrow \pi \bar{K}$ decays, *i.e.* the $\bar{B} \rightarrow \rho \bar{K}$, $\bar{B} \rightarrow \pi \bar{K}^*$, $\bar{B} \rightarrow \rho \bar{K}^*$ decays. These decays were analysed *e.g.* in [311, 312, 318], but the experimental data on these decays does not allow yet to clarify the picture. We point out here that there could be as well other interesting decays of the B_s meson to look for, namely the $\bar{B}_s \rightarrow \phi \pi$ and the $\bar{B}_s \rightarrow \phi \rho$ decays, which could give more insight. Contrary to the $\bar{B} \rightarrow \pi \bar{K}$ decays these modes are pure isospin-breaking transitions, and therefore they are expected to be very small within the Standard Model [313], with a branching ratio of order a few 10^{-7} [67, 179]. Their structure is very simple and they are the only non-leptonic B decays which are dominated by electroweak penguin contributions, so that they are very sensitive to new physics in this sector. Besides, they cannot be calculated by means of flavour symmetries, so that the QCD factorization method is here essential. While there are not yet experimental analyses on these decays, our work shows that their branching ratio could be 2 to 4 times larger than in the Standard Model, if new physics in the electroweak penguin sector is present. If confirmed, this prediction can help in clarifying the discrepancies found in the other decay modes.

6.2 Analysis of the $\bar{B} \rightarrow \pi \bar{K}$ modes

In recent years tensions were found, at the $\sim 2 \sigma$ level, between theoretical predictions and experimental results, in the $\bar{B} \rightarrow \pi \bar{K}$ decays. These differences, however, have decreased since the first time they were found, so that the picture is not yet clear, in order to claim for new physics. As a starting point, therefore, our aim is to review the theoretical prediction of QCD factorization for these decay modes, comparing them with the current experimental status. The main features of these decays can be understood in the approximation defined in [67], in which one neglects the annihilation contribution proportional to λ_u , all the electroweak penguin terms except $\alpha_{3,\text{EW}}^c$ and all the electroweak annihilation contributions. The first assumption is justified because the annihilation terms proportional to λ_u are strongly CKM suppressed with respect to the ones proportional to λ_c ; this amounts to set $\beta_1 = \beta_2 = \beta_3^u = \beta_4^u = 0$. Similarly, all the neglected terms in the electroweak penguin sector are strongly suppressed with respect to $\alpha_{3,\text{EW}}^c$. With this

approximation, the amplitudes of the $\bar{B} \rightarrow \pi \bar{K}$ decays read

$$\begin{aligned}
\mathcal{A}_{B^- \rightarrow \pi^- \bar{K}^0} &= A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c \left(1 + \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \frac{\hat{\alpha}_4^u}{\hat{\alpha}_4^c} e^{-i\gamma} \right), \\
\sqrt{2} \mathcal{A}_{B^- \rightarrow \pi^0 K^-} &= A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c \left(1 + \frac{A_{\bar{K}\pi} \frac{3}{2} \alpha_{3,\text{EW}}^c}{A_{\pi \bar{K}} \hat{\alpha}_4^c} \right. \\
&\quad \left. + \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \frac{A_{\pi \bar{K}} (\alpha_1 + \hat{\alpha}_4^u) + A_{\bar{K}\pi} \alpha_2}{A_{\pi \bar{K}} \hat{\alpha}_4^c} e^{-i\gamma} \right), \\
\mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ K^-} &= A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c \left(1 + \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \frac{(\alpha_1 + \hat{\alpha}_4^u)}{\hat{\alpha}_4^c} e^{-i\gamma} \right), \\
\sqrt{2} \mathcal{A}_{\bar{B}^0 \rightarrow \pi^0 \bar{K}^0} &= -A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c \left(1 - \frac{A_{\bar{K}\pi} \frac{3}{2} \alpha_{3,\text{EW}}^c}{A_{\pi \bar{K}} \hat{\alpha}_4^c} \right. \\
&\quad \left. + \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \frac{A_{\pi \bar{K}} \hat{\alpha}_4^u - A_{\bar{K}\pi} \alpha_2}{A_{\pi \bar{K}} \hat{\alpha}_4^c} e^{-i\gamma} \right), \tag{6.1}
\end{aligned}$$

where the various symbols were defined in chapter 2. We recall that $\hat{\alpha}_4^c = \alpha_4^c + \beta_3^c$, so that, with the approximation above, β_3^c is the only annihilation contribution entering the decay amplitudes. In (6.1) we have written the decay amplitudes in such a way as to collect the dominant common penguin contribution, and we have explicitly indicated the terms proportional to the weak angle γ . In this way it is immediate to rewrite the amplitudes in terms of the following ratios:

$$\begin{aligned}
r_T &\equiv - \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \frac{\alpha_1}{\hat{\alpha}_4^c}, & r_C &\equiv - \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \frac{R_{\pi \bar{K}} \alpha_2}{\hat{\alpha}_4^c}, \\
r_P &\equiv \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \frac{\hat{\alpha}_4^u}{\hat{\alpha}_4^c}, & r_{\text{EW}} &\equiv \frac{R_{\pi \bar{K}} \frac{3}{2} \alpha_{3,\text{EW}}^c}{\hat{\alpha}_4^c}, \tag{6.2}
\end{aligned}$$

where $R_{\pi \bar{K}} \equiv A_{\bar{K}\pi}/A_{\pi \bar{K}}$. Explicitly, (6.1) becomes

$$\begin{aligned}
\mathcal{A}_{B^- \rightarrow \pi^- \bar{K}^0} &= A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c (1 + r_P e^{-i\gamma}), \\
\sqrt{2} \mathcal{A}_{B^- \rightarrow \pi^0 K^-} &= A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c \left(1 + r_{\text{EW}} - (r_T + r_C - r_P) e^{-i\gamma} \right), \\
\mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ K^-} &= A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c (1 - (r_T - r_P) e^{-i\gamma}), \\
\sqrt{2} \mathcal{A}_{\bar{B}^0 \rightarrow \pi^0 \bar{K}^0} &= -A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c \left(1 - r_{\text{EW}} + (r_C + r_P) e^{-i\gamma} \right). \tag{6.3}
\end{aligned}$$

The utility of this notation is that the common penguin amplitude often drops out when considering ratios of physical observables. Moreover, the coefficients r_i can be related directly to physical quantities because they are physically meaningful, contrary to the

flavour amplitudes α_i whose absolute phase cannot be determined. The coefficients r_i are in general small; r_T , r_C , and r_P are small because of the CKM suppression factor, which is of order $\lambda^2 R_b/(1 - \lambda^2) \sim \lambda^2 \simeq 0.05$. Among these coefficients, r_T is the largest, and represents the colour-allowed tree-to-penguin ratio. In QCD factorization it has the smallest uncertainty, because the coefficient α_1 is proportional at tree level to the large short-distance coefficient C_1 , while the one-loop correction, which is the major source of uncertainty, is proportional to the smaller coefficient C_2 . This is opposite to r_C , which represents the colour-suppressed tree-to penguin ratio. In case of the coefficient α_2 in fact the role of C_1 and C_2 are interchanged, so that the one-loop correction is much more important, but contains the uncertainties from the Gegenbauer moments, the masses m_s and m_c , and the divergent twist-three contribution to the spectator scattering correction, parameterized by the phenomenological parameter X_H . In our analysis this term will be the main source of uncertainty. The penguin ratio r_P is the smallest one, because of the QCD factorization prediction $\alpha_4^u \simeq \alpha_4^c$, and it can often be neglected with respect to the other ratios. Finally, the electroweak-to-QCD penguin ratio is not CKM suppressed, but it is of the same order as r_T because of the small electroweak short-distance coefficients with respect to the QCD penguin ones. Using the inputs in table 2.2, we obtain

$$\begin{aligned}
r_T &= 0.18_{-0.05}^{+0.06} + 0.007_{-0.055}^{+0.020}i, \\
r_C &= 0.07_{-0.05}^{+0.04} - 0.016_{-0.035}^{+0.027}i, \\
r_P &= 0.018_{-0.003}^{+0.005} + 0.004_{-0.001}^{+0.002}i, \\
r_{EW} &= 0.14_{-0.04}^{+0.05} + 0.005_{-0.043}^{+0.015}i.
\end{aligned} \tag{6.4}$$

These patterns are useful to better understand the origin of QCD factorization predictions. In table 6.1 we compare the theoretical predictions and the experimental results for a series of observables useful to our analysis. Considering first the branching ratios, one notes that, because of their large uncertainties, it is often more useful to consider ratios or sums of them (“sum rules”), which are suggested by the isospin symmetries which relate the amplitudes in the Standard Model. In this way one minimizes the theoretical uncertainties and highlights possible discrepancies originated by (isospin-breaking) new contributions. In this respect, it is useful to consider the following ratios [19, 55, 314, 315],

$$\begin{aligned}
R_c &\equiv 2 \frac{\text{Br}(B^- \rightarrow \pi^0 K^-) + \text{Br}(B^+ \rightarrow \pi^0 K^+)}{\text{Br}(B^- \rightarrow \pi^- K^0) + \text{Br}(B^+ \rightarrow \pi^+ K^0)} \\
&\simeq 1 + |r_{EW}|^2 + |r_T|^2 + |r_C|^2 + 2\text{Re}(r_{EW}) + 2\text{Re}(r_T r_C^*) \\
&\quad - 2\text{Re}(r_T + r_C) \cos \gamma - 2\text{Re}(r_{EW}(r_T^* + r_C^*)) \cos \gamma + \dots, \\
R_n &\equiv \frac{1}{2} \frac{\text{Br}(\bar{B}^0 \rightarrow \pi^+ K^-) + \text{Br}(B^0 \rightarrow \pi^- K^+)}{\text{Br}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) + \text{Br}(B^0 \rightarrow \pi^0 K^0)} \\
&\simeq 1 + |r_T|^2 - |r_{EW}|^2 - |r_C|^2 - 2\text{Re}(r_T) \cos \gamma + 2\text{Re}(r_{EW}) - 2\text{Re}(r_C) \\
&\quad + 2\text{Re}(r_{EW} r_C^*) \cos \gamma - 4\text{Re}(r_T) (\text{Re}(r_{EW}) - \text{Re}(r_C)) \cos \gamma + \dots,
\end{aligned}$$

$$\begin{aligned}
R &\equiv 2 \frac{\Gamma(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) + \Gamma(B^- \rightarrow \pi^0 K^-)}{\Gamma(B^- \rightarrow \pi^- \bar{K}^0) + \Gamma(\bar{B}^0 \rightarrow \pi^+ K^-)} \\
&\simeq 1 + |r_{\text{EW}}|^2 + |r_{\text{C}}|^2 - \text{Re}(r_{\text{EW}} r_{\text{T}}^*) \cos \gamma - \text{Im}(r_{\text{EW}} r_{\text{T}}^*) \sin \gamma \\
&\quad - 2\text{Re}(r_{\text{EW}} r_{\text{C}}^*) \cos \gamma - 2\text{Im}(r_{\text{EW}} r_{\text{C}}^*) \sin \gamma + \text{Re}(r_{\text{T}} r_{\text{C}}^*) + \dots,
\end{aligned} \tag{6.5}$$

which have small uncertainties. In the Standard Model, R_c and R_n are expected to be a bit larger than one, see table 6.1, while R is expected to be close to 1, since the r_i parameters appear only in quadratic form. The difference of these predictions with respect to the experimental results have decreased during the years and now they agree within the experimental and theoretical errors, even if the central values are still different.

More interesting are the discrepancies one finds when considering the CP asymmetries. Parameterizing the Standard Model amplitude as

$$\mathcal{A}(\bar{B} \rightarrow f) \propto 1 + d e^{-i\gamma}, \tag{6.6}$$

the direct CP asymmetries reads

$$\begin{aligned}
A_{\text{CP}} &\equiv \frac{\text{Br}(\bar{B} \rightarrow \bar{f}) - \text{Br}(B \rightarrow f)}{\text{Br}(\bar{B} \rightarrow \bar{f}) + \text{Br}(B \rightarrow f)} \\
&= \frac{2 \text{Im}(d) \sin \gamma}{1 + 2\text{Re}(d) \cos \gamma + |d|^2} \sim 2 \text{Im}(d) \sin \gamma,
\end{aligned} \tag{6.7}$$

where the last expansion is allowed by the smallness of the parameter d , which is always doubly CKM suppressed, as one can see explicitly by comparing (6.6) with (6.3). Using (6.7), one obtains

$$\begin{aligned}
A_{\text{CP}}(B^- \rightarrow \pi^- \bar{K}^0) &\sim 2 \text{Im}(r_{\text{P}}) \sin \gamma, \\
A_{\text{CP}}(B^- \rightarrow \pi^0 K^-) &\sim -2 \text{Im} \left(\frac{r_{\text{T}} + r_{\text{C}}}{1 + r_{\text{EW}}} \right) \sin \gamma, \\
A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^+ K^-) &\sim -2 \text{Im}(r_{\text{T}}) \sin \gamma, \\
A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &\sim 2 \text{Im} \left(\frac{r_{\text{C}}}{1 - r_{\text{EW}}} \right) \sin \gamma.
\end{aligned} \tag{6.8}$$

The corresponding numerical results are collected in table 6.1. The result for $A_{\text{CP}}(B^- \rightarrow \pi^- \bar{K}^0)$ is a well known prediction of QCD factorization and it tells us that, because of the smallness of r_{P} , $A_{\text{CP}}(B^- \rightarrow \pi^- \bar{K}^0) \sim 0$. More generally, in view of the comments we wrote in section 2.2.3, 2.3.5, the important prediction here is not the precise value of the CP asymmetries, but the fact that they are all predicted to be small, not exceeding the 10%, because of the absence of a mechanism able to generate large strong rescattering phases. This generates a conflict with the experimental data, in particular when considering the following difference between direct CP asymmetries:

$$\begin{aligned}
\Delta A_{\text{CP}} &= A_{\text{CP}}(B^- \rightarrow \pi^0 K^-) - A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^+ K^-) \\
&\sim -2 \left[\text{Im} \left(\frac{r_{\text{C}}}{1 + r_{\text{EW}}} \right) - \text{Im} \left(\frac{r_{\text{T}} r_{\text{EW}}}{1 + r_{\text{EW}}} \right) \right] \sin \gamma,
\end{aligned} \tag{6.9}$$

which, as a consequence of (6.8), it is predicted to be small, while the experimental result is seven times larger. Taking errors into account, this makes a deviation of 1.5σ , which is not yet a strong indication of deviation from the Standard Model, despite the large difference in the central values, because of the large error on the theoretical prediction. The picture becomes more unclear, when $A_{CP}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$ is taken into account: this is because, according to (6.8), (6.9), in the Standard Model one predicts a correlation $\Delta A_{CP} \sim -A_{CP}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$ ¹, which is not seen in the experimental data, see table 6.1, because the value shown there is a correlated average of the opposite result from Babar and Belle, [335, 336]:

$$A_{CP}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) = \begin{cases} (-0.13 \pm 0.13 \pm 0.3), & \text{Babar} \\ (0.14 \pm 0.13 \pm 0.6), & \text{Belle} \end{cases} \quad (6.10)$$

The experimental picture is completed by the time dependent CP asymmetry S_{CP} , which is defined by

$$\frac{\text{Br}(\bar{B}^0(t) \rightarrow f) - \text{Br}(B^0(t) \rightarrow f)}{\text{Br}(\bar{B}^0(t) \rightarrow f) + \text{Br}(B^0(t) \rightarrow f)} \equiv S_{CP} \sin(\Delta m_B t) - C_{CP} \cos(\Delta m_B t), \quad (6.11)$$

where $C_{CP} = -A_{CP}$ is the direct CP asymmetry, up to a sign. In terms of the parametrization (6.6) we have

$$S_{CP} \simeq \sin 2\beta + 2\text{Re}(d) \cos 2\beta \sin \gamma, \quad (6.12)$$

which gives

$$S_{CP}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) \simeq \sin 2\beta + 2\text{Re}\left(\frac{r_C}{1 - r_{EW}}\right) \cos 2\beta \sin \gamma. \quad (6.13)$$

In this case, the theoretical prediction has a $\sim 1\sigma$ deviation from the experimental value.

Given this analysis, the observed discrepancies are just “tensions”, which could be accounted for in the Standard Model, considering the uncertainties of the QCD factorization approach. One can, however, take these results at face value and interpret them as an indication of new physics contributions. Many works in the literature have analysed this possibility, [298–310], with the conclusion that the data could be explained better in a scenario with new physics in the electroweak penguin sector, with a large new weak phase. To see this, let’s assume that a new contribution with the structure of the electroweak penguin, but with a new weak phase δ_z arises, $r'_{EW} e^{-i\delta_z}$. Then the $\bar{B} \rightarrow \pi \bar{K}$ amplitudes get modified according to

$$\begin{aligned} \mathcal{A}_{B^- \rightarrow \pi^- \bar{K}^0} &= A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c (1 + r_P e^{-i\gamma}), \\ \sqrt{2} \mathcal{A}_{B^- \rightarrow \pi^0 K^-} &= A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c \left(1 + r_{EW} - (r_T + r_C - r_P) e^{-i\gamma} + r'_{EW} e^{-i\delta_z} \right), \\ \mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ K^-} &= A_{\pi \bar{K}} \lambda_u^{(c)} \hat{\alpha}_4^c (1 - (r_T - r_P) e^{-i\gamma}), \\ \sqrt{2} \mathcal{A}_{\bar{B}^0 \rightarrow \pi^0 \bar{K}^0} &= -A_{\pi \bar{K}} \lambda_c^{(s)} \hat{\alpha}_4^c \left(1 - r_{EW} + (r_C + r_P) e^{-i\gamma} - r'_{EW} e^{-i\delta_z} \right). \end{aligned} \quad (6.14)$$

¹the second term in (6.9) is much smaller than the first one, because of the product of the two small factors $r_T r_{EW}$.

Observable	Theory	Experiment
$\text{Br}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$	$5.48^{+4.43}_{-3.05}$	$9.8^{+0.6}_{-0.6}$
$\text{Br}(\bar{B}^0 \rightarrow \pi^+ \bar{K}^-)$	$13.4^{+9.4}_{-6.6}$	$19.4^{+0.6}_{-0.6}$
$\text{Br}(B^- \rightarrow \pi^0 \bar{K}^-)$	$9.18^{+5.71}_{-4.18}$	$12.9^{+0.6}_{-0.6}$
$\text{Br}(B^- \rightarrow \pi^- \bar{K}^0)$	$15.0^{+10.6}_{-7.5}$	$23.1^{+1.0}_{-1.0}$
$R_c(\pi K)$	$1.23^{+0.24}_{-0.20}$	$1.12^{+0.07}_{-0.07}$
$R_n(\pi K)$	$1.22^{+0.28}_{-0.22}$	$0.99^{+0.07}_{-0.07}$
$R(\pi K)$	$1.03^{+0.03}_{-0.02}$	$1.07^{+0.05}_{-0.05}$
$A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$	$-0.023^{+0.056}_{-0.076}$	$-0.010^{+0.10}_{-0.10}$
$A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^+ \bar{K}^-)$	$-0.006^{+0.109}_{-0.038}$	$-0.098^{+0.012}_{-0.012}$
$A_{\text{CP}}(B^- \rightarrow \pi^0 \bar{K}^-)$	$0.020^{+0.132}_{-0.050}$	$0.050^{+0.025}_{-0.025}$
$A_{\text{CP}}(B^- \rightarrow \pi^- \bar{K}^0)$	$0.005^{+0.008}_{-0.003}$	$0.009^{+0.025}_{-0.025}$
ΔA_{CP}	$0.026^{+0.053}_{-0.049}$	$0.148^{+0.027}_{-0.028}$
$S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$	$0.80^{+0.06}_{-0.08}$	$0.57^{+0.17}_{-0.17}$

Table 6.1: Theoretical versus experimental results for the $\bar{B} \rightarrow \pi \bar{K}$ decays. The experimental data are taken from [44]. The original results can be found in [325–336].

The amplitudes contain a new weak phase, and they can be written in general as

$$\mathcal{A}(\bar{B} \rightarrow f) \propto 1 + d e^{-i\gamma} + d_N e^{-i\varphi}. \quad (6.15)$$

In terms of (6.15), the direct and the time dependent CP asymmetries now read

$$\begin{aligned} A_{\text{CP}} &= \frac{2 \text{Im}(d) \sin \gamma + 2 \text{Im}(d_N) \sin \varphi + 2 \text{Im}(dd_N^*) \sin(\gamma - \varphi)}{1 + 2 \text{Re}(d) \cos \gamma + 2 \text{Re}(d_N) \cos \varphi + 2 \text{Re}(dd_N^*) \cos(\gamma - \varphi) + |d|^2 + |d_N|^2} \\ &\sim 2 \text{Im}(d) \sin \gamma + 2 \text{Im}(d_N) \sin \varphi, \end{aligned} \quad (6.16)$$

$$\begin{aligned} S_{\text{CP}} &= \frac{\sin 2\beta + 2 \text{Re}(d) \sin(\gamma + 2\beta) + 2 \text{Re}(d_N) \sin(\varphi + 2\beta)}{1 + 2 \text{Re}(d) \cos \gamma + 2 \text{Re}(d_N) \cos \varphi + 2 \text{Re}(dd_N^*) \cos(\gamma - \varphi) + |d|^2 + |d_N|^2} \\ &+ \frac{2 \text{Re}(dd_N^*) \sin(\gamma + \varphi + 2\beta) + |d|^2 \sin(2\gamma + 2\beta) + |d_N|^2 \sin(2\varphi + 2\beta)}{1 + 2 \text{Re}(d) \cos \gamma + 2 \text{Re}(d_N) \cos \varphi + 2 \text{Re}(dd_N^*) \cos(\gamma - \varphi) + |d|^2 + |d_N|^2} \\ &\sim \sin 2\beta + 2 \text{Re}(d) \cos 2\beta \sin \gamma + 2 \text{Re}(d_N) \cos 2\beta \sin \varphi. \end{aligned} \quad (6.17)$$

Using (6.16), (6.17) the direct CP asymmetries in (6.8) become

$$\begin{aligned} A_{\text{CP}}(B^- \rightarrow \pi^- \bar{K}^0) &\sim 2 \text{Im}(r_P) \sin \gamma, \\ A_{\text{CP}}(B^- \rightarrow \pi^0 \bar{K}^-) &\sim -2 \text{Im} \left(\frac{r_T + r_C}{1 + r_{\text{EW}}} \right) \sin \gamma + 2 \text{Im}(r'_{\text{EW}}) \sin \delta_z, \\ A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^+ \bar{K}^-) &\sim -2 \text{Im}(r_T) \sin \gamma, \\ A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &\sim 2 \text{Im} \left(\frac{r_C}{1 - r_{\text{EW}}} \right) \sin \gamma - 2 \text{Im}(r'_{\text{EW}}) \sin \delta_z. \end{aligned} \quad (6.18)$$

As a consequence, the difference ΔA_{CP} reads

$$\Delta A_{\text{CP}} \sim -2 \left[\text{Im} \left(\frac{r_C}{1 + r_{\text{EW}}} \right) - \text{Im} \left(\frac{r_T r_{\text{EW}}}{1 + r_{\text{EW}}} \right) \right] \sin \gamma + 2 \text{Im}(r'_{\text{EW}}) \sin \delta_z. \quad (6.19)$$

The main point here is that, contrary to the Standard Model electroweak contribution, the new one is not suppressed by r_T , so that a large new weak phase could indeed give a significant contribution to ΔA_{CP} . Moreover, the correlation with $A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$ is maintained, so that this scenario predicts also a large (negative) direct CP asymmetry for $\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$, in accordance with the Babar result. At this point, however, we can note the drawback of this hypothesis: the new contribution always appears together with r_C , so that, in principle, similar results could be achieved in the Standard Model, if an enhancement of the colour-suppressed tree-to-penguin ratio is present, as suggested by the large branching ratios $\text{Br}(\bar{B} \rightarrow \pi^0 \pi^0)$ and $\text{Br}(\bar{B} \rightarrow \pi^0 \bar{K}^0)$. We recall, however, that in QCD factorization it is anyway difficult to get such a large phase. An aid in clarifying this ambiguity could be given by the time-dependent CP asymmetry: one has

$$S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) \simeq \sin 2\beta + 2 \text{Re} \left(\frac{r_C}{1 - r_{\text{EW}}} \right) \cos 2\beta \sin \gamma - 2 \text{Re}(r'_{\text{EW}}) \cos 2\beta \sin \delta_z. \quad (6.20)$$

The “hadronic hypothesis” of an enhanced r_C leads to a large value of $S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$, close to one [310]; on the other hand, the hypothesis of new physics in the electroweak sector can contribute in lowering S_{CP} to values closer to the experimental data, depending on the sign of $\text{Re}(r'_{\text{EW}})$ and $\sin(\delta_z + 2\beta)$. Finally, we observe that since r_C and r_{EW} give the same contribution to R_c , R_n and R , both hypotheses considered here bring them to values closer to the experimental results, so that they cannot add much information for the solution of this ambiguity.

6.3 Consequences on other decay modes

6.3.1 The $\bar{B}_s \rightarrow \phi\pi$, $\bar{B}_s \rightarrow \phi\rho$ decays

Because of the present inconclusive status of the $\bar{B} \rightarrow \pi\bar{K}$ decays, the only way one has in order to sort out the puzzle is to analyse other decays where new physics in the electroweak sector could have a sizeable effect. On one hand, this means to consider the $\bar{B} \rightarrow \rho\bar{K}$, $\bar{B} \rightarrow \pi\bar{K}^*$ and $\bar{B} \rightarrow \rho\bar{K}^*$ decays, because they have the same flavour structure as the $\bar{B} \rightarrow \pi\bar{K}$ decays, but, in particular in case of the PV , VV decays, the dominant penguin contribution $\hat{\alpha}_4^c$ is smaller, so that they are more sensitive to modifications in the other terms of the amplitudes. On the other hand, the B_s decays $\bar{B}_s \rightarrow \phi\pi$, $\bar{B}_s \rightarrow \phi\rho$ are dominated by the electroweak penguin amplitude $\alpha_{3,\text{EW}}$ and they appear as the golden modes to probe new physics in the electroweak sector. The structure of these decays is very simple; in the approximation described at the beginning of section 6.2 their amplitude reads

$$\mathcal{A}_{\bar{B}_s \rightarrow \phi M_2} = \frac{A_{\phi M_2}}{\sqrt{2}} \left(\lambda_u^{(s)} \alpha_2(\phi M_2) + \frac{3}{2} \lambda_c^{(s)} \alpha_{3,\text{EW}}(\phi M_2) \right), \quad (6.21)$$

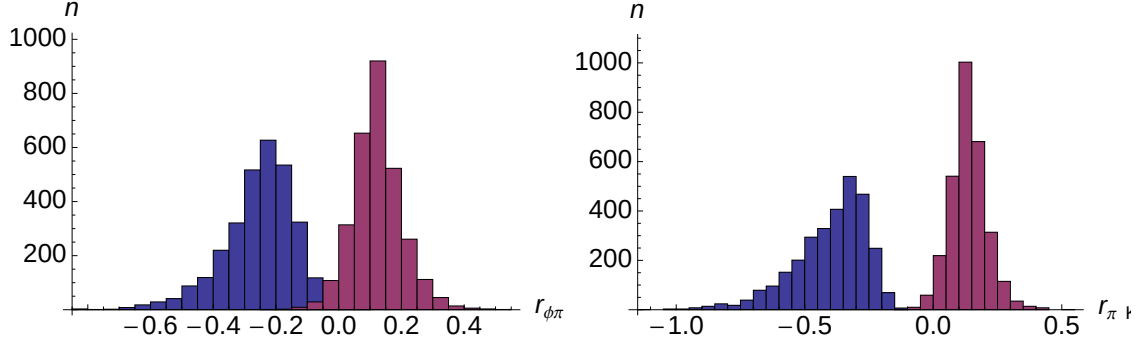


Figure 6.1: Ratios $r_{\phi\pi}$ and $r_{\pi K}$. Real part, blue, and imaginary part, red.

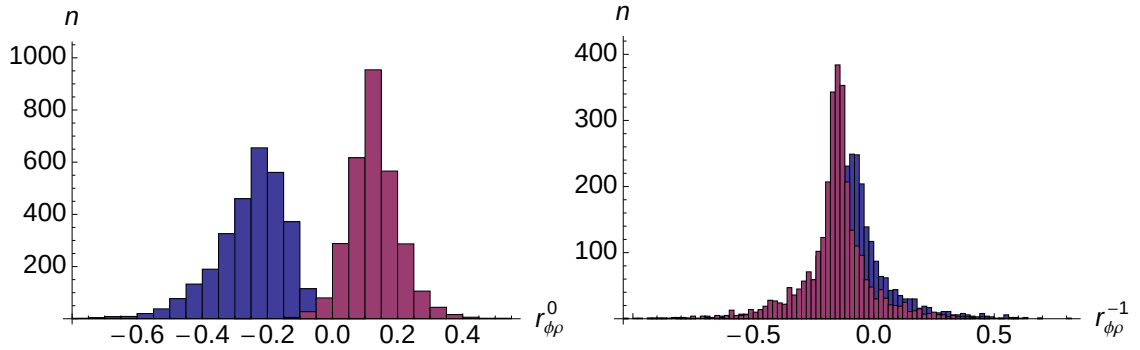


Figure 6.2: Ratios $r_{\phi\rho}^0$ and $r_{\phi\rho}^{-1}$. Real part, blue, and imaginary part, red.

where $M_2 = \pi, \rho$ depending on the decay considered. In the VV case, an index h for the helicity amplitudes has to be added to $\mathcal{A}$ and the flavour amplitudes α_i in (6.21).

In the Standard Model one finds

$$\begin{aligned} \text{Br}(\bar{B}_s \rightarrow \pi^0 \phi) &= 0.15_{-0.04}^{+0.11} \cdot 10^{-6}, & A_{\text{CP}}(\bar{B}_s \rightarrow \pi^0 \phi) &= 0.27_{-0.62}^{+0.49}, \\ \text{Br}(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.43_{-0.11}^{+0.28} \cdot 10^{-6}, & A_{\text{CP}}(\bar{B}_s \rightarrow \pi^0 \phi) &= 0.18_{-0.61}^{+0.53}. \end{aligned} \quad (6.22)$$

Moreover, it will be useful to consider the branching ratio of the decay into longitudinally polarized vector mesons ρ and ϕ ,

$$\text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi) = 0.36_{-0.09}^{+0.26} \cdot 10^{-6}, \quad (6.23)$$

as well as the ratios

$$\frac{\text{Br}(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} = 2.33_{-0.08}^{+0.10}, \quad \frac{\text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} = 2.80_{-0.25}^{+0.37}, \quad (6.24)$$

in which the form factor cancels, and which have relatively small uncertainties compared to the branching ratios in (6.22). Besides the electroweak flavour amplitude, from (6.21) we see that these decays receive a contribution from α_2 , too, in the same way it happens for the $\bar{B} \rightarrow \pi \bar{K}$ decays. Once again, therefore, given the large uncertainty on this

coefficient, one has to face the possibility of having an enhanced amplitude from both a new electroweak contribution, or from an enhanced hadronic contribution to α_2 . In order to deal with this ambiguity, we focus on the QCD factorization prediction for the ratio of the two parts of the amplitudes: defining

$$r_{\phi M_2}^h \equiv \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \frac{2}{3} \frac{\alpha_2^h}{\alpha_{3,\text{EW}}^{c,h}}, \quad (6.25)$$

one finds

$$\begin{aligned} \text{Re}(r_{\phi\pi}) &= -0.41_{-0.37}^{+0.41}, & \text{Im}(r_{\phi\pi}) &= 0.13_{-0.30}^{+0.30}, \\ \text{Re}(r_{\pi K}) &= -0.58_{-0.35}^{+0.43}, & \text{Im}(r_{\pi K}) &= 0.15_{-0.22}^{+0.23}, \\ \text{Re}(r_{\phi\rho}^0) &= -0.39_{-0.35}^{+0.39}, & \text{Im}(r_{\phi\rho}^0) &= 0.13_{-0.28}^{+0.29}, \\ \text{Re}(r_{\phi\rho}^{-1}) &= -0.20_{-0.48}^{+0.45}, & \text{Im}(r_{\phi\rho}^{-1}) &= -0.15_{-0.43}^{+0.43}. \end{aligned} \quad (6.26)$$

(6.26) tells us that the part of the amplitude proportional to α_2 can never be larger than the electroweak contribution $\alpha_{3,\text{EW}}$, within hadronic (and CKM) uncertainties of QCD factorization. This result is confirmed by a parameter space scan: in figure 6.1 we show on the left the real (blue) and imaginary part (red) of $r_{\phi\pi}$, for 3000 points in the space of the QCD factorization input parameters: the scattered points range in the error intervals of (6.26). For comparison, in (6.26) we have included the value of the same ratio for the $\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$ decay, which appears also in the right plot of figure 6.1. In this case the ratio is a bit larger, so that the effect of a hadronically enhanced α_2 could be more relevant. We note, however, that this is true only for the real part of the ratio, confirming our expectation that it is difficult to get large soft phases in QCD factorization. In any case, because of the different parameters involved in the $\bar{B}_s \rightarrow \pi\phi$ decay, we observe a clear dominance of the electroweak flavour amplitude. In figure 6.2 we show the same ratio for the $\rho\phi$ final state. While the situation is similar for the longitudinal helicity amplitude, a slightly larger uncertainty arises in case of the negative one, and this reflects the well known problem that the transverse helicity amplitudes do not factorize at leading order (see discussion at the end of section 2.2.2), resulting in a larger uncertainty. In figure 6.3 we show the branching ratios $\text{Br}(\bar{B}_s \rightarrow \pi^0\phi)$, $\text{Br}(\bar{B}_s \rightarrow \pi^0\rho)$ for the same points: one can see that they range in the intervals $\text{Br}(\bar{B}_s \rightarrow \pi^0\phi) \in [0.10, 0.30] \cdot 10^{-6}$, $\text{Br}(\bar{B}_s \rightarrow \pi^0\rho) \in [0.35, 0.90] \cdot 10^{-6}$, in agreement with the QCD factorization errors reported in (6.22). This analysis allows us to conclude that a new electroweak contribution of the same order as the Standard Model one, $\alpha'_{3,\text{EW}} \simeq \alpha_{3,\text{EW}}$, could easily enhance the branching fraction of these decays by a factor ~ 4 , which is not possible to obtain from an enhanced α_2 , within QCD factorization. The resulting branching ratios then would be of order $\sim 10^{-6}$ and therefore easy to observe.

6.3.2 $\bar{B} \rightarrow \rho\bar{K}$, $\bar{B} \rightarrow \pi\bar{K}^*$ and $\bar{B} \rightarrow \rho\bar{K}^*$ decays

These decays have the same flavour structure as the $\bar{B} \rightarrow \pi\bar{K}$ decays, but the dominant penguin amplitude $\hat{\alpha}_4^c$ is predicted to be smaller. Naively, this is because a_6 vanishes at

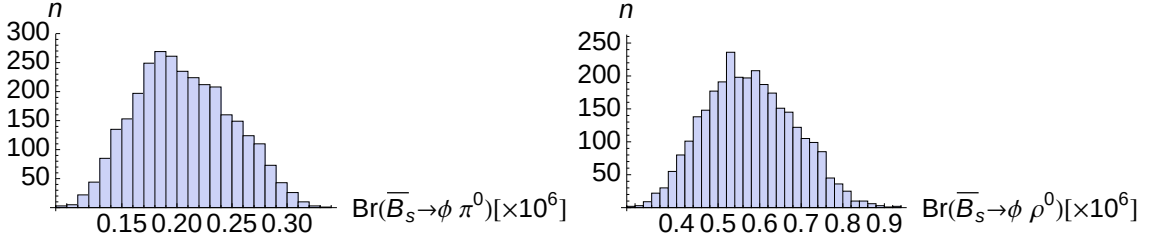


Figure 6.3: Branching ratios $\text{Br}(\bar{B}_s \rightarrow \phi \pi^0)$, $\text{Br}(\bar{B}_s \rightarrow \phi \rho^0)$.

tree level when $M_2 = V$. As a consequence, effects of the other flavour amplitudes are enhanced. Quantitatively, one can estimate the expected enhancement by inspecting the value taken by the ratios r_i in case of these decays, whose decay amplitudes are given by (6.1), (6.3) with obvious substitutions of the π and K indices. Within the input values of table 2.2 we obtain²

$$\begin{aligned}
 r_T(\rho \bar{K}) &= -0.42_{-0.27}^{+0.22} - 0.16_{-0.16}^{+0.38}i, \\
 r_C(\rho \bar{K}) &= -0.23_{-0.14}^{+0.18} - 0.02_{-0.09}^{+0.20}i, \\
 r_P(\rho \bar{K}) &= 0.019_{-0.003}^{+0.006} + 0.002_{-0.002}^{+0.002}i, \\
 r_{EW}(\rho \bar{K}) &= -0.41_{-0.24}^{+0.22} - 0.16_{-0.16}^{+0.37}i,
 \end{aligned} \tag{6.27}$$

$$\begin{aligned}
 r_T(\pi \bar{K}^*) &= 0.40_{-0.12}^{+0.38} + 0.05_{-0.30}^{+0.13}i, \\
 r_C(\pi \bar{K}^*) &= 0.10_{-0.10}^{+0.12} - 0.02_{-0.11}^{+0.07}i, \\
 r_P(\pi \bar{K}^*) &= 0.018_{-0.004}^{+0.007} + 0.006_{-0.002}^{+0.006}i, \\
 r_{EW}(\pi \bar{K}^*) &= 0.27_{-0.08}^{+0.24} + 0.03_{-0.20}^{+0.09}i,
 \end{aligned} \tag{6.28}$$

$$\begin{aligned}
 r_T^0(\rho \bar{K}^*) &= 0.54_{-0.20}^{+0.45} - 0.30_{-0.30}^{+0.41}i, \\
 r_C^0(\rho \bar{K}^*) &= 0.13_{-0.17}^{+0.22} - 0.15_{-0.19}^{+0.19}i, \\
 r_P^0(\rho \bar{K}^*) &= 0.017_{-0.006}^{+0.006} + 0.006_{-0.004}^{+0.006}i, \\
 r_{EW}^0(\rho \bar{K}^*) &= 0.45_{-0.19}^{+0.36} - 0.25_{-0.25}^{+0.35}i.
 \end{aligned} \tag{6.29}$$

The ratios are now larger, as anticipated. Note also that in case of $\bar{B} \rightarrow \rho \bar{K}$ decays the ratios r_T , r_C and r_{EW} have opposite sign with respect to the other two cases. Starting from these results, one can deduce the theoretical predictions for the ratios R_c , R_n , R and the CP asymmetries collected in tables 6.2, 6.3, 6.4. The addition of a new electroweak amplitude would modify the decay amplitudes according to (6.14) and the CP asymmetries according to (6.18), (6.19), (6.20), with obvious replacement of $\pi \bar{K}$ with

²in case of $\bar{B} \rightarrow \rho \bar{K}^*$ the superscript 0 is added, to indicate that we refer to ratios of the longitudinal amplitude coefficients.

Observable	Theory	Experiment
$\text{Br}(\bar{B}^0 \rightarrow \rho^0 \bar{K}^0)$	$3.14^{+3.72}_{-1.62}$	$5.4^{+0.9}_{-1.0}$
$\text{Br}(\bar{B}^0 \rightarrow \rho^+ \bar{K}^-)$	$4.29^{+6.15}_{-2.59}$	$8.6^{+0.9}_{-1.1}$
$\text{Br}(B^- \rightarrow \rho^0 \bar{K}^-)$	$1.58^{+2.43}_{-1.04}$	$3.81^{+0.48}_{-0.46}$
$\text{Br}(B^- \rightarrow \rho^- \bar{K}^0)$	$3.61^{+5.94}_{-2.43}$	$8.0^{+1.5}_{-1.4}$
$R_c(\rho K)$	$0.88^{+0.46}_{-0.30}$	$0.95^{+0.24}_{-0.19}$
$R_n(\rho K)$	$0.68^{+0.28}_{-0.22}$	$0.80^{+0.20}_{-0.15}$
$R(\rho K)$	$1.21^{+0.30}_{-0.20}$	$1.12^{+0.18}_{-0.17}$
$A_{\text{CP}}(\bar{B}^0 \rightarrow \rho^0 \bar{K}^0)$	$0.006^{+0.198}_{-0.105}$	$0.01^{+0.20}_{-0.20}$
$A_{\text{CP}}(\bar{B}^0 \rightarrow \rho^+ \bar{K}^-)$	$0.22^{+0.19}_{-0.51}$	$0.15^{+0.06}_{-0.06}$
$A_{\text{CP}}(B^- \rightarrow \rho^0 \bar{K}^-)$	$0.39^{+0.34}_{-1.04}$	$0.37^{+0.11}_{-0.11}$
$A_{\text{CP}}(B^- \rightarrow \rho^- \bar{K}^0)$	$0.009^{+0.005}_{-0.015}$	$-0.12^{+0.17}_{-0.17}$
ΔA_{CP}	$0.17^{+0.17}_{-0.54}$	$0.22^{+0.13}_{-0.13}$
$S_{\text{CP}}(\bar{B}^0 \rightarrow \rho^0 \bar{K}^0)$	$0.45^{+0.20}_{-0.14}$	$0.54^{+0.18}_{-0.21}$

Table 6.2: Theoretical versus experimental results for the $\bar{B} \rightarrow \rho \bar{K}$ decays, taken from [44]. The original results can be found in [337–348].

$\rho \bar{K}$, $\pi \bar{K}^*$ and $\rho \bar{K}^*$. One could therefore analyse the experimental results in tables 6.2, 6.3, 6.4, to see if this data, too, can be accomodated better within a scenario with new physics in the electroweak sector. Since the purpose of this chapter is to focus first on the B_s decays, we leave a more detailed analysis of the $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ modes for a future work. Nevertheless, we will consider these decays as constraints on the new physics scenario described in section 6.4, as explained in section 6.5.

6.4 A viable scenario: the flavour-changing Z^0 penguin

The only way to perform a more quantitative analysis is to consider a specific new physics scenario. In principle, one could assume that new physics is likely to appear in one single operator, (“dominance of an individual new physics operator”, [295]), but this is indeed not the case in some of the most common extensions of the Standard Model. Despite the fact that we have performed also this type of analysis, we prefer to focus here on a specific new physics scenario. We will show that in this case one finds most of the times specific signatures and correlations, which would not appear from a model independent analysis in which only one operator is varied.

There are various models in which new physics is expected to appear in the electroweak penguin sector of the theory. In generic supersymmetric models, for instance, gluino box and penguin diagrams can contribute to the electroweak penguin operators; in chapter 5 we have seen that this is e.g. the case in the CMM model, if the condition $m_{\tilde{u}} \neq m_{\tilde{d}}$ is satisfied at the high scale. Another possibility is to consider the existence of additional

Observable	Theory	Experiment
$\text{Br}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^{*0})$	$1.40^{+1.22}_{-1.44}$	$2.4^{+0.7}_{-0.7}$
$\text{Br}(\bar{B}^0 \rightarrow \pi^+ \bar{K}^{*-})$	$4.98^{+3.21}_{-3.98}$	$10.3^{+1.1}_{-1.1}$
$\text{Br}(B^- \rightarrow \pi^0 \bar{K}^{*-})$	$4.25^{+2.25}_{-2.94}$	$6.9^{+2.3}_{-2.3}$
$\text{Br}(B^- \rightarrow \pi^- \bar{K}^{*0})$	$5.23^{+3.33}_{-4.49}$	$9.9^{+0.8}_{-0.9}$
$R_c(\pi K^*)$	$1.62^{+1.18}_{-0.50}$	$1.39^{+0.49}_{-0.48}$
$R_n(\pi K^*)$	$1.77^{+2.40}_{-0.66}$	$2.15^{+0.91}_{-0.54}$
$R(\pi K)$	$1.11^{+0.25}_{-0.10}$	$0.91^{+0.24}_{-0.24}$
$A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^{*0})$	$-0.03^{+0.20}_{-0.38}$	$-0.15^{+0.12}_{-0.12}$
$A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^+ \bar{K}^{*-})$	$-0.09^{+0.49}_{-0.22}$	$-0.23^{+0.08}_{-0.08}$
$A_{\text{CP}}(B^- \rightarrow \pi^0 \bar{K}^{*-})$	$-0.02^{+0.39}_{-0.18}$	$0.04^{+0.29}_{-0.29}$
$A_{\text{CP}}(B^- \rightarrow \pi^- \bar{K}^{*0})$	$0.009^{+0.021}_{-0.005}$	$-0.038^{+0.042}_{-0.042}$
ΔA_{CP}	$0.07^{+0.15}_{-0.16}$	$0.27^{+0.30}_{-0.30}$

Table 6.3: Theoretical versus experimental results for the $\bar{B} \rightarrow \pi \bar{K}^*$ decays, taken from [44]. The original results can be found in [337–348].

Observable	Theory	Experiment
$\text{Br}(\bar{B}^0 \rightarrow \rho^0 \bar{K}^{*0})$	$1.75^{+1.55}_{-0.81}$	$5.6^{+1.6}_{-1.6}$
$\text{Br}(\bar{B}^0 \rightarrow \rho^+ \bar{K}^{*-})$	$4.47^{+3.39}_{-1.81}$	0^{+12}_{-0}
$\text{Br}(B^- \rightarrow \rho^0 \bar{K}^{*-})$	$4.03^{+2.82}_{-1.77}$	$0^{+6.1}_{-0}$
$\text{Br}(B^- \rightarrow \rho^- \bar{K}^{*0})$	$4.63^{+3.36}_{-1.54}$	$9.2^{+0.8}_{-0.9}$
$A_{\text{CP}}(\bar{B}^0 \rightarrow \rho^0 \bar{K}^{*0})$	$-0.24^{+0.32}_{-0.18}$	$0.09^{+0.19}_{-0.19}$
$A_{\text{CP}}(\bar{B}^0 \rightarrow \rho^+ \bar{K}^{*-})$	$0.33^{+0.11}_{-0.38}$	—
$A_{\text{CP}}(B^- \rightarrow \rho^0 \bar{K}^{*-})$	$0.35^{+0.18}_{-0.34}$	$0.20^{+0.32}_{-0.29}$
$A_{\text{CP}}(B^- \rightarrow \rho^- \bar{K}^{*0})$	$0.002^{+0.013}_{-0.006}$	$-0.01^{+0.16}_{-0.16}$

Table 6.4: Theoretical versus experimental results for the $\bar{B} \rightarrow \rho \bar{K}^*$ decays, taken from [44]. The original results can be found in [337–348].

“horizontal” gauge symmetries, which would give rise to the presence of extra Z bosons, which could mediate new four-quark FCNC. Similarly, extra gauge bosons arise in models of extra dimensions. In all these scenarios the new contribution to the electroweak penguin operators arises from integrating out the new heavy particles, and due to the dimension six of the four-quark operators one expects the new contribution to scale like $1/M_X^2$, where M_X^2 represents the mass of the new particles appearing in the loops. Another interesting possibility is to have a model with a $Z\bar{s}b$ coupling. Such a possibility arises, for instance, in models with a fourth generation, or in models with an extra gauge symmetry, through the Z - Z' mixing. For a more exhaustive list of models where a new $Z\bar{s}b$ coupling could arise, we refer to [316]. This scenario is particularly interesting, because the $Z\bar{s}b$ coupling has dimension four, so that its magnitude is expected to be proportional to some symmetry breaking terms, but it does not need to contain any $1/M_X$ suppression. In other words, it is formally non-decoupling, even if it should be noted that the requirement of naturalness in the size of the $SU(2)_L \times U(1)_Y$ breaking terms suggests that also the adimensional coupling above should decouple in the limit $M_X/M_Z \rightarrow \infty$. Because of the simplicity of this scenario, which can be described with the introduction of only two new coupling constants, we will focus in the following on this model, analysing quantitatively the modifications predicted in the $\bar{B} \rightarrow \pi \bar{K}$ and the $\bar{B}_s \rightarrow \phi \pi, \phi \rho$ decays.

6.4.1 The model

We start with a brief description of the model, following closely [288]. The flavour-changing $Z\bar{d}_i d_j$ couplings can be accounted for by the effective Lagrangian

$$\mathcal{L}_Z^{eff} = -\frac{g}{4\cos\theta_W} \sum_{I \neq J} \bar{d}_I [\kappa_L^{IJ} \gamma^\mu (1 - \gamma_5) + \kappa_R^{IJ} \gamma^\mu (1 + \gamma_5)] d_J Z_\mu, \quad (6.30)$$

where I, J are generation indices, as usual. Here we are interested in the two (complex) couplings κ_L^{bs} and κ_R^{bs} . Since the flavour violating couplings are expected to be small, the flavour-diagonal couplings of the Z bosons are to leading order the same as in the Standard Model. As a consequence, at the electroweak scale one can write the following effective hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\kappa_L^{bs} (\bar{s}b)_{V-A} + \kappa_R^{bs} (\bar{s}b)_{V+A} \right] \sum_q [C_L^q (\bar{q}q)_{V-A} + C_R^q (\bar{q}q)_{V+A}], \quad (6.31)$$

where

$$\begin{aligned} C_L^u &= \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W, & C_R^u &= -\frac{2}{3} \sin^2 \theta_W, \\ C_L^d &= -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W, & C_R^d &= \frac{1}{3} \sin^2 \theta_W. \end{aligned} \quad (6.32)$$

These effective couplings can be matched with the standard form of the four-quark operators appearing the weak effective Hamiltonian. One obtains corrections to the Standard Model short-distance coefficients as well as new contributions to the mirror operators:

$$\delta C_3(\mu_Z) = -\frac{\kappa_L^{bs}}{6(-\lambda_t^{(s)})} \quad \tilde{C}_5(\mu_Z) = -\frac{\kappa_R^{bs}}{6(-\lambda_t^{(s)})}$$

	SM	$\delta C_i/C_i$	$\tilde{C}_i/C_i$
C_1	1.08126	0.0	0.0
C_2	-0.19031	0.0	0.0
C_3	0.01369	$-0.230 \frac{\kappa_L^{bs}}{6.9 \cdot 10^{-4}}$	$-0.006 \frac{\kappa_R^{bs}}{6.9 \cdot 10^{-4}}$
C_4	-0.03565	$-0.024 \frac{\kappa_L^{bs}}{6.9 \cdot 10^{-4}}$	$-0.001 \frac{\kappa_R^{bs}}{6.9 \cdot 10^{-4}}$
C_5	0.00870	$-0.012 \frac{\kappa_L^{bs}}{6.9 \cdot 10^{-4}}$	$-0.297 \frac{\kappa_R^{bs}}{6.9 \cdot 10^{-4}}$
C_6	-0.04188	$-0.004 \frac{\kappa_L^{bs}}{6.9 \cdot 10^{-4}}$	$0.023 \frac{\kappa_R^{bs}}{6.9 \cdot 10^{-4}}$
C_7	-0.00009	$+28.209 \frac{\kappa_L^{bs}}{6.9 \cdot 10^{-4}}$	$-94.744 \frac{\kappa_R^{bs}}{6.9 \cdot 10^{-4}}$
C_8	0.00047	$-2.046 \frac{\kappa_L^{bs}}{6.9 \cdot 10^{-4}}$	$6.872 \frac{\kappa_R^{bs}}{6.9 \cdot 10^{-4}}$
C_9	-0.00972	$-1.000 \frac{\kappa_L^{bs}}{6.9 \cdot 10^{-4}}$	$0.298 \frac{\kappa_R^{bs}}{6.9 \cdot 10^{-4}}$
C_{10}	0.00173	$-1.280 \frac{\kappa_L^{bs}}{6.9 \cdot 10^{-4}}$	$0.382 \frac{\kappa_R^{bs}}{6.9 \cdot 10^{-4}}$

Table 6.5: Short-distance coefficients at the scale m_b .

$$\begin{aligned}
\delta C_7(\mu_Z) &= -\frac{2}{3} \frac{\kappa_L^{bs} \sin^2 \theta_W}{(-\lambda_t^{(s)})} & \tilde{C}_7(\mu_Z) &= \frac{2}{3} \frac{\kappa_R^{bs} \cos^2 \theta_W}{(-\lambda_t^{(s)})} \\
\delta C_9(\mu_Z) &= \frac{2}{3} \frac{\kappa_L^{bs} \cos^2 \theta_W}{(-\lambda_t^{(s)})} & \tilde{C}_9(\mu_Z) &= -\frac{2}{3} \frac{\kappa_R^{bs} \sin^2 \theta_W}{(-\lambda_t^{(s)})}.
\end{aligned} \tag{6.33}$$

Having the values of the short-distance coefficients at the high scale $\mu_Z \sim m_W$, it is straightforward to obtain their values at the low scale $\mu_b \sim m_b$ by means of the RGE. Since the strong and the electromagnetic interactions are chirally invariant, the Standard Model four-quark operators and the corresponding mirror operators have the same evolution under the RGE. Here we use the two-loop anomalous dimension, following [53, 65], as described briefly in section 2.3.2. Because of the mixing between different operators, at the low scale all the QCD penguin and the electroweak penguin short-distance coefficients are modified. Similarly, QCD and electroweak mirror coefficients are generated. The former are controlled by the parameter κ_L^{bs} , while the latter depend on κ_R^{bs} . In table 6.5 we provide the ratios $\delta C_i/C_i$, $\tilde{C}_i/C_i$ at the scale m_b . The normalization of the $\kappa_{L,R}^{bs}$ coefficient is fixed in order to have a new contribution of order one to the coefficient C_9 . The largest new contribution arises for the electroweak coefficients, in particular in case of δC_9 and $\tilde{C}_7$.

6.4.2 Hadronic matrix elements for $\bar{B} \rightarrow M_1 M_2$

The evaluation of the hadronic matrix elements is straightforward, using the results of chapter 2 for the Standard Model operators and the results of chapter 4 for the mirror operators. Since the mirror operators have the same flavour structure as the corresponding operators in the Standard Model, their contribution to the amplitude can be obtained

$\alpha_{3,\text{EW}}^p$	$\tilde{\alpha}_{3,\text{EW}}^p$	
$a_9^p + \delta a_9^p - a_7^p - \delta a_7^p$	$-\tilde{a}_9^p + \tilde{a}_7^p$	if $M_1 M_2 = PP$
$a_9^p + \delta a_9^p + a_7^p + \delta a_7^p$	$\tilde{a}_9^p + \tilde{a}_7^p$	if $M_1 M_2 = PV$
$a_9^p + \delta a_9^p - a_7^p - \delta a_7^p$	$\tilde{a}_9^p - \tilde{a}_7^p$	if $M_1 M_2 = VP$
$a_9^p + \delta a_9^p + a_7^p + \delta a_7^p$	$-\tilde{a}_9^p - \tilde{a}_7^p$	if $M_1 M_2 = V^0 V^0$
$a_9^p + \delta a_9^p + a_7^p + \delta a_7^p$	$-f_{\pm}^{M_1} (\tilde{a}_9^p + \tilde{a}_7^p)$	if $M_1 M_2 = V^{\pm} V^{\pm}$

Table 6.6: Contribution to the dominant electroweak penguin amplitude in the modified Z^0 scenario

adding a $\tilde{\alpha}_i$ coefficient for each α_i appearing in the transition operators in (2.37) and (2.51). The relation between the mirror penguin amplitudes $\tilde{\alpha}_{3,4}$ and the coefficient $\tilde{a}_{3,4,5,6}$ has been given in (4.30). The relation between $\tilde{\alpha}_{3\text{EW},4\text{EW}}$ and $\tilde{a}_{7,8,9,10}$ can be easily deduced, and reads

$$\tilde{\alpha}_{3\text{EW}}^p(M_1 M_2) = \begin{cases} -\tilde{a}_9^p(M_1 M_2) + \tilde{a}_7^p(M_1 M_2), & \text{if } M_1 M_2 = PP, \\ \tilde{a}_9^p(M_1 M_2) + \tilde{a}_7^p(M_1 M_2), & \text{if } M_1 M_2 = PV, \\ \tilde{a}_9^p(M_1 M_2) - \tilde{a}_7^p(M_1 M_2), & \text{if } M_1 M_2 = VP, \\ -\tilde{a}_9^p(M_1 M_2) - \tilde{a}_7^p(M_1 M_2), & \text{if } M_1 M_2 = V^0 V^0, \\ -f_{\pm}^{M_1} (\tilde{a}_9^p(M_1 M_2) + \tilde{a}_7^p(M_1 M_2)), & \text{if } M_1 M_2 = V^{\pm} V^{\pm}, \end{cases}$$

$$\tilde{\alpha}_{4\text{EW}}^p(M_1 M_2) = \begin{cases} -\tilde{a}_{10}^p(M_1 M_2) - r_{\chi}^{M_2} \tilde{a}_8^p(M_1 M_2), & \text{if } M_1 M_2 = PP, \\ \tilde{a}_{10}^p(M_1 M_2) + r_{\chi}^{M_2} \tilde{a}_8^p(M_1 M_2), & \text{if } M_1 M_2 = PV, \\ \tilde{a}_{10}^p(M_1 M_2) - r_{\chi}^{M_2} \tilde{a}_8^p(M_1 M_2), & \text{if } M_1 M_2 = VP, \\ -\tilde{a}_{10}^p(M_1 M_2) + r_{\chi}^{M_2} \tilde{a}_8^p(M_1 M_2), & \text{if } M_1 M_2 = V^0 V^0, \\ f_{\pm}^{M_1} (-\tilde{a}_{10}^p(M_1 M_2) + r_{\chi}^{M_2} \tilde{a}_8^p(M_1 M_2)), & \text{if } M_1 M_2 = V^{\pm} V^{\pm}. \end{cases} \quad (6.34)$$

The NLO expression for the coefficients a_i has been given in (4.33), and the various terms involved in these expressions have been discussed afterwards. With these results at hand, it is easy to derive the amplitudes for the decays of interest.

6.4.3 Phenomenology of non-leptonic B decays

Given that there are two independent free parameters in the model, we can consider three simple scenarios, namely $\kappa_L^{bs} \neq 0, \kappa_R^{bs} = 0$; $\kappa_L^{bs} = 0, \kappa_R^{bs} \neq 0$; and $\kappa_L^{bs} = \kappa_R^{bs} \neq 0$. In the last scenario we fix $\kappa_L^{bs} = \kappa_R^{bs}$ in order to have only one complex free parameter for each model. Our purpose is to fit these complex parameters from the $\bar{B} \rightarrow \pi \bar{K}$ decays, and to study the effects in the B_s and other decays. We parameterize the complex parameters as $\kappa_i^{bs} = z \exp(-i\delta_z)$, and fit z and δ_z .

Each scenario presents specific signatures, which allows one to distinguish easily among the three possibilities. The interesting point here is that κ_L^{bs} and κ_R^{bs} involve new physics

with opposite chirality: κ_L^{bs} rules $b \rightarrow s$ left-handed transitions, as in the Standard Model, while κ_R^{bs} involves $b \rightarrow s$ right-handed transitions. The operators contributing to the two transitions are related by a parity transformation, which involve the following relation between their matrix elements [177]:

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = -(-)^{P_{M_1 M_2}} \langle M_1 M_2 | \tilde{Q}_i | \bar{B} \rangle, \quad \Rightarrow \quad a_i(M_1 M_2) \propto C_i^{\text{SM}} + \delta C_i - (-)^{P_f} \tilde{C}_i, \quad (6.35)$$

where $P_{M_1 M_2}$ is the parity of the final state. The relations in (6.35) are at the bottom of the different sign patterns between (6.34) and (2.44), which in turn produce different effects, when the two cases $\kappa_L^{bs} \neq 0$ or $\kappa_R^{bs} \neq 0$ are considered. In table 6.6 we show explicitly the effects of (6.35) on the electroweak amplitude coefficients $\alpha_{3\text{EW}}$ and $\tilde{\alpha}_{3\text{EW}}$, where we denote by a_i the Standard Model contribution, by δa_i the new contribution proportional to κ_L^{bs} and by $\tilde{a}_i$ the new contribution proportional to κ_R^{bs} . The information contained in this table becomes clearer, when the values of the short-distance coefficients collected in table 6.5 are taken into account. This additional information, which is strictly model-dependent, tell us that the main contributions come from a_9 , δa_9 and from $\tilde{a}_7$. Moreover, δa_7 is small, but it gives a contribution which is comparatively larger than the corresponding Standard Model coefficient a_7 . With these remarks at hand we can draw the following predictions: in case of new left-handed physics ($\kappa_L^{bs} \neq 0$), the new contribution δa_9 has always the same sign patterns with respect to the Standard Model contribution; one expects therefore a similar effect in all the PP , PV , VP , VV decays, unless some “accidental” enhancement occurs, like the one expected for PV , VV decays due to the absence of scalar currents. In case of right-handed new physics, instead, ($\kappa_R^{bs} \neq 0$), the main new contribution $\tilde{a}_7$ has different sign patterns in the PP , PV and in the VP , VV decays: this means that, if *e.g.* one fits a phase which gives a destructive interference in the PP decays, the new contribution will interfere constructively in case of VP , VV decays and the effects of new physics will be enhanced. This is exactly the case for the $\bar{B}_s \rightarrow \phi\pi, \phi\rho$ decays, but also for the $\bar{B} \rightarrow \pi\bar{K}^*$ decays³. Considering now δa_7 , we note it contributes with opposite sign to VP and VV decays. This means that this contribution will interfere constructively in case of *e.g.* the $\bar{B}_s \rightarrow \phi\pi$ decay, and destructively for the $\bar{B}_s \rightarrow \phi\rho$ decay, or viceversa. In case of right handed new physics, the same effect is obtained through $\tilde{a}_9$, which has similar patterns. These effects can be tested clearly by means of the ratios in (6.24). Finally, it remains to consider the case in which $\kappa_L^{bs} = \kappa_R^{bs} \neq 0$ (parity symmetric new physics). Because of the patterns of the κ_L^{bs} and κ_R^{bs} discussed above, we immediately deduce that this scenario is not interesting for the $\bar{B}_s \rightarrow \phi\pi, \phi\rho$ (and $\bar{B} \rightarrow \pi\bar{K}^*$) decays, because the new contributions have the same magnitude, and they cancel, so that no (or very small) modifications arise. In this case, some interesting information may be deduced instead from the $\bar{B} \rightarrow \rho\bar{K}$ decays.

In Figs. 6.4-6.6 we give the results for the branching ratio of the $\bar{B}_s$ decays in the three scenarios discussed here. It is easy to observe that for a new electroweak contribution around two times the corresponding Standard Model amplitude one gets an order one enhancement of the branching ratios, depending on the phase δ_z . The dependence on the

³the electroweak amplitude is a function of $\alpha_{3,\text{EW}}(\bar{K}^*\pi)$.

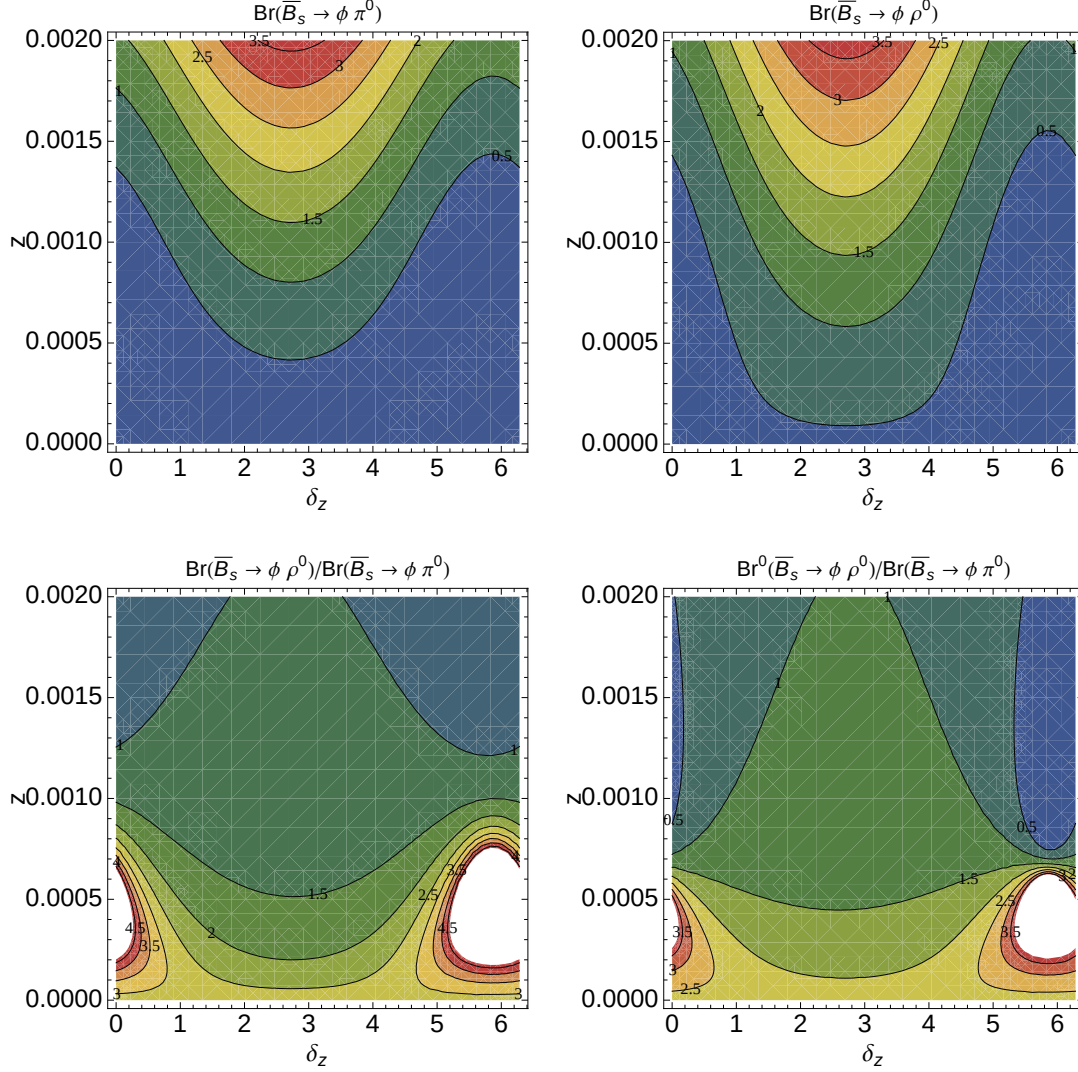


Figure 6.4: Relevant observables for the $\bar{B}_s$ decay modes in the first scenario, $\kappa_L^{bs} \neq 0$, $\kappa_R^{bs} = 0$.

phase and the results for the two ratios $\text{Br}^{(0)}(\bar{B}_s \rightarrow \rho^0 \phi)/\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)$ agrees with the qualitative prediction derived in this section.

6.4.4 Constraints from semi-leptonic B decays

The modified Z^0 scenario affects also the semi-leptonic B decays $\bar{B} \rightarrow X_s l^+ l^-$, $\bar{B} \rightarrow X_s \nu \bar{\nu}$, which have to be taken into account in our analysis. We focus in particular on $\bar{B} \rightarrow X_s e^+ e^-$, which has recently been observed, [349–351] and which provides the strongest constraints. We follow here [288] and assume that the Standard Model contribution is negligible with respect to the new one. The effective Hamiltonian for this process reads

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\kappa_L^{bs} (\bar{s}b)_{V-A} + \kappa_R^{bs} (\bar{s}b)_{V+A} \right] \sum_q [C_L^e (\bar{e}e)_{V-A} + C_R^e (\bar{e}e)_{V+A}], \quad (6.36)$$

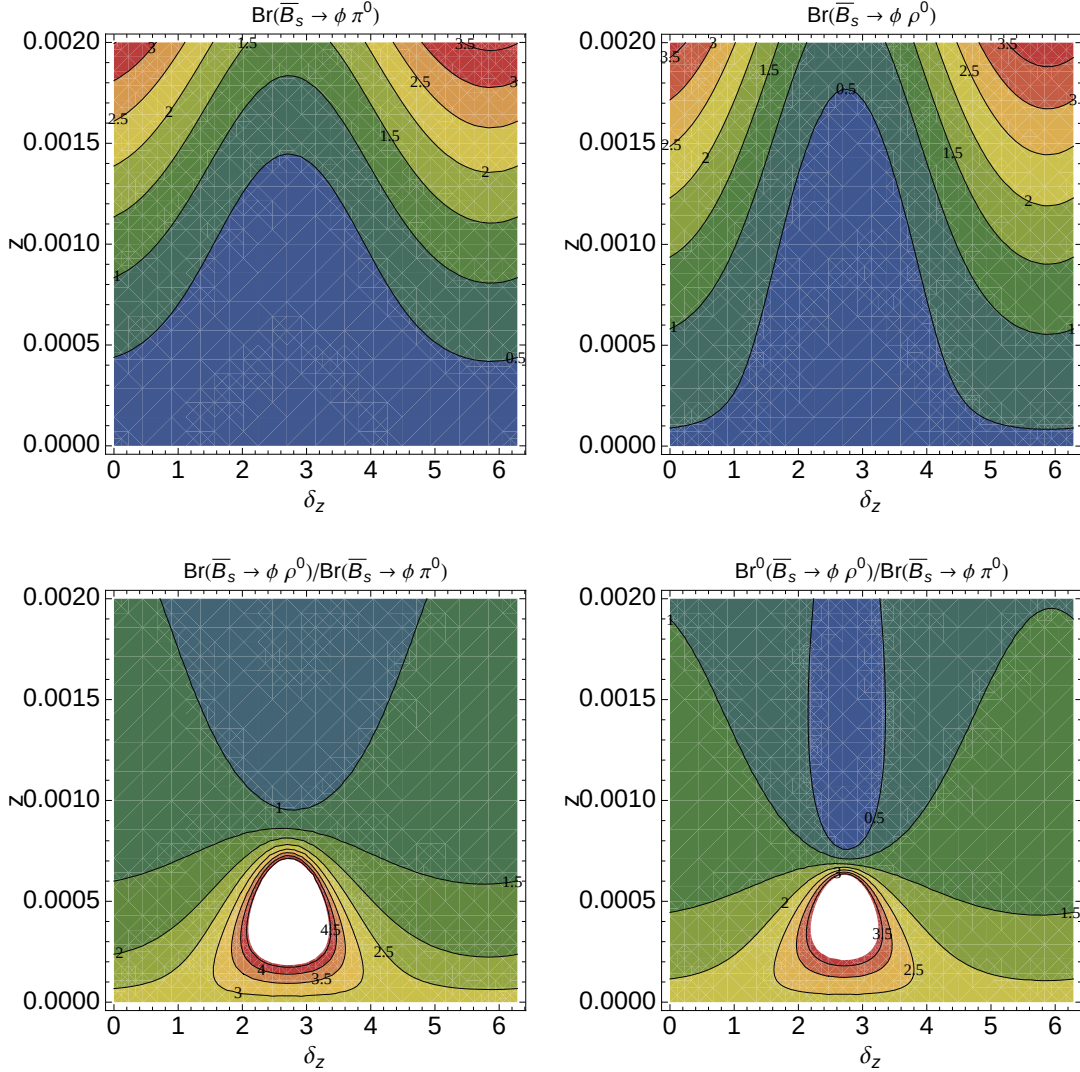


Figure 6.5: Relevant observables for the $\bar{B}_s$ decay modes in the second scenario, $\kappa_L^{bs} = 0$, $\kappa_R^{bs} \neq 0$.

where

$$C_L^e = -\frac{1}{2} + \sin^2 \theta_W, \quad C_R^e = \sin^2 \theta_W. \quad (6.37)$$

Normalizing the decay rate to the semi-leptonic rate $\Gamma(\bar{B} \rightarrow X_c e^- \bar{\nu}_e)$ one obtains

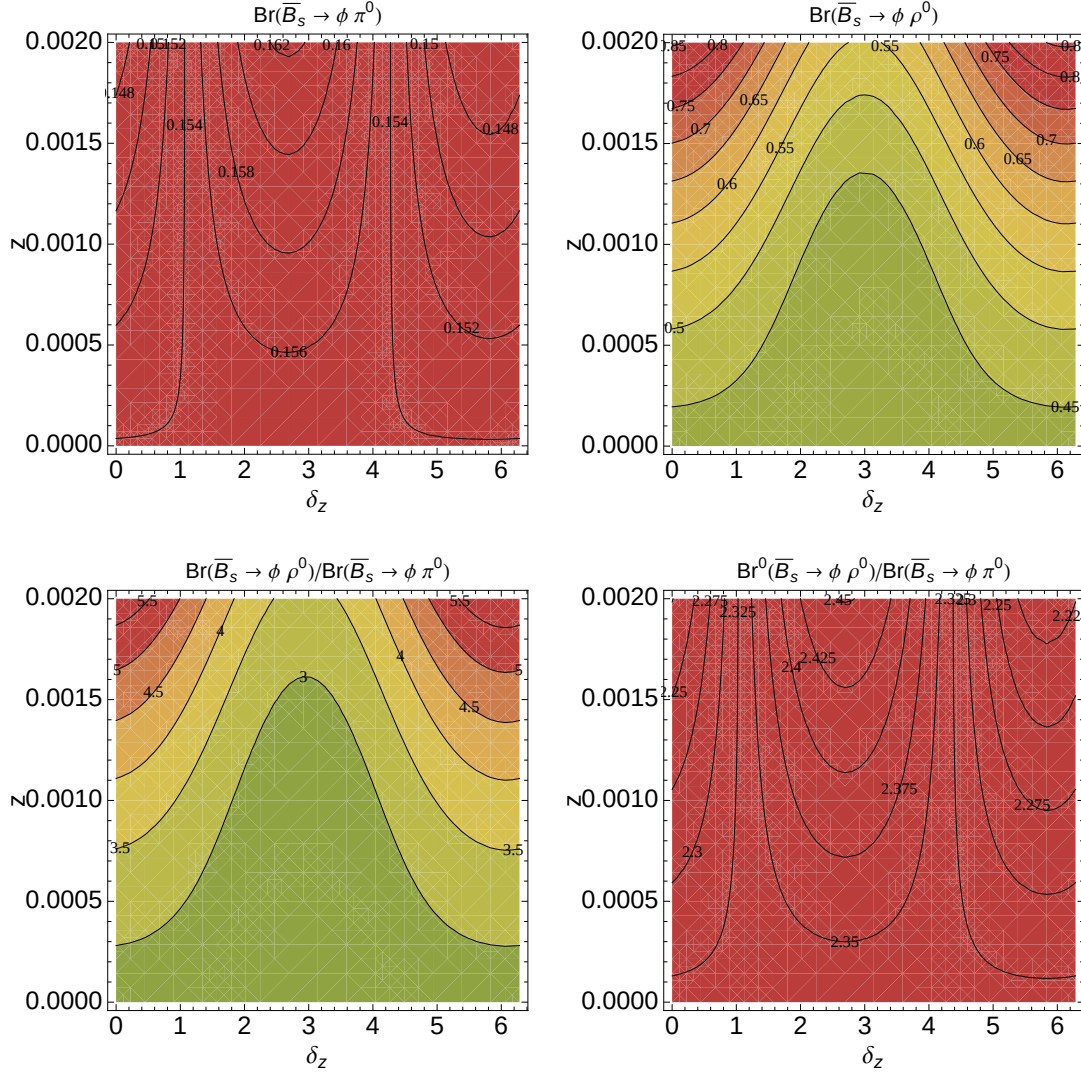
$$\frac{\Gamma(B \rightarrow X_s e^+ e^-)}{\Gamma(B \rightarrow X_c e^- \bar{\nu}_e)} = \frac{|\kappa_L^{bs}|^2 + |\kappa_R^{bs}|^2}{f(m_c/m_b)|V_{cb}|^2} ((C_L^e)^2 + (C_R^e)^2) \approx 157 \left(|\kappa_L^{bs}|^2 + |\kappa_R^{bs}|^2 \right), \quad (6.38)$$

where $f(m_c/m_b) \approx 0.5$ is the phase space factor for the semi-leptonic decay $\Gamma(\bar{B} \rightarrow X_c e^- \bar{\nu}_e)$, and $|V_{cb}| \approx 0.04$. The measured values for the decays considered are [8]

$$\text{Br}(B \rightarrow X_s e^+ e^-) = 4.7_{-1.3}^{+1.3} \cdot 10^{-6}, \quad (B \rightarrow X_c e^- \bar{\nu}_e) = 0.104_{-0.04}^{+0.04}, \quad (6.39)$$

at 3σ we get the constraint

$$3.4 \cdot 10^{-4} \leq \sqrt{|\kappa_L^{bs}|^2 + |\kappa_R^{bs}|^2} \leq 7.6 \cdot 10^{-4}, \quad (6.40)$$



6.5 Fitting the data: numerical results

6.5.1 The fit

The idea of the fit is quite simple: we calculate, within the new physics model, the expectation value for a set of observables, which are therefore a function of the parameters $\kappa_{L,R}^{bs}$, *i.e.* of z and δ_z . We compare then the value obtained at each point in the z - δ_z plane with the experimental measurements. The points in the z - δ_z plane for which the experimental and the theoretical results are closest are more likely to be realized, *i.e.* they represent the parameters according to which the theoretical prediction describes best the experimental measurements. Technically, this comparison is realized by evaluating at each point the χ^2 function, defined as

$$\sum_i \frac{(x_{i\text{theo}} - x_{i\text{exp}})^2}{\sigma_{i\text{exp}}^2}, \quad (6.41)$$

where $x_{i\text{theo}}$ represents the theoretical prediction of a physical observable like a branching ratio, and $x_{i\text{exp}}$ represents the corresponding experimental measurement. $\sigma_{i\text{exp}}$ represents the experimental error, *i.e.* the 1σ experimental uncertainty: $x_{i\text{exp}} \pm \sigma_{i\text{exp}}$.

The non-trivial part of the analysis is the implementation of the theoretical error, for which we follow [324]. The point is that the use of the χ^2 function in the fit assumes a gaussian distribution of the variables involved. This is indeed the case for the experimental measurements, which are assumed to be distributed like a gaussian around the central value $x_{i\text{exp}}$, with standard deviation $\sigma_{i\text{exp}}$. The same assumption cannot be made for the theoretical estimates. They depend in fact on a series of input parameters, like the form factors, the decay constants, *etc.* which have a flat distribution around the central value; actually, the meaning of the theoretical error is only that the corresponding observable can take any value inside the error range, which by itself does not imply any distribution at all. According to [324], to which we refer for further details, the problem of combining theoretical and experimental errors can be solved by considering the following χ_2^2 :

$$\chi^2 = \sum_i \begin{cases} \frac{(|x_{i\text{exp}} - x_{i\text{theo}}| - \sigma_{i\text{theo}})^2}{\sigma_{i\text{exp}}^2} & \text{if } |x_{i\text{exp}} - x_{i\text{theo}}| > \sigma_{i\text{theo}}, \\ 0 & \text{otherwise.} \end{cases} \quad (6.42)$$

Since the theoretical error range is asymmetric in case of the observables of interest, we generalize (6.42) to

$$\chi^2 = \sum_i \begin{cases} \frac{(x_{i\text{theo}} - \sigma_{i\text{theo,inf}} - x_{i\text{exp}})^2}{\sigma_{i\text{exp}}^2} & \text{if } x_{i\text{exp}} < (x_{i\text{theo}} - \sigma_{i\text{theo,inf}}), \\ \frac{(x_{i\text{exp}} - (x_{i\text{theo}} + \sigma_{i\text{theo,sup}}))^2}{\sigma_{i\text{exp}}^2} & \text{if } x_{i\text{exp}} > (x_{i\text{theo}} + \sigma_{i\text{theo,sup}}), \\ 0 & \text{otherwise,} \end{cases} \quad (6.43)$$

where $\sigma_{i\text{theo,sup}}$ and $\sigma_{i\text{theo,inf}}$ represent the theoretical upper and lower error respectively, $(x_{i\text{theo}})_{-\sigma_{i\text{theo,inf}}}^{+\sigma_{i\text{theo,sup}}}$. Using 6.43 it is possible to define confidence levels (CLs) by means of the function

$$\text{CL} = \frac{1}{\sqrt{2^{N_{\text{dof}}}} \Gamma(N_{\text{dof}}/2)} \int_{\Delta\chi^2(y_{\text{mod}})}^{\infty} e^{-t/2} t^{N_{\text{dof}}/2-1} dt, \quad (6.44)$$

where y_{mod} represents the theoretical parameters upon which the function χ^2 depends, like the form factors, quark masses, *etc.*, and the function $\Delta\chi^2$ represent the normalized χ^2 after its minimum has been subtracted: $\Delta\chi^2 = \chi^2 - \min(\chi^2)$. N_{dof} are the degree of freedom of the problem; in this case $N_{\text{dof}} = 2$. Using this input in (6.44) we obtain

$$\text{CL} = e^{-\frac{\Delta\chi^2(y_{\text{mod}})}{2}}. \quad (6.45)$$

Setting $\text{CL} = 1 - 68.27/100$, $\text{CL} = 1 - 95.45/100$, $\text{CL} = 1 - 99.73/100$, $\text{CL} = 1 - 99.99/100$ we individuate respectively the 1σ , 2σ , 3σ and 5σ confidence level. In our χ^2 we include the ratios R_c , R_n , R , the CP asymmetries $A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$, $A_{\text{CP}}(B^- \rightarrow \pi^- \bar{K}^0)$, $S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$ and the CP asymmetry difference ΔA_{CP} .

Here we are interested in fitting the parameters z , δ_z from the experimental information on the $\bar{B} \rightarrow \pi \bar{K}$ decays, and to look for consequences in the $\bar{B}_s$ decays. We consider therefore the other decays discussed in section 6.3.2 as constraints: this is necessary since, as explained there, large modifications are expected in these decays, which already rule out regions of the z - δ_z plane. In more detail, we set

$$CL = \begin{cases} CL & \text{if } \begin{cases} (x_{i\text{theo}} + \sigma_{i\text{theo, sup}}) > (x_{i\text{exp}} - 2\sigma_{i\text{exp, inf}}) \\ \vee (x_{i\text{theo}} - \sigma_{i\text{theo, inf}}) < (x_{i\text{exp}} + 2\sigma_{i\text{exp, sup}}), \end{cases} \\ 0 & \text{otherwise,} \end{cases} \quad (6.46)$$

i.e. we consider a 2σ constraint on these decays. Finally, we perform two fits: one in which we don't take into account the constraint from semi-leptonic decays, and one in which we add the constraint from (6.40), considering it as an additional theoretical input in the χ^2 function.

6.5.2 Results

$$\kappa_L^{bs} \neq 0, \quad \kappa_R^{bs} = 0$$

In figure 6.7 we show the result of the fit without (left) and with (right) the semi-leptonic constraint. In figure 6.8 we show the parameter space region excluded by the $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ decays, according to (6.47). These modes gives some constraints on the first fit, but none in case of the second one, because the semi-leptonic decay provides the strongest bound, as expected. The result of the first fit is

$$z = 9.6 \cdot 10^{-4}, \quad \delta_z = -90.7^\circ, \quad (6.47)$$

which corresponds to the following value for the observables in the $\bar{B} \rightarrow \pi \bar{K}$ modes:

$$\begin{aligned} R_c &= 1.18_{-0.18}^{+0.23}, & A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= -0.03_{-0.08}^{+0.16}, \\ R_n &= 1.19_{-0.20}^{+0.25}, & S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 0.34_{-0.20}^{+0.18}, \\ R &= 1.02_{-0.02}^{+0.04}, & \Delta A_{\text{CP}} &= 0.06_{-0.16}^{+0.08}. \end{aligned} \quad (6.48)$$

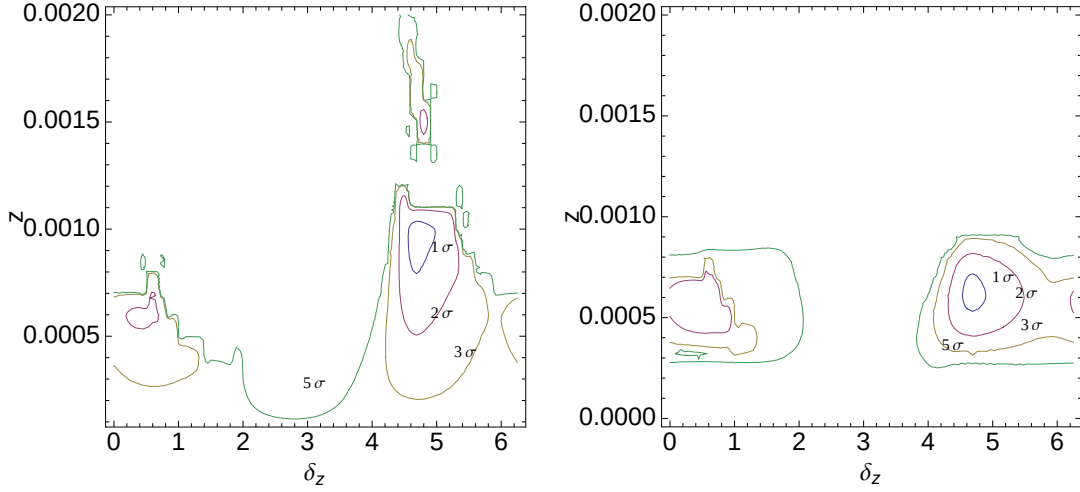


Figure 6.7: Confidence level regions in the z - δ_z plane, according to the first scenario considered, without (left) and with (right) the constraint from semi-leptonic decay.

One observes a better agreement with the experimental data. Based on this result the predictions for the $\bar{B}_s$ decays are

$$\begin{aligned}
 \text{Br}(\bar{B}_s \rightarrow \phi\pi^0) &= 0.47^{+0.50}_{-0.23}|_{\text{theo}}^{+0.60}_{-0.35}|_{\text{fit}} \cdot 10^{-6}, \\
 \text{Br}^0(\bar{B}_s \rightarrow \rho^0\phi) &= 0.48^{+0.56}_{-0.26}|_{\text{theo}}^{+0.37}_{-0.38}|_{\text{fit}} \cdot 10^{-6}, \\
 \text{Br}(\bar{B}_s \rightarrow \phi\rho^0) &= 0.60^{+0.56}_{-0.21}|_{\text{theo}}^{+0.43}_{-0.35}|_{\text{fit}} \cdot 10^{-6}, \\
 \frac{\text{Br}(\bar{B}_s \rightarrow \phi\rho^0)}{\text{Br}(\bar{B}_s \rightarrow \phi\pi^0)} &= 1.29^{+0.41}_{-0.23}|_{\text{theo}}^{+0.72}_{-0.40}|_{\text{fit}}, \\
 \frac{\text{Br}^0(\bar{B}_s \rightarrow \phi\rho^0)}{\text{Br}(\bar{B}_s \rightarrow \phi\pi^0)} &= 1.03^{+0.21}_{-0.18}|_{\text{theo}}^{+1.22}_{-0.33}|_{\text{fit}}.
 \end{aligned} \tag{6.49}$$

These results have to be compared with the Standard Model prediction (6.22), (6.23) and (6.24). The branching ratio of $\bar{B}_s \rightarrow \phi\pi^0$ is enhanced by a factor 3 with respect to the Standard Model, while there is no sizeable modification in the $\bar{B}_s \rightarrow \phi\rho^0$ decay. On one hand, this is an unfortunate circumstance, because the latter is easier to measure in experiments. On the other hand, being able to measure both decays would give the possibility of inspecting the ratios of the two decay rates, which, according to (6.49), differ significantly from the Standard Model prediction. This difference is indeed an effect of δa_7 , which contributes to the two decay amplitudes with opposite sign, as explained in section 6.4.3. In (6.49) the theoretical error is calculated in the standard way, summing in quadrature all the errors from each theoretical variable; the “fit error” is obtained by calculating the maximum and the minimum value taken by the observable inside the 2σ region individuated by the fit (Fig. 6.9), and subtracting the central value of the observable.

Taking into account the constraints from semi-leptonic decays, one obtains a smaller

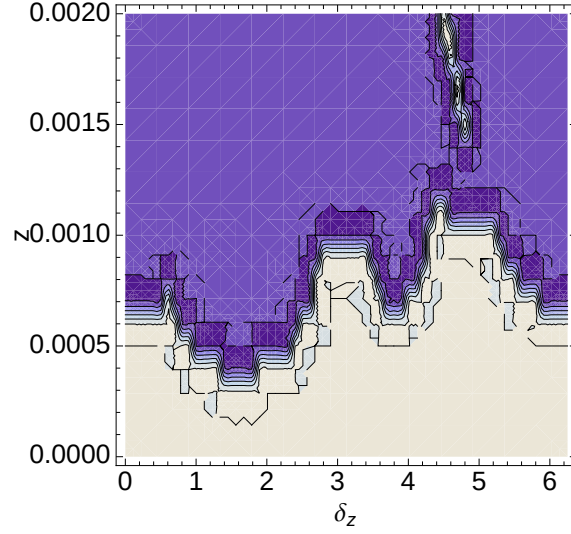


Figure 6.8: Region (blue) excluded by the experimental data of the $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ decays.

new contribution:

$$z = 6.1 \cdot 10^{-4}, \quad \delta_z = -91.0^\circ; \quad (6.50)$$

which gives

$$\begin{aligned} R_c &= 1.18_{-0.19}^{+0.23}, & A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= -0.03_{-0.06}^{+0.10}, \\ R_n &= 1.23_{-0.22}^{+0.27}, & S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 0.53_{-0.13}^{+0.11}, \\ R &= 1.01_{-0.01}^{+0.02}, & \Delta A_{\text{CP}} &= 0.05_{-0.10}^{+0.07}, \end{aligned} \quad (6.51)$$

and the prediction for the $\bar{B}_s$ decays reads

$$\begin{aligned} \text{Br}(\bar{B}_s \rightarrow \pi^0 \phi) &= 0.23_{-0.13}^{+0.28}|_{\text{theo}} {}^{+0.19}_{-0.20}|_{\text{fit}} \cdot 10^{-6}, \\ \text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.35_{-0.16}^{+0.39}|_{\text{theo}} {}^{+0.20}_{-0.27}|_{\text{fit}} \cdot 10^{-6}, \\ \text{Br}(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.44_{-0.15}^{+0.40}|_{\text{theo}} {}^{+0.18}_{-0.23}|_{\text{fit}} \cdot 10^{-6}, \\ \frac{\text{Br}(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} &= 1.91_{-0.51}^{+0.99}|_{\text{theo}} {}^{+5.05}_{-0.46}|_{\text{fit}}, \\ \frac{\text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} &= 1.50_{-0.29}^{+0.42}|_{\text{theo}} {}^{+1.65}_{-0.38}|_{\text{fit}}. \end{aligned} \quad (6.52)$$

As a consequence, modifications in the $\bar{B}_s$ modes are smaller and the decays are probably difficult to observe.

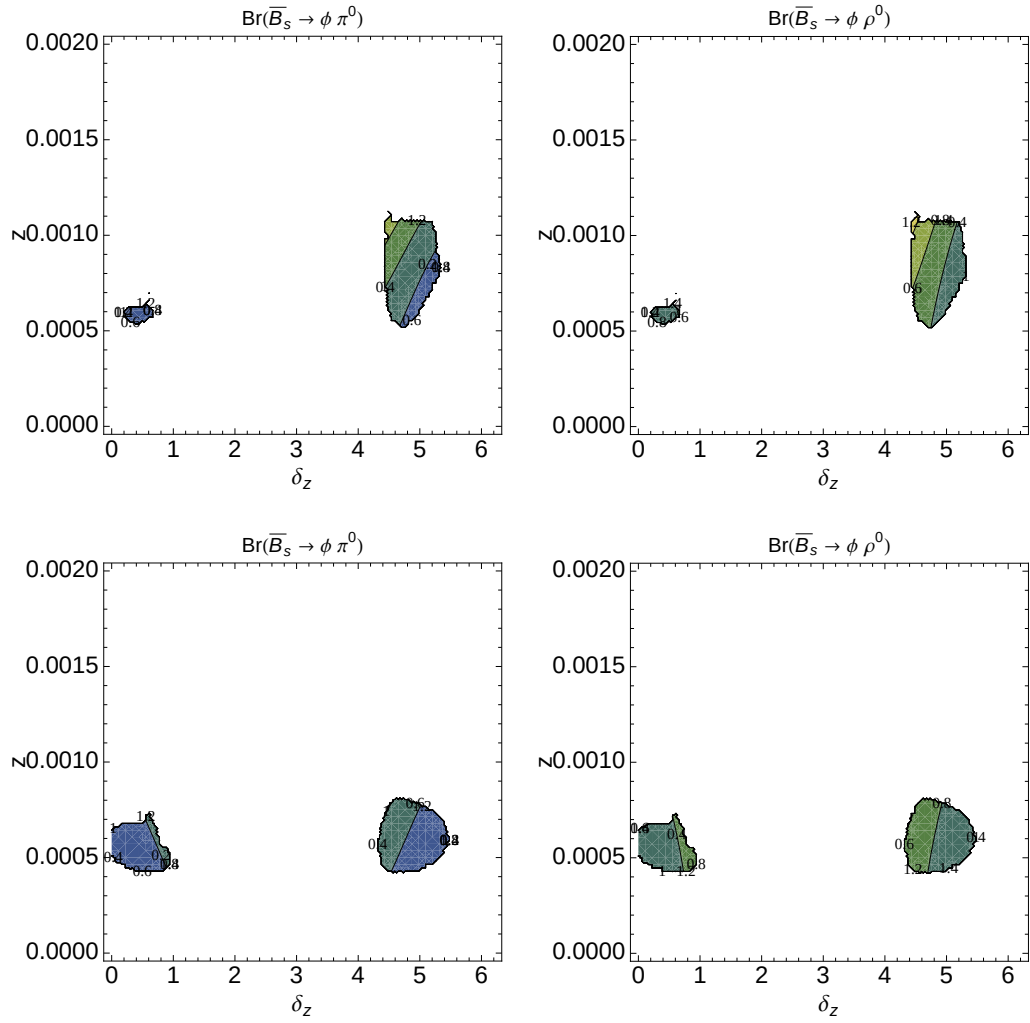


Figure 6.9: Branching ratios of the B_s decays inside the 2σ region, without (up) and with (low) the constraint from semi-leptonic decay.

$$\kappa_L^{bs} = 0, \kappa_R^{bs} \neq 0$$

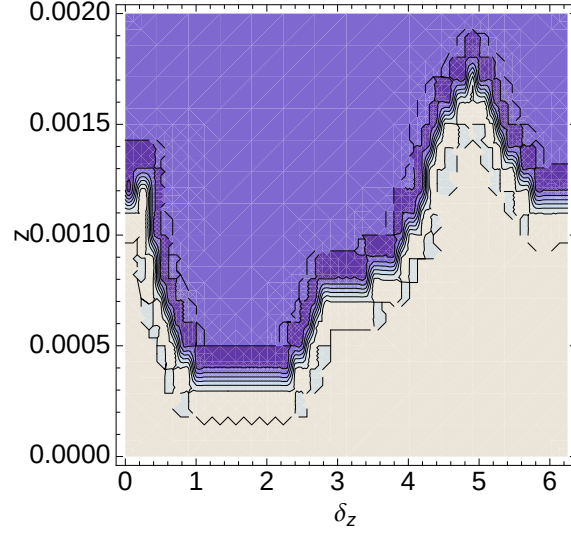


Figure 6.10: Region (blue) excluded by the experimental data of the $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ decays, second scenario.

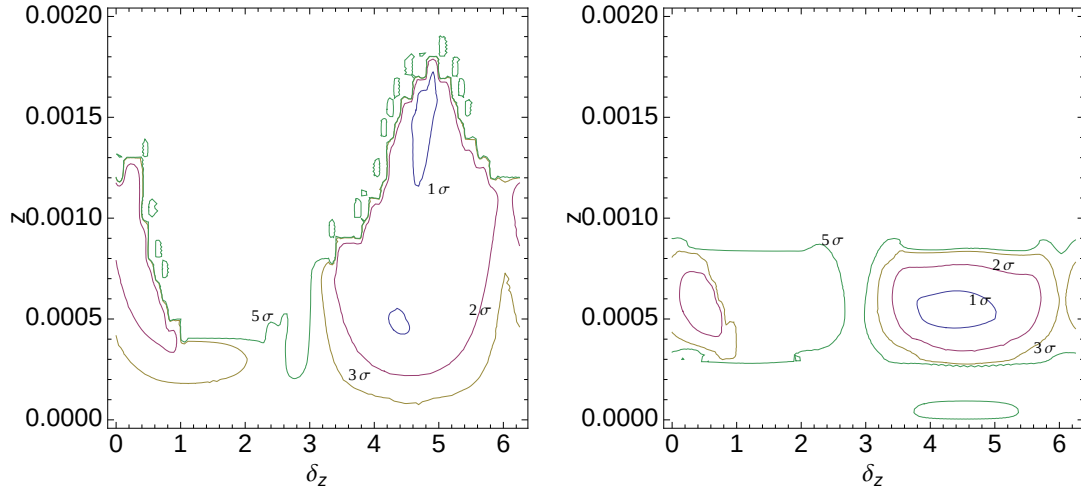


Figure 6.11: Confidence level regions in the z - δ_z plane, according to the second scenario considered, without (left) and with (right) the constraint from semi-leptonic decay.

In case of right-handed new physics the fit of the $\pi \bar{K}$ system points to larger values of the coupling κ_R^{bs} . Constraints from the $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ decays are consequently more important (Fig. 6.10), and they rule out a significant region in the z - δ_z plane, which would be favoured by the $\bar{B} \rightarrow \pi \bar{K}$ decays. After this constraint has been taken into account, the result of the fit, shown in Fig. 6.11, is

$$z = 16 \cdot 10^{-4}, \quad \delta_z = -87.3^\circ, \quad (6.53)$$

which is larger than the outcome of the previous scenario. The predicted values of the observables in the $\pi\bar{K}$ system are

$$\begin{aligned} R_c &= 1.15^{+0.22}_{-0.17}, & A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= -0.16^{+0.34}_{-0.12}, \\ R_n &= 1.20^{+0.24}_{-0.20}, & S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 0.39^{+0.28}_{-0.33}, \\ R &= 1.00^{+0.06}_{-0.04}, & \Delta A_{\text{CP}} &= 0.23^{+0.14}_{-0.50}. \end{aligned} \quad (6.54)$$

The predicted value for the B_s decays are

$$\begin{aligned} \text{Br}(\bar{B}_s \rightarrow \pi^0 \phi) &= 1.97^{+1.16}_{-0.27}|_{\text{theo}}^{+0.18}|_{\text{fit}} \cdot 10^{-6}, \\ \text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi) &= 1.93^{+1.12}_{-0.33}|_{\text{theo}}^{+0.32}|_{\text{fit}} \cdot 10^{-6}, \\ \text{Br}(\bar{B}_s \rightarrow \rho^0 \phi) &= 2.08^{+1.12}_{-0.34}|_{\text{theo}}^{+0.32}|_{\text{fit}} \cdot 10^{-6}, \\ \frac{\text{Br}(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} &= 1.05^{+0.08}_{-0.07}|_{\text{theo}}^{+0.97}|_{\text{fit}}, \\ \frac{\text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} &= 0.97^{+0.07}_{-0.07}|_{\text{theo}}^{+0.67}|_{\text{fit}}. \end{aligned} \quad (6.55)$$

In this case much larger branching ratios are obtained, around one order of magnitude larger than in the Standard Model. This is a consequence of two factors: on one hand, the larger value obtained for $|\kappa_R^{bs}|$; on the other hand, the fact that in VP decays the new contribution interferes constructively with the Standard Model amplitude, for the value δ_z found by the fitting procedure. Given such a large deviation, these results should be analysed with more care, in particular a complete analysis of the $\bar{B} \rightarrow \rho\bar{K}, \pi\bar{K}^*, \rho\bar{K}^*$ decays should be performed, even if they were considered as constraints in the fit above, because large deviations are expected in those channels, too.

When the semi-leptonic decay data are added to the fit, however, we find again strong bounds on the $|\kappa_R^{bs}|$ coupling, which is constrained to be three times smaller than the previous result:

$$z = 5.4 \cdot 10^{-4}, \quad \delta_z = -110.2^\circ. \quad (6.56)$$

The outcome in the $\pi\bar{K}$ system is

$$\begin{aligned} R_c &= 1.23^{+0.24}_{-0.20}, & A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= -0.07^{+0.11}_{-0.07}, \\ R_n &= 1.30^{+0.33}_{-0.27}, & S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 0.68^{+0.09}_{-0.12}, \\ R &= 1.02^{+0.03}_{-0.03}, & \Delta A_{\text{CP}} &= 0.09^{+0.07}_{-0.17}, \end{aligned} \quad (6.57)$$

and in case of the B_s decays we obtain

$$\begin{aligned} \text{Br}(\bar{B}_s \rightarrow \pi^0 \phi) &= 0.33^{+0.20}_{-0.10}|_{\text{theo}}^{+0.51}|_{\text{fit}} \cdot 10^{-6}, \\ \text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.50^{+0.31}_{-0.15}|_{\text{theo}}^{+0.60}|_{\text{fit}} \cdot 10^{-6}, \\ \text{Br}(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.58^{+0.32}_{-0.14}|_{\text{theo}}^{+0.61}|_{\text{fit}} \cdot 10^{-6}, \end{aligned}$$

$$\frac{\text{Br}(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} = 1.73_{-0.17}^{+0.36}|_{\text{theo}} {}_{-0.34}^{+0.30}|_{\text{fit}},$$

$$\frac{\text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} = 1.48_{-0.14}^{+0.18}|_{\text{theo}} {}_{-0.28}^{+0.16}|_{\text{fit}}. \quad (6.58)$$

The $\bar{B} \rightarrow X_s l^+ l^-$ decay limits the possibility of having a large enhancement of the $\bar{B}_s \rightarrow \phi \pi^0, \phi \rho^0$ branching ratios. Eventually, an indication of deviation from the Standard Model could be given by the ratio of the two decay rates. The effects found for $\kappa_R^{bs} \neq 0$ are in any case larger with respect to the scenario with $\kappa_L^{bs} \neq 0$, also when the semi-leptonic decay is taken into account.

$$\kappa_L^{bs} = \kappa_R^{bs} \neq 0$$

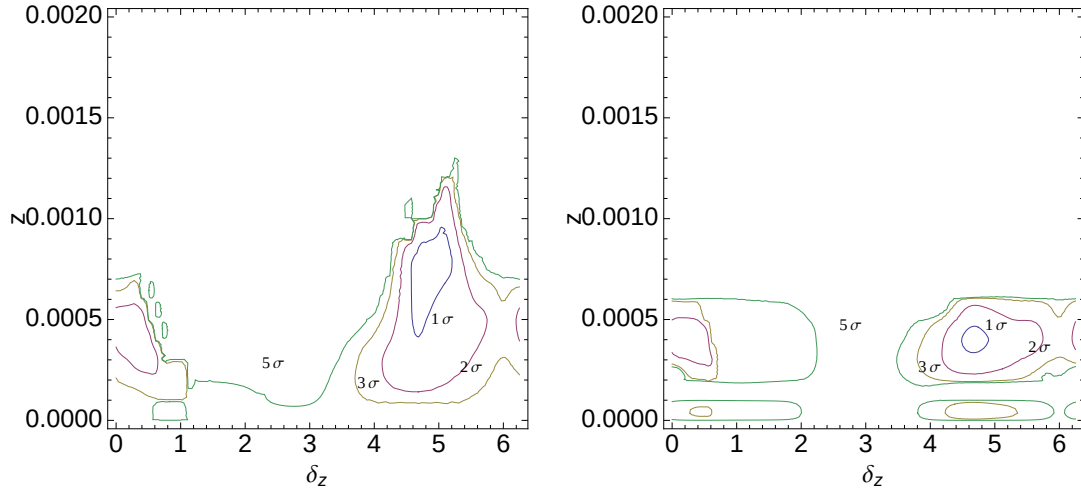


Figure 6.12: Confidence level regions in the z - δ_z plane, according to the third scenario considered, without (left) and with (right) the constraint from semi-leptonic decay.

This case is not particular interesting in view of modifications in the B_s decays. Nevertheless, we report here the results for completeness. We find

$$z = 6.4 \cdot 10^{-4}, \quad \delta_z = -89.7^\circ. \quad (6.59)$$

Confidence level regions as well as constraints from $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ decays are shown in figures 6.12, 6.13. Then

$$\begin{aligned} R_c &= 1.17_{-0.18}^{+0.23}, & A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= -0.09_{-0.10}^{+0.25}, \\ R_n &= 1.18_{-0.20}^{+0.24}, & S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 0.33_{-0.27}^{+0.23}, \\ R &= 1.02_{-0.03}^{+0.05}, & \Delta A_{\text{CP}} &= 0.13_{-0.31}^{+0.10}, \end{aligned} \quad (6.60)$$

and the $\bar{B}_s \rightarrow \phi \pi^0, \phi \rho^0$ branching ratios read

$$\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi) = 0.15_{-0.04}^{+0.11}|_{\text{theo}} {}_{-0.002}^{+0.001}|_{\text{fit}} \cdot 10^{-6},$$

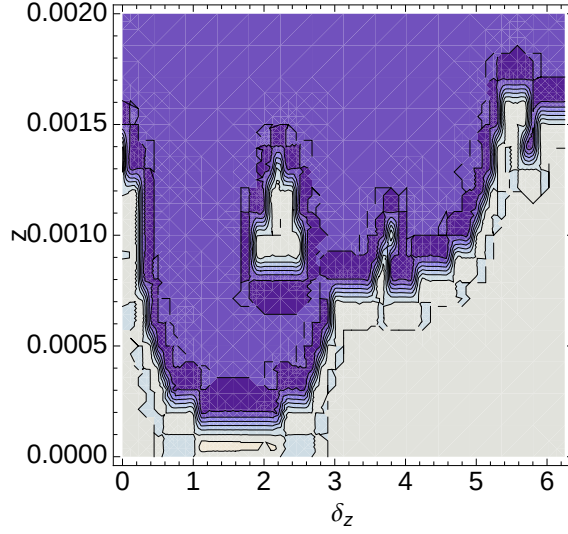


Figure 6.13: Region (blue) excluded by the experimental data of the $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ decays, third scenario.

$$\begin{aligned}
 \text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.35_{-0.10}^{+0.26}|_{\text{theo}} 0.004_{-0.01}^{+}|_{\text{fit}} \cdot 10^{-6}, \\
 \text{Br}(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.47_{-0.11}^{+0.27}|_{\text{theo}} {}^{+0.10}_{-0.03}|_{\text{fit}} \cdot 10^{-6}, \\
 \frac{\text{Br}(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} &= 3.05_{-0.40}^{+0.54}|_{\text{theo}} {}^{+0.73}_{-0.19}|_{\text{fit}}, \\
 \frac{\text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} &= 2.32_{-0.09}^{+0.11}|_{\text{theo}} {}^{+0.01}_{-0.04}|_{\text{fit}}.
 \end{aligned} \tag{6.61}$$

Taking into account the semi-leptonic decay, we get

$$z = 4.0 \cdot 10^{-4}, \quad \delta_z = -91.9^\circ, \tag{6.62}$$

so that

$$\begin{aligned}
 R_c &= 1.19_{-0.19}^{+0.22}, & A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= -0.07_{-0.08}^{+0.15}, \\
 R_n &= 1.24_{-0.22}^{+0.27}, & S_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 0.52_{-0.17}^{+0.14}, \\
 R &= 1.01_{-0.02}^{+0.03}, & \Delta A_{\text{CP}} &= 0.09_{-0.19}^{+0.08},
 \end{aligned}$$

and the $\bar{B}_s \rightarrow \phi \pi^0, \phi \rho^0$ decays read

$$\begin{aligned}
 \text{Br}(\bar{B}_s \rightarrow \pi^0 \phi) &= 0.15_{-0.04}^{+0.11}|_{\text{theo}} {}^{+0.001}_{-0.001}|_{\text{fit}} \cdot 10^{-6}, \\
 \text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.36_{-0.10}^{+0.26}|_{\text{theo}} {}^{+0.002}_{-0.0005}|_{\text{fit}} \cdot 10^{-6}, \\
 \text{Br}(\bar{B}_s \rightarrow \rho^0 \phi) &= 0.45_{-0.11}^{+0.27}|_{\text{theo}} {}^{+0.03}_{-0.003}|_{\text{fit}} \cdot 10^{-6}, \\
 \frac{\text{Br}(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} &= 2.91_{-0.31}^{+0.44}|_{\text{theo}} {}^{+0.20}_{-0.03}|_{\text{fit}},
 \end{aligned}$$

$$\frac{\text{Br}^0(\bar{B}_s \rightarrow \rho^0 \phi)}{\text{Br}(\bar{B}_s \rightarrow \pi^0 \phi)} = 2.32_{-0.09}^{+0.11}|_{\text{theo}} {}_{-0.02}^{+0.005}|_{\text{fit}}. \quad (6.63)$$

6.6 Conclusion

In this chapter we performed a new physics analysis following a complementary approach with respect to the work described in chapters 4 and 5. Instead of analysing non-leptonic B decay in the context of a specific new physics model, we started analysing the experimental results, looking for actual discrepancies with respect to the theoretical prediction. The motivation for this work were a series of recent papers [298–310], where it was pointed out, using flavour symmetries of QCD for the evaluation of the decay amplitudes, that discrepancies with the experimental measurement suggest the presence of new physics in the electroweak penguin sector of the theory.

As a first step, we performed again the study of the $\bar{B} \rightarrow \pi \bar{K}$ decays, in which the discrepancies were found, and updated the analysis by considering the most recent experimental results and using QCD factorization for the evaluation of the amplitudes. The outcome is that the discrepancies have decreased in the latest experimental averages, and now they could be accounted for in the Standard Model, assuming an enhanced colour-suppressed tree contribution, possibly with the exception of the direct CP asymmetry difference $\Delta A_{\text{CP}} = A_{\text{CP}}(B^- \rightarrow \pi^0 K^-) - A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^+ K^-)$, whose experimental average deviates $\sim 1.5\sigma$ from the theoretical prediction. Given the difficulties of obtaining a large rescattering phase in QCD factorization, which could be at the origin of the large ΔA_{CP} observed, and given some inconsistencies in the experimental data, we tried the road to consider this result as an indication of new physics in the electroweak penguin sector, and we looked for consequences in other decays.

In this context, modifications are expected in particular in the $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ decays, which have the same flavour structure as the $\bar{B} \rightarrow \pi \bar{K}$ decays, and in the $\bar{B}_s \rightarrow \phi \pi, \phi \rho$ decays, which are electroweak penguin dominated. We focused in particular on the latter, because their amplitude is very simple and their branching ratio small; the observation of these decays with a branching ratio 2-to-4 times larger than their Standard Model expectation would be a clear signal of new physics in the electroweak sector.

In order to make quantitative predictions, we considered an actual model in which one has new physics in the electroweak sector, namely the modified Z^0 penguin scenario. We took into account both the possibilities of having left-handed and right-handed $\bar{s}bZ^0$ transitions, and we estimated the value of the corresponding couplings by minimizing the differences between theoretical predictions and experimental data in the $\bar{B} \rightarrow \pi \bar{K}$ decays. The results obtained were used to estimate the branching ratios of the B_s decays. An important point was the inclusion in the analysis of the $\bar{B} \rightarrow \rho \bar{K}, \pi \bar{K}^*, \rho \bar{K}^*$ and the semi-leptonic $B \rightarrow X_s l^+ l^-$ decays as constraints on the free parameters. While the former give weak constraints, the latter was found to give the strongest bound: taking it into account, only small deviations are expected in the B_s decays.

More in general, because of the interference patterns we found that the $B \rightarrow VP, VV$ decays are more sensitive to right-handed new physics than to left-handed one. Given that the bare results of the fit points to a new large contribution in the electroweak

sector, of order 1-to-2 times the electroweak amplitude of the Standard Model, it would be interesting to extend this analysis to other new physics scenarios, in which constraints from semi-leptonic observables are weaker, such as models with extra gauge symmetries, or supersymmetric models.

Chapter 7

Conclusion

In this thesis we have considered various aspects of non-leptonic B decays. These processes are very interesting because they allow us to test various aspects of the Standard Model, and also to look for new physics. The decays are driven at the quark level by the transitions of the b quark into lighter quarks, and therefore their analysis allows us to extract the matrix elements of the CKM matrix. Even if this task is made nontrivial by the dominant effects of low-energy QCD interactions, nowadays many different strategies to address this problem have been elaborated, and, in the end, this turns out into a better understanding of low-energy strong interactions, too. The comprehension of QCD interaction in non-leptonic B decays allows us not only to extract with better accuracy the CKM matrix elements, but also to perform new physics searches. This is because the virtual transitions that convert the b quark into lighter ones occur at the electroweak scale, and we have various reasons to believe that new physics should manifest at scales just above it.

Our work focused on aspects of non-leptonic decays both in the Standard Model and beyond. After a general introduction, we have devoted chapter 2 to an exhaustive description of QCD factorization, *i.e.* the method used in this thesis to calculate the decay amplitudes of these processes. The successive chapters focus on our work.

More in detail, chapter 3 concerns a study of B decays into charmonium states and a light meson. This work is an extension of [125], and was motivated by previous papers [191,192], which found that QCD factorization does not work for these decays even at leading order in the Λ_{QCD}/m_b expansion. We have revisited the problem, showing that factorization can be achieved, if the bound state scales of charmonium are considered. The latter are in fact perturbative in the heavy quark limit, and this allowed us to develop a formalism, based on effective field theory techniques, in which the charmonium states are treated as non-relativistic bound states. In this framework one has to consider charmonium production through colour-octet operators, whose inclusion allows us to absorb the divergencies found in the matrix elements of colour-singlet operators, which were the only ones included in previous analyses. The numerical analysis performed afterwards shows that for reasonable choices of the input parameters the main features of the data, *i.e.* large corrections to (naive) factorization and suppression of the χ_{c2} and h_c final states, can be reproduced, even though the suppression of χ_{c2} is not as strong as seen in the data. The formalism developed for this analysis may be of some interest for a better comprehension

of other non-leptonic B decays, because some of the divergences arising in the $B \rightarrow HM_1$ decays are endpoint divergences which appear in the evaluation of the spectator scattering. These divergences afflict also *e.g.* non-leptonic B decays into two light-mesons and it is not known how to address them in the general case. In our work we have provided a first example, in which endpoint divergences can be removed by absorbing them into the matrix elements of colour-octet operators. In chapter 3 we have extended the work presented in [125] by including the case of S -wave, adding the twist-3 contribution to the spectator scattering, and providing a full numerical analysis including these new results.

The successive chapters contain analyses of non-leptonic B decays in models of physics beyond the Standard Model. In chapter 4 and 5 we have considered two specific models, in order to test the sensitivity of these decays to new physics. In more detail, in chapter 4 we have reported the results obtained in [126], where we considered the MSSM with large $\tan\beta$, an interesting minimal flavour violating theory, which however can exhibit large deviations from the Standard Model. In chapter 5 we have considered an interesting supersymmetric grand unified scenario, in which effects of the large neutrino-mixing angles appear in the mixing among right-handed quarks. In both cases we find that effects in non-leptonic B decays are small, when constraints from leptonic decays in the first case, and from the particle spectrum and lepton flavour violation in the second case, are taken into account.

Finally, in chapter 6 we report about the ongoing work [128] in which we took the opposite approach, *i.e.* we looked for actual discrepancies between theory predictions and experimental results, which are found to suggest the possibility of having new physics in the electroweak penguin sector of the theory. We considered therefore an actual model in which new physics arises in this sector, namely the modified Z^0 penguin scenario, and we fitted the flavour-changing couplings of the model from the experimental data of the $\bar{B} \rightarrow \pi\bar{K}$ decays. Our interest was primarily to look for effects in other decays, such as the $\bar{B} \rightarrow \rho\bar{K}$, $\bar{B} \rightarrow \pi\bar{K}^*$ and $\bar{B} \rightarrow \rho\bar{K}^*$ decays, and the B_s modes $\bar{B}_s \rightarrow \phi\pi$, $\bar{B}_s \rightarrow \phi\rho$. In chapter 6 we focus principally on the latter, taking the former as constraints. We find that sizeable modification in these decays can arise, even if they are reduced consistently when constraints from semi-leptonic decays are considered. Since there are other scenarios, such as models with extra gauge bosons, where these constraints do not apply, we concluded that further analyses on these decays could give interesting results.

Appendix A

Effective Field Theory Review

Effective field theories are a powerful tool in quantum field theory, since they provide a systematic formalism for the analysis of multi-scale problems. This is particularly important in QCD, where the value of the running coupling $\alpha_s(\mu)$ can change significantly between different energy scales. The purpose of this appendix is to review the effective field theories that are useful in the treatment of the heavy-quark systems considered in this thesis.

The application of effective field theory to heavy-quark system is mandatory, since they present a natural separation of scales, $m_Q \gg \Lambda_{\text{QCD}}$, where m_Q represents the mass of the heavy quarks. In particular we consider both mesons formed by an asymmetric configuration $Q\bar{q}$, where Q represents the heavy quark and $\bar{q}$ a light antiquark (such as the B mesons), and systems formed by a pair of heavy quark-antiquark $Q\bar{Q}$ (such as the charmonium states, H). The two systems present quite different features, since the latter develops weak-coupling bound states in the heavy-quark limit, while the former don't. Moreover, the small expansion parameter for the two system is different, being $\lambda = (\Lambda_{\text{QCD}}/m_b)^{1/2}$ for the first, and v for the second, the velocity of the heavy quark. Indeed, the two systems are described by two different effective field theories, the heavy-quark effective field theory (HQET) for the $Q\bar{q}$ system, and the non-relativistic (NR) QCD for the $Q\bar{Q}$ states.

The decay of heavy-quarks usually leads to a set of light energetic hadrons, in the rest frame of the decaying meson. This provides again the presence of two well separated scales, given by the energy of the light mesons, $E \sim m_Q/2$, and the hadronic scale Λ_{QCD} . The elaboration of an effective field theory describing light energetic mesons is however nontrivial, because only some of the components of the light mesons momenta p^μ are large, while $p^2 \approx 0$. In contrast to HQET, integrating out those large momentum components leads to a nonlocal theory, that does not admit a local operator product expansion (OPE). The generalization of the OPE to this case has been achieved with the so-called soft-collinear effective field theory, (SCET), which deals with light energetic mesons.

SCET, in the two formulations I and II that involve different momentum scales, is at the basis of the theoretical understanding of non-leptonic B decays. In the following we provide a short review of their fields content and scaling, as well as other fundamental features. HQET, that is required for the treatment of the B meson, is automatically

Momentum scaling	Terminology
$(1, 1, 1)$	hard
$(\lambda, \lambda, \lambda)$	semi-hard
$(1, \lambda, \lambda^2)$	hard-collinear
$(\lambda^2, \lambda^2, \lambda^2)$	soft
$(1, \lambda^2, \lambda^4)$	collinear

Table A.1: Terminology for the various momentum modes relevant to exclusive B decays, as defined in [150]. The momentum components are given as $(n_+p, p_\perp, n_-p)$, but mass dimension has to be restored by multiplying with appropriate factors of m_b .

included. The exposition will follow closely the formulation of SCET given in [84, 150]; we will not consider here the hybrid momentum-position space formulation elaborated in [81, 83]. In the subsequent section we provide also the basic information about NRQCD, that is necessary for the theoretical treatment of the charmonia systems considered in chapter 3. More extensive review of NRQCD can be found in [121–124].

A.1 SCET_I

Fields

In table A.1 we enumerate the relevant momentum regions appearing in exclusive B decays, according to their scaling with respect to the small parameter $\lambda = (\Lambda_{\text{QCD}}/m_b)^{1/2}$. The SCET_I Lagrangian and operators are built from hard-collinear light quark fields ξ_C , hard-collinear gluon field A_C and soft light quarks, heavy quarks and gluon fields, denoted by q_s , h_v , A_s , respectively. It corresponds to a theory in which only the hard momentum region is integrated out. No distinction is made between hard-collinear and collinear momenta, and this is the reason why we use here the label C to indicate a generic (hard)-collinear momentum, instead of hc.

For illustration we focus only on one collinear sector, defined by the momenta decomposition

$$p^\mu = (n_+p)\frac{n_-^\mu}{2} + p_\perp^\mu + (n_-p)\frac{n_+^\mu}{2}, \quad (\text{A.1})$$

so that the individual components of a collinear momentum have the following scaling behaviour,

$$n_+p \sim m_b, \quad p_\perp \sim m_b\lambda, \quad n_-p \sim m_b\lambda^2, \quad (\text{A.2})$$

such that $p^2 \sim m^2\lambda^2$ as required. Here $n_\pm$ are the usual light-like vectors, $n_-^2 = n_+^2 = 0$, $n_+n_- = 2$. In the following we set $m_b = 1$ in scaling relation such as A.2, understanding that the dimension of any quantity is restored by inserting the appropriate power of m_b .

A soft field varies significantly only over distances of order $x \sim 1/\lambda^2$, so derivatives acting on these fields scale as

$$\partial_\mu \phi_s \sim \lambda^2 \phi_s, \quad (\text{A.3})$$

where we have indicated by ϕ_s a generic soft field. Hard-collinear fields have significant variations over $n_-x \sim 1$, and $x_\perp \sim 1/\lambda$; derivatives acting on them scale therefore as

$$n_+\partial\phi_C \sim \phi_C, \quad \partial_\perp\phi_C \sim \lambda\phi_C, \quad n_-\partial\phi_C \sim \lambda^2\phi_C. \quad (\text{A.4})$$

We first want to derive scaling rules for the fields; this is read off from the corresponding propagators in momentum space. In case of the hard-collinear quark field, it can be decomposed into two fields, containing the two large and small components of the original spinor. One has

$$\psi_C = \xi(x) + \eta(x), \quad \xi(x) = \frac{\not{n}_-\not{n}_+}{4}\psi_C(x), \quad \eta(x) = \frac{\not{n}_+\not{n}_-}{4}\psi_C(x), \quad (\text{A.5})$$

where $(\not{n}_\mp\not{n}_\pm)/4$ are projection operators, and $\not{n}_-\xi = \not{n}_+\eta = 0$. The propagator for the ξ field then reads

$$\langle 0|T\xi(x)\bar{\xi}(y)|0\rangle = \frac{n_-n_+}{4}\langle 0|T\psi_C(x)\bar{\psi}_C(y)|0\rangle \frac{n_+n_-}{4} = \int \frac{d^4p}{(2\pi)^4} \frac{\not{n}_-}{2} \frac{in_+p}{p^2 + i\epsilon} e^{-ip(x-y)}. \quad (\text{A.6})$$

For hard-collinear momentum p the r.h.s is of order λ^2 ; from this and the analogous equation for η with n_+ and n_- interchanged, one finds

$$\xi \sim \lambda, \quad \eta \sim \lambda^2. \quad (\text{A.7})$$

The small η field is usually integrated out from the Lagrangian. In a similar way one obtains that a soft quark field scale as

$$q_s \sim \lambda^2, \quad (\text{A.8})$$

while in case of hard-collinear and soft gluon fields one has

$$n_+A_C \sim 1, \quad A_\perp \sim \lambda, \quad n_-A_C \sim \lambda^2, \quad A_s^\mu \sim \lambda^2; \quad (\text{A.9})$$

the scaling of hard-collinear and soft gluons is the same as the corresponding momentum components, and this assure the homogeneity of the covariant derivatives, $iD_C^\mu\phi_C = (i\partial^\mu + gA_C^\mu)\phi_C$, $iD_s^\mu\phi_s = (i\partial^\mu + gA_s^\mu)\phi_s$. It remains to consider the field for the heavy quark, whose interactions with hard collinear fields are integrated out. Since the heavy quark interacts only with soft gluons, it is appropriate to remove the large component $m_b v$ of a heavy quark with momentum $p_B = m_b v + k$, as in HQET, introducing the field

$$h_v(x) = \frac{1 + \not{v}}{2} e^{im_b v \cdot x} Q(x), \quad (\text{A.10})$$

which varies significantly only over distances $x \sim 1/\lambda^2$, since k is soft. The momentum space propagator is $1/(vk)$, so

$$h_v \sim \lambda^3 \quad (\text{A.11})$$

as for the massless soft quark field.

To conclude this section, we provide the scaling rule for the integration element d^4x , which is needed for the derivation of Feynman rules from the effective action. It can be shown that in case of an interaction involving hard-collinear fields, or both hard-collinear and soft fields, the scaling is $d^4x \sim 1/\lambda^4$, while in case of soft fields the scaling is $1/\lambda^8$.

Gauge transformation

The theory has a residual gauge invariance from the gauge functions $U(x)$, that can themselves be classified as hard-collinear or soft. The proper gauge transformation under which the theory stays invariant can be individuated noting that soft fields cannot transform under hard-collinear gauge transformation, because this would turn them into hard-collinear fields; the latter instead can be multiplied by both hard-collinear and soft $U(x)$. We refer to [84] for a throughout discussion about the derivation of the residual gauge transformation under which the theory is invariant; we summarize here the result, which reads

$$\begin{aligned}
 \text{hard-collinear:} \quad & A_C \rightarrow U_C A_C U_C^\dagger + \frac{i}{g} U_C [D_s, U_C^\dagger], & \xi &\rightarrow U_C \xi, \\
 & A_s \rightarrow A_s, & q &\rightarrow q, \\
 \text{soft:} \quad & A_C \rightarrow U_s A_C U_s^\dagger, & \xi &\rightarrow U_s \xi, \\
 & A_s \rightarrow U_s A_s U_s^\dagger + \frac{i}{g} U_s [\partial, U_s^\dagger], & q &\rightarrow U_s q,
 \end{aligned} \tag{A.12}$$

Lagrangian

The derivation of the SCET_I Lagrangian is rather technical. One of the important results is that the Lagrangian derived from tree-level matching is not renormalized at any order in α_s . The usual approach is to derive first a Lagrangian that is gauge invariant, but whose terms are not homogeneous in λ . In a successive step each term is expanded in powers of λ , so that the Lagrangian can be written in the end as a sum of terms with definite scaling in λ . We will sketch this procedure only for the collinear part of the Lagrangian, since this allows us to show some important elements entering the SCET formalism, like the Wilson lines. The Feynman rules derived from the SCET_I Lagrangian can be used to calculate diagrams like the one in fig. 2.13.

The starting point is the Lagrangian for massless quark

$$\mathcal{L}_C = \bar{\psi}_C (i \not{D} + i\epsilon) \psi_C, \tag{A.13}$$

where ψ_C is the four-component spinor introduced in A.5, and the covariant derivative contains the sum of the hard-collinear and the soft gluon field. Inserting the decomposition A.5, one obtains

$$\mathcal{L}_C = \bar{\xi} \frac{\not{n}_+}{2} i n_- D \xi + \bar{\eta} \frac{\not{n}_-}{2} i n_+ D \eta + \bar{\xi} (i \not{D}_\perp + i\epsilon) \eta + \bar{\eta} (i \not{D}_\perp + i\epsilon) \xi, \tag{A.14}$$

where the $i\epsilon$ prescription is indicated explicitly, to show that the prescription from the original Lagrangian is attached to the transverse derivative. The subsequent step is to integrate out the small field η . Since the Lagrangian is quadratic this amounts to solving the classical equation of motion, which gives

$$\eta(x) = -\frac{\not{n}_+}{2} \frac{1}{i n_+ D + i\epsilon} i \not{D}_\perp \xi(x). \tag{A.15}$$

The result is highly non-local, due to the inverse differential operator acting on the hard-collinear quark field. This point is at the basis of the difference between SCET and other

effective field theories that admit a local OPE [78]. One can see this comparing A.15 with the similar solution for the small component of the heavy quark field H_v in HQET; in this case¹,

$$H_v = \frac{1}{2m_b + iv \cdot D} i\not{D}_\perp h_v = \frac{1}{2m_b} \sum_{n=0}^{\infty} \left(-\frac{iv \cdot D}{2m_b} \right)^n i\not{D}_\perp h_v, \quad (\text{A.16})$$

i.e., since the derivative in the denominator produces a power of the residual momentum $v \cdot k$, the result is “almost local”, and it can be expanded in an infinite series of local operators (OPE). In SCET however this is not the case. The solution comes from the introduction of Wilson lines

$$W(x) = P \exp \left(ig \int_{-\infty}^0 ds n_+ A(x + sn_+) \right), \quad (\text{A.17})$$

where P denotes path ordering, and $A = A_C + A_s$. These objects obey the useful relations

$$in_+ D W = W in_+ \partial, \quad \frac{1}{in_+ D + i\epsilon} = W \frac{1}{in_+ \partial + i\epsilon} W^\dagger, \quad (\text{A.18})$$

and this allows one to turn the inverse covariant derivative into an ordinary derivative, i.e. an integral, through the relation

$$\frac{1}{in_+ \partial + i\epsilon} \phi(x) = -i \int_{-\infty}^0 ds \phi(x + sn_+). \quad (\text{A.19})$$

The result is

$$\eta(x) = -\frac{\not{n}_+}{2} \frac{1}{in_+ D + i\epsilon} i\not{D}_\perp \xi(x) = i \frac{\not{n}_+}{2} W(x) \int_{-\infty}^0 ds \left[W^\dagger i\not{D}_\perp \xi \right] (x + sn_+). \quad (\text{A.20})$$

Inserting A.20 into A.14 one simply obtains

$$\mathcal{L}_C = \bar{\xi}(x) in_- D \frac{\not{n}_+}{2} \xi(x) + i \int_{-\infty}^0 ds \left[\bar{\xi} i \overleftarrow{\not{D}}_\perp W \right] (x) \left[W^\dagger i\not{D}_\perp \frac{\not{n}_+}{2} \xi \right] (x + sn_+), \quad (\text{A.21})$$

here $\overleftarrow{\not{D}}$ indicates a covariant derivative acting to the left. This is almost the final expression for the hard-collinear Lagrangian, except that it does not respect a proper power counting, since terms of different order in λ are mixed up. In order to avoid double counting, it is important to expand all objects in the effective Lagrangian such that the power counting is consistent. For instance, the different components of the covariant derivative acting on hard-collinear fields scale like

$$iD^\mu = i\partial^\mu + g_s A_{hc}^\mu + g_s A_s^\mu \sim (1, \lambda, \lambda^2) + (1, \lambda, \lambda^2) + (\lambda^2, \lambda^2, \lambda^2). \quad (\text{A.22})$$

While for $n_- D$ all the three contributions are of order λ^2 , for $n_+ D$ and for $D_\perp$ the contributions involving the soft gluon field are power suppressed and should be neglected at leading order. Similarly, in the definition of the Wilson line W the contribution of the soft gluon field in the exponent is power suppressed and should be neglected, i.e. $W = W_C + O(\lambda)$. Finally, in interactions with hard-collinear fields, soft fields must

¹we do not enters into the details of HQET, for which we refer to [79,80].

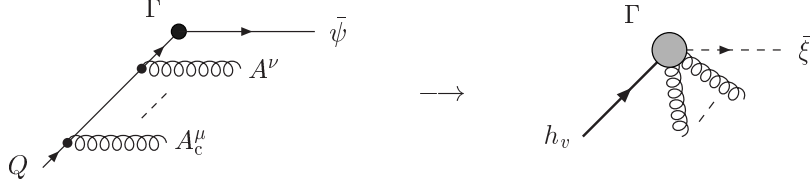


Figure A.1: Tree level matching of the heavy quark current, picture taken from [84]

be multi-pole expanded in order to ensure a proper power counting. This can be seen considering e.g. the phase factor associated with the coupling of a hard-collinear field with incoming momentum p_C to a soft field with incoming momentum p_s , producing a hard-collinear field with outgoing momentum p'_C :

$$S_{\text{int}} \ni \int d^4x \phi'_C(x) \phi_C(x) \phi_s(x) \sim \int d^4x e^{i(p'_C - p_C - p_s) \cdot x} \phi'_C(0) \phi_C(0) \phi_s(0). \quad (\text{A.23})$$

The combined momentum in the exponent scales like a hard-collinear momentum, since $(p'_C - p_C - p_s)^\mu \sim (1, \lambda, \lambda^2)$. A leading contribution to the action comes therefore from $x^\mu \sim (\lambda^{-2}, \lambda^{-1}, 1)$. However, the phase factor involving the soft momentum can be Taylor expanded:

$$e^{-ip_s \cdot x} = e^{-ip_{s+} \cdot x_-} (1 - ip_{s\perp} \cdot x_\perp - ip_{s-} \cdot x_+ + \dots), \quad (\text{A.24})$$

where the terms inside the brackets scale like 1, λ , and λ^2 , respectively. It follows that in interactions with hard-collinear fields we should expand soft fields as

$$\phi_s(x) = (1 + x_\perp \cdot \partial_\perp + \dots) \phi_s(x_-). \quad (\text{A.25})$$

Using these results, the Lagrangian A.21 can be rewritten as

$$\begin{aligned} \mathcal{L}_C = & \bar{\xi}(x) i n \cdot D_C(x) \xi(x) + \bar{\xi}(x) g_s n_- \cdot A_s(x_-) \xi(x) \\ & + \left(\bar{\xi} i \not{D}_{C\perp} W_C \right)(x) \frac{\not{n}_+}{2} i \int_{-\infty}^0 dt \left(W_C^\dagger i \not{D}_{C\perp} \xi \right)(x + s n_+). \end{aligned} \quad (\text{A.26})$$

It is easy to check that each term in A.25 scales like λ^4 , which when integrated with the measure $d^4x \sim \lambda^{-4}$ gives a leading contribution to the action.

At leading power there are no terms in the Lagrangian containing interactions of soft quark fields with hard-collinear fields, but there are interactions among soft and hard-collinear gluons. In principle, one can derive the higher order terms and include the soft part of the Lagrangian. Since the final result is rather lengthy, we will not report here a complete expression for the SCET_I Lagrangian, for which we refer instead to [84]. In the following we prefer to focus on the derivation of the heavy-to-light current, that enters in the definition of the leading operators (2.63).

Heavy-to-light current

Weak interaction processes are mediated by flavour-changing neutral currents, which in QCD have the general form $J = \bar{\psi} \Gamma Q$, where Q is a heavy quark, ψ a light quark and

Γ denotes some Dirac structures. Such a transition is at the basis of exclusive B decays, and it is therefore important to obtain an expression for this current in SCET. At this purpose, the first point to note is that the emission of an hard-collinear gluon from the heavy quark puts it off-shell by an amount of order m_b^2 . These states do not appear in SCET_I, and must be integrated out. The matching process is shown in fig. A.1, and it correspond to the evaluation in position space of the current

$$\begin{aligned} J_{\text{QCD}}(x) &= \bar{\psi}(x) \Gamma \sum_{n=0}^{\infty} \frac{1}{i\partial - m} \left(-g\mathcal{A}(x) \right) \cdots \frac{1}{i\partial - m} \left(-g\mathcal{A}_c(x) \right) Q(x) \\ &= e^{-imv \cdot x} \bar{\psi} \Gamma \left(1 - \frac{1}{i\mathcal{D} - m(1 - \psi)} g\mathcal{A}_c \right) Q_v. \end{aligned} \quad (\text{A.27})$$

The task is to derive an expression of A.27 in terms of only the fields entering the SCET_I Lagrangian, and to expand in the end in powers of λ . The calculation is rather technical and we refer for this to [84]. The final result is that the heavy-to-light current can be organized as

$$\begin{aligned} [\bar{\psi}(x) \Gamma Q(x)]_{\text{QCD}} &\rightarrow e^{-imv \cdot x} \left\{ J^{(A)} + J^{(B)} \right\} \\ &= e^{-imv \cdot x} \left\{ J^{(A0)} + J^{(A1)} + J^{(A2)} + J^{(B1)} + J^{(B2)} \right\}, \end{aligned} \quad (\text{A.28})$$

where 0, 1, 2... represent the power in λ of the various terms, and the split of the current in the A and B parts is done for convenience, because the two terms are separately invariant under the reparametrization of $n_{\pm}$, as explained in [84]. Expressions for these terms are given in [84] up to order λ^2 ; here we provide the expression for the leading order of the type A and B currents, that enter directly in the definition of the leading SCET_I operators 2.63:

$$\begin{aligned} J^{(A0)} &= \bar{\xi} \Gamma W_C h_v, \\ J^{(B1)} &= -\bar{\xi} \Gamma \frac{\not{n}_-}{2m} i\mathcal{D}_{\perp C} W_C h_v. \end{aligned} \quad (\text{A.29})$$

A.2 SCET_{II}

Fields

In SCET_{II} hard and hard-collinear modes are integrated out. This means that the initial and final states appearing in the theory can have invariant mass only of order λ^4 , and therefore it is suitable for the description of exclusive decays. (SCET_I, that admits initial and final states with invariant mass of order λ^2 , can be used to describe inclusive processes, or as an intermediate matching step between QCD and SCET_{II}). The theory contains collinear light quark fields ξ_c , collinear gluon fields A_c , and soft light quark, heavy quark and gluon fields, denoted respectively q_s , h_v and A_s . The scaling of the fields can be read off as before from the corresponding propagator in momentum space; soft fields have the same scaling as in SCET_I, while collinear fields scale as

$$\xi_c = \frac{\not{n}_- \not{n}_+}{4} \psi_c \sim \lambda^2, \quad n_+ A_c \sim 1, \quad A_{\perp c} \sim \lambda^2, \quad n_- A_c \sim \lambda^4. \quad (\text{A.30})$$

In this case the integration measure in the action is always of order $1/\lambda^8$.

Soft and collinear fields have significant variations over different length scales in the n_- and n_+ directions; consequently, they have to be multipole-expanded in products of soft and collinear fields, as in case of SCET_I. The multipole expansion we need here is however different, due to the different scaling of the collinear fields. In this case the definition is

$$\begin{aligned}\phi_s(x) &= \phi_s(x_-) + \frac{n_- x}{2} [n_+ \partial \phi_s](x_-) + \dots, \\ \phi_c(x) &= \phi_c(x_+) + \frac{n_+ x}{2} [n_- \partial \phi_c](x_+) + \dots,\end{aligned}\tag{A.31}$$

where

$$x_- \equiv n_+ x \frac{n_-}{2} + x_\perp, \quad x_+ \equiv n_- x \frac{n_+}{2} + x_\perp.\tag{A.32}$$

Gauge transformation

The theory is invariant under collinear and soft gauge transformations, defined as the restriction of gauge functions $U(x)$ to the corresponding spatial variations.

The implementation of gauge transformations in the effective theory is not unique, since field redefinitions or applications of the field equations can be used to alter the gauge-transformation properties. Considering that in SCET_{II} collinear and soft fields decouple at leading power in the λ expansion, and that the product of a collinear and a soft field gives an hard-collinear momentum mode, so that general soft gauge transformations acting on collinear fields (and vice versa) are not allowed, a natural choice for the gauge transformation is [150]

$$\begin{aligned}\xi_c &\rightarrow U_c \xi_c, \quad A_c \rightarrow U_c A_c U_c^\dagger + \frac{i}{g} U_c [\partial, U_c^\dagger], \\ h_v &\rightarrow U_s h_v, \quad q_s \rightarrow U_s q_s, \quad A_s \rightarrow U_s A_s U_s^\dagger + \frac{i}{g} U_s [\partial, U_s^\dagger].\end{aligned}\tag{A.33}$$

Lagrangian

The derivation of the effective Lagrangian is again rather technical, but straightforward. It can be organized as a two matching steps from QCD to SCET_I, and then to SCET_{II}. Here we focus on the main features of the effective Lagrangian, referring to the literature [87, 88, 150, 151] for the complete results. The main feature of the SCET_{II} Lagrangian is the decoupling of soft and collinear interactions at leading order in the λ expansion, so that the one has

$$\mathcal{L}^{(0)} = \mathcal{L}_s + \mathcal{L}_c,\tag{A.34}$$

with

$$\begin{aligned}\mathcal{L}_c &= -\frac{1}{2} \text{tr} (F_{\mu\nu c} F_c^{\mu\nu}) + \bar{\xi}_c \left(i n_- D_c + (i \not{D}_\perp - m) \frac{1}{i n_+ D_c} (i \not{D}_\perp + m) \right) \frac{\not{n}_+}{2} \xi_c, \\ \mathcal{L}_s &= -\frac{1}{2} \text{tr} (F_{\mu\nu s} F_s^{\mu\nu}) + \bar{q}_s (i \not{D}_s - m) q_s + \mathcal{L}_{\text{HQET}},\end{aligned}\tag{A.35}$$

with $m \sim \lambda^2$ the mass of the light quarks. Soft-collinear interactions arise only at order λ^2 ; we refer to [150] for an expression of higher order terms. The main point is that these higher-order interactions do not enter in the dynamics of exclusive B decays: this is because scattering processes of the type $s + c \rightarrow s + c$ do not occur (the transition is from the B meson, composed only by soft states, to light collinear mesons), and processes $s + s \rightarrow c + c$ are forbidden by momentum conservation in the $n_\mp$ direction. In case of exclusive B decays therefore all the soft-collinear interactions are restricted to the heavy-to-light current. At subleading order also the collinear and soft Lagrangian $\mathcal{L}_c, \mathcal{L}_s$ in A.34 receive corrections, that arise from integrating out heavy-quark loops.

Heavy-to-light current

The heavy-to light current

$$J(x) = e^{-im_b vx} [\bar{\psi} \Gamma \mathcal{Q}] (x) \quad (\text{A.36})$$

is derived in SCET_{II} writing the heavy- and light-quarks in terms of the various momentum modes

$$\begin{aligned} \psi &= \xi_c + \eta_c + \xi_{hc} + \eta_{hc} + q_s, \\ \mathcal{Q} &= \left(1 + \frac{i \not{D}_s}{2m_b}\right) h_v - \frac{1}{n_- v} \frac{\not{n}_-}{2m_b} (g \not{A}_{\perp c} + g \not{A}_{\perp hc}) h_v + O(\lambda^4 h_v), \end{aligned} \quad (\text{A.37})$$

where now the presence of the hard-collinear and collinear fields is considered explicitly, and η_c, η_{hc} represent their small components. The heavy-to-light current is obtained integrating out the hard collinear quark field first, thus obtaining an expression for ψ in terms of soft, collinear fields, and hard-collinear gluon fields. The result is expanded in power of λ , so that one obtains ψ as

$$\psi = \xi_c + \eta_c + \xi_{hc} + \eta_{hc} + q_s \equiv \psi^{(2)} + \psi^{(3)} + \psi^{(4)} + \psi^{(5)} + \dots; \quad (\text{A.38})$$

ψ has been calculated up to order λ^5 , with e.g.

$$\begin{aligned} \psi^{(2)} &= \xi_c, \\ \psi^{(3)} &= \left(1 + \frac{1}{in_- \partial} g \not{A}_{\perp c} \frac{\not{n}_-}{2}\right) q_s, \end{aligned} \quad (\text{A.39})$$

while for higher order terms we refer to [150]. The solution for ψ is subsequently inserted into the gluon field equation, so that the hard-collinear gluon is expressed in terms of soft and collinear fields. The final result for the heavy-to-light current can be expressed as

$$J(x) = e^{-im_b vx} \left[J_{\text{eff}}^{(0)}(x) + J_{\text{eff}}^{(1)}(x) + J_{\text{eff}}^{(2)}(x) + J_{\text{eff}}^{(3)}(x) + \dots \right], \quad (\text{A.40})$$

where the superscript indicates the λ suppression with respect to the leading term. One has e.g.

$$\begin{aligned} J_{\text{eff}}^{(0)} &= \bar{\xi}_c W_c \Gamma Y_s^\dagger h_v, \\ J_{\text{eff}}^{(1)} &= \bar{q}_s Y_s \not{A}_{\perp c} \frac{1}{in_- \partial} \frac{\not{n}_-}{2} \Gamma Y_s^\dagger h_v, \end{aligned} \quad (\text{A.41})$$

where W_c , Y_s are collinear and soft Wilson lines,

$$\begin{aligned} W_c(x) &= U_c^\dagger(x) = P \exp \left(ig \int_{-\infty}^0 ds n_+ A_c(x + sn_+) \right), \\ Y_s^\dagger(x) &= U_s(x) = P \exp \left(ig \int_0^\infty dt n_- A_s(x + tn_-) \right), \end{aligned} \quad (\text{A.42})$$

necessary to have gauge invariant expressions. For higher order terms we refer again to [150]. The SCET_{II} heavy-to-light current can be used to investigate the factorization properties of the heavy-to-light form factor at large recoil. In particular, considering the matrix element $\langle \pi | J_{\text{eff}}^{(n)} | \bar{B} \rangle$, one can see that neither $J_{\text{eff}}^{(0)}$, neither $J_{\text{eff}}^{(1)}$ have the right quantum number to give a matrix element different from zero. Going to higher order in the expansion one finds that the first term which has the right quantum number appears in $J_{\text{eff}}^{(3)}$, and has the form

$$- \left(\frac{g^2}{(in_+ \partial)(in_- \partial)} \bar{q}_s Y_s \gamma^{\mu_\perp} T^A W_c^\dagger \xi_c \right) \frac{1}{n_- v} \bar{\xi}_c W_c \Gamma \frac{\not{n}_-}{2m_b} \gamma_{\mu_\perp} T^A Y_s^\dagger h_v, \quad (\text{A.43})$$

which can be identified with the tree diagram, where a transverse gluon is exchanged between the heavy quark and the spectator quark; the inverse differential operator is the hard-collinear gluon propagator in position space. With these results one is able to reproduce the well known scaling of the heavy-to-light form factor 2.9, 2.10, [148]: one finds $J_{\text{eff}}^{(3)} \sim \lambda^8$ and, restoring the correct mass dimension and using the power counting for the external states as in 2.60, we obtain

$$\langle \pi(p') | J(0) | B(p) \rangle \approx \langle \pi(p') | J_{\text{eff}}^{(3)}(0) | B(p) \rangle \sim \lambda^3 \sim m_b \left(\frac{\Lambda}{m_b} \right)^{3/2}. \quad (\text{A.44})$$

A.3 Non-relativistic QCD

Fields and Lagrangian

Meson formed by a heavy-quark-antiquark pair, $\bar{Q}Q$, such as the charmonium states H formed by a $\bar{c}c$ pair, or as the bottomonium states $\bar{b}b$ are best described in the framework of non-relativistic QCD (NRQCD), or in its further elaboration of potential non-relativistic QCD (PNRQCD). These effective field theories exploit the hierarchy of scales $m_Q \gg m_Q v \gg m_Q v^2$, which originates from the fact that in the meson rest frame the heavy quark and antiquark are non-relativistic and move with velocity $v \ll 1$. $m_Q v$ and $m_Q v^2$ represent the scales of the typical heavy quark momentum and kinetic energy, respectively. The inverse of the momentum scale, $1/(m_Q v)$ represents the size of the meson, while $m_Q v^2$ represents also its binding energy. Typical values of v are $v = 0.3$ for the charmonium states and $v = 0.1$ for bottomonium states, so that the assumptions about the scale hierarchy are very good for the $\bar{b}b$ systems and they are still reasonable for the $\bar{c}c$ systems considered here.

NRQCD factorizes the large scale m_Q and the only assumption needed is $m_Q \gg \Lambda_{\text{QCD}}$. It does not deal with the large scale hierarchy $m_Q v \gg m_Q v^2$; for this reason, it has

to be considered as the first matching step to the potential NRQCD, which is the most appropriate framework in which perturbative heavy $\bar{Q}Q$ systems, *i.e.* systems which satisfy $m_Q v^2 \gg \Lambda_{\text{QCD}}$, are studied. In this regime the quark-antiquark pair develops bound states driven by the dominance of the static Coulomb force, a feature that is not considered explicitly in NRQCD and which is at the basis of our charmonium treatment in chapter 3.

Integrating out the heavy mass scale m_Q leads to the Lagrangian of NRQCD, which in the heavy meson rest frame reads

$$\begin{aligned}
\mathcal{L}_{\text{NRQCD}} = & \psi^\dagger \left(iD^0 + \frac{\mathbf{D}^2}{2m_Q} \right) \psi + \frac{1}{8m_Q^3} \psi^\dagger \mathbf{D}^4 \psi \\
& - \frac{d_1 g_s}{2m_Q} \psi^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} \psi + \frac{d_2 g_s}{8m_Q^2} \psi^\dagger (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}) \psi \\
& + \frac{d_3 i g_s}{8m_Q^2} \psi^\dagger \boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \psi + \dots \\
& + \text{antiquark terms} \\
& + \sum_{\Gamma} \frac{d_{\Gamma}}{m_Q^2} (\psi^\dagger \Gamma \chi) (\chi^\dagger \Gamma \psi) + \dots + \mathcal{L}_{\text{light}}, \tag{A.45}
\end{aligned}$$

where ψ is a four-component spinor representing the heavy quark, and χ represents the heavy antiquark. The $d_i(\mu)$ are effective couplings which can be calculated in perturbation theory, since $\alpha_s(m_c) \simeq 0.24$. By adding more operators the full scattering amplitude can be reproduced to any accuracy in an expansion in α_s and v . The NRQCD Lagrangian is similar to the HQET Lagrangian, except that here we have an antiquark field clone of the heavy-quark field and a quark-antiquark sector which is absent in HQET. The couplings in the single heavy-quark sector are identical to the ones in HQET to all order in α_s , provided the same factorization prescription is used. However, the power counting is in general different. For instance, the kinetic energy term $\partial^2/(2m_Q)$ is of the same order as $i\partial_0$, because non-relativistic $\bar{Q}Q$ systems have momenta of order $m_Q v$ and energies of order $m_Q v^2$. Actually, because the existence of these two scales, the matching process is more involved than in HQET. This is because if one fixes the cut-off of the theory Λ several times $m_Q v$, he has to calculate the effective coupling with a full dependence on v , that is much more complicate with respect to the final aim of expanding the amplitude in powers of v . On the other hand, choosing the cut-off $\Lambda \gg m_Q$ instead of $m_Q \gg \Lambda \gg m_Q v$ gives wrong results in dimensional regularization, and QCD is not matched correctly onto NRQCD. This difficulty is related exactly to the existence of the two scales $m_Q v$ and $m_Q v^2$.

A correct expansion can be derived using the method of regions [76,77], which explains how to construct an expansion in powers of v of an on-shell scattering amplitude of heavy-quarks with momentum $m_Q v$ and gluons with momentum $m_Q v^2$. One first has to identify the relevant momentum regions in the loop integrals, which follow from the singularity structure of the Feynman integrand. For non-relativistic scattering of heavy quarks one

finds four momentum regions [123],

$$\begin{aligned}
\text{hard (h): } l_0 &\sim \mathbf{l} \sim m_Q, \\
\text{semi-hard (sh): } l_0 &\sim \mathbf{l} \sim m_Q v, \\
\text{potential (p): } l_0 &\sim m_Q v^2, \mathbf{l} \sim m_Q v, \\
\text{soft (s): } l_0 &\sim \mathbf{l} \sim m_Q v^2.
\end{aligned} \tag{A.46}$$

Both, heavy-quarks and gluons can be hard, semi-hard and potential, but only gluons (and eventually light quarks) can be soft². The threshold expansion is constructed by writing a Feynman diagram in QCD as a sum of terms which follow from dividing each loop momentum integral into these four regions. The expansion rule is that in every region one performs a Taylor expansion in the quantities which are small in that region. These rules can in the end be reformulated as an effective Lagrangian. The hard subgraphs give the dimensionally regularized couplings of the NRQCD Lagrangian, but the distinction of semihard, potential and soft momentum implies a manipulation of the NRQCD lagrangian that is not evident from A.45.

After the hard region is integrated out, the remaining integrals can still be semi-hard, potential or soft. Since semi-hard heavy-quarks and gluons do not appear as external lines for non-relativistic systems, they can be integrated out. This constitutes the second matching step, as it is summarized in the scheme below, which leads to the construction of the PNRQCD Lagrangian:

$$\begin{array}{ccc}
\mathcal{L}_{\text{QCD}}[Q(h, sh, p), g(h, sh, p, s)] & \mu > m_Q & \\
\downarrow & & \\
\mathcal{L}_{\text{NRQCD}}[Q(hs, p), g(hs, p, s)] & m_Q v < \mu < m_Q & \\
\downarrow & & \\
\mathcal{L}_{\text{PNRQCD}}[Q(p), g(s)] & \mu < m_Q v &
\end{array}$$

In general the PNRQCD Lagrangian can be written as

$$\mathcal{L}_{\text{PNRQCD}} = \mathcal{L}'_{\text{NRQCD}} + \mathcal{L}_{\text{non-local}}, \tag{A.47}$$

where $\mathcal{L}_{\text{non-local}}$ collects all non-local interactions. The local interactions are exactly those of NRQCD, but the interpretation is different, because only potential heavy quarks and soft gluons are left over, and the interaction terms have definite velocity scaling rules. A potential quark propagator in coordinate space scales as v^3 , and therefore a quark field in PNRQCD scales as $v^{3/2}$. A soft gluon field scales as v^2 .

We analyse now the relevant term in $\mathcal{L}_{\text{PNRQCD}}$. The leading order term scales as v^5 and it is obtained by integrating out potential gluon exchange between a quark and an

²in the NRQCD literature the semi-hard and soft region are defined respectively soft and ultrasoft. Here we employ the notation semi-hard and soft to match the notation adopted in the thesis, where ultrasoft regions are never introduced.

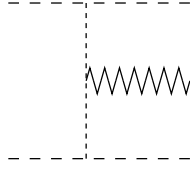


Figure A.2: NRQCD graph which generates the mixed non-local-soft interaction in A.50, taken from [123]. Long- and short-dashed lines represent respectively potential quarks and gluons. The zig-zag line represents a soft gluon.

antiquark at tree level. The unperturbed PNRQCD is

$$\begin{aligned} \mathcal{L}_{\text{PNRQCD}}^0 = & \psi^\dagger \left(i\partial^0 + \frac{\partial^2}{2m_Q} \right) \psi + \chi^\dagger \left(i\partial^0 - \frac{\partial^2}{2m_Q} \right) \chi \\ & + \int d^3\mathbf{r} \left[\psi^\dagger T^A \psi \right] (\mathbf{r}) \left(-\frac{\alpha_s}{r} \right) \left[\chi^\dagger T^A \chi \right] (0). \end{aligned} \quad (\text{A.48})$$

One can see that the Coulomb interaction is leading and cannot be treated as a perturbation. In general, it is possible to rewrite the PNRQCD lagrangian in terms of a “tensor field” $[\psi \otimes \chi](t, \mathbf{R}, \mathbf{r})$, which depends on the center of mass $\mathbf{R}$ and relative coordinate $\mathbf{r}$. The leading order Lagrangian describes the free propagation of particles with mass $2m_Q$ in the center of mass frame. The propagation of $[\psi \otimes \chi](t, \mathbf{R}, \mathbf{r})$ in its relative coordinate is given by the Coulomb Green function of a particle with reduced mass $m_Q/2$. Corrections of order $(\alpha_s/v)^n$ in the calculation of diagrams with Coulomb Green functions need to be resummed to all orders; the remaining terms can be treated as perturbation in v and α_s to the leading order Lagrangian. Even if the local interactions in A.47 are the same as those in NRQCD, they have to be multipole expanded, in order to take into account the expansion rules for soft gluons: when a soft gluon with momentum l connects to a quark line with momentum $k - l/2$ for the incoming and $k + l/2$ for the outgoing quark line, the quark-gluon vertex and the quark propagator have to be expanded in $l/k \simeq v$. This implies e.g.

$$\left[\psi^\dagger A^i \partial^i \psi \right] (x) \equiv \psi^\dagger(x) A^i(t, 0) \partial^i \psi(x) + \psi^\dagger(x) (x^j \partial^j A^i(t, 0)) \partial^i \psi(x) + \dots, \quad (\text{A.49})$$

and similarly for all the other terms in the Lagrangian. Skipping the detail of the calculation, for which we refer to [123], we quote the result for the leading soft interactions terms, which follows from the multipole expansion of the gauge field terms in $\psi^\dagger(iD^0 + \mathbf{D}^2/(2m_Q))\psi$ and from the inclusion of a non-abelian non-local term which comes from matching the graphs in figure A.2 on PNRQCD. Up to order $v^{13/2}$ one has

$$\begin{aligned} \mathcal{L}_s = & g_s [\psi^\dagger T^A \psi](x) A^{0A}(t, 0) + g_s [\psi^\dagger T^A \psi](x) x^i \partial^i A^{0A}(t, 0) \\ & - \frac{ig_s}{2m_Q} [\psi^\dagger (\overleftarrow{\partial}^i - \overrightarrow{\partial}^i) T^A \psi](x) A^{iA}(t, 0) + \text{antiquark terms} \\ & - \int d^3\mathbf{r} \left[\psi^\dagger T^B \psi \right] (x + \mathbf{r}) \left[\chi^\dagger T^C \chi \right] (x) \left(-\frac{\alpha_s}{r} \right) g_s f^{ABC} r^i A^{iA}(t, 0). \end{aligned} \quad (\text{A.50})$$

The first term scales like $v^{1/2}$ relative to the leading v^5 order terms in the PNRQCD lagrangian; the other interaction terms scale like $v^{3/2}$. Using $[x^i, \boldsymbol{\partial}^2] = -2\partial^i$ and the equation of motion at leading order in v , which includes the Coulomb potential, the $v^{3/2}$ interactions combine into the chromo-electric dipole operator (up to higher order terms) which is responsible for the soft-gluon interaction that converts the colour-octet green function in section 3.4.2 to the colour-singlet one. Introducing the ultrasoft covariant derivative $D^0 = \partial^0 - ig_s A^0(t, 0)$, we obtain

$$\begin{aligned} \mathcal{L}_{\text{PNRQCD}} = & \psi^\dagger(x) \left(iD^0 + \frac{\boldsymbol{\partial}^2}{2m_Q} - g_s x^i E^i(t, 0) \right) \psi(x) + \text{antiquark terms} \\ & + \int d^3\mathbf{r} \left[\psi_a^\dagger \psi_b \right] (x + \mathbf{r}) V_{ab;cd}(r, \partial^i) \left[\chi_c^\dagger \chi_d \right] (x), \end{aligned} \quad (\text{A.51})$$

where

$$V_{ab;cd}(r, \partial^i) = T_{ab}^A T_{cd}^A \cdot \left(-\frac{\alpha_s}{r} \right) + \delta V_{ab;cd}(r, \partial^i). \quad (\text{A.52})$$

This Lagrangian is invariant under soft gauge transformation $U(t, 0)$.

Appendix B

Higgs sector in the MSSM with large $\tan \beta$

In this appendix we review the Higgs boson couplings to quarks in the MSSM with large $\tan \beta$, in order to provide the elements for the derivation of the effective Hamiltonian (4.1).

The MSSM presents important loop effects in the large $\tan \beta$ limit, because in some cases the loop factors get multiplied by the large ratio of the two Higgs vacua, $v_u/v_d \equiv \tan \beta$. These effects are investigated more clearly in the decoupling limit, i.e. one considers the mass scale of the superpartners M_{SUSY} to be somewhat larger than the electroweak scale, $\mu_W \sim 2M_W$. In practice, this approximation works for M_{SUSY} as low as a few hundred GeV, while hierarchy problems require $M_{\text{SUSY}} \leq \mathcal{O}(10 \text{ TeV})$. Under this assumption one can integrate out the heavy superpartners and derive an effective low energy theory, whose couplings are obtained matching them with the full MSSM before electroweak symmetry breaking, i.e. in the limit of unbroken $SU(2) \times U(1)$ symmetry. The low energy theory is described by a two-Higgs doublet model (2HDM), in which the up and down Higgs bosons are coupled to both up and down quarks.

Focusing on the Higgs boson couplings to quarks, integrating out the particles in the loops of figure B.1 one derives the following effective Lagrangian:

$$\mathcal{L}_{\text{eff}}^d = -\epsilon_{ij} H_i^{(d)} \bar{d} \cdot (\mathbf{Y}_d + \Delta_d \mathbf{Y}_d) \cdot P_L \cdot q_j - H_i^{(u)*} \bar{d} \cdot \Delta_u \mathbf{Y}_d \cdot P_L \cdot q_j + \text{h.c.}, \quad (\text{B.1})$$

with $P_{R/L} = (1 \pm \gamma_5)/2$. The coupling $\Delta_u \mathbf{Y}_d$ arises at the one loop level, and it is not enhanced by itself. However, when decomposing the Higgs doublets in (B.1) into their physical states,

$$\begin{aligned} H_1^{(d)} &= \frac{v_d}{\sqrt{2}} + \frac{1}{\sqrt{2}} (c_\alpha H^0 - s_\alpha h^0 + i s_\beta A^0 - i c_\beta G^0), \\ H_2^{(u)*} &= \frac{v_u}{\sqrt{2}} + \frac{1}{\sqrt{2}} (s_\alpha H^0 - c_\alpha h^0 - i c_\beta A^0 - i s_\beta G^0), \end{aligned} \quad (\text{B.2})$$

the down quark masses receive large $\tan \beta$ enhanced correction to their tree level value, $m_{d_J} = -v_d/\sqrt{2} y_{d_J}$, due to the large ratio of the vacuum expectation values [240]: defining the physical masses as $\bar{m}_{d_J}$, one has

$$\bar{m}_{d_J} = m_{d_J} (1 + \tan \beta \Delta_u \mathbf{Y}_d). \quad (\text{B.3})$$

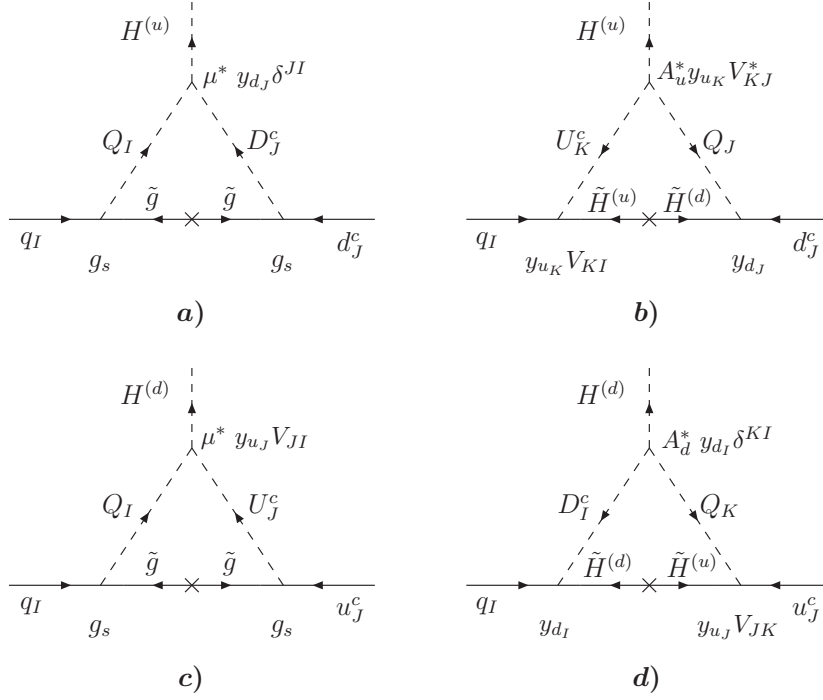


Figure B.1: Vertex corrections in the $SU(2) \times U(1)$ symmetry limit, taken from [251]. Diagrams a) and b) give rise to corrections $(\Delta_u \mathbf{Y}_d)^{JI}$. Corrections $(\Delta_d \mathbf{Y}_d)^{JI}$ are generated by similar diagrams with outgoing $H^{(u)}$ replaced by incoming $H^{(d)}$ and factors μ^* and A_u^* in diagrams a) and b) replaced by A_d and μ , respectively. Diagrams c) and d) give rise to corrections $(\Delta_d \mathbf{Y}_u)^{JI}$. Corrections $(\Delta_u \mathbf{Y}_u)^{JI}$ are generated by similar diagrams with outgoing $H^{(d)}$ replaced by incoming $H^{(u)}$ and factors μ^* and A_d^* in diagrams c) and d) replaced by A_u and μ , respectively.

This relation is at the origin of the $H\bar{d}d$ coupling enhancement by one power of $\tan \beta$, when the Yukawa couplings y_{d_J} are expressed in term of the physical masses:

$$y_{u_J} = 2^{3/4} G_F^{1/2} \bar{m}_{u_J}, \quad y_{d_J} = -2^{3/4} G_F^{1/2} \frac{\bar{m}_{d_J} \tan \beta}{1 + \tilde{\epsilon}_J \tan \beta}, \quad y_{l_J} = -2^{3/4} G_F^{1/2} \bar{m}_{l_J} \tan \beta. \quad (\text{B.4})$$

An additional enhancement is found in the flavour-changing $H\bar{d}d$ coupling, [248, 251]. Without repeating the derivation, for which we refer to these papers, we report here the result, which can be expressed introducing the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -G_F^{1/2} 2^{1/4} \left\{ [X_d^S]^{JI} \bar{d}_J P_L d_I S^0 + \text{h.c.} \right\}, \quad (\text{B.5})$$

where $S^0 = h^0, H^0, A^0, G^0$ represents the physical Higgs fields. This Lagrangian include the diagonal and off-diagonal $H\bar{d}d$ interaction expressed in [251] in (2.32), (2.33); our normalization of the coefficients $[X_d^S]^{JI}$ differs from the one given in [251], since we pull out a factor $G_F^{1/2} 2^{1/4}$, for later purposes. The explicit form of the coefficients $[X_d^S]^{JI}$

relevant for B -physics is given by

$$[X_d^S]^{JJ} = \frac{\bar{m}_{dJ}}{1 + \tilde{\epsilon}_J \tan \beta} \tan \beta x_{\text{diag}}^S, \quad (\text{B.6})$$

where

$$x_{\text{diag}}^S = \{c_\alpha + \tilde{\epsilon}_J s_\alpha, -s_\alpha + \tilde{\epsilon}_J c_\alpha, i\}, \quad (\text{B.7})$$

respectively for H^0 , h^0 , A^0 (the G^0 coupling is suppressed); the coupling $(JI) = (13)$, (23), (31), (32) read then

$$[X_d^S]^{JI} = \frac{\bar{m}_{dJ} V_{\text{eff}}^{3J*} V_{\text{eff}}^{3I} \epsilon_Y y_t^2}{(1 + \tilde{\epsilon}_3 \tan \beta)(1 + \epsilon_0 \tan \beta)} \tan^2 \beta x_{\text{off-diag}}^0, \quad (\text{B.8})$$

with

$$x_{\text{off-diag}}^S = \{s_{\alpha-\beta}, c_{\alpha-\beta}, -i\}, \quad (\text{B.9})$$

(here the G^0 coupling cancels); in case of $(JI) = (12), (21)$ one has

$$[X_d^S]^{JI} = \bar{m}_{dJ} V_{\text{eff}}^{3J*} V_{\text{eff}}^{3I} \frac{(1 + \tilde{\epsilon}_3 \tan \beta)^2}{(1 + \tilde{\epsilon}_0 \tan \beta)^4} \epsilon_Y y_t^2 \tan^2 \beta x_{\text{off-diag}}^S. \quad (\text{B.10})$$

Note that in previous equations V_{eff}^{IJ} indicates the physical CKM matrix elements, since, similarly to the definition of the masses, the tree-level CKM matrix elements of the MSSM lagrangian get modified by one loop corrections too, [248, 251]. These expressions are equivalent to the ones given in (3.22), (3.26)-(3.29) in [251], apart from the normalization factor mentioned above.

Using the effective couplings defined above, it is possible to construct effective four-quark operators combining two flavour changing couplings from (B.8) and (B.10), giving rise to “double penguin” operators, or a flavour changing and a flavour conserving one from (B.6), giving rise to “penguin” transitions. The various operators can be collected in the following effective Hamiltonian, which corresponds to the four-quark interactions represented in the first diagram of figure 4.1:

$$\begin{aligned} \mathcal{H}_{\text{eff}} = \sqrt{2} G_F \sum_{S=H^0, h^0, A^0} \frac{-1}{M_S^2} \left\{ [X_d^S]^{JI} [X_d^S]^{LK} \bar{d}_J P_L d_I \bar{d}_L P_L d_K \right. \\ \left. + [X_d^{S\dagger}]^{JI} [X_d^S]^{LK} \bar{d}_J P_R d_I \bar{d}_L P_L d_K \right\} + \text{h.c.}; \end{aligned} \quad (\text{B.11})$$

the short-distance coefficients (4.4) are derived matching the term proportional to the operator $\bar{d}_J P_R d_I \bar{d}_L P_L d_K$, with the appropriate flavour content, to the term containing the operator Q_{13}^q in (4.1).

The interactions among the neutral Higgs fields $S^0 = h^0, H^0, A^0, G^0$ and up-type quarks don't receive sizeable corrections, and are given by

$$\mathcal{L}_{\text{eff}} = -G_F^{1/2} 2^{1/4} \left\{ [X_u^S]^J \bar{u}_J P_L u_J S^0 + \text{h.c.} \right\}, \quad (\text{B.12})$$

with

$$\begin{aligned}
\left[X_u^{H^0}\right]^J &= \bar{m}_{u_J} s_\alpha, \\
\left[X_u^{h^0}\right]^J &= \bar{m}_{u_J} c_\alpha \\
\left[X_u^{A^0}\right]^J &= \bar{m}_{u_J} (i) \left(-\tan^{-1} \beta + \epsilon'_0 + \epsilon'_Y y_{d_J}^2\right), \\
\left[X_u^{G^0}\right]^J &= \bar{m}_{u_J} (-i) s_\beta.
\end{aligned} \tag{B.13}$$

(note that a correction similar to the one in the 3rd term is present also in the 1st and 2nd, but in those cases it is negligible). One obtains penguin-like four-quark operators joining a flavour changing coupling from (B.8) and (B.10) with the couplings in (B.12). The corresponding effective Hamiltonian reads

$$\begin{aligned}
\mathcal{H}_{\text{eff}} = \sqrt{2} G_F \sum_{S=H^0, h^0, A^0} \frac{-1}{M_S^2} \left\{ \left[X_d^S\right]^{JI} \left[X_u^S\right]^K \bar{d}_J P_L d_I \bar{u}_K P_L u_K \right. \\
\left. + \left[X_d^{S\dagger}\right]^{JI} \left[X_u^S\right]^K \bar{d}_J P_L d_I \bar{u}_K P_L u_K \right\} + \text{h.c.}, \tag{B.14}
\end{aligned}$$

and comparing with (B.8), (B.10) and (B.12) it is possible to see that the short distance coefficients are enhanced only by a factor of $\tan^2 \beta$, and therefore they are not included in the analysis of chapter 4.

The interactions among the charged Higgs fields $S^+ = H^+, G^+$ and quarks are given by

$$\mathcal{L}_{\text{eff}}^{S^+} = G_F^{1/2} 2^{1/4} \left\{ \left[P_{RL}^S\right]^{JI} \bar{u}_J P_L d_I S^+ + \left[P_{LR}^S\right]^{JI} \bar{u}_J P_R d_I S^+ \right\} + \text{h.c.}; \tag{B.15}$$

again, we follow the definition in (2.39), (2.40) of [251], except for the factor $G_F^{1/2} 2^{1/4}$ that is taken out. There is only one coefficient which is $\tan \beta$ enhanced, namely

$$\left[P_{LR}^H\right]^{JI} = \sqrt{2} V_{\text{eff}}^{JI} \frac{\bar{m}_{d_I} \tan \beta}{1 + \tilde{\epsilon}_J \tan \beta}. \tag{B.16}$$

This definition is equivalent to the one in (3.47) of [251], divided by the factor $G_F^{1/2} 2^{1/4}$. The short-distance coefficients in (4.6) are derived matching the operator Q_{11}^p in (2.15) with

$$\mathcal{H}_{\text{eff}} = \sqrt{2} G_F \frac{-1}{M_H^{+2}} \left\{ \left[P_{LR}^S\right]^{JI} \left[P_{LR}^S\right]^{LK*} \bar{u}_J P_R d_I \bar{d}_K P_R u_L \right\} + \text{h.c.}. \tag{B.17}$$

Appendix C

The Chang-Masiero-Murayama model

C.1 The basic idea: Yukawa unification and dimension 5 operators

Grand unified theories develop from the idea of unifying the gauge and matter fields of the Standard Model in simpler and more fundamental representations, as suggested by the running of the gauge coupling constants. In these theories lepton and quark interactions are usually related. The purpose of the CMM model is to construct a phenomenologically viable theory, *i.e.*, able to catch the main features of the neutrino sector, such as the large solar and atmospheric mixing angles, and to look for effects in the quark sector, in such a way to probe the underlying grand unified model itself.

In order to fit neutrino physics, the CMM model is based on a $SO(10)$ group, because in this framework right-handed neutrinos are naturally accommodated and the light Standard Model neutrinos generated through the seesaw mechanism. $SO(10)$ breaks, in an intermediate step, to $SU(5)$, which finally breaks to the Standard Model Gauge groups, $\mathcal{G}_{SM}$. Before coming to a description of the symmetry breaking chain in the full model we prefer to focus on the fundamental properties at the basis of the flavour structure of the theory: the unification of right-handed down-quark and left handed charged leptons Yukawa coupling and the way one takes into account that this unification works well only for the third generation. These features can be described in the context of $SU(5)$.

In a minimal $SU(5)$, the down-quark singlet d^{c1} and the lepton doublet L are embedded in a $\bar{5}$ representation, while the quark doublet Q , the up-quark and the electron singlet u^c e^c are embedded in the 10; moreover, the two Higgs doublets of the MSSM are embedded into two separate quintet fields:

$$\begin{aligned}\mathbf{10}_I &= (Q_I, u_I^c, e_I^c), \\ \bar{\mathbf{5}} &= (d_I^c, L_I),\end{aligned}$$

¹the theory contains only left-handed fields, so that *e.g.* $d^c \equiv d_R$, $u^c \equiv u_R$, and so on.

$$\begin{aligned}\mathbf{5}_H &\equiv H = (\mathbf{3}_H, H_u), \\ \bar{\mathbf{5}}_H &\equiv \bar{H} = (\bar{\mathbf{3}}_H, H_d);\end{aligned}\tag{C.1}$$

SU(5) is broken to $\mathcal{G}_{\text{SM}}$ by a Higgs field in the adjoint representation, $\mathbf{24}_H \equiv \Sigma$, whose VEV is proportional to the hypercharge, $\langle \Sigma \rangle = \sigma \text{diag}(2, 2, 2; -3, -3)$. In this model, the Yukawa couplings read

$$W_Y = \mathbf{Y}_1^{IJ} \epsilon_{abcde} \mathbf{10}_I^{ab} \mathbf{10}_J^{cd} H^e + \mathbf{Y}_2^{IJ} \mathbf{10}_I^{ab} \bar{\mathbf{5}}_{Ja} \bar{H}_b,\tag{C.2}$$

where $a, b, \dots$ are SU(5) indices, and I, J flavour indices. The second coupling is at the origin of the well-known unification of down-quark and charged lepton Yukawa couplings $\mathbf{Y}_{d,e}$: if $\mathbf{Y}_{d,e}$ are defined in such a way that the weak doublets are on the left and the singlets on the right, one has

$$\mathbf{Y}_d = \mathbf{Y}_e^\top,\tag{C.3}$$

at the high scale $M \sim 10^{17}$ GeV. This relation works well for the third generation, but not for the lighter ones. The problem can be solved by including corrections, which are generated by higher-dimensional Yukawa operators, suppressed by powers of the Plank scale. With the particle content considered here, there are two operators of mass dimension five contributing to the down-quark and charged lepton masses, namely

$$\mathbf{Y}_{\sigma 1}^{IJ} \mathbf{10}_I^{ab} \bar{\mathbf{5}}_{Ja} \frac{\Sigma_b^c}{M_{\text{Pl}}} \bar{H}_c + \mathbf{Y}_{\sigma 2}^{IJ} \mathbf{10}_I^{ab} \bar{\mathbf{5}}_{Jc} \frac{\Sigma_b^c}{M_{\text{Pl}}} \bar{H}_a;\tag{C.4}$$

when SU(5) is broken, one has to consider in (A.4) the VEV σ of Σ , and the relation (A.3) is modified to

$$\mathbf{Y}_d = \mathbf{Y}_e^\top + 5 \frac{\sigma}{M_{\text{Pl}}} \tilde{\mathbf{Y}}_{\sigma 2}.\tag{C.5}$$

Now the down-quark and the charged leptons Yukawa matrices cannot be diagonalized simultaneously anymore; instead, in the basis where the charged leptons are diagonal, one has

$$\mathbf{U}_l \mathbf{D}_d \mathbf{U}_r^\dagger = \mathbf{D}_e + 5 \frac{\sigma}{M_{\text{Pl}}} \tilde{\mathbf{Y}}_{\sigma 2},\tag{C.6}$$

where $\mathbf{D}_i$ denotes the diagonal Yukawa matrices, and $\mathbf{U}_l$ and $\mathbf{U}_r$ are rotation matrices for the down-quark fields. The good agreement of the bottom and tau masses at the GUT scale indicates that the matrices $\mathbf{U}_l, \mathbf{U}_r$ have a nontrivial 1-2 block only,

$$\mathbf{U}_l, \mathbf{U}_r \sim \begin{pmatrix} * & * & 0 \\ & * & 0 \\ 0 & 0 & 1 \end{pmatrix}\tag{C.7}$$

The effect of this additional rotation may be seen only in observables involving the first and second generation and in [287] it was found to be severely constrained by the precise measurements in K and B_d physics. In case of non-leptonic B decays, they affect only the $\Delta D = 1$ transitions, and their contribution is negligible. Our analysis in chapter 5 is therefore focused on the effects of the large b_R - s_R mixing, that affect the $\Delta S = 1$ decays. Having analysed the central ideas at the basis of the flavour sector of the theory, we turn now to describe how they are implemented in the CMM model.

C.2 The model

The model is based on a $SO(10)$ group. Being $SO(10)$ a real group, one has to work a bit to obtain complex representations for the matter fields, which in the end can be accomodate into $\mathbf{16}$ spinor representations. All the matter fields, together with the right-handed neutrinos are embedded into one $\mathbf{16}_I$, namely

$$\mathbf{16}_I = (\mathbf{10}_I, \bar{\mathbf{5}}_I, \mathbf{1}_I) = ((Q_I, u_I^c, e_I^c), (d_I^c, L_I), N_I), \quad (C.8)$$

where we have indicated the successive decompositions into $SU(5)$ and then into the Standard Model multiplets. The chain of symmetry breaking goes as follows: a pair of spinor Higgs fields, $\mathbf{16}_H + \bar{\mathbf{16}}_H$, breaks $SO(10)$ to $SU(5)$, which is then broken to $\mathcal{G}_{SM}$ by an adjoint field $\mathbf{45}_H = (\mathbf{24}_H, \mathbf{10}_H, \bar{\mathbf{10}}_H \mathbf{1}_H)$: in fact, both the $SU(5)$ adjoint $\mathbf{24}_H \equiv \Sigma$ and the singlet $\mathbf{1}_H$ acquire a VEV, being the latter (denoted by $v_0 \sim 10^{17}$ GeV) one order of magnitude larger than the former, ($\sigma \sim 10^{16}$ GeV). $\mathcal{G}_{SM}$ is in the end broken by the Standard model Higgs field, contained in two $\mathbf{10}_H = (*, \mathbf{5}_H) = (*, (\mathbf{3}_H, H_u))$, $\mathbf{10}'_H = (\bar{\mathbf{5}}_H, *) = ((\mathbf{3}_H, H_d), *)$ representations. The Yukawa couplings of the model read

$$W_Y = \mathbf{16}_I \mathbf{Y}_1^{IJ} \mathbf{16}_J \mathbf{10}_H + \mathbf{16}_I \mathbf{Y}_2^{IJ} \mathbf{16}_J \frac{\mathbf{45}_H \mathbf{10}'_H}{M_{Pl}} + \mathbf{16}_I \mathbf{Y}_N^{IJ} \mathbf{16}_J \frac{\bar{\mathbf{16}}_H \bar{\mathbf{16}}_H}{M_{Pl}}. \quad (C.9)$$

The CMM model is non-minimal, in the sense that one introduces two Higgs representations, the $\mathbf{10}_H$ and $\mathbf{10}'_H$, to give mass to the up and down quarks, respectively, so that there is no top-bottom Yukawa unification. Moreover, the down-quark Yukawa couplings originate from a dimension 5 operator, which, as discussed before, stand for a series of inequivalent effective operators in which the down quarks are coupled to both the adjoint field Σ and the singlet of $\mathbf{45}_H$, so that the Yukawa matrix $\mathbf{Y}_2$ has to be understood symbolically. This choice is necessary in order to have a nontrivial flavour structure. The matrices $\mathbf{Y}_1$ and $\mathbf{Y}_N$ are in fact symmetric, and in the CMM model one hypothesizes they can be chosen diagonal (this assumption is motivated by the observed values for fermion masses and mixing and it might be a result of family symmetries.). The remaining matrix $\mathbf{Y}_2$ therefore has to be asymmetric, in order to allow for fermion mixing, and it contains all the flavour information. The magnitude of the second mass term is determined by the VEV of the $SU(5)$ -singlet component of $\mathbf{45}_H$, v_0 , so that the strong hierarchy between the t and b, τ masses follows from the v_0/M_{Pl} suppression factor (the model does not require a large $\tan \beta$). The smaller VEV σ , which is proportional to the hypercharge, is important for the modification of the light generation Yukawa couplings, as already seen.

In the basis where $\mathbf{Y}_1$ and $\mathbf{Y}_N$ are simultaneously diagonal the superpotential can be rewritten as

$$W_Y = \mathbf{16}_I \mathbf{D}_1^{IJ} \mathbf{16}_J \mathbf{10}_H + \mathbf{16}_I (\mathbf{V}_q^* \mathbf{D}_2 \mathbf{V}_\ell^\top)^{IJ} \mathbf{16}_J \frac{\mathbf{45}_H \mathbf{10}'_H}{M_{Pl}} + \mathbf{16}_I \mathbf{D}_N^{IJ} \mathbf{16}_J \frac{\bar{\mathbf{16}}_H \bar{\mathbf{16}}_H}{M_{Pl}}, \quad (C.10)$$

where the matrices D_i indicate diagonal matrices as before. The second term more explicitly reads

$$(Q, e^c)^\top (\mathbf{V}_q^* \mathbf{D}_2 \mathbf{V}_\ell^\top) (d^c, L) \frac{\mathbf{45}_H \mathbf{10}'_H}{M_{Pl}}, \quad (C.11)$$

and therefore V_q and V_ℓ can be identified respectively with $\mathbf{V}_{\text{CKM}}$ and $\mathbf{V}_{\text{PMNS}}$, up to phases. As discussed in section C.1, the breaking of $\text{SU}(5)$ by the $\mathbf{45}_H$ Higgs field modifies the relation (C.3) to (C.5), more precisely, one obtains

$$\mathbf{Y}_d = \mathbf{Y}_e^\top + 5 \frac{\sigma}{v_0} \tilde{\mathbf{Y}}_\sigma. \quad (\text{C.12})$$

With these corrections, the large atmospheric mixing angle cannot be translated any more directly to the quark sector. The CKM matrix diagonalizes $\mathbf{Y}_d \mathbf{Y}_d^\dagger$, whereas the PMNS matrix diagonalizes $\mathbf{Y}_e \mathbf{Y}_e^\dagger$. The effect of the additional term in (C.12) can be taken into account parameterizing it with the additional rotation introduced in (C.6), so that the diagonalization of the down-quark Yukawa matrices read

$$\mathbf{Y}_d = \mathbf{V}_q^* \mathbf{D}_d \mathbf{R}_d, \quad (\text{C.13})$$

with

$$\mathbf{R}_d = (\mathbf{U}_r \mathbf{V}_\ell)^\top, \quad (\text{C.14})$$

with U_r a unitary matrix. Here we follow the parametrization of [287], *i.e.*

$$\mathbf{U}_r = \begin{pmatrix} U_{11} & U_{12} & 0 \\ U_{21} & U_{22} & 0 \\ 0 & 0 & e^{i\phi_4} \end{pmatrix} = \begin{pmatrix} \cos \theta e^{i\phi_1} & -\sin \theta e^{i(\phi_1 - \phi_2 + \phi_3)} & 0 \\ \sin \theta e^{i\phi_2} & \cos \theta e^{i\phi_3} & 0 \\ 0 & 0 & e^{i\phi_4} \end{pmatrix}, \quad (\text{C.15})$$

with $\theta \in [0, \pi/2]$. Moreover, we assume the tribimaximal form of the leptonic mixing matrix, *i.e.* we assume the mixing angles $\theta_{12} = \arcsin(1/\sqrt{3}) \sim 35^\circ$, $\theta_{13} = 0^\circ$, and $\theta_{23} = 45^\circ$. In the up-basis one can absorb six phases from V_d , so that one can identify $V_q = V_{\text{CKM}}$. The same cannot be done for V_ℓ , because the phases would only be moved from the down-quark Yukawa matrix to the down-squark soft breaking masses. One needs therefore to parameterize V_ℓ with six phases; explicitly, for $\theta_{13} \neq 0$, one has

$$\mathbf{V}_\ell = \begin{pmatrix} \sqrt{\frac{2}{3}} c_{13} e^{i\alpha} & \frac{1}{\sqrt{3}} c_{13} e^{i\alpha_2} & s_{13} e^{i(\delta + \alpha_3)} \\ e^{i\alpha_4} \left(-\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} s_{13} e^{-i\delta} \right) & e^{i(-\alpha_1 + \alpha_2 + \alpha_4)} \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} s_{13} e^{-i\delta} \right) & \frac{1}{\sqrt{2}} c_{13} e^{i(-\alpha_1 + \alpha_3 + \alpha_4)} \\ e^{i\alpha_5} \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} s_{13} e^{-i\delta} \right) & e^{i(-\alpha_1 + \alpha_2 + \alpha_5)} \left(-\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} s_{13} e^{-i\delta} \right) & \frac{1}{\sqrt{2}} c_{13} e^{i(-\alpha_1 + \alpha_3 + \alpha_5)} \end{pmatrix}, \quad (\text{C.16})$$

where $c_{13} \equiv \cos \theta_{13}$, $s_{13} \equiv \sin \theta_{13}$. In $\theta_{13} = 0$, the phase δ drops out from V_ℓ . From (C.14) we obtain, for $\theta_{13} = 0$,

$$\mathbf{R}_d = \frac{1}{\sqrt{6}} \cdot \begin{pmatrix} 2U_{11} e^{i\alpha_1} - U_{12} e^{i\alpha_4} & 2U_{21} e^{i\alpha_1} - U_{22} e^{i\alpha_4} & e^{i(\phi_4 + \alpha_5)} \\ \sqrt{2} e^{i\alpha_2} (U_{11} + U_{12} e^{i(\alpha_4 - \alpha_1)}) & \sqrt{2} e^{i\alpha_2} (U_{21} + U_{22} e^{i(\alpha_4 - \alpha_1)}) & -\sqrt{2} e^{i(\phi_4 - \alpha_1 + \alpha_2 + \alpha_5)} \\ \sqrt{3} U_{12} e^{i(-\alpha_1 + \alpha_3 + \alpha_4)} & \sqrt{3} U_{22} e^{i(-\alpha_1 + \alpha_3 + \alpha_4)} & \sqrt{3} e^{i(\phi_4 - \alpha_1 + \alpha_3 + \alpha_5)} \end{pmatrix}. \quad (\text{C.17})$$

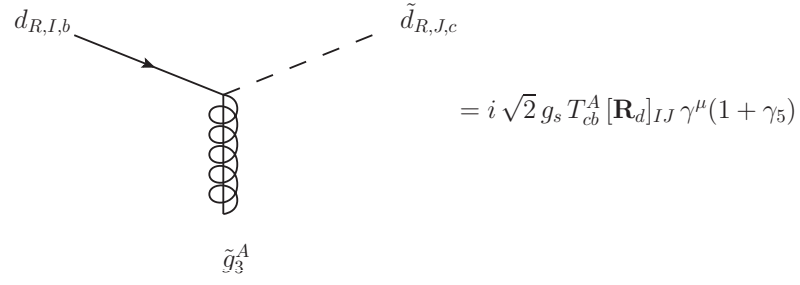


Figure C.1: *The flavour-changing quark-squark-gluino coupling.*

The effects of $\mathbf{R}_d$ become physical due to the induced mixing in the down-squark soft breaking masses, as shown in (5.2). As a consequence, one has flavour-changing squark-quark-gluino vertices, whose Feynman rules are reported in figure C.1.

Bibliography

- [1] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531.
- [2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.
- [3] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5** (1967) 32 [JETP Lett. **5** (1967 SOPUA,34,392-393.1991 UFNAA,161,61-64.1991) 24].
- [4] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **87** (2001) 091801 [arXiv:hep-ex/0107013].
- [5] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **87** (2001) 091802 [arXiv:hep-ex/0107061].
- [6] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **93** (2004) 131801 [arXiv:hep-ex/0407057].
- [7] Y. Chao *et al.* [Belle Collaboration], Phys. Rev. Lett. **93** (2004) 191802 [arXiv:hep-ex/0408100].
- [8] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.
- [9] G. W. Bennett *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **89** (2002) 101804 [Erratum-ibid. **89** (2002) 129903] [arXiv:hep-ex/0208001].
- [10] G. W. Bennett *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **92** (2004) 161802 [arXiv:hep-ex/0401008].
- [11] G. W. Bennett *et al.* [Muon G-2 Collaboration], Phys. Rev. D **73** (2006) 072003 [arXiv:hep-ex/0602035].
- [12] J. C. Hardy and I. S. Towner, Phys. Rev. Lett. **94** (2005) 092502 [arXiv:nucl-th/0412050].
- [13] R. Fleischer, Phys. Lett. B **365**, 399 (1996) [arXiv:hep-ph/9509204].
- [14] R. Fleischer and T. Mannel, Phys. Rev. D **57** (1998) 2752 [arXiv:hep-ph/9704423].
- [15] R. Fleischer, Eur. Phys. J. C **6** (1999) 451 [arXiv:hep-ph/9802433].
- [16] M. Neubert and J. L. Rosner, Phys. Lett. B **441** (1998) 403 [arXiv:hep-ph/9808493].

- [17] M. Neubert and J. L. Rosner, Phys. Rev. Lett. **81** (1998) 5076 [arXiv:hep-ph/9809311].
- [18] M. Neubert, Nucl. Phys. Proc. Suppl. **86** (2000) 477 [arXiv:hep-ph/9909564].
- [19] A. J. Buras and R. Fleischer, Eur. Phys. J. C **11** (1999) 93 [arXiv:hep-ph/9810260].
- [20] M. Neubert, JHEP **9902**, 014 (1999) [arXiv:hep-ph/9812396].
- [21] M. Gronau and D. London, Phys. Rev. Lett. **65** (1990) 3381.
- [22] I. Dunietz and R. G. Sachs, Phys. Rev. D **37** (1988) 3186 [Erratum-ibid. D **39** (1989) 3515].
- [23] M. Gronau and D. London., Phys. Lett. B **253** (1991) 483.
- [24] M. Gronau and D. Wyler, Phys. Lett. B **265** (1991) 172.
- [25] I. Dunietz, Phys. Lett. B **270** (1991) 75.
- [26] R. Aleksan, I. Dunietz and B. Kayser, Z. Phys. C **54** (1992) 653.
- [27] R. Fleischer and I. Dunietz, Phys. Lett. B **387** (1996) 361 [arXiv:hep-ph/9605221].
- [28] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. **78** (1997) 3257 [arXiv:hep-ph/9612433].
- [29] I. Dunietz, Phys. Lett. B **427** (1998) 179 [arXiv:hep-ph/9712401].
- [30] A. F. Falk and A. A. Petrov, Phys. Rev. Lett. **85** (2000) 252 [arXiv:hep-ph/0003321].
- [31] D. London, N. Sinha and R. Sinha, Phys. Rev. Lett. **85** (2000) 1807 [arXiv:hep-ph/0005248].
- [32] M. Diehl and G. Hiller, Phys. Lett. B **517** (2001) 125 [arXiv:hep-ph/0105213].
- [33] J. P. Silva, A. Soffer, L. Wolfenstein and F. Wu, Phys. Rev. D **67** (2003) 036004 [arXiv:hep-ph/0209274].
- [34] Y. Nir and H. R. Quinn, Phys. Rev. Lett. **67**, 541 (1991).
- [35] A. E. Snyder and H. R. Quinn, Phys. Rev. D **48** (1993) 2139.
- [36] J. P. Silva and L. Wolfenstein, Phys. Rev. D **49** (1994) 1151 [arXiv:hep-ph/9309283].
- [37] Y. Grossman and H. R. Quinn, Phys. Rev. D **58** (1998) 017504 [arXiv:hep-ph/9712306].
- [38] J. Charles, Phys. Rev. D **59** (1999) 054007 [arXiv:hep-ph/9806468].
- [39] R. Fleischer, Phys. Lett. B **459** (1999) 306 [arXiv:hep-ph/9903456].

- [40] M. Gronau, E. Lunghi and D. Wyler, Phys. Lett. B **606** (2005) 95 [arXiv:hep-ph/0410170].
- [41] M. Beneke, M. Gronau, J. Rohrer and M. Spranger, Phys. Lett. B **638** (2006) 68 [arXiv:hep-ph/0604005].
- [42] J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. C **41**, 1 (2005) [arXiv:hep-ph/0406184].
- [43] M. Bona *et al.* [UTfit Collaboration], JHEP **0610**, 081 (2006) [arXiv:hep-ph/0606167].
- [44] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex].
- [45] M. Artuso *et al.*, Eur. Phys. J. C **57** (2008) 309 [arXiv:0801.1833 [hep-ph]].
- [46] M. Antonelli *et al.*, arXiv:0907.5386 [hep-ph].
- [47] R. Fleischer, Phys. Rept. **370** (2002) 537 [arXiv:hep-ph/0207108].
- [48] A. J. Buras, arXiv:hep-ph/0505175.
- [49] R. Fleischer, arXiv:hep-ph/0608010.
- [50] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343.
- [51] H. D. Politzer, Phys. Rev. Lett. **30** (1973) 1346.
- [52] S. Kluth, Rept. Prog. Phys. **69** (2006) 1771 [arXiv:hep-ex/0603011].
- [53] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125 [arXiv:hep-ph/9512380].
- [54] A. J. Buras, arXiv:hep-ph/9806471.
- [55] M. Gronau and J. L. Rosner, Phys. Rev. D **59** (1999) 113002 [arXiv:hep-ph/9809384].
- [56] Y. Grossman, Z. Ligeti, Y. Nir and H. Quinn, Phys. Rev. D **68** (2003) 015004 [arXiv:hep-ph/0303171].
- [57] J. D. Bjorken, Nucl. Phys. Proc. Suppl. **11** (1989) 325.
- [58] M. J. Dugan and B. Grinstein, Phys. Lett. B **255** (1991) 583.
- [59] Z. Ligeti, M. E. Luke and M. B. Wise, Phys. Lett. B **507** (2001) 142 [arXiv:hep-ph/0103020].
- [60] D. Fakirov and B. Stech, Nucl. Phys. B **133** (1978) 315.
- [61] N. Cabibbo and L. Maiani, Phys. Lett. B **73** (1978) 418 [Erratum-ibid. B **76** (1978) 663].
- [62] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C **34**, 103 (1987).

- [63] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999) [arXiv:hep-ph/9905312].
- [64] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **591**, 313 (2000) [arXiv:hep-ph/0006124].
- [65] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **606**, 245 (2001) [arXiv:hep-ph/0104110].
- [66] M. Beneke and M. Neubert, Nucl. Phys. B **651**, 225 (2003) [arXiv:hep-ph/0210085].
- [67] M. Beneke and M. Neubert, Nucl. Phys. B **675**, 333 (2003) [arXiv:hep-ph/0308039].
- [68] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. D **72**, 098501 (2005) [arXiv:hep-ph/0411171].
- [69] A. V. Efremov and A. V. Radyushkin, Phys. Lett. B **94** (1980) 245. Efremov:1979qk, Lepage:1980fj
- [70] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22** (1980) 2157.
- [71] Y. Y. Keum, H. n. Li and A. I. Sanda, Phys. Lett. B **504** (2001) 6 [arXiv:hep-ph/0004004].
- [72] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D **63** (2001) 054008 [arXiv:hep-ph/0004173].
- [73] Y. Y. Keum, H. n. Li and A. I. Sanda, AIP Conf. Proc. **618** (2002) 229 [arXiv:hep-ph/0201103].
- [74] Y. Y. Keum and A. I. Sanda, Phys. Rev. D **67** (2003) 054009 [arXiv:hep-ph/0209014].
- [75] Y. Y. Keum and A. I. Sanda, *In the Proceedings of 2nd Workshop on the CKM Unitarity Triangle, Durham, England, 5-9 Apr 2003, pp WG420* [arXiv:hep-ph/0306004].
- [76] M. Beneke and V. A. Smirnov, Nucl. Phys. B **522** (1998) 321 [arXiv:hep-ph/9711391].
- [77] V. A. Smirnov, Springer Tracts Mod. Phys. **177** (2002) 1.
- [78] M. Neubert, arXiv:hep-ph/0512222.
- [79] M. Neubert, Phys. Rept. **245** (1994) 259 [arXiv:hep-ph/9306320].
- [80] A. V. Manohar and M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **10** (2000) 1.
- [81] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63** (2001) 114020 [arXiv:hep-ph/0011336].

- [82] C. W. Bauer and I. W. Stewart, Phys. Lett. B **516** (2001) 134 [arXiv:hep-ph/0107001].
- [83] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **65** (2002) 054022 [arXiv:hep-ph/0109045].
- [84] M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, Nucl. Phys. B **643** (2002) 431 [arXiv:hep-ph/0206152].
- [85] R. J. Hill and M. Neubert, Nucl. Phys. B **657** (2003) 229 [arXiv:hep-ph/0211018].
- [86] M. Beneke and T. Feldmann, Phys. Lett. B **553** (2003) 267 [arXiv:hep-ph/0211358].
- [87] T. Becher, R. J. Hill and M. Neubert, Phys. Rev. D **69** (2004) 054017 [arXiv:hep-ph/0308122].
- [88] T. Becher, R. J. Hill, B. O. Lange and M. Neubert, Phys. Rev. D **69** (2004) 034013 [arXiv:hep-ph/0309227].
- [89] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. **87** (2001) 201806 [arXiv:hep-ph/0107002].
- [90] C. W. Bauer, B. Grinstein, D. Pirjol and I. W. Stewart, Phys. Rev. D **67** (2003) 014010 [arXiv:hep-ph/0208034].
- [91] J. Chay and C. Kim, Nucl. Phys. B **680**, 302 (2004) [arXiv:hep-ph/0301262].
- [92] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004) [arXiv:hep-ph/0401188].
- [93] T. Feldmann and T. Hurth, JHEP **0411** (2004) 037 [arXiv:hep-ph/0408188].
- [94] C. W. Bauer, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **74** (2006) 034010 [arXiv:hep-ph/0510241].
- [95] C. M. Arnesen, Z. Ligeti, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **77** (2008) 054006 [arXiv:hep-ph/0607001].
- [96] T. Becher and M. Neubert, Phys. Rev. Lett. **97** (2006) 082001 [arXiv:hep-ph/0605050].
- [97] T. Becher, M. Neubert and B. D. Pecjak, JHEP **0701** (2007) 076 [arXiv:hep-ph/0607228].
- [98] M. Neubert, arXiv:0706.2136 [hep-ph].
- [99] T. Becher, M. Neubert and G. Xu, JHEP **0807** (2008) 030 [arXiv:0710.0680 [hep-ph]].
- [100] V. Ahrens, T. Becher, M. Neubert and L. L. Yang, Phys. Rev. D **79** (2009) 033013 [arXiv:0808.3008 [hep-ph]].

- [101] T. Becher and M. Neubert, Phys. Rev. Lett. **102** (2009) 162001 [arXiv:0901.0722 [hep-ph]].
- [102] T. Becher and M. Neubert, JHEP **0906** (2009) 081 [arXiv:0903.1126 [hep-ph]].
- [103] T. Becher and M. Neubert, Phys. Rev. D **79** (2009) 125004 [arXiv:0904.1021 [hep-ph]].
- [104] V. Ahrens, T. Becher, M. Neubert and L. L. Yang, Eur. Phys. J. C **62** (2009) 333 [arXiv:0809.4283 [hep-ph]].
- [105] M. Beneke, A. P. Chapovsky, A. Signer and G. Zanderighi, Phys. Rev. Lett. **93** (2004) 011602 [arXiv:hep-ph/0312331].
- [106] M. Beneke, A. P. Chapovsky, A. Signer and G. Zanderighi, Nucl. Phys. B **686** (2004) 205 [arXiv:hep-ph/0401002].
- [107] M. Beneke, N. Kauer, A. Signer and G. Zanderighi, Nucl. Phys. Proc. Suppl. **152** (2006) 162 [arXiv:hep-ph/0411008].
- [108] M. Beneke, P. Falgari, C. Schwinn, A. Signer and G. Zanderighi, Nucl. Phys. B **792** (2008) 89 [arXiv:0707.0773 [hep-ph]].
- [109] S. Actis, M. Beneke, P. Falgari and C. Schwinn, Nucl. Phys. B **807** (2009) 1 [arXiv:0807.0102 [hep-ph]].
- [110] M. Beneke, P. Falgari and C. Schwinn, arXiv:0907.1443 [hep-ph].
- [111] S. Fleming, A. H. Hoang, S. Mantry and I. W. Stewart, Phys. Rev. D **77** (2008) 074010 [arXiv:hep-ph/0703207].
- [112] S. Fleming, A. H. Hoang, S. Mantry and I. W. Stewart, Phys. Rev. D **77** (2008) 114003 [arXiv:0711.2079 [hep-ph]].
- [113] C. W. Bauer and M. D. Schwartz, Phys. Rev. Lett. **97** (2006) 142001 [arXiv:hep-ph/0604065].
- [114] C. W. Bauer and M. D. Schwartz, Phys. Rev. D **76** (2007) 074004 [arXiv:hep-ph/0607296].
- [115] C. W. Bauer, S. P. Fleming, C. Lee and G. Sterman, Phys. Rev. D **78** (2008) 034027 [arXiv:0801.4569 [hep-ph]].
- [116] C. W. Bauer, A. Hornig and F. J. Tackmann, Phys. Rev. D **79** (2009) 114013 [arXiv:0808.2191 [hep-ph]].
- [117] M. Bona *et al.*, arXiv:0709.0451 [hep-ex].
- [118] T. Kageyama, AIP Conf. Proc. **842** (2006) 1064.

- [119] T. Browder, M. Ciuchini, T. Gershon, M. Hazumi, T. Hurth, Y. Okada and A. Stocchi, JHEP **0802** (2008) 110 [arXiv:0710.3799 [hep-ph]].
- [120] T. E. Browder, T. Gershon, D. Pirjol, A. Soni and J. Zupan, arXiv:0802.3201 [hep-ph].
- [121] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125 [Erratum-ibid. D **55** (1997) 5853] [arXiv:hep-ph/9407339].
- [122] M. Beneke, arXiv:hep-ph/9703429.
- [123] M. Beneke, arXiv:hep-ph/9911490.
- [124] N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. **77** (2005) 1423 [arXiv:hep-ph/0410047].
- [125] M. Beneke and L. Vernazza, Nucl. Phys. B **811** (2009) 155 [arXiv:0810.3575 [hep-ph]].
- [126] M. Beneke, X. Q. Li and L. Vernazza, Eur. Phys. J. C **61** (2009) 429 [arXiv:0901.4841 [hep-ph]].
- [127] L. Vernazza, in preparation.
- [128] L. Hofer, U. Nierste, D. Scherer and L. Vernazza, in preparation.
- [129] S. P. Martin, arXiv:hep-ph/9709356.
- [130] J. Rosiek, arXiv:hep-ph/9511250.
- [131] S. Heinemeyer, W. Hollik and G. Weiglein, Phys. Rept. **425**, 265 (2006) [arXiv:hep-ph/0412214].
- [132] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) [arXiv:hep-ph/0207036].
- [133] A. J. Buras, Acta Phys. Polon. B **34** (2003) 5615 [arXiv:hep-ph/0310208].
- [134] R. N. Mohapatra, arXiv:hep-ph/9911272.
- [135] A. Masiero, S. K. Vempati and O. Vives, arXiv:0711.2903 [hep-ph].
- [136] M. Beneke and S. Jager, Nucl. Phys. B **751** (2006) 160 [arXiv:hep-ph/0512351].
- [137] M. Beneke and S. Jager, Nucl. Phys. B **768** (2007) 51 [arXiv:hep-ph/0610322].
- [138] A. Jain, I. Z. Rothstein and I. W. Stewart, arXiv:0706.3399 [hep-ph].
- [139] V. Pilipp, arXiv:0709.0497 [hep-ph].
- [140] V. Pilipp, Nucl. Phys. B **794** (2008) 154 [arXiv:0709.3214 [hep-ph]].
- [141] G. Bell, Nucl. Phys. B **795** (2008) 1 [arXiv:0705.3127 [hep-ph]].

- [142] G. Bell, arXiv:0705.3133 [hep-ph].
- [143] G. Bell, arXiv:0902.1915 [hep-ph].
- [144] M.Beneke, T. Huber and X. q. Li, in preparation.
- [145] G. Bell, M.Beneke, T. Huber and X. q. Li, in preparation.
- [146] X. q. Li and Y. d. Yang, Phys. Rev. D **72** (2005) 074007 [arXiv:hep-ph/0508079].
- [147] X. q. Li and Y. d. Yang, Phys. Rev. D **73** (2006) 114027 [arXiv:hep-ph/0602224].
- [148] M. Beneke and T. Feldmann, Nucl. Phys. B **592** (2001) 3 [arXiv:hep-ph/0008255].
- [149] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **67** (2003) 071502 [arXiv:hep-ph/0211069].
- [150] M. Beneke and T. Feldmann, Nucl. Phys. B **685** (2004) 249 [arXiv:hep-ph/0311335].
- [151] B. O. Lange and M. Neubert, Nucl. Phys. B **690** (2004) 249 [Erratum-ibid. B **723** (2005) 201] [arXiv:hep-ph/0311345].
- [152] M. Beneke, Y. Kiyo and D. s. Yang, Nucl. Phys. B **692** (2004) 232 [arXiv:hep-ph/0402241].
- [153] R. J. Hill, T. Becher, S. J. Lee and M. Neubert, JHEP **0407** (2004) 081 [arXiv:hep-ph/0404217].
- [154] T. Becher and R. J. Hill, JHEP **0410** (2004) 055 [arXiv:hep-ph/0408344].
- [155] G. G. Kirilin, arXiv:hep-ph/0508235.
- [156] M. Beneke and D. Yang, Nucl. Phys. B **736** (2006) 34 [arXiv:hep-ph/0508250].
- [157] M. Neubert and B. Stech, Adv. Ser. Direct. High Energy Phys. **15** (1998) 294 [arXiv:hep-ph/9705292].
- [158] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015 [arXiv:hep-ph/0406232].
- [159] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014029 [arXiv:hep-ph/0412079].
- [160] P. Ball, G. W. Jones and R. Zwicky, Phys. Rev. D **75** (2007) 054004 [arXiv:hep-ph/0612081].
- [161] V. M. Braun and I. E. Filyanov, Z. Phys. C **44** (1989) 157 [Sov. J. Nucl. Phys. **50** (1989 YAFIA,50,818-830.1989) 511.1989 YAFIA,50,818].
- [162] V. M. Braun and I. E. Filyanov, Z. Phys. C **48** (1990) 239 [Sov. J. Nucl. Phys. **52** (1990 YAFIA,52,199-213.1990) 126].
- [163] P. Ball, V. M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. B **529** (1998) 323 [arXiv:hep-ph/9802299].

- [164] A. G. Grozin and M. Neubert, Phys. Rev. D **55** (1997) 272 [arXiv:hep-ph/9607366].
- [165] V. M. Braun, D. Y. Ivanov and G. P. Korchemsky, Phys. Rev. D **69** (2004) 034014 [arXiv:hep-ph/0309330].
- [166] A. Khodjamirian, T. Mannel and N. Offen, Phys. Lett. B **620** (2005) 52 [arXiv:hep-ph/0504091].
- [167] S. J. Lee and M. Neubert, Phys. Rev. D **72** (2005) 094028 [arXiv:hep-ph/0509350].
- [168] A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D **70**, 094002 (2004) [arXiv:hep-ph/0407226].
- [169] V. M. Braun and A. Lenz, Phys. Rev. D **70**, 074020 (2004) [arXiv:hep-ph/0407282].
- [170] P. Ball and R. Zwicky, Phys. Lett. B **633**, 289 (2006) [arXiv:hep-ph/0510338].
- [171] P. Ball, V. M. Braun and A. Lenz, JHEP **0605** (2006) 004 [arXiv:hep-ph/0603063].
- [172] P. Ball and G. W. Jones, JHEP **0703** (2007) 069 [arXiv:hep-ph/0702100].
- [173] P. Ball, V. M. Braun and A. Lenz, JHEP **0708** (2007) 090 [arXiv:0707.1201 [hep-ph]].
- [174] V. M. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006) [arXiv:hep-lat/0606012].
- [175] P. A. Boyle, M. A. Donnellan, J. M. Flynn, A. Juttner, J. Noaki, C. T. Sachrajda and R. J. Tweedie [UKQCD Collaboration], Phys. Lett. B **641**, 67 (2006) [arXiv:hep-lat/0607018].
- [176] A. L. Kagan, Phys. Lett. B **601** (2004) 151 [arXiv:hep-ph/0405134].
- [177] A. L. Kagan, arXiv:hep-ph/0407076.
- [178] M. Beneke, J. Rohrer and D. Yang, Phys. Rev. Lett. **96**, 141801 (2006) [hep-ph/0512258].
- [179] M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B **774** (2007) 64 [arXiv:hep-ph/0612290].
- [180] H. Y. Cheng and K. C. Yang, Phys. Rev. D **78**, 094001 (2008), arXiv:0805.0329 [hep-ph].
- [181] M. Bartsch, G. Buchalla and C. Kraus, arXiv:0810.0249 [hep-ph].
- [182] M. Beneke, Phys. Lett. B **620** (2005) 143 [arXiv:hep-ph/0505075].
- [183] H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev. D **73** (2006) 014017 [arXiv:hep-ph/0508104].
- [184] H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev. D **77** (2008) 014034 [arXiv:0705.3079 [hep-ph]].

- [185] H. Y. Cheng and K. C. Yang, Phys. Rev. D **76** (2007) 114020 [arXiv:0709.0137 [hep-ph]].
- [186] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, arXiv:0902.4446 [hep-ph].
- [187] J. H. Munoz and N. Quintero, arXiv:0905.1526 [hep-ph].
- [188] J. Chay and C. Kim, arXiv:hep-ph/0009244.
- [189] H. Y. Cheng and K. C. Yang, Phys. Rev. D **63**, 074011 (2001) [arXiv:hep-ph/0011179].
- [190] Z. z. Song, C. Meng and K. T. Chao, Eur. Phys. J. C **36**, 365 (2004) [arXiv:hep-ph/0209257].
- [191] Z. z. Song and K. T. Chao, Phys. Lett. B **568**, 127 (2003) [arXiv:hep-ph/0206253].
- [192] Z. Z. Song, C. Meng, Y. J. Gao and K. T. Chao, Phys. Rev. D **69**, 054009 (2004) [arXiv:hep-ph/0309105].
- [193] T. N. Pham and G. h. Zhu, Phys. Lett. B **619**, 313 (2005) [arXiv:hep-ph/0412428].
- [194] C. Meng, Y. J. Gao and K. T. Chao, Commun. Theor. Phys. **48**, 885 (2007) [arXiv:hep-ph/0502240].
- [195] C. H. Chen and H. N. Li, Phys. Rev. D **71** (2005) 114008 [arXiv:hep-ph/0504020].
- [196] C. Meng, Y. J. Gao and K. T. Chao, arXiv:hep-ph/0506222.
- [197] C. Meng, Y. J. Gao and K. T. Chao, arXiv:hep-ph/0607221.
- [198] C. Bobeth, B. Grinstein and M. Savrov, Phys. Rev. D **77**, 074007 (2008) [arXiv:0712.1953 [hep-ph]].
- [199] G. T. Bodwin, X. Garcia i Tormo and J. Lee, Phys. Rev. Lett. **101**, 102002 (2008) [arXiv:0805.3876 [hep-ph]].
- [200] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **46**, 1914 (1992) [arXiv:hep-lat/9205006].
- [201] G. T. Bodwin, E. Braaten, T. C. Yuan and G. P. Lepage, Phys. Rev. D **46**, 3703 (1992) [arXiv:hep-ph/9208254].
- [202] M. Beneke, F. Maltoni and I. Z. Rothstein, Phys. Rev. D **59**, 054003 (1999) [arXiv:hep-ph/9808360].
- [203] M. Beneke, G. A. Schuler and S. Wolf, Phys. Rev. D **62**, 034004 (2000) [arXiv:hep-ph/0001062].
- [204] F. Abe *et al.* [CDF Collaboration], Phys. Rev. Lett. **76**, 2015 (1996).

- [205] F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **58**, 072001 (1998) [arXiv:hep-ex/9803013].
- [206] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **88**, 031802 (2002) [arXiv:hep-ex/0111069].
- [207] D. E. Acosta *et al.* [CDF Collaboration], Phys. Rev. D **66**, 052005 (2002).
- [208] F. Fang *et al.*, Phys. Rev. Lett. **90**, 071801 (2003) [arXiv:hep-ex/0208047].
- [209] K. Abe *et al.* [BELLE Collaboration], Phys. Rev. D **67**, 032003 (2003) [arXiv:hep-ex/0211047].
- [210] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **69**, 071103 (2004) [arXiv:hep-ex/0310015].
- [211] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **92**, 241802 (2004) [arXiv:hep-ex/0401035].
- [212] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **70**, 011101 (2004) [arXiv:hep-ex/0403007].
- [213] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **94**, 141801 (2005) [arXiv:hep-ex/0412062].
- [214] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **94**, 171801 (2005) [arXiv:hep-ex/0501061].
- [215] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **72**, 072003 (2005) [Erratum-ibid. D **74**, 099903 (2006)] [arXiv:hep-ex/0507004].
- [216] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **72**, 051101 (2005) [arXiv:hep-ex/0507012].
- [217] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **72**, 052002 (2005) [arXiv:hep-ex/0507025].
- [218] B. Aubert [BABAR Collaboration], arXiv:hep-ex/0507074.
- [219] N. Soni *et al.* [Belle Collaboration], Phys. Lett. B **634**, 155 (2006) [arXiv:hep-ex/0508032].
- [220] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **96**, 052002 (2006) [arXiv:hep-ex/0510070].
- [221] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **74**, 032003 (2006) [arXiv:hep-ex/0605003].
- [222] F. Fang *et al.*, Phys. Rev. D **74**, 012007 (2006) [arXiv:hep-ex/0605007].

- [223] R. Kumar *et al.* [Belle Collaboration], Phys. Rev. D **74**, 051103 (2006) [arXiv:hep-ex/0607008].
- [224] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **74**, 071101 (2006) [arXiv:hep-ex/0607050].
- [225] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **76**, 092004 (2007) [arXiv:0707.1648 [hep-ex]].
- [226] A. Szczepaniak, E. M. Henley and S. J. Brodsky, Phys. Lett. B **243**, 287 (1990).
- [227] B. Melic, Phys. Rev. D **68** (2003) 034004 [arXiv:hep-ph/0303250].
- [228] Z. G. Wang, L. Li and T. Huang, Phys. Rev. D **70**, 074006 (2004) [arXiv:hep-ph/0311296].
- [229] B. Melic, Phys. Lett. B **591**, 91 (2004) [arXiv:hep-ph/0404003].
- [230] B. A. Kniehl, A. Onishchenko, J. H. Piclum and M. Steinhauser, Phys. Lett. B **638** (2006) 209 [arXiv:hep-ph/0604072].
- [231] M. Beneke, Y. Kiyo and A. A. Penin, Phys. Lett. B **653**, 53 (2007) [arXiv:0706.2733 [hep-ph]].
- [232] M. Beneke and Y. Kiyo, Phys. Lett. B **668**, 143 (2008) [arXiv:0804.4004 [hep-ph]].
- [233] G. Bell and T. Feldmann, Nucl. Phys. Proc. Suppl. **164**, 189 (2007) [arXiv:hep-ph/0509347].
- [234] E. Bagan, P. Ball and V. M. Braun, Phys. Lett. B **417**, 154 (1998) [arXiv:hep-ph/9709243].
- [235] N. Brambilla *et al.* [Quarkonium Working Group], arXiv:hep-ph/0412158.
- [236] A. V. Manohar and I. W. Stewart, Phys. Rev. D **76**, 074002 (2007) [arXiv:hep-ph/0605001].
- [237] W. S. Hou, Phys. Rev. D **48**, 2342 (1993).
- [238] C. Hamzaoui, M. Pospelov and M. Toharia, Phys. Rev. D **59**, 095005 (1999) [hep-ph/9807350].
- [239] K. S. Babu and C. F. Kolda, Phys. Rev. Lett. **84**, 228 (2000) [hep-ph/9909476].
- [240] M. S. Carena, D. Garcia, U. Nierste and C. E. M. Wagner, Nucl. Phys. B **577** (2000) 88 [arXiv:hep-ph/9912516].
- [241] P. H. Chankowski and L. Slawianowska, Phys. Rev. D **63**, 054012 (2001) [hep-ph/0008046].
- [242] C. S. Huang, W. Liao, Q. S. Yan and S. H. Zhu, Phys. Rev. D **63**, 114021 (2001) [Erratum-ibid. D **64**, 059902 (2001)] [hep-ph/0006250].

- [243] G. Degrandi, P. Gambino and G. F. Giudice, JHEP **0012**, 009 (2000) [hep-ph/0009337].
- [244] M. S. Carena, D. Garcia, U. Nierste and C. E. M. Wagner, Phys. Lett. B **499**, 141 (2001) [arXiv:hep-ph/0010003].
- [245] C. Bobeth, T. Ewerth, F. Krüger and J. Urban, Phys. Rev. D **64**, 074014 (2001) [hep-ph/0104284].
- [246] U. Nierste, arXiv:hep-ph/0107333.
- [247] A. Dedes, H. K. Dreiner and U. Nierste, Phys. Rev. Lett. **87**, 251804 (2001) [arXiv:hep-ph/0108037].
- [248] G. Isidori and A. Retico, JHEP **0111**, 001 (2001) [arXiv:hep-ph/0110121].
- [249] C. Bobeth, A. J. Buras, F. Kruger and J. Urban, Nucl. Phys. B **630**, 87 (2002) [arXiv:hep-ph/0112305].
- [250] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Phys. Lett. B **546**, 96 (2002) [arXiv:hep-ph/0207241].
- [251] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B **659**, 3 (2003) [arXiv:hep-ph/0210145].
- [252] A. G. Akeroyd and S. Recksiegel, J. Phys. G **29**, 2311 (2003) [arXiv:hep-ph/0306037].
- [253] G. Isidori and P. Paradisi, Phys. Rev. D **73**, 055017 (2006) [arXiv:hep-ph/0601094].
- [254] M. S. Carena, A. Menon, R. Noriega-Papaqui, A. Szykman and C. E. M. Wagner, Phys. Rev. D **74**, 015009 (2006) [arXiv:hep-ph/0603106].
- [255] G. Isidori and P. Paradisi, Phys. Lett. B **639**, 499 (2006) [arXiv:hep-ph/0605012].
- [256] A. Freitas, E. Gasser and U. Haisch, Phys. Rev. D **76**, 014016 (2007) [arXiv:hep-ph/0702267].
- [257] M. S. Carena, A. Menon and C. E. M. Wagner, Phys. Rev. D **76**, 035004 (2007) [arXiv:0704.1143 [hep-ph]].
- [258] M. Albrecht, W. Altmannshofer, A. J. Buras, D. Guadagnoli and D. M. Straub, JHEP **0710**, 055 (2007) [arXiv:0707.3954 [hep-ph]].
- [259] G. Isidori, arXiv:0710.5377 [hep-ph].
- [260] U. Nierste, S. Trine and S. Westhoff, Phys. Rev. D **78**, 015006 (2008), arXiv:0801.4938 [hep-ph].
- [261] M. Beneke, P. Ruiz-Femenia and M. Spinrath, arXiv:0810.3768 [hep-ph].
- [262] L. Hofer, U. Nierste and D. Scherer, arXiv:0907.5408 [hep-ph].

- [263] A. J. Buras, M. Misiak and J. Urban, Nucl. Phys. B **586** (2000) 397 [arXiv:hep-ph/0005183].
- [264] O. Brein, Comput. Phys. Commun. **170** (2005) 42 [arXiv:hep-ph/0407340].
- [265] A. J. Buras, T. Ewerth, S. Jager and J. Rosiek, Nucl. Phys. B **714** (2005) 103 [arXiv:hep-ph/0408142].
- [266] J. F. Cheng and C. S. Huang, Phys. Lett. B **554**, 155 (2003) [arXiv:hep-ph/0207177].
- [267] S. Baek, Phys. Rev. D **67**, 096004 (2003) [arXiv:hep-ph/0301269].
- [268] J. F. Cheng, C. S. Huang and X. h. Wu, Phys. Lett. B **585**, 287 (2004) [arXiv:hep-ph/0306086].
- [269] J. F. Cheng, C. S. Huang and X. H. Wu, Nucl. Phys. B **701**, 54 (2004) [arXiv:hep-ph/0404055].
- [270] P. K. Das and K. C. Yang, Phys. Rev. D **71** (2005) 094002 [arXiv:hep-ph/0412313].
- [271] C. S. Huang, P. Ko, X. H. Wu and Y. D. Yang, Phys. Rev. D **73**, 034026 (2006) [arXiv:hep-ph/0511129].
- [272] J. F. Cheng, Y. N. Gao, C. S. Huang and X. H. Wu, Phys. Lett. B **647**, 413 (2007) [arXiv:hep-ph/0612116].
- [273] Y. L. Wu and C. Zhuang, Phys. Rev. D **75**, 115006 (2007) [arXiv:hep-ph/0701072].
- [274] H. Hatanaka and K. C. Yang, Phys. Rev. D **77**, 035013 (2008) [arXiv:0711.3086 [hep-ph]].
- [275] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100**, 101802 (2008) arXiv:0712.1708 [hep-ex].
- [276] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **77** (2008) 011107, arXiv:0708.2260 [hep-ex].
- [277] Belle Collaboration, arXiv:0809.3834 [hep-ex].
- [278] D. Chang, A. Masiero and H. Murayama, Phys. Rev. D **67** (2003) 075013 [arXiv:hep-ph/0205111].
- [279] S. Jager and U. Nierste, Eur. Phys. J. C **33**, S256 (2004) [arXiv:hep-ph/0312145].
- [280] S. Jager, Ph.D thesis, TUM Munich, 2003.
- [281] S. Jager and U. Nierste, arXiv:hep-ph/0410360.
- [282] S. Jager, arXiv:hep-ph/0505243.
- [283] M. Knopf, Diploma thesis, Karlsruhe, March 2008.

- [284] C. Scherrer, Diploma thesis, Karlsruhe, March 2008.
- [285] W. Mertens, Diploma thesis, Karlsruhe, April 2008.
- [286] S. Jager, M. Knopf, W. Mertens, U. Nierste, C. Scherrer and S. Wiesenfeldt, in preparation.
- [287] S. Trine, S. Westhoff and S. Wiesenfeldt, arXiv:0904.0378 [hep-ph].
- [288] Y. Grossman, M. Neubert and A. L. Kagan, JHEP **9910** (1999) 029 [arXiv:hep-ph/9909297].
- [289] B. Pontecorvo, Sov. Phys. JETP **7**, 172 (1958) [Zh. Eksp. Teor. Fiz. **34**, 247 (1957)].
- [290] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.
- [291] A. Lenz and U. Nierste, JHEP **0706** (2007) 072 [arXiv:hep-ph/0612167].
- [292] U. Nierste, arXiv:0904.1869 [hep-ph].
- [293] S. Baek and D. London, Phys. Lett. B **653** (2007) 249 [arXiv:hep-ph/0701181].
- [294] M. Imbeault, S. Baek and D. London, Phys. Lett. B **663** (2008) 410 [arXiv:0802.1175 [hep-ph]].
- [295] T. Feldmann, M. Jung and T. Mannel, JHEP **0808**, 066 (2008) [arXiv:0803.3729 [hep-ph]].
- [296] S. Baek, C. W. Chiang and D. London, Phys. Lett. B **675** (2009) 59 [arXiv:0903.3086 [hep-ph]].
- [297] S. Baek, C. W. Chiang, M. Gronau, D. London and J. L. Rosner, Phys. Lett. B **678** (2009) 97 [arXiv:0905.1495 [hep-ph]].
- [298] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C **32** (2003) 45 [arXiv:hep-ph/0309012].
- [299] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Phys. Rev. Lett. **92** (2004) 101804 [arXiv:hep-ph/0312259].
- [300] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Nucl. Phys. B **697** (2004) 133 [arXiv:hep-ph/0402112].
- [301] S. Mishima and T. Yoshikawa, Phys. Rev. D **70** (2004) 094024 [arXiv:hep-ph/0408090].
- [302] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Acta Phys. Polon. B **36** (2005) 2015 [arXiv:hep-ph/0410407].
- [303] C. S. Kim, S. Oh and C. Yu, Phys. Rev. D **72** (2005) 074005 [arXiv:hep-ph/0505060].

- [304] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C **45** (2006) 701 [arXiv:hep-ph/0512032].
- [305] M. Gronau and J. L. Rosner, Phys. Rev. D **74** (2006) 057503 [arXiv:hep-ph/0608040].
- [306] M. Gronau and J. L. Rosner, Phys. Lett. B **644** (2007) 237 [arXiv:hep-ph/0610227].
- [307] R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C **51** (2007) 55 [arXiv:hep-ph/0702275].
- [308] R. Fleischer, Eur. Phys. J. C **52** (2007) 267 [arXiv:0705.1121 [hep-ph]].
- [309] C. S. Kim, S. Oh and Y. W. Yoon, Phys. Lett. B **665** (2008) 231 [arXiv:0707.2967 [hep-ph]].
- [310] R. Fleischer, S. Jager, D. Pirjol and J. Zupan, Phys. Rev. D **78** (2008) 111501 [arXiv:0806.2900 [hep-ph]].
- [311] Q. Chang, X. Q. Li and Y. D. Yang, JHEP **0809** (2008) 038 [arXiv:0807.4295 [hep-ph]].
- [312] C. W. Chiang and D. London, arXiv:0904.2235 [hep-ph].
- [313] R. Fleischer, Phys. Lett. B **332** (1994) 419.
- [314] H. J. Lipkin, Phys. Lett. B **445** (1999) 403 [arXiv:hep-ph/9810351].
- [315] A. J. Buras and R. Fleischer, Eur. Phys. J. C **16** (2000) 97 [arXiv:hep-ph/0003323].
- [316] G. Buchalla, G. Hiller and G. Isidori, Phys. Rev. D **63** (2001) 014015 [arXiv:hep-ph/0006136].
- [317] V. Barger, L. Everett, J. Jiang, P. Langacker, T. Liu and C. Wagner, arXiv:0902.4507 [hep-ph].
- [318] Q. Chang, X. Q. Li and Y. D. Yang, JHEP **0905** (2009) 056 [arXiv:0903.0275 [hep-ph]].
- [319] V. Barger, L. L. Everett, J. Jiang, P. Langacker, T. Liu and C. E. M. Wagner, arXiv:0906.3745 [hep-ph].
- [320] T. G. Rizzo, Phys. Rev. D **59** (1999) 015020 [arXiv:hep-ph/9806397].
- [321] U. Egede, T. Hurth, J. Matias, M. Ramon and W. Reece, JHEP **0811** (2008) 032 [arXiv:0807.2589 [hep-ph]].
- [322] W. Altmannshofer, P. Ball, A. Bharucha, A. J. Buras, D. M. Straub and M. Wick, JHEP **0901** (2009) 019 [arXiv:0811.1214 [hep-ph]].
- [323] W. Altmannshofer, A. J. Buras, D. M. Straub and M. Wick, JHEP **0904** (2009) 022 [arXiv:0902.0160 [hep-ph]].

- [324] A. Hocker, H. Lacker, S. Laplace and F. Le Diberder, Eur. Phys. J. C **21** (2001) 225 [arXiv:hep-ph/0104062].
- [325] D. M. Asner *et al.* [CLEO Collaboration], Phys. Rev. D **53** (1996) 1039 [arXiv:hep-ex/9508004].
- [326] A. Bornheim *et al.* [CLEO Collaboration], Phys. Rev. D **68** (2003) 052002 [Erratum-ibid. D **75** (2007) 119907] [arXiv:hep-ex/0302026].
- [327] P. Chang *et al.* [Belle Collaboration], Phys. Lett. B **599** (2004) 148 [arXiv:hep-ex/0406075].
- [328] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **75** (2007) 012008 [arXiv:hep-ex/0608003].
- [329] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **97** (2006) 171805 [arXiv:hep-ex/0608036].
- [330] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **98** (2007) 181804 [arXiv:hep-ex/0608049].
- [331] K. Abe *et al.*, Phys. Rev. Lett. **99** (2007) 121601 [arXiv:hep-ex/0609015].
- [332] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **76** (2007) 091102 [arXiv:0707.2798 [hep-ex]].
- [333] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **78** (2008) 012004 [arXiv:0803.4451 [hep-ex]].
- [334] B. Aubert *et al.* [BABAR Collaboration], arXiv:0807.4226 [hep-ex].
- [335] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **79** (2009) 052003 [arXiv:0809.1174 [hep-ex]].
- [336] I. Adachi *et al.* [Belle Collaboration], arXiv:0809.4366 [hep-ex].
- [337] C. P. Jessop *et al.* [CLEO Collaboration], Phys. Rev. Lett. **85** (2000) 2881 [arXiv:hep-ex/0006008].
- [338] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **98** (2007) 051803 [arXiv:hep-ex/0608051].
- [339] A. Garmash *et al.* [Belle Collaboration], Phys. Rev. D **75** (2007) 012006 [arXiv:hep-ex/0610081].
- [340] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **76** (2007) 011103 [arXiv:hep-ex/0702043].
- [341] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **78** (2008) 052005 [arXiv:0711.4417 [hep-ex]].

- [342] B. Aubert *et al.* [BABAR Collaboration], arXiv:0905.3615 [hep-ex].
- [343] E. Eckhart *et al.* [CLEO Collaboration], Phys. Rev. Lett. **89** (2002) 251801 [arXiv:hep-ex/0206024].
- [344] R. Godang *et al.* [CLEO Collaboration], Phys. Rev. Lett. **88** (2002) 021802 [arXiv:hep-ex/0101029].
- [345] K. Abe *et al.* [BELLE-Collaboration], Phys. Rev. Lett. **95** (2005) 141801 [arXiv:hep-ex/0408102].
- [346] A. Garmash *et al.*, Phys. Rev. Lett. **96** (2006) 251803 [arXiv:hep-ex/0512066].
- [347] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **97** (2006) 201801 [arXiv:hep-ex/0607057].
- [348] B. Aubert *et al.* [BABAR Collaboration], arXiv:0808.0700 [hep-ex].
- [349] S. Glenn *et al.* [CLEO Collaboration], Phys. Rev. Lett. **80** (1998) 2289 [arXiv:hep-ex/9710003].
- [350] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **93** (2004) 081802 [arXiv:hep-ex/0404006].
- [351] M. Iwasaki *et al.* [Belle Collaboration], Phys. Rev. D **72** (2005) 092005 [arXiv:hep-ex/0503044].

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