PROCEEDINGS of the
23rd International Congress on Acoustics

9 to 13 September 2019 in Aachen, Germany

Empirical Study of Decentralized Multi-Channel Active Noise Control Based on the Genetic Algorithm

Guoqiang Zhang[1]; Jiancheng Tao[2]; Xiaojun Qiu[1]

[1] University of Technology Sydney, Australia

[2] Nanjing University, China

ABSTRACT

In an active noise control (ANC) system, computational complexity is one major concern when designing practical control algorithms. One approach to reducing computational complexity is to apply a decentralized control scheme rather than the centralized scheme. The decentralized scheme attempts to control a number of ANC subsystems independently, where for simplicity, one subsystem consists of one loudspeaker and one error microphone. Our recent published article has shown theoretically that a decentralized two-channel ANC system can achieve the same noise reduction performance as the centralized one with guaranteed convergence in the frequency domain. In this work, we attempt to extend the results from two-channel case to N (N>1) channel case. The challenge sits in finding N complex numbers that could properly shape the eigenvalues of an $N \times N$ matrix for each frequency bin towards guaranteed convergence. Due to the problem complexity, we conduct empirical study by using the genetic algorithm (GA). Simulation results on the channel numbers of 4, 6, and 12 demonstrate that the resulting decentralized ANC controller is also able to achieve the same noise reduction performance as the centralized controller.

Keywords: ANC, decentralized control

1. INTRODUCTION

Active noise control (ANC) techniques have been successfully exploited to remove or mitigate sound noise in many applications, such as designing ANC systems for headphone applications [1], reducing acoustic noise in magnetic resonance (MR) imaging [2], and creating a quiet zone around listener ears [3]. The basic principle of an ANC system is to introduce a set of secondary sources to interfere destructively with the primary sound by using multiple channels of loudspeakers and error microphones, where each channel consists of one pair of loudspeaker and error microphone. To be able to cancel the sound noise, a central controller is commonly employed to collect and process signals from all the error microphones, and then make proper adjustments to the loudspeakers using the filtered-x LMS algorithm [4], referred to as the centralized control scheme. In general, the noise reduction performance of the system improves along with an increasing number of channels at the cost of high computational complexity and expensive wiring, which is therefore unscalable and preventing it from broad applicability.

In recent years, it has attracted increasing attention on reducing the computational complexity of the centralized controller for a large-scale ANC system in the sound and acoustic community. Two schemes have been proposed in the literature to address the above issue. The first one is to introduce the framework of distributed computing in the ANC system [5], [6], [7]. Specifically, [5] treats an ANC system as a ring network where each secondary source is taken as one node in the network. The computational burden is then distributed across all the secondary sources by performing incremental computation sequentially over the ring network. At each iteration, the computation unit at each secondary source receives, updates and then transmits the control signal of all the secondary sources by interacting with neighbours. The method achieves the same noise reduction performance as the centralized controller at the cost of high transmission bandwidth and potential slow tracking speed. The work of [6] and [7] extend [5] by utilizing a diffusion strategy in the update expressions of the

[1] {guoqiang.zhang, xiaojun.qiu}@uts.edu.au
[2] jctao@nju.edu.cn

control signal.

A decentralized control scheme aims to reduce both the computational complexity and expensive wiring of the central controllers (see [8], [9]). The basic idea is to treat a large scale ANC system as a combination of small-size ANC subsystems, where each subsystem, for example, can be taken as a single-channel ANC. Each subsystem adjusts its own control signal only based on its associated error microphone(s). Due to the inherent coupling between the secondary sources and error microphones, it remains challenging to design effective and robust control methods for the multiple subsystems. The work of [8] proposed and analyzed a feedforward decentralized control method by studying matrix eigenvalues in the frequency domain, where the matrix is determined by transfer functions of the secondary paths of the whole system. It is found that if the inputs of the secondary sources are constrained to be sufficiently small in magnitude, the control method is guaranteed to converge. However, small inputs implicitly degrade the noise reduction performance, which is practically unattractive. The recent work [9] conducted extensive simulations and experiments for the control method of [8] and drew a number of insightful observations.

Recently, we have reconsidered decentralized two-channel control in the frequency domain [10]. In this situation, it is shown in [10] that properly designed decentralized control can theoretically achieve the same noise reduce performance as the centralized controller without enforcing small inputs of the secondary sources as in [8]. Considering each frequency bin, the basic idea of [10] is to find two complex values that can push the eigenvalues of a 2×2 matrix to the right complex domain (i.e., the two eigenvalues having positive real parts). A procedure is provided in [10] for finding the two complex values for each frequency bin. Later on, the authors in [11] attempted to extend our results to decentralized N-channel ANC (N>1, see Fig. 1). They proposed to find N complex values such that all the eigenvalues of the corresponding $N\times N$ matrix for each frequency bin are equal to 1, which are then transformed to solving N nonlinear equations. However, it is unclear if a solution exists or not. It also remains open to explicitly solve the nonlinear equations to obtain the N optimal complex values.

In this paper, we reconsider decentralized N-channel ANC based on the genetic algorithm (GA) [12]. Our motivation is that the problem formulation in [11] for the N-channel case might be unnecessarily strict, making it difficult to search for the optimal solution even if it exists. In this work, we intend to find N complex values for each frequency bin such that all the eigenvalues of the associated $N\times N$ matrix are pushed to the right complex domain rather than being pushed to 1 as in [11]. By relaxing the problem formulation, it would more likely lead to the existence of a solution for the problem. Due to the problem complexity, we apply GA to compute the N complex values for each $N\times N$ matrix. A special nonconvex optimization problem in terms of the maximum and minimum eigenvalue phases is constructed to facilitate the usage of the GA method. Simulation results on the channel numbers of 4, 6, and 12 for an ANC system attached to a rectangular enclosure with a baffled opening (see the simulation setup in Section 4) demonstrate that the resulting decentralized ANC controller is also able to achieve the same noise reduction performance as the centralized controller.

2. Multi-Channel ANC Signal Model

Consider a primary sound disturbance generated by a point monopole at a single frequency. We exploit an ANC system to mitigate or remove the disturbance in the same environment. The considered ANC system consists of N (N>1) secondary sources and N error microphones, where each secondary source and its collocated error microphone form a single-channel ANC subsystem (see Fig. 1). As the considered disturbance is from one frequency, all the signals can be represented as single complex numbers. We omit the frequency index here for concision.

Based on the above guidelines, the complex steady state signal at the ith error microphone can be represented as

$$e_i = d_i + \sum_{j=1}^{N} z_{ij}\, u_j, \tag{1}$$

where d_i represents the disturbance from the primary source at the ith error microphone, u_j is the complex input to the jth secondary source, and z_{ij} represents the complex response from the jth secondary source to the ith error microphone. Eq. (1) can be further expressed into a compact vector form as

$$\boldsymbol{e} = \boldsymbol{d} + \boldsymbol{Z}\boldsymbol{u}, \text{ where } \boldsymbol{Z} = \begin{pmatrix} z_{11} & \cdots & z_{1N} \\ \vdots & \ddots & \vdots \\ z_{N1} & \cdots & z_{NN} \end{pmatrix}. \tag{2}$$

The $N\times N$ complex matrix $\boldsymbol{Z}$ captures the joint response of the N secondary sources at the N error

microphones.

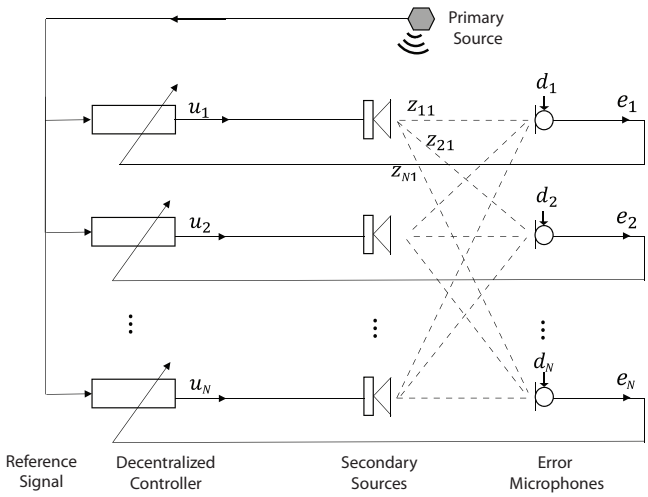

Figure 1 – Demonstration of a decentralized multi-channel ANC system, where each channel consists of a secondary source and its collocated error microphone. The parameters z_{vs}, s,$v = 1,2 \cdots, N$, denote the frequency response of the acoustic path from s to v, and d_s represents the noise disturbance at microphone index s.

The objective of a controller is to minimize the error vector $\boldsymbol{e}$ by adjusting the input vector $\boldsymbol{u}$ properly. For the case that $\boldsymbol{Z}$ is nonsingular, the optimal input vector $\boldsymbol{u}^\star$ can be computed by minimizing the sum of squared error signals directly, given by [8]

$$\boldsymbol{u}^\star = \arg\min_{\boldsymbol{u}} \left(\frac{1}{2}\boldsymbol{e}^H\boldsymbol{e}\right) = -\boldsymbol{Z}^{-1}\boldsymbol{d}, \tag{3}$$

where the superscripts $(\cdot)^H$ and $(\cdot)^{-1}$ denote conjugate transpose and matrix inversion, respectively. It is immediate that when $\boldsymbol{u} = \boldsymbol{u}^\star$, the error signal $\boldsymbol{e} = \boldsymbol{0}$, which completely cancels out the primary noise. When the matrix $\boldsymbol{Z}$ is singular, a common strategy for computing $\boldsymbol{u}^\star$ is to introduce a quadratic penalty function of $\boldsymbol{u}$ into the objective function. One can then easily derive the optimal solution of $\boldsymbol{u}^\star$ accordingly [8].

3. Towards Optimal Decentralized ANC Control

In this section, we first describe the optimization problem to allow for optimal decentralized control of an *N*-channel ANC system. A decentralized controller is optimal if it is able to achieve the same noise reduction performance as the centralized controller. We then explain how to solve the optimization problem using the genetic algorithm (GA).

3.1 Problem formulation

We start with the iterative update expression for an *N*-channel decentralized controller. Suppose its input to the *N* secondary sources at iteration k is $\boldsymbol{u}(k)$. The input signal $\boldsymbol{u}(k+1)$ at next iteration $k+1$ can be computed by following the steepest descent algorithm as

$$\boldsymbol{u}(k+1) = \boldsymbol{u}(k) - \mu\boldsymbol{C}\boldsymbol{e}(k), \tag{4}$$

where $\boldsymbol{C}$ =diag($[c_1, \dots, c_N]$) is a diagonal matrix to be determined, and $\mu > 0$ is the step-size of the steepest descent algorithm. Plugging the expression (2) for $\boldsymbol{e}$ at iteration k into (4) yields

$$\boldsymbol{u}(k+1) = [\boldsymbol{I} - u\boldsymbol{C}\boldsymbol{Z}]\boldsymbol{u}(k) - \mu\boldsymbol{C}\boldsymbol{d}, \tag{5}$$

where the matrix $\boldsymbol{Z}$ is assumed to be nonsingular. It can be shown that the iterates in (5) converge to a fixed point if and only if all the eigenvalues of the matrix in the square bracket have modulus less than one. By using algebra, it can be concluded that as long as the real parts of all the eigenvalues of $\boldsymbol{C}\boldsymbol{Z}$ are positive, a sufficient small step-size u would allow the iterates in (5) to converge. It is not difficult to show that the fixed point of (5) after convergence can be expressed as

$$\boldsymbol{u}(\infty) = -(\boldsymbol{C}\boldsymbol{Z})^{-1}\boldsymbol{C}\boldsymbol{d} = -\boldsymbol{Z}^{-1}\boldsymbol{d}, \tag{6}$$

which coincides with the optimal solution in (3), thus reaching the optimal noise reduction performance.

The above analysis suggests that the key step to ensure convergence of (4) or (5) is to find a proper

diagonal matrix $\boldsymbol{C}$ such that all the eigenvalues of the matrix product $\boldsymbol{CZ}$ are located in the right complex domain. The above problem can be formulated mathematically as

$$\boldsymbol{C}^* = \arg\min_{\boldsymbol{C}} 1 \qquad \text{subject to } \boldsymbol{C} = \text{diag}([c_1, \dots, c_N]) \text{ and } \lambda_{i,Re}(\boldsymbol{CZ}) > 0 \quad \forall i = 1, \dots, N, \tag{7}$$

where 1 represents a constant function, and $\lambda_{i,Re}(\cdot)$ denotes the real part of the ith eigenvalue of a matrix. Eq. (7) defines a nonconvex optimization problem in terms of the N complex numbers $\{c_1, c_2, \dots, c_N\}$ embedded in the diagonal matrix $\boldsymbol{C}$. It is noted from (7) that there are in total N nonlinear constraints, one for each matrix eigenvalue. That is, the number of free parameters is equal to the number of constraints. When N=2, it is clear from [10] that there exists $\boldsymbol{C}^*$ such that $\lambda_{i,Re}(\boldsymbol{C}^*\boldsymbol{Z}) \geq 0$ for $i = 1,2$. For the more general case N>2, it remains open to show the existence of a solution in (7).

We note that the optimal solution for $\boldsymbol{C}$ in (7) is not unique if it exists. One can simply multiply a positive scalar to $\boldsymbol{CZ}$, which would not change the sign of the eigenvalues. Alternatively, one can also twist an optimal solution a bit by adding a sufficiently small noise without changing the sign of the eigenvalues. In general, it is preferable to find an optimal solution for $\boldsymbol{C}$ such that the resulting eigenvalues are far away from the left complex domain, which would lead to fast algorithmic convergence.

Remark: The traditional decentralized controller in [8, 9] directly sets $\boldsymbol{C}$ to be $\boldsymbol{C} = \boldsymbol{Z}_d^H$, where $\boldsymbol{Z}_d$ is a diagonal matrix obtained by keeping only the diagonal elements of $\boldsymbol{Z}$. Empirical studies on an ANC system with 2, 6, and 15 channels in [9] demonstrate that the above special setup of $\boldsymbol{C}$ does not always converge, where the failure cases are due to the negative real parts of the eigenvalues of $\boldsymbol{Z}_d^H\boldsymbol{Z}$.

3.2 On shaping matrix eigenvalues using the genetic algorithm

In this subsection, we consider solving the nonconvex optimization problem of (7) using GA [12]. It is well known that GA is quite flexible and can be used to solve highly nonlinear and nonconvex optimization problems. Differently from the traditional optimization methods such as the steepest descent algorithm, GA does not rely on any gradient information. Instead, it creates a population of solution candidates. At each iteration, the candidates undergo a series of bio-inspired operations such as mutation, crossover and selection to produce new candidates. Only the candidates that produce low functional costs survive and the rests are eliminated to be able to evolve towards a satisfactory solution.

For the nonconvex problem (7), it is, in general, quite difficult to compute the gradient of the eigenvalues of $\boldsymbol{CZ}$ w.r.t. $\boldsymbol{C}$. This prevents it from employing traditional optimization methods for solving (7). In principle, GA can be used for solving (7) without difficulty due to its flexibility and generality. One property of GA is that given limited computation time, it may not reach the global optimal solution. For our problem (7), a solution that satisfies all the N inequality constraints is sufficient.

3.2.1 On utilizing GA

To be able to employ GA for solving (7), we first need to perform problem reformulation. A constant objective function 1 in (7) cannot be directly used in GA. To do so, we first introduce a number of notations. Given a complex number $c = c_r + jc_i$, we use $|c|$ and $\angle c \in [-\pi, \pi]$ to denote its amplitude and phase, respectively. To facilitate computation by GA, we set $\boldsymbol{C}$ in (7) to be a product of two diagonal matrices where one matrix is $\boldsymbol{Z}_d^H$, which can be expressed as

$$\boldsymbol{C} = \text{diag}(\boldsymbol{a})\boldsymbol{Z}_d^H, \tag{8}$$

where $\boldsymbol{a}$ is an N-dimensional complex vector to be optimized. Next we define two functions $\theta_{\max}(\boldsymbol{a}) \in [-\pi, \pi]$ and $\theta_{\min}(\boldsymbol{a}) \in [-\pi, \pi]$ to be

$$\theta_{\max}(\boldsymbol{a}) = \max_i \angle\, \lambda_i(\text{diag}(\boldsymbol{a})\boldsymbol{Z}_d^H\boldsymbol{Z}) \quad \text{and} \quad \theta_{\min}(\boldsymbol{a}) = \min_i \angle\, \lambda_i(\text{diag}(\boldsymbol{a})\boldsymbol{Z}_d^H\boldsymbol{Z}), \tag{9}$$

where the maximization and minimization in (9) are taken over the phase values of all the N eigenvalues of the matrix $\text{diag}(\boldsymbol{a})\boldsymbol{Z}_d^H\boldsymbol{Z}$.

We now construct a new optimization problem in terms of $\theta_{\max}(\boldsymbol{a})$ and $\theta_{\min}(\boldsymbol{a})$ as

$$\boldsymbol{a}^* = \arg\min_{\boldsymbol{a}} \left[g(\boldsymbol{a}) = \left((\theta_{\max}(\boldsymbol{a}))^4 \times 1_{\theta_{\max}(\boldsymbol{a})>0} + (\theta_{\min}(\boldsymbol{a}))^4 \times 1_{\theta_{\min}(\boldsymbol{a})<0}\right)\right], \tag{10}$$

subject to

$$b_u \geq |a_i| \geq b_l \qquad \forall i = 1, \dots, N \tag{11}$$

$$2\pi \geq \angle a_i \geq 0 \qquad \forall i = 1, \dots, N \tag{12}$$

where $1_{(\cdot)}$ is an indicator function. The two parameters b_u and b_l are the imposed upper and lower bounds on the magnitude of elements of $\boldsymbol{a}$. In practice, the lower bound b_l should be selected to be positive (i.e., b_l>0) to ensure that the overall matrix $\text{diag}(\boldsymbol{a})\boldsymbol{Z}_d^H\boldsymbol{Z}$ is nonsingular.

From a high level perspective, the optimization problem (8)-(12) is formulated to search for a solution

of $\boldsymbol{a}$ such that the phases of all the eigenvalues of the matrix $\mathrm{diag}(\boldsymbol{a})\boldsymbol{Z}_d^H\boldsymbol{Z}$ are close to 0 as much as possible. This is realized by penalizing the 4th order of the maximum and minimum eigenvalue phases $\theta_{\max}(\boldsymbol{a})$ and $\theta_{\min}(\boldsymbol{a})$ if necessary. If the solution space of (7) is not empty, one can alternatively solve the problem (8)-(12) to obtain a solution to (7).

In this work, we apply GA to solve the nonconvex problem (8)-(12). The objective function $g(\boldsymbol{a})$ will be used to evaluate the quality of each solution candidate generated by GA at each iteration, where only those candidates that produce small functional values are kept to give birth to new child candidates. The amplitude-phase constraints (11) -(12) are imposed on $\boldsymbol{a}$ to facilitate the computation of GA.

3.2.2 Solution refinement after the GA procedure

We note that the global optimal solution $\boldsymbol{a}^*$ to (8)-(12) should have the following property:

$$\theta_{\max}(\boldsymbol{a}^*) + \theta_{\min}(\boldsymbol{a}^*) = 0.$$

However, in practice, the GA method may not produce the global optimal solution given limited time and computational resources. It is highly likely that the output $\hat{\boldsymbol{a}}_{GA}$ of GA would lead to $\theta_{\max}(\hat{\boldsymbol{a}}_{GA}) + \theta_{\min}(\hat{\boldsymbol{a}}_{GA}) \neq 0$. That is, the maximum and minimum phases $\theta_{\max}(\hat{\boldsymbol{a}}_{GA})$ and $\theta_{\min}(\hat{\boldsymbol{a}}_{GA})$ are not symmetric around the real axis of the complex domain.

Based on the above analysis, we can refine the solution $\hat{\boldsymbol{a}}_{GA}$ by performing a rotation operation to the eigenvalues of $\mathrm{diag}(\hat{\boldsymbol{a}}_{GA})\boldsymbol{Z}_d^H\boldsymbol{Z}$ such that the resulting minimum eigenvalue phase is the negative of the maximum eigenvalue phase. The rotation operation can be easily achieved by multiplying a scalar $e^{j\phi}$ to $\hat{\boldsymbol{a}}_{GA}$, where the phrase ϕ can be computed as

$$\phi = -(\theta_{\max}(\hat{\boldsymbol{a}}_{GA}) + \theta_{\min}(\hat{\boldsymbol{a}}_{GA}))/2. \tag{13}$$

Based on (8)-(13), we can obtain the final estimate $\hat{\boldsymbol{C}}_{GA}$ for $\boldsymbol{C}$ in (7), which can be represented as

$$\hat{\boldsymbol{C}}_{GA} = e^{j\phi} \cdot \mathrm{diag}(\hat{\boldsymbol{a}}_{GA})\boldsymbol{Z}_d^H, \tag{14}$$

where $\hat{\boldsymbol{a}}_{GA}$ is computed by solving (8)-(12) using the GA method, and ϕ is obtained from (13).

4. Simulation Results

4.1 Simulation setup

We follow a similar simulation setup to that in [9, 10] to evaluate the proposed GA-based method for decentralized *N*-channel ANC. A monopole sound source is located inside a rectangular enclosure with *N* channels of loudspeakers and microphones placed near a baffled opening (see Fig. 2 for the setup of *N*=6, 12). The enclosure dimensions are chosen as $L_x = 0.432$ m, $L_y = 0.670$ m, and $L_z = 0.598$ m. The opening of the enclosure at $z = L_z$ is assumed to be embedded in an infinite baffle. The primary source is placed at $r_s = (0.12, 0.12, 0.16)$ m with a strength of $q_0 = 1\times10^{-4} m^3/s$.

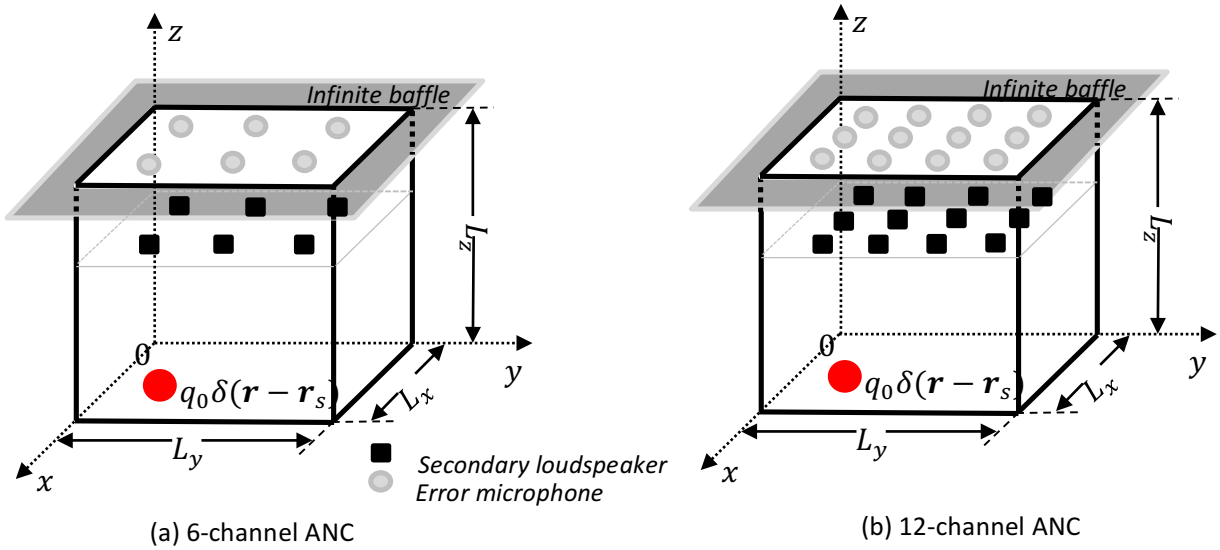

Figure 2 - A monopole sound source inside a rectangular enclosure with a baffled opening.

The channel numbers of $N = 4, 6, 12$ were studied in the simulation. For each *N*-channel ANC, the *N* error microphones were evenly placed on the open surface at $z = L_z$ while the *N* loudspeakers were evenly placed right below the error microphones at the vertical level $z = L_z - 0.1\,m$. We consider noise

reduction performance at 100 evenly sampled frequency bins from 5 Hz to 500 Hz: $f = 5\text{ Hz}, 10\text{ Hz}, \ldots, 500\text{ Hz}$. As the GA method needs to be applied for each ANC setup at each frequency bin, 300 $\boldsymbol{Z}$ matrices were tested in total using our proposed method. The default implementation of the GA method in Matlab R2016a was used in our simulations. The two bounds in (11) were set to be $b_u = 20$ and $b_l = 0.5$, respectively.

For performance comparison, we tested both the conventional decentralized controller considered in [8, 9], and our newly proposed decentralized controller. While the convergence of our proposed controller is characterized by the real parts of the eigenvalues of $\widehat{\boldsymbol{C}}_{GA}\boldsymbol{Z}$ (see Section 3) for each frequency bin, the convergence of the conventional controller in [8, 9] is determined by the eigenvalues' real parts of $\boldsymbol{Z}_d^H\boldsymbol{Z}$. Therefore in the following, we mainly study the eigenvalue properties of the two matrices $\widehat{\boldsymbol{C}}_{GA}\boldsymbol{Z}$ and $\boldsymbol{Z}_d^H\boldsymbol{Z}$ across different frequency bins over the ANC channel numbers of 4, 6, and 12.

4.2 On stability of the proposed decentralized control method

Fig. 3 shows the minima of the eigenvalues' real parts and their signs for the two matrices $\boldsymbol{Z}_d^H\boldsymbol{Z}$ and $\widehat{\boldsymbol{C}}_{GA}\,\boldsymbol{Z}$ for the channel numbers of 4, 6, 12, where it is noted that $\boldsymbol{Z}_d^H\boldsymbol{Z}$ and $\widehat{\boldsymbol{C}}_{GA}\,\boldsymbol{Z}$ are extracted from the decentralized controller of [8, 9] and our proposed controller, respectively. It is clear from the figure that for each channel number, our proposed method manages to shape the eigenvalues of $\widehat{\boldsymbol{C}}_{GA}\,\boldsymbol{Z}$ properly over all the considered frequency bins, ensuring that the real parts of all the eigenvalues are positive. As a result, the proposed control method is able to achieve the same noise reduction performance as the centralized one. This indicates that the GA method is effective for finding a satisfactory solution $\widehat{\boldsymbol{C}}_{GA}$ for each matrix $\boldsymbol{Z}$ measured from the considered ANC setup.

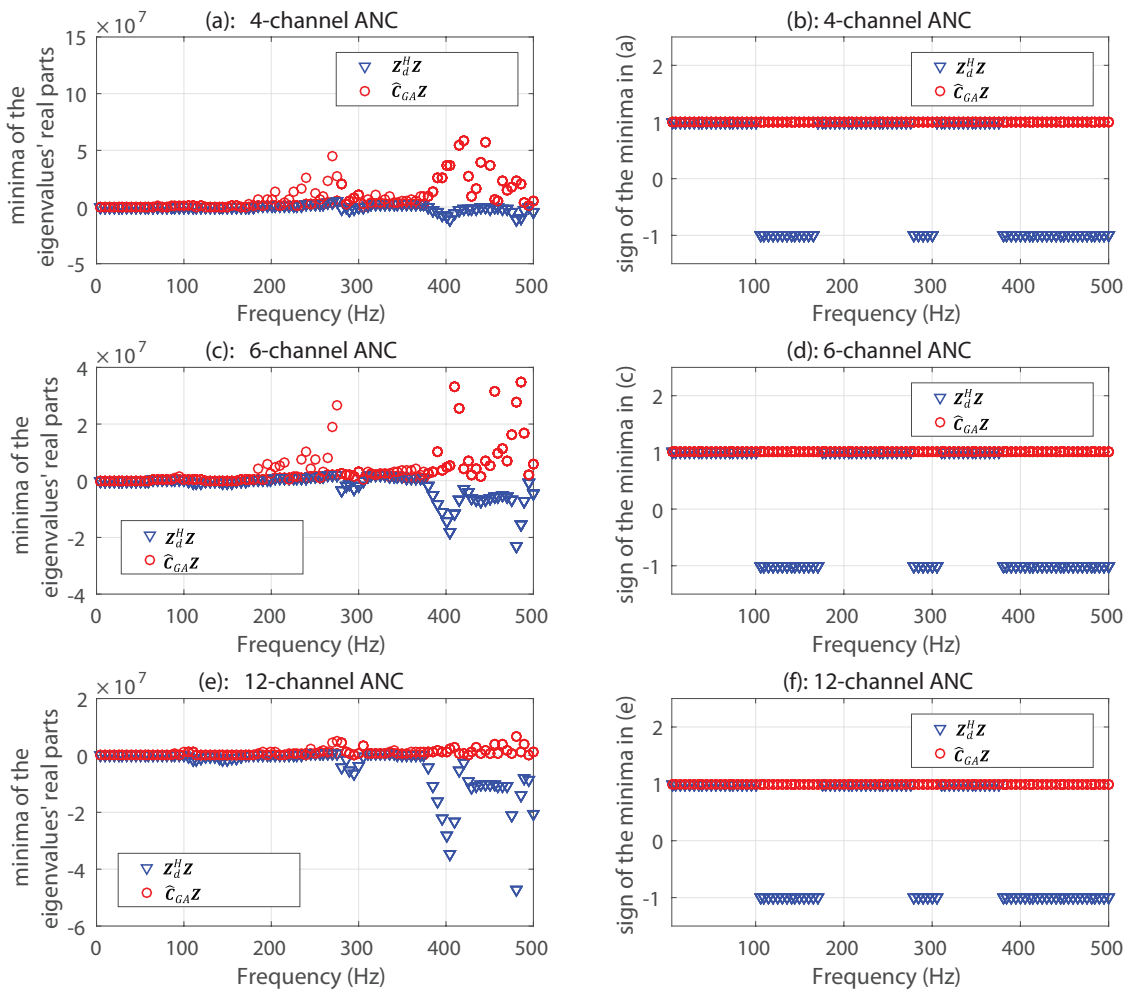

Figure 3 - The minima of the eigenvalues' real part and their signs. (a)-(c)-(e): the minima of the eigenvalues' real parts of $\boldsymbol{Z}_d^H\boldsymbol{Z}$ and $\widehat{\boldsymbol{C}}_{GA}\,\boldsymbol{Z}$. (b)-(d)-(f): the signs of the eigenvalues' real part minima of $\boldsymbol{Z}_d^H\boldsymbol{Z}$ and **("1" and "-1" stand for positive and negative respectively)**

By contrast, the conventional decentralized method in [8, 9] would diverge at the frequency bins where the minima of the eigenvalues' real part are negative. To fix the convergence issue in this situation, the noise reduction performance of the conventional decentralized method has to be sacrificed by imposing constraints on the magnitude of the input to the loudspeakers (see [8, 9] for details).

4.3 Comparison of eigenvalue distributions

In this subsection, we study the eigenvalue distributions of $\boldsymbol{Z}_d^H\boldsymbol{Z}$ and $\widehat{\boldsymbol{C}}_{GA}\boldsymbol{Z}$ in the complex domain at two frequencies of 350 Hz and 500 Hz. Our primary motivation to do so is to reflect the controllability of the GA method on the resulting eigenvalues of $\widehat{\boldsymbol{C}}_{GA}\boldsymbol{Z}$ qualitatively. The two frequencies were chosen carefully so that the minima of the eigenvalues' real parts of $\boldsymbol{Z}_d^H\boldsymbol{Z}$ at 350 Hz and 500 Hz are negative and positively, respectively, (see Fig. 3). To facilitate comparison between the two matrices $\boldsymbol{Z}_d^H\boldsymbol{Z}$ and $\widehat{\boldsymbol{C}}_{GA}\boldsymbol{Z}$, the eigenvalues of each matrix were normalized by division over its largest eigenvalue magnitude.

Fig. 4 shows the normalized eigenvalues of the two matrices for the two considered frequencies across the channel number of 4, 6, and 12. For the simple case of 350 Hz, it is seen that the GA method managed to squeeze the phases of all eigenvalues of $\widehat{\boldsymbol{C}}_{GA}\boldsymbol{Z}$ around 0 for all three channel numbers while the phases of $\boldsymbol{Z}_d^H\boldsymbol{Z}$ are scattered in the right complex domain. At 500 Hz, the eigenvalue phases of the matrix $\widehat{\boldsymbol{C}}_{GA}\boldsymbol{Z}$ are pushed in a relatively small phase region by the GA method. The above properties suggest that the reformulated optimization problem (8) -(12) is effective, which makes the GA method work as expected.

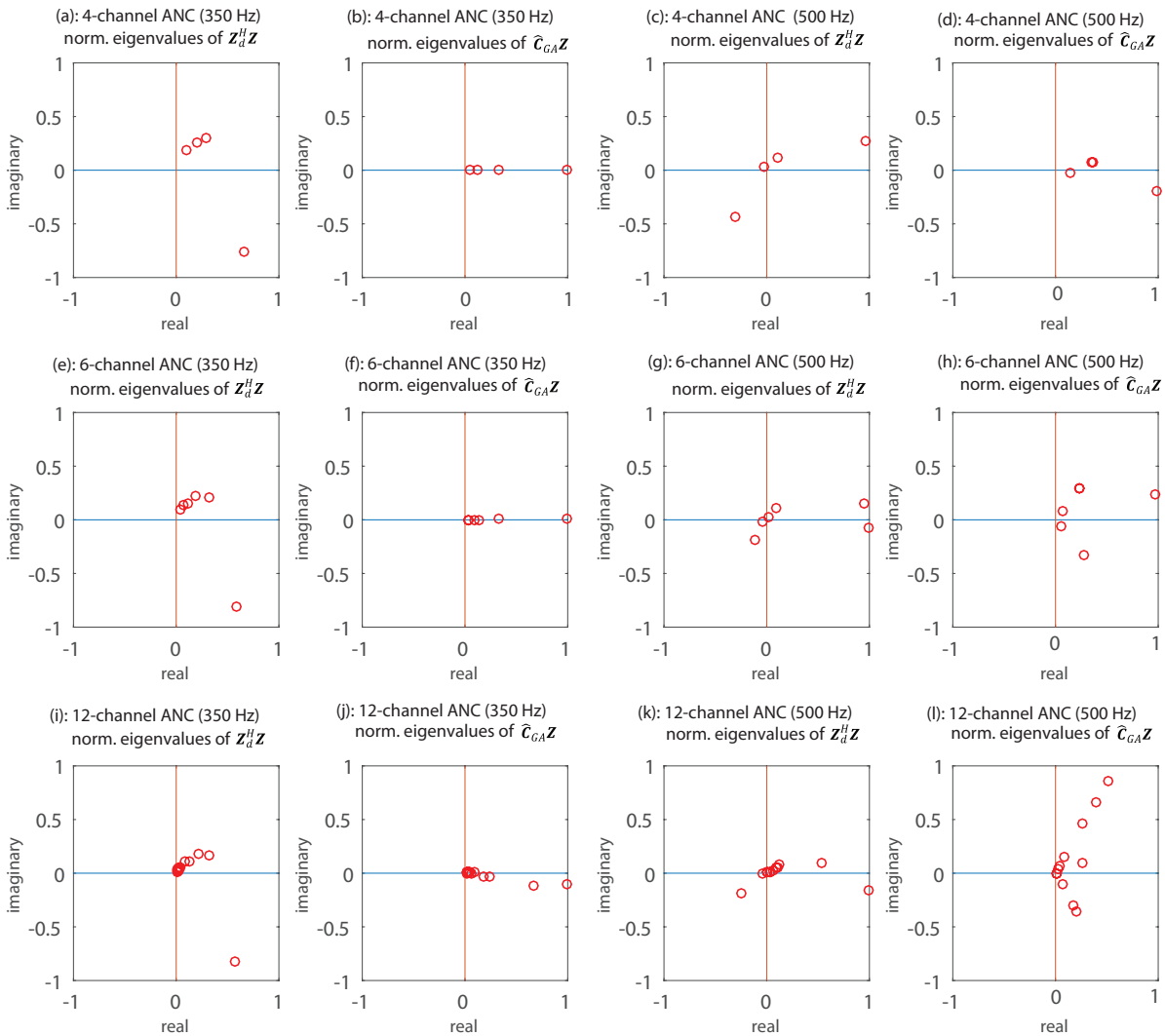

Figure 4 – Visualization of the normalized eigenvalues of $\boldsymbol{Z}_d^H\boldsymbol{Z}$ and $\widehat{\boldsymbol{C}}_{GA}\boldsymbol{Z}$ at two frequencies of 350 Hz and 500 Hz over the ANC channel numbers of 4, 6, and 12.

5. CONCLUSIONS

In this paper, we have studied decentralized control of an N-channel ANC system in the frequency domain. As an extension of our previous two-channel work [10], we considered shaping the eigenvalues of an $N\times N$ matrix for each frequency bin by using the GA method. To be able to apply the GA method, we have built a special nonconvex optimization problem in terms of the maximum and minimum eigenvalue phases. Simulation results for the ANC channel numbers of 4, 6, and 12 over a semi-open rectangular enclosure suggest that the GA method indeed is able to effectively push all the eigenvalues of the $N\times N$ matrix to the right complex domain, thus ensuring the convergence of the corresponding decentralized controller to reach the optimal noise reduction performance.

The research conducted in this paper is empirical. It still remains open to find out the existence of a solution for an arbitrary $N\times N$ matrix extracted from a real N-channel ANC system that can lead to optimal decentralized ANC control.

In addition, the GA method would become increasingly heavy as the channel number N increases. To allow for decentralized control of a large-scale ANC system (e.g., several hundred channels), it is desired to develop advanced optimization methods other than the GA method that can properly shape the eigenvalues of the $N\times N$ matrix with low complexity.

ACKNOWLEDGEMENTS

This research was supported under the Australian Research Council's Linkage Project funding scheme (LP140100987, Integrated Passive and Active Control of Humming Noise from KCGMs Haul Trucks).

REFERENCES

1. Kuo S, Mitra S, Gan W-S. Active noise control system for headphone applications. IEEE Trans. Control Systems Technology, vol. 14, no. 2, pp. 331–335, 2006.
2. Chen C, Chiueh T-D, Chen J-H. Active cancellation system of acoustic noise in MR imaging. IEEE Trans. Biomedical Engineering, vol. 46, no. 2, pp. 186–191, 1999.
3. Diego M-D, Gonzalez A, Ferrer M. Pinero G. An adaptive algorithms comparison for real multichannel active noise control. 12th European Signal Processing Conference, 2004.
4. Kosaka T, Elliott S-J, Boucher C-C. A novel frequency domain filtered-x LMS algorithm for active noise reduction. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), 1997, pp. 403–406.
5. Ferrer M, Diego M, Pinero G, Gonzalez A. Active noise control over adaptive distributed networks. signal processing, vol. 107, pp. 82–95, 2015.
6. Antonanzas C, Ferrer M, Gonzalez A, Diego M-D, Pinero G. Diffusion algorithm for active noise control in distributed networks. 22nd International Congress on Sound and Vibration, 2015.
7. Chu Y, Mak C-M. Diffusion control for multi-channel ANC systems using Filtered-X LMS algorithms. 24th International Congress on Sound and Vibration, 2017.
8. Elliott S-J and Boucher C-C. Interaction between multiple feedforward active control systems. IEEE Trans. Speech and Audio Processing, vol. 2, no. 4, pp. 521–530, 1994.
9. Tao J, Wang S, Qiu X, Pan J. Performance of an independent planar virtual sound barrier at the opening of a rectangular enclosure. Applied Acoustics, vol. 105, pp. 215–223, 2016.
10. Zhang G, Tao J, Qiu X, Burnett I. Decentralized two-channel active noise control for single frequency by shaping matrix eigenvalues. IEEE/ACM Trans. Audio, Speech, and Language Processing, 27(1):44-52, 2019.
11. An G, Cao Y, Liu B. Optimized decentralized adaptive control of noise and vibration for periodic disturbances. The Journal of the Acoustical Society of America, pp. 275-280, 2018.
12. Goldberg D-E. Genetic Algorithms in Search, Optimization and Machine Learning. Addison-Wesley Longman Publishing Co., 1989.