Structural behaviour of origami-based folded shells made of carbon textile reinforced concrete

Homam Spartali, Josef Hegger, Rostislav Chudoba

Lehrstuhl und Institut für Massivbau,
RWTH Aachen University,
Mies-van-der-Rohe Str. 1, 52074 Aachen, Germany

hspartali@imb.rwth-aachen.de

Abstract

Carbon textile reinforced concrete is a composite material made of a cementitious matrix and carbon fabrics that is being increasingly used in different structural applications. The unique properties of carbon reinforcement, such as its light weight, corrosion resistance and superior tensile strength, enable thin, material-efficient, and high-performance structural elements. However, achieving a thin-walled structural element with high material strength utilization requires a proper distribution of the material in the spatial form considering boundary conditions and loading. In addition, the fabrication of such thin-walled geometries is often cumbersome and not economical due to their spatial complexity, that requires the construction of single-use formwork. An approach to alleviate this problem is by exploiting the folding kinematics of waterbomb origami tessellation, which, when parameterized, provides high geometric flexibility generating a wide range of spatial forms by folding a flat, tessellated surface. Moreover, the parameterization provides the ability to adjust structural properties, such as cross-sectional height, curvature and density of the folds in the spatial shape yielding structures with increased stiffness. This paper presents the preliminary results of a test series of thin folded waterbomb-based shells made of carbon concrete with a span of 1.8 m, tested with a three-point bending setup. The shells were produced by folding concrete in the fresh state (*fold-in-fresh* method) using a formwork with foldable surface provided with waterbomb tessellation. The fabrication procedure and the structural behaviour of the tested shells are presented. Both tested shell variants, with 1 cm and 2 cm thickness, showed excellent load-bearing capacity, almost 100 times higher than the shells' own weight. This illustrates the potential of folded thin-walled carbon concrete elements as high-performance structural components with effective production method using a reusable foldable formwork.

Keywords: textile reinforced concrete, carbon concrete, thin-walled elements, folded shells, origami, waterbomb, foldable formwork, structural

Kurzfassung

Carbontextilbewehrter Beton ist ein innovativer Verbundwerkstoff aus einer zementhaltigen Matrix und Carbongeweben, der aufgrund seiner attraktiven Merkmale zunehmend in Bauanwendungen eingesetzt wird. Die einzigartigen Eigenschaften der Carbonbewehrung, wie geringes Gewicht, Korrosionsbeständigkeit und überragende Zugfestigkeit, ermöglichen dünne, materialsparende und leistungsstarke Bauteile. Um ein dünnwandiges Element mit hoher Ausnutzung der Materialfestigkeit zu erreichen, ist jedoch eine geeignete Verteilung des Materials in der räumlichen Form unter Berücksichtigung von Randbedingungen und Belastungen erforderlich. Darüber hinaus ist die Herstellung solcher dünnwandigen Geometrien aufgrund ihrer räumlichen Komplexität, die den Bau von Einwegschalungen erfordert, oft umständlich und nicht wirtschaftlich. Ein Ansatz zur Vermeidung dieses Problems ist die Ausnutzung der Faltkinematik der Waterbomb-Origami-Tesselierung, die, wenn sie parametrisiert ist, eine hohe geometrische Flexibilität bietet und durch das Falten einer flachen, tesselierten Oberfläche ein breites Spektrum an räumlichen Formen ermöglicht. Darüber hinaus bietet die Parametrisierung die Möglichkeit, konstruktive Eigenschaften wie die Querschnittshöhe, die Krümmung und die Dichte der Falten in der räumlichen Form einzustellen, wodurch Strukturen mit erhöhter Steifigkeit entstehen. In diesem Beitrag werden die vorläufigen Ergebnisse einer Versuchsreihe von dünnen gefalteten Schalen aus Carbonbeton mit einer Spannweite von 1,8 m, die mit einem Drei-Punkt-Biegeversuch getestet wurden, vorgestellt. Die Schalen wurden durch Falten von Beton im frischen Zustand (*fold-in-fresh*

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Methode) unter Verwendung einer Schalung mit faltbarer Oberfläche hergestellt, die mit einer Waterbomb-Tessellierung versehen ist. Das Herstellungsverfahren und das Tragverhalten der geprüften Schalen werden vorgestellt. Die beiden getesteten Schalenvarianten mit einer Dicke von 1 cm und 2 cm zeigten eine sehr hohe Tragfähigkeit, die fast das 100-fache des Eigengewichts der Schalen beträgt. Dies verdeutlicht das Potenzial von gefalteten dünnwandigen Carbonbetonbauteilen als Hochleistungsbauteile mit einer effektiven Herstellungsmethode unter Verwendung einer wiederverwendbaren faltbaren Schalung.

Keywords: Textilbeton, Carbonbeton, dünnwandige Elemente, gefaltete Schalen, Origami, faltbare Schalung, Waterbomb, Tragverhalten

1 Introduction

The construction sector, being one of the pivotal contributors to escalating CO2 emissions, necessitates the exploration of sustainable solutions that cover the entire construction lifecycle, from material production to actual construction, followed by operation and maintenance. Despite the fact that the design of structural elements, including geometry and material selection, contributes significantly to the waste of materials [1], massive, material-inefficient reinforced concrete elements are currently still widely used. However, in recent decades, the development of high-performance corrosion-resistant reinforcement types such as carbon-fibre-reinforced polymers (CFRP) opened up a new design space for material-efficient thin-walled structural elements [2–6]. Nonetheless, adaptive design strategies, that ensure rapid, customizable prototyping and manufacturing of such innovative elements are still needed.

One concept that might serve the systematic design and efficient fabrication of thin-walled carbon concrete elements is to exploit the folding geometries and kinematics of the Japanese paper-folding art known as origami, in particular the waterbomb pattern. Waterbomb is a famous origami pattern with six facets meeting at one vertex point in the centre, which proved to provide significant form flexibility and attractive structural properties when tessellated and folded [7–12]. An origami tessellation is a folding pattern created by repeating a compatible smaller base pattern. Various geometries for thin-walled elements including shells, walls and slab systems are obtainable by changing the length and folding parameters of the waterbomb base pattern [7]. Furthermore, by utilizing the flat foldability and folding kinematics of a waterbomb tessellation, three-dimensional elements can be efficiently produced by transforming a planar surface into the target spatial geometry using a foldable formwork. In this way, a reusable, customizable, and modular production process becomes possible. Additionally, high-performance elements with material-efficient designs are achievable [9].

In this paper, the first results of experimental investigations of thin origami-based carbon concrete shells are introduced. First, the derivation of the waterbomb geometry and the production using the previously introduced *fold-in-fresh* method [13] utilizing the principles of rigid origami are first presented, followed by the experimental results and final conclusions. The introduced test series is part of broader experimental and numerical studies aiming to provide a systematic analysis, design, and production framework of folded carbon concrete elements with optimized structural behaviour. The results of this study will not only assist in refining our design and manufacturing approaches, but will also provide a basis for the validation of numerical models supporting the nonlinear modelling of TRC shell elements.

2 Experimental investigations

2.1 Materials

The shells have been produced using a fine concrete with the product name CARBOrefit® from the company PAGEL®. A complete set of concrete specimens was prepared with the production of each shell, and the specimens were tested on the test day of the corresponding shell. Each set consisted of six cylinders with 15 cm diameter, three of which with 30 cm height used for the compressive strength and Young's modulus, and another three with 20 cm height used for the splitting tensile strength. In addition, three compressive strength cubes (15 cm × 15 cm × 15 cm) and three standard fine concrete prisms (4 cm × 4 cm × 16 cm) were produced, see Figure 1 (right). The prisms were used to determine flexural tensile strength according to DIN EN 196-1 [14]. The mean values for cylindrical and cubic compressive strength, E-modulus, splitting tensile strength, and flexural tensile strength at an average

Figure 1 The unimpregnated carbon reinforcement on the left and concrete specimens set on the right

age of 41 days were as follows: $f_{\mathrm{cm,cyl}} = 68.2$ MPa, $f_{\mathrm{cm,cu}} = 81.5$ MPa, $E_{\mathrm{cm}} = 24805$ MPa, $f_{\mathrm{ctm,sp}} = 2.9$ MPa, $f_{\mathrm{ctm,pr}} = 7.2$ MPa.

An unimpregnated carbon textile reinforcement from the company Saertex® was used to provide the flexibility needed for the folding process, see Figure 1 (left). The reinforcement consists of an equidistant grid of rovings with a spacing of 10 mm in both directions and is comparable to the "MAG05-00" reinforcement used in [15], where the following properties from roving tensile tests were reported: $f_{\mathrm{tm}} = 1732$ MPa, $E_{\mathrm{t}} = 123880$ MPa.

2.2 Experimental program and shell geometry

The folded concrete shells featured in this paper constitute the initial findings from a broader experimental program aimed at exploring different materials and shell variations. In particular, we study the behaviour of five folded shells: three shells have a thickness of 1 cm with a reinforcement layer centrally positioned in the cross section, while the other two have a thickness of 2 cm with two reinforcement layers placed 5 mm from both the top and bottom surfaces of the shell. The geometry of the shells represents a folded state of a waterbomb pattern with two waterbomb cells in the longitudinal direction. The length parameters a, b, c of the base waterbomb cell forming the folding pattern, and the folding angle γ (see Figure 2) are selected to yield an optimized load-capacity in analogy to the approach described in [9]. All shells have an effective span of 180 cm and an effective height of 30 cm in the final folded state. The base waterbomb cell, the complete folding pattern (tessellation), and the final geometry of the folded shells with the two cross-section variants are shown in Figure 2. The shells are denoted in the format "S#-TxRy", where (#) is the shell number and (x), (y) are replaced by the shell thickness in cm and the number of the reinforcement layers, respectively, see Table 1.

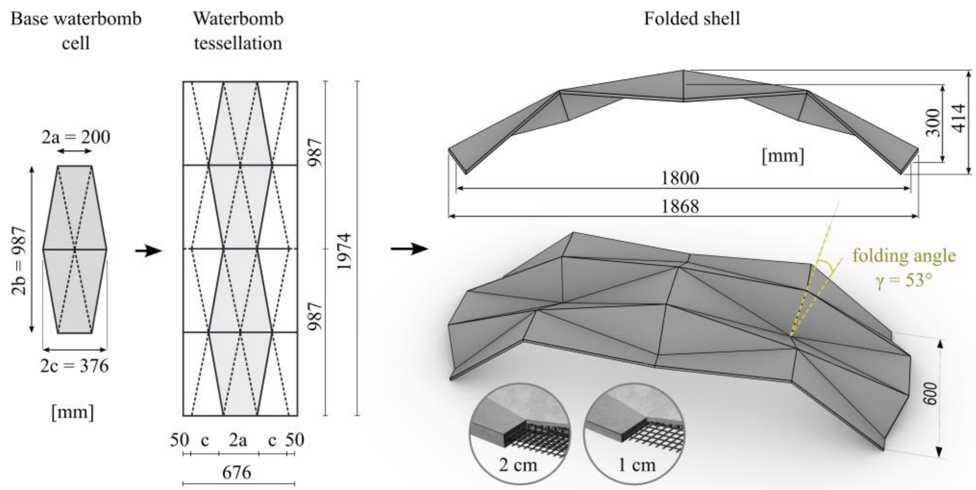

Figure 2 Folding pattern and final folded form of the tested shells

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2.3 Shell production

The shells were produced using the *fold-in-fresh* method introduced in [13], however, employing steel sheets instead of wooden plates for the production of the foldable surface. First, the foldable surface corresponding to the waterbomb tessellation shown in Figure 2, is made of 1.5 mm thick laser-cut steel sheets connected with hinges. Angle profiles are mounted on the surrounding steel sheets of the tessellated surface to prevent concrete from flowing to the sides, and the voids along the fold lines are sealed with transparent tape. Afterwards, the tessellated surface is placed in a prefabricated wooden box with vertical supports from the inside ensuring the right form is obtained after folding. The production of a shell starts by placing the tessellated surface over steel profiles placed across the wooden box near the top surface. Then, for 1 cm thick shells, two layers of concrete are cast, separated by one layer of reinforcement, and for 2 cm thick shells, three layers of concrete are cast, separated by two layers of reinforcement. The surface of the formwork is then folded into the desired shape while the concrete is still fresh. Finally, the formwork is left in place until the concrete hardens and is stiff enough to remove the formwork. The production, including the folding process, is illustrated in Figure 3 for a shell with 1 cm thickness and one reinforcement layer.

2.4 Test setup, loading, and instrumentation

The shells were tested using a three-point bending setup with fixed supports on the sides and a concentrated load applied using a wide, thick steel plate placed in the middle of the shell, as shown in Figure 4. The displacement-controlled load was applied at a rate of 1 mm/min and 3 mm/min in case of loading and unloading, respectively. Two prefabricated steel supports with adaptable steel angles were used at both ends of the shell to prevent horizontal and vertical movement. In addition, cement mortar was applied between the steel angles and the surface of the shell prior to each test to avoid local stress concentrations along the ends of the shell and to ensure maximum fixation.

Three linear variable differential transformers (LVDTs) were mounted in the middle of the shell, one directly below the load application point, and two on the sides, as shown in Figure 4. Other LVDTs were also used to measure the width of the transverse cracks on the midspan sides, transverse deformations due to unfolding, and possible support displacement. Additionally, a digital image correlation (DIC) speckle pattern was applied to one half of the shell surface to track cracks and strains development using a 3D ARAMIS® DIC system.

3 Experimental results and discussion

3.1 Summary of results

As a measure of production quality control, the target thickness of 1 cm and 2 cm is checked by measuring the actual thickness at 8 points along the outer edges of each shell. The average values of the 8 measurements for each shell are given in Table 1 with the corresponding number of reinforcement layers in the shell cross section. Despite the various measures taken to achieve a stiff and complete fixation at the ends of the shell, a small displacement was unavoidable, however, it was measured and reported in Table 1.

During the tests, the cracks on both sides of the shell developed in a similar manner to the point of failure, which always occurred on one side of the shell. The failure side and the maximum load capacity F_{max} with the corresponding deflection w_{max} obtained by the central LVDT at the middle of the shell are

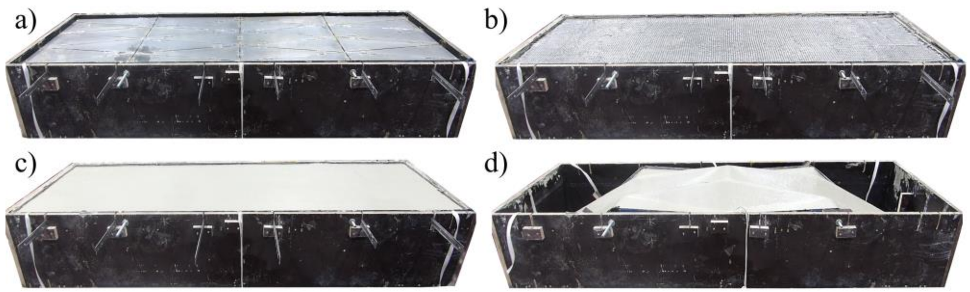

Figure 3 Production process using the *fold-in-fresh* method

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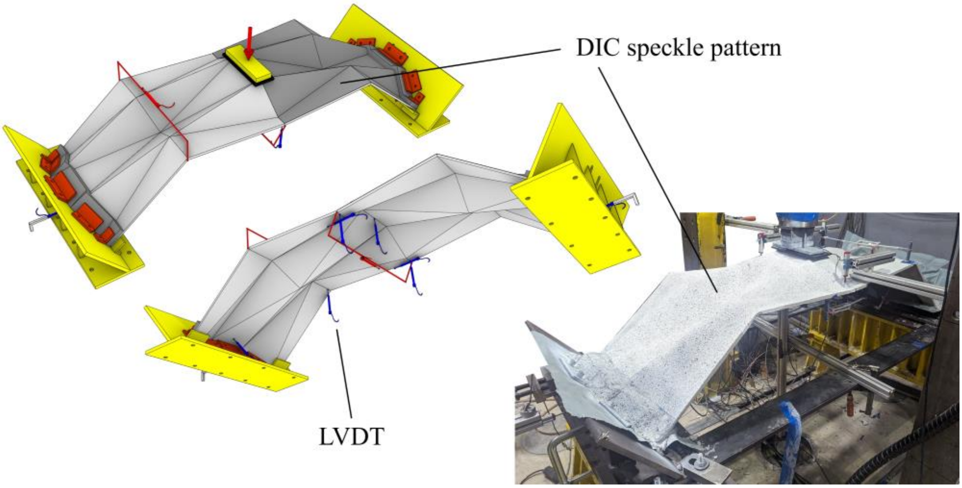

Figure 4 Test setup showing the measurement instrumentation, loading and the fixed supports

also given in Table 1. As the table shows, a maximum load capacity up to 31.7 kN was obtained for the shells with 1 cm thickness. This value is almost 100 times higher than the self-weight, which is approximately 0.3 kN, indicating a high structural efficiency accompanied with relatively low maximum deflection. A comparable structural efficiency was also obtained for the 2 cm thick shells with 2 layers of reinforcement, reaching a value of up to 59.9 kN for the shell with a self-weight of about 0.6 kN.

As expected, a larger scatter was obtained for the maximum load of the 1 cm thick shells compared to the 2 cm thick shells due to the higher sensitivity of the thin section to the deviations in geometric and material properties. In addition, larger displacement is observed in the supports for the thicker shells because of the higher horizontal reactions corresponding to the higher maximum load F_{max}.

Table 1: Results summary of the five tested shells

Shell	Thickness		Num. of reinf. layers	Failure side	F_{max}	w_{max}	Support displacement	
	[mm]		[-]	[-]	[kN]	[mm]	[mm]	
	Target	Measured					Right	Left
S1-T1R1	10	10.5	1	right	31.7	14.3	2.1	2.6
S2-T1R1	10	10.9	1	left (DIC)	22.7	9.0	0.9	1.4
S3-T1R1	10	10.7	1	right	25.4	10.8	1	1.7
S4-T2R2	20	20.7	2	right	58.1	26.1	4.1	2.7
S5-T2R2	20	21.1	2	left (DIC)	59.9	20.7	3.9	3.1

3.2 Load-deflection curves

The load-deflection curves with the deflection measured at the central LVDT in the shell midspan are provided in Figure 5 for the five tested shells. During the test, shells S2, S3, and S5 were fully unloaded and reloaded to quantify the stored elastic energy, the plastic energy, and the damage. The unloading and reloading of the shells S2 and S3 show an almost identical path indicating a consistent behaviour for the 1 cm thick shells. It is to be mentioned that the initial, relatively large drop in the S2 and S3 curves at a load level around 8 kN is caused by the initiation of the transverse crack on the side of the

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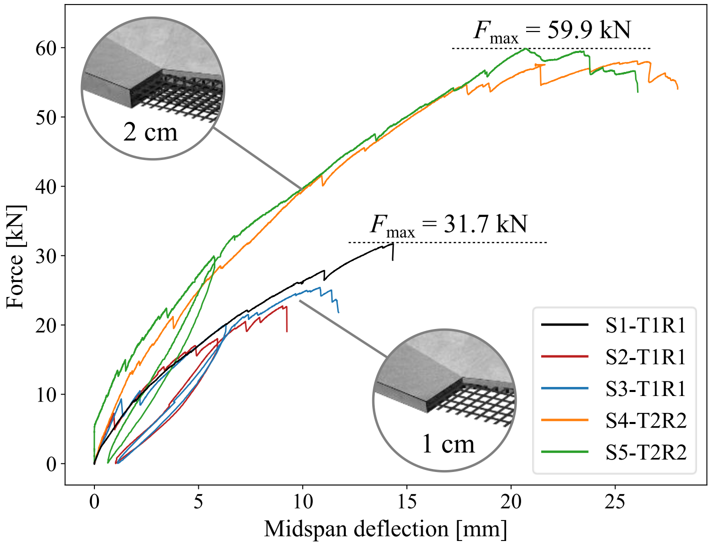

Figure 5 Load-deflection curves of the five tested shells

shell at mid-span, as shown in Figure 6 (top). Since this transverse crack developed in shell S1 during shell installation on the test setup, a corresponding drop in the curve is missing, which explains the difference in the initial stiffness between the shell S1 and S2, S3.

The 2 cm shells had a maximum load capacity and stiffness almost twice that of the 1 cm shells. Also, the load-deflection curves of the shells S4 and S5 exhibit similar response, as shown by the comparable stiffness and maximum load capacity, indicating consistent quality of the shell fabrication and the fixed supports. The 1 cm thick shells showed a larger scatter in load capacity as mentioned earlier, however, the curves show a qualitatively similar stiffness in addition to the consistent unloading behaviour.

3.3 Failure mechanisms and crack patterns

The maximum principal strains obtained from the DIC measurements are illustrated in Figure 6, showing the crack development at three different loading levels until complete failure. The measurements in the figure are provided for the shells S2 and S5 that had a decisive failure on the DIC side.

For the other 1 cm thick shells (S1, S3), a failure mechanism similar to that of shell S2 was observed, where the concrete failed in the valley zone to the side of the shell, mostly due to compression, see Figure 6 (bottom left). Despite the wide transverse cracks that developed in the midspan, the cracking pattern of these shells showed a domination of longitudinal cracks that eventually led to failure. The two shells with a thickness of 2 cm showed a different failure mechanism compared to the thinner shells. In this case, a section located to the side of the load application point failed under concrete compression as shown in Figure 6 (bottom right). In contrast to the thinner shells, the crack pattern of these shells was dominated by various distributed transverse cracks. The cracks that appeared first to the left of the loading point in Figure 6 (top right) propagated initiating the failure of the section near the loading point. In both observed failure mechanisms, no major rupture of the textile reinforcement has been detected.

4 Conclusions

Initial investigations of a test series of folded carbon concrete shells have been introduced in this paper. The production process using the *fold-in-fresh* method and the experimental results of the three-point bending tests are presented and discussed. The form of the shells and the production concept are based on the well-known origami waterbomb tessellation and the rigid origami theory. The conclusions of the current study can be summarized as follows:

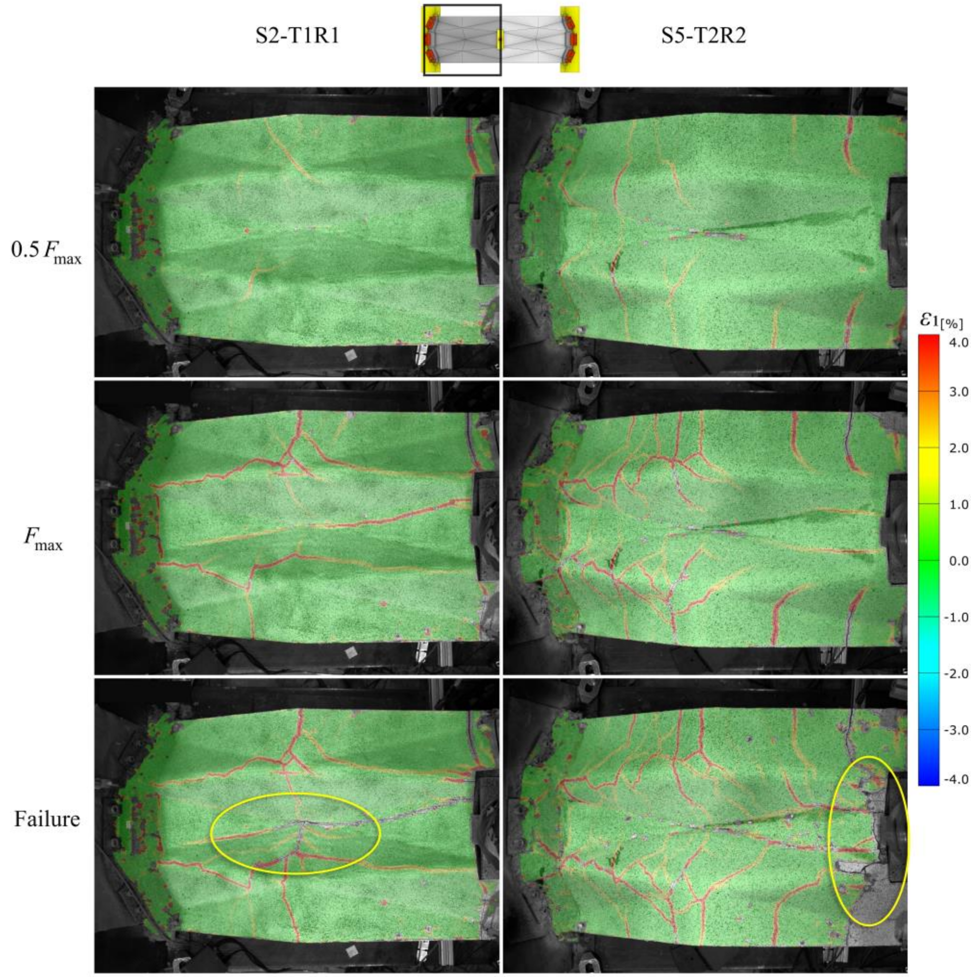

Figure 6 DIC measurements of the maximum principal strains showing crack development until failure for shells S2 and S5

- The *fold-in-fresh* method, based on rigid origami concepts, proved to be a practical fabrication method enabling the production of thin-walled carbon concrete elements with various cross-sectional configurations, i.e., different thickness and number of reinforcement layers, using a single reusable formwork.
- The use of origami waterbomb tessellations provides high flexibility in the folded 3D form and supports modularity through the compatibility of the waterbomb cells that form the global geometry.
- Although, multiple measures have been conducted to ensure fully fixed supports at shell ends during testing, a minor displacement was unavoidable. However, the displacement on both supports was measured and reported with the test results.
- The load-deflection curves indicated a high load capacity up to 100 times the self-weight for both shell variants. The 2 cm thick shells with two reinforcement layers showed a stiffness, load capacity, and maximum deflection almost twice that of the 1 cm thick shells with one reinforcement layer.
- Two different failure mechanisms were observed for the two shell variants. The 1 cm thick shells failed in the valley zone on the side of the shell, with mainly longitudinal

cracks, while the 2 cm thick shells showed compression failure near the loading point with pronounced and widespread transverse cracks.

The presented results demonstrate the potential of folded thin-walled carbon textile reinforced concrete elements as material efficient and high-performance structural elements. The folded geometry provides a structural behaviour with improved stiffness and load-bearing capacity, and the fabrication process using waterbomb origami patterns allows a rapid, modular, and economic production of these elements. Further experimental and numerical investigations on folded waterbomb elements will be presented in subsequent research contributions, providing a comprehensive framework for the development of efficient carbon concrete structural members.

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6 References

[1] *Ajayi, S.O.*; *Oyedele, L.O.:* Critical design factors for minimising waste in construction projects: A structural equation modelling approach, *Resources, Conservation and Recycling*. 137 (2018) 302-313. doi:10.1016/j.resconrec.2018.06.005.

[2] *Sharei, E.*; *Scholzen, A.*; *Hegger, J.*; *Chudoba, R.:* Structural behavior of a lightweight, textile-reinforced concrete barrel vault shell, *Composite Structures*. 171 (2017) 505-514. doi:10.1016/j.compstruct.2017.03.069.

[3] *Scholzen, A.*; *Chudoba, R.*; *Hegger, J.:* Thin-walled shell structures made of textile-reinforced concrete: Part I: Structural design and construction, *Structural Concrete*. 16 (2015) 106-114. doi:10.1002/suco.201300071.

[4] *Scholzen, A.*; *Chudoba, R.*; *Hegger, J.*; *Will, N.:* Leichte Dachschalen aus Carbonbeton: Fertigteilproduktion, experimentelle Untersuchungen und Anwendungspotenzial, *Beton- und Stahlbetonbau*. 111 (2016) 663-675. doi:10.1002/best.201600030.

[5] *Preinstorfer, P.*; *Huber, P.*; *Huber, T.*; *Kromoser, B.*; *Kollegger, J.:* Experimental investigation and analytical modelling of shear strength of thin walled textile-reinforced UHPC beams, *Engineering Structures*. 231 (2021) 111735. doi:10.1016/j.engstruct.2020.111735.

[6] *Bielak, J.*; *Schmidt, M.*; *Hegger, J.*; *Jesse, F.:* Structural Behavior of Large-Scale I-Beams with Combined Textile and CFRP Reinforcement, *Applied Sciences*. 10 (2020) 4625. doi:10.3390/app10134625.

[7] *Chudoba, R.*; *Niemeyer, A.C.*; *Spartali, H.*; *Robertz, D.*; *Plesken, W.:* Description of the origami waterbomb cell kinematics as a basis for the design of thin-walled oricrete shells, içinde: S.A. Behnejad, G.A.R. Parke, O.A. Samavati (Ed.), *IASS Annual Symposium 2020/21 and Surrey 7th, Inspiring the Next Generation, 23–27 August 2021, Guildford, UK*, 2021. doi:10.5281/zenodo.6759486.

[8] *Robertz, D.*; *Spartali, H.*; *Chudoba, R.*; *Plesken, W.*; *Niemeyer, A.C.:* Semi-symmetric origami waterbomb cell kinematics and tessellation for the design of thin-walled folded shells, içinde: *Proceedings of the IASS Annual Symposium 2022*, Beijing, China, 2022: ss. 2314-2324.

[9] *Spartali, H.*; *van der Woerd, J.D.*; *Hegger, J.*; *Chudoba, R.:* Stress redistribution capacity of textile-reinforced concrete shells folded utilizing parameterized waterbomb patterns, içinde: *Proceedings of the IASS Annual Symposium 2022*, Beijing, China, 2022: ss. 96-106.

[10] *Zhao, Y.*; *Endo, Y.*; *Kanamori, Y.*; *Mitani, J.:* Approximating 3D surfaces using generalized waterbomb tessellations, *Journal of Computational Design and Engineering*. 5 (2018) 442-448. doi:10.1016/j.jcde.2018.01.002.

[11] *Ma, J.*; *Feng, H.*; *Chen, Y.*; *Hou, D.*; *You, Z.:* Folding of Tubular Waterbomb, *Research*. 2020 (2020) 1-8. doi:10.34133/2020/1735081.

[12] *Zhao, Y.*; *Li, S.*; *Zhang, M.*; *Zeng, L.*; *Yang, Y.*; *Kanamori, Y.*; *Mitani, J.:* Computational design methods for cylindrical and axisymmetric waterbomb tessellations, *Computer Aided Geometric Design*. 91 (2021) 102037. doi:10.1016/j.cagd.2021.102037.

[13] *van der Woerd, J.D.*; *Chudoba, R.*; *Bonfig, C.*; *Hegger, J.:* Construction of a vault using folded segments made out of textile reinforced concrete by fold-in-fresh, içinde: *Interfaces: architecture.engineering.science (IASS) Symposium. published on USB stick*, Hamburg, 2017.

[14] DIN EN 196-1:2016-11, Prüfverfahren für Zement_- Teil_1: Bestimmung der Festigkeit; Deutsche Fassung EN_196-1:2016, *Beuth Verlag GmbH*, t.y. doi:10.31030/2482416.
[15] *Molter, M.:* Zum Tragverhalten von textilbewehrtem Beton, 1. Aufl, *Eigenverl*, Aachen, 2005.