

Numerical study on energy transmission for soft materials and metamaterials structures by structural intensity method

Rong Huang¹; Ziqian Li¹; Yiheng Xue¹; Zishun Liu¹;

¹ International Center for Applied Mechanics, State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiao Tong University, China

ABSTRACT

In this work, we studied the energy transmission and dynamic performance of multiple structures from the aspect of energy flow by using structure intensity (SI) method. Compared with the traditional vibration or dynamic analysis methods, the SI method considers the internal force and displacement in the structures, and can accurately reflect the effect of the excitation source, the damper or virtual damper locations and the energy flow on the structures. In this study, numerical simulation system was built to represent different structures under different loading and damping cases. We get the corresponding results of structural intensity distribution and plot the SI streamline for each case. We find that the structure intensity distribution can be significantly affected by adjusting the material parameters and setting damper in specific locations. Based on the SI energy distribution of the engineering materials, soft materials and metamaterials structures, we studied the methods to how to reduce the vibration of the structure or identify the potential damage positions by using its main energy propagation paths. In addition, the effect of structural damping on the energy flow transmission performance of metamaterials structures is discussed for swelling process. We hope that our works could provide researchers new ideas of vibration control for similar new smart structures in potential industry applications.

Keywords: Structure Intensity, metamaterials, energy flow

1. INTRODUCTION

The study on structural intensity of vibration and vibro-acoustic characteristic in different structures has been investigated extensively. The concept of structural intensity was first introduced as a quantifier for the structural vibro-acoustic analysis in 1970s by Noiseux.(1) Analogous to the concept of acoustic intensity in acoustic medium, the structural intensity can be expressed as the result of stress and the velocity, a direction sum of the scalar product of the force vectors and their corresponding velocity vectors. Pavic developed the formulation of structural intensity and applied it to complex structure entity, which led to the growing interest to this field for the next few decades in applied mechanics community.(2) Hambric developed the computational approach of 3D structural intensity through the finite element method.(3) Gavric and Pavic evaluated the structural intensity fields of a rectangular plate to identify the source and the sink of the energy flow.(4) Xu et al.(5) and Liu et al.(6) firstly successfully introduced the structural intensity streamline concept into plate structures. Furthermore, Liu et al.(6) theoretically derived the formulations for structural intensity streamline according to the concept of fluid mechanics streamline. The structural intensity streamline can be used to clearly display the structural intensity flow paths, which provide the visualization expression of the energy sources and sinks in structures.

The structural intensity streamline concept was then introduced into different engineering applications. Guo et al.(7) studied the SI distribution of beam structures under random vibration. Liu et al.(8) applied the SI methods into medical field to study the vibrational characteristics of human body snoring identification. In noise control aspect, Liu et al.(9) adopted the SI method to propose a basic method of active and passive interior noise control for box structures, which would be used for vehicle cabins. Ang et al.(10) presented a potential for controlling cabin noise of automobiles and armoured vehicles. Besides, the SI streamline concept can also be helpful for the detection of cracks. For

¹ zishunliu@mail.xjtu.edu.cn

example, Tian et al. (11) applied the SI method to detect crack of offshore platform. Lee et al. (12) studied the diversion of energy flow near crack tips of a vibrating plate. Overall, the study in structural intensity method still has a great potential and can be applied into multiple complex engineering problems.

This study is mainly an extension of our previous work for the structure intensity method. We will investigate the potential application for structure intensity method in the study of fracture and fatigue problems of soft materials (such as hydrogel), metamaterials structures. The SI method may be useful to investigate their vibration characteristics and failure mode under multiple loading cases with different structures. We will first propose the vibration mode method to calculate the structural intensity of cylindrical shell structures theoretically and investigate the vibro-acoustic behaviors of cylindrical shell structures. Then the SI distribution for notched hydrogel plate will be briefly introduced. Currently, the fracture and fatigue problems of hydrogel soft materials become the new research trend, what is the SI distribution when the dynamic loading is applied? The SI distributions of a notched hydrogel plate and a kind of metamaterials structures are provided in this short summary paper. For notched hydrogel plate, our results demonstrate the effect of Mullins-effect for hydrogel. For metamaterials structure, we found that the SI distribution is significantly changed during deformation process.

2. RESULTS

2.1 Using SI approach to characterize vibro-acoustic behavior of the cylindrical shell structures

2.1.1 Formulation of structural intensity by vibration mode shapes

From our study, we found the structural intensity is an infinite weighted sum of function of vibration mode shapes. Simplify the detailed formulation to weight coefficients about load conditions and distribution functions of vibration mode shapes, the SI of the cylindrical shell structure along axial direction and circumferential direction can be further expressed as follow:

$$I_x = -(\omega_0/2)C \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} f_{mn} \cdot f_{rs} \cdot X_{mnr s}(x, \theta) \quad (1)$$

$$I_\theta = -(\omega_0/2)C \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} f_{mn} \cdot f_{rs} \cdot \Theta_{mnr s}(x, \theta)$$

where C is model coefficient; ω_0 is exciting frequency; $f_{mn} \cdot f_{rs}$ are weight coefficients; $X_{mnr s}, \Theta_{mnr s}$ are distribution functions; m, n, r, s are wave numbers. The wave numbers indicates the number of waves on the vibration shapes and represents a definite modal. We use (m, n) to represent the modality of a force condition, using (r, s) for a displacement of the modal. Finally, the (m, n, r, s) can represent a modal state of the structural intensity component. In this equation, the weight coefficient components show the load conditions and reflect the weight coefficient of time.

The structural intensity streamlines are a family of curves that are instantaneously tangent to the vector of the structural intensity. For steady vibration, the structural intensity streamlines are the vibrational power flow paths and can effectively assess vibrational sources and sinks. The structural intensity streamline in the thin cylindrical shell surface can be defined as (6):

$$d\mathbf{r} \times \mathbf{I}(\mathbf{r}, t) = 0 \quad (2)$$

where r is particle position; I is structural intensity vector. The structural intensity vectors that located on the streamlines are perpendicular to r and parallel to dr. For the cylindrical shell, the differential equation describing is:

$$\frac{dx}{I_x} + \frac{Rd\theta}{I_\theta} = 0. \quad (3)$$

Every particular point that locate on the cylindrical shell corresponds with a structural intensity vector, and the vector is tangential to a streamline at this point. Therefore the group of streamlines can describe the power flow travel paths on the cylindrical shell.

2.1.2 Vibro-acoustic behavior of the cylindrical shells

Once we get the above formulation, we can calculate the structural intensity for different loading case. If we make the loop-integration of structural intensity and applied it on the internal surface of vibrating cylindrical shell, which is the coupling surface of vibration and sound, structural intensity energy can be obtained. And at mean time the volume integral of internal acoustic intensity is considered as sound energy in sound field. Once we get this two-energy value, we find that the internal acoustic pressure field has the close connection with structural intensity in the interface of structural-acoustic coupling. From figure 1, we know that the structural intensity energy and the radiation sound energy almost follows the same trend when varying damping cases and exaction frequencies. Therefore, it provides a method to control the sound radiation energy for a vibrating structure by changing its structural intensity energy.

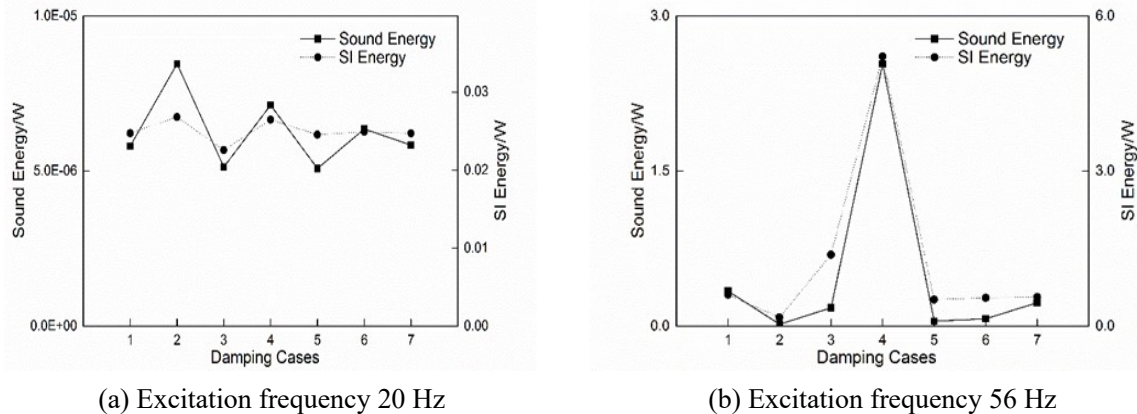


Figure 1 Structural intensity and sound energy trends in various cases

Based on this result, we can further provide a method which can reduces the acoustic energy through properly putting damper position and redistributing the structural intensity in the cylinder shell is developed. For cylinder shell structures, there are some eddies pattern in the structural intensity streamline, and these eddies have potential of collecting energy. When the damper is put in the eddy position, the stored energy will be released and a new sink appears. According to source and sink positions, we can modify the structural intensity distribution so as to reduce the acoustic energy of cylinder shell structure. (13)

2.2 Vibration performance of slotted plate by using SI method

We have also investigated the SI distribution of a given simply supported slotted plate under sinusoidal force loading condition. The effect of slotted depth on SI distribution of slotted plate and its vibration control method has been investigated. It is easily found that the slot depth can significantly affect the SI distribution of the slotted plate. According to the SI distribution, setting the position of the damper close to the excitation source can obviously affect the vibration performance of the slotted plate. As an alternative to traditional dampers, the ease and effectiveness of using a constrained hydrogel layer have been considered to dissipate energy. The simulation shows that hydrogel materials can effectively reduce the maximum stress and maximum displacement of the slotted plate. This highlights the potential applications of the hydrogel in vibration control systems, and it proves that hydrogel layers can be applied to dissipate energy. (14)

2.3 Using SI to detect Hydrogel crack propagation for fatigue test case

We established a hydrogel model to simulate pure shear fatigue test, and the size of crack tip is 0.4mm. For dynamic analysis, we taken time period as 1s. During this time step, the model will be stretched twice as shown Figure 2.

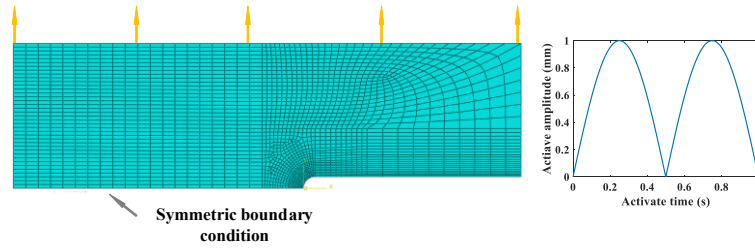


Figure 2 The schematic of hydrogel crack model. The model is a 1/2 up-down symmetric model.

The SI streamline for every time frame is shown in Figure 3a-c. The streamline distributions vary dramatically at a different time frame. At $t=0.15s$ the distribution of streamline is gentle. But at highly stretched state, streamline distribution becomes complex, and a virtual source and a virtual sink of streamline will appear. Besides, SI streamline can represent the energy flow within material. In Figure 3a-b, the applied excitation does work to the hydrogel system, hence we can see that the energy flows from upper line to the lower line and the crack tip. In Figure 3c, the model is restoring its deformation and releasing stored elastic energy, thus the energy is flowing out of the hydrogel model.

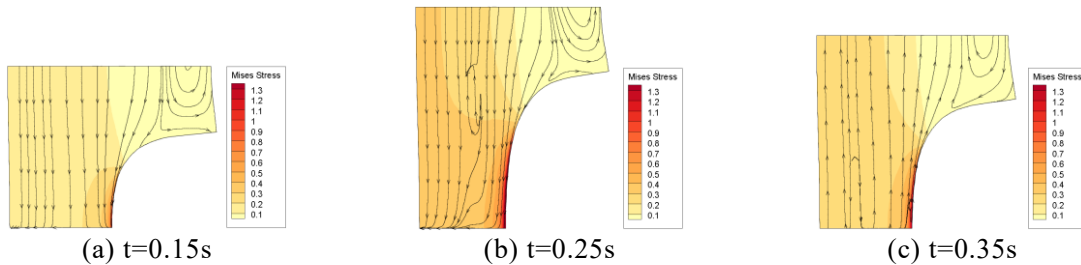


Figure 3 The distribution of SI streamline at different time frame. (a) $t=0.15s$, (b) $t=0.25s$, there are a virtual source and a virtual sink (c) $t=0.35s$, there is also a virtual source.

2.4 Energy flow transformation for metamaterial structure under dynamic loading during swelling of hydrogel

We also build a metamaterial structure as shown in Figure 4. In this case, we first use Static-General step to simulate the process of hydrogel swelling. For different swelling state, we then analyze their SI fields.

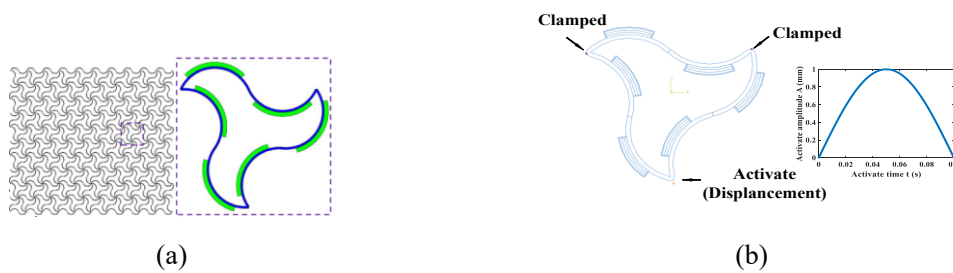


Figure 4. (a) The schematic of the metamaterial structure and its representative volume element. (b) The boundary condition in unit cell model.

As shown in Figure 4a, the green area can absorb water and then swell. We select two equilibrium states (before swelling and swollen state) in the Static-General step. Then we export the models of these two states into a new case with Dynamic analysis. Here, the time period is 0.1s and we hold the two upper corners and apply sinusoid excitation at the lower corner as shown in Figure 4b. From Figure 5, we plot the SI streamlines for different swelling state. We can easily see that the SI streamline distributions are much more complicated than that in hydrogel swelling for normal structure. It is as the result of the complex structure of the metamaterial deformation. Besides, in this case, there are much more vortexes with metamaterial. We also show the effect of the Mullins effect on SI field. We find that viscoelasticity indeed influences the SI distribution for soft materials, that is, Mullins effect can weaken vortexes within the material and result in energy dissipation. This finding

may be useful for further metamaterial structure control.

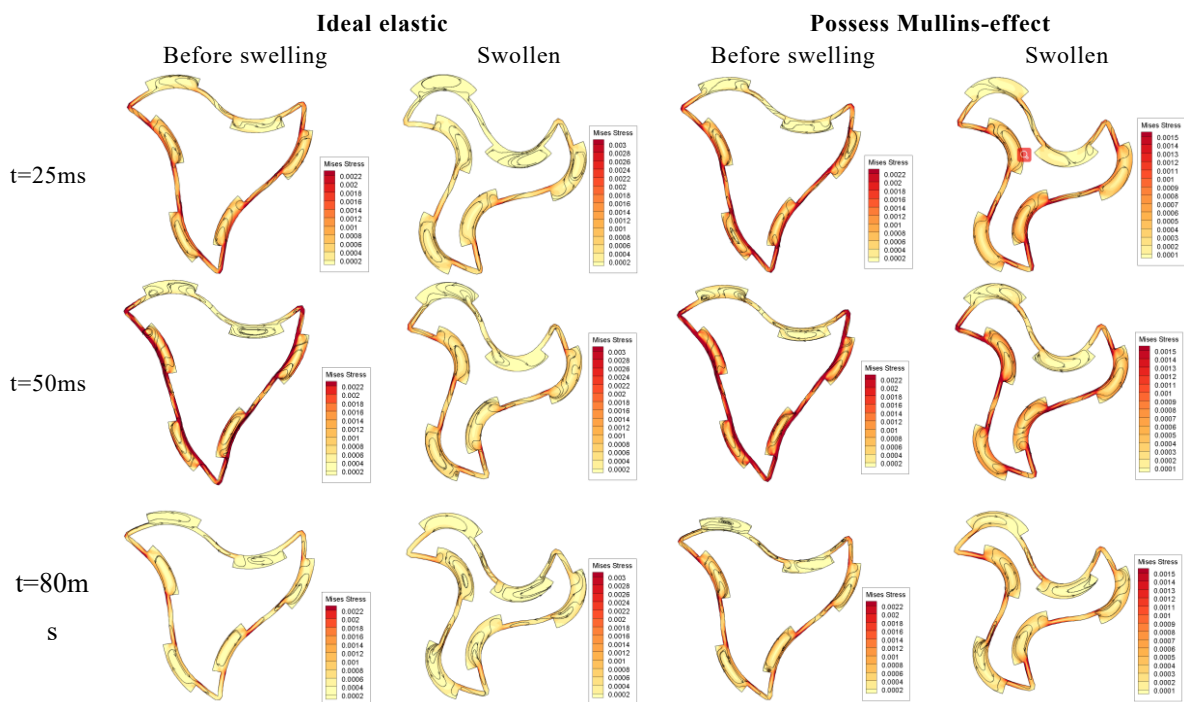


Figure 5 The distribution of SI streamlines on the metamaterial structure.

3. CONCLUSIONS

This paper summarized some SI potential applications for vibrational structures, fracture of soft materials and metamaterial structure. For regular structure such as cylinder shell and plate structures, we found there are some eddies pattern, which have potential of collecting energy in the structural intensity. Therefore, the vibration energy can be adjusted by setting dampers to create new sinks in SI streamline and release stored energy. The SI streamline can also clearly demonstrate the energy distribution and dissipation pattern for soft material in fracture mode as well as metamaterial structure under complex loading conditions. This could provide researchers new ideas when analyzing deformation mechanisms and failure mode for similar smart structures (hydrogel and shape memory polymers) in potential industry applications.(15, 16)

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