

Design of a Brake Test Bench and the Experimental and Numerical Investigation of Friction Pairings

Konstruktion eines Bremsenprüfstandes und die experimentelle und numerische Untersuchung von Reibpaarungen

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PREFACE

The work described in this doctoral dissertation was carried out as a teaching and scientific assistant between the years 2012 and 2017 at the Institute of General Mechanics of the RWTH Aachen University. In this period of my occupational career I gained profound knowledge in the design and manufacturing process, the experimental measurements and the finite element simulations. However, during the years and the numerous projects many people have crossed my path and were contributed in many ways to the realization of this work. I would like to sincerely thank all of them.

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Zusammenfassung - Summary

Die Ziele der vorliegenden Arbeit sind, die Fortschritte der numerischen Simulation von Reibpaarungen mit dem aktuellen Stand der Technik zu kombinieren. Hierzu wird das stark gekoppelte, multiphysikalische Problem entkoppelt und anhand von Python-Skripten automatisiert. Die Formulierung des eulerschen/lagrangeschen Simulationsansatzes wird in diesem Modell implementiert. Die numerische Simulation von Reibpaarungen wie z.B. die Reibpaarung Bremsscheibe-Bremsbelag wird anhand von CAD-Modellen oder selbstgezeichneten Modellen durchgeführt. Ausgehend von den Anfangsbedingungen und notwendigen Annahmen werden alle wesentlichen physikalischen Eigenschaften in der Modellierung berücksichtigt, die aus der Struktur, den sich bewegenden und interagierenden Reibpaarungen, des thermischen und mechanischen Materialverhaltens resultieren. Das entwickelte Simulationsmodell ist somit in der Lage, alle relevanten Nichtlinearitäten des stark gekoppelten, multiphysikalischen Problems abzubilden. Der Rechenaufwand oder die Simulationsdauer wird durch die Implementierung des eulerschen/lagrangeschen Simulationsansatzes enorm gesteigert. Um die praktische Anwendbarkeit des Modells unter Beweis zu stellen, werden mehrere Prüfstände entwickelt. Der Schwungmassenprüfstand ermöglicht die Untersuchung von Reibpaarung durch Einbringung von hohen Energien unter nahezu realen Gesichtspunkten. Der Tribometer ermöglicht die Validierung von Simulationsmodellen aufgrund von genau definierten Parametern, hinsichtlich der Geometrie der Reibpaarungen und den Belastungen. Eine erweiterte und ergänzende Messtechnik ermöglicht, eine eindeutige Zuordnung des Reibverhaltens von Reibpaarung und ihren Ursachen zu erlangen.

The goal of this contribution is to present the modular designed test bench with the advanced measurement system for friction systems. The design and construction process and the installation of the measurement system is the main part of this work. Another part of this work is the design of the tribometer for the validation of numerical models with well defined variables such as the normal force, the rotational speed and the friction pairs. The design of the combined brake, which consists of a friction and an eddy-current brake closed the gap to the electro mobility. The efficient numerical treatment of the decoupled thermomechanical problem is carried out by the finite element method, where the overall solution procedure and the algorithmic implementation of the steady-state algorithm is described. The practical applicability of the steady-state algorithm is adapted to the real experiments of brake actions. Finally, the particulate matter and the wear behaviour of a friction system is measured with a Fast Mobility Particle Sizer. Due to the designed test benches and the numerical implementation, the gap of numerical models and the validation with the experiments is closed.

Motivation

Die Zuverlässigkeit und Funktionsfähigkeit der Bremsanlage bei Flugzeugen, Schienen-, Last- und Kraftfahrzeugen hat eine sehr hohe Wichtigkeit für die Passagiere und das Vertrauen in dieser Art der Fortbewegung. Durch die Effizienzsteigerung der letzten Jahre kommen Strukturteile an ihre Belastungsgrenzen, insbesondere tribologische Strukturteile (z.B. Bremsscheiben, Bremsbeläge), die eine thermomechanische Beanspruchung erfahren. Die Bremsanlage

ist ein sehr wichtiges und in den letzten Jahren viel untersuchtes System. Bremsen befinden sich in allen kinematischen und dynamischen Systemen so in Kraftfahrzeugen, Schienenfahrzeugen, Flugzeugen und Industrieanlagen. Das wichtigste Dimensionierungsmerkmal einer Bremsscheibe ist die Fähigkeit der Dissipation kinetischer Energie. Hierbei ist z.B. die maximale Geschwindigkeit eines Hochgeschwindigkeitszuges von 574 km/h bei einer Masse von 500 Tonnen und einer Geschwindigkeit von 300 km/h eines Automobils bei 1,5 Tonnen zu nennen. Dennoch sind die Betriebsbedingungen an die Belastungsgrenzen des Bremssystems gekoppelt. Hierbei wird die Funktionszuverlässigkeit der Bremsscheibe durch eine Reihe von Faktoren gemindert, da wären Risse, Verschleiß und metallurgische Transformationen zu erwähnen. Die Problematik der Rissentstehung von Bremsscheiben spielt industriell wie auch wissenschaftlich eine sehr wichtige Rolle. Industrielle Gesichtspunkte wären die Sicherheit der Umgebung und Passagiere, die Verkürzung der Instandhaltungszeiten und Kosten. Unter den wissenschaftlichen Gesichtspunkten wäre die Analyse des Systems Bremsscheibe-Bremsbelag hinsichtlich tribologischer, multiphysikalischer und multiskalen Aspekte zu nennen. Desweiteren kommt die Berücksichtigung der sich verändernden Kontaktverhältnisse hinzu. Die Ursache dieser Aspekte ist ein tribologisches und thermomechanisches Phänomen. Die Kenntnis der durch die Reibung in der Kontaktstelle zwischen Bremsscheibe und Bremsbelag induzierte Wärme und dementsprechend die Temperatur ist essentiell für die Dimensionierung des Bremssystems. Die Temperaturen, die in der Kontaktstelle entstehen, verursachen thermische Dehnungen und können zu lokalisierten Temperaturgradienten führen. Ein Problem, welches die Leistungsfähigkeit von Bremssystemen einschränkt. Das Phänomen der lokalisierten Temperaturgradienten kann zum frühzeitigen Ausfall der Bremsscheibe durch thermische Wechselbeanspruchung (Erwärmung und Abkühlung), welche Zug-, Druckspannungen induzieren und somit zur thermischen Ermüdung führen. Frühzeitiger Verschleiß und Effizienzverlust des Bremsbelags und Bremsquietschen ist das Resultat. Die Klassifizierung der lokalisierten Temperaturgradienten für Kraft- und Schienenfahrzeuge wurden von Andersson & Knapp [5] und Dufrenoy & Weichert [41] durchgeführt.

Die Simulation der gesamten Reibpaarung benötigt einen sehr hohen Zeit- und Vernetzungsaufwand. Des Weiteren werden Prüfstände mit erweiterter Messtechnik benötigt für die Validierung des Simulationsmodells und der eindeutigen Zuordnung von Ursache und Wirkung der Reibpaarungen. Aus diesem Grund wird in dieser Arbeit der eulersche/lagrangesche Simulationsansatz verwendet und ermöglicht einen enormen verringerten Zeit- und Vernetzungsaufwand. Die Validierung des Simulationsmodells erfolgt mit Hilfe von selbstgebauten Prüfständen mit erweiterter Messtechnik, die idealisierte und realitätsnahe Betriebsbedingungen ermöglichen.

Zielsetzung und Vorgehensweise

In der vorliegenden Arbeit wird ein Simulationsmodell und die Konstruktion von Bremsenprüfstände betrachtet. Das Simulationsmodell basiert auf den Steady-State Algorithmus, welche das drehen der Bremsscheibe durch einen Materialfluss simuliert. Das thermomechanische Problem wird in dem Simulationsmodell entkoppelt, um Konvergenzprobleme hinsichtlich der Kontaktformulierung zu vermeiden. Der Algorithmus ermöglicht ein schnelleres Berechnen der Temperaturen und Spannungen. Simulationsmodelle, welche die Temperaturen der Bremsscheibe berechnen, werden in [140], [139], [136], [123], [110], [102], [40], [3] und [135] beschrieben. Sowohl FE Ansätze als auch analytische Ansätze zur Beschreibung des thermomechanischen Verhaltens von Reibpaarungen werden in [137], [105], [107], [106], [99], [98], [97], [96] erläutert. Der Inhalt

dieser Arbeit konzentriert sich hierbei auf die Konstruktion eines Schwungmassenprüfstandes, eines Tribometers und einer kombinierten Bremse. Kommerzielle Schwungmassenprüfstände beinhalten eine fixe Maschineneinheit, welche keine Möglichkeit bieten eines modularen Aufbaus. Die Preise für modular aufgebaute Bremsenprüfstände sind sehr teuer und beinhalten einen intensiven Austausch mit dem Hersteller. Der konstruierte Schwungmassenprüfstand ist einerseits modular aufgebaut und mit einer für die Grundlagenforschung und Industrie entsprechenden Messtechnik ausgestattet. Dies ermöglicht je nach Fragestellung einen dementsprechenden Aufbau hinsichtlich des Schwungmassenprüfstandes und der Messtechnik. Die Leistung und die Größe des Schwungmassenprüfstandes ist begrenzt durch die örtlichen Begebenheiten und den modularen Aufbau. Die Einhaltung der ISO und DIN Normen wurden in der Konstruktionsphase zur Grundlage gelegt. Die Installation und Synchronisation der Messtechnik ist das Hauptaugenmerk des Schwungmassenprüfstandes. Ein weiterer Prüfstand für die Validation von Simulationsmodellen ist das Pin-on-disc Tribometer. Die einstellbaren Parameter für die Versuchsreihen sind die Normalkraft und die Rotationsgeschwindigkeit der Scheibe. Durch diesen einfachen, robusten Aufbau des Tribometers ist es möglich komplizierte Simulationsmodelle zu validieren. Die kombinierte Bremse ist ein Bremssystem, welches die Forderungen der Elektromobilität erfüllen kann. Sie besteht aus einer Wirbelstrombremse und einer klein dimensionierten konventionellen Reibbremse. Erste Ansätze zur Einführung im PKW-Bereich wird in der Arbeit von Gay [52] gezeigt. Das Konzept der Wirbelstrombremse ist nicht neu, aber durch die Kombination mit der Reibbremse ein interessantes Lösungskonzept für die Elektromobilität. Der Abschluss dieser Arbeit wird durch die Untersuchung der Eigenschaften des Verschleißes gebildet. Die Partikelkonzentration und Partikelanzahl

Gliederung der Arbeit

In *Kapitel 1* wird die Arbeit eingeleitet und die Reibpaarung Bremsscheibe-Bremsbelag vorgestellt. Des Weiteren wird ein umfangreicher Überblick über den Stand der Forschung hinsichtlich der Simulationsmodelle dargestellt und erläutert.

Die kontinuumsmechanischen Grundlagen der euler/lagrangeschen Simulationsansatzes und der verwendeten Materialgesetze inklusive der Kinematik, der Bilanzgleichungen und des Entropieprinzips werden in *Kapitel 2* dargestellt. Eine verständliche Einführung in der infinitesimalen inelastischen Deformation von Festkörpern wird gegeben.

Das eulersche/langrangesche Simulationsmodell wird in *Kapitel 3* eingeführt. Der Algorithmus zur Berechnung der Temperaturen und Spannungen wird näher erläutert und relevante Konstitutivgleichungen auf Basis eines allgemeinen Festkörpers vorgestellt.

In *Kapitel 4* werden die konstruierten Prüfstände dargestellt. Die Möglichkeiten zu simulierenden Massen, Geschwindigkeiten, Energien in Abhängigkeit der Anzahl der Schwungmassen werden für den Schwungmassenprüfstand erläutert. Das Funktionsprinzip des Tribometers wird ebenfalls dargestellt. Des Weiteren wird die erweiterte Messtechnik zur Bestimmung der Ursachen ausführlich beschrieben.

Die Validierung der Simulationsergebnisse erfolgt anhand der konstruierten Prüfstände in *Kapitel 5*.

In *Kapitel 6* wird das Verschleißverhalten und die Partikelentstehung beschrieben. Die Auswirkung der Verschleißpartikel auf den menschlichen Organismus wird ebenfalls erfasst.

Eine abschließende Zusammenfassung und Diskussion wird im *letzten Kapitel* gegeben, wobei mögliche Weiterentwicklungen und Anwendungen in anderen Ingenieurdisziplinen, wie z. B. der Biomechanik, gezielt angesprochen werden.

Nomenclature

The nomenclature throughout this thesis follows the conventions and symbols that are commonly used in modern tensor calculus, such as in the books and lectures by de Boer [34] and Markert [90], [92], [89], [88]. Specific formulae and rules of tensor algebra and analysis are described in [34], [90], [92], [89], [88].

Conventions

Index and suffix conventions

$i, j, k, l, \dots$	indices as sub- or superscripts range from 1 to N ; usually $N = 3$ in the 3-d physical space of our experience
$(\cdot)_k(\cdot)^k = \sum_k (\cdot)_k(\cdot)^k$	Einstein's summation convention, i.e., summation over indices that appear twice unless stated otherwise
$(\cdot)_e, (\cdot)_i$	indicate the elastic and inelastic parts associated with an inelastic solid deformation

Kernel conventions

$(\cdot)$	place holder for kernel quantity
$s, t, \dots$ or $\sigma, \tau, \dots$	scalars (0th-order tensors)
$\mathbf{s}, \mathbf{t}, \dots$ or $\boldsymbol{\sigma}, \boldsymbol{\tau}, \dots$	vectors (1st-order tensors)
$\mathbf{S}, \mathbf{T}, \dots$ or $\boldsymbol{\sigma}, \boldsymbol{\tau}, \dots$	2nd-order tensors
$\overset{n}{\mathbf{S}}, \overset{n}{\mathbf{T}}, \dots$ or $\overset{n}{\boldsymbol{\sigma}}, \overset{n}{\boldsymbol{\tau}}, \dots$	n th- or higher-order tensors
$\mathbf{s}, \mathbf{t}, \dots$ or $\mathbf{S}, \mathbf{T}, \dots$	general column vectors ($n \times 1$ and general matrices ($n \times m$))

Selected tensor operations

$(\cdot)^T$	transpose of vectors, tensors or matrices
$\overset{n}{\mathbf{S}} \overset{s}{\mathbf{T}} = \overset{n-s}{\mathbf{R}}$ with $n \geq s$	contracting (inner) tensor product (linear mapping)
$\overset{n}{\mathbf{S}} \cdot \overset{n}{\mathbf{T}} = r$	scalar (dot) product (special contracting tensor product)
$\overset{n}{\mathbf{S}} \otimes \overset{s}{\mathbf{T}} = \overset{n+s}{\mathbf{R}}$	dyadic (vector space) tensor product

Symbols

Symbol	Unit	Description
α	[1/°C]	expansion coefficient
δ	[°]	azumith angle
ϵ_{klm}	[-]	<i>Levi-Cevita</i> permutation symbol
η	[Ns/m ²]	solid shear viscosity constant
η	[J/K kg]	mass-specific entropy
η	[J/K m ³ s]	volume-specific direct entropy production
λ	[W/(mK)]	heat conduction coefficient
μ	[-]	coefficient of friction
ν	[-]	<i>Poisson's</i> ratio
ρ	[kg/m ³]	density
σ	[N/m ²]	stress tensor
τ	[N/m ²]	stress tensor
ξ	[J/m ² Ks ^{$\frac{1}{2}$}]	thermal effusivity
ε	[-]	strain tensor
χ_i	[J/(s ^{1/2} m ² K)]	emissivity of the i-th body
ω_i	[1/s]	angular velocity of the i-th body
Θ	[°]	angle
Θ	[kgm ²]	mass inertia tensor
Ω	[sr]	solid angle
c_p	[J/kgK]	heat capacity
d	[m]	diameter
$\mathbf{e}_k$	[-]	hardness
l	[m]	length
r	[m]	radius
r	[J/kg s]	volume-specific direct entropy production
p	[-]	heat partition
p	[N/m]	line load
p	[N/m ²]	pressure

t	[s]	time
$\mathbf{u}$	[m]	displacement vector
v	[m/s]	velocity
v_r	[m/s]	relative velocity
w	[m]	deflexion
$\mathbf{x}$	[m]	actual position vector
$\dot{\mathbf{x}}$	[m/s]	velocity vector
$\ddot{\mathbf{x}}$	[m ² /s]	acceleration vector
A_0	[m ²]	reference surface
$\mathcal{B}$		body
D	[Nm ³]	bending stiffness
E	[N/m ²]	elasticity constant
F_N	[N]	normal force
$\mathbf{F}$	[-]	deformation tensor
H	[N/m ²]	hardness
$H\epsilon$	[J/kg]	mass-specific internal energy
$\mathbf{I}$	[-]	unit tensor
J	[-]	<i>Jacobian</i> determinant
M_T	[Nm]	thermal torque
N	[-]	number of micro contacts
N_r	[N]	force in radial direction
N_Θ	[N]	force in circumference direction
R	[J/(K mol)]	universal gas constant
R_e	[N/m ²]	yield strength
Re	[-]	<i>Reynolds</i> number
$\mathcal{S}$		surface of $\mathcal{B}$
S_d	[m ²]	friction surface of the disc
S_p	[m ²]	friction surface of the pad
T	[K]	temperature
$\mathbf{X}$	[m]	reference position vector

List of Abbreviations

3D	Three-dimensional
ASM	Asynchronous Motor
CARB	California Air Resources Board
CCD	Charge-Coupled Device
CMOS	Complementary Metal Oxide Semiconductor
DAQ	Data Acquisition
DoF	Degree of Freedom
DSP	Digital Signal Processor
EMC	Electromagnetic compatibility
EMR	End of measuring range
FE	Finite Element
FMPS	Fast Mobility Particle Sizer
FOV	Field of View
FPA	Focal Plane Array
FSO	Full Scale Output
HFOV	Horizontal Field of View
IFAS	Institute for Fluid Power Drives and Controls
IFOV	Instantaneous Field of View
IR	Infrared
MCMM	Multi Control Multifunctional Modul
MDTD	Minimum Detectable Temperature Difference
MMR	Midrange
MRTD	Minimum Resolvable Temperature Difference
MTTF	Mean Time To Failure
NAO	Non Asbestos Organic
NETD	Noise Equivalent Temperature Difference
PDE	partial differential equation

PM	Particulate Matter
PWD	Progressive Waviness Distortion
QWIP	Quantum Well Infrared Photodetector
RMS	root mean square
ROIC	Readout Integrated Circuit
RTSC	Real Time Surface Compensation
SMR	Start of measuring range
SRF	Slit Response Function
TRC	Transitoire dans le Repère du Chargement
TTL	Transistor-Transistor Logic
VFOV	Vertical Field of View

1 Introduction

The research field of brake systems is a highly interesting topic due to the consideration of diverse disciplines. The complexity of the problem and the overlapping of different research fields, such as tribology, thermomechanics, dynamics and vibro-acoustics, are necessary to understand or gain a proper insight of the brake phenomena. Another important field is the installation of the measurement technology and the design of the brake test benches. The real temperature distribution in the disc brake is shown with a high speed IR camera. The correct handling and the understanding of the physics of IR images are necessary. In summary, it can be said, that this research field is a highly interesting due to the consideration of diverse disciplines and complicated topic, where the researcher is able to obtain knowledge in diverse disciplines.

1.1 Motivation

It is quite obvious that the main part, beside the power train, in a transport vehicle is the braking. There are several grave accidents in the world with many dead. 1972, the brakes of a train fail in a strong gradient, which carries pilgrim in Mexico. More than 230 people died and 1300 were injured [103]. 1994, due to defective brakes a train derailed and crashed into a gorge in Angola. 300 people died [121]. 1999, a driver kills three children due to defective brakes [38]. 2015, a bus accident causes a accident due to failure of the brakes. One person died and 25 were injured [37]. Grigoratos and Martini [55] shows in different investigations that in urban environments, the total non-exhaust traffic related PM10 emissions contains of 55% by mass of brake wear. Furthermore, the brake wear according to the total traffic related PM10 emissions lies up to 21% by mass. This can be measured in high frequented cities, while in highways the braking is lower. Beside of the discussion of the exhaust emissions, the attention should be placed at the brake, clutch and tyre wear. The acceleration can be achieved with several methods but the problem remains how to decelerate planes, trains, trucks and cars. There are several brake mechanisms such as the eddy-current brakes [6], [51], [80], [94], compressed air brakes, electromagnetic brakes and disc brakes [1], [20], [12], [64], [78], [110]. The reliability and security of the brake system is essential. The security of the passenger is an important aspect for the designer, which defines the success of the product. One of the most used brake systems is the brake disc. The in-situ tests are one possibility to test the brake system. The fundamental understanding is necessary for high performance brakes. An appropriate test bench with an advanced measuring system and an effective simulation model can accelerate the design and production process. The validation of the simulation models with the test benches is an appropriate process of understanding the contact behaviour and the distribution of the temperature of the brake disc and pad [30], [35], [40], [42], [69], [70], [87], [109], [111], [110]. Another point of view is the consideration of the contact behaviour on the nanoscale. There are several commercial test benches with the constraint of a non-modular design and low measurement technology. The requirements of the fundamental research and the test conditions necessitate a modular test bench. Furthermore, the measurement technology has to be adapted to the test bench and meet the requirements of the mathematical models. The combination of

the modular test bench and the measurement technique enable a unique test bench for both the fundamental research and the industrial application. The FE analysis of the brake disc and pad with the coupled approach is time-consuming. Therefore, the numerical algorithm of the thermomechanical model of brake disc and pad is calculated with the steady-state algorithm. The algorithm uncouples the coupled thermomechanical problem to achieve good calculation times. The thesis is composed of a design of a modular flywheel test bench, which can be used for different friction systems, such as clutches. The advanced measurement system is required to measure the required variables for the validation of the numerical model. The modular flywheel test bench and the measurement system is designed to meet the demands of research. The commercial test benches are not modular designed and the measurement system did not fulfil the requirements of research. An uncoupled thermomechanical algorithm is used to improve the FE calculation time of a true to scale brake disc and pad. A tribometer is designed to validate the numerical model. The advantage of the tribometer is the little changeable variables, which it is appropriate for the validation of the numerical model. The measurements of the particulate matter shows the size and concentration of the particles of a commercial brake pad and a grey cast iron disc.

1.2 State of the Art

Dry running brakes are very reliable due to the lawful response to external stresses and are used in security devices. The impetus of trains enables the movement of high masses with specified speeds. Therefore, the first security measurement is the brake system which warrants the security of the passengers, goods and the environment. There are several types of brake systems namely air brakes, vacuum brakes, electro-pneumatic brakes and disc brakes. The friction based brake systems remains the most reliable brake systems among emergency brakes. The braking distance of a TGV at a speed of 320 km/h is almost 3000 m. Here, we can notice the reliability of the brake system in cases of emergency. The kinetic energy will be dissipated proportionally into the brake disc and the brake pad. This induced dissipation of energy and the high brake period provoke a high change in temperature. These high changing temperatures could conduct macroscopic cracks which can lead to failure of brake discs and accidents [30], [35], [43], [77], [84], [134]. In the work of Wicker [131] four different brake pads in actual operation are investigated. The four pads are different in terms of physical characteristics, mechanical properties and geometries (pins location and fixation on the support). In all cases, a crack network appears very quickly on the friction surface of the brake disc. During the operation two of the four brake pads cause long cracks on brake discs. This can be explained by the thermal fatigue in association with friction. It can be easily seen that the investigation is a multi-physic and multi-scale problem. Moreover, the interaction between the brake disc and the brake pad, is in regards to the metallurgic transformation, the hair cracks, the oxidations, and the not sufficiently understood wear. Furthermore, the prediction of the thermal load is difficult due to its dependency on time and space. A crack prediction under these conditions is very difficult. The author tries to answer the influences of the brake pads on the brake disc, the definition of the load criteria and the determination of the parameters that abet cracks in regard to the load criteria. Moreover, it is considered the general problem of the brake in regard to the aspects of temperature and thermomechanics in the microstructure and macrostructure (multi-scale). The observed crack network can be distinguished in radially-oriented or in closed cells. The crack networks obtained are mainly radial with some circumferential ramifications.

Macroscopic cracks are mainly radial and several centimetres long. The retired brake discs of the trains are judged according to the distance between the free end of the brake disc and the ends of the cracks. Four types of cracks can be distinguished. Lineally radial cracks do not reach to the outer or inner side of the brake disc (Type I). Radial cracks do not reach to the outer or inner side of the brake disc, the appearance is wild and crooked with other cracks (Type II). Radial cracks, which reach the outer side of the brake disc (Type III), and long radial cracks, which start at the outer or inner side of the brake disc and show a progressive crack growth due to fatigue and brittle failure (Type IV). The most observed crack types are I and II, on the contrary to crack types III and IV, which rarely occur. The maximum depth of a macroscopic crack is nearly 15 mm (brake disc thickness 45 mm). The reason of the crack growth can be explained with the level of material cleanliness. The brake discs of the test bench show a crack network with mainly radial cracks and ramifications in circumference direction. The origin of this kind of crack network is the thermal fatigue. This crack network can be observed in several applications, namely friction brakes [102], [12], [84], press molds [112], forming tools [26], [128], [43], [85], rolling shafts and exhaust manifolds. In addition, thermal fatigue and successive braking result in high alternating temperatures. The prediction of cracks is realised with a test program that simulates the real brake action. This program consists of three brake actions. They enable the reproduction of cracks on the full scale test bench, which were observed on the retired brake discs. The test bench consists of a power unit and flywheels which simulate the inertia. The brake discs are made of low alloy steel and the brake pads are sintered with ceramic particles embedded in a metal matrix from different suppliers. The nine pins of the brake pad are on a pad holder. The brake actions are carried out with brake pads at each side of the brake disc. An example of brake pads, which are used in the industry, is shown in figure 1.1.



Figure 1.1: (a) One half of a sintered brake pad of a train, (b) One half of an organic brake pad of a train

The position and diameter of the pins of the four brake pads are different in terms of the bending stiffness of the pad holder. There are several works which investigate the influence of the brake pads on the brake discs due to the damage and cracks [39], [56], [35]. The damage is divided into two stages, the first stage describes the fast appearance of hair cracks and the second stage describes the occurrence of macroscopic cracks. A further distinguishing feature is the wear behaviour. Certain brake pads cause a macroscopic crack on the brake discs. The

two peaks of the profile of the wear can be explained with the dissipated heat flux along the radial direction. Figure 1.2 shows the length of the friction contact as a function of the brake disc radius. The right figure shows an example of the contact length in regard of the radius of different brake pads. Through these non-uniform friction paths the outcome shows non-uniform wear and heat flux. Taking into consideration the wear of the brake pads, they also play an important role in the costs and maintenance intervals.

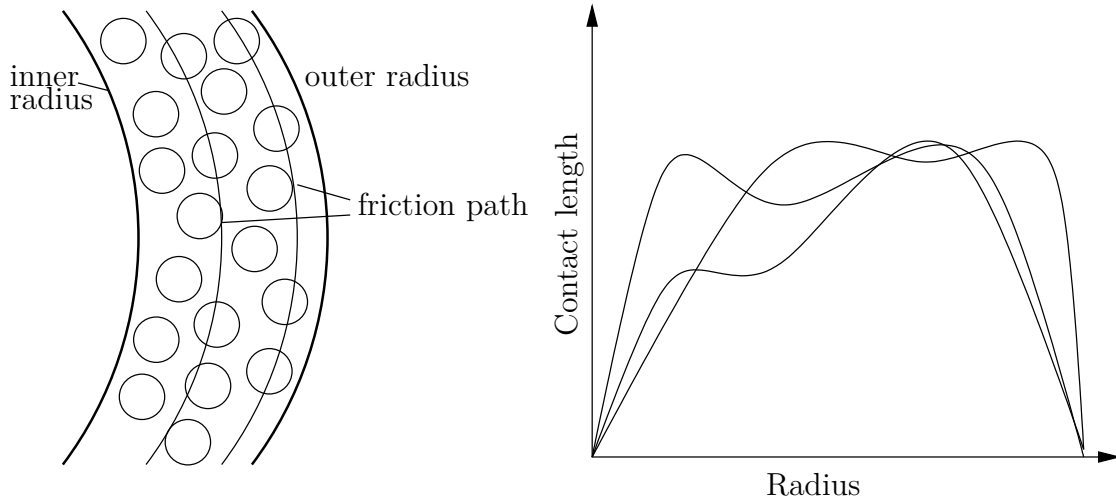


Figure 1.2: Friction path of a sintered brake pad

[56] describes the appearance of cracks and hair cracks. The observed cracks/hair cracks are developed after a simulated 12 month use of the brake disc on a test bench, this means nearly 12000 brakes. In these experiments, it can be seen that different crack networks on the brake disc depend on the used brake pad. In figure 1.3 radial cracks can be seen. It shows the crack network of a tested brake disc in cooperation with the Institute for Fluid Power Drives and Controls (IFAS). It is shown that the brake pads have a great influence on the appearance of crack networks. The appearance depends hence on the thermomechanical load and it can be deduced that the brake pads causes different thermal mechanical loads on the brake disc. Acknowledgement to "IFAS" and the scientific assistant Dipl.-Ing. Matthias Petry for the cooperation.

The brake disc is a low steel alloy with the name 28CrMoV5-08. The brake disc is produced by the forging process and has a composition of 0.28 % carbon, 1.25 % chrome, 0.8 % molybdenum, and 0.2-0.4 % vanadium. The microstructure of the brake disc is a bainitic structure with high mechanical properties such as a yield stress $R_e = 970$ MPa at 20 °C and 600 MPa at 600 °C. The composition of the microstructure remains stable till up to 600 °C [117]. Three zones can be distinguished: a martensitic zone in the near of the surface with a fine structure, an intermediate zone with larger grains and a zone without any transformations. The first zone can be explained by the cooling and heating of the surface while the brake is in action. During the first brake, the fast cooling of the surface changes the austenite structure to the martensitic structure on the edges of the grains. Hence, as a result, the size of the grains becomes smaller. The measurement shows that the maximum hardness lies at the surface and decreases with a depth of 4 mm under the surface, where it has the same hardness as an unloaded brake disc [56]. Wicker [131] observed brake discs with macroscopic cracks and hair cracks with regard to the length and depth. The macroscopic cracks reached a length of 75 mm and a maximum



Figure 1.3: Haircracks on the friction surface of a brake disc of a high speed train (Acknowledgement to the IFAS for the good cooperation)

depth of 15 mm. The origin of these cracks is the thermal fatigue, which occurs with each brake action. The thermal load has a higher influence on the brake disc than the mechanical load (pressure) applied by the brake pads. It was possible to remake the cracks on the brake discs in a full scale test bench, which were observed in the real use of the high speed trains. Two of the four used brake pads cause cracks after a simulated use of one year. Through these experiments it can be said that the brake pads have an influence on the crack behaviour of the brake disc. According to the work of [56], the crack behaviour of a brake pad depends on the wear.

Tribology

In the tribology, we consider bodies which are in contact. In the contact area, there are several phenomena that are interesting, such as the hydrodynamic behaviour, the friction and the wear. The model of the third-body is developed by Godet [53], [54] and further developed by Berthier [22]. The third-body model is shown in figure 1.4. The first bodies are the brake disc and the brake pad and the contact area is the so-called third body. The theory of contact depends on the tribology, the third-body model, the adaptation of the velocity and the tribological loop.

The third body is a thin coat that is few micrometers thick. In this thin coat, the velocity is accommodated. Moreover, the coat is heterogenously made of the two other bodies and a transmitter of the contact forces. Generally, there are two static coats at the surface of the first two bodies as well as in the third body where the particles move and arrange. The phenomena of two plane coats and a coat of powder (third body) were observed by several researchers [116], [104], [47], [46]. The two plane coats of the brake disc and the brake pad (first bodies) ensure the adaptation of the velocity. It was formed from the deposits of the brake pad (fibres, ceramic particles), which build a wear barrier on the brake disc and are compressed under the pressure. The third bodies (powder coats) stimulate the formation of plane coats, which increases when the plane coats fragment and the debris follow the path of the third body. [4] reveals that a powder coat and a trace of the slide direction have developed. The tribological loop and the different debris of the third body is introduced by several authors ([22] and [36]). Figure 1.5

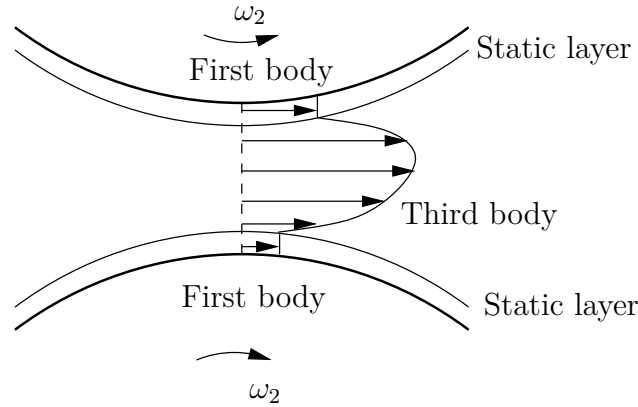


Figure 1.4: Third-body model

shows the tribological loop of the brake pad and the compounding of the third body [36]. The source debris consists of the particles of the two first bodies (1). The particles move in the contact area and develop the first debris (2). The particles either constitute a powder coat or they accumulate to an obstacle (4). Then they are compressed (3) to form a flat plate (5). Under pressure and oxidation, the flat plate can fragment (6). This debris forms the secondary source flow. The external flow collects the particles outside the contact area and reintegrates the particles through the rotation of the brake disc (9). The wear flow is the loss of the particles in the recirculation flow (10). The aspects of the physical-chemical behaviour in the contact area play a decisive role regarding the temperature determination of the third body (oxidation, loss of mass) [27]. The comprehension of the third body behaviour plays an important role regarding the macroscopic behaviour of the brake. It has significant influence in respect to the coefficient of friction, wear, temperature and vibration. The third body helps to understand the complexity of the braking. However, the high heterogeneity of the third body in regard to the thermal gradient, contact pressure and the material can force us to make oversimplifications for the analysis of the macroscopic braking behaviour. An example for an oversimplification is to consider just the coefficient of friction and the thermal contact resistance in order to consider the mechanical and thermal behaviour of the contact area.

Heat Flux

The dissipated energy in heat is almost 90% and the generation of heat in the contact area is not well understood. Bardon [19] distinguished two types of heat sources. The generation of heat in the volume through the third body due to abrasion of the surface in cause of high shear forces and the generation of heat in the surface through the asperities of the contact. The understanding of the generation of heat at the scale of the contact is very difficult in terms of accessibility of the measurement instrumentation into the contact area. Therefore, it is considered as the transformation of the mechanical energy into the thermal energy on the macroscopic level. The power of the dissipation of energy can be calculated with macroscopic parameters. The deceleration is described by *Coulomb's* law. The distribution of the heat flux cannot be equal because the friction pair (brake disc and brake pad) consists of different materials with different mechanical and thermal properties. The coefficient of distribution of the heat flux p between the brake pad and the brake disc describes the percentage of the total

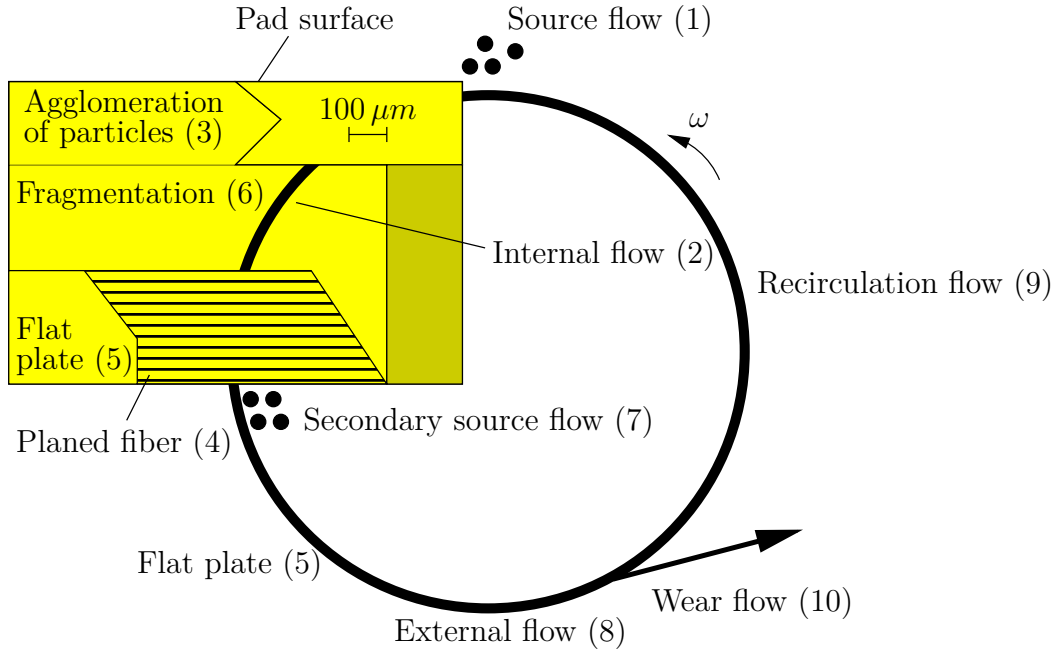


Figure 1.5: Tribological loop of a brake pad

incoming of heat flux at each solid. Based on [129] and Archard [8], the coefficient is determined by assuming smooth, perfect and static contact. Moreover, the same temperature distribution is between the two solids. The coefficient p is constant and dependent on the contact surface S_i and the thermal effusivity ξ for each solid. The formulation of p is

$$p = \frac{S_d \xi_d}{S_d \xi_d + S_p \xi_p}, \quad (1.1)$$

where

$$\xi = \sqrt{\lambda \rho c_p} \quad (1.2)$$

describes the heat input capability of a solid. λ , ρ , and c_p are the heat conduction, the density, and the heat capacity, respectively. The determination of the properties of the third body is very difficult and requires a high measuring effort. In summary, the generation of heat is very complex and not well understood. An assumption is made regarding the heat flux distribution between the two solids. The distribution is constant and depends on the surface and the thermal effusivity. The distribution and the absolute value of the heat flux depend on the inhomogeneity of the surface and can conduct to the formation of hot spots in the contact. This can be observed in a larger scale as hot spots. This phenomenon can be observed in several friction components such as clutches and brakes. The causes of these phenomena are difficult to determine because there are different couplings of scales and physic-chemical phenomena. There are the thermomechanical coupling, heat transfer, temperature-dependent material properties, friction, dynamics of the brake disc and brake pad and wear. Barber [13], [17], [14], [15], [18] shows that the contact conditions vary due to the asperities in the contact area and this conducts to localised thermal deformations [13]. Finally, it can be said that a coupling between the thermal deformation, wear and the changing contact area exists. These

conduct to the variation of the contact pressure and the formation of localised temperatures. The localised temperatures cause temperature gradients through the volume, which generate thermal deformation by the thermal gradients and also alternating stresses due to the heating and cooling of the brake disc. These thermomechanical loads conduct to thermal fatigue. Moreover, the thermal gradient can cause plastic deformation and the above explained damages. The permanent deformations can conduct to plasticity, change of the microstructure, modified contact area, vibrations, drop of the coefficient of friction and non-uniform wear. These aspects are very important to the safety of the passengers and are very important for the science as well as for the industry. For high speed trains, there are different thermal gradients [39], which depend on the brake power, angular velocity and energy. Anderson and Knapp [5] suggested a classification of the thermal gradients observed in the car brake disc according to their size, temperature and duration. It is not possible to take the same classification because the boundary conditions are different. Table 1.1 shows the brake characteristics between the car and railway brakes in the case of a full braking at 100 km/h.

Application	Energy per disc [kJ]	Brake time [s]	Mean power [kW]	Surface power [kW/m ²]	Diameter [mm]
Automobile	165	4	41.3	1640	227
Railway	1550	22.2	69.4	170	640

Table 1.1: Comparison between the automobile and railway brake discs

The dissipated energy and the velocity of the railway brake are much greater than of the car brake. The geometry and the size of the brake pads are different. Moreover, the pressure of the railway brake (max. 0.5 MPa) is much smaller than the brake of the automobile (max. 8 MPa). The high energy and the low pressure abet the deformation of the brake calliper and the formation of the thermal gradients.

Thermal gradients

Thermal gradients are an invisible phenomenon, which can be shown only with an IR camera. Several researches try to describe the hot spot phenomenon [5], [28], [42], [59], [68], [69], [72], [110], [111], [118], [119], [123], [140]. Figure 1.6 shows the different types of hot spots that can be detected with a high speed IR camera.

Thermographs are carried out on a line under a fixed angular position of the disc. Acquisition of the angular velocity is done simultaneously with the testing bench. The conversion of the data to polar coordinates reconstitutes an image of the disc for each revolution. These experimental results illustrate the following classification from Dufrenoy and Weichert [40], which differs from those of Anderson and Knapp [5] and is shown in table 1.2. Asperity (a) type corresponds to rapid temperature rising briefly on very small areas of the rubbing surface. They result from discrete asperity contacts. Gradients on hot bands (b) correspond to small contact sites which appear along a single rubbing path. Hot bands (c) appear as reduced contact friction areas of the pad in the radial direction. They are seen on the disc as narrow rings of high temperatures in the direction of sliding. They can move along the radial direction during braking, depending on the evolution of the bearing surface. Macroscopic hot spots (d) are large

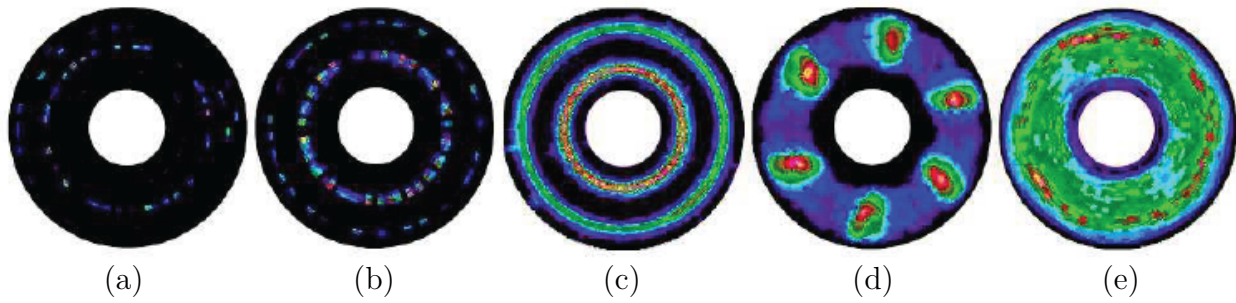


Figure 1.6: Hot spot classification by Dufrenoy and Weichert [Reprinted from [111] by permission of the publisher (Taylor & Francis Ltd, <http://www.tandf.co.uk/journals>)]

	Thermal gradient	Size [mm]	Temperature [°C]	Period
(a)	Asperity	< 1	up to 1200	< 1 ms
(b)	Gradients on hot bands	5-20	650-1000	0.5 – 10 s
(c)	Hot bands	5-50	up to 800	> 10 s
(d)	Macroscopic hot spots	40-110	up to 1100	> 10 s
(e)	Regional hot spots	80-200	20-300	> 10 s

Table 1.2: Classification of the hot spots in railway disc brakes

thermal gradients regularly distributed on the discs surface. They are fixed on the disc and appear in buckling pattern. This phenomenon reduces drastically the contact surface area with high temperatures. Regional hot spots (e) are low thermal gradients on the whole surface of the disc, due to inhomogeneous cooling. Such distributions appear at the end of braking due to thermal diffusion. This classification slightly differs from the automotive one proposed by Anderson and Knapp [5]. This is due to the differences in disc size, geometry, energy dissipation and braking duration. As noticed by Anderson and Knapp, some metallurgic transformations may occur, especially martensite formation at hot spot site. Martensite occurs due to high temperature above the austenitic phase level followed by a rapid cooling and may produce severe local strains due to the associated increase of specific volume. Cracks at the periphery of hot spots may initiate during cooling after the martensite formation. The most critical hot spots correspond to types (b), (c), and (d). Type (a) corresponds to short times and type (e) to a conventional heat diffusion that should normally appear when hot spotting is reduced. It is now well known that hot bands (type c) are linked to the reduced contact area of the pad, depending on the pad design, thermal dilatations and wear of the friction material. Some models have been developed in the literature that shows good agreement with experiments when introducing these phenomena [8-10]. Macroscopic hot spots (type d) correspond to a global distortion of the disc with a waviness shape. This will be shown later in this thesis. Some authors have shown that this waviness deformation is progressive when the disc is subjected to high energy dissipation [110]. Gradients on hot bands or focal hot spots (type b) are supposed to be associated to frictionally excited thermoelastic instabilities (TEIs) [139]. The exact conditions of appearance of these localisations are still debated. It is not discussed here and readers could refer to the proposed references, but it shows the great importance of getting experimental data of thermal

heating. The hot bands and the macroscopic hot spots are well studied. The peculiarity of the TGV brakes is that the macroscopic hot spots remain at the same place in all brakes. Panier [109] showed that the macroscopic hot spots are the most damaging for the disc. They occur generally for high braking power and high velocity but also for lighter braking applications. Practically, a reduced number of macroscopic hot spots per side is commonly observed. Various analyses, with simultaneous measurements on both faces of the disc, have shown that they are located alternatively on both sides of the disc. This asymmetrical distribution indicates a waviness deformation pattern of the whole disc. Complementary analyses have shown that this pattern is generally definitively marked by plastic deformation and hard to remove. However, successive measurements show that hot spots migrate in an angular direction. It is probably due to the asymmetry of the contact with friction leading to a non-symmetric heating of hot spots which is higher at the leading edge of the pad. Due to this asymmetric contact, plastic flow occurs modifying slowly the shape of the waviness deformation. This analysis is consistent with the direction of rotation of the macroscopic hot spots migration which is opposite to the direction of rotation of the disc. The number of hot spots in the case of macroscopic hot spots is generally associated to the ratio of the disc mean perimeter to the pad contact length. But, in some analysis, this correspondence is not strictly respected. However, one should consider the effective contact length that could be significantly reduced compared to the apparent one due to thermal dilatations, non-uniform contact pressure, etc. It may relate to a higher number of hot spots than the strict ratio between the mean perimeter of the disc and the apparent contact length. As previously described, macroscopic hot spots correspond to the last stage of disc deformation, when the dissipated energy is high. Consequently, their appearance is linked to the rigidity of the disc, to its geometry and to the fastening device. Following this reasoning, the pad type should normally have less influence. Such results show that IR thermograms for four separated discs equipped with different pads (the pads differ in terms of rigidity) and for high energy braking. For each pad type macroscopic hot spots occur, with variations in terms of shape and gradient. It shows that even if the disc is already deformed by previous braking with a waviness shape, the macroscopic hot spots could be less or more intense giving various thermal gradient types. It shows that for such braking, the pad may significantly influence the thermal loading. As this loading is strongly linked with the thermomechanical load (thermal stresses), it may influence the risk of damage. The hot bands are circular hot rings on the disc, which occur as one type of thermoelastic instability and were explained by Barber [17]. Ostermeyer [105] explained well the oscillating radius that were also observed in railway disc brakes by Dufrenoy [41]. [41] explains with a 2D axisymmetric model of the disc the pressure distribution which takes into consideration the dilatation and the wear of the brake action. It was considered that the real geometry of the disc and the non-uniform radial distribution of the contact and as well as the non-uniform distribution of the pressure at the back of the brake pad. The sliding speed changes along the radius of the friction ring and therefore the heat flux is non-uniform. It describes the evolution of the located temperature on the disc by the stop braking. At the beginning, the pressure is uniform but after a certain time it develops two hot bands due to the change of the geometry of the pad and the changing pressure. This localisation can be observed on the center of the pad due to dilatation. Furthermore, the wear provokes a displacement of the contact area. At the end, the temperature decreases with the decreasing velocity and therefore the dilatation becomes very small. Hence, an interplay between the wear and the dilatation can be observed. This model cannot be considered as an explanation of the hot bands for all disc brakes but it delivers a good reproduction of the hot bands. Another well explained thermomechanical phenomenon is the macroscopic hot spot. This phenomenon

can be explained by the theory of the thermoelastic instabilities as well as the buckling of the disc. Moreover, the work of Panier [110] describes the formation of the macroscopic hot spot subjected to the boundary conditions and the material properties. The theory of the thermoelastic instability considers certain conditions of the velocity, wear and pressure. Due to the generation of heat between the two solids they suffer thermoelastic deformations and influence the pressure distribution in the contact area. In the friction system, Barber [17] shows the numerical models to predict hot spots in automobile brakes; but they are not applicable to railway brake discs. Other authors worked on this field such as Zagrodzki [139] and Yi [138]. In the thesis of Panier [108], the thermoelastic instability is used to predict the macroscopic hot spots on the brake discs of a TGV and found the critical speed to be 30 km/h by the formation of 12 macroscopic hot spots. These results do not coincide with the experimental observations. The several limits of this approach are considered by Dufrenoy [40], there is mention that he considered the whole geometry of the brake disc and the finite dimension of the brake pad. However they play a role for the structural deformation and the evolution of the perturbation. The stability is conditioned by the critical velocity but the experiments of Panier [108] show that the velocity does not have an influence on the formation of the macroscopic hot spots. The main parameter is the level of the temperature. The determination of a single instability is considered and not the evolution of the deformation. This approach remains in the elastic regime and the macroscopic hot spots correspond to the structural deformation of the disc. But it needs to consider the inelastic behaviour of the disc to get the level of the temperature in the located contact zone. Hence, the approach of the thermoelastic instability is not usable to predict the macroscopic hot spots on the brake disc of a high speed train. Another attribute of the macroscopic hot spots is the asymmetric periodical circulation on the both faces. This can be considered as the structural buckling phenomenon of the disc. The thermal load generates compression in circumferential direction and supports the development of the buckling. Here, it can be referred to the work of Henrik and Fan [59], [48] which considers the elastic buckling of the disc under thermal loads. This approach delivers permanent deformations on the disc and also after the cooling of the disc. Furthermore, it is necessary to consider the plastic buckling under the thermal loads but it makes the simulation complicated due to the nonlinear behaviour of the disc. These approaches were used in Panier's works [108]. A comparison of the behaviour of the disc in regard to the elastic and plastic buckling was described and shows for the elastic buckling a maximum temperature of 19000 °C with 5 bulges. The maximum temperature for the plastic buckling is 10500°C with 6 bulges. From that, it can be deduced that the approach of an elastic and plastic buckling is not applicable to predict the formation of macroscopic hot spots.

Progressive Waviness Distortion

The literature shows that the thermoelastic instability as well as the buckling of the disc cannot predict the formation of macroscopic hot spots. Panier et al. [110] proposed a solution to predict the formation of macroscopic hot spots. They considered the transient heating and the changing velocity. Moreover, the elastoplastic behaviour of the disc is considered. Finally, there are no predefined perturbations but the boundary conditions (load and geometry) are lifelike used. In the Progressive Waviness Distortion (PWD), there are three steps to predict the macroscopic hot spots: the two wave disc bending, the transition from two waves to a higher number of waves and the hot spots stabilisation. The first step considers an isotropic

homogeneous disc with uniform thickness h . The disc is loaded with a transverse contact load $p(r, \theta)$ and a temperature rise $T(r, \theta, z)$, which is induced by friction. In matters of the classical theory of plates, the deflection w of the plate is small compared to the thickness h . The cross-sections remain before and after the deformation perpendicular to the plates middle surface. Moreover the change in the length is negligible. The equilibrium equation (lateral load and forces in the plane) can be written as

$$D\nabla^4 w - \frac{1}{r} \frac{\partial}{\partial r} \left(r N_r \frac{\partial w}{\partial r} \right) - \frac{N_\theta}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \nabla^2 M_T - p = 0, \quad (1.3)$$

where ∇^4 , ∇^2 , D , w , M_T , N_r and N_θ are the two-dimensional biharmonic operator, the Laplace operator, the bending stiffness, the lateral deflection, the thermal bending torque, the radial forces and the tangential force, respectively. The bending stiffness and the thermal moment are defined by

$$D = \frac{Eh^3}{12(1 - \nu^2)} \quad (1.4)$$

and

$$M_T = \frac{E\alpha}{1 - \nu} \int_{-\frac{h}{2}}^{\frac{h}{2}} T z \, dz, \quad (1.5)$$

where E , ν , h and α are the Young's modulus, the Poisson ratio, the thickness of the disc and the thermal expansion coefficient, respectively. The boundary conditions for the equation 1.3 are the clamping on the inner radius $w(r = R_{in}) = 0$ and $\left(\frac{\partial w}{\partial r}\right)_{r=R_{in}} = 0$. The outer radius is free, where the bending moment and shear resultants normal to the boundaries are $M_n(r = R_{out}) = 0$ and $V_n(r = R_{out}) = 0$. The thermal moment M_T is vanished if the temperature is axially symmetric. In this case there are only in-plane forces and consequential planar deformations. A buckling of the disc through large thermal induced compressive stresses can be neglected based on the dimension of the brake disc. Finally, only the thermal moment can induce lateral deformation. The TGV brake disc is asymmetrical mounted on a hub and therefore, the temperature distribution is also asymmetrical and that means that $M_T \neq 0$. In step 2 the thermal loading on the brake disc is high enough that the two wave disc bending becomes stationary through the plastic deformation. Due to these conditions, it develops two skew-symmetric thermal gradients per side. As it is mentioned in step 1, these thermal gradients induce a thermal moment which induce a further two wave disc bending. In step 3 the occurrence of the six hot spots can be calculated and the wave order does not change. Due to the plastic yielding, the hot spots are stationary. In reference to the PWD theory, hotspots arise at subsequent brakes as a result of effects of plastic deformation at the same place. Concomitant measurements of the brake disc deformation during braking were not made, so that a comparison of the model concept to the experimental results regarding wave development is not possible. In summary, it can be said that the importance of the energy, the brake time in conjunction with a low pressure, and a non ventilated geometry of the disc of the high speed train favours the formation of macroscopic hot spots on the both faces. The literature reveals

that the number of the macroscopic hot spots on the both faces of the brake disc of the TGV lies between 5 and 7 and they are asymmetric. Furthermore, the macroscopic hot spots are stationary and move opposite to the rotation of the brake disc during the brake action. They can appear in the middle or on the exterior and interior radius of the disc. The rigidity and the circumference of the brake pad have an important influence in regard to the number and thermal intensity of the macroscopic hot spots. Before the formation of the macroscopic hot spots, the hot bands and the gradients on hot bands (small hot spots on the hot bands) appear. The formation of the hot bands are explained with the non-uniform initial pressure distribution in the contact area of the brake pad and the transmission of the force at the backside of the brake pad. The migration of the hot bands between the interior and exterior radius can be explained with the phenomenon of the thermomechanical buckling, which is adapted from the coupling between wear and thermal dilatation. The gradients on hot bands corresponds to numerous hot spots appearing with more or less regular shape and more or less well distributed on the surface. Contrary to the macroscopic hot spots, they are not always fixed on the disc surface and can move easier. The importance of considering the gradients on hot bands and the cracking damage is described in [131]. The work of Wicker [131] deals with the problem of cracks and squeals on brake discs of high speed trains. The main objective of the work is the amelioration of the design of the brake pads concerning squeal and cracks. The work of Wong [132] considers the problem of cracks very detailed. In the literature, there are several models presented which determine the thermal loads numerically in the contact area (disc-pad) that have different rate of complexity. Several authors published axisymmetric 2-D models of the system disc-pad [32], [33], [68]. The models of Zagrodzki [139], [140], Choi [29], Tirovic [124], Dufrenoy [39] and Nguyen-Tajan [102] enable to simulate just the hot bands. Hence, to predict the hot spots, it is necessary to consider a 3-D model. Furthermore, several authors consider conventional finite element models [124], [68], [28], [50] which enable to determine a nonuniform pressure field and conduct to bending deformation of the disc. Indeed, the elastic behaviour of the disc cannot predict the formation of hot spots on a high speed brake disc as shown above. In addition, the difficulty in these models lies in the simulation of the rotation of the brake disc. To obtain reliable results of a numerical FE simulation, it is necessary to generate a fine mesh. However, this leads to high calculation periods and shows that the root of the problem lies in the thermal resolution. Floquet [49] used a hybrid FFT-FEM model, which enables to impinge a certain value under the surface of the disc. It is considered a 2-D axisymmetric model, which can predict thermal gradients due to rotation of the disc. Nonetheless, it needs several numbers of calculations and the model becomes very difficult for a non axisymmetric geometry of the disc. In the frame of automobile brakes, Nguyen-Tajan [102] made an Eulerian approach, which considers the variable loads during a rotation. This algorithm allows the evolution of the load subjected as a function of the time and the variation of the rotation speed of the disc. However, the application of this approach on the railway brake disc takes a high computation period due to the brake time and 3-D geometry of the brake disc. Moreover, in these models the pressure between the brake disc and the brake pad is not updated during the brake action. The pressure in the contact area is calculated at the initial stage and then assumed as constant during the whole braking. Due to the long brake period of railway braking, it is necessary to consider the variation of the pressure. The approach of Tirovic [124], Dufrenoy [39] and Panier [108] consists of the application of the transient thermal load on a stripe of the disc, which corresponds to a 2-D axisymmetric model. The generated heat flux under the brake pad is uniform during a turn on a stripe of the brake disc. During the remaining time, a cooling effect such as convection is considered. Now it is possible to design a diagram or a figure, which determines the temperature

distribution as a function of the time and position. The methods of the determination of the 3-D temperature distribution, such as FFT-FEM, *Eulerian* approach and 3-D/2-D models are very complicated to predict the hot spots in a circumferential direction due to the non-uniform contact area during the brake action.

Researched Hypotheses

Another consideration of the models, which predict the formation of hot spots, was the work of Sarda [118]. In his thesis, he made a classification of hot spots and showed a validation method concerning the cause of the hot spot formation. The models were investigated concerning falsifiable predictions and experimentally verified. The occurrence of hot spots on disc brakes is a subjective perception and it depends on the observer. The observation of the thermal gradients (hot spots) could differ with regards to the size, number of the hot spots, temperature peaks, and the shape of the hot spots. He tried to explain the appearance of hot spots on brake discs in an objective view. A further aspect of his work [118] is the simultaneous measurement of the temperature distribution and the thermal expansion of the brake disc to analyse the formation of the hot spots. Moreover, in the literature it is not mentioned if the experimental results are reproducible, and he uses these results as the basis for his work to try to give reproducible results. The aim of the work [118] is therefore, a better understanding of the formation of hot spots on brake discs in practical operational conditions and to develop a cause-and-effect chain to avoid the formation of hot spots. The literature shows several models to predict hot spots but the validation of the numerical results with the experiments is not conducted. The results of these models were used to verify the results in the experiments. A further aspect of the hot spots in the case of automobile brake discs is the dynamical behaviour of the disc and the surrounding components. Due to the reversible thermal distortions on the brake disc, a forced vibration is induced, which is called hot judder. The cold judder phenomenon is due to waviness and roughness of disc surface, misalignment of axis (Surface Run Out) and elastic distortion. Further reading is shown in [83], [63], [65], [64], [46]. The analysis of the literature reveals that there are several models which differ in the basic assumptions, but they provide the possibility to predict hot spots. The models to predict hot spots are not approved or merely approved on the basis of few experiments. Hence, Sarda takes model predictions and validates them experimentally. The models are valid if the predictions coincide with the results of the experiment and disprove if they are not coinciding with the results of the experiment. If the results of the experiment do not coincide with the model concerning a certain aspect, the model is only disproved for the considered aspect. The reliability is secured due to the validation of the measurement process and the repetition of experiments under the same conditions. The objectivity is ensured based on *Fourier's* transformation because of the periodicity of the thermal gradients, deformation of the brake disc and the brake torque. A good knowledge of the brake system concerning the tendency to form hot spots is necessary for the formulation of hypotheses. In addition, it is important to know under which conditions hot spots occur. There are, as mentioned, several parameters which influence the formation of hot spots such as the velocity, pressure, type of braking, geometry etc. Furthermore, it is important to check if hot spots occur regularly, equidistantly distributed or phase lagged on the circumference of the friction ring of the brake disc. Moreover, it is necessary to investigate in the experiments the reproduction of the attributes like the amplitude of the variations in temperature and the number of the hot spots under the same conditions. The checked hypotheses of the formation

of hot spots by successive braking in the work of Sarda [118] are given as follows.

Hypothesis 1.1: According to the PWD theory [108] hot spots occur always at the same place due to plastic deformation by successive braking. The hypothesis is falsified if the phase lag between two hot spots in circumference direction is greater or smaller than $\pm \frac{\pi}{6}$.

Hypothesis 1.2: It exists a clear relation between the positions of the occurrence of hot spots by successive braking which is described by a phase lag. The hypothesis is falsified if there is no relation beyond the tolerance range of $\pm \frac{\pi}{6}$.

Verification of the hypothesis 1.1 and 1.2: The positions of the hot spots by successive braking have a phase lag of almost 180° , so that it is observed a cyclic alternation of the positions of the hot spots. Hence, the hypothesis 1.1 is falsified and the hypothesis 1.2 is proved.

Conclusion: Memory effects of the brake disc exist through the cyclic alternation of the hot spots. The PWD theory is not applicable to the investigated system.

According to the theory of the thermoelastic instabilities the distance between two hot spots in circumferential direction is always smaller than the length of the brake pad.

Hypothesis 2.1: The reduction of the arc length of the brake pad increases the number of the hot spots. The hypothesis is falsified if the number of the hot spots is in spite of bisecution of the arc length of the brake pad in the tolerance range of ± 1 .

Verification of the hypothesis 2.1: Braking under the same conditions and a reduction of the arc length of the brake pad by 20 % and 50 % gives the same number of the hot spots compared to the complete arc length of the brake pad. The hypothesis is falsified.

Conclusion: A change of the arc length of a brake pad does not change the number of the hot spots. The model which predicts the change of the number of the hot spots in function of the arc length is not applicable on the brake system of Sarda [118].

Hypothesis 2.2: The distance between two hot spots is always smaller than the arc length of the brake pad. The hypothesis is falsified if the distance between two hot spots is greater than the arc length of the brake pad.

Verification of the hypothesis 2.2: The brake pads with the arc length of 23° reveal a formation of 10 hot spots. The mean distance between two hot spots in the experiment is 36° and thereby greater than the arc length of the brake pad. The hypothesis is falsified.

Conclusion: The model of the thermoelastic instability which is based on the hypothesis 2.2 is not applicable for the considered brake system.

According to the PWD theory, the formation of hot spots is initiated due to a local temperature jump.

Hypothesis 3: Before the formation of a periodical temperature variation on the friction path of the brake disc in circumferential direction, it is necessary that a single hot spot occurs. The hypothesis is falsified if an initial hot spot does not occur.

Verification of the hypothesis 3: The analysis of the temperature distribution on the surface of the brake disc cannot observe an initial temperature increase before the simultaneous formation of the hot spots. The hypothesis is falsified.

Conclusion: An initial process of the formation of hot spots due to a local temperature increase is not observed.

The assumption in the literature is that the brake disc has a wave shape in the condition of developed hot spots. The number of hot spots coincides with the number of wave crests. The following hypothesis reveals the correlation between the formation of hot spots and the brake disc formation.

Hypothesis 4.1: The brake disc has a wave shape by the developed hot spots. The hypothesis is falsified if the amplitude spectrum is unchanged while braking.

Hypothesis 4.2: The number of the hot spots coincides with the number of the wave crests. The hypothesis is falsified if the dominant order of the amplitude spectrum of the deformation of the brake disc and the number of the hot spots in the circumferential direction do not coincide.

Hypothesis 4.3: The position of the hot spots and the wave crests coincide. The hypothesis is falsified if the hot spots do not occur at the wave crests or they have a differing phase lag of $\pm \frac{\pi}{6}$.

Verification of the hypotheses 4.1-4.3: The developed hot spots coincide with the undulated deformation of the brake disc. The position of the hot spots and the deformation of the brake disc have almost the same phase lag. The hypotheses 4.1-4.3 are falsified, respectively.

Conclusion: The results of the experiments reveal the coupling of the undulated deformation of the brake disc and the hot spots. The hypotheses 4.1-4.3 are true.

Hypothesis 4.4: The formation of the hot spots and the waves occurs at the same time. The hypothesis is falsified if the amplitude of the temperature variation and the topography of the brake disc have a discrepancy of at least five turns at the beginning of the formation of the dominant number of Hot Spots.

Verification of the hypothesis 4.4: An undulated deformation of the surface of the brake disc before the formation of the temperature variation is observed. The deformation has the same dominant order as well as the phase lag of the hot spots. Brake pads with higher compressibility deform the brake disc waved without formation of hot spots. The hypothesis 4.4 is falsified.

Conclusion: The conformity of the number and position (phase lag) of the hot spots and the wave crests reveals a coupling between the increase of the topography and the increase of the friction power at the positions of the hot spots. The undulated deformation of the brake disc is the cause of the occurrence of temperature variations in circumferential direction.

From the hypothesis 4.4 follows the falsification or the validity of the hypotheses 4.5 and 4.6, respectively. The hypothesis 4.4 is falsified.

Hypothesis 4.5: The formation of the hot spots occurs before the formation of the surface waviness of the brake disc which are of the same order.

Verification of the hypothesis 4.5: The brake disc is thermally charged at the same place on both sides with a stationary heat source. An undulated deformation of the brake disc is not observed. The hypothesis 4.5 is falsified.

Hypothesis 4.6: The formation of a surface waviness of the brake disc is observed before the temperature changes, which has the same order.

Verification of the hypothesis 4.6: The hypothesis is true.

Through the experiments, which are conducted by Sarda [118], it can be said that no model can predict the hot spots in the experiments.

Thermoelastic Instabilities

Barber [13], [14], [16], [15], [139],[2], [138], [137], [66] has proposed an approach to predict hot spots. He considered local changes of temperature due to friction in the contact area between two sliding solids and named them thermoelastic instabilities. The basic assumption is that friction systems (brake pad-brake disc) in the real brake action are not plane but they are in contact in small contact points and may depend on surface roughness. It can be said that the friction layer changes not only due to the global load which is applied due to velocity, forces and time. Also the varying local loads are changing and therefore the friction layer and the temperature. He divides the contact in many small contact elements, which are in contact, and describes the periodical change of temperature due to local thermal damage in the contact elements. At the beginning, a contact element participates in the friction force transfer. In this contact element, the friction force transfer is increasing over time and it becomes always hotter. The temperature increases in this contact element. Due to this temperature rise, the contact element loses shear strength and the friction stress becomes higher. The friction stress decreases very fast and therefore, the temperature decreases in this contact element. After a while the same process occurs in the neighbour contact element, and a cyclic process starts. In the work of Lee and Barber [82], they show that up to a certain thermal load, the local temperatures change periodically and alternate in an opposite manner. This periodical change of temperature in the contact elements disappears if the contact elements are overcharged. They describe the thermoelastic instabilities in circumference direction in two stages. A slow migration of the instabilities in circumferential direction and a radial migration of the temperature; migration in circumference direction is not observed in this stage. The transition from the first to second stage of the thermoelastic instability occurs at a critical temperature or velocity. Under this critical temperature, it occurs only in the first stage. These cyclic changes of the thermal instabilities depend on the properties of the friction materials.

Analytical Temperature Calculation

In the contact area between brake pad and brake disc, a transformation of energy or dissipation of energy takes place. The mechanical energy is dissipated to thermal energy, which increases the temperature of the friction pairing. This temperature increase causes a change of the temperature-dependent material properties of friction materials. The determination of mean temperatures on the brake disc or brake pad can be easily done with thermocouples under the surfaces of the friction materials. However, the measurement of temperatures on hot spots is carried out with an infrared camera because the place of the hot spots (flash temperatures) are randomly distributed on the friction surface. The calculation of the flash temperatures according to Blok [23], [24], which considers a line contact, and Archard [8], which considers a point contact, provide maximum temperatures in the contact area. Due to temperature-dependent material properties and locally varying friction coefficients, results are badly provided for flash temperatures. The model is refined by Kuhlmann-Willsdorf [75], which considers microcontacts. The consideration of these microcontacts delivers good results. For the calculation of the mean temperature increase and the flash temperature, it is necessary to estimate the number of the microcontacts with [74]

$$N = \frac{4F_N}{\pi d^2 H(T)}, \quad (1.6)$$

where d , $H(T)$ and F_N are the diameter of the microcontact, the hardness at the hot spot and the normal force, respectively. Under the assumption that the heat is distributed equally to the friction pairing in a sliding system, the temperature increase is according to Ashby [11]

$$T_v - T_0 = \frac{\mu F_N v}{A_0} \left(\frac{1}{\frac{\lambda_1}{l_1} + \frac{\lambda_2}{l_2}} \right). \quad (1.7)$$

T_v , T_0 , λ_i , μ , v , A_0 , and l_i are the mean temperature increase, the temperature of the system, the thermal conductivity, the friction coefficient, the velocity, the area and the length of the contact geometry, respectively.

Microstructural Point of View

Poeste [114] investigated changes of the microstructure and residual stresses on brake discs and brake pads. The thermal and mechanical loads have a great influence on the brake disc and therefore, it places emphasis on the brake disc. He charged the samples in his experiments with three natures of load: mechanical, thermal and thermomechanical. In the mechanical load case, the brake disc and brake pad are charged with different contact pressures, rotation speeds, total braking times and load cycles by low friction energy. On the contrary, the thermomechanical load is applied with high friction energy and corresponds to the real use. The thermal load is carried out on a thermal shock test bench, wherein the samples were heated with induction coil and then refrigerated under air. Several heating cycles were carried out to obtain different load cases. Moreover, the organic bounded brake pads were investigated in the same process as the brake discs. The result in his work is an increase in the hardness in the hot spot area. In the hot spot area, a hardness of 700 HV and a formation of martensite and retained austenite is observed. Outside the friction layer or heat-affected zone, the hardness lies at 250 HV, an increase of 40 HV. He demonstrates with the help of diffraction pattern and optical microscopy austenite on a brake disc sample in the contact area. The conditions for the formation of γ -phase (gamma ferrite) and martensite of a pearlitic-ferritic microstructure is a temperature above 713 °C. The real temperature peaks of the hot spot were investigated through a time-temperature-transformation diagram. The metallographic consideration of the hot spot area shows a martensite structure due to carbonisation in this area. A microscopic and X-ray diffraction analysis found in the border zone of the retained austenite martensite. These martensite grow preferred in high carbon areas. Under the martensite structure and near the hot spot area there is a bainitic structure. The reason for this structure is a lower cooling rate in regard to the martensite and retained austenite structure, which follows the time-temperature-transformation diagram. The reason of the formation of the martensitic structure in the hot spot area is the achievement of temperatures above the transformation temperature 713 °C. Due to the phase transformations, the volume increases 4% with operating temperatures under 700 °C. In the friction system brake disc and pad a complicated residual stress state of the brake disc is observed, namely grinding residual stresses, thermal residual stresses and transformation residual stresses in confined macroscopic space. Residual stress analysis shows a phase-specific high tensile strength of 400 MPa for the retained austenite and a residual compressive stress of -300 MPa for the martensite in the hot spot area. Moreover, the residual stresses and the structural change of the brake disc due to hot spots is greater than those in the brake pad. The

retained austenite is observed only in surface area up to $10 \mu\text{m}^2$. The martensitic structure with the bainitic structure as transition to the ferritic-pearlitic structure extends to $500 \mu\text{m}$. The residual stresses due to hot spots is observed until 3 mm in the depth of the ferrite as thermal stresses. In summary, these mentioned residual stresses and the local change of hardness offer the formation of micro residual stresses. The conjunction of micro residual stresses and cyclic thermomechanical loads of the brake action can induce the formation of cracks or crack expansion in the brake discs. The shape of the martensite needle as well as the possible degradation of the lattice strains by plastic deformation of the austenite can induce cracks too. The martensite is characterised by the diffusionless lattice shearing, which introduces lattice distortions in the material. Thus, the increase in volume and shape change due to the formation of martensite can lead to a variation in thickness of the brake disc and to the judder phenomenon.

1.3 Goal & Overview

The goal of this work is to develop test benches to fulfil the research requirements and the validation of the numerical model. The flywheel test bench has a modular structure and provide the capability to change the structure fast for certain research activities. The flexibility and the advanced measurement equipments of the test benches allow to measure the required data for the validation of the numerical model. Furthermore, the developed numerical model based on the uncoupled steady-state algorithm reduces the simulation time and obtain a better knowledge of the friction behaviour of friction pairs.

To begin with, in *Chapter 2*, the theoretical fundamentals of continuum mechanics are briefly reviewed including the concept of the kinematics, the governing balance laws, and the entropy principle for solids. In this context, the particular focus is on a comprehensible introduction of the infinitesimal inelastic solid kinematics in the framework of the natural basis representation. The numerical treatment of the friction pair brake disc and brake pad and the steady state algorithm is shown in *Chapter 3*. In this regard, the required weak formulations of the governing balance relations and their linearisation are presented, followed by appropriate techniques for the discretisation in the space and the time domain including a convenient strategy for the treatment of the inelastic evolution equations. Finally, the temperature computation of the friction pairing is described. In *Chapter 4*, in order to demonstrate its practical applicability, test benches are designed. The designed flywheel test bench with the advanced measurement system is introduced. Furthermore, the tribometer and a combined brake are introduced. The comparison of the numerical and experimental results of the temperature distribution of the brake disc and brake pad are shown in *Chapter 5*. Finally, the particulate matter and wear behaviour is described in *Chapter 6*. A final conclusion and discussion is given in the last Chapter, including some statements on the conceivability of further developments and the future potential.

2 Theoretical Fundamentals

A brief review of the basic concepts of continuum mechanics are shown in this chapter. The description of the material behaviour with numerical methods provide a deep and broad insight into mechanical structures. Particular attention should therefore be paid to the kinematics, the governing balance laws and the entropy principle. The formulations and derivations are based on the works of [44], [90], [93], [92], [89], [34]. The experimental results show that for the brake disc the deformations are small strain compared to the dimensions of the brake system. The measured deformation of ventilated and non-ventilated brake discs are between 0.01 mm and 0.1 mm.

2.1 Introduction

The research of the material behaviour of complicated engineered and loaded pieces requires a powerful mathematical tool to describe the stress and strain state. It is important to understand and apply the common used and advanced constitutive models. There exist two possibilities to describe material behaviour under loading. The phenomenological and the microscopic material model. In the last decade, it can be observed that there is an increase in the research of nano-material models. In this work, a rheological model will be used.

2.2 Kinematics

The kinematics describe the movement of a material body $\mathcal{B}$. The kinematic of a deformable body consists of rigid motions, which can be superimposed with deformations of the body. The deformation are relative changes of material points of the body. The movement of a material point can be explained in *Lagrangian* or *Eulerian* coordinates. The material points can be described in a fixed *Cartesian* coordinate system with the origin $\mathcal{O}$. All points of the body can be identified with a position vector $\mathbf{X}_i$ in the reference configuration. The *Lagrangian* motion ($\mathbf{X} = \mathbf{x}_0 = \mathbf{x}(t_0)$) can be described with

$$\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t), \quad (2.1)$$

where the mapping function $\boldsymbol{\chi}(\mathbf{X}, t)$ describes the mapping of the reference to the current configuration. The spatial (*Eulerian*) description with

$$\mathbf{X} = \boldsymbol{\chi}^{-1}(\mathbf{x}, t), \quad (2.2)$$

where the mapping function $\boldsymbol{\chi}^{-1}(\mathbf{x}, t)$ describes the tracing of a material point, which occupies the position $\mathbf{x}$ in the current configuration to its reference configuration. The *Lagrangian* approach describes the properties of a material point $\mathbf{X}$ and the *Eulerian* approach describes the properties of the location, which the material points are occupying. If the equation of

motion of a material point is given, a change from the *Lagrangian* to the *Eulerian* approach and vice versa is possible:

$$\mathbf{x}(\mathbf{X}, t) \Leftrightarrow \mathbf{X}(\mathbf{x}, t). \quad (2.3)$$

The displacement vector of a material point is given by

$$\mathbf{u} = \mathbf{x} - \mathbf{X}. \quad (2.4)$$

The time derivatives of velocity and acceleration in the *Lagrangian* approach can be calculated with

$$\dot{\mathbf{x}} = \frac{d}{dt}\boldsymbol{\chi}(\mathbf{X}, t), \quad \ddot{\mathbf{x}} = \frac{d^2}{dt^2}\boldsymbol{\chi}(\mathbf{X}, t). \quad (2.5)$$

The total time derivatives in the *Eulerian* approach are

$$\frac{D}{Dt}\phi = \frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi, \quad (2.6)$$

where $\frac{\partial \phi}{\partial t}$ is the local time derivative of the location $\mathbf{x}$ of the material point and $\mathbf{v} \cdot \nabla \phi$ the temporal change of the properties of the material point. The kinematic motion of a material point in a body is shown in figure 2.1.

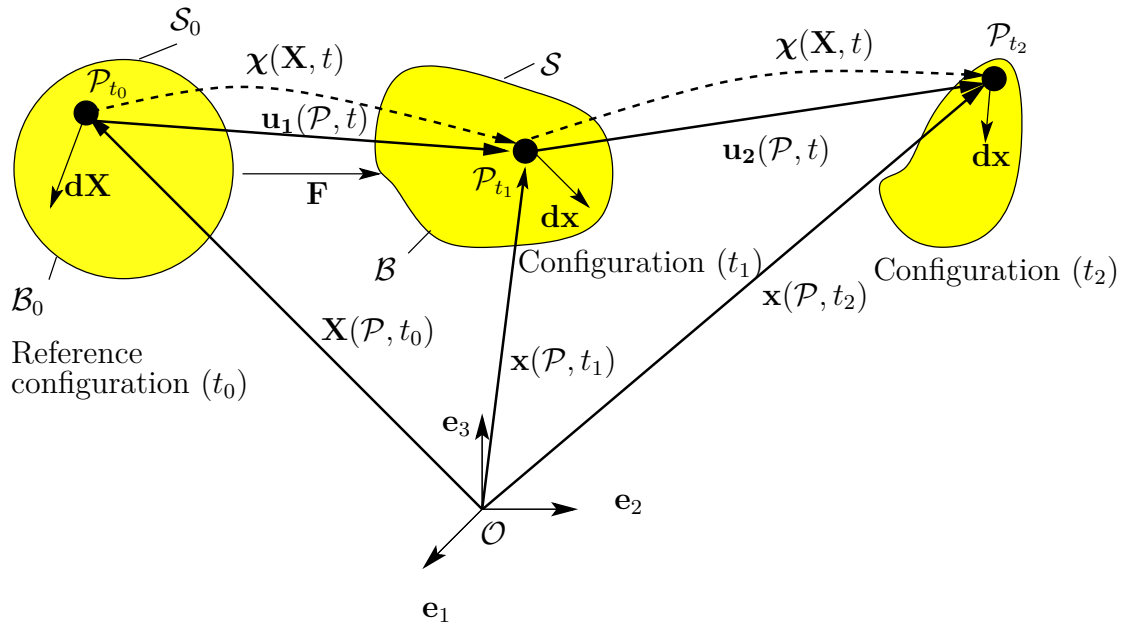


Figure 2.1: Kinematic motion

Hence, the velocity of the material point $\mathcal{P}$ during the motion is

$$\dot{\mathbf{x}}(\mathbf{X}, t) = \frac{\partial \boldsymbol{\chi}(\mathbf{X}, t)}{\partial t}. \quad (2.7)$$

The map $\chi(\mathcal{P}, t)$ describes for each time t the occupied place of the material point. The mapping is invertible with

$$\mathbf{X} = \chi^{-1}(\mathbf{x}, t) = \mathbf{x} - \mathbf{u}(\chi^{-1}(\mathbf{x}, t), t). \quad (2.8)$$

The spatial velocity or the velocity field is calculated by

$$\mathbf{v}(\mathbf{x}, t) = \dot{\chi}^{-1}(\mathbf{x}, t), t) \quad (2.9)$$

and defines the velocity of the material point positioned at $\mathbf{x}$ at time t . The equations 2.7 and 2.9 describe the velocity of a material point. The equation 2.7 depends on the position of a material point and the time. The equation 2.9 depends on the spatial position and the time. A general time-dependent field is defined as a material field, when it depends on the material point. A general time-dependent field is defined as a spatial field, when it depends on the spatial position.

2.2.1 Material Deformation Gradient

The velocities and accelerations of material points are now calculable with the *Lagrangian* and *Eulerian* approach. The introduction of the two-field tensor $\mathbf{F}$ enables the description of the local behaviour of the body. The *Lagrangian* and *Eulerian* deformation gradient are

$$\mathbf{F} := \frac{\partial \chi(\mathbf{X}, t)}{\partial \mathbf{X}} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} = \text{Grad } \mathbf{x}, \quad \mathbf{F}^{-1} := \frac{\partial \chi^{-1}(\mathbf{x}, t)}{\partial \mathbf{x}} = \frac{\partial \mathbf{X}}{\partial \mathbf{x}} = \text{grad } \mathbf{X}. \quad (2.10)$$

The deformation gradient with regards to the displacement vector is

$$\mathbf{F} = \mathbf{I} + \text{Grad } \mathbf{u}. \quad (2.11)$$

The determinant of the deformation gradient is described as

$$J := \det \frac{\partial \chi(\mathbf{X}, t)}{\partial \mathbf{X}} = \det \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \neq 0, \quad (2.12)$$

where J is the Jacobian. There exists a reversibly unambiguous relationship between $\mathbf{X}$ and $\mathbf{x}$, if $J \neq 0$. The determinant of the deformation tensor shows locally the volume change ratio:

$$J = \det \mathbf{F} > 0. \quad (2.13)$$

The *Jacobian* is for $J > 0$ a shrunk/extended volume element, $J = 0$ a shrinkage on a material point $dV=0$, and $J < 0$ a recovering dV on the opposite material side (non-physical). The deformation gradient tensor $\mathbf{F}$ maps the infinitesimal material line $d\mathbf{X}$ into $d\mathbf{x}$, which can be described also with the deformation tensor $\mathbf{F}$, which maps $d\mathbf{X}$ into $d\mathbf{x}$. For the reference configuration the deformation gradient and its determinant are

$$\mathbf{F}(t_0) = \mathbf{I}, \quad \det \mathbf{F}(t_0) = 1. \quad (2.14)$$

In the reference configuration, there is no deformation or loading action, therefore the deformation gradient is the unity tensor. For other configurations as the reference configuration and with loading action the Jacobian has to be $J > 0$. The transformation of line elements from the reference configuration to the actual configuration is shown in figure 2.1. $\mathbf{X}$ and $\mathbf{x}$ are position vectors of the material points at the time t_0 and t . The vectors $d\mathbf{X}$ and $d\mathbf{x}$ determine the material line elements in the body. The deformation gradient describes the rigid motion and the relative deformation of the line element $d\mathbf{X}$:

$$d\mathbf{x} = \mathbf{F} d\mathbf{X} \quad (2.15)$$

The deformation gradient is in general an unsymmetrical tensor. Furthermore, the deformation gradient describes the relation of surface and volume elements in the reference and current configuration. The transformation of a surface element from the reference configuration to the actual configuration is

$$d\mathbf{a} = (\mathbf{F}d\mathbf{X}_1) \times (\mathbf{F}d\mathbf{X}_2) = \text{cof } \mathbf{F} d\mathbf{A}. \quad (2.16)$$

The transformation of a volume element from the reference configuration to the current configuration is

$$dv = [(\mathbf{F}d\mathbf{X}_1) \times (\mathbf{F}d\mathbf{X}_2)] \cdot (\mathbf{F}d\mathbf{X}_3) = \det \mathbf{F} dV. \quad (2.17)$$

The determination of the deformations of a body, which introduce forces in the body without rigid body motions are necessary for the formulation of constitutive models. Therefore, a separation of rigid body motions and deformations of the deformation gradient is necessary. The deformation tensor can be separated in regard of the used constitutive model. The decomposition of the deformation gradient into an elastic and inelastic part is

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_i. \quad (2.18)$$

The deformation gradient, which describes the total movement is inappropriate for the consideration of deformations, due to the consideration of rigid motion of the deformation tensor. Therefore, a decomposition of the deformation tensor in a rigid motion tensor and a deformation tensor is necessary. The polar decomposition of the deformation tensor contains the proper orthogonal tensor $\mathbf{R}$, which describes the rigid rotation and the symmetric positive definite tensors $\mathbf{U}$, and $\mathbf{V}$, respectively the right and left stretch tensor, which describe the elongation along the principal axis.

$$\mathbf{F} = \mathbf{R} \mathbf{U} = \mathbf{V} \mathbf{R} \quad (2.19)$$

The orthogonal tensor has the properties $\mathbf{R}^T = \mathbf{R}^{-1}$, and the symmetric right and left stretch tensor $\mathbf{U}^T = \mathbf{U}$, $\mathbf{V}^T = \mathbf{V}$. A further property for proper orthogonal tensor is that, $\mathbf{R}^T \mathbf{R} = \mathbf{I}$, and $\det \mathbf{R} = 1$ is.

2.2.2 Strain

The simulation of the deformation or the measured deformation are small compared to the dimension of the brake disc. Hence, the displacement gradient is small. The Cauchy-Green tensors can be determined with a concrete representation. The square of the deformed length of a material line is

$$||d\mathbf{x}||^2 = d\mathbf{x} \cdot d\mathbf{x} = d\mathbf{X} \cdot \mathbf{F}^T \mathbf{F} d\mathbf{X} =: d\mathbf{X} \cdot \mathbf{C} d\mathbf{X}. \quad (2.20)$$

To determine the strain tensors, the square of the reference length of the material line is required:

$$||d\mathbf{X}||^2 = d\mathbf{X} \cdot d\mathbf{X} = d\mathbf{x} \cdot \mathbf{F}^{T-1} \mathbf{F}^{-1} d\mathbf{x} = d\mathbf{x} \cdot \mathbf{B}^{-1} d\mathbf{x}. \quad (2.21)$$

The subtraction of the length of the material line of the current configuration from the reference configuration is conducted

$$\begin{aligned} ||d\mathbf{x}||^2 - ||d\mathbf{X}||^2 &= d\mathbf{X} \cdot \mathbf{C} d\mathbf{X} - d\mathbf{X} \cdot d\mathbf{X} = d\mathbf{X} \cdot (\mathbf{C} - \mathbf{I}) d\mathbf{X}, \\ ||d\mathbf{x}||^2 - ||d\mathbf{X}||^2 &= d\mathbf{x} \cdot d\mathbf{x} - d\mathbf{x} \cdot \mathbf{B}^{-1} d\mathbf{x} = d\mathbf{x} \cdot (\mathbf{I} - \mathbf{B}^{-1}) d\mathbf{x}. \end{aligned} \quad (2.22)$$

This subtraction provides the right and left *Cauchy-Green* deformation tensor

$$\begin{aligned} \mathbf{C} &= \mathbf{F}^T \mathbf{F} = \mathbf{U}^2 = \mathbf{I} + \text{grad } \mathbf{u} + \text{grad } \mathbf{u}^T + \text{grad } \mathbf{u} \text{ grad } \mathbf{u}^T, \\ \mathbf{B} &= \mathbf{F} \mathbf{F}^T = \mathbf{V}^2 = \mathbf{I} + \text{grad } \mathbf{u} + \text{grad } \mathbf{u}^T + \text{grad } \mathbf{u} \text{ grad } \mathbf{u}^T. \end{aligned} \quad (2.23)$$

The inverse of the right and left *Cauchy-Green* tensor can be described with the following equations:

$$\mathbf{C}^{-1} = \mathbf{F}^{-1} \mathbf{F}^{T-1}, \quad \mathbf{B}^{-1} = \mathbf{F}^{T-1} \mathbf{F}^{-1}. \quad (2.24)$$

Due to the assumption of small deformations, the second-order term is neglected (linearisation), so that the right and left *Cauchy-Green* deformation tensors are given as

$$\mathbf{C} \approx \mathbf{B} \approx \mathbf{I} + \text{grad } \mathbf{u} + \text{grad } \mathbf{u}^T. \quad (2.25)$$

The introduced *Cauchy-Green* deformation tensors have the disadvantage that for a non-deformed body the deformation tensors become unity tensors. Therefore, the introduction of an engineering strain, which produces a zero tensor for the case of no deformation is convenient. Hence, two alternative formulations of the strain tensors are provided. The *Green-Lagrange* strain tensor $\mathbf{E}$ can be described with

$$\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I}). \quad (2.26)$$

The *Eulerian Green-Lagrange* strain tensor $\boldsymbol{\varepsilon}$ for small deformations is

$$\mathbf{E} \approx \boldsymbol{\varepsilon} = \frac{1}{2}[\text{grad } \mathbf{u} + \text{grad } \mathbf{u}^T]. \quad (2.27)$$

The *Eulerian Green-Lagrange* strain tensor for small strains can be split in an elastic and inelastic strain tensor

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_i. \quad (2.28)$$

The second tensor is the *Almansi* strain tensor

$$\mathbf{A} = \frac{1}{2}(\mathbf{I} - \mathbf{B}^{-1}). \quad (2.29)$$

These two strain tensors show for an undeformed body a strain measure of zero. The factor of one-half is required for the downward compatibility to the engineering strain. Both tensors can be transferred to each other via a contravariant push-forward and pull-back transformation as illustrated in figure 2.2. Furthermore, the proved transport theorems shift second-order tensors from the reference to the current configuration.

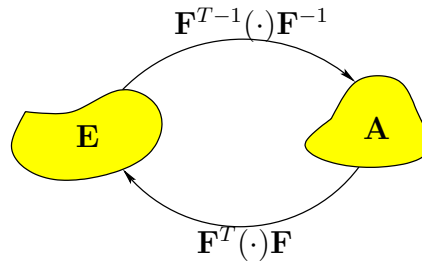


Figure 2.2: Contravariant forward and backward transport mechanisms of the strain measures at reference and current configurations.

Further and detailed description of the deformation and strain tensors and the corresponding rates can be found in Markert [90], [89], [88], Truesdell and Noll [125], Holzapfel [61] or Ehlers [45].

2.2.3 Strain Rates

The deformation gradient describes the local deformations for line, surface and volume elements. The spatial velocity gradient tensor describes their rate of change. The spatial velocity gradient tensor $\mathbf{L}$ is defined by the total time derivative of the deformation gradient as

$$\dot{\mathbf{F}} = \frac{d\mathbf{F}}{dt} = \frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{X}} = \text{Grad } \dot{\mathbf{x}} = \frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{X}} = \mathbf{L}\mathbf{F}. \quad (2.30)$$

Therefore, the spatial velocity gradient is

$$\mathbf{L} := \text{grad } \dot{\mathbf{x}} = \frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{x}} = \frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{X}} \frac{\partial \mathbf{X}}{\partial \mathbf{x}} = \dot{\mathbf{F}}\mathbf{F}^{-1}. \quad (2.31)$$

The split of the velocity gradient $\mathbf{L}$ into a symmetric and skew-symmetric part or rate deformation tensor $\mathbf{D}$ and spin tensor $\mathbf{W}$ yields

$$\mathbf{D} = \text{sym } (\mathbf{L}) = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T) = \mathbf{D}^T, \quad (2.32)$$

$$\mathbf{W} = \text{skew } (\mathbf{L}) = \frac{1}{2}(\mathbf{L} - \mathbf{L}^T) = \mathbf{W}^T. \quad (2.33)$$

The spin tensor $\mathbf{W}$ describes a rigid rotation. The rate of change of line, surface and volume elements are

$$(\mathbf{dx})^\cdot = \mathbf{L} \, d\mathbf{X}, \quad (2.34)$$

$$(\mathbf{da})^\cdot = (\text{div } \mathbf{v} \, \mathbf{I} - \mathbf{L}^T) \, d\mathbf{A}, \quad (2.35)$$

$$(dv)^\cdot = \text{div } \mathbf{v} \, dV. \quad (2.36)$$

The elastic and plastic strain rate tensors of small deformations is

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}_e + \dot{\boldsymbol{\varepsilon}}_i. \quad (2.37)$$

In summary, the deformation gradient transforms elements from the reference configuration to the actual configuration. The spatial velocity gradient determines the rate of change of the elements in the actual configuration.

2.3 Stress Concept

The mechanical interaction of a deformed body $\mathcal{B}$ can be described with the body loadings and the surface loadings. The body loadings $\mathbf{b}$ are acting from a distance such as the gravity or electrical fields on each volume element of the body $\mathcal{B}$. The surface loadings $\mathbf{t}$ or contact loadings are acting on the surface $\mathcal{S}$ and are applied immediately on the boundary $\partial\mathcal{B}$. The resulting load of a volume element is

$$\mathbf{f} = \underbrace{\int_B \rho \, \mathbf{b} \, dv}_{\mathbf{f}_v} + \underbrace{\int_S \mathbf{t} \, da}_{\mathbf{f}_a}. \quad (2.38)$$

One of the fundamental theorems in continuum mechanics is the *Cauchy* theorem, which relates the stress vector $\mathbf{t}$ to a stress tensor $\mathbf{T}(\mathbf{x}, t)$ in regard of a normal vector $\mathbf{n}$:

$$\mathbf{t}(\mathbf{x}, t, \mathbf{n}) = \mathbf{T}(\mathbf{x}, t) \mathbf{n}. \quad (2.39)$$

Herein, the normal vector $\mathbf{n}$ is the orientation of the cut surface or the outward oriented unit surface normal. The *Cauchy* stress tensor describes the stress state of a material point $\mathbf{x}$ at the related area element da . The *Cauchy* stress tensor is related to the surface element in the actual configuration. The incremental surface force $d\mathbf{f}$ related to the *Cauchy* stress tensor and to the current and referential area is

$$d\mathbf{f} = \mathbf{t} \, da = \mathbf{T} \underbrace{d\mathbf{a}}_{\mathbf{n} \, da} = \mathbf{T} \operatorname{cof} \mathbf{F} \, d\mathbf{A} = \underbrace{\overbrace{\mathbf{T}(\det \mathbf{F})}^{\boldsymbol{\tau}} \mathbf{F}^{T-1}}_{\mathbf{P}} \, d\mathbf{A}. \quad (2.40)$$

The *Cauchy* stress tensor, or often referred to as the true stress tensor, describes the current force to the current oriented area element $d\mathbf{a}$. The *Kirchhoff* stress $\boldsymbol{\tau}$, which is a weighted form of the *Cauchy stress*, is defined as a spatial tensor field. The *Cauchy* stress appears together with $J = \det \mathbf{F}$:

$$\boldsymbol{\tau} = J \mathbf{T}. \quad (2.41)$$

While the 1st *Piola-Kirchhoff* stress (nominal or engineering stress) is a two-field tensor, which relates the current force to the referential undeformed area element $d\mathbf{A}$:

$$\mathbf{P} = \det \mathbf{F} \mathbf{T} \mathbf{F}^{T-1}. \quad (2.42)$$

Finally, the 2nd *Piola-Kirchhoff* stress $\mathbf{S}$ describes the acting force in the reference configuration to the reference area element:

$$\mathbf{S} = \mathbf{F}^{-1} \mathbf{P} = \mathbf{F}^{-1} \boldsymbol{\tau} \mathbf{F}^{T-1}. \quad (2.43)$$

For infinitesimal strains, commonly $\boldsymbol{\sigma} = \mathbf{T}$ the *Cauchy* stress tensor $\mathbf{T}(\mathbf{x}, t)$ can be used for the constitutive and numerical modelling and the balance relations. Therefore, it is not necessary to transform the configurations.

2.4 Master Balance Principle

In the following, the master balance principles are introduced. Several volume-specific scalar and vector quantities are balanced with a set of material independent equations, such as the conservation of mass, momentum, angular momentum, energy and entropy. For a detailed description of the master balance relations, the reader is referred to Haupt [57], Ehlers [45] and Markert [90], [89], [88]. The balance equations can be generalised with ψ and $\boldsymbol{\psi}$ for certain densities of volume-specific scalar and vector valued quantities. Then, the general balance relations take the global form [57], [45], [90], [89], [88]:

$$\begin{aligned}
\frac{d}{dt} \int_{\mathcal{B}} \Psi \, dv &= \int_{\mathcal{S}} \boldsymbol{\phi} \cdot \mathbf{n} \, da + \int_{\mathcal{B}} \sigma \, dv + \int_{\mathcal{B}} \hat{\Psi} \, dv, \\
\frac{d}{dt} \int_{\mathcal{B}} \boldsymbol{\Psi} \, dv &= \int_{\mathcal{S}} \boldsymbol{\Phi} \mathbf{n} \, da + \int_{\mathcal{B}} \boldsymbol{\sigma} \, dv + \int_{\mathcal{B}} \hat{\boldsymbol{\Psi}} \, dv \quad (*).
\end{aligned} \tag{2.44}$$

Herein, $\boldsymbol{\phi} \cdot \mathbf{n}$ and $\boldsymbol{\Phi} \mathbf{n}$ describe the scalar- and vector-valued densities of the effluxes of the quantity through the surface $\mathcal{S}$. σ and $\boldsymbol{\sigma}$ describe the supply terms (external source) of the physical quantity. $\hat{\Psi}$ and $\hat{\boldsymbol{\Psi}}$ describe the production of the mechanical quantity due to couplings of the body $\mathcal{B}$ with its environment. After some mathematical manipulations the local (differential) form is

$$\begin{aligned}
\dot{\Psi} + \Psi \operatorname{div} \dot{\mathbf{x}} &= \operatorname{div} \boldsymbol{\phi} + \sigma + \hat{\Psi}, \\
\dot{\boldsymbol{\Psi}} + \boldsymbol{\Psi} \operatorname{div} \dot{\mathbf{x}} &= \operatorname{div} \boldsymbol{\Phi} + \boldsymbol{\sigma} + \hat{\boldsymbol{\Psi}} \quad (**),
\end{aligned} \tag{2.45}$$

where $\nabla \cdot (\cdot)$ describes the divergence operator and $\nabla(\cdot)$ the spatial gradient. Proceeding from the axiomatically introduced balance principles, one identifies the quantities in (*) and (**) according to the table 2.1.

Balance equations	$\Psi, \boldsymbol{\Psi}$	$\sigma, \boldsymbol{\sigma}$	$\hat{\Psi}, \hat{\boldsymbol{\Psi}}$	$\phi, \boldsymbol{\Phi}$
Conservation of mass	ρ	0	0	$\mathbf{0}$
Momentum	$\rho \dot{\mathbf{x}}$	$\rho \mathbf{b}$	$\mathbf{0}$	$\mathbf{T}$
Angular momentum	$\mathbf{x} \times (\rho \dot{\mathbf{x}})$	$\mathbf{x} \times (\rho \mathbf{b})$	$\mathbf{0}$	$\mathbf{x} \times \mathbf{T}$
Energy	$\rho \varepsilon + \frac{1}{2} \dot{\mathbf{x}} \cdot (\rho \dot{\mathbf{x}})$	$\dot{\mathbf{x}} \cdot (\rho \mathbf{b}) + \rho r$	0	$\mathbf{T} \dot{\mathbf{x}} - \mathbf{q}$
Entropy	$\rho \eta$	σ_η	$\hat{\eta}$	ϕ_n

Table 2.1: Master balance principles

In the table 2.1, volume-specific scalar- and vector-valued quantities are referred to physical quantities. These are the density ρ , the momentum $\rho \dot{\mathbf{x}}$, the angular momentum $\mathbf{x} \times (\rho \dot{\mathbf{x}})$, the internal energy ε , the heat influx vector $\mathbf{q}$, the external heat supply r , the entropy η , the efflux of entropy ϕ_n , the external entropy supply σ_η and the non-negative entropy production $\hat{\eta} \geq 0$, which considers the irreversibility of the thermodynamical process. With the table 2.1 the balance equations can be determined. The conservation of mass for a closed system in local form is

$$0 = \dot{\rho} + \rho \operatorname{div} \dot{\mathbf{x}}. \tag{2.46}$$

Newton's second law describes the change of the momentum within a time. The left hand side and the right hand side of equation 2.47 defines the time-dependent change of the volume-specific momentum and the acting loadings:

$$\rho \ddot{\mathbf{x}} = \operatorname{div} \mathbf{T} + \rho \mathbf{b}. \quad (2.47)$$

Compared to the momentum, the angular momentum represents the change within time by the acting torques. The momentum can be used for translational movements and the angular momentum for rotational movements. The corresponding local form is

$$\mathbf{x} \times (\rho \ddot{\mathbf{x}}) = \mathbf{x} \times (\operatorname{div} \mathbf{T} + \rho \mathbf{b}). \quad (2.48)$$

Inserting the equation 2.47 into equation 2.48 leads to

$$\mathbf{0} = \mathbf{I} \times \mathbf{T} \Leftrightarrow \mathbf{T} = \mathbf{T}^T, \quad (2.49)$$

which is described as the *Boltzmann* axiom and which shows that the *Cauchy* stress tensor is a symmetric tensor. The stored energy in a body $\mathcal{B}$ can be achieved on the one hand with the volume-specific kinetic energy and on the other hand with the volume-specific internal energy. The internal energy describes the stored energy due to chemical energy and deformation. The kinetic energy is achieved by the motion of a body. The first principle of thermodynamics represents the conversation of energy

$$\rho \dot{\varepsilon} = \mathbf{T} \cdot \mathbf{L} - \operatorname{div} \mathbf{q} + \rho r, \quad (2.50)$$

where $\dot{\varepsilon}$ is the internal energy rate. The product of $\mathbf{T} \cdot \mathbf{L}$ can be simplified to $\mathbf{T} \cdot \mathbf{D}$ due to the fact, that $\mathbf{T} \cdot \mathbf{W} = 0$. The first principle of thermodynamics describes that the energy in a closed system cannot be lost. A further aspect, which is not considered is the direction of the energy flow. This describes the second principle of thermodynamics, for which the entropy or the irreversibility of the entropy production is considered. The second principle of thermodynamics is

$$\rho \dot{\eta} \geq \operatorname{div} \phi_n + \sigma_n. \quad (2.51)$$

In order to formulate admissible constitutive relations for the description of specific material behaviour evaluation of the entropy principle provides thermodynamic restrictions. The variables in equation 2.51 are $\phi_n = -\frac{1}{\theta} \mathbf{q}$ and $\sigma_n = \frac{\rho r}{\theta}$, where $\dot{\eta}$ and $\theta > 0$ are the entropy rate and the absolute Kelvin's temperature, respectively. The entropy principle then takes the form

$$\rho \dot{\eta} + \operatorname{div} \left(\frac{\mathbf{q}}{\theta} \right) - \frac{\rho r}{\theta} \geq 0. \quad (2.52)$$

Next, by formal introduction of the Helmholtz free energy density

$$\psi := \varepsilon - \theta \eta \quad (2.53)$$

and use of the local energy balance and $\operatorname{div} \frac{\mathbf{q}}{\theta} = \frac{1}{\theta} \operatorname{div} \mathbf{q} - \frac{1}{\theta^2} \mathbf{q} \cdot \nabla \theta$ the entropy principle can be rephrased in the most common *Clausius-Duhem* representation

$$\mathbf{T} \cdot \mathbf{L} - \rho (\dot{\psi} + \dot{\theta} \eta) - \frac{1}{\theta} \mathbf{q} \cdot \nabla \theta \geq 0. \quad (2.54)$$

The heat influx vector given by the *Fourier* law is thermodynamically consistent with $k_\theta > 0$

$$\mathbf{q} = -k_\theta \nabla \theta. \quad (2.55)$$

Assuming isothermal processes with $\theta = \text{const.}$, the above inequality reduces to the so-called *Clausius-Planck* inequality

$$D_{int} = \mathbf{T} \cdot \mathbf{L} - \rho \dot{\psi} \geq 0, \quad (2.56)$$

which actually represents the non-negative internal dissipation of a purely mechanical process. The balance principles are applicable for any continuum body. The material or constitutive models have to satisfy fundamental principles, namely thermodynamic determinism, material objectivity and material symmetry. The thermodynamic determinism implies the knowledge of the histories of $\mathbf{F}$, θ , and $\nabla \theta$. The material objectivity describes the independence of the material response in regard to the observer. The material symmetry describes the rotations of the reference configuration for which the response functional remains unchanged. The constitutive equations, which satisfy the above three principles, may depend on historical variables. The introduction of thermodynamics with internal variables is an alternative concept for which a finite number of state variables are necessary. The thermodynamic state can be completely determined with the state variables at any time for a given point. The thermodynamic state at a point can be determined with the instantaneous values of the deformation gradient $\mathbf{F}$, the temperature θ , the temperature gradient $\nabla \theta$ (t), and a set of scalar, vectorial, and tensorial internal variables $\boldsymbol{\alpha}$. The definition of the thermodynamic potential or the stress constitutive equation is determined with the free energy $\psi(\mathbf{F}, \theta, \boldsymbol{\alpha})$ and the rate of change

$$\dot{\psi} = \frac{\partial \psi}{\partial \mathbf{F}} \cdot \dot{\mathbf{F}} + \frac{\partial \psi}{\partial \theta} \dot{\theta} + \frac{\partial \psi}{\partial \boldsymbol{\alpha}_k} \dot{\boldsymbol{\alpha}}_k. \quad (2.57)$$

Therefore, the constitutive equations can be determined with the *Clausius-Duhem* inequality

$$\left(\boldsymbol{\sigma} \mathbf{F}^{-T} - \rho \frac{\partial \psi}{\partial \mathbf{F}} \right) \cdot \dot{\mathbf{F}} - \rho \left(\eta + \frac{\partial \psi}{\partial \theta} \right) \dot{\theta} - \rho \frac{\partial \psi}{\partial \boldsymbol{\alpha}_k} \dot{\boldsymbol{\alpha}}_k - \frac{1}{\theta} \mathbf{q} \cdot \nabla \theta \geq 0. \quad (2.58)$$

The thermodynamical forces can be defined for each internal variable as $\mathbf{A}_k = \rho \frac{\partial \psi}{\partial \boldsymbol{\alpha}_k}$ and have to satisfy the *Clausius-Duhem* inequality

$$-\mathbf{A} \dot{\boldsymbol{\alpha}} - \frac{1}{\theta} \mathbf{q} \cdot \nabla \theta \geq 0. \quad (2.59)$$

It is necessary for ensuring equation 2.59, the existence of a scalar-valued, convex function with respect to $\mathbf{A}$ and $\nabla \theta$, non-negative, zero-valued at the origin potential.

2.5 Material parameters

The coefficient of thermal expansion α describes the change of size of a solid due to a change of temperature. The linear expansion in a solid is

$$\alpha = \frac{||\Delta\epsilon||}{||\epsilon|| \Delta T}, \quad (2.60)$$

where $||\Delta\epsilon||$, $||\epsilon||$, and ΔT are the current strain tensor, the initial strain tensor, and the temperature increase, respectively. The mean temperature of a brake disc can be calculated by

$$\Delta q = c m \Delta T, \quad (2.61)$$

where q , c , and m are the brake power, the heat capacity, and the mass of the brake disc, respectively. The area or volume expansion coefficient relates the change in area or volume dimension to a change in temperature. The heat transfer h coefficient or film coefficient describes the ability of a fluid, gas or solid to conduct or divert energy from a surface of a material, typically by convection between a fluid and a solid

$$\dot{q} = h A \nabla(T_s - T_\infty), \quad (2.62)$$

where $\dot{q}$, A , T_∞ , and T_s are the heat flux, the heat transfer area, the temperature of the fluid, and the temperature of the solid, respectively. Another important material property is the thermal conductivity λ which describes the heat transfer across a material with a high thermal conductivity to a material with a low thermal conductivity. The heat transfer through conduction in the brake disc is

$$\dot{q} = -\lambda A \nabla(T - T_s), \quad (2.63)$$

where λ and T are the heat conduction tensor and the temperature at the surface, respectively. The thermal conductivity λ describes the energy transport and is a second-order tensor. The thermal conductivity of insulating materials is low and for heat sinks high. Equalising equations 2.62 and 2.63 yields to the *Nusselt* number Nu

$$Nu = \frac{h l}{\lambda} = \frac{\frac{\partial(T_s - T)}{\partial x}}{\frac{(T_s - T_\infty)}{l}}. \quad (2.64)$$

The *Nusselt* number is the ratio of convection to conductive heat transfer normal to the surface. The thermal diffusivity a describes the temporal change of the spatial distribution of the temperature by heat conduction as a result of a temperature difference:

$$a = \frac{\lambda}{\rho c}. \quad (2.65)$$

3 Numerical Treatment

The numerical simulation of the temperature distribution of the brake pad and brake disc is a time-consuming calculation. Today's process of power is very high compared to the past. The strong coupled problem is simulated with an uncoupled analysis of the temperature simulation of the brake pad and the brake disc in a sequential analysis. Nguyen-Tajan's [102] model provides a time-consuming calculation of the temperature with the disadvantage of using only axially symmetric components. The simulation of the brake disc, an axially symmetric component, can be calculated very fast. The simulation of the whole brake system (brake disc, brake calliper, brake pad, etc.) requires a high amount of calculation power due to the increasing of the elements and the complicated geometries. A simulation methodology is introduced to close the gap of the Transitoire dans le Repère du Chargement (TRC) model and the simulation of non axially symmetric components. The steady-state algorithm describes the numerical calculation in the reference point of the load, instead of the *Lagrangean* approach, where the numerical calculation is described in the reference point of the structure. The advantages are the fixed load and a refined fixed mesh of the load area and a coarse one in the other regions. The simulation of the material flow from the reference point of the load is described in *Eulerian* coordinates. Considering this, the material flows through the fixed mesh and the rotating brake disc is described in regards to the kinematic description in a pure spatially manner. The advantage of this approach is a faster calculation of the thermomechanical analysis of the brake disc and pad. The simulation in *Lagrangean* approach is conducted with the mesh or the discretisation of the brake disc, which rotates relatively to the brake pad. Another simulation approach in the *Lagrangean* approach is the rotation of the brake pad around the brake disc. With this simulation approach, many revolutions are simulated. In summary, the simulation in *Lagrangean* description for coupled thermomechanical problems is a time consuming and complicated task. The calculation time of a thermomechanical problem for a brake disc and pad takes more than one week on an 8 core Intel processor, 3.2 GHz clock rate, 16 GB RAM computer with the ABAQUS version 6.14. In the *Lagrangean* approach, the mesh of a high stressed brake disc becomes distorted, for which the accuracy of the results decreases. The use of adaptive meshing procedures are computationally expensive and are still not fully developed for 3-D problems. Due to this time consuming simulation, an alternative methodology should be used.

3.1 The Finite Element Method

The determination of the material behaviour with well defined constitutive equations and the finite element method is a powerful tool. The solution of the coupled partial differential equation (PDE) can be conducted for certain problems [62], [133], [141]. The numerical methods are essential for solving problems, which cannot be solved analytically. The finite element method is the most popular used numerical method. A finite element analysis can be divided in two general steps, the time discretisation of the underlying constitutive initial value problem and the finite element discretisation. Furthermore, the commercial software ABAQUS 6.14 consists of the pre-processor, the solver and the post-processor. The pre-processor defines the geometry,

the initial and boundary conditions and the constitutive equations. The solver calculates the physical quantities, which are defined in the pre-processor. Finally, the post-processor is the visualisation tool of the calculated numerical results.

3.1.1 Time Discretisation

The integral representation of the governing equations for the numerical treatment of the finite element implementation is conducted with local coordinates $\boldsymbol{\xi}$ of the element. The equations should be reformulated with regards to the local coordinates $\boldsymbol{\xi}$ of the element. This can be conducted by the Gauss quadrature method, cf. Zienkiewicz [141]. Using this numerical integration technique and the geometrical relation of the local coordinates $\boldsymbol{\xi}$ to the global coordinates $\mathbf{x}$ via $\boldsymbol{\xi} \rightarrow \mathbf{x} = h^e(\boldsymbol{\xi})$, an integral can be approximated as follows:

$$\int_{\Omega_e} f(\mathbf{x}) \, d\mathbf{x} = \sum_{k=1}^{N_k} f(\mathbf{h}^e(\boldsymbol{\xi}_k)) J_e(\boldsymbol{\xi}_k) w_k. \quad (3.1)$$

Therein, N_k is the number of integration (Gauss) points $\boldsymbol{\xi}_k$, w_k is a certain weight factor and J_e denotes the Jacobian determinant. For further discussion of the time integration and the solution procedure of the global system of equations, the interested reader is referred to the works by Zienkiewicz and Taylor [141] and Markert [90].

3.1.2 Spatial Discretisation

The numerical solution of an IBVP requires the discretisation of the weak form of the governing equations in space and time. For simplicity purpose, here, a spatial discretisation is carried out by the FEM, whereas a time integration is implemented by the implicit Euler method. For more comprehensive information about the finite element method, the readership is referred to Zienkiewicz and Taylor [141], Segerlind [120], Hughes [62] among others. The FEM is a numerical algorithm used to define an approximate solution of differential equations describing the different physical processes. To be more precise, the domain Ω is approximated by a spatially discretised domain Ω^h , which is represented by the union of non-overlapping subdomains Ω_e

$$\Omega \approx \Omega^h = \bigcup_{e=1}^{N_e} \Omega_e, \quad (3.2)$$

where N_e is the number of subdomains. The corresponding number of nodes is denoted as N_n . Correspondingly, the infinite dimensional trial space $\mathcal{S}_{(\cdot)}(t)$ and test space $\mathcal{T}_{(\cdot)}$ could be transformed into the corresponding discrete spaces $\mathcal{S}_{(\cdot)}^h(t)$ and $\mathcal{T}_{(\cdot)}^h$, where the discrete function is defined as

$$\mathbf{u}(\mathbf{x}, t) \approx \mathbf{u}^h(\mathbf{x}, t) = \bar{\mathbf{u}}(\mathbf{x}, t) + \sum_{j=1}^{N_n} \phi_{\mathbf{x}}^j(\mathbf{x}) \mathbf{u}^j(t) \in \mathcal{S}_{\mathbf{u}}^h(t). \quad (3.3)$$

Therein, $\bar{\mathbf{u}}^h$ represent the *Dirichlet* boundary conditions. The functions $\phi_{\mathbf{u}}^j = \{\phi_{\mathbf{u}1}^j, \dots, \phi_{\mathbf{u}d}^j\}$ are

the global basis functions of the respective trial functions, which correspond to the set of the nodal points N_n and are normalised such that

$$\phi_{\mathbf{x}}^j = 0, \quad \text{if } \mathbf{x} \in \bigcup_{e \in N_{ej}} \Omega_e, \quad j = 1, \dots, N_n, \quad (3.4)$$

$$\phi_d^j(\mathbf{x}_i) = \delta_i^j, \quad i, j = 1, \dots, N_n, \quad (3.5)$$

where δ_i^j is the *Kronecker* symbol. The nodal value of the primary variable or the so-called degrees of freedom (DOF) $\mathbf{u}^j$ depend only on time t , while the basis functions depend only on the spatial position $\mathbf{x}$.

In order to derive the discrete test functions, the *Bubnov-Galerkin* approximate method is applied. In the framework of this method, the same global basis functions are used for trial and test functions. Moreover, in contrast to the discrete trial functions, the discrete test functions disappear at the Dirichlet boundaries and are given in the following form:

$$\delta \mathbf{u}(\mathbf{x}) \approx \delta \mathbf{u}^h(\mathbf{x}) = \sum_{j=1}^{N_n} \phi_{\mathbf{u}}^j(\mathbf{x}) \delta \mathbf{u}^j \in \mathcal{T}_{\mathbf{u}}^h. \quad (3.6)$$

The whole procedure of the spatial discretisation leads to a system of linear independent equations for each unknown nodal quantity (each DOF).

3.2 Return-Mapping Algorithm

The constitutive law for the stress can be defined with the free energy potential

$$\psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_i, \boldsymbol{\alpha}), \quad (3.7)$$

where $\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon}_i$, and $\boldsymbol{\alpha}$ are the total strain, the inelastic strain, and the internal variables, respectively. The yield function and the flow potential ($f = \psi$) are equal for the used plasticity model. In regard to equation 2.28, the free energy can be describe as

$$\psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_i, \boldsymbol{\alpha}) = \psi_e(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_i) + \psi_i(\boldsymbol{\alpha}) \quad (3.8)$$

where $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\alpha}$ are internal variables. According to the equation 2.54 and the linear elastic material behaviour, the elastic part of the free energy is

$$\rho \psi_e(\boldsymbol{\varepsilon}_e) = \frac{1}{2} \boldsymbol{\varepsilon}_e \mathbf{E}^4 \boldsymbol{\varepsilon}_e, \quad (3.9)$$

where $\mathbf{E}^4$ is the isotropic elasticity tensor. The elastic law can be described by

$$\boldsymbol{\sigma} = \mathbf{E}^4 \boldsymbol{\varepsilon}_e. \quad (3.10)$$

The introduction of the yield function f is required to determine the elastic and inelastic deformation. Therefore, the yield function depends on the stress tensor $\boldsymbol{\sigma}$ and the hardening thermodynamical forces $\mathbf{A}$. The elastic behaviour is achieved if $f(\boldsymbol{\sigma}, \mathbf{A}) < 0$. In contrary, the inelastic behaviour is achieved if $f(\boldsymbol{\sigma}, \mathbf{A}) = 0$. The evolution of the internal variables, such as the inelastic strain and the hardening variables $\boldsymbol{\alpha}$, are calculated by

$$\dot{\boldsymbol{\epsilon}}_i = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}(\mathbf{x}, t)} \quad (3.11)$$

and

$$\dot{\boldsymbol{\alpha}} = -\lambda \frac{\partial f}{\partial \mathbf{A}}(\mathbf{x}, t), \quad (3.12)$$

where λ , $\frac{\partial f}{\partial \boldsymbol{\sigma}}$, and $\frac{\partial f}{\partial \mathbf{A}}(\mathbf{x}, t)$ are the plastic multiplier, the flow vector, and the generalised hardening modulus respectively. The occurrence of the evolution of the inelastic strain and the internal variables are defined by the loading and unloading conditions (*Karush-Kuhn-Tucker* condition)

$$f \leq 0 \quad \lambda \geq 0 \quad \dot{f}\lambda = 0. \quad (3.13)$$

The determination of the plastic multiplier λ requires $\dot{f}\lambda = 0$, the start point of plasticity ($f = 0$). The derivation of the yield function is

$$\dot{f} = \frac{\partial f}{\partial \boldsymbol{\sigma}} \dot{\boldsymbol{\sigma}} + \frac{\partial f}{\partial \mathbf{A}} \dot{\mathbf{A}}. \quad (3.14)$$

The stress rate according to the equation 2.28 is

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}^4(\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}_i) = \mathbf{E}^4 \left(\dot{\boldsymbol{\epsilon}} - \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}(\mathbf{x}, t)} \right). \quad (3.15)$$

The equation 3.14 can be rewritten according to equation 3.12 and $\mathbf{A} = \rho \frac{\partial \Phi_i}{\partial \boldsymbol{\alpha}}$ to

$$\dot{f} = \frac{\partial f}{\partial \boldsymbol{\sigma}} \mathbf{E}^4(\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}_i) + \frac{\partial f}{\partial \mathbf{A}} \rho \frac{\partial^2 \phi_i}{\partial \boldsymbol{\alpha}^2} \lambda \frac{\partial f}{\partial \mathbf{A}}(\mathbf{x}, t). \quad (3.16)$$

Hence, the plastic multiplier can be calculated by

$$\lambda = \frac{\frac{\partial f}{\partial \boldsymbol{\sigma}} \mathbf{E}^4 \dot{\boldsymbol{\epsilon}}}{\frac{\partial f}{\partial \boldsymbol{\sigma}} \mathbf{E}^4 \frac{\partial f}{\partial \boldsymbol{\sigma}} - \frac{\partial f}{\partial \mathbf{A}} \rho \frac{\partial^2 \phi_i}{\partial \boldsymbol{\alpha}^2} \frac{\partial f}{\partial \mathbf{A}}(\mathbf{x}, t)}. \quad (3.17)$$

The stress rate under plastic flow can be calculated according to equation 3.15 and 3.17 is

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}_{ei}^4 \dot{\boldsymbol{\epsilon}}, \quad (3.18)$$

where the elasto-plastic continuum tangent is a symmetric tensor

$$\mathbf{E}_{ei}^4 = \mathbf{E}_e^4 - \frac{\mathbf{E}^4 \frac{\partial f}{\partial \boldsymbol{\sigma}} \otimes \mathbf{E}^4 \frac{\partial f}{\partial \boldsymbol{\sigma}}}{\frac{\partial f}{\partial \boldsymbol{\sigma}} \mathbf{E}^4 \frac{\partial f}{\partial \boldsymbol{\sigma}} - \frac{\partial f}{\partial \mathbf{A}} \rho \frac{\partial^2 \psi_i}{\partial \boldsymbol{\alpha}^2} \frac{\partial f}{\partial \mathbf{A}}}. \quad (3.19)$$

The integration of the rate or evolution equations (constitutive equations) can be done with a numerical algorithm. The elastic predictor-corrector algorithm is used to solve the initial value problem of the elasto-plastic equations for complex strain behaviour. The algorithm consists of two sequentially steps, the elastic trial and the plastic corrector step or return-mapping algorithm. Hence, the first step is assumed to be in the elastic domain and the elastic trial strain is

$$\boldsymbol{\varepsilon}_{e,n+1}^{trial} = \boldsymbol{\varepsilon}_{e,n} + \Delta \boldsymbol{\varepsilon}, \quad (3.20)$$

where $\boldsymbol{\varepsilon}_e, n$ and $\Delta \boldsymbol{\varepsilon}$ are strain at the time t_n and the increment change of the strain, respectively. The elastic trial of the internal variables is

$$\boldsymbol{\alpha}_{n+1}^{trial} = \boldsymbol{\alpha}_n, \quad (3.21)$$

where $\boldsymbol{\alpha}_n$ is the internal variable at the time t_n . Therefore, the elastic trial stress and elastic trial hardening force are

$$\boldsymbol{\sigma}_{e,n+1}^{trial} = \rho \frac{\partial f}{\partial \boldsymbol{\varepsilon}} \bigg|_{n+1}^{trial}, \quad (3.22)$$

$$\mathbf{A}_{e,n+1}^{trial} = \rho \frac{\partial f}{\partial \boldsymbol{\alpha}} \bigg|_{n+1}^{trial}. \quad (3.23)$$

The equations 3.22 and 3.23 have to satisfy

$$f^{trial} = f(\boldsymbol{\sigma}_{n+1}^{trial}, \mathbf{A}_{n+1}^{trial}) \leq 0. \quad (3.24)$$

The update of the variables is done, when the elastic trial state lies within the elastic domain or on the yield surface. If it is not an elastic state, the second step, the return-mapping will determine the solution. Therefore, the elastic strain, the internal variables, and the plastic multiplier $\Delta \gamma > 0$ has to be solved

$$\boldsymbol{\varepsilon}_{e,n+1} = \boldsymbol{\varepsilon}_{e,n+1}^{trial} - \Delta \gamma \frac{\partial f}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}_{n+1}, \mathbf{A}_{n+1}), \quad (3.25)$$

$$\boldsymbol{\alpha}_{n+1} = \boldsymbol{\alpha}_{n+1}^{trial} - \Delta \gamma \frac{\partial f}{\partial \mathbf{A}}(\boldsymbol{\sigma}_{n+1}, \mathbf{A}_{n+1}), \quad (3.26)$$

$$0 = f(\boldsymbol{\sigma}_{n+1}, \mathbf{A}_{n+1}). \quad (3.27)$$

3.3 General Formulation of the Steady-State Algorithm

The mesh for the *Eulerian* approach is stationary and the material flows through the mesh. The main point in the steady-state algorithm [102], [100], [122] is the description of large displacements and small deformations. The calculation in the reference point of the loading, in *Eulerian* coordinates, the distortions of the elements are small and the mesh does not have to be updated and remains fixed throughout the calculation. The thermomechanical load is generally fixed and therefore, the mesh can be fined in certain regions. The other region can be coarsely meshed, which economises the memory space and calculation time. The history of the loads and the rotational velocity of the disc is calculated from the reference point of the load instead from the structure. The material flow is described in *Eulerian* coordinates. The advantage of this algorithm is the choice of the reference point on the load and negligible small deformations. This guarantees a fixed mesh for the whole simulation. The algorithm is based on the following assumptions: small deformation, elastoplastic constitutive law for the disc, *Fourier's* conduction law, the existence of coupling between the thermal constraint and the heat dissipated by the plastic deformation. Moreover, the heat production due to elastic deformation is neglected. The steady-state algorithm is described in the coordinate system, where the load is. The material derivative for scalar, vectorial, and tensorial quantities is

$$\dot{\Phi}(\mathbf{x}, t) = \frac{D}{Dt}\Phi(\mathbf{x}, t) = \frac{\partial \Phi}{\partial t}(\mathbf{x}, t) + \nabla \Phi(\mathbf{x}, t) \mathbf{v}(\mathbf{x}, t). \quad (3.28)$$

The velocity $\mathbf{v}$ of a material point is composed of

$$\mathbf{v}(\mathbf{x}, t) = \mathbf{v}_r(\mathbf{x}, t) - \mathbf{V}(\mathbf{x}, t), \quad (3.29)$$

where $\mathbf{v}(\mathbf{x}, t)$, $\mathbf{v}_r(\mathbf{x}, t)$, and $\mathbf{V}(\mathbf{x}, t)$ describe the velocity of a material point in the coordinate system of the load, the velocity of that point in the coordinate system of the structure, and the velocity of the material point or of the coordinate system of the load in regard to the coordinate system of the structure, respectively. The material point is described with the position vector $\mathbf{x}$ and the time t . Due to the assumption of small deformations in the domain $\mathcal{B}$, the velocity $\mathbf{v}_r(\mathbf{x}, t)$ is negligible compared to $\mathbf{V}(\mathbf{x}, t)$ and the total derivative of Φ becomes

$$\dot{\Phi} = \frac{\partial \Phi}{\partial t}(\mathbf{x}, t) - \nabla \Phi(\mathbf{x}, t) \cdot \mathbf{V}(\mathbf{x}, t). \quad (3.30)$$

In the domain $\mathcal{B}$ with the surface $\partial \mathcal{B}$ the mechanical load is composed of the volume force $\mathbf{f}_v$ in $\mathcal{B}$, surface force $\mathbf{f}_a$ on $\mathcal{S}$, and displacement $\mathbf{u}_a$ imposed on $\mathcal{S}$. The thermal load consists of internal heat production r in $\mathcal{B}$, imposed heat flow $\mathbf{q}_a$ on $\mathcal{S}$, imposed temperature T_a on $\mathcal{S}$. The description of the thermomechanical equations or the master balances are time-independent for the steady state algorithm. The momentum balance for the determination of the displacement vector is

$$\begin{aligned}
\operatorname{div} \boldsymbol{\sigma}(\mathbf{x}, t) + \mathbf{f}(\mathbf{x}, t) &= \rho \frac{D^2 \mathbf{x}}{Dt^2} = \rho \frac{D}{Dt} \left(\frac{\partial \mathbf{x}}{\partial t} - \boldsymbol{\nabla} \mathbf{x} \cdot \mathbf{V}(\mathbf{x}, t) \right) \\
&= \rho \left[\frac{\partial}{\partial t} \left(\frac{\partial \mathbf{x}}{\partial t} - \boldsymbol{\nabla} \mathbf{x} \cdot \mathbf{V}(\mathbf{x}, t) \right) - \boldsymbol{\nabla} \left(\frac{\partial \mathbf{x}}{\partial t} - \boldsymbol{\nabla} \mathbf{x} \cdot \mathbf{V}(\mathbf{x}, t) \right) \cdot \mathbf{V} \right] \\
&= \rho \left[\frac{\partial^2 \mathbf{x}}{\partial t^2} - 2 \boldsymbol{\nabla} \left(\frac{\partial \mathbf{x}}{\partial t} \right) \cdot \mathbf{V}(\mathbf{x}, t) - \boldsymbol{\nabla} \mathbf{x} \left(\frac{\partial \mathbf{V}(\mathbf{x}, t)}{\partial t} \right) + \right. \\
&\quad \left. + \boldsymbol{\nabla} (\boldsymbol{\nabla} \mathbf{x} \cdot \mathbf{V}(\mathbf{x}, t)) \cdot \mathbf{V}(\mathbf{x}, t) \right], \tag{3.31}
\end{aligned}$$

where $\boldsymbol{\sigma}$ are the stresses. Furthermore, the energy balance for the determination of the temperature with internal variables is

$$\begin{aligned}
&\lambda \Delta T(\mathbf{x}, t) - \rho c \frac{\partial T(\mathbf{x}, t)}{\partial t} + \rho c \boldsymbol{\nabla} T(\mathbf{x}, t) \cdot \mathbf{V}(\mathbf{x}, t) = \\
&-r - \boldsymbol{\sigma} \cdot \left(\frac{\boldsymbol{\varepsilon}_i(\mathbf{x}, t)}{\partial t} - \boldsymbol{\nabla} \boldsymbol{\varepsilon} \cdot \mathbf{V}(\mathbf{x}, t) \right) + \mathbf{A}_k \cdot \left(\frac{\partial \boldsymbol{\alpha}_k(\mathbf{x}, t)}{\partial t} - \boldsymbol{\nabla} \boldsymbol{\alpha}_k(\mathbf{x}, t) \cdot \mathbf{V}(\mathbf{x}, t) \right), \tag{3.32}
\end{aligned}$$

where λ , ρ , c , T , r , $\boldsymbol{\varepsilon}_i$, $\mathbf{A}_k$, and $\boldsymbol{\alpha}_k$ are the thermal conductivity, the density, the specific heat capacity, the temperature, the internal heat production, the strain tensor, the inelastic strain tensor, the thermodynamical forces, and the internal variables, respectively.

3.4 Steady-State Algorithm

The steady state algorithm [127], [86], [101], [100], [102], [122] is developed for the repeating loadings. The assumption is the stationarity of the load. Here, the load is for the current cycle constant but can change for the next cycle in regards to the applied loads. In regards to the equations (3.31) and (3.32), the time derivatives and the time-dependent variables vanish. In one calculation pass, the dependence on time is freed. The total time derivative becomes

$$\dot{\Phi} = -\boldsymbol{\nabla} \Phi(\mathbf{x}, t) \cdot \mathbf{V}(\mathbf{x}, t). \tag{3.33}$$

The momentum balance equation to determine the displacement vector is

$$\operatorname{div} \boldsymbol{\sigma}(\mathbf{x}, t) + \mathbf{f}(\mathbf{x}, t) = \rho \frac{D^2 \mathbf{x}}{Dt^2} = \rho \boldsymbol{\nabla} (\boldsymbol{\nabla} \mathbf{x} \cdot \mathbf{V}(\mathbf{x}, t)) \cdot \mathbf{V}(\mathbf{x}, t). \tag{3.34}$$

The elastoplastic behaviour is described with

$$\boldsymbol{\sigma}(\mathbf{x}, t) = \mathbf{E}^4 [\boldsymbol{\varepsilon}(\mathbf{x}, t) - \boldsymbol{\varepsilon}_i(\mathbf{x}, t) - \alpha(T(\mathbf{x}, t) - T_0(\mathbf{x}))\mathbf{I}] + \boldsymbol{\sigma}_0(\mathbf{x}), \tag{3.35}$$

where $\boldsymbol{\sigma}_0(\mathbf{x})$ is the stress tensor at the position $\mathbf{x}$ and initial time. The decomposition of the strain is

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_i. \quad (3.36)$$

The total strain tensor can be determined with the displacement vector

$$\boldsymbol{\varepsilon}(\mathbf{x}, t) = \frac{1}{2} \left(\nabla \mathbf{u}(\mathbf{x}, t) + \nabla^T \mathbf{u}(\mathbf{x}, t) \right). \quad (3.37)$$

The plastic flow law, which is required for the return mapping algorithm, is

$$\dot{\boldsymbol{\varepsilon}}_i = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}(\mathbf{x}, t)} = -\nabla \boldsymbol{\varepsilon}_i(\mathbf{x}, t) \mathbf{V}(\mathbf{x}, t) \quad (3.38)$$

with $\lambda f = 0$, $\lambda \geq 0$, $f \leq 0$. Moreover, the evolution function of the internal variables is

$$\dot{\boldsymbol{\alpha}}_k = -\lambda \frac{\partial f}{\partial A_k}(\mathbf{x}, t) = -\nabla \boldsymbol{\alpha}_k(\mathbf{x}, t) \mathbf{V}(\mathbf{x}, t). \quad (3.39)$$

The derivation of the heat equation requires the first law of thermodynamics

$$\rho \dot{\epsilon} = \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} + r - \operatorname{div} \mathbf{q}, \quad (3.40)$$

where $\dot{\epsilon}$ are the time derivation of the internal energy. The heat influx can be described with *Fourier's law*

$$\mathbf{q}(\mathbf{x}, t) = -k_\Theta \nabla \Theta(\mathbf{x}, t). \quad (3.41)$$

If we choose the specific free energy as a function of $\psi = \psi(T, \boldsymbol{\varepsilon}_i, \mathbf{V}_k)$ and therefore the entropy, the stress tensor and the thermodynamical forces can be calculated:

$$s = -\frac{\partial \psi}{\partial T}, \quad (3.42)$$

$$\boldsymbol{\sigma} = \rho \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}_e}, \quad (3.43)$$

$$A_k = \rho \frac{\partial \psi}{\partial \alpha_k}. \quad (3.44)$$

With the equations 3.42-3.44 the free energy is

$$\dot{\psi} = \frac{\partial \psi}{\partial T} \dot{T} + \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}_e} \cdot \dot{\boldsymbol{\varepsilon}}_e + \frac{\partial \psi}{\partial \alpha_k} \dot{\alpha}_k = -s \dot{T} + \frac{1}{\rho} \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}}_e + \frac{1}{\rho} A_k \dot{\alpha}_k. \quad (3.45)$$

The thermal internal energy is therefore,

$$\begin{aligned} \dot{u} = T\dot{s} - s\dot{T} + \frac{1}{\rho}\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}}_e + \frac{1}{\rho}A_k\dot{\alpha}_k + s\dot{T} - \frac{1}{\rho}\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} = T\dot{s} + \frac{1}{\rho}\boldsymbol{\sigma} \cdot (\dot{\boldsymbol{\varepsilon}}_e - \dot{\boldsymbol{\varepsilon}}_i) + \\ + \frac{1}{\rho}A_k\dot{\alpha}_k = T\dot{s} - \frac{1}{\rho}\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}}_i + \frac{1}{\rho}A_k\dot{\alpha}_k. \end{aligned} \quad (3.46)$$

The derived heat equation with neglected terms of thermomechanical coupling $\dot{u} = T\dot{s} = c\dot{T}$ reads

$$r + k \Delta T = \rho c \dot{T} - T(\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}}_e + A_k\dot{\alpha}_k) - \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}}_i + A_k\dot{\alpha}_k. \quad (3.47)$$

The discretised equations can be simplified for the internal thermal energy $u = u_0 + c(T - T_0)$ and the derivative with respect to the temperature $\frac{\partial u}{\partial T} = c$. The derivative of the heat flux vector with respect to the temperature is $\frac{\partial \mathbf{q}}{\partial T} = \mathbf{0}$. Moreover, the derivatives of the internal thermal energy and the heat flux vector with respect to the temperature are $\frac{\partial u}{\partial(\nabla T)} = 0$ and $\frac{\partial \mathbf{q}}{\partial(\nabla T)} = -k\mathbf{I}$. Taking into consideration the steady state algorithm, the heat equation is

$$\begin{aligned} k \Delta T(\mathbf{x}, t) - \rho c \dot{T} = -r(\mathbf{x}, t) - \boldsymbol{\sigma}(\mathbf{x}, t) \cdot \dot{\boldsymbol{\varepsilon}}_i(\mathbf{x}, t) + \mathbf{A}_k(\mathbf{x}, t) \cdot \dot{\boldsymbol{\alpha}}_k(\mathbf{x}, t) = -r(\mathbf{x}, t) - \\ - \boldsymbol{\sigma}(\mathbf{x}, t) \cdot (-\nabla \boldsymbol{\varepsilon}_i(\mathbf{x}, t) \mathbf{V}(\mathbf{x}, t)) + A_k(\mathbf{x}, t) \cdot (-\nabla \alpha_k(\mathbf{x}, t) \mathbf{V}(\mathbf{x}, t)). \end{aligned} \quad (3.48)$$

The elasticity domain defined by elastoplastic model with kinematic and isotropic flow in the form of *von Mises*

$$f(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_i) = \sqrt{(\mathbf{s} - H\boldsymbol{\varepsilon}_i) \cdot (\mathbf{s} - H\boldsymbol{\varepsilon}_i)} - \sigma_y(\bar{\boldsymbol{\varepsilon}}_i(t)), \quad (3.49)$$

where the deviator of $\boldsymbol{\sigma}$ is $\boldsymbol{\sigma}^D = \boldsymbol{\sigma} - \frac{1}{3}(\text{tr } \boldsymbol{\sigma})\mathbf{I}$. The work hardening module H describes the ratio of the stress and the strain after the yield stress. Furthermore, the cumulated plastic deformation describes the summation of the plastic deformation $\bar{\boldsymbol{\varepsilon}}_i(t) = \bar{\boldsymbol{\varepsilon}}_i(0) + \int_t \sqrt{\dot{\boldsymbol{\varepsilon}}_i(\tau) \cdot \dot{\boldsymbol{\varepsilon}}_i(\tau)} d\tau$ and $\sigma_y(\bar{\boldsymbol{\varepsilon}}_i(t))$ describes the yield stress according to the changing plastic deformation or the hardening rule. The numerical solution of the mechanic problem for elastoplastic material in the form of *von Mises* with kinematic flow is based on [122]. The plastic multiplier is calculated according to

$$\lambda = \frac{1}{2\mu + H} \frac{\|\boldsymbol{\xi}^*\| - \sigma_y}{\Delta t}, \quad (3.50)$$

where μ , H , $\|\boldsymbol{\xi}^*\|$, and σ_y are the shear modulus, the hardening module, the return mapping algorithm, and the yield stress, respectively. The return-mapping algorithm is

$$\boldsymbol{\xi}^* = \mathbf{s}_0 - H\boldsymbol{\varepsilon}_{i0} + 2\mu\Delta\mathbf{e} - (2\mu + H)\Delta\boldsymbol{\varepsilon}_i(n). \quad (3.51)$$

Then, if $\|\boldsymbol{\xi}^*\| \leq \sigma_y$, there is no plastic deformation

$$\boldsymbol{\varepsilon}_i = \boldsymbol{\varepsilon}_{i0} + \Delta\boldsymbol{\varepsilon}_i(n). \quad (3.52)$$

If $\|\boldsymbol{\xi}^*\| > \sigma_y$, there is plastic deformation. With the plastic flow rule

$$\dot{\boldsymbol{\varepsilon}}_i = \lambda \left(\frac{\partial f}{\partial \boldsymbol{\sigma}} \right), \quad (3.53)$$

the new inelastic strain is calculated with

$$\boldsymbol{\varepsilon}_i = \boldsymbol{\varepsilon}_{i0} + \Delta\boldsymbol{\varepsilon}_i(n) + \frac{1}{2\mu + H} \left(1 - \frac{k}{\|\boldsymbol{\xi}^*\|} \right) \boldsymbol{\xi}^*. \quad (3.54)$$

The stress tensor is determined with

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_0 + \mathbf{E}^4 [\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}_i - \alpha\Delta T \mathbf{I}]. \quad (3.55)$$

The continuum tangent matrix for the convergence criteria can be calculated to assure the quadratic convergence. If there is no plastic deformation

$$\frac{\partial \Delta\boldsymbol{\sigma}}{\partial \Delta\boldsymbol{\varepsilon}} = \mathbf{E}^4. \quad (3.56)$$

If there is plastic deformation

$$\frac{\partial \Delta\boldsymbol{\sigma}}{\partial \Delta\boldsymbol{\varepsilon}} = \mathbf{E}^4 - \frac{4\mu^2}{2\mu + H} \left(\mathbf{I}^4 - \frac{1}{3} \mathbf{I} \otimes \mathbf{I} \right) \left[\frac{\boldsymbol{\xi}^* \otimes \boldsymbol{\xi}^*}{\|\boldsymbol{\xi}^*\|^2} + (\|\boldsymbol{\xi}^*\| - k) \left(\frac{\mathbf{I}^4}{\|\boldsymbol{\xi}^*\|} - \frac{\boldsymbol{\xi}^* \otimes \boldsymbol{\xi}^*}{\|\boldsymbol{\xi}^*\|^3} \right) \right]. \quad (3.57)$$

The calculation of the steady-state algorithm (pass by pass) for an elastoplastic material [122] is described below. At the beginning, the simulation starts with an initialisation of the thermomechanical variables, such as the initial total and inelastic strain, the initial stress and the initial temperature along each current line. Then, ΔT as temperature change increases for the current time increment is added. The stress and strain increase for the initial step is defined to zero. At the first step, the stress and strain increase are calculated with regards to the thermomechanical equations for the current increment. In the second step, the total strain increment for the next increment ($j + 1$) is calculated. The third step checks the plasticity of the solid for the next increment with the return algorithm. If the stress is smaller than the yield stress, an elastic deformation is observed and the strain increment is added for the next increment. If the stress is larger than the yield stress, an elastic and plastic increment is added for the next increment. Finally, the stress for the next increment is calculated with regards to the elastic and plastic increment and the temperature increment. The fourth step checks the convergence with the continuum tangent matrix. If the algorithm is convergent, the algorithm starts again with the initialisation. If not, the algorithm starts at the first step with a smaller time increment.

0. Given at time t : $\{\boldsymbol{\sigma}_0, \boldsymbol{\varepsilon}_0, \boldsymbol{\varepsilon}_{i0}, T_0\}$ along each current line
 Given at time $t + \Delta t$: $\{\Delta T\}$
 Initialisation: $\{\Delta \boldsymbol{\sigma}_j, \Delta \boldsymbol{\varepsilon}_j, \Delta \boldsymbol{\varepsilon}_{i,j}\} = \{0,0,0\}$
1. Given at time $t_j = t + \Delta t_j$ for increment j : $\{\Delta \boldsymbol{\sigma}_j, \Delta \boldsymbol{\varepsilon}_j, \Delta \boldsymbol{\varepsilon}_{i,j}\}$
2. Calculate the global equation and obtain $\Delta \boldsymbol{\varepsilon}_{j+1}$
3. Plastic test $\boldsymbol{\xi}_{j+1}^* = \boldsymbol{s}_0 - H \boldsymbol{\varepsilon}_{i0} + 2\mu \Delta \boldsymbol{\varepsilon}_{j+1} - (2\mu + H) \Delta \boldsymbol{\varepsilon}_{i,j+1}(n)$
 If $\|\boldsymbol{\xi}_{j+1}^*\| \leq k \rightarrow$ Elastic increment: $\boldsymbol{\varepsilon}_{i,j+1} = \boldsymbol{\varepsilon}_{i0} + \Delta \boldsymbol{\varepsilon}_{i,j+1}(n)$
 Else $\rightarrow$ Plastic increment: $\boldsymbol{\varepsilon}_{i,j+1} = \boldsymbol{\varepsilon}_{i0} + \Delta \boldsymbol{\varepsilon}_{i,j+1}(n) + \frac{1}{2\mu + H} \left(1 - \frac{k}{\|\boldsymbol{\xi}^*\|}\right) \boldsymbol{\xi}^*$
 Finally calculate: $\boldsymbol{\sigma}_{j+1} = \boldsymbol{\sigma}_0 + \mathbf{E}^4 \cdot [\Delta \boldsymbol{\varepsilon}_{j+1} - \Delta \boldsymbol{\varepsilon}_{i,j+1} - \alpha \Delta T \mathbf{I}]$
4. Convergence Test $\frac{\partial \Delta \boldsymbol{\sigma}}{\partial \Delta \boldsymbol{\varepsilon}}$
 Convergence: go to 0
 No convergence: go to 1

3.5 Comparison of Different Algorithms

The different steady-state algorithms and the TRC algorithm are compared in table 3.1. The TRC algorithm is explained in [102]. The stress calculation and the yield criteria are for all algorithms the same. The algorithms differ in the summation of the inelastic strains. Furthermore, each algorithm is applicable for a certain problem. The definition of the boundary conditions and the understanding of the limits of the algorithms are necessary for appropriate numerical solution. The steady-state algorithm pass by pass is applicable for a certain number of rotations. Moreover, the steady-state algorithm direct is used for a high number of rotations. The steady-state algorithm periodic can be used for the periodical repeating of the load without the changing of the amplitude of the load.

TRC	Steady-State (pass by pass)	Steady-State (direct)	Steady-State (periodic)
Integrate the history of the load (the evolution of heat flow, contact surface)	In every passage, the load is considered as constant and moves at a constant velocity	If the load cycles are identical and the structure is finally adapted or accommodated.	For a periodic structure with a moving load, the adjacent element could be vacant

$$\boldsymbol{\sigma}(\mathbf{x}, t) = \boldsymbol{\sigma}_0 + \mathbf{E}^4 \cdot [\boldsymbol{\varepsilon}(\mathbf{x}, t) - \boldsymbol{\varepsilon}_i(\mathbf{x}, t) - \alpha(T(\mathbf{x}, t) - T_0(\mathbf{x}))\mathbf{I}]$$

$$\mathbf{q}(\mathbf{x}, t) = -k \nabla T(\mathbf{x}, t)$$

$$f(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_i) = \sqrt{(\mathbf{s} - H\boldsymbol{\varepsilon}_i) : (\mathbf{s} - H\boldsymbol{\varepsilon}_i)} - k(\bar{\boldsymbol{\varepsilon}}_i(t))$$

$$\begin{aligned} \frac{\boldsymbol{\varepsilon}_i - \boldsymbol{\varepsilon}_{i0}}{\Delta t} + \frac{\boldsymbol{\varepsilon}_{i0} - \boldsymbol{\varepsilon}_{i0}(n)}{\Delta x} \mathbf{V} & \frac{\boldsymbol{\varepsilon}_{i0} - \boldsymbol{\varepsilon}_{i0}(n)}{\Delta x} \mathbf{V} = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} & \frac{\boldsymbol{\varepsilon}_{i0} - \boldsymbol{\varepsilon}_{i0}(n)}{\Delta x} \mathbf{V} = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} & \Psi(x, t) = \Psi(x+X, t+T) \\ & = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} & \boldsymbol{\varepsilon}_i = \boldsymbol{\varepsilon}_{i0} + \Delta \boldsymbol{\varepsilon}_i(n) + \dots & \boldsymbol{\varepsilon}_i = \boldsymbol{\varepsilon}_{i0} + \Delta \boldsymbol{\varepsilon}_i(n) + \dots & \boldsymbol{\varepsilon}_{i,j} = \boldsymbol{\varepsilon}_{i,ini} + \Delta \boldsymbol{\varepsilon}_{i,j-1} + \dots \\ \boldsymbol{\varepsilon}_i = (1 - \frac{V\Delta t}{\Delta x}) \boldsymbol{\varepsilon}_{i0} & & \text{Stop the calculation when} & & \\ + \frac{V\Delta t}{\Delta x} \boldsymbol{\varepsilon}_{i0}(n) + \dots & & \boldsymbol{\varepsilon}_i(-\infty) = \boldsymbol{\varepsilon}_i(+\infty) & & \end{aligned}$$

can be used for all the problems, especially for those where the velocity and amplitude of the load should be simulated precisely. Calculation is quicker than the classical algorithm in Abaqus

Less calculations compared to TRC and being useful when simulating a great number of cycles

Much quicker than passage by passage when searching the stabilising response of a structure after a great number of cycles

Quicker calculation of the periodic structure

Table 3.1: Comparison of different algorithms

3.6 Simulation of a Brake Disc

The simulation of the coupled problem, such as the friction system brake disc and pad, is an intricate work for the determination of the stress and temperature distribution of the brake disc and pad. The contact definition and the high heat fluxes generate convergence problems. Furthermore, the simulation time is excessively high. Only a couple of hours are needed for the simulation of uncoupled problems by using the *Eulerian* method, where as, a couple of days is needed whenever the *Lagrangian* method is used. The final simulation with the steady-state algorithm requires a simulation time of five hours for a 40 s braking. The developed *Python* scripting offers the possibility of an automatically simulation of the friction pair. The input parameters are the normal force, the velocity, the brake time and the geometries of the friction pair. Moreover, the mesh in the steady-state algorithm, which is based on the *Eulerian* approach is fixed and the material flows through the elements. This approach is generally used in fluid mechanics due to large displacement and small deformations. Furthermore, the force is applied on the top of the pad and can only move perpendicular to the contact surface. The heat flow entering into the brake disc is generated by friction

$$\|\mathbf{q}(t)\| = \mu \|\boldsymbol{\sigma}(t)\| \|\mathbf{v}(t)\|, \quad (3.58)$$

where μ , $\|\mathbf{v}(t)\|$, and $\|\boldsymbol{\sigma}(t)\|$ are the local coefficient of friction, which changes during the brake process, the magnitude of the velocity of the brake disc, and the magnitude of the pressure between the brake disc and pad, which also varies due to the coefficient of friction and temperature, respectively. The heat flow is determined from the kinetic energy of the dissipated vehicle during the braking. The dissipated kinetic energy during the braking can be calculated with

$$E_c = \frac{1}{2} \boldsymbol{\omega}(t) \cdot \boldsymbol{\Theta} \boldsymbol{\omega}(t), \quad (3.59)$$

where $\boldsymbol{\omega}$, $\boldsymbol{\Theta}$ are the angular velocity, and the mass moment of inertia tensor, respectively. Moreover, the velocity of rotation decreases linearly with the time t

$$\boldsymbol{\omega}(t) = \boldsymbol{\omega}_0 \left(1 - \frac{t}{t_B}\right), \quad (3.60)$$

where $\boldsymbol{\omega}_0$ and t_B are the initial angular velocity of the brake disc and the brake time, respectively. Herein, the heat flow into the brake disc with respect to area and time is

$$\|\mathbf{q}(t)\| = \frac{dE_c(t)}{dt} = 2 \frac{E_c}{S} \left(\frac{t}{t_B^2} - \frac{1}{t_B} \right). \quad (3.61)$$

The brake disc is defined by an axis-symmetric model, which is then symmetrically swivelled to define the 3D model of the brake disc. The revolution of the axisymmetric model can be conducted for one model for different angles, which permits to refine the mesh in the radial and tangential direction of the contact area. The restriction of this kind of simulation is just the application of axisymmetric models and applying the brake pad after the revolution of the brake disc. The mass flow of the material through the mesh is defined in the thermal analysis. The standard implicit analysis is conducted in *Abaqus* 6.14. The user subroutine USMASFL is used for the calculation of the mass flow rate. The mass flow describes the material, which is flowing per area and time through a controlled volume. The subroutine is called for each node and is calculated by

$$\dot{\mathbf{m}}(t) = \rho \mathbf{v}(t) = \rho \boldsymbol{\omega}(t) \times \mathbf{r}(t), \quad (3.62)$$

where ρ , $\mathbf{v}$, $\boldsymbol{\omega}(t)$, and $\mathbf{r}(t)$ are the density, the velocity, the angular velocity, and the radius, respectively. The simulation is conducted with the same characteristics as the experiment in [69], [70], [71]. The experiment and simulation are conducted with an initial speed of 2200 rpm, a force of 850 N and a brake time of 40 s. The friction coefficient is 0.35. The material properties of the brake pad and disc are shown in tables 3.2 and 3.3.

The simulation methodology is shown in figure 3.1. The simulation starts with an initialisation script, which defines the initial parameters of the simulation, such as the input of the brake parameters (initial speed, brake time, brake force) and the internal variables such as the time passed, the current speed and the step time. After this initialisation, the mechanical analysis is conducted to calculate the normal forces in the contact area and the deformation of the geometry of the pad. After this mechanical analysis, the normal forces in the contact area and the pad geometry are extracted. Then, the heat fluxes are calculated according to the normal

Temperature T [°C]	20	200	400	600
Density ρ [kg/m ³]	2480	2400	2200	1700
Youngs modulus E [Gpa]	2,2	1,8	1,2	0.75
Poissons ratio ν [-]	0.306	0.312	0.320	0.337
Specific heat c [J/kgK]	770	920	960	900
Thermal conductivity λ [W/mK]	14.5	13.2	12	9.2
Thermal expansion coefficient α [1/K]	$8 \cdot 10^{-6}$	$8 \cdot 10^{-6}$	$8 \cdot 10^{-6}$	$8 \cdot 10^{-6}$

Table 3.2: Material properties of the pad

Temperature T [°C]	20	150	450	600	750
Density ρ [kg/m ³]	7850	7850	7850	7850	7850
Youngs modulus E [Gpa]	210	195	177	133	100
Temperature T [°C]	0	200	400	600	800
Poisson ratio ν [-]	0.29	0.29	0.29	0.29	0.29
Specific heat c [J/kgK]	440	510	570	630	700
Thermal conductivity λ [W/mK]	47.3	44.1	39.3	32.9	25.0
Thermal expansion coefficient α [1/K]	$10.8 \cdot 10^{-6}$	$11.8 \cdot 10^{-6}$	$13.5 \cdot 10^{-6}$	$15.8 \cdot 10^{-6}$	$18.5 \cdot 10^{-6}$

Table 3.3: Material properties of the disc

forces and the friction coefficient. The calculated heat flux is applied in the axisymmetric thermal analysis for the brake disc and the thermal analysis for the pad. The results of the thermal analyses of the brake pad and the brake disc is the temperature; however, in addition to the temperature, the new pad geometry is regularly updated for the next loop. If the brake time is achieved, the simulation stops and if not, the next loop is started.

The calculation of the time increment or the time for a certain position of the brake disc is shown below. The angular displacement of the material flow for one turn per loop is $\varphi = 2\pi$ except for the last one. The time t_c of the current round can be determined by

$$\int_{t_p}^{t_p+t_c} \omega dt = \int_{t_p}^{t_p+t_c} \omega_c - \omega_0 \frac{t}{t_B} dt = \varphi, \quad (3.63)$$

where t_p , ω_c , ω_0 , t_B are the time passed, the corresponding angular velocity, the initial angular velocity, and the total brake time, respectively. After the integration of equation 3.63, the time increment can be calculated by

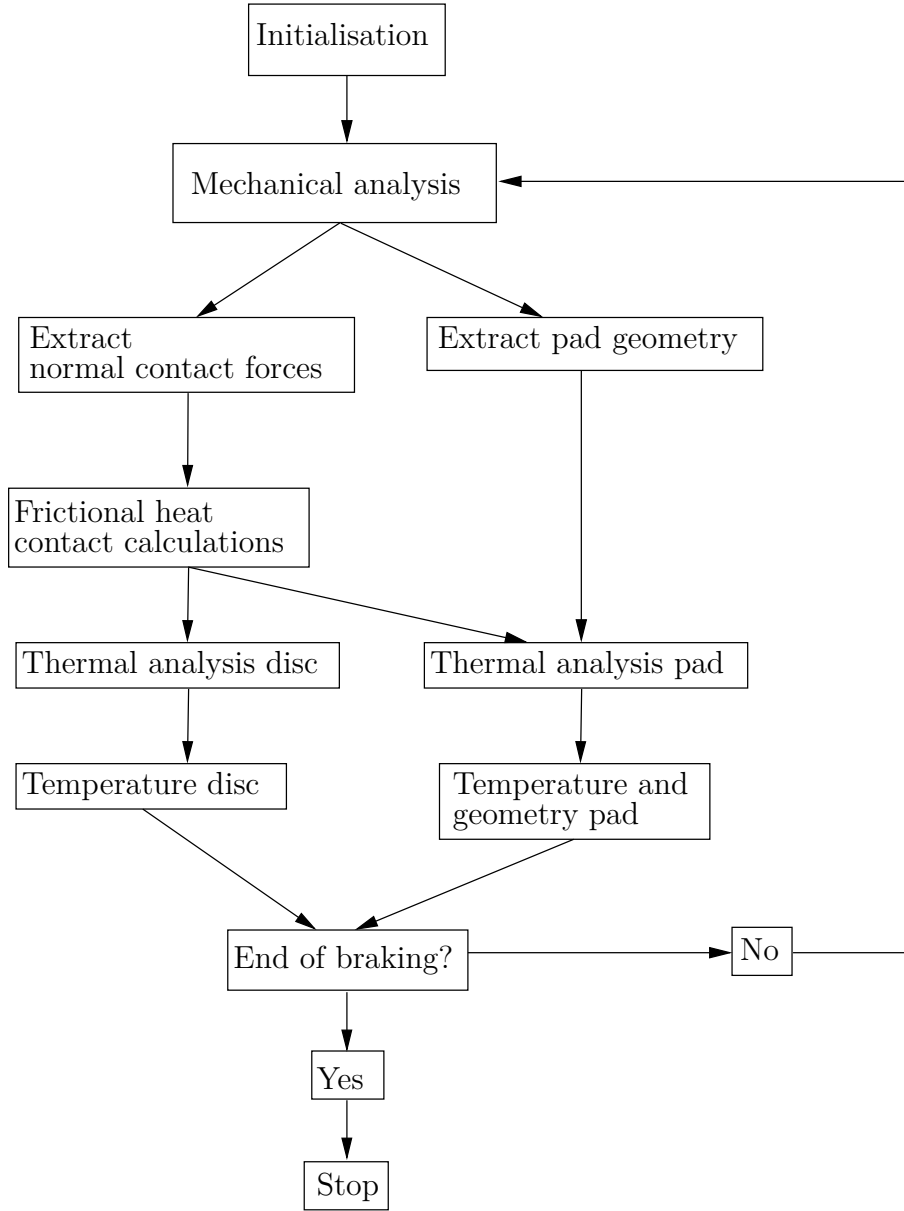


Figure 3.1: Simulation methodology of the steady-state algorithm

$$t_c^2 - 2 t_B \left(\frac{2 \omega_c}{\omega_0} - 1 \right) t_c + 2 t_B \frac{\varphi}{\omega_0} = 0. \quad (3.64)$$

The application of the completing the square is

$$\Leftrightarrow \left(t_c - t_B \left(\frac{2 \omega_c}{\omega_0} - 1 \right) \right)^2 = \left(t_B \left(\frac{2 \omega_c}{\omega_0} - 1 \right) \right)^2 - 2 t_B \frac{\varphi}{\omega_0}. \quad (3.65)$$

After solving the equation 3.64, there is at least one solution if φ in equation 3.66 satisfies the inequality

$$\left(t_B \left(\frac{2\omega_c}{\omega_0} - 1\right)\right)^2 - 2t_B \frac{\varphi}{\omega_0} \geq 0 \quad \Leftrightarrow \quad \varphi \leq t_B \left(\frac{2\omega_c^2}{\omega_0} - 2\omega_c + \frac{1}{2}\omega_0\right). \quad (3.66)$$

Accordingly, the solutions of the equation 3.64 are

$$t_{c1,c2} = t_B \left(\frac{2\omega_c}{\omega_0} - 1\right) \pm \sqrt{\left(t_B \left(\frac{2\omega_c}{\omega_0} - 1\right)\right)^2 - 2t_B \frac{\varphi}{\omega_0}}. \quad (3.67)$$

There are two solutions of the equation 3.67, which can be determined correctly by a case-by-case analysis. The first solution is

$$t_p + t_{c1} = \frac{\omega_c}{\omega_0} t_B - \sqrt{\left(t_B \left(\frac{2\omega_c}{\omega_0} - 1\right)\right)^2 - 2t_B \frac{\varphi}{\omega_0}} \leq t_B \quad (3.68)$$

and the second solution is

$$t_p + t_{c2} = \frac{\omega_c}{\omega_0} t_B + \sqrt{\left(t_B \left(\frac{2\omega_c}{\omega_0} - 1\right)\right)^2 - 2t_B \frac{\varphi}{\omega_0}} \geq t_B. \quad (3.69)$$

With $t_p + t_{c1} \leq t_B$, the correct time step of the certain round t_{c1} is given by

$$t_{c1} = t_B \left(\frac{2\omega_c}{\omega_0} - 1\right) - \sqrt{\left(t_B \left(\frac{2\omega_c}{\omega_0} - 1\right)\right)^2 - 2t_B \frac{\varphi}{\omega_0}}. \quad (3.70)$$

The angular displacement of the last round is

$$\int_{t_p}^{t_B} \omega_c - \omega_0 \frac{t_B - t}{t_B} dt = \frac{\omega_c^2 t_B}{2\omega_0}, \quad (3.71)$$

where the time passed can be determined with $t_B - t_p = \frac{\omega_c}{\omega_0} t_B$. The total number of turns is 733,33. The determination of the number of turns for a certain loop is determined with regards to the simulation time for one loop. The angular speed of the brake disc is decreasing and therefore, the time for the increasing loops is increasing. The simulation time for the uncoupled thermomechanical simulation for a single processor is 5 h, which can be decreased to 3 h with multi-processing. Moreover, it should be considered as the real acting forces on the element for the calculation of the applied heat flux on the elements. The temperature distribution of the simulation of the brake disc is shown in figure 3.2.

The simulation results show a good agreement with the experimental results. The maximum temperature is at the outer diameter 309 °C. The formation of hot band could be well observed. The steady-state algorithm with the python scripting shows a good simulation results with fast computation time. The simulation methodology or the steady-state algorithm can be extended by adding film conditions on the brake disc and pad. The calculation of the wear should be considered for a better simulation. Plasticity or viscoplasticity should be used for the material behaviour of the brake disc and pad. In summary, the simulation methodology reduces the computation time from days to several hours. Furthermore, the reduction of the computation

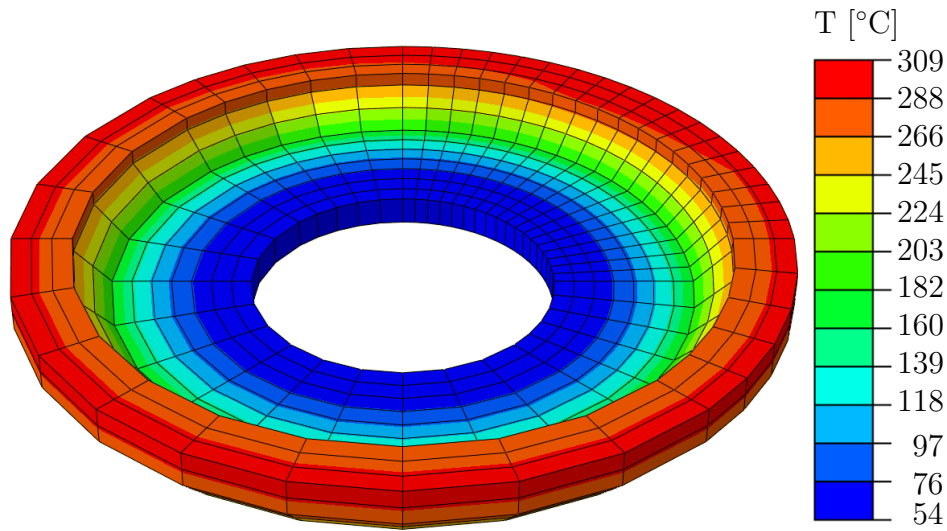


Figure 3.2: Temperature distribution of the brake disc

time can be achieved with the increasing of the CPU's. The best computation time is achieved with 2 CPU's. The used workstation is a Dell Precision T7500 with an operatin system of CentOS 6, which consists of 8 Xeon E5620 CPU with a clock rate of 2400 MHz and 12288 MB RAM. An increase in the CPU number leads to an ineffective computation time because *Abaqus* tries to share the computation between the CPU's. Another possibility of the reduction of the computational time is the multiprocessing, which allows to launch two computations at the same time. In this case, it is used for the thermal analysis of the brake disc and pad. The effectiveness is very high by analyses of the same order of magnitude. It allows the duration of the simulation to be reduced by 2 hours, from 5 to 3 hours.

4 Design of Test Benches

The design process of a plant or a machine relies on the need to meet the customer's requirements, as well as, development and production time and local conditions. Another important point in the design process is the itemisation of the requirements specification. The performance of the flywheel test bench depends on the local boundary conditions. Furthermore, the flywheel test bench is installed on the first floor, and consequentially the load capacity of the ceiling has to be considered. Due to these boundary conditions, the diameter of the flywheels and the power of the electric motor is limited. The weight of the clamping plate also reduces the load capacity of the ceiling. The dimension of the clamping plate is 3000 mm $\times$ 1250 mm and has a weight of 2200 kg. The load capacity of the ceiling is 1000 kg/m². The dimension of the test bench room is 4550 mm $\times$ 4200 mm. The control room has a dimension of 4550 mm $\times$ 2000 mm. Through these local conditions and the load capacity of the ceiling, the performance of the flywheel test bench is limited. The unique selling point of the flywheel test bench is the modularity and the installed measurement technology. The purchase of a flywheel test bench starts at 250000 € and could reach the million Euro range.

4.1 Brake Components

The engine power and the vehicle's mass play a vital role in providing a good acceleration performance, as a result a higher break performance is applied in deceleration. Components, because vibration is suppressed, are non-suspended masses. The unsprung masses are dangerous because of the resonance phenomena of the components. The excitation of high masses of non-suspended assemblies require a higher period of time to return to the rest position. The brake system has a long leverage on the springs and therefore, strong forces are applied on a high mass. There are several components in the car or train, which are unsprung. The non-suspended components in the car are the brake system, the wheel, the rim, the axle shaft, the wheel bearing, the damper, the spring, the anti-roll bar, and the axle guide. Their masses act directly on the ground and are non-suspended. The car body is suspended and therefore, the unevenness of the roadway surface cannot be totally detected. The increase of the non-suspended masses must be balanced with the suspended car body. Therefore, decreasing the total mass of the unsprung components, enhances the driving characteristics of a vehicle. The reduction of the weight of the brake system force can be achieved with the consideration of the compound brake discs. The compound brake discs are lighter and insensitive in regard to the thermal load compared to the conventional brake discs. The compound brake discs consists of a grey cast iron friction ring and an aluminium brake disc pot. They are connected with moulded stainless steel pins. The advantage of the separation besides the weight improvement, is the compensation of the thermal expansion and the resulting thermal stresses. The compound brake discs, provide a good performance in regard to the thermal loads due to the internal ventilation and cooling hole. A disadvantage is the strength of the brake disc pot, which can be deformed by mounting the brake discs and the wheel rim. The brake test bench is used to test the brake performance of a vehicle. The brake performance is one of the most important parts of a vehicle. So brake performance testing has a special meaning in the process of designing a

vehicle.

Nowadays, the brake system of a vehicle can be divided into two types; the drum brake system and the disc brake system. The drum brake is used for financial reasons, self reinforcement, and the combined parking brake. The disc brake has better cooling option. The advantages of drum brakes are the relative simple structure and low costs. The disadvantages are the complexity for inspection, the low brake force and problems with heat dissipation. The drum brakes are usually used in a small vehicle, which has relatively low requirements for the brake force and brake stability. A disc brake has the ability to overcome all of the drum brake's disadvantages. A disc brake can provide a high brake force and the friction heat can also be smoothly dissipated. A disc brake has also the ability to provide a high brake force as a result the friction heat is smoothly dissipated (only if the dissipation is a result of the high brake force). The brake stability is superior to the drum brake. This is why, the automotive industry uses mostly this type of brakes. For the testing and the evaluation of the disc brake, a brake test bench is particularly needed. One of the core problems of such an inertia brake tester is how to simulate the inertia of a real vehicle on the brake test bench. According to the different ways of simulating the inertia on the test bench, there are two main types, the mechanical inertia and the electric inertia test bench. The mechanical inertia test bench is the most common brake tester. For the mechanical inertia test bench, the inertia moment of the vehicle is simulated by several flywheels. Due to the changing of the amount of flywheels or the size of the flywheels, the inertia moment will be changed. The electric inertia test bench consists of a data acquisition and controlling system. The motor torque, as a resisting torque, simulates the inertia of a running vehicle. With the data acquisition and controlling system, the inertia moment can be accurately simulated. The detailed description of the designed test benches and the advanced measurement system is shown in section 4.2. The design of a test bench requires a good comprehension of the whole system. It follows the consideration of the components of a brake system, which are involved in the brake action. It is obvious that it is not possible to consider all components in detail, in order to describe the real behaviour of the brake components. However, most of the assumptions are taken into consideration. At first, it is necessary to know, which brake components are important to consider and which are sufficient to make a good assumption. The resistance caused by driving is not strong enough to decelerate the vehicle, making the brakes a necessary tool in decelerating the speed of a vehicle. In reverse of the power train, the brakes are dissipating mechanical energy in heat. The acting forces and moments during braking are greater in comparison to when a vehicle is in motion. Retarding brake, parking brake and holding brake are different types of braking. The conventional brakes are the retarding brake and the parking brake. The holding brake is placed in the cardan-shaft drive. The schematic representation of a standard brake system in a car is illustrated in figure 4.1.

The brake of the vehicle consists of a brake pedal, a brake force booster, a brake master cylinder, a brake disc, a brake pad, a brake force limiter and brake pipes. The schematic representation of the brake system and the corresponding forces are shown in figure 4.2.

The actuating force F_{act} is generated by the driver, which moves the brake pedal. Due to the leverage of the actuating force at the brake pedal, an increased actuating force F'_{act} is detected at the brake force booster. Hereby, the brake pressure is applied by the brake master cylinder F_{master} , the actuating force F'_{act} and the auxiliary force F_{aux} of the brake force booster. The brake pedal moves the brake master cylinder supported by the brake force booster. The actuating force of the brake pedal is transferred in hydraulic pressure. The brake pipes conduct

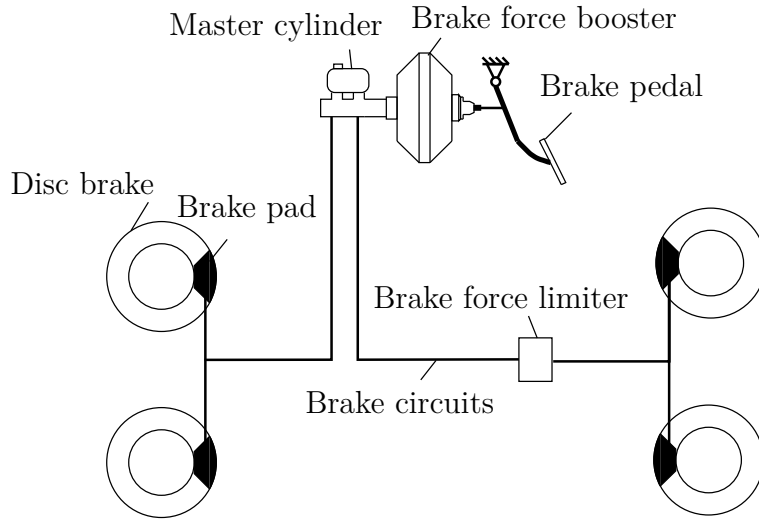


Figure 4.1: Brake system of a car

the hydraulic pressure to all wheel cylinders, where they generate the clamping force F_{clamp} . The clamping force generates due to friction and the brake pads a tangential force F_t on the brake disc and pad. The brake force $F_{B,\text{wheel}}$ on the tyre footprint is generated due to the brake moment in the brake disc, where $F_{W,i}$ describes the axle load of the vehicle at the ground for the front or rear axle. The brake force of a tyre footprint is

$$F_{B,\text{wheel}} = F_{\text{act}} i_{\text{ratio}} C_{\text{idn}} \frac{r}{R_{\text{dyn}}}. \quad (4.1)$$

The actuation force on the brake master cylinder is $F_{\text{master}} = F'_{\text{act}} + F_{\text{aux}}$, which consists of the actuating force and the auxiliary force. The clamping force

$$F_{\text{clamp}} = F_{\text{act}} i_{\text{ratio}} \quad (4.2)$$

depends on the actuating force and the external reinforcement ratio. The tangential force is

$$F_t = F_{\text{act}} i_{\text{ratio}} C^*, \quad (4.3)$$

where C^* is the brake identifier. The ratio can be rewritten as $i_{\text{ratio}} = F_{\text{clamp}}/F_{\text{act}}$. Therefore, the brake identifier is $C^* = F_t/F_{\text{clamp}}$. Because of the many brake components, the interrelationship of F_{act} , F_t and F_{clamp} can be determined with the total reinforcement of the brake system

$$i_{\text{total}} = i_{\text{ratio}} C^*. \quad (4.4)$$

The boundary conditions for the effective brake radius and the dynamic radius is $r/R_{\text{dyn}} = 0.33 - 0.5$. The brake identifier depends strongly on the sliding friction μ_{sliding} . The ratio i_{ratio} is restricted by the anatomical maximum pedal move of 180 mm, the brake calliper path and the loss of the brake system. The maximum actuating force can not exceed 500 N, but a larger brake force can be achieved with a brake force booster. Furthermore, the ratio depends on the

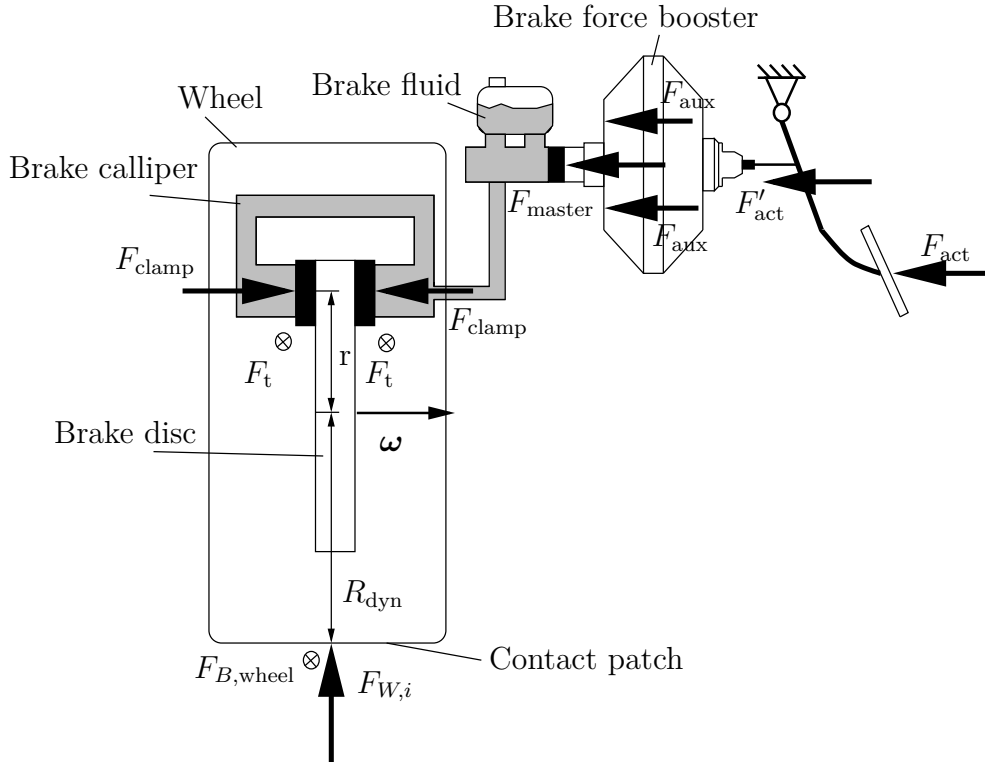


Figure 4.2: Forces acting in the brake system

air path of the brake calliper, the compression of the brake fluid, the expansion of the brake pipes, the sealing elements, and the thermal and mechanical deformation of the components of the brake system. The swerving of the rear axle has to be suppressed. Therefore, the rear axle portion of the load of the car is

$$\phi = \frac{F_{W,r}}{F_W} = \frac{l_f}{l}, \quad (4.5)$$

where $F_{W,r}$, F_W , l_f , and l are the front axle load, the total weight of the car, the distance of the centre of gravity to the rear axle, and the wheelbase, respectively. The wheelbase l in regard to the height of the centre of gravity h_c is

$$\psi = \frac{h_c}{l}. \quad (4.6)$$

Therefore, the braking ratio can be expressed by

$$z = \frac{F_{B,f}}{F_W} + \frac{F_{B,r}}{F_W}, \quad (4.7)$$

where $F_{B,f}$ and $F_{B,r}$ are the front brake force of the wheel at the ground and the rear brake force of the wheel at the ground, respectively. The brake ratio considers the dynamic axle load distribution from the rear axle to the front axle [25]. The friction or brake force of a wheel can be calculated with the friction coefficient in regard to *Coulomb's* law by

$$F_B = \mu_i F_W. \quad (4.8)$$

In relation to equation 4.5, 4.6, and 4.7 the related brake forces of the front and rear axles are

$$\frac{F_{B,i}}{F_G} = \mu_i \frac{F_{G,i}}{F_G} = \mu_i (1 - \phi + Z \psi), \quad (4.9)$$

where the index i describes the front or rear axle of the car. During braking, an additional torque due to the mass inertia of the vehicle is applied

$$\mathbf{M}_I = \mathbf{\Theta} \dot{\boldsymbol{\omega}}, \quad (4.10)$$

where $\mathbf{\Theta}$ and $\dot{\boldsymbol{\omega}}$ are the mass inertia tensor and the angular acceleration. The mass inertia in the braking is applied on the front axle, whereas the rear axle is unloaded. Due to the suspending of the vehicle, the front of the vehicle is declined. The reverse effect is observed in the acceleration. This effect can be used to increase the cornering force in the cornering. The cornering force depends on the brake or acceleration torque and the centrifugal force. In figure 4.3, the applied forces and torques for acceleration and deceleration are shown.

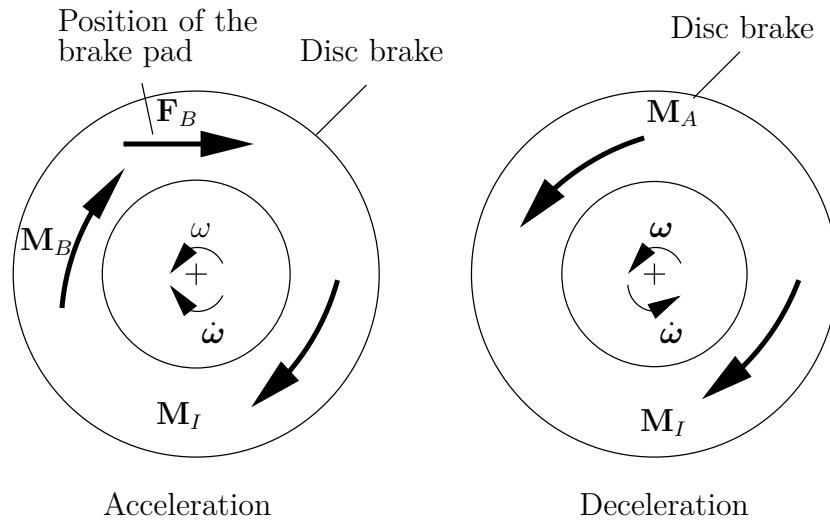


Figure 4.3: Brake load

The generated forces has to be transferred to the brake pad, which are conducted with brake pipes. The brake circuit distribution or the brake pipes are divided mostly in two independent circuits. The reason for this kind of distribution is the failure of the brake system due to outlet of the brake fluid. The possibilities of the brake circuit distribution with two brake circuits and the advantages and disadvantages are shown in the table 4.1.

The split of the braking force application into two brake circuits require a brake master cylinder, which can transfer the brake force into the two brake circuits. The mostly used brake master cylinder are the graduated master cylinder and the tandem master cylinder. The advantage of the tandem master cylinder is its low price and, as for the graduated master cylinder, its advantages is the remaining correlation of the actuating brake force and the deceleration by the

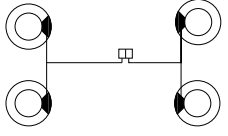
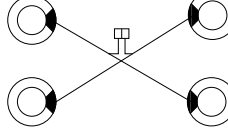
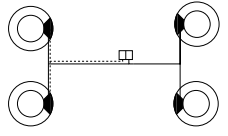
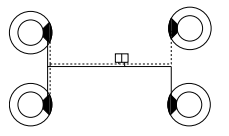
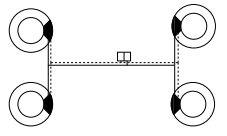
Front/Rear axle split	Diagonal split	Front Rear/Front axle split	Front axle and Rear wheel/ Front axle split	Four wheel split
				
Advantage: Better than the handbrake	Advantage: One front wheel	Advantage: Two front wheels	Advantage: Three wheels	Advantage: Four wheels
Disadvantage: No front wheels, just rear wheels	Disadvantage: Moment over the normal axis, just two wheels	Disadvantage: Just two wheels	Disadvantage: Small moment over the normal axis	Disadvantage: Cost

Table 4.1: Brake circuits (- 1st circle, - - 2nd circle)

failure of one brake circuit. The transmission of the actuating force is conducted hydraulically or pneumatically. The advantages of the hydraulically transmission is its low weight and the smaller space requirement. The unproblematic expansion possibilities for further brake applications is a positive plus of the pneumatically transmission. The supporting force of the brake force booster is generated on the working piston by the low pressure of the suction pipe of the motor. Vacuum pumps are necessary for diesel engines due to the insufficient vacuum. The pressure is controlled by the valves in the brake force booster, which guarantees a constant supporting force. The brake system with brake discs requires the brake calliper with the two brake pads. There exist two types of brake callipers, the fixed calliper and the floating or sliding calliper. The schematic view of the callipers are illustrated in figure 4.4. The advantages of the fixed calliper are its price and its low space requirement on the wheel rim side. The disadvantage are the disturbance of the contact pressure due to friction at the wheel rim side. The pros and cons for the floating calliper are the uniformly distributed contact pressure and the high price.

Operating Parameters

The operating parameters, which influence the braking action, are the kinetic energy, the brake power, and the braking time of the vehicle. The equation of the kinetic energy is

$$E_{kin} = \frac{1}{2} \boldsymbol{\omega}^T \cdot \boldsymbol{\Theta} \boldsymbol{\omega} = \frac{1}{2} \boldsymbol{v}^T \cdot \boldsymbol{m} \boldsymbol{v}, \quad (4.11)$$

where $\boldsymbol{\omega}$, $\boldsymbol{\Theta}$, $\boldsymbol{v}$, and $\boldsymbol{m}$ are the angular velocity, the mass inertia tensor, the velocity, and the mass matrix, respectively. The braking power can be calculated by

$$P = \mu(\boldsymbol{v}) \boldsymbol{F}_N \cdot \boldsymbol{v}, \quad (4.12)$$

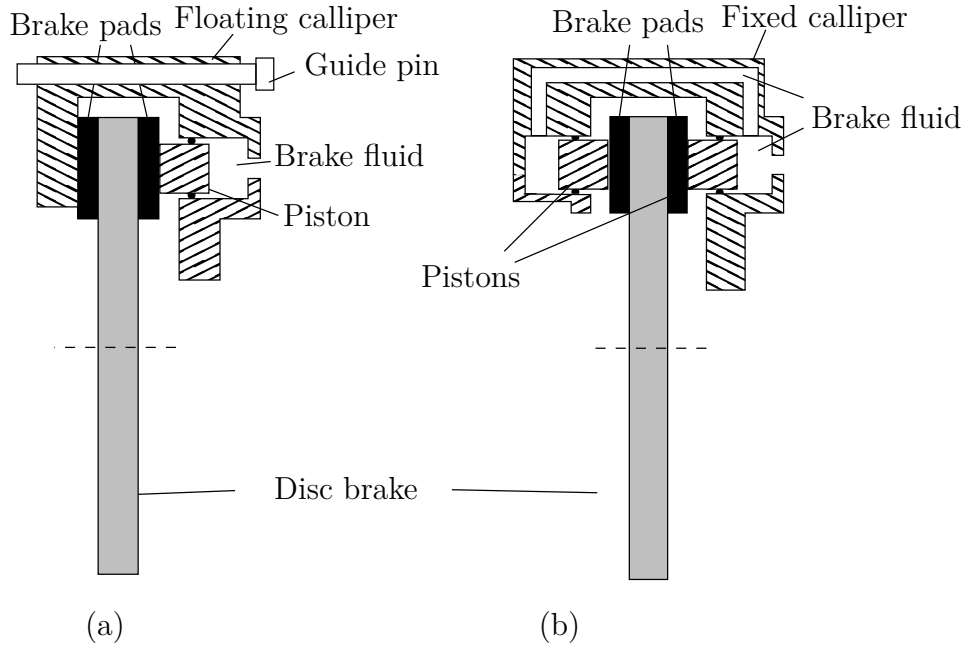


Figure 4.4: Brake calliper

where $\mu(\mathbf{v})$ is the coefficient of friction, $\mathbf{v}$ the velocity, and $\mathbf{F}_N$ the normal force charged at the brake pad. The total braking time t_B is the deceleration, where the vehicle reaches the required velocity. The braking deceleration can be determined by

$$a = \frac{v_2^2 - v_1^2}{2s}, \quad (4.13)$$

where v_2 , v_1 , and s are the start velocity, the end velocity, and the braking distance, respectively. In the literature, it is also written in multiple or fraction of the acceleration of gravity [25]. For the calculation of the operating parameters it is necessary to introduce several parameters [73]. On the mean friction radius $\mathbf{r}_m$, the resultant friction force $\mathbf{F}_R(t)$ is applied under the condition that the friction surface is the same for the coefficient of friction $\mu(t)$ and the pressure distribution $\sigma_n(t)$. Therefore,

$$\|\mathbf{r}_m\| = \frac{\int_{R_i}^{R_a} \int_0^{2\pi} r^2 d\varphi dr}{\int_{R_i}^{R_a} \int_0^{2\pi} r d\varphi dr} = \frac{2}{3} \frac{R_a^3 - R_i^3}{R_a^2 - R_i^2}, \quad (4.14)$$

where R_a and R_i are the outer and inner diameter of the friction area of the brake disc, respectively. The current resultant friction force $\mathbf{F}_t(t)$ can be calculated by means of the brake torque

$$\|\mathbf{F}_t(t)\| = \frac{\|\mathbf{M}_B(t)\|}{\|\mathbf{r}_m\|} = \mu(t) \|\sigma_n(t)\| A(t), \quad (4.15)$$

and the current friction coefficient $\mu(t)$ is calculated by

$$\mu(t) = \frac{\|\mathbf{F}_t(t)\|}{\|\mathbf{F}_n(t)\|} = \frac{\|F_T(t)\|}{\|\boldsymbol{\sigma}_n(t)\| A(t)}. \quad (4.16)$$

The mean friction work by area can be calculated with

$$\|\mathbf{q}\| = \frac{1}{2A} \boldsymbol{\omega}^T \cdot \boldsymbol{\Theta} \boldsymbol{\omega}. \quad (4.17)$$

This expression 4.17 is the dissipated mechanical energy due to the braking action, where $\boldsymbol{\Theta}$ is the mass of inertia tensor, the $\boldsymbol{\omega}$ the rotation speed and A the friction area. The mean friction power by area is

$$\|\dot{\mathbf{q}}\| = \mu \|\boldsymbol{\sigma}_n\| \|\mathbf{v}\|, \quad (4.18)$$

where $\|\boldsymbol{\sigma}_n\|$ is the magnitude of the normal pressure, $\|\mathbf{v}\|$ the magnitude of the friction velocity and μ the coefficient of friction. These equations are not considering local behaviour. The real local parameters differ from the global view. The local observation is combined by the imperfection of the two surfaces, the heterogeneous thermal expansion due to asperities of the two surfaces, the tribology effects (third-body model) in the contact area, the physicochemical behaviour of the contact pair and the micro structural transformation of the surfaces.

Brake Pad

The brake pad has to ensure the decrease or total mobility of the moving objects. This diminution of velocity as well as the kinetic energy, produces thermal energy due to the first law of thermodynamics. The dissipation is usually made by friction materials. A good friction material is chosen due to the behaviour of thermal conductivity, coefficient of friction, low usage (wear rate) and costs. In the majority of cases, brake pads consist of metallic materials in order to improve their wear resistance (lifespan), thermal diffusivity and strength. These metallic materials in the brake pads may be composed of copper, steel, iron, brass, bronze and aluminium in the form of fibres or particles. These fibres or particles have an influence on friction and wear (lifespan-longitivity) due to their hardness. There is not a concrete explanation about the relation between metallic material types and their behaviour on friction, wear and thermal expansion. Hence, the production of the brake pads is still based on trial and error. The friction pairing consists of the brake disc, the brake pad, the third-body layer, the bonding layer and the brake pad holder. All these components have different properties compared to the virgin condition concerning the introduced stresses. The third-body (see chapter 1.2) is several μm thin and important for the accommodation of the velocity. Moreover, the third-body ensures that the virgin brake pad is not damaged due to the high thermal load. The figure 4.5 shows the front (figure 4.5 a) and backside (figure 4.5 b) of a sintered TGV brake pad. The position and the dimension of the brake pad pins and the brake pad holder may differ for different suppliers [131].

There are several types of friction materials that can be bought of the market. Some examples are: natural friction pads (leather, wood), sintered friction pads, organic friction pads (Non Asbestos Organic (NAO)), semi-metallic friction pads, low-metallic friction pads, ceramic



(a) TGV brake pad pins

(b) TGV brake pad holder

Figure 4.5: TGV brake pad

friction pads (air planes, formula) and paper friction pads. The natural friction pad is used in clutches and have their name from the fact that they are made in the same technique as paper. The most used friction materials in railway brakes are organic and sintered brake pads. The organic friction pad is produced by hot pressing and the brake pads are usually made of composite materials. The components are listed in table 4.2.

Components	Material	Function
Structural material	Metallic fibres, carbon, glass	Mechanical resistance
Additives	Graphite, metallic sulphides, alumina and silica	Lubrication and stabilisation of friction materials or to promote cleaning of the disc surfaces through grinding
Fillers	silicon base oxide and barium sulphate	improve the production cost
Bonds	Phenolic resins, elastomer	Bonding and the fulfilment of the pad materials, forming a thermally stable compound, where sometimes anti-noise materials can be added

Table 4.2: Structural components of an organic brake pad

The sintered friction pad is produced by powder metallurgy. It consist of a metallic matrix, iron, silica and graphite. The silica increases the coefficient of friction whereas iron increases the wear resistance and coefficient of friction. Bronze is the base metal in the metallic matrix

due to good thermal conductivity. Due to the metallic matrix, the friction pad has higher resistance to heat and wear [67]. A comparison to the behavior of an organic and sintered brake pad is made by Esswein Junior et al. [67]. They show that the sintered brake pad has a lower wear rate and a higher coefficient of friction than the organic brake pad. These advantages could be explained by the good ductility of the metallic matrix since it increases the wear resistance, as well as, the crystal SiO_2 in the sintered brake pad that gives a higher friction coefficient. The wear rate decreased as result of the process parameters as pressure to obtain higher hardness. The porosity in the sintered brake pad is higher than in the organic brake pad. This causes temperature to increase because the pores work as insulators in the pad and they make the heat conduction difficult. The advantages of the pores explain the authors due to material roughness and therefore the increase of the coefficient of friction in the contact area. Moreover, the pores help the abrasive particles to remain in the brake pad. The mass loss can be explained due to the material composition and the different melting temperatures. Moreover, the thermal conductivity can be increased due to the pores in sintered brake pads. The result depends on the friction pairing and therefore it is not possible to give a general statement. The compressibility is the relation of the applied normal force on the brake pad material and the resulting displacement in the direction of the normal force. The important factor for the compressibility is the thickness of the brake pad and the thermal and mechanical properties. The influence of different rigidities and lengths of the brake pad of a high speed train on hot spots developments are investigated experimentally by Panier et al [111]. The study shows a classification and explanation of the formation of the thermal gradients. These experiments are conducted on a full-scale test bench. They showed that the formation of the gradients on hot bands correlate with the geometry of the pins of the pad. Moreover, they presented a precise condition of the formation of the macroscopic hot spots, which strongly depends on the thermal loading such as the dissipated energy and the maximal braking power. Furthermore, they show that the decreasing of the pad stiffness decreases the temperature level and increases the size of the macroscopic hot spots. Another researcher was Steffen [123], who investigated the influence of the thickness of a brake pad on an automobile brake disc using a flywheel test bench. He considered brake pads thickness to be between 4 and 12 mm. The formation of hot spots occurs only under 8 mm. Another correlation between the number of hot spots and the thickness of brake pads was not observed.

Brake Disc

The brake disc must absorb and store the heat and then it must immediately release it again at the end of the braking action. Moreover, the disc must have a good wear resistance and the ability to resist corrosion. The technical demands are highly specific and include heat capacity, sufficient strength, high temperature strength and high thermal shock resistance. The economic demands are small brake discs as well as low costs. The avoiding of the brake fade phenomenon is an essential aspect, which can be observed in repeated and sustained braking such as the downhill drive. This phenomenon is accompanied by a reduction in the braking performance due to the heating of the friction material or the boiling of the brake fluid. The problems of brake discs are the environmental influences such as water, dirt and salt. The increase of the heat removal for brake discs can be achieved with internal ventilation. Steffen [123] investigated two brake discs of different suppliers, where the geometry and dimension was the same. He has observed a different behaviour of hot spots. The composition, measurements of the hardness

and the analysis of the polished micrograph section of the brake disc cannot reveal the causes of the different behaviour of the formation of hot spots in brake discs. Probably, this different behaviour can be described due to the residual stresses induced in the manufacturing process. It is obvious that the material properties play a very important role on the formation of hot spots. The mostly used material for brake discs is grey cast iron, whereby also spheroidal cast and ferrous alloy can be observed. Furthermore, the compound brake discs are two-piece brake discs, which reduces the weight of the brake disc. The two-piece brake disc is composed of two different materials, the aluminium hub and the steel friction ring. The non-suspended mass is reduced due to the aluminium hub and a better heat conduction is achieved. The integral brake discs are casted by two different materials. The integral brake discs show good radial thermal expansion, whereby hot spots can be avoided. In materials with high thermal diffusivity, heat moves rapidly through it, because the material conducts heat quickly relative to its volumetric heat capacity or thermal bulk. The heat capacity c describes the required heat to change the temperature of a body by a given amount. Heat capacity is an extensive property and if it is necessary to obtain the specific heat capacity independent of the size (intensive property), it has to be divided by mass. Moreover, it is important to note that density and Young's modulus are functions of time and temperature. As we have seen, the brake disc and the brake pad must have a sufficient damping property in regard to the thermal conductivity and the heat capacity to compensate or to balance local inhomogeneities. This would prevent the formation of hot spots. In consideration of these material properties, grey cast iron brake discs have a better thermal absorption as steel brake discs. It is obvious that the geometry of brake discs has a high influence on the formation of hot spots. More on this in the next section. Steffen [123] considered the influence of the brake disc thickness on the formation of hot spots. Hot spots occur under 10 mm brake disc thickness. The number of hot spots do not change with brake disc thickness. Moreover, the outer diameter of the brake disc was increased under constant brake pad thickness, initial temperature and inner diameter. The results show that the number of hot spots increases with the outer diameter from 11 to 13.

Contact Area

In this section, the friction and wear behaviour in the contact area between brake disc and brake pad is considered. There are many researchers who work in this field and explain in details all effects in the contact area. The work of Czichos is already known in that area [31]. He shows a classification of the wear modes and describes the wear mechanisms in the contact area:

- Adhesion and shear: Destruction of the adhesion bonding which occurs through molecular interactions while the contact of friction pair.
- Plastic deformation: Deformation of the contact and therefore dissipation of energy.
- Furrowing or grooving or groove forming: Friction mechanism in which the two first bodies have different hardness and because of this, it is possible for the roughness of the first body to reach its peak, or the wear particles might enter the softer first body easily.
- Elastic hysteresis and damping: Energy transfer due to lattice vibration, migration of dislocations and sound emission.

- Surface fatigue: Occurs due to cyclic charged load on the contact surfaces, cracks growing due to cyclic normal and tangential forces.
- Abrasion: In the tribology contact one of the first bodies is harder and rougher than the counterpart or if particles are impressed in the softer first body.
- Adhesion: welding of debris of a first body on the other first body due to molecular interactions.
- Tribochemical reaction: Chemical reactions between the two first bodies mostly in connection with the air. This causes changes in the contact area. These chemical reactions can have good or bad influence concerning wear behaviour.

Another important point is the running-in program of the brake pad. It is necessary to run-in the brake pad before the application. As mentioned in the chapter 1.2, the third body plays an important part in the friction pairing. It protects the first bodies against high temperatures and can act as a dry lubrication. In this case, it is better to begin with the real application of the brake pads after a recovery program. For more information see [115] and [126]. In the work of Panier et al. [111] he investigated experimentally the behaviour of the formation of the hot spots on a railway brake disc in regards of a varying brake pad perimeter. The angle of the origin brake pad was 66° and the shorter brake pad 44° . The shorter brake pad was one third shorter. The experiments were carried out between 60 and 90 km/h and the dissipated energy between 1.4 and 9.4 MJ, resulting in the formation of hot bands. At a velocity starting at 90 km/h, hot spots are formed. They found out that 6 hot spots were formed with the original brake pad and 9 with the shorter brake pad. They concluded that with a one third shorter brake pad, the number of hot spots increases by half as much again.

4.2 Inertia Dynamometer

The flywheel test bench is used to simulate the load of a vehicle during the brake action on the brake disc. Furthermore, the flywheels simulate the charging of the vehicle or the different masses of the vehicles. The modularity of the test bench is an important point. The test bench is designed for the experimental investigations of brake systems. The changeover of the flywheel test bench to a clutch test bench can be conducted very simple. The requirements for this test bench is a second electric motor, which is placed at the opposite side of the first motor. The boundary conditions are the load capacity of the ceiling and the test bench room dimensions. The general structure of the flywheel test bench is shown in the figure 4.6.

The flywheel test bench consists of the Data Acquisition (DAQ), the frequency converter, the Asynchronous Motor (ASM), the test bench and the sensor system. The DAQ controls the input and output of the flywheel test bench. The DAQ consists of a data acquisition PCIe card and a computer. The frequency converter controls the rotational speed of the ASM, which generates a variable alternating voltage in frequency and amplitude from a fixed alternating voltage. The ASM drives the flywheels and provides the braking energy. The test bench consists of the flywheels, the shafts, the clutches and the bearings. The test bench is designed modular. The main item of the flywheel system is the sensor system. The schematic view of the test bench with the flywheels, the shafts, the clutches and the bearings is illustrated in figure 4.7.

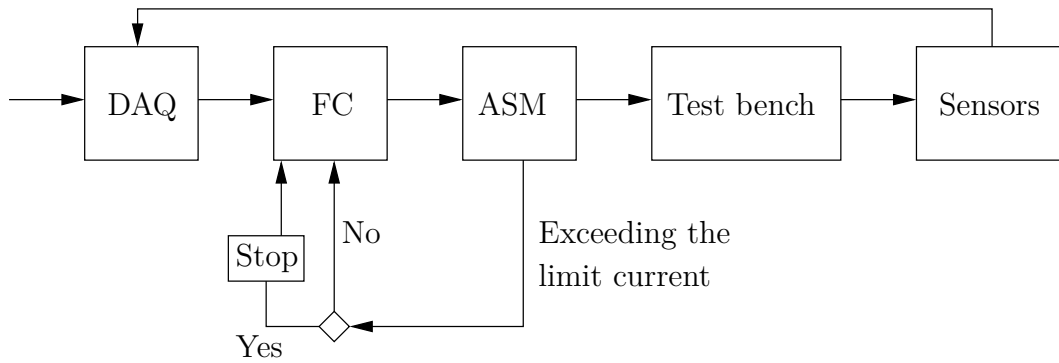


Figure 4.6: Control loop of the flywheel test bench

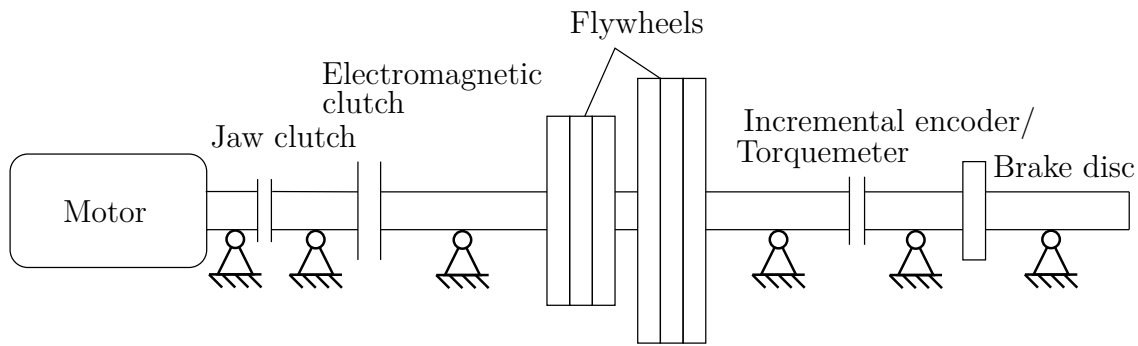


Figure 4.7: Schematic view of the flywheel test bench

The experimental investigations are conducted on the designed flywheel test bench at the Institute of General Mechanics. The test bench is being driven with a 30 kW squirrel-cage ASM. Several weights of vehicles can be simulated by the coupling and the disengaging of the geometrically graduated flywheels. With the maximum rotational speed of the ASM at a 1460 rpm, certain vehicle velocities in regard to the wheel rim combination of the vehicle type can be simulated. The maximum speed of a car with a tyre 165/70 R14, which can be simulated on the test bench is 161 km/h. The brake disc is attached to two flanges at the end of the test bench. The brake calliper is bolted at the machine bed for the transmission of the occurring brake torque. The measuring system consists of several sensors. The temperature measurement is conducted with a high speed IR camera, pyrometer and thermocouples. The temperature is measured by an IR camera and is conducted with infrared reflective mirrors on both sides of the brake disc. The high speed measurement of the thermal image is achieved in the line scan modus and can reach frequencies up to 13000 Hz. The deformation measurement of the brake disc is conducted with stationary, non-contact and high-frequency resolution capacitive sensors and a laser triangulation sensor. The capacitive sensor is placed in a unit of measurement, which can detect for a certain radius, the deformation of the brake disc on one side. The movement measurement of the brake calliper is performed with the 3-D motion tracking. The measurement of the rotational speed of the brake disc is measured with an incremental encoder of 2048 pulses per revolution. Force sensors are used to detect the brake forces and the brake fluid pressure. Detailed information about the measurement system can be found in the section 4.2.5 or in the appendix B.

4.2.1 Asynchronous Motor

In the brake test bench the electric motor is used to provide the power in order to rotate the shafts and the flywheels with a certain rotation speed. The power of the motor must be suitable with the chosen system of inertias. There exist three rotating electric engines: the direct current machine, the synchronous machine and the asynchronous machine. The advantages of the direct current machine are the simple and the low cost of the rectifier, the high control dynamic and the high power density. The disadvantage is the high maintenance cost. The advantages of the asynchronous machine are the high rotational speed and the power. The advantage of the synchronous machine is the coupling of the speed or the angular position to the operating frequency. The ASM consists of the stator and the rotor, which are responsible for generation of the torque or the rotational speed. The power supply is achieved by the junction plate. The stator consists of the stator lamination core and the stator winding. The stator winding is spatially distributed over the entire circumference. The stator winding and the lagging are shown in figure 4.9. The winding consists of three parts, which are arranged as a function of the number of poles (120° , 60° , 30°). The individual windings are separated with a lagging. The spooling ends can be connected in a star or a delta connection. The star connection, the delta connection and the star-delta switch are three possibilities for the connection of the ASM to the main power supply. The ASM can work at the voltages 330 V and 660 V. The star connection is used for the higher voltage and the delta connection for the lower one. Figure 4.8 describes the structure of the asynchronous machine. The rotor carries in contrary to the synchronous motor a three-phase winding. This is conducted either by a short-circuited with the squirrel-cage or with slip rings or contact brushes, which are led outwards.

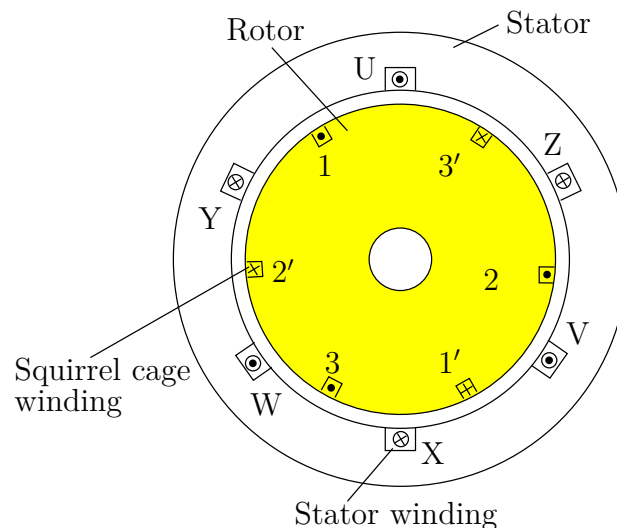


Figure 4.8: Structure of the asynchronous machine

The stator winding and the lagging are illustrated in figure 4.9.

The stator winding is placed in the winding grooves of the stator lamination core. The winding grooves and the overlapping of the windings are illustrated in figure 4.10.

There are two types of the ASM, the slip ring rotor and the squirrel-cage motor. The three-phase development of the slip ring rotor is in the stator and rotor. The winding end of the rotor are on the commutator ring, which can be wired. The three-phase development of the



Figure 4.9: Stator winding and the lagging

squirrel-cage motor is in the stator and the rotor. The rotor consists of several bars, which are connected to the end of the bars with short circuit rings. The rotor winding for the squirrel-cage rotor is realised with rotor bars. In the notches of the squirrel-cage rotor there are instead of windings bars of copper or aluminium. The rotor consists of the rotor lamination stack, the bars, the shaft, the bearings and the short circuit ring. The rotor lamination stack is pressed on the shaft. The rotor is supported in the stator. The lamination stack enables a proper conduction of the magnetic field but conducts no current in the longitudinal direction of the bars. In figure 4.11, the squirrel-cage rotor (cylindrical cage) with the laminated sheet package and the bars are illustrated.

The working principle of the ASM is the alternating current of three mutually shifted out-of-phase sinusoidal voltages with the same amplitude. The three-phase asynchronous motor is an induction machine. The stator consists of a three-phase winding on the extent. The rotor has also a three-phase winding (squirrel-cage rotor), which is short-circuited. The rotor current is induced from the stator to the rotor after the transformation principle. The magnetic flux is transformed to the rotor via the air gap. There is a slip of the rotational speed between the stator and the rotor rotating magnetic field. The inrush current of the stator windings generates for each winding a magnetic field. The magnetic field of the stator rotates with 50 Hz or 3000 rpm. The changing magnetic field of the stators is applied to the bars when the inrush current flows. Due to this changing magnetic field, a voltage is induced in the bars. The short circuit rings short-circuit the bars and a short-circuit current flows through the bars, which generates at each bar a magnetic field. The electromagnetic induction describes the movement of a conductor in a magnetic field or the change of the magnetic field in a current flow conductor. The movement of an electric conductor in a permanent magnetic field, which is perpendicular to the conductor, generates a voltage in the conductor. The origin is that a force, the *Lorentz*

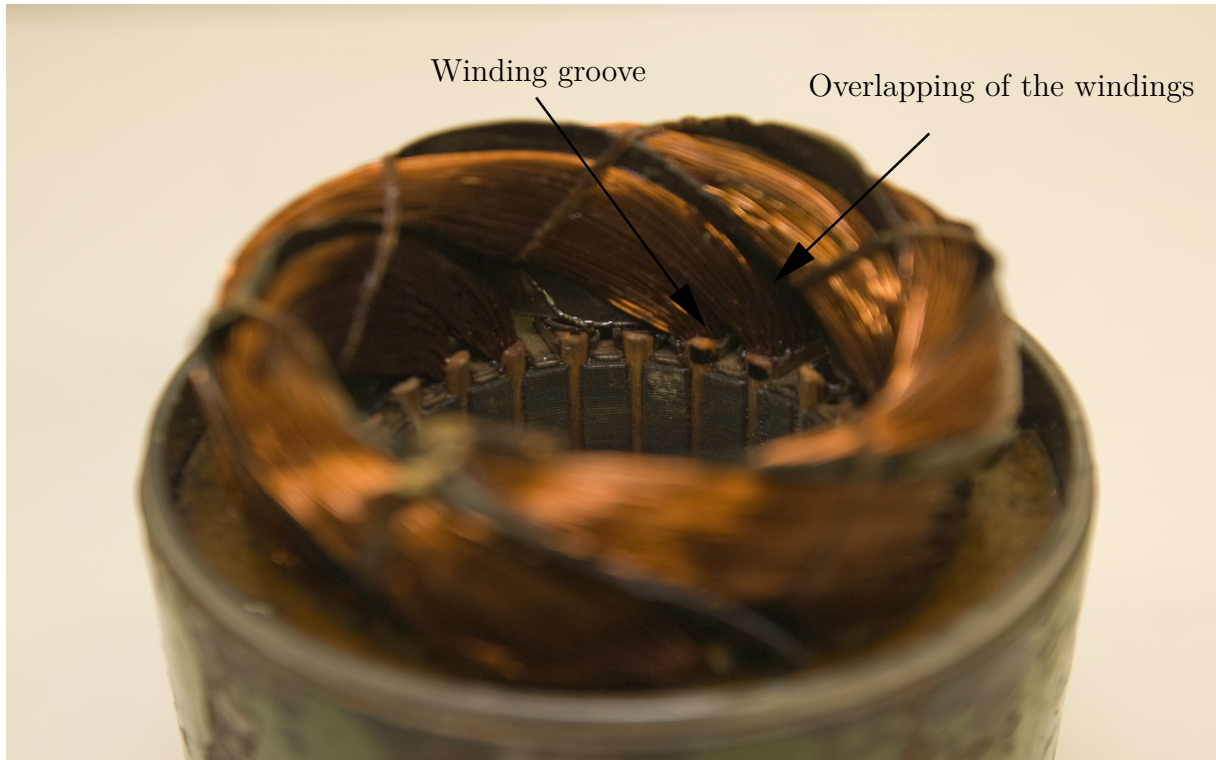


Figure 4.10: Overlapping of the windings and the winding groove

force, moves the electrons in a certain direction. Due to the movement of the electrons, a positive and negative pole is formed in the conductor and the voltage can be measured. The induction voltage can be increased with a higher velocity, higher conductor loops and a higher magnetic field. The increase of the magnetic field can be achieved with merging of the conductor loops into a coil or using magnetic materials as a coil body. The changing magnetic field comes from the three-phase system. The resultant voltage vector rotates and draws a co-rotating 90° magnetic field behind him. The rotor follows the magnetic field. The magnetic field of the stator interconnects through the air gap with the rotor. The rotating stator field with the higher velocity, pulls the magnetic field of the rotor. The higher the rotational speed of the rotor is, the smaller is the induced voltage. The magnetic field of the rotor becomes weaker due to the lower short-circuit current. The rotation of the rotor of the ASM is guaranteed when there is a slip between the magnetic field of the stator and the rotor. In summary, the rotating magnetic field in the stator induces a certain voltage inside the rotor. Due to the generated current and the magnetic field, a torque is applied on the rotor. There exist three possibilities to adjust the rotational speed, the gearbox, the change of input frequency and the pole-changing. The used adjustment of the speed is done with a frequency converter and is explained below. The speed adjustment for the ASM is necessary for the smooth accelerating of the flywheels or for the simulation of the brake application at different velocities. The optimal working range of the asynchronous motor is below the synchronous speed n_s . In this point, the motor has a high rotational speed stiffness under the load. The starting torque M_s is smaller than the maximum torque $M_{\max}$. The torque is increasing with the rotational speed n up to the maximum torque $M_{\max}$. The synchronous speed is the rotational speed of the stator magnetic field, which is nearly achieved only in the unloading condition. Due to the increasing load, the slip becomes higher. The slip can be calculated with

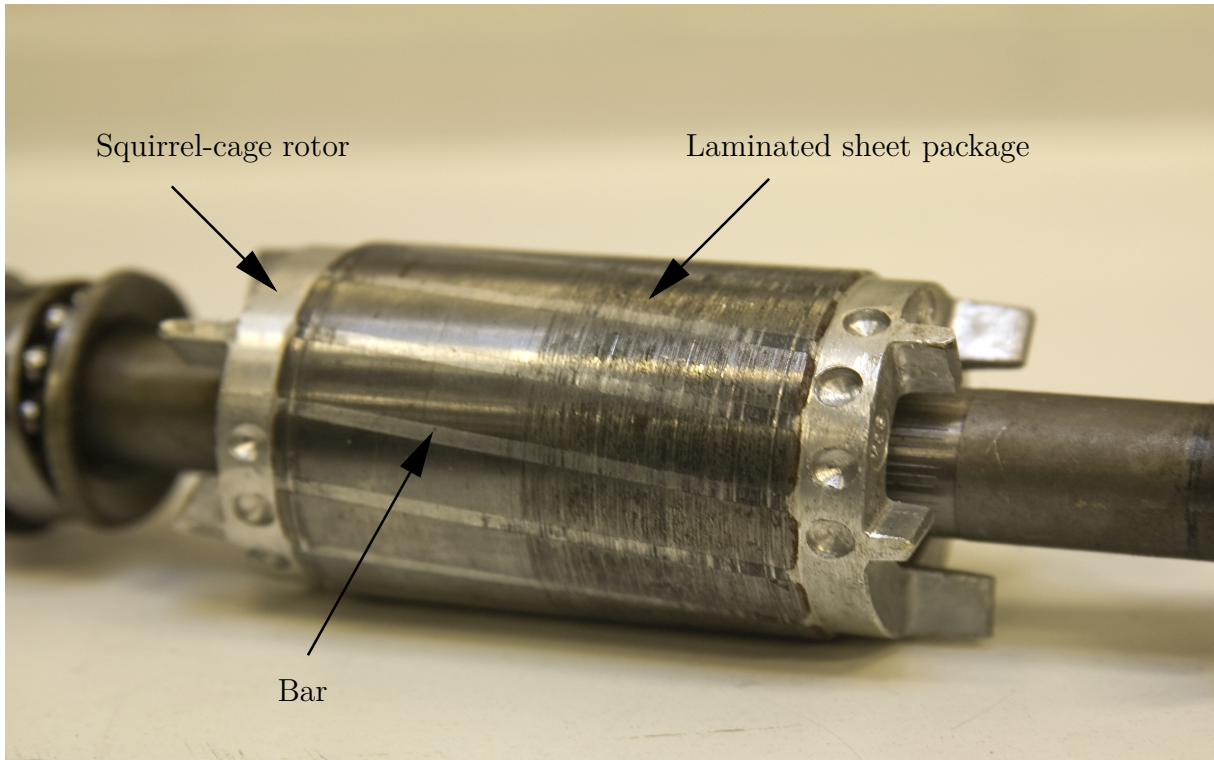


Figure 4.11: Squirrel-cage rotor

$$s = \frac{n_s - n}{n_s}, \quad (4.19)$$

where n_s and n are the synchronous speed and the rotational speed, respectively. The synchronous speed depends on the frequency of the input voltage f and the number of pairs of poles p :

$$n_s = \frac{f}{p}. \quad (4.20)$$

A disadvantage of this type of motor is the low starting torque and the high starting current, which can reach fivefold of the current in the rating. The torque characteristic of the asynchronous motor can be defined with the *Kloßian* equation, which describes the brake torque according to the slip or the rotational speed. The *Kloßian* equation reads

$$\frac{M}{M_{\max}} = \frac{2}{\frac{s}{s_{\max}} + \frac{s_{\max}}{s}}, \quad (4.21)$$

where M , $M_{\max}$, s , and $s_{\max}$ are the torque depending on the slip or rotational speed, the maximum achievable torque, the slip between the magnetic field of the stator and rotor, and the maximum slip at the maximum torque. In the *Kloßian* equation, two characteristic areas can be observed:

- $|s| \ll s_{\max} : M \approx 2 \frac{M_{\max} s}{s_{\max}}$ (line),
- $|s| \gg s_{\max} : M \approx 2 \frac{M_{\max} s_{\max}}{s}$ (hyperbola).

The rotational speed close to the synchronous speed is described by the line. The rotational speed is described by the hyperbola in the speed-torque curve. In other words, before the maximum torque the ratio of the torque and the rotational speed or slip is a hyperbola and after the maximum torque the ratio of the torque and the rotational speed or slip is a line. Figure 4.12 shows the torque/speed characteristics and the rotor current characteristic curve of an asynchronous machine.

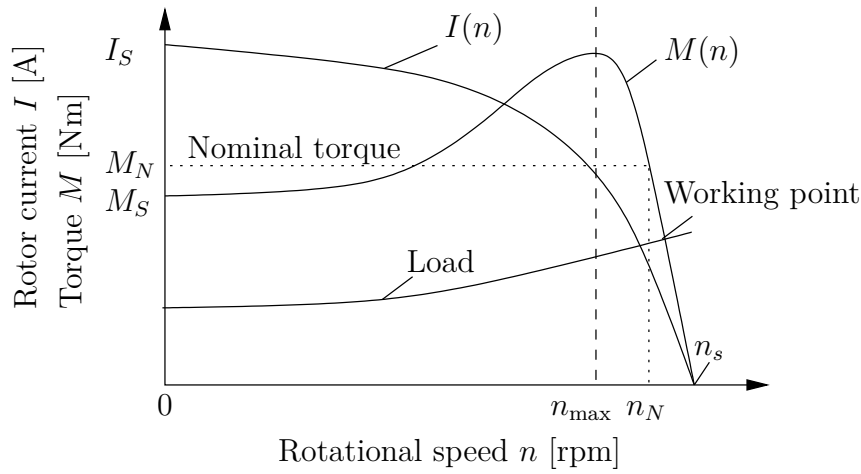


Figure 4.12: Torque/speed characteristics and rotor current characteristic curve of an asynchronous machine

The start process of the electric engine of a torque M_S is developed by taking a stator current I_S from the main power supply. With increasing rotational speed, the motor torque is also increasing up to the maximum torque at $n_{\max} = n_s(1 - s_{\max})$. In this acceleration, the stator current decreases very slow. The further acceleration up to the nominal speed can be observed with the decreasing of the torque and the stator current to the nominal state. The motor torque has to be 30% larger than the torque of the work machine for a safe run-up. To reduce the starting current of the asynchronous motor connected at the utility frequency, there is a possibility to install a star-delta starting of the stator field. But with a decreasing current, the torque is also decreasing. Both ends of the bars have an influence on the torque characteristics of the asynchronous motor. The rotational speed can be changed after the equation 4.20 due to the frequency of the stator voltage, and the number of pairs of poles. With a changing frequency a continuously change of the rotational speed is guaranteed instead of the number of pairs of poles. The specifications of the asynchronous motor (ASM) are shown in table 4.3.

The starting current can reach 6.5 times of the nominal current, which is 377 A. The maximum torque is 490 Nm:

$$M_{\max} = M_N \cdot \lambda = 196 \text{ Nm} \cdot 2.5 = 490 \text{ Nm}. \quad (4.22)$$

The synchronous speed n_s of the used ASM is

Power	Nominal torque	Nominal current at 400 V	Starting current $\frac{I_s}{I_N}$	Maximum torque $\frac{M_{\max}}{M_N}$	Nominal speed	Number of pairs of poles
30 kW	196 Nm	58 A	6.5	2.5	1460 rpm	2

Table 4.3: Specifications of the ASM

$$n_s = \frac{f}{p} = 1500 \text{ rpm.} \quad (4.23)$$

The slip of the magnetic field of the rotor and stator can be calculated with equation 4.19. The nominal slip of the ASM is $s_n = (n_s - n_N)/n_s = (1500 - 1460)/1500 = 0.0267$. The equation 4.21 describes the relation between torque and slip or rotational speed. The maximum slip at the maximum torque can be calculated by

$$s_{\max} = s_N(\lambda + \sqrt{\lambda^2 - 1}). \quad (4.24)$$

Substituting all the parameters into equation 4.24, the maximum slip $s_{\max}$ is 0.1279. In figure 4.13, the torque-speed or torque-slip characteristic at 50 Hz of the used asynchronous motor is shown.

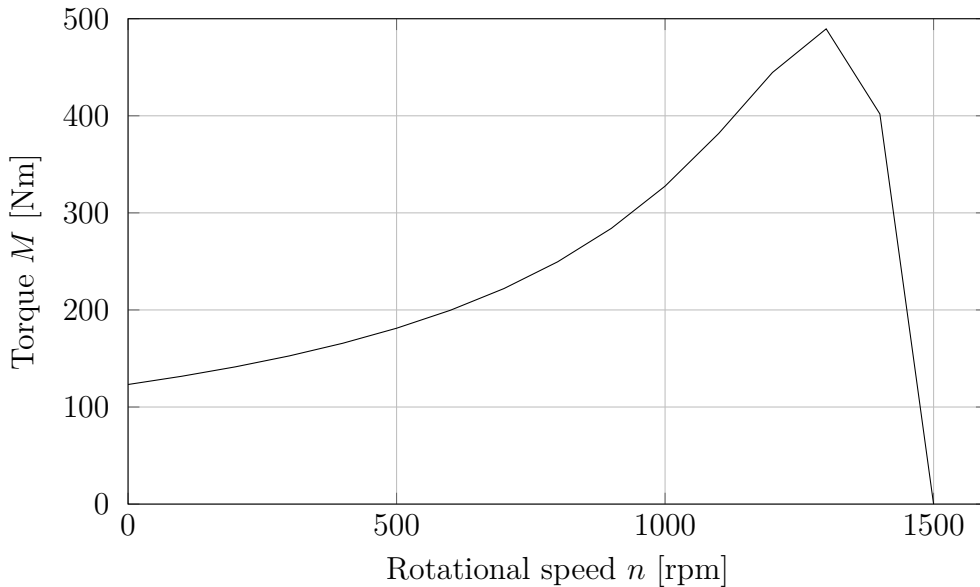


Figure 4.13: Torque-speed curve of the 30 kW motor at 50 Hz

The torque M_N at the nominal speed $n_N = 1460$ rpm is 196 Nm, which confirms the motor parameters in table 4.3. The hyperbola region is left of the maximum torque. The torque is increasing with the rotational speed. In this diagram, the motor does not show a pull-up torque. The ASM drives according to the characteristic curve. This means that the ASM drives over several undesired operating points. One of these points is the region of the pull-up torque. The acceleration of the power train requires a higher driving torque than the counter torque

or load. In the operating point there is an equilibrium of the driving torque and the counter torque. The change of the rotational speed at the operating point changes the driving torque immediately. The change of the driving torque in the unstable operating points can generate a large change in the rotational speed. The flywheels or the weight of the test bench represents a constant counter torque. This means there exists more than one operating point with the same torque. The best operating point is the point with the highest mechanical power, whereas the other operating points dissipate a lot of the electrical power in heat. The avoidance of these undesirable operating points can be conducted with a frequency converter. In figure 4.14, the torque-speed characteristics for different frequencies of the used motor is shown, in which they are adjusted with the frequency converter.

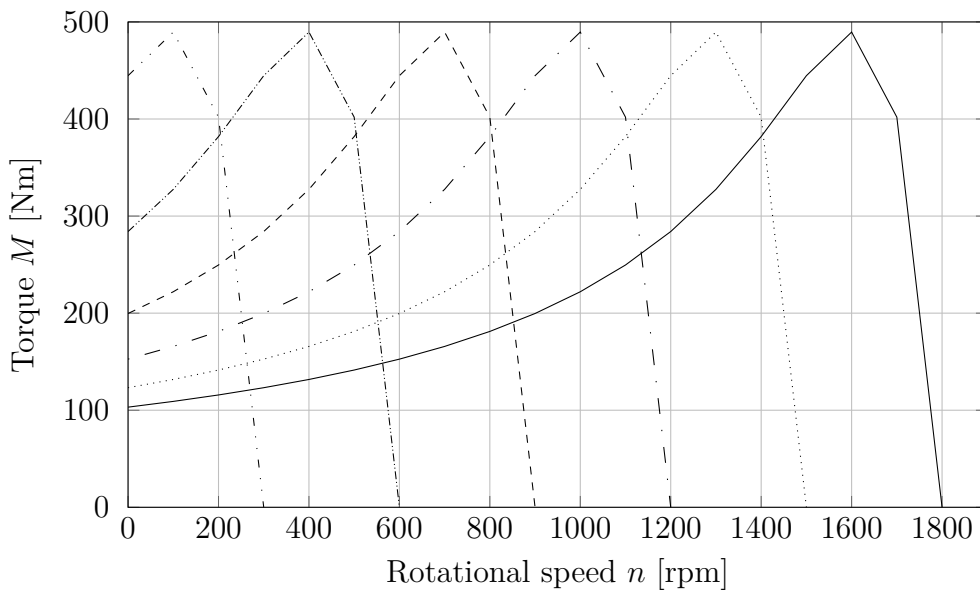


Figure 4.14: Torque-speed characteristics $M - n$ for a 4-pole asynchronous motor controlled by a frequency converter for different frequencies: 60 Hz (—), 50 Hz (.....), 40 Hz (· -), 30 Hz (- - -), 20 Hz (- · - ·), 10 Hz (- · · ·).

The acceleration time of the motor (equation 4.25) is a function consisting of the driving torque, the friction torque, the inertia moment of the flywheels and the final rotational speed:

$$t_{\text{acc}} = f(M, M_f, \Theta, n). \quad (4.25)$$

The rotational speed can be calculated by $n = n_s(1 - s)$. The derivative of the rotation speed after the slip is

$$\frac{dn}{ds} = -n_s. \quad (4.26)$$

The angular momentum on the shaft, which is loaded by the electric motor and the flywheels is

$$M_M - M_L = \Theta_{\text{res}} \frac{d\omega}{dt}. \quad (4.27)$$

The non-constant torque which accelerates the rotating masses can be calculated by the difference between the motor torque and the load torque. The run-up time can be determined with the angular momentum

$$t_{\text{acc}} = \int_{n_a}^{n_i} \frac{\Theta_{\text{res}}}{M_M(n) - M_L(n)} 2\pi \, dn. \quad (4.28)$$

Finally, the acceleration time t_{acc} can be calculated by inserting the equations 4.26 and 4.21 in equation 4.28

$$t_{\text{acc}} = \int_{s_e}^{s_a} \frac{\Theta_{\text{res}}}{\frac{2 M_{\text{max}}}{\frac{s}{s_{\text{max}}} + \frac{s_{\text{max}}}{s}} - M_L} 2\pi n_s \, ds. \quad (4.29)$$

The run-up can be calculated analytically if the torques are of analytical functions or the term $1/(M_M(n) - M_L(n))$ can be integrated analytically. Hence, the determination of the run-up time has to be divided in the speed-torque curve in small steps, so that the motor torque and the load torque can be expressed as constants. Due to the split of the run-up time of the ASM and the constant acceleration torque $M_{B,i} = M_M(n) - M_L(n)$, it is possible to calculate the step times with

$$t_{\text{acc},i} = \frac{\Theta_{\text{res}}}{M_{B,i}} 2\pi (n_{e,i} - n_{a,i}). \quad (4.30)$$

The run-up time of the ASM can then be calculated by the summation of the step times $t_A = \sum_i^n t_{A,i}$. Due to the fact, that the acceleration moment up to the maximum torque is independent of the rotational speed, the run-up curve of the ASM is a straight line. The run-up time for flywheels can be calculated analytically if the *Kloßian* equation of the motor is given. With $n = n_1(1 - s) \Rightarrow dn = -n_1 \, ds$ the run-up time is

$$t_A = \Theta_{\text{res}} \frac{2\pi n_1}{2 M_{\text{max}}} \left(s_{\text{max}} \ln \frac{s_a}{s_e} + \frac{s_a^2 - s_e^2}{2 s_{\text{max}}} \right) \quad (4.31)$$

for stator resistance $R_1 = 0$ and the counter load $M_L = 0$. The figure 4.15 shows the equation 4.29, which describes the relationship of the acceleration time of the electric motor for several inertias with respect to the slip and the rotational speed.

Another very important point is the heat development in the ASM or the exact heat development in the rotor circuit during the acceleration time. The heat, which is dissipated in the rotor circuit, can be calculated with

$$Q = 2\pi n_s \int_{t_s}^{t_e} s M_M(s) \, dt. \quad (4.32)$$

The heat which is converted can be calculated by inserting all values for $M_L = 0$. The angular momentum can be described as

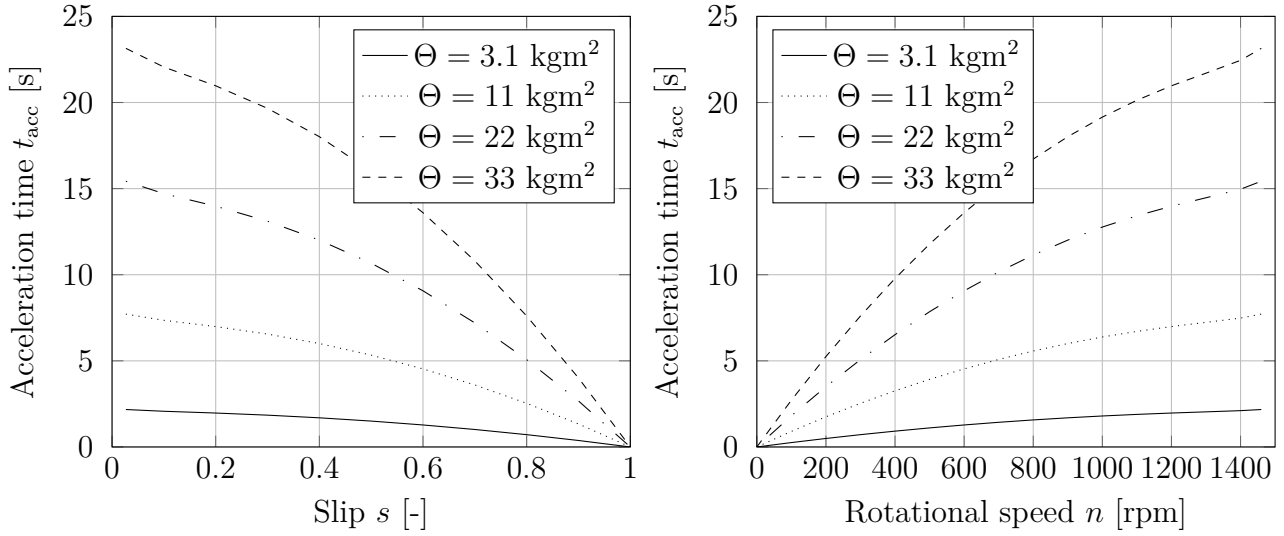


Figure 4.15: Acceleration time of several inertias

$$M_M(s) = \Theta_{\text{res}} \frac{d\omega}{dt} = \Theta_{\text{res}} 2 \pi \frac{dn}{dt} = -\Theta_{\text{res}} 2 \pi n_1 \frac{ds}{dt}. \quad (4.33)$$

The differential of the time is

$$dt = \frac{-\Theta_{\text{res}} 2 \pi n_1}{M_M(s)} ds \quad (4.34)$$

and inserted in the equation 4.32 gives

$$Q_2 = \int_{s_a}^{s_e} \Theta_{\text{res}} (2 \pi n_1)^2 s ds. \quad (4.35)$$

The heat dissipation in equation 4.35 of the rotating masses depends on the kinetic energy $E_k = \frac{1}{2} \Theta_{\text{res}} (2 \pi n_1)^2$ of the rotating masses in the synchronous speed. Hence, the dissipation of heat in an ASM for a run-up of flywheels is

$$Q_2 = E_k (s_a^2 - s_e^2). \quad (4.36)$$

In the equation 4.36, the dissipation of heat in the rotor circuit depends on the kinetic energy of the rotating masses and the slip difference. In the figure 4.16, the heat development with respect to the rotational speed is shown.

The dissipation of the heat in the rotor circuit for a run-up of flywheels from the idleness ($s_a = 1$) to the synchronous speed ($s_e = 0$) is equal to the kinetic energy (E_k) of the flywheels. It is obvious that the heat dissipation in the rotor circuit is independent from the run-up time. The advantage of the slip ring motor is that the heat dissipation is separated in the rotor winding, hence the series resistance can be cooled better. The cooling of the ASM is achieved with a ventilator and with the finned package in which the stator lamination stack is. The heat dissipation must be controlled in a reasonable range, in this way the temperature of the

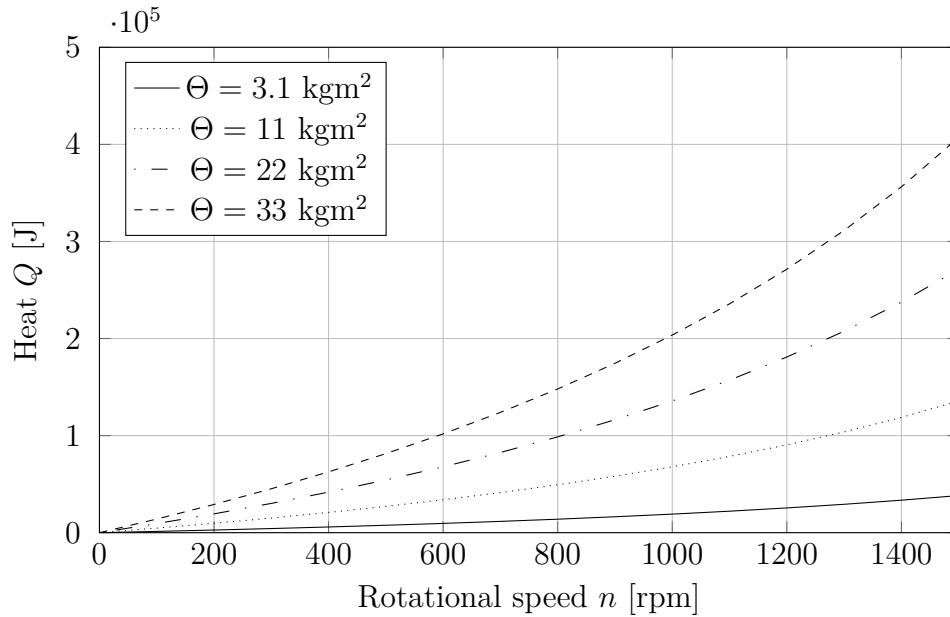


Figure 4.16: Heat development in the motor

motor will not be too high, so the components (electrical insulation) will not be damaged. The electric motor is divided in several heat categories. Power is lost inside the electric motor due to the formation of heat. In the rated operation, the electric motor is in a steady state, that is the heat output and input are equal. That means, the heat input depends on the charge state and the heat output depends on the cooling conditions. The maximum allowable temperature for electrical insulation is 155°C . Other important components in regard to the heat development are the bearings. The bearings in the electric motor are grooved ball bearings with local lubrication. If the maximum bearing temperature is achieved, a special hot bearing grease is needed. The bearing shields protect the grooved ball bearings of wear and dust. The asynchronous motor can work in three operating modes. For the sake of completeness, any asynchronous motor can be used for generating electricity or better converting energy. In the range $-\infty < n < 0$ the motor works as a brake, in the range $0 < n < n_s$ as a motor, and in the range $n_s < n < +\infty$ as a generator. The torque/speed characteristic for the motor and generator is shown in figure 4.17.

4.2.2 Frequency Converter

There exist two types of frequency converters, the direct frequency converter and the frequency converter with an intermediate circuit. The direct frequency converters are used only in the high power application. The used frequency converter is a frequency converter with an intermediate circuit. The working principle of a frequency converter is the conversion of a fixed alternating voltage in an alternating voltage variable in frequency and amplitude. The used frequency converter consists of the control electronics and the power section. The schematic representation of the power section is shown in the figure 4.18.

The power section of the frequency converter is composed of the rectifier package, the direct current voltage intermediate circuit and the alternating-current converter. The input voltage is demodulated by the rectifier package and consists of an uncontrolled three-phase bridge circuit.

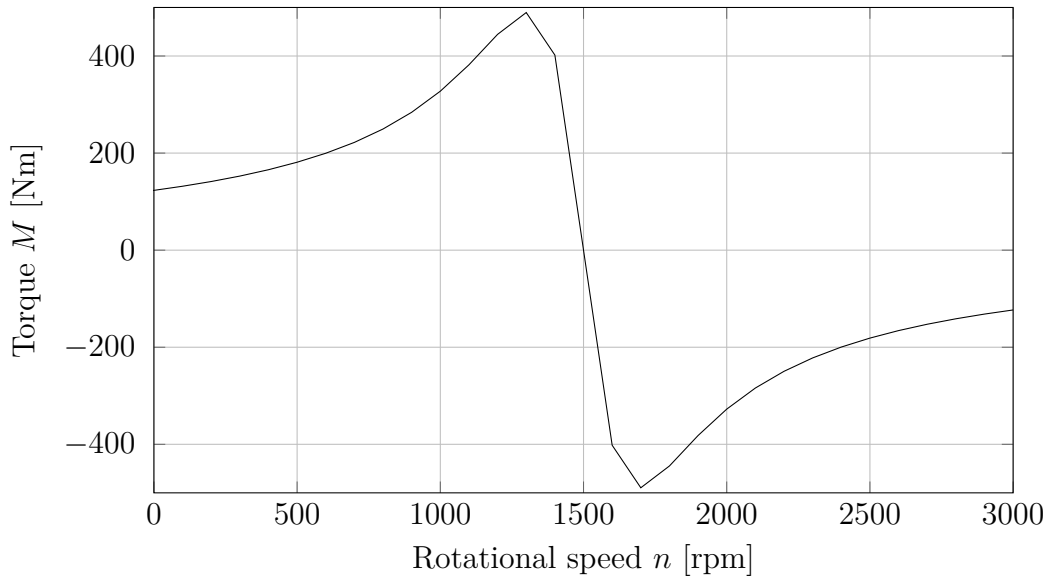


Figure 4.17: Torque-speed curve of the 30 kW motor

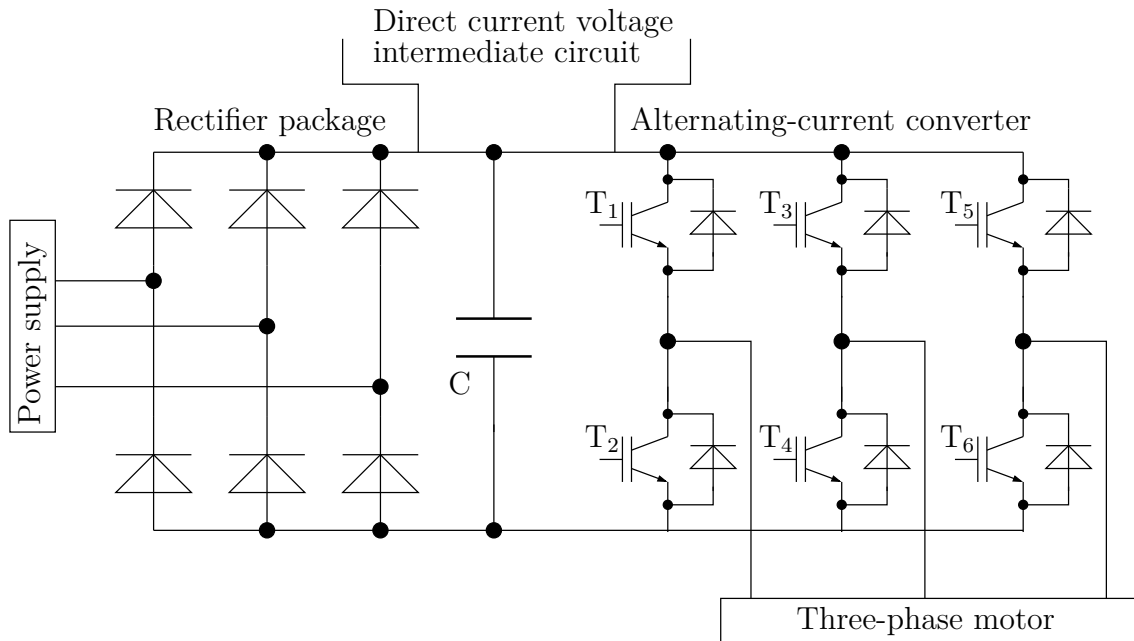


Figure 4.18: Schematic view of the power section of the frequency converter

It converts the incoming alternating voltage in a pulsating direct-current voltage, which is smoothed by the capacitor in the direct current voltage intermediate circuit. A capacitor in the direct current voltage intermediate circuit holds the rectified input voltage stable. The voltage in the direct current voltage intermediate circuit is converted in a clocked periodic form by the alternating-current converter. The theoretical background is the pulse duration modulation. The alternating-current converter consists of three bridge sections with two transistors and diodes for each bridge section. The output of the alternating-current converter is three-phase pulsating alternating voltage. The communication with the data acquisition system and the asynchronous motor is conducted over the control electronics. The used frequency converter

consists of a brake chopper, which can be used with a brake resistance for a four-quadrant operation. The four-quadrant operation enables the braking and accelerating of the ASM in both directions. The deceleration or the kinetic energy of the deceleration of the power transmission is stored in the intermediate circuit capacity and dissipated into heat. The torque of the electric motor can be calculated with

$$M = \frac{P}{\omega} = \frac{\sqrt{3} \eta U I \cos \varphi}{\frac{f}{p}} = \frac{k U I}{f}, \quad (4.37)$$

where P , ω , η , U , I , $\cos \varphi$, f , p , and k are the power, the angular velocity, the efficiency factor, the voltage, the current, the power factor, the frequency, the pole pair number, and the machine constant, respectively. Hereby, the torque is proportional to

$$M \propto \frac{U I}{f}. \quad (4.38)$$

The U/f control of the frequency converter describes the ratio of the voltage and the frequency. This constant ratio determines the magnetic flux density of the electric motor and guarantees an optimal torque of the electric motor. Moreover, a change of the frequency of 1 Hz changes the output voltage of the frequency converter of U/f [V]. The output voltage of the frequency converter in the U/f control remains constant till up to 50 Hz, while it generates a constant torque. Frequencies increase because of the lagging of the windings. The frequency converter can maintain a constant U/f ratio over only a certain frequency. This range is described as the field weakening range due to the fact that the magnetic flux density is decreasing. The attenuated magnetic field generates a low torque and decreases with a frequency of $\frac{1}{f}$. The U/f ratio can be increased with the consideration of the star-delta connection. The requirements are the capability of the frequency converter to supply the required current and the lagging is designed for the higher voltage. The asynchronous motor is connected over the electromagnetic clutch with the flywheels. The experiments can be conducted with different brake cycles. It can be distinguished in stop braking, repeated braking and successive braking. In the stop braking, the asynchronous motor accelerates the flywheels to a certain rotational speed and are then separated over the electromagnetic clutch. After the separation of the asynchronous motor and the power train, the inertia is decelerated to 0 rpm. In the repeated braking, the flywheels are decelerated with the brake disc and then accelerated with the asynchronous motor for a certain number of cycles. For the successive braking, the flywheels are decelerated with the brake disc and then kept of a constant velocity with the asynchronous motor. The asynchronous motor requires a huge torque for the two last brake cycles to keep or accelerate the rotating flywheels. If the required torque of the asynchronous motor cannot accelerate or keep the flywheels at a constant velocity, the rotor of the asynchronous motor revolves. This can change the work principle of the asynchronous machine from a motor to a generator. The mechanical energy of the flywheels is converted into electrical energy. The intermediate circuit of the frequency converter stores the brake power. If the brake power is higher than the power loss of the frequency converter, the voltage in the intermediate circuit increases very fast. The frequency converter is shut down, when the voltage exceeds the internal voltage limiter of the frequency converter. This can be avoided with the degradation of energy in the motor, the installation of a brake chopper circuit or with unit to feed back into the electrical network.

4.2.3 Design of the Flywheel Test Bench

The test bench must fulfil several requirements. The main requirements are:

- the whole length must be as short as possible,
- the cost must be as low as possible,
- the inertia moment of the test bench must be large and changeable,
- the weight must not exceed the load capacity of the ceiling,
- the installation of an advanced measurement system.

The most difficult requirement is the design of the flywheels. On the market there are several designs for the simulation of the inertias. There are test benches, for which the inertia moment of the flywheels can be changed. The inertia moment of the flywheels can be changed through a complex mechanical lifting structure. The inertia moment of the flywheels can be changed through the torque control of the motor, that means using the resistance torque of the motor to simulate the inertia of the vehicle. During the acceleration process there are two main loads on the shaft. One is the torque generated by the asynchronous motor and another one is the bending moment by the gravity of the flywheels. In this test bench there are in total 6 flywheels. The small flywheels, which are circular rings, have an inner diameter of 145 mm and an outer diameter of 600 mm. The large flywheels have an inner diameter of 145 mm and an outer diameter of 750 mm. The mass of the small flywheel is 72 kg and for the large flywheel 116 kg. The maximum mass of the flywheels and the shaft is 610 kg. The adjustable moments of inertia of the flywheels test bench are listed in table 4.4.

No. of small flywheel	No. of large flywheel	Θ_{total} [kgm ²]	No. of small flywheel	No. of large flywheel	Θ_{total} [kgm ²]
0	0	0	2	0	6,2
0	1	7,9	2	1	14,1
0	2	15,8	2	2	22
0	3	23,7	2	3	29,9
1	0	3,1	3	0	9,3
1	1	11	3	1	17,2
1	2	18,9	3	2	25,1
1	3	26,8	3	3	33

Table 4.4: Combination of flywheels

The production of the flywheels requires a compliance with geometric dimensioning and tolerancing. Through the compliance of the geometric dimensioning and tolerancing and the

composition of the raw material, a collection of materials is observed area by area. The unbalanced masses are detectable through vibrations by the rotation of the component. The vibrations are generated due to centrifugal forces. The balancing procedure is described in the standard specification DIN ISO 1940-1:2004-04. The centrifugal forces can be calculated by

$$\mathbf{F} = m \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}), \quad (4.39)$$

where m , r , and Ω are the unbalanced mass, the radius of the unbalanced mass, and the angular velocity, respectively. The unbalance is divided in a static unbalance and a dynamic unbalance. The static unbalance can be measured in the standstill. By rotating the flywheel with the applied unbalanced mass, the unbalanced mass on the flywheel moves downwards. The centrifugal forces of the unbalanced mass is applied at the centre of gravity. These forces generate additional rotating bearing reactions, which can destroy the bearing. The unbalanced mass can be removed by adding the same mass on the opposite side or by removing of the unbalanced mass. The adding of the mass on the opposite side can be applied variably on the opposite side of the radius. The correction mass can be calculated by

$$u_c r_c = u_u r_u \Rightarrow u_c = \frac{r_u}{r_c} u_u, \quad (4.40)$$

where u_c , r_c , u_u , and r_u are the correction mass, the correction radius, the unbalanced mass, and the radius of the unbalanced mass, respectively. The dynamic unbalance is composed of the static and couple unbalance. Before explaining the dynamic unbalance, the couple unbalance has to be explained. In the couple unbalance, the two unbalanced masses are located opposite and in a distance of each other. The rotor is statically unbalanced. By rotating the rotor, a torque is generated due to the two opposite centrifugal forces. This torque applies an additional bearing torque. The quality of the balancing of flywheels or rotors can be described with the balance quality G . The balance quality enables the comparison of the balance quality for different rotors. The balance quality depends on the rotational angular velocity ω , the unbalance U and the eccentricity of the mass e

$$G = e \Omega = \frac{U}{m} \Omega = \frac{u r}{m} \Omega. \quad (4.41)$$

The balance quality is lower for heavy rotors. The heavier the rotor is, the higher is the remaining unbalanced mass. The transmission shaft is unbalanced due to the balancing of the whole system, shaft and flywheels together. In figure 4.19, the mounted flywheels and shaft on the balancing machine are shown.

The necessity of the balancing can be shown in a small example. The flywheels and the shaft are manufactured with compliance of the geometric dimensioning and tolerancing. The milling and turning machines have maximum achievable tolerances. It is assumed that the straightness of the machine has a maximum tolerance of 0.1 mm. This tolerance is also reflected on the work piece. The calculation is done with the large flywheel. This means, a line can be observed from the work piece zero at the inner diameter of 145 mm to the outer diameter at 750 mm. The additional mass at the outer diameter 750 mm for a circular ring for 1° is 379.9 g. Furthermore, the homogeneity of the flywheel must be taken into account. The maximum removed unbalanced mass of a flywheel was approximately 350 g. The inertia moment is



Figure 4.19: Balancing of the flywheels and the shaft on a Schenck balancing machine (thanks to Thouet GmbH)

variable from 0 to 33 kgm². In this test bench, the kinetic energy of a car is replaced by the rotating energy of the flywheels. Based on this, the weight of the car can be calculated. By neglecting the rolling resistance, rotational energy of the rotating parts and the air drag, the equality of the energy can be assumed. The rotational energy is equal to the kinetic energy of the car

$$E = \frac{1}{2} \omega^T \cdot \Theta \omega = \frac{1}{2} v^T \cdot m_V v. \quad (4.42)$$

The simulated mass of the car is an important criterion for the performance of the flywheel test bench. The simulated vehicle mass for certain rotational speeds and vehicle velocities in regard to the moment of inertia are described. The simulated mass for a brake disc of the vehicle m_V on the test bench can be calculated with the number of the installed flywheels, the rotational speed n and the velocity of the vehicle v_V . The maximum mass for a wheel or brake disc of a car which can be simulated on the test bench is shown in table 4.5.

The figure 4.20 describes the total mass of a vehicle with regards to the moment of inertia. The vehicle velocity is assumed to be $v_V = 100$ km/h. The curves are plotted for different rotational speeds of the power train. The maximum vehicle mass is achieved with the rotational speed of 1460 rpm, the vehicle velocity of 100 km/h and the moment of inertia of 33 kgm². The maximum mass of the vehicle is approximately 1000 kg. This means at one brake a brake mass of 1000 kg is applied. For reasons of driving stability, the braking force is distributed on the four wheels in function of the vehicle weight. The front brake power must always be higher than the rear. Normally, the ratio between the front braking force and the rear braking force is 2-2,5 [25]. This means, 70 % of the weight is applied on the front wheels, while about 30 % burdens on the back ones.

The figure 4.21 illustrates the introduced energy into the brake system with respect to the moment of inertia for certain rotational speeds. The maximum energy is achieved with the

J	$n = 365$ rpm	$n = 730$ rpm	$n = 1095$ rpm	$n = 1460$ rpm
[kgm ²]	$v_V = 50$ km/h	$v_V = 75$ km/h	$v_V = 125$ km/h	$v_V = 175$ km/h
	m_V	m_V	m_V	m_V
3.1	23.5 kg	41.7 kg	33.8 kg	30.7 kg
7.9	59.8 kg	106.4 kg	86.2 kg	78.1 kg
11	83.8 kg	148.1 kg	120 kg	108.8 kg
15.8	119.7 kg	212.7 kg	172.3 kg	156.3 kg
22	166.6 kg	296.2 kg	239.9 kg	217.6 kg
26.8	203 kg	360.8 kg	292.3 kg	265.1 kg
33	249.9 kg	444.3 kg	359.9 kg	326.4 kg

Table 4.5: Simulated masses on the test bench

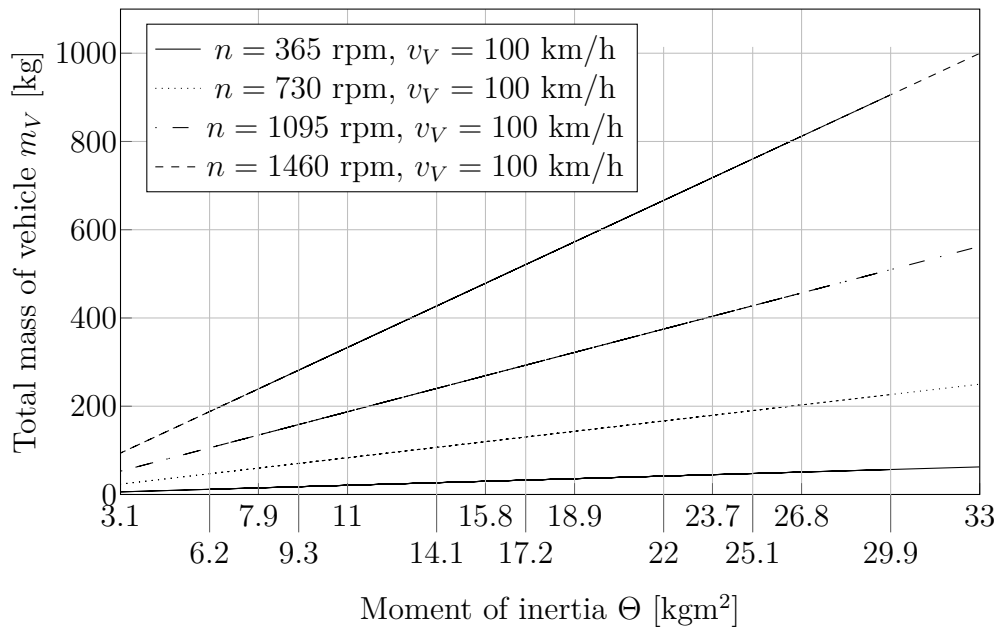


Figure 4.20: Total mass of a vehicle

rotational speed of 1460 rpm and the moment of inertia of 33 kgm². The maximum energy is 386 kJ. The lower the rotational speed, the lower is the introduced energy. It is obvious that the doubling of the rotational speed quadruples the energy.

The introduced energy according to the rotational speed for fixed moments of inertia is shown in figure 4.22.

The curves show a quadratic characteristic. The total power of the introduced energy from 0 to 1460 rpm can be calculated by energy over the rotational speed. The static and dynamic loads of the experiments are transferred on the transmission shaft. The bending moments of

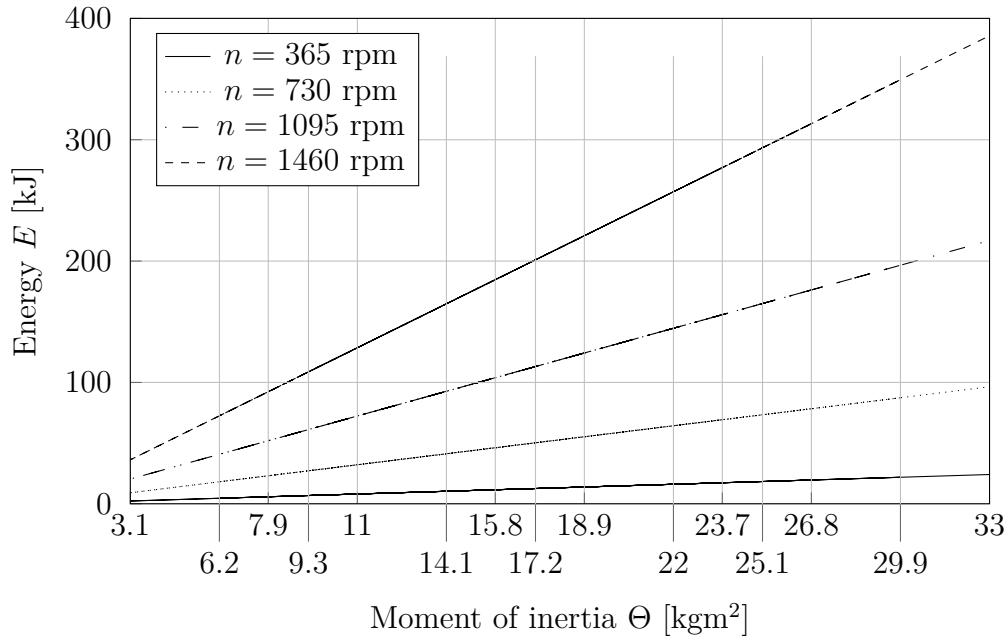


Figure 4.21: Energy

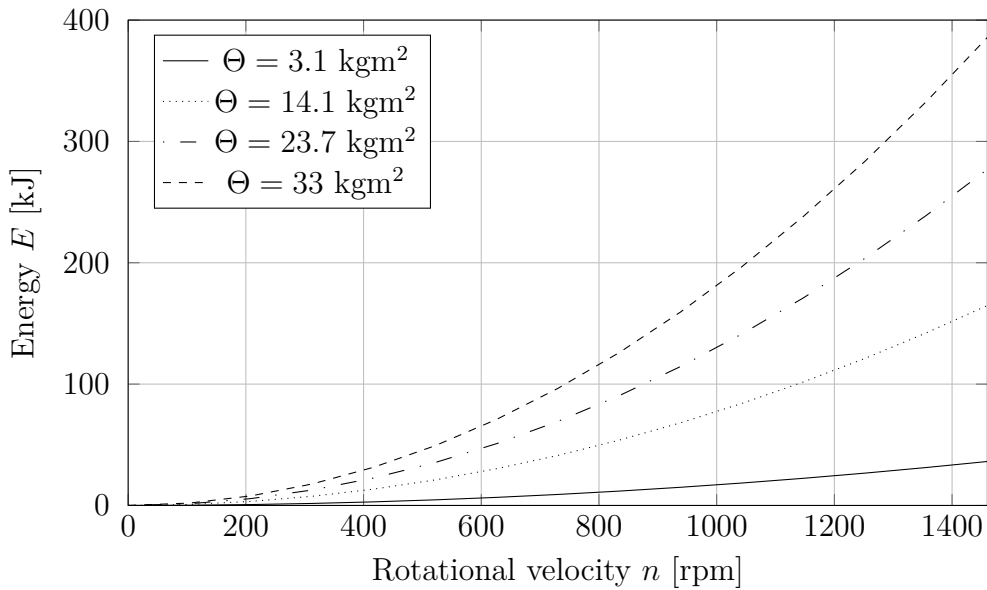


Figure 4.22: Energy rotational velocity

the flywheels, the bearing reactions and torsional moments are applied on the transmission shaft. The torsional moments are applied through the electromagnetic clutch and the braking. The maximum coupling torque of the electromagnetic clutch is 640 Nm. The brake torque can be calculated according to the following description. During the acceleration and deceleration process, the gravity of the flywheels produces a bending moment. In the deceleration process, the torque is included due to the brake action. In order to get the torque load, the brake force must be known at first. The two-side linear mechanical force generated by the calliper is $F_{\text{cal}} = 2p_{\text{cal}}A_{\text{cal}}$, where F_{cal} , A_{cal} and p_{cal} are the mechanical force generated by the calliper on one side, the effective area of the calliper hydraulic piston, and the hydraulic pressure on the

piston, respectively. The friction force on the brake pad can be described with $F_f = \mu F_{cal}$, where F_f and μ are the friction force and the friction coefficient, respectively. The brake moment is $M_B = F_f r_{mean}$, where r_{mean} is the mean friction radius of the brake disc. The verification of the safety against failure or the calculation of the load capacity of shafts is conducted with the standard specification DIN 743. The DIN 743 is composed of four parts. The first part shows the general calculation step of the calculation of load capacity of shafts. The second part deals with the calculation of the theoretical stress concentration factors and fatigue notch factors. The third part determines the strength of materials. The last part describes the fatigue limit, the endurance limit and the equivalently damaging continuous stress. The methodology of calculation of the load capacity of shafts is illustrated in figure 4.23. The bearing reactions of the shaft are 2690 N and 2930 N. The weights of the two flywheels packages are 2155 N and 3466 N. Furthermore, the weight of the shaft is neglected, which is small compared to the weight of the flywheels. The shear stresses due to shear forces are neglected because they are small compared to the bending moment and the torsional stresses. The maximum bending moment with the full installed flywheels is 809 Nm. The torsional stress consists of the coupling torque and the brake torque. The brake torque depends on the mean radius of the friction force and the friction force. The brake torque of a brake disc for which the braking is applied with two brake pads is twice. The brake torque with a mean radius of 0.25 m, a brake pressure of 40 bar, a brake pad area of 4275 mm² and a coefficient of friction of 0.4 is 3420 Nm. The calculation of the safety against fatigue fracture requires the calculation of strengths and stresses. These require the calculation of the total influence factor. The total influence factor is composed of the shape, the kind of stressing, the roughness and the hardening of the shaft. The safety calculation is conducted for critical regions of the shaft. The shoulders, the keyways and the circumferential groove for the retaining ring are all found in the critical regions of the shaft. After this safety calculation, a Finite Element (FE) analysis is carried out. The applied loads on the shaft in the model are the brake torque of 3420 Nm and the weight of the flywheels 2155 N and 3466 N. The bearings are simulated as fixed bearings. The goal of the simulation is to avoid plastic deformation and therefore a linear elastic model is adapted. The maximum displacement is 15 μ m and the maximum *von-Mises* stress at the point of application of the brake torque is 372 N/mm². The application of the standard specification DIN 743 and the FE analysis shows the fulfilment of the fatigue strength and the finite life fatigue strength of the designed shaft.

The used electromagnetic clutch provides the coupling torque of the ASM and the transmission shaft. The reason for the choice of the electromagnetic clutch is the dry running. The clutch does not need oil, pumps and a cooling system, which makes the test bench expensive and complex. The electromagnetic clutch provides a coupling torque of 640 Nm. The magnet coil is supplied with 104 DCV, which is generated by a half-wave rectifier. The electromagnetic clutch consists of three parts, the rotor, the magnet coil and the flanged hub. The contact of oil or grease with the friction surface must be avoided otherwise the coupling torque will be reduced. The nominal coupling torque of 640 Nm is achieved after the run-in of the clutch. The voltage supply to the magnetic coil generates a magnetic field. The flanged hub is attracted to the rotor with the friction pad. The coupling torque is transmitted due to friction. The torque is transmitted from the ASM via the flanged hub and the rotor to the transmission shaft. The decoupling is achieved by separation of the power supply to the coil. Thereby the membrane spring of the flanged hub is drawn back to the shaft and the connection between the ASM and shaft is separated. The electromagnetic clutch provides a good solution with

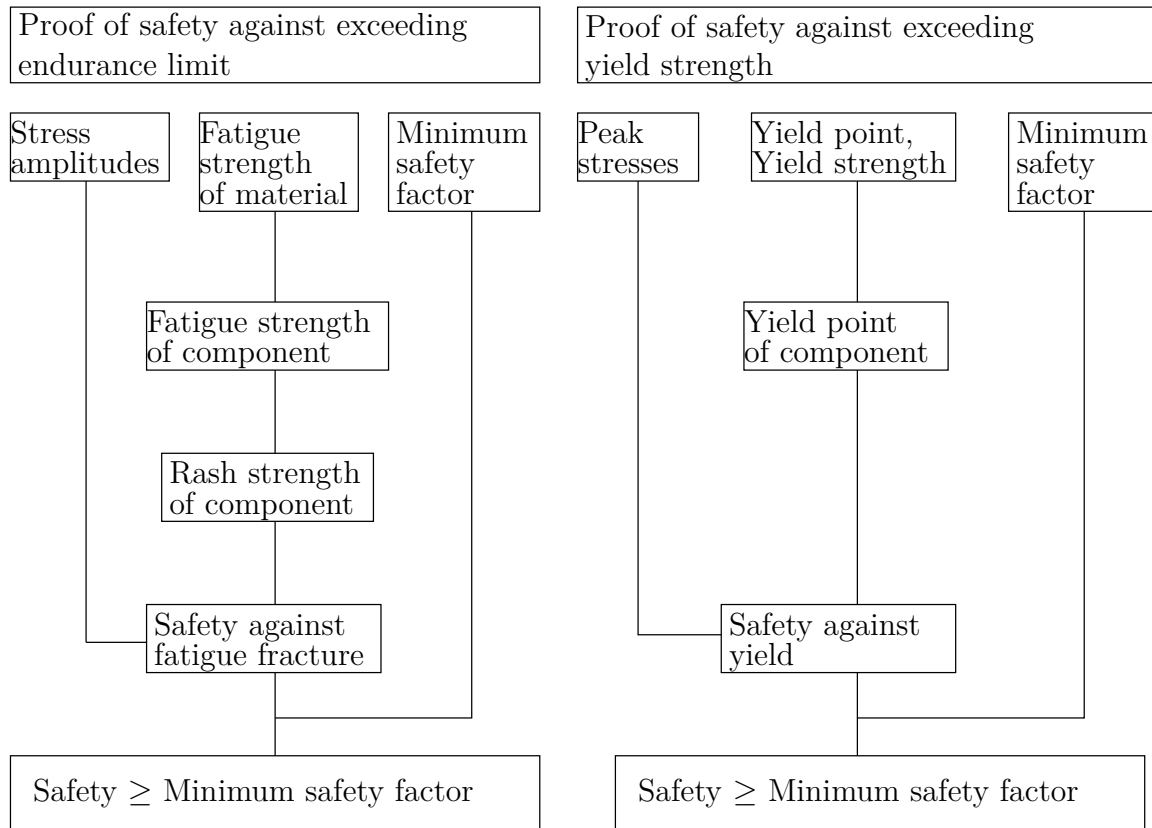


Figure 4.23: Methodology of calculation of the load capacity of shafts

the disadvantage of a high tolerance in regards to the shaft misalignment and centre offset. The permitted shaft misalignment and the centre offset are 0.1 mm and 0.25 mm, respectively. The danger of a failure of the flywheel test is obvious on closer inspection of the loads. The maximum energy of the flywheel test bench is 386 kJ. The maximum mass of the shaft and the flywheels is 610 kg. The maximum rotational speed is 1460 rpm. Therefore a predetermined breaking point between the brake disc and the transmission shaft is installed. The jaw clutch is the predetermined breaking point with a maximum torque of 2560 Nm. The jaw clutch fails before the transmission shaft fails. Another advantage of the jaw clutch is the inclusion of tolerance errors. The maximum allowable radial and axial shaft misalignment is 0.48 mm and 3.0 mm, respectively. The angular misalignment of the shafts is 1.2° . Figure 4.24 shows the flywheel test bench.

The brake force is applied over the geared motor. The geared motor consists of a direct current motor and a worm gearing. The geared motor has a travelling distance of 150 mm. The maximum holding force of the geared motor is 3000 N. A potentiometer is installed for the determination of the position of the crank. The geared motor is controlled by the data acquisition system. The geared motor moves the piston of the main brake cylinder, which generates a hydraulic pressure in the cylinder. The brake fluid is transported over the brake-hose pipe into the cylinder of the brake calliper. The generated pressure in the cylinder of the brake calliper moves the piston of the brake calliper and generates the brake pressure or the brake force between the brake disc and the brake pad. In figure 4.25 the brake force generation is shown.

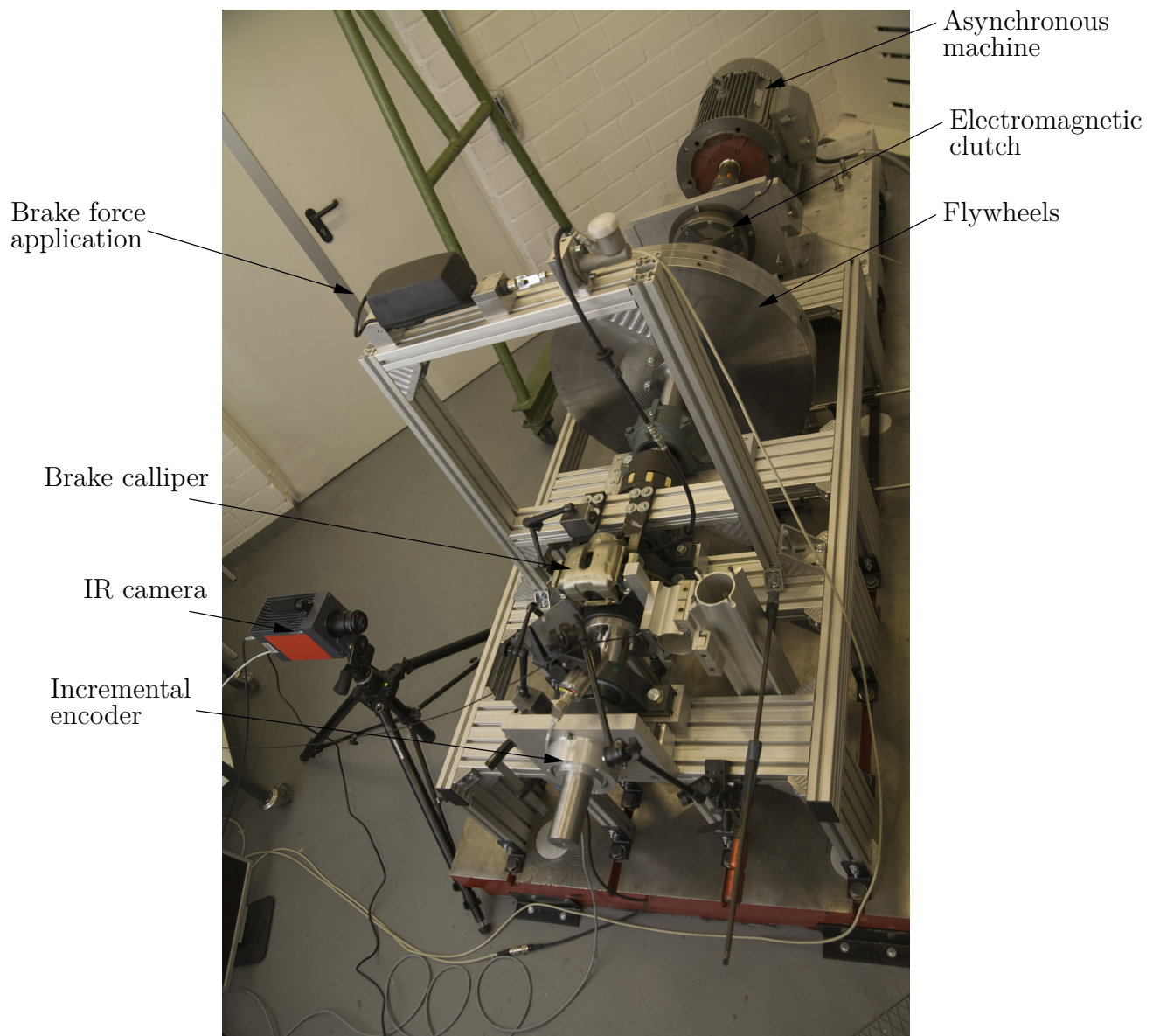


Figure 4.24: Flywheel test bench

4.2.4 Data Acquisition System

The DAQ is the main component of the test bench. It controls all incoming and outgoing data. Moreover, the DAQ system is protected from interfering signals, mains-borne disturbance and unwanted harmonics. The integer multiples of the fundamental frequency are the unwanted harmonics, which are generated by the electrical power supply. This makes the DAQ system complicated, because there are several components with different working behaviour. Another important issue is the frequency converter and the three-phase asynchronous motor, which generate unwanted harmonics. In conclusion, the Electromagnetic compatibility (EMC) consists of the conducted emission and radiated emission. The emission can be divided in the galvanic couple, the capacitive couple, the inductive coupling and the electromagnetic coupling. The galvanic couple can be observed, when two circuits are connected over a common conductor. The capacitive couple occur when two circuits are connected on the same reference ground. In

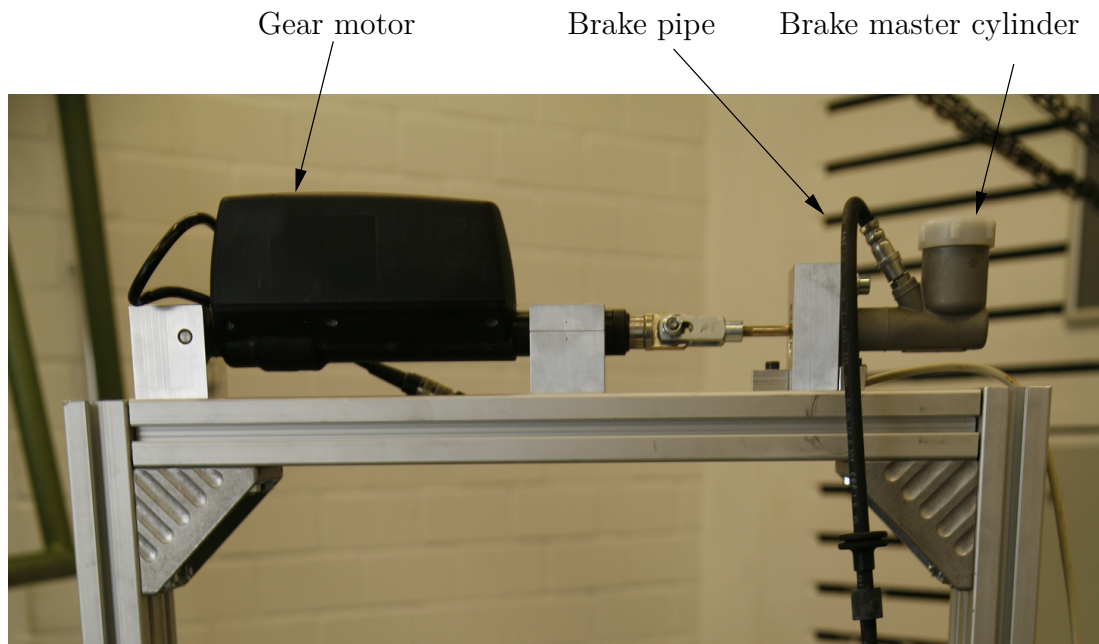


Figure 4.25: Brake force generation

this case, the oscillation is induced over the connected conductors. Currents are induced in the lines when a magnetic field is applied on a current-carrying conductor. Finally, the electromagnetic coupling is observed when an electromagnetic energy is irradiated by a conductor and detected by the current circuit. The electromagnetic coupling is mostly observed by frequency converters and electrical power supply. The frequency converter switches high voltages in μs range with high voltage amplitudes between 500-1000 V and high currents. The disturbance source of the frequency converter is the alternating-current converter with the direct voltage intermediate circuit, which generates a pulse-shaped output voltage with short rise and fall time. This voltage is grounded over the capacity couple in the electric motor cable and generates a common mode current. This common mode current flows back to the frequency converter and generates the interferences. These can be avoided, when the shielded cable on the motor side is connected with the crankcase and the shielded cable on the frequency side is connected with the case of the frequency converter. In this case, the common mode current flows over the common mode capacitor to the intermediate circuit. Another possibility to avoid the interferences is the installation of a filter. The shielding plays an important role for the measuring of the signals. The shielding increases the noise immunity and decreases the emitted interference. The shielding has to be connected at a large-area on the cases of the component and on both sides with the cases. The data acquisition system controls the frequency converter, the measuring system and the emergency stop. The DAQ and the control of the flywheels test bench and its components (ASM, brake calliper, ventilator, etc.) are conducted with NI LabVIEW 2015 software and the NI PCIe-6321 hardware. The software provides the possibility to develop the measuring system faster and automate several measurements. Furthermore, it enables the integration of a variety of measuring instruments, which will be explained in section 4.2.5. Following this, data from almost any device can be detected. The reconfigurable hardware and the LabVIEW software supports the modular design of the test bench with the possibility of a faster development of an automated measuring system or a modified control of the test bench. The used PCIe-6321 card has 16 analogue inputs. The maximum analogue input sampling rate is 250 kS/s. Furthermore,

it consists of 2 analogue outputs and 24 digital inputs and outputs. The output signals are provided with an update rate of 900 kS/s. The maximum digital input and output rate is 1 MHz. The possibility of triggering with digital signals is given. The ADC and DAC resolution for the analogue input and output is 16 bits. The switching units and the interface adapter are illustrated in the figures 4.26 and 4.27. The switching units energise the components with 230 VAC, 5 VDC and 24 VDC. The capacitive sensor, and the pyrometer are supplied with 24 VDC, which measures the displacement and the punctual temperature of the brake disc, respectively. The relays R1.0, R1.1, R1.6, R1.5, R2.4 and R2.6 are supplied with 5 VDC to control the components, such as the electromagnetic clutch, the geared motor, the frequency converter and the Multi Control Multifunctional Modul (MCMM) of the infrared camera. The 230 VAC energise the power supply units of 5 and 24 VDC. The interface adapter in the figure 4.26 consists of the 68-pin VHDCI connector and the pins of the input and output signals. The 68-pin VHDCI connector connects the computer or the PCIE-6321 with the measurement and control system. The pins provide the acquisition and the output of signals. Seven of the digital I/O of the interface adapter are connected with an array of seven NPN Darlington transistors. These Darlington transistors are used for the voltage feed of 24 V for the relays R1, R2 and R3. The Darlington transistors can be switched with a maximum support current of 0.5 A and the maximum output voltage is 50 V. The digital output signal P1.0 closes the relay R1.0 which then closes the relay R2. Thereby, the electromagnetic clutch is supplied by the half-wave rectifier with a direct current of 104 VDC. The voltage feed of the geared motor is realised with a relay connection. This has to enable the clockwise and anticlockwise rotation of the geared motor. A voltage metering is conducted for the determination of the position of the piston of the main brake cylinder. This is necessary for the determination of the brake pressure. The digital output signal of P1.1 closes the relay R1.1 which closes the relay R3. This relay connection supplies the gear motor with voltage. The digital output signal P1.6 determines the clockwise and anticlockwise rotation of the gear motor.



Figure 4.26: Control unit I of the test benches

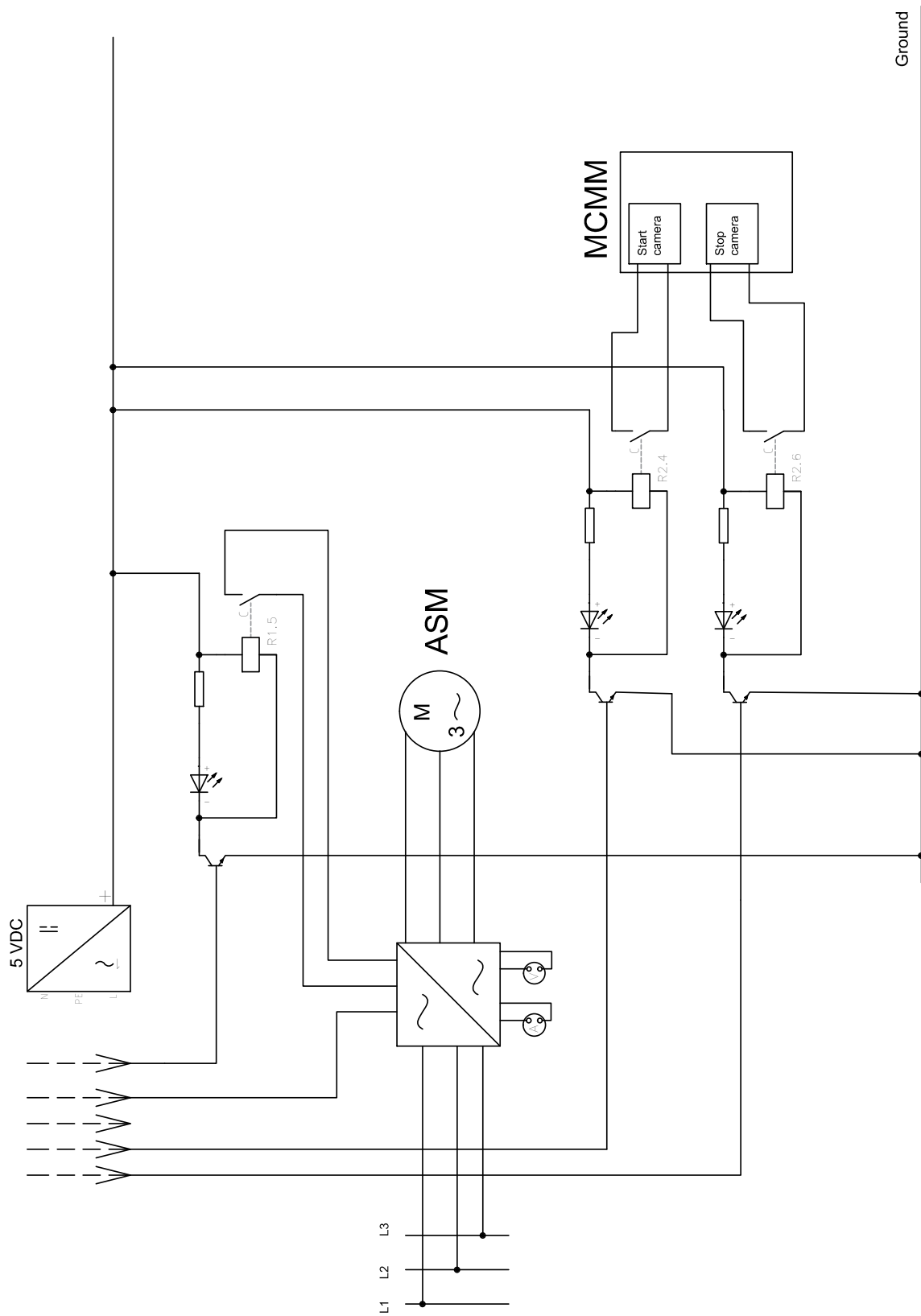


Figure 4.27: Control unit II of the test benches

The PCIe-6321 controls the frequency converter which supplies the ASM. The digital signal output P1.5 set the clockwise and anticlockwise rotation of the ASM. The analogue output signal AO0 supplies the pin for the frequency adjustment with a voltage between 0 and 10 V. This is realised with a linear ratio between voltage and frequency. The start and stop of the measurement of the thermograms of the IR camera is conducted with the digital output signal P2.4 and P2.6. The relays R2.4 and R2.6 are connected with the MCMM and close the circuit. The voltage feed of the ASM is ensured with the frequency converter which is connected with the power supply system. The frequency dependent voltage is measured and displayed over the LabVIEW software and the PCIe-6321. The measurement of the frequency dependent voltage is used to control with the incremental encoder the current speed. The current dependent current is used to monitor the output current of the frequency converter. The measured current with the PCIe-6321 is between 0 and 20 mA which represents the linear ratio of the output current of the frequency converter. The emergency stop of the flywheel test bench is shown in figure 4.28. The electrical power supply is regulated over the cam switch. The emergency switches are connected with the contactor, which connects the control boxes with the public power supply. The opening of one of the emergency switches opens the contactor and disconnect the control boxes with the public power supply. The caution lights are switched on during the experiments or the power on of the control boxes. The caution lights are supplied by the transformer and the relay.

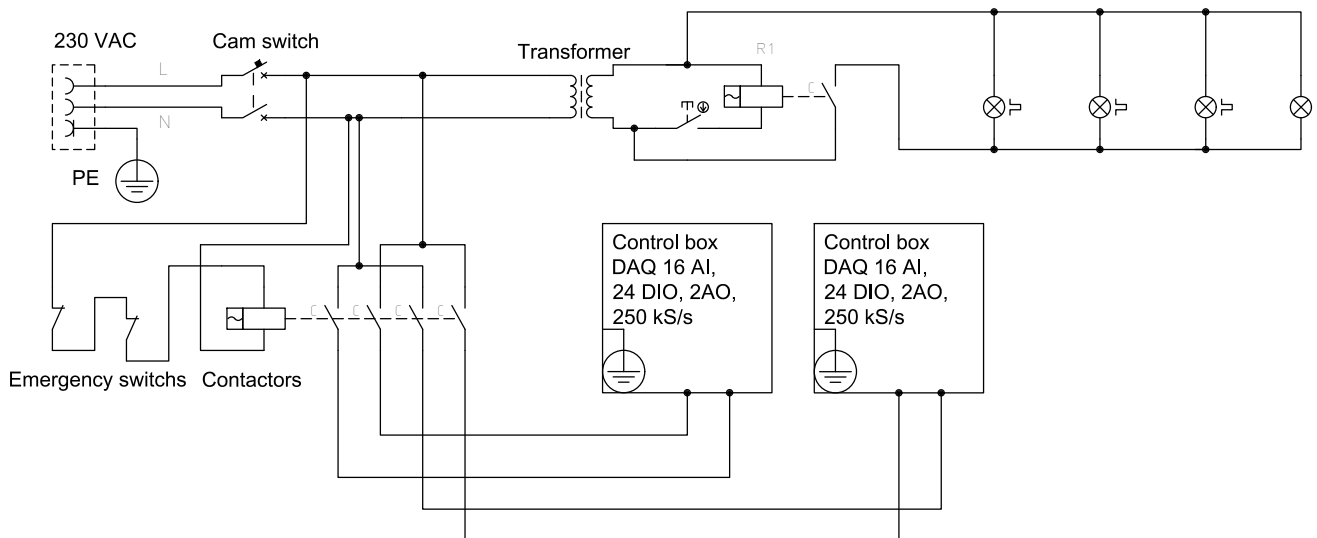


Figure 4.28: Emergency stop of the test benches

4.2.5 Measuring System

The goal of the design of the flywheel test bench is the installation of the sensor technology. The fundamental research and industrial research of the brake system can be conducted on the test bench. In the fundamental research, there are several important fields of research. Cases in point of relevance here are the deformation measurement of the brake disc and brake pad for the validation of material models to describe the material behaviour. Furthermore, the understanding of the tribological behaviour and its description is a challenging task. Suitable

sensor systems such as thermocouples, IR camera, load cells and microsections, can help for a better understanding of the tribological phenomena. The three-dimensional measurement of the movements of the brake calliper in space can offer possibilities for the brake force development or the deformation of the brake disc during the experiment. The deformation measurement is conducted with high speed capacitive sensors. They offer the possibility to detect the hot spot phenomenon. The main measurement system is the high speed IR camera which measures the temperature distribution. The synchronisation and interaction with the other sensors and the flywheel test bench has to be programmed. The proper understanding of the infrared and the detecting system is indispensable. In all respects, the understanding of the contexts and the peculiarities of the sensors is a very important task. Subsequently, the sensors which are used for the test benches are introduced and described.

Capacitive Sensor

The measurement of the displacement is conducted with the capacitive method. The used capacitive distance measurement is based on the principle of the parallel plate capacitor. The displacement measurement consists of two devices of of Micro-Epsilon GmbH,

1. the single-channel measurement electronic DT6350 and
2. the CS2 cylindrical sensor.

The signal conditioning electronics are in the single-channel measurement system. The used digital signal processor achieves a sample rate of up to 50 kHz bandwidth at the analogue output. The schematic representation is shown in figure 4.29. The single-channel measurement system consists of the Digital Signal Processor (DSP), the oscillator and the demodulator. The DSP calculates the output signal. The oscillator provides to the sensor a constant and amplitude-stable sinusoidal current of 250 kHz. The demodulator amplifies and demodulates the distance-dependent measuring signal.

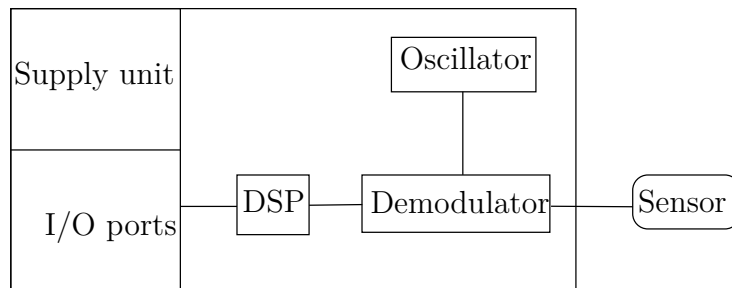


Figure 4.29: Capacitive evaluation unit

The single-channel measurement system evaluates the reactance X_c of the plate capacitor, which changes strictly in proportion to the distance

$$X_C = \frac{1}{j \omega C}, \quad (4.43)$$

where j , ω , C are the imaginary number, the angular frequency, and the capacity, respectively. The capacity can be calculated according to

$$C = \epsilon_r \epsilon_0 \frac{A}{d}, \quad (4.44)$$

where ϵ_r , ϵ_0 , A , and d are the relative dielectric constant of the dielectric medium, the absolute permittivity of vacuum, the electrode surface, and their distance, respectively. The measurement principle needs the conductive targets for which the sensor and targets opposite form the two plate electrodes. The capacitive sensor operates as an ideal plate-type capacitor, where the sensor and the brake disc represent the plate electrodes. The distance is measured by the flow of a constant alternating current through the capacitive sensor, where the amplitude of the alternating voltage on the sensor is proportional to the distance of the sensor and the brake disc. The alternating voltage is demodulated, amplified and outputted as an analogue signal. The output signal of the measuring signal is achieved for electrically conductive target materials without any additional electronic linearisation.

The distance can be calculated with

$$d = \frac{X_c}{\text{Constant}} = \frac{\text{Constant}}{\text{Capacity } C}. \quad (4.45)$$

The capacitive sensor is realised as guard ring capacitor to achieve the required accuracy and linearisation. This type of a capacitor eliminates the inhomogeneous field at the edge of the capacitor during the measurement. The schematic view of the capacitive sensor is illustrated in Fig. 4.30. The capacitor consists of three main parts, the ground, the screening electrode, and the measuring electrode. The screening electrode is required for the protection of the measuring electrode due to disturbing signals.

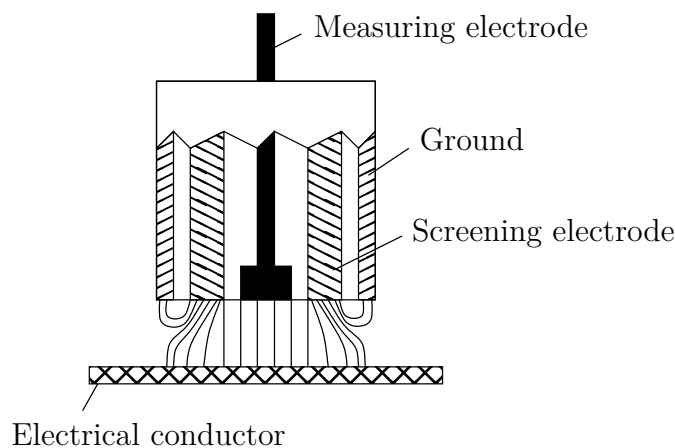


Figure 4.30: Capacitive sensor

The used capacitive sensor has a measurement range of 2 mm. The measurement area is related to the measuring range. For the used sensor with a measuring range of 2 mm, a minimum area of 17 mm is needed for the target object. The operating temperature of the sensor is between -50 °C and 200 °C. Due to the radiation and convective cooling, the capacitive sensor can be used for higher temperatures. This has to be considered for the measurement of the brake disc

in the conducted experiments. The temperature of the brake disc can reach temperatures higher than 200 °C and therefore the temperature stability of the sensor has to be investigated. The temperature stability of the sensor is $\pm 0.01\%$ of the Full Scale Output (FSO) per degree Celsius. One can use a reduced or extended range but at the expense of accuracy. The resolution of a dynamic measurement at 8.5 kHz is 40 nm. The controller consists of one channel with a static and dynamic resolution of 0.005% FSO, and 0,1 % FSO at 50 kHz. The limit frequency is adjustable. The control offers the possibility to synchronise with other controllers. The measuring range of the sensor can be changed to half measuring range or double measuring range with full output signal. The output signal in regard to the measuring range is shown in figure 4.31.

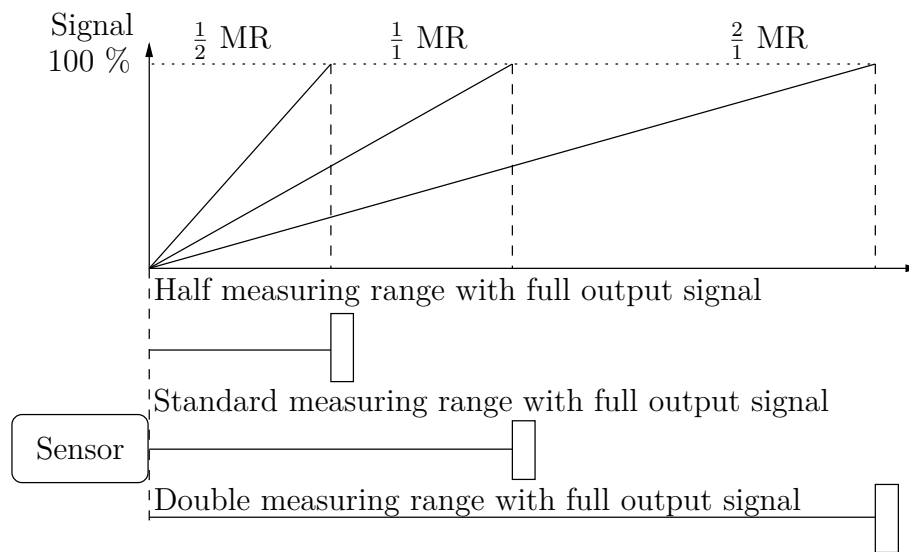


Figure 4.31: Measuring range of the capacitive sensor

The advantage of this kind of measurement principle is a high bandwidth for fast measurements, wear free, and no interfering force on the target. The sensitivity or linearity is not changing by slight changes in the conductivity or magnetic properties. A disadvantage is that the capacitive sensor measures only for electrically conductive materials and small target and uneven surfaces cause a non-linear characteristic.

Laser-Optical Sensor

A further displacement measurement is the laser triangulation. The measurement is conducted with the optical laser displacement measurement. The used optical laser sensor is an ILD 2200-20 of Micro-Epsilon GmbH with a measuring range of 20 mm and a resolution of 3 μm . The optical triangulation modulates a visible point of light onto the measuring object. It consists of a laser diode, a filter, a multi-lens optics, an optical receiving system, the Charge-Coupled Device (CCD)-array with Real Time Surface Compensation (RTSC), and a signal conditioning electronics. A visible light spot is projected onto the brake disc. Position sensitive elements detect the diffusion fraction of the reflected light and the change of the position of the light spot is focussed by the receiving elements, which have a certain angle according to the optical axis of the laser beam. Due to the change of the position, the distance is calculated according to an angle function. The intensity of the CCD signal of the diffuse reflection is measured in

real time. This enables to compensate the intensity fluctuation during the measurement. The signal is detected for a wide reflection factor. The laser triangulation operates in a measuring range. The Start of measuring range (SMR) and End of measuring range (EMR) define the measuring range and is illustrated in figure 4.32. The reflected laser beam contains a horizontal and vertical angle.

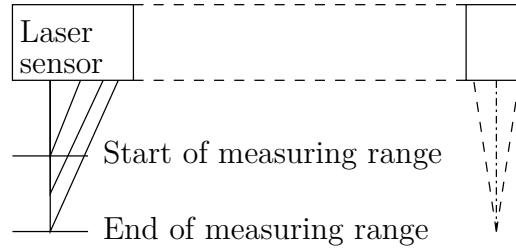


Figure 4.32: Reflection of a laser beam

The semiconductor laser diode emits visible red light with a wavelength of 670 nm and operates in a pulsed mode. The pulse frequency is the same as for the measuring frequency. The maximum achievable frequency with the used sensor is 50 kHz. The pulse duration depends on the measured object and can be set from 1 to 80 μ s. The principle composition of the signal conditioning electronic of the laser-optical sensor is shown in figure 4.33. The signals of the CCD-line are conditioned and converted into digital signals. The DSP provides the output signals and they are converted into analogue signals for the DAQ.

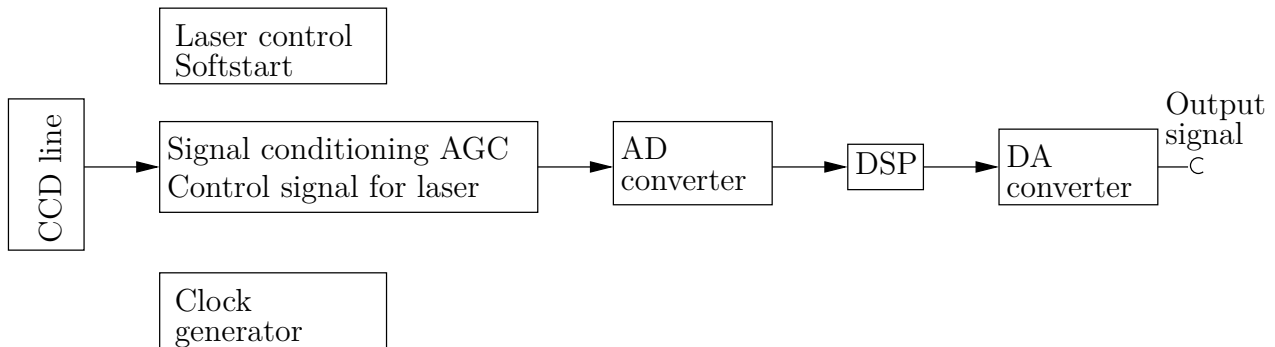


Figure 4.33: Schematic view of the Laser-Optical Sensor

The laser-optical sensor has a measurement range of 50 mm, whereas the start and end of the measuring range are 45 and 95 mm. The linearity is smaller than 0.03% of the FSO and the resolution is 0.0015% of the FSO. The maximum measuring rate is 20 kHz. The light spot diameter is for the SMR 215 μ m, the Midrange (MMR) 80 μ m and the EMR 215 μ m. This spot diameter has to be taken into account for the positioning of the spot diameter on the brake disc. The influence of other light sources and the inhomogeneous surface can be reduced by a small spot diameter. The output signal of ± 5 V has to be calibrated. This is conducted with a device, which consists of a micrometer screw. The reflection phenomenon of the laser beam is described in the section 4.2.5. The measurement of μ m or sub- μ m range requires a stable and vibration-free mounting of the sensor. For high reflecting surfaces, the tilt angle of a brake disc of 5° has a disturbing effect. The tilt angle of the disc between 5° and 15° leads to an approximately distance change of 0.12-0.2% of the FSO. As in the thermography, the surface behaviour of the brake disc plays an important role for the reflection of the laser

beam. The advantage of this kind of measurement is the purely trigonometrical calculation of the distance and therefore is appropriate for moving objects. The disadvantage of this sensor is its application for small distances.

3-D Movement

The movement of non-contacting parts like the brake calliper can be conducted through tracking of moving objects in the Three-dimensional (3D) space. The used device is nexonar 3D IR-Tracking of nexonar GmbH. With this kind of measurement device, it is possible to measure the orientation, distance, acceleration, velocity and the angle. The tracking system provides the absolute- and relative measurement. The maximum achievable frame rate of 162 Hz is provided through infrared. The measurement of the movement is realised with the transient time measurement of infrared pulses between transmitter and receiver. The measurement system consists of the infrared markers and the infrared camera of 6 Degree of Freedom (DoF) which operates as the receiver. The markers are placed on the measurement device. The 3-D position is calculated by infrared diodes of the tracker in the picture. The infrared tracker can be placed to the infrared camera in a range of 600 mm and 5000 mm. The trackers require a direct line of sight to the infrared camera. The setting of the coordinate system of the infrared tracker devices can be changed. This means, for a single camera the coordinate system refers to the used camera. Furthermore, the coordinate system can refer to a defined coordinate system and relative coordinate system in regard to three selected markers.

Force and Torque

The force measurement is used to calculate the coefficient of friction between the brake disc and the brake pad. The used force sensor is a U3 force sensor with a force rate of 5 kN of HBM GmbH. The torque measurement is conducted with the T22 torque transducer of HBM GmbH with the maximum torque rate of 1000 Nm. Furthermore, the comparison of the brake pressure in the brake pipes and the normal force between the brake disc and the brake pad is conducted for the normal force development. The force measurement can be divided in a direct and indirect force measurement. The direct measurement is an intrinsic measurement where the charge, resistance or the inductance change in the sensor with respect to the applied force. There are several direct force sensors, such as piezoelectric, piezoresistive and magnetoelastic sensors. The applied force on the piezoelectric sensor causes a charge transfer in the crystalline material. The forces disturb the equilibrated charge, which generates a current. The measured charge can be calculated according to

$$Q = k F, \quad (4.46)$$

where k , and F are the piezoelectric constant, and the applied force on the sensor, respectively. The piezoelectric sensor is suitable for dynamic measurements instead of static measurements. The piezoresistive sensor changes the specific resistance of the semiconductor crystal depending on the applied load. The applied force changes the valence-conduction band gap. Therefore, the movement of the electrons is changed and the resistance is changed too. In summary, the piezoresistive sensor is a high sensitive sensor with a high temperature dependence. The magnetoelastic sensor changes the magnetic field of the ferromagnetic body by application of

the load. The shape is changed by application of the load and therefore, the magnetic field is changed. The sensor consists of an inductor and the body, where the inductor generates a voltage by changing of the magnetic field. The indirect force measurement is divided in the vibrating-wire, strain gauge and the inductive sensor. The vibrating-wire force sensor determines the applied force in regards of the vibration of the chord. The tight chord when excited changes its frequency as a result of the applied force. The strain gauge uses the change of the resistance to determine the elongation. The principle behind the strain gauge is the elasticity theory where the applied force causes an elongation. The strain gauges are placed on the measurement object, two in force direction and two perpendicular to the force direction. The maximum detection is achieved in a bridge circuit, where the resistance of the strain gauges in force direction is small and the resistance of the strain gauges perpendicular to the force direction is large. The change of the resistance of the strain gauge transducers is proportional to the applied force. The inductive force sensors measure the applied force with an inductive range sensor, which is attached at a spring diaphragm. The tracer pin moves in the conductor, which generates a voltage. The voltage of the inductor and the feeding voltage can be measured and the applied force can be calculated according to the sum of the voltages. The force can be measured in one or three directions with one force sensor. The used force measurement device consists of a plug-in amplifier, a connecting plate and strain gauge transducer force sensors. The rated value of the force sensor is achieved by 10 V. There are two plug-in amplifiers used, the direct current measuring amplifier for strain gauges with a maximum measurement frequency of 10 kHz and the carrier frequency amplifier with a frequency of 600 Hz for strain gauge transducers.

Incremental Encoder

The simulations and the analysis of the experiments require the detection of the angular changes of the brake disc. The used incremental encoder is an incremental encoder ITD 75 A 4 2048 T NI KR1 S 75 with 2048 pulse per rotation of Baumer GmbH. Furthermore, Transistor-Transistor Logic (TTL) signals are necessary for synchronisation of the IR camera with the brake disc rotation. The measure principles can be divided in a direct and indirect measurement. For the direct measurement acquisition, the measured variable is compared with an equivalent reference magnitude. As for the indirect measurement acquisition, the measured variable is converted into a physical quantity and from this physical quantity a measured value is determined. By the installation of a measuring system, the fault influence must be taken into account. That would be for example, the temperature, pitch error, angle error, and elastic deformation of the shaft. The measured value acquisition is divided into analogue and digital. The analogue measured value acquisition correlate directly with the measured variable. The digital measured value acquisition subdivides the measured value in equally spaced sections. The measured value is then calculated by their multiples. Another distinctive feature is the relative and absolute measured value acquisition. For the incremental measured value acquisition, the periodically measuring signals are counted and visualised. A reference signal or a coded signal can be installed to determine the absolute position. The analogue measuring system are per definition absolute measuring systems. After this short explanation of the measuring systems, the installed rotation speed control counter is introduced. For the synchronisation of the IR camera and the high resolution of the photoelectric scanning systems an incremental encoder is selected. The incremental encoder uses an optical scanning principle. The main parts of an incremental

encoder are the graduated disc and the scanner. The detection is realised either with the reflected-light method or the transmitted light method

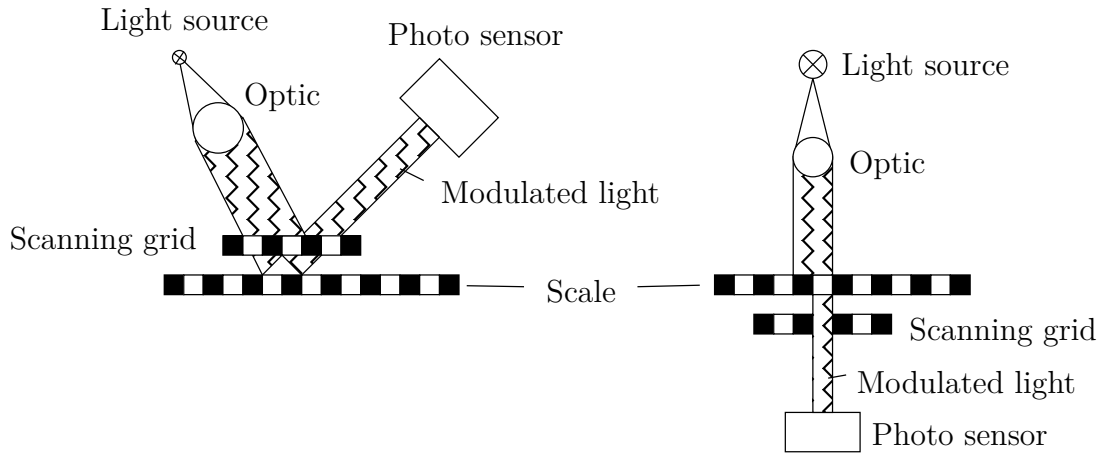


Figure 4.34: Reflected-light method and the transmitted light method

The reflected-light method consists of alternating reflecting and non-reflecting areas. The transmitted light method consists of transparent and opaque areas. The scanner is composed of the light source, the scanning grid, and the photo detector. The photo detector detects the modulated light and converts the light intensity in electrical signals, which can be used to measure the rotation. The output of the photo elements are sinusoidal signals, which are converted into square wave signals in the evaluation unit. The schematic diagram of the incremental encoder is shown in figure 4.35.

The properties of the used incremental encoder are

- output in TTL, and
- pulses per revolution are 2048 with a zero pulse.

The disadvantage of incremental encoders is the necessity of a reference point. Due to the law of geometrical optics, the minimum pitch is $10\text{ }\mu\text{m}$. To increase the resolution, the interpolation of the sinusoidal signals is necessary. The accuracy grade depends on the bearing, the scanner, the disc, and the temperature distribution of the incremental encoder.

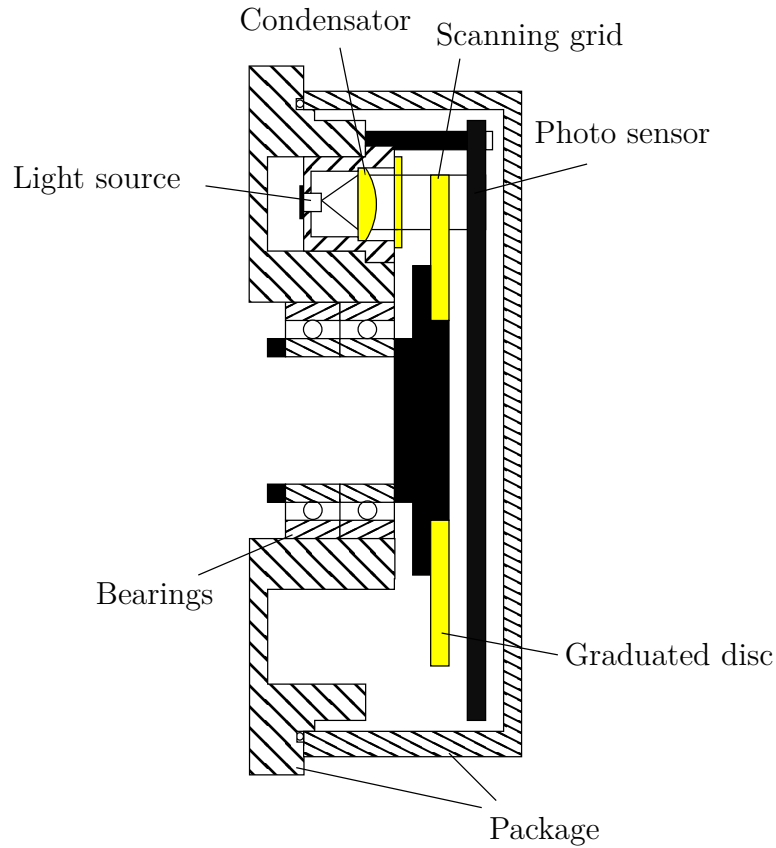


Figure 4.35: Schematic view of the incremental encoder

Pyrometer

The used pyrometer IMPAC IGA 6/23 Advanced of LumaSense GmbH measuring range of 50 to 1000 °C is a fast infrared measuring sensor for non-contact temperature measurement. The measurement principle is based on detecting the thermal radiation. The pyrometer consists of the optics and the detector. The optic focuses the thermal radiation onto the detector. The detector outputs a signal, which is equivalent with the detected temperature. The measured temperature can be calculated with

$$M = \sigma \epsilon T^4, \quad (4.47)$$

where σ , ϵ , and T are the *Stefan-Boltzmann* constant, the emissivity, and the temperature. The spectral range of the pyrometer is between 2 and 2.6 μm . The minimum span is 50 °C, and a resolution of 0.1 °C or 0.0015% of the adjustable temperature range is achieved. The analogue output is conducted with a discretisation of 16 bit. The pyrometer has the possibility to adjust the emissivity, the transmittance, and the exposure time. The integration time of the pyrometer is 0.0005 s or the sampling rate is 2000 Hz. The temperature range is between 50 and 1000 °C. Due to the noise performance of the pyrometer, a satisfactory measurement is achieved, which is approximately 70 °C and above. The measurement uncertainty is 0.3% of the temperature range. The detection of at least 90% of the irradiance depends on the measuring distance and the spot size diameter. The correlation of the measuring distance and the spot size diameter are given from the manufacturer. For smaller measuring distances, the spot size

diameter can be calculated by $M_2 = \frac{a_2}{a}(M - D) + D$ and for longer measuring distances by $M_1 = \frac{a_1}{a}(M + D) - D$. As seen in the figure 4.36, the spot size diameter before and after $\emptyset M$ increases. The larger the distance after $\emptyset M$ between pyrometer and object, the larger the spot size diameter. The smaller the distance before $\emptyset M$ between pyrometer and object, the larger the spot size diameter.

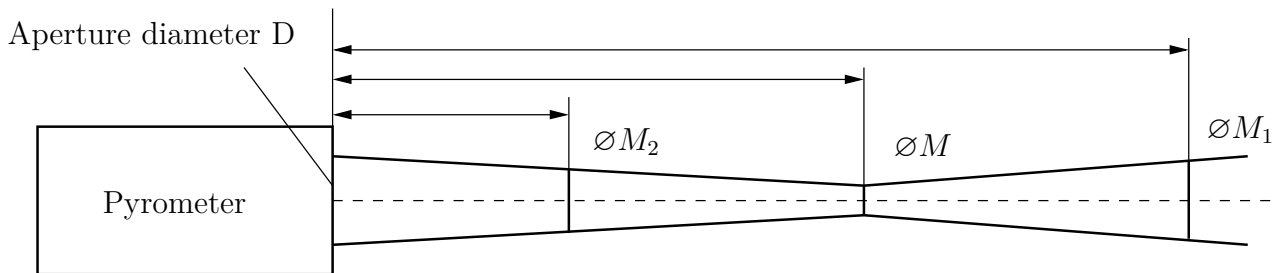


Figure 4.36: Measurement range of the pyrometer

Thermocouple

The measurement of steady state temperatures in solids can be conducted with thermocouples. The thermocouples can be placed on the surface of the measured object to measure the surface temperature or inside of the measured object to measure the temperature in regard to the borehole depth. The attached thermocouple on the surface of the measured object has to be well bonded to the surface or in a shallow borehole to reduce the thermal contact resistance. The measurement errors, which can occur are:

1. the ambient temperature is lower than of the measured object
2. the ambient temperature is greater than of the measured object.

For the first one, the thermocouple measures lower temperatures due to the fact that the heat transfer from the surface towards the thermocouple is higher. Due to this heat transport temperature gradients can be observed. For the second one, the thermocouple conducts the heat from the ambient to the solid. The observed temperature errors for imbedded thermocouples are the raising or lowering of the temperature of the solid. Furthermore, the temperature difference of the adhesive and the solid has to be considered, for a correct measuring.

The advantages and disadvantages are shown in table 4.6

Advantages	Disadvantages
Simple, only two wires, non-sensitive to shock and vibration, applicable for high temperatures, small size and high response rate, economical	Needs physical contact, can be damaged and contaminated, wear on rotating components, acts like a heat sink or heat source

Table 4.6: Advantages and disadvantages of thermocouples

The Seebeck effect describes the voltage output or the thermoelectric voltage of the thermocouples. Two dissimilar conductors with one junction at temperature T show a voltage at the open end of the conductors. The measured voltage or temperature difference can be calculated by

$$U = s_{A,B} dT, \quad (4.48)$$

where $s_{A,B}$ is the Seebeck coefficient for the materials A and B . The Seebeck coefficient itself depends on the temperature. By applying a current between the two wires, a reversed mechanism can be observed; the *Peltier* effect. Due to this effect, a heat sink or heat source according to the current flow can be generated

$$\dot{Q} = \pi_{A,B} I. \quad (4.49)$$

$\pi_{A,B}$ are the material constants.

The temperature range and the application conditions depends on the material composition of the thermocouples. In the DIN EN 60584-1:2014-07 the electromotive force and the *Seebeck* coefficients are listed. The used thermocouple is of the type K. The temperature range according to DIN EN 60584-1:2014-07 is between -270 °C and 1300 °C.

Infrared Thermography

The IR thermography is a non-invasive procedure to determine the temperature distribution for a thermal radiation emitting device. The imaging infrared camera is a relative new technology, which was developed in the middle of the last century. The use of this procedure can be found in several applications, e.g. monitoring, modernising insulation of buildings, research and development and quality control. In the following, the synchronisation of the IR camera and the test bench, the physical fundamentals, the correct application and the different ranges of application are described. The purchased camera is a high speed IR camera ImageIR 5300 of InfraTec GmbH. The synchronisation and the installation of the IR detection of the brake disc is shown in figure 4.37.

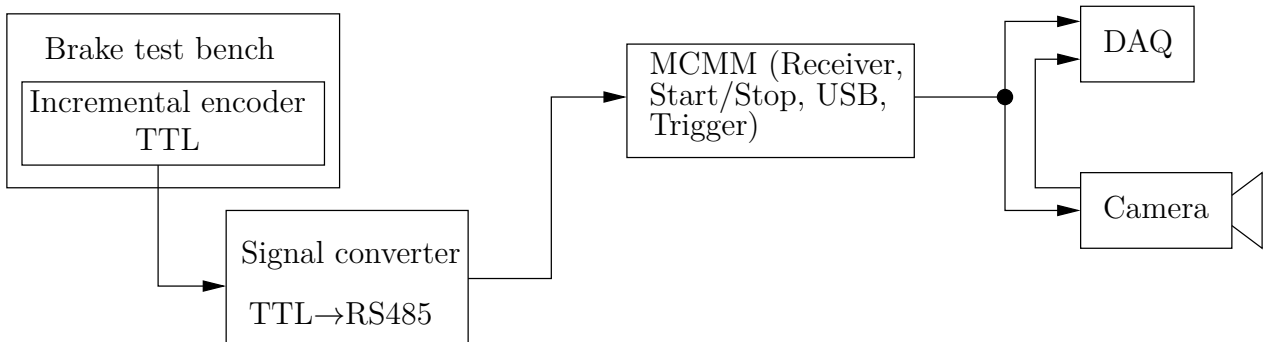


Figure 4.37: Measurement of IR radiation

The synchronisation of the position of the brake disc with the frame rate of the IR camera requires an incremental encoder, a signal converter, a MCMM, a DAQ system and an IR

camera. The position of the brake disc is determined with the incremental encoder, which is placed at the power transmission. The TTL output signal of the incremental encoder is converted to a RS485 signal. This signal is supplied into the MCMM, which synchronises the thermograms (IR images) with the position of the brake disc. The output of the MCMM is the synchronised thermogram, which can be saved in the DAQ system. The understanding of the physical fundamentals is essential for the correct measurement of temperatures for rotating or not rotating objects. The infrared measurement is based on the principle of the transport of heat due to radiation, which is emitted by each body above the absolute temperature over 0 K in thermodynamic temperature scale. The electromagnetic spectrum can be divided in an ultraviolet, visible and an infrared radiation. The wavelength range of the infrared radiation is between 780 nm and 10^6 nm. The frequency range is between 385 THz and 0.3 THz. The spectral range of infrared radiation is divided between a close, a medium and far one. The classification of the radiation can be deduced from the standard specification DIN 5031-7 and is shown in the table 4.7.

Radiation	Symbol	Wavelength λ [nm]	Frequency ν [THz]	Wave number σ [mm ⁻¹]		
Infrared radiation	Near-IR Mid-IR Far-IR IR	NIR	IR-A	780 – 1400	385 – 215	1300 – 700
			IR-B	1400 – 3000	215 – 100	700 – 330
		IR-C	MIR	$3000 - 5 \cdot 10^4$	100 – 6	330 – 20
			FIR	$5 \cdot 10^4 - 10^6$	6 – 0.3	20 – 1

Table 4.7: Classification of the IR radiation according to DIN 5031-7

The focus of this work lies on the spectral range of the infrared radiation which is used in thermography cameras. The range of the infrared radiation over the whole spectrum is illustrated in figure 4.38.

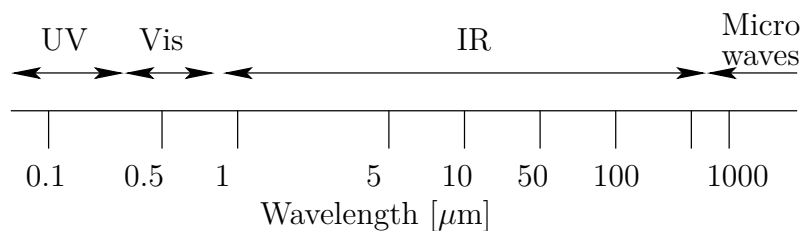


Figure 4.38: Electromagnetic spectrum

The following physical fundamentals are taken from the book Infrared Thermal Imaging of M. Vollmer and M. Vollmer and K.-P. Mollmann [130]. The radiation of all objects consists of absorption, transmission and reflection. The atmosphere, IR windows, camera optics and surroundings are influencing the detection of infrared images. Several parameters can adjust IR images and the details of the IR images and quantitative results of the IR images. Wavelength, temperature, angular dependence of emissivity, reflections, filters, shadows, optical properties and items between camera and objects have a strong influence on the quantitative analysis and

interpretation of IR images. Furthermore, the temperature span, range and level, and colour palette is influencing the plotted images. The emissivity and size of objects, relative humidity, external optics temperature and transmission, and ambient temperature are influencing IR images, which are generated from the detector. IR radiations are electromagnetic waves, which can be described by the spatial and transient periodicity, the wavelength λ and the frequency f . The IR wave consists of an electric and magnetic field, which can be polarised by filters. The IR spectrum is subdivided in the long-wave (LW) field from 7 to 14 μm , the mid wave (MW) field from 3 to 5 μm and the short wave (SW) field from 0.9 to 1.7 μm [130]. The IR radiation is a straight line, which can be described by the laws of reflection and refraction. Therefore, the law of reflection states that the angle of incidence is equal to the reflected ray

$$\alpha = \alpha'. \quad (4.50)$$

Another important issue is the condition of the surfaces. Specular and diffuse reflection can be observed for rough surfaces. The law of refraction states that

$$n_1 \sin \alpha = n_2 \sin \alpha', \quad (4.51)$$

where n_i describes the index of refraction of the materials. The index of refraction depends on the wavelength. With the equations 4.50 and 4.51, the direction of radiation can be calculated. IR radiation is opaque for solid objects, in which the emission refers to the surface of the objects. A surface element dA emits thermal radiation in regard to a solid angle in the specific direction of the detector. The emissive power M describes the total energy flux $d\phi$ of a surface element dA , which emits radiation in the hemisphere

$$M = \frac{d\phi}{dA}. \quad (4.52)$$

This energy cannot be detected completely from the detector. The contrary of the emissive power is the irradiance E , which describes the received energy flux for a surface area dA

$$E = \frac{d\phi}{dA}. \quad (4.53)$$

The spectral density

$$\phi_\lambda = \frac{d\phi}{d\lambda} \quad (4.54)$$

describes the radiant power dependent on the wavelength. Another important variable is the solid angle $d\Omega$, which characterises the non-uniform emittance of the radiation in the hemisphere. In space, any direction can be determined with the azimuth angle φ and the zenith angle δ . The description of the solid angle according to a normal area element dA_{nor} is shown in the figure 4.39.

The irradiate area, which is normal to a point of the emitting surface, can be calculated with

$$d\Omega(\delta, \varphi) = \sin \delta \, d\delta \, d\varphi = \frac{dA_{\text{nor}}}{R^2}. \quad (4.55)$$

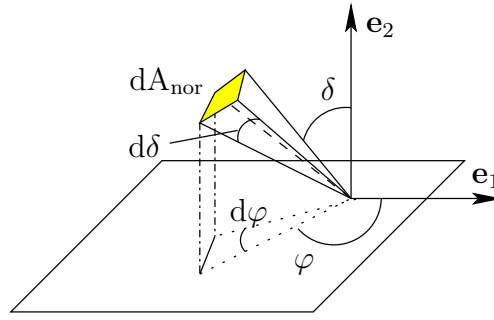


Figure 4.39: Description of the solid angle

With the solid angle $d\Omega$, the radiant intensity I and the radiance L can be calculated by

$$I = \frac{d\phi}{d\Omega}, \quad (4.56)$$

and

$$L = \frac{d^2\phi}{\cos \delta d\Omega dA}. \quad (4.57)$$

The radiance L describes the radiant power per unit of projected source angle and per unit solid angle. With this important expression it is possible to calculate the total radiant power ϕ , the excitance M , and the radiant intensity I . The factor $\cos \delta dA$ describes the projection of the emitting surface. The fundamental law of radiometry

$$d^2\phi = \frac{L_1 \cos \delta_1 \cos \delta_2}{r^2} dA_1 dA_2 \quad (4.58)$$

specifies the emitting total radiant power of the finite area A_1 reflected by the finite area A_2 . The equation 4.58 is derived by $d^2\phi = L_1 \cos \delta_1 dA_1 d\Omega_2$, where L_1 is the radiance from dA_1 , and $d\Omega_2$ is the solid angle under which dA_2 is seen from dA_1 . The solid angle $d\Omega_2$ can be described with $\cos \delta_2 dA_2/r^2$. The total incident radiant power reflected by the surface A_2 from the emitting surface A_1 can be calculated by

$$\phi_{12} = L_1 \int \int_{A_1, A_2} \frac{\cos \delta_1 \cos \delta_2}{r^2} dA_1 dA_2. \quad (4.59)$$

A further important parameter is the view factor F_{ij} , which describes the quotient of the total emitting radiant power $\phi_1 = \pi L_1 A_1$ of the surface A_1 into the hemisphere and the ratio of ϕ_{12} of the surface A_2 :

$$F_{12} = \frac{\phi_{12}}{\phi_1} = \frac{1}{\pi A_1} \int \int_{A_1, A_2} \frac{\cos \delta_1 \cos \delta_2}{r^2} dA_1 dA_2. \quad (4.60)$$

The description of the view factor can be explained vice versa, where F_{12} describes the portion of ϕ_1 that is intersected by A_2 . Based on this, it follows that

$$A_1 F_{12} = A_2 F_{21}. \quad (4.61)$$

For the calibration of the IR cameras or the theoretical explanation of thermal radiation, there is a construct necessary, which is called **black body**. The black body emits the maximum energy for a certain wavelength and temperature. The radiance of a black body does not depend on the direction but on the wavelength. Another property is that black bodies absorb every incident radiation. The spectral excittance of a black body for a given temperature and wavelength interval is

$$M_\lambda(T) d\lambda = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{\frac{h c}{\lambda k T}} - 1} d\lambda, \quad (4.62)$$

where $h = 6,626 \cdot 10^{-34}$ Js, λ , $c = 2,998 \cdot 10^8$ m/s, $k = 1,381 \cdot 10^{-23}$ J/K and T are the *Planck's* constant, the wavelength, the speed of light in vacuum, the *Boltzman* constant, and the absolute temperature, respectively. Therefore, the radiance can be determined by

$$L_\lambda(T) = \frac{M_\lambda(T)}{\pi}. \quad (4.63)$$

The black body radiance spectra for various temperatures is shown in figure 4.40.

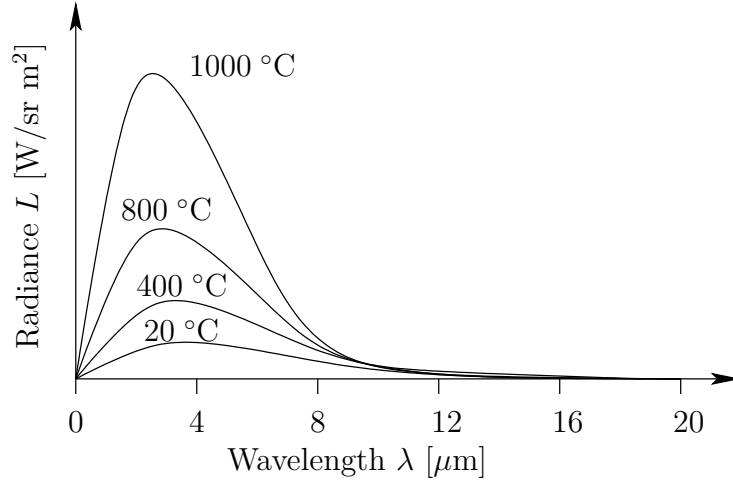


Figure 4.40: Black body radiance spectra for various temperatures

It can be observed that the radiance increases with temperature for a fixed wavelength. The spectra are continuous and the spectral region depends on the temperature. The maximum of the temperature at a certain wavelength can be determined by

$$\frac{dM_\lambda(T)}{d\lambda} = 0, \quad (4.64)$$

which leads to the *Wien's* displacement law

$$\lambda_{\max} T = 2897,8 \mu\text{m K}. \quad (4.65)$$

The excitance of the black body source can be calculated with the *Stefan-Boltzmann* law

$$M(T) = \int_0^\infty M_\lambda(T) d\lambda = \int_0^\infty M_\nu(T) d\nu = \sigma T^4, \quad (4.66)$$

where $\sigma = 5,76 \cdot 10^{-12} \text{ W/m}^2 \text{ K}^4$ is the *Stefan-Boltzmann* constant. The emitted total radiance of a black body is M/π . The black body radiation function $F_{(0 \rightarrow \lambda)}$ describes the quotient of the emission or radiation from 0 to λ and the total radiation from 0 to ∞ :

$$F_{(0 \rightarrow \lambda)} = \frac{\int_0^\lambda M_\lambda d\lambda}{\int_0^\infty M_\lambda d\lambda}. \quad (4.67)$$

IR cameras are detecting radiation for a certain spectrum due to the used detector and material properties. The equation 4.66 has no analytical solutions for certain wavelength values, which makes the definition of the black body radiation function necessary. The figure 4.41 shows the black body radiation function of equation 4.66 or the fractional excitance of a black body.

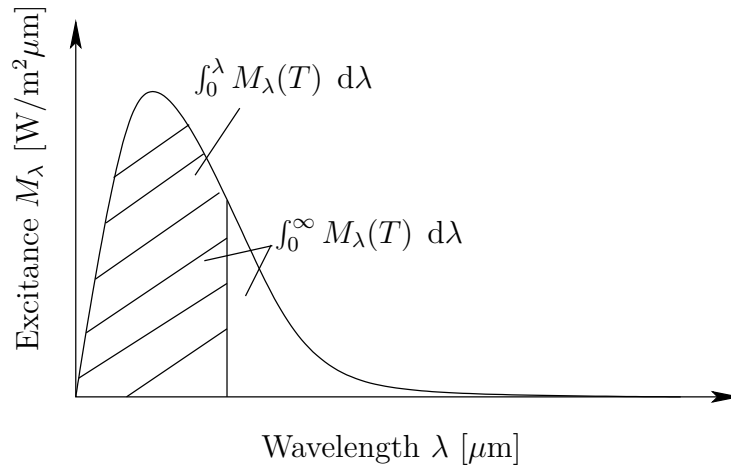


Figure 4.41: Fractional excitance of a black body

Hence, the black body radiation function can be calculated for any wavelength intervals

$$F_{(\lambda_1 \rightarrow \lambda_2)} = F_{(0 \rightarrow \lambda_2)} - F_{(0 \rightarrow \lambda_1)}. \quad (4.68)$$

The emitted radiation for a wavelength interval according to the *Stefan-Boltzmann* law is

$$\int_{\lambda_1}^{\lambda_2} M_{\lambda BB}(T_{BB}) d\lambda = F_{(\lambda_1 \rightarrow \lambda_2)} \sigma T^4. \quad (4.69)$$

For the calibration of the IR cameras, it is necessary to know, which input power is needed for the detection element for a 1 K temperature difference. The detecting spectrally integrated radiant power of the IR camera is

$$\phi_{BB \rightarrow \text{cam}} = M_{BB}(T_{BB}) \frac{r_{BB}^2 r_{\text{cam}}^2 \pi}{R^2}, \quad (4.70)$$

where r_{BB} , r_{cam} , and R are the circular radius of the black body radiator, the lens radius of the IR camera, and the distance of the black body radiator and the IR camera, respectively. The use of an infrared camera shows a possibility to detect very small temperature changes depending on the detector type. In the figure 4.42, the fractional change of the incident radiance $\Delta L/L$ due to a fractional change of the temperature $\Delta T/T$ is plotted. It describes the fractional change of the excittance of a blackbody.

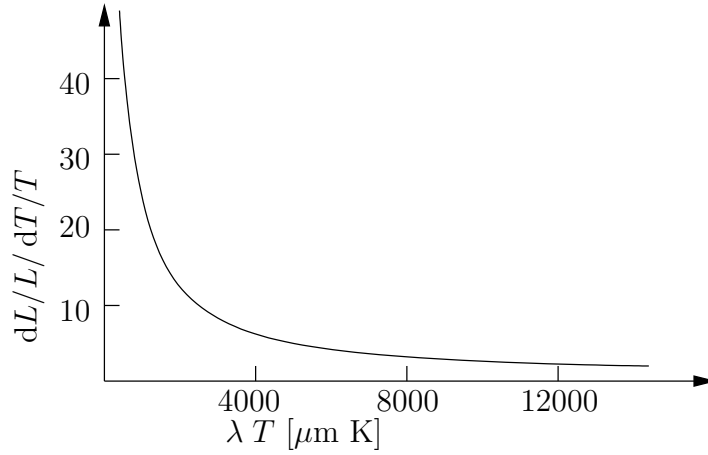


Figure 4.42: Fractional change of the excittance of a blackbody

Due to the proportionality of the radiance and excittance $M = \frac{d\phi}{dA} = \int L \cos \delta d\Omega$, the equation 4.62 is deviated and rearranged with relation to T

$$\frac{\frac{dM_\lambda(T)}{M_\lambda(T)}}{\frac{dT}{T}} = \frac{c_2}{\lambda T} \frac{e^{\frac{c_2}{\lambda T}}}{e^{\frac{c_2}{\lambda T}} - 1}, \quad (4.71)$$

where $c_2 = hc/k = 14387.7 \mu\text{m K}$. For $\lambda T < 2898 \mu\text{m K}$ the exponential part of the equation 4.71 can be simplified as

$$\frac{\frac{dM_\lambda(T)}{M_\lambda(T)}}{\frac{dT}{T}} \approx \frac{c_2}{\lambda T}. \quad (4.72)$$

In the equation 4.73,

$$\frac{\frac{dM_{MW}(T)}{M_{MW}(T)}}{\frac{dT}{T}} = \frac{\frac{c_2}{\lambda_{MW} T}}{\frac{c_2}{\lambda_{LW} T} \frac{e^{\frac{c_2}{\lambda_{LW} T}}}{e^{\frac{c_2}{\lambda_{LW} T}} - 1}} \frac{\frac{dM_{LW}(T)}{M_{LW}(T)}}{\frac{dT}{T}}, \quad (4.73)$$

one can observe that for a change of the temperature, a larger fractional change in MW IR radiation than for LW IR radiation happens. Due to this fact, MW cameras have better

pronounced dependence of signal on object temperatures. Another difference between MW and LW IR cameras is that the slope for MW IR cameras is much steeper for the fractional signal change with fractional temperature change. The **emissivity** ε describes the influencing issues, i.e. surface conditions of the detecting object with regards to the ideal black body. Moreover, the emissivity ε describes the ratio of the radiation of an object to that of a black body at the same temperature. The emissivity can be described in different radiometric quantities such as the spectral directional emissivity (equation 4.74), the spectral hemispherical emissivity (equation 4.75) with $M = \int_{\text{hemisphere}} L \cos \delta \, d\Omega$, the directional total emissivity (equation 4.76) and the total hemispherical emissivity (equation 4.77). It holds

$$\varepsilon(\lambda, \delta, \varphi, T) = \frac{L(\lambda, \delta, \varphi, T)}{L_{BB}(\lambda, T)} \quad (4.74)$$

$$\varepsilon(\lambda, T) = \frac{M(\lambda, \delta, \varphi, T)}{M_{BB}(\lambda, T)} \quad (4.75)$$

$$\varepsilon(\delta, \varphi, T) = \frac{L(\delta, \varphi, T)}{L_{BB}(T)} \quad (4.76)$$

$$\varepsilon(T) = \frac{M(T)}{M_{BB}(T)} = \frac{M(T)}{\sigma T^4}. \quad (4.77)$$

The emissivity factors lie in the range $0 \leq \varepsilon \leq 1$. The values of the emissivity of some surfaces such as the ideal black body and the gray body are $\varepsilon = 1$ and $\varepsilon = 0.7$, respectively. For both the emissivity does not change with the wavelength. The third group are the selective emitters, which depend on the wavelength and the boundary conditions. With the *Kirchhoff's* law it is possible to calculate the emissivity. It describes that the amount of radiation absorbed by an object is equal to the amount of radiation that is emitted

$$\varepsilon = \alpha, \quad (4.78)$$

where α represents for the absorptivity. The total radiation consists of a reflected radiation ϕ_R , transmitted radiation ϕ_T through the object and absorbed radiation ϕ_A within the object

$$\phi_0 = \phi_R + \phi_T + \phi_A. \quad (4.79)$$

The incident radiation is composed by

$$1 = R + t + \alpha, \quad (4.80)$$

where R , α , and t are the reflected radiation, the absorptivity and the transmitted radiation, respectively. The reflected radiation can consist of a directly reflected, and diffuse reflected radiation. The emissivity can be estimated if the reflectivity and the transmissivity are given. According to *Kirchhoff's* law and equation 4.80, the emissivity can be estimated by

$$\varepsilon = 1 - R - t. \quad (4.81)$$

Another point is that the emissivity ε depends on the material, surface structure and regular geometry. The variation of the observation direction, the wavelength, and the temperature affect also the emissivity. Most non-metallic materials are gray emitters with high emissivity values. Metals have low emissivity values. The emissivity can change for the identical surface depending on the surface behaviour (oxidation, roughness etc.). Figure 4.43 illustrates a schematic representation of the black body radiance and the real surface radiance.

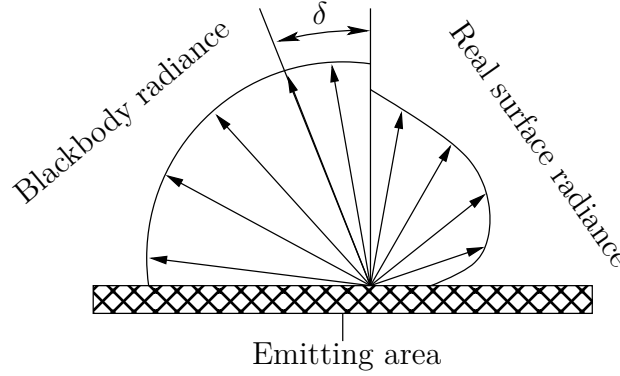


Figure 4.43: Comparison of an angular independent radiance of a black body and a directionally dependent radiance of a real surface

The blackbodies are emitting radiation isotropically. Real surfaces are emitting radiation depending on the viewing angle. The maximum radiation from a surface is detected in normal direction ($\delta = 0^\circ$). The emissivity depends on the angle of observation in regards to the surface normal. The effect of the surface structure can be shown with the Leslie cube. As aforementioned, the emissivity depends on the temperature, wavelength, and the viewing angle. In the figure 4.44, a horizontal macroscopic surface with a microscopic polished V-groove surface is presented. A superposition of the not uniform roughness and the viewing angle has to be considered for IR imaging. Grooves and cavities can increase the detecting radiation due to reflection of the radiation. The understanding of the reflection effects in the grooves can be explained as follow. The emissivity of the point 1 of the groove with an angle of 60° for direct radiation is characterised by $\varepsilon_d = \varepsilon(60^\circ)$. The emitted radiation from point 2 and reflected from point 1 is calculated by $\varepsilon_{21} = \varepsilon(0^\circ)R(60^\circ) = \varepsilon(0^\circ)[1 - \varepsilon(60^\circ)]$. The emitted radiation from the point 1 in direction to point 2, reflected back to point 1 and then reflected to the normal direction is $\varepsilon_{121} = \varepsilon(0^\circ)R(0^\circ)R(60^\circ) = \varepsilon(60^\circ)[1 - \varepsilon(0^\circ)][1 - \varepsilon(60^\circ)]$. The total emissivity is $\varepsilon_{\text{total}} = \varepsilon_d + \varepsilon_{21} + \varepsilon_{121}$.

Due to this example, it can be shown that for rougher surfaces the emissivity is higher than for polished surfaces. As a summary, one can say, regular surface structures have a strong influence on the observation angle. The emissivity depends also on the wavelength. If the material is wavelength dependent and the emissivity changes in the IR camera spectral range, narrow band filters have to be used to guarantee constant emissivities for the spectral range. Another point is the temperature dependency of the emissivity. For practical use, it has to be guaranteed that the emissivity is not changing with the temperature. As discussed above, the emissivity has to be known. There are several methods for measuring the emissivity, which is in normal direction and integrated over the selected wavelength range of the IR camera. The tape method is used if the emissivity of the bounded tape on the material is well known. The emissivity is adjusted until the temperature of the tap is achieved on the material. The calibration of the IR image can

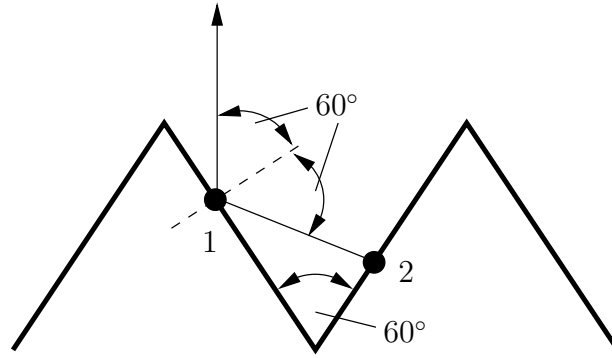


Figure 4.44: Horizontal macroscopic surface with a microscopic polished V-groove surface

be done also with thermocouples. The condition for the use of the thermocouples is, that the heat capacity of the thermocouple is smaller than that of the detecting object. For calibrating the IR cameras, there are blackbody radiators made of high-emissivity materials with surface structures that stabilise the temperature. The physical basic is the *Kirchhoff's* law (equation 4.78). Therefore, the material of the blackbody radiator has to have a high absorptivity. The blackbody radiators consist of cavities, which diffuse scattering. A further aspect is that the infrared radiation has to pass through several objects until it reaches the detector. These **optical materials** can emit additional radiation, which can distort the thermal radiation of the detecting object. Another important point is the patency of the thermal radiation in the considered spectral range. The transmittance of the thermal radiation through a solid material is accompanied by the reflection at flat surfaces, the back scattering of rough surfaces, the forward scattering of rough surfaces, the absorption in the solid material and the emission. The transmitted portion of thermal radiation, which is expressed as a function of the radiance and the travelled distance d , can be calculated with the *Bouguer-Lambert-Beer* law

$$T(\lambda, d) = \frac{I(\lambda, d)}{I(\lambda, 0)} = e^{-\gamma(\lambda) d}. \quad (4.82)$$

$\gamma(\lambda) = \alpha(\lambda) + \sigma(\lambda)$ describes the reduction of the thermal radiation due to transmittance, where α and σ are absorption of radiation and scattering (change of direction of radiation), respectively. For gases the reduction coefficient is

$$\gamma_{\text{gas}}(\lambda) = n (\alpha(\lambda) + \sigma(\lambda)), \quad (4.83)$$

where n is the volume concentration of the gas (number of particles per volume). The reduction coefficient for solid materials in non-scattering materials is related to the imaginary part of the index of refraction $n = n_1 + i n_2$, which is characterised by

$$\gamma_{\text{solid}}(\lambda) = \alpha(\lambda) = \frac{4\pi n_2}{\lambda}. \quad (4.84)$$

In summary, transmittance depends on the concentration of the absorbing gas, the relative humidity and the distance of object and camera. For camera lenses the reflection has to be calculated. For non-absorbing materials the total transmission can be calculated with $T_{\text{total}} = L_{\text{trans}}/L_0$. The reflectivity for a single interface is $R = L_R/L_0$, where L_R is the reflection loss.

The transmittance factor for non-absorbing materials can be approximately calculated in regard to several reflections at the interfaces with

$$T_{\text{slab}} = (1 - R)^2(1 + R^2 + R^4 + R^6 + \dots). \quad (4.85)$$

The exact calculation of the transmittance is done by

$$T = \frac{2n_B}{n_B^2 + 1}. \quad (4.86)$$

For absorbing materials the transmission in air (n_{1A}) is

$$T_{\text{slab}}(\lambda, d) = \frac{16n_1^2 e^{-\frac{4\pi n_2 d}{\lambda}}}{[(n_1 + 1)^2 + n_2^2]^2 - [(n_1 - 1)^2 + n_2^2]^2 e^{-\frac{8\pi n_2 d}{\lambda}}}, \quad (4.87)$$

where d is the thickness of the slab. The camera lenses used in the IR camera consist of Germanium. The transmission coefficient is approximately $T \approx 0.46$. Water with a thickness of 1 mm suppresses any infrared radiation transmission in the spectral range between 2 μm and 16 μm . The transmission losses are as mentioned before due to reflection losses from air to the material. To avoid these losses, the lenses are treated with anti-reflection coatings. These coatings can modify optical properties. The idea behind this coating is to use the phenomenon of interference. The electromagnetic waves can be superimposed or cancelled if they meet each other at the same time and location. The electromagnetic wave is cancelled if one wave is shifted by half a wavelength. The electromagnetic wave is superimposed if the two waves are oscillating in phase. The total amplitude is larger than the two amplitudes of the superimposed waves. The square of the amplitude of the wave is the total transported energy. In these cases, it is observable that depending on the propagation directions, the interferences reduce or increase the flux with the condition of energy conservation. The used lenses consist of the lens material and the coatings or Anti-Reflection (AR) coatings. The electromagnetic waves or the IR radiations are reflected at the AR-coating and the lens material. The two reflections can interfere and based on the different reflection positions a phase shift is induced. The phase shift depending on the film thickness and the refraction can be used in such a way that the reflected IR radiation is totally suppressed. In the terms of equal amplitudes and that the film thickness or the coating layer is $\lambda/4$ of the IR radiation wavelength, the reflections of the lenses can be suppressed. Conversely, this also applies to superimposed IR radiation. The influencing parameters for transmitted and reflected radiation for the optics are the index of refraction of the lens material, the index of refraction of the film material, the thickness of the film material, the angle of the incidence and the wavelength of the IR radiation. Multilayer coatings (bandpass filters) are used for wavelength ranges. The multilayer coating can consist of few up to several hundreds of layers. These layers can be used as high-pass or low-pass filters to suppress undesirable radiations such as background radiations. These spectral filters are used to limit the wavelength range of an IR camera. There are short-pass, long-pass, bandpass, and narrow bandpass filters. The physics behind these filters is, that they reflect and transmit by interference effect the incident radiation. The main part of an IR camera is the infrared detector, which converts thermal radiation into electrical signals. There are two types of infrared detectors, the photon detector and the thermal detector. Photon detectors

are single-step transducers. The physical background is that the absorbed photons from the infrared radiation change the concentration or the mobility of the free charge carries in the detector element. The absorbed photons generate an additional photocurrent or an electrical resistance. Thermal detectors are two-step transducers. The absorbed radiation changes the temperature of the material. Due to the temperature change, an electrical output in regard to the physical property of the material is generated. There are several parameters, which describe the working performance of an IR camera or detector. The responsive area A_D of a single pixel of the Focal Plane Array (FPA) is for the used camera $30 \mu\text{m} \times 30 \mu\text{m}$. The time constant τ describes the response time of the detector. $R_\lambda^U = U_{\text{Signal}}/\phi_{\text{radiation}}$ or $R_\lambda^I = I_{\text{Signal}}/\phi_{\text{radiation}}$ is the spectral responsivity, which characterises the ratio of the detector signal to the spectral radiant flux on the detector area. $U_N/\sqrt{\Delta f}$ or $I_N/\sqrt{\Delta f}$ is the root mean square (RMS)-noise voltage or current density. It describes the detector output noise voltage or current with respect to the square root of the applied bandwidth $\sqrt{\Delta f}$ of the signal measurement. The spectral noise equivalent power $\text{NEP}_\lambda = U_N/R_\lambda^U = I_N/R_\lambda^I$ is the necessary radiant power on the detector area to generate a signal-to-noise ratio of unity. The specific spectral detectivity $D_\lambda^* = \sqrt{A_D \Delta f}/\text{NEP}_\lambda$ is the spectral noise equivalent power independent of the detector area and bandwidth of signal detection. The temperature of the detector is T_D . The D_λ^* -value is an experimentally determined value, which characterises the performance of radiation detectors. The maximum temperature difference for a blackbody radiator, which can be detected for a given wavelength, is given by

$$\Delta T_{BB} = \left(\frac{\partial \phi_{BB}}{\partial T_{BB}} \right)^{-1} \Delta \phi_{BB}. \quad (4.88)$$

The limit of the detectable radiant power difference $\Delta \phi_{BB}$ is the NEP. The Noise Equivalent Temperature Difference (NETD) describes the minimum detectable temperature difference of the detector, which depends on the square root of the detector area, the bandwidth of signal detection, and the specific spectral detectivity:

$$\text{NETD} = \left(\frac{\partial \phi_{BB}}{\partial T_{BB}} \right)^{-1} \frac{\sqrt{A_D \Delta f}}{D_\lambda^*}. \quad (4.89)$$

Furthermore, the NETD depends on the optics of the camera, and parameters, which are influencing the incident radiation. The NETD for a wavelength dependent specific detectivity within the wavelength range $\lambda_1 \leq \lambda \leq \lambda_2$ is

$$\text{NETD} = \frac{1}{\text{FOV} \int_{\lambda_1}^{\lambda_2} \tau_{\text{optics}}(\lambda) D_\lambda^*(\lambda) \frac{\partial M_{\lambda BB}(\lambda, T_{BB})}{\partial T_{BB}} d\lambda} \sqrt{\frac{\Delta f}{A_D}}, \quad (4.90)$$

where FOV and $\tau_{\text{optics}}(\lambda)$ are the field of view related to the hemisphere Field of View (FOV) and the spectral dependent reduction factor of camera optics, respectively. The thermal detectors convert the incident radiation to processable signals in two steps. The rise of the detector temperature due to the electromagnetic radiation depends on the absorptance $\alpha = \Phi_{\text{absorbed}}/\Phi_{\text{incident}}$ of the detector. The transferred radiation $\Phi_0 = \Phi(T_{\text{object}}) - \Phi(T_{\text{detector}})$ to the detector consists of the received radiation from the object and the emitted radiation of the detector. The energy conversion for thermal detectors can be described by

$$\alpha\Phi_0 = C_{th} \frac{d\Delta T}{dt} + G_{th}\Delta T, \quad (4.91)$$

where C_{th} and G_{th} are the heat capacity and heat conductance G_{th} , respectively. The heat conductance consists of conduction, convection and radiation. The factor $\alpha\Phi_0$ increases the temperature of ΔT in the detector. The change of the radiation, e.g. due to a square-wave pulse, shows an exponential rise and decay with the time constant τ . Figure 4.45 describes for a square-wave pulse of radiation, the time dependent temperature change.

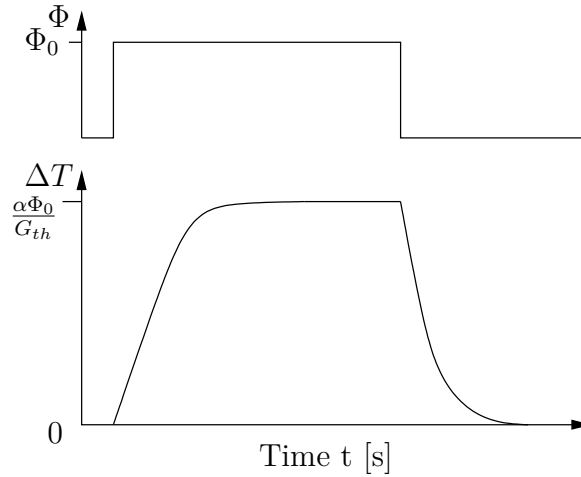


Figure 4.45: Temperature change of thermal detector for square-wave pulse radiation

The solution of the differential equation 4.91 is

$$\Delta T = \frac{\alpha\Phi_0}{G_{th}} \left(1 - e^{-\frac{t}{\tau}}\right). \quad (4.92)$$

The time constant τ describes the response time of the thermal detector and depends on the heat capacity and heat conductance

$$\tau = \frac{C_{th}}{G_{th}}. \quad (4.93)$$

For $t \rightarrow \infty$ the temperature change can be written as

$$\Delta T = \frac{\alpha\Phi_0}{G_{th}}. \quad (4.94)$$

Therefore, the decay period is

$$\Delta T = \frac{\alpha\Phi_0}{G_{th}} e^{-\frac{t}{\tau}}. \quad (4.95)$$

In consequence of these equations, a fast and sensitive thermal detector with a small time constant is achieved with low heat capacity and heat capacitance. The detection of the radiation by using temperature-dependent material properties (e.g. temperature dependence of the

electrical resistance) is used by bolometers. The bolometer changes the electrical resistance of a material by an absorbed radiation. The electrical resistance $R(T)$ is calculated by the temperature coefficient

$$\beta = \frac{1}{R} \frac{\partial R}{\partial T}. \quad (4.96)$$

A change of ΔT changes the resistance to

$$\Delta R = \frac{\partial R}{\partial T} \Delta T = \overline{\beta R \Delta T}, \quad (4.97)$$

where $\overline{\beta R \Delta T}$ is the average value for ΔT . In the steady-state condition, the resistance change is according to equation 4.94:

$$\Delta R = \frac{\beta R \alpha \Phi_0}{G_{th}}. \quad (4.98)$$

With the measured signal $U_{\text{signal}} = I \Delta R$ and the steady-state bias current I for the bolometer, the responsivity equation is

$$R_{\lambda}^U = \frac{\beta I R \alpha(\lambda)}{G_{th}}. \quad (4.99)$$

The used micro bolometer are semiconductors of vanadium oxide VO_x or amorphous silicon $\alpha\text{-Si}$ with $\beta \approx 3\% \text{ K}^{-1}$. An increasing current causes self-heating and increases the responsivity of the bolometer. Due to self-heating, a resistance decrease can be observed. The NEP according to the noise mechanism and the current heating for the bolometer can be calculated by

$$\text{NEP} = \sqrt{4k T_D^2 G_{th} \left(1 + \frac{I^2 R}{T_D G_{th}} \right) \Delta f + \frac{4k T_D G_{th}^2 \Delta f}{\beta^2 I^2 R}}. \quad (4.100)$$

The first term in the root describes the temperature fluctuation noise (increasing NEP with current heating) and the second term describes the decreasing NEP with increasing current (increasing signal response). The minimum NEP for the optimum current flow is obtained by differentiating the NEP in regard to the current. Therefore, the optimum bolometer current is

$$I(\text{NEP}_{\min}) = \sqrt{\frac{G_{th}}{|\beta| R}}. \quad (4.101)$$

Therefore, the minimum NEP is

$$\text{NEP}_{\min} = \sqrt{4k T_D^2 G_{th} \left(1 + \frac{2}{T_D |\beta|} \right) \Delta f}. \quad (4.102)$$

In equation 4.102, it can be observed that the temperature coefficient β , the heat conductance G_{th} and the detector temperature T_D are the optimisation parameters for the NEP. The affecting

parameters for IR cameras with bolometer arrays are the three mentioned above. Therefore, the bolometers are working under vacuum and room temperature. The remaining heat transfer processes are radiation and conduction. Figure 4.46 describes the working principle of the bolometer.

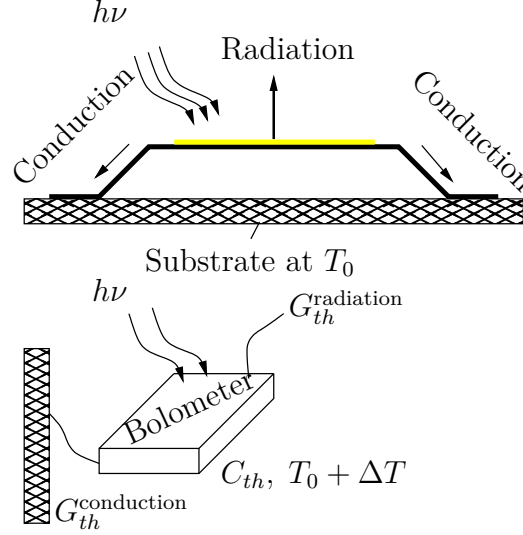


Figure 4.46: Schematic view of the heat transfer in a bolometer

The total thermal conductance is

$$G_{th} = G_{th}^{radiation} + G_{th}^{conduction}. \quad (4.103)$$

The radiant heat exchange is described by

$$G_{th}^{radiation} = \frac{d(A_D \sigma T_D^4)}{dT} = 4 A_D \sigma T_D^3 \quad (4.104)$$

for unit emissivity and hemispherical FOV. The heat conduction is estimated by

$$G_{th}^{conduction} = \lambda \frac{A_{cross}}{l}, \quad (4.105)$$

where λ , A_{cross} , and l are the thermal conductivity of the holder of the bolometer membrane, the cross-sectional area, and the length of the holder of the bolometer, respectively. The theoretical minimum NETD and the limit of the specific detectivity D^* for neglected heat conduction and only thermal fluctuation noise are

$$NEP_{ideal} = \left(\sqrt{A_D \sigma k T_D^5 \Delta f} \right)^{\frac{1}{4}}, \quad (4.106)$$

$$D_{ideal}^* = \left(\frac{1}{\sqrt{\sigma k T_D^5}} \right)^{\frac{1}{4}}. \quad (4.107)$$

The smaller the holder of the bolometer, the higher is the filling factor of a FPA. For an absorption of a long-wavelength infrared radiation, a $\lambda/4$ resonant cavity is used. The bolometer is connected over the metal studs with a Complementary Metal Oxide Semiconductor (CMOS). The Readout Integrated Circuit (ROIC) couples to a FPA, which read the individual electrical FPA outputs. The ROIC is a transducer, which transforms small detector signals to a large measurable output voltage. The next detector, which is mostly used for high speed and research cameras and also installed in the IR camera, converts the absorbed radiation directly into a change of the electronic energy distribution. The detector is a semiconductor and changes the free charge carrier concentration by incident radiation, the so called internal photo-electrical effect. The internal structure of semiconductors consist of energy band and energy gaps. An energy threshold ΔE has to exceed the quantum energy of the photons $E = h\nu$ to detect radiation in a semiconductor. The reached energy threshold excites an electronic transition between the valence and the conduction band. The excitation processes can be described with the intrinsic photo-effect and the extrinsic photo-effect. The intrinsic photo-effect describes the excitation of the electrons into the conduction bands by leaving holes in the valence band. The photons are exciting the electrons with different energies $h\nu$. The extrinsic photo-effect is the excitation of electrons or holes from an impurity level to the conduction or the valence band. The spectral sensitivity region is limited by a cut-off wavelength $\lambda_{\text{cut-off}}$ due to the energy threshold ΔE of the photon detectors. The cut-off wavelength $\lambda_{\text{cut-off}}$ according to the threshold ΔE is

$$\Delta E = h \nu_{\text{cut-off}} = \frac{hc}{\lambda_{\text{cut-off}}} \rightarrow \lambda_{\text{cut-off}} = \frac{hc}{\Delta E}. \quad (4.108)$$

Figure 4.47 describes the energy band structure of a solid, where E_C , E_V , and $h\nu_1$ are the upper edge of the valence band, the lower edge of the conduction band, and the quantum energy of the photon, respectively.

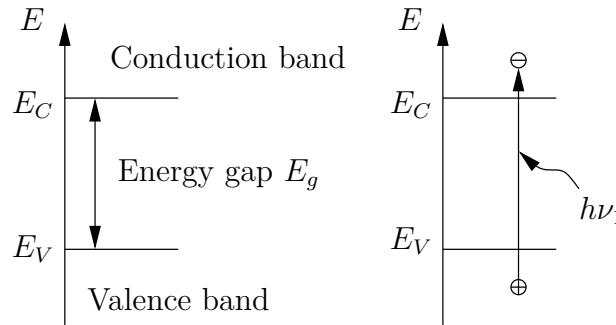


Figure 4.47: Energy band structure of a solid

How one can see in the equation 4.108, the responsivity and the specific detectivity depending on the wavelength. The photon detector counts the incident photons. The photo detector generates a signal in regards to the incident photons every time. The quantum flux $Z_{\text{radiation}}$ according to the incident radiative power $\Phi_{\text{radiation}}$ is

$$Z_{\text{radiation}} = \frac{\Phi_{\text{radiation}}}{h \nu} = \lambda \frac{\Phi_{\text{radiation}}}{hc}. \quad (4.109)$$

Therefore, the signal current of a photon detector is

$$I_{\text{signal}} = e \eta Z_{\text{radiation}} = \frac{e \eta \lambda \Phi_{\text{radiation}}}{h c}, \quad (4.110)$$

where η and $e = 1,602 \cdot 10^{-19}$ C are the quantum efficiency, which describes the number of free charge carriers per incident photon and the elementary charge, respectively. The responsivity is

$$R_{\lambda}^I = \frac{I_{\text{signal}}}{\Phi_{\text{radiation}}} = \frac{e \eta \lambda}{h c} \rightarrow \frac{I_{\text{signal}}}{Z_{\text{radiation}}} = e \eta. \quad (4.111)$$

In equation 4.111, it can be observed that the responsivity depends linearly on the wavelength. There exist different noise sources, which have to be considered to get processable signals. By a current flow, the movement of individual free charge carriers (electrons or holes) are observed. These free charge carriers cross barriers. This happens not uniform but is a stochastic process. Due to this cross of barriers, fluctuations of current flow are observed. The shot noise can be calculated by

$$I_N = \sqrt{2e I_{\text{signal}} \Delta f} = \sqrt{\frac{2e^2 \eta \lambda \Phi_{\text{signal}} \Delta f}{h c}}. \quad (4.112)$$

The signal noise limit $\text{NEP}_{SL}(\lambda)$ is

$$\text{NEP}_{SL}(\lambda) = \frac{2h c \Delta f}{\eta \lambda} = \frac{2E_{\text{photon}}}{\eta} \Delta f. \quad (4.113)$$

According to equation 4.113 the signal noise limit depends on the photon energy. The specific spectral detectivity $D^*(\lambda)$ in regard to the detector area is

$$D^*(\lambda) = \frac{\eta \lambda}{2h c} \sqrt{\frac{A_D}{\Delta f}}. \quad (4.114)$$

The equations 4.112-4.114 show the ideal condition without background radiation. The influence of the background radiation on the photon detector is shown in equation 4.115, which contains an additional noisy detector current due to the background radiation:

$$I_N = \sqrt{2e I_{\text{signal}} \Delta f} = \sqrt{\frac{2e^2 \eta \lambda (\Phi_{\text{signal}} + \Phi_{\text{background}}) \Delta f}{h c}}. \quad (4.115)$$

The specific spectral detectivity D_{BLIP}^* for background limited infrared photo-detection is

$$D_{BLIP}^* = \sqrt{\frac{\eta \lambda}{2h c \left(\frac{\Phi_{\text{background}}}{A_D} \right)}} = \sqrt{\frac{\eta \lambda}{2h c E_{\text{background}}}}, \quad (4.116)$$

where $E_{\text{background}}$ is the background radiation. The photon flux $Z_{\text{background}}$ of the background radiation from a hemisphere is

$$Z_{\text{background}} = A_D \int_0^{\lambda_{\text{cut-off}}} M_{\lambda}(\lambda, T_{\text{background}}) \frac{\eta \lambda}{h c} d\lambda. \quad (4.117)$$

To avoid additional noise signals due to the heating of the photon detector, the photon detector has to be cooled to decrease the noise and to increase the sensitivity of the photon detector. The cooling is necessary due to the fact that the equilibrium concentration of the free carriers excited by thermal radiation is larger than the non-equilibrium carrier concentration excited by the thermal radiation. The energy of photons depends on the wavelength. The incident radiation has to be smaller than the cut-off wavelength to generate free charge carriers. According to the Boltzmann equation, the probability W of thermal carrier excitation is

$$W = \text{const. } e^{\frac{\Delta E}{k T}}. \quad (4.118)$$

The probability depends on the temperature. The lower the temperature, the lower is the charge carrier concentration. There are several types of photon detectors such as the semiconductor detector, the photoconductor, the photodiode, the Schottky barrier detector and the Quantum Well Infrared Photodetector (QWIP). The photoconductor consists of a single uniform semiconductor material. The absorbed thermal radiation creates free charge carriers and is measured by the current-voltage characteristic. The applied voltage or current increases the sensitivity of the photoconductor. Due to the higher sensitivity, a higher heat dissipation of the detector is observed. The highest heat dissipation increases the noise behaviour and decreases the D^* value. The photoconductors are not suitable for thermal imaging due to long time constants for high sensitivity. The **photodiodes** are semiconductors with a p-n junction. The output signal is a current or voltage. The energy gap defines the cut-off wavelength. The FPA of the used IR camera consists of the semiconductor MCT (Mercury-Cadmium-Telluride $\text{Hg}_{(1-x)}\text{Cd}_x\text{Te}$). The cut-off wavelength depends on the composition of the used semiconductor material for LW. In the MW region, the MCT and InSb (indium antimonide) are equal. The InSb is the most sensitive detector. The energy gaps depend only on the temperature and the cut-off wavelength at $T = 77 \text{ K}$ is $5.5 \mu\text{m}$. The incident radiation on a photodiode causes a current flow to a readout circuit. The photodiodes can provide a high time constant or integration time. The **Schottky barrier detector** is a semiconductor with a variety of metal films. Schottky barriers consist of silicon. This guarantees an integration of the infrared sensor array with the readout circuit. The Schottky barrier detectors have low defect rates, good homogeneity, and are cheap. The **Quantum well infrared photodetector** consists of heterostructures. It is possible through crystal growth to create a combination of different semiconductor materials with different band gaps. The band edge distribution can be varied by different semiconductor materials and layer thicknesses. Due to this variation of semiconductor materials, alternating quantum wells and barriers for electrons and holes are produced. The thermally excited charge carriers decrease with decreasing temperature. This detector type is suitable for narrowband applications because of the spectral range from 3 to $20 \mu\text{m}$. The radiometric chain describes all incident radiation on the detector. In the table 4.8, the advantages of the detectors are listed.

The total radiation power can be described by

$$\Phi_{\text{total}} = \tau_{\text{atm}} \epsilon \Phi_{\text{object}}^{\text{bb}} + \tau_{\text{atm}} (1 - \epsilon) \Phi_{\text{amb}}(T_{\text{amb}}) + (1 - \tau_{\text{atm}}) \Phi_{\text{amb}}(T_{\text{amb}}). \quad (4.119)$$

The total radiation power consists of the radiant power of the object through the atmosphere,

Bolometer	Photon detector
Practical application	Higher sensitivities
Less expensive	Higher time resolution
Low sensitivities/detectivities	Larger responsivity/detectivity
Broad/flat spectral response curve	Spectral response varying with wavelength
Slow response time	Time constants up to $1\mu s$
No change of the integration time	Smaller NETD values
Frame rate cannot be changed	Higher frame rates
Sequential readout of the detector lines	Snapshot operation

Table 4.8: Advantages of the detectors

the reflected radiant power through the atmosphere of the surroundings, and the atmospheric transmittance radiation. The radiation for the detector signal has to be integrated of the wavelength of the detector. The IR camera is composed of several components. The important parts of a IR camera are:

- Optics
- Detector
 - Filter
 - Detector cooling
 - Signal amplification
 - Digitisation
- Sensors and Actuators
 - Camera temperature
 - Flag
 - Lens recognition
- Electronics
 - Calculations
 - Corrections
- User interface
 - User control
 - Display
 - I/O ports

There exist two concepts of detecting radiation, the scanning and the starring system. The scanning system generates the image as a function of time row by row. The starring system projects the image onto all pixels of the detector array. The image in the scanning system is built up sequentially. The Instantaneous Field of View (IFOV) of the detector is moved between the FOV of the camera. Mirrors are used to scan the objects. The use of one line of the detector can be used in the line modus to increase the line frequency. The line frequency of the used IR camera is 13000 Hz. In this case the line modus is used for moving object. To achieve high frame rates, a line modus is necessary for the detection of radiation in brakes. The detectors are arranged in a matrix of columns and rows in the starring systems or FPAs. The FPA consists of the infrared sensor and the ROIC. The FPA is an infrared radiation sensitive material and the ROIC is made of silicon. The ROIC is necessary to realise the signal readout and the signal processing such as signal amplification and integration, multiplexing, and A/D conversion. The advantages of the detection systems are illustrated in the table 4.9.

Scanning system	Starring system
Moving objects	Whole FOV
Radiometric accuracy	Signal-noise ratio
No integration of two materials	Higher frame rates at higher pixel numbers

Table 4.9: Advantages of the detecting systems

The advantage for the scanning detection system is the measurement of radiation for moving objects, where the radiometric accuracy is achieved due to the detection of all image pixels by one detector element. There is no complex joining of two materials such as for the starring system. The advantage for the starring system is that the whole FOV is covered simultaneously for the entire frame time. Further advantages are the improvement of the signal-noise ratio and higher frame rates at higher pixel numbers. The IR images depend on the FPA dimension and the calibration of the FPA. The non-uniformity correction (NUC) is a calibration process for the different individual detectors. The FPA consists of multiple detectors and each of these detectors has its own signal responsivity and signal offset. This means, that for the same incident radiant power different signal outputs are observed. The IR image becomes unrecognisable if the difference of the response behaviour of the detectors is too large. The NUC consists of the detection of the differing gains or possible signal offsets and the absolute temperature calibration. The NUC procedure consists of

1. a signal offset correction for a given object temperature,
2. a correction of the detector signal dependence on the object temperature and
3. an offset correction for identical response curves of the different detectors.

The NUC can be done for a certain temperature (one-point NUC) or for a temperature range (two-point NUC). The NUC procedure is described in figure 4.48.

The calibration process (NUC) for an IR camera has to be done periodically and for a certain range of the detector saturation signal. In the FPA and in screens there exist bad or non-working pixels. These bad pixels are corrected electronically by weighted average values of the

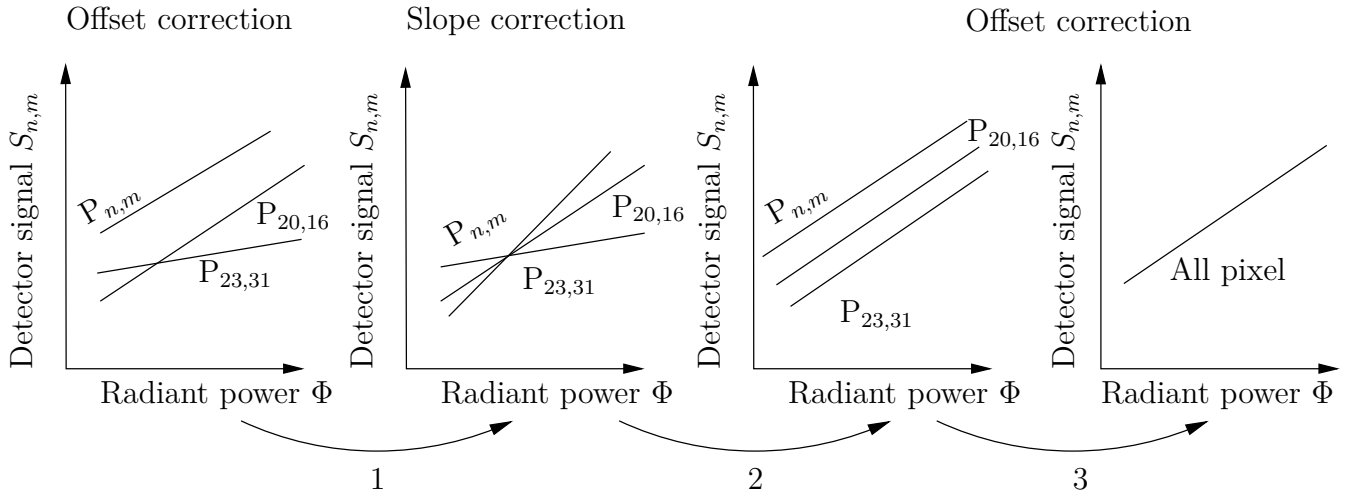


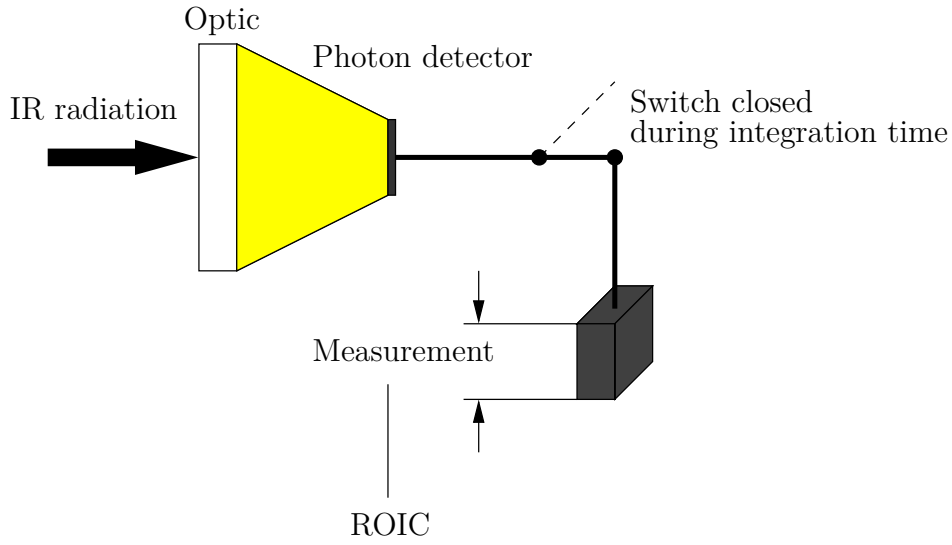
Figure 4.48: Non-uniformity correction

neighbouring pixels. The snapshot mode of the photon detectors allows to capture all pixels from a scene simultaneously, due to the fact of the direct conversion of radiation into electrical signals and the small time constant. The pixel charges a capacitor for a defined integration time. The integration time of the detectors are given by the thermal time constant of the detector (see equation 4.93). The readout by the ROIC is carried out either integration then read or integration while read mode. Because of these properties, the IR cameras with photon detectors can be triggered. Figure 4.49 describes the signal processing of the photon detector and the thermal detector, respectively.

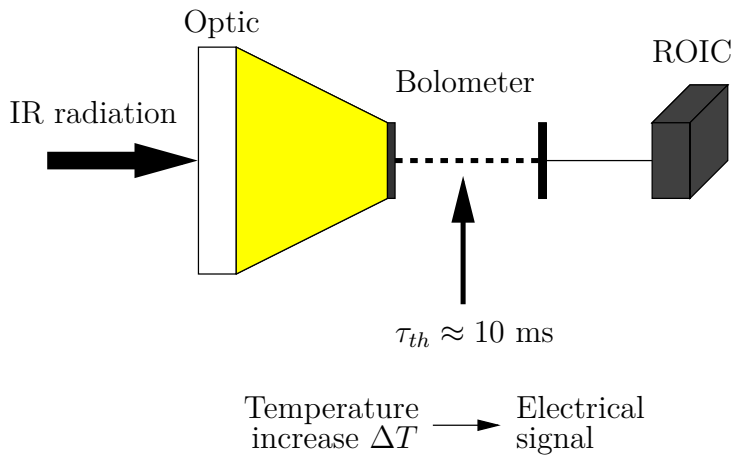
The constraints of the frame rate are not due to the photon detectors itself but due to the speed of data transfer and storage. The integration time of the used IR camera can be adjusted between $1 \mu\text{s}$ and $20000 \mu\text{s}$ in $1 \mu\text{s}$ steps. An important device beside the detector is the detector cooling to stabilise the temperature of the detector. The bolometers are using *Peltier* elements, which are using thermoelectric cooling/heating effects. A schematic representation of the *Peltier* element is illustrated in figure 4.50. Direct current flows through two different semiconductors and through thermoelectric materials, which then convert current into temperature. The different energy levels (either p-type or n-type) of the conduction bands of the semiconductors causes a cold and a hot side. Bolometer with *Peltier* elements or thermoelectric elements should not work at ambient temperature to avoid temperature changes of the bolometer detector sensitivity with changing ambient temperature. The maximum temperature change between the two sides is for a single stage cooler 70 K. A three-stage cooler can reach a maximum temperature difference of 170 K.

The photon detectors are cooled down at 77 K and the cooling increases with the wavelength. The used IR camera uses a *Stirling* cooler, which uses cooling and heating of a gas with a regenerator of high heat capacity. The *Stirling* cooler removes heat from the FPA to a heat sink. The efficiency is low but due to the low heat capacity and low heat loss, it fulfils the condition for photon detectors. The Mean Time To Failure (MTTF) of the *Stirling* cooler is 8000 h, which determines the service life of the IR camera. A schematic representation of the *Stirling* cooler is shown in figure 4.51.

A further limitation for the image quality is the chromatic aberration of the lenses. The figure 4.52 shows an ideal system, and two realistic situations with the fuzziness due to optics.



a) Signal processing in a photon detector



b) Signal processing in a bolometer or thermal detector

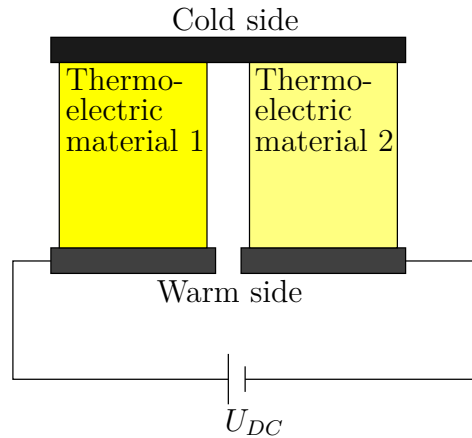
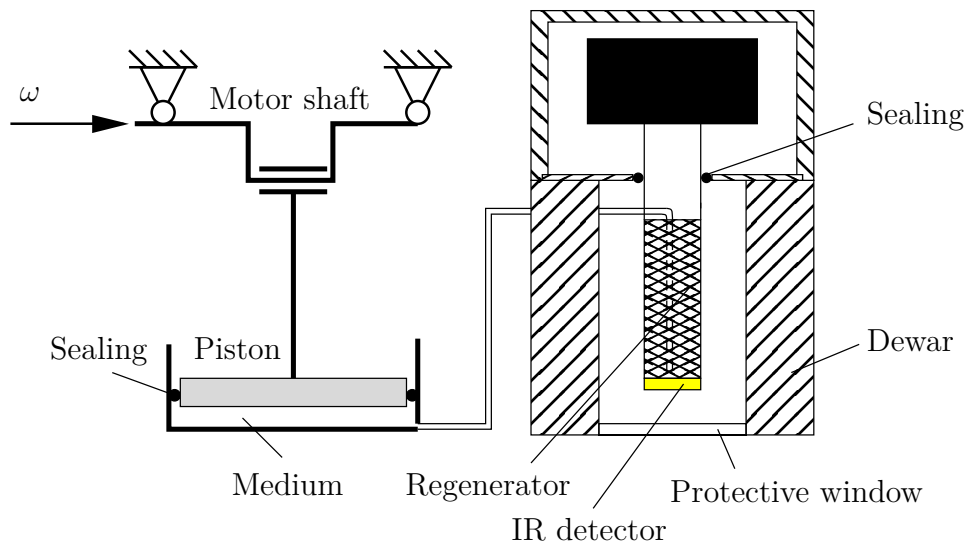
Figure 4.49: Signal processing of detectors

Aberration scatters the incident radiation of the object on the FPA or object. Based on the aberration of the lenses, the spatial resolution is decreased. The FOV is the detectable area or angle of the IR camera from an object. The principle illustration of the FOV of a IR camera is shown in figure 4.53.

As seen in the figure 4.53, the FOV depends on the camera lenses and the FPA dimensions. The FOV can be calculated based on geometrical optics. The detectable angle of a lens can be calculated by

$$FOV = 2 \arctan \left(\frac{b}{2f} \right), \quad (4.120)$$

where b and f are the detector length and the focal length, respectively. The length of the object area can be calculated for a given distance between the lens and the object according to

Figure 4.50: Working principle of *Peltier* elementsFigure 4.51: Working principle of *Stirling* cooler

$$x = 2D \tan \left(\frac{FOV}{2} \right). \quad (4.121)$$

The FPA consists of a rectangular shape, hence the FOV consists of a Horizontal Field of View (HFOV) and a Vertical Field of View (VFOV). The detectable object area can be changed with the dimension of the FPA, the distance between the lens and the object, and the lens (focal length). The close-up lenses illustrate far objects closely. The label of close-up lenses 50/100 describes the minimum distance of 100 mm by a horizontal object size of 50 mm. This label and the number of pixels can show the reflected object size for one pixel. Problems which can be observed due to reflection of the detector can be described by the Narcissus effect. The Narcissus effect is the reflection of the camera lens, window or an object (polished plate) for which the IR camera detects the reflection of itself. This can be avoided by changing the viewing angle of the object or IR camera. The dynamic range of an IR camera is between 12 and 16 bits, which are 4096 or 655369 signal levels. In the camera software, it is possible to fix a level and a span in the temperature range of the IR camera. The visualisation of the image or of the span range is illustrated mostly in 8-bit scale with 256 colours. In the following section, the important

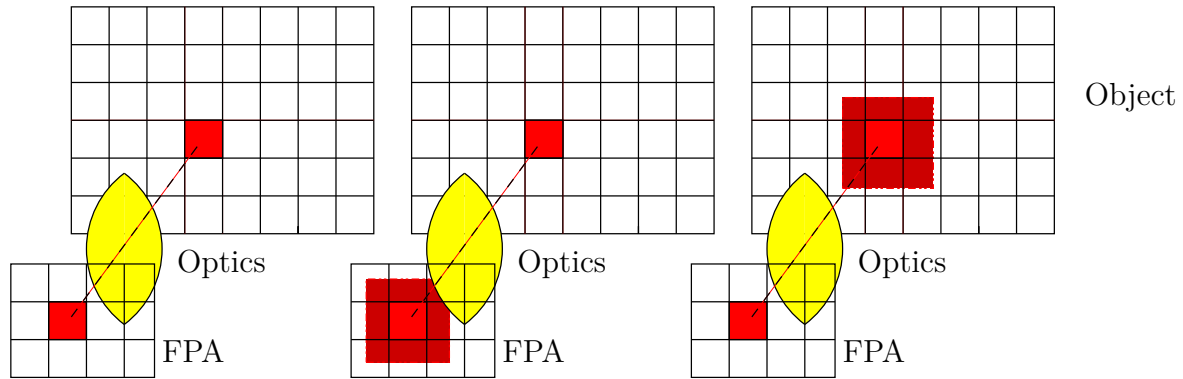


Figure 4.52: Consideration of optics aberrations

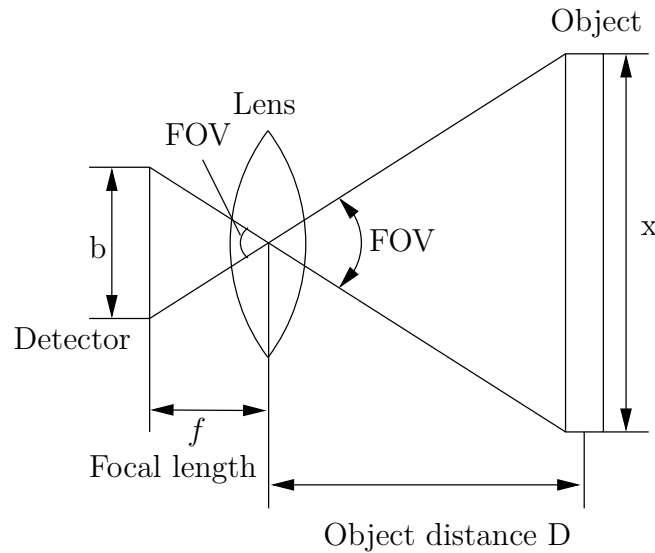


Figure 4.53: Determination of the FOV

camera performance parameters are discussed. The NETD shows the thermal sensitivity or the temperature resolution of the IR camera. The NETD is calculated by the system noise and the signal transfer function. The **IFOV** describes the angle over which one pixel of the detector is detecting incident radiation of an object. The **Slit Response Function (SRF)** describes the diffraction of the optics, which can be determined by a normalised dependence of the system response to an object with variable slits width. The **Minimum Resolvable Temperature Difference (MRTD)** describes the minimum temperature between the test patterns on a black-body background. The MRTD depends on the NETD and the IFOV of the IR camera. The **Minimum Detectable Temperature Difference (MDTD)** describes the ability of the thermal imaging system to detect a small size object for maximum level and span. The time resolution is described with the **frame rate** and the **integration time**. As mentioned before, the integration time can be adjusted for photon detectors but not for bolometers or thermal detectors. The integration time is very important for high-speed thermography. High frame rates are achieved with both detector types. But the temperature resolution for moving objects needs a high integration time. The measurement system of the flywheel test bench is shown in figure 4.54.

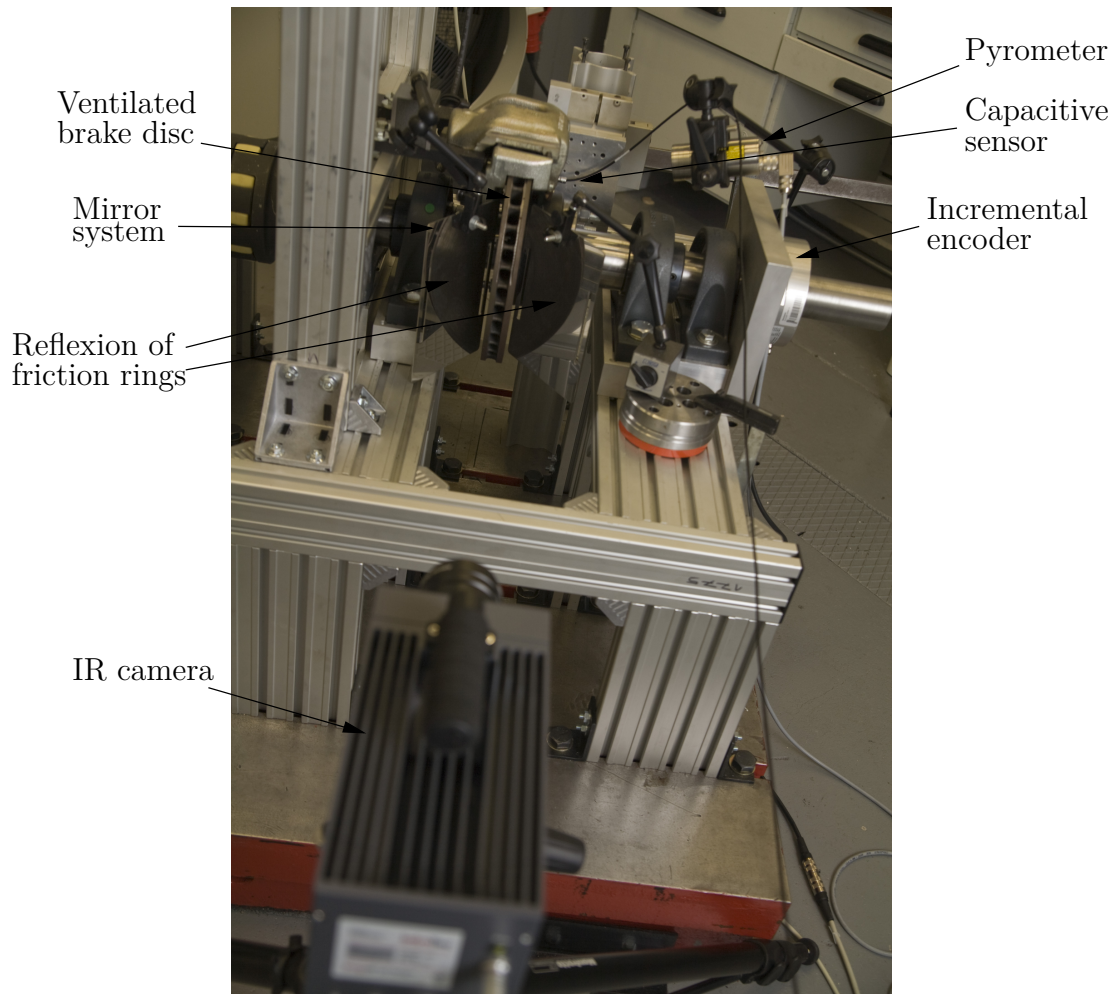


Figure 4.54: Measurement system of the flywheel test bench

4.3 Tribometer

The validation of the mathematical models or the understanding of certain phenomena requires a test bench with few modifiable variables. Besides the flywheel test bench an idealised test bench such as the tribometer has been designed. This enables the realisation of a well defined experiment. The designed tribometer is a pin-on-disc tribometer, which consists of a fixed pin in contact with a rotating disc. The load is applied on the pin. The test reading delivers the coefficient of friction, the contact forces and wear. The coefficient of friction is determined by the ratio of the frictional force to the normal force on the pin. The measurement results have to be considered in regard to the accurate test conditions. The designed pin-on-disc tribometer is shown in figure 4.55.

The pin-on-disc is composed of the three-phase asynchronous motor, the disc, the pin, the masses and the support device. The electric motor has a power of 2.2 kW. The nominal torque and rotational speed is 14.8 Nm and 1420 rpm, respectively. The disc is fixed on the ASM and the geometry of the pin is arbitrary and depends on the test conditions. Furthermore, the masses superimpose the load over the lever arm on the pin. The following test variables can be changed: the rotational speed of the disc, the normal force, which is applied over the masses,

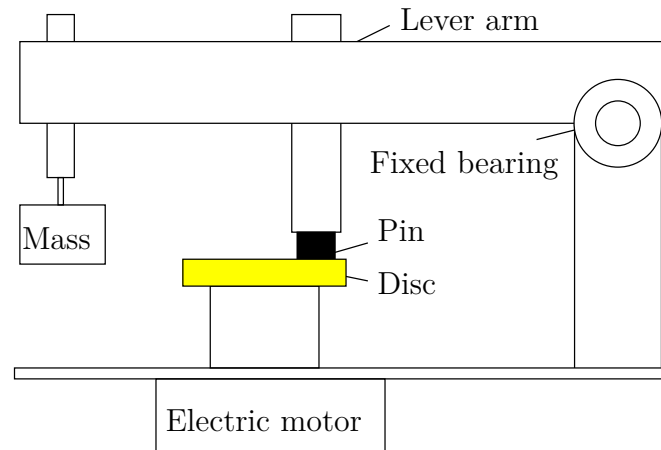


Figure 4.55: Pin-on-disc test bench at the IAM

and the friction pairing. These test conditions and the small number of modifiable variables makes the pin-on-disc appropriate for the validation of material models and the understanding of the behaviour of the friction pairings.

4.4 Combined Brake

In this section a combined braking system is introduced, which consists of an eddy-current and a friction brake for automotive braking applications. The possibilities and a constructive design of the combined brake system are illustrated and discussed in details. The investigation captures the speed-current characteristics of the motor and the eddy-current brake. The demand and the support of the governments to change from combustion cars to electric cars are considered environmental friendly aspects. The wear of disc brakes, which results into brake dust can cause visual disorder, heart and lung diseases, bronchitis, and cancer. Another important point is the safety of passengers, which depends upon the reliability of brakes. The performance of a combined brake system depends on a concerted brake behaviour. The collateral security guaranties a high braking efficiency. The eddy-current brake provides a braking without friction and wear. The use of the eddy-current brake has been the subject of a few publications over the last decade. Gay [51], [52] described the analysis of the physics of the eddy-current brake with a two-dimensional analytical model and a three-dimensional finite element analysis. Furthermore the numerical results are validated with experiments. Anwar [6] presented an anti-lock braking algorithm for an ECB, which controls the brake torque. Maurya et al. [94] show an analytical description of an ECB for high speed. The possibilities of a combined brake system in regards to electric cars is shown. The experiments are performed by using an in-house designed test bench. The experimental apparatus, which is shown in figures 4.56 and 4.57, consists of an asynchronous induction motor, a non-engaging coupling, an eddy-current brake, a friction brake, a brake disc, and a tachogenerator.

The motor is an asynchronous motor with a power of 0.55 kW. The asynchronous motor is a 6-pole motor with a rate torque of 5.68 Nm at 50 Hz. The maximum torque is 17.61 Nm. To control the rotational speed, the 6-pole asynchronous motor is connected with a frequency converter, which changes the frequency of the motor and is proportional to the frequency of

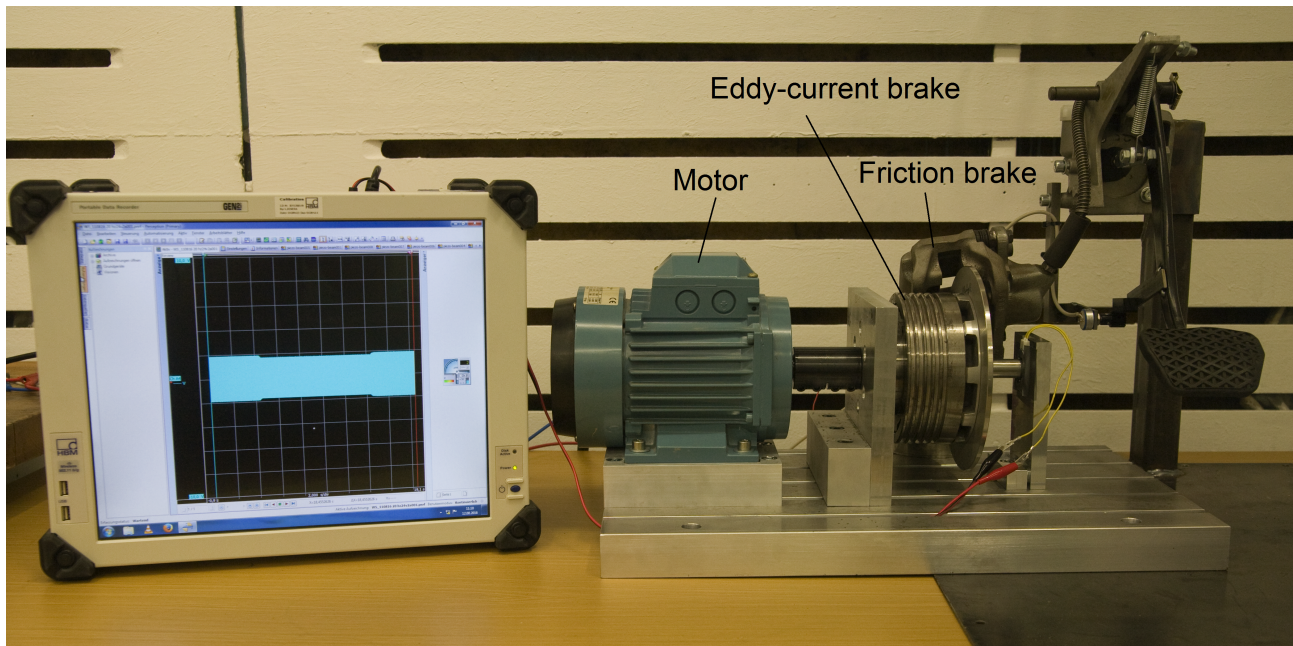


Figure 4.56: Eddy-current and friction brake test bench at the IAM

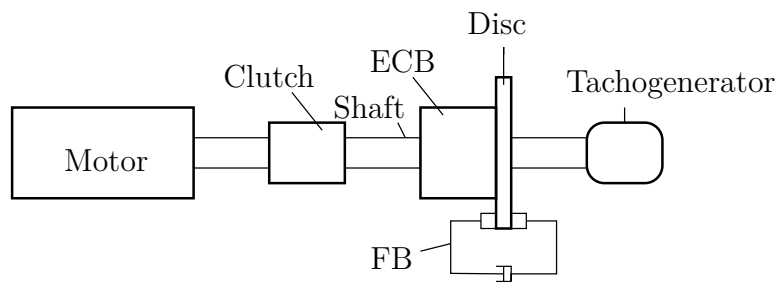


Figure 4.57: Schematic representation of the eddy-current and friction brake

the changed rotational speed. The torque-speed characteristics of the asynchronous motor for different frequencies or rotational speeds are shown in figure 4.58. It is apparent that the torque-speed characteristics of the asynchronous motor for frequencies from 10 Hz to 60 Hz is in ascending order from left to right.

In the figure 4.58 it can be observed that the maximum torque remains constant for all frequencies except for the frequency of 10 Hz. The slope of the torque-speed characteristic after the maximum torque is also constant. The pull-up torque before the maximum torque changes with frequency. For low frequencies, the maximum torque is reached earlier except for the torque-speed characteristic for 10 Hz. The deceleration of the asynchronous motor is conducted with an eddy-current brake. The eddy-current brake works frictionless and free of wear. The used eddy-current brake has a power of 48 W with a maximum current of 2 A. The applied torque is directly transmitted by electro-dynamic forces. These forces or the applied brake torque depends on the current and voltage input of the eddy-current brake, the rotational speed, and the thermal dissipation loss. The eddy-current brake provides a maximum brake torque of 5.6 Nm at a current of 2 A, a voltage of 24 V and a rotational speed of 550 rpm. The brake consists of the coil former and the anchor ring, which are concentrically supported. The magnetic field

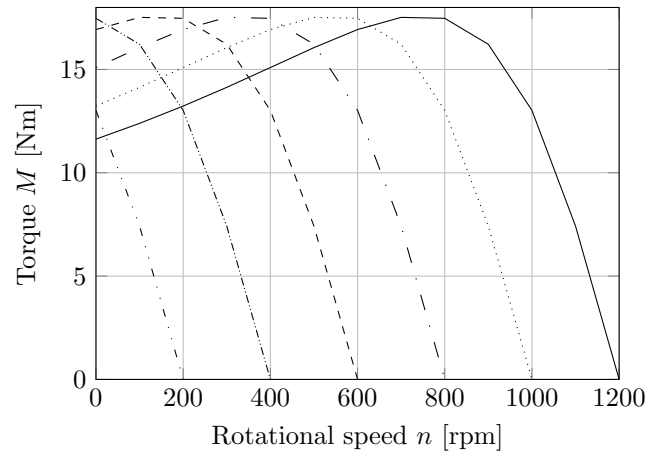


Figure 4.58: Torque-speed characteristics $M - n$ for a 6-pole asynchronous motor controlled with a frequency converter for different frequencies from 10 Hz to 60 Hz in ascending order from left to right.

is generated through the coil former, anchor ring, and the gap by applying direct current on the coil former. The eddy currents are generated in the anchor ring, which causes the braking. The brake torque size depends on the field coil and the slip speed. The braking effects depend on the electrical conductivity, the direction and strength of the magnetic field, the surface of the disc, and the speed. Materials with higher electrical conductivity are observed to decelerate faster. The highest braking effect is achieved with a magnetic field, which is perpendicular to the disc. Small surfaces have a lower braking effect compared to large surfaces. The effect is very slow for a discontinuous surface due to the fact that the eddy-currents cannot be formed. The braking effect depends on the relative velocity of the rotating disc and the magnetic field. The eddy-current brake decelerates a system but cannot be used as a fixing brake. The advantage of eddy-current brakes are the absence of mechanical contact (no wear), change of friction coefficients and air gap, and torque change during braking. For the measuring of the rotational speed, a tachometer is used. The tachogenerator is essentially an electric generator, which detects the rotational speed proportional to a voltage. The advantages of the tachogenerator are the simple application, the unnecessary carbon brushes, and the unnecessary power supply. The disadvantage of the tachogenerator is that very slow rotations cannot be detected. The experiments are conducted after the calibration of the tachogenerator. For this, a stroboscope and a rotation speed control counter is used, so that two independent measuring systems are used. The stroboscope is a non-contact measurement that allows to measure rotation speeds. The object should be visible for all 360° of rotation and axisymmetric. Following this, a mark has to be placed on the object as a reference point. The correct speed is shown by the stroboscope if the reference point is stationary. But mistakes can happen here, because one can observe images at harmonics of the actual speed. The actual speed can be observed at $1/2$, $1/3$, $1/4$, etc. of the actual speed. The rotation speed control counter measures the rotation speed with the mechanical attachments. The calibration curves of the tachogenerator are shown in figure 4.59.

The rotational speed n in regards to the frequency f for the stroboscope and the rotation speed control counter is shown on the left y-axis. The output voltage of the tachogenerator in regard to the frequencies is shown on the right y-axis. One can observe that the curves of the stroboscope and the rotation speed control counter are superposed. Due to the fact

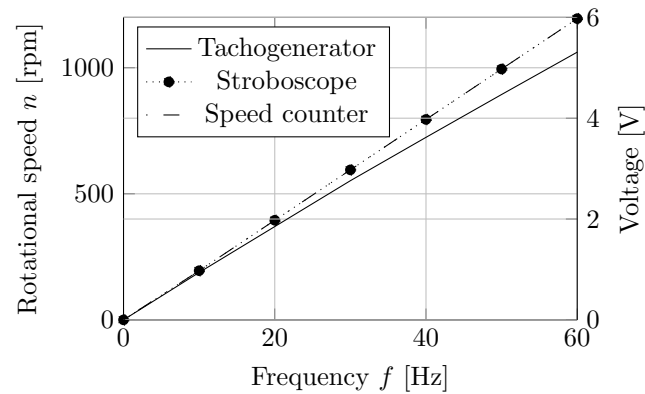


Figure 4.59: Calibration of the tachogenerator with the stroboscope and the rotation speed control counter

that the asynchronous motor is unloaded, the rotational speed of the asynchronous motor is approximately the synchronous speed. The tachogenerator shows a linear behaviour in regard to the frequencies. For the detection of the output voltage a data recorder with a maximum sample rate of 1 MS/s is used. The application of the brake torque is conducted on the one hand with the eddy-current brake and on the other hand with the conventional friction brake. The brake diagram is shown in figure 4.60.

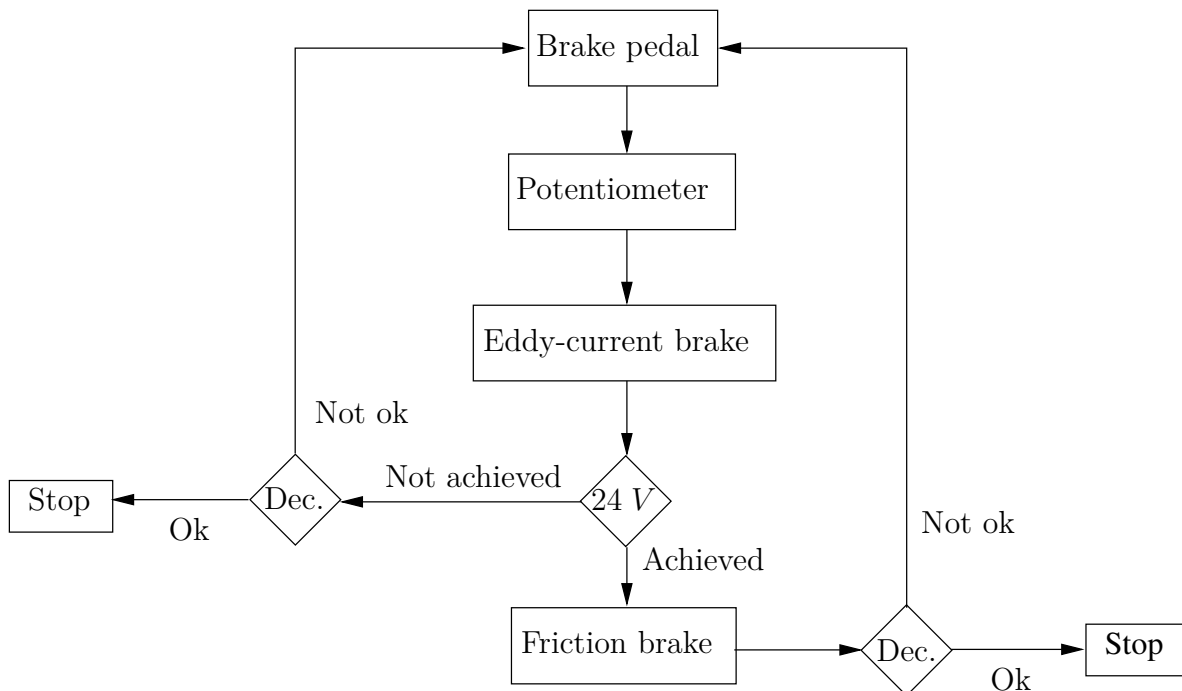


Figure 4.60: Brake sequence of the eddy-current and friction brake

The initiation of the brake action is carried out with the brake pedal, the potentiometer, the master cylinder, and the brake calliper. The movement of the brake pedal changes the voltage of the potentiometer in regard to the position of the brake pedal. If the maximum voltage of the potentiometer is reached, and the brake pedal is moving, the conventional friction brake is

activated. If the brake pedal is moving in the reverse direction and the conventional friction brake is relieved, the voltage of the potentiometer is changed from the maximum voltage to the voltage which depends on the brake pedal position. The braking curves of the experiments are shown in figures 4.61-4.65. As aforementioned, the tachogenerator generates an alternating current voltage for which the maximum voltage is proportional to the rotational speed. Figure 4.61 depicts the maximum rotational speed of 195 rpm before the braking and the braking ramp, which starts at 0.3 s. The rotational speed with the applied eddy-current brake is 174 rpm. After 0.8 s, the friction brake is applied. Figure 4.62 shows the braking ramp at the starting rotational speed of 395 rpm. The braking starts at 0.2 s and the decelerated rotational speed is 357 rpm. After 0.75 s, the friction brake is applied.

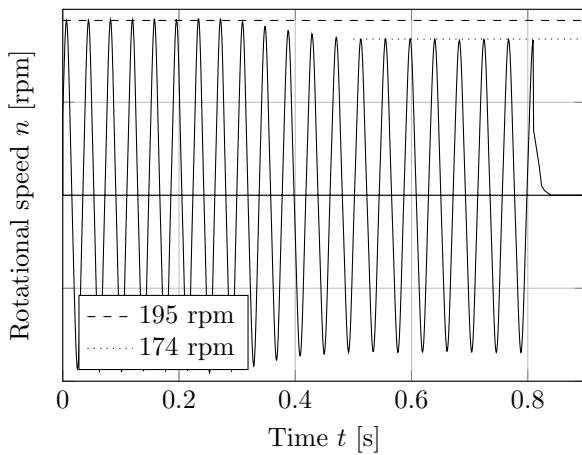


Figure 4.61: Brake curve 1; Braking from 195 rpm to 174 rpm

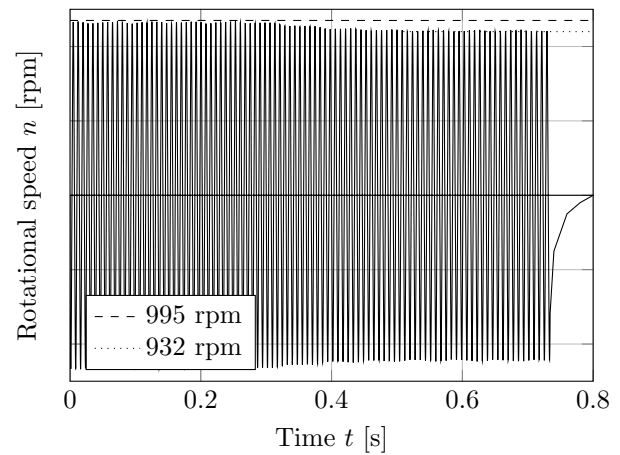


Figure 4.62: Brake curve 2; Braking from 395 rpm to 357 rpm

Figure 4.63 illustrates the braking from 595 rpm to 554 rpm, which starts at 0.35 s. After 0.75 s, the friction brake is applied. Figure 4.64 shows the braking from 795 rpm to 753 rpm, which starts at 0.3 s. After 0.78 s the friction brake is applied.

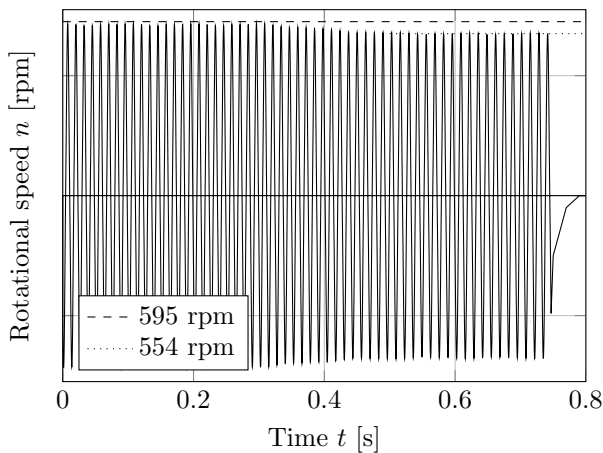


Figure 4.63: Brake curve 3; Braking from 595 rpm to 554 rpm

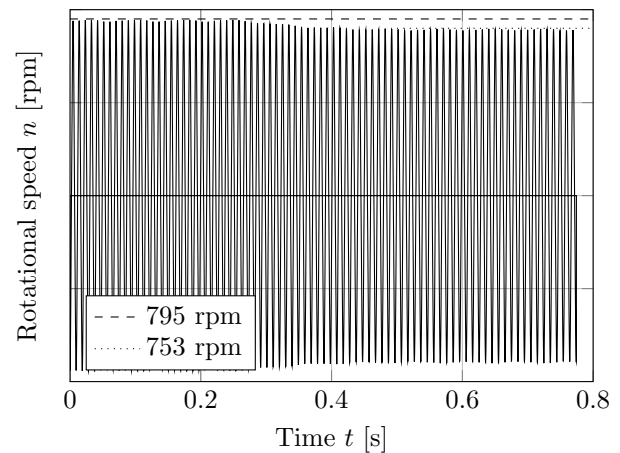


Figure 4.64: Brake curve 4; Braking from 795 rpm to 753 rpm

The last braking curve of the experiments is shown in figure 4.65. The maximum rotational speed before the braking is 995 rpm. The braking starts at 0.3 s and the rotational speed is 932 rpm

rpm. After 0.75 s the friction brake is applied. In figure 4.66, the difference of the rotational speed before and after the braking is shown. It can be observed that for the eddy current brake, the brake torque increases linearly with the rotational speed. The braking torque can be adjusted via the exciting current and the eddy-current brake operates smoothly without superimposed torques.

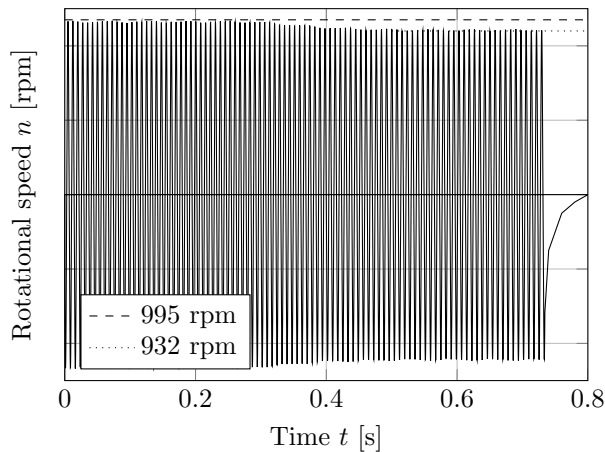


Figure 4.65: Brake curve 5; Braking from 995 rpm to 932 rpm

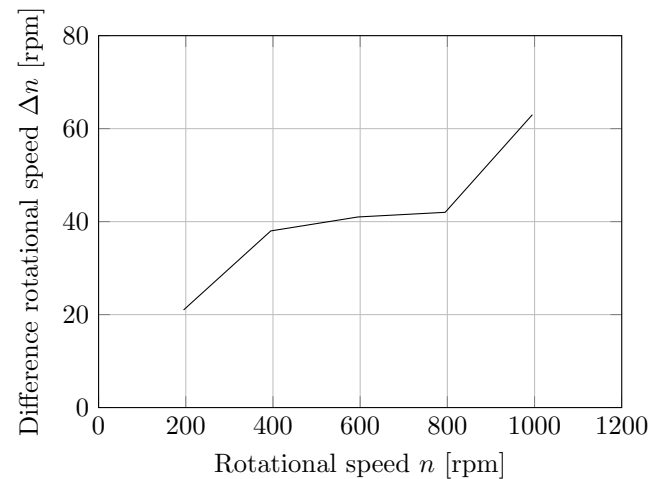


Figure 4.66: Brake behaviour of the eddy-current brake

The combined brake system has an environmental and convenience advantage. The experiments show a promising possibility for a combined brake system. Two brake torques were applied on the disc brake. The braking ramps were observed for all rotational speed. The reason for the slow decreasing of the rotational speed is that the motor was unloaded. The inertia of the whole system is very low. It was also shown that the brake torque increases with respect to the rotational speeds. The reason for a combined brake system is that an eddy-current brake can not be used as a fixing brake. The future work can include experiments with higher inertia and the measurement of the temperature development of the eddy-current brake, which is the most influencing effect on the brake efficiency. A further perspective could be to analyse the brake efficiency of a more powerful eddy-current brake on the full-scale test bench at the IAM.

5 Application to Friction Pairings

In this chapter, several applications of simulations and experiments are described. The first application is the standard simulation of the hot spot formation of a brake disc based on an uncoupled simulation model of Lamjahdy et al. [79]. The numerical results are compared with the work of Kasem [71] to compare the used numerical methodology with other experimental results. In this case, the experimental results of Kasem [71] are compared with our numerical methodology. The second application is the simulation of the temperature and wear behaviour of a disc brake [78]. The simulation is conducted with the steady-state algorithm of the section 3.3 and validated with the experimental results. The simulation time with the steady-state algorithm is decreased from 48 hours in the standard simulation to 6 hours in the steady state simulation. Furthermore, the experiments are conducted on the pin-on-disc tribometer.

5.1 Simulation of Hot Spots

Generally, hot spotting describes the development of thermal localisations and can lead to early damage, early wear, pad performance loss, and squeal noise [5], [42]. The aim of the present study is to improve a disc-pad transient numerical model by the use of a decoupled thermomechanical method with the consideration of the wear behaviour. It is based on full 3D thermomechanical calculations taking disc rotation into account. The braking process in high speed applications is a high dynamic and transient process, in which kinetic energy is dissipated. The generated heat energy causes localised thermal gradients on the brake disc. These thermal gradients also called hot spots are above the temperature of the phase transformation from austenite to martensite. These transformations have irreversible effects like modifications in the crystallographic structure, volume, and hardness of the brake disc (see section 1.2). The identification and better understanding of these thermal gradients may augment the durability of the brake system, and the security of the passengers. To understand the high transient and dynamic process, a deep insight into the thermomechanical problem is required. The mentioned thermal gradients can cause cracks in the brake disc, fading and vibrations as well as failure. The thermomechanical behaviour of the brake system poses a coupled multi-physics and multi-scale problem (cf. Markert [91], [93]), which needs to be simplified because of the high number of process variables. There are several approaches such as the thermoelastic instability, progressive waviness distortion, and uncoupled methods, which may describe the hot spotting phenomena, but each of them has advantages and disadvantages. Here, an uncoupled approach to describe the onset of macroscopic hot spots is applied. Due to the superposition of the results of the mechanical and thermal analysis, we can show the formation of hot spots in an acceptable time frame by a high dynamic and transient model using a linear approach (cf. [91]). The boundary value problem is a reduced scale system of a brake system. The system consists of a single piston, which consists of the same material as the brake pads and a steel disc. The mechanical and thermal subsystem solutions are staggered and reinserted at certain time increments. The heat flux is calculated externally between the mechanical and thermal analysis and is used in the thermal analysis. The output of the thermal analysis and the temperature are inserted in the mechanical analysis. The simulation is conducted with

the FEM software Abaqus 6.14 and a *Fortran* script (FORTRAN-77). The simulation starts with an initialisation step, which is conducted to apply a reference temperature due to the dependence of the temperature of the properties of the materials. The second step consists of the mechanical analysis. The mechanical analysis is based on a quasi-static analysis during single rotation of the disc. The extracted normal forces of the contact area are required for the calculation of the heat flux and the displacement of the contact nodes. Subsequent, the heat flux is calculated. The contact nodes of the disc and the pad at each step of the mechanical analysis and for each turn of the thermal analysis are calculated. Due to the different materials of the brake disc and pad, the heat input is different. The heat partition can be calculated after Vernotte [129] (see section 1.2). After the calculation of the heat flux and the stop of the mechanical analysis, the thermal analysis starts to calculate the temperature distribution of the brake disc and pad. The updated geometries, which are deformed in the mechanical analysis are imported in the thermal analysis. The corresponding calculated heat flux of the angle increment in the mechanical analysis is applied in the thermal analysis during the time increment taking into consideration the rotation speed and the angle increment. After the thermal analysis and the calculation of the wear, the calculated temperature of all the nodes are inserted. Before starting the calculation of the wear or the displacement of the nodes, the associated area of the contact nodes are determined. The ratio of the areas and forces are

$$\frac{S_N}{S_{\text{pad}}} = \frac{F_N}{F}, \quad (5.1)$$

where S_N , S_{pad} , F_N , and F are the area associated to the node, the contact area of the pad, the force associated to the node, and the applied force on the pad, respectively. The simulation model is shown in the figure 5.1.

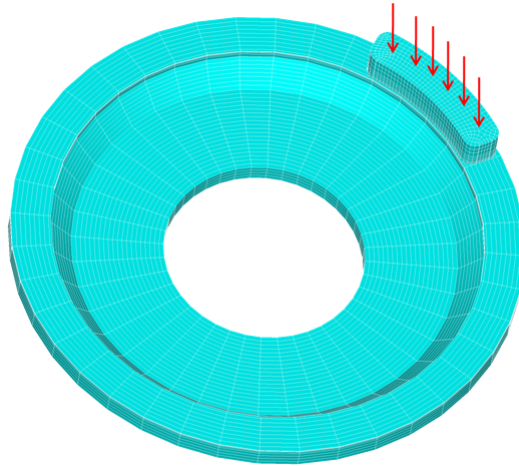


Figure 5.1: Model of the brake disc and brake pad

The wear coefficient is determined by figure 5.2, where an average temperature is required. This temperature is determined by use of the initial temperature T_i and the final temperature T_f :

$$T_{\text{mean}} = \frac{T_i + T_f}{2}. \quad (5.2)$$

The determination of the wear coefficient according to the temperature of the pad is conducted for the composite material HD671 according to figure 5.2.

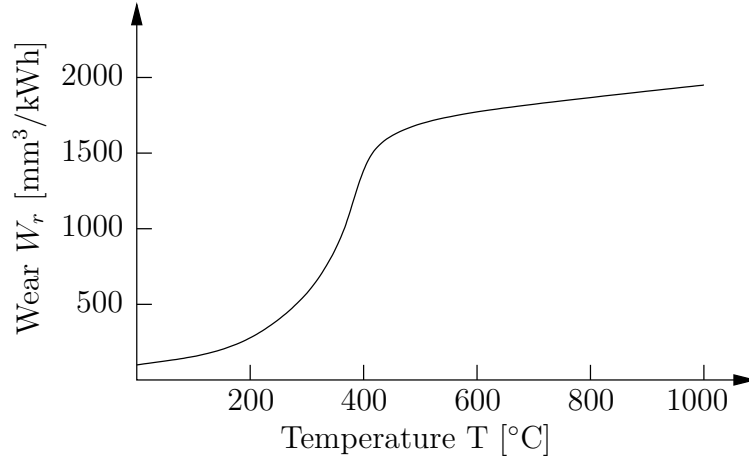


Figure 5.2: Determination of the wear coefficient

The consideration of the wear is conducted with the position of the node. The wear is implemented in the model in regard to the change of the height of the node, which represents the wear and is calculated by

$$h_n = \frac{W_r E_N}{S_N}. \quad (5.3)$$

Herein, W_r , E_N , and S_N are the wear coefficient, the energy transmitted to the node, and the area associated to a node, respectively. The energy is calculated after the multiplication of the heat flux with the duration. The total energy is the sum of the energies of all the nodes

$$E_{\text{total}} = \sum_{i=1}^n (||\mathbf{q}_i|| \Delta t_i), \quad (5.4)$$

where ϕ_i , and t_i are the heat flux corresponding to one increment and the time corresponding to one increment, respectively. Finally, the coordinates of all that nodes are modified to obtain the new geometry of the pad. The material properties according to the temperature of the friction pair are shown in table 5.1.

The braking starts at 2200 rpm and decreases linearly after 40 s to 0 rpm. The experimental results of Kasem [71] are compared below. The simulated inertia is 5.1 kgm². The disc is composed of C48 steel and has an average friction radius of 100 mm, friction width of 16 mm and a thickness of 22 mm. The pad is made of a bean-shaped organic matrix composite with a width of 16 mm, a thickness of 15 mm and an angle of 39°. The numerical results of the formation of hot spots can be observed in the figure 5.3.

Kasem [69], [70], [71] has realised a sequence of tests for the detection of macroscopic hot spots. The experiments are conducted on a tribometer with variable inertia in the Laboratoire de Mecanique de Lille. The disc is driven by an electric motor with a rotational speed of 2200 rpm and is decelerated after 40 s to 0 rpm. The brake disc and pad are the same as in the simulation. The disc surface temperature is measured with an IR camera. The test 7 is used for

Material	Temperature [°C]	Thermal expansion [$\times 10^{-6}$ 1/°C]	Thermal conductivity [W/mK]	Specific heat [J/kgK]	Young's modulus [GPa]	Poisson Ratio [-]	Density [kg/m ³]
HD671	20	8	14.5	770	2, 2	0.306	2480
	200	8	13.2	920	1, 8	0.312	2400
	400	8	12	960	1, 2	0.320	2200
	600	8	9.2	900	0.75	0.337	1700
Disc	20	10.8	47.3	440	210	0.29	7850
	200	11.8	44.1	510	195	0.29	7850
	400	13.5	39.3	570	177	0.29	7850
	600	15.8	32.9	630	133	0.29	7850
	800	18.5	25	700	100	0.29	7850

Table 5.1: Material properties of the composite brake pad and brake disc

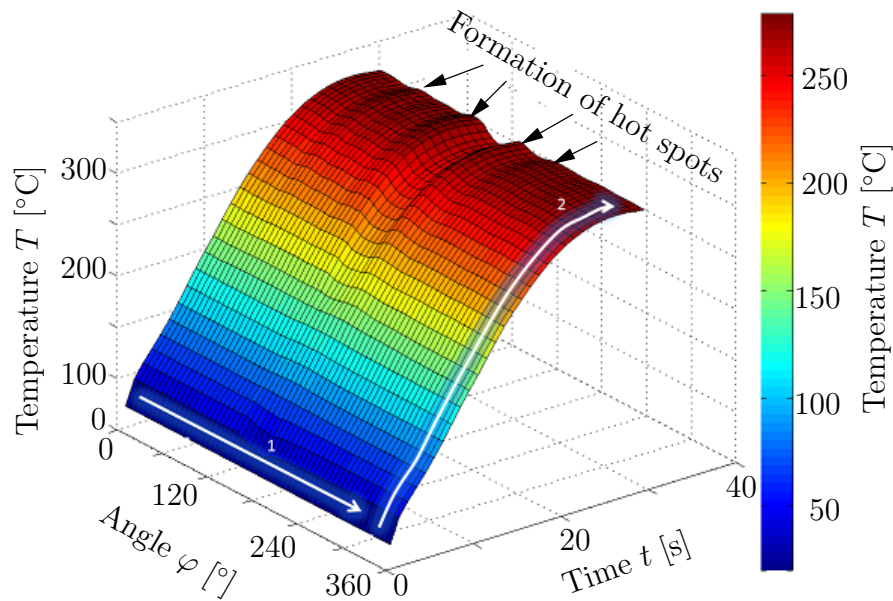


Figure 5.3: Formation of macroscopic hot spots.

the simulation. During the experiments of the work of Kasem the generation of macroscopic hot spots was observed. After the first seconds, a hot band appears, which follows the formation of macroscopic hot spots. The temperature level of the experimental and numerical results are equal. The simulation methodology shows a good agreement of the experiments and the numerical simulation. The experimental results are shown in Kasem [71]. The disadvantage of this numerical model is the calculation time, which requires 2 days to finish the simulation. The used hard- and software is shown in 5.2.

This time consuming uncoupled thermomechanical simulation requires an improvement and an alternative numerical treatment. In the next section, the used steady-state algorithm and the new simulation methodology are explained with a thermomechanical wear analysis, which is validated with the experimental results.

Software	Computer	Simulation time
Abaqus 6.14	Dell Precision T7500 (8 Xeon E5620 CPU 2400 MHz)	approx. 48 hours
FORTRAN-77	12288 MB RAM	
Centos 6		

Table 5.2: Used hard- and software

5.2 Experimental and Numerical Wear Analysis on the Tribometer

The main aim of this work is to investigate experimentally and numerically dry sliding contact and wear of disc brakes using different brake pad materials. Experiments are performed by using the pin-on-disc test bench. The fully coupled thermomechanical simulation is based on frictional heat contact calculations and wear simulations according to *Achard's* wear law. A comparison between experimental and numerical investigations is carried out based on different input parameters, such as load on pin and rotational speed of disc. *Achard's* wear law has been used previously by researchers for calculating wear in general sliding contacts. Podra and Andersson [113] calculated sliding wear with the FEM by using *Achard's* wear law. Molinari. et al. [95] used *Achard's* wear law to calibrate and validate finite element models of dry sliding wear of metals. Hegadekotte et al. [58], Benabdallah and Olender [21] used *Achard's* wear law for calculating wear in their respective works. Wear in general sliding contact is a complex phenomenon which depends upon many factors, such as thermal contact properties, load, braking time etc. Laguna-Cormacho et al. [77] investigated the high sliding and abrasive wear of the brake disc pad surfaces, which lead to the formation of friction layers and friction films that determine the friction behaviour of automotive brakes. Wear depends on several parameters such as the load, the rotational speed and the time. In low temperature regions, wear occurs because of adhesion, abrasive wear and fatigue. In high temperature regions, it occurs because of thermally activated processes. There are two types of aerosols [60]. The primary aerosol which are introduced directly into the atmosphere and the secondary aerosols particles which are introduced into the atmosphere by chemical reactions of gaseous components. Brake wear is a primary aerosol and is generated as a result of erosion between brake disc and brake pad. As a result, the concentrations of aerosols in the environment is increasing, which is bringing adverse effects on environment and human health. Therefore, it is important to understand the factors affecting the erosion rate of brakes. The aim is to validate the numerical model with the advanced simulation methodology of the temperature distribution and the mass lost with the experimental results. The experimental apparatus with the used pin-on-disc tribometer and the test procedure at the IAM is explained below. Subsequently, the numerical model and the results are described. Experimental investigations are carried out at the laboratory by using the pin-on-disc test bench. The experimental set-up is explained and shown in the section 4.3. The experimental set-up consists of a stationary cylindrical pin with a length of 20 mm and a diameter of 28 mm fixed against the rotating disc. The desired load is applied on the stationary pin. Different materials brass and bronze are used for investigation as stationary pin. The measurement equipment consists of a thermocouple type K, which is

used for the measurement of the temperature in the pin 2 mm over the contact surface. The measurement of the temperature of the disc is done with a pyrometer. The temperature range is between 50 °C and 1000 °C. The pyrometer is aligned on the reference circle (see figure 5.13). Experiments are performed by varying input parameters such as the load on the pin and the rotational speed of the disc. The three conducted experiments are shown in table 5.3.

Test	Material	Rotational speed [rpm]	Load [kg]
1	Brass	820	110
2	Brass	820	140
3	Bronze	1120	110

Table 5.3: Conducted experiments on the pin-on-disc tribometer

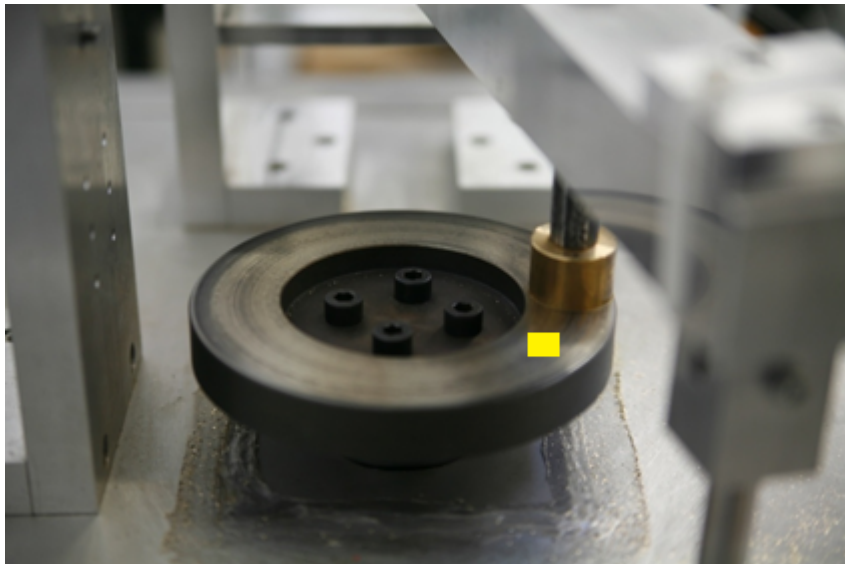


Figure 5.4: Measure point on the disc

The accuracy of experimental results depends greatly on having a proper contact between brake disc and brake pad. In order to achieve proper contact, the surface at the interface between the brake disc and brake pad is run in for an half an hour or if 90% of the contact surface is worn out before performing actual experiments. After the run-in and the development of the third body or the friction layer of the friction pair the system is cooled down to 30 °C. The initial temperature for each test is 30 °C. Figure 5.5 illustrates the rotational speed of the brake disc during the experiments. The rotation of the disc starts with an applied load on the disc and the application of the load is kept till the experiment is done. The conducted experiments are done for 820 and 1120 rpm.

The acceleration time t_{acc} and the deceleration time t_{dec} are approximately identical. The brake time t_B with the accelerating and decelerating time is 120 s. During the braking, the temperature of the disc and the pad are measured. The measured point and the experimental apparatus are shown in figure 5.6. After the braking, the cooling of the disc and the pin in air

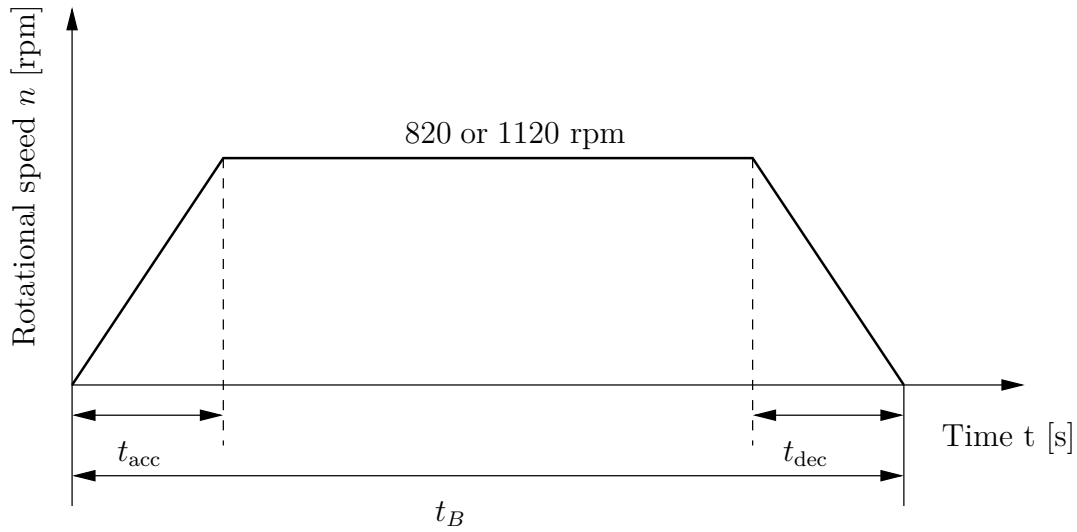


Figure 5.5: Rotational speed of the brake disc during the experiments

is measured for 60 s. The measurement data are recorded synchronously with a data recorder with a sample rate of 1000 Hz. The simulation methodology is developed by using the steady-state algorithm. The flow chart of the simulation methodology as shown in figure 5.6 consists of a series of solution steps combined with additional calculation steps.

The overall simulation methodology consists of three main steps, the mechanical analysis, the thermal analysis and the wear analysis. In the first step, the normal contact pressure at the interface of brake disc and pad is extracted from the mechanical analysis, which is being used for the frictional heat contact calculations. The frictional heat increment is calculated at the interface of the brake disc and pin and is used as a point heat flux boundary condition for the thermal analysis in the second step. The generated heat is equal to the total work done by the friction force (cf. equation 4.18).

In the subsequent step, *Archard's* wear law is implemented for calculating wear. For the calculation of the mass lost in the simulation, it is necessary after *Archard's* wear law [9], [7], [10],

$$V = \frac{W_c F_n s}{H}, \quad (5.5)$$

to determine the wear coefficient or the wear rate to calculate the mass lost, where V , W_c , H , F_n , and s are the volume lost, wear coefficient, hardness, normal force, and the sliding distance, receptively. The wear rate is calculated then with

$$W_r = \frac{W_c}{H} = \frac{V_{\text{lost}}}{s F_n}. \quad (5.6)$$

The experimental results to determine the wear rate are given in table 5.4.

The mass lost is then calculated by measuring the weight of the specimen before and after the experiments. Volume lost is found out by using the following equation

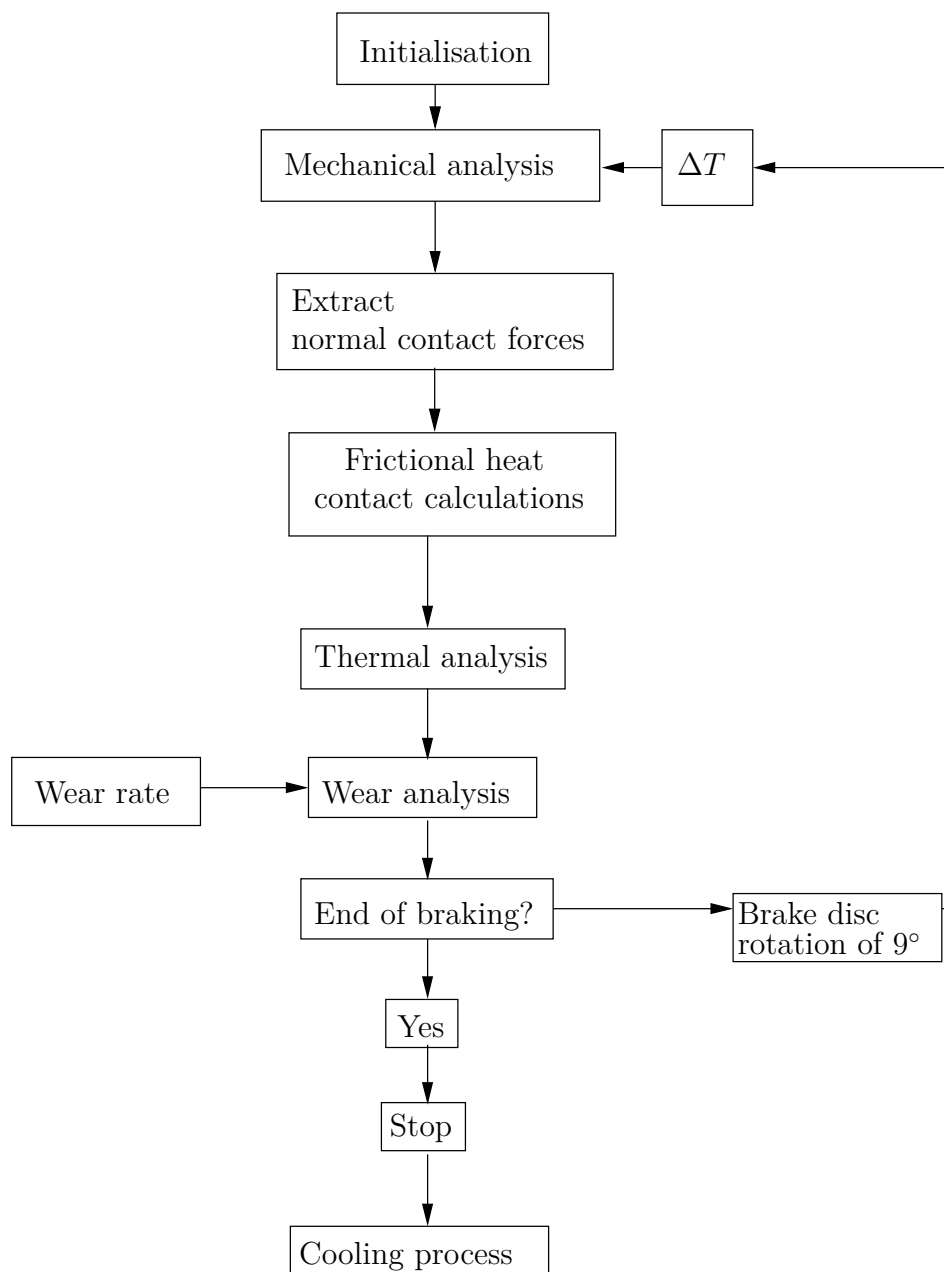


Figure 5.6: Numerical methodology

Test	Mass lost [kg]	Volume lost [m ³]	Wear rate W_r [mm ³ /J]
1	0.0006	$3.45 \cdot 10^{-8}$	$1.86 \cdot 10^{-4}$
2	0.0007	$4.6 \cdot 10^{-8}$	$3.61 \cdot 10^{-4}$
3	0.00065	$3.61 \cdot 10^{-8}$	$1.42 \cdot 10^{-4}$

Table 5.4: Experimental results of the pin-on-disc tribometer

$$V_{\text{lost}} = m_{\text{lost}} \rho, \quad (5.7)$$

where ρ is the density. The nodal wear increments are calculated with

$$\Delta h_i = p_{ci} W_r v \Delta t, \quad (5.8)$$

where p_{ci} is the nodal contact pressure at the node i , v is the sliding velocity, W_r is the specific wear rate and Δt is the time increment. The worn mass is calculated by using the following equation

$$V_{\text{lost}} = h_i A, \quad (5.9)$$

where A is the apparent area of contact of the friction pair. The mass lost can be calculated by applying equation 5.7. The used material properties of the friction pair are shown in table 5.5.

Material	Thermal expansion [$\times 10^{-6}$ 1/ $^{\circ}\text{C}$]	Thermal conductivity [W/mK]	Specific heat [J/kgK]	Young's modulus [MPa]	Poisson ratio [-]	Density [kg/m ³]	Emissivity [-]	Heat transfer coefficient [W/m ² K]
GG-25	11.7	46.5	490	117000	0.26	7200	0,7	45
Brass	18.7	144	380	117000	0.33	8930	0,7	55
Bronze	18	55	435	100000	0.34	8300	0,7	65

Table 5.5: Material properties of the friction pairs

The simulation methodology is assigned with the maximum revolution of the disc. If the maximum revolution is not reached, it will proceed to the next iteration. The increase in temperature during the thermal analysis is then supplied to the mechanical analysis in the next iteration. After the braking, a cooling step of the friction pair is done. The cooling of the friction pair is based upon the heat transfer through conduction, convection and radiation. The temperature gradient is the main driving force for the heat conduction. Whenever a gradient in temperature exists, there exists a conductive heat flow. *Fourier's* law is used for calculating the heat conduction. *Fourier's* law states that the heat transfer through a material is proportional to the negative gradient of the temperature and perpendicular to the area (cf. 3.41).

The heat transferred by the motion of a fluid is called heat convection. There are two main types of heat convection, the forced convection, which is caused by the forced fluid flow induced by means of external forces, e.g. due to a pump or a fan and the natural or free convection, which is caused by the motion of the fluid induced by temperature variations in the fluid. Forced convection is a faster and efficient method compared to natural or free convection because significant amount of heat can be transferred via forced convection. The Newton law of cooling, which describes the heat transfer per unit surface through convection, is expressed as

$$\mathbf{q}_{\text{conv}} = \mathbf{h} \Delta T, \quad (5.10)$$

where $\mathbf{q}_{\text{conv}}$ and $\mathbf{h}$ are the heat transferred due to convection, and heat transfer coefficient, respectively. The heat radiation occurs because of the transfer of internal energy in the form of electromagnetic waves. *Stefan-Boltzmann* derived a law that relates the heat flow rate emitted or absorbed from an object.

$$M = \varepsilon \sigma A (T^4 - T_0^4), \quad (5.11)$$

where M , ε , σ , A , T , and T_0 are the heat flow rate or exitance emitted or absorbed, the emissivity, the *Stefan-Boltzmann* constant $5.670 \cdot 10^{-8} \text{ W/m}^2\text{K}^4$, the surface area of the object emitting or absorbing thermal radiation, the absolute temperature of the object emitting or absorbing thermal radiation, and the absolute temperature of the surrounding environment. The geometry and FE mesh of the pin-on-disc model of the numerical model is shown in figure 5.7. The model is fixed at the bottom side with a degree of freedom of rotation around the axial axis. The pin is fixed at the top side with a degree of freedom of displacement around the axial axis. The load is applied at the top side.

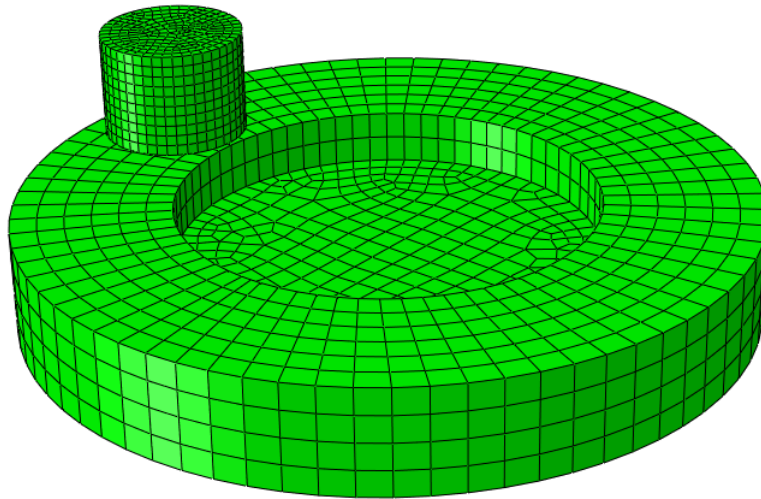


Figure 5.7: Geometry and FE mesh of the pin-on-disc model

Experimental and numerical results are compared and validated based upon the load on the pin and the rotational speed of the brake disc. Good agreement is observed between experimental and numerical results (see figures 5.8-5.10). The inputs of the model are the rotational speed of the brake disc n , the load on the pin F_n , the wear rate W_r , the total braking time t_B and the cooling time. The outputs are the temperature rise in brake disc and brake pad as a result of braking, the wear depth at each contacting node, the total material removed after every iteration and the temperature decrease as a result of the cooling process. In figures 5.8-5.10, experimental results, numerical results, and the temperature variations for the measured points for brake pad and brake disc with respect to the time for the test cases brass and bronze are shown. One can observe a good agreement of the numerical and experimental results. The left and the right side of the peak of the curves are the heating phase and the cooling phase, respectively. The pyrometer starts to measure at a temperature of 50°C . Good agreement is observed between the experimental and numerical investigation. A difference of 22°C is observed for the brake pad and 2°C for the brake disc for the test case brass 1 at the peak temperature. A difference of 34°C is observed for the brake pad and 2°C for the brake disc for

the test case brass 2 at the peak temperature. A difference of 18°C is observed for the brake pad and 3°C is observed for the brake disc for the test case bronze 1 at the peak temperature. The comparison between figures 5.8, 5.9, and 5.10 shows that an increase of the load on the pin and the rotational speed of the brake disc plays a vital role in the overall temperature distribution predicted by the model. Comparison between figures 5.8 and 5.9 shows that by keeping the rotational speed and material constant and increasing the load on the pin from 165 N to 240 N, the peak temperature of the brake pad is increasing from 275°C to 350°C and the peak temperature of the brake disc is increasing from 175°C to 218°C .

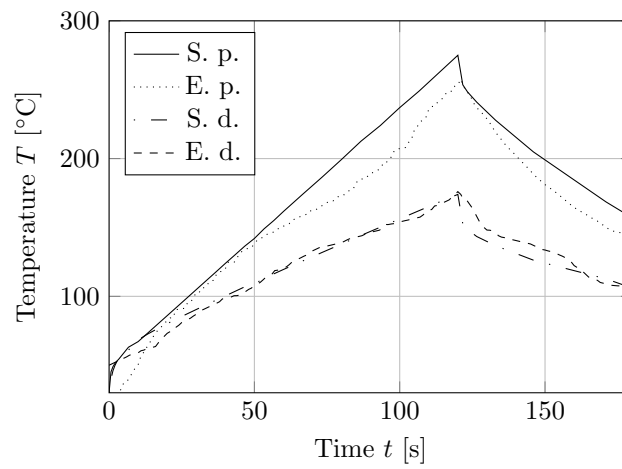


Figure 5.8: Brass 1, $n = 820$ rpm, $F_n = 165$ N, Validation of the experimental and simulation results, (S. p.: Simulation results of the pad, E. p.: Experimental results of the pad, S. d.: Simulation results of the disc, E. d.: Experimental results of the disc)

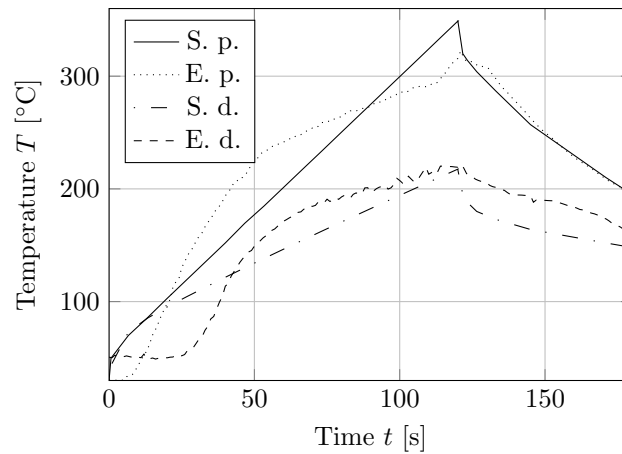


Figure 5.9: Brass 2, $n = 820$ rpm, $F_n = 240$ N, Validation of the experimental and simulation results, (S. p.: Simulation results of the pad, E. p.: Experimental results of the pad, S. d.: Simulation results of the disc, E. d.: Experimental results of the disc)

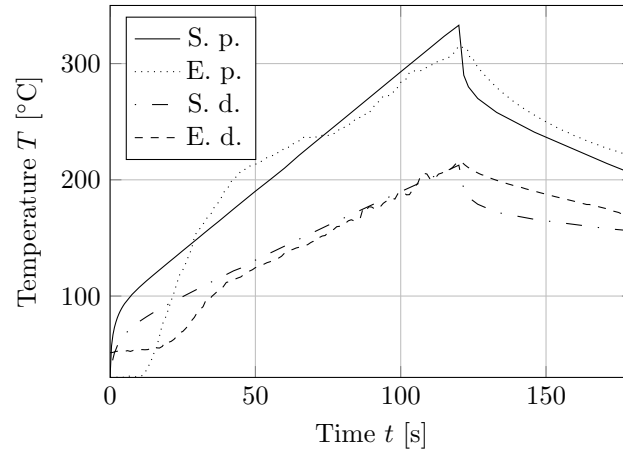


Figure 5.10: Bronze 1, $n = 1120$ rpm, $F_n = 165$ N, Validation of the experimental and simulation results, (S. p.: Simulation results of the pad, E. p.: Experimental results of the pad, S. d.: Simulation results of the disc, E. d.: Experimental results of the disc)

The peak temperatures of the experimental and numerical results in the figures 5.8-5.10 are shown in table 5.6.

Test pair	Friction	Max. temperature experiment [°C]	Max. temperature simulation [°C]
Brass 1	Pin	253	275
	Disc	176	174
Brass 2	Pin	315	349
	Disc	219	217
Bronze 1	Pin	315	333
	Disc	216	213

Table 5.6: Peak temperatures of the friction pairs

In table 5.7, the error of the measured and calculated mass lost is shown. The error lies between 0 and 14.29 %.

Test	Experimental mass lost [g]	Simulated mass lost [g]	Error [%]
Brass 1	0.6	0.6	0
Brass 2	0.7	0.8	14.29
Bronze 1	0.65	0.69	6.15

Table 5.7: Mass lost of the pins

The experiments carried out with the pin-on-disc test bench show the heating and the cooling process of a simulated brake pad and brake disc. Numerical investigation is done by the application of the FEM using Abaqus. The generated simulation methodology is capable to calculate the temperature distribution, the wear depth and the volume of material lost at the interface of the brake disc and pad as a result of braking. The numerical investigation is carried out with respect to the experiments. The results show a good agreement between experimental and numerical investigations. It is possible with a good agreement to get an approximation of the wear lost with *Archard's* wear law. With a linear wear model, it is possible to get a good agreement of the measured and simulated temperature distribution of the disc and pad. The different materials show a different temperature peak for the different loading conditions. Error is a fate of every experiment. Possible sources of error in the experimental investigation includes vibrational effects, human and instrumental errors. Possible area of future works includes the usage of temperature dependent material properties and coefficients of friction, the introduction of a (visco)plastic material model, and the experimental measurement of the coefficient of friction at the interface of brake disc and brake pad by the use of force sensors.

5.3 Brake disc analysis with the flywheel test bench

The experimental study of the thermomechanical behaviour of an automotive disc brakes is shown. The goal of this analysis is to show the capability and the performance check of the designed flywheel test bench. Therefore, the designed fly wheel test bench shows the capability for a better understanding of the thermomechanical effects of disc brakes by use of an advanced measurement system. Temperature measurements with a high speed infrared camera and a pyrometer have been carried out on the rubbing surface of brake discs and brake pads on a flywheel test bench. The mechanical behaviour is determined with a high-speed detecting capacitive sensor. Based on the advanced measurement system, a detailed description of the occurring phenomena is possible. From these experimental investigations, a scenario of brake cycles is conducted. The understanding of the brake system with regards to its failure performance of the brake system is an essential point and important for the safety of passengers and the environment. Therefore, the understanding and the validation of numerical models has to be conducted with experimental results. The experimental results are essential and require a modular test bench, which fulfils the requirements of the research. Moreover, the test bench has to be designed in a modular way to change easily the experimental arrangement. Furthermore, the measurement equipment has to fulfil the requirements for the validation of the numerical models. There exists commercial flywheel test benches, which are optimised for standard industrial test protocols rather than for dedicated research. The designed test bench tries to fill this gap. The advantage of commercial test benches are the high power, the high rotational speed and the high inertia. The disadvantage are the high price, the limited modularity and the low measurement technology.

The designed flywheel test bench has a power of 30 kW, which is supplied by a three-phase asynchronous motor. The maximum rotational speed is 1460 rpm. Moreover, the maximum mass of the six flywheels is 609 kg. Hence, the maximum inertia is 33 kgm² and can be changed with regards to the six mounted flywheels. The cooling of the brake disc or the simulation of the fair wind is realised with a ventilator with a maximum velocity of 100 km/h. Furthermore, the test bench consists of an advanced measurement system. A high speed IR camera ImageIR 5300

of InfraTec GmbH with a mirror system enables the detection of the temperature distribution on both sides of the brake disc. The maximum achievable frame rate is 13 kHz. A pyrometer measures the punctual temperature on the brake disc. Furthermore, a high speed capacitive sensor with a DT6350 one-channel electronic and CS2 cylindrical sensor with a measuring range of 2 mm of Micro-Epsilon GmbH measures the deformation of the rotating brake disc with a maximum sample rate of 50 kHz in a high resolution of several nanometres. The measurement system is shown in Figure 5.11. The figure depicts the pyrometer IMPAC IGA 6/23 Advanced of LumaSense GmbH measuring range of 50 to 1000 °C, the capacitive sensor, the mirror system and the incremental encoder ITD 75 A 4 2048 T NI KR1 S 75 with 2048 pulse per rotation of Baumer GmbH for the synchronisation of the IR camera. All tests are conducted on this flywheel test bench. In particular, a drag braking operation is simulated. The brake time is 420 s and the initial temperature is 25 °C. Furthermore, the friction pair is a commercial brake system of a luxury class passenger car. The diameter and the thickness of the ventilated brake disc are 325 mm and 30 mm, respectively. The brake calliper and the organic brake pad are fixed at the support structure.

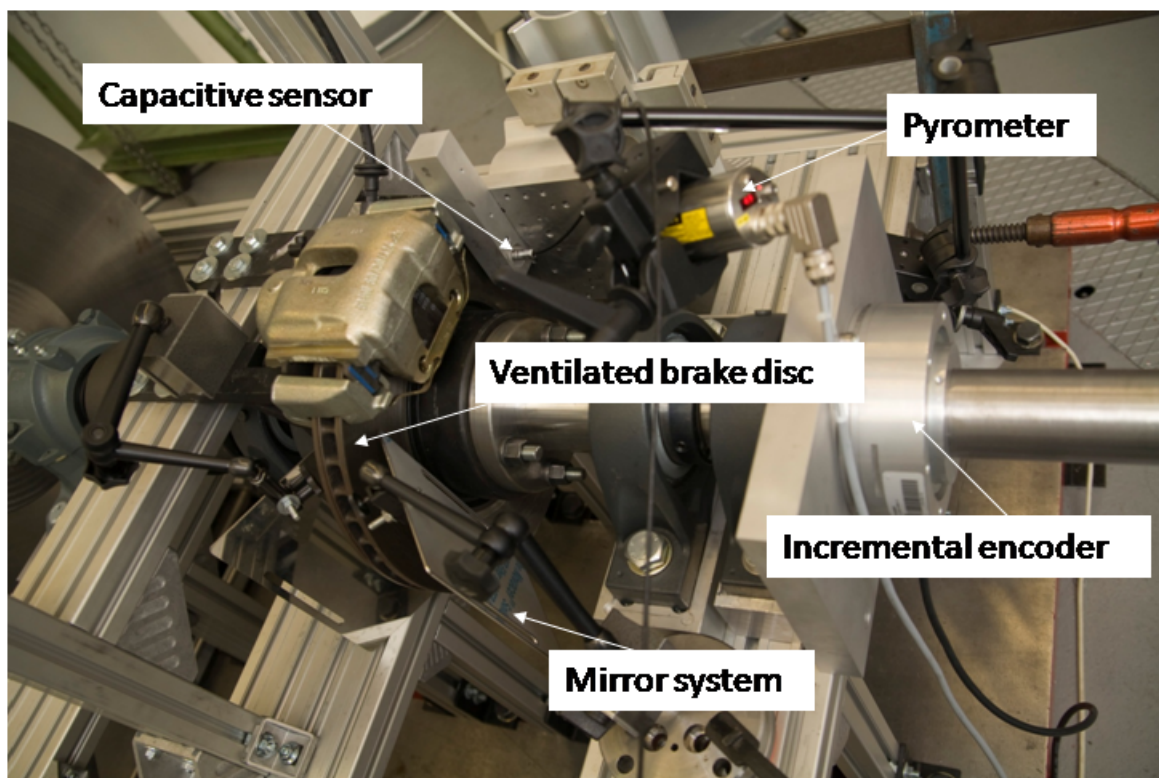


Figure 5.11: Flywheel test bench at the IAM.

Figure 5.12 illustrates the deformation behaviour of the brake disc. The capacitive sensor is placed at the inner diameter of the brake disc. One can observe a deformation of around 3 μm for more than 400 s. The vibration in the measured curve can be explained due to the cooling fins, which generates hot spots. Another explanation of the vibration is the unknown initial structure of the brake disc. The initial deflection of the brake disc is 23 μm , which is in the normal range. The maximum allowable deflection of the brake disc in a car is 50 μm . Furthermore, the brake disc shows a linear increasing deflection. This can be explained by the yielding of the mounting of the brake disc with the supporting structure, which requires further

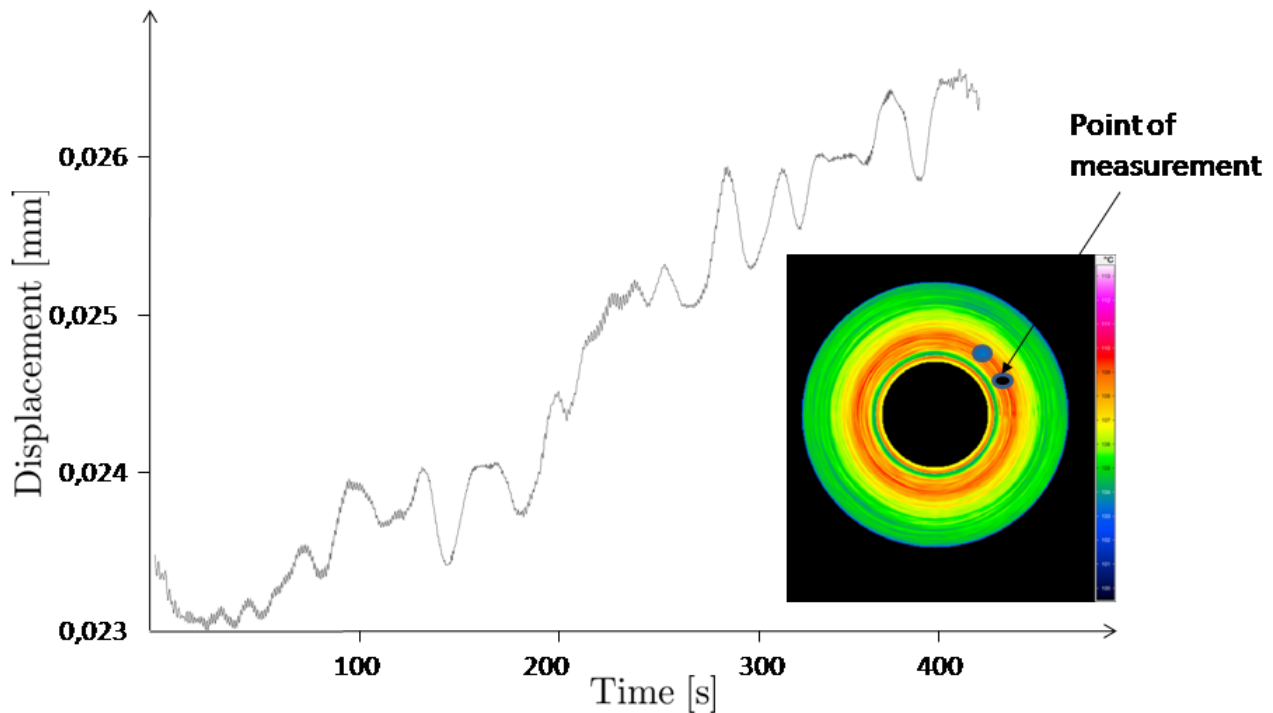


Figure 5.12: Displacement measurement of the brake disc.

investigations. Figure 5.13 depicts the punctual temperature of the brake disc at the same radius where the capacitive sensor is placed. The pyrometer starts to measure the temperature from 50 °C. The pyrometer detects the infrared radiation of the emitting surface of the brake disc according to the Stefan-Boltzmann law. The brake disc shows a high thermal capacity and the increase in temperature can be observed after 300 s (Figure 5.13). The maximum achievable temperature is 100 °C. Figure 5.14 shows the temperature distribution of the two sides of the brake disc measured with the high-speed infrared camera. It shows the temperature distribution at the beginning, middle and end of the braking. The emitted infrared radiation of the two sides of the brake disc is reflected with a mirror system to the infrared camera. It shows the formation of a hot band at the inner diameter of the brake disc, which increases in the width over the time. The infrared camera shows the same temperature level. Furthermore, the formation of a second hot band at the outer diameter of the brake disc can be observed.

The designed flywheel test bench at the IAM provides good performance results in regard to the installed measurement system. The test bench is designed in a modular way to fulfil the requirements of the changing boundary conditions. The next step is the investigation of the brake cycles and the influence on the brake disc. Another very important point is the analysis of the brake system in regards to the electro mobility and the autonomous driving. Moreover, an analysis of the electronic stability control in regard to the brake-by-wire can be conducted. For further investigations, an acoustic camera will be acquired for the analysis of the NVH of the brake system.

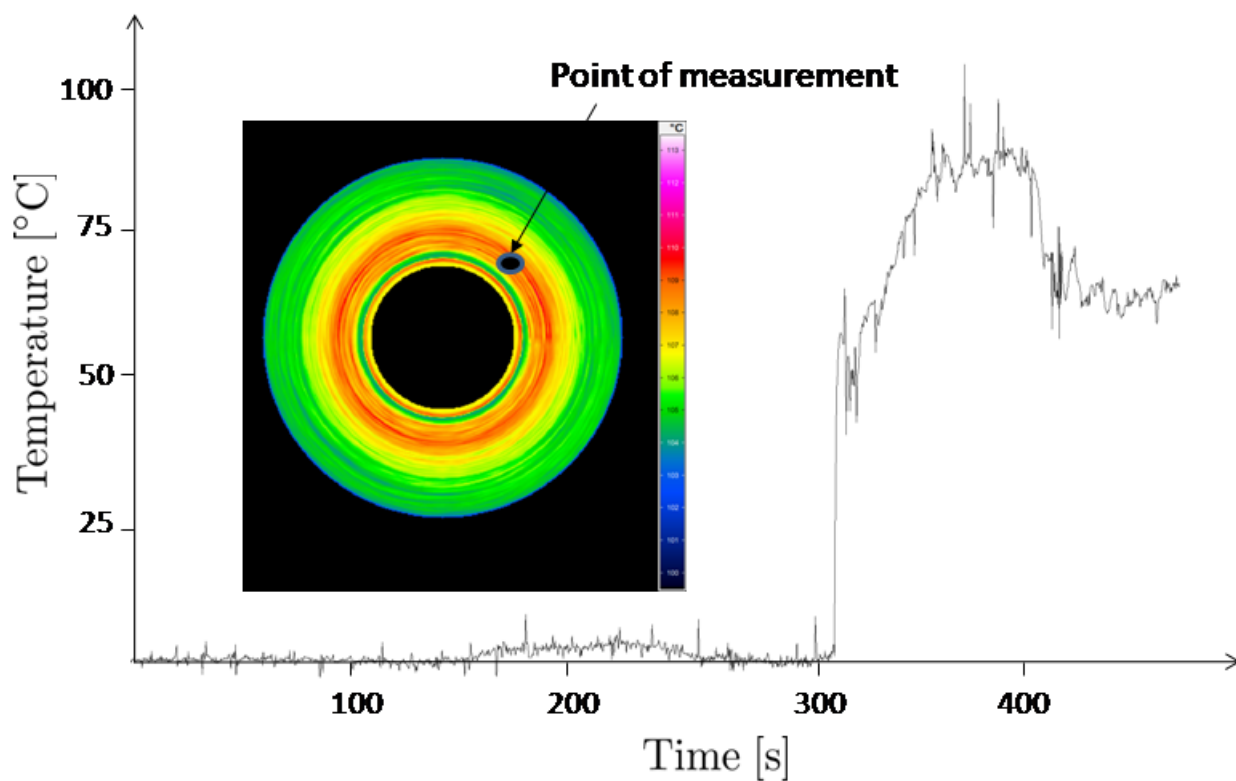


Figure 5.13: Punctual temperature measurement.

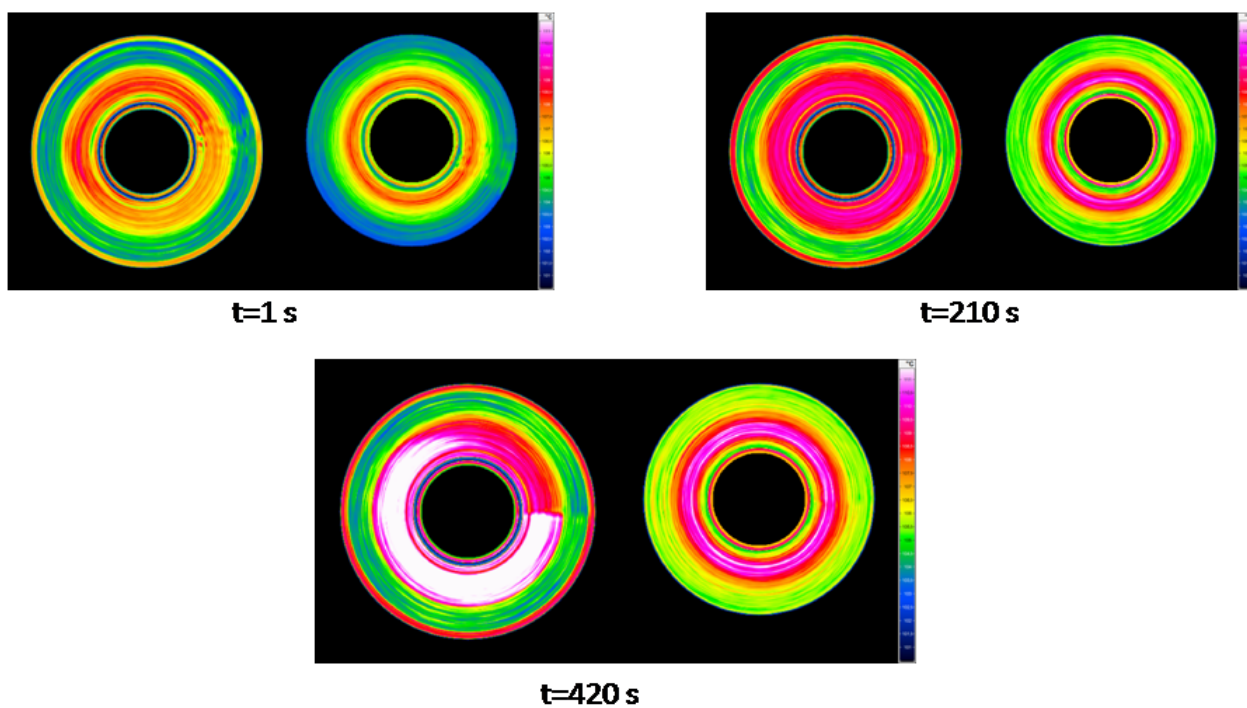


Figure 5.14: Temperature distribution of two sides of the brake disc at different times.

6 Particulate Matter and Wear

The Particulate Matter (PM) plays an important role for the health-related quality of life. The PM is not just a technical but also a political issue. The protection of the environment is an important point in the agenda of the parties in the industrialised countries. The PM plays also an important role for the newly industrialising country. The main reason for the protection of the environment in the newly industrialising country is not only the shortage of resources, the climate change or natural catastrophe, but also the reduced quality of life due to the smog. The smog in the largest cities in China make the sighting of the sky and the stars impossible. The PM becomes also very important for the expansionary policy of the companies. The unconditional introduction of the diesel engine of the Volkswagen AG into the american market led to the dieselgate. The emission values of the nitrogen oxides NO_x of the diesel engines was 30 times higher than the regulation of the California Air Resources Board (CARB). The filtering of PM smaller than $10\text{ }\mu\text{m}$ in the nasopharynx, which is not sufficient for fine particles. Therefore, a big part of the PM gets into the lungs whenever it is inhaled. Particles smaller than $0.1\text{ }\mu\text{m}$ reach into the pulmonary vesicles and are removed very slowly or not at all [81]. For further information about the aerosol particles, the interested reader is referred to the works by Hinds [60] and Kulkarni [76]. The following physical fundamentals are taken from the book Aerosol Technology of W. C. Hinds [60].

6.1 Introduction

Aerosol is defined as the simplest form of a collection of solid or liquid particles suspended in a gas. Some of the examples of aerosols are resuspended soil particles, smoke from power generations, photo-chemically formed particles, salt particles and ice particles or atmospheric clouds of water droplets. The properties of aerosols depends on the size of the particles and their concentrations in the suspended medium. The study of the aerosol principles is important because of its toxic, global warming and human health effects. The study of the aerosol technology in regard to properties, behaviour and physical principles is important for the application of the measurement and control. Aerosol technology is used in the field of occupational hygiene, air pollution control, inhalation toxicology, atmospheric physics, chemistry and radiological health. The primary aerosols are introduced directly into the atmosphere and the secondary aerosols are introduced into the atmosphere by chemical reactions of gaseous components. Important parameters for characterising the behaviour of aerosols are the particle size, the shape and the density. The mathematical description of the behaviour of the aerosols depends on the particle size. Therefore, a microscopic approach with characterised properties on an individual particle has to be developed. The average properties can then be estimated by integrating over the size distribution. The suspended gas interacts with particles and influences them. Particle motion in the suspended gas is resisted by the particle size. The aerosol particles share energy with gas molecules and they exhibit the *Brownian* motion. Hence, the temperature, the pressure, the mean free path, the diffusion and the viscosity of a gas are manifestations of the gas molecules. The temperature is a measure of the kinetic energy of the molecules. The pressure represents the force of molecular impacts on the container wall. The mean free path is the

average path length that a particle travels in a flowing material without impact with other particles. The diffusion describes the transfer of the molecular mass. The viscosity represents the transfer of momentum by molecular motion. The difference between gases and molecules are that gases have a larger number of molecules, the molecules are small compared to the distance between them and the molecules are rigid spheres travelling in straight lines between elastic collisions. The properties temperature, pressure, viscosity, mean free path, diffusion and thermal conductivity are derived in terms of billiard balls [60]. The ideal gas law is given as

$$p V = n R T, \quad (6.1)$$

where p , V , n , R , and T are the pressure, the volume, the amount of substance of gas, the universal gas constant, and the temperature, respectively. It is derived by using *Boyle's*, *Charles's* and *Avogadro's* law. An alternate way is also used for the derivation of *Boyle's* law. The measuring of the RMS value of molecular velocity of a gas is

$$c_{rms} = \frac{3 R T}{2 M}, \quad (6.2)$$

where M , R and T are the molecular weight of the gas $M = m N_a$, the gas constant, and the temperature, respectively. The *Avogadro* constant N_a indicates how many particles are contained in one mole of the respective substance. The mean free path can be calculated with the following equation

$$\lambda = \frac{\bar{c}}{n_z}, \quad (6.3)$$

where $\bar{c}$ and n_z are the average distance travelled in 1 s and the average number of collisions of a molecule in 1 s, respectively. The viscosity transfers the molecular momentum from a faster moving layer to a slower moving layer. According to *Newton's* law of viscosity, the frictional force between fluid layers moving at different velocities is given by

$$F = \frac{\eta A U}{y}, \quad (6.4)$$

where A , U , η and y are the area of the layers, the constant velocity of upper layer, the viscosity and distance between two layers, respectively. The diffusion describes the transfer of mass through another in the absence of fluid flow. The diffusion flux J transfers continuity per unit time through a unit perpendicular to the direction of diffusion under the influence of the concentration gradient $\frac{dC}{dx}$. *Fick's* first law of diffusion is

$$J = -D_{ba} \frac{dC}{dx}, \quad (6.5)$$

where the diffusion coefficient $D_{ba} = \frac{2}{3 \pi^{\frac{3}{2}} n d_m^2} \sqrt{\frac{R T}{M}}$ and d_m describe the diffusion of gas b in air and the diameter of a gas molecule, respectively [60].

6.2 Particle Form and Size

Some of the examples of non-spherical particles are cubic (sea salt particle), cylindrical (bacteria and fibres), single crystals, clusters of spheres, crushed materials and agglomerated particles. For the calculation of the drag force and the settling velocity for non-spherical particles, a correction factor is applied to *Stoke's* law. The dynamic shape factor accounts the effect of the form of the particle in regard to its motion,

$$\chi = \frac{F_D}{3 \pi \eta v d_e}, \quad (6.6)$$

where d_e , F_D , v , and η are the equivalent volume diameter, the drag force, the velocity, and the viscosity, respectively. The terminal settling velocity is given by

$$v_{TS} = \frac{\rho_p d_e^2 g}{18 \eta \chi}, \quad (6.7)$$

where ρ_p and g are the particle density and the gravitational constant. The aerodynamic diameter d_a is an equivalent diameter of the spherical particle with the density of 1 g/cm^3 , which has the same settling velocity as the particle. Another equivalent diameter is the *Stokes* diameter d_s , which has the same density and settling velocity as the particle. The terminal settling velocity is therefore for the introduced equivalent diameters

$$v_{TS} = \frac{\rho_p d_e^2 g}{18 \eta \chi} = \frac{\rho_0 d_a^2 g}{18 \eta} = \frac{\rho_b d_s^2 g}{18 \eta}, \quad (6.8)$$

where ρ_0 , and ρ_b are the standard particle density (1 g/cm^3), and the density of the bulk material. In figure 6.1, an irregular particle, an aerodynamic and a *Stokes'* equivalent sphere are shown. They have the same settling velocity, but with different shapes and densities.

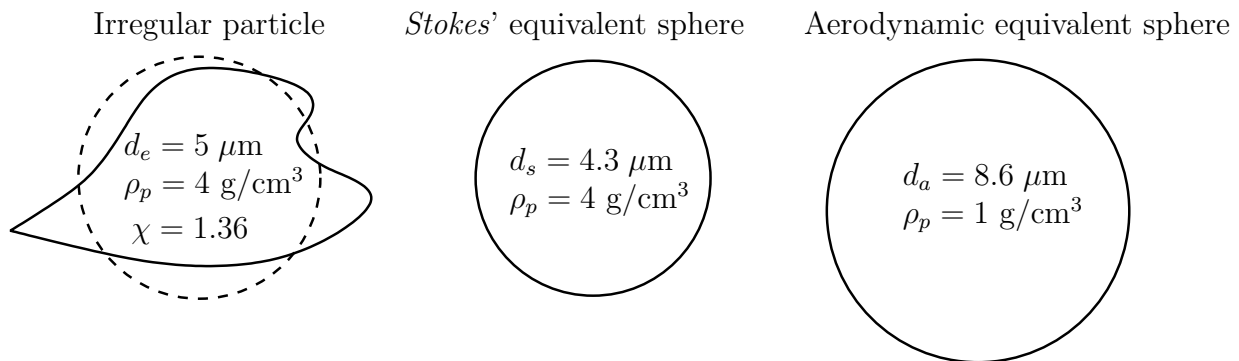


Figure 6.1: Equivalent spheres

The calculation of the atmosphere diameter can be calculated according to the particle diameter d_p , the standard density ρ_0 and the particle density ρ_p :

$$d_a = d_p \sqrt{\frac{\rho_p}{\rho_0}}. \quad (6.9)$$

The determination of the aerosol concentration or the change of the aerosol concentration in regard to the time and volume is very complicated. Therefore, two idealised models of the settling is described, the stirred and the tranquil settling. The reality lies between these two models. For the stirred settling the concentration is uniform throughout the chamber. Diffusion, resuspension and deposition on the walls are assumed to be neglected. Particle settling velocity is superimposed on the vertical components of the convective velocity. For the tranquil settling, there is no motion of gas and each particles motion is solely its gravitational settling in the stagnant air mass. Diffusion is neglected and the particles will settle with the same velocity v_{TS} . The determination of the particle size distribution, the mass concentration, the moment distributions, the moment averages and the log-normal distributions are necessary for the description of the particles. The representation of particles in mono-disperse aerosols is conducted with the particle diameter. The range of the particle size for poly-disperse aerosols is over two or more orders of magnitude. Therefore, it is necessary to characterise these size distributions by statistical means for the wide size ranges. The properties of the particles are strongly dependent on particle size. The used statistical tools are the moment averages, which frequently measures the quantity that is proportional to the particle size raised to power such as surface area or mass. The moment distribution describes the mass distribution as a function of the particle size. It gives the fraction of the total number of particles in any size range. The distribution of mass gives the fraction of total mass of the particles contributed by particles in any size range. The log-normal distributions is selected empirically to fit the wide range and skewed shape of most aerosol size distributions. The normal distribution is rarely used to describe particle size distribution because most aerosols exhibit a skewed shape [60].

6.3 Particle Motion

The analysis of uniform particle motion is important because aerosol particles reach to constant velocity almost immediately. There are two laws applicable, the *Newton* resistance law (applicable when the *Reynolds* number is greater than 1000) and the *Stokes* law (applicable when the *Reynolds* number is smaller than 1). Between these two regions there is a transition region. In the *Newton* resistance law, the force resisting of motion of a sphere passing through a gas is derived by *Newton*. The rate of change of momentum is equal to the force required to move the sphere through the gas

$$F_D = K \rho_g \frac{\pi}{4} d^2 v^2 = C_D \rho_g \frac{\pi}{8} d^2 v^2, \quad (6.10)$$

where K , ρ_g , C_D , d , and v are the constant proportionality, the density of the gas, the drag coefficient, the diameter of the particle, and the velocity of the particle, respectively.

In the figure 6.2, the drag factor with the three regions is illustrated. The dimensionless drag factor is constant for spheres with a *Reynolds* number $Re > 1000$ but changes for $Re < 1000$. Newton's resistance law is valid when inertial forces larger than the viscous forces. For aerosol motion mostly *Stokes'* law is used because of its movements at low *Reynolds* numbers due to fact that aerosols have smaller diameter and velocities. In *Stokes'* regions, viscous forces are higher than the inertial forces. It is derived from *Newton's* second law of motion in which body forces, pressure and viscous forces are included. The following assumptions are made for the derivations of *Stokes'* law. The inertial forces are negligibly small compared to the viscous

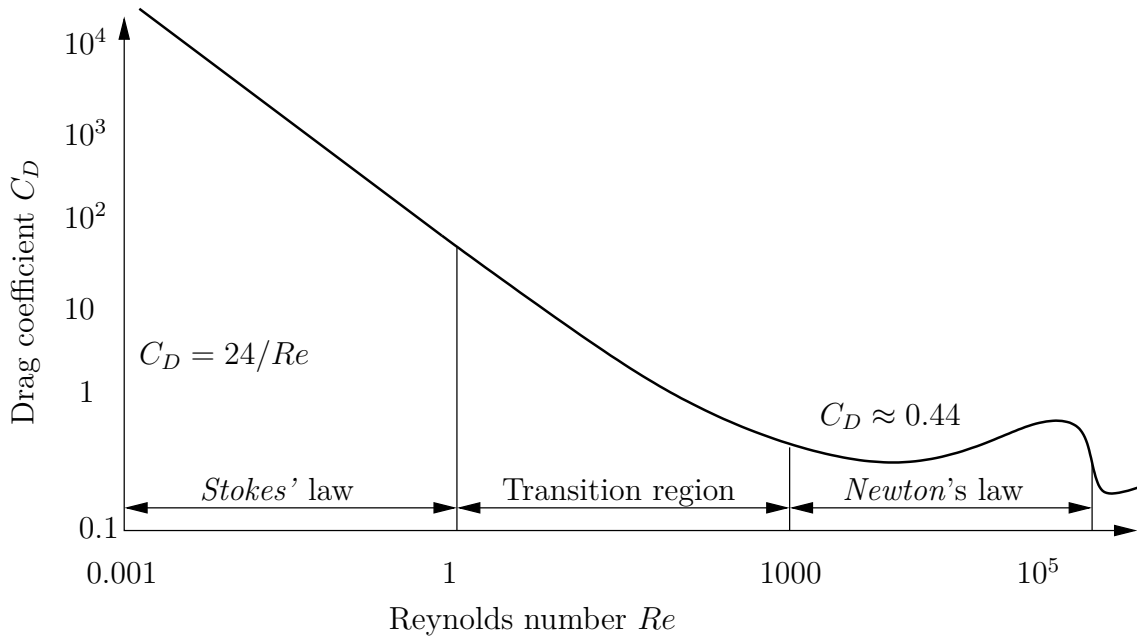


Figure 6.2: Drag factor

forces. The elimination of the higher-order terms in the *Navier-Stokes* equations, which yields to linear equations. The fluids are incompressible and there are no walls or other particles nearby. The motion of the particle is constant and the particles are rigid spheres. The fluid velocity at the particle surface is zero. The equations for the forces acting at any point in the fluid surrounding a spherical particle are integrated into a normal force

$$F_n = \pi \eta v d \quad (6.11)$$

and a tangential force over the surface of the particle

$$F_t = 2 \pi \eta v d. \quad (6.12)$$

Combined together, the resisting forces on the sphere is

$$F_D = 3 \pi \eta v d, \quad (6.13)$$

where η , v , and d are the viscosity, the velocity, and the diameter of the particle, respectively. The determination of the velocity of an aerosol particle undergoing gravitational settling in still air is

$$v_{TS} = \frac{\rho_P d^2 g}{18 \eta} \quad \text{for } d > 1 \mu\text{m} \quad \text{and} \quad Re < 1, \quad (6.14)$$

where ρ_P , d , g , and η are the density of the particle, the diameter of the particle, the gravitational constant and the viscosity, respectively. The drag force F_D of the air on the particle is equal and opposite to the gravitational force $F_D = F_G = mg$. In a centrifugal force field, which can be observed in the braking, the settling velocity is given by

$$v_{TC} = \frac{\rho_g d^2 a_c}{18 \eta}, \quad (6.15)$$

where a_c is the centrifugal acceleration of the particle. Another variable to describe the particle mobility is the ratio of the terminal velocity of a particle to the steady force producing that velocity

$$B = \frac{v}{F_D} = \frac{1}{3 \pi \eta d} \quad \text{for } d > 1 \mu\text{m}. \quad (6.16)$$

Therefore, the terminal velocity is

$$v_{TS} = F_G B. \quad (6.17)$$

For particles less than $1 \mu\text{m}$ the slip correction factor is necessary to describe that these particles do not settle faster. The *Stokes* drag force with the correction factor C_c is

$$F_D = \frac{3 \pi \eta v d}{C_c}, \quad (6.18)$$

where $C_c = 1 + \frac{2.52 \lambda}{d}$. For particles smaller than $0.1 \mu\text{m}$ the following slip correction factor is used

$$C_c = 1 + \frac{\lambda}{d} \left[2.34 + 1.05 e^{(-0.39 \frac{d}{\lambda})} \right], \quad (6.19)$$

where λ is the mean free path, which describes the path without collision with other particles. Besides the uniform particle motion, the irregular motion of particles in still air is described by the *Brownian* motion. The particles are affected in their motion due to the gas molecules. Another transport mechanism is the diffusion, where the particles are transported from a higher concentration region to a lower concentration region. The diffusion is characterised by the particle diffusion coefficient D . The higher D , the higher is the *Brownian* motion and the faster is the mass transfer in the concentration gradient. According to *Fick's* law the flux in regards to the gradient of the number of particles is

$$J = -D \frac{dn}{dx}, \quad (6.20)$$

where the diffusion coefficient is given by $D = \frac{k T C_c}{3 \pi \eta d} = k T B$. The k and T are the *Boltzmann* constant and the temperature, respectively. The diffusion force on the particles is

$$F_{\text{diff}} = \frac{3 \pi \eta v d}{C_c}. \quad (6.21)$$

The mean velocity, which depends on the particle density, is

$$\bar{c} = \sqrt{\frac{8 k T}{\pi m}} = \sqrt{\frac{48 k T}{\pi^2 \rho_p d^3}}. \quad (6.22)$$

The particle mean free path (λ_p) in the *Brownian* motion is the travelled mean distance of a particle before it changes its direction. The λ_p is the stopping distance of a particle at the mean velocity. The particle mean free path is

$$\lambda_p = \tau \bar{c}, \quad (6.23)$$

where τ is the particle size dependent parameter, which describes the relaxation time. The particle mean free path depends on the particle density. Furthermore, the diffusion coefficient can also be expressed in regard to the particle mean free path and the mean velocity

$$D = \frac{\pi}{8} \lambda_p \bar{c}. \quad (6.24)$$

The description of the straight line acceleration under constant and varying forces and the curvilinear motion is important for aerosol collection mechanisms (filter, lung, cascade impactor). The relaxation time of a particle describes the time to change the velocity to a new condition of forces. The relaxation time is

$$\tau = m B = \frac{\rho_p d^2 C_c}{18 \eta} = \frac{\rho_0 d_a^2 C_c}{18 \eta}, \quad (6.25)$$

where m , B , ρ_p , d , η , and d_a are the particle mass, the particle mobility, the particle density, the particle diameter, the viscosity, and the atmospheric diameter, respectively. According to *Newton's* second law the straight line particle acceleration at any time after released in still air ($t = 0$) and in a gravitational field is

$$v(t) = v_{TS}(1 - e^{-\frac{t}{\tau}}). \quad (6.26)$$

The stopping distance of this particle can be calculated according to

$$x(t) = v_f t - (v_f - v_0) \tau (1 - e^{-\frac{t}{\tau}}). \quad (6.27)$$

The stopping distance can also be written as $s = Bm v_0$ and is used for characterising the curvilinear motion and determining how far particles can be thrown during various mechanical operations (sawing, grinding) within the *Stokes* region. The situation becomes difficult to analyse for higher diameter particles because the proportionality between drag force and the velocity changes with the velocity. The position of a particle at any time instant along the x and y axe can be found by using the following equations

$$\begin{aligned} x(t) &= v_0 \tau (1 - e^{-\frac{t}{\tau}}), \\ x(t) &= v_{TS} t - v_{TS} \tau (1 - e^{-\frac{t}{\tau}}). \end{aligned} \quad (6.28)$$

This calculation can be done analytically only for very simple geometries, such as flow around a spherical or cylindrical obstacle. The curvilinear motion is characterised by the dimensionless *Stokes* number

$$\text{Stk} = \frac{S}{d_c} = \frac{\tau U_0}{d_c} \quad \text{for } Re_0 < 1, \quad (6.29)$$

where U_0 and d_c are the undisturbed air velocity well away from the cylinder and the diameter of the cylinder, respectively. The *Stokes* number is defined as the ratio of the stopping distance of a particle to a characteristic dimension of the obstacle. For a *Stokes* number > 1 the particles continue to move in straight line when the gas turns. For a *Stokes* number < 1 the particle follows the gas streamlines perfectly. This is used for the measurement of the properties of the particle. The inertial impactor is a special case of curvilinear motion that finds extensive application in the collection and measurement of aerosol particles. Particles whose inertia exceeds a certain value impact onto the impactor plate while smaller ones pass on. The cascade impactors collect all particles that escape. The sampling of the aerosol gives information about the particle size distribution. The principle is to measure the weight of the filter before and after the impaction. Several impactors are arranged in series in order of decreasing the cut-off size. The largest cut-off size is the first impactor. Liquid in the impactor adheres while solid particles may adhere or may bounce back, so in order to reduce this bouncing effects, the impaction plate is coated with a thin film of oil or grease. The impaction plate is replaced by a collection probe to avoid bounce and overloading of the particles. The losses of the particles are larger in particle size near the cut-off size. The losses occur on the lip of the collection probe and are larger for liquid particles than for solid particles [60].

6.4 Interaction of Particles

The adhesion of the aerosol particles to any surface is very high. This characteristic distinguishes them from gas molecules and from millimetre sized particles. They adhere and form agglomerates. Filtration and other particle collection methods are based on this behaviour. Adhesion is a complicated phenomenon and studies are conducted on ideal surfaces. The adhesive forces consist of *Van-der-Waals* forces, electrostatic forces and the forces from the surface tension of absorbed liquid. These forces are affected by the material shape, the surface roughness, the film size of the particle, the material roughness, the contamination of the surface, the relative humidity, the temperature, the duration of contact and the initial contact velocity. The *Van-der-Waals* force arises for the long range force and the short range force because of the chemical bonds and the absorbed layers of water and organic molecules. The resulting adhesive force is given by

$$F_{\text{adh}} = \frac{A d}{12 x^2}, \quad (6.30)$$

where A , d , and x are the *Hamaker* constant, the particle diameter, and the separation distance. The electrostatic force arises because of the random movement of electrons in any material, which creates momentary areas of concentrated charge called dipoles. At any instant, these dipoles include complementary dipoles in neighbouring material which in turn produce

attractive forces. The attractive electrostatic force is given by

$$F_E = \frac{K_E q^2}{x_q^2}, \quad (6.31)$$

where K_E , q and x_q are the constant of proportionality, the amount of charge on a particle, and the separation distance of the surfaces, respectively. The force from surface tension of the absorbed liquid is an attractive force between a particle and a surface, which is created by the surface tension of the liquid drawn into the capillary space at the point of contact. This force is

$$F_s = 2 \pi \gamma d, \quad (6.32)$$

where γ is the surface tension of the liquid. The experimental measurement of the force required to separate a particle from a surface can be measured by using a fibre micro-balance, a centrifugal force, vibrations or air currents. The empirical expression of the adhesive force is

$$F_{\text{adh}} = 0.063 d[1 + 0.009 RH], \quad (6.33)$$

where RH is the relative humidity in %. The detachment of the particles require a force to detach particles from a surface. The force can be measured by subjecting the particles to a centrifugal force normal to the surface in a centrifuge and determining the rotational speed required to detach the particles. The resuspension detaches a particle from a surface and transports it away from the surface. The collection of the particles are conducted with filters. Filters are used for aerosol sampling and most commonly for air cleaning. They are simple, versatile and favourable for collecting samples of aerosol particles. The particles are removed by a fibrous filter when they collide and attach to the surface of the fibres. The efficiency of a filter is calculable

$$E = \frac{N_{\text{in}} - N_{\text{out}}}{N_{\text{in}}}, \quad (6.34)$$

where N_{in} and N_{out} are the total number of incoming and outgoing particles. The particle penetrations decrease exponentially with increasing filter thickness. The value depends on particle size, face velocity, solidity and fibre size [60].

6.5 Phase Changes and Electrical Properties of Aerosols

The formation and growth of aerosol particles by condensation is the principle method of aerosol production in nature and the most important mass-transfer process between the gas phase and the particulate phase. Supersaturated vapour is initiated by the presence of small particles or ions that serve for particle formation. The reverse of growth by condensation is the closely related process of evaporation. The formation and early growth are complicated and incompletely understood. The homogeneous nucleation is the formation of particles from a supersaturated vapour without the assistance of condensation of nuclei or ions. The attractive

forces between molecules such as *Van-der-Waals* forces will lead to the formation of molecular clusters. Clusters are formed continuously but are unstable and continuously disintegrated. The growth by condensation is formed by a stable droplet. The droplet has passed a threshold and will grow by condensation. The nucleated condensation is a process of particle formation and growth that is promoted by the presence of condensation of nuclei or ions. It occurs at supersaturated conditions and droplets that are stable in size. It is the primary mechanism for the formation of clouds in the atmosphere. The evaporation is the reverse of condensation. More molecules will leave the particle surface than arrive. There is no threshold as in condensation. The electrostatic effect is the force exerted on a charged particle in an electrostatic field. Most aerosol particles carry some electric charge and some may be highly charged. For highly charged particles the electrostatic force can be thousands of times greater than the force of gravity [60].

6.6 Measurement of the Particles

The ability to collect representative samples of particles is necessary for the analysis. The samples shall accurately represent the airborne particles in the concentration and the number of the concentration. The isokinetic sampling measures from a moving aerosol stream a representative sample of aerosol, which enters the inlet of a sampling tube. This sampling is used for tube samples in a windy environment. The inlet axis of the sample of a thin walled tube or probe is aligned parallel to the gas streamline. The gas velocity is entering the probe and the free stream velocity of the inlet is equivalent. There is no distortion of the streamlines just upstream of the inlet. In isokinetic sampling, there is no particle loss at the inlet regardless of particle size or inertia. A reason for an anisokinetic sampling of particles is the failure to sample isokinetically. This results in a distortion of the size distribution and a biased estimate of the concentration due to the particle inertia in the region of curved streamlines near the inlet. Depending on the conditions, the sample may contain an excess or a deficiency of large particles. Error occurs because particles have so much inertia that they continue in a straight line as the gas curves into the inlet. If particles have no inertia, they will be no sampling error because they will follow the gas streamlines perfectly. In the sampling of particles from still air their occur two sources of error: the settling velocity of the particles and the particle inertia. The bias occurs when the inlet velocity is low and the inlet tube faces upward. When the sampling flow rate is zero, the collection of the settling particles will lead to an infinite sampling error. The sampling tube facing down corresponds to an underestimation of the concentration. The measurement of mass concentrations is conducted in two types of measurement methodologies. The direct measurement is sensing particle mass or inertia by gravimetric analysis. The indirect measurement relies on a property such as light scattering. The transport loss of the particles consist of the efficient entry into the sampling inlet, the particle lost in the tubing and fitting in the inlet, the collection or measuring devices. Another problem is the coagulation. Wherein aerosol particles collide with one another due to a relative motion between them and adhere to form larger particles. The thermal coagulation is a relative motion between particles in the *Brownian* motion. The kinematic coagulation is the relative motion between particles due to external force,s e.g. gravity, electrical forces or aerodynamic effects. The mono-disperse coagulation is a thermal coagulation of mono-disperse spherical particle with $d_p > 0.1 \mu\text{m}$. The particles adhere with every collision and the particle size changes slowly. The poly-disperse coagulation is very complex and no explicit solution exists [60].

6.7 Measurement of Brake Dust

The goal of the investigation of the brake dust is to measure the particle number concentration and the particle size of superfine particles. The experiments are conducted on the tribometer (Section 4.3). The friction pairing consists of a grey cast iron (GG-25) and a commercial brake pad. The properties of the brake disc are illustrated in table 6.1.

Material	Thermal expansion [$\times 10^{-6}$ 1/ $^{\circ}\text{C}$]	Thermal conductivity [W/mK]	Specific heat [J/kgK]	Young's modulus [MPa]	Poisson's ratio [-]	Density [kg/m ³]	Yield stress [N/m ²]	Heat transfer coefficient [W/m ² K]
GG-25	11.7	46.5	490	117000	0.26	7200	165	45

Table 6.1: Material properties of the brake disc

The organic brake pad consists of more than 30 ingredients and the thermal and mechanical properties are unknown. The measurements are conducted with a TSI Fast Mobility Particle Sizer (FMPS) 3091 spectrometer. The wide size range of the spectrometer is from 5.6 to 560 nm. The FMPS enables the dynamic measurement of the brake dust. The schematic view of the FMPS of the "Institut für Arbeits-, Sozial- und Umweltmedizin" is shown in figure 6.3. Acknowledgement to "Institut für Arbeits-, Sozial- und Umweltmedizin" and the physicist Dr. phil. nat. Peter Brandt for the good cooperation.

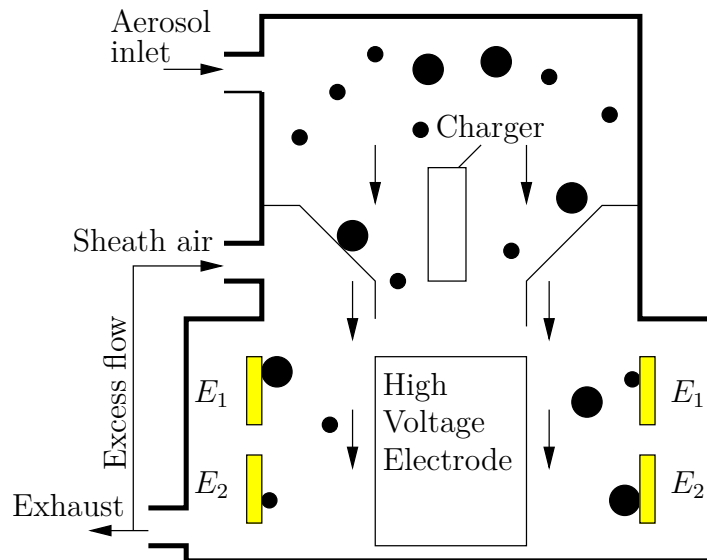


Figure 6.3: FMPS spectrometer

The aerosol inlet sucks the aerosols with 10 L/min continuously. After the inlet, a charger charges the particles positively. Due to the filtered sheath air, the particles flow between the high voltage electrode and the electrometers. The electrode generates an electric field, which pushes the particles towards the electrometers. The electrical mobility of the particles is higher or lower in regard to the contact position on the electrometer. The several electrometers enable the detection of different particle sizes simultaneously. The brake time is 30 min. The experiments are conducted at two different days. The first measurement is conducted without a run-in of the friction pairing. The second measurement is conducted after the first measurement, which can

be interpreted as a run-in. Therefore, the friction pairing is run-in for the second measurement. After each experiment, a cooling time of one hour is conducted. The experimental test bench is shown in figure 6.4. The test bench consist of the tribometer (for more informations see section 4.3), the TSI FMPS and the ventilator. The ventilator is installed to provide an air stream, where the particles follow the air stream to the inlet of the FMPS. The second measurement is conducted without the ventilator. A filter is installed at the inlet of the ventilator and the articles are aspirated by the FMPS with 10 L/min. The experiments start at 0 rpm. The rotational speed of the brake disc is 1120 rpm. The acceleration and deceleration time are approximately identical. The load between the brake disc and the brake pad is applied before the rotation of the brake disc. The applied load is 140 N and is conducted with masses.

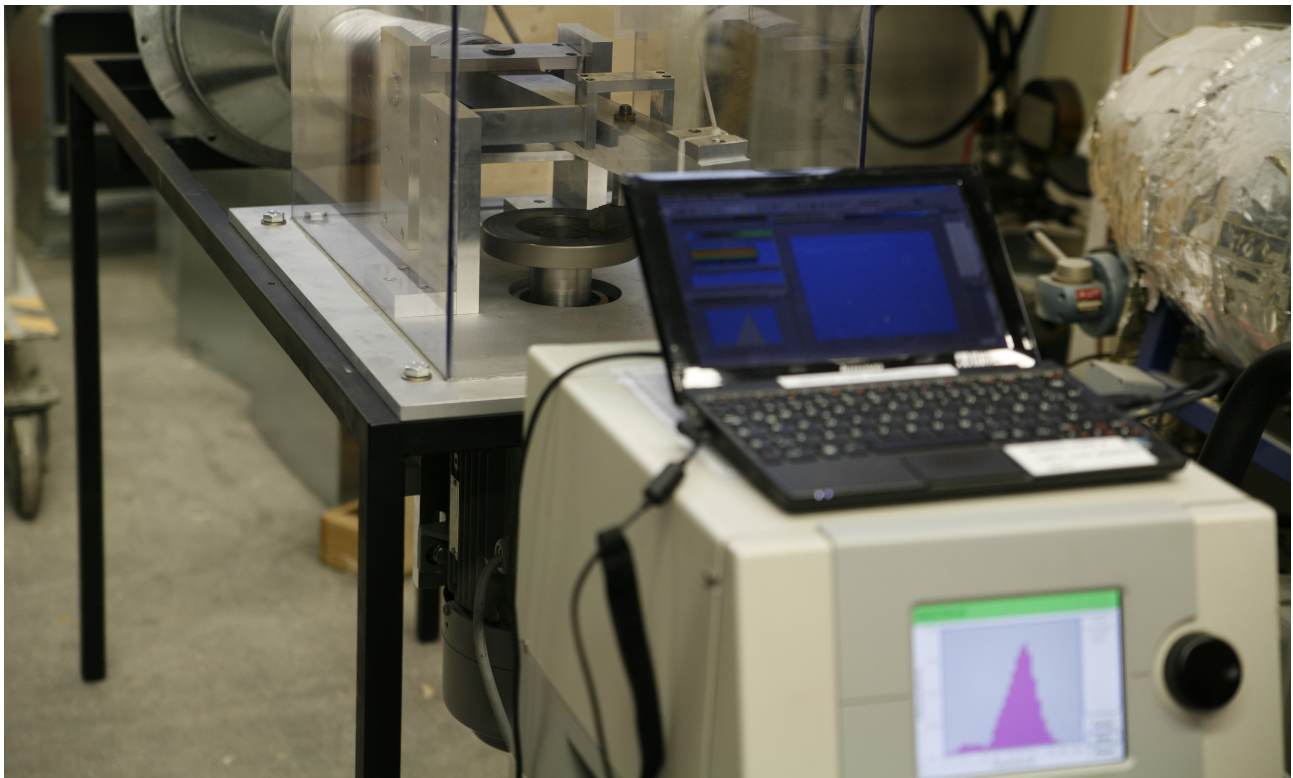


Figure 6.4: Brake dust test bench

The trends of the detected curves are different. The particle number concentration of the two measurements are shown in figure 6.5.

The first measurement with the two runs shows the particle emission at a period of latency of 420 s. A continuous increase of the particle number concentration can be observed. The maximum number of concentration is achieved at 1100 s at almost 10^6 cm^{-3} . In figure 6.6, the second measurement with four runs in an enlarged representation of figure 6.5 shows irregular trends. It can be observed an instantaneous increasing in the concentration but the absolute number of concentration is one order smaller than the first measurement.

In the figures 6.7 - 6.12, the particle size in regard to the time and the particle number concentration are shown. In the figures 6.7 and 6.8 the particle size according to the time and the particle number concentration of the first measurement and first run is illustrated. In the figure 6.7, the particle size of the highest particle concentration $1.4 \cdot 10^5 \text{ cm}^{-3}$ is between 64 and 110 nm at the end of the braking. The particle size at the concentration of $4 \cdot 10^4 \text{ cm}^{-3}$

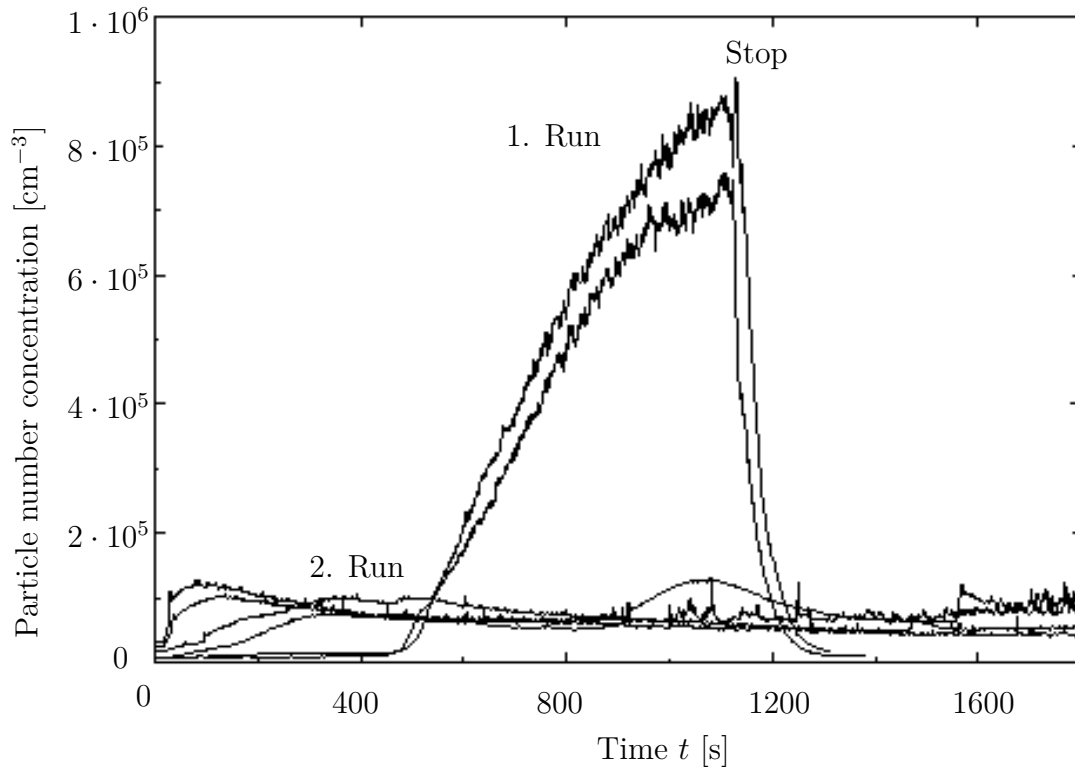


Figure 6.5: Particle number concentration of two measurements

is between 30 and 178 nm. At the end of the braking, a concentration of $4 \cdot 10^4 \text{ cm}^{-3}$ of the superfine particles (6 nm-30 nm) is detected.

The maximum particle number concentration is detected earlier. A shifting of the concentration is observable. At 1700 s a higher particle number concentration of the superfine particles are measured. The superfine particles are observable for a higher time period and in the same particle size range of the first run of the first measurement.

In the figures 6.9 - 6.12, the particle size in regard to the time and the particle number concentration of the four runs in the second measurement is shown. In the figure 6.9, there are three regions, begin, middle and end of the braking with a high particle concentration at the same particle size level. At the end of the braking, superfine particles between 5.6 nm and 10 nm are observable.

In the figure 6.10, the particle number concentration of the second run of the second measurement shift at the beginning of the braking. The particle size level is between 87 nm and 320 nm. The maximum particle number concentration of $2.51 \cdot 10^5 \text{ cm}^{-3}$ is achieved in the time interval between 300 s and 800 s.

In the figure 6.11, several spots of high particle number concentration are observable. At the beginning of the braking, a high particle number concentration between 100 nm and 178 nm for a time period of 400 s is detected. At the time of 1200 s a high concentration of particles in the range of 150 nm and 178 nm is measured. At the end of the braking, a particle number concentration of $2.27 \cdot 10^5 \text{ cm}^{-3}$ is detected between 5.6 nm and 10 nm for a time period of 100 s.

In the figure 6.12, the maximum particle number concentration is observed at the beginning of

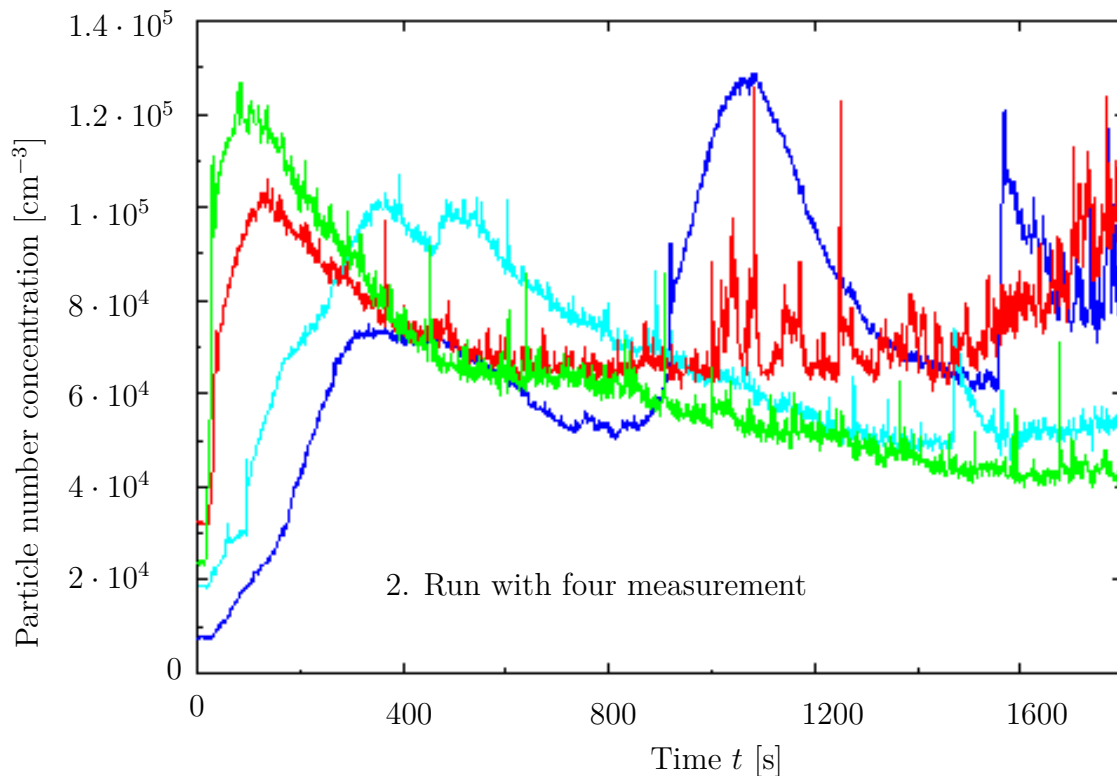


Figure 6.6: Particle number concentration of the second measurement in an enlarged representation

the braking. The maximum particle number concentration of $2.86 \cdot 10^5 \text{ cm}^{-3}$ for a time period of 400 s is measured. Superfine particles are not measured. After the 400 s, the particle number concentration remains constant over the whole braking time.

The measurements of the brake dust show an irregular behaviour. The reason of this irregular behaviour can be the obstruction of the aerosol inlet. The cleaning of the TSI FMPS reveals this assumption. The range of the particle sizes of the spectrometer is from 5.6 to 560 nm. The detected PM contains larger particles than the particle size range of the spectrometer. The different initial conditions also can be a reason for the irregular behaviour. Further investigations of the brake dust require several measurement devices. The measurement of large particles is necessary. Furthermore, a filter system for the TSI FMPS is indispensable. The aerosol inlets and the tube of the electrometers were covered with a black layer. The emerging question was the origin of this black layer. A scanning electron microscope would provide clarity if the black layer contains of grey cast iron or brake pad material. Another important point is the retention of the boundary conditions. A deep investigation of the particulate matter or the brake dust with the tribometer and the flywheel test bench would provide a better understanding of the particle generation and transport. Further perspectives would be a clinical study of the influence of the particulate matter or the brake dust on the human organism. In the point of view of a mechanical engineer the consideration of the particles would provide an insight into the contact behaviour or tribology.

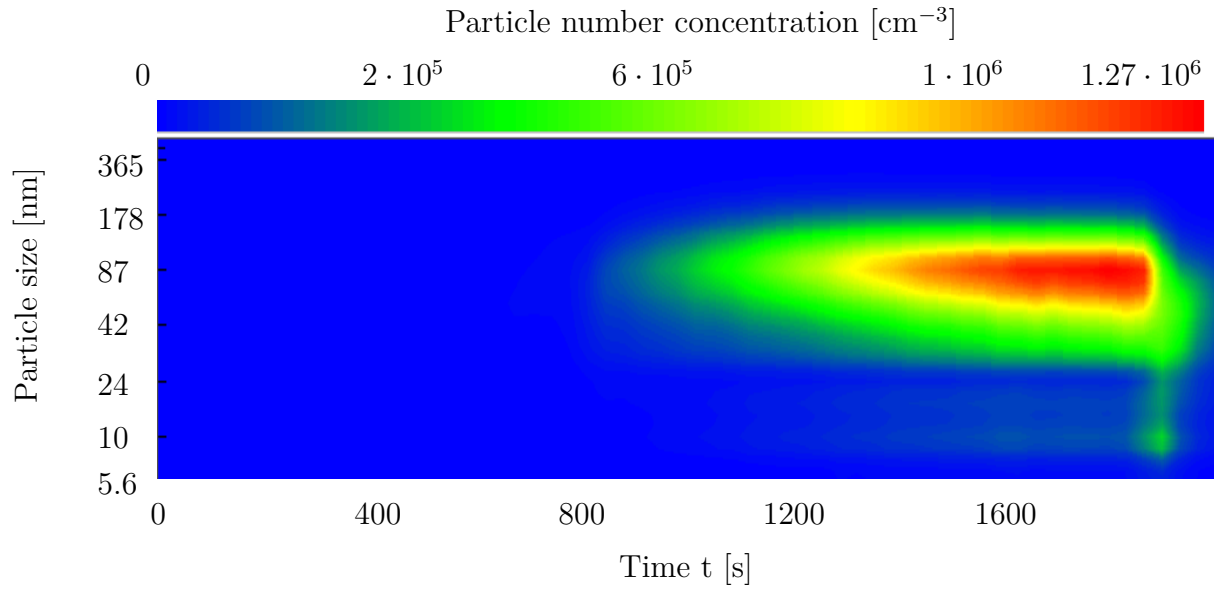


Figure 6.7: Particle size of the first run of the first measurement

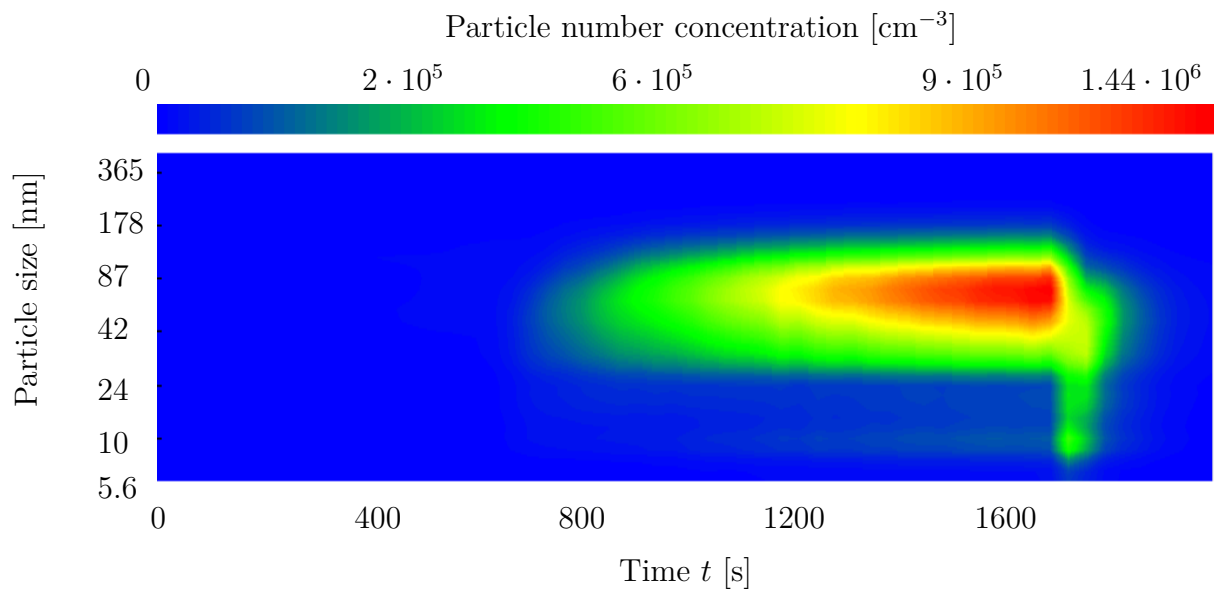


Figure 6.8: Particle size of the second run of the first measurement

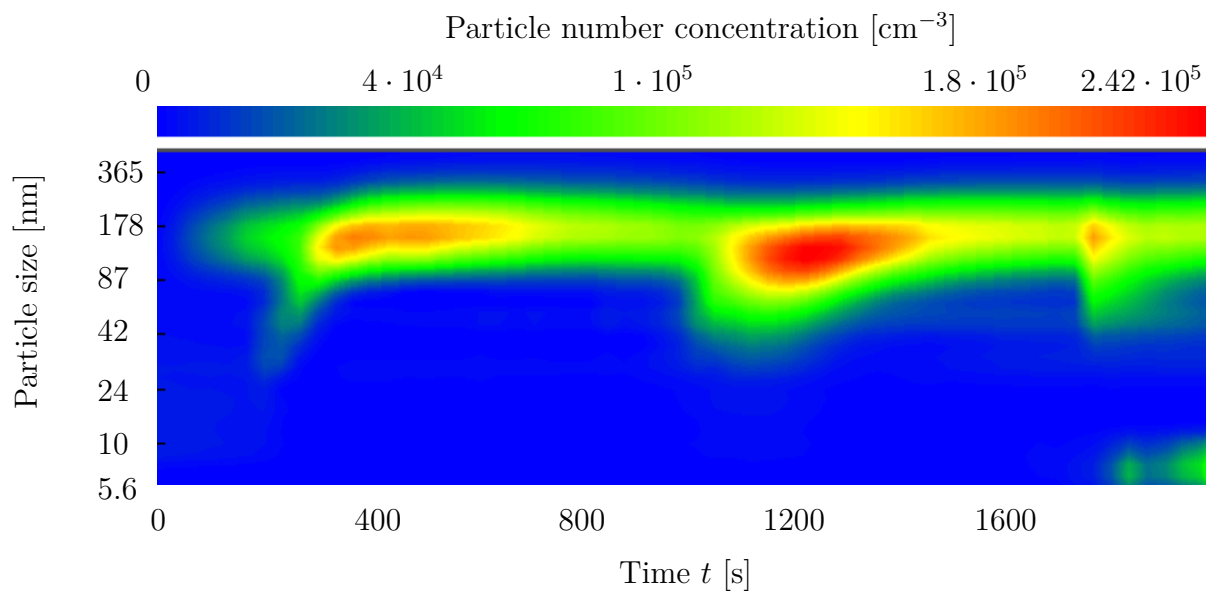


Figure 6.9: Particle size of the first run of the second measurement

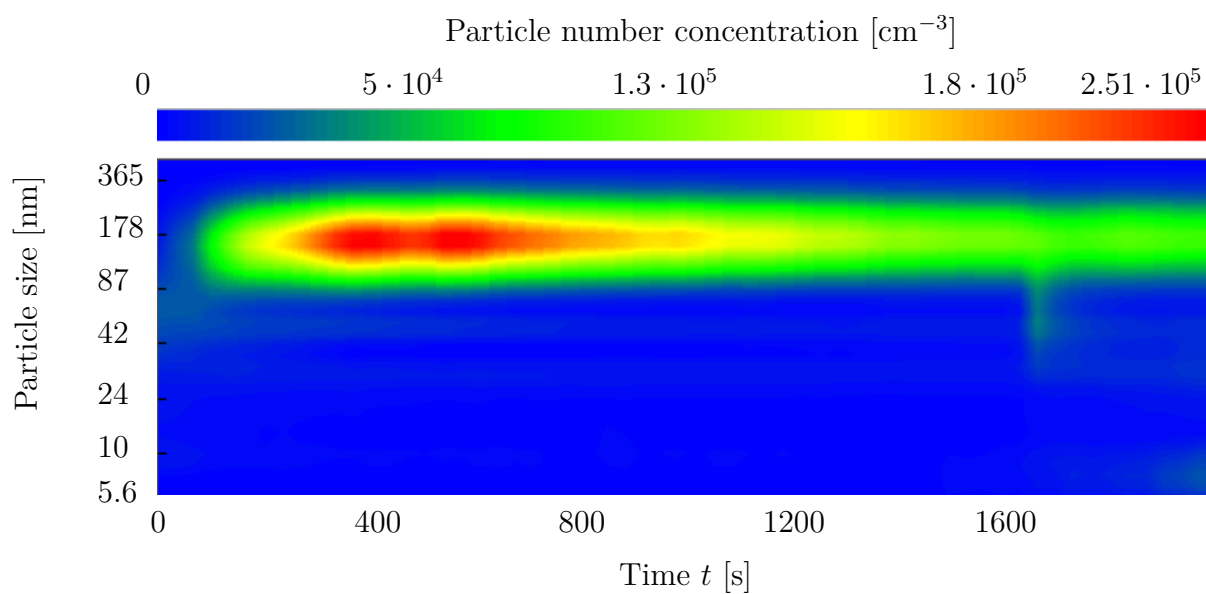


Figure 6.10: Particle size of the second run of the second measurement

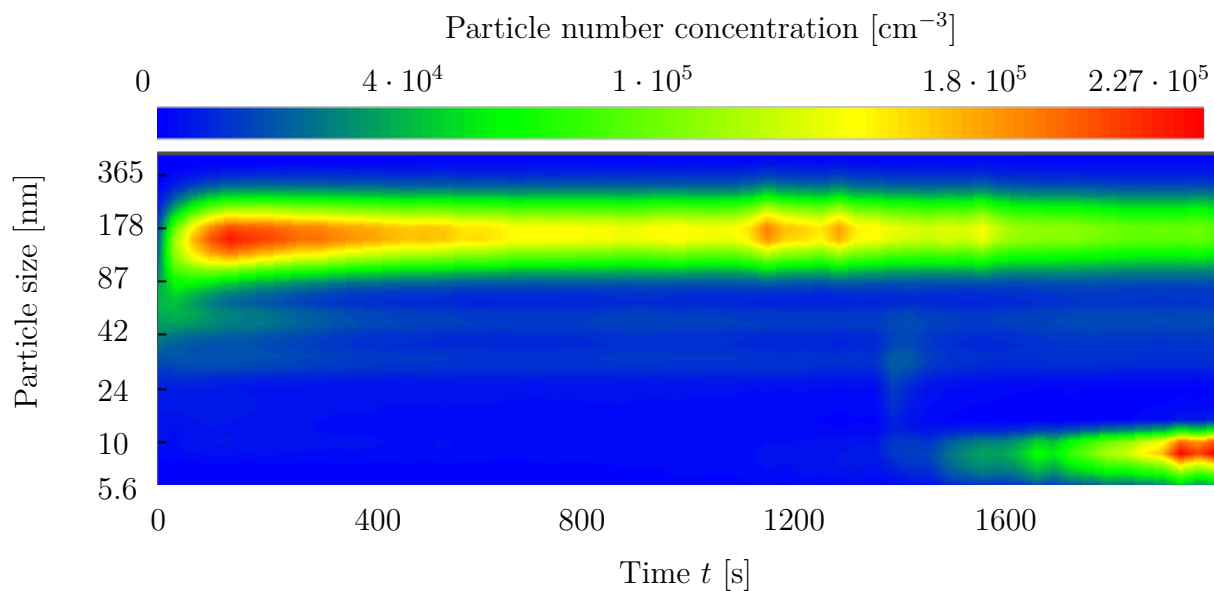


Figure 6.11: Particle size of the third run of the second measurement

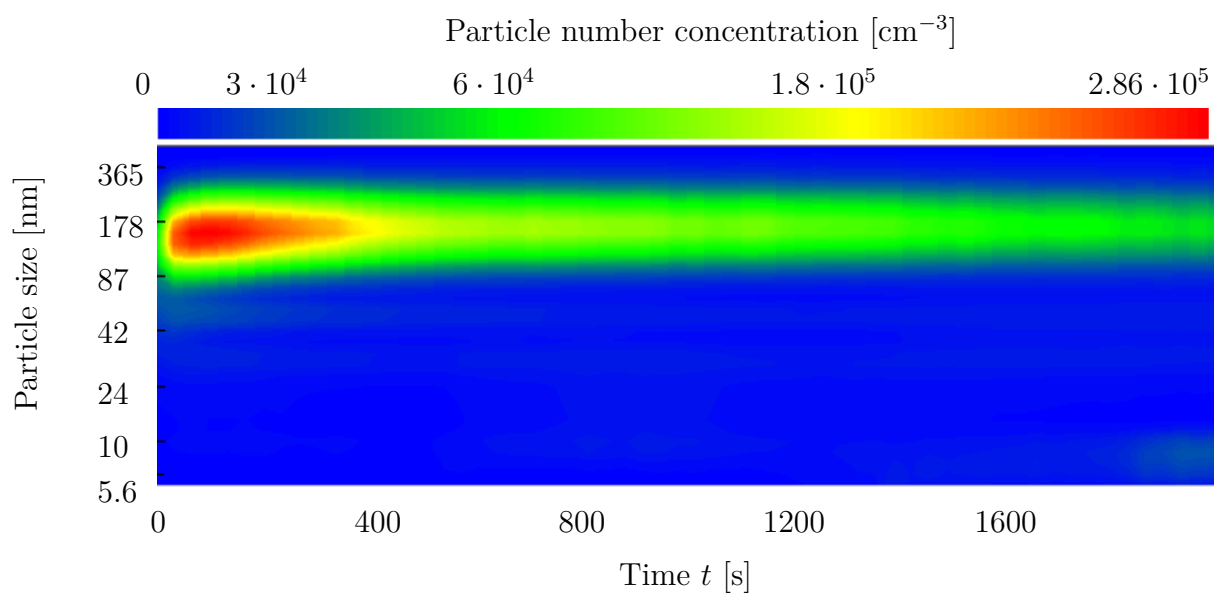


Figure 6.12: Particle size of the fourth run of the second measurement

7 Conclusions

7.1 Summary

The goal of this contribution was to present the modular designed test bench with the advanced measurement system for friction systems. The design and construction process and the installation of the measurement system was the main part of this work. Another part of this work was the design of the tribometer for the validation of numerical models with well defined variables such as the normal force, the rotational speed and the friction pairs. The design of the combined brake, which consists of a friction and an eddy-current brake closed the gap to the electro mobility. The efficient numerical treatment of the decoupled thermomechanical problem was carried out by the finite element method, where the overall solution procedure and the algorithmic implementation of the steady-state algorithm were described. The practical applicability of the steady-state algorithm was adapted to the real experiments of brake actions. Finally, the particulate matter and the wear behaviour of a friction system was measured with a Fast Mobility Particle Sizer. Due to the designed test benches and the numerical implementation, the gap of numerical models and the validation with the experiments was closed.

7.2 Conclusion and Discussion

Proceeding from a comprehensible introduction of continuum mechanics, a short explanation of the required theoretical background is shown. The theoretical fundamentals are required for the understanding and the application of the finite element method and the used steady-state algorithm. It started with the kinematic description of deformable material bodies. Furthermore, the material deformation gradient, the strain and the strain rates are introduced. Thereafter, the stress concept and the different references are explained. With the master balance principle, the thermodynamical consistency of the constitutive models can be verified.

The development of a simulation methodology, which improves the calculation time for thermomechanical problems is introduced. The steady-state algorithm is appropriate for repeating loadings. An example of a brake simulation, which consists of a brake disc and a brake pad is validated with experimental results. In this example, a good agreement of the numerical and experimental results are observed.

The new designed flywheel test bench is a full scale test bench, which is in a modularly way designed. The modularity offers the possibility to respond to questions concerning research. Moreover, the test bench is composed of an advanced measurement system. The modularity and the measurement system enable a deep understanding of frictional systems. The investigation of the whole brake system and the components enable the improvement of each components separately or the brake system as a whole. Another designed test bench is the tribometer, which is appropriate for the calibration and validation of numerical models. The combined brake is a suitable test bench, which can be used for the electro mobility.

The simulation methods, in particular the steady-state algorithm, is validated with the experiments. The algorithm and the simulation methodology reveal a good agreement regarding the simulated temperature and the wear analysis. The calculation time is decreased from a couple of days to several hours.

The particulate matter and the wear behaviour are described in regard to the particles of the braking. The particle number concentration and the particle size are analysed and described with a Fast Particle Mobility Particle Sizer.

In conclusion, the present work delivers the modular designed flywheel test bench with advanced measurement system and the numerical treatment of friction systems, where the overall combination defines a system for the reasearch and development of friction systems.

7.3 Future Work

Due to the designed flywheel test bench and the presented steady-state simulation methodology, a good combination of numerical and experimental mechanics are presented. The modularity of the flywheel test bench offers the possibility to analyse different friction systems. The investigation of the friction system clutch requires only several handles for the changeover. The test benches provide the possibility of a deep understanding of friction systems in regard to the macroscopic, microscopic and nano scale. The designed tribometer shows the first step in the validation of the numerical models. The final validation of the numerical model with the flywheel test bench offers a tool for the industry to reduce the development costs. The investigation of the composition of organic and sintered brake pads in regard to the coefficient of friction, temperature behaviour and wear can be conducted on the flywheel test bench. Furthermore, it is possible to analyse the whole brake system, such as the brake pedal, the brake master cylinder, the brake pipes, the brake force booster, the brake calliper, the brake pad holder, the brake pad and the brake disc. This is possible due to the advanced measurement system, which enables the measurement of different variables such as the temperature, the forces, the deformations, the movement, the vibrations, and the acoustics. Moreover, the investigation of electromechanical braking also referred to brake-by-wire can be conducted. The designed flywheel test bench offers a possibility of analysing the whole brake system.

References

- [1] A. Belhocine and M. Bouchetara. Thermomechanical behaviour of dry contacts in disc brake rotor with a grey cast iron composition. *Transactions of the Indian Institute of Metals*, 65:231–238, 2012.
- [2] L. Afferrante, M. Ciavarella, and J. R. Barber. Sliding thermoelastodynamic instability. *Proceedings of the Royal Society of London A*, 462:2161–2176, 2006.
- [3] I. Ahmed and Y. Fatouh. Controlling the vehicle disc brake squeal using a 10 dof model. *Global Advanced Research Journal of Engineering, Technology and Innovation*, 1:236–251, 2012.
- [4] Y. J. Ahn, E. Bertocchi, and J. R. Barber. Shakedown of coupled two-dimensional discrete frictional systems. *Journal of the Mechanics and Physics of Solids*, 56:3433–3440, 2008.
- [5] A. E. Anderson and R. A. Knapp. Hot spotting in automotive friction systems. *Wear*, 135:319–337, 1990.
- [6] S. Anwar. An antilock-braking algorithm for an eddy-current-based brake-by-wire system. *IEEE Transactions on Vehicular Technology*, 56:1100 – 1107, 2007.
- [7] J. F. Archard. Contact and rubbing of flat surface. *Journal of Applied Physics*, 24:981–988, 1953.
- [8] J. F. Archard. The temperature of rubbing surfaces. *Wear*, 2:438–455, 1959.
- [9] J. F. Archard. Wear theory and mechanisms. *Wear control handbook*, 1:35–80, 1980.
- [10] J. F. Archard and W. Hirst. The wear of metals under unlubricated conditions. *Proceedings of the Royal Society, A*, 236:397–410, 1956.
- [11] M. F. Ashby, J. Abulawi, and H. S. Kong. Temperature maps for frictional heating in dry sliding. *Tribology Transactions*, 34:577–587, 1991.
- [12] F. Bagnoli, F. Dolce, and M. Bernabei. Thermal fatigue cracks of fire fighting vehicles gray iron brake discs. *Eng*, 16:152–163, 2009.
- [13] J. R. Barber. The influence of thermal expansion on the friction and wear process. *Wear*, 10:155–159, 1967.
- [14] J. R. Barber. The transient thermoelastic contact of a sphere sliding on a plane. *Wear*, 59:21–29, 1980.
- [15] J. R. Barber. Thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. *Journal of Applied Mechanics*, 51:636–640, 1984.
- [16] J. R. Barber, J. Dundurs, and M. Comninou. Stability considerations in thermoelastic contact. *Journal of Applied Mechanics*, 47:871–874, 1980.

- [17] J.R. Barber. Thermoelastic instabilities in the sliding of conforming solids. *Proceedings of the Royal Society of London A*, 312:381–394, 1969.
- [18] J.R. Barber. Thermoelasticity and contact. In *Thermal Stresses*, 1999.
- [19] J. P. Bardon. Physical basis of the thermal interface boundary-conditions for an imperfect contact between 2 sliding media. *Revue générale de thermique*, 33 (386):85–91, 1994.
- [20] A. Belhocine and M. Bouchetara. Thermomechanical coupling analysis of a disc brake rotor. *Journal of Failure Analysis and Prevention*, 12:697–706, 2012.
- [21] H. Benabdallah and D. Olender. Finite element simulation of the wear of polyoxymethylene in pin-on-disc conguration. *Wear*, 261:1213–1224, 261.
- [22] Y. Berthier. Maurice godet’s third body. *Elsevier Science*, 1:21–30, 1996.
- [23] H. Blok. Theoretical study of temperature rise at surfaces of actual contact under oiliness lubricating conditions. *Proceedings of the General Discussion on Lubrication & Lubricants*, 2:222–235, 1937.
- [24] H. Blok. The dissipation of frictional heat. *Applied Science*, A:151–181, 1954.
- [25] B. Breuer and K. H. Bill. *Bremsenhandbuch*. Springer Vieweg, 2012.
- [26] O. Brucelle and G. Bernhart. Methodology for service life increase of hot forging tools. *Journal of Materials Processing Technology*, 87:237–246, 1999.
- [27] A.-L. Bulthé. *Caractérisation expérimentale du contact frottant disque garniture sous sollicitations sévères de freinage. Prise en compte des interactions tribologie, thermique et physico-chimie*. PhD thesis, Ecole Centrale de Lille, 2006.
- [28] H. Cho and C. Cho. Prediction of hot spots by correlating finite element analysis and measurement for an automotive disk brake. *Tribology Transactions*, 51:609–620, 2008.
- [29] J.-H. Choi and I. Lee. Finite element analysis of transient thermoelastic behaviors in disk brakes. *Wear*, 257:47–58, 2004.
- [30] M. Collignon, A.-L.Cristol, P.Dufrénoy, Y.Desplanques, and D. Balloy. Failure of truck brake discs: A coupled numerical-experimental approach to identifying critical thermo-mechanical loadings. *Tribology International*, 59:114–120, 2013.
- [31] H. Czichos and K.-H. Habig. *Tribologie-Handbuch*. Vieweg+Teubner Verlag | GWV Fachverlage GmbH, Wiesbaden, 2010.
- [32] A. J. Day and T. P. Newcomb. The dissipation of frictional energy from the interface of an annular disc brake. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 198:201–209, 1984.
- [33] A. J. Day, M. Tirovic, and T. P. Newcomb. Thermal effects and pressure distributions in brakes. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 205:199–205, 1991.
- [34] R. de Boer. *Vektor- und Tensorrechnung für Ingenieure*. Springer-Verlag, 1982.

- [35] G. Degallaix, P. Dufrénoy, J. Wong, P. Wicker, and F. Bumbieler. Failure mechanisms of tgv brake discs. *Key Engineering Materials Vols.*, 345-346:697–700, 2007.
- [36] Y. Desplanques, O. Roussette, G. Degallaix, R. Copin, and Y. Berthier. Analysis of tribological behaviour of pad-disc contact in railway braking part 1. laboratory test development, compromises between actual and simulated tribological triplets. *Wear*, 262:582–591, 2006.
- [37] dpa. Defekte Bremsen womöglich Ursache für tödlichen Unfall. *Spiegel Online*, 2015.
- [38] dpa-AFX. Volvo wegen fahrlässiger Tötung verurteilt. *Spiegel Online*, 2008.
- [39] P. Dufrénoy. *Etude du comportement thermomécanique des disques de frein vis-à-vis des risques de défaillance*. PhD thesis, Université des Sciences et Technologies de Lille, 1995.
- [40] P. Dufrénoy. Two-/three-dimensional hybrid model of the thermomechanical behaviour of disc brakes. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 218:17–30, 2004.
- [41] P. Dufrénoy and D. Weichert. Prediction of railway disc brake temperatures taking the bearing surface variations into account. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 209:67–76, 1995.
- [42] P. Dufrénoy and D. Weichert. Experimental analysis of hot spotting in sliding systems. *Encyclopedia of Thermal Stresses*, 1:1–12, 2013.
- [43] R. Ebara and K. Kubota. Failure analysis of hot forging dies for automotive components. *Engineering Failure Analysis*, 15:881–893, 2008.
- [44] W. Ehlers. *Ergänzung zu den Vorlesungen Technische Mechanik und Höhere Mechanik*. Universität Stuttgart, 2009/10.
- [45] W. Ehlers. *Foundations of multiphasic and porous materials. In Porous Media: Theory, Experiments and Numerical Applications*. Springer-Verlag, Berlin 2002.
- [46] M. Eriksson, F. Bergman, and S. Jacobson. On the nature of tribological contact in automotive brakes. *Wear*, 252:26–36, 2002.
- [47] M. Eriksson and S. Jacobson. Tribological surfaces of organic brake pads. *Tribology International*, 33:817–827, 2000.
- [48] X. Fan and H. Lippmann. Elastic-plastic buckling of plates under residual stress. *Advances in engineering plasticity and its applications*, 1:95–100, 1996.
- [49] A. Floquet and M. C. Dubourg. Realistic braking operation simulation of ventilated disk brakes. *ASME Journal of Tribology*, 118:466–472, 1996.
- [50] C. H. Gao, J. M. Huang, X. Z. Lin, and X. S. Tang. Stress analysis of thermal fatigue fracture of brake disks based on thermomechanical coupling. *Journal of Tribology*, 129:536–543, 2007.
- [51] S. E. Gay. Analysis and experimental testing of a permanent magnet eddy-current brake. *IEEE Vehicule Power and Propulsion Conference*, 1:756–765, 2005.

- [52] S. E. Gay. *Contactless Magnetic brake for automotive applications*. PhD thesis, Texas A& M University, 2005.
- [53] M. Godet. The third-body approach: A mechanical view of wear. *Wear*, 100:437–452, 1984.
- [54] M. Godet. Third-bodies in tribology. *Wear*, 136:29–45, 1990.
- [55] Theodoros Grigoratos and Giorgio Martini. Brake wear particle emissions: a review. *Environmental Science and Pollution Research International*, 22:2491–2504, 2015.
- [56] R. Hamam. *Simulation de la propagation des fissures par fatigue dans les toiles des roues de train sous chargement à amplitude variable*. PhD thesis, Ecole Normale Supérieure de Cachan, 2006.
- [57] P. Haupt. *Continuum Mechanics and Theory of Materials*. Springer-Verlag, Berlin 2000.
- [58] V. Hegadekatte, S. Kurzenhauser, N. Huber, and O. Kraft. A predictive modeling scheme for wear in tribometers. *Wear*, 41:1020–1031, 2008.
- [59] S. Hernik. Modeling fgm brake disk against global thermoelastic instability (hot-spot). *ZAMM*, 89:88–106, 2009.
- [60] W. Hinds. *Aerosol Technology: Properties, Behavior, and Measurement of Airborne Particles*. John Wiley & Sons, 1999.
- [61] G. A. Holzapfel. *Nonlinear Solid Mechanics: A Continuum Approach for Engineering*. John Wiley and Sons, Chichester 2001.
- [62] T. J. R. Hughes. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Dover Publications, 2000.
- [63] H. Jacobsson. *Brake Judder*. PhD thesis, Machine and Vehicle Systems Chalmers University of Technology Göteborg Schweden, 2001.
- [64] H. Jacobsson. Aspect of disc brake judder. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 217:419–430, 2003.
- [65] H. Jacobsson. Disc brake judder considering instantaneous disc thickness and spatial friction variation. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 217:325–342, 2003.
- [66] Y. H. Jang, H. Cho, and J.R. Barber. The thermoelastic hertzian contact problem. *International Journal of Solids and Structures*, 46:4073–4078, 2009.
- [67] J. A. L. Esswein Junior, F. E. Arrieche, and L. Schaeffer. Analysis of wear in organic and sintered friction materials used in small wind energy converters. *Materials Research*, 3:269–273, 2008.
- [68] T.K. Kao and J.W. Richmond. Brake disc hot spotting and thermal judder: an experimental and finite element study. *International Journal of Vehicle Design*, 23:276–296, 2000.

- [69] H. Kasem, J.F. Brunel, P. Dufrénoy, M. Siroux, and B. Desmet. Thermal levels and subsurface damage induced by the occurrence of hot spots during high-energy braking. *Wear*, 270:355–364, 2011.
- [70] H. Kasem, P. Dufrénoy, and Y. Desplanques. Relationships between surface thermal gradients and disc distortion during stop-braking with high energy dissipation. *Tribology Letters*, 48:169–181, 2012.
- [71] H. Kasem, J.-F. Witz, P. Dufrénoy, and Y. Desplanques. Monitoring of transient phenomena in sliding contact application to friction brakes. *Tribology Letters*, 1:1–8, 2013.
- [72] J. Kim, B.-C. Goo, S.-C. Yoon, and S.-T. Kwon. Thermographic investigation of hot spots in railway brake discs. *Key Engineering Materials*, 385-387:669–672, 2008.
- [73] C. Kleinlein. *Beschreibung von Reibpaarungen in trocken laufenden Kupplungen und Bremsen durch ihre global und lokalen Reibungs- und Verschleißigenschaften*. PhD thesis, Technische Universität Berlin, 2005.
- [74] D. Kuhlmann-Wilsdorf. Flash temperatures due to friction and joule heat at asperity contacts. *Wear*, 105:187 – 198, 1985.
- [75] D. Kuhlmann-Wilsdorf. Energy minimization of dislocations in low-energy dislocation structures. *physica status solidi (a)*, 104:121–144, 1987.
- [76] P. Kulkarni, P. A. Baron, and K. Willeke. *Aerosol Measurement: Principles, Techniques, and Applications*. John Wiley & Sons, 2011.
- [77] J. R. Laguna-Camacho. A study of the wear mechanisms of disk and shoe brake pads. *Engineering Failure Analysis*, 56:348–359, 2015.
- [78] A. Lamjahdy, J. Ali, and B. Markert. Simulation of the temperature and wear behaviour of a disc brake. *PAMM*, 16:217–218, 2016.
- [79] A. Lamjahdy, J. F Brunel, B. Lamure, P. Dufrenoy, D. Weichert, and B. Markert. The cyclic thermomechanical coupled problem of thermal gradients in friction railway disc brakes. *PAMM*, 14:461–462, 2014.
- [80] A. Lamjahdy and B. Markert. Possibilities of a combined eddy current and friction brake system. submitted, 2016.
- [81] Landesamt für Natur, Umwelt und Verbraucherschutz Nordrhein-Westfalen. *Gesundheitliche Wirkungen von Feinstaub und Stickstoffdioxid im Zusammenhang mit der Luftreinhalteplanung*, 2010.
- [82] K. Lee and J.R. Barber. An experimental investigation of frictionally-excited thermoelastic instability in automotive disk brakes under a drag brake application. *Journal of Tribology*, 116:409–414, 1994.
- [83] M. Schumann and H. Winner. Analysemethode zur Beurteilung des ungleichförmigen Bremsscheibenverschleißes an Pkw-Scheibenbremsen. *Fortschritt Berichte-VDI*, 635, 2006.

- [84] T. J. Mackin. Thermal cracking in disc brakes. *Engineering Failure Analysis*, 9:63–76, 2002.
- [85] V. Mailliot, A. Fissolo, G. Degallaix, and S. Degallaix. Thermal fatigue crack networks parameters and stability: an experimental study. *International Journal of Solids and Structures*, 42:759–769, 2005.
- [86] M. H. Maitournam. *Formulation et resolution numerique des problemes thermoviscoplastiques en regime permanent*. PhD thesis, These de l'Ecole Nationale des Ponts et Chaussees, 1989.
- [87] D. Majcherczak and P. Dufrénoy. Dynamic analysis of a disc brake under frictional and thermomechanical internal loading. *Applied Mechanics*, 75:497–512, 2006.
- [88] B. Markert. Porous media mechanics. Lecture 2016.
- [89] B. Markert. Selected topics of inelasticity theory. Lecture 2015/2016.
- [90] B. Markert. *Porous media viscoelasticity with application to polymeric foams*. PhD thesis, Institute of Applied Mechanics (Continuum Mechanics), University of Stuttgart, 2005.
- [91] B. Markert. A survey of selected coupled multifield problems in computational mechanics. *Journal of Coupled Systems and Multiscale Dynamics*, 1:22–48, 2013.
- [92] B. Markert. *Mechanik II (Strength of Materials)*. Institute of General Mechanics RWTH Aachen University, 2016.
- [93] B. Markert. *Weak or Strong On Coupled Problems in Continuum Mechanics*. Habilitation, Institute of Applied Mechanics (Continuum Mechanics), University of Stuttgart, 2010.
- [94] V. K. Maurya. Eddy current braking embedded system. *International Journal of Applied Engineering and Technology*, 1:104–113, 2011.
- [95] J.F. Molinari, R. Radovitzky M. Ortiz, and E.A. Repetto. Finite-element modeling of dry sliding wear in metals. *Engineering Computations*, 18:592–609, 2001.
- [96] M. Mueller and G.P. Ostermeyer. A cellular automaton model to describe the three-dimensional friction and wear mechanism of brake systems. *Wear*, 263:1175–1188, 2007.
- [97] H. Nakata, K. Kobayashi, M. Kajita, and C.H. J. Chung. A new analysis approach for motorcycle brake squeal noise and its adaptation. *SETC*, 1:1–11, 2001.
- [98] M.P. Natarajan and B. Rajmohan. Heat dissipation and temperature distribution of brake liner using steady state analysis. *Applied Mechanics and Materials*, 249-250:712–717, 2013.
- [99] T. P. Newcomb. Transient temperatures attained in disk brakes. *Journal of Applied Physics*, 10:339–340, 1959.
- [100] Q. S. Nguyen. *Stability and Nonlinear Solid Mechanics*. John, 2000.
- [101] Q. S. Nguyen and M. Rahimian. Mouvement permanent d’une fissure en milieu elasto-plastique. *Journal de Mecanique appliquee*, 5(1):95–120, 1981.

- [102] T. M.-L. Nguyen-Tajan. *Modélisation thermomécanique des disques de frein par une approche eulérienne*. PhD thesis, Ecole Polytechnique, 2002.
- [103] Pablo Noemi and Jaime Mendoza. El trenazo, en saltillo 1972...!!! <http://saltillodelrecuerdo.blogspot.de/2009/04/el-trenazo.html>, 2009.
- [104] W. Österle and I. Urban. Friction layers and friction films on pmc brake pads. *Wear*, 257:215–226, 2004.
- [105] G.-P. Ostermeyer and G. Matthias. Hot banding with oscillating radii: models including wear and temperature. In *Eurobrake 2012*, 2012.
- [106] G.P. Ostermeyer. On the dynamics of the friction coefficient. *Wear*, 254:852–858, 2003.
- [107] G.P. Ostermeyer and M. Mueller. Dynamic interaction of friction and surface topography in brake systems. *Tribology International*, 39:370–380, 2006.
- [108] S. Panier. *Etude théorique et expérimentale des points chauds dans les organes de friction*. PhD thesis, Université des Sciences et Technologies de Lille, 2002.
- [109] S. Panier, P. Dufrénoy, and P. Brémond. Infrared characterization on thermal gradients on disc brakes. In *13th International Conference on Thermal Engineering and Thermogrammetry*, 2003.
- [110] S. Panier, P. Dufrénoy, J.F. Brunel, and D. Weichert. Progressive waviness distortion: a new approach of hot spotting in disc brakes. *Journal of Thermal Stresses*, 28:47–62, 2005.
- [111] S. Panier, P. Dufrénoy, and D. Weichert. An experimental investigation of hot spots in railway disc brakes. *Wear*, 256:764–773, 2004.
- [112] A. Perssona, S. Hogmarkb, and J. Bergström. Thermal fatigue cracking of surface engineered hot work tool steels. *Surface & Coatings Technology*, 191:216–227, 2005.
- [113] P. Podra and S. Andersson. Simulating sliding wear with finite element method. *Tribology International*, 32:71–81, 1999.
- [114] T. Poeste. *Untersuchungen zu reibungsinduzierten Veränderungen der Mikrostruktur und Eigenspannungen im System Bremse*. PhD thesis, Technische Universität Berlin, 2005.
- [115] W. A. Rakowski. The surface layer of friction plastics. *Wear*, pages 21–27, 1980.
- [116] O. Roussette, Y. Desplanques, and G. Degallaix. Représentativité thermique d’essais tribologiques à échelle réduite. *C. R. Mécanique*, 331:343–349, 2003.
- [117] H. Samrout. Fatigue behaviour of 28crmov5-08 steel under thermomechanical loading. *International Journal of Fatigue*, 20(8):555–563, sep 1998.
- [118] A. Sardá. *Wirkungskette der Entstehung von Hotspots und Heißrubbeln in Pkw-Scheibenbremsen*. PhD thesis, Technischen Universität Darmstadt, 2009.
- [119] A. Sardá, M. Haag, H. Winner, and M. Semsch. Experimentai investigation of hot spots and thermal judder. In *SAE International*, 2008.

- [120] L. J. Segerlind. *Applied Finite Element Analysis*. John Wiley and Sons, 1984.
- [121] Peter Semmens. *Katastrophen auf Schienen. Eine weltweite Dokumentation*. Transpress Verlag, 1996.
- [122] J. C. Simo and T. J. R. Hughes. *Computational Inelasticity*. Springer, 2000.
- [123] T. Steffen. *Untersuchung der Hotspotbildung bei PKW-Bremsscheiben*. PhD thesis, Universität der Bundeswehr Hamburg, 1998.
- [124] M. Tirovic and A. J. Day. Disc brake interface pressure distributions. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 205:137–146, 1991.
- [125] C. A. Truesdell and W. Noll. *The non-linear field theories of mechanics*. In *Handbuch der Physik, Vol. III*. Springer Verlag, Berlin 1965.
- [126] P. Tsang, M. Jacko, and S. Rhee. Comparison of chase and inertial brake dynamometer testing of automotive friction materials. *Wear*, pages 107–114, 1985.
- [127] K. van Dang, G. Inglebert, and J. M. Proix. Sur un nouvel algorithme de calcul de structure élastoplastique en régime stationnaire. *3ème colloque "Tendances actuelles en calcul de structures"*, pages 387–400, 1985.
- [128] V. Velay. *Modélisation du comportement cyclique et de la durée de vie d'aciers à outils martensitiques*. PhD thesis, Ecole des Mines de Paris, 2003.
- [129] P. Vernotte. *Calcul numérique, calcul physique, application à la thermocinétique*. En vente au Service de documentation et d'information technique de l'aéronautique, 1956.
- [130] M. Vollmer and K.-P. Mollmann. *Infrared Thermal Imaging: Fundamentals, Research and Applications*. Wiley, 2010.
- [131] P. Wicker. *Influence des garnitures de frein sur les sollicitations thermiques des disques tgv et conséquences sur les risques de fissuration*. PhD thesis, Ecole Centrale de Lille, 2009.
- [132] J. Wong. *Analyse de l'endommagement par fatigue thermique et modélisation du comportement thermomécanique de couples disques-garnitures de type TGV*. PhD thesis, Ecole Centrale de Lille, 2007.
- [133] P. Wriggers. *Nonlinear Finite Element Methods*. Springer, 2008.
- [134] Z. Yang, J. Han, W. Li, Z. Li, L. Pan, and X. Shi. Analyzing the mechanisms of fatigue crack initiation and propagation in crh emu brake discs. *Engineering Failure Analysis*, 2099:1–14, 2013.
- [135] A. Yevtushenko and E. Ivanyk. Determination of temperatures for sliding contact with applications for braking systems. *Wear*, 206:53–59, 1997.
- [136] A. Yevtushenko and M. Kuciej. Temperature and thermal stresses in a pad/disc during braking. *Applied Thermal Engineering*, 30:354–359, 2010.

-
- [137] Y.-B. Yi, J. R. Barber, and P. Zagrodzki. Eigenvalue solution of thermoelastic instability problems using fourier reduction. *Proceedings Royal Society London A*, 456:2799–2821, 2000.
 - [138] Y.-B. Yi, S. Du, J.R. Barber, and J.W. Fash. Effect of geometry on thermoelastic instability in disk brakes and clutches. *Journal of Tribology*, 121:661–666, 1999.
 - [139] P. Zagrodzki, K. B. Lam, E. Al Bahkali, and J. R. Barber. Nonlinear transient behavior of a sliding system with frictionally excited thermoelastic instability. *Journal of Tribology*, 1123:699–708, 2001.
 - [140] P. Zagrodzki and S. A. Truncone. Generation of hot spots in a wet multidisk clutch during short-term engagement. *Wear*, 254:474–491, 2003.
 - [141] O. C. Zienkiewicz and R. L. Taylor. *The Finite Element Method*. Butterworth-Heinemann, 2005.

A Ground Plot

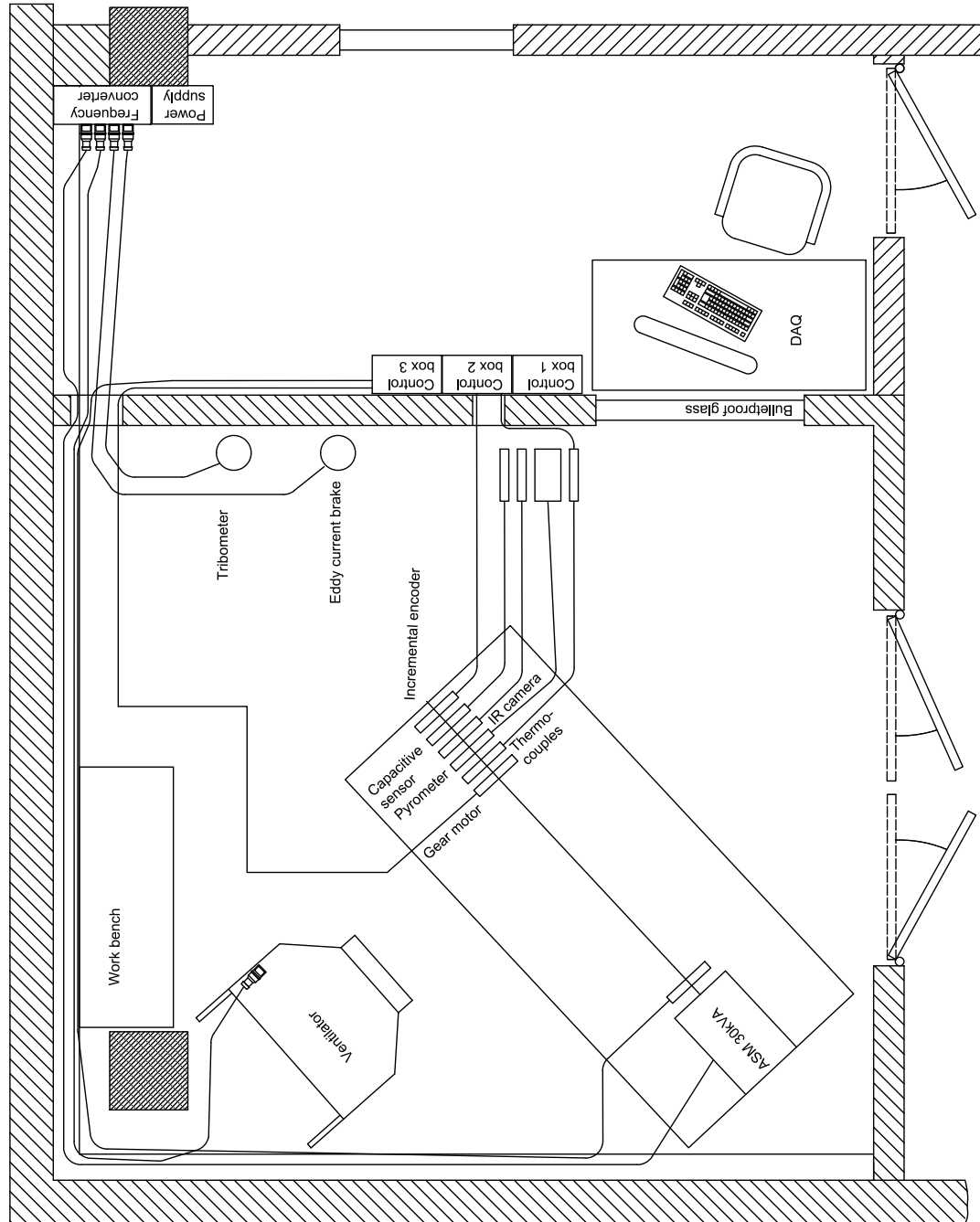


Figure A.1: Ground plot of the flywheel test bench and the wiring

B System Description

Description	Device/Software	Specification
Brake test bench	Flywheel test bench	$P_{\max} = 30 \text{ kW}$, $n_{\max} = 1460 \text{ rpm}$
Thermogram	Infrared camera	320×256 Pixels, MWIR $3 - 5 \mu\text{m}$, NETD 20 mK , 14 bit, Full frame up to 450 Hz , Line frame up to 13000 Hz
	Software	IR frame up to 8000 Hz (320×2 Pixel), Triggered recording, Line mode, Detection of both sides with mirror function
Deformation measurement	Capacitive sensor	Measuring range $1\text{-}4 \text{ mm}$, Linearity $1 \mu\text{m}$
	Controller	Cut-off frequency 50 kHz , Resolution $0,1\% \text{ FSO}$
Movement measurement	3D Motion Tracking	Accuracy in $X = 0.2 \text{ mm}$, $Y = 0.5 \text{ mm}$, $Z = 0.2 \text{ mm}$, Frame rate 162 Hz
Pyrometer	Spot temperature measurement	Wavelength $2 - 2.6 \mu\text{m}$, min. detection time $0,5 \text{ ms}$
Thermocouple	Temperature measurement in the measuring object	NiCr-Ni, Temperature range $-270\text{-}1300 \text{ }^{\circ}\text{C}$
DAQ	I/O of signals	16 AI, 24 DIO, 2AO, Frame rate 250 kS/s
	Software	LabVIEW, DIAdem
Incremental encoder	Speed measurement	2048 pulses per revolution, Optical sensing, Zero signal
Force sensor	Force measurement before and after the brake master cylinder	Measurement frequency 10 kHz , Max. measurable force 5 kN
Torque transducer	Torque measuring shaft	Rated torque 1000 Nm , Rated speed 9000 rpm

Table B.1: System description of the flywheel test bench and the measurement system

Curriculum Vitae

Personal Data:

Name: Abdelkrim Lamjahdy
Date of birth: February 05, 1985
Place of birth: Hamm, Germany
Nationality: German
Marital status: married

Work Experience:

09/2017 - now Product Design and Test Engineer (Veoneer-Nissin Brake Systems)

University Education:

03/2012 - 8/2017 Scientific assistant and Ph.D. student at the Institute of General Mechanics (IAM) at the RWTH Aachen University, Germany
10/2005 - 03/2012 Dipl.-Ing. in Mechanical Engineering at RWTH Aachen University, Germany

Internship and military service:

09/2010 - 10/2010 internship, Access e.V, Aachen, Germany
04/2010 - 05/2010 internship, Pallas GmbH & Co. KG, Aachen, Germany
08/2006 - 09/2006 internship, Salzgitter Mannesmann Line Pipe GmbH, Hamm, Germany
06/2005 - 08/2005 internship, Beumer GmbH & Co. KG, Beckum, Germany
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Education:

1995 - 2004 Friedensschule, Secondary and Higher Secondary School, Hamm, Germany
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