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How the distance and radius of two circular loudspeaker arrays affect sound field reproductions and directivity controls

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Abstract
We previously proposed methods for sound field reproduction and directivity control using two parallel circular loudspeaker arrays (2CLA). The results showed that 2CLAs have a higher potential than a single circular loudspeaker array when reproducing a virtual sound source outside the array or producing a high directivity beam. In addition, we found that the performance of the 2CLA varies with the distance between the two arrays and their radii. In this paper, we discuss how the two factors affect the performance of a 2CLA. We also investigate the influences of either factor with the other fixed through computer simulations. The results of the sound field reproduction show that the reproducing accuracy decreases when the distance between the two arrays increases, whereas the radii of the 2CLA do not affect the results if no constraint has been set to the filter gain. By contrast, if the filter gain is constrained to 0 dB, the reproducing accuracy improves when the radii increase. For directivity controls with filter gain constraints, the performance improves when larger radii have been selected, whereas a large distance may lead to spatial aliasing.
Keywords: Sound Field Reproduction, Directivity Control, Loudspeaker Array, 2CLA

1 INTRODUCTION

Array signal processing in audio using multiple microphones or loudspeakers is an important technique for sound field reproduction and directivity control.

Sound field reproduction is a technique applied to reproduce an entire sound field using loudspeaker arrays. Based on the Huygens-Fresnel principle, a method for reproducing the sound field with multiple loudspeakers was proposed(1). Later, methods such as a wave field synthesis(2; 3) and Ambisonics(4; 5) were proposed and improved. This sound field reproduction technique can deal with problems in immersive audio, personal audio, and noise canceling systems.

Directivity control, however, is a technique used to emphasize the sound to radiate in a particular directivity pattern. Major studies have focused on forming a beam to the target direction, which is also known as beamforming. Methods such as delay-and-sum and the least squares method have been proposed(6). In addition, another technique called an adaptive beamformer can solve specific problems with an optimized performance(7). In applications, a directivity control technique can also be applied for immersive audio, personal audio, and noise canceling systems as well.

Conventional studies in sound field reproduction require large systems and are difficult to realize. Directivity control using a small system also experiences problems at low frequencies. Recently, the rapid development of virtual reality and next-generation vision technology has increased the need for compact acoustic reproduction systems. Thus, array signal processing using loudspeaker arrays is still an advanced technology.

In general, studies on loudspeaker arrays typically use a linear, circular, or spherical loudspeaker array(3). Although regular configurations provide simple properties and easy calculations, such arrays incur problems in generating complex sound fields. Attempting an irregular configuration, our prior studies on sound field reproduction(8) and directivity control(9) show that the use of a 2CLA model can provide a better performance than a single circular array. However, the manner of which the distance and radius, which determine the shape of the 2CLA, affect the performance of a sound field reproduction or a directivity control remains unknown.

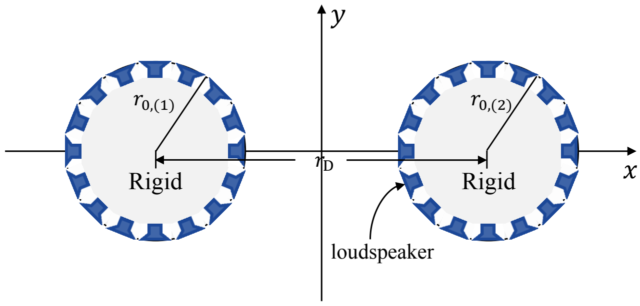


Figure 1. Configuration of the 2CLA. This is a special 2CLA where the centres of both circular arrays are located along the x-axis and are symmetric to the origin with distance r_D, radius $r_{0,(1)}$, and $r_{0,(2)}$, and in which the numbers of loudspeakers of both arrays are also the same.

In this study, we aim to reveal the relation between the performance and each of these two factors in a sound field reproduction and a directivity control .

2 TWO CIRCULAR LOUDSPEAKER ARRAY

The 2CLA is defined as a pair of circular loudspeaker arrays placed in a two-dimensional sound field, where both arrays are considered as a rigid circular baffle with point sources located on it. Figure 1 shows an example of the 2CLA. Note that the 2CLA can also be considered as a pair of infinite cylinder baffles parallel to each other in three-dimensional sound field, with line sources located on the baffles. Multiple scattering occurs when placing two rigid circular baffles in the sound field, which means that the sound field of a 2CLA can be more complex than that of a CLA. This may provide the additional possibility for a sound system to reproduce a complex target sound field or a complex directivity pattern.

However, the complexity of a sound field also means that the transfer function of a 2CLA should be different from that of a CLA. An analytical transfer function that includes multiple reflections between the arrays is required to apply array signal processing techniques or investigate the properties of a 2CLA. Studies on multiple scattering have determined radiation patterns for two parallel rigid cylinder baffles(10; 11; 12); in addition, our work also shows a method for deriving the transfer function of a 2CLA(8). Truncating efficiently small elements and reorganizing the series into a product of the matrices and vectors, this transfer function can be computed rapidly using the matrix products.

Using the transfer function above, we previously proposed a sound field reproduction method using a 2CLA(8). Moreover, we proposed a method for directivity control using 2CLA(9). Comparing the performance of a 2CLA to that of a CLA, the 2CLA provides a better performance at low frequencies.

In our prior studies, we found a phenomenon in which the performance of the 2CLA changes if the distance between the two arrays is altered. In addition, we believe that the 2CLA has the same property as a CLA, the performance of which is associated with the radius of the array. Although the results indicate that a 2CLA may provide a better performance than a CLA with the same radius, the influence on such performance when the radius or length of the 2CLA changes remains unknown. Looking at the transfer function in (8), it is easy to see that the distance between two circular arrays, and the radius of each circular array, affect the value of the transfer function. This means that they also have an influence on the results of the sound field reproduction and the directivity control of a 2CLA.

3 CASE 1: SOUND FIELD REPRODUCTION

In this section, we introduce how the distance and radius factor of a 2CLA affect the performance in terms of a sound field reproduction. The sound field with a monopole source is set as the original sound field. This technique is also known as a virtual source reproduction. We use the same reproduction method, called a pressure matching method, as in our prior study(8), and compared the performance of systems with different radii or distances.

3.1 Pressure Matching Method

A circular microphone array encircling the 2CLA is set as the control points, and it is considered that no source exists outside this circle in the sound field. Based on the Kirchhoff-Helmholtz integral equation(13), it is possible to reproduce the sound field outside the circle by reproducing the sound pressure and particle velocity on the circle. Furthermore, a simple source formulation(13) can be applied for this situation, and thus the sound field can be synthesized by reproducing the sound pressure on a continuous circle boundary. By sampling the circle using discrete control points, the sound field is then able to be reproduced by matching the sound pressure at the control points. Let $P(\boldsymbol{r})$ denote the original (target) sound pressure at the control point located at $\boldsymbol{r}$, and $G(\boldsymbol{r}|\boldsymbol{r}_l)$ denote the transfer function from the loudspeaker located at $\boldsymbol{r}_l$ to the same control point. The driving function d_l of the loudspeaker for sound pressure matching should satisfy the following equation:

$$P(\boldsymbol{r}) = \sum_{l=1}^{L} G(\boldsymbol{r}|\boldsymbol{r}_l) d_l, \tag{1}$$

where L represents the number of loudspeakers, and the angular frequency ω is omitted in this paper for convenience. Listing $P(\boldsymbol{r})$ into a vector $\mathbf{P}$, and rewriting the right part of Eq.(1) into the product of the matrix and vector, it is possible to match the sound pressure at every control point using the following equation:

$$\mathbf{P} = \mathbf{G}\mathbf{d}. \tag{2}$$

The vector $\mathbf{d}$ denotes the driving functions of each loudspeaker, which can be therefore computed using the least squares method. To constrain the filter gain $W_0 = 10\log_{10}||\mathbf{d}||_2^2$, we use a least squares method with L2 regularization:

$$\mathbf{d} = \frac{\mathbf{G}^{\mathrm{H}}\mathbf{P}}{\mathbf{G}^{\mathrm{H}}\mathbf{G} + \lambda \mathbf{I}}, \tag{3}$$

where λ represents the regularization parameter.

Furthermore, for a virtual source reproduction, the target sound field can be described analytically. The sound pressure of a two-dimensional monopole source located at $\boldsymbol{r}_{\mathrm{s}}$ can be expressed as follows:

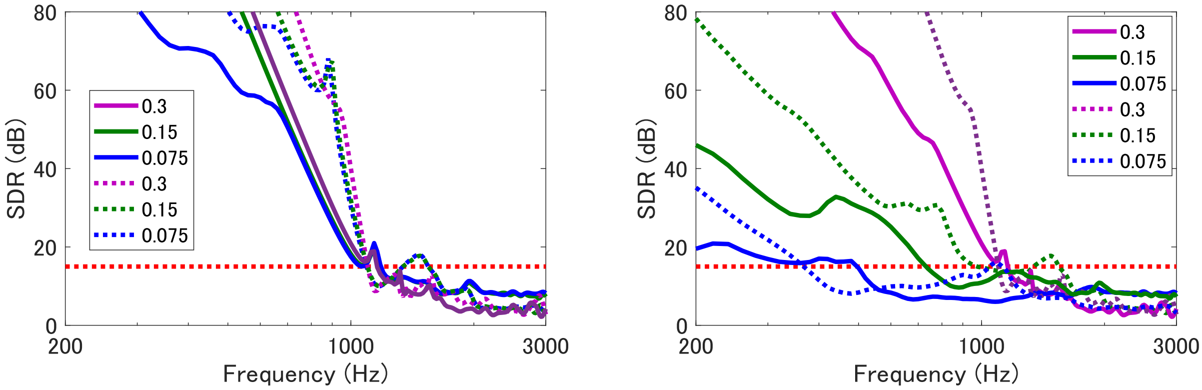
$$P(\boldsymbol{r}) = \frac{j}{4} H_0^{(2)}(k|\boldsymbol{r} - \boldsymbol{r}_{\mathrm{s}}|). \tag{4}$$

3.2 Experiments

Several computer simulations were conducted to verify how the radius and distance factor affect the performance of a virtual source reproduction. We used a 30-channel 2CLA (with 15 loudspeakers set to each circular array) with a rigid baffle. The radius of the two circular array is described as $r_{0,(1)}$ and $r_{0,(2)}$, respectively, and the distance between the center of the two arrays is described as r_{D}. In this study, we altered the radii of both arrays simultaneously such that this radius factor is set to r_0 while $r_{0,(1)} \triangleq r_{0,(2)} \triangleq r_0$. The center of the two arrays was set along the x-axis and symmetric to the origin. A total of 144 control points were set uniformly on a circle with a radius of 1.5 m. We attempted to reproduce two target sound fields where the target virtual source was set at $\boldsymbol{r}_{\mathrm{s}} = (0\ \mathrm{m}, 0.25\ \mathrm{m})$ and $\boldsymbol{r}_{\mathrm{s}} = (0\ \mathrm{m}, 0.5\ \mathrm{m})$, respectively.

We evaluated the performance using the signal-to-distortion ratio (SDR).

$$\mathrm{SDR}\ [\mathrm{dB}] = 10\log_{10} \frac{\int_{\Omega} |P(\boldsymbol{r})|^2 d\boldsymbol{r}}{\int_{\Omega} |P(\boldsymbol{r}) - \hat{P}(\boldsymbol{r})|^2 d\boldsymbol{r}}, \tag{5}$$



(a) without regularization (b) constant filter gain

Figure 2. The performance results in SDRs with different radius factors r_0. The legends in the figures represent the radius factor in [m]. The solid and dotted lines represent the results for the target source at (0, 0.5 m) and (0, 0.25 m), respectively. The red dotted line represents the threshold for an SDR of 15 dB. The distance factor in this experiment is fixed as $r_D = 1$ m. (a) The results for the regularization parameter is $\lambda = 0$ and (b) the results when the filter gain is constrained at $W_0 = 0$ dB.

where $\hat{P}(\boldsymbol{r})$ represents the sound pressure in the reproduced sound field, and Ω represents the exterior area of the circle of the control points. A greater SDR means that the system performance in reproducing the sound field is better. Moreover, we considered that a sound field is perceptually reproduced if the SDR exceeds a threshold of 15 dB(14).

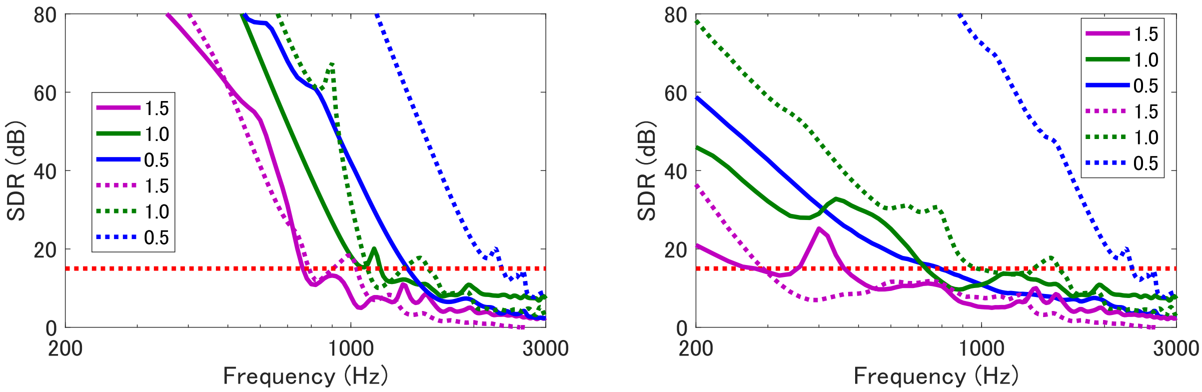
We chose three radius factors r_0 of 0.075, 0.15, and 0.3 m to investigate the affect of this factor. Figure 2 shows the results of this experiment. Figure 2(a) with $\lambda = 0$ shows the results in which the system provides the best performance without considering the filter gain; whereas Fig. 2(b) with $W_0 = 0$ dB shows the results under practical use. The distance factor in this experiment is fixed. Each radius factor results in a close SDR in Fig. 2(a), whereas Fig. 2(b) shows that the SDR increases with r_0. At higher frequencies, none of the systems can reproduce the sound field, and thus the comparison under a high frequency is considered meaningless.

We also chose three distance factors r_D of 0.5, 1, and 1.5 m for the investigation. Figure 3 shows the results of this experiment, where (a) and (b) have the same purpose as in 2. In addition, the radius factor in this experiment is fixed. The SDR increases whereas r_D decreases in both (a) and (b), although an exception exists in (b): an array with a r_D of 1 m has a higher SDR than an array with a r_D of 0.5 m within approximately 400-700 Hz. A comparison at higher frequencies is also ignored in this experiment.

3.3 Discussion

The results in Fig. 2 show that the radius factor has a slight effect on the performance of a 2CLA system when the filter gain is not considered. Instead, larger arrays may perform better than smaller arrays, whereas the spatial aliasing problem must be considered at higher frequencies. This means that for the radius factor, a 2CLA has a property close to that of a CLA, which also performs better at a greater radius. By contrast, Fig. 3 indicates that the distance factor and the performance of a sound field reproduction have a negative correlation, regardless of whether the filter gain is considered or not. The free space between two arrays is thought to cause a spatial aliasing problem, whereas linear loudspeaker arrays continuously face the aliasing problem.

We considered whether the reflection has a negative effect on the performance in a room(15). However, the results indicate that a larger radius with a shorter distance performs better, whereas in this situation a reflection between the arrays is larger than that with a smaller radius and longer distance. Hence, the reflection may have a positive effect, or only slightly effect on the performance.



(a) without regularization (b) constant filter gain

Figure 3. The performance results in SDRs with different distance factors r_D. The legends in the figures represent the distance factor in [m]. The solid and dotted lines represent the results for the target source at (0, 0.5 m) and (0, 0.25 m) respectively. The red dotted line represents the threshold for an SDR of 15 dB. The radius factor in this experiment is fixed as $r_0 = 0.15$ m. (a) The results when the regularization parameter is $\lambda = 0$, and (b) the results when the filter gain is constrained at $W_0 = 0$ dB.

4 CASE 2: DIRECTIVITY CONTROL

In this section, we introduce how the distance and radius of a 2CLA affect the performance in terms of directivity control. We used the pressure matching method in our prior study(9) to reproduce narrow beams. However, the pressure matching method cannot ensure a flat frequency response at the control points. Hence, a method with constraint points is used in this study.

4.1 Minimum Variance Distortionless Response Beamforming

To reproduce a directivity pattern with a flat response at the control points, we use a method call the Minimum Variance Distortionless Response (MVDR)(7) for beamforming. Differing from the least squares method(16), this method can ensure a fixed level and a flat frequency response of the output signal, and achieves a higher robustness than the least squares method. The method minimizes the total power at the suppression points while setting a strict constraint at the constraint points. Denoting $\mathbf{G}$ as the transfer function matrix for the suppression points and $\mathbf{C}$ as the transfer function matrix for points that should satisfy a strict constrained sound pressure vector $\mathbf{f}$, the problem can be expressed as follows:

$$\min_{\mathbf{w}} \quad \mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w} \tag{6}$$

$$\text{subject to} \quad \mathbf{C}^{\mathrm{H}}\mathbf{w} = \mathbf{f}, \tag{7}$$

where $\mathbf{w}$ represents the beamforming filter and $\mathbf{R} = \mathbf{G}^{\mathrm{H}}\mathbf{G}$. This optimization problem could be solved using the method of Lagrange multipliers. Furthermore, by adding a L2 regularization to the equation, a constraint for the filter gain can be set for practical use. The filter including the regularization can be calculated as

$$\mathbf{w} = \frac{\mathbf{R}_{\mathrm{REG}}^{-1}\mathbf{C}}{\mathbf{C}^{\mathrm{H}}\mathbf{R}_{\mathrm{REG}}^{-1}\mathbf{C}}\mathbf{f}, \tag{8}$$

where $\mathbf{R}_{\mathrm{REG}} = \mathbf{G}^{\mathrm{H}}\mathbf{G} + \lambda\mathbf{I}$ and λ represents the regularization parameter.

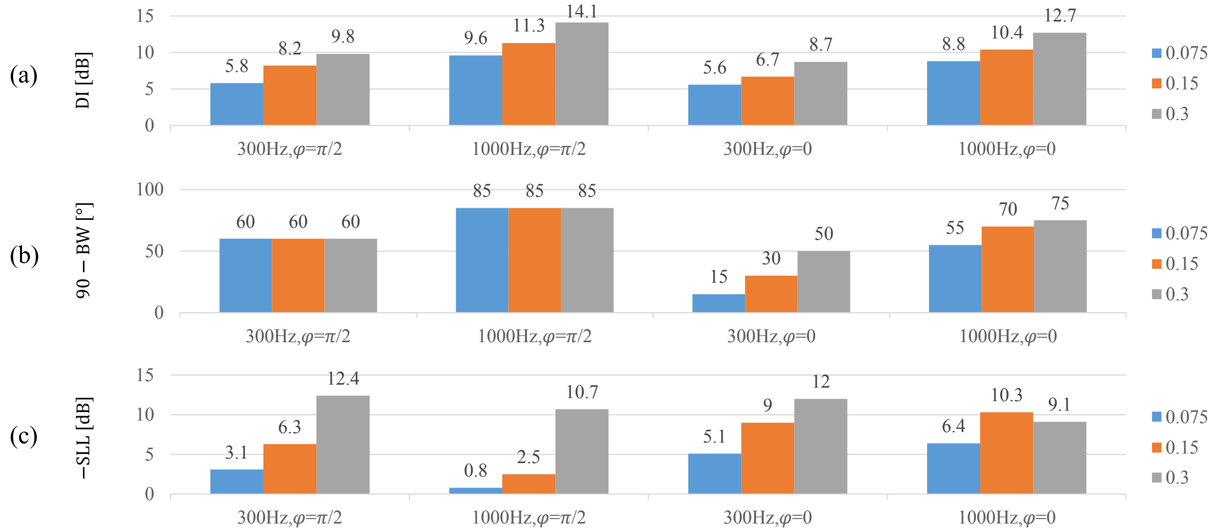


Figure 4. The performance of a 2CLA in terms of directivity control with different radius factors r_0. The legends in the figures represent the radius factor in [m], whereas the distance factor used in this experiment is fixed as $r_D = 1$ m. Data on the left side represent the results for $\varphi = \pi/2$, whereas those on the other side represent the results for $\varphi = 0$. For convenience, all characteristics in this figure are set as positively correlated with the performance of the directivity control. Accordingly, (a) shows the results of the DI in [dB], (b) shows the results of $(90°-\text{BW})$ in [°], and (c) shows the results of $-\text{SLL}$ in [dB].

4.2 Experiments

Computer simulations were conducted to verify how the radius and distance affect the performance of a directivity control. We used the same 30-channel 2CLA as described in Sec. 3.2. The same number of control points, i.e. 144, were set uniformly along a circle with a radius of 2 m for directivity control. One constraint point was set at $\varphi = 0$ or $\varphi = \pi/2$ (which was the first or 37th point of the array), whereas the other 143 points were set as the suppression points. This two directions of the constraint point were selected for testing the performance of the 2CLA along its major and minor axes. The sound pressure at the constraint point was strictly set to 1 whereas the filter gain was constrained at $W_0 = 0$ dB.

Three characteristics were selected to evaluate the performance of the directivity control. The directivity index (DI) is a widely used characteristic for evaluating the directivity patterns, and is expressed as

$$\text{DI [dB]} = 10\log_{10}\frac{2\pi||P_\varphi||^2}{\int_0^{2\pi}||P_\phi||^2 d\phi}, \tag{9}$$

where $||P_\phi||^2$ represents the radiation power in the direction ϕ. In addition, the beam width (BW) was used to evaluate the narrowness of the beam, and is defined as the angle at which the power attenuates to half of that at the look direction. The side lobe level (SLL) was also applied in this study to evaluate whether there was a powerful side lobe or if spatial aliasing occurred. The SLL describes the maximum level of the side lobe, which is a relative value in reference to the level of the look direction.

We conducted simulations at 300 and 1,000 Hz to investigate the performance of the 2CLA within the low- to mid-frequency range, whereas a high-frequency reproduction is considered difficult for the 2CLA(9). The radius factor r_0 was set to the same value as used in the experiments described in Sec. 3.2, and the distance factor was also fixed. Figure 4 shows the results when the radius factor is altered. The results in Fig. 4(a) indicate that the DI, which also represents the power ratio of the look direction to other directions, increases when r_0 increases. The left side of Fig. 4(b) indicates that the BW is not affected by the radius factor when $\varphi = \pi/2$,

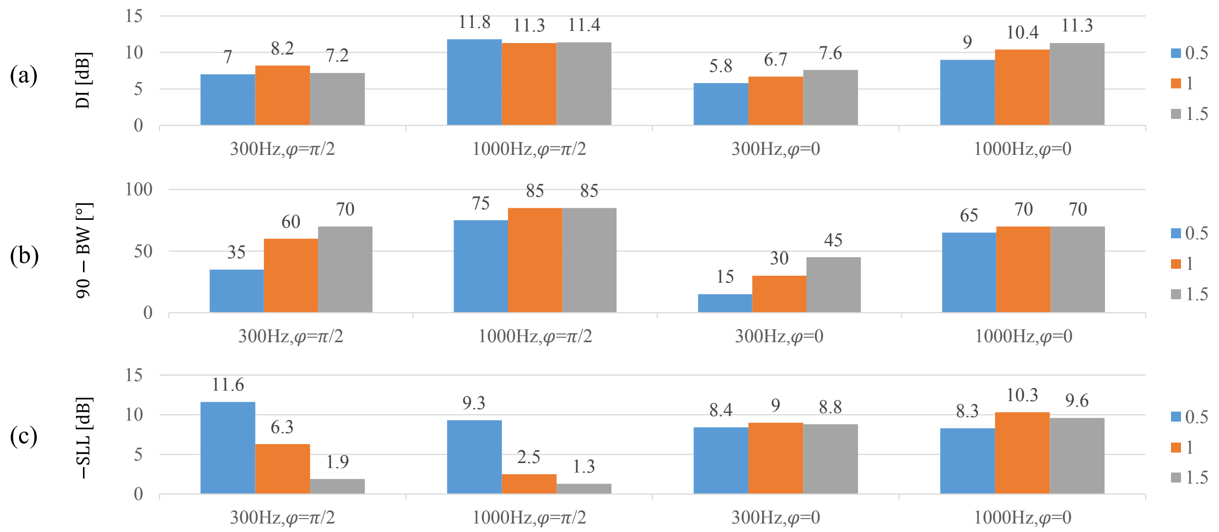


Figure 5. The performance of 2CLA in terms of directivity control with different distance factors r_D. The legends in the figures represent the distance factor in [m], whereas the radius factor in this experiment is fixed as $r_0 = 0.15$ m. Data on the left side represent the results for $\varphi = \pi/2$, whereas those on the other side represent the results for $\varphi = 0$. For convenience, all characteristics in this figure are set as positively correlated to the performance of the directivity control. Accordingly, (a) shows the results of the DI in [dB], (b) shows the results of ($90^\circ - \text{BW}$) in [°], and (c) shows the results of $-\text{SLL}$ in [dB].

whereas the right side indicates that an array with a larger r_0 has a narrower BW when $\varphi = 0$. Fig. 4(c) shows that the SLL decreases when r_0 increases in both directions. However, an exception at 1,000 Hz and $\varphi = 0$ indicates that an array of $r_0 = 0.15$ m performs at a lower SLL than an array of $r_0 = 0.3$ m.

Simulations for different distance factors r_D were also conducted using the same factors described in Sec. 3.2. Figure 5 shows the results of the simulations. For the left side of the figure, when $\varphi = \pi/2$, the DI is only slightly affected by r_D, whereas BW and SLL show opposite results. The 2CLA performs better for the BW and worse for the SLL when the distance between the arrays increases. The right side representing $\varphi = 0$ indicates that the 2CLA performs better for the DI and BW when r_D increases. However, SLL is slightly affected by the distance factor when $\varphi = 0$.

4.3 Discussion

The results in Fig. 4 indicate that a greater radius factor simply provides a better performance in terms of directivity control using an MVDR beamformer, whereas the results for $\varphi = \pi/2$ indicate that the radius factor may be of no help with the main lobe in this direction. By contrast, the results in Fig. 5 turn out to be quite complex. With $\varphi = 0$, the performance improves with a longer distance. This can be explained by the array displacement toward the look direction leading to a better performance even if only one CLA is present. Instead, a longer distance with $\varphi = \pi/2$ results in smaller BW and larger SLL, which means that the directivity pattern may have a narrow main lobe with a high-level side lobe. This problem may have the same explanation of the spatial aliasing that described in Sec 3.3.

After all, the radius and distance factors affect the performance in terms of the directivity control with a different trend as that found in a sound field reproduction. In addition, a difference in performance based on direction was shown in this experiment. This indicates that a further investigation is necessary for various methods and directions.

5 CONCLUSIONS

It was confirmed that the radius and distance factors of a 2CLA both affect the performance of a sound field reproduction or a directivity control. The radius factor has a positive effect on the performance when the filter gain is constrained, which means it affects the easiness of the sound control. However, the distance factor has different effects based on the direction, where a longer distance improves the sound control along the major axis direction and worsens the performance along the minor axis direction.

Future studies to explain the reasons for the results presented in this paper are required. Furthermore, the results of such studies may be used to discover a better configuration of loudspeaker arrays for use in sound field reproduction or directivity control.

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