Identification and synthesis of linear-quadratic regulator for digital control of electrohydraulic steering system

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The paper presents an optimal reference tracking algorithm for electrohydraulic steering systems which is based on multivariable system identification, linear quadratic control and Kalman filtering for state estimation. A laboratory test-bench composed of electrohydraulic-steering unit (EHSU), steering cylinder, 32-bit microcontroller, steering wheel and joystick supports experimental work. Traditional approach for reference tracking in steering usually is based on classical control algorithms such digital PI regulator or non-digital hydro-mechanical feedback. In contrast the control theory suggests advanced control techniques, which can take into account multivariable nature of the process. In this way a higher closed-loop performance can be achieved.

Keywords: Multivariable identification, linear-quadratic regulator, Kalman filter, electrohydraulic steering system
Target audience: Systems, Mobile Hydraulics

1 Introduction

In modern mobile machines, the proportional electrical control of the steering system is used due to the need for remote control via GPS. In addition, the mechanical steering with variable steering ratio from the steering wheel to the machine's steer axle is often a sought after function to improve driver productivity and comfort [1,2]. This leads to the need for an effective integrated control system which should ensure the quality behavior of the entire electrohydraulic system [3].

In the case of remote control via controller, the behavior of the machine depends heavily on embedded software regulator. Quality of control for this case can be improved if more accurate plant model is used. Model-based approach is widely applied in modern control system design. This approach is also useful in software engineering for design of real-time signal processing systems. In mathematical modeling always there is a trade-off between complexity and accuracy of the model [4]. Usually complex models are appropriate for plant behavior analysis but they are inconvenient to design control algorithm.

Steering control systems are necessary because loading torques acting upon steering axle may disturb steering performance. Also the mathematical model cannot take into account all physical details because it would become impractically complex [5]. In industry there are two common control strategies - feedback interconnection and hierarchical (or cascade) subordination of feedback loops. A wide known fact is that more than 90 % of all the industrial feedback control loops are derivatives of PID algorithm [6,7]. The reason is not only the intuitive idea behind PID but also because it has proved robust to small model uncertainties. However PID controller tuning becomes a difficult tasks in case of many inputs many outputs (MIMO) plant. For MIMO case the control theory suggests many advanced control techniques, which can take into account multivariable nature of the process. Such practical approved control technique is linear quadratic Gaussian regulator (LQG) that involves linear quadratic regulator (LQR) and uses state estimates obtained by Kalman filter [8,4]. The LQG algorithm takes into account not only multivariable process nature but also the influence of noises to the plant dynamics.

The main objective of this work is to present the designed system for control of electrohydraulic steering system that is implemented in low speed mobile machines. The goal of control algorithm is to achieve fast transient response without overshooting and static error in whole working range. To achieve this aim first a multivariable dynamical plant model is estimated by identification procedure. The model obtained is validated by various statistical tests. The multivariable LQR regulator with integral action and Kalman filter are designed. Appropriate software which is implemented in 32-bit microcontroller is developed. Experimental results are presented which confirm that the control system achieves the prescribed performance.

The paper is organized as follows: section 2 presents designed experimental setup, in section 3 the results from is multivariable system identification are presented, section 4 shows design of linear-quadratic controller with Kalman filter and in section 5 some experimental results are given.

2 Experimental system layout

Authors have developed a laboratory hydraulic test equipment for EHSU type OSPEC200 LSRM, taking into account current technical specifications from the manufacturer [3]. Figure 1 shows hydraulic schematics of the test bench system, described in detail in [9].

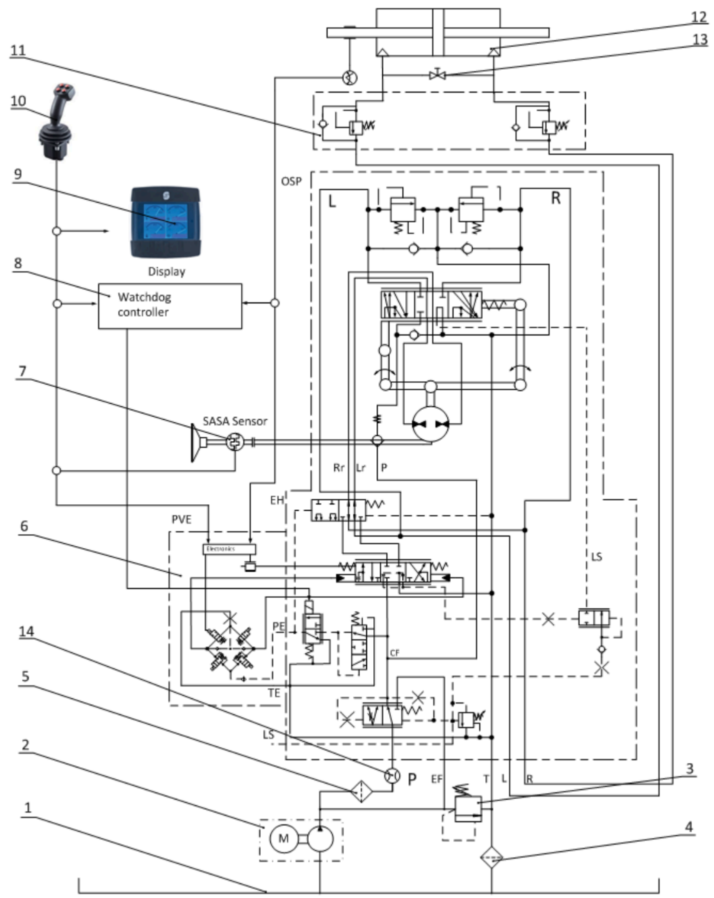

Figure 1: Hydraulic schematic of EHSU test-bench with pressure loading subsystem.

A digital control system has been developed in which an electronic joystick is configured for an input device. It consists of an electronic joystick, a digital LQG regulator supplying control voltage to the PVE module, which in turn control the PVE built-in electrohydraulic proportional valve. The electrohydraulic proportional valve (6) determines the direction of movement of the executive hydraulic cylinder (12), by feeding a working fluid to one of both chambers. The plant output is a cylinder piston position that is measured through a sensor. The measured sensor signal is used in feedback and is compared with the reference signal.

Digital control systems is based on a controller type MC012-022 and electronic joystick type JS6000 (Danfoss platform for mobile applications), by which operated module with two-way, two-position valves connected in parallel - PVE.

In the programming environment (PLUS + 1 Guide) [10] controller MC012-022 is created and a program that calculates a parameters of control signal. The experimental system (Fig.1) consists of EHSU type OSPE 200 and symmetric servo cylinder connected by pipelines through L and R.

The developed test bench system is in accordance with the current requirements for the testing of electro-hydraulic steering devices with different pressure loads. Pressure loading system is composed of a hydraulic block with over-center valves (pos. 11, Fig. 1), which are connected to the both chambers of the servo-cylinder.

3 Multivariable system identification

To determine the mathematical model of electrohydraulic steering system one may apply physical modeling or identification [5,11]. Physical modeling requires profound of knowledge about physics of the plant and a lot of a priori information such as various characteristics, values of specific constants and hydraulic resistances. Due to the lack of a priori information, in this study a numerical model obtained by identification procedure is used. Another reason to use this approach is that in addition to the description of plant dynamics the noise model is obtained. This model can be used to design an appropriate optimal filter such as Kalman filter. Thus the goal of identification is to obtain a linear black box model which sufficiently well describes electrohydraulic steering system dynamics and noises in wide working range. To obtain such model first the open loop identification experiments, according to scheme shown in Figure 2, is designed. The sample time of $T_s = 0.05s$ is chosen, that is sufficiently small. To provide persistent excitation to the plant input a random binary signal (RBS) is applied. It is obtained from filtered through relay white Gaussian noise. The amplitude of RBS is chosen to be ± 2500. In this manner the whole working range of input signal is used. The plant dynamics can be represented by single input two outputs model in which the input is control signal, the first output is the difference between measured pressures in right and left chambers and the second output is the measured position of cylinder.

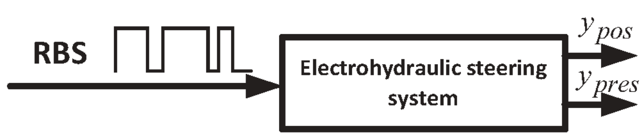

Figure 2: Scheme of open loop identification experiment

Before use the input-output data for identification the constant values should be removed. From the centered measured data two data sets are formed. First of them is used for model estimation and second - for model validation. They are depicted in Figures 3-4. The excitation level of identification input signal is 500. This means that up to 500 parameters can be estimated from estimation data set. The identification procedure starts with estimation of state space model with free parameterization

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + K_v v(k) \\ y(k) &= Cx(k) + Du(k) + v(k) \end{aligned}, \qquad (1)$$

where $x(k) = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$ is a state vector, $u(k)$ is the input signal, $y(k) = \begin{bmatrix} y_{pres} & y_{pos} \end{bmatrix}^T$ is the output vector , $v(k)$ is a model disturbance (residual) and $A, B, C, D.K_v$ are the matrices with appropriate dimensions.

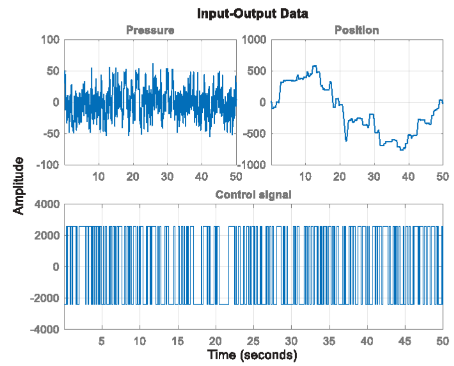

Figure 3: Estimation data set

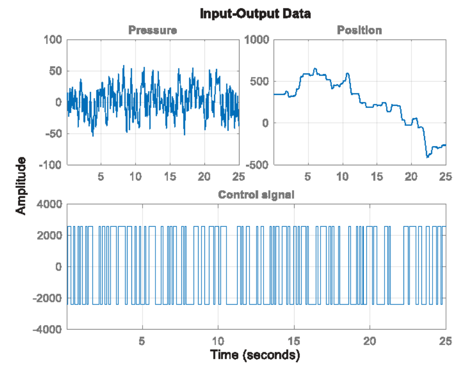

Figure 4: Validation data set

It is chosen to estimate a state space model (1) because it has a form which can be directly used for Kalman filter design with *Matlab* [12,13]. Assuming that the possible model order is between 1 and 5, the model set of five state space models is formed. After estimation of these models by prediction error method [5], the validation tests are performed. The model of order 3 is chosen, because it is a simplest model from model set that passes the validation procedure. The parameters values of estimated model are

$$A = \begin{bmatrix} 0.8769 & -0.3987 & 0.3986 \\ 0 & 0 & 1 \\ -0.1666 & -0.5099 & 1.509 \end{bmatrix}, B = \begin{bmatrix} -0.0043 \\ 0.0011 \\ 0.0021 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, K_v = \begin{bmatrix} 0.1112 & -0.06214 \\ -0.09525 & 1.55 \\ -0.2003 & 1.897 \end{bmatrix} \qquad (2)$$

The comparison between measured pressure and cylinder position (according validation data set) and model outputs is presented in Figure 5. The value of FIT between measured pressure and model pressure is 56.12% and the one between the measured position and model position is 76.5%. These results mean that estimated model captures sufficiently well plant dynamics. The results from whitening and independence tests of residuals are shown in Figure 6. The frequency response of estimated high order finite impulse response (FIR) model between control signal and residuals along with 99% confidence region is depicted in Figure 7. As can be seen from Figure 6 the noise model is adequate and there is not significant correlation between input and residuals. This result is confirmed again from the test presented in Figure 7(it shows that there is not significant dynamics between input signal and residuals in the whole interested frequency range).

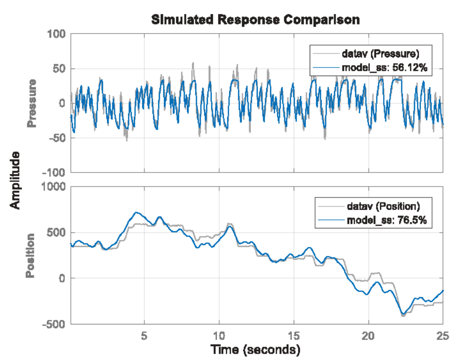

Figure 5: Model outputs and measured outputs

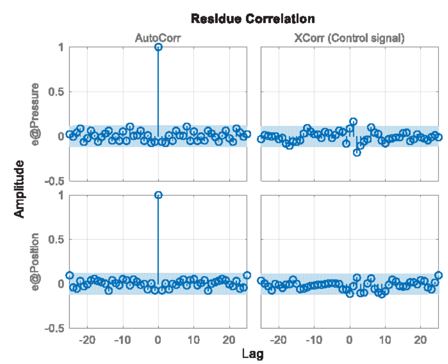

Figure 6: Residual test of estimated model

As a result from identification a 3-th order state space model is obtained. It describes sufficiently well the both a plant and a noise dynamics. The plant dynamic model will be used for LQR controller design whereas the noise model will be used for Kalman filter design in next Section.

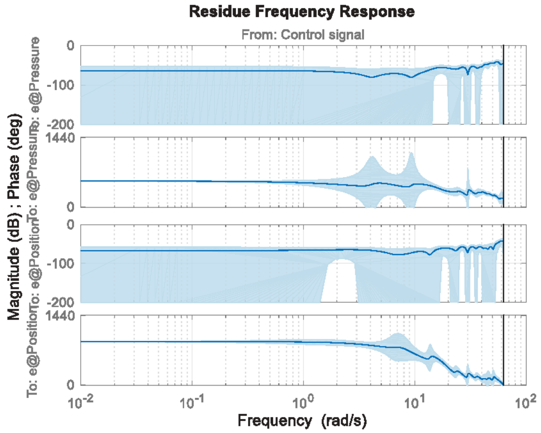

Figure 7: Residuals to input signal frequency response

4 Design of linear-quadratic regulator

The structure scheme of control system with LQR controller and Kalman filter is shown in Figure 8. To ensure sufficiently well reference tracking an LQR controller with integral action is designed. The design is done on the basis of deterministic part of estimated model (2) which is extended with an extra state x_i . This extra state is discrete time integral of position error

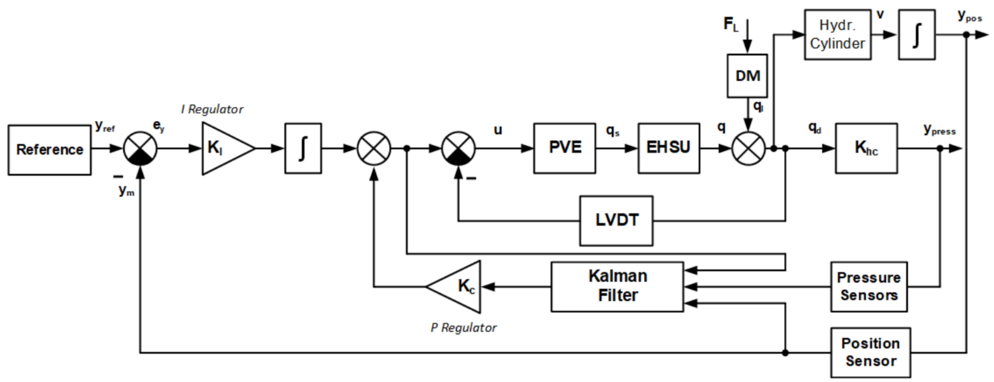

Figure 8: LQR with Kalman filtering closed-loop schematic

$$x_i(k+1)=x_i(k)+T_se_y(k)=x_i(k)+T_s(y_{ref}(k)-y_{pos}(k))\,, \tag{3}$$

where $y_{ref}(k)$ is the reference. Thus, combining the deterministic part of equation (2) and equation (3) one obtains the augmented system

$$\begin{aligned}\bar{x}(k+1)&=\bar{A}\bar{x}(k)+\bar{B}u(k)+\bar{G}y_{ref}(k),\\ y(k)&=\bar{C}\bar{x}(k),\end{aligned} \tag{4}$$

where

$$\bar{x}(k)=\begin{vmatrix}x(k)\\x_i(k)\end{vmatrix},\bar{A}=\begin{vmatrix}A&0\\-T_sC&1\end{vmatrix},\bar{B}=\begin{vmatrix}B\\0\end{vmatrix},\bar{C}=\begin{vmatrix}C&0\end{vmatrix},\bar{G}=\begin{vmatrix}0\\T_s\end{vmatrix}.$$

The optimal control law is obtained in the form

$$u(k)=-\bar{K}\bar{x}(k),\bar{K}=[K_c\quad -K_i] \tag{5}$$

where K_c is the proportional term matrix gain and K_i is the integral term gain. The controller matrix $\bar{K}$ is obtained from minimization of quadratic performance index

$$J(u)=\sum_{k=0}^{\infty}\bar{x}^T(k)Q\bar{x}(k)+u^T(k)Ru(k)\,, \tag{6}$$

where Q and R are positive definite matrices chosen to ensure acceptable transient response of the closed-loop system. The optimal feedback matrix $\bar{K}$ is determined by [4]

$$\bar{K}=(R+B^TPB)^{-1}B^TPA\ , \tag{7}$$

Where P is the positive definite solution of the discrete-time matrix algebraic Riccati equation

$$A^TPA-P-A^TPB(R+B^TPB)^{-1}B^TPA+Q=0\,. \tag{8}$$

The optimal controller matrix is obtained for $Q=\begin{bmatrix}7\times10^4&0&0&0\\0&10^4&0&0\\0&0&10^4&0\\0&0&0&0\end{bmatrix}$ and $R=5000$.

Since the state $x(k)$ of system (2) is not accessible, the optimal control law (6) is implemented as

$$u(k)=-K_c\hat{x}(k)+K_ix_i(k)\,, \tag{9}$$

where $\hat{x}(k)$ is estimate of $x(k)$ .It is obtained by discrete time Kalman filter

$$\hat{x}(k+1)=A\hat{x}(k)+Bu(k)+K_f(y(k+1)-CBu(k)-CA\hat{x}(k))\,. \tag{10}$$

The filter matrix K_f is determined as

$$K_f=D_fC^T(CDC^T+10^{-4}I_2)^{-1}, \tag{11}$$

where I_2 is second order unit matrix and matrix D_f is the positive semi-definite solution of Riccati equation

$$AD_fA^T-D_f-AD_fC^T(CDC^T+10^{-4}I_2)^{-1}CD_fA^T+K_vD_vK_v^T=0\,. \tag{12}$$

The matrix $D_v=\begin{bmatrix}108.97&0\\0&27.44\end{bmatrix}$ is the variance of noise $v(k)$.

5 Experimental results

Experiments with the designed LQR regulator require specific implementation technique in target microcontroller MC012-022. The controller interconnection can be represented in equivalent single matrix vector formulas.

$$\begin{pmatrix} \hat{x}(k+1) \\ x_i(k+1) \\ u(k+1) \end{pmatrix} = \begin{pmatrix} A-CA & 0 & B-CB \\ -T_S C & 1 & 0 \\ K_c & -K_i & 0 \end{pmatrix} \begin{pmatrix} \hat{x}(k) \\ x_i(k) \\ y(k) \end{pmatrix} \tag{13}$$

$$y(k) = \left(y_{ref}(k) \quad y_{press}(k) \quad y_{pos}(k) \right)^T \tag{14}$$

Danfoss programming environment supports visual dataflow programming (like in Simulink or LabView) and textual programming options. Our choice for the present project is visual programming style. Hence Figure 9 shows implementation of vector matrix multiplication in PLUS 1 IDE with fixed-point arithmetic. The blocks on the figure relate to one of two functions – resource and control.

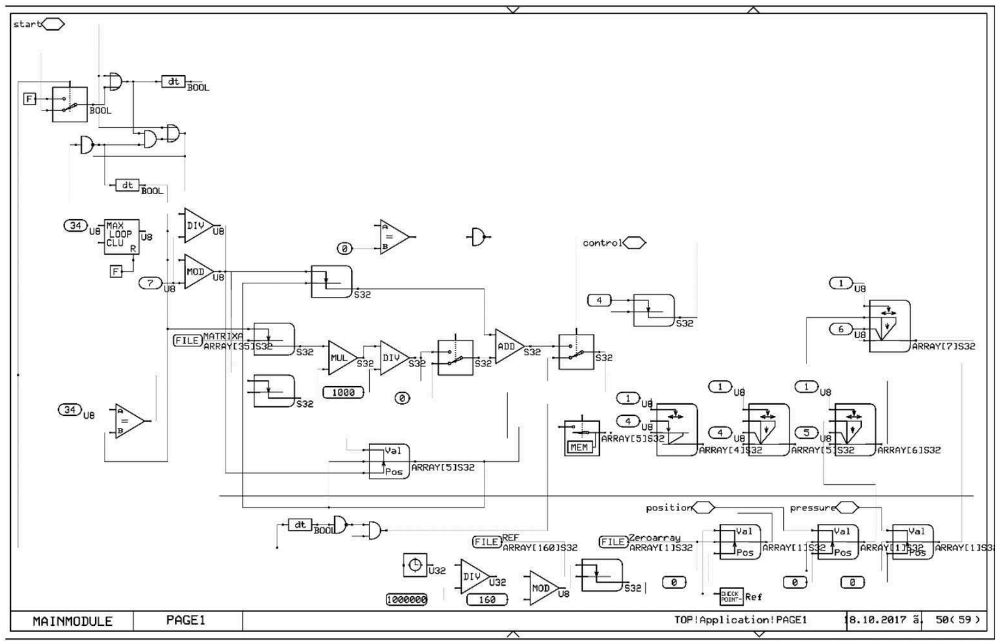

Figure 9: LQR with Kalman filtering controller implementation

For example multiplication and summation blocks have resource function while index counter and register reset Boolean switch bock have resource function within the system. A start signal is triggered in the beginning of the sample period to execute the calculation in the next step. The index counter increments through indices of the matrix which is represented as one dimensional row-stacked array. Based on the current index corresponding elements from the matrix and the vector are selected to be multiplied. Since they are represented as fixed point numbers with scaling 1000 after multiplication the result is divided by 1000. Multiplication results corresponding to the given matrix row are accumulated within register formed by sumator element and positive feedback. Accumulator reset is triggered by the increment of the current row index. Rest of the blocks controls formation of the input vector array.

Figure 10 shows experimental response of the cylinder piston during periodic step reference trajectory in both directions relative to the central position. Recorded reaction of the cylinder piston is aperiodic with setting time around 35 seconds for 1/3 of the piston stroke. This is acceptable for low speed steering of heavy duty machines. During the motion to the final position the piston stops for a certain amount of time in some intermediate positions. The reason is relatively large dead zone of EHSU with PVE module due to its constructive specifics.

There is zero error in steady state. Transitional processes are of an aperiodic nature, without overshoot. The quality of the transition processes is maintained when cylinder piston moving in both directions.

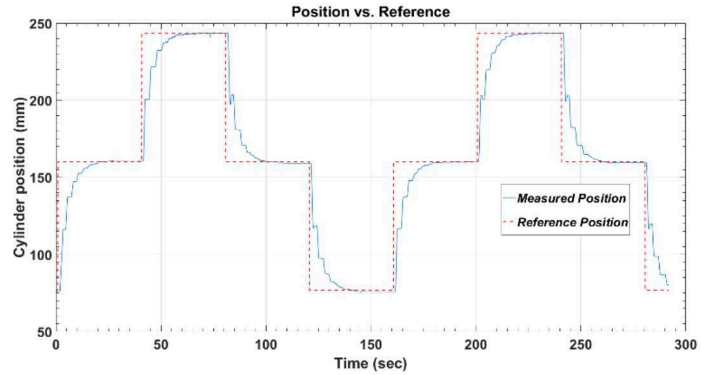

Figure 10: Step response of closed-loop system

Figure 11 shows the measured control signal supplied by the controller to the PVE module during the experiment. The control signal approaches its maximum value during the transition process, indicating that the output reacts with the maximum possible performance boost [14]. The noise level in the control signal is low, indicating a high accuracy of the measuring sensor [15] for the cylinder position. This in turn translates as a quality of the closed-loop system.

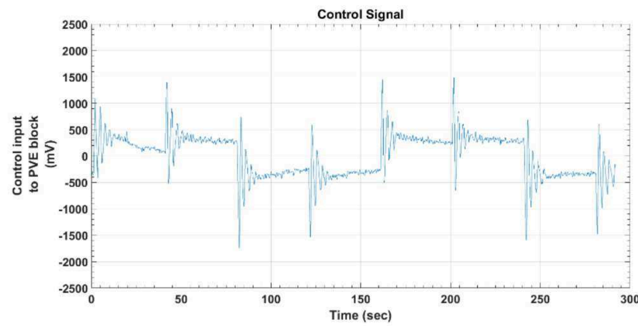

Figure 11: Control signal to PVE block

Figure 12 shows the dynamic pressure variation in the two chambers of the executive servo-cylinder, and Figure 13 shows the difference from them. Experimental studies were performed at a fixed setting of the loading pressure (0.5MPa) set by the load system based on a hydraulic block with over-center valves (pos.11, Fig.1). This system makes it possible to realize different pressure loads in the two chambers of the servo-cylinder.

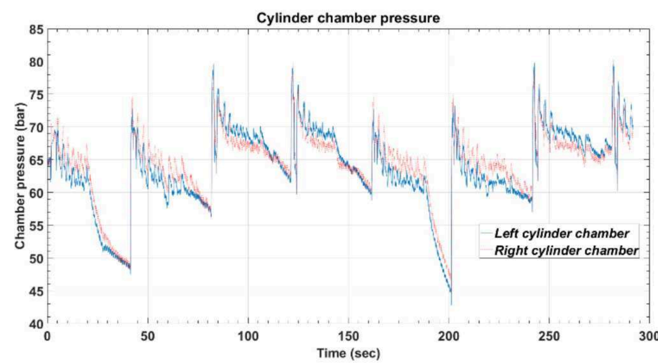

Figure 12: Cylinder chamber pressure

The variable pressure load affects the closed system as a low-frequency output disturbance. The results show the low sensitivity of the system to it. It`s an important quality because there is no need to re-set the system at different loads. This insensitivity occurs at the expense of the increased power of the control signal.

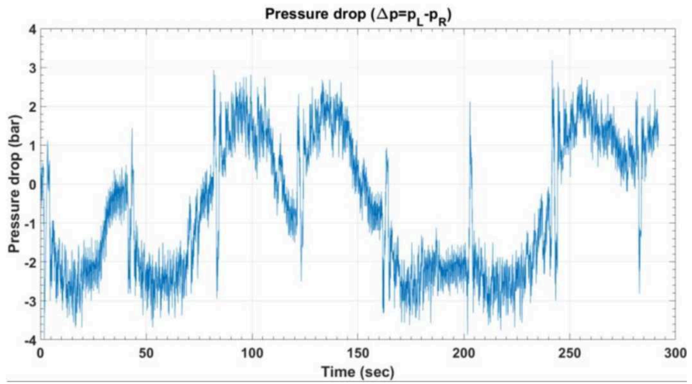

Figure 13: Pressure drop between cylinder chambers

6 Conclusion

The main result of the paper is a developed system for LQG control of electrohydraulic steering system that is implemented in low speed mobile machines. A two output one input discrete time stochastic plant model is obtained by identification procedure. This model is validated by statistical tests and is used to design of LQR controller and Kalman filter. The multivariable LQR regulator with integral action and Kalman filter are designed. Appropriate software which is implemented in 32-bit microcontroller is developed. The results from experiment with developed by authors laboratory setup confirm control system performance. The embedded control system achieves the prescribed requirements: fast transient response without overshooting and static error in whole working range.

7 Acknowledgements

Current research was supported by the funding contract № DM07/7 with National Science Fund Bulgaria. Also the authors are thankful to conference organization committee for their invitation.

Nomenclature

Variable	*Description*	*Unit*
x	State vector	[-]
y	Output vector	[-]
v	Model disturbance (residual)	[-]
K_c	Proportional term matrix gain	[-]
K_I	Integral term matrix gain	[-]
K_f	Filter matrix	[-]
T_s	Sample time	[s]
u	Input signal	[-]
A, B, C, D	Matrices with appropriate dimensions	[-]
Q, R	Matrices chosen to ensure acceptable transient response of the closed-loop system	[-]

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