

Impact of Network Parameters Uncertainties on Distribution Grid Power Flow

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Abstract—Many distribution grid management functions rely upon power flow algorithms to analyse the behaviour of the grid under specific conditions. The knowledge of the grid parameters is usually the starting point for the definition of the models behind power flow algorithms. However, in the distribution system scenario, several factors can affect the level of confidence with which these parameters are known. This paper aims at investigating the main sources of uncertainty in the modelling of distribution lines, taking into account the specific characteristics of the distribution system, and it studies the impact of those uncertainties on the power flow results. The goal is to identify which factors can potentially bring severe degradations of the power flow results, so that the distribution grid model can be improved accordingly or their impact can be duly considered when evaluating the results.

Index Terms—Power Flow, Distribution Network Analysis, Distribution Grid Model, Power System Simulation, Network Impedance, Line Parameters Uncertainties.

I. INTRODUCTION

The transformation of distribution systems into active and complex networks calls for the development of suitable tools for distribution network analysis and for grid management and control [1]. Many of the available tools rely on power flow algorithms to assess the behaviour of the distribution grid in specific scenarios or to determine possible control actions to be performed. Hosting capacity studies for the integration of renewable energy sources or real-time applications, like voltage control, network topology reconfiguration or storage management, are only some examples of functions that can integrate power flow procedures in their core [2], [3]. The reliability of the power flow algorithm and of its results is thus of paramount importance for undertaking correct decisions and having an efficient management and control of the grid.

Based on these considerations, in recent years, several research efforts focused on the development of power flow algorithms tailored to the distribution system. Proposed algorithms mainly provide improved models of the distribution grid, usually to deal with specific characteristics of the system, such as its three-phase nature, the presence of highly unbalanced conditions, the role of the neutral wire or the impact brought by the grounding system [4]–[6]. A common assumption for

all these models is to have the exact knowledge of the line parameters. However, different factors can affect the level of confidence with which these parameters are known. In the literature, the impact of the network parameters uncertainties is often disregarded or not duly evaluated. Probabilistic power flow algorithms are sometimes used to evaluate the range in which the power flow results can vary, but these approaches mainly focus on the variability associated to the operating conditions (uncertainty in the power consumption or generation) [7], [8]. In other cases, the presence of the network parameters uncertainties is instead considered, but it is modelled as a random variation around a reference value [9], which does not allow taking into account the more severe effects brought by possible correlations or systematic errors. Correlations or mean errors can indeed arise due to different reasons. In [10], the effects of conductors' temperature is integrated in the power flow model: neglecting this aspect is an example of possible systematic errors. To address this issue, in [11], an interval power flow algorithm has been proposed, which allows also dealing with the cumulative effects of network uncertainties. This solution gives a more realistic picture of the possible variations of the power flow output, but the characteristics of the network uncertainties are not analysed in detail.

This paper aims at performing a more detailed analysis of the network parameters uncertainties and at investigating the potential impact of such uncertainties on the distribution grid power flow results. The first goal is to identify the main sources of uncertainty in the modelling of the distribution lines. To this purpose, the relationships behind the definition of the line parameters are analysed in order to identify the root causes of uncertainty. Based on this, the second objective is to analyse the potential impact of such uncertainties on the power flow results. As previously mentioned, different uncertainty sources can present different uncertainty characteristics. This work points out the detrimental effects of possible correlations and systematic errors and proves that these are relevant aspects that must be considered when evaluating the overall impact of network uncertainties.

II. MODELLING OF DISTRIBUTION GRIDS

From a modelling perspective, distribution grids significantly differ from the transmission power systems due to the possible presence of two-phase and single-phase laterals and

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of highly unbalanced conditions. This leads to the use of three-phase grid models, where mutual impedances must be duly taken into account, and to the necessity of integrating in the model the neutral and the ground return path where the unbalanced current flows [12].

In the *pi*-model of a three-phase line with neutral conductor, mutual components of inductance and capacitance exist among all the conductors of the line (here including also the neutral). The self and mutual terms of the series impedance are usually computed using the Carson's equations [12]. These equations allow integrating in the impedance the effects given by the possible circulation of unbalanced currents through the ground. Applying the Carson's equations to each conductor of the line (see [13] for more details), a *primitive* impedance matrix $\hat{\mathbf{Z}}_p$ having the following form can be obtained (eq. (1)):

$$\hat{\mathbf{Z}}_p = \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & \hat{z}_{an} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & \hat{z}_{bn} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & \hat{z}_{cn} \\ \hat{z}_{na} & \hat{z}_{nb} & \hat{z}_{nc} & \hat{z}_{nn} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{Z}}_{ii} & \hat{\mathbf{Z}}_{in} \\ \hat{\mathbf{Z}}_{ni} & \hat{\mathbf{Z}}_{nn} \end{bmatrix} \quad (1)$$

where $\hat{z}_{jj}$, with $j = \{a, b, c, n\}$, is the self impedance of conductor j , and $\hat{z}_{jk}$, with $j, k = \{a, b, c, n\}$ and $j \neq k$, is the mutual impedance between conductors j and k . These terms can be calculated as shown in eq. (2) and (3):

$$\hat{z}_{jj} = r_j + k_r f + j k_{l1} f \cdot \left(\ln \frac{1}{GMR_j} + k_{l2} + 0.5 \ln \frac{\rho}{f} \right) \quad (2)$$

$$\hat{z}_{jk} = k_r f + j k_{l1} f \cdot \left(\ln \frac{1}{D_{jk}} + k_{l2} + 0.5 \ln \frac{\rho}{f} \right) \quad (3)$$

where r_j and GMR_j are the resistance and the Geometric Mean Radius (GMR) of the conductor, D_{jk} is the distance between conductors j and k , f is the frequency of the system, ρ is the earth resistivity and k_r , k_{l1} and k_{l2} are constant values that depend on the used measurement units ([12] gives their values to obtain the impedance in Ω/mile).

The primitive impedance matrix can be partitioned in the sub-matrices indicated in (1), where $\hat{\mathbf{Z}}_{ii}$ is the sub-matrix associated to the three phases of the grid, $\hat{\mathbf{Z}}_{in}$ and $\hat{\mathbf{Z}}_{ni}$ are the vectors with the mutual terms between phases and neutral, and $\hat{\mathbf{Z}}_{nn}$ is the self impedance of the neutral. Under the hypothesis of multi-grounded neutral, the primitive impedance matrix can be transformed into an equivalent 3×3 phase impedance matrix $\mathbf{Z}_{abc}$ by means of the Kron reduction (eq. (4)):

$$\mathbf{Z}_{abc} = \hat{\mathbf{Z}}_{ii} - \hat{\mathbf{Z}}_{in} \cdot (\hat{\mathbf{Z}}_{nn})^{-1} \cdot \hat{\mathbf{Z}}_{ni} \quad (4)$$

Using the phase impedance matrix $\mathbf{Z}_{abc}$ computed in (4), the voltage drop between the terminal nodes m and n of a line can be calculated as follows in eq. (5):

$$\begin{bmatrix} v_{n,a} \\ v_{n,b} \\ v_{n,c} \end{bmatrix} = \begin{bmatrix} v_{m,a} \\ v_{m,b} \\ v_{m,c} \end{bmatrix} - \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \cdot \begin{bmatrix} i_{mn,a} \\ i_{mn,b} \\ i_{mn,c} \end{bmatrix} \quad (5)$$

where $v_{s,j}$ with $s = \{m, n\}$ and $j = \{a, b, c\}$ is the voltage at phase j of bus s , while $i_{mn,j}$ is the current flowing at the phase j of the line.

A similar process can be performed to determine the shunt admittance matrix resulting due to the capacitive coupling among conductors and between them and the ground. A primitive potential matrix $\hat{\mathbf{P}}_p$, with a form similar to the primitive impedance matrix in (1), is first computed. The self and mutual potential coefficients ($\hat{p}_{jj}$ and $\hat{p}_{jk}$, respectively) can be calculated via eq. (6) using the method of conductors images (see [12] for more details):

$$\hat{p}_{jj} = k_c \cdot \ln \frac{S_{jj}}{R_j} \quad \text{and} \quad \hat{p}_{jk} = k_c \cdot \ln \frac{S_{jk}}{D_{jk}} \quad (6)$$

where S_{jj} is the distance between conductor j and its image, S_{jk} is the distance between conductor j and the image of conductor k , R_j is the radius of conductor j , and k_c is a constant value that depends on the used measurement units ([12] gives its value to calculate the coefficients in $\text{mile}/\mu\text{F}$).

From the primitive matrix $\hat{\mathbf{P}}_p$, using the Kron reduction, the phase potential matrix $\mathbf{P}_{abc}$ can be obtained. The capacitance matrix $\mathbf{C}_{abc}$ of the line is the inverse of the potential matrix $\mathbf{P}_{abc}$. Such capacitance is finally used to calculate the associated 3×3 shunt admittance matrix $\mathbf{Y}_{abc} = j2\pi f \cdot \mathbf{C}_{abc}$. In the *pi*-model, the shunt admittance is divided between start and end node of the line and it brings an injection of reactive current at the beginning and the end of the line. As an example, at the starting node m of a line between nodes m and n , it is:

$$\begin{bmatrix} i_{mn,a}^{in} \\ i_{mn,b}^{in} \\ i_{mn,c}^{in} \end{bmatrix} = \begin{bmatrix} i_{mn,a} \\ i_{mn,b} \\ i_{mn,c} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} y_{aa} & y_{ab} & y_{ac} \\ y_{ba} & y_{bb} & y_{bc} \\ y_{ca} & y_{cb} & y_{cc} \end{bmatrix} \cdot \begin{bmatrix} v_{m,a} \\ v_{m,b} \\ v_{m,c} \end{bmatrix} \quad (7)$$

where $i_{mn,j}^{in}$ is the current entering the line at phase j , and y_{jk} with $j, k = \{a, b, c\}$ is the generic term of the shunt admittance matrix $\mathbf{Y}_{abc}$.

III. NETWORK PARAMETERS UNCERTAINTIES

Eq. (5) and (7) are the relationships linking the electrical quantities of the distribution grid to its network parameters. In the following, possible sources of uncertainties for the resistive, inductive and capacitive terms at the basis of the line model are discussed.

A. Series impedance

For the resistive terms, the main source of uncertainty is the resistance itself of the conductors. Such uncertainty $u_r\%$ appears in the self components of the primitive impedance matrix $\hat{\mathbf{Z}}_p$ and directly propagates to the self terms of the phase matrix $\mathbf{Z}_{abc}$. Due to the Kron reduction, the uncertainty of the neutral resistance also propagates to the mutual resistances and the reactive components of $\mathbf{Z}_{abc}$. The propagation of the uncertainties to $\mathbf{Z}_{abc}$ usually leads the resistive terms of $\mathbf{Z}_{abc}$ to have a relative uncertainty quite close to the starting uncertainty $u_r\%$, while the resulting uncertainties for the reactive terms are quite lower. It is worth noting that in distribution grids the resistive terms are not negligible as in transmission (the ratio R/X can be close to 1 or even higher for low voltage grids). Hence, the resistance uncertainties can have an important impact on the grid voltage.

As for the inductances, their uncertainty mainly depends on the original uncertainties $u_G\%$ for the GMR of the conductors and $u_D\%$ for the distance D among them (for the self and mutual inductance, respectively). The uncertainty contributions given by variations of frequency or of earth resistivity generally have, instead, a limited impact. Similarly to the previous case, when moving from the primitive to the phase impedance matrix, the application of the Kron reduction leads $u_G\%$ and $u_D\%$ to bring uncertainties to all the entries of the matrix. However, since in (2) and (3) GMR and D appear within a logarithmic term, the uncertainty propagation process attenuates the starting uncertainties. Generally, the relative uncertainties of the impedance terms in $\mathbf{Z}_{abc}$ are thus lower than the starting uncertainties $u_G\%$ and $u_D\%$.

As an example, Table I shows the relative uncertainty of the self and mutual impedance terms in $\mathbf{Z}_{abc}$ due to $u_r\%$, $u_D\%$ and $u_G\%$ for the overhead line configuration “1” of the IEEE 123-bus grid in [14]. Results are obtained via 50000 Monte Carlo iterations and show the average uncertainty for the self and mutual terms of resistance R and reactance X . While these results refer to a specific line configuration, they are representative of a trend that can be found also in other cases: the uncertainty in the conductors resistance has a large impact on all the resistive terms, while the uncertainty on the spacing characteristics of the line brings only slight effects, mainly on the self reactance term.

Beyond the random deviations from benchmark values, possible correlations or systematic errors can further affect the voltage profile calculation. Correlations in the $\mathbf{Z}_{abc}$ terms could exist, for example, among the line sections of a same feeder having same configuration and installation characteristics. A possible cause of systematic errors is instead the ageing of the conductors. Ageing can lead to an increased resistance of the conductors due to annealing, corrosion and other factors [15]. The increased conductor resistance can be modelled as a systematic error with an additional uncertainty on top to create the interval where the actual resistance is expected to be. Another cause of systematic errors is the variation of the conductors resistance due to the temperature, which is:

$$r_c = r_0 \cdot [1 + \alpha (T_c - T_0)] \quad (8)$$

where r_c is the actual resistance of the conductor at its current temperature T_c , r_0 is the resistance at a reference temperature T_0 and α is a temperature coefficient of resistance for the used conductor material. The temperature variations of the conductors can be determined using ad hoc steady-state or dynamic thermal models [16]. A simple steady-state model is:

$$r_c(T_c)I^2 + Qs = k_{qr} (T_c^4 - T_a^4) + k_{qc} (T_c - T_a) \quad (9)$$

where Qs is the heat given to the conductor by the solar radiation, T_a is the ambient temperature, and k_{qr} and k_{qc} are constant values (for given weather and conductor characteristics) associated to the convection and radiation heat emission from the conductor. Through (9), it is possible to calculate the temperature T_c of a conductor carrying a given current I , which in turns allows the estimation of the actual resistance

TABLE I
EXAMPLE OF PHASE IMPEDANCE MATRIX UNCERTAINTY

Impedance term	Starting uncertainty		
	$u_r\% = 10\%$	$u_G\% = 10\%$	$u_D\% = 10\%$
R_{jj}	8.60%	0.13%	0.30%
R_{jk}	6.63%	0.91%	1.46%
X_{jj}	0.65%	1.16%	1.00%
X_{jk}	1.60%	0.49%	3.30%

r_c via (8). As a consequence, for a given loading scenario, the error made by considering the standard resistance r_0 in the model can be estimated. It is worth noting that the dependence of the resistance from its temperature is usually neglected in the power flow algorithms available in common power system tools [17]. However, this aspect can play a significant role in distribution grids, where large line resistances exist.

B. Shunt admittance

The main sources of uncertainty for the capacitance are the geometrical characteristics of the conductor and the spacing configuration of the line (see (6)). Due to the specific steps needed to obtain the capacitance matrix $\mathbf{C}_{abc}$ (Kron reduction and matrix inversion), the propagation of the uncertainties to the final capacitance is not easily traceable. Monte Carlo simulations on different line configurations showed that the relative uncertainty for the self capacitance is usually lower than the starting uncertainties, while the mutual capacitance can assume larger uncertainties. In general, however, the current injections brought by shunt admittances in distribution grids are very small (due to the low voltage levels). For this reason, shunt admittances are often neglected in the distribution line models. The most relevant aspect to be investigated thus becomes the effect of the systematic errors introduced when disregarding the capacitive current injections.

C. Line length

The calculation of both the series impedance and the shunt admittance matrices shown in Section II gives the associated terms per unit of length. For the model of a specific line, these values thus have to be multiplied to the length of the line itself. The uncertainty $u_L\%$ on the length L of the line is therefore an additional source of uncertainty for the model. Using the uncertainty propagation law and indicating with $u_E\%$ the percentage uncertainty for a generic entry E of the series impedance or the shunt admittance matrix (as resulting because of the uncertainty sources previously described), the following overall uncertainty $u_{E,tot\%}$ can be found:

$$u_{E,tot\%} = \sqrt{u_{E\%}^2 + u_{L\%}^2} \quad (10)$$

Eq. (10) shows that the line length uncertainty $u_L\%$ has a large impact on the overall impedance (or admittance) uncertainty and it puts a lower boundary for it ($u_{E,tot\%} \geq u_{L,tot\%}$). Moreover, since all the impedance and admittance terms are multiplied to the same length L , $u_L\%$ introduces correlations among the errors of the impedance and admittance of the same line. This correlation increases for growing levels of $u_{L\%}/u_{E\%}$, tending to 1 (full correlation) when $u_{L\%} \gg u_{E\%}$.

IV. IMPACT OF NETWORK UNCERTAINTIES

Distribution power flow algorithms integrate the line model and the equations presented in Section II to compute the voltage profile of the grid given certain load and generation scenarios. Usually, at distribution level, the main substation is assumed as the slack bus of the grid and it takes fixed values of voltage magnitude and angle. All the other nodes are generally considered as load or generation buses and, thus, they are represented as PQ nodes or, in more detailed algorithms, as ZIP nodes. Focusing on the analysis of the voltage profile of the grid, the voltage phasor $\mathbf{v}_n$ resulting at a generic node n can be written as in eq. (11):

$$\mathbf{v}_n = \mathbf{v}_1 - \sum_{k \in \Gamma} \mathbf{Z}_{abc,k} \cdot \mathbf{i}_k \quad (11)$$

where $\mathbf{v}_1$ is the three-phase voltage at the slack bus, $\mathbf{i}_k$ and $\mathbf{Z}_{abc,k}$ are the three-phase current and phase impedance matrix at the generic branch k , and Γ is the set of branches in the path between slack bus and node n .

Looking at the voltage at a generic phase Ψ of node n , assuming (without loss of generality) that the phase-angle of the slack bus voltage $v_{1,\Psi}$ is zero, and converting (11) into its rectangular form, the following eq. (12) holds:

$$\begin{aligned} v_{n,\Psi}^{re} + jv_{n,\Psi}^{im} = v_{1,\Psi}^{re} - \sum_{k \in \Gamma} \sum_{\phi \in \{a,b,c\}} [R_{k,\Psi\phi} i_{k,\phi}^{re} - X_{k,\Psi\phi} i_{k,\phi}^{im} \\ + jX_{k,\Psi\phi} i_{k,\phi}^{re} + jR_{k,\Psi\phi} i_{k,\phi}^{im}] \end{aligned} \quad (12)$$

where $R_{k,\Psi\phi}$ and $X_{k,\Psi\phi}$ are the resistance and reactance terms of the phase impedance matrix at branch k , the superscripts *re* and *im* denote the real and imaginary components of the represented electrical quantities and j is the imaginary operator.

From (12), the voltage magnitude in terms of the network parameters can be derived as shown in eq. (13):

$$|v_{n,\Psi}| = \sqrt{v_{n,\Psi}^{re2} + v_{n,\Psi}^{im2}} \quad (13)$$

where:

$$v_{n,\Psi}^{re} = v_{1,\Psi}^{re} - \sum_{k \in \Gamma} \sum_{\phi \in \{a,b,c\}} R_{k,\Psi\phi} i_{k,\phi}^{re} - X_{k,\Psi\phi} i_{k,\phi}^{im} \quad (14)$$

$$v_{n,\Psi}^{im} = - \sum_{k \in \Gamma} \sum_{\phi \in \{a,b,c\}} (X_{k,\Psi\phi} i_{k,\phi}^{re} + R_{k,\Psi\phi} i_{k,\phi}^{im}) \quad (15)$$

Using the uncertainty propagation law, the uncertainty resulting on the voltage magnitude $|v_{n,\Psi}|$ due to the network parameter uncertainties can be approximated as shown in eq. (16) (the imaginary branch current $i_{k,\phi}^{im}$ is also considered as a variable since it is directly affected by the shunt admittance):

$$\begin{aligned} u_{|v_{n,\Psi}|} = \left[\sum_{k \in \Gamma} \sum_{\phi \in \{a,b,c\}} \left(\frac{\partial |v_{n,\Psi}|}{\partial R_{k,\Psi\phi}} \right)^2 u_{R_{k,\Psi\phi}}^2 + \left(\frac{\partial |v_{n,\Psi}|}{\partial X_{k,\Psi\phi}} \right)^2 u_{X_{k,\Psi\phi}}^2 \right. \\ \left. + \left(\frac{\partial |v_{n,\Psi}|}{\partial i_{k,\phi}^{im}} \right)^2 u_{i_{k,\phi}^{im}}^2 + u_{corr} \right]^{1/2} \end{aligned} \quad (16)$$

with u_{corr} being the contribution brought by possible correlations among the network errors, which is given by the sum of all the mixed partial derivatives written as in eq. (17):

$$u_{corr}(a, b) = \frac{\partial |v_{n,\Psi}|}{\partial a} \frac{\partial |v_{n,\Psi}|}{\partial b} \rho_{ab} u_a u_b \quad (17)$$

with $a, b = \{R_{k,\Psi\phi}, X_{k,\Psi\phi}, i_{k,\phi}^{im}\}$ and $a \neq b$, and with ρ_{ab} being the correlation factor between the generic variables a and b . As visible in (16), correlations have the potential either to increase or to decrease the resulting uncertainty of the voltage depending on the overall effect of the components $u_{corr}(a, b)$. For this reason, when present, correlations need to be duly considered.

The relationship in (16) is only an approximation of the voltage uncertainty resulting from power flow calculations. Nevertheless, it can help to have a rough evaluation of the expected impact of different network parameter uncertainties. To this purpose, the sensitivity of the voltage magnitude to the different parameter uncertainties can be evaluated via the associated derivative terms given in (18), (19) and (20):

$$\frac{\partial |v_{n,\Psi}|}{\partial R_{k,\Psi\phi}} = \frac{-v_{n,\Psi}^{im} i_{k,\phi}^{im} - v_{n,\Psi}^{re} i_{k,\phi}^{re}}{|v_{n,\Psi}|} \approx -i_{k,\phi}^{re} \quad (18)$$

$$\frac{\partial |v_{n,\Psi}|}{\partial X_{k,\Psi\phi}} = \frac{v_{n,\Psi}^{re} i_{k,\phi}^{im} - v_{n,\Psi}^{im} i_{k,\phi}^{re}}{|v_{n,\Psi}|} \approx i_{k,\phi}^{im} \quad (19)$$

$$\frac{\partial |v_{n,\Psi}|}{\partial i_{k,\phi}^{im}} = \frac{v_{n,\Psi}^{re} X_{k,\Psi\phi} - v_{n,\Psi}^{im} R_{k,\Psi\phi}}{|v_{n,\Psi}|} \approx x_{k,\phi} \quad (20)$$

where the last simplification in each of the above equations is obtained considering the approximations $v_{n,\Psi}^{im} \approx 0$ and $v_{n,\Psi}^{re} \approx |v_{n,\Psi}|$, which in general hold for most of the distribution grids. From (18)-(20), it is possible to infer:

- since at distribution level R/X ratios are generally close to 1 and the grid is operated with high power factors, it is $i_{k,\phi}^{re} > i_{k,\phi}^{im}$ (note that the slack bus angle for phase Ψ was set equal to zero). This implies that the uncertainties of the resistive terms in Z_{abc} will cause relatively larger effects on the voltage magnitude than the reactance terms.
- the variations of the reactive current $i_{k,\phi}^{im}$ determined by the capacitance uncertainty are generally small, and lead to even smaller effects on the voltage magnitude profile.

V. SIMULATION RESULTS

In this Section, the considerations reported in Sections III and IV are assessed via ad hoc simulations performed on the three-phase IEEE 123-bus grid presented in [14]. For the sake of simplicity, the grid has been modified by removing voltage regulators and renumbering the nodes as shown in Fig. 1.

A first test has been performed to analyse the sensitivity of the voltage magnitude to the different parameter uncertainties. To this purpose, Monte Carlo simulations have been conducted (with 10000 iterations) to statistically evaluate the resulting uncertainty of the voltage profile given by a power flow algorithm having in input (alternatively) resistance, reactance and admittance terms of the phase matrices of each line

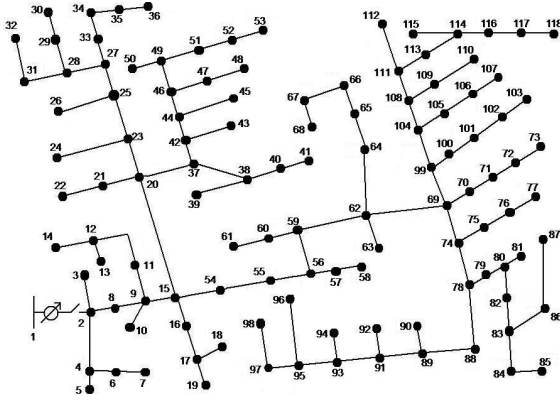


Fig. 1. IEEE 123-bus distribution grid.

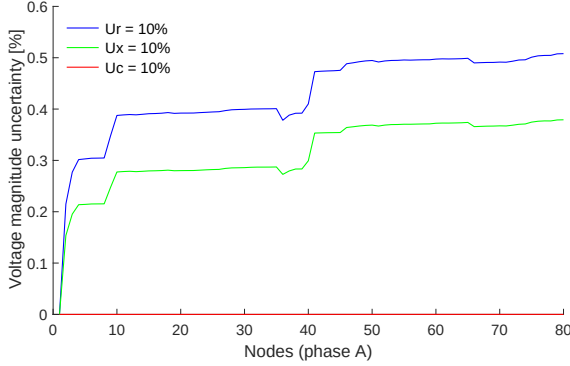


Fig. 2. Voltage magnitude uncertainty due to network parameters uncertainty.

extracted with a 10% uncertainty (with Gaussian distribution) around the nominal value. Fig. 2 shows the obtained results. The voltage output of the power flow algorithm has a level of uncertainty that exceeds 0.5% when the uncertainty is applied to the resistance terms: this proves that network parameter uncertainties can potentially undermine the correctness of the power flow results. As anticipated in Section IV, the resistance is the parameter that affects more the voltage magnitude profile (note that the lines in the considered grid also have R/X ratios smaller than 1). Vice versa, possible uncertainties on the capacitance basically do not bring any effect. The role of the capacitance has been also investigated by comparing the power flow results obtained when considering or neglecting the shunt admittance in the grid model. The found differences on the voltage profile were almost null, with maximum voltage deviations lower than 0.001%. While these results are specific for the considered grid and assumptions must be carefully evaluated for each scenario (e.g. networks with underground cables have larger capacitance), the results found for the 123-bus grid justify the common practice of neglecting the shunt admittance in the distribution grid model.

Another parameter introducing uncertainties in the grid model is the length of the lines. For testing the impact of this parameter, Monte Carlo tests have been conducted considering a 10% uncertainty for the length of each line provided in input to the power flow algorithm. In addition, the effects of possible correlations have been also investigated. To this end, different levels of correlation have been introduced among the errors

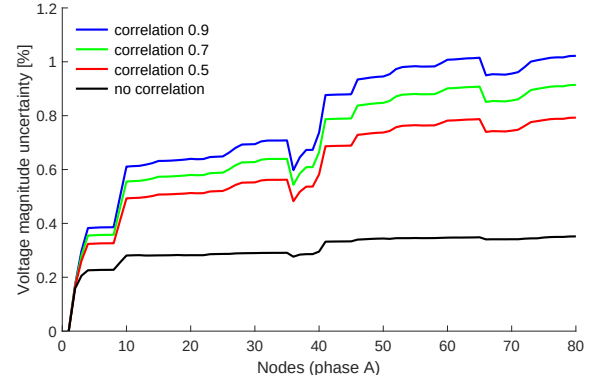


Fig. 3. Voltage magnitude uncertainty due to lines length uncertainty.

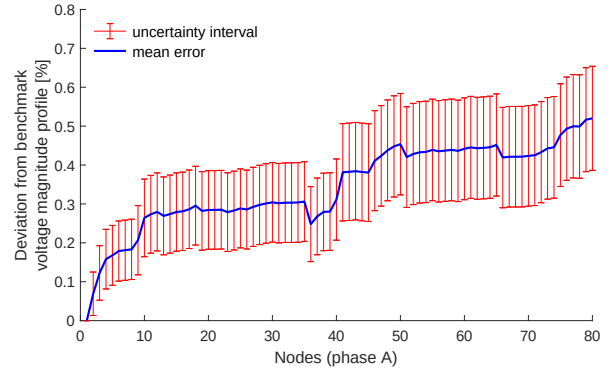


Fig. 4. Voltage magnitude deviation from benchmark profile due to simulated ageing of conductors.

considered for the length of all the three-phase branches of the grid. Results shown in Fig. 3 prove that possible uncertainties on the knowledge of the section lengths can significantly affect the level of confidence of the power flow outputs. Moreover, as expected, correlations are proved to further amplify the uncertainties of the power flow results, thus calling for a careful evaluation of the network uncertainty characteristics.

Beyond the general uncertainties unavoidably existing in the input parameters, systematic errors that are not duly considered can also affect the accuracy of power flow algorithms. In Section III, the ageing of conductors and the electrical variations due to the operating temperature of the conductors were suggested as possible sources of systematic errors. Both these aspects potentially introduce errors in the definition of the resistive terms of the series impedance (which were shown to be the most important factor of degradation for the voltage magnitude profile of the distribution grid). The possible increase of the line resistances due to the ageing of the network has been analysed by considering a growth of the conductor resistance between 8% and 12% (with a Gaussian uncertainty within these boundaries). The resistances of the conductors were randomly extracted according to this uncertainty characteristic and used in input to the power flow algorithm during a Monte Carlo simulation. The results obtained from this power flow algorithm were then statistically evaluated and compared to those of another power flow algorithm where the ageing effects were neglected. Fig. 4 shows the difference of the voltage magnitude results obtained in the two cases. The

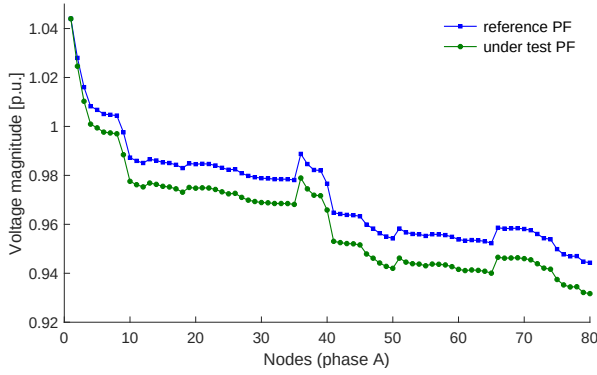


Fig. 5. Voltage magnitude profile with and without the update of the conductor resistance as a function of its operating temperature.

increased resistance clearly leads to larger voltage drops in the lines and for this reason the voltage differences increase when moving along the feeders. An average deviation of the voltage up to 0.5% has been obtained at the last nodes of the feeders. However, when considering the confidence interval associated to the obtained level of voltage uncertainty, the difference between the voltage results of the two power flow scenarios gets close to 0.7% (the voltage error had a Gaussian distribution around the obtained mean deviation).

As for the systematic errors introduced by the resistance variations linked to the operating temperature of the conductors, a simulation has been run comparing the results of a power flow algorithm using the standard values of resistance with those of a second power flow algorithm where the line resistances were updated according to the conductor temperatures. In this second power flow, the relationships presented in Section III have been used to compute first the conductor temperature (for each branch) and then to update the resistance values (as a function of such temperature), given the current temporarily estimated at each iteration of the power flow. Fig. 5 shows the voltage magnitude profiles obtained in the two cases for the phase A of the system (which is the most loaded one). In general, the operating temperature of the conductors leads the line resistances to grow and, therefore, larger voltage drops are obtained along the lines when taking the thermal effects into account. In the considered scenario, this determines a lower voltage at the nodes, with deviations in the obtained profiles up to 1.3%. Such result proves that considering (or neglecting) the thermal behaviour of the conductors can drastically affect the power flow results. This mismatch in the power flow results is an important aspect to take into account, above all when power flow-based algorithms are used to trigger management or control functions that require reliable information for their proper operation.

VI. CONCLUSIONS

This paper presented an analysis aimed at identifying the sources of uncertainty for the distribution grid models and at investigating their impact for the power flow results. The first outcome of the performed analysis is that line resistances are

the network parameters that can lead to the largest impact on the voltage profile. The uncertainties associated to the knowledge of the resistances have thus to be duly taken into account since they have the potential to drastically affect the power flow results, undermining their reliability. The second main outcome is that systematic and correlated errors, like those associated to the ageing of the lines or to the variations of resistance due to the operating temperature of the conductors, can significantly amplify the impact of the network uncertainties on the power flow results. Therefore, these characteristics have to be duly considered when evaluating the impact of network uncertainties. While these aspects are generally disregarded at transmission level and in the common tools for power system analysis, their role in the distribution systems has been proved to be relevant and calls for a much more careful consideration of the associated impacts.

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