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Generalized method of describing acoustic duct-like system as a multi-port

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ABSTRACT

The paper presents general method of analysing acoustic mufflers or other duct-like systems assuming propagation of a multimode wave and applying the multiport method. Apart from the main propagating tube and expansion chambers the analysed device may contain segments of annular duct (concentric or without the central tube) branch resonators, cavities and cavity diaphragms. Systems of such complicated geometrical structure have been widely analysed under the assumption that the wave propagating through any element is the plane wave. Thus the problem could have been analysed by means of the two-dimensional transfer matrices (or others equivalents) derived from conditions of conservation of the acoustic pressure and the volume velocity across each discontinuity. However, the assumption of the pure plane propagation across every element of the acoustic system imposes strong limitations on the excitation frequency or the radius of the duct segments as the Helmholtz number should not exceed the value of 1.84 which represents the non-dimensional cut-on frequency of the first circumferential mode. If the excitation and the system are axially symmetric the limited value of the Helmholtz number raises to 3.83. The analysis is carried out by means of the scattering matrix which can finally be transformed into transmission matrix.

Keywords: Acoustical multi-port, Scattering matrix, Multimode wave, Muffler

1. INTRODUCTION

The aim of the paper is to present a general method of analysing acoustic systems of complicated geometry assuming propagation of a multimode wave and applying the multiport method. This analysis will be carried out within the linear theory which allows not only to split the system into a network of sub-systems but also to apply the procedure consisting of subsequently reducing the range of the admittance or the scattering matrix describing the system as a whole. In the carried out analysis each sub-system (multi-port) corresponds to a certain wave transmission path. The procedure can be applied step by step to each of the internal sub-systems leading finally to a matrix representing only the external ports, most frequently the tail and the outlet pipe. Thus the paper presents general method to analyse the acoustic mufflers' or other duct-like systems to which the multi-port method can be efficiently applied.

Apart from the main propagating duct and expansion chambers the analysed device may contain segments of annular duct (concentric or not with the central tube) branch resonators, cavities and cavity diaphragms. Systems of such complicated geometrical structure have been widely analysed under the assumption that the wave propagating through any element is the plane wave [ref] and neglecting the generation of some other duct modes at the duct discontinuity (junction). Thus the problem might have been analysed by means of the two-dimensional matrices (impedance, admittance, scattering, transmission or others equivalents) derived from conditions of conservation of the acoustic pressure and the volume velocity across each discontinuity. It is well known that the assumption of the plane wave propagation across every element of the acoustic system imposes strong limitations on the excitation frequency or the radius of the duct segments as the Helmholtz number should not exceed the value 1.84 which represents the non-dimensional cut-on frequency of the first circumferential mode. If the excitation and the system are axially symmetric the limited value of the Helmholtz number raises to 3.83. However, this approach applied to the simplest muffler composed only of one expansion chamber led to significant errors in calculating the transmission loss (TL) for frequencies considerably lower than the limited ones [1]. The source of this discrepancy is an insufficient adjustment of the boundary condition on the junction being a consequence of applying the conditions of conservation of the acoustic volume velocity across each discontinuity. The presented method in which at a certain point of the calculations the sub-systems are analysed separately allows fulfilling the boundary condition with prescribed accuracy without significant increase of the computational costs.

The first attempt was to model the muffler design in the low frequency limit (only the plane wave propagation) as a two-port in analogy to the four-pole electric circuit [2-4]. In this case the analogy can be applied straightforwardly. More recent was application of the electric network method to model muffler design assuming propagation of a multimode wave [4-6] which introduced a necessity to define what will be called the acoustic multi-port. Abom [8] and some other authors [2-7] define the acoustic multi-port depending on the number of state variables which completely describe the state of a certain black box. Thus, the definition is not related to the number of the connected ducts and the topology of the acoustic system. This approach has several advantages, one of which is that it provides information on the number of degrees of freedom *i.e.* on the size of the matrix representing relationship between ingoing and outgoing waves (input and output variables, in general). Another possibility is to relate the definition to the number of joints (straight ducts most frequently) [9] which is compatible with the topology of the acoustic system and also is independent of the carried out frequency range analysis. This approach will be applied subsequently

The analysis is carried out by means of the scattering matrix which can be finally transformed into the transmission matrix to calculate the transmission loss.

2. APPLICATION OF THE THEORY OF THE MULTI-PORTS TO ACOUSTICS

2.1 Electrical and acoustical multi-ports

The multiport method applied in acoustic origins from electric network analysis, in which a port connects the poles of a black box and describes the relation between incident and outgoing waves represented by the state variables. Most frequently the chosen state variables are the electric current and the voltage in electrical multi-ports and the sound pressure and the acoustic velocity in acoustical. In the electric circuits analysis the number of poles corresponds to the number of the degrees of freedom represented by flowing electric currents each of which needs a separate wire (joint in topological representation) and is compatible with the number of joints appearing in an acoustical system as far as propagation of the plane wave only is considered (the so-called low frequency approximation.

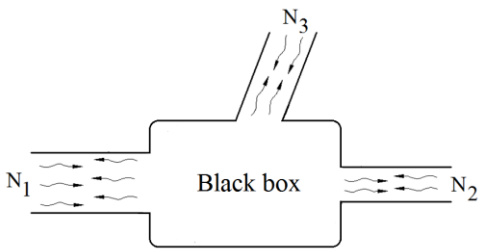

Figure 1 – Acoustic 3-port with joints of different size, $N=N_1+N_2+N_3$ port in acousto-electrical analogies

However substantial differences originate from the fact that in general many modes can propagate in a duct depending on the duct size (dimensions in rectangular, radius in cylindrical ect.) and the excitation frequency. For example, for a port with 3 connected joints represented by ducts of differentiated geometry, the total number of modes can be $N=N_1+N_2+N_3$, depending additionally on the frequency. Thus, within the straightforwardly applied acoustical-electrical analogies it will be called N-port, with N depending on frequency. It seems that in the sound field analysis the number of joints is essential, as it determines the geometry (topological representation) of the system under consideration, and that is why we have introduced the concept of an acoustic n-port defined by the number of ducts coupled to the tested element.

As a result a selected acoustical sub-system connected to some others by n-joints (ducts) will be called *the acoustical n-port* independently on the radiuses of different ducts and the frequency of excitation.

2.2 Basic assumptions

An acoustic system can be analysed as an acoustic multi-port (Fig. 2) under the following assumptions:

1. The phenomena fulfil the assumptions of linearity.
2. Subsequent multi-ports (sub-systems) are connected by duct-like elements of a constant cross sections called joints.
3. It allows describing the acoustic pressure as a superposition of duct modes which means decomposition into a series of eigenfunctions.
4. The wave propagating in a joint can be decomposed into a wave going out of the black box element and going into it.
5. The joints are long enough to consider only the propagating waves at a selected cross section *i.e.* neglect the near field effects appearing close to the system discontinuity.
6. The selected duct cross section fulfilling the above assumptions is called a port.
7. There is a certain freedom in choosing the position of the port as the change in its localizations will result only in a change of phase of the state variables.
8. At a port the wave can be described by a one column vector of modal sound pressures and acoustic velocities (impedance or admittance approach) or, equivalently by the vectors of the ingoing and outgoing modal sound pressures (scattering matrix approach **S**).

In what fallows the scattering matrix approach will be applied.

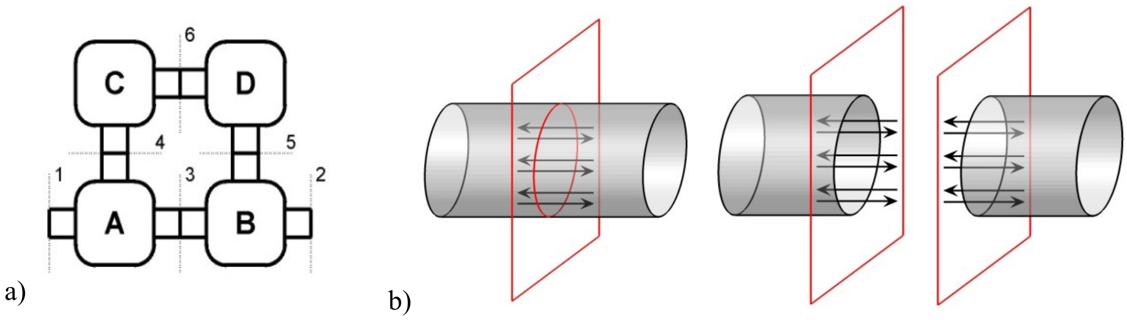

Figure 2 – a) Topological representation of an exemplary acoustical systems: A, B, C, D – sub-systems (multi-ports), A and B – three-ports, C and D –two-ports, joints with marked ports: 1 and 2 – external, 3, 4, 5 and 6 – internal; b) at a given port the acoustic pressure of each wave mode outgoing from one sub-system equals the acoustic pressure of the corresponding in-going mode of the consecutive sub-system.

3. THE SCATTERING MATRIX APPLIED TO ACOUSTICAL SYSTEM COMPOSED OF MULTI-PORTS

3.1 Theoretical basis

Let us consider an acoustical system composed of some multi-ports as presented in Fig.2. Our goal is to analyse its property by means of the scattering matrix. The final result is to derive the dependence between the state variables of the external ports (1and 2 in Fig. 2). In the scattering matrix approach it means the relation between the modal acoustic pressures of the ingoing and outgoing waves. In many technical applications, such as muffler design it allows to predict the transmission loss and analyse its variation due to changes in the muffler geometry.

In general, considering sources present within the acoustical system the modal sound pressures of the outgoing wave modes ${}^{out}\boldsymbol{P}$ depends on the scattering matrix $\boldsymbol{S}$ and the pressures of the internal sound sources ${}^{sr}\boldsymbol{P}$

$$ {}^{out}\boldsymbol{P} = \boldsymbol{S} \times {}^{in}\boldsymbol{P} + {}^{sr}\boldsymbol{P} \tag{1} $$

where ${}^{out}\boldsymbol{P}, {}^{in}\boldsymbol{P}, {}^{sr}\boldsymbol{P}$ are represented by one column matrices composed of modal pressure amplitudes at a selected duct cross sections where the phenomena of the near field can be omitted.

$$ {}^{in}\boldsymbol{P} = \begin{bmatrix} \begin{bmatrix} {}^{in}\boldsymbol{P}_1 \end{bmatrix} \\ \begin{bmatrix} {}^{in}\boldsymbol{P}_2 \end{bmatrix} \\ \begin{bmatrix} {}^{in}\boldsymbol{P}_3 \end{bmatrix} \end{bmatrix} \qquad {}^{out}\boldsymbol{P} = \begin{bmatrix} \begin{bmatrix} {}^{out}\boldsymbol{P}_1 \end{bmatrix} \\ \begin{bmatrix} {}^{out}\boldsymbol{P}_2 \end{bmatrix} \\ \begin{bmatrix} {}^{out}\boldsymbol{P}_3 \end{bmatrix} \end{bmatrix} \tag{2} $$

The procedure leading to the final result relies, as will be presented below, on reducing step by step the dimension of the scattering matrix representing the system as a whole by subsequently eliminating the selected ports.

Let us assume that in the analysed system we can distinguish two sub-systems which constitute a five-port and are connected by a joint in a form of a straight duct merging port 2 with port 4. The presented method relies on reducing the range of the multi-port to a lower number by joining port 2 with port 4.

Next step is denoting these ports as *a* and *b* and the corresponding modal sound pressure matrices by $\boldsymbol{P_a}$ and $\boldsymbol{P_b}$. Applying the relations and introducing new symbols to better distinguish the selected joint

$$ {}^{in}\boldsymbol{P}_a = {}^{out}\boldsymbol{P}_b = \boldsymbol{P}_A \qquad {}^{in}\boldsymbol{P}_b = {}^{out}\boldsymbol{P}_a = \boldsymbol{P}_B \tag{3} $$

equation Eq. 1 takes the form

$$ \underbrace{\begin{bmatrix} {}^{out}\boldsymbol{P}_1 \\ {}^{out}\boldsymbol{P}_B \\ {}^{out}\boldsymbol{P}_3 \\ {}^{out}\boldsymbol{P}_A \\ {}^{out}\boldsymbol{P}_5 \end{bmatrix}}_{{}^{out}\boldsymbol{P}} = \underbrace{\begin{bmatrix} \boldsymbol{S}_{1,1} & \boldsymbol{S}_{1,a} & \boldsymbol{S}_{1,3} & \boldsymbol{S}_{1,b} & \boldsymbol{S}_{1,5} \\ \boldsymbol{S}_{a,1} & \boldsymbol{S}_{a,a} & \boldsymbol{S}_{a,3} & \boldsymbol{S}_{a,b} & \boldsymbol{S}_{a,5} \\ \boldsymbol{S}_{3,1} & \boldsymbol{S}_{3,a} & \boldsymbol{S}_{3,3} & \boldsymbol{S}_{3,b} & \boldsymbol{S}_{3,5} \\ \boldsymbol{S}_{b,1} & \boldsymbol{S}_{b,a} & \boldsymbol{S}_{b,3} & \boldsymbol{S}_{b,b} & \boldsymbol{S}_{b,5} \\ \boldsymbol{S}_{5,1} & \boldsymbol{S}_{5,a} & \boldsymbol{S}_{5,3} & \boldsymbol{S}_{5,b} & \boldsymbol{S}_{5,5} \end{bmatrix}}_{\boldsymbol{S}} \times \underbrace{\begin{bmatrix} {}^{in}\boldsymbol{P}_1 \\ {}^{in}\boldsymbol{P}_A \\ {}^{in}\boldsymbol{P}_3 \\ {}^{in}\boldsymbol{P}_B \\ {}^{in}\boldsymbol{P}_5 \end{bmatrix}}_{{}^{in}\boldsymbol{P}} + \underbrace{\begin{bmatrix} {}^{sr}\boldsymbol{P}_1 \\ {}^{sr}\boldsymbol{P}_a \\ {}^{sr}\boldsymbol{P}_3 \\ {}^{sr}\boldsymbol{P}_b \\ {}^{sr}\boldsymbol{P}_5 \end{bmatrix}}_{{}^{sr}\boldsymbol{P}} \tag{4} $$

At this stage it is possible to calculate $\boldsymbol{P_A}$ and $\boldsymbol{P_B}$ performing the following matrix operations:

$$ \begin{bmatrix} \boldsymbol{P}_B \\ \boldsymbol{P}_A \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_{a,a} & \boldsymbol{S}_{a,b} \\ \boldsymbol{S}_{b,a} & \boldsymbol{S}_{b,b} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{P}_A \\ \boldsymbol{P}_B \end{bmatrix} + \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{F} \end{bmatrix} \tag{5} $$

where:

$$ \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{F} \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_{a,1} & \boldsymbol{S}_{a,3} & \boldsymbol{S}_{a,5} \\ \boldsymbol{S}_{b,1} & \boldsymbol{S}_{b,3} & \boldsymbol{S}_{b,5} \end{bmatrix} \times \begin{bmatrix} {}^{in}\boldsymbol{P}_1 \\ {}^{in}\boldsymbol{P}_3 \\ {}^{in}\boldsymbol{P}_5 \end{bmatrix} + \begin{bmatrix} {}^{sr}\boldsymbol{P}_a \\ {}^{sr}\boldsymbol{P}_b \end{bmatrix} \tag{6} $$

and so

$$ \begin{bmatrix} \boldsymbol{S}_{a,a} & \boldsymbol{S}_{a,b} - \boldsymbol{I} \\ \boldsymbol{S}_{b,a} - \boldsymbol{I} & \boldsymbol{S}_{b,b} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{P}_A \\ \boldsymbol{P}_B \end{bmatrix} = - \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{F} \end{bmatrix} \tag{7} $$

The above considerations lead to the algorithm composed of some steps described below.

3.2 Algorithm for reducing the range of the scattering matrix

In what fallows the algorithm for merging two ports of an acoustical system is derived. The algorithm consists of the subsequent calculations:

- Construction of the $\boldsymbol{U}$ matrix such as

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{S}_{a,a} & \boldsymbol{S}_{a,b} - \boldsymbol{I} \\ \boldsymbol{S}_{b,a} - \boldsymbol{I} & \boldsymbol{S}_{b,b} \end{bmatrix} \tag{8}$$

- Derivation of the inverse matrix $\boldsymbol{T} = -\boldsymbol{U}^{-1}$ divided into segments corresponding to matrix $\boldsymbol{S}$ segments

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{T}_{a,a} & \boldsymbol{T}_{a,b} \\ \boldsymbol{T}_{b,a} & \boldsymbol{T}_{b,b} \end{bmatrix} = -\boldsymbol{U}^{-1} \tag{9}$$

- Modification of the sub-matrices of the $\boldsymbol{S}$ matrix according to the formula

$$\begin{aligned} \boldsymbol{S}'_{i,k} = \boldsymbol{S}_{i,k} & +\boldsymbol{S}_{i,a} \times \boldsymbol{T}_{a,b} \times \boldsymbol{S}_{b,k} \\ & +\boldsymbol{S}_{i,a} \times \boldsymbol{T}_{a,a} \times \boldsymbol{S}_{a,k} \\ & +\boldsymbol{S}_{i,b} \times \boldsymbol{T}_{b,b} \times \boldsymbol{S}_{b,k} \\ & +\boldsymbol{S}_{i,b} \times \boldsymbol{T}_{b,a} \times \boldsymbol{S}_{a,k} \end{aligned} \tag{10}$$

where both indices i and k do not take values a and b $i \neq a,\ i \neq b$ and $k \neq a,\ k \neq b$.

- Modification of the source sub-matrices ${}^{sr}\boldsymbol{P}$ according to the formula

$$\begin{aligned} {}^{sr}\boldsymbol{P}'_{i} = {}^{sr}\boldsymbol{P}_{i} & +\boldsymbol{S}_{i,a} \times \boldsymbol{T}_{a,b} \times {}^{sr}\boldsymbol{P}_{b} \\ & +\boldsymbol{S}_{i,a} \times \boldsymbol{T}_{a,a} \times {}^{sr}\boldsymbol{P}_{a} \\ & +\boldsymbol{S}_{i,b} \times \boldsymbol{T}_{b,b} \times {}^{sr}\boldsymbol{P}_{b} \\ & +\boldsymbol{S}_{i,b} \times \boldsymbol{T}_{b,a} \times {}^{sr}\boldsymbol{P}_{a} \end{aligned} \tag{11}$$

where $i \neq a,\ i \neq b$.

- Finally the formula for the multi-port of a range reduced by two (from a five-port to a three-port) described by the scattering matrix is obtained

$$\underbrace{\begin{bmatrix} {}^{out}\boldsymbol{P}_1 \\ {}^{out}\boldsymbol{P}_3 \\ {}^{out}\boldsymbol{P}_5 \end{bmatrix}}_{{}^{out}\boldsymbol{P}} = \underbrace{\begin{bmatrix} \boldsymbol{S}'_{1,1} & \boldsymbol{S}'_{1,3} & \boldsymbol{S}'_{1,5} \\ \boldsymbol{S}'_{3,1} & \boldsymbol{S}'_{3,3} & \boldsymbol{S}'_{3,5} \\ \boldsymbol{S}'_{5,1} & \boldsymbol{S}'_{5,3} & \boldsymbol{S}'_{5,5} \end{bmatrix}}_{\boldsymbol{S}'} \times \underbrace{\begin{bmatrix} {}^{in}\boldsymbol{P}_1 \\ {}^{in}\boldsymbol{P}_3 \\ {}^{in}\boldsymbol{P}_5 \end{bmatrix}}_{{}^{in}\boldsymbol{P}} + \underbrace{\begin{bmatrix} {}^{sr}\boldsymbol{P}'_1 \\ {}^{sr}\boldsymbol{P}'_3 \\ {}^{sr}\boldsymbol{P}'_5 \end{bmatrix}}_{{}^{sr}\boldsymbol{P}'} \tag{12}$$

The above presented algorithm can be applied to any two ports represented by matrices $\boldsymbol{S}_{\boldsymbol{A}}$ and $\boldsymbol{S}_{\boldsymbol{B}}$, not necessarily connected at the first stage. In the first step the scattering matrix comprising both ports has to be constructed

$$\boldsymbol{S}_{\boldsymbol{AB}} = \begin{bmatrix} \boldsymbol{S}_{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_{\boldsymbol{B}} \end{bmatrix} \tag{13}$$

The next step consists of merging these two ports applying the above algorithm.

4. THE SCATTERING MATRIX OF SOME MUFFLERS' ELEMENTS

The elements encountered most frequently in the mufflers' design are straight ducts of given length and different cross-section, hard or finite impedance lids, duct outlets – baffled or unflanged etc. In a straight duct the sound pressure of a mode n can be divided at a given cross section into parts representing waves going upstream and downstream.

4.1 Duct terminations as an acoustical one-port

If the duct is terminated by a lid of a given acoustic impedance the termination can be analysed as an acoustical one-port with the scattering matrix

$$S_{nk} = \left(\frac{y_n - Y}{y_n + Y}\right) \delta_{nk} \tag{14}$$

where y_n is the modal admittance $y_n = k_{z,n}k^{-1}$, and Y is the non-dimensional acoustic admittance of the lid surface, and δ_{nk} is the Kronecker delta function, which means that the matrix $\boldsymbol{S}$ is diagonal.

For an infinite duct or a duct terminated with non-reflecting surface or anecoic termination the scattering matrix $\boldsymbol{S} = \boldsymbol{0}$ as there are no out-going waves.

For an open end of a duct the scattering matrix is composed of the reflection and transformation coefficients, which have been derived analytically for a circular flanged [10] or unflanged [11, 12] duct.

4.2 Straight duct of a given length as an acoustical two-port

In a straight duct the sound pressure of a mode n can be divided at a given cross section into parts representing waves going upstream and downstream

$$P_n(z) = P_n^+(z) + P_n^-(z) \tag{15}$$

where

$$P_n^+(z) = P_n^+ e^{-ik_{z,n}z} \tag{16}$$

and P_n^+ is the mode amplitude, and $k_{z,n}$ is the axial wave number. Thus the scattering matrix for a duct of length d is

$$\boldsymbol{S} = \begin{bmatrix} 0 & \mathrm{diag}[\exp(-ik_{z,n}d)] \\ \mathrm{diag}[\exp(-ik_{z,n}d)] & 0 \end{bmatrix} \tag{17}$$

The values of the axial wave numbers $k_{z,n}$ are obtained solving the wave equation with adequate boundary condition and are known for ducts of cylindrical symmetry, plain or annular, with all three kinds of the boundary condition, and for ducts of rectangular and equilateral triangle cross-section. For more complicated shapes the scattering matrix has to be calculated by means of some numerical methods such as FEM, BEM or similar.

4.3 Junction of pipes with different cross-sections as an acoustical multi-port

Figure 3 – An example of an acoustical multi-port

Junction of pipes with different cross-sections constitutes an acoustical multi-port. Junction of two pipes of different radiuses constitute an acoustical two-port, while Fig. 3 represents an acoustical three-port consisting of a tail pipe A and an annular duct C connected to a larger pipe B.

The boundary condition of the sound pressure and the normal component of the acoustic velocity continuity have to be fulfilled across the junction, which means

$$p_L(\boldsymbol{x}) = p_R(\boldsymbol{x}), \qquad v_L(\boldsymbol{x}) = v_R(\boldsymbol{x}), \tag{18}$$

where $\boldsymbol{x}$ represents a point on a junction cross-section, indices L and R describe the left and the right side of it and the relation is fulfilled at each point of the junction.

For the acoustical three-port in Fig. 3 it reads

$$\mathrm{v_L}(\boldsymbol{x}) = \begin{cases} \mathrm{v_A}(\boldsymbol{x}) & \boldsymbol{x} \in A \\ \mathrm{v_C}(\boldsymbol{x}) & \boldsymbol{x} \in C \\ 0 & \boldsymbol{x} \in D \end{cases} \tag{19}$$

assuming that surface D is an acoustically hard surface.

The above-mentioned boundary conditions can be meet applying to the acoustic pressure and velocity the mode matching method (MMM) [13]. At first, both quantities should be distributed into modes in the orthogonal function database (eigenfunctions is the best choice, but also other complete set of orthogonal functions will lead to the result). Next step is to apply linear transformation to calculate the unknown distribution coefficients. This in turn allows to calculate the amplitudes of modes propagating to and from the multi-port and ultimately to determine the scattering matrix [9, 14].

The scattering matrix of multi-ports of irregular geometry can be calculated by means of numerical methods or experimentally.

5. APPLICATION TO MUFFLERS ANALYSIS

The above described algorithm will be applied to a muffler presented in Fig. 4 and discussed previously in the literature [1]

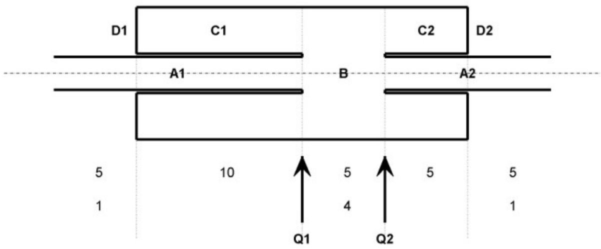

Figure 4 – Muffler with extended inlet and outlet pipes

The muffler is composed of elements such as: plain (A1 and A2 and B) and annular (C1 and C2) circular ducts, ducts hard terminations (D1 and D2) and junctions (Q1 and Q2) at which the mode matching method (MMM) has to be applied [9, 14] to fulfil the boundary conditions. The inlet pipe is extended into the expansion chamber by one second of its length *l*=0.4 m and the outlet pipe by one fourth, the radiuses are 0.04 m and 0.16 m, respectfuly (*cf.* [1], page 324).

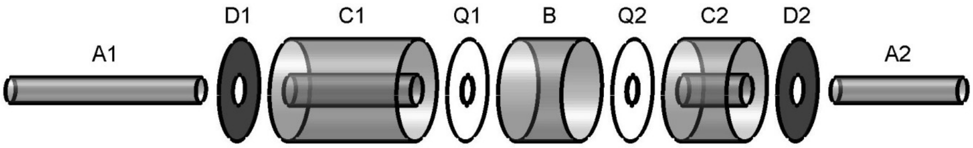

Figure 5 – Muffler elements disconnected

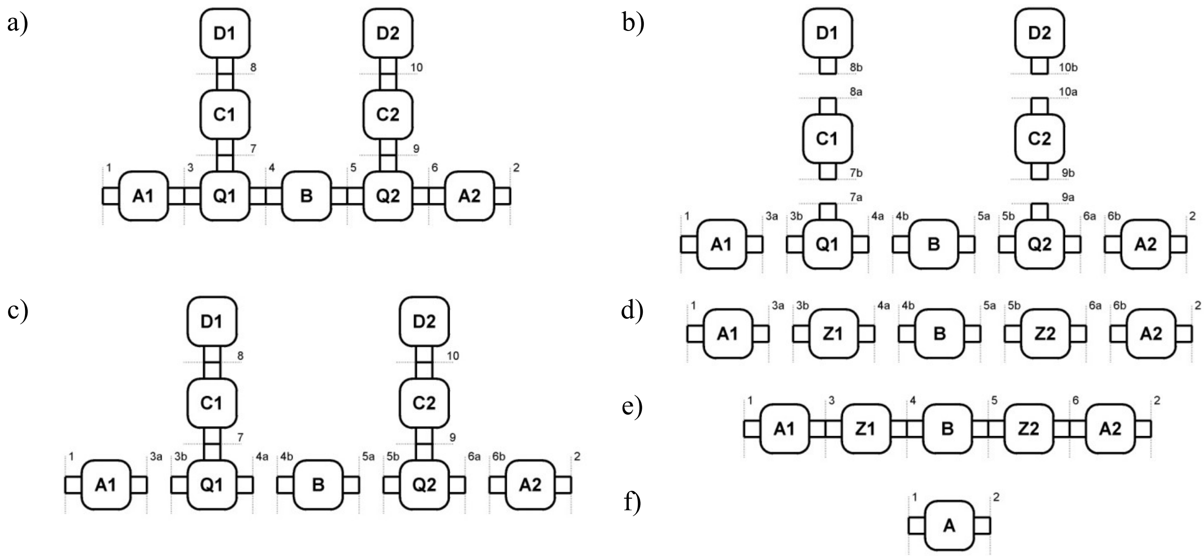

Figure 6 – Calculations of the scattering matrix step by step: a) topological scheme of the muffler presented in Fig. 5; b) the system decomposed into separate multi-ports of known scattering matrix; c) and d) and e) consecutive steps of reducing the number of ports by applying the procedure presented in Section 3; f) final result – the system reduced to an acoustical two-port of determined scattering matrix

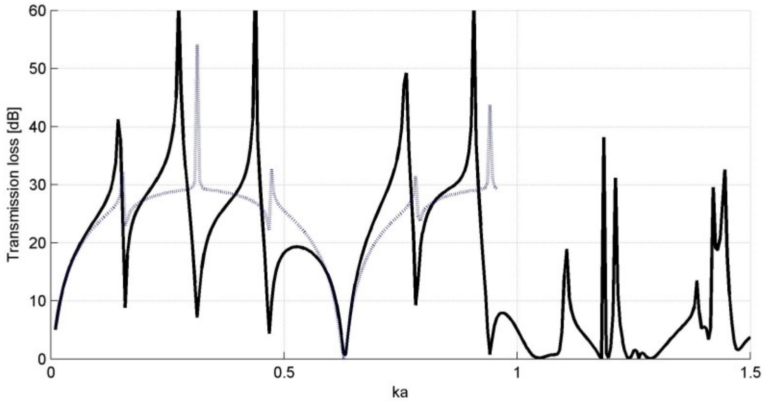

Figure 7 – Transmission loss of a muffler as in Fig. 4 calculated according to the presented algorithm (continuous line) and presented by Munjal (*cf.* [1], page 324, dotted line) assuming the plane wave incident to the left-hand tail pipe;

6. Conclusions

The presented method is a general way of describing acoustic duct-like systems as multi-ports. It can be applied to devices composed of many sub-systems not necessarily constituting a cascade. The procedure can be implemented step by step applying the scattering matrices already derived and the mode matching method at junctions. For acoustical system of more complicated geometry for which the reflection coefficients have not been calculated the problem can be solved by means of the numerical methods. The scattering matrix can be also derived experimentally.

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