TRusT: A Two-stage Robustness Trade-off approach for the design of decentralized energy supply systems

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Abstract

The design of decentralized energy supply systems is a complex task and thus best addressed by mathematical optimization. However, design problems typically rely on uncertain input data, such as future energy demands or prices. Still, conventional optimization models are usually deterministic and thus neglect uncertainties. For this reason, the deterministic optimal solution is in general suboptimal or even infeasible. Robust design methods are available to guarantee security of energy supply, however, they usually lead to significant additional costs. In this work, we show that energy supply systems with guaranteed secure energy supply are not expensive per se. For this purpose, we propose the ***T****wo-stage* ***R****ob***u****stnes****s*** ***T****rade-off (TRusT)* approach. The TRusT approach considers the trade-off between expected costs in the nominal scenario and costs in the worst case while guaranteeing

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security of energy supply. Thereby, the TRusT approach identifies balanced robust energy supply systems which are cost-efficient in both the daily business and the worst case. The TRusT approach can be applied and solved efficiently. In a case study, we identify robust design options which ensure security of energy supply at low additional costs. Hence, the TRusT approach is a suitable tool to design cost-efficient and secure energy systems.
Keywords: decentralized energy supply systems, robust optimization, bi-objective optimization, mixed integer linear programming (MILP), system design

1. Introduction

Decentralized energy supply systems (DESS) are commonly designed using deterministic mathematical optimization models [1]. Deterministic optimization models, however, rely on the assumption of *perfect foresight*. Perfect foresight implies that input parameters (e. g., energy demands, prices for gas and electricity, or efficiency of the equipment) are known with certainty, when the optimization is performed. If the actual conditions differ from the deterministic parameters considered in the optimization, the obtained solutions will usually become suboptimal or even infeasible, as shown for general optimization problems [2] as well as for typical engineering problems, such as unit assignment [3], production planning [4], and DESS [5].

For DESS, infeasibility implies insufficient energy supply, for example, if the actual energy demand is higher than assumed in the design of the DESS. In practice, a pragmatic approach is often employed to aim for a reliable energy supply: Peak loads are taken into account in the optimization when considering average monthly values [6], typical days [7] or other typical periods [8]. Another pragmatic but expensive way to increase the reliability of energy supply is to install redundant equipment [9]. While security of energy supply can be achieved still assuming perfect foresight but with extraordinary high peak loads, the resulting system will be over-conservative: The peak loads are assumed to occur for certain and a corresponding energy system is built. Thus, the uncertain nature of demands has to be considered in the optimization to identify a cost-efficient robust energy supply system. The definition of a *robust energy supply system* is not used consistently in literature [10]. We expect a robust energy supply system to cover uncertain energy demands without regarding failure of equipment.

The contribution of our work is to show that a robust design which *ensures* security of energy supply can be cost-efficient at the same time. For this purpose, we introduce the ***T****wo-stage* ***R****ob****u****stnes****s*** ***T****rade-off (TRusT)* approach. The advantages of the TRusT approach are twofold: First, the approach highlights the trade-off between nominal costs and worst-case costs for the robust design. The nominal costs represent costs obtained when uncertainties are neglected and perfect foresight is assumed. Second, the TRusT approach guarantees security of energy supply. Existing approaches often make compromises: The reliability of the system is reduced in order to receive a less expensive design, as shown in the following literature review.

If probability distributions of the uncertain parameters are available, stochastic optimization [11] can be applied. However, probability distributions are not known in general and have to be estimated. The uncertainty in the estimated probability distribution then impacts on the actual robustness of the design [12].

Robust optimization overcomes the need for uncertain probability distributions. A robust solution considers every possible scenario (i. e., every possible combination of parameter values) without relying on its probability of occurrence (for a review see [13]). *Strictly robust optimization* [14] ensures the feasibility of a solution for each scenario while minimizing costs for the most expensive scenarios. The concept of strictly robust optimization was introduced for linear programs by Soyster [14] in 1973 and restated by Ben-Tal and Nemirovski [15] in 1999. The resulting strictly robust optimal solutions are in general very conservative and, thus, expensive compared to the nominal costs. In order to reduce the degree of conservatism of robust solutions, the so-called Γ-*robustness* was introduced by Bertsimas and Sim [16]. Herein, not all uncertain parameters vary at the same time. The level of uncertainty Γ limits the number of parameters varying simultaneously and represents the degree of conservatism. For linear problems, the resulting Γ-robustness program can be transformed into a linear program. Robust optimization approaches have been applied successfully to the operation of decentralized energy supply systems (DESS): Dong et al. [17] use the Γ-robustness to derive a fuzzy radial linear programming model for planning robust energy management systems with environmental constraints. Akbari et al. [18] employ Γ-robustness to optimize operation. Additionally, they determine the Γ-robust structure and sizing of the installed heating and cooling components, considering renewable energy and storage. Renewable energy technologies in the design of energy systems are also considered by Moret et al. [19]. They classify the uncertain parameters to define adequate ranges of their variation. In general, Γ-robustness can only ensure security of energy supply if the conservativeness level Γ is set to its upper bound. However, using the upper bound for the level Γ leads to strictly robust optimization and thus unnecessarily expensive solutions which are not relevant for practical applications. Reducing the degree of conservatism in any way, however, compromises security of energy supply. This compromise can be measured by a robustness index analyzing the trade-off between investments and achievable targets [20].

The *minimax regret approach* (e. g., [21]) aims to identify a solution with low risk, characterized by the so-called *regret*. For each scenario, the regret is defined as the deviation between the occurring costs and the minimal possible costs for the scenario. The maximal possible regret is then minimized. To decrease the conservatism in minimax regret, Yokoyama et al. [22] allow a violation of the energy balance constraints and introduce a penalty term for the unmet energy demands in the objective function. The minimax regret approach is also successfully applied by Dong et al. [23] to determine power generation and capacity expansion for uncertain demands.

The design of decentralized energy supply systems can also be interpreted as a *two-stage optimization problem*. Two-stage problems are characterized by two kinds of variables [24]: *First-stage variables* have to be fixed in the beginning (so-called *here-and-now variables*). *Second-stage variables* (*wait-and-see variables*) can be revised later when knowledge on the scenario is available. In the design of energy systems, first-stage variables commonly correspond to design variables that determine which components should be installed at which capacity. The second stage defines the operation of the installed components. The concept of two stages has been introduced into robust optimization as *adjustable robustness* (also called *adaptive robustness*) [24]. The original concept was further extended by Thiele et al. [25]. Bertsimas et al. [26] propose a two-stage unit-commitment problem taking into account the failure of components. They consider unit commitment as first stage and dispatch as second stage. Adaptive robustness can be combined with a conservatism scaling factor to achieve less conservative solutions [3]. Applying adaptive robustness leads to complex problems which are hard to solve. Thus, special solving strategies are necessary, such as Bender's decomposition [27] and outer approximation [28]. Two-stage problem formulations are used frequently in stochastic optimization [11]. Recent advances in two-stage programming are reviewed by Grossmann et al. [29]. Applications to building energy systems [30] and to utility system optimization [5] showed that uncertainties have to be considered already at the design stage. In order to analyze the trade-off between the system economy and system failure, risk preferences of decision makers are introduced in stochastic approaches [31].

Most robust design approaches thus aim to be less conservative than strictly robust optimization, in order to obtain less expensive solutions. In return, however, they cannot guarantee security of energy supply. In this work, we propose the *Two-stage Robustness Trade-off (TRusT)* approach which identifies robust design options ensuring security of energy supply. Cost-efficient design options can be chosen by analyzing the trade-off between nominal costs and robust costs. Thus, the proposed TRusT approach allows the designer to find solutions which are both, robust *and* cost-efficient.

Our approach builds upon the classical concept of strictly robust optimization stated by Soyster et al. [14]. Robust optimization can be efficiently applied to DESS design using MILP optimization. We extend the concept of strict robustness and introduce the TRusT approach. The TRusT approach recognizes the two-stage nature of DESS design: A two-stage bi-objective optimization is employed studying the nominal cost and the strictly robust costs simultaneously. Both objective functions employ *identical* first-stage variables representing design decisions. The second-stage variables, however, are adapted *separately* for the nominal and the robust objective function. Thus, the design decisions ensure that the system is feasible for every scenario. Operation can thus be adapted to each specific case, e. g., the nominal or worst-case scenario.

A related *bi-criteria approach to robust optimization* has recently been introduced by Chassein and Goerigk [32] for *single*-stage problems with uncertainties in the objective function. In their approach, the nominal and the robust objective function are minimized. Schöbel [33] generalizes the concept of *light robustness* [34] which selects solutions with the highest robustness among all solutions within a certain range of the nominal optimal objective function value. They show that the trade-off between robustness and nominal quality yields promising solutions. For stochastic programming, a similar concept has recently been proposed [35]: A stochastic analysis is performed for the expected and worst-case costs of residential cogeneration systems.

We apply the TRusT approach to a DESS design problem formulated deterministically in our earlier work [6]. The DESS design problem considers decisions on structure, unit sizing, and operation in a deterministic *mixed-*

integer linear program (MILP). In this work, we assume uncertainties in energy prices and energy demands.

The resulting TRusT problem is formulated as bi-objective MILP. Thus, the problem is efficiently solvable without implementing special solution algorithms and the approach can easily be applied to DESS design problems. Thereby, the TRusT approach increases the acceptance for practical applications. Furthermore, we show that the identified strictly robust optimal design of the DESS guarantees security of energy supply and still can be implemented at low additional costs. Thus, the results have significant practical relevance.

This work is structured as follows: In section 2, we introduce the idea and the mathematical formulation of the TRusT approach. The TRusT approach is applied to a real-world problem in section 2. A brief summary and conclusions are given in section 4.

2. Two-stage Robustness Trade-off (TRusT) for decentralized energy supply systems

In section 2.1, we explain the idea of the TRusT approach before introducing the formulation for strictly robust optimization in section 2.2 which is employed in the TRusT approach (in section 2.3). In section 2.4, we state the problem formulation of the TRusT approach for DESS. In the following, we always refer to strictly robust optimization, and hence, we use the terms *robust* and *strictly robust* synonymously.

2.1. Idea

The idea of the Two-stage Robustness Trade-off (TRusT) approach is introduced for the design of decentralized energy supply systems (DESS). In DESS optimization, we consider three decision-levels: structure, sizing, and operation [1] (Fig. 1). Allowing for uncertainties in the energy demands has an impact not only on the operation of the DESS but also on the structure and sizing. We incorporate uncertainties by interpreting DESS optimization as a two-stage problem: The first-stage variables De correspond to the design variables which determine structure and sizing. The second-stage variables Op fix the operation and can be adapted to the occurring combination of uncertain parameters, called *scenario*.

The *robust* objective function $\phi^{rob/rob}$ minimizes the worst possible costs while the constraints must be fulfilled for every possible scenario: This leads to a robust design De^{rob} and robust operation Op^{rob}, and thus to the classical problem of strictly robust optimization [14] for DESS design. The strictly robust optimization is represented by the right branch in Fig. 1. The resulting robust optimal solution is generally conservative. Conservative solutions are often desired for energy supply systems, as the availability of DESS is mandatory [36]. However, there is a trade-off between the degree of conservatism of the design and the resulting costs. Thus, the TRusT approach complements strictly robust optimization by taking into account also the *nominal scenario* (left branch in Fig. 1). The nominal scenario corresponds to the best available prediction for the uncertain parameters which would have been employed in the deterministic problem with assumed perfect foresight.

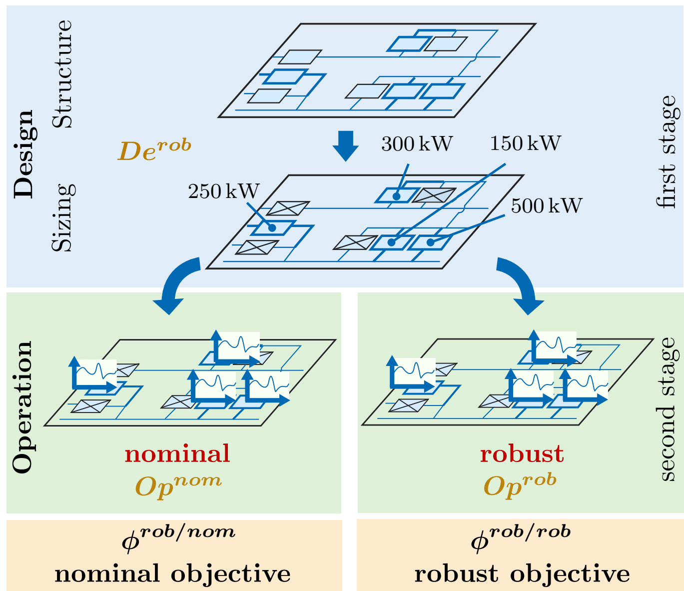

Figure 1: Illustration of the TRusT approach based on bi-objective optimization with objectives $\phi^{rob/nom}$ and $\phi^{rob/rob}$. The nominal objective $\phi^{rob/nom}$ considers the robust design determined by first-stage variables De^{rob} and nominal operation represented by the second-stage variables Op^{nom}; the robust objective $\phi^{rob/rob}$ considers robustness in both stages, i. e., De^{rob} and Op^{rob}.

The bi-objective TRusT approach integrates the nominal evaluation into the strictly robust concept by optimizing the nominal objective function $\phi^{rob/nom}$ *and* the robust objective function $\phi^{rob/rob}$. However, not only the robust objective function $\phi^{rob/rob}$ but also the nominal objective function $\phi^{rob/nom}$ employs a robust design De^{rob} to ensure security of energy supply. Hence, using the nominal objective $\phi^{rob/nom}$, a robust design De^{rob} must be found which is also cost efficient for the nominal scenario. The two objective functions evaluate the performance of the *same* robust design De^{rob} in the nominal and in the worst-case scenario. Thus, the TRusT approach identifies the trade-off between conservativeness and costs.

2.2. Strictly robust optimization

The TRusT approach is based on strictly robust optimization. Hence, this section gives a brief introduction to the concept of strictly robust optimization.

Assuming that potentially uncertain parameters ξ are known with certainty, an optimization problem $(\mathcal{P}_\xi)$ can be defined by

$$
\begin{aligned}
(\mathcal{P}_\xi) \quad \min \quad & f(x,\xi) \\
\text{s.t.} \quad & F(x,\xi) \geq 0 \\
& x \in \mathbb{R}^n,
\end{aligned}
$$

where $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ and $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^M$ specify the objective function and the feasible region, respectively.

In real life, the parameters ξ in the optimization are often uncertain. Each occurring set of uncertain parameters is called a *scenario*. The union of all scenarios is represented by the *uncertainty set* $\mathcal{U}$. Taking all uncertain parameters into account leads to an *uncertain optimization problem* [15] defined by the family

$$(\mathcal{P}_\mathcal{U}) \coloneqq ((\mathcal{P}_\xi), \xi \in \mathcal{U}).$$

Problem $(\mathcal{P}_\xi)$ is the deterministic optimization problem resulting for one particular scenario $\xi \in \mathcal{U}$, i. e., for each uncertain parameter, a certain value is assumed. The problem $(\mathcal{P}_\mathcal{U})$ contains all sub-problems $(\mathcal{P}_\xi)$, thus, considering all scenarios of the uncertainty set $\mathcal{U}$: The uncertain problem $(\mathcal{P}_\mathcal{U})$ represents the union of all sub-problems $(\mathcal{P}_\xi)$ over all scenarios.

The *robust counterpart* $(\mathcal{P}_{\mathcal{RC}})$ of the uncertain problem $(\mathcal{P}_{\mathcal{U}})$ minimizes the maximal objective function value possible while the constraints of each Problem $(\mathcal{P}_{\xi})$ are forced to hold for all scenarios [15]:

$$
\begin{aligned}
(\mathcal{P}_{\mathcal{RC}}) \quad \min \quad & \sup_{\xi\in\mathcal{U}} f(x,\xi) & \\
\text{s.t.} \quad & F(x,\xi) \geq 0 & \forall \xi \in \mathcal{U} \\
& x \in \mathbb{R}^n. &
\end{aligned}
$$

For purpose of solvability, we consider the equivalent formulation:

$$
\begin{aligned}
(\mathcal{P}^{\tau}_{\mathcal{RC}}) \quad \min \quad & \tau & \\
\text{s.t.} \quad & f(x,\xi) \leq \tau & \forall \xi \in \mathcal{U} \\
& F(x,\xi) \geq 0 & \forall \xi \in \mathcal{U} \\
& x \in \mathbb{R}^n, \tau \in \mathbb{R}. &
\end{aligned}
$$

The equivalence holds because $(\mathcal{P}^{\tau}_{\mathcal{RC}})$ minimizes the auxiliary variable τ, which is an upper bound for the original objective function $f(x,\xi)$ no matter which scenario occurs [15]. An optimal solution of $(\mathcal{P}_{\mathcal{RC}})$ is *strictly robust optimal* for the uncertain problem $(\mathcal{P}_{\mathcal{U}})$.

In the robust counterpart $(\mathcal{P}_{\mathcal{RC}})$, the constraints $F(x,\xi) \geq 0$ are satisfied for each scenario $\xi \in \mathcal{U}$, which ensures the feasibility of the solution for the whole range of uncertainties. The objective determines the maximal cost over all scenarios. Thus, the resulting optimal solution is feasible for all scenarios and minimizes the worst-case costs.

2.3. The TRusT approach

The TRusT approach applies bi-objective optimization to a two-stage uncertain problem. Two-stage formulations divide the variables x of the optimization problem $(\mathcal{P}_{\xi})$ into first-stage variables and second-stage variables.

The bi-criteria optimization problem of the TRusT approach (see section 2.1) employs identical first-stage variables u^{rob} for both objective functions. The second stage is determined by the variables v^{nom} and v^{rob} for nominal and robust objective, respectively. The corresponding *TRusT problem* $\left(\mathcal{P}_{TRusT}\right)$ is defined by

$$\left(\mathcal{P}_{TRusT}\right) \qquad \min_{u^{rob},v^{nom},v^{rob}} \begin{pmatrix} f\left(u^{rob},v^{nom},\hat{\xi}\right) \\ \sup_{\xi\in\mathcal{U}} f\left(u^{rob},v^{rob},\xi\right) \end{pmatrix}$$
$$\text{s. t.} \quad F\left(u^{rob},v^{rob},\xi\right) \geq 0 \qquad\qquad \forall \xi \in \mathcal{U}$$

with uncertainty set $\mathcal{U} \subseteq \mathbb{R}^m$, and functions $f : \mathbb{R}^n \times \mathcal{U} \to \mathbb{R}$ and $F : \mathbb{R}^n \times \mathcal{U} \to \mathbb{R}^M$. The objective function $f\left(u^{rob},v^{nom},\hat{\xi}\right)$ corresponds to the nominal objective $\phi^{rob/nom}$ and $\sup_{\xi\in\mathcal{U}} f\left(u^{rob},v^{rob},\xi\right)$ to the robust objective $\phi^{rob/rob}$, respectively (see Fig. 1). For DESS design, first-stage variables u^{rob} correspond to the design De^{rob} and second-stage variables v^{nom} and v^{rob} correspond to the operation Op^{nom} and Op^{rob}. The Pareto front of this problem is called *TRusT curve* and contains all efficient solutions of problem $\left(\mathcal{P}_{TRusT}\right)$, i. e., solutions which cannot be improved for both objective functions simultaneously (for a detailed introduction to multi-objective optimization see [37]). The supremum in problem $\left(\mathcal{P}_{TRusT}\right)$ can be substituted by the auxiliary variable τ in the same way as in problem $(\mathcal{P}^{\tau}_{\mathcal{RC}})$.

The proposed TRusT approach provides the trade-off between the nominal objective function $f\left(u^{rob},v^{nom},\hat{\xi}\right)$ and the robust objective function $\sup_{\xi\in\mathcal{U}} f\left(u^{rob},v^{rob},\xi\right)$. Both objective functions employ the same robust first-stage variables u^{rob} for the structure, whereas the second-stage variables v^{nom} and v^{rob} are adjusted independently.

2.4. Problem formulation for decentralized energy supply systems

To apply the TRusT approach to design DESS, we consider the total annualized costs TAC as objective function

$$
\begin{aligned}
TAC = \sum_{t \in L} \Big[\Delta t_t \Big(& p^{gas} \cdot \dot{U}_t^{gas,buy}(De, Op) \\
& + p^{el,buy} \cdot \dot{U}_t^{el,buy}(De, Op) \\
& - p^{el,sell} \cdot \dot{V}_t^{el,sell}(De, Op) \Big) \Big] \\
& + \sum_{k \in K} \left(\frac{1}{PVF} + p_k^m \right) \cdot I_k(De),
\end{aligned}
$$

where p^{gas}, $p^{el,buy}$ and $p^{el,sell}$ denote the energy prices for gas as well as purchased and sold electricity. Energy conversion units $k \in K$ are modeled as generic converters of an input energy flow $\dot{U}_t$ (e. g., gas or electricity) to an output energy flow $\dot{V}_t$ (e.g., heating, cooling, electricity) in time step $t \in L$ for period Δt_t. Components k are contained in set of technologies K which, in this work, includes absorption chillers AC, compression chillers CC, boilers B, and combined heat and power engines CHP. The indexes *buy* and *sell* denote if an energy flow is purchased or sold, respectively. The present value factor PVF is used to annualize investment costs $I_k(De)$ [38]. The factor p_k^m expresses the maintenance costs of unit k as a share of the investment costs.

Commonly, demands and energy prices are uncertain in DESS design problems. Heating, cooling, and electricity demands are denoted by $\dot{E}^{heat}$, $\dot{E}^{cool}$, and $\dot{E}^{el}$. Let $\hat{\xi} = \left(\hat{p}^{gas}, \hat{p}^{el,sell}, \hat{p}^{el,buy}, \hat{\dot{E}}^{heat}, \hat{\dot{E}}^{cool}, \hat{\dot{E}}^{el} \right)$ define the nominal scenario, which corresponds to the parameters which would have been employed in the deterministic problem with perfect foresight. Usually, demands and energy prices fluctuate within some range. For this reason, we assume *interval-based uncertainty*:

$$\mathcal{U} = \Bigg\{ \left(\tilde{p}^{gas}, \tilde{p}^{el,sell}, \tilde{p}^{el,buy}, \tilde{\dot{E}}^{heat}, \tilde{\dot{E}}^{cool}, \tilde{\dot{E}}^{el} \right) \Bigg|$$

$$\tilde{p}^{gas} = \hat{p}^{gas}(1+pg),\ pg \in [-\varepsilon^{pg}, \varepsilon^{pg}];$$

$$\tilde{p}^{el,sell} = \hat{p}^{el,sell}(1+pe),$$

$$\tilde{p}^{el,buy} = \hat{p}^{el,buy}(1+pe),\ pe \in [-\varepsilon^{pe}, \varepsilon^{pe}];$$

$$\tilde{\dot{E}}_t^{heat} \in \left[\max\left\{0, \hat{\dot{E}}_t^{heat} - \varepsilon_t^{\dot{E}h}\right\}, \hat{\dot{E}}_t^{heat} + \varepsilon_t^{\dot{E}h}\right],$$

$$\tilde{\dot{E}}_t^{cool} \in \left[\max\left\{0, \hat{\dot{E}}_t^{cool} - \varepsilon_t^{\dot{E}c}\right\}, \hat{\dot{E}}_t^{cool} + \varepsilon_t^{\dot{E}c}\right],$$

$$\tilde{\dot{E}}_t^{el} \in \left[\max\left\{0, \hat{\dot{E}}_t^{el} - \varepsilon_t^{\dot{E}e}\right\}, \hat{\dot{E}}_t^{el} + \varepsilon_t^{\dot{E}e}\right],$$

$$t \in L \Bigg\}.$$

Herein, the uncertainty of each parameter is expressed by an upper and a lower bound of the variation around the nominal value [15]. For some time steps, the demand might be smaller than the assumed variation $\varepsilon_t^{\dot{E}\cdot}$. To exclude negative demands, the demands are further restricted to positive values. For electricity prices, the variation of the prices for purchasing $p^{el,buy}$ and feeding in electricity $p^{el,sell}$ are coupled by the parameter pe because their price levels are correlated.

When stating the TRusT problem for design of DESS, we are potentially confronted with two problems: First, the problem is intrinsically infeasible and, second, there is an infinite number of constraints. Thus, the problem would not be solvable. In the following, we show why these two problems occur and how we can adapt the TRusT problem.

According to strictly robust optimization, we enforce feasibility of a solution for *every* scenario in the TRusT problem: The solution has to be feasible for the full range of uncertain parameters. As demands are assumed to be uncertain, robust operation would have to cover the full range of demands *at the same time* which is impossible. The energy balances

$$\begin{aligned}
&\sum_{k \in B \cup CHP} \dot{V}_{kt}(De^{rob}, Op^{rob}) - \sum_{k \in AC} \dot{U}_{kt}(De^{rob}, Op^{rob}) = \tilde{\dot{E}}_t^{heat} && \forall t \in L, \forall \xi \in \mathcal{U} \\
&\sum_{k \in AC \cup CC} \dot{V}_{kt}(De^{rob}, Op^{rob}) = \tilde{\dot{E}}_t^{cool} && \forall t \in L, \forall \xi \in \mathcal{U} \\
&\sum_{k \in CHP} \dot{V}_{kt}^{el}(De^{rob}, Op^{rob}) - \sum_{k \in CC} \dot{U}_{kt}^{el}(De^{rob}, Op^{rob}) \\
&\qquad + \dot{U}_t^{el,buy}(De^{rob}, Op^{rob}) - \dot{V}_t^{el,sell}(De^{rob}, Op^{rob}) = \tilde{\dot{E}}_t^{el} && \forall t \in L, \forall \xi \in \mathcal{U}
\end{aligned} \tag{2.1}$$

cannot be satisfied for every possible scenario ξ of the uncertainty set $\mathcal{U}$ at the same time. As a result, the robust solution space is empty. Mathematically, this is due to the equality constraints in the energy balances which cannot be satisfied for two or more different levels of demand at the same time. Hence, we relax the equality constraints to inequality constraints, such that every energy demand can be fulfilled, while overproduction is allowed. Thereby, we ensure that *at least* the required energy demand is satisfied. As a result, load cases might arise which cannot be covered exactly. In this case, overproduction occurs, e. g., if a demand is smaller than the minimal part load of the installed components. This problem arises if overproduction is cheaper than installing a smaller unit to cover the demand without overproduction. To reduce overproduction in our model, we require that the robust design has to be able to cover the nominal and the smallest demands *exactly*. Hence, we add the following constraints to the TRusT problem $\left(\mathcal{P}_{TRusT}\right)$

stated in section 2.3:

$$F\left(u^{rob}, v^{nom}, \hat{\xi}\right) = 0$$
$$F\left(u^{rob}, v^{lb}, \underline{\xi}\right) = 0 \tag{2.2}$$

with $\underline{\xi}$ being the scenario containing the lower bounds of the uncertain demands. The auxiliary operation variables v^{lb} guarantee that the equality-constraints can be satisfied, without influencing the objective functions. These additional constraints remove overproduction in most load cases.

Due to the interval-based uncertainty, we are still confronted with the problem of having an infinite number of constraints. For each scenario $\xi \in \mathcal{U}$, the relaxed energy balances must be fulfilled. Thus, an infinite number of constraints has to be considered. This problem can be solved since the uncertain values on the right-hand side, i. e., the energy demands $\tilde{\dot{E}}_t^{heat}$, $\tilde{\dot{E}}_t^{cool}$, and $\tilde{\dot{E}}_t^{el}$, can be replaced by their upper bounds. Thereby, redundant constraints are eliminated and the resulting constraints are given by

$$\begin{aligned}
&\sum_{k\in B\cup CHP} \dot{V}_{kt}(De^{rob}, Op^{rob}) - \sum_{k\in AC} \dot{U}_{kt}(De^{rob}, Op^{rob}) \geq \hat{\dot{E}}_t^{heat} + \varepsilon_t^{\dot{E}h} && \forall t \in L\\
&\sum_{k\in AC\cup CC} \dot{V}_{kt}(De^{rob}, Op^{rob}) \geq \hat{\dot{E}}_t^{cool} + \varepsilon_t^{\dot{E}c} && \forall t \in L\\
&\sum_{k\in CHP} \dot{V}_{kt}^{el}(De^{rob}, Op^{rob}) - \sum_{k\in CC} \dot{U}_{kt}^{el}(De^{rob}, Op^{rob})\\
&\qquad + \dot{U}_t^{el,buy}(De^{rob}, Op^{rob}) - \dot{V}_t^{el,sell}(De^{rob}, Op^{rob}) \geq \hat{\dot{E}}_t^{el} + \varepsilon_t^{\dot{E}e} && \forall t \in L.
\end{aligned} \tag{2.3}$$

The problem still contains an infinite number of constraints induced by the robust objective function: In order to eliminate the supremum, the auxiliary variable τ is introduced (see section 2.3). The auxiliary variable τ limits the total annualized costs for every scenario $\xi \in \mathcal{U}$:

$$\sum_{t\in L}\Big[\Delta t_t\Big(\tilde{p}^{gas}\cdot \dot{U}_t^{gas,buy}(De^{rob},Op^{rob}) + \tilde{p}^{el,buy}\cdot \dot{U}_t^{el,buy}(De^{rob},Op^{rob}) - \tilde{p}^{el,sell}\cdot \dot{V}_t^{el,sell}(De^{rob},Op^{rob})\Big)\Big] + \sum_{k\in K}\left(\frac{1}{PVF}+p_k^m\right)\cdot I_k(De^{rob}) \leq \tau \quad \forall \xi \in \mathcal{U}.$$

Finally, we eliminate the resulting infinite number of constraints by inserting the upper bound for the gas price (see Eq. (2.4)). This is exact because the highest price corresponds to the highest cost. The uncertain electricity costs for purchasing and selling energy vary simultaneously because the price levels are correlated. As revenue from electricity sales reduce total annualized costs, not only the upper bound but also the lower bound of the electricity prices has to be taken into account. Hence, the following constraints have to be considered:

$$\sum_{t\in L}\Big[\Delta t_t\Big(\hat{p}^{gas}(1+\varepsilon^{pg})\cdot \dot{U}_t^{gas,buy}(De^{rob},Op^{rob}) + \hat{p}^{el,buy}(1+pe)\cdot \dot{U}_t^{el,buy}(De^{rob},Op^{rob}) - \hat{p}^{el,sell}(1+pe)\cdot \dot{V}_t^{el,sell}(De^{rob},Op^{rob})\Big)\Big] + \sum_{k\in K}\left(\frac{1}{PVF}+p_k^m\right)\cdot I_k(De^{rob}) \leq \tau \quad \forall\, pe \in \{-\varepsilon^{pe},\varepsilon^{pe}\}\,. \tag{2.4}$$

This formulation involves two constraints: one for high and one for low electricity prices with $pe = -\varepsilon^{pe}$ and $pe = \varepsilon^{pe}$, respectively.

By eliminating the supremum and reducing the constraints to a finite number, all constraints and the robust objective function are reformulated to mixed-integer linear relations. The nominal objective function is already a mixed-integer linear function, so no reformulation is necessary. The resulting problem is a bi-objective mixed-integer linear problem (MILP) and can be solved efficiently with common concepts for multi-objective optimization [37]. For reference, the full problem is stated in Appendix A.

3. Case study

In section 3.1, we describe a real-world decentralized energy supply system (DESS) design problem of an industrial park. The problem considers uncertainties for the deterministic problem introduced in our previous work [6]. All solutions are computed with CPLEX 12.6.0.1 to machine accuracy.

3.1. Real-world problem

An industrial park is considered comprising of 6 building complexes. The buildings include production facilities, laboratories as well as office space. The existing energy supply system of the industrial park comprises 2 boilers, 1 combined heat and power (CHP) engine, and 2 compression chillers (see Fig. 3). Furthermore, the industrial park is divided into 2 areas *Site A* and *Site B*. Due to existing infrastructure, the cooling systems of the areas cannot be connected. Thus, 2 independent cooling systems are installed. The heating system is connected and can supply heating to both, Site A and Site B.

We consider energy demands and prices for gas and electricity to be uncertain. The prices, all load profiles, and the corresponding uncertainties are deduced from real data of previous years. The total heating and cooling demands and their uncertainties are shown in Fig. 2.

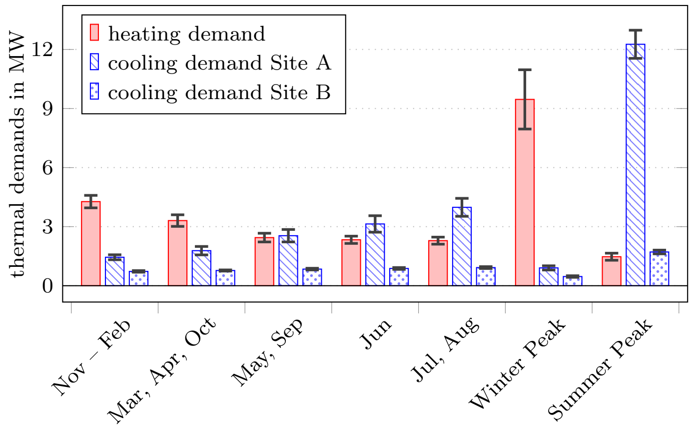

Figure 2: Heating and cooling demands of building complexes on Site A and Site B with uncertainties indicated by error bars

The load cases represent aggregated months with similar load profiles. The deterministic model must be able to cover both the aggregated demands *and* the peak loads to ensure sufficient capacities of the energy supply system (see [6]). Peak loads are always considered, independent of taking uncertainties into account or not. For the TRusT approach, we consider uncertainties for *all* load cases, i. e., also peak loads. The deviation between minimal and maximal cooling demand is 5.2 GWh/a. This range corresponds to 19.4 % of the nominal total annual cooling demand. For the heating demand, minimal and maximal values differ by 4.6 GWh/a, i. e., 16.2 % of the nominal total annual heating demand. The total electricity demand lies within 34.6 GWh/a and 64.9 GWh/a.

Gas and electricity can be purchased from the public grids at a price of $p^{gas} = 5$ ct/kWh and $p^{el,buy} = 16$ ct/kWh, respectively. Electricity that is not used on-site can be fed in at a price of $p^{el,sell} = 10$ ct/kWh. The uncertainties of the gas and electricity price correspond to 40 % and 46 % of the original values, respectively. The cash flow time is assumed to be 10 years.

3.2. Optimal solutions for single-objective optimization

If perfect foresight is assumed, i. e., only the nominal scenario is considered, optimal total annualized costs are $(TAC^{nom})^* = 5.92\,\text{Mio.}\,€/\text{a}$. The corresponding optimal solution requires the installation of 2 CHP engines, 1 absorption chiller, and 1 compression chiller on Site A (see Fig. 3). From the existing energy system, 1 boiler and 1 compression chiller are retained. On Site B, 1 absorption chiller is installed. This structure is referred to as *reference structure* in the following. However, if energy prices rise, the nominal optimal solution will become suboptimal. Even worse, the solution will be infeasible if the demands increase according to the uncertainties (see Fig. 2). As a result, the energy demands could not be covered.

Security of energy supply is ensured for all ranges of uncertainty by the single-objective robust problem. The resulting robust optimal solution is structurally identical to the nominal optimal solution (see Fig. 3), but employs much lager units. Thus, the robust optimal total annualized costs are $(TAC^{rob})^* = 10.63\,\text{Mio.}\,€/\text{a}$, an increase of 80 % compared to the nominal optimal value. However, the robust optimal solution considers the worst-case scenario for the demands. Therefore, the robust solution supplies a significantly lager amount of energy than the solution of the nominal problem. Even more importantly, the strictly robust problem assumes higher prices for the energy purchased. Thus, there is no common basis for a direct comparison of robust and nominal optimal costs, $(TAC^{rob})^*$ and $(TAC^{nom})^*$, respectively. To provide a sound comparison and to find an adequate trade-off between nominal quality and robustness, we apply the bi-objective TRusT approach.

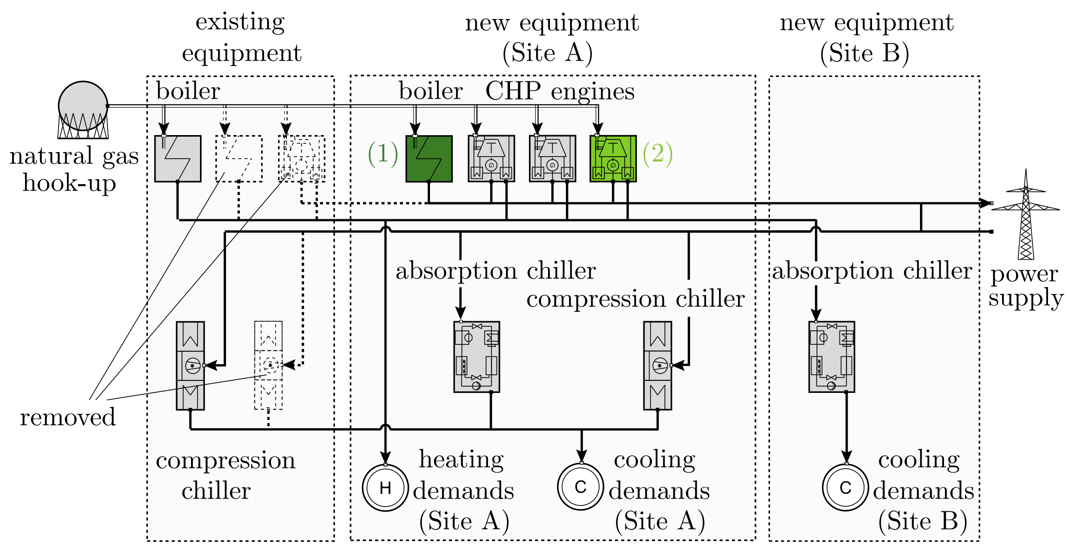

Figure 3: Optimal structure for both single-objective nominal and single-objective robust solution depicted in light gray (referred to as reference structure); additional dark green and green components (marked by ”(1)” and ”(2)”, respectively) are referred to in section 3.3 in Fig. 4: “(1)” only in solutions with “×”-marks, “(2)” only in solutions marked by “∗”

3.3. Two-stage Robustness Trade-Off curve

The <u>T</u>wo-stage <u>R</u>ob<u>u</u>stnes<u>s</u> <u>T</u>rade-off (TRusT) approach provides insight on the true costs related to robust energy supply systems. The TRusT problem for the real-world example described in section 3.1 is solved with the augmented ε-constraint method [39]. Here, in contrast to the single-objective nominal problem, *both* objective functions are based on robust design variables. To emphasize this difference, we will refer to as *robust feasible* costs if the reference might not be clear. The *TRusT curve* of the total annualized costs is the Pareto front of the robust feasible nominal total annualized costs $TAC^{rob/nom}$ and the robust feasible robust total annualized costs $TAC^{rob/rob}$. The TRusT curve of the real-world DESS design problem is shown in Fig. 4.

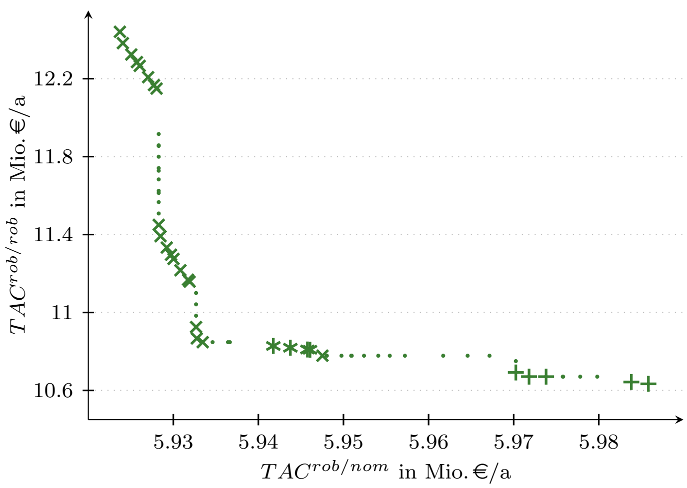

Figure 4: TRusT curve of the real-world problem; values corresponding to the reference structure shown in Fig. 3 (light gray components) are marked by "×", solutions with 1 more boiler than contained in the reference structure are marked with "∗" (and shown in dark green and marked with "(1)" in Fig. 3), solutions marked by "+" contain 1 additional CHP engine (shown in green and marked with "(2)" in Fig. 3)

The anchor point on the top left-hand side represents the minimal robust feasible nominal total annualized costs $(TAC^{rob/nom})^*$: The solution is based on robust design variables, i. e., the design is feasible for all demands, while determining the operation for the nominal scenario. The second anchor point on the right-hand side corresponds to the minimal robust feasible robust total annualized costs $\left(TAC^{rob/rob}\right)^*$. This solution considers not only a robust design but also robust operation. Additionally, each design can exactly fulfill both the nominal demands as well as the lower bounds of the demands (Eq. (2.2)). The TRusT curve has kinks at 5.928 Mio. €/a and 5.933 Mio. €/a, respectively, where a remarkable decrease of the robust feasible robust total annual costs can be observed. The costs decrease because the sizing of the absorption chiller on Site A increases along with a decrease of the sizing of the compression chiller. In combination with excess heat of a CHP engine, an absorption chiller is more efficient than a compression chiller.

Fig. 5 shows the robust design De^{rob} of the anchor points and of the solution at the kink with costs of $TAC^{rob/nom} = 5.933\,\text{Mio.}\,€/\text{a}$ which provides a good trade-off.

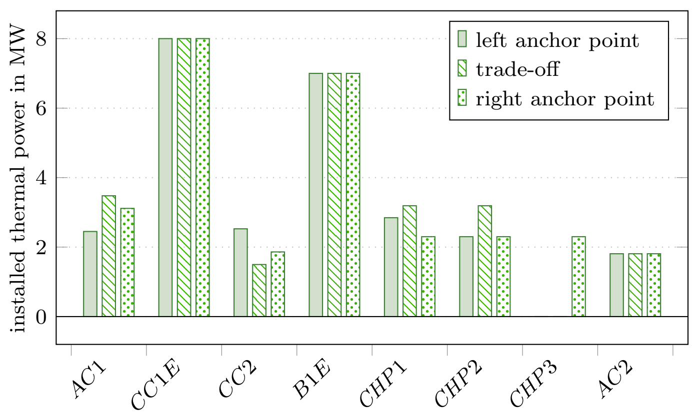

Figure 5: Robust design D^{rob} of three Pareto-optimal solutions in terms of installed thermal power for robust feasible nominal optimal total annualized costs $(TAC^{rob/nom})^*$ (left anchor point), for the solution at the kink (see text for details) with costs of $TAC^{rob/nom} = 5.933\,\text{Mio.}\,€/\text{a}$ (trade-off), and for robust feasible robust optimal total annualized costs $(TAC^{rob/rob})^*$ (right anchor point): $AC1$, $AC2$ are absorption chillers, $CC1E$, $CC2$ are compression chillers, $B1E$ is a boiler, $CHP1$, $CHP2$, $CHP3$ are combined heat and power engines; components ending with "E" are already existing components with a fixed size; only $AC2$ is placed on Site B.

There are only small variations in the sizing of the components. The main difference is that the robust optimal design De^{rob} with minimal robust costs $(TAC^{rob/rob})^*$ selects 1 additional CHP engine and, thus, provides more heat. Consequently, absorption chiller $AC1$ which runs on heat is larger and compression chiller $CC2$ is smaller compared to the robust optimal design for minimal nominal costs $(TAC^{rob/nom})^*$.

In Fig. 4, all values of the robust feasible nominal total annualized costs $TAC^{rob/nom}$ lie within a range of only 1 %. Thus, even installing the robust optimal design implies only low costs $TAC^{rob/nom}$ for the nominal conditions. The maximal deviation to the single-objective $(TAC^{nom})^*$ is only 1.2 %. The nominal solution represents a lower bound for all solutions including solutions from other design approaches that include uncertainties. Approaches that compromise on the security of energy supply might lead to a more cost-efficient design than the TRusT solutions. However, the upper bound of the potential savings is only 1.2 % of the robust costs which corresponds to 70 000 €/a. Here, a reason for the small increase of the costs for a robust design is that peak loads are already considered in the deterministic problem. The robust design does not cause a further large increase of the costs. Thus, using peak loads in conventional optimization results in similar additional costs as a robust design. However, the conventional design does *not* ensure security of energy supply. The comparison of the nominal optimal total annualized costs $(TAC^{nom})^*$ with the costs $(TAC^{rob/nom})^*$ of a robust system for the nominal conditions is more adequate than examining the deviation between nominal optimal costs $(TAC^{nom})^*$ and the robust optimal costs $(TAC^{rob})^*$ which is 80 %, as shown in section 3.2. The comparison of the nominal optimal costs $(TAC^{nom})^*$ with the robust optimal costs $(TAC^{rob})^*$ does not provide the sound basis for a comparison of the designed energy system because the single-objective robust problem assumes higher demands and higher prices. The higher energy prices would also increase the costs of the nominal optimal design. Furthermore, the nominal optimal design is not feasible for all scenarios, thus, a lack of energy supply may arise for several scenarios if the nominal optimal solution was implemented.

For the robust designs from the TRusT curve, the range is quite small for the nominal costs, whereas the robust costs vary significantly: The range of the robust total annualized costs $TAC^{rob/rob}$ is 17 % (see Fig. 4). This implies, a robust design that is cost efficient for the nominal scenario can cause significantly higher costs if an unexpected scenario occurs.

The case study shows that the TRusT approach provides robust designs at low additional costs. The constraints (2.3) and (2.2) ensure that every robust design can cover at least any demand in the uncertainty interval and it is able to cover exactly the nominal demands and the lower bound of each demand. The solutions at kinks of the TRusT curve should be preferred because they provide a good trade-off: Slightly higher costs for the nominal scenario lead to significantly lower costs in the robust case. It is remarkable that the full range of robust Pareto-optimal design is not expensive, when operated for the nominal scenario. Even the maximal deviation to the nominal optimal total annualized costs $(TAC^{nom})^*$ is only 1.2 %.

3.4. Sensitivity analysis of the TRusT curve

In the previous sections, we considered the uncertainty interval to be explicitly known. However, in real life, lower and upper bounds are, in general, unknown themselves. Thus, we analyze the sensitivity of the total annualized costs with respect to the bounds of the demand uncertainties. In the following, we vary the lower and upper bounds of the uncertainty intervals for the demands by scaling the uncertainty size $\varepsilon_t^{\dot{E}h}$, $\varepsilon_t^{\dot{E}c}$, and

$\varepsilon_t^{\dot{E}e}$, for each time step $t \in L$ with the *uncertainty-scaling factor* ω. E. g., the resulting uncertainty interval for the heating demand is now given by $\left[\max\left\{0, \hat{\dot{E}}_t^{heat} - \omega \cdot \varepsilon_t^{\dot{E}h}\right\}, \hat{\dot{E}}_t^{heat} + \omega \cdot \varepsilon_t^{\dot{E}h}\right]$.

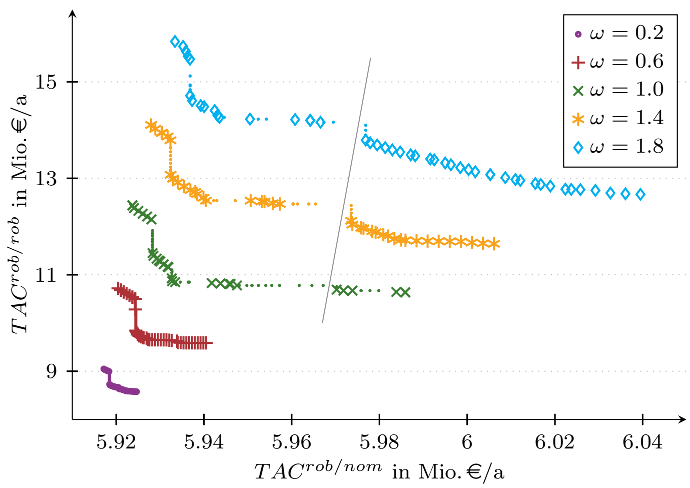

Figure 6: Sensitivity of the TRusT curve with respect to the uncertainty-scaling factor ω; solutions for $\omega \geq 1$ contain 3 instead of 2 CHP engines on the right-hand side of the gray line

We vary the size of the uncertainty significantly ($\omega \in \{0.2, 0.6, 1, 1.4, 1.8\}$). Still, the Pareto-optimal solutions contain only small deviations in the structure: For instance, 3 instead of 2 CHP engines are contained in the solutions for $\omega \geq 1$ (on the right-hand side of the gray line in Fig. 6).

For a high uncertainty-scaling factor ω, the robust design is still cheap for the operation in the nominal scenario: The maximal robust feasible nominal total annualized costs $TAC^{rob/nom}$ are only 2.1 % higher than the nominal optimal total annualized costs of $(TAC^{nom})^* = 5.92$ Mio. €/a. In contrast, robust feasible nominal optimal solutions are expensive for the robust case, especially for large uncertainty-scaling factor ω, e. g., for $\omega = 1.8$, the deviation for different robust designs of the robust costs $TAC^{rob/rob}$ is about 25 % which corresponds to about 3.2 Mio. €/a. For different uncertainty-scaling factors ω, the TRusT curve also allows for selecting less conservative solutions providing good trade-offs, as already observed in section 3.3. Combining the TRusT approach with the sensitivity analysis shows that trade-off solutions at kinks appear for all considered uncertainty-scaling factors in the analyzed case study.

4. Conclusions

We show how robust design of energy supply systems can provide both cost-efficient solutions *and* security of energy supply. For this reason, we propose the Two-stage Robustness Trade-off (TRusT) approach to integrate uncertainties of input parameters into the design of decentralized energy supply systems: The bi-objective approach minimizes both the *nominal* and the *strictly robust* objective function. Decentralized energy supply system (DESS) optimization with uncertain input parameters can be interpreted as a two-stage problem: The first-stage variables correspond to structure and sizing, i. e., the design, of the installed units. A *robust design* has to satisfy all uncertain demands to ensure availability of the DESS. The second-stage variables determine the operation of the units. We incorporate the two-stage nature of DESS in the TRusT approach: Second-stage variables are determined separately for the nominal and the robust objective function, while the strictly robust first-stage variables are shared.

The TRusT approach can be applied easily for interval-based uncertainty, as the TRusT problem can be reformulated with low effort as bi-objective mixed-integer linear program. The resulting *TRusT curve* represents the Pareto front of the nominal (i. e., expected) and robust objectives employing identical robust first-stage variables. Thus, we obtain information about how the same robust design performs in everyday business and in the worst case. The trade-off between nominal and worst-case costs helps the decision-maker to reach a final design decision: Preferable solutions are located at the kinks of the TRusT curve.

The case study considers uncertain demands and prices and minimizes the total annualized costs. Applying the TRusT approach shows that implementing a strictly robust design implies only low additional costs for the nominal operation. The maximal nominal costs employing a robust design lie only 1.2 % above the single-objective nominal optimal costs. In contrast, the robust total annualized costs turn out to be high for a nominal optimal robust design if the parameters differ from the nominal scenario. The sensitivity study shows that even for high uncertainties the strictly robust design is still cost-efficient for the nominal scenario.

Applying the TRusT approach to DESS design problems allows the designer to identify a robust solution with low additional costs. The designed system covers uncertain demands and guarantees security of energy supply for the considered scenarios. Thus, the availability of the DESS is ensured. Our investigations show that security of energy supply and cost-efficient energy supply are not mutually exclusive, but can be integrated using the TRusT approach leading to robust and cost-efficient designs of decentralized energy supply systems. The proposed approach is easy to apply and the resulting optimization problem can efficiently be solved. As a result, the TRusT approach allows to include robustness aspects in practical energy design problems in an easy and effective way.

Acknowledgments

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Appendix A. TRusT problem formulation for a decentralized energy supply system

We present the TRusT problem for a decentralized energy supply system (DESS). The TRusT problem is proposed in section 2.4. The application to DESS is based on a deterministic model introduced in our previous work [6].

The nominal and robust total annualized costs are minimized:

$$\min_{De^{rob},Op^{nom},Op^{rob},\tau} \begin{pmatrix} \sum_{t\in L} & \Big[\Delta t_t\Big(\hat{p}^{gas}\cdot \dot{U}_t^{gas,buy}(De^{rob},Op^{nom}) \\ & \quad +\hat{p}^{el,buy}\cdot \dot{U}_t^{el,buy}(De^{rob},Op^{nom}) \\ & \quad -\hat{p}^{el,sell}\cdot \dot{V}_t^{el,sell}(De^{rob},Op^{nom})\Big)\Big] \\ & \quad + \sum_{k\in K}\left(\frac{1}{PVF}+p_k^m\right)\cdot I_k(De^{rob}) \\ \tau & \end{pmatrix}.$$

The auxiliary variable τ restricts the robust objective function:

$$\sum_{t\in L}\Big[\Delta t_t\Big(\hat{p}^{gas}(1+\varepsilon^{pg})\cdot\dot{U}_t^{gas,buy}(De^{rob},Op^{rob})$$
$$+\hat{p}^{el,buy}(1-\varepsilon^{pe})\cdot\dot{U}_t^{el,buy}(De^{rob},Op^{rob})$$
$$-\hat{p}^{el,sell}(1-\varepsilon^{pe})\cdot\dot{V}_t^{el,sell}(De^{rob},Op^{rob})\Big)\Big]$$
$$+\sum_{k\in K}\left(\frac{1}{PVF}+p_k^m\right)\cdot I_k(De^{rob})\leq\tau$$
$$\sum_{t\in L}\Big[\Delta t_t\Big(\hat{p}^{gas}(1+\varepsilon^{pg})\cdot\dot{U}_t^{gas,buy}(De^{rob},Op^{rob})$$
$$+\hat{p}^{el,buy}(1+\varepsilon^{pe})\cdot\dot{U}_t^{el,buy}(De^{rob},Op^{rob})$$
$$-\hat{p}^{el,sell}(1+\varepsilon^{pe})\cdot\dot{V}_t^{el,sell}(De^{rob},Op^{rob})\Big)\Big]$$
$$+\sum_{k\in K}\left(\frac{1}{PVF}+p_k^m\right)\cdot I_k(De^{rob})\leq\tau.$$

For all $t\in L$, maximal energy demands have to be covered at least:

$$\sum_{k\in B\cup CHP}\dot{V}_{kt}(De^{rob},Op^{rob})-\sum_{k\in AC}\dot{U}_{kt}(De^{rob},Op^{rob})\geq\hat{\dot{E}}_t^{heat}+\omega\cdot\varepsilon_t^{\dot{E}h}$$
$$\sum_{k\in AC\cup CC}\dot{V}_{kt}(De^{rob},Op^{rob})\geq\hat{\dot{E}}_t^{cool}+\omega\cdot\varepsilon_t^{\dot{E}c}$$
$$\sum_{k\in CHP}\dot{V}_{kt}^{el}(De^{rob},Op^{rob})-\sum_{k\in CC}\dot{U}_{kt}^{el}(De^{rob},Op^{rob})$$
$$+\dot{U}_t^{el,buy}(De^{rob},Op^{rob})-\dot{V}_t^{el,sell}(De^{rob},Op^{rob})\geq\hat{\dot{E}}_t^{el}+\omega\cdot\varepsilon_t^{\dot{E}e}.$$

For all $t\in L$, nominal and minimal demands have to be fulfilled exactly:

$$\sum_{k\in B\cup CHP}\dot{V}_{kt}(De^{rob},Op^{nom})-\sum_{k\in AC}\dot{U}_{kt}(De^{rob},Op^{nom})=\hat{\dot{E}}_t^{heat}$$
$$\sum_{k\in AC\cup CC}\dot{V}_{kt}(De^{rob},Op^{nom})=\hat{\dot{E}}_t^{cool}$$
$$\sum_{k\in CHP}\dot{V}_{kt}^{el}(De^{rob},Op^{nom})-\sum_{k\in CC}\dot{U}_{kt}^{el}(De^{rob},Op^{nom})$$
$$+\dot{U}_t^{el,buy}(De^{rob},Op^{nom})-\dot{V}_t^{el,sell}(De^{rob},Op^{nom})=\hat{\dot{E}}_t^{el}$$

$$
\begin{aligned}
&\sum_{k\in B\cup CHP} \dot{V}_{kt}(De^{rob}, Op^{lb}) - \sum_{k\in AC} \dot{U}_{kt}(De^{rob}, Op^{lb}) = \hat{\dot{E}}_t^{heat} - \omega\cdot\varepsilon_t^{\dot{E}h}\\
&\sum_{k\in AC\cup CC} \dot{V}_{kt}(De^{rob}, Op^{lb}) = \hat{\dot{E}}_t^{cool} - \omega\cdot\varepsilon_t^{\dot{E}c}\\
&\sum_{k\in CHP} \dot{V}_{kt}^{el}(De^{rob}, Op^{lb}) - \sum_{k\in CC} \dot{U}_{kt}^{el}(De^{rob}, Op^{lb})\\
&\qquad + \dot{U}_t^{el,buy}(De^{rob}, Op^{lb}) - \dot{V}_t^{el,sell}(De^{rob}, Op^{lb}) = \hat{\dot{E}}_t^{el} - \omega\cdot\varepsilon_t^{\dot{E}e}.
\end{aligned}
$$

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