Thinking beyond the monetary objectives of modern portfolio theory – a tri-dimensional optimization model for thematic investments leading to practical advice

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List of abbreviations

99BM	benchmark of 99 portfolios
CAPM	capital asset pricing model
CBN	concentration-based naivety
CSR	corporate social responsibility
ESG	environment, social, governance
ETF	exchange-traded fund
MAUT	multi-attribute utility theory
MaxDD	maximum drawdown
MCDA	multi-criteria decision analysis
MCDM	multi-criteria decision making
MVO	minimum variance optimization
PBN	portfolio-based naivety
SBN	stock-based naivety
SRI	socially responsible investment
VaR	Value at Risk
WMP	world market portfolio

List of symbols

$\boldsymbol{C}$	covariance matrix
E	return of a portfolio
$\hat{E}$	return of a core tailor-made satellite portfolio
E_{CS}	return of a core satellite portfolio
E_{Tri}	return of a tri-criterion portfolio
F_z	distribution function of variable z
$H(x)$	Herfindahl index of an allocation vector $\boldsymbol{x}$
i	index variable denoting individual assets
IN	in-sample estimation window
j	index variable denoting individual assets
M	number of megatrend portfolios
m	number of stocks included in a thematic portfolio
N	number of assets
n	number of stocks included in a conventional portfolio
OUT	out-of-sample testing window
$\mathbb{R}$	set of all real numbers
S	number of different samples
t	proportion of the thematic portfolio in a core satellite portfolio
t_t	proportion of the thematic portfolio that the tailor-made portfolio is optimized for
T	thematic proportion of a portfolio
$\hat{T}$	thematic proportion of a core tailor-made satellite portfolio
T_{CS}	thematic proportion of a core satellite portfolio
T_{Tri}	thematic proportion of a tri-criterion portfolio

V	volatility
$\hat{V}$	volatility of a core tailor-made satellite portfolio
V_{CS}	volatility of a core satellite portfolio
V_{Tri}	volatility of a tri-criterion portfolio
$\boldsymbol{x}$	solution vector
$\boldsymbol{x_C}$	allocation vector of a conventional portfolio
x_i	investment proportion of asset i
$\boldsymbol{x_T}$	allocation vector of a thematic portfolio
$\widehat{\boldsymbol{x_T}}$	allocation vector of a tailor-made thematic satellite portfolio
$\boldsymbol{y}$	solution vector for a two-portfolio decision
y_1	relative share of the conventional portfolio in a core satellite portfolio
y_2	relative share of the thematic portfolio in a core satellite portfolio
$\boldsymbol{y_{CBN}}$	solution vector for a two-portfolio decision with concentration-based naivety
$\boldsymbol{y_{PBN}}$	solution vector for a two-portfolio decision with portfolio-based naivety
$\boldsymbol{y_{SBN}}$	solution vector for a two-portfolio decision with stock-based naivety
z	random variable

δ	relative yield improvements
δ_{TM}	relative risk improvements
$\boldsymbol{\mu}$	vector of asset returns
μ_i	return of asset i
ρ_{ij}	correlation of two assets *i* and *j*
σ_i	standard deviation of returns of asset *i*
τ_i	binary parameter labeling thematic assets

Summary

The way financial decisions are understood and handled has changed drastically over time so that new financial tools are needed to capture the multi-dimensionality of these decisions and their developments (Zopounidis et al. 2018). In the context of this dissertation, these decisions relate to portfolio decisions of private investors that allocate an investment budget to various financial products. While modern portfolio theory has traditionally focused on wealth-related objectives, i.e., return and risk, recent developments in the area of thematic investments seem to go beyond these limits, as they can no longer be displayed in conventional models (Markowitz 1952, Aouni et al. 2018).

Ethical concerns, personal interests, or the conformity with personal convictions motivate investors to include non-monetary objectives in their decisions. So-called thematic funds exploit this expansion of objectives to advertise their allocations. Thematic investors thereby follow a modified core satellite strategy in which conventional funds ensure diversification whereas supplemented satellite funds accommodate investors' additional non-monetary interests. Both portfolios are separately allocated ignoring beneficial inter-portfolio correlation effects. However, modern portfolio theory has originally tried to optimize these correlation effects (Markowitz 1952). Consequently, efficiency of core satellite portfolios can only be achieved by chance.

This study provides more specific theoretical foundation and quantifies the efficiency of core satellite portfolio solutions. The efficiency of thematic core satellite investing is evaluated by stating a three-dimensional model for thematic investments and by comparing both portfolio solutions. Furthermore, this study develops two approaches to reduce the inefficiency of a core satellite strategy. One addresses fund providers with the idea of tailoring thematic funds to conventional ones. The other approach addresses private investors providing pragmatic heuristics for their two-portfolio decision.

1 Financial decision making has changed and thematic investments increase their economic significance

Sound financial decision making contributes to increasing financial wealth. Therefore, financial resources are allocated to grow and at the same time the risks associated with an investment are controlled. However, in recent years, strategies have been observed that cannot be explained by only monetary incentives anymore. The way financial decisions are understood and handled has changed drastically (Zopounidis et al. 2018). Investments are now also driven by the idea of changing company behavior, demonstrating morality, and profiting from evolving megatrends that will change society (Giammattei 2014, Riedl and Smeets 2017, Amel-Zadeh and Serafeim 2018). These interests and motivations have been recognized by financial institutions and fund providers that open new funds under the name of thematic investments. These thematic investments are advertised as new opportunities to individualize portfolios and to reflect investors' beliefs on future developments. However, scientific research has yet to adequately address these developments and to provide the necessary background and models to display these investments.

This study aims at providing further theoretical foundation, getting insights to investing behavior and the efficiency of different underlying strategies, and ultimately at giving practical advice on how to improve the status quo of thematic investing. More specifically, first, the efficiency of the status quo portfolio solutions is evaluated by stating a three-dimensional model for thematic investments. Second, an innovation for thematic products is presented that improves the value of thematic funds to private investors by tailoring them to conventional funds. Hence, it presents a guideline for fund providers to improve their products. Third, practical advice for thematic investors will be given by stating and evaluating concrete heuristics to allocate their thematic portfolios.

To comprehend the implications of those goals in the context of thematic investments and to clearly understand the importance of this still fast-growing topic in the area of finance, a thorough introduction to the topic is be given. When analyzing the relevant drivers for thematic investments, the main motivations for investors will determine the understanding. Hence, in the following, the importance and drivers of thematic investments will be demonstrated, and the underlying strategy will be derived from the investing behavior. The relevant literature is reviewed and gaps in research are identified. This literature review leads to detailed research questions that are answered and discussed.

Overviewing the field of thematic investments, two major areas of interest are identified: morals and megatrends. A trend to ethical, socially responsible, and moral conviction compliant investments is observed. This area is commonly known and referred to by different names: socially responsible investments (SRI), or sustainable investments that consider environment, social, governance (ESG), or corporate social responsibility (CSR) criteria. Picking one of these investment types to demonstrate the economic significance that these fields have arrived at, in 2018, the volume of investments under professional management that considers ESG aspects has reached $12 trillion in the United States alone (US SIF Foundation 2018).

Thematic investments are not limited to ethical criteria. So-called megatrend investments are advertised as opportunities to be part of the next upcoming trends. Megatrend investments try to capture on the one hand long-term socio-economic developments, i. e., climate change, demographic change, or rapid urbanization, and on the other hand, technology-driven trends that often focus on a technological break-through and disruptive technologies. The characteristics of technological trends are associated to, e. g., big data, artificial intelligence, and application-oriented themes like cyber-security.

Besides these two major thematic developments, several additional themes have arisen, too, that target investors can identify with. Such themes focus on e-sports, aerospace, or any sector that an investor relates to as themes can break sectoral boundaries but do not have to. However, while, at the first glance, the motivations of these themes might still also be financially, the complete set of fundamental objectives and underlying drivers of the investors still remains hidden. To get a more structured and profound understanding of investors' objectives, further investigation is needed.

In the following, deeper insights into thematic investors will be given. This section bases on a recent publication that categorizes the underlying motivations and shows how they could be dealt with in portfolio consulting (Methling and Nitzsch 2019a). In traditional portfolio theory, it is assumed that there are homogeneous expectations in the market with regard to the return and risk expectations of all assets, i.e., the market is efficient in the long term (Fama 1970). Accordingly, there should be no possibility of achieving risk-adjusted excess returns through, e. g., megatrend investments. In addition, it is assumed that an investor is only interested in maximizing her wealth while taking into account the maximum tolerable risks. An investment that expands this set of objectives and meets sustainability criteria is not intended at this level. In an ideal-typical textbook view of the Capital Asset Pricing Model (CAPM), it follows immediately that every investor should adhere to a strategy that invests a risky portion of the investment in the maximum diversified world market portfolio (WMP) (Sharpe 1964). Such a WMP strategy would only leave room for individuality to the extent that the division between the safe investment (e.g., an investment in government bonds of the highest credit rating) and the WMP depends on the individual risk attitude.

In principle, however, investors seem to be willing (albeit partially triggered by the financial industry) to engage in solutions other than the CAPM and thus not to invest in the WMP. For example, many investors do not diversify as broadly as theory requires and are primarily invested in domestic stocks (Tesar and Werner 1995). Others want to contribute their convictions to their own portfolio and voluntarily limit themselves to sustainable or ethical investments. Another group of investors sees certain megatrends and expects to achieve higher returns with thematic investments (Beal et al. 2005, Lewis and Mackenzie 2017).

In order to explain the deviations of reality from a WMP investment strategy, a systematic and clean answer must be given to the question of what motivates investors to deviate from the WMP strategy and to choose other individual, thematic strategies. Figure 1 demonstrates the outline of the following sub-sections and overviews the motivations for thematic investing that are explained in more detail below.

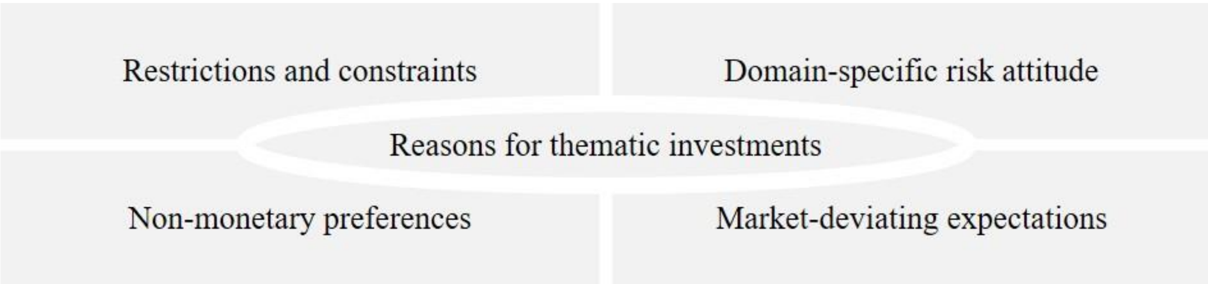

Figure 1: *Overview of motivations for thematic investing (Methling and Nitzsch 2019a).*

1.1 Restrictions and constraints

The first category of restrictions and constraints includes constellations in which investors want or need to see their thematic interest taken into account under all circumstances. These investors regularly act as employees or on behalf of institutions that have anchored these restrictions for very different reasons.

The Christian church institutions represent a large group of corresponding institutions in Germany, which categorically exclude a number of business fields in their investment decisions. These exclusions include, e. g., investments in tobacco, alcohol and armaments businesses, with varying formulations between the dioceses, religious communities and non-profit organisations as well as Christian universities and hospitals (Louche et al. 2012). Similar restrictive conditions apply, e. g., to Jewish or Islamic organizations that reflect religionspecific beliefs (Union for Reform Judaism 1997, Warde 2000, Louche et al. 2012). Thus, Islamic investment funds must be managed Sharia-compliant and controlled by Sharia supervision, which also leads to the exclusion of alcohol- and tobacco-related companies (Zaher and Kabir Hassan 2001).

Many institutions outside the religious spectrum also include restrictions in their investment guidelines. For example, a large proportion of pension funds are restricted to certain regions and thus turn away from the world market portfolio (Brough and Shepperson 2019). A popular example is the Norwegian State Pension Fund, which has set itself an ethical framework based on national convictions and excludes companies involved in the construction of weapons of mass destruction as investment objects (Norwegian Ministry of Finance 2014). In a similar spirit, the Irish sovereign wealth fund joined the Fossil Fuel Divestment movement (Fossil Fuel Divestment Act, 2018). This movement promotes the withdrawal of all investments in companies with businesses in and around the extraction, processing and distribution of fossil fuels. A portion of the global market portfolio can no longer be invested as part of such a strategy, so that the portfolio and the investment decision are limited.

1.2 Non-monetary preferences

In the second category, non-monetary preferences, consideration should primarily be given to ethical principles or sustainability criteria, which are increasingly and rather generally subsumed under the term ESG investments in the financial sector. The desire to align one's own investments with appropriate criteria is growing and seems to be more than just a fashionable trend (Lewis and Mackenzie 2017). This is seen, e. g., by the fact that the ESG market of professionally managed assets grew by more than 37% to $12 trillion between early 2016 and early 2018 (US SIF Foundation 2016, 2018). However, the difference to the first category is that the investors described in this category see an added value in the ethical orientation, but do not want or have to force it at any price. They often hold both ethical and unethical parts in their portfolios and thus put both financial and non-monetary added values into perspective (Lewis 2001).

Within this category, intrinsic and self-exposing motives can be found for moral behavior (Bénabou and Tirole 2006, Ariely et al. 2009). These motives may be real convictions of investors, which they would like to see reflected in the investment. By doing so, investors want to identify with the investment and increase loyalty (Webley et al. 2001, Forster 2017). Investors may be interested in the idea that their investments can contribute to positive social change and change entrepreneurial behavior (Beal et al. 2005, Amel-Zadeh and Serafeim 2018). In any case, they are rewarded by the feeling that they have done something good, the warm glow giving (Andreoni 1989, 1990). The orientation for such a feeling of well-being is generally strongly influenced by the personal values of the investors. These values are initially shaped by culture, society and personality and in any case influence the attitude and behavior of the individual (Rokeach 1973). An investment behavior that deviates from one's own values and the resulting preferred behavior would, conversely, be perceived as unpleasant. This cognitive dissonance, which results from the inconsistency of these preferences, triggers a strong motivation to dissolve the dissonance and to adapt behavior (Festinger 1957, Frey and Gaska 1998).

The motivation for an investment can also arise from the desire to be seen in a certain way if the investment is sufficiently communicated, as a study by Riedl and Smeets in 2017 shows on investors' reasons for holding socially responsible mutual funds (Riedl and Smeets 2017). This self-portrayal is ultimately a consequence of the need for social recognition and a desire for status and reputation (Glazer and Konrad 1996). Despite the resulting social pressure, the observability of the decision can further have a positive influence on the utility of decision-makers (Cappelen and Tungodden 2017). It is often difficult to separate this motive from the desire to identify with one's own investment. As long as the investor is prepared to forgo returns or take higher risks for this recognition, belonging to category 2 is a perfect fit in any case.

Category 2 is not limited to ESG preferences only. For example, there are other themes such as investments that are associated with a specific brand name, from which investors can draw added value and benefit alone from the positive non-financial characteristics of the company, or topics that investors perceive in a playful context (Anand and Cowton 1993, Giammattei 2014). Playful gambling situations create tension and excitement, which are further linked to thrill, adventure and excitement (Zuckerman 1971, Boyd 1976). This entertainment can motivate investors to trade on average twice as much as other investors (Dorn et al. 2008, Dorn and Sengmueller 2009). Even Black (1986) attributed the amount of noise trading to potential gambling. This field does not focus exclusively on an investment topic of gambling, such as funds for investment in gambling and casinos, but on each topic, in which investors experience non-monetary advantages through individual perception. In summary, the positive emotions which an investor associates with the investment and which thereby increase the benefit of a thematic investment are the cross-topic drivers. The underlying personal values can be as dynamic as fashion and fads in financial markets (Bondt and Thaler 1995).

1.3 Domain-specific risk attitude

Referring to the third category, domain-specific risk attitude, there is empirical evidence that personal risk attitudes are not independent of the context of the risk situation (Weber et al. 2002). Some people consciously take very high risks, e. g., in leisure sports, but hesitate to invest a small part of their financial investment in an equity fund. In a financial context, this domain-dependence of the risk attitude shows up in different mental accounts of the investor (Shefrin and Statman 2000). Some tend to avoid risk more if money is saved for an important investment objective, e.g., retirement provision, but show an increased risk appetite in a mental account that is supposed to exploit potentials, such as when buying lottery tickets (Statman 1999). Lottery tickets, e. g., demonstrate a changing risk behavior in gaming situations that can be caused by the desire for thrills and different risk attitudes (Deck et al. 2014).

The domain dependency of this risk attitude is not only related to a differentiation of the investment goals or the mental accounts. Similarly, the degree of information or perceived ambiguity, mood or suggested time window are also factors that influence the investor's specific risk perception and thus the personal risk attitude in this context (Ellsberg 1961, Jacobs-Lawson and Hershey 2005, Conte et al. 2013, Drichoutis and Nayga 2013, Campos-Vazquez and Cuilty 2014). Depending on how a decision and thus a risk is presented, this leads to different perceptions of the risk (Tversky and Kahneman 1981, Johnson et al. 1993). In concrete terms, this means that the risk of different investment themes can be perceived differently by investors which subsequently influences behavior. Thus, investors can deviate from an efficient allocation from a proportionate investment in the world market portfolio, as they evaluate the uncertainty or perceived ambiguity in a topic differently on the basis of their own previous knowledge and expertise. In addition, the topic itself can also trigger emotions in them that lead to riskier behavior or an extended time horizon of a topic geared to long-term changes can also lead to a higher risk tolerance. In summary, there are a number of factors that can influence risk perception and thus personal risk attitudes with regard to certain thematic investments.

1.4 Market-deviating expectations

In the fourth category, market-deviating expectations, the firstly stated assumptions need to be questioned. The assumptions of the ideal-typical portfolio textbook theory mentioned in the introduction also include homogeneous expectations of all market participants with regard to the expected return opportunities for all securities. In such a world, there would be no reason to speculatively invest against market expectations. Reality, however, draws a different picture.

A large part of stock trading is speculative, as can be seen, e. g., from the ratio of stock holdings to stock trading. Global share trading of approximately 142 trillion US dollars exceeds the total shareholding of 77 trillion US dollars in 2018, which leads to an average holding period of around six and a half months per share (WFE, 2019). Investors do not only want to profit from interests and dividends of the companies, but also expect price increases and are convinced that they can predict them better than the market.

In some cases, the assessment of being smarter than the market is founded. Information-based motivated rational insiders can exploit their information superiority and, within a non-scalable framework, achieve higher returns compared to the general market development (Jaffe 1974, Finnerty 1976). These investments are compatible with the weak efficient market hypothesis (Kyle 1985).

However, in 2012, Fenzl and Pelzmann analyze aggregate financial market behavior and show that the transition from an actual information advantage to a mere overestimation of one's own possibilities and information is not clearly evident for individual investors (Fenzl and Pelzmann 2012). On the one hand, patterns in investment behavior are often only apparent at an aggregated investor level, resulting in exuberant market movements in the short term. On the other hand, these can often no longer be attributed to individual triggers, since investors are not always aware of the socially and psychologically contagious processes and do not notice how their decisions are influenced (Fenzl and Pelzmann 2012).

A further explanation, which is partly complementary, can be found in behavioral research, which identifies deviations in rationality derived from human behavior, e.g., in the form of overestimation of one's self, as the cause of excessive trade (Shleifer 2000). Many empirical studies support the existence of this systematic phenomenon, also known as overconfidence, which is expensive for investors (Odean 1998). Further systematic evaluation distortions, so-called bias factors, must be addressed by investors (Montibeller and Winterfeldt 2015). In this respect, bias effects are highly relevant for the emergence of thematic preferences, e.g., due to differences in cognitive availability caused by media reporting. Even advertising a topic in the media can exaggerate and manipulate investors' expectations. Investors explicitly overestimate the value of private information they have developed themselves (Barberis and Thaler 2003). Furthermore, investors resort to heuristics such as simulation heuristics and availability heuristics to save cognitive capacities, orient themselves to the media reporting and herd behavior mentioned above, and are guided by the retrospective explanation of past price movements as well as an expectation of the subsequent reward in purchasing behavior. (Tversky and Kahneman 1974, Kahneman and Tversky 1981, Andreassen 1987, Lux 1995, Devenow and Welch 1996, Hirshleifer 2001, Peterson 2002, Barberis and Thaler 2003, Stephan and Kiell 2017). Summarizing the motives of this fourth category, it can be stated that there are a number of factors in human behavior that influence personal assessments of return opportunities for specific thematic investments.

2 Recent developments and the status quo in research

In the following, it is analysed to what extent motivations for thematic investing have yet been captured in research and how they are displayed in financial models. Furthermore, the framework of this study will be stated by the delimitation to existing work starting with modern portfolio theory. Modern portfolio theory originated in 1952, when Harry Markowitz introduced his work on portfolio selection (Markowitz 1952). To his understanding, the process of allocating an optimal portfolio is divided into two separate parts. First, an investor has to state individual beliefs about the developments of future stock prices and their correlations to each other. Second, based on these beliefs, the investor chooses the best fitting portfolio allocation.

In classical portfolio theory, the preferences of investors are represented in a Bernoulli utility function, which axiomatically builds on the theoretical work of Neumann and Morgenstern (Neumann et al. 1953, Bernoulli 1954). Such a utility function refers solely to the monetary effects of an investment and takes into account both height and risk preferences (Nitzsch 2017). In a simplified form, this preference model can be written as a return-risk preference function which is used by default in portfolio theory to determine efficient portfolios within the framework of Markowitz optimization. It is based on the assumption of a risk-averse investor seeking return (Markowitz 1952, 1996). However, what has been stated as a necessary condition has become common understanding of investors' objectives (Markowitz 1996, Steuer and Na 2003, Steuer et al. 2013). Based on these objectives, the expected return of a portfolio is measured as the mean expected return of the particular stocks weighted by their relative share in the portfolio according to the portfolio allocation. Diversification is a means to reduce the deviations of portfolio returns; therefore, the second objective is measured, e. g., with the variance of portfolio returns.

In this two-dimensional model, portfolios can be evaluated by dominance criteria. A portfolio weakly dominates another one if there is no other portfolio with either a higher expected return and maximum the same variance, nor a portfolio with a smaller variance and at least the same expected return. Furthermore, a portfolio that is not being dominated by any other one can be called efficient. The so-called efficient frontier shows all the efficient portfolios in a two-dimensional plot with regard to their risk/ return evaluation.

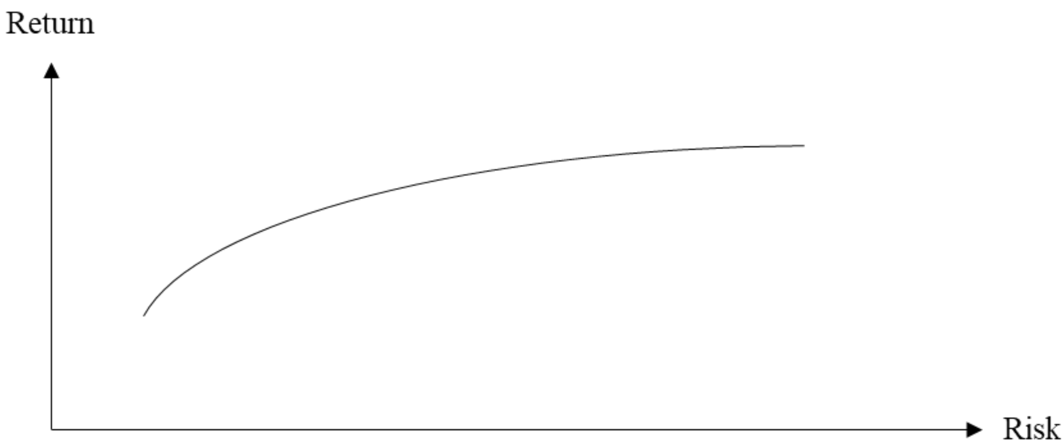

Figure 2: *Efficient frontier of return/ risk optimized portfolios showing the maximum theoretical return of a portfolio for any given constraining risk level.*

The shape of the efficient frontier shown in Figure 2 is a result of the relationships between different stocks. While the expected return of the portfolio is independent of these correlations, as a statistical measure of this relationship, the risk evaluation is not (Markowitz 1952). Assuming two stocks that are negatively correlated and one stock is below average compared to its expected return, it is more likely that the other stock will outperform. Therefore, it is not only relevant to choose the assets with the best individual risk/ return evaluations, but also to regard their correlation effects so that diversification can be increased (Markowitz 1952).

Investors with thematic preferences that can be assigned to the first category "restrictions and constraints" can be well integrated into this model of thinking. Thus, while retaining the investor's two-dimensional preference model, only the investment universe is to be limited to those securities that are not excluded via the restrictions. Depending on the degree of restriction, this can reduce the financial performance of the investment, an improvement is not possible according to the textbook view. Furthermore, Steuer, Qi, and Hirschberger (2007) as well as Drut (2010) have evaluated the sensitivity of the efficient frontier to additional objectives like social responsibility. They conclude that including such additional criterion significantly changes the efficient frontier as, e. g., negative screenings that exclude unethical stocks can only reduce the asset universe.

However, in the case of investors with reasons from the other three categories, explicit extensions of the two-dimensional preference model are required. Hence, Steuer and Na (2003) have provided a very broad bibliographic study about multi-criteria decision making (MCDM) in finance that has been updated in 2015 by Zopounidis et al. (Zopounidis et al. 2015). According to those reviews, portfolio optimization is the most studied area in these MCDM in finance, especially with multi-objective optimization and goal programming (Zopounidis et al. 2015). Most of these models only focus on multidimensional models of risk including risk-related measures and objectives. Nevertheless, there are also some approaches in the literature that expand the model with additional investor-specific value drivers besides risk (Spronk and Hallerbach 1997, Bana e Costa and Soares 2004, Ehrgott et al. 2004, Steuer et al. 2006, Xidonas et al. 2012). Furthermore, there are studies on how to include such value drivers in specific situations, i. e. ethical investments (see Barracchini 2004, Ballestero et al. 2012, Utz et al. 2014, Gasser et al. 2017). With reference to the general guidelines for setting up multidimensional value or utility functions, the highest possible fundamentality of the considered objectives should be ensured. Therefore, it is to be assessed which kind of motivations and corresponding categories are to be displayed in such a multi-criteria model.

Fundamental objectives have an independent meaning for the decision maker and must be distinguished from instrumental objectives, which are only named because they have a positive effect on another (fundamental) objective, but are essentially "worthless" in themselves (Keeney and Raiffa 1993, Keeney 1996). Should investors, e. g., express the aim of maximizing the proportion of ethical products in their portfolio, this could be seen as a fundamental goal as long as these investors derive their own added value from the ethical value orientation of their portfolio. If, however, they only express this goal because they implicitly expect higher returns or lower risks but do not recognize any added value, the only fundamental objectives would be return and risk. The proportion of ethical products would only be a means to another end and thus an instrumental goal.

This fundamental objective characteristic is undisputed for the second category "non-monetary preferences". Thus, in section 1, it was demanded as a characteristic feature of this category that investors are prepared to accept monetary losses. If one considers the maximization of the expected return and the minimization of the risk to be taken as two objectives of a multidimensional preference model, then the model can be expanded by the dimension of the intrinsic thematic preference.

At this point, the framework of the dissertation is stated. While additional motivations for thematic investments are found in practice and literature, this disseration focuses only on an extension with one added fundamental objective that summarizes all non-monetary preferences that are set against risk and return in a trade-off. Additional potentially information- or bias-based motivations or those that are based on the individual investor's risk perception of particular assets are excluded. In this reduced model, multi-attribute utility theory (MAUT) is applicable. Several studies have already shown that applying MAUT to financial decision making improves understanding of investor behaviour (Bana e Costa and Soares 2004, Ehrgott et al. 2004, Xidonas et al. 2012).

Additionally, a methodology is needed to solve portfolio decision problems. First, in two-dimensional space, the discrete construction method converts one of the objectives into a constraint and solves the optimization problem in several steps. Since Haimes in 1971 it has been commonly referred to as the ε-constraint method (Haimes 1971). The efficient frontier presented in Figure 3 shows the maximum return portfolio for any given restrictive risk level with a chosen accuracy.

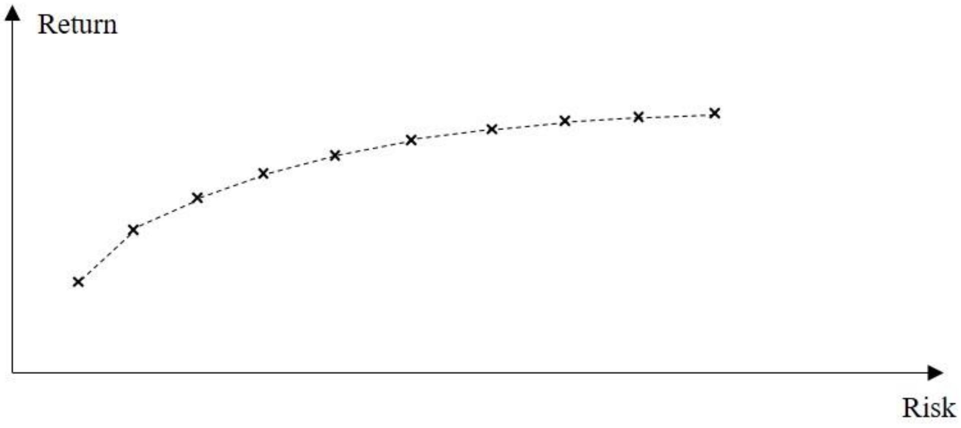

Figure 3: *Efficient frontier of ten portfolios with maximized returns for given maximum risk levels.*

Second, in 2010, Anagnostopoulos and Mamanis have presented an integration of a third dimension, i. e., minimizing the number of assets (Anagnostopoulos and Mamanis 2010). The efficient solution space can be approximated to a three-dimensional mesh grid by connecting the particular dots as demonstrated in Figure 4. Each dot represents a maximum return portfolio for a given restriction set of a maximum risk evaluation and a maximum number of assets.

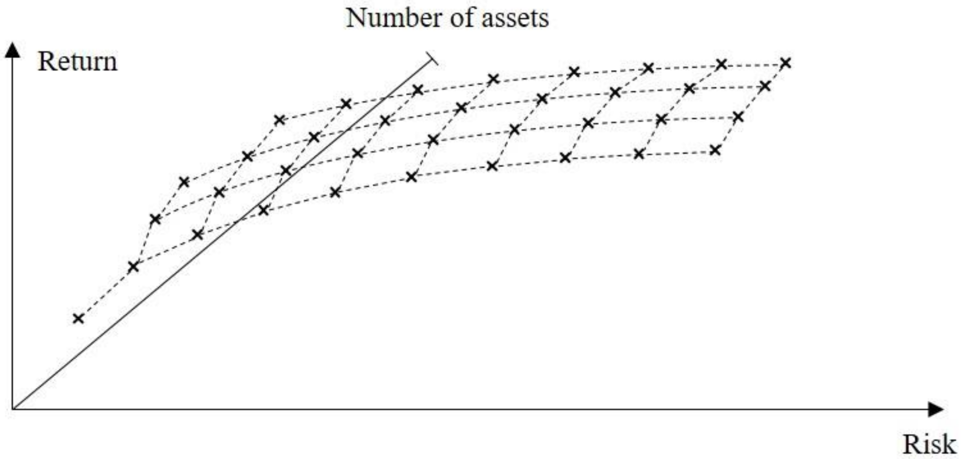

Figure 4: *Visualization of the dotted surface of efficient portfolios with maximized returns for given risk constraints and maximum numbers of assets.*

Three years later, an algorithm was presented that calculated the actually curved platelets of the efficient surface (Hirschberger et al. 2013). The development from algorithms for solving the efficient frontier to models and algorithms that solve problems with three dimensions is discussed in an overview of Steuer, Wimmer, and Hirschberger (2013).

Having provided models for the academic handling of this multidimensional decision problem, the practical handling, i. e., private investors' investing strategies, is still to be enlightened. A commonly used strategy handles thematic interests by supplementing thematic portfolios, e. g., thematic exchange-traded funds (ETF), to conventional portfolios (Magoon 2009, Bérubé et al. 2014, Marchioni et al. 2016). Those investors hold, e. g., a worldwide diversified ETF to capture the global economic development and attach a satellite portfolio like, e. g., a thematic ETF to follow their personal interest. For instance, ethical investors often hold both ethical and unethical parts in their portfolios (Lewis 2001, Lewis and Mackenzie 2017). This strategy resembles a core satellite strategy with diversified core portfolios that are conventionally allocated by considering financial criteria, and thematic satellite portfolios that are attached to reflect convictions, personal interests, or any imaginable additional motivation. Hence, thematic investors deviate from textbook finance suggesting a relative investment in the WMP while research and financial models have yet not been adequately developed to better meet investors' needs.

3 Research gaps

The literature review shows several approaches that investigate investment performance in the context of additional objectives. However, on the one hand they do not sufficiently differentiate between monetary and non-monetary motivations of objectives, and on the other hand they do not consider the strategy of core satellite investing. Some studies analyse the relationship between CSR and stock performance or profitability (Alexander and Buchholz 1978, Aupperle et al. 1985, Friede et al. 2015). Furthermore, several studies evaluate the fund performance of

SR funds (Hamilton et al. 1993, Renneboog et al. 2008, Gil-Bazo et al. 2010, Revelli and Viviani 2013). However, whether the additional objective is pursued by financial motivations (socially responsible companies could be managed more sustainably and thus lead to better stock performances) or driven by non-monetary preferences is neither quantified nor the possible added value from the non-monetary motivation is set against. More importantly, these studies do not consider that often SR funds are not held exclusively but as supplements to conventional ones.

The differences between academic research and practical implementations lead to significant research gaps. First of all, even application-oriented studies that evaluate performances with regard to additional objectives like ESG ignore the fact that investors do not always hold tri-dimensionally optimized portfolios. In contrast, ethical investors often hold two separate parts in their portfolios, one for ethical and one for unethical parts, with different investment goals (Lewis 2001, Lewis and Mackenzie 2017). The effects of this presumably inefficient strategy have yet not been investigated. Furthermore, a generalized framework for thematic investments that would include not only ethical or financial objectives has not been stated.

Moving from a theoretical and academic perspective to a more practical perspective including the parties of an investment process, i. e., fund providers and investors, further room for improvement can be found. Fund providers are aware of the core satellite strategy stating that the core part of the portfolio typically makes up the greater part of the investment compared to the thematic satellite portfolio (Marchioni et al. 2016). However, they offer one-for-all solutions for the most common investment themes, no matter the corresponding conventional portfolios although they know that their thematic products do not make up a whole portfolio, but are supplemented to conventional portfolios. Nevertheless, no research steps have been taken to benefit from this strategy by anticipating correlation effects with conventional portfolio to improve core satellite portfolios' performances.

Focusing on investors, pragmatic strategies that are suitable for private investors have not been considered by scientific research, yet. Some studies expand the two-dimensional risk and return framework (Steuer et al. 2005, Utz et al. 2015, Gasser et al. 2017). However, these models do not only need financial expertise, data availability and ultimately private investors' willingness to work with complex models, but also suitable solving algorithms (Steuer et al. 2006, Hirschberger et al. 2013). Therefore, investors that need to weight their two portfolio parts would need to draw on complex optimization models and adapt them to a two portfolio case, but a pragmatic aid that would be truly useful for private investors has never been regarded.

To sum it up:

1: The inefficiency of the thematic core satellite model has not yet been investigated, neither is there a clear model or framework defined to do so.

2: Core satellite investing misses out beneficial correlation effects that could be exploited even by fund providers, however, nobody yet stated a guideline or provided a proof-of-concept work.

3: Private investors are left alone with the question of how to weight the two parts in their portfolios as no research has adequately focused on the two portfolio decision, yet.

4 Goals of this study

The ultimate goal of this study is to support the understanding and developments of thematic investments by first, finding a thematic framework to evaluate the inefficiency of core satellite portfolios, second, use it to improve thematic products, and third, aid investors in thematic portfolio choices.

In the previous sections, it has been demonstrated how recent studies integrate additional objectives besides risk and return into their portfolio models but that they do not sufficiently differentiate between non-monetarily motivated objectives. This shows that the performance analyses do not provide the necessary information for thematic investors. Furthermore, investors follow a strategy of core satellite portfolios that has not been considered, yet. Therefore, this study will use multi-criteria decision analysis (MCDA) methods, i. e., multi-objective programming to build a general thematic framework based on literature that considers non-monetarily motivated additional objectives besides risk and return with the goal to establish a model for further research. Hence, by displaying both efficient portfolio solutions and solutions based on a core satellite strategy, insights into the core satellite strategy will be given. The question 'how efficient are thematic core satellite portfolio solutions?' will be answered and influencing variables will be identified.

Thematic investments attract the attention of both private investors as well as fund providers. Considering the to be demonstrated lack of efficiency regarding core satellite portfolio solutions, suggestions will be given for both parties to unilaterally improve core satellite portfolio solutions. Hence, a proof-of-concept work will be provided for tailoring thematic portfolios to conventional core portfolios that enables fund providers to independently reduce the inefficiency of core satellite investments. These improved tailor-made thematic portfolios profit from beneficial correlation effects with conventional portfolios. This will increase the overall performance of the core satellite portfolio at the expense of less efficient stand-alone thematic portfolios.

After having provided a direction for improvements for fund providers, support will be given to private investors in terms of practical and truly pragmatic solutions for their reduced portfolio allocation problem of how to weight the two individual parts in their portfolios. The research analysis has shown that all concepts that consider multiple criteria in portfolio decisions either require expert knowledge or even complex portfolio models and solving algorithms. Therefore, based on this lack of research an additional call for research is stated that looks for portfolio decision rules that are applicable for private investors. The goal is to provide competitive and pragmatic decision rules to strengthen portfolio choices. Figure 5 briefly summarizes how the underlying publications of this dissertation address the different goals.

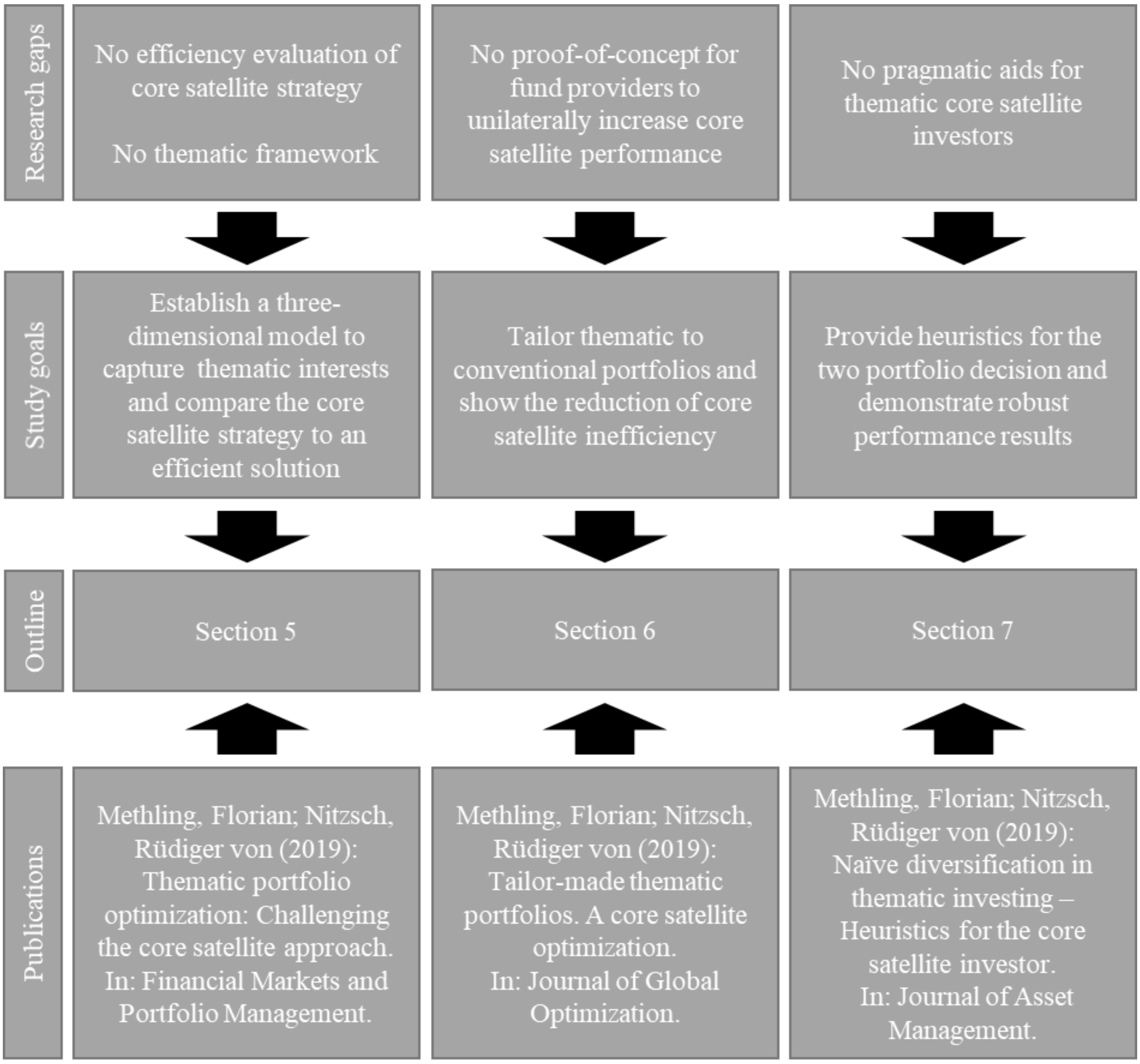

Figure 5: *Outline of this study showing how the publications provided content for the different sections to answer the research questions.*

5 Tri-dimensional thematic portfolio optimization

So far, an increased economic significance of thematic exchange-traded funds is observed providing investors with opportunities to consider more than just financial objectives. Hence, these investors gain a non-monetary added value by including thematic fractions in their portfolios. Traditional portfolio optimization models only target financial criteria and investors' needs remain unmet. Therefore, this section presents parts of the study "Thematic portfolio optimization: Challenging the core satellite approach" to suit these investors' needs (Methling and Nitzsch 2019d). This study uses a tri-criterion thematic portfolio optimization model as an overall framework. In a two-part analysis with tradable ETFs on the one hand and a simulation with 250,000 draws and 1,750,000 portfolio optimizations performed on the other hand, the status quo of thematic core satellite investing is compared to the tri-criterion model.

5.1 Theoretical model – transition from bi-criterion to tri-criterion

In 1952, Markowitz introduced the portfolio selection problem considering optimization in a mean-variance portfolio model (Markowitz 1952). This has become the foundation of modern portfolio theory. Therefore, it is used as the bi-criterion foundation of the thematic portfolio model.

In this original model, a risk-averse investor is assumed that focuses on two investment goals: maximizing the expected investment return E, and minimizing risk V which is measured by the standard deviation of the investment returns. In a considered selection problem, a portfolio is allocated by investing in N different assets. Each asset $i \in \{1, \ldots, N\}$ is described by the expected return μ_i and the standard deviation of returns σ_i, and its correlation with other assets $i, j \in \{1, \ldots, N\}$ is denoted by ρ_{ij}. The N-dimensional solution vector $\boldsymbol{x}=[x_1, \ldots, x_N]^{\mathrm{T}} \in \mathbb{R}^{\mathrm{N}}$ specifies the investment proportion x_i in each asset. The a posteriori formulation of the bi-criterion optimization problem is given by Eqs. (1)-(3)*.

* This is not to be understood as quasi code and conforms as a notation for simplicity, e. g., to Ehrgott et al. (2004), Steuer et al. (2007), Hirschberger et al. (2013). Optimization can be performed by, e. g., maximizing return under the restriction of a given maximum risk level.

$$max \qquad E = \sum_{i=1}^{N} \mu_i x_i \tag{1}$$

$$min \qquad V = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i \sigma_j x_i x_j \rho_{ij}} \tag{2}$$

$$s.\ t. \qquad \sum_{i=1}^{N} x_i = 1, \qquad x_i \geq 0 \tag{3}$$

Following Keeney and Raiffa (1993) an objective hierarchy should be complete, operational, decomposable, non-redundant and minimal. As long as non-monetary objectives are not displayed in a portfolio model, completeness is not achieved. Therefore, the two-dimensional objective hierarchy is expanded according to Hirschberger et al. (2013). In the following, the additional objective is a non-monetary one measured by the proportion of thematic assets in a portfolio. The third criterion is not to be further unbundled and is understood as the union of all non-monetary interests of an investor.

The binary parameter τ_i labels compliant assets while the criterion T denotes the relative proportion of these assets in a portfolio that conform to the investor's convictions and interests. The transition from a bi-criterion to a tri-criterion a posteriori portfolio optimization problem† leads to Eqs. (4)-(7):

$$max \qquad E = \sum_{i=1}^{N} \mu_i x_i \tag{4}$$

$$min \qquad V = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i \sigma_j x_i x_j \rho_{ij}} \tag{5}$$

$$max \qquad T = \sum_{i=1}^{N} \tau_i x_i \tag{6}$$

$$s.\ t. \qquad \sum_{i=1}^{N} x_i = 1, \qquad x_i \geq 0 \tag{7}$$

Accordingly, the efficient frontier is expanded to an efficient surface that shows all efficient portfolios with a maximum return for given maximum risk levels and minimum thematic proportions as shown in Figure 6.

† Same as (2). Optimization by holding two objective functions as restrictions.

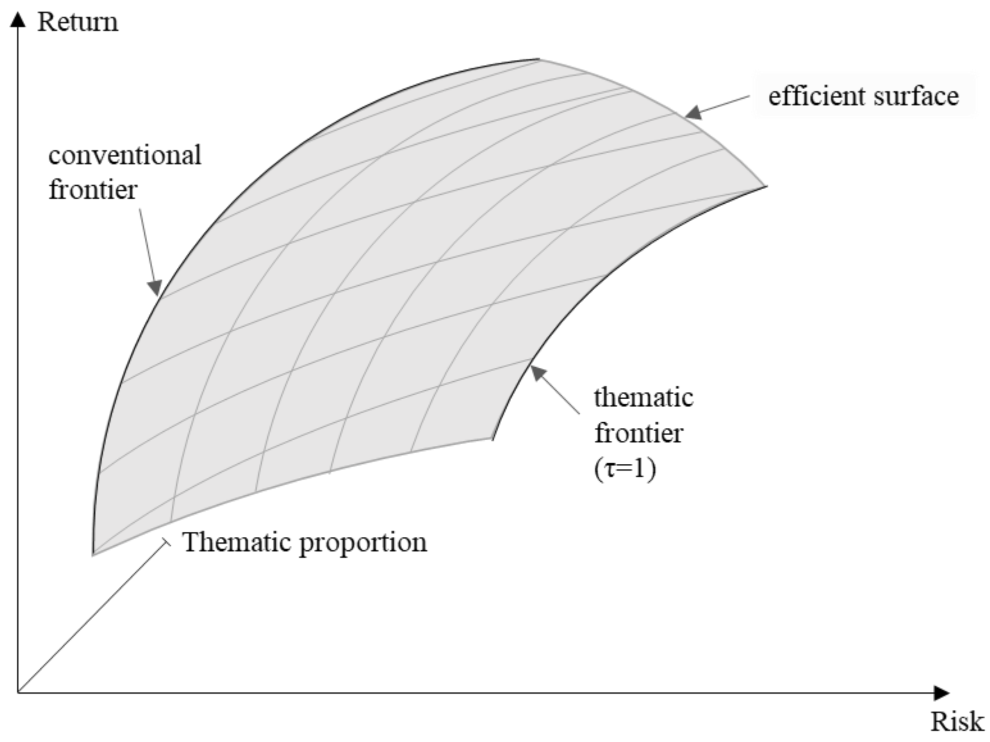

Figure 6: *Visualization of the efficient surface with three objectives: maximize return, minimize risk, and maximize thematic proportion.*

5.2 General methodology to evaluate core satellite inefficiency

The status quo of thematic core satellite portfolios is to be questioned and the inefficiency that results from disregarding the correlation effects between the two portfolios is to be quantified. Furthermore, the results of the inefficiency analysis need to be lead back to critical variables describing the two portfolios, i. e., the core and the satellite portfolio, and their relationship. Therefore, benchmark portfolios for the status quo need to be stated which are further optimized by the use of the tri-dimensional model to achieve empirical results.

The database is described in the following. Thematic ETFs are selected in a web search and identified by name and fund description. Ten conventional indexes complete the data set of 100 thematic ETFs. Due to the short history of thematic products this study focuses on the years 2016 and 2017. Data have been obtained from Thomson Reuters Eikon. Estimations for returns are computed as the averages of daily stock returns and their standard deviations are assumed as risk estimations. The database is constructed as the union of these indexes and the 100 thematic ETFs including a total of 3,891 stocks out of 47 different countries of exchange, 11 Sectors, 24 Industry Groups, 68 Industries, and 157 Sub-Industries.

The first approach considers the selection of 100 thematic ETFs and combines them with four conventional index-based funds. The second approach uses a simulation to increase the data sample of 400 combinations. It creates new artificial thematic and conventional ETFs by randomly drawing sub-samples from the database.

Each portfolio, thematic ETF, conventional ETF, or random sub-sample, is optimized regarding the Sharpe-ratio (Sharpe 1994). Therefore, each individual portfolio is the best performing one in terms of return per risk. Each combination of a thematic and a conventional portfolio with a thematic share $t \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ defines one benchmark status quo portfolio. These benchmarks are compared to the tri-dimensional portfolios.

Tri-dimensional portfolios use the thematic proportion T and the standard deviation V of a benchmark core satellite portfolio as restrictions for a return optimization. The variable δ (Eq. (8)) describes yield enhancements by the relative difference between the estimated returns of the benchmark (E_{CS}) and the tri-dimensional portfolios (E_{Tri}):

$$\delta(t) = (E_{Tri}(t) - E_{CS}(t))/E_{CS}(t) \tag{8}$$

Subsequently, a correlation analysis is conducted to provide further information on the dependence of yield increases on critical variables.

5.3 Core satellite portfolios reduce efficiency

The results show that optimizing tri-criterion portfolios leads to a mean yield enhancement of 5.67 % for tradable thematic ETFs and 6.23 % when considering the random-sample of the simulations. Hence, an assumed portfolio with a one year portfolio return E_{CS} of 8 % could be improved to a tri-dimensional portfolio with a return E_{Tri} of 8.2128 %. This leads to an absolute yield enhancement of 21.28 basis points for a portfolio that allocates minimum the same thematic proportion and maximum the same risk. The main finding of the correlation analysis shows that this improvement is negatively affected by a strong positive correlation between the thematic and the conventional portfolio.

5.4 Further research is needed to improve thematic core satellite investments

Currently, thematic interests of private investors are met in a core satellite strategy with thematic ETFs as satellite portfolios that capture the thematic interests of their investors. Motivated by the raising interest in thematic investments and the lack of scientific research with regard to optimization models that include non-monetary objectives, this study expands the work of Markowitz from 1952 in accordance with Hirschberger et al. (2013). Generalizing the subsequent work of, e. g., Utz, Wimmer, and Steuer (2015), an overall framework is established that not only includes sustainable investments, but non-monetary interests in general. Hence, this study proves and quantifies the assumed and theoretically justified inefficiency of the status quo of core satellite investments in thematic portfolio optimization. Results depend on the relative share that is spent in the thematic portfolio so that different amounts were considered within the methodology. Based on this framework and the empirical methodology that compares benchmark core satellite portfolios to efficient tri-dimensional portfolios, the correlation analysis shows that the correlation of the thematic portfolio with the conventional one significantly affects the efficiency of the resulting core satellite portfolio. Especially for very exotic themes that only few assets belong to and which only weakly correlate with the conventional portfolio, the tri-dimensional model increases returns up to 46.88 %.

The results are subject to the following reservations: short time frames and historical data as future estimations limit the significance of the results and are only partly mitigated with regard to DeMiguel et al. (2009a; 2009b) who justified the approach. Transaction costs are not considered but reduce the yield enhancements as tri-dimensional portfolios require the purchase of a large amount of individual assets in comparison to buying packages of assets, e. g., in the form of ETFs, in the core satellite approach. Furthermore, the applicability of a tri-dimensional portfolio optimization for private investors is highly questionable due to limited resources.

Nevertheless, two major implications of this study results are enlightened in the following. First, the correlation of thematic portfolios to conventional portfolios affects the efficiency of the whole core satellite portfolio, which can be targeted in the allocation of the individual portfolio and, second, the relative share spent in the thematic portfolio affects the efficiency as well without the need to reallocate the whole portfolio or establishing a tri-dimensional framework. The first implication is an opportunity for fund providers to improve the status quo of core satellite investments. When thematic ETFs become more tailored to the conventional portfolio and more particular in general, the correlation effects between the two portfolios can be exploited and core satellite investments improved. This call for research is answered in the next section. The second implication enables investors to improve their core satellite portfolios themselves by choosing the optimal relative thematic share. Therefore, pragmatic and useful heuristics will be developed in section 7.

6 Tailor-made thematic portfolios

The status quo in thematic investing resembles a core satellite strategy. Diversified conventional portfolios build the core of an investment while thematic satellite portfolios capture the non-monetary interests of private investors by investing in megatrends, social responsibility, or any imaginable theme. In this model, both portfolios are allocated separately. However, as already shown, this leaves correlation effects between the two portfolios unexploited which leads to inefficient core satellite portfolios. In order to reduce this inefficiency a call for research is stated that addresses fund providers to adapt their thematic products to this promising effect. To answer this call for research this section builds on the findings of the publication “Tailor-made thematic portfolios: A core satellite optimization” (Methling and Nitzsch 2019c). Thematic products that are tailored to specific conventional portfolios have the potential to improve the performance of core satellite portfolios without increasing private investors efforts or higher transaction costs. At the expense of less efficient thematic products more efficient core satellite portfolios can be allocated. Further analysis sees more concentrated thematic portfolios to be the key to higher core satellite performances.

6.1 Theoretical model – how to tailor thematic ETFs

The individual portfolio selection problem of a private investor that has already picked a conventional as well as a thematic portfolio with allocation vectors $\boldsymbol{x_C}$ and $\boldsymbol{x_T}$ given exogenously by the providers is based on the earlier developed tri-dimensional framework. It can be reduced to choosing an investment amount $t \in P = \{t \in \mathbb{R} \mid 0 \leq t \leq 1\}$ for the thematic satellite portfolio (Amenc et al. 2004). Hence, the a posteriori formulation can be stated with $\boldsymbol{C}$ as the covariance matrix of asset returns $\boldsymbol{\mu}$:

$$max \qquad E(t) = \boldsymbol{\mu}'[(1-t)\,\boldsymbol{x_C} + t\,\boldsymbol{x_T}] \tag{9}$$

$$min \qquad V^2(t) = [(1-t)\,\boldsymbol{x_C} + t\,\boldsymbol{x_T}]'\,\boldsymbol{C}\,[(1-t)\,\boldsymbol{x_C} + t\,\boldsymbol{x_T}] \tag{10}$$

$$max \qquad T(t) = \boldsymbol{\tau}'\,[(1-t)\,\boldsymbol{x_C} + t\,\boldsymbol{x_T}] \tag{11}$$

The basic idea of tailor-made thematic portfolios builds on using the tri-dimensional framework to optimize a tri-dimensional portfolio. This portfolio uses an assumed proportion (1-*t*) of the allocation vector of the conventional portfolio and is only allowed to increase the shares of thematic assets (Eqs. (12)-(17)). Subsequent relative subtraction of the conventional portfolio from the tri-dimensional portfolio results in an allocation that needs to be standardized to suit as an allocation vector of the tailor-made thematic portfolio.

$$max \qquad E = \boldsymbol{\mu}'\boldsymbol{x} \tag{12}$$

$$min \qquad V^2 = \boldsymbol{x}'\,\boldsymbol{C}\boldsymbol{x} \tag{13}$$

$$max \qquad T = \boldsymbol{\tau}'\boldsymbol{x}, \tag{14}$$

$$s.\ t. \qquad \boldsymbol{x}'\,\boldsymbol{1} = 1, \tag{15}$$

$$x_i\tau_i \geq x_{C_i}(1-t)\tau_i\ \forall\ i = 1, \dots, N \tag{16}$$

$$x_i(1-\tau_i) = x_{C_i}(1-t)(1-\tau_i)\ \forall\ i = 1, \dots, N\ . \tag{17}$$

The problem can be solved when two of the objective functions are held as restrictions. Subtracting $(1-t)\boldsymbol{x_C}$ from solution vector $\widehat{\boldsymbol{x}}$ and afterwards standardizing the vector generates the allocation of the new tailor-made thematic satellite portfolio $\widehat{\boldsymbol{x_T}}$.

6.2 General methodology to quantify the effects of tailoring thematic portfolios

In accordance to the earlier approach of comparing core satellite portfolios to tri-dimensional portfolios, this study uses a similar approach. Benchmark portfolios are stated that are compared to portfolios that are optimized within the new model. Explicitly, this means that conventional portfolios as well as thematic portfolios are chosen from a dataset, both defined by tradable ETFs as well as randomly drawn subsamples. Afterwards, they are optimized with regard to the Sharpe ratio and combined with different specific relative shares of the thematic satellite portfolio. These portfolios suit as benchmarks and are compared to core satellite portfolios that include tailor-made thematic satellite portfolios instead of Sharpe ratio optimized ones. Results both quantify the performance improvements in terms of relative risk reduction in comparison to an efficient tri-dimensional portfolio as well as describe the changes that were made within the thematic satellite portfolios. Eq. (18) defines the measure to quantify the relative risk reduction for different relative portfolio weights of the thematic portfolio. *t* defines the relative proportion of the thematic portfolio and t_t indicates the relative proportion that the tailor-made portfolio was initially optimized for. The efficient tri-criterion portfolio characteristics are denoted by V_{Tri}, E_{Tri}, and T_{Tri}. The characteristics of the two core satellite portfolios are measured by $\hat{V}$, $\hat{E}$, and $\hat{T}$ for a combination with a tailor-made thematic satellite and V_{CS}, E_{CS}, and T_{CS} for a combination with a separately optimized satellite.

$$\delta_{TM}(t) = \frac{V_{CS}(t) - \hat{V}(t, t_t)}{V_{CS}(t) - V_{Tri}(t)} \tag{18}$$

As Figure 7 shows, tailor-made thematic satellites will be allocated with higher volatilities, but enable core satellite portfolios with the same return $\hat{E} = E_{CS} = E_{Tri}$ and thematic proportion $\hat{T} = T_{CS} = T_{Tri}$ as the tri-dimensional portfolio and the benchmark core satellite portfolio but can reduce volatility in comparison to the benchmark core satellite portfolio.

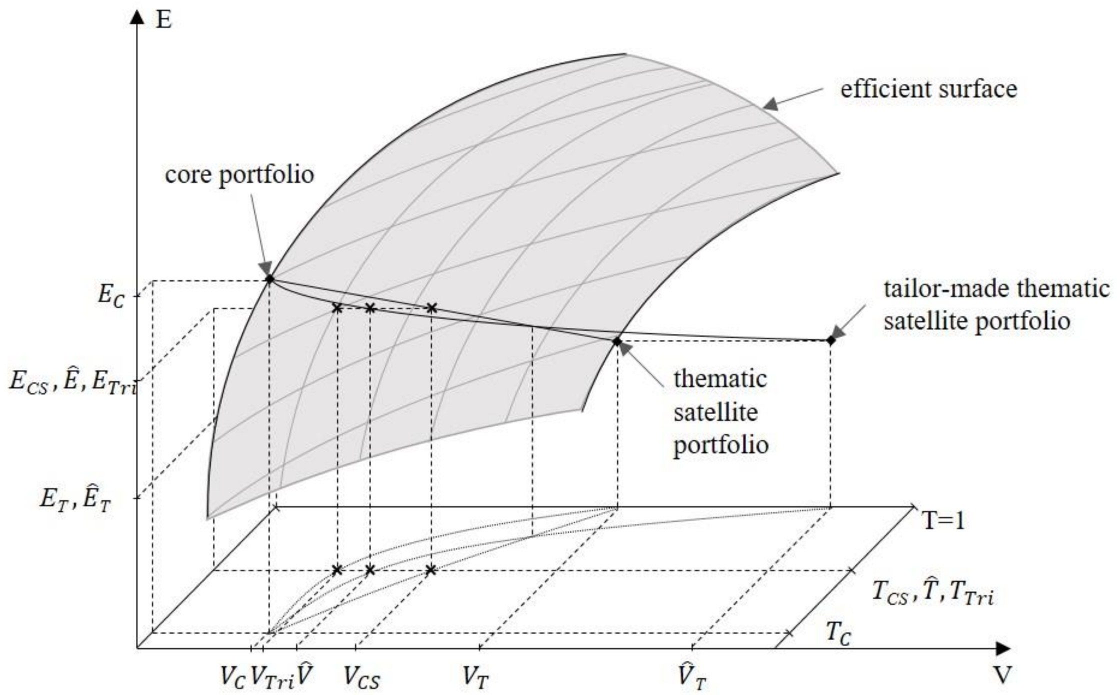

Figure 7: *Efficient tri-criterion portfolio volatility V_{Tri}, volatilities $\hat{V}$ and V_{CS} of core satellite portfolios with a tailor-made thematic satellite and a separately optimized satellite for given estimated return $\hat{E} = E_{CS} = E_{Tri}$ and thematic proportion $\hat{T} = T_{CS} = T_{Tri}$.*

6.3 Tailor-made thematic portfolios increase core satellite efficiency at the cost of satellite efficiency

Results of 100 tradable thematic ETFs in combination with four different conventional ETFs show that tailor-made thematic portfolios reduce core satellite inefficiency on average by $\delta_{TM} = 11.74$ % while being 4.55 % more volatile themselves. The results of the simulation are even more pronounced and show 16.99 % relative inefficiency reduction for the core satellite portfolio with 2.22 % higher volatility as an offset. In general, the efficiency of a core satellite portfolio can be increased by a targeted efficiency reduction of the tailor-made satellite portfolio.

Additional findings support the earlier findings showing that tailoring satellites to core portfolios leads to higher concentrations in portfolio allocations. These tailored portfolios have increased weighted amounts of assets less correlated with the core portfolio.

6.4 Tailor-made thematic portfolios enable a higher portfolio concentration

Summarizing the implications of this study, there is one major limitation that needs to be overcome, but several positive outlooks. The problem of the idea of tailoring thematic products to conventional ones lies in the trade-off. Improving the performance of the core satellite portfolio can, to a certain extent, only be achieved by reducing the efficiency of the satellite portfolio. Although an investor should always be interested in the overall performance of the investment, mental accounting and advertisements that can only display the single product properties will be limiting factors.

When fund providers overcome these problems by smarter advertisements that are tailored to conventional products as well, new opportunities will arise. Bundles of financial products can be marketed, that use the correlation effects between several products without the need to create a new product for every combination of conventional and thematic portfolio, targeted thematic proportion, and risk evaluation. As the analysis has shown, tailor-made thematic products can increase performance for a range of thematic proportions that can display both risk appetite as well as thematic interest.

Furthermore, more narrowly defined, particular themes can be developed that capture very exotic themes. When they are combined and most importantly advertised and understood as parts in a core satellite portfolio, the high individual risks can be compensated. This motivates further research and increase the amounts of innovations in financial product development.

7 Improving core satellite portfolios by the use of heuristics

So far, the need for further research has been quantified by comparing core satellite portfolios to efficient ones, and a framework for subsequent studies has been developed. Furthermore, trendsetting ideas have been analysed under a fund provider's view. To examine ways for private investors to independently support their investment strategies, the following section will give pragmatic support for their investment decision. As shown above, a thematic core satellite investor who has already picked a conventional portfolio and a thematic satellite to supplement has reduced the portfolio selection problem to only one variable, i. e., the proportion of the thematic satellite portfolio. The question: how to weight the satellite in comparison to the core portfolio?

This section demonstrates the main findings of the study "Naïve diversification in thematic investing: heuristics for the core satellite investor" to provide answers to the question above (Methling and Nitzsch 2019b). Although it has already been shown that correlation effects are crucial for the efficiency of thematic portfolios and the use of tri-dimensional models, complex factor models and sophisticated research are not applicable to private investors. Therefore, heuristics are investigated with regard to their usability as well as competitiveness. Starting with naïve diversification that allocates an equal part to all available asset, further developments include the different stock amounts in the two portfolios and the portfolio concentration to determine investment rules.

7.1 Theoretical model – how to aid thematic investors with naïve diversification

> *"A man should always place his money, one third into land, a third into merchandise and keep a third in hand"* ‡

Derived from this 1,500 years old allocation rule, naïve diversification was generalized to

$$\boldsymbol{y} = \begin{pmatrix} 1/N \\ \vdots \\ 1/N \end{pmatrix} \tag{19}$$

representing an N-dimensional investment vector $\boldsymbol{y}$ that equally allocates the budget to all available assets.

The most intuitive adaption of the general case to the specific portfolio selection problem of the thematic core satellite investor leads to a portfolio-based naivety. Portfolio-based naivety (PBN) equally allocates the investment budget to both portfolios, i. e., y_1=y_2=50 % with y_1 being the relative share spent in the conventional portfolio and y_2 the relative share of the thematic portfolio.

$$\boldsymbol{y_{PBN}} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \tag{20}$$

However, naïve diversification should minimize exposure to different stocks, but not portfolios. Hence, a second adaption considers the two portfolios as equally weighted and invests proportionally to the numbers of stocks included in the conventional and the thematic portfolio.

‡ Talmud - Bava Metzia 42a

The solution vector of stock-based naivety (SBN) is given as follows – n and m denote the numbers of stocks included in the conventional and the thematic portfolio.

$$\boldsymbol{y_{SBN}} = \begin{pmatrix} \frac{n}{n+m} \\ \frac{m}{n+m} \end{pmatrix} \tag{21}$$

As most of the thematic and conventional ETFs that investors could consider are not equally weighted, actual exposure to stocks can be better minimized by considering the concentration of the two portfolio allocations. Hence, the Herfindahl index is used as a measure of concentration that sums up the squared relative shares x_i of the allocation vectors $\boldsymbol{x_C}$ and $\boldsymbol{x_T}$ of the conventional and the thematic portfolio. Therefore, concentration-based naivety (CBN) invests inversely proportional to the concentration, i. e., the Herfindahl index of the two portfolios (Eq. (22)).

$$\boldsymbol{y_{CBN}} = \begin{pmatrix} \frac{H(x_T)}{H(x_C)+H(x_T)} \\ \frac{H(x_C)}{H(x_C)+H(x_T)} \end{pmatrix} \tag{22}$$

7.2 General methodology to compare different core satellite allocations

A rolling window is used in accordance to DeMiguel et al. (2009a; 2009b). Shown in Figure 8, from 01/10/2015 to 31/12/2018, in-sample windows of 250 trading days and out-of-sample windows of 500 trading days are shifted by including in each subsequent sample 15 later days and dropping 15 earlier days. By combining 10 conventional and 50 thematic ETFs in each of the samples, the method leads to 11,000 data points for comparing the different heuristics.

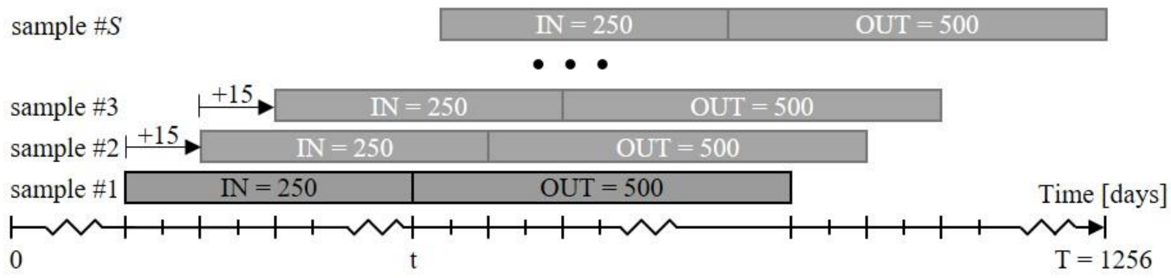

Figure 8: *Visualization of the rolling window approach for in-sample estimation windows IN and out-of-sample testing windows OUT resulting in S different samples.*

The heuristics are not only compared to each other but also to two different benchmark portfolios. As naïve diversification has originally been established to provide risk control, the allocation rules are compared with respect to three different risk measures calculated for the out-of-sample windows, i. e., volatility, Value at Risk ($VaR_{95\,\%}$) and maximum drawdown (MaxDD).

Volatility V is measured as already mentioned via the standard deviation of daily returns (see Eq. (2) above). The $VaR_{95\,\%}$ is defined by using the out-of-sample period as an estimation for a discrete distribution function F_Z and computing the estimated loss that a random variable z will not exceed in 95 % of cases. The maximum drawdown is calculated as the largest difference of a portfolio value peak to a subsequent portfolio value low during the out-of-sample window.

One of the two benchmarks, 99BM, is stated as an abstract of the total number of considerable investment allocations. Here, 99 different relative shares of the thematic portfolio are considered with y_1 = [1 %, 2 %, …, 99 %] while a heuristic is assumed to be competitive if it leads to a smaller risk evaluation than at least 50 of the benchmark portfolios. The other benchmark is based on a minimum variance optimization (MVO) that uses daily returns of the in-sample period as estimations and assumes them to reoccur in the future. Again, heuristics are assumed to be competitive if they result in smaller risk evaluations.

7.3 Heuristics are competitive

Table 1 summarizes the findings of this study. In general, a minimum variance optimization shows the smallest risks. However, considering the volatility measure as an example, no dominance is observed because the mean volatilities of the heuristics are in a range of only plus one standard deviation and MVO could only achieve smaller volatilities in 57.61 % (in comparison to SBN) and 54.21 % (in comparison to CBN) of cases.

	PBN	SBN	CBN	99 BM	MVO
Volatility	12.49 %	11.8 %	11.77 %	17.34 %	11.65 %
Value at Risk	-1.26 %	-1.18 %	-1.18 %	-1.8 %	-1.17 %
MaxDD	13.05 %	12.19 %	12.24 %	17.78 %	11.97 %

Table 1: *Mean yearly volatility approximated from daily volatility with factor √250, mean Value at Risk 95 %, and mean maximum drawdown (MaxDD) of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*

Robustness checks consider different time frames, thematic families, conventional indexes, and bullish as well as bearish market scenarios. All of them support the main findings with regard to competitiveness.

7.4 Pragmatic aids direct a promising way

While the previous sections developed a thematic framework and pointed a way for further development in fund providers' research, this study gives private investor practical advice. Within their specific portfolio selection problem of choosing a relative weight of the thematic part in the core satellite portfolio, heuristics have been developed based on naïve diversification. Besides competitiveness, one main goal of this study was to give practical advice that is suitable for private investors no matter their financial literacy or resources.

With regard to the portfolio-based naïve strategy that always invests 50 % of the investment budget in the two portfolios, applicability is given as there are no calculations or resources required. However, the effort increases when the different numbers of stocks included need to be taken into account. These numbers are provided within a fund prospectus and should be easily understood by private investors. The required effort increases even further with regard to the calculation of Herfindahl indexes that are needed for the last heuristic of concentration-based naivety. Herfindahl indexes are yet not provided by fund providers so that they need to be calculated based on the constituent weights of an ETF. This effort has individually to be set against the diversification benefits depending on the resources that an investor is willing to invest in the portfolio decision.

Hence, some truly useful tools are provided for private investors that enable them to autonomously strengthen their core satellite portfolio without excessive efforts. These results provide long-time needed advice that has yet been missing besides research that has mostly been done and written for financial experts.

8 Implications

The motivation of this study was to improve the understanding of thematic investments by describing them in an adequate model and to improve the status quo by advising fund providers and investors. Considering the particular goals of this study, MCDA methods were to be used to find a general thematic framework that enabled a display of recent developments in finance with regard to thematic investments. Being able to model thematic investments, further analysis should quantify the inefficiency of the status quo strategy of core and thematic satellite portfolios. Addressing fund providers, a new direction of tailoring thematic portfolios to conventional portfolios was to be set. With regard to private investors competitive and pragmatic portfolio allocation rules for the two portfolio case were to be presented to strengthen their portfolio choices.

A thematic core satellite strategy allocates inefficient portfolios in comparison to an elaborate optimization that finds optimal solutions in three-dimensional space. However, presumably mostly due to the required effort, data availability, and transaction costs for setting up a portfolio that requires the purchase of several stocks instead of only two funds, the status quo still remains a core satellite strategy. By considering the three-dimensional model, it becomes clear that a thematic interest reduces the expected financial performance of a portfolio. Hence, a non-monetary thematic interest that is justified with an additional non-monetary interest (i.e., no insider information, risk perception or market-deviating expectations) has to be set against financial interests in a clear trade-off. As a thematic interest adds an additional objective to the optimization or reduces the solution space by focusing especially on thematic stocks, an investor has to accept and understand the trade-off. Nevertheless, thematic investments are usually advertised by monetary incentives which demonstrates the lack of clarification and information.

Two implications follow immediately. First, investors need to be educated so that they understand this trade-off between monetary and non-monetary interests and address it pro-actively. Second, the benefits of a core satellite strategy need to be compared to the reduction in financial performance. A benefit is the reduction of set-up costs, i. e., both the reduced research effort for a core satellite weighting decision instead of a decision problem in a three dimensional model and the smaller cost to purchase and allocate an initial portfolio. Further benefits might arise from the added value of a separate display of the thematic and the conventional portfolio performance and the potentially better divisibility of investment budget when investing in a fund instead of stocks.

Considering the implications of the proof-of-work concept of tailor-made thematic portfolios, results provide interesting insights. On the one hand, tailor-made thematic portfolios enable a more efficient use of the core satellite strategy. Thus, it directly improves the status quo with regard to private investors' portfolios and performances. On the other hand, these thematic portfolios profit from beneficial correlation effects as they are tailored to a conventional portfolio. Consequently, at the cost of the individual expected performance, tailor-made thematic portfolios can be allocated with higher concentrations and larger individual risks. This opens up opportunities to target more specific themes and to differentiate these portfolios from others. A very popular example for thematic ETFs that try to capture society changing megatrends are ETFs focused on artificial intelligence. Currently, the individual evaluation of an investment with regard to appropriate risks does not generally allow for very specific investment themes due to the cluster risks. However, by tailoring the ETF to a conventional one and understanding it as a bundle, these ETFs can also capture the underlying macro trends like autonomous mobility or even more specialised micro trends that are driven by recent developments, e. g., in radio or light detection and ranging systems.

Stretching the boundaries of possible implications, more specified thematic ETFs could even help to reduce the negative effects that are usually associated to broad basket ETFs. The broader the basket that the ETF invests in, the less the individual company stocks are evaluated and chosen by reason. This investment stream leads to overvaluations of stocks that are only bought because they are in a bundle with high-performance stocks which ultimately increases financial risks and the likelihood of financial bubbles. This can be reduced when thematic ETFs become more specialised to small basket portfolios so that they do not pull too many stocks in their price movements. While this hypothesis is not sufficiently supported by the analyses of this dissertation, it constitutes a strong call for further research.

This research also leads to directly applicable practical recommendations for private investors. Thus, important implications are drawn from the results considering heuristic strategies for the core satellite investor. Benartzi and Thaler (2001) have demonstrated that some investors show a tendency to follow a naïve strategy by investing equal amounts in all available assets. Based on this strategy for the two portfolio case of thematic core satellite investments, results show that even small adaptions, e. g., to the number of stocks included in the two portfolios can reduce the portfolio risk. Further, more elaborate heuristics that also consider the concentration of the two portfolios by the Herfindahl index can even be more promising. This opens up opportunities for private investors that do not have the resources, the capabilities, or the interest for elaborate optimizations to take responsibility for their investment and to define their allocations at least more sophistically than by only spending an equal part in any asset.

9 Limitations

The implications of this study have to be reflected with regard to limitations of the models. In the following, limitations are discussed that consider both theoretical as well as practical reservations. On the one hand, theoretical reservations arise within the optimization of a model that is highly sensitive to the input data. On the other hand, practical reservations arise by holding assumptions that simplify real world problems to display them in a model.

Optimization problems need data as inputs. To define a model for the optimization problem, especially in a portfolio selection problem, estimations about future developments need to be given. These estimations can be based on historical data, simulations, or educated guesses and beliefs of financial experts. As the solutions of mean-variance optimizations are highly sensitive to the input parameters, the validity of input data is fundamentally important. Therefore, the data should be as reliable and objective as possible. Using financial experts as data providers might lead to good estimations. However, the tendency of individuals to overestimate recent influences limits the objectivity of the results and brings a hardly measurable bias into the data impossible to quantify. Simulations in comparison use simplified models of the developments and estimate more parameters with smaller influences. For example, the development of a stock could be estimated by considering the interest rates of the national banks, the consumer price index and a sentiment analysis of investors. This methodology would reduce the impact of a particular bias but would also require a new model that itself has certain limitations and influences the results.

The results and drawn implications are computed by the use of historical data as the remaining option of the aforementioned ones. Price returns are observed and assumed to reoccur in the future. Therefore, they serve as estimations for future price developments and correlations between these developments. These data are needed as inputs for the different models that find optimal solutions for a problem that has implications for the future but is optimized in the past. While the objectivity of the inputs is given, validity is given only to a certain extent. Trends in the past can be followed in the future, but it can also be the other way around. Nevertheless, in the first two parts of the study, only the efficiency of solutions is compared for different models that have the same objectives. Hence, by using the same input data for all models and only comparing the efficiency but not the performance in a subsequent time frame, the implications stay the same.

This transferability does not apply to implications with regard to the robustness of these solutions. In research, a common consensus criticizes that mean-variance optimization does not only maximize the efficiency of solutions but also the effects of estimation errors (Michaud 1989). Therefore, an efficient solution is not to be misunderstood as the portfolio that leads to the best performance with regard to the objectives. Small deviations in the estimations can lead to significant differences in the portfolio weights and the performance of a portfolio. While the first two studies were initially set up to quantify only the inefficiency of model solutions, in the third study, the performance of strategies were about to be compared. Hence, optimized solutions are compared in subsequent time frames to check how strongly their performances deviate by unforeseeable price developments. Within a number of robustness checks, the influence of, e. g., different market scenarios and time frames are tested so that the effects are better understood. Nevertheless, the short history of thematic products did only allow short time frames so that even the robustness checks need to be subject to reservations.

In conclusion, the implications of the studies do strongly depend on the quality of the input data. Hence, data collection is one important cornerstone of not only modern portfolio theory but also portfolio optimization in general. However, this dissertation focuses on the second part of the portfolio selection problem that determines portfolio allocations based on already collected data. Nevertheless, both parts, data collection and portfolio optimization, are connected and further research needs to investigate the effects in more detail.

The database of the studies is biased by investor attention in terms of market volume. The considered conventional portfolios are based on indexes that are highly traded, and thematic ETFs could only be taken into account if especially media coverage stated an initial interest so that a thematic fund was opened in the first place. These limitations were partly reduced by using an additional approach. A simulation randomly defined new sub-samples to reduce the bias and create new unbiased artificial ETFs. However, the extent to what a random selection of an already biased sample can reduce these limitations can also only be evaluated in further research.

With regard to displaying real world problems in a simplified model a first limitation describes the exclusion of transaction costs that arise when allocating an initial portfolio by purchasing individual products. However, disregarding transaction costs has different effects within the different studies. In the first model, the exclusion has the biggest impact on the results as the number of assets that need to be invested in vary the most. A core satellite portfolio is allocated by purchasing two portfolios, i. e., two different funds. In comparison, allocating a tri-dimensional portfolio as given by optimization requires buying single stocks. Assuming that the cost of purchasing a product is the sum of a fixed and a variable cost, transaction costs would increase drastically and reduce the performance of a portfolio. Therefore, transaction costs need to be included in further research. Furthermore, advanced solving algorithms would be needed to solve the expanded portfolio selection problem, as it is not only a quadratic problem, but becomes also a mixed integer one to consider fixed purchase costs of stocks. Thus, computation time and complexity are expected to increase strongly. The same effect needs to be considered with regard to the idea of tailor-made thematic portfolios. While private investors still face transaction costs twice when purchasing a thematic and a conventional fund, fund providers would need to calculate different costs. The study shows that tailor-made thematic portfolios usually allocate less assets and are concentrated within these assets more strongly. Consequently, there is no particular additional implication for the situation of private investors, but fund providers could in fact reduce their fixed costs of opening the funds, as they would not need to purchase so many different stocks in the first place. On the downside, running the fund could be more expensive as a higher concentration of a fund could require costly rebalancing more often. Hence, the effects for fund providers with regard to transaction costs need to be further investigated. Limitations with regard to transaction costs in the third study can be disregarded as they would equally reduce the performance of both two-portfolio strategies.

Notwithstanding, the effects of assumptions describing private investors' attitude and behaviour need to be discussed in more detail. The third study works with the smallest assumptions only stating that risk measured by volatility, Value at Risk, and maximum drawdown is not in favour of investors' objectives and is to be reduced. This assumption is in line with commonly assumed investor models, i. e., a risk-averse investor. However, the study works without a concrete preference function as the considered criteria only serve as measures for some undesired risk. It is questionable if less risk is always preferred over more risk. At least, this assumption does not hold in general. There are investors, that, e. g., trade for entertainment and enjoy risky propositions (Dorn and Sengmueller 2009). There are also investors imaginable that understand risk as a means to attract attention. Such themes that have a great media coverage could lead to a desired social change or simply to attract more investors to give impetus to the price development. Of course, the fundamental objective of such investors could still be to reduce risk, but this just leads to a second assumption of these studies: investors are aware of their preferences and act accordingly. Therefore, the conditional applicability of these assumptions to actual investors' attitude and behaviour limits the implications.

This assumption of awareness is also highly relevant when it comes to implementing tailor-made thematic portfolios that are introduced in the second study. To make tailor-made portfolios competitive in comparison to regular thematic portfolios, investors need to understand their own fundamental objectives so that effects of biases like mental accounting can be reduced. Private investors have a tendency to evaluate different investments separately although the most interesting evaluation should consider the whole portfolio. How far this will limit the practical application is hard to estimate. Either prototyping or further investigation with investor surveys need to enlighten this field.

Currently, thematic ETFs are mostly advertised by financial promises although an additional constraint such as only investing in theme-related stocks can only reduce the financial efficiency of a conventional portfolio solution. Consequently, many misunderstandings arise when it comes to investors' objectives or investors that try to understand their objectives. This gives a strong limitation to the results of this study as applicability is based on understanding. Therefore, the results of this study can only be applied to situations in which private investors are fully aware of their trade-off between monetary and non-monetary interests and follow their thematic interests as a fundamental objective.

10 Outlook

Further research has to address the fundamental objectives of investors. The two dimensional paradigm of modern portfolio theory of considering only the returns and risks of different investments has served its time. New developments are needed to display the multiple facets of portfolio decisions and include all the relevant aspects based on investors' investment goals. Non-monetary objectives need to be further investigated and better understood and more tools need to be provided for private investors. Considering their non-monetary objectives, indexes and measures must be developed so that the objectives can be displayed in more detail. Only then, the added value of equally evaluated assets in terms of financial criteria can be differentiated.

Furthermore, the case of a thematic core satellites strategy with multiple thematic satellites needs to be investigated. Thus, the inefficiency of core satellites solutions in comparison to an optimal solution can be quantified and point the direction for further research. It could be analysed if multiple satellites are capable of replacing a conventional core portfolio and how many thematic satellites would be needed to provide the same diversification that only conventional portfolios provide so far. Different themes are likely to be differently suited for diversification as the findings imply that correlation effects are highly responsible for efficiency increases. Therefore, thematic investments need to be further understood and characteristics to be identified. This may lead to more effective thematic investments by using correlation effects on a thematic level.

Combining correlations on a thematic level and addressing the aforementioned need of collecting valid data as inputs for optimization models, an interesting research field could rise up by evaluating correlation data not by historical time series, but actual causal relationships. Historical data has proven to lack on validity and reliability. Hence, trend following thematic portfolios present opportunities to estimate correlations by considering the different effects that the underlying socio-economic trends have on each other and on society. Methods of scenario analysis like cross impact balances seem to be very promising. By estimating if a megatrend like demographic change positively or negatively affects rapid urbanization future scenarios could be identified that are more likely to happen than others. Using this information of identified likely future scenarios, correlations could be calculated that provide the necessary input data for optimizing an M-dimensional allocation vector with M being the number of different megatrend portfolios considered. Sound reasoning could replace retrograde analyses of past data. This can improve the understanding of allocation vectors and could even enable private investors without any financial literacy to get insights into their portfolios and to provide the necessary input parameters themselves. They only needed to estimate the effects that their targeted thematic portfolios respectively their underlying trends have on each other.

Besides investments in megatrends, non-monetary objectives of investors need to be better understood so that portfolios can be better optimized. This requires fundamental and generic objective hierarchies and methods to determine the trade-offs and preferences that best reflect the interests of individual investors. First of all, best practices for the financial services industry need to be evaluated on how to differentiate monetary and non-monetary objectives of investors. Driven by overconfidence or media attention biases, only to name a few, investors need to understand the effects of focusing on a thematic asset sub-sample. Financial consultants need to know how to handle those biases and how to deal with truly non-monetary thematic interest with underlying fundamental objectives.

To close the circle to the introduction of this study: The way financial decisions are understood and handled has changed drastically over time so that new financial tools are needed to capture the multi-dimensionality of these decisions and their developments (Zopounidis et al. 2018). MCDA researchers lead the development in research to build the theoretical groundwork for displaying and understanding financial/ non-financial trade-offs (Aouni et al. 2018). However, more research is needed to follow these developments and to adequately address the needs of all parties in financial decision making.

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Paper 1: Thematic Portfolio Optimization - Challenging the Core Satellite Approach

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Thematic portfolio optimization – challenging the core satellite approach

In recent years, thematic exchange-traded funds (ETF) have increased in economic significance. Investors in thematic ETFs have more than just financial objectives and gain a non-monetary added value from a thematic portion in their portfolios. Therefore, traditional portfolio optimization models which target only financial criteria cannot suit these investors' needs anymore. Nevertheless, to count in their thematic interests, investors adapt a core satellite strategy in which conventional core portfolios and thematic satellite portfolios are combined. Thus, these portfolios are separately optimized without further considering inter-portfolio correlation effects. Since modern portfolio theory has originally been established to, *inter alia*, optimize these correlation effects, portfolios can only be efficient by chance. Therefore, this study targets the correlation effects between conventional and thematic portfolios and uses a tri-criterion thematic portfolio optimization model as an overall framework. Throughout a two-part analysis with tradable ETFs and a simulation with 250,000 draws and 1,750,000 portfolio optimizations performed, the status quo is compared to the tri-criterion model. Quantifying the suboptimality, simulation results show a mean portfolio improvement of 6.23 % measured as relative yield enhancement. Further analysis concludes that the more narrowly a theme is being defined and the more particular it is relative yield enhancements can increase up to 46.88 %.

Keywords: Portfolio management; Thematic investing; Portfolio optimization; Finance; Multiple criteria analysis

JEL codes: G11 Portfolio Choice and Investment Decision; G24 Investment Banking; G4 Behavioral Finance

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1 Introduction

Modern portfolio theory goes back to Markowitz' portfolio selection problem in 1952, and still primarily focuses on financial incentives. In contrast, behavioral finance teaches that investors have more than just rational and solely financial objectives and can therefore not be satisfied within mean-variance portfolio optimization and conventional funds (Barberis and Thaler 2003; Kahneman and Riepe 1998). Non-monetary objectives are experimentally proven by Webley, Lewis, and Mackenzie (2001) and need to be taken into account. Therefore, this study finds a theoretical model that includes non-monetary objectives. Furthermore, it demonstrates that these objectives are still inefficiently handled within the common strategy of core satellite investing.

In recent years, investors have attempted to address non-monetary interests in so-called thematic investment funds. These thematic funds are supplemented to conventional ones, resulting in a strategy similar to a core satellite approach (Marchioni et al. 2016). Actually, in a core satellite strategy, a diversified core portfolio ensures said diversification while the satellite portfolio aims for an uncorrelated return surplus.

In comparison, the derived thematic investment strategy also manifests investors' convictions and non-monetary interests within the satellite portfolio (Magoon 2009, Bérubé et al. 2014). Up to now, the most economically relevant thematic interests are environmental, social or corporate governance (ESG) aspects. In the United States alone, the corresponding market size has grown significantly to $12 trillion of all investment under professional management in 2018 (US SIF Foundation 2018). Lewis (2001) has shown that many ethical investors follow the aforementioned strategy by allocating both ethical and unethical parts in their portfolio.

Besides ethical investors, the amount of megatrend investors is rising, too. These investors, e. g., try to anticipate the effects of disruptive technologies, demographic change, or rapid urbanization and track the corresponding economic development with their portfolios

(Giammattei 2014; Marchioni et al. 2016). To this end, megatrend exchange-traded funds (ETF) like "Disruptive Technologies ETF" or "Ageing Population ETF" are managed with volumes as high as $1 billion[§]. While these themes are clearly advertised to satisfy monetary goals, they are also motivated by additional interests. As these investments are associated to a strong belief in the long-term economic impact, investors experience a higher confidence in their portfolio (Forster 2017).

Personal brand perception based on personal experiences with a company's products can influence investors' decisions, too. Therefore, Anand and Cowton (1993) have investigated additional dimensions of utility functions to understand positive non-financial company values. All these different motivations and dimensions of utility considered, thematic investing is a strategy that supports investors' convictions by allocating adequate assets to a portfolio (Bérubé et al. 2014).

However, the combination of two portfolios in the core satellite strategy, one conventional and one thematic, can only be accepted as a pragmatic solution to take into account the thematic interests of investors. Potential advantageous inter-portfolio correlations are not yet regarded respectively exploited. Consequently, the thematic core satellite portfolios could only be efficient by chance.

Therefore, this study addresses the following questions: Is there a portfolio model that optimally meets investors' needs? And proceeding from this thematic portfolio model, how efficient are the status quo solutions of core satellite portfolios in comparison?

This study founds on the tri-criterion model developed by Hirschberger et al. (2013). It is generalized to a thematic tri-criterion portfolio optimization model to suite as an overall framework. In addition, it allows the quantification of financial performance enhancements

[§] Data obtained from Thomson Reuters Eikon.

compared to the current solutions of the thematic bi-criterion model adopted from the core satellite strategy.

Results prove the inefficiency of thematic core satellite investing by promising relative yield enhancements of more than 5 % on average while leading them back to critical variables of the two portfolios. On the one hand the particularism of a theme as well as its individualism and on the other hand the general amount of assets taken into account are key drivers for the inefficiency of the status quo. The more exotic a theme is and the more specific it is defined the more the tri-criterion model can improve the performance. Furthermore, the amount of conventional assets strengthens the need for more complex models.

2 Theoretical model and hypotheses

The traditional mean-variance portfolio optimization model has been introduced by Markowitz in 1952. It is the foundation on which the conventional and the thematic portfolios are allocated within the thematic core satellite model. In the original model, a risk-averse investor has two investment goals. The first one is the maximization of the investment's expected return μ. The second goal is to minimize risk measured as the standard deviation σ or the variance σ^2 of returns. Eq. (1) shows an a priori approach of this optimization. Alpha and beta indicate the investor's preference parameters for return (α) and risk (β).

$$max \qquad \alpha\mu - \beta\sigma^2 \tag{1}$$

A risk-averse and return-maximizing investor, as considered, is only described by positive preference parameters (Markowitz 1996). The number of available assets is denoted by n. Each n-dimensional solution vector $x=[x_1, \ldots, x_n]^T \in \mathbb{R}^n$ indicates the investment proportion x_i in each asset $i \in \{1, \ldots, n\}$; μ_i and σ_i denote the expected return and the standard deviation of asset

$i \in \{1, \ldots, n\}$, and ρ_{ij} represents the correlation between the returns of assets $i, j \in \{1, \ldots, n\}$. This results in the following a posteriori formulation of the bi-criterion optimization problem[**].

$$max \qquad \mu = \sum_{i=1}^{n} \mu_i x_i \tag{2}$$

$$min \qquad \sigma = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j x_i x_j \rho_{ij}} \tag{3}$$

$$s.\ t. \qquad \sum_{i=1}^{n} x_i = 1, \qquad x_i \geq 0 \tag{4}$$

Modern portfolio theory focuses on finding the investor-specific optimal trade-off between increasing return and accepting risk to maximize its utility. However, the underlying objective hierarchy should be complete, operational, decomposable, non-redundant and minimal (Keeney and Raiffa 1993). As non-monetary objectives are not considered in traditional portfolio theory, completeness cannot be given. Therefore, the two-dimensional objective hierarchy is to be expanded.

Several studies already focus on multi-criteria portfolio optimization (Baker and Haslem 1974, Spronk and Hallerbach 1997, Bana e Costa and Soares 2004, Ehrgott et al. 2004, Steuer et al. 2006). Stone (1973) as well as Konno and Suzuki (1995), e. g., study the third moment of asset returns and develop mean-variance-skewness models while others expand the traditional bi-criterion model by considering additional criteria like social responsibility to introduce non-monetary objectives (Ballestero et al. 2012, Hirschberger et al. 2013, Utz et al. 2014, Gasser et al. 2017). Furthermore, the “suitable-portfolio investor” was introduced by Steuer, Qi, and Hirschberger in 2007.

However, while all these studies consider multiple criteria, none of them provide an abstraction to an overall and unifying thematic objective. Therefore, in the following, portfolio optimization is expanded according to Hirschberger et al. (2013). The additional non-monetary

[**] This is not to be understood as quasi code. Optimization can be performed by, e. g., maximizing return under the restriction of a given maximum risk level. This notation for simplicity conforms, e. g., to Ehrgott et al. (2004), Steuer et al. (2007), Hirschberger et al. (2013).

criterion measures the proportion of thematic assets in the portfolio. These thematic assets are compliant with the investor's convictions and suit non-monetary intentions of the investor. Therefore, they can be categorized into individual themes. To comply with the minimalism of the objective hierarchy, the third criterion is not to be further unbundled. It is understood as the union of all non-monetary interests of an investor. Under the criterion τ, a portfolio can now also be evaluated according to the relative proportion of assets that conform to the investor's convictions and interests. τ_i is a binary variable that labels these thematic assets. An a priori expression for the three-dimensional maximization problem of one portfolio is given by Eq. (5).

$$max \qquad \alpha\mu - \beta\sigma^2 + \gamma\tau \tag{5}$$

Gamma denotes the belonging thematic preference parameter and will only be considered positive. However, in further research, a negative value can be taken into account. This would enable an implementation of the tri-criterion optimization model onto excluding portfolios and would suit investors trying to avoid investments into specific themes like tobacco or weapons. The focus on positive preference parameters follows the idea of an investor having an added value of a positively defined thematic portfolio.

The a posteriori tri-criterion portfolio optimization problem[††] is stated considering the new criterion τ as follows:

$$max \qquad \mu = \sum_{i=1}^{n} \mu_i x_i \tag{6}$$

$$min \qquad \sigma = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j x_i x_j \rho_{ij}} \tag{7}$$

$$max \qquad \tau = \sum_{i=1}^{n} \tau_i x_i \tag{8}$$

$$s.t. \qquad \sum_{i=1}^{n} x_i = 1, \qquad x_i \geq 0 \tag{9}$$

Using the binary variable τ_i, investors can label any asset as thematic. As long as they do not install a theme that simply includes assets with a specific estimation of return it is assumed that

[††] Same as (2). Optimization by holding two objective functions as restrictions.

thematic interests are not correlated with financial objectives. Therefore, the thematic proportion of a portfolio is the third investment goal; its objective function is the second linear one. It is assumed that the thematic proportion is additive which conforms to several studies on ethical scores (Barracchini 2004, Drut 2010, Ballestero et al. 2012, Utz et al. 2015, Gasser et al. 2017). There is no evidence that claimed a difference in usability.

Besides, e. g., analytical methods, the bi-criterion formulation of the traditional model can be solved using the *e*-constraint method (Haimes 1971). Thus, an efficient frontier is calculated piecewise by maximizing return for any given risk restriction. The foundation of this methodology is the dominance criterion. A portfolio that is characterized by a non-dominated criterion vector can further be called efficient, independent of investors' (positive) preference parameters (Steuer et al. 2008). Hence, the expansion with a third objective leads to an efficient surface.

In comparison to the two-dimensional frontier, this three-dimensional surface can be calculated by regarding another restriction like allocating at least a specified thematic proportion. By piecewise toughen the restrictions and each time maximizing the expected return the surface is calculated for any given accuracy.[‡‡] A multi parametric algorithm that precisely computes the non-dominated curved surface platelets of three objectives (two linear, one quadric in accordance with this study) is devised by Hirschberger et al. (2013).

[‡‡] An exemplary surface is being presented in Appendix A.

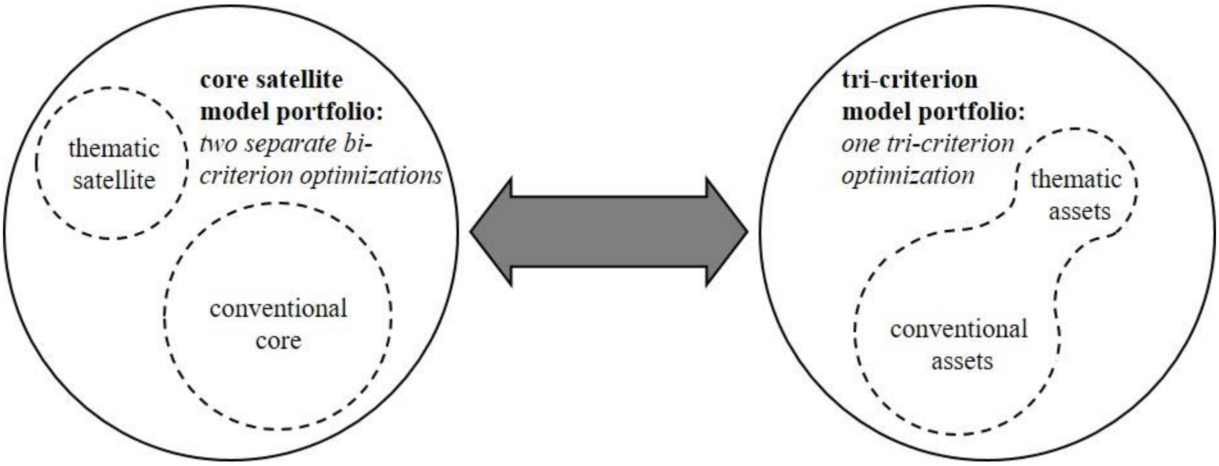


Fig. 1: *Visualization of the core satellite model as a combination of two separately return/ risk optimized portfolios and the tri-criterion model which allocates one portfolio regarding three criteria including a thematic criterion.*

In Figure 1, the tri-criterion model portfolio is contrasted with the core satellite model portfolio. As it is visualized, the tri-criterion model considers the correlations between the conventional and the thematic assets. Hence, the portfolios calculated by the tri-criterion model are efficient so that even in the worst case scenario, the criterion vector of the tri-criterion portfolio cannot be dominated by the ones computed with the thematic core satellite model. Therefore, the first hypothesis is stated:

H_1) *the tri-criterion model increases financial portfolio performance significantly.*

A significant performance improvement implies a relative yield enhancement of 1 %, 3 %, or 5 % in 95 % of cases. This hypothesis is about to found the call for research regarding the inefficiency of the status quo in thematic investing.

The following hypotheses are stated to investigate the relationship between improvements and various parameters. Hence, the second hypothesis concerns the definition of a theme:

H_2) *the tri-criterion model increases financial performance more, the more narrowly a theme is defined and thus, the less of the potential assets are classified as thematic.*

A narrow definition reduces the thematic asset universe thus probably affecting the efficiency of the whole portfolio.

In addition, the general amount of assets is considered, too:

H_3) *the tri-criterion model increases financial performance more, the more conventional assets are considered.*

This improvement is achieved by exploiting advantageous correlation effects between assets that have not been equally weighted in the thematic core satellite model. A larger amount of conventional assets increases the possibilities for the tri-criterion model to find assets that have beneficial correlation effects with the thematic ones.

This leads to the assumption that the correlation between the core and the satellite portfolio is also crucial for the added value of the tri-criterion model. Hence, the last hypothesis concerns the inter-portfolio correlation:

H_4) *the tri-criterion model increases financial performance more, the less the thematic portfolio correlates with the conventional one.*

A small correlation of the thematic portfolio with the conventional one indicates that the thematic restriction hinders the performance more and leads to a less efficient solution.

3 Data

A selection of 100 thematic ETFs is considered in which eleven thematic families are identified by their name and fund description. The thematic ETFs themselves are selected through a web search, initially focusing on funds called thematic ETFs. As most thematic ETFs have only recently been opened, the database is expanded to include ETFs that are referred to as thematic in, e. g., grey literature.[§§] In Table 1, the second and the third column count the thematic ETFs within their thematic families and show the average numbers of stocks included. These stocks are understood as thematic investment universes. In these universes, fund providers optimize allocations based on their individual beliefs about estimated returns, risks and correlations

[§§] The complete collection of assets can be found in ESM_1.

between these stocks. Since there is no interest in analyzing financial fund performances, stocks being defined as thematic (or respectively unthematic) are allocated by optimization based on historical data. However, historical data can actually only be used to a limited extent to predict future returns (DeMiguel et al. 2007). Nevertheless, the same data are used to optimize solutions and only the efficiency of these solutions compared to each other is considered. Therefore, this does not limit the results of the study.

With regard to the quantity and the age of most thematic ETFs, this paper focuses on the years 2016 and 2017 and generates data by computing daily stock returns of the observation periods. These data have been obtained from Thomson Reuters Eikon. Stocks without a full history of prices in 2016 respectively 2017 or that had no positive share in the ETF on 31/12 have been removed. The averages of daily stock returns are used as return estimations while their standard deviations are calculated as risk estimations. The average numbers of different Countries of Exchange (C) as well as Sectors (S), Industry Groups (IG), Industries (I) and Sub-Industries (S-I) that the stocks belong to, based on Global Industry Classification Standard (GICS) (MSCI 2016), are described in columns four to eight.

thematic family	ETFs	stocks	C	S	I G	I	S-I
agriculture	5	60	18	5	6	9	12
consumer	17	33	7	4	6	7	10
energy	4	33	4	4	6	8	14
financials	9	33	6	3	4	6	8
healthcare	8	52	7	5	8	12	16
infrastructure	11	64	12	6	8	12	15
metals	6	29	8	4	6	7	9
natural resources	10	76	13	5	6	10	17
renewable energy	8	40	10	5	6	9	15
technology	12	37	6	5	7	10	12
water	5	40	9	4	5	6	8
miscellaneous	5	65	3	8	13	21	26
summary			9	5	7	10	13

Table 1: *Constituent summary of thematic ETFs: amounts of different categories that the stocks belong to considering Country (C), Sector (S), Industry Group (I G), Industry (I) and Sub-Industry (S-I).*

Furthermore, ten conventional indexes*** complete the data set of 2,641 thematic stocks with 1,250 additional conventional stocks. The database as the union of these indexes and the 100 thematic ETFs includes 3,891 stocks out of 47 different countries of exchange, 11 Sectors, 24 Industry Groups, 68 Industries, and 157 Sub-Industries as listed in Table 2 with each the three maximum shares as reported in 2016.

	top 1	share	top 2	share	top 3	share
Country of Exchange	USA	41.93 %	Japan	10.78 %	UK	6.11 %
Sector	Industrials	14.15 %	Financials	13.39 %	Consumer Discretionary	13.08 %
Industry Group	Materials	10.78 %	Capital Goods	9.5 %	Energy	6.8 %
Industry	Oil, Gas & Consumable Fuels	5.75 %	Banks	5.37 %	Metals & Mining	4.65 %
Sub-Industries	Diversified Banks	3.37 %	Packaged Foods & Meats	3.22 %	Internet Software & Services	2.22 %

***Table 2:** Simulation data – summary of constituents and top categories with belonging shares.*

4 Empirical methodology

In the adapted core satellite model, two portfolios, a thematic and a conventional one, are separately optimized. The union of both portfolios describes the status quo of thematic investing. It is used as a benchmark and compared with the efficient solution of the tri-criterion portfolio by the estimated return. Furthermore, Figure 2 shows the two-part analysis which will be introduced at the end of this section.

*** DAX, Dow Jones Global Titans 50, Financial Times Stock Exchange 100 Index, The Global Dow, MSCI All Country World Index, MSCI World, Nikkei 225, Russell 1000 Index, Standard & Poor's 500, STOXX Europe 600.

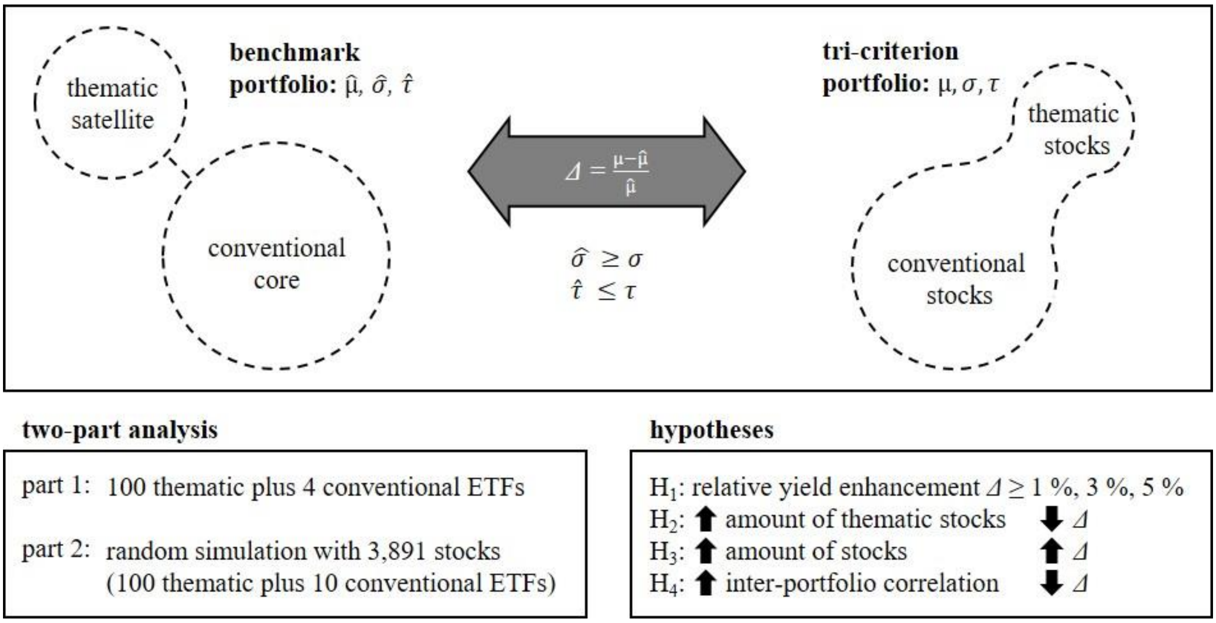


Fig. 2: *Abstract of the empirical methodology that compares the status quo throughout the benchmark portfolio with the efficient solution of the tri-criterion portfolio considering return μ, risk σ, and thematic proportion τ.*

First, the benchmark is to be calculated and one efficient portfolio solution has to be chosen by rule. Therefore, the two portfolios are allocated with a maximized Sharpe ratio as can be seen in Eq. (10). The Sharpe ratio measures the relation between excess return and standard deviation (Sharpe 1994). Risk free assets are not taken into account so that the benchmark for excess return is assumed to be zero. Hence, both portfolios are not being dominated by any other in two-dimensional space within their stock universes. Eqs. (10)–(11) show the a posteriori formulation for the one-period conventional portfolio optimization problem. The maximization problem for the thematic portfolio is described and solved by including the additional constraint Eq. (12).

$$max \qquad S = \mu/\sigma = \sum_{i=1}^{n} \mu_i x_i \; / \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j x_i x_j \rho_{ij}} \tag{10}$$

$$s.\,t. \qquad \sum_{i=1}^{n} x_i = 1,\ x_i \geq 0 \text{ with solution vector } x_C \tag{11}$$

$$\sum_{i=1}^{n} \tau_i x_i = 1 \text{ with solution vector } x_T \tag{12}$$

Eq. (12) ensures that only thematic stocks are allocated in the thematic portfolio. The union of both portfolios defines the benchmark solution depending on the amount $t \in [0, 1]$ invested in the thematic portfolio. Combining the vectors x_T and x_C that indicate the investment proportions in each stock within the thematic respectively the conventional portfolio, this results in the benchmark's solution vector $\hat{x}(t)$ (Eq. (13)) with the estimated return $\hat{\mu}(t)$ (Eq. (14)).

$$\hat{x}(t) = t * x_T + (1 - t) * x_C \tag{13}$$

$$\hat{\mu}(t) = t * \mu_T + (1 - t) * \mu_C \tag{14}$$

The estimated risk $\hat{\sigma}(\hat{x})$ (Eq. (15)) and the derived thematic proportion $\hat{\tau}(\hat{x})$ (Eq. (16)) complete the benchmark solution, that is described by $\hat{\mu}$, $\hat{\sigma}$ and $\hat{\tau}$.

$$\hat{\sigma}(\hat{x}) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j x_i x_j \rho_{ij}} \tag{15}$$

$$\hat{\tau}(\hat{x}) = \sum_{i=1}^{n} \tau_i x_i \tag{16}$$

The thematic proportion $\hat{\tau}$ is not to be mixed up with the thematic ETF fraction t. Even in the conventional portfolio, there can be a thematic proportion allocated so that $\hat{\tau} \geq t$. The tri-criterion model is to be restricted by the share of thematic stocks in the union to dominate the benchmark in all three criteria.

The three-dimensional optimization problem is stated to maximize return under the restrictions of allocating at least the thematic proportion $\hat{\tau}$ and maximum the risk level $\hat{\sigma}$. These restrictions are taken into account by Eqs. (18) and (19):

$$max \qquad \mu = \sum_{i=1}^{n} \mu_i x_i \tag{17}$$

$$s.\,t. \qquad \sum_{i=1}^{n} \tau_i x_i \geq \hat{\tau}, \tag{18}$$

$$\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j x_i x_j \rho_{ij}} \leq \hat{\sigma}, \tag{19}$$

$$\sum_{i=1}^{n} x_i = 1, \qquad x_i \geq 0 \tag{20}$$

To compare both models, a new variable is defined. δ (Eq. (21)) describes the relative difference between the estimated returns[†††] of the allocated portfolios in the two models:

[†††] Calculations have been done comparing risk levels, too. Improvement results are qualitatively the same.

$$\delta(t) = (\mu(t) - \hat{\mu}(t))/\hat{\mu}(t) \tag{21}$$

Relative yield enhancements are preferred over absolute yield enhancements, so that the quantified improvements of the tri-criterion model do not depend on return estimations or the particular risk restriction. Since the thematic proportion can vary, values $t \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ are considered and the average of $\delta(t)$ is described by Δ.[‡‡‡] The maximum amount of t=50 % is justified because the conventional portfolio is assumed to be the key part of the investment and should therefore not be smaller than the thematic fraction (Marchioni et al. 2016).

The analysis is divided into two parts as already shown in Figure 2. At first, 100 tradable thematic ETFs define each different themes within the stock universe. Each time, the stock universe is set up as the union of one thematic ETF's stocks and one conventional ETF's stocks. The conventional core portfolio is allocated by optimizing the total stock universe. Within the subset of thematic stocks, which were initially allocated in the thematic ETF, optimization defines the explicit allocation of the thematic satellite portfolio.

A sample of 100 results is small for hypothesis tests because critical values to reject a considered alternative hypothesis are very tough. Furthermore, it is no sample size that justifies universal implications and weakens the significance of a correlation analysis. Therefore, a second approach is chosen in addition.

The second approach uses a simulation to increase the amount of data. Within the given data set, the considered thematic ETFs equal 100 possible declarations of thematic subsets of a total stock universe. These can be understood as 100 possible draws, while the amount of possible subsets for further themes is larger. A simulation that imitates subjective criteria by randomly defining stocks in the specific stock universe as thematic can draw further thematic subsets.

[‡‡‡] Results show that the mean peak of conceived δ is being calculated for thematic proportions higher than t=50 %.

Hence, this simulation is divided into 25 sub simulations (considering different amounts of conventional and thematic stocks) with each 10,000 draws. In each draw, a number of $n \in \{200, 400, 600, 800, 1000\}$ stocks is picked randomly from the database of 3,891 stocks presented in section Data. The first $nT \in \{20, 40, 60, 80, 100\}$ stocks are further defined as thematic so that $\tau_i = 1$ for $i \in \{1, \dots, nT\}$ and $\tau_i = 0$ for $i \in \{nT+1, \dots, n\}$.

The probability of picking the same stock universes twice or more often can be neglected.[§§§] Each time portfolios are optimized and the relative yield enhancement is calculated by using the tri-criterion model instead of the bi-criterion core satellite model.

5 Results

Some exemplary results are shown in Figure 3 for the development of δ depending on different proportions t invested in the thematic portfolio. In this example, an aggregated Δ of 2.66 % is computed. Hence, an assumed one year portfolio return $\hat{\mu}$ of 8 % could be increased to a return μ of 8.2128 % with an absolute yield enhancement of 21.28 basis points. The whole three-dimensional surface and the five portfolios compared in this example can be found in Appendix A.

[§§§] Smallest amount of different draws within a simulation >3.87e368.

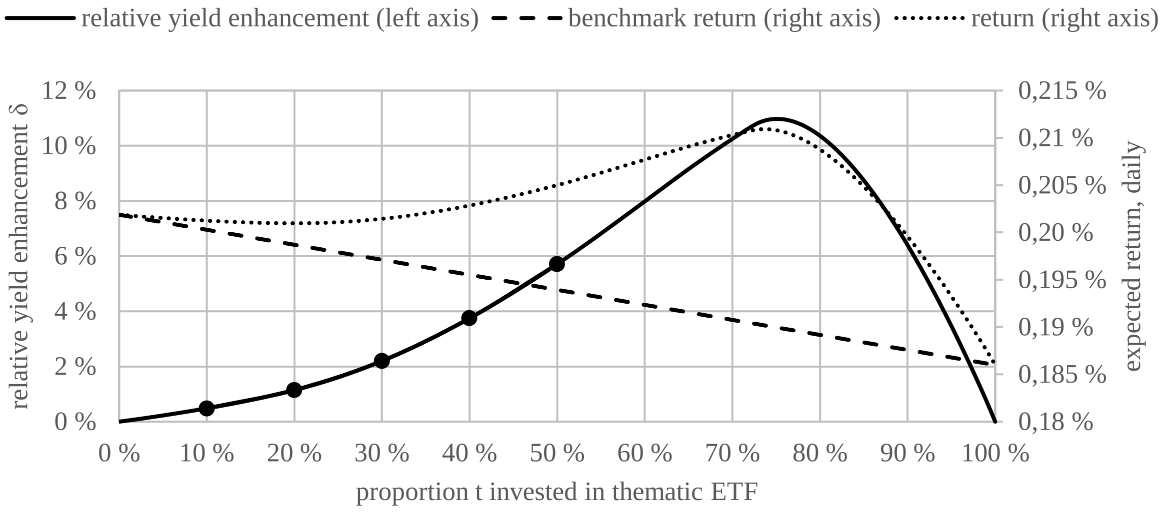


Fig. 3: *Returns of the union of a Sharpe ratio optimized thematic (Robotics & A.I. Thematic ETF) and conventional ETF (S&P 500) with the conceived yield enhancement of using the tri-criterion model depending on the proportion t invested in the thematic ETF. Return and risk are computed by mean and standard deviation of daily stock returns in 2016.*

In general, calculation results with 100 tradable thematic ETFs that supplement conventional ones are shown in Table 3 and Table 4. Relative yield enhancement as a measure of performance improvement is computed by optimizing the expected return of the tri-criterion portfolio under the restrictions given by the benchmark solution of the united, but separately optimized core satellite portfolios. A mean yield enhancement of 5.14 % in 2016, respectively 6.19 % in 2017, is achieved over all 400 combinations of conventional and thematic ETFs. The last three columns count the combinations that have exceeded the corresponding threshold.

	mean Δ	min Δ	max Δ	correlation	$\Delta \geq 1$ %	$\Delta \geq 3$ %	$\Delta \geq 5$ %
MSCI World	9.3 %	1.73 %	40.54 %	52.66 %	100**	95	84
Russel 1000	4.08 %	0.56 %	27.46 %	65.37 %	96	46	22
S&P 500	3.01 %	0.38 %	20.89 %	63.92 %	94	44	19
Stoxx 600	4.15 %	0.5 %	30.24 %	65.79 %	94	30	12

Table 3: *Relative yield enhancements (Δ) for combinations with each 100 thematic ETFs in 2016. Significance levels $\alpha^* = 0.05$ and $\alpha^{**} = 0.01$ calculated with binomial testing rejecting the alternative hypothesis of 95 % maximum probability.*

	mean Δ	min Δ	max Δ	correlation	$\Delta \geq 1$ %	$\Delta \geq 3$ %	$\Delta \geq 5$ %
MSCI World	11.73 %	2.44 %	47.05 %	45.71 %	100**	99*	96
Russel 1000	4.87 %	0.63 %	24.77 %	60.43 %	99*	57	24
S&P 500	3.85 %	0.82 %	20.55 %	57.11 %	99*	46	16
Stoxx 600	4.32 %	0.65 %	24.05 %	58.66 %	96	52	20

Table 4: *Relative yield enhancements (Δ) for combinations with each 100 thematic ETFs in 2017. Significance levels $\alpha^* = 0.05$ and $\alpha^{**} = 0.01$ calculated with binomial testing rejecting the alternative hypothesis of 95 % maximum probability.*

Results show the inefficiency of the status quo compared to the tri-criterion model. However, in 2016, only for one conventional ETF the alternative hypothesis $H_{1,a}$, which states that no more than 95 % of combinations show a relative yield enhancement of at least Δ=1 %, can be rejected significantly. Significance is determined by binomial tests on a theoretically expected binomial distribution with a maximum probability of 95 % to exceed a particular delta. The results of the year 2017 show nearly the same results, but four combinations exceed the threshold.

In comparison, the results of the simulation show that the calculated financial performance improvements differ per sub simulation from a minimum average yield enhancement of 1.22 % to a maximum of 13.25 %. A mean Δ of 6.23 % for the whole simulation at a standard deviation of 3.74 % confirms efficiency improvements. Table 5 shows the hypothesis tests with the significance level on which the corresponding alternative hypothesis can be rejected in each simulation. For each of the 25 simulations, the result cells show the strongest of the three variations of hypothesis H_1 (1 %, 3 % or 5 %) whose alternative hypothesis can be rejected significantly and the mean yield enhancement Δ.[****]

[****] Full histograms can be found in Appendix B.

	20	40	60	80	100
200	1 %**; 5.21 %	1 %**; 3.31 %	--; 2.36 %	--; 1.7 %	--; 1.22 %
400	3 %**; 7.92 %	1 %**; 5.59 %	1 %**; 4.39 %	1 %**; 3.58 %	1 %**; 2.99 %
600	5 %**; 9.9 %	3 %**; 7.24 %	3 %**; 5.91 %	1 %**; 5.02 %	1 %**; 4.33 %
800	5 %**; 11.64 %	3 %**; 8.71 %	3 %**; 7.26 %	3 %**; 6.24 %	3 %**; 5.52 %
1,000	5 %**; 13.25 %	5 %**; 10.0 %	3 %**; 8.45%	3 %**; 7.38 %	3 %**; 6.58 %

Table 5: *First entry: Hypothesis tests - significance levels α^* = 0.05 and α^{**} = 0.01 calculated with binomial testing rejecting the alternative hypothesis of at least the specific improvement in maximum 95 % of cases; second entry: mean relative yield enhancements. Different amounts of conventional and thematic (as a subset) stocks are considered.*

In three of the 25 sub simulations, none of the first alternative hypotheses can be rejected on a significance level of α^*=0.05. Hypothesis H_1 considering a yield improvement of at least 5 % can only be confirmed indirectly for four of the 25 sub simulations, which have few thematic stocks.

These results let assume that the beliefs about the dependence of Δ on the amount of thematic stocks in the stock universe can be supported. Hence, hypotheses H_2-H_4 are further investigated in the following correlation analysis. The analysis starts again with the 100 tradable thematic ETFs for the years 2016 and 2017.

	n. core stocks	n. thematic stocks	correlation	Δ
n. core stocks	1	0.049	-0.3551**	0.4391**
n. thematic stocks	0.049	1	0.548**	-0.3122**
correlation	-0.3551**	0.548**	1	-0.6938**
Δ	0.4391**	-0.3122**	-0.6938**	1

Table 6: *Correlation analysis for calculations with thematic ETFs for the year 2016 on significance levels α^* = 0.05 and α^{**} = 0.01.*

	n. core stocks	n. thematic stocks	correlation	Δ
n. core stocks	1	0.0845	-0.3199**	0.4885**
n. thematic stocks	0.0845	1	0.517**	-0.2681**
correlation	-0.3199**	0.517**	1	-0.7936**
Δ	0.4885**	-0.2681**	-0.7936**	1

Table 7: *Correlation analysis for calculations with thematic ETFs for the year 2017 on significance levels α^* = 0.05 and α^{**} = 0.01.*

The correlation analysis in Table 6 and Table 7 shows significant correlations of Δ with the number of thematic (n. thematic stocks) as well as conventional stocks (n. core stocks) and the correlations (correlation) between their portfolios. The more stocks are taken into account and the less of them are thematic, the higher the relative yield enhancement gets which supports hypotheses H_2 and H_3. Furthermore, the fourth hypothesis H_4 considering a negative dependence of Δ on the inter-portfolio correlation is confirmed, too. Obviously, this inter-portfolio correlation increases with the relative share of thematic stocks in the stock universe. This multicollinearity is to be further investigated within the greater data sample of the simulation in the second part of the analysis.

Table 8 shows the results of the correlation analysis from the simulation. The correlation analysis is expanded with partial correlation coefficients because, as can be seen in the first correlation analysis, the explanatory variables condition each other. The partial correlation coefficient measures the linear relationships' direction between one variable and the relative yield enhancement and controls the effect of the two other variables.

	n. core stocks	n. thematic stocks	correlation
correlation of Δ	0.5945**	-0.495**	-0.8029**
partial correlation of Δ	0.4158**	-0.2078**	-0.5177**

Table 8: *Correlation analysis for relative yield enhancements Δ from the simulation.*

The correlation analysis strengthens the results of the first analysis computed throughout the ETFs. Both hypotheses H_2 and H_3 concerning the correlations between relative yield enhancement and the amounts of conventional respectively thematic stocks are supported. The less thematic stocks and the more conventional stocks are taken into account, the higher the relative yield enhancement gets. Furthermore, as expected, a strong positive correlation between the thematic and the conventional portfolio affects the resulting improvement of the tri-criterion model negatively which confirms hypothesis H_4.

In conclusion, a mean yield enhancement of 5.67 % for tradable thematic ETFs and 6.23 % within the simulations have been achieved by optimizing the tri-criterion portfolio. This shows that the current investing strategy of core and satellite portfolios needs to increase performance about 6 % to serve as an efficient solution. The average yield enhancement for tradable ETFs is lower than within the simulation. Since thematic ETFs have yet only been opened for quite conventional themes they have a strong correlation with the market. The correlation analysis has shown that this weakens potential yield enhancements.

6 Discussion

The results are robust against assumptions in the empirical methodology. Three of them are further investigated. Hence, we show that the inefficiency in practice can only emphasize the need for research more strongly, since the assumptions have weakened solely the tri-criterion model.

First, it is assumed that investors target their thematic interest with the amount invested in thematic stocks. In practice, they might not consider the thematic share in the core portfolio and target their thematic interest only with the amount invested in the thematic satellite. Within the empirical methodology the thematic shares in the core portfolios are considered, too. This tightens the constraints for the efficient solution of the tri-criterion model and ensures that the benchmark solution is weakly dominated in all three criteria. Consequently, the solution is on the safe side and the results without the assumptions cannot be weaker, but stronger.

Second, it is assumed that the thematic stocks are a subset of the stocks taken into account to allocate the conventional portfolio. In practice, a conventional portfolio could be built without considering all the thematic stocks which differs to this assumption. Hence, in practice, a conventional portfolio and a financially dominated thematic portfolio with some exclusive stocks could be allocated. Both portfolios would be defined by their expected return and risk. If some of the exclusive stocks in the thematic portfolio would be outperforming, an improvement measure δ of the tri-criterion optimization could be discontinuous for $\lim_{t \to 0} \delta$. An investor without any thematic interest would only take the conventional stocks into account.

The moment the investor has a thematic interest, the usable stock universe for a tri-criterion optimization would increase and allocate a portfolio with possibly a higher estimated return and a lower risk. This would increase financial portfolio performance although a restriction ($t > 0$) was added. This is to be prevented. Additionally, the focus of this paper lies on the correlation effects between the two portfolios that are not considered in the traditional model. Hence, performance improvement was only to be explained by these effects and not by an increased amount of stocks taken into account.

Third, the additional simulation approach itself could be criticized for picking thematic stocks randomly. Thus, excessive systematic clump risks are not regarded. Referring to Giammattei (2014) thematic investing is not restricted by traditional boundaries, which is supported by the data section showing said diversification on sector and industry levels. Therefore, clump risks may arise randomly, but a systematically excessive risk due to a thematic investing strategy must be rejected. Furthermore, the simulation approach strengthens the analysis of the hypotheses by providing more significant results on a wider range of possible stock universes and stock numbers that could not be investigated by focusing on tradable ETFs only.

The assumptions are necessary to enable a quantification of inefficiency in practical models, but have always approximated the results on the safe side. Therefore, inefficiency in practice can only be greater which further strengthens the importance and proves the significance of further research in thematic portfolio optimization.

7 Conclusion

This study was motivated by currently used investment strategies and the raising interest in thematic investing. Adapting a core satellite strategy, fund providers open thematic ETFs that investors use as supplements to conventional portfolios. Thus, the portfolios are separately optimized which leads, as assumed, to dominated criterion vectors and at that affects efficiency.

In literature, there has been no study that either evaluated the inefficiency of such handling or presented a model that suited investors' needs more.

To close this gap in research, a tri-criterion portfolio optimization model has been considered. Expanding the work of Markowitz from 1952 in line with Hirschberger et al. (2013), and generalizing further developments of, e. g., Utz, Wimmer, and Steuer (2015), who worked with the special case of a sustainable investment, an overall framework for non-monetary interests has been established. Portfolios are optimized within three dimensions that count *thematic interest* as an objective next to *return* and *risk*. Thus, efficient portfolios are allocated whose criterion vectors are not dominated and span an efficient surface in investors' three interest dimensions. This enables quantifying the inefficiency of the status quo and follows the motivation of this study to demonstrate the need for further research with these results. Furthermore, the relationship with the inter-portfolio correlation as well as the numbers of stocks considered should be examined.

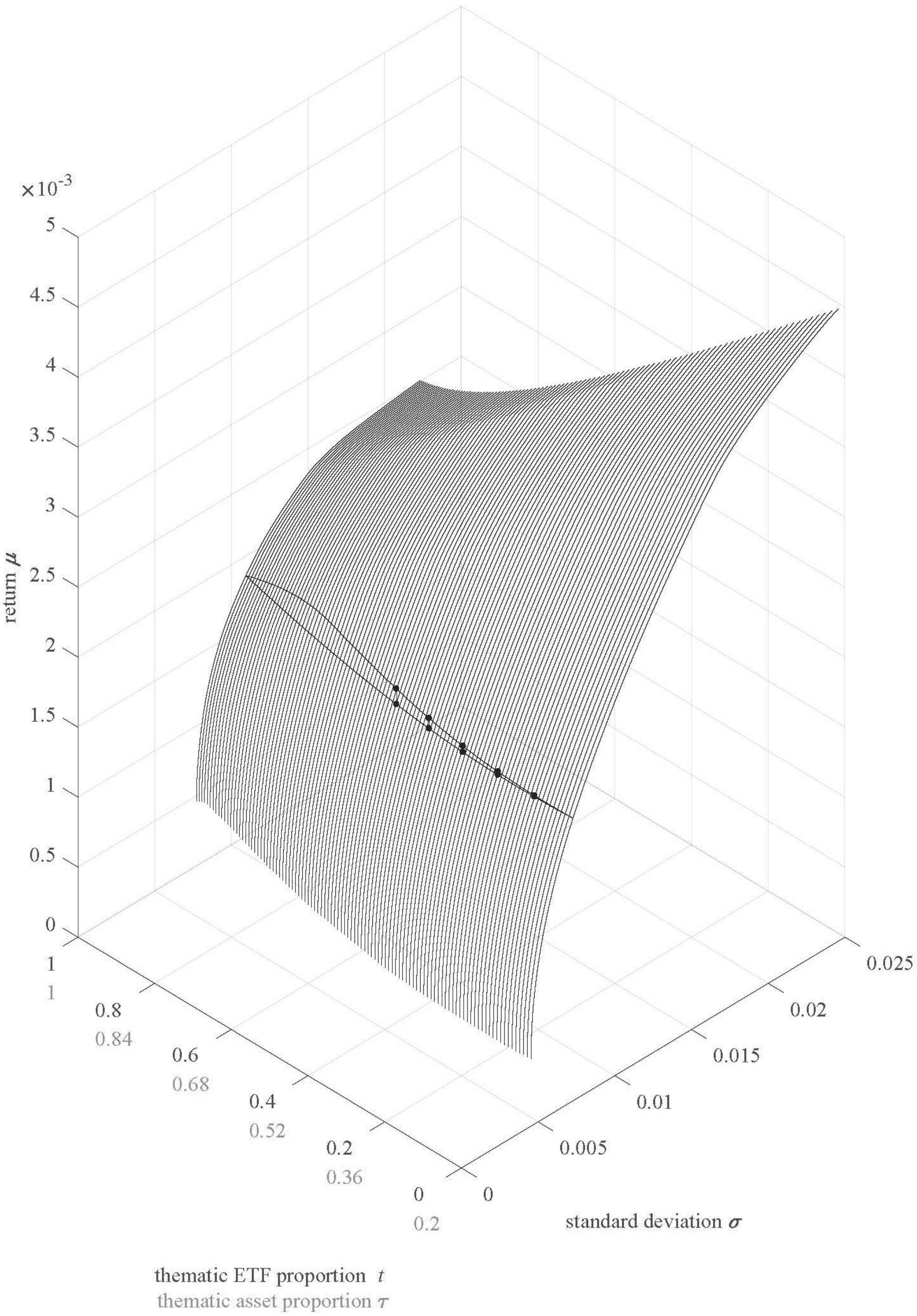
Allocations of the core satellite model have been separately optimized within their conventional respectively thematic stock universes so that their union stated the status quo of thematic investing and a benchmark for the tri-criterion model. Two of the benchmark defining criteria, thematic proportion and risk, have been used as restrictions for the maximization of the expected return in the efficient solution. This latter criterion has been compared and yield enhancements have been documented as model improvements. A correlation analysis has shown that the amounts of conventional and thematic stocks as well as the correlations between the two separately optimized portfolios are significant for the inefficiency of the core satellite model. The less thematic stocks are considered and the less they correlate with the conventional portfolio, the greater the inefficiency becomes.

Working with very specific thematic interests that only few stocks belong to, the three-dimensional approach relatively increased expected return on a range up to 46.88 %. Mean relative yield enhancements of 5.67 % for thematic ETFs and 6.23 % within the simulation have been achieved by optimizing the tri-criterion portfolio. The difference arises since thematic ETFs have yet only been opened for quite conventional themes. Therefore, they have a strong correlation with the market which weakens potential yield enhancements. When thematic ETFs become more individual and more particular, correlations between the two portfolios will get smaller so that relative yield enhancements of the tri-criterion model will increase.

Proceeding from this study, private as well as institutional investors should evaluate their investment goals. Having very specific interests that differ from traditional models' objectives and are targeted in supplement portfolios, the strategy should be questioned. The less the thematic satellites correlate with their diversified core portfolios the more they should take the tri-criterion model into consideration. In order to support the practical use and to increase the sensitivity to the interests of investors, continuous scales for the thematic evaluation of a stock need to be investigated. Therefore, a methodology is required that counts in investors' preferences and measures the thematic impact of a stock. Hence, a better understanding of investors' preferences and objectives is necessary to cluster multiple satellites respectively themes in one portfolio. At the moment, the model is presented as an overall framework and is formulated a posteriori to ensure independency of investors' preferences. Further research has to evaluate such effects on yield enhancements. Therefore, a priori models have to be further analyzed.

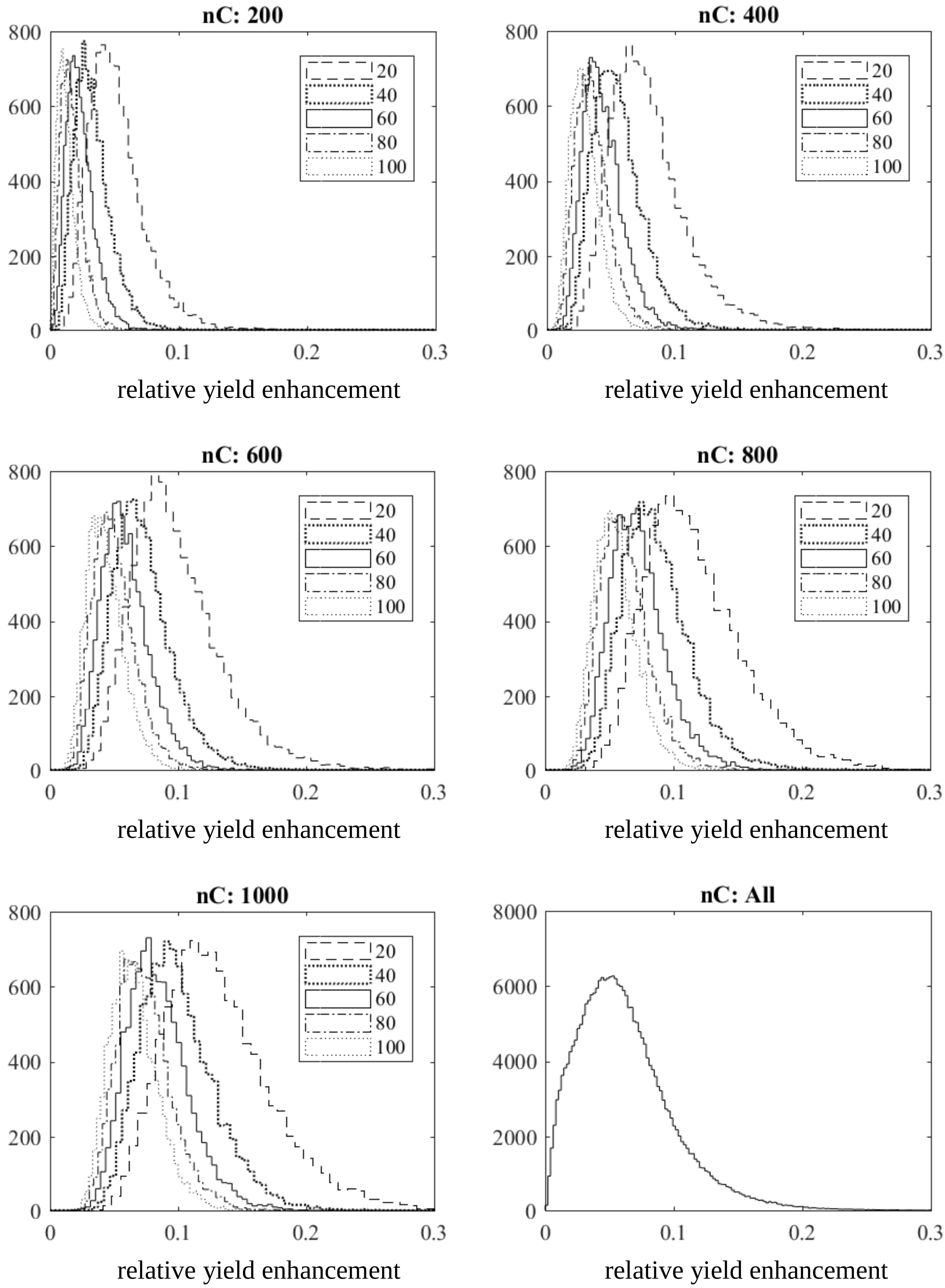
Nevertheless, this study has quantified the inefficiency of the status quo in thematic portfolio optimization. It has shown that improvements are necessary and how they depend on the numbers of stocks taken into account as well as the correlations of themes with the market. The current state of research does yet not reflect the increase in economic significance of thematic ETFs which will motivate further studies.

Appendix A: Three-dimensional surface and compared portfolios



Thematic stocks are defined by a Robotics & A.I. thematic ETF in combination with a S&P 500 ETF. Return and risk are computed by mean and standard deviation of daily stock returns in 2016. The points represent the portfolios that are compared for $t \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ and the lines connect them to portfolios of the tri-criterion model on the efficient surface and indicate the relative yield enhancement.

Appendix B: Simulation histograms



Histograms show the results of the simulation with 3,891 stocks and a random evaluation of thematic stocks for different amounts of conventional stocks nC using daily stock returns of the year 2016. Within the histograms, the different lines show each 10,000 results concerning different amounts of thematic stocks. Histogram “nC: All” summarizes the 250,000 results of the five previous histograms.

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Paper 2: Tailor-made thematic portfolios - a core satellite optimization

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Tailor-made thematic portfolios – a core satellite optimization

In recent years, thematic exchange-traded funds have arisen and broaden the narrow thinking of purely monetary driven investment decisions. Ethical concerns, a stronger belief in society-shaped trends, or the conformity with personal convictions motivate investors to include non-monetary objectives in so called thematic funds.

Thematic investors follow a modified core satellite strategy in which conventional funds ensure diversification. Investors' non-monetary interests manifest within additional satellite funds. Both portfolios are separately allocated so that inter-portfolio correlation effects are not considered and inefficient core satellite portfolios are allocated.

However, in order to reduce the inefficiency of such a core satellite strategy, this study addresses fund providers and develops the idea of tailoring thematic funds to conventional ones. An empirical investigation shows that by easing the efficiency constraint for the satellite portfolio, correlation effects between the portfolios can be exploited. At the expense of an average relative volatility increase of 4.55 %, the inefficiency of core satellite portfolios can be reduced by an average of 11.74 % compared to volatilities of efficient tri-criterion portfolios.

Further analysis of these thematic products shows that tailored products can also be more concentrated within a given theme. This opens up new opportunities for fund providers to become more involved in various thematic trends and at the same time achieve better performance for satellite core investors.

Multi criteria decision making; Portfolio management; Thematic investing; Finance;

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1 Introduction

Modern portfolio theory has always focused primarily on monetary goals (Markowitz 1952). Restricted by a maximum risk level or the individual risk appetite of a specific investor, portfolio optimization allocates portfolios with a maximum expected return. It is assumed that the considered investor, who must be satisfied with such a simplified approach, is only interested in the financial results of an investment. Motivations like ethical concerns (Anand and Cowton 1993), interests in megatrends (Giammattei 2014, Marchioni et al. 2016) or the gamification (Sironi 2016) of an investment are not regarded. In recent years, this two-dimensional narrow thinking of a risk/ return-oriented optimization has been called into question to improve this unsatisfactory situation. Furthermore, the underlying maxims waver as new technologies bring up new opportunities (Zopounidis et al. 2018).

With these new opportunities, thematic investing and more complex investor models have arisen that do not only focus on financial objectives. In contrast, thematic investing assigns portfolios that take into account investors' personal beliefs and ethical concerns, their confidence in socio-economic trends in megatrend investments, or their interest in trading on various themes (Bérubé et al. 2014). Robotics and Artificial Intelligence, Aging Society and Social Responsibility are some of the best known examples of this development. They have emerged with the corresponding exchange-traded funds (ETFs) trying to meet the particular interests by designing new passive indices (Forster 2017). Hence, they respond to the needs of investors to take into account their individual objectives and personal convictions. In this way, they create added value for investors in new dimensions of utility. Therefore, thematic portfolio optimization is understood as an extension of the original portfolio selection problem (Methling and Nitzsch 2018). In addition to financial goals, it also considers non-monetary objectives as another criterion.

Several studies already focus on multi-criteria portfolio management. In recent years, the variety of criteria has increased significantly (Aouni et al. 2018). Steuer, Qi, and Hirschberger account for social responsibility next to multiple financial criteria (2005, 2007, 2008). Hence, they present a framework that Ballestero et al. continue leading to a financial-ethical bi-criteria model (2012). Such a bi-criteria model that considers social responsibility can also be implemented by using positive screening to first reduce the asset universe. In such a socially responsible subset, Liern, Pérez-Gladish, and Méndez-Rodríguez use mean-variance optimization to allocate socially responsible portfolios in two-dimensional space (Zopounidis and Doumpos 2017).

In three-dimensional space, including an additional linear objective (like social responsibility) on top of estimated return and risk, Hirschberger et al. present an exact method to calculate the efficient surface's curved platelets (2013). Hence, such a tri-dimensional model enables a comparison of efficient solutions considering different objective ratios that Gasser, Rammerstorfer, and Weinmayer presented in 2017 [2017]. All these developments improved evaluation methods to determine the efficiency of multi-criteria portfolios. Nevertheless, these theoretical findings have to be adapted to find more practical relevance and to improve actual investment behavior.

In contrast, actual investment behavior, regarding the status quo in thematic investing, does not yet state three-dimensional optimization problems. Investors use heuristics like a modified core satellite strategy to account for their thematic interests. A core satellite investment usually consists out of a passively managed diversified core portfolio and an actively managed satellite portfolio striving for out-performance (Amenc et al. 2004). In the modified core satellite investment, investors' convictions manifest within thematic satellite funds that are supplemented to diversified core portfolios (Magoon 2009). Both portfolios are separately allocated without considering the inter-portfolio correlation. Consequently, the core satellite portfolio combination as a whole can only be efficient by chance (Methling and Nitzsch 2018). However, the efficient solution of the aforementioned tri-criterion optimization solely serves as a theoretical one for both private investors as well as fund providers. On the one hand, private investors had to spent more effort in ordering several individual stocks which consumes excess returns by increasing transaction costs. On the other hand, fund providers trying to directly provide tri-criterion funds would need to open one particular fund for each combination of thematic interest, conventional core portfolio, and individual preference weights. Nevertheless, there are two more possibilities: Either fit conventional funds to thematic ones, or the other way around. As conventional funds usually address a broader range of investors, creating tailor-made thematic products fitting to a specific conventional core product like one based on an MSCI World index or a particular pension or savings plan looks more promising. In recent years, the market for international pension and savings plans has been expanding sharply (Brough and Shepperson 2019). Investors in these plans are bound to portfolio allocations and can individualize their portfolios only by supplementing additional products. Such products can be tailor-made thematic investment funds or thematic exchange-traded funds and tracker funds that are based on a customized thematic index. Therefore, this study addresses fund providers and index providers as well as their combined process of fund allocation.

The goal of this study is to evaluate potential inefficiency reductions regarding core satellite portfolios. By reducing the constraint of allocating thematic portfolios separately and taking into account the allocation of a conventional portfolio, tailor-made thematic funds and customized indexes to be followed by thematic ETFs can be optimized. Making inter-portfolio correlation effects available for consideration in the optimization process, tailor-made thematic satellite portfolios can have the potential to increase the performance of the whole core satellite portfolio for the investor. In the following, this improvement is quantified while additional hypotheses regarding the concentration and the correlation of the satellite portfolio's allocation will improve the understanding of this handling.

In the following section, the relevant literature is presented and the theoretical model starting with Markowitz is described accounting for further developments in research (Markowitz 1952, Steuer et al. 2007, Hirschberger et al. 2013, Gasser et al. 2017, Methling and Nitzsch 2018). A short description of the used data set goes into the actual description of the problem statements that are solved to suit the research questions. The results of the empirical methodology are presented afterwards. The last section summarizes the key findings and presents an outlook for further research.

2 Theoretical model

In a review of Multiple Criteria Decision Making related literature in finance, Zopounidis et al. show that most studies focus on portfolio optimization (2015). Commonly known as modern portfolio theory, starting with Markowitz' portfolio selection problem, a risk-averse investor seeking to increase the investment's return is assumed (Markowitz 1952). This expected return E is denoted by a linear objective function. Taking risk measured as the volatility V or the variance V^2 of returns into account, either an additional quadric objective function or a quadric constraint is stated.

Efficiency is understood as the impossibility of improving any portfolio's criterion without weakening another. Therefore, the efficient frontier, plotting any portfolio with a not dominated criterion vector, shows the maximum estimated return for any given restrictive risk level of the traditional portfolio selection problem. Eq. (1) states the a priori formulation of this optimization problem including investors' preference parameters for return (α) and risk (β). As Markowitz considers a risk-averse and return-maximizing investor only positive preference parameters are taken into account (1996).

$$max \qquad \alpha E - \beta V^2 \tag{1}$$

Hence, the a posteriori formulation can be stated for the bi-criteria optimization problem with $\boldsymbol{C}$ as the covariance matrix of asset returns $\boldsymbol{\mu}$.[††††] Additional restrictions for the *n*-dimensional solution vector $\boldsymbol{x}$ considering a set of feasible solutions *F* are indicated by Eq. (4).

$$max \qquad E(\boldsymbol{x}) = \boldsymbol{\mu}'\boldsymbol{x} \tag{2}$$

$$min \qquad V^2(\boldsymbol{x}) = \boldsymbol{x}'\boldsymbol{C}\,\boldsymbol{x} \tag{3}$$

$$s.\,t. \qquad \boldsymbol{x} \in F \tag{4}$$

The notation has already been expanded to take e. g. social responsibility or sustainability into account (Steuer et al. 2007, Utz et al. 2015, Gasser et al. 2017). A more general approach understands the third dimension as the union of a considered investor's non-monetary interests (Methling and Nitzsch 2018). These interests are summarized within investment themes. The binary vector $\boldsymbol{\tau}$ marks assets that a fund or index provider defines as related to such a theme. The weighted sum of these assets *T*, accounting for the thematic preference parameter *γ*, describes the added value *γT* in Eq. (5). Thematic investors try to maximize the relative share of thematic assets within their portfolio. Therefore, only positive preference parameters are still considered.

$$max \qquad \alpha E - \beta V^2 + \gamma T \tag{5}$$

The tri-criterion a posteriori description of the problem is stated as follows:

$$max \qquad E(\boldsymbol{x}) = \boldsymbol{\mu}'\boldsymbol{x} \tag{6}$$

$$min \qquad V^2(\boldsymbol{x}) = \boldsymbol{x}'\boldsymbol{C}\,\boldsymbol{x} \tag{7}$$

$$max \qquad T(\boldsymbol{x}) = \boldsymbol{\tau}'\,\boldsymbol{x} \tag{8}$$

$$s.\,t. \qquad \boldsymbol{x} \in F \tag{9}$$

[††††] This is only a notation for simplicity in line with several studies and is not be understood as quasi code Ehrgott et al. (2004), Steuer et al. (2007), Hirschberger et al. (2013).

Assuming that thematic investors use a core satellite strategy, their optimization problem is reduced to finding the optimal amount $t \in P = \{t \in \mathbb{R} \mid 0 \leq t \leq 1\}$ spent in the thematic satellite portfolio. Therefore, the allocations for a conventional portfolio $\boldsymbol{x_C}$ and a thematic portfolio $\boldsymbol{x_T}$ are given exogenously by the providers. Investors' selection problem is stated as follows:

$$max \qquad E(t) = \boldsymbol{\mu}'[(1-t)\,\boldsymbol{x_C} + t\,\boldsymbol{x_T}] \tag{10}$$

$$min \qquad V^2(t) = [(1-t)\,\boldsymbol{x_C} + t\,\boldsymbol{x_T}]'\,\boldsymbol{C}\,[(1-t)\,\boldsymbol{x_C} + t\,\boldsymbol{x_T}] \tag{11}$$

$$max \qquad T(t) = \boldsymbol{\tau}'\,[(1-t)\,\boldsymbol{x_C} + t\,\boldsymbol{x_T}] \tag{12}$$

$$s.\,t. \qquad t \in P \tag{13}$$

Conventional core portfolios are not provided to suit non-monetary interests but to ensure diversification, to track the market, or in comparison, to strive for a return surplus. Therefore, these portfolios are regarded as given parameters. The goal is to optimize thematic satellite funds that are tailored to conventional core portfolios and to decrease the inefficiency of core satellite portfolios with regard to the interests of retail investors.

> Therefore, the efficiency of a core satellite portfolio increases through targeted efficiency reduction of the satellite portfolio.

This statement will be supported with empirical evidence while additional hypotheses are stated to improve the understanding of this approach. Efficiency increases are achieved by either increasing the return or reducing the risk of the portfolio. Both the increase in efficiency of the core satellite portfolio as well as the decline in satellite efficiency are quantified in terms of risk respectively diversification to support the theoretical conclusion. In this study, efficiency is defined considering portfolios' criterion vectors. Therefore, in the following, the a posteriori formulation is used and preference weights are no further considered.

Diversification considering intra-portfolio correlation depends on the prediction of variance and covariance (Markowitz 1952). More general implications can be investigated regarding forecast-independent portfolio concentration. The relations and differences of the two measures regarding concentration as the opposite of naïve diversification are well studied (Evans and Archer 1968, Statman 1987, Tang 2004). Since the thematic satellite portfolio no longer has to be assessed for itself, the maxim of risk diversification is being questioned. Hence, to what extent these tailor-made satellite portfolios not only deviate from diversification but also from naïve diversification is tested considering the following hypothesis H1:

H1: Tailoring satellites to core portfolios leads to higher concentrations in portfolio allocations.

Tailoring satellites to particular core portfolios allows the evaluation of assets regarding their core correlation. Knowing that small correlations reduce portfolio risk, the second hypothesis shows how far this additional information is used within the allocation optimization:

H2: Tailoring satellites to core portfolios increases the weighted amount of assets less correlated with the core portfolio.

3 Empirical methodology

In this methodology, a status quo solution for the core satellite investment is stated in order to compare it with the use of tailor-made thematic portfolios. In the status quo solution, conventional core portfolios as well as thematic satellite portfolios are separately allocated. The unified core satellite portfolio is compared with a core satellite portfolio including the same core portfolio but a tailored satellite portfolio. Results show performance improvements computed as inefficiency reductions relative to an efficient tri-criterion portfolio benchmark.

3.1 Data

The database of this study considers an amount of 100 thematic ETFs and four conventional ETFs that offer themselves as core portfolios. Estimations regarding assets' returns, risks and their covariance are computed using daily asset returns of 2016. Data have been obtained from

Thomson Reuters Eikon. On average, the considered thematic ETFs allocate 46 assets from 5 Sectors, 7 Industry Groups, 10 Industries and 13 different Sub-Industries based on Global Industry Classification Standard (GICS) (MSCI 2016).

The performances of different funds depend on the particular portfolio allocation, that is built on different predictions and assumptions, and the underlying timeframe. Therefore, only ETF constituents but not their actual allocations are used as there is no interest in evaluating fund providers or their prediction qualities. Out-of-sample analyses are also out of scope for this study as it focuses on the efficiency of different investment models' solutions and not on prediction optimization. However, whether suboptimal solutions can randomly achieve higher outcomes is an interesting question, but shall be answered in a proceeding study also comparing the robustness and sensitivity of both portfolio models. Therefore, in this study, portfolios are allocated based on the same historical data, predictions and assumptions. Hence, tradable ETFs are only considered to define investment themes and assets universes.

The union of all these ETFs' assets (3,826) taken into account founds the data base for a second simulative methodology. Thematic ETFs are different exotic and do not only seem to be arbitrary as they can actually be defined by considering random-looking subjective criteria. However, most thematic ETFs are currently only opened for very traditional themes. Therefore, a simulation in which a random draw defines new thematic subsets is necessary to increase the data sample by imitating such individual criteria so that the effects of the full range of possible investment themes can be investigated. Investors' asset universes, from which they are drawn, are defined with the same method. Afterwards, the corresponding portfolios are allocated.

3.2 Status quo portfolio

First, the status quo portfolio solution is to be specified. The allocations of the presented ETFs are usually volume based respectively market capitalization weighted. Therefore, the performance of such allocations as well as the results of the study would be strongly dependent on the used data set. In particular, in comparison to tailor-made portfolios that are optimized through the use of the data set, it could not be distinguished whether performance improvements occurred because tailor-made portfolios were tailored to the data set or to the conventional funds. Therefore, this study uses optimization as an idealization of portfolio allocations to reduce this dependency.

As the range of efficient solutions for both portfolios is very broad, particular solutions are allocated by considering the maximum Sharpe ratio. The Sharpe ratio (S), leaving risk free assets out of consideration, measures the relation between excess return and standard deviation (Sharpe 1966). Hence, both portfolios are not being dominated by any other in two-dimensional space within their asset universes. Eqs. (14)–(19) show the a posteriori formulation which furthermore ensures that the investment amount is fully invested and short selling is restricted. $\boldsymbol{x}$ is stated as the union of all n assets taken into account so that thematic assets are always a subset of the whole considered asset universe and are indicated by binary vector $\boldsymbol{\tau}$. The set of feasible solution is limited to $F_r = \{\boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{0} \leq \boldsymbol{x} \leq \boldsymbol{1}\}$ to forbid short-selling. Eq. (15) ensures *inter alia* the exclusive use of thematic assets only.

$$\max \qquad S(\boldsymbol{x}) = \frac{E(x)}{V(x)} \tag{14}$$

$$s.\,t. \qquad \boldsymbol{x}'\boldsymbol{\tau} = 1\ , \tag{15}$$

$$\boldsymbol{x} \in F_r\ \ , \tag{16}$$

These equations lead to the solution vector $\boldsymbol{x_T}$ for the thematic satellite portfolio with the corresponding return E_T, volatility V_T and thematic proportion T_T. According to this, the core portfolio described by return E_C, volatility V_C and thematic proportion T_C is optimized with

$$max \qquad S(\boldsymbol{x}) = \frac{E(x)}{V(x)} \ , \tag{17}$$

$$s.\ t. \qquad \boldsymbol{x}'\mathbf{1} = 1 \ , \tag{18}$$

$$\boldsymbol{x} \in F_r \ . \tag{19}$$

The status quo solution vector $\boldsymbol{x_{CS}}$ is denoted by the linear combination of both portfolios' solution vectors $\boldsymbol{x_C}$ and $\boldsymbol{x_T}$ depending on the relative invested amount $t \in P = \{t \in \mathbb{R} \mid 0 \leq t \leq 1\}$ in the thematic portfolio

$$s.\ t. \qquad \boldsymbol{x_{CS}} = (1 - t)\boldsymbol{x_C} + t\boldsymbol{x_T} \tag{20}$$

$$t \in P \ . \tag{21}$$

The status quo solution is characterized by return E_{CS}, variance V_{CS}^2 respectively volatility V_{CS} and thematic proportion T_{CS}:

$$E_{CS} = E(\boldsymbol{x_{CS}}) = \boldsymbol{\mu}'\boldsymbol{x_{CS}} \ , \tag{22}$$

$$V_{CS}^2 = V^2(\boldsymbol{x_{CS}}) = \boldsymbol{x}_{\boldsymbol{CS}}' \, \boldsymbol{C} \, \boldsymbol{x_{CS}} \ , \tag{23}$$

$$T_{CS} = \ T(\boldsymbol{x_{CS}}) = \boldsymbol{\tau}'\boldsymbol{x_{CS}} \ . \tag{24}$$

3.3 Tailoring satellite portfolios to core portfolios

Two of the criteria, that characterize the solution of the status quo core satellite portfolio, define the constraints for the optimization of the core satellite portfolio including a tailor-made satellite portfolio. Eq. (25) describes the minimization of the core satellite portfolio's risk while ensuring at least the status quo's estimated return (Eq. (26)) and its thematic share (Eq. (27)). Eq. (30) ensures that the relative invested amounts in the core portfolio are not reduced. Eq. (29) allows only to increase thematic shares based on the initially allocated shares within the core portfolio. Parameter $t_t \in P = \{t_t \in \mathbb{R} \mid 0 \leq t_t \leq 1\}$ denotes the targeted relative investment amount in the thematic portfolio for which it is optimized.

$$\textit{min} \qquad V^2 = \boldsymbol{x}' \boldsymbol{C} \boldsymbol{x} \tag{25}$$

$$\textit{s. t.} \qquad \boldsymbol{\mu}'\boldsymbol{x} \geq E_{CS} , \tag{26}$$

$$\boldsymbol{\tau}'\boldsymbol{x} \geq T_{CS} , \tag{27}$$

$$\boldsymbol{x}' \mathbf{1} = 1 , \tag{28}$$

$$x_i \tau_i \geq x_{C_i}(1 - t_t)\tau_i \; \forall \; i = 1, \ldots, n \tag{29}$$

$$x_i(1 - \tau_i) = x_{C_i}(1 - t_t)(1 - \tau_i) \; \forall \; i = 1, \ldots, n . \tag{30}$$

The problem is solved by $\widehat{\boldsymbol{x}}$ and the corresponding core tailor-made satellite portfolio's risk is denoted by $\widehat{V^2}$ and its return $\hat{E}$ and thematic proportion $\hat{T}$.

Subtracting $(1 - t_t)\boldsymbol{x_C}$ from $\widehat{\boldsymbol{x}}$ and afterwards standardizing the vector generates the allocation of the new tailor-made thematic satellite portfolio $\widehat{\boldsymbol{x_T}}$. The corresponding return $\widehat{E_T}$ and risk $\widehat{V_T^2}$ are calculated with Eqs. (31)–(32).

$$\widehat{E_T} = \boldsymbol{\mu}'\widehat{\boldsymbol{x_T}} \tag{31}$$

$$\widehat{V_T^2} = \widehat{\boldsymbol{x_T}}' \boldsymbol{C} \widehat{\boldsymbol{x_T}} \tag{32}$$

3.4 Tri-criterion benchmark portfolio allocation

A tri-criterion allocation that is not restricted by the constraint of a separate optimization of the two portfolio parts is computed as follows (Methling and Nitzsch 2018):

$$\textit{min} \qquad V^2 = \boldsymbol{x}' \boldsymbol{C} \boldsymbol{x} \tag{33}$$

$$\textit{s. t.} \qquad \boldsymbol{\mu}'\boldsymbol{x} \geq E_{CS} , \tag{34}$$

$$\boldsymbol{\tau}'\boldsymbol{x} \geq T_{CS} , \tag{35}$$

$$\boldsymbol{x}' \mathbf{1} = 1 , \tag{36}$$

$$\boldsymbol{x} \in F_r . \tag{37}$$

The tri-criterion allocation $\boldsymbol{x}_{\boldsymbol{Tri}}$ belongs to a non-dominated criterion vector in three-dimensional space. The efficient solution further described by return E_{Tri}, variance V_{Tri}^2 and thematic proportion T_{Tri} serves as a benchmark for the efficiency comparison.

3.5 Efficiency measures

All portfolios are compared with such an efficient solution defined by the additional constraints and further characterized by the tri-criterion portfolio's volatility V_{Tri} (see Figure 1). In the following, risk is compared by the use of the volatility of asset returns.

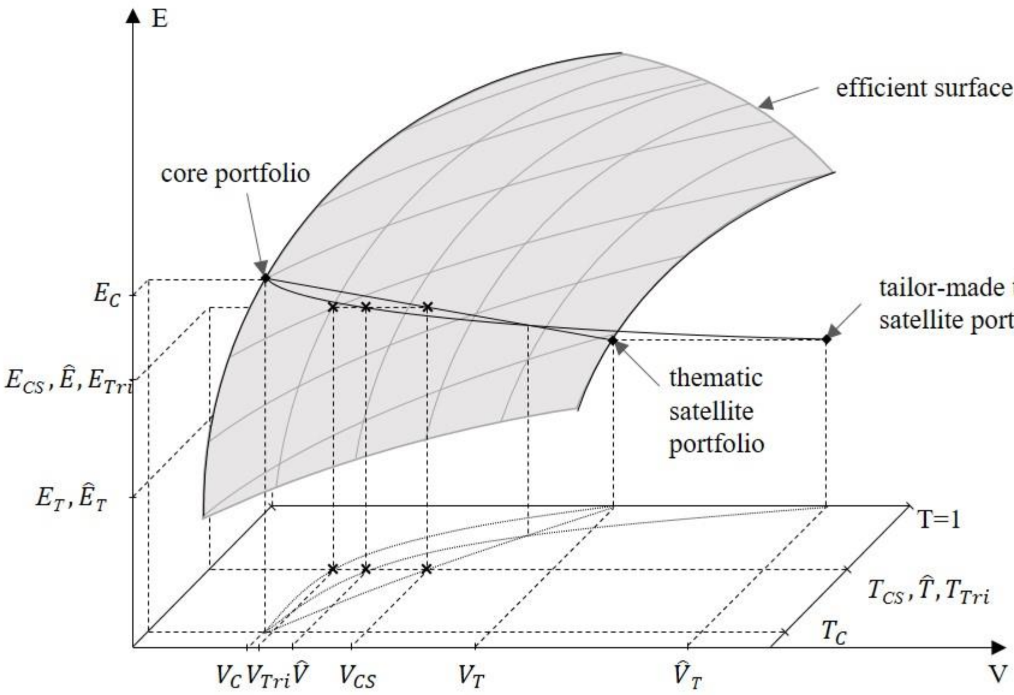


Figure 1: *Efficient tri-criterion portfolio's volatility V_{Tri}, volatilities $\hat{V}$ and V_{CS} of core satellite portfolios with a tailor-made thematic satellite and a separately optimized satellite for given estimated return $\hat{E} = E_{CS} = E_{Tri}$ and thematic proportion $\hat{T} = T_{CS} = T_{Tri}$.*

Regarding the research question and all parts that will be taken into account during the discussion of the results, the comparison parameters are described in the following. Investors can spend different relative amounts *t* in a thematic satellite portfolio that itself can be optimized for different relative amounts t_t. Therefore, various portions have to be considered for both the targeted invested amount t_t which the satellite was initially optimized for as well as the amount *t* actually invested. Hence, five different investment scenarios are defined as an abstract of the total amount of possible investment scenarios. This abstract considers five relative shares t Є {10 %, 20 %, 30 %, 40 %, 50 %} spent in the thematic satellite portfolio. The maximum amount of t = 50 % is justified because the conventional core portfolio is assumed to be the key part of an investment (Marchioni et al. 2016). Therefore, it should not be smaller than the thematic fraction. A more detailed approach could be directly optimized considering the integration of all possible thematic investment amounts. It is not believed that the potentially higher accuracy justifies the increase in optimization complexity. Therefore, the aforementioned abstract is used which optimizes the satellite portfolio once for each of the five considered amounts t_t. Thereafter, each portfolio is tested with all five investment proportions t. Performance improvements in terms of efficiency increases respectively inefficiency decreases are measured as risk reductions δ relative to the maximum achievable risk reduction of an efficient tri-criterion solution:

$$\delta(t) = \frac{V_{CS}(t) - \hat{V}(t,t_t)}{V_{CS}(t) - V_{Tri}(t)} \qquad (38)$$

Comparing five different targeted invested amounts t_t that the satellite is optimized for, the maximum average of the relative inefficiency reductions in the five investment scenarios defines the best targeted invested amount. The average relative inefficiency reduction considering a thematic satellite portfolio that is optimized for the said targeted invested amount is further referenced to as Δ. The maximum Δ = 1 represents an efficient solution in three-dimensional space. That is the target value of the optimization.

The study focuses on comparing risk and risk optimized satellite portfolios, as otherwise the core satellite portfolios would match only the original portfolio's thematic evaluation for different relative invested amounts t. Therefore, they needed to be compared using two criteria or at least risk-adjusted returns; an unnecessary complication that is avoided.

Hence, risk is compared on a satellite portfolio level by comparison parameter R in Eq. (39) regarding a relative increase in risk:

$$R = \frac{\hat{V}_T - V_T}{V_T}. \qquad (39)$$

This shows to what extent the satellite has to be weakened – in regard to diversification and efficiency - to improve the core satellite portfolio. As already mentioned in the second section of this study, diversification is measured in two ways. In addition to the forecast-dependent standard deviation of returns, naïve diversification respectively concentration is compared by comparison parameter *H* as the difference of the two portfolios' Herfindahl indexes (Woerheide and Persson 1993):

$$H = \widehat{\boldsymbol{x_T}}'\widehat{\boldsymbol{x_T}} - \boldsymbol{x}'_{\boldsymbol{T}}\boldsymbol{x_T} \,. \qquad (40)$$

The last comparison parameter *K* denotes the difference between the mean weighted correlation of the new thematic satellite portfolio's assets respectively the original thematic portfolio's assets with the conventional portfolio. This shows if on average assets with a higher or a lower correlation with the core portfolio are allocated. The *n*-dimensional vector $\boldsymbol{\rho}$ denotes the correlation coefficients of assets with the core portfolio.

$$K = \widehat{\boldsymbol{x_T}}'\boldsymbol{\rho} - \boldsymbol{x}'_{\boldsymbol{T}}\boldsymbol{\rho} \,. \qquad (41)$$

3.6 Analysis

In the following, the two-parted approach that is used to compute comparison results is described. In a first approach, assets of both tradable thematic and conventional ETFs define asset universes wherein the status quo solutions as well as the tailor-made satellite solutions are optimized and compared. ETFs based on indexes like the MSCI World serve as surrogates for conventional core portfolios while themes defined by thematic ETFs are understood as subsets of these asset universes. Therefore, the particular allocation of the core portfolio is optimized within the union of both ETFs' assets. The thematic satellite is optimized within the subset of the thematic ETF's assets. Afterwards, the tailor-made thematic portfolio is optimized and tailored to the core portfolio. The tri-criterion solution that minimizes risk under the given constraints serves as a benchmark. The different measures are computed for any combination of given core and satellite ETFs.

In a second approach, a simulation randomly defines new thematic subsets of different considered assets universes of all assets (3,826) taken into account to imitate further investment themes. The simulation is divided into 25 different sub-simulations with each different amounts of assets for the satellite nT = [20, 40, 60, 80, 100] respectively the core portfolio nC = [200, 400, 600, 800, 1,000] taken into account. Hence, an amount nC of assets is initially drawn from the whole data sample to represent an asset universe. Afterwards an amount nT of assets is drawn from this universe to define the subset of thematic assets. Each time 1,000 of these draws create a sub-sample for the particular sub-simulation. Within these sub-samples the portfolios are optimized and the comparison parameters are computed. The simulation is necessary to increase significance and to better understand the effects of the full range of possible subjective criteria. The results are shown in the following section.

4 Results

The results of 100 thematic funds tailored to four different core portfolios are shown in Table 1. Mean Δ shows the average risk reduction relative to the maximum achievable risk reduction considering a tri-criterion benchmark portfolio under the given constraints. The core satellite portfolio's inefficiency is reduced by an average of 11.74 %. In contrast, a relative increase in volatility (R) of 4.55 % has to be set against.

With regard to the first hypothesis H1 that tailor-made funds are more concentrated within their allocations, the data support this assumption. In total, 242 satellite portfolio allocations of the

400 computed allocations perceive an increase in concentration by being tailored to a conventional portfolio.

The second hypothesis H2 states that tailor-made thematic funds reduce the average correlation of assets with the core portfolio. Within the data sample 301 of all 400 portfolio allocations reduce their mean correlation with the core portfolio. Therefore, both alternative hypotheses considering a 50 % maximum probability of increasing concentration respectively decreasing correlation can be rejected on a significance level of α = 0.001 using binomial testing.

	mean *Δ*	mean *R*	*H*	*K*
MSCI World	7.38 %	4.15 %	55	74***
Russel 1000	11.71 %	4.2 %	65**	80***
S&P 500	13.11 %	4.92 %	63**	74***
Stoxx 600	14.77 %	4.91 %	59*	73***

Table 1: *Mean inefficiency reductions Δ for combinations with each 100 thematic ETFs; mean relative risk increases (R) of the thematic satellite portfolios; amounts of comparison parameters that support the assumptions considering a concentration increase (H), and a correlation decrease (K) on significance levels α*** = 0.001, α** = 0.01 and α* = 0.05 calculated with binomial testing rejecting an alternative hypothesis of 50 % maximum probability.*

Quantifying results of the simulation are shown in Table 2. On average, 16.99 % inefficiency reduction is achieved by a mean relative risk increase of 2.22 % in the thematic satellite portfolio.

	nT=20	*nT*=40	*nT*=60	*nT*=80	*nT*=100
nC=200	15.91 %; 2 %	19.44 %; 1.84 %	20.12 %; 1.45 %	19.79 %; 1.02 %	18.56 %; 0.71 %
nC=400	13.9 %; 2.3 %	17.84 %; 2.43 %	19.38 %; 2.26 %	20.33 %; 2.06 %	21.34 %; 1.85 %
nC=600	10.95 %; 2.03 %	15.96 %; 2.56 %	17.98 %; 2.57 %	19.47 %; 2.48 %	20.9 %; 2.38 %
nC=800	10.37 %; 2.05 %	14.49 %; 2.57 %	17.08 %; 2.71 %	18.24 %; 2.7 %	19.94 %; 2.74 %
nC=1,000	9.26 %; 1.97 %	13.08 %; 2.45 %	15.36 %; 2.66 %	16.91 %; 2.81 %	18.08 %; 2.85 %

Table 2: *Mean inefficiency reductions Δ; mean relative risk increases R of the thematic satellite portfolios for different amounts of conventional assets nC and the subset of thematic assets nT with each 1,000 draws arranged as Δ; R.*

In the following, in Table 3, data points from the data sample of each 1,000 draws are shown that meet the corresponding threshold in regard to the two hypotheses. The first entry shows the amount of thematic portfolios with increased concentrations (*H*) respectively that support hypothesis H1. The second entry counts satellites with average allocated assets that correlate less with the core portfolio (*K*) so that they support hypothesis H2. In 98 % of cases, satellite concentration is increased and in 92 % of cases, mean core-correlation of the portfolio's assets is reduced. Consequently, both hypotheses are assumed to hold.

	nT=20	nT=40	nT=60	nT=80	nT=100
nC=200	953; 854	988; 833	980; 781	993; 722	993; 688
nC=400	965; 943	985; 947	988; 923	995; 900	995; 906
nC=600	936; 940	989; 970	992; 970	995; 964	997; 955
nC=800	949; 961	982; 979	986; 982	998; 984	997; 987
nC=1,000	943; 967	972; 983	989; 990	988; 994	990; 996

Table 3: *Counts of comparison parameters that support the assumptions considering a concentration increase H, and a correlation decrease K for different amounts of conventional assets nC and the subset of thematic assets nT with each 1,000 draws arranged as H; K all significant on significance level α = 0.001 calculated with binomial testing rejecting a 50 % maximum probability.*

5 Conclusion

Motivated by the rising interest in thematic investing and the increasing economic significance of thematic investments, this study uses multi-objective portfolio optimization to improve current investment strategies. Not only focusing on monetary goals, recent developments in research - that also question traditional investor models - are combined with the current thematic investing strategy of core and thematic satellite portfolios. As inter-portfolio correlation effects are not yet considered, there is still plenty of scope for operations research and optimization (Methling and Nitzsch 2018).

Therefore, this study addresses fund and index providers to follow the idea of tailor-made thematic funds tailored to conventional core portfolios. Thus, overall efficiency can be increased by easing the constraint of the economic optimality of satellite portfolios when they are separately optimized. The empirical work has supported the introductory motivation and points out a way to make core satellite investing more efficient.

In a two-parted analysis, thematic ETF portfolios were first tailored to conventional ones. On average, the inefficiency of the whole core satellite portfolio was reduced by 11 %, measured as ceteris paribus relative volatility reduction. To achieve this improvement, satellite volatility was increased by less than 5 %. Furthermore, the mean correlation of assets with the core portfolio was decreased in more than 75 % of cases. Thematic satellites were thereby more strongly concentrated within their allocations.

The second approach used a random definition of new database subsets to imitate subjective thematic criteria. Within these subsets, further portfolios were created. The simulation also supported the idea of tailor-made funds as well as the hypotheses. Therefore, by easing the restriction on allocating efficient satellites, portfolios are allowed that are more concentrated in their themes and use more exotic or rather less core-correlated assets. This reduced core satellite inefficiency by almost 17 %. Consequently, it is stated that a targeted reduction of thematic portfolio efficiency increases the performance of a thematic core satellite investment.

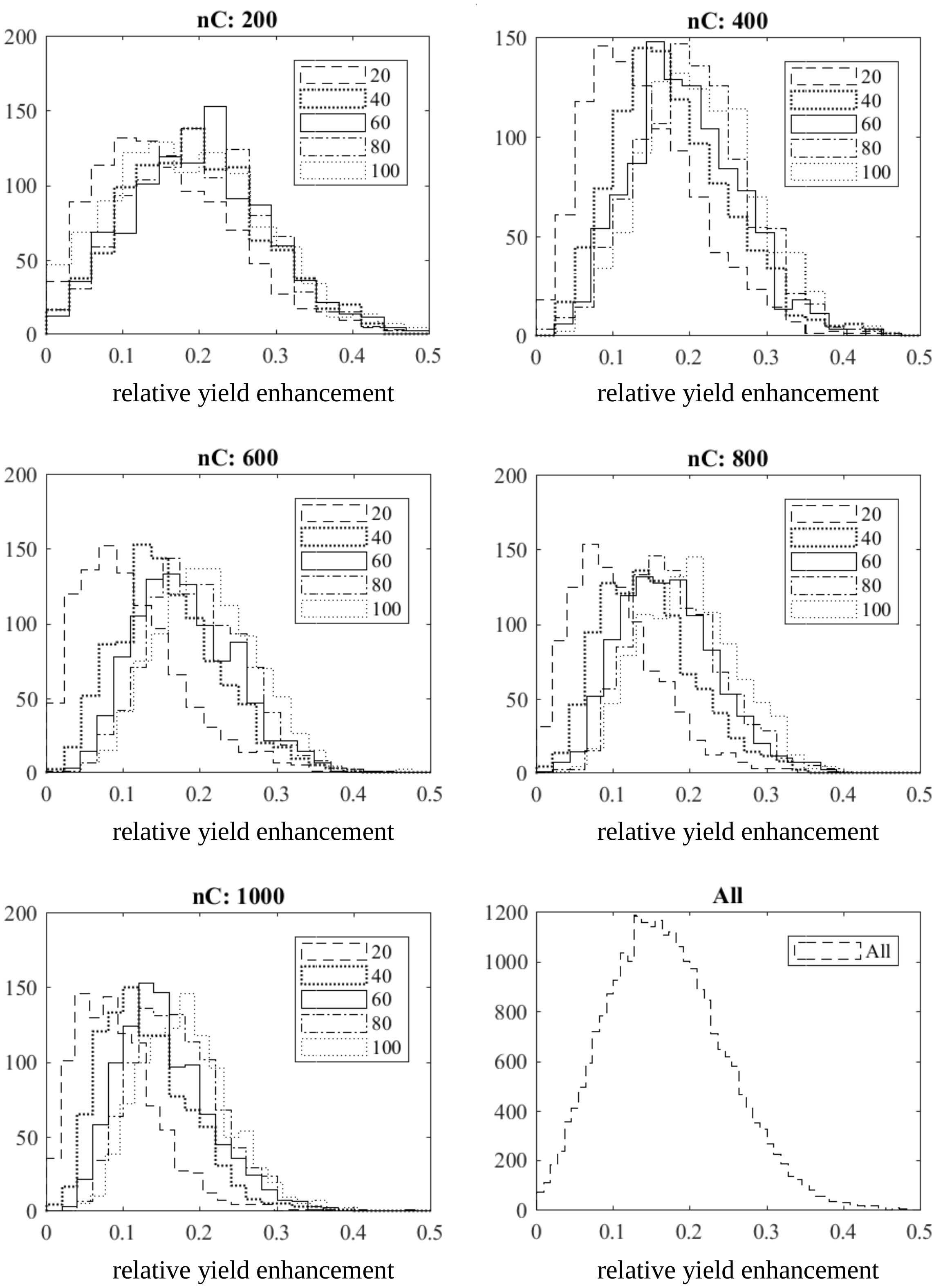
Here, it should be noted that the quantitative difference of 6 % between the results of the two-parted analysis occurs because a significant proportion of tradable thematic ETFs is currently still motivated by monetary interests in their brochures. Therefore, core and satellite portfolios are relatively similar so that two financially efficient portfolios do not leave much room for customization. The more exotic and individual themes are composed, the greater the differences between the portfolios will be and results of tradable ETFs will resemble the simulative results. Especially for such ordinary themes, a tri-criterion optimization provides strong solutions, as tailor-made thematic funds can only achieve the smaller part of inefficiency reductions with regard to the efficient benchmark. However, the larger improvements that can be achieved by directly investing into a tri-criterion portfolio are offset by additional allocation costs.

In contrast to the limitations and the alternative approach of tri-criterion optimization, the implementation in practice is one big advantage of tailor-made thematic funds and a two-stage core satellite approach. No new distribution channels have to be implemented and no additional transaction costs have to be compensated or offset. Investors can still follow their two-portfolio strategy and profit from the benefits regarding optimality of solutions.

Hence, it is the fund and index providers that have to adjust and to improve their current thematic products to account for their customers' needs. Consequently, this is where further research is needed, and in accordance with Aouni et al., it is MCDA research that forms the basis for such developments (Aouni et al. 2018).

In proceeding studies, heuristics could be developed to identify promising assets suitable for tailor-made thematic funds without running completely prediction based optimization models. Correlations that are founded on market knowledge could be strong enough to identify such assets within a theme. Therefore, evaluation methods for the definition of thematic asset universes need to be developed. Furthermore, deeper insights into investors' investment motivations have to be provided in order to meet thematic/ financial trade-offs, especially when different thematic interests are taken into account.

Appendix: Simulation histograms



Histograms show the results of the simulation with 3,826 assets and a random evaluation of thematic assets for different amounts of conventional assets nC. Within the histograms, the different lines show each 1,000 results concerning different amounts of thematic assets. Histogram “All” summarizes the 25,000 results of the five previous histograms.

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Paper 3: Naïve diversification in thematic investing – Heuristics for the core satellite investor

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Naïve diversification in thematic investing – Heuristics for the core satellite investor

In recent years, thematic exchange-traded funds (ETF) have given core satellite strategies a new impetus. Thematic investing attempts to participate in certain trends, or to serve any conceivable subjective interest such as ethics and sustainability by supplementing the corresponding ETFs to conventional ones. Hence, the question arises how to weight the thematic satellite in relation to the diversified core portfolio. Complex research and factor models are hardly suitable for private investors and the short history of thematic products would not provide reliable information anyway. Therefore, this study develops naïve diversification for thematic core satellite investors and provides three heuristics. The first strategies focus on portfolio and stock amounts, the later considers minimum concentration as an allocation rule based on the Herfindahl index. The heuristics prove to be useful and competitive to provide diversification regarding volatility of portfolio returns compared to a minimum variance optimization in out-of-sample tests. Hence, this study offers some pragmatic and truly practical aid for thematic investors.

Thematic investing; Heuristics; Naïve diversification; Core satellite investing

1 Introduction

Recently, new financial products have emerged, in particular exchange-traded funds (ETF), which are no longer advertised as solely monetary performance providers for financial experts. Rather, these specific ETFs are promoted as opportunities for all to participate in upcoming trends and reflect personal interests and convictions (Forster 2017). Therefore, these so-called thematic ETFs attract the attention not only of institutional, but also of private investors.

In the moment, the biggest market for thematic products targets environmental, social and governance (ESG) criteria. In the United States alone, an amount of $12 trillion of assets under professional management now considers ESG aspects (US SIF Foundation 2018). Perhaps against the intuition, many ethically motivated investors do not focus on such products exclusively but supplement them to a conventional portfolio (Lewis 2001). A strategy that is also observed across other investments themes and is best described as a variation of a core satellite strategy.

In the original understanding of the core satellite strategy, a diversified core portfolio is the foundation of a portfolio. Actively managed portfolios are supplemented to provide excess returns while controlling risk exposure through the core portfolio. A thematic core satellite strategy in comparison also provides risk control and diversification with a conventional portfolio, but adds satellite portfolios to obtain a thematic exposure (Magoon 2009, Bérubé et al. 2014).

The reasons for private investors to divert from conventional products are manifold. First, investors try to meet their ethical, social, or religious conviction (Webley et al. 2001, Beal et al. 2005, Riedl and Smeets 2017). Second, they perceive risks within specific themes differently (see, e. g., domain-specific risk attitudes as investigated by Tversky and Kahneman 1981, Shefrin and Statman 2000, Weber et al. 2002), and third, they can see their individual market expectations – driven by overconfidence or not – exploited within a thematic ETF (Odean 1998, Zaher and Kabir Hassan 2001, Louche et al. 2012, Giammattei 2014).

Megatrend oriented thematic ETFs, e. g., are promoted as low cost opportunities to get exposure to long-term evolving socio-economic trends that also provide some dynamism to a portfolio (Marchioni et al. 2016). These ETFs include their investors' beliefs about future developments, such as motivated within the third category of individual market expectations. Furthermore, they allow investors to intuitively understand the rationale of the portfolio allocation which could change the investor's risk perception within this theme, as mentioned in the second category. Such opportunities have yet only been provided to wealthy customers by individual consultants and portfolio managers.

In this multi-dimensional solution space, investors focus on thematic criteria besides Markowitz' paradigm of return maximization and risk reduction (Markowitz 1952, 1956, 1996). An overview of different multi-criteria decision making methodologies is provided by Xidonas et al. (2012). Hence, multiple objectives have already been considered in finance literature and their effects on the efficient frontier of portfolio allocations have been studied (Steuer et al. 2005, 2007). Efficient surfaces in three-dimensional space regarding one additional criterion can also be calculated (Hirschberger et al. 2013). Furthermore, the added value for thematic investors who optimize their portfolios instead of following the aforementioned core satellite strategy has already been quantified and proves the need for further research (Methling and Nitzsch 2019).

Nevertheless, while these studies undoubtedly provide academic value the practical utility for private investors is restricted. An elaborate optimization of a thematic portfolio by purchasing individual assets is very costly, and ETFs are not customizable for private investors. Furthermore, complex research and factor models are hardly suitable for private investors and the short history of thematic products would not provide reliable information anyway.

Therefore, pragmatic and truly practical solutions need to be provided to help thematic investors that follow a core satellite strategy with a conventional and a thematic fund.

Thematic investors address a simple question: how to weight the satellite in relation to the core portfolio?

Heuristics, stemming from the Greek word *heurisko* (to find, discover), could find an answer. The heuristic approach of naïve diversification is reconsidered and adapted to the two portfolio choice of thematic investors. In conjunction with earlier research, this study further develops naïve diversification based on deeper insights into the two portfolios. Hence, allocation rules are established that also consider the amounts of different stocks within the funds as well as the concentrations of the two specific allocations based on the Herfindahl index.

Furthermore, the identified heuristics are compared to benchmark portfolios also including minimum variance optimizations. As naïve diversification as well as minimum variance optimization focus on risk reduction, the allocations are compared regarding the volatility of portfolio returns. Portfolio returns themselves are not considered in this comparison, as the results would not be as reliable and highly sensitive to time series noise due to the short history of thematic products respectively the sample frames. Competitiveness is proven while also demonstrating the heuristics' added value for private investors in regard to effort and complexity.

In the following section, the transition from naïve diversification to further more complex naïve diversification strategies is shown while discussing the required effort for private investors who consider these heuristics. The subsequent section presents the empirical methodology including a description of considered data as well as the research design to quantify the competitiveness of the individual strategies. After the presentation of the results, robustness tests for different risk measures and several variations of the data sample are performed in the discussion section. The last section shows key findings and introduces future research streams.

2 Theoretical model

In 1968, Evans and Archer quantified the general effect of an increased number of assets held in a portfolio on the reduction of portfolio volatility. In the following, Wagner and Lau (1971) supported the finding that an increased number of securities rapidly improved portfolio diversification and suggested that even small investors bundled their investments to ensure broad diversification. This opened a debate with various contributions discussing the required amounts of different assets, as Statman (1987) concluded the minimum amount of stocks held by an investor to be at least 30-40. A list of finance and investment textbooks as well as the corresponding recommended amounts of stocks to reduce diversifiable risk is provided by Tang (2004) who focuses on the marginal diversification rates of additional stocks. Differences are led back to research designs and portfolio allocations. Therefore, the following subsections focus on specifying portfolio allocation decisions within the model as well as strategies to determine the explicit allocations.

2.1 Portfolio allocation decisions in the model

In general, portfolio allocations are denoted by the investment vector $\boldsymbol{x}$ that is chosen out of a set of feasible solutions F. In the considered situation, a core satellite investment in different ETFs, there are three portfolio allocations. First, two exogenously given allocations of a thematic and a conventional ETF, that are explained shortly referring to index ETFs and smart-beta ETFs. Second, the particular allocation that results from combining these two portfolios, which is presented afterwards.

2.1.1 Two exogenous portfolio allocations

The two portfolio allocation vectors of the conventional core portfolio and the thematic satellite portfolio are defined by $\boldsymbol{x_C} \in F_C$ and $\boldsymbol{x_S} \in F_S$ as relative portfolio weights. In this study, we focus on positive portfolio weights as most thematic ETF restrict short sales. Therefore, the two portfolio allocation vectors are given as follows considering n conventional stocks and m thematic stocks:

$$\boldsymbol{x_C} \in F_C = \{\boldsymbol{x_C} \in \mathbb{R}^n \mid 0 \leq x_i \leq 1 \wedge |\boldsymbol{x}| = 1\} \quad (1)$$

$$\boldsymbol{x_S} \in F_S = \{\boldsymbol{x_S} \in \mathbb{R}^m \mid 0 \leq x_i \leq 1 \wedge |\boldsymbol{x}| = 1\} \quad (2)$$

Such allocation vectors can track any favored index, a customized index or follow its own strategy. Well known indexes like S&P 500 or DAX30 list assets by their market capitalization, customized indexes are calculated continuously by index providers to build individual benchmarks, and recently occurred smart-beta ETFs assume value-drivers to signal buy and sell orders. No matter the type of ETF, the explicit allocation that an investor perceives is given exogenously.

2.1.2 Explicit portfolio allocation

An investor's decision space is limited to allocating the investment amount into the different portfolios. For a thematic core satellite investor who has already picked a thematic portfolio to be supplemented to its core portfolio the investment decision reduces to one decision variable for a given investment amount.

The particular investment is defined by vector $\boldsymbol{y}$. In the following, $y_1 \in [0, 1]$ denotes the investment share spent in the conventional core portfolio. The relative amount spent in the thematic satellite portfolio is denoted by $y_2 = 1 - y_1$.

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (3)$$

$$\boldsymbol{y} \in F = \{\boldsymbol{y} \in \mathbb{R}^2 \mid 0 \leq y_1 \leq 1 \wedge \mathrm{y}_2 = 1 - \mathrm{y}_1\} \quad (4)$$

This is where investors are in need of practical aids to determine the portfolio allocation. To provide this support, different approaches are investigated in the following while questioning their practicability for private investors.

2.2 Naïve diversification

> *"A man should always place his money, one third into land, a third into merchandise and keep a third in hand"* ‡‡‡‡

This rule is more than 1,500 years old and goes back to the Talmud advising to hold money in three parts, a simple form of diversification. In recent decades, lots of research has devoted to that, trying to quantify the effects of the proposed diversification and to compare it to optimization models (Tang 2004, DeMiguel et al. 2009b, DeMiguel et al. 2009a). In textbook finance, the 1,500 years old heuristic allocation rule is systematically explained (Levy and Sarnat 1972). Hence, naïve portfolio diversification describes allocations that invest equal fractions $1/N$ in all N available assets.

$$\boldsymbol{y} = \begin{pmatrix} 1/N \\ \vdots \\ 1/N \end{pmatrix} \tag{5}$$

Benartzi and Thaler (2001) demonstrated the usage of this rule in various situations and attracted a lot of attention by investigating investor behavior in saving plans. This study has proven the applicability of naïve diversification for private investors. In the following, the strategy is further developed and investigated.

‡‡‡‡ Talmud - Bava Metzia 42a

2.2.1 Stock-based naivety (SBN)

As naïve portfolio diversification tries to minimize exposure in different stocks the naïve strategy is to be adapted to the amounts of stocks in the two portfolios. This heuristic assumes that each portfolio itself is allocated by naïve diversification respectively equally weighted so that the relative amount allocated in each stock is the same. Consequently, exposure is minimized by Eq. (6).

$$\min: \max\left(\frac{y_1}{n}, \frac{y_2}{m}\right) \tag{6}$$

This problem is solved by:

$$\boldsymbol{y_{SBN}} = \begin{pmatrix} \dfrac{n}{n+m} \\ \dfrac{m}{n+m} \end{pmatrix} \tag{7}$$

representing the allocation vector of the stock-based naïve diversification strategy (SBN). The effort for private investors is limited to calculating ratios of the numbers of stocks included in a portfolio that can be read off an index or fund prospectus. However, in the following, a strategy is presented that even reduces this effort as well as a more elaborate but more promising strategy.

2.2.2 Portfolio-based naivety (PBN)

Considering the two portfolios that the core satellite investor has to relatively weight the situation can still be more simplified. Having only N=2 assets to choose from the investment vector of the portfolio-based naïve diversification strategy (PBN) is defined by Eq. (8). The explicit allocations of the two portfolios are not further considered and an equal amount is spent in the two portfolios.

$$\boldsymbol{y_{PBN}} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \tag{8}$$

This strategy is the simplest form of naive diversification with the least effort.

2.2.3 Concentration-based naivety (CBN)

A more elaborate strategy can be developed accounting for portfolio concentration because most portfolios are actually not naively diversified as it is assumed in the first strategy. Motivated by the amount of studies focusing on diversification and the required minimum numbers of securities, Woerheide and Persson (1993) looked for diversification measures that do not assume evenly distributed stock portfolios. The most promising measure was found to be the complement of the Herfindahl index. In the following, this index is developed to an investment rule. It is assumed that a less concentrated portfolio is less exposed to particular risks. This goes in line with Meucci (2010) who defines a well-diversified portfolio as one "not heavily exposed to individual stocks". Concentration is measured by the Herfindahl score of the two portfolios and is defined by Eq. (9):

$$H(\boldsymbol{x}) = \sum x_i^2 \tag{9}$$

Hence, the concentration of the portfolio combined out of the two portfolios is calculated as follows:

$$H(y_1, y_2) = \sum_{i=1}^{n} \left(x_{C_i} * y_1\right)^2 + \sum_{j=1}^{m} \left(x_{S_j} * y_2\right)^2 \tag{10}$$

The minimum core satellite concentration portfolio is defined by the local minimum:

$$0 = \frac{\partial H(y_1)}{\partial y_1} = 2y_1 \sum_{i=1}^{n} x_{C_i}^2 + 2y_1 \sum_{j=1}^{m} x_{S_j}^2 - 2 \sum_{j=1}^{m} x_{S_j}^2 \tag{11}$$

$$\Leftrightarrow y_1 = \frac{\sum_{j=1}^{m} {x_{S_j}}^2}{\sum_{i=1}^{n} {x_{C_i}}^2 + \sum_{j=1}^{m} {x_{S_j}}^2} = \frac{H(x_S)}{H(x_C) + H(x_S)} \tag{12}$$

The investment vector of the concentration-based naïve diversification strategy (CBN) is defined by:

$$y_{CBN} = \begin{pmatrix} \frac{H(x_S)}{H(x_C) + H(x_S)} \\ \frac{H(x_C)}{H(x_C) + H(x_S)} \end{pmatrix} \tag{13}$$

The most elaborate naïve diversification strategy requires not only calculating ratios, but Herfindahl scores of the portfolios, that are usually not provided within a fund prospectus. Nevertheless, in comparison to a minimum variance optimization that requires large amounts of data and complex computation of correlation matrixes, the Herfindahl index can be calculated with any pocket calculator with data provided by the index or fund provider. Therefore, practicability for private investors is assumed to be given.

3 Empirical methodology

The three developed heuristics PBN, SBN and CBN are tested in a rolling window approach against two benchmarks. First, the general added value is investigated by comparison to a situation where an investor has no additional information and chooses a strategy of allocating a fixed amount in the thematic satellite. Second, the benchmark of a minimum variance portfolio is considered to compare the heuristics to an optimization-based approach. The research design assumes a buy-and-hold strategy so that transactions costs, assuming thematic and conventional ETFs to be equally expensive, and turnover volume are not relevant.

3.1 Data

The data base considers 10 different conventional index ETFs[§§§§] as well as 50 different thematic ETFs identified by their name and fund description. The thematic fund selection process focused on a web search and was expanded to not only look for funds called thematic

[§§§§] DAX30, DJ Global Titans 50, FTSE 100, Global Dow, Nikkei 225, S&P 500, STOXX600, Russell 1000, MSCI ACWI, MSCI World.

ETF but also those that have been referred to as thematic in, e. g., grey literature. Due to the short history of thematic products this paper takes data from 2014 to 2018 into account. Although this limits the data base to 5 years, as Figure 1 demonstrates the data base includes bullish and bearish market situations so that the heuristics can be investigated during different market scenarios.

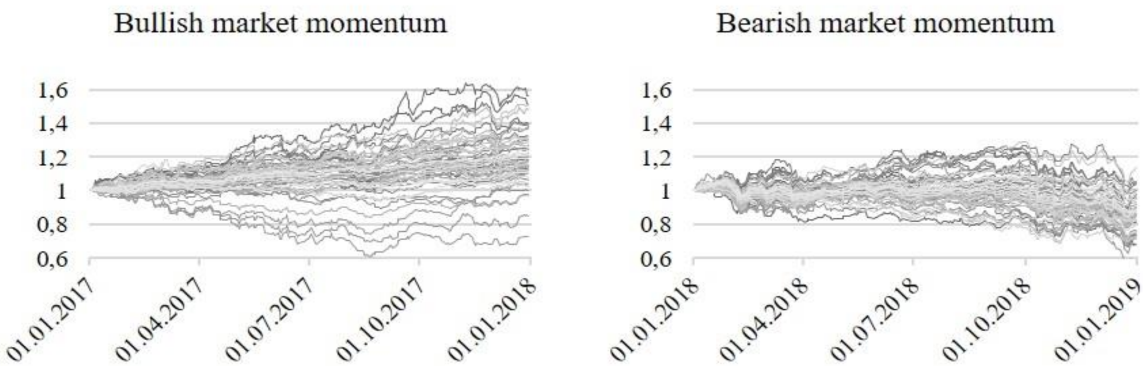

Figure 1: *Daily prices of considered thematic and conventional ETFs of the years 2017 and 2018.*

These data have been obtained from Thomson Reuters Eikon. The underlying distribution of daily asset returns with a mean of 0.01 % and a standard deviation of 1.18 % is negatively skewed (skewness: -0.27) and leptokurtic with fatter tails (kurtosis: 6.91) so that the Anderson-Darling test rejected the null-hypothesis of a normally distributed data sample (significance level <0.01%). Nevertheless, this is not a necessary assumption within the theoretical model as we use the variance of portfolio returns only as a diversification measure and do not assume an explicit preference model.

3.2 Methodology

Referring to DeMiguel et al. a “rolling-sample” respectively “rolling window” approach is well suited for a comparison of different strategies (2009a, 2009b). In a $T=1256$ trading days-long dataset of asset returns, the estimation window includes $IN = 250$ days. From day t on daily asset returns of the previous IN days are calculated to estimate future returns. In addition, fund information - stock amounts and constituent weights - of day *t-1* are provided so that the necessary parameters for the considered strategies are stated. In the following, these parameters are used to determine the allocation vectors of the portfolios. The daily returns of the portfolios are computed during the out-of-sample period of the following *OUT = 500* days including day *t*. As illustrated in Figure 2 the windows shift forwards by each time adding 15 later days to and dropping the 15 oldest days of the estimation and the evaluation sample ending with the last sample identified by day $t = T - OUT+1$.

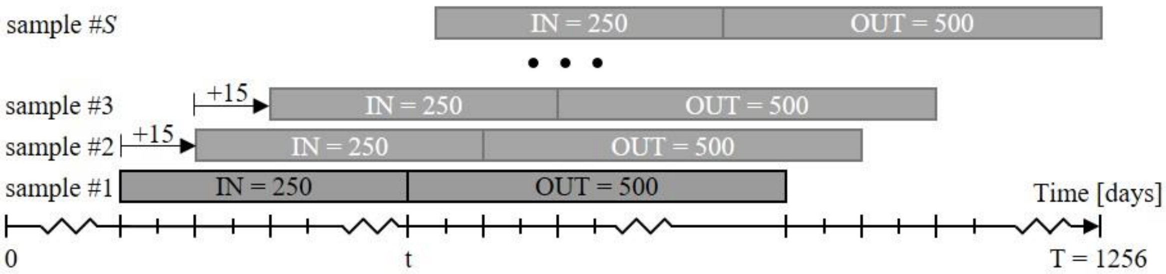

Figure 2: *Visualization of the rolling window approach for in-sample estimation windows IN and out-of-sample testing windows OUT resulting in S different samples.*

To show the effects of the provided diversification strategies benchmarks as well as a risk measure need to be defined. Risk is compared regarding the volatility of daily portfolio returns throughout the testing window. A heuristic is assumed to be useful and to provide reliable diversification if the heuristic's portfolio shows less volatility than the benchmark's portfolio. Volatility measures the variation respectively deviation of a trading price from the average given a particular time frame. Eq. (14) provides the mean $\bar{r}$ of daily returns r_t in the considered time frame.

$$\bar{r} = \frac{1}{OUT} \sum_{i=t}^{t+OUT-1} r_i \tag{14}$$

Volatility *V* is measured as the standard deviation of daily returns (Eq. 15):

$$V = \sqrt{\frac{1}{OUT-1}\sum_{i=t}^{t+OUT-1}(r_i - \bar{r})^2} \tag{15}$$

3.3 Benchmarks

In the following, different benchmarks are considered. In the first comparison, the heuristics have to prove the general added value compared to a sample of 99 benchmarks. In the second comparison, the heuristics are challenged against an optimization-based allocation.

3.3.1 An abstract of the total amount of different portfolio allocations

First, the heuristics are compared to all imaginable core satellite portfolio allocations. As an abstract of the total amount of different allocations, 99 benchmark portfolios are considered. A heuristic that leads to less volatility than at least 50 benchmarks is assumed to be useful for the specific core satellite combination. Each benchmark's investment vector is defined by Eq. (16) with y_1 = [1 %, 2 %, …, 99 %]. Results are presented under the abbreviation 99BM.

$$\boldsymbol{y} = \binom{y_1}{1-y_1} \tag{16}$$

3.3.2 Minimum variance optimization

Second, the heuristics are compared to a minimum variance optimization. Minimum variance optimizations take information from the estimation window and assume them to reoccur in the future. Calculating the standard deviation σ_i and the correlations of asset returns $\rho_{i,j}$ within the estimation window of the previous *IN* days, a minimum variance vector is optimized:

$$\min \sigma(\boldsymbol{x}) = \min \sqrt{\sum\sum x_i x_j \sigma_i \sigma_j \rho_{i,j}} \tag{17}$$

Eq. (17) can be shortened for the two portfolio case:

$$\min \sigma(\boldsymbol{y}) = \min \sqrt{y_1^2\sigma_1^2 + y_2^2\sigma_2^2 + 2y_1y_2\sigma_1\sigma_2\rho_{1,2}} \tag{18}$$

The minimum core satellite volatility portfolio is defined as follows:

$$0 = \frac{\partial\sigma(y_1)}{\partial y_1} = 2y_1\sigma_1^2 - 2\sigma_2^2 + 2y_1\sigma_2^2 + 2(1-2y_1)\sigma_1\sigma_2\rho_{1,2}$$

$$\Leftrightarrow y_1 = \frac{\sigma_2^2 - \sigma_1\sigma_2\rho_{1,2}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{1,2}} \tag{19}$$

This study focuses on purchasing thematic ETFs and restricts short-selling. Therefore, a negative analytical solution $y_1 < 0$ respectively a solution $y_1 > 1$ is to be rejected. Hence, the investment vector is defined by Eq. (20) for three different scenarios:

$$\boldsymbol{y}(\sigma_1, \sigma_2, \rho_{1,2}) = \begin{cases} \begin{pmatrix} 0 \\ 1 \end{pmatrix} & for\ \ \rho_{1,2} > \dfrac{\sigma_2}{\sigma_1} \\ \begin{pmatrix} \dfrac{\sigma_2^2 - \sigma_1\sigma_2\rho_{1,2}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{1,2}} \\ \dfrac{\sigma_1^2 - \sigma_1\sigma_2\rho_{1,2}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{1,2}} \end{pmatrix} & for\ \ \rho_{1,2} < \min\left(\dfrac{\sigma_2}{\sigma_1}, \dfrac{\sigma_1}{\sigma_2}\right) \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} & for\ \ \rho_{1,2} > \dfrac{\sigma_1}{\sigma_2} \end{cases} \tag{20}$$

This benchmark is compared under the name “MVO”. This benchmark not only requires access to data but also a certain financial literacy to understand the optimization. An optimization that is highly sensitive to the provided input and rarely expresses the investor’s personal view (Black and Litterman 1992). Neither the access to data, nor the understanding of optimizations and the resulting allocations can be taken for granted regarding private investors that only try to individualize their portfolio by gaining some thematic exposure. Therefore, the practicability of optimization models is more than questionable.

4 Results

The empirical analysis shows the competitiveness of the different strategies. From 01/10/2015, an in-sample window of 250 days and an out-of-sample window of 500 days is chosen with a sample shift of 15 days. The strategies are compared using 10 conventional and 50 thematic ETFs allocated as provided by fund respectively index providers resulting in 11,000 data points. In Table 1, starting with the portfolio-based naïve diversification strategy (PBN) the development to stock-based naïve diversification (SBN) shows a reduction of volatility during the time frame. In 65.92 % of cases, SBN perceived less volatility which was only even more reduced by concentration-based diversification (CBN). CBN showed less volatility than SBN in 54.95 % of cases.

	SBN	CBN
PBN	65.92 %	67.05 %
SBN	--	54.95 %

Table 1: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison. Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*

In Table 2, the performances of the naïve strategies are compared to the benchmarks 99BM and MVO, representing 99 different portfolios and a minimum variance optimization. While both the SBN as well as CBN show better results than 99BM, MVO can still outperform both strategies over the whole data sample. Nevertheless, no dominance is observed.

	PBN	SBN	CBN
99BM	49.01 %	77.88 %	84.35 %
MVO	17.05 %	42.39 %	45.79 %

Table 2: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*

In means of absolute values, Table 3 shows mean volatilities for the whole data set of the different strategies. Measured as yearly volatility, the portfolio-based, stock-based as well as the concentration-based diversification strategies show mean volatilities in the range of plus one standard deviation (1.56 %) compared to the MVO data sample. To determine if the heuristics as well as benchmarks lead to significantly different volatility distributions, two-sample Kolmogorov-Smirnov tests were performed pairwise. Each hypothesis test rejected the null hypothesis that samples had been drawn from the same underlying distribution at the 5 % significance level. However, while the MVO strategy shows the smallest average volatility, no dominance can be observed. Consequently, summarizing both tables' empirical analyses, naïve diversification strategies prove to be competitive.

	PBN	SBN	CBN	99 BM	MVO
Volatility	12.49 %	11.8 %	11.77 %	17.34 %	11.65 %

Table 3: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*

5 Discussion

In accordance with the presented results, DeMiguel et al. (2009a; 2009b) compared naïve diversification strategies and optimization-based mean-variance portfolios also concluding no dominance regarding portfolio performance. Nevertheless, there are also studies that present different results saying optimization would outperform naïve diversification and differences were to be traced back to research designs (Kirby and Ostdiek 2012).

To mitigate such doubts, the robustness of results is discussed in the following. Several variations of the considered data set and time frames are investigated. In Table 4, the robustness checks are summarized while full tables are provided in the Appendix.

	PBN	SBN	CBN	99BM	MVO
in-250/out-500	12.49 %	11.8 %	11.77 %	17.34 %	11.65 %
in-250/out-250	12.58 %	11.87 %	11.85 %	17.45 %	11.66 %
in-500/out-500	12.02 %	11.35 %	11.32 %	16.92 %	11.19 %
bullish	9.14 %	8.13 %	8.16 %	13.9 %	8.07 %
bearish	15.18 %	14.79 %	14.76 %	20.95 %	14.45 %
consumer	12.00 %	11.69 %	11.55 %	16.01 %	11.66 %
nat. resources	11.8 %	11.67 %	11.51 %	15.34 %	11.65 %
renew. energy	13.2 %	11.95 %	12.01 %	20.43 %	11.79 %
technology	12.87 %	11.95 %	11.89 %	18.78 %	12.01 %
Dax30	13.27 %	13.45 %	13.55 %	17.36 %	13.20 %
S&P500	12.48 %	10.91 %	11.02 %	17.36 %	10.88 %
MSCI World	11.74 %	10.39 %	10.33 %	17.34 %	10.33 %

Table 4: *Mean yearly volatility approximated from daily volatility with factor $\sqrt{250}$ of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios for different time frames, market momentums, thematic families, and particular core portfolios.*

In this initial analysis, first, different time frames are investigated. In the first three lines of Table 4, results of the variation of the in-sample and out-of-sample windows, one time reducing the out-of-sample window and one time extending the in-sample-window, are shown. Not surprisingly, the mean volatility of the minimum variance portfolio is reduced by an extended estimation window. Looking at the naïve diversification strategies, there is a reduction of portfolio volatility, too, which reduces the justification of an estimation-based optimization approach, as the naïve diversification strategy should not be affected by an extended estimation time frame. Therefore, additional timeframes are considered.

One timeframe is especially interesting when it comes to thematic investments motivated to diversify the portfolio in terms of latent risks not yet covered in the conventional portfolio. Hence, in line 4 and 5, a bullish and a bearish market momentum are considered that focus on the years 2017 and 2018. The bearish market scenario of 2018 was unexpected as the expectations are based on mean returns from the bullish year 2017. Nevertheless, MVO is again the most promising strategy, dominance is not observed, and the earlier shown general results are supported.

Thematic ETFs can also be clustered in different thematic families so the results are investigated with regard to these categories. In lines 6 to 9 of Table 4, thematic families consumer, natural resources and technology show the smallest volatility when using concentration-based naivety, technology shows even less volatility for stock-based naivety compared to a minimum variance optimization.

To see the effects of portfolio size, three particular core portfolios are investigated separately. In lines 10 to 12 of Table 4, mean volatilities for a Dax30, a S&P 500, and a MSCI World ETF each combined with 50 thematic ETFs allocated with the different strategies are shown. All strategies show a reduction of volatility with an increased portfolio size. SBN and CBN lead to less volatility than MVO in 41.55 % to 56.18 % of cases proving consistency (see Tables A17 to A22).

All variations of input parameters and the resulting empirics never show a dominance of any strategy. 99BM seems to be the most ineffective strategy to reduce portfolio volatility and PBN is also mostly outperformed by the other strategies. SBN and CBN are at least competitive to MVO.

In the following, a robustness check considering the benchmark 99BM of 99 different portfolios is provided. In Table 5, results for a time frame of 1066 days starting from 01/10/2015 with sample shifts of 15 days and in-sample windows of 250 days respectively an out-of-sample window of 500 days is shown for a variation of the benchmark now considering only 49 portfolios. One could argue that a thematic satellite portfolio should not claim more than 49 % of the invested amount to still be the satellite part of the portfolio. Therefore, 49 benchmarks with $y_2 = [0.01, 0.02, \ldots, 0.49]$ are considered for which a heuristic is assumed to be useful if it leads to less volatility than at least 25 benchmarks. Again, SBN and CBN lead to less volatility than 99/49BM.

	PBN	SBN	CBN
49 benchmarks	28.95 %	58.27 %	66.05 %

Table 5: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 49 different invested amounts. Column wise it shows how often the heuristic leads to less volatility.*

Another robustness check considers different measurements of risk. Besides volatility, risk can be measured, e. g., by the use of the Value at Risk (VaR) or the maximum drawdown (MaxDD) of a portfolio in a specified time period. Again, the same test design is considered using a 250 days in-sample and a 500 days out-of-sample period with a rolling window of 15 days after each calculation starting from 01/10/2015 resulting in 11,000 data points.

The VaR defines an estimated loss that is not to be exceeded given a particular confidence probability $(1-\alpha)$. In Eq. (21), a random variable z and its cumulative distribution function F_Z are given.

$$VaR_{1-\alpha} = \inf_{z} \{z \in \mathbb{R} : F_Z(z) \geq 1-\alpha\}, 0 < \alpha < 1 \tag{21}$$

To determine this measure, returns of the out-of-sample window are used to approximate a discrete distribution function. Simplifying further analysis daily returns are sorted in ascending order and the $\alpha = 5\ \%$ smallest integer of the return sample is identified as the $\text{VaR}_{95\ \%}$. Table 6 shows the results of the comparison mirroring the initial results.

	PBN	SBN	CBN
99 benchmarks	56.95 %	77.51 %	80.84 %
MVO	24.9 %	47.26 %	47.86 %

Table 6: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to a better Value at Risk than the corresponding strategy in the left-most column.*

In Table 7, mean results of the $\text{VaR}_{95\ \%}$ for the different strategies and the aforementioned time frames are presented. No significant difference between stock-based and concentration-based naïve diversification and the MVO portfolio is observed. All strategies outperform the 99 benchmark portfolio.

	PBN	SBN	CBN	99BM	MVO
Value at Risk	-1.26 %	-1.18 %	-1.18 %	-1.8 %	-1.17 %

***Table 7:** Mean Value at Risks of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN), the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*

The third considered risk measure is the maximum drawdown. The maximum drawdown describes the maximum loss in a given time frame that an investor could face. Eq. (22) defines the largest drawdown over all imaginable intervals within the time frame always from a price (P) peak to a subsequent price low.

$$MDD(P, t_0, t_1) = \sup_{t \in [t_0, t_1]} \left[\sup_{s \in [t_0, t]} P(s) - P(t) \right] \qquad (22)$$

Table 8 shows the results of the different strategies. They are less pronounced compared to the VaR and the volatility of the portfolios but also show no clear dominance.

	PBN	SBN	CBN
99 benchmarks	37.65 %	64.36 %	69.36 %
MVO	29.35 %	41.59 %	42.12 %

***Table 8:** Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to a smaller maximum drawdown than the corresponding strategy in the left-most column.*

The mean maximum drawdowns of the different strategies mirror again the initial results but the stock-based diversification strategy has a smaller mean maximum drawdown compared to the concentration-based strategy as presented in Table 9.

	PBN	SBN	CBN	99BM	MVO
MaxDD	13.05 %	12.19 %	12.24 %	17.78 %	11.97 %

***Table 9:** Mean maximum drawdowns (MaxDD) of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*

All robustness checks considered, taking into account different time frames, bullish and bearish market momentums, different thematic families, particular conventional portfolios, a variation of benchmarks, and different risk measures a tendency in favor of a minimum variance optimization occurs but the stock-based as well as the concentration-based naïve diversification strategy prove to be competitive. Therefore, the discussion of the robustness checks supports the initial results and proves consistency.

6 Conclusion

This study was motivated by the rising interest in thematic investments and the lack of pragmatic and truly useful aids for thematic core satellite investors. Addressing those who think beyond the risk/ return paradigm of modern portfolio theory by taking ethical aspirations, their personal conviction and believes as well as their individual interests into account, this study provides competitive heuristics for portfolio allocations.

Based on naïve diversification, the first heuristic invests proportionally to the numbers of stocks included in the two portfolios. In this strategy, private investors need to obtain explicit amounts of stocks held in the two ETFs and calculate their ratios to determine portfolio allocations. An effort that can even be simplified in the second heuristic by ignoring stock amounts only regarding the two portfolios as assets to be equally weighted. In contrast, a third more elaborate development uses the Herfindahl index to consider different portfolio concentrations. Allocation fractions are thereby determined by the inverse ratio of the two portfolios' Herfindahl indexes. As this index is usually not provided by ETF or index providers, private investors must compute it themselves. An increased effort compared to the initial naïve diversification strategy but still limited with regard to the required amount of data needed for optimization models that confront private investors with larger complexities.

Results show that more elaborate naïve diversification approaches are required to provide competitiveness in regard to a minimum variance optimization. Both the stock-based as well as the concentration-based allocation rule appear to be useful as the optimized portfolios show lower volatilities in no more than 57 % respectively 54 % of cases. Mean volatilities do not exceed optimized portfolios' volatilities in a range of more than one standard deviation.

Nevertheless, due to the short history of thematic products the strategies could not be compared over decades with holding periods of more than 2 years, and future latent risks that investors might try to cover with the thematic ETF still remain undetected and unconsidered. Therefore, the conclusions of the findings need to be subject to reservations. Nevertheless, especially when the required amount of data and effort for private investors is taken into account, the two strategies are very helpful as no elaborate optimization model is needed. In cases of new products that do not have a long history of prices that can be used to calculate correlation matrixes, naïve diversification is one tool still usable without an increased effort. Furthermore, the short in-sample periods more or less define the problem of the status quo as investors also face these issues. Short out-of-sample periods can only be expanded in later research but the presented results are more than promising.

Another limitation of this study is the focus on risk performances of portfolios only. Stock returns have proven to be much more unpredictable and errors in return estimations manipulate results many times more. Especially regarding the short holding periods and out-of-sample windows, return measurements would bring to much randomness into the results which justifies the approach. Nevertheless, this problem can be overcome in later research.

But, later research is exactly what private investors at the moment cannot wait for. In recent time, thematic core satellite investors were in need of allocation rules and practical aids that have now been provided as some truly useful tools for any investor no matter the financial literacy and the amount of provided price history of the preferred product.

APPENDIX

Different time frames

	PBN	SBN	CBN
99 benchmarks	47.29 %	76.39 %	82.31 %
MVO	17.84 %	42.18 %	42.99 %

Table A1: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
t(01.10.2015) = 1; T = 1066; IN = 250; OUT = 250, Delta = 15, 19000 data points

	PBN	SBN	CBN	99BM	MVO
Volatility	12.58 %	11.87 %	11.85 %	17.45 %	11.66 %

Table A2: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
t(01.10.2015) = 1; T = 1066; IN = 250; OUT = 250, Delta = 15, 19000 data points

	PBN	SBN	CBN
99 benchmarks	49.2 %	78.29 %	84.89 %
MVO	14.71 %	40.21 %	42.28 %

Table A3: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
t(01.01.2016) = 1; T = 1002; IN = 500; OUT = 500, Delta = 15, 8500 data points

	PBN	SBN	CBN	99BM	MVO
Volatility	12.02 %	11.35 %	11.32 %	16.92 %	11.19 %

Table A4: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
t(01.01.2016) = 1; T = 1002; IN = 500; OUT = 500, Delta = 15, 8500 data points

	PBN	SBN	CBN
99 benchmarks	39.8 %	84.8 %	91.2 %
MVO	19.6 %	44.8 %	43.6 %

Table A5: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
Bullish market momentum: t(01.01.2017) = 1; T = 500; IN = 250; OUT = 250

	PBN	SBN	CBN	99BM	MVO
Volatility	9.14 %	8.13 %	8.16 %	13.9 %	8.07 %

Table A6: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
Bullish market momentum: t(01.01.2017) = 1; T = 500; IN = 250; OUT = 250

	PBN	SBN	CBN
99 benchmarks	53.8 %	69 %	74.8 %
MVO	7.6 %	35.8 %	35.8 %

Table A7: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
Bearish market momentum: t(01.01.2018) = 1; T = 500; IN = 250; OUT = 250

	PBN	SBN	CBN	99BM	MVO
Volatility	15.18 %	14.79 %	14.76 %	20.95 %	14.45 %

Table A8: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
Bearish market momentum: t(01.01.2018) = 1; T = 500; IN = 250; OUT = 250

Different thematic families

In the following, the used data set and design are identified by:

t(01.10.2015) = 1, T = 1066, IN = 250, OUT = 500, Delta = 15

	PBN	SBN	CBN
99 benchmarks	54.01 %	77.19 %	86.07 %
MVO	17.77 %	49.46 %	55.79 %

Table A9: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
*Thematic family**:** Consumer - 11 different tETFs, 2420 data points*

	PBN	SBN	CBN	99BM	MVO
Volatility	12.00 %	11.69 %	11.55 %	16.01 %	11.66 %

Table A10: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN), the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
*Thematic family**:** Consumer - 11 different tETFs, 2420 data points*

	PBN	SBN	CBN
99 benchmarks	57.9 %	75.85 %	84.66 %
MVO	19.72 %	48.92 %	55 %

Table A11: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
*Thematic family**:** Natural resources - 8 different tETFs, 1760 data points*

	PBN	SBN	CBN	99BM	MVO
Volatility	11.8 %	11.67 %	11.51 %	15.34 %	11.65 %

Table A12: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN), the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
*Thematic family**:** Natural resources - 8 different tETFs, 1760 data points*

	PBN	SBN	CBN
99 benchmarks	30.91 %	90.09 %	93.91 %
MVO	2.64 %	26.64 %	33.91 %

Table A13: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
*Thematic family**:** Renewable Energy - 5 different tETFs, 1100 data points*

	PBN	SBN	CBN	99BM	MVO
Volatility	13.2 %	11.95 %	12.01 %	20.43 %	11.79 %

Table A14: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN), the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
*Thematic family**:** Renewable Energy - 5 different tETFs, 1100 data points*

	PBN	SBN	CBN
99 benchmarks	36.97 %	87.05 %	94.39 %
MVO	26.44 %	46.74 %	54.09 %

Table A15: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
*Thematic family**:** Technology - 6 different tETFs, 1320 data points*

	PBN	SBN	CBN	99BM	MVO
Volatility	12.87 %	11.95 %	11.89 %	18.78 %	12.01 %

Table A16: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN), the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
*Thematic family**:** Technology - 6 different tETFs, 1320 data points*

Different conventional cores

In the following, the used data set and design are identified by:

t(01.10.2015) = 1, T = 1066, IN = 250, OUT = 500, Delta = 15

	PBN	SBN	CBN
99 benchmarks	81.55 %	74.36 %	71.91 %
MVO	11.36 %	56.18 %	54.73 %

Table A17: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
Conventional core: Dax30

	PBN	SBN	CBN	99BM	MVO
Volatility	13.27 %	13.45 %	13.55 %	17.36 %	13.20 %

Table A18: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
Conventional core: Dax30

	PBN	SBN	CBN
99 benchmarks	16.91 %	88 %	92.82 %
MVO	24.73 %	45.18 %	43.27 %

Table A19: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
Conventional core: S&P 500

	PBN	SBN	CBN	99BM	MVO
Volatility	12.48 %	10.91 %	11.02 %	17.36 %	10.88 %

Table A20: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
Conventional core: S&P500

	PBN	SBN	CBN
99 benchmarks	24.27 %	82.91 %	86.82 %
MVO	26.45 %	41.55 %	47.73 %

Table A21: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 99 different invested amounts (99BM) and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
Conventional core: MSCI 1600

	PBN	SBN	CBN	99BM	MVO
Volatility	11.74 %	10.39 %	10.33 %	17.34 %	10.33 %

Table A22: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the 99 benchmark portfolios (99BM) and minimum variance optimization (MVO) portfolios.*
Conventional core: MSCI 1600

Different benchmark

For the following Table the used data set and design are identified by:

t(01.10.2015) = 1, *T* = 1066, *IN* = 250, *OUT* = 500, Delta = 15

	PBN	SBN	CBN
49 benchmarks	28.95 %	58.27 %	66.05 %
MVO	17.05 %	42.39 %	45.79 %

Table A23: *Portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) in comparison to the benchmarks of 49 different invested amounts and a minimum variance optimization (MVO). Column wise it shows how often the strategy leads to less volatility than the corresponding strategy in the left-most column.*
1-49 %

	PBN	SBN	CBN	49BM	MVO
Volatility	12.49 %	11.8 %	11.77 %	12.84 %	11.65 %

Table A24: *Mean yearly volatility approximated from daily volatility with factor √250 of portfolio-based (PBN), stock-based (SBN) and concentration-based naivety (CBN) as well as the variation of 49 benchmark portfolios (49BM) and minimum variance optimization (MVO) portfolios.*

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