Translation Dynamics in Holistic Analysis of Functional Human-body System

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Introduction

The core tenet in system fault diagnostics is that behavior of the whole emerges from activity of functional components and coordination of their interaction. This motivates a *drill-down* approach in functional analysis of complex systems [1, 2].
Manned spacecraft, aircraft carriers, submarines, and the socio-biological system of human body with person's actions all impress by their complexity of interwoven detail as soon as one can look behind their encasement, and full comprehension of functional structures and of output in response to stimuli input will often deem impossible. A systematic approach may then start from construction principles that let the apparatus fulfill its mission: for example, components that allow the plane to fly (wings, propulsion), to take off and to land (wheels), and to carry goods and passengers (cabin). Next, the set of vital prerequisites, as energy supply, wiring, cabin air, to mention a few. Finally, all equipment, including human operators, to pilot the plane's operations – have orientation, decide on direction, and interact with outside world. Apart from disrupted material integrity of whole plane, review of these items for clues is witnessed, for example whenever an Airbus or Boeing disappears on a scheduled passenger flight over the oceans. Speculating about malfunction of said *productivity* sub-systems, *vital functions* sub-systems, *operational functions* sub-systems illustrates a scaling understanding that is equally seen in a doctor's scrutinizing first view of a patient as totality of person 'encased' in body system before deciding on an entry point for diagnostic work-up.

Definition 1 *Highly complex systems intended by design for purposeful action are called* production systems.

Principle 1 *Production systems share a* tri-partite functional decomposition *of their ensemble, the Whole, into* vital functions*,* productivity functions*, and* operational functions*, referred to as* wirk-components*, meaning* components for effectuation*, in German more precisely* Komponenten des Bewirkens.

Corollary 1 *The tri-partite functional decomposition is then* canonical *among all production systems.*

The decomposition is exhaustive and non-overlapping. Its components are referred to as *canonical wirk-components*.

Principle 2 *At any moment, the behavioral activity of the Whole is a complete translation of* interaction *between its three canonical* wirk-components *and their functional properties currently prevailing.*

These principles are to apply throughout though systems differ substantially in material realization and in designated environment. The present exposition then describes an equally canonical *configuration of effectuation dynamics*, a *wirkgefüge* independent of its physical realization, that first energizes the cooperation of *canonical wirk-components* - the *logical units* at tri-partite "function-level no.1" - and then translates this to whole-body "function-level 0" aggregate dynamics.
The configuration of control couplings that permits the human body system to energize required action of its *canonical wirk-components* in any situation and that is conceptually equally independent of its material realization, is referred to as *schaltgefüge*, which is considered as the mission of *Wiener* [3]-sense *"kybernetik"* in [4]. By their independence of material realization, *axiomatic wirkgefüge* and *kybernetik schaltgefüge* then require each other, which explains the level of abstraction in the present exposition.

Materials and Methods

Prerequisites

The design of *axiomatic wirkgefüge* exploits intuitive concepts from electric direct-current circuits: tension (voltage), accumulated charge, capacity, resistance, supply and consumer power (wattage) demand, that are basic – not to suggest that physical electricity is actually involved.

Definition 2 *The* axiomatic wirkgefüge *consists of a ubiquitous* source *with potential difference (voltage)* U_0*, and for every single logical unit, of a* supply condenser *of* capacity *C for buffer storage of* supply charge $Q_s(t)$ *by time t, a* resistor *with* resistance $R_s(t)$ *for passage control of* supply current *(amperage)* $I_s(t)$ *to said condenser, and of another condenser for charge consumption, conceived as a* mirror *of the supply-part condenser, with current voltage* $U_c(t)$ *generated by its current charge* $Q_c(t)$ *as source for the demand part, another* resistor *with* resistance $R_d(t)$ *for passage control of* demand current *(amperage)* $I_d(t)$ *to an end consumer with* demand wattage $P_e(t)$.

Principle 3 (Sluggishness) *The exposition focuses on almost instantaneous charge increments and decrements on supply and demand part condensors, respectively, in any incremental time interval* $[t, t+dt[$*,* $t > 0$*; accumulated charges Q are assumed continuous functions* $(Q(t))_{t>0}$ *with derivative* $(I(t))_{t>0}$ *everywhere. Voltages, resistances, wattages, however are assumed to lag behind, shown in their pre-t values,* $U(t-)$*,* $R(t-)$*,* $P(t-)$*. They can then be* *assumed to have currently fixed values during an arbitrarily small amount of time while accumulated charge on the condensors changes.*

Remark 1 *The concept of a "mirror" condenser is introduced for use in the postponed control regimen, in particular to control access to available supply charge.*

Assumption 1 Ohm*'s law and* Kirchhoff*'s rules as known for direct electric current circuits do apply in the* axiomatic wirkgefüge.

Remark 2 *For human body system, source voltage may be thought of being generated by the cellular system, the body's material base layer, while effectuation dynamics affect only the activity on function levels* above *the cellular system. That all functional activity is realized by ubiquitous living cells may 'close the loop' for the circuit.*

Generic Single Circuit

The *axiomatic wirkgefüge* of Def. 2 applies; see Fig. 1 for illustration.

Single-circuit supply-part

Define the charge supply part through quadruple $[U_0,(R_s(t))_{t>0},C,(Q_s(t))_{t\geq 0}]$. To determine supply current $I_s(t)$ explicitly, note that ohmic resistance $R_s(t)$ implies a voltage drop of $U_s^\star(t) = R_s(t)I_s(t)$ that leaves an effective supply voltage $U_s(t)$ at the condenser of $U_s(t) = U_0 - U_s^\star(t)$, that generates t-prevalent charge $Q_s(t)$ on the condenser through $Q_s(t) = CU_s(t)$. While $U_s(t) = U_0$ when $R_s(t) = 0$, $R_s(t) > 0$ permits re-arrangement such that $I_s(t) = U_0/R_s(t-) - Q_s(t)/[CR_s(t-)]$, and, by $dQ_s(t) = I_s(t)dt$, a supply-charge increment during $[t,t+dt[$ of

$$dQ_s(t) = \frac{U_0C - Q_s(t)}{CR_s(t-)}dt \tag{1}$$

yields the first-order differential equation

$$U_0C = \dot{Q}_s(t)R_s(t-)C + Q_s(t) \tag{2}$$

with $Q_s(0) = 0$. For constant resistance $R_s(t) \equiv R_s^\star > 0$, a solution of (2) is

$$Q_s(t) = U_0C\left(1 - e^{-\frac{1}{R_s^\star C}t}\right). \tag{3}$$

Single circuit demand-part

With Q_c differentiable, the quadruple

$$[(Q_c(t))_{t\geq 0},C,(R_d(t))_{t>0},(P_e(t))_{t>0}] \tag{4}$$

defines the charge demand part according to end-consumer power demand $P_e(t)$; demand-circuit current is $I_d(t) = \dot{Q}_c(t)$, voltage drop at *demand-control resistor* of resistance $R_d(t)$ then is $U_d(t-) = R_d(t-)\dot{Q}_c(t)$. With source voltage $U_c(t) = Q_c(t)/C$ for consumption according to t-prevalent condenser charge $Q_c(t)$, effective voltage $U_e(t)$ at

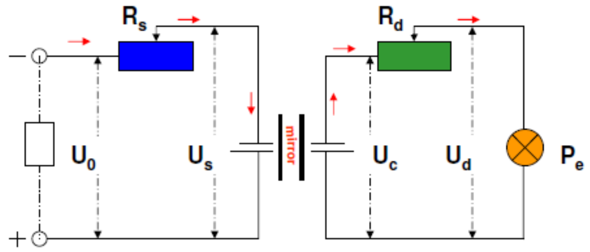

Figure 1: The motivation for the generic single-circuit according to Def. 2 is illustrated as if it were a pair of electric circuits.

the *end consumer* is then $U_e(t) = U_c(t) - U_d(t)$ with ohmic resistance $R_e(t-) = U_e(t-)/\dot{Q}_c(t)$ and *power* (wattage) $P_e(t)$, $P_e(t-) = U_e(t-)\dot{Q}_c(t)$, at any time $t > 0$. While $U_e(t) = U_c(t)$ when $R_d(t) = 0$, $R_d(t) > 0$ permits rearrangement that gives a first-order quadratic differential equation in Q_c,

$$\dot{Q}_c^2(t) - \dot{Q}_c(t)\frac{Q_c(t)}{CR_d(t-)} + \frac{P_e(t-)}{R_d(t-)} = 0, \tag{5}$$

which implies a decremental change in charge on the demand-part condenser. that depends on t-prevalent condenser charge $Q_c(t)$, end-consumer's pre-t-power (wattage) demand $P_e(t-)$, and pre-t demand-current control resistance $R_d(t-)$, for a fixed capacity C.

When one absorbs the demand-control resistor's and the end-consumer's ohmic resistances into one resistor according to their serial connection, $R_.(t) = R_d(t) + R_e(t)$, one obtains the linear first-order differential equation in $Q_c(t)$,

$$\dot{Q}_c(t) = -\frac{Q_c(t)}{CR_.(t-)}. \tag{6}$$

Results

A Triple Circuit

The triple-circuit *wirkgefüge* will have one copy of the *generic single-circuit* in Sect. "Generic Single Circuit" for each *Wirkkomponente*, set in a parallel connection which implies common source voltage U_0 for each while total current and moved charges split at the branching points into the partial circuits, see Fig. 2.

For each replication identified by a subscript i, for $i = 1,2,3$, one has (1) now as

$$dQ_{si}(t) = \frac{1}{R_{si}(t-)C_i}\left(U_0C_i - Q_{si}(t-)\right)dt. \tag{7}$$

and (5) as

$$dQ_{ci}(t) = -\frac{Q_{ci}(t)dt}{C_iR_{ci}(t-)} \tag{8}$$

$$\dot{Q}_{ci}(t)Q_{ci}(t) = R_{di}(t-)C_i\dot{Q}_{ci}^2(t) + C_iP_{ei}(t-)\,. \tag{9}$$

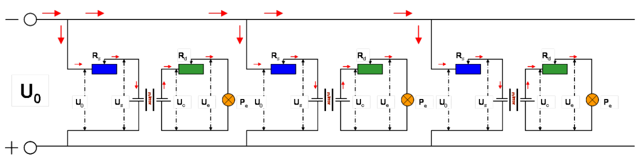

Figure 2: Schematic triple-circuit in parallel connection of three replicates of generic circuit in Fig. 1.

Up-scaling: triple- to single-circuit

For clarity of exposition, it is helpful to use superscripts FL0 and FL1 in order distinguish the appropriate function levels in notation, whenever both function levels are involved in a statement.

Up-scaling supply-parts

Here, it has to be shown that the configuration permits to re-obtain (2) from the triple in (7).
Capacities and t-prevalent charges on the three supply-part condensors add up as much as currents according to *Kirchhoff*'s rule, $C_{\cdot} = C_1 + C_2 + C_3$, $Q_{s\cdot}(t) = Q_{s1}(t) + Q_{s2}(t) + Q_{s3}(t)$, $I_{s\cdot}(t) = I_{s1}(t) + I_{s2}(t) + I_{s3}(t)$, while for supply-control resistances $((R_s)^{-1})_{\cdot}(t) = R_{s1}^{-1}(t) + R_{s2}^{-1}(t) + R_{s3}^{-1}(t)$ holds.

Theorem 1 *The* FL1 *triple-circuit supply-parts defined by*

$$[C_i^{\mathrm{FL1}}, Q_{si}^{\mathrm{FL1}}(t), R_{si}^{\mathrm{FL1}}(t)]_{i=1,2,3;t>0} \tag{10}$$

translate into a FL0 *single-circuit supply-part*

$$[C^{\mathrm{FL0}}, Q_s^{\mathrm{FL0}}(t), R_s^{\mathrm{FL0}}(t)]_{t>0} \tag{11}$$

by summation of capacities, charges, and inverse resistances,

$$\begin{aligned} C^{\mathrm{FL0}} &= C_{\cdot}^{\mathrm{FL1}}, & (12)\\ Q_s^{\mathrm{FL0}}(t) &= Q_{s\cdot}^{\mathrm{FL1}}(t), t>0, & (13)\\ (R_s^{\mathrm{FL0}})^{-1}(t) &= (R_s^{\mathrm{FL1}})_{\cdot}^{-1}(t), t>0. & (14) \end{aligned}$$

Proof: Rewriting (7) for each i,

$$dQ_{si}(t) = \frac{U_0}{R_{si}(t-)} - \frac{Q_{si}(t)dt}{R_{si}(t-)C_i} \tag{15}$$

and observing that $Q_{si}(t) = U_{si}(t)C_i$ for the supply-part condensers and $U_{si}(t-)/R_{si}(t-) = I_{si}(t)$, for every i, summation of (7) over $i = 1,2,3$ yields indeed

$$dQ_{s\cdot}(t) = \frac{1}{C_{\cdot}}(R_s)_{\cdot}^{-1}(t-)\left(U_0 C_{\cdot} - Q_{s\cdot}(t)\right)dt \tag{16}$$

Up-scaling demand-parts.

It now has to be shown that the configuration permits to re-obtain (5) from the triple in (9), too.
Capacities C_i and t-prevalent charges $Q_{ci}(t)$ on the three demand-part (mirror) condensors, and end-consumers' power demands (wattages) $P_{ei}(t)$ are first added, $C_{\cdot} = C_1 + C_2 + C_3$, $Q_{c\cdot}(t) = Q_{c1}(t) + Q_{c2}(t) + Q_{c3}(t)$, and $P_{e\cdot}(t) = P_{e1}(t) + P_{e2}(t) + P_{e3}(t)$.
One then has to construct a demand-part circuit at FL0 with the desired properties that arise from summations across respective circuits at FL1. To organize the train of thought, a general lemma is helpful.

Lemma 1 *For any single circuit with condenser of capacity C and t-prevalent charges $Q(t), t \geq 0$, and with a consumer of power demand $P(t), t \geq 0$, there exists a resistor of resistance $R(t), t \geq 0$, such that*

$$R(t) = \frac{\frac{Q(t)}{C} - \frac{P(t)}{\dot{Q}(t)}}{\dot{Q}(t)} \tag{17}$$

for any t, $\dot{Q}(t) > 0$, $t \geq 0$.

Proof: On the condenser, t-prevalent charges $Q(t)$ imply t-prevalent voltages $U(t)$ and $U(t) = Q(t)/C$ for any $t \geq 0$. Voltage drop $U_P(t)$ at t according to a t-prevalent power demand $P(t)$ is $U_P(t-) = P(t-)/\dot{Q}(t)$, for any $t \geq 0$. For any residual voltage $U(t) - U_P(t) \geq 0$, there is a resistor in the circuit which implies a voltage drop $U_R(t) = U(t) - U_P(t)$ at t; its resistance $R(t)$ is then $R(t-) = U_R(t-)/\dot{Q}(t)$, for any $t \geq 0$. Together, these results prove the Lemma.
It remains to identify FL0 capacity, charges, powers from the FL1 triple-circuits summaries, first, and then to apply the Lemma in order to finalize the up-scaling.

Theorem 2 *The* FL1 *triple-circuit demand-parts defined by*

$$[C_i^{\mathrm{FL1}}, Q_{ci}^{\mathrm{FL1}}(t), R_{di}^{\mathrm{FL1}}(t), P_{ei}^{\mathrm{FL1}}(t)]_{i=1,2,3;t>0} \tag{18}$$

translate into a FL0 *single-circuit demand-part*

$$[C^{\mathrm{FL0}}, Q_c^{\mathrm{FL0}}(t), R_d^{\mathrm{FL0}}(t), P_e^{\mathrm{FL0}}(t)]_{t>0} \tag{19}$$

by summations of capacities, charges, and power demands,

$$\begin{aligned} C^{\mathrm{FL0}} &= C_{\cdot}^{\mathrm{FL1}}, & (20)\\ Q_c^{\mathrm{FL0}}(t) &= Q_{c\cdot}^{\mathrm{FL1}}(t), t>0, & (21)\\ P_e^{\mathrm{FL0}}(t) &= P_{e\cdot}^{\mathrm{FL1}}(t), t>0; & (22) \end{aligned}$$

and a derived resistance

$$R_d^{\mathrm{FL0}}(t-) = \frac{\frac{Q_c^{\mathrm{FL0}}(t)}{C^{\mathrm{FL0}}} - \frac{P^{\mathrm{FL0}}(t-)}{\dot{Q}_c^{\mathrm{FL0}}(t)}}{\dot{Q}_c^{\mathrm{FL0}}(t)} \tag{23}$$

for the demand-control resistor.

Proof: With $C = C_{\cdot}^{\mathrm{FL1}}$, $Q(t) = Q_{c\cdot}^{\mathrm{FL1}}(t)$, and $P(t) = P_{e\cdot}^{\mathrm{FL1}}(t)$, the assertion follows from Lemma 1.

Remark 3 *Note that (23) is the quadratic first-order differential equation (5).*

Discussion

Methodology

An *axiomatic wirkgefüge* is described as a mathematical framework for translation of dynamics from canonical tripartite "function-level no.1" to whole-body "function-level

0" aggregate. It splits the aggregate's functional activity according to the number of the aggregate's constituent logical units and distributes its charge according to their respective demands; but it also re-combines individual units' dynamics into the dynamics of their aggregate whole. This points to its use at other function levels: One may consider the liver organ as an aggregate whole at "function-level no. n" for some positive number n, say, and involve its *wirk-components* as logical units on "function-level no. $(n+1)$", taking care that the decomposition is exhaustive and non-overlapping. In this way, one can start to work downwards from any logical unit in a functional hierarchy; to work upwards, however, one has to take into account *all* logical-unit "neighbors" on the same function level before one can upscale to upper level dynamics. Hence, drill-down appears to be the preferable direction.

The mathematics of *axiomatic wirkgefüge* is deliberately simple, and may seem too simplistic to be relevant, at first sight. However, it rapidly builds up complexity when decomposing functional aggregates into logical units way down - long before one gets to the bottom of 10^{14} living cells, each a burn chamber and base functional unit. It then reminds one of the wheat-chessboard problem.

This exposition merely describes the "firing" in the sense of energy transfer dynamics; specification of a control regimen of dynamics in single and in triple circuits is postponed. The differential equations were hence left without solutions, as the side conditions that are necessary to obtain specific solutions depend on an adequate "wiring" in the sense of a *kybernetik schaltgefüge* that permits the human body system to energize required action of its *axiomatic wirkgefüge* in *any* situation for preservation of its integrity and survival. Their common independence of physical realization is expected to permit to design both configurations by analogy with better understood complex engineered production systems.

Medical Perspective

Partition of whole-body's power demand into its canonical *wirk-components* - vital, productional, operational - marks an entry point for investigation of partitional variation in different stressing (by exposures or interventions) scenarios and of any discrepancy in power-demand partitioning from so-called "normal" population in various disorders; disablement, impairment, somatic or psychic chronic disease can be expected to imply partition shifts in either productivity (musculo-skeletal locomotor function, sexual reproductivity), or operational ability (communication with body's outside world, self-steering one's operations), or vital-functions activity (e.g. respiration, circulation, immune response, nervous control, hormonal regulation, food intake, digestion, waste ejection), and possibly in more than one. Detectable as deviations from "normal" whenever it manifests itself in person's behavior, evidence of persistent shifted partioning may also be of interest in a context of unspecific screening for power-demand anomalities in human body system.

Specifically, among the first would be investigations in patients with manifest movement disorders and Parkinson's disease, as well as in patients in different stages of progressive recovery, functional rehabilitation, work and social-life re-integration, and life-balance restoration after an ischemic stroke with moderate to severe neurological deficits [5]; for healthy-subject controls, athletes will be first choice in these investigations.

The practical implementation poses challenging problems regarding technology for taking specific measurements of *wirk-components* power demands and methodology of its application. Relative assessments will be in foreground. Whole-body power demands when "sleeping", "awake lying still" of 77 cal/h, "sitting at rest", "standing relaxed", "walking slowly", and "running" of 65 cal/h, 77 cal/h, 100 cal/h, 105 cal/h, 200 cal/h, and 570 cal/h, respectively, [6], give first clues, e.g. to net walking-power demand against background upright posture, of 95 cal/h. (Note that an interface for exchange with outside world in total energy balances is not included.)

Conclusions

The axiomatic construct suggests a path into multi-scale modeling of human body system *wirkgefüge*, coherent by design, and progress will be made from top to bottom. This way, impact from health-compromising exogenous factors can be integrated at any level, but applicability in clinical context is still to be confirmed.

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