

Reduction of bearing load capacity due to measured wall slip

Tobias Corneli*, Maximilian M. G. Kuhr* and Peter F. Pelz*

Institut für Fluidsystemtechnik, Technische Universität Darmstadt,
Otto-Berndt-Str. 2, D-64278 Darmstadt, Germany*
E-Mail: peter.pelz@fst.tu-darmstadt.de

The presented work investigates the temperature dependence of the Navier slip boundary condition and the related reduction of load capacity of a bearing. In part (i), the Navier slip boundary condition is discussed and a modified Reynolds equation, including slip, is derived. Based on this modified Reynolds equation, the pressure distribution and the load capacity of a slider bearing are obtained. Part (ii) presents the Darmstadt Slip Length Tribometer, utilized for measuring the slip length of technical rough surfaces. Part (iii) shows the temperature dependent results of the slip length measurements and the effect on the load capacity of the slider bearing in comparison to the standard no slip boundary condition.

Keywords: Fundamentals, Journal Bearing, Sealing Technology, Slider bearing

Target audience: Fundamentals, Pumps, Valves, Seals

1 Introduction

Since the beginning of the 20th century, hydraulic sealings and journal bearings [1-3] are designed employing Reynolds lubrication theory. The Reynolds lubrication theory presumes the no slip boundary condition at the liquid-solid interface. Recent studies conducted by the authors show, that the so far assumed no slip boundary condition at the liquid-solid interface is not valid for most fluid power applications; cf. [4]. This effects the prediction of leakage flow and frictional behaviour of sealing systems as well as bearing capacity of journal bearings. Thus, considering slip at the liquid-solid interface is important for the design of hydraulic components.

The concept of wall slip was already discussed by Navier [5] and Stokes [6], when deriving the momentum equation for Newtonian fluids in the 19th century. Stokes favours the no slip boundary condition and justifies his hypothesis by a good agreement of the theory with experimental investigations of Poiseuille [7]. In contrast, Navier [5] formulates the slip boundary condition, with the slip velocity being the product of the shear rate and the slip length. Recent experimental studies of the authors at the Technische Universität Darmstadt show that the slip length is for oil/steel of the order of magnitude of 100 nm with a strong temperature dependence.

Regardless of this discussion, the no slip boundary condition is established over the centuries, based on the insufficient measurement techniques. However, in many technically important applications of fluid power technologies wall slip is not negligible. This is the case if the quotient of slip length and typical flow geometry is less than 10^{-3} . Thus, for typical hydraulic systems is reasonable to consider slip, if the gap geometries are of the order of magnitude of 10 μm .

With the exception of the slip length measurement results provided by the authors [4], existing investigations and test methods are limited to material pairings far from hydraulic applications. The deployed liquids commonly are water or aqueous solutions [8-21] and pure hydrocarbons [15-19, 22]. Investigated solids generally are glass [8, 10, 11, 14, 18-21, 23] and mica [9, 15-17, 22]. These surfaces are smooth at the atomic level with a typical maximum square mean roughness of 2 nm. Typical surfaces of hydraulic applications however have a mean roughness of the order of magnitude of 100 nm.

Replacing the liquid does not pose a challenge when applying existing slip length measuring methods for hydraulic applications. However, solids [8-23] and in particular the surface topographies cannot be varied as desired. Because surface roughness is quantified by means of integral methods. The DIN EN ISO 4287: 2010-07 standard specifies the required integral length for measuring the surface roughness. The characteristic length of the slip length measurement should be at least one order of magnitude larger than the length for determining the surface roughness, since otherwise local disturbances affect the slip length measurement. Above mentioned slip length measuring methods have a characteristic length from 10 μm to 100 μm . Thus, they are limited to surfaces with a roughness of the order of magnitude of 1 nm.

In order to be able to measure systematic slip lengths for technical systems of hydraulic applications the Darmstadt Slip Length Tribometer (DSLT) [24] is developed at the Chair of Fluid Systems of the Technische Universität Darmstadt in cooperation with the Fluid Power Association of the VDMA. This tribometer enables measuring slip lengths for technically rough surfaces with a mean roughness of 10 nm to 400 nm. Furthermore, for the first time it is possible to measure the slip length as a function of temperature, which was out of focus in earlier studies. For hydraulic applications, this temperature impact on the slip length is of major relevance due to typical operating conditions of hydraulic systems.

This article provides novel insights on the influence of slip on a hydraulic bearing system. It is organized as follows. In part (i), the Navier slip boundary condition is introduced and a modified Reynolds equation, including slip, is derived. Based on this modified Reynolds equation, the pressure distribution and the load capacity of a slider bearing are obtained. Part (ii) presents the Darmstadt Slip Length Tribometer, utilized for measuring the slip length of technical rough surfaces. Part (iii) shows the temperature dependent results of the slip length measurements and the effect on the load capacity of the slider bearing in comparison to the standard no slip boundary condition. The paper closes with a summary and a conclusion.

2 Navier slip boundary condition and Reynolds equation

This section provides the theoretical fundamentals of the slip length and the application to the well-known Reynolds equation. Afterwards, the modified equation is solved analytically for a slider bearing.

2.1 Navier's slip boundary condition

Interesting is the train of thought, which Navier causes to formulate the slip boundary condition in 1822 [25]: Navier interprets the processes at the wall as a dynamic equilibrium between the shear force of the liquid at the wall $\mu \partial u / \partial n$ and the wall-parallel adhesive forces. The adhesive forces are proportional to the slip velocity u_s , conforming to Stoke's law. Hence the balancing yields

$$u_s \cdot \text{const} = \mu \frac{\partial u}{\partial n} \quad (1)$$

Helmholtz [26] interprets the constant in Navier's relationship by means of dimensional analysis as a length, the nowadays called slip length λ

$$\lambda := \frac{\text{const}}{\mu} \quad (2)$$

Thus, (1) yields the purely kinematic form

$$u_s = \lambda \frac{\partial u}{\partial n} \quad (3)$$

known today, no longer revealing Naviers original thought and dynamic nature of the boundary condition. Figure 1 illustrates the geometrical interpretation of the slip length by means of a simple shearing flow example. It shows the velocity profiles for no slip boundary condition (grey) and Navier slip boundary condition (black). The lower surface is fixed while the upper surface moves with constant velocity U at a distance h . In the case of no-slip, the

velocity of the liquid molecules at the wall is identical to the wall velocity. In the case of slip, there is a relative velocity between the wall near molecules of the liquid and the wall.

The relative velocity at the fixed wall u_{s1} is greater than zero and the relative velocity at the moving wall u_{s2} reduces the liquid velocity relative to the wall velocity U . Extrapolating the velocity profile down to zero and up to the surface velocity U yields the slip length as the perpendicular distance from the surface to the boundaries of the extrapolated velocity profile. Due to this, Helmholtz's [26] geometric interpretation of an apparent gap opening due to wall slip is deduced. Slip velocity and shear rate are proportional and the proportionality constant is the slip length.

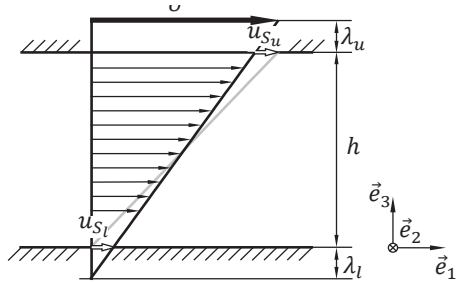


Figure 1: Simple shearing flow for no-slip (grey) and slip (black).

The slip length is a characteristic quantity of tribology, characterizing each tribological system consisting of a liquid and a solid. The slip length depends on the molecular weight and the additives of the liquid, the surface material, the surface topography and the temperature. For identical solid-state pairings the slip length remains constant, $\lambda_l = \lambda_u = \lambda$. This relationship is used in the experiment in section 5 for measuring the slip length.

2.2 Generalized Reynolds equation taking Navier's slip boundary condition into account

So far, the Reynolds equation is derived and applied only assuming no-slip boundary condition. Since wall slip affects both the leakage flow of seals and the bearing capacity of plain bearings, it is necessary to solve the Reynolds equation while taking the dynamic slip boundary condition into account. Within this section the generalized Reynolds equation incorporating wall slip is derived in five steps.

Starting from the well-known assumptions of the Reynolds equation (cf. [27]), i.e. $\alpha Re \ll 1$, the Navier Stokes equations simplify to the linear differential equations

$$\frac{\partial p}{\partial x_1} = \mu \frac{\partial^2 u_1}{\partial x_3^2}; \quad \frac{\partial p}{\partial x_2} = \mu \frac{\partial^2 u_2}{\partial x_3^2}; \quad \frac{\partial p}{\partial x_3} = 0. \quad (4)$$

Integrating the linear differential equation twice provides the general solution of the velocity distribution in the lubrication gap. Applying Navier's slip boundary condition for the velocities u at the fixed wall ($y = 0$) and the moving wall ($y = h$)

$$u_i(x_3 = 0) = \lambda_l \frac{\partial u_i}{\partial x_3} \Big|_{x_3=0}, \quad (5)$$

$$u_i(x_3 = h) = U_i - \lambda_u \frac{\partial u_i}{\partial x_3} \Big|_{x_3=h(x_i)}. \quad (6)$$

yields the special solution of the velocity distribution for ($i = 1, 2$). Substituting the integration constants using the boundary conditions from Equation (5) and (6) provides the solution to the boundary value problem, with $h = h(x_i, t)$,

$$u_i(x_3, t) = \frac{1}{2\mu} \frac{dp}{dx_i} \frac{x_3^2(h + \lambda_l + \lambda_u) - x_3(h^2 + 2h\lambda_u) - 2h\lambda_l\lambda_u - h^2\lambda_l}{h + \lambda_l + \lambda_u} + U_i \frac{(x_3 + \lambda_l)}{h + \lambda_l + \lambda_u}. \quad (7)$$

The volume flow per unit depth in the x_i - direction, $i = 1, 2$ is obtained by integrating the velocity distribution across the lubrication gap height

$$q_i(t) = \frac{U_i h}{2} \frac{1 + 2\lambda_l/h}{1 + \lambda_l/h + \lambda_u/h} - \frac{h^3}{12\mu} \frac{\partial p}{\partial x_i} \frac{1 + 4\lambda_l/h + 4\lambda_u/h + 12\lambda_l\lambda_u/h^2}{1 + \lambda_l/h + \lambda_u/h}. \quad (8)$$

Still the Couette term and the Poiseuille term are superimposed. This is because the equation of motion (4) and the boundary condition (5), (6) are linear, due to negligible inertia for $\alpha Re \ll 1$. The continuity equation in integral form (integrated from $0 < x_3 < h(x_i, t)$) yields for incompressible fluids

$$\frac{\partial h}{\partial t} + \frac{\partial q_i}{\partial x_i} = 0. \quad (9)$$

Inserting (8) into (9) yields the here for the first time given generalized Reynolds equation, being a Poisson type partial differential equation

$$\frac{\partial}{\partial x_i} \left[\frac{h^3}{\mu} \frac{dp}{dx_i} \left(1 + \frac{4\lambda_l}{h} + \frac{4\lambda_u}{h} + 12 \frac{\lambda_l \lambda_u}{h^2} \right) \right] = \frac{\partial}{\partial x_i} \left[6U_i h \left(1 + 2 \frac{\lambda_l}{h} \right) \right] + \frac{\partial h}{\partial t}. \quad (10)$$

Equation (10) is the well-known Reynolds equation for plane flows, but now taking wall slip into account. Integrating this equation gives the pressure distribution within the fluid film.

2.3 Slider Bearing

In this section, the pressure distribution in the plane lubrication gap of a slider bearing is investigated, taking Navier's slip boundary condition into account; cf. Figure 2. The upper wall is inclined by the angle α with respect to the x - axis and moves at the constant velocity U . The liquid is pulled into the narrowing gap, leading to a pressure increase. In sliding bearing technology, such an arrangement is also known as a slider bearing. So far, the load-bearing capacity for this arrangement is only examined for the kinematic no-slip boundary condition. In the following, the influence of wall slip is considered as well.

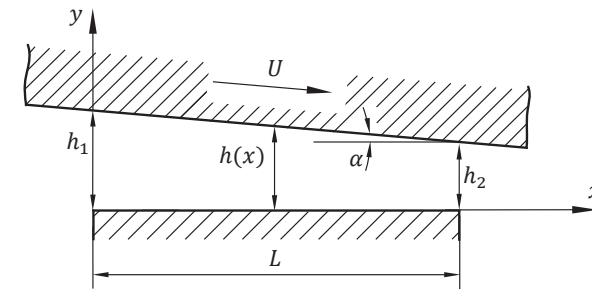


Figure 2: Geometry of the slider bearing.

Deriving the bearing capacity of the slider bearing, it is assumed for simplicity that both the fixed and moving wall are of the same material, thus the slip length is identical at both solid-liquid interfaces; $\lambda_l = \lambda_u = \lambda$. Under this assumption and for quasi-stationary flow, the modified Reynolds equation (10) reads

$$\frac{d}{dx} \left[\frac{h^3}{\mu} \frac{dp}{dx} \left(1 + 8 \frac{\lambda}{h} + 12 \frac{\lambda^2}{h^2} \right) \right] = \frac{d}{dx} \left[6Uh \left(1 + 2 \frac{\lambda}{h} \right) \right]. \quad (11)$$

Integration of (11) leads to

$$\frac{h^3}{\mu} \frac{dp}{dx} \left(1 + 8 \frac{\lambda}{h} + 12 \frac{\lambda^2}{h^2} \right) = 6U\mu \left(1 + 2 \frac{\lambda}{h} \right) + C_1. \quad (12)$$

The integration constant C_1 is determined at the location $x = \bar{x}$ of maximum pressure, where $dp/dx = 0$, giving the pressure gradient depending on the gap height $h(x)$ and the gap height at maximum pressure $\bar{h}$

$$\frac{dp}{dx} = 6\mu U \frac{1 - \bar{h}/h}{h^2 + 8\lambda h + 12\lambda^2}. \quad (13)$$

With the gap height $h(x) = h_1 - \alpha x$ and substituting $dx = -dh/\alpha$, the pressure distribution yields

$$p(h(x)) = \frac{6\mu U}{\alpha} \int \frac{1 - \bar{h}/h}{h^2 + 8\lambda h + 12\lambda^2} dh, \quad (14)$$

with the two integrals

$$\int \frac{dh}{h^2 + 8\lambda h + 12\lambda^2} = \frac{1}{4\lambda} [\ln(h + 2\lambda) - \ln(h + 6\lambda)] + C, \quad (15)$$

$$\int \frac{\bar{h}}{h(h^2 + 8\lambda h + 12\lambda^2)} dh = \frac{\bar{h}}{24\lambda^2} [2 \ln h - 3 \ln(h + 2\lambda) + \ln(h + 6\lambda)] + C. \quad (16)$$

The integration constant C is determined by applying the boundary condition $p(x = 0) = p(h_1) = 0$. The position of the maximum pressure $\bar{h}$ is determined by applying the condition $p(x = L) = p(h_2) = 0$, resulting in

$$\bar{h} = 6\lambda \frac{\ln\left(\frac{h_2 + 6\lambda}{h_1 + 6\lambda}\right) - \ln\left(\frac{h_2 + 2\lambda}{h_1 + 2\lambda}\right)}{2 \ln\left(\frac{h_2}{h_1}\right) + \ln\left(\frac{h_2 + 6\lambda}{h_1 + 6\lambda}\right) - 3 \ln\left(\frac{h_2 + 2\lambda}{h_1 + 2\lambda}\right)}. \quad (17)$$

Hence, the pressure distribution is obtained as

$$p(h) = \frac{\mu U}{4\alpha\lambda^2} \left[2\bar{h} \ln\left(\frac{h}{h_1}\right) - 3(\bar{h} + 2\lambda) \ln\left(\frac{h + 2\lambda}{h_1 + 2\lambda}\right) + (\bar{h} + 6\lambda) \ln\left(\frac{h + 6\lambda}{h_1 + 6\lambda}\right) \right]. \quad (18)$$

Integrating the pressure distribution from Equation (18) gives the load capacity of the slider bearing

$$F(h) = \frac{\mu U}{4\alpha^2\lambda^2} \left[2h_2\bar{h} \ln\left(\frac{h_1}{h_2}\right) - (3h_2\bar{h} + 6h_2\lambda + 6\bar{h}\lambda + 12\lambda^2) \ln\left(\frac{h_1 + 2\lambda}{h_2 + 2\lambda}\right) + (h_2\bar{h} + 6h_2\lambda + 6\bar{h}\lambda + 36\lambda^2) \ln\left(\frac{h_1 + 6\lambda}{h_2 + 6\lambda}\right) \right]. \quad (19)$$

The dimensionless load capacity is here the Sommerfeld number

$$So = \frac{F(h)\alpha^2}{\mu U} = \frac{1}{4\lambda^2} \left[2h_2\bar{h} \ln\left(\frac{h_1}{h_2}\right) - (3h_2\bar{h} + 6h_2\lambda + 6\bar{h}\lambda + 12\lambda^2) \ln\left(\frac{h_1 + 2\lambda}{h_2 + 2\lambda}\right) + (h_2\bar{h} + 6h_2\lambda + 6\bar{h}\lambda + 36\lambda^2) \ln\left(\frac{h_1 + 6\lambda}{h_2 + 6\lambda}\right) \right]. \quad (20)$$

3 Darmstadt Slip Length Tribometer: Function and measuring principle

This section gives the function and measuring principle of the Darmstadt Slip Length Tribometer (DSLTL). The DSLTL is an indirect measuring method for quantifying wall slip. The slip length is determined by the measurement of an integral quantity and a suitable model. The integral measured quantity is the friction torque between rotating and the stationary disk depending on the gap height. As a suitable model, the Reynolds equation is used, considering Navier's slip boundary condition.

The DSLTL is a classical plate-plate tribometer (cf. Figure 3) measuring the friction torque transmitted from the rotating disk through the liquid film of height h to the stationary disk. The torque is measured with a reaction torque sensor with a response threshold less than 0.03 mNm at the stationary disk. The distance is measured by means of capacitive distance sensors with a resolution of 4 nm. These sensors are integrated directly into the stationary disk. In order to allow a cardanic self-levelling of the two disks relative to each other, the lower one is supported by a jewel bearing. The adjustment of the lubrication gap is achieved by the axial spring stiffness and the feed pressure of the test liquid.

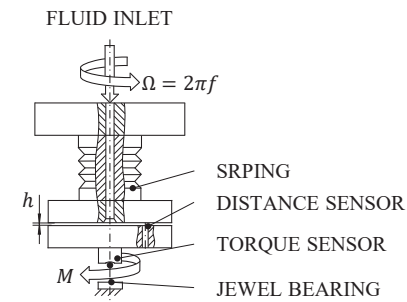


Figure 3: Principle sketch of the slip length tribometer, with a disk diameter of 64 mm.

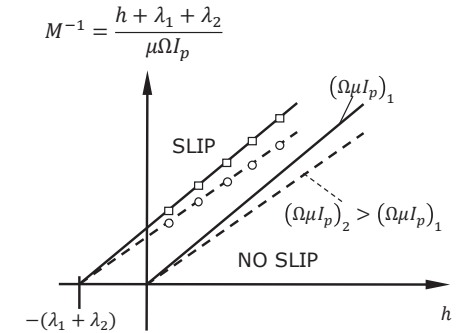


Figure 4: Inverse friction torque depending on the gap height for slip and no slip boundary condition.

In the lubrication gap, the pressure flow in radial direction and the drag flow in circumferential direction are superimposed. For small gap heights, the Reynolds number is of the order of magnitude of 0.1 and the tilt angle of the disks is smaller than 0.001° . Thus the Reynolds equation (8) for wall slip can be used, giving the Couette velocity profile $u(r, y) = \Omega r(y + h)/(h + \lambda_1 + \lambda_2)$. With the velocity profile in the circumferential direction, the friction torque is determined by integrating the shear stresses. The inverse of the friction torque

$$M^{-1} = \frac{h + \lambda_1 + \lambda_2}{\mu \Omega I_p} \quad (21)$$

is a linear equation. With the polar moment of area $I_p = \int_A r^2 dA$, the sum of the slip lengths at the stationary and the rotating disk can be obtained by determining the x-axis intercept for the curve of the equation.

Figure 4 illustrates schematically the relationship of equation (21) for Newtonian fluids. The inverse friction torque depends linearly on the gap height. As the no-slip boundary condition holds true, the linear curve of the inverse friction torque intersects the coordinate origin. As the rotational frequency of the rotating disks increases, the slope of the straight-line equation decreases. If wall slip occurs at the liquid-solid interface, the curve intersects the x-axis in the negative. This distance of the x-axis intercept to the coordinate origin is equal to the sum of the two slip lengths at the stationary and the rotating disk. As in the case of no slip, the slope of the linear curve decreases with increasing rotational frequency of the rotating disk, however the point of intersection with the abscissa remains unchanged. Thus, the slip length for Newtonian fluid is independent of the shear rate.

In the conducted experimental studies, gap height and friction torque are measured; cf. markers in Figure 4. The slip length is obtained by determining the x-axis intercept of the best-fit line, which is obtained by means of the

least square fit method. This linear extrapolation is necessary since the measuring method is an indirect measuring method. However, this is not considered a disadvantage since the slip length is not a function of the gap height, as the linearity of the inverse friction torque shows. Rather, the systematic measurement over a gap height change of 10 μm offers two advantages: (i) the plausibility of the individual measurements is examined over a wide measuring range and (ii) the shear rate varies by an order of magnitude during the measurement, thus a shear rate independence can be verified at once.

Two planar disks made of surface hardened stainless steel (steel type 1.8519) with a diameter of 64 mm form the lubrication gap. Due to the hardened surface it is possible to manufacture a surface flatness of 30 nm with a mean roughness of 10 nm to 100 nm by means of lapping. The disks utilized for the conducted studies have a mean roughness of 10 nm to allow comparison with the slip lengths of smooth surfaces published in literature. The diameter of the measuring disks constitutes the decisive advantage of the DSLT in comparison to the so far used measuring methods. As discussed in the introduction, the measurement geometries in the earlier used measuring devices are too small to quantify slip lengths for technically rough surfaces.

Technical surfaces are quantified according to DIN EN ISO 4287: 2010-07 using integral quantities, such as arithmetic mean roughness or average roughness. DIN EN ISO 4288: 1997 specifies the measuring section averaging the surface roughness. For technically rough surfaces with a mean roughness of the order of 100 nm, a single measuring section of 800 μm is required. The characteristic lengths of so far used measuring geometries for slip length measurement vary from 1 to 100 μm . These characteristic lengths are smaller than the lengths over which parameters for characterizing surface topographies are integrated. Based on this fact, it is reasonable to distinguish slip length measuring methods into local and integral measuring methods based on the effective length of the measuring geometry.

Measuring methods whose characteristic measuring geometry is smaller or of the same order of magnitude as the effective length that characterizes the surface roughness are considered as local measuring methods. Measuring methods whose characteristic measuring geometry is at least one order of magnitude greater than the characteristic length that quantifies the surface roughness are considered as integral methods. This ensures that the measurement is integrated via local effects and the slip length represents an average over the rough surface, analogous to the surface roughness itself.

4 Results

This section presents the results of the conducted slip length measurements utilizing the DSLT. Additionally, the influence of the slip length on the load bearing capacity of the slider bearing is discussed. The following section is subdivided into two subsections. First, slip length measurements at constant temperature are presented, depicting that wall slip exists in hydraulic systems. Due to the fact, that hydraulic systems are not operated at constant temperature, it is necessary for system design to know the slip length depending on the temperature. Thus, the authors verified an Arrhenius relation for the thermal characteristic of the slip length; cf. [4]. Second, with these temperature-dependent slip length, the influence of the slip length on the load capacity of a slider bearing is quantified.

4.1 Slip length measurements

Figure 5 shows the measurement of the inverse torque as a function of varying gap heights, showing the $M^{-1} - h$ - curve for an alpha-olefin 6 at constant temperature of 29.9 °C and constant rotational frequency of 2 Hz. The symbols mark the individual measurement points at which the torque was measured depending on the gap height. The different colors of the markers characterize the repeated measurements. Overall, the figure shows 20 measurement series. The best fit linear regression curve discussed in Figure 3 is determined individually for each measurement series and the slip length is determined from the intersection with the x-axis; cf. Figure 4.

Figure 6 gives a detailed view at the intersection of the regression curves with the x-axis. The measured slip length for an alpha-olefin at 29.9 °C averages to $\lambda = 540$ nm. The measurements can be repeated with a standard deviation

of $\sigma = 50$ nm. Due to the 20 repetitions, the statistical uncertainty is reduced by the factor $t/\sqrt{n}$ (student-t-distribution). The statistical uncertainty of the measured value with $n = 20$ repetitions and a confidence interval of 95% is less than 24 nm and thus less than 5% of the measured mean value. The systematic errors are dominated by the distance measurement.

The distance measurement is critical for two reasons: On the one hand, the permittivity of the test liquid is temperature-dependent and on the other hand, the sensor position changes as a function of the temperature relative to the gap surface. This means that the distance sensor has to be calibrated for each test fluid depending on the temperature. This is done by means of interferometric layer thickness measurements as an absolute reference.

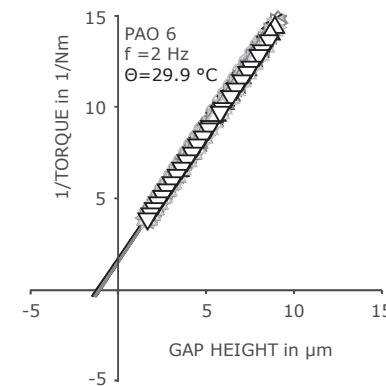


Figure 5: Slip length measurements for PAO 6 at $\theta = 29.9$ °C and constant rotational speed of 2 Hz. The figure shows 20 measurement runs. Each run is marked by a symbol. Only the last one marked by triangles is visible.

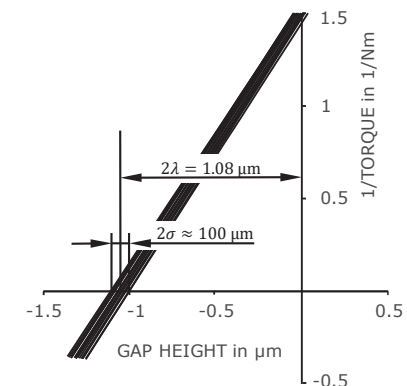


Figure 6: Detailed view of the intersections with the x-axis.

4.2 Bearing capacity of the slider bearing

Figure 7 gives the pressure distribution in a lubrication gap for the slip as well as for the no slip boundary condition applying Equation (18). The geometry of the used slider bearing is shown in Figure 2. The gap height h_1 is 20 μm , the tilt angle α of the upper plate is 0.286° and this plate is moved at constant velocity U of 0.3 m/s. The utilized fluid is an alpha-olefin with a dynamic viscosity μ of 0.039 Pa·s and a slip length λ of 540 nm at 29.9 °C. The quotient of slip length and typical flow geometry h_1 is thus 0.027. Figure 7 exhibits, that the peak pressure is reduced due to slip by approximately 25 %. Integrating the pressure along the horizontal directions yields the load capacity per unit depth, which is also reduced by approximately 25%.

Hydraulic applications operate in a wide temperature range. Thus the thermal behaviour of the load capacity is of major interest. The temperature dependent dynamic viscosity and the temperature dependent slip length are obtained by means of Arrhenius relations; cf. [4]. Using these Arrhenius relations, Figure 8 gives the temperature depending load capacity of the above mentioned slider bearing. The presented results clearly show that difference in load capacity per unit depth for slip and no slip decreases with increasing temperature. This is reasonable since the activation energy for wall slip is smaller than the activation energy for shearing.

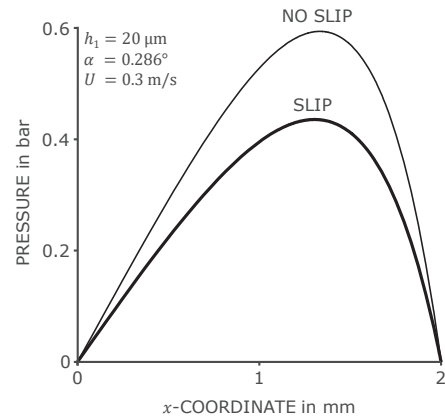


Figure 7: Pressure distribution in the lubrication gap for a polyalpha-olefin with a dynamic viscosity of 0.039 Pa·s and a slip length of 540 nm at 29.9 °C.

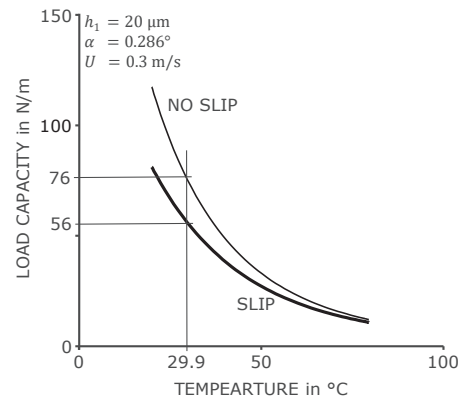


Figure 8: Load capacity per unit depth of the slider bearing depending on the temperature for a polyalpha-olefin with a dynamic viscosity of 0.039 Pa·s and a slip length of 540 nm at 29.9 °C.

5 Summary

The presented article provides for the first time the application of the slip length at hydraulic systems. As hydraulic system, a typical slider bearing is considered, quantifying the influence of wall slip by means of the pressure distribution and the load capacity.

At first, the slip boundary condition is introduced following the dynamic considerations of Navier. Based on this slip boundary condition the Reynolds equation considering wall slip is derived. Integrating the Reynolds equation gives the pressure distribution as well as the load capacity of a slider bearing.

For measuring the slip length of hydraulic applications the Darmstadt Slip Length Tribometer (DSLTT) is used. The slip length for tribological system steel/oil/steel at a temperature of 29.9 °C is measured and accounts for 540 nm. The influence of slip on the slider bearing is quantified by means of pressure distribution and load capacity, showing that the consideration of slip reduces the load capacity of the considered geometry by approximately 25%.

6 Acknowledgements

The authors thank the Research Association for Fluid Power of the German Engineering Federation VDMA for its financial support. Special gratitude is expressed to the participating companies and their representatives in the accompanying industrial committee for their advisory and technical support.

Nomenclature

Variable	Description	Unit
F	force per unit length	M T^{-2}
f	frequency	T^{-1}
h	height	L
$\bar{h}$	height at maximum pressure	L
M	torque	$\text{L}^2 \text{M T}^{-2}$
p	pressure	$\text{L}^{-1} \text{M T}^{-2}$
q	volume flow	$\text{L}^3 \text{T}^{-1}$
U	velocity of the slider bearing	L T^{-1}
u	velocity	L T^{-1}
α	angle	*
θ	temperature	Θ
λ	slip length	L
μ	dynamic viscosity	$\text{L}^{-1} \text{M T}^{-1}$
Ω	angular velocity	T^{-1}

References

- [1/] Trutnovsky, K., *Die Dichtung bewegter Maschinenteile*, in *Berührungsdichtungen: An ruhenden und bewegten Maschinenteilen*. 1975, Springer Berlin Heidelberg: Berlin, Heidelberg. p. 189-271.
- [2/] Müller, H.K., *Abdichtung bewegter Maschinenteile. Funktion-Gestaltung-Berechnung-Anwendung*. 1990: Medienverlag U. Müller.
- [3/] Hopf, L., et al., *Abhandlungen über die hydrodynamische Theorie der Schmiermittelreibung*. 1927: Akademische Verlagsgesellschaft.
- [4/] Pelz, P.F. and T. Corneli, *The Activation Energy for Wall Slip*. Manuscript submitted for publication, 2017.
- [5/] Navier, M., *Sur les lois du Mouvement des Fluides*. Mémoires de l'Académie royale des Sciences de l'Institut de France, 1822. 27(5): p. 414-419.
- [6/] Stokes, G.G., *On the Theories of the Internal Friction of Fluids in Motion, and of the Equilibrium and Motion of Elastic Solids*, in *Mathematical and Physical Papers*, G.G. Stokes, Editor. 2009, Cambridge University Press: Cambridge. p. 75-129.
- [7/] Poiseuille, J.L., *Comptes rendus 11*. Mémoires des Savants Etrangers 9, 1840.
- [8/] Vinogradova, O.I. and G.E. Yakubov, *Dynamic Effects on Force Measurements. 2. Lubrication and the Atomic Force Microscope*. Langmuir, 2003. 19(4): p. 1227-1234.
- [9/] Craig, V.S.J., C. Neto, and D.R.M. Williams, *Shear-dependent boundary slip in an aqueous Newtonian liquid*. Phys. Rev. Lett., 2001. 87(5): p. 054504.

- /10/ Bonaccurso, E., M. Kappl, and H.-J. Butt, *Hydrodynamic force measurements: boundary slip of water on hydrophilic surfaces and electrokinetic effects*. Phys. Rev. Lett., 2002. **88**(7): p. 076103.
- /11/ Bonaccurso, E., H.-J. Butt, and V.S.J. Craig, *Surface roughness and hydrodynamic boundary slip of a Newtonian fluid in a completely wetting system*. Phys. Rev. Lett., 2003. **90**(14): p. 144501.
- /12/ Cheng, J.T. and N. Giordano, *Fluid flow through nanometer-scale channels*. Physical Review E, 2002. **65**(3): p. 031206.
- /13/ Choi, C.-H. and C.-J. Kim, *Large Slip of Aqueous Liquid Flow over a Nanoengineered Superhydrophobic Surface*. Phys. Rev. Lett., 2006. **96**(6): p. 066001.
- /14/ Churaev, N.V., V.D. Sobolev, and A.N. Somov, *Slippage of liquids over lyophobic solid surfaces*. Journal of Colloid and Interface Science, 1984. **97**(2): p. 574-581.
- /15/ Zhu, Y. and S. Granick, *Rate-dependent slip of Newtonian liquid at smooth surfaces*. Phys. Rev. Lett., 2001. **87**(9): p. 096105.
- /16/ Zhu, Y. and S. Granick, *Limits of the Hydrodynamic No-Slip Boundary Condition*. Phys. Rev. Lett., 2002. **88**(10): p. 106102.
- /17/ Zhu, Y. and S. Granick, *Apparent slip of Newtonian fluids past adsorbed polymer layers*. Macromolecules, 2002. **35**(12): p. 4658-4663.
- /18/ Cottin-Bizonne, C., et al., *Nanorheology: An investigation of the boundary condition at hydrophobic and hydrophilic interfaces*. The European Physical Journal E, 2002. **9**(1): p. 47-53.
- /19/ Cottin-Bizonne, C., et al., *Boundary Slip on Smooth Hydrophobic Surfaces: Intrinsic Effects and Possible Artifacts*. Phys. Rev. Lett., 2005. **94**: p. 056102.
- /20/ Meinhart, C.D., S.T. Wereley, and J.G. Santiago, *PIV measurements of a microchannel flow*. Experiments in Fluids, 1999. **27**(5): p. 414-419.
- /21/ Tretheway, D.C. and C.D. Meinhart, *Apparent fluid slip at hydrophobic microchannel walls*. Physics of Fluids, 2002. **14**(3): p. L9-L12.
- /22/ Zhu, Y. and S. Granick, *No-slip boundary condition switches to partial slip when fluid contains surfactant*. Langmuir, 2002. **18**(26): p. 10058-10063.
- /23/ Huang, P., J.S. Guasto, and K.S. Breuer, *Direct measurement of slip velocities using three-dimensional total internal reflection velocimetry*. Journal of Fluid Mechanics, 2006. **566**: p. 447-464.
- /24/ Corneli, T., G. Ludwig, and P.F. Pelz. *Slip length of the tribo system steel-polyalphaolefin-steel determined by a novel tribometer*. in *International Fluid Power Conference*. 2016. Dresden.
- /25/ Goldstein, S. and A.R. Council, *Modern developments in fluid dynamics: an account of theory and experiment relating to boundary layers, turbulent motion and wakes*. 1938: The Clarendon Press.
- /26/ Helmholtz, H. and G. von Piotrowski, *Über Reibung tropfbarer Flüssigkeiten*. Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe, 1860. **40**: p. 607-661.
- /27/ Spurk, J.H. and N. Aksel, *Strömungslehre.: Eine Einführung in die Theorie der Strömungen*. 2006: Springer.